

Introduction to fluid mechanics

Fluid mechanics is concerned with the behaviour of fluid at rest (fluid static) & in motion (fluid dynamic) and the interaction of fluid with other fluids or solids at the boundaries.

Fluid is a substance, might be liquid or gas, that deforms continuously (flow) when subjected to a shear stress.

Solids & Liquids

From Microscopic point of view

Solid: large intermolecular forces with small spacing between molecules.

Liquid: the intermolecular forces are smaller than for solids with bigger spacing between molecules.

From Fluid Mechanics Perspective

When shear stress is subjected to a substance:

- If the substance deforms continuously (flow), it is a fluid
- If the substance experiences a small deformation δl (strain), it is a solid

Fluid Mechanics is divided into several categories:

- **Hydrodynamics:** deals with the flow of incompressible fluids (especially water) at low speed such as hydro power plants (Dams).
- **Aerodynamics:** deals with the flow of compressible fluids (especially air) over bodies such as aircrafts, rockets and cars.
- **Gas dynamics:** deals with the flow of fluids (especially gases) that undergoes significant density change such as the flow of natural gas through nozzles at high speed, for example the flow of natural gases in to the combustion chamber of gas turbine.



Property of Fluids

Density (ρ): The ratio of the mass of a fluid to its volume (kg/m^3). It characterizes a mass of the fluid system.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

The density of the water is 1000 kg/m^3 .

The variations in pressure and temperature generally have only a small effect on the value of ρ for liquid. In contrast to liquid, the density of a gas is strongly influenced by both pressure and temperature.

Specific weight (γ): The ratio of the weight of a fluid to its volume (N/m^3). It characterizes the weight of a fluid system.

$$\gamma = \frac{\text{weight of fluid}}{\text{Volume of fluid}}$$

$$\gamma = \frac{m \times g}{v} = \frac{\rho \times v \times g}{v} = \rho \times g$$

The specific weight of the water is $\gamma_{\text{water}} = \rho_{\text{water}} \times g = 1000 \times 9.81 = 9810 \text{ N/m}^3$.

Specific Gravity (SG): The specific gravity of a fluid is the ratio of fluid density to the density of standard fluid (Water for liquids & air for gases).

$$SG = \frac{\text{density of fluid}}{\text{density of water}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{\gamma_{\text{liquid}}}{\gamma_{\text{water}}}$$

This give us an indication about the heaviness of the Liquids

- If $SG > 1$, The liquid is heavier than the water
- If $SG < 1$, The liquid is lighter than the water

Specific volume (v): is defined as the volume of a fluid occupied by a unit mass. Thus, it is the reciprocal of the density.

$$v = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\text{density}} = \frac{1}{\rho}$$

Ex) 200 litres of a certain oil weights 180 kg. Calculate the specific weight, specific gravity and specific volume of it.

Ex) Calculate the density, specific weight and weight of one litre of petrol of specific gravity equals to 0.7.

Compressibility and bulk modulus

Compressibility (C) shows how easily can the volume of a fluid change when the pressure changes.

Bulk Modulus K: A property that is commonly used to characterize compressibility. It is defined as the ratio of pressure change to volumetric strain.

$$K = \frac{\text{pressure change}}{\text{volumetric strain}} = -\frac{\Delta P}{\Delta V/V} \quad \text{or} \quad \frac{\Delta P}{\Delta \rho/\rho}$$

The negative sign means that the increase in pressure will cause a decrease in volume. Compressibility is reciprocal of bulk modulus.

For most purposes, liquids can be considered as incompressible fluids. For example, it would require a pressure of 220 bar to compress a unit volume of water by 1%.

Ideal Gas Law

Gases are highly compressible in comparison to liquids, changes in gas volume (and hence density) can be directly related to changes in pressure and temperature through the ideal gas equation:

$$PV = mRT \quad \text{or} \quad P = \rho RT \quad \dots \quad (\text{Ideal Gas Law})$$

P: Absolute pressure ($P_{\text{gauge}} + P_{\text{atm}}$)

T: Absolute temperature ($T \text{ (K)} = T \text{ (}^\circ\text{C)} + 273$)

m: Mass

ρ : Density

R: Gas constant, (Air: $287 \frac{\text{J}}{\text{Kg} \cdot \text{K}}$)

Ex) A compressed air tank has a volume of 0.3 m³. When the tank is filled with air at a gage pressure of 0.2 Mpa, determine the density of the air and the weight of air in the tank. Assume the temperature is 30 C and the atmospheric pressure is 101.3 Kpa.

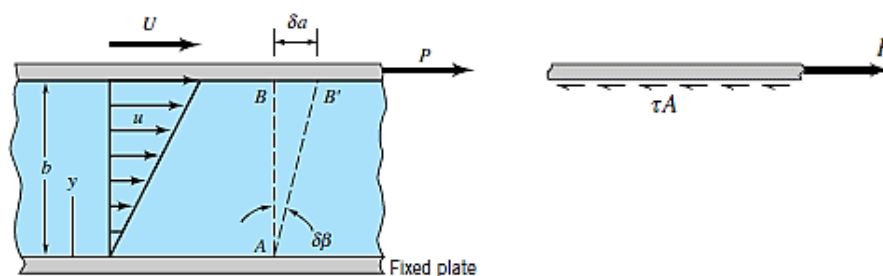


Viscosity

The properties of density and specific weight are measures of the “heaviness” of a fluid. However, these properties are not enough to describe the behavior of the fluid. For example, two fluids such as water and oil can have approximately the same value of density but they behave differently when they flow. This is because they have different viscosity values.

Viscosity: is the property of a fluid which offers resistance to shear of one layer of fluid over another adjacent layer. Simply, it is fluid's resistance to flow.

To determine the viscosity, consider a hypothetical experiment in which a fluid is placed between two very wide parallel plates as show below:



Assumptions:

1. The bottom plate is rigidly fixed while the upper plate is free to move.
2. no-slip condition, *i.e.* fluid “sticks” to the solid boundaries.

A fluid at rest has no shearing forces. When the force P is applied to the upper plate. Shear stresses are developed and the fluid is now in motion. Thus, the particles of the fluid move relative to each other at different velocities and as follow:

- The fluid in contact with the bottom fixed plate has a zero velocity.
- The fluid in contact with the upper plate moves with the plate velocity U ,

This means that The fluid between the two plates moves with velocity that vary linearly as follow:

$$u = U y/b$$

U: plate velocity, u: Fluid velocity
b: Film Thickness, y: distance between fluid layers

From the figure, we can conclude that the higher the shear stress the higher the fluid velocity.

Thus the shear stress (τ) and the velocity gradient ($\frac{du}{dy}$) can be related with a relationship:

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}, \text{ Newton Law of Viscosity}$$

μ : The viscosity of the liquid and it is highly affected by the temperature.

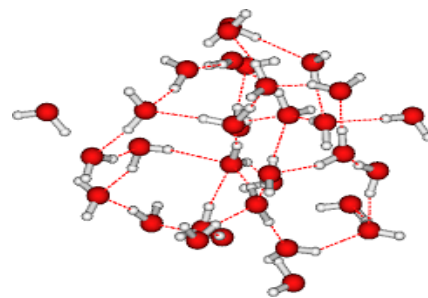
$$\mu_{\text{Water}} = 0.001 \text{ Pa.s} = 1 \text{ mPa.s}$$

Viscosity & Temperature

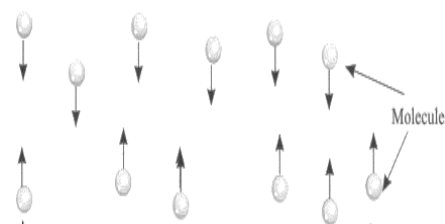
The viscosity of liquids **decreases** with the **increase** of temperature while the viscosity of gases **increases** with the temperature increase.

This difference in the effect of temperature on the viscosity of liquids and gases can again be traced back to the **difference in molecular Structure**.

The liquid molecules are **closely spaced**, with strong cohesive forces between molecules. The resistance to relative motion between adjacent layers of fluid (viscosity) is related to these **intermolecular forces**. As the temperature increases, these cohesive forces are reduced and hence result in reduction in viscosity.



In gases, the molecules are widely spaced and intermolecular forces negligible. In this case resistance to relative motion arises due to the exchange of momentum of gas molecules between adjacent layers. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases.

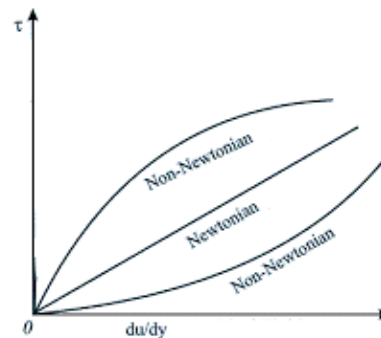


Molecular momentum transfer: is the drag forces between the molecules.

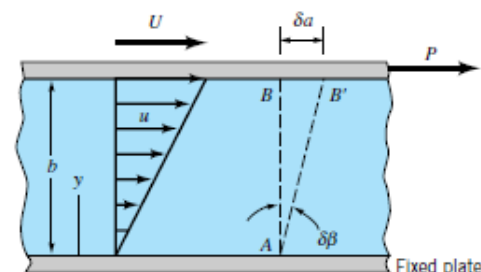
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Newtonian and non-Newtonian fluids

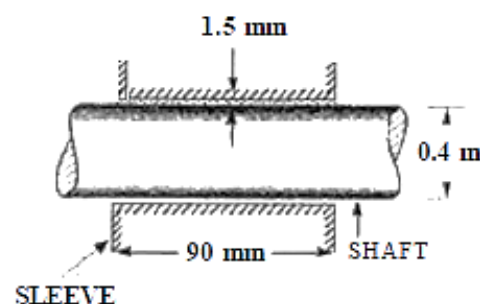
- **Newtonian Fluids:** The shearing stress is linearly related to the velocity gradient. Fortunately, most common fluids, both liquids and gases are Newtonian.
- **Non-Newtonian Fluids:** the shearing stress is not linearly related to the velocity gradient.



Ex) As shown in the figure, certain oil ($\mu = 1.5 \text{ N} \cdot \text{s}/\text{m}^2$) is used for lubrication purposes. The distance (b) between the plates is 6 mm. what is the minimum force (p) required to move the plate at 5.5 m/s. knowing that the area of the upper plate is 0.01 m^2 .



Ex) The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is $0.6 \text{ Pa}\cdot\text{s}$. The shaft is of diameter 0.4 m and rotates at 190 r.p.m. Calculate the power lost due to shear stress knowing that the sleeve length is 90 mm and thickness of the oil film is 1.5 mm.



Viscosity Grade (Multi – Grade Oil)

In most vehicles, the temperature range the oil is exposed to can be wide, ranging from cold temperatures in the winter before the vehicle is started up to hot operating temperatures when the vehicle is fully warmed up. Thus, the oil will have high viscosity when cold and a lower viscosity at the engine's operating temperature.

The difference in viscosities is too large between the extremes of temperature. To bring the difference in viscosities closer together, special polymer additives called viscosity index improvers (VIIs) are added to the oil. These additives are used to make the oil a multi-grade oil.

The Society of Automotive Engineering (SAE) designation for multi-grade oils includes two viscosity grades. For example, 10W-40 designates a common multi-grade oil. The first number (10W) represents oil's viscosity at cold temperature and the second number (40) describes its viscosity at 100 °C. Note that both numbers are grades and not viscosity values. The viscosity value equivalent to each grade is as follow

SAE 10 = 65 m Pa. sec

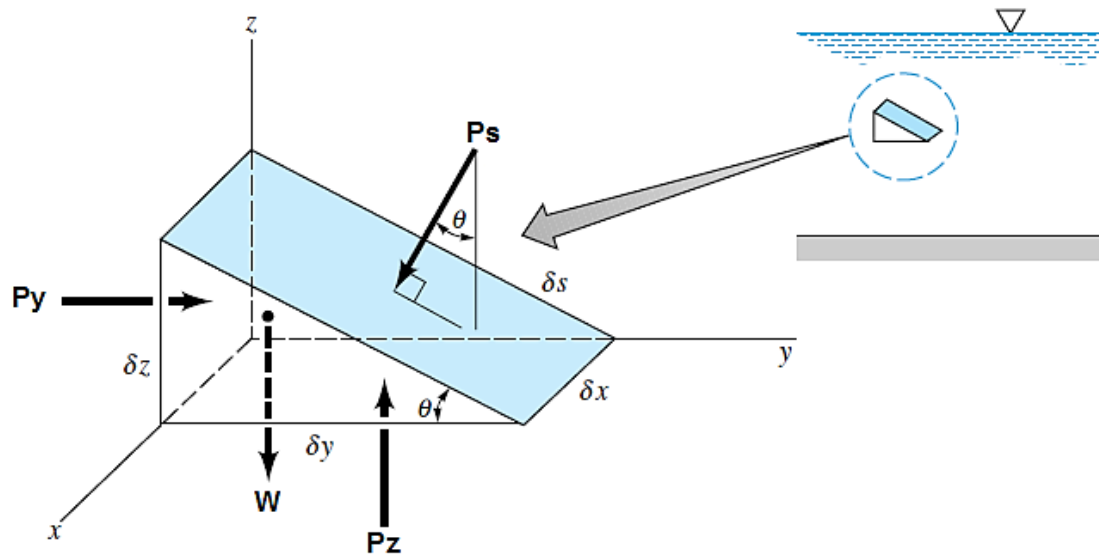
SAE 40 = 320 m Pa. sec

Fluid Static

The fluid is either at rest or moving in such a manner so that there is no relative motion between adjacent (متجاورة) particles. In both cases, there will be no shearing stress in the fluid and the only forces that develop on the surfaces of the particles will be due to the pressure.

Pascal's Law (Pressure at a Point)

'The pressure acting on a point in a fluid at rest is the same in all directions'. To prove that, consider a **small triangular slice** of fluid within a fluid mass.



Since the fluid is at rest (there are no shearing stresses), the only external forces acting on the slice are due to the pressure (P_y , P_z and P_s) and the weight (W).

Applying Newton's 2nd law of motion ($F = m \cdot a$) in the y and z directions respectively:

$$\sum F_y = m \cdot a_y \rightarrow P_y \delta x \delta z - P_s \delta x \delta s \sin \theta = \rho v a_y$$

$$P_y \delta x \delta z - P_s \delta x \delta s \sin \theta = \rho \frac{\delta z \delta y \delta x}{2} a_y \quad 1.1$$

$$\sum F_z = m \cdot a_z \rightarrow P_z \delta x \delta y - P_s \delta x \delta s \cos \theta - \gamma v = \rho v a_z$$

$$P_z \delta x \delta y - P_s \delta x \delta s \cos \theta = (\rho a_z + \gamma) \frac{\delta z \delta y \delta x}{2} \quad 1.2$$

$$\delta y = \delta s \cos \theta, \quad \delta z = \delta s \sin \theta$$

Substitute in 1.1 and 1.2 yield



$$P_y \delta x \delta s \sin \theta - P_s \delta x \delta s \sin \theta = \rho \frac{\delta s \delta y \delta x \sin \theta}{2} a_y$$

$$P_y - P_s = \rho \frac{\delta y}{2} a_y \quad 1.3$$

$$P_z \delta x \delta s \cos \theta - P_s \delta x \delta s \cos \theta = (\rho a_z + \gamma) \frac{\delta z \delta x \delta s \cos \theta}{2}$$

$$P_z - P_s = (\rho a_z + \gamma) \frac{\delta z}{2} \quad 1.4$$

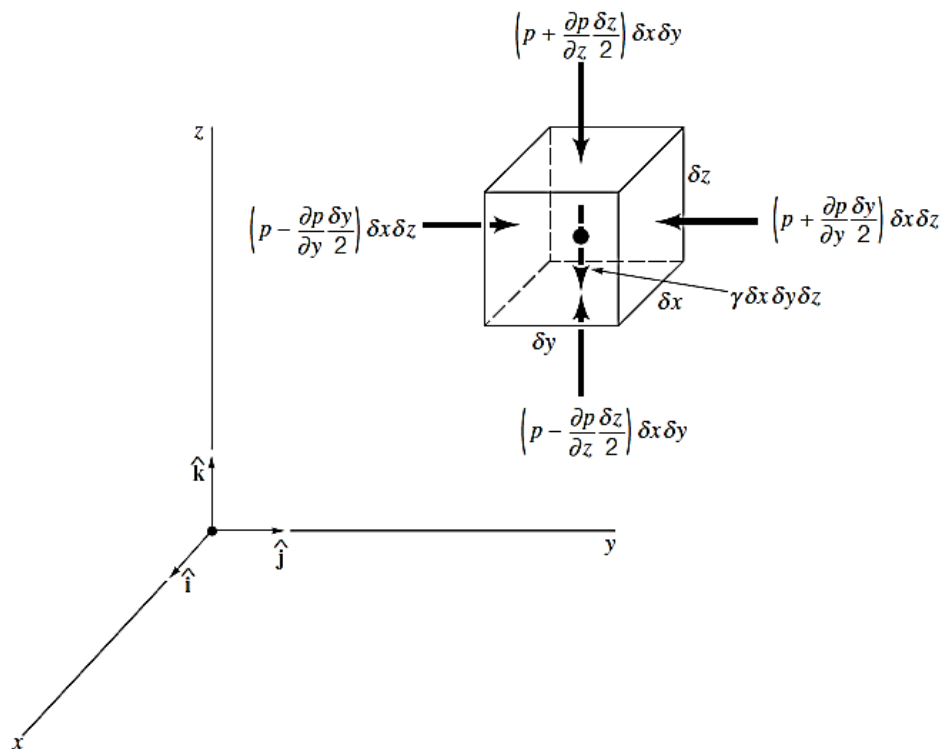
Since we are really interested in what is happening at a point, therefore $\delta x \delta y \delta z$ are **really small** and can be considered **zero** (1.3&1.4) which gives:

$$P_y = P_z = P_s$$

The angle was arbitrarily chosen so we can conclude that the pressure at a point in a fluid at rest is **independent of direction** as long as there are no shearing stresses present.

Hydrostatic Pressure Law

To answer the question of how the pressure changes from point to point, consider a rectangular element of fluid as follow:



The pressure at the center of the element is P , then the average pressure on the various faces can be expressed in Taylor series expansion as shown above.

The resultant surface force in the y direction is:

$$F_y = \left(p - \frac{\partial p}{\partial y} \frac{\delta y}{2} \right) \delta x \delta z - \left(p + \frac{\partial p}{\partial y} \frac{\delta y}{2} \right) \delta x \delta z$$

$$F_y = -\frac{\partial p}{\partial y} \delta x \delta y \delta z$$

Similarly, for the x and z directions the resultant surface forces are:

$$F_x = -\frac{\partial p}{\partial x} \delta x \delta y \delta z \quad F_z = -\frac{\partial p}{\partial z} \delta x \delta y \delta z$$

The resultant surface force acting on the element can be expressed in vector form as

$$\mathbf{F}_R = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$\mathbf{F}_R = -\left(\frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k} \right) \delta x \delta y \delta z$$

Also, the weight of the element can be expressed in vector form as:

$$\mathbf{W} = -\gamma \delta x \delta y \delta z \mathbf{k}$$

Newton's second law, applied to the fluid element, can be expressed as:

$$\sum \mathbf{F} = m \mathbf{a}$$

$$-\left(\frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k} \right) \delta x \delta y \delta z - \gamma \delta x \delta y \delta z \mathbf{k} = \rho \delta x \delta y \delta z \mathbf{a}$$

$$-\left(\frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k} \right) - \gamma \mathbf{k} = \rho \mathbf{a} \quad \text{For a fluid at rest, } a = 0$$

$$-\left(\frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k} \right) = \gamma \mathbf{k}$$

$$\frac{\partial p}{\partial x} = 0 \quad \frac{\partial p}{\partial y} = 0 \quad \frac{\partial p}{\partial z} = -\gamma$$

These equations show that the pressure **does not** depend on **x or y directions**. Thus, when we move from point to point in a horizontal plane, the pressure does not change and it **depends** only on **z direction**.

Since p depends only on **Z direction**, the equation can be written as an ordinary differential equation.

$$\frac{dp}{dz} = -\gamma$$

This is the fundamental equation for fluids at rest that can be used to determine how pressure changes with elevation:

$$\int_{P_0}^{P_1} dp = -\gamma \int_{Z_0}^{Z_1} dz$$

$$P_1 - P_0 = -\gamma (Z_1 - Z_0)$$

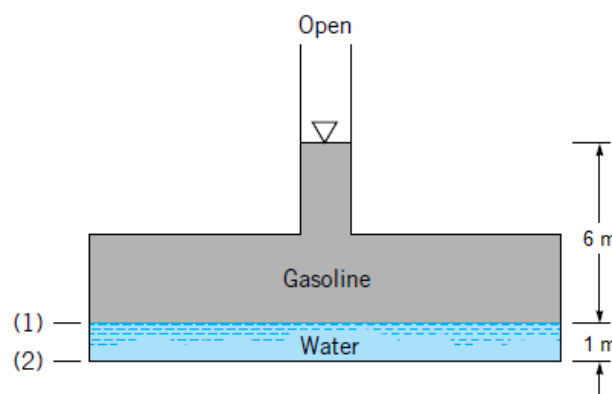
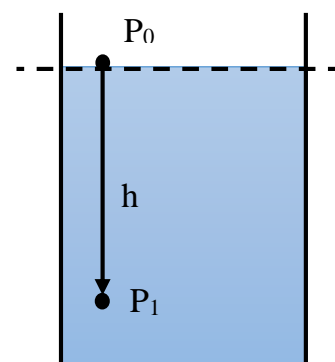
$$P_1 = P_0 - \gamma (-h - 0)$$

$$P_1 = P_0 + \gamma h$$

P_0 can be taken as zero (open to atmosphere)

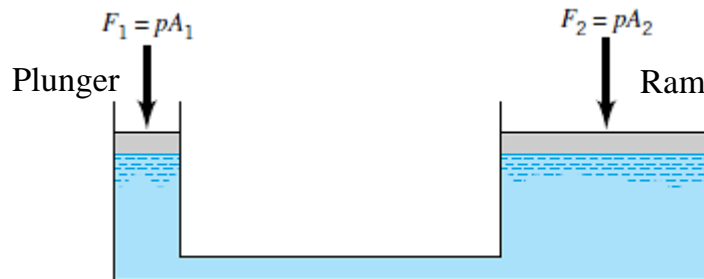
$$P_1 = \gamma h$$

Ex) Because of a leak in a buried gasoline storage tank, water has seeped in to the depth. If the specific gravity of the gasoline is 0.68. Determine the gauge pressure at the gasoline-water interface (1) and at the bottom of the tank (2).



Application on Hydrostatic Pressure Law

The equality of pressures at same elevations throughout a system is important for the operation of hydraulic jacks, lifts and presses.



The pressure p acting on the faces of both pistons is the same. Therefore, the Ram area (A_2) can be made much larger than Plunger area (A_1) and hence a large mechanical advantage can be developed.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

That is, a small force applied at the smaller piston can be used to develop a large force at the larger piston.

Ex) A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 10 kN. Find the force required at the plunger.

H.W) Can we create a hydraulic hand jack that can be used to lift heavy weights (say 1000 kN) by simply making A_2 way much bigger than A_1 ?



Pressure (Gauge, Absolute and Atmospheric)

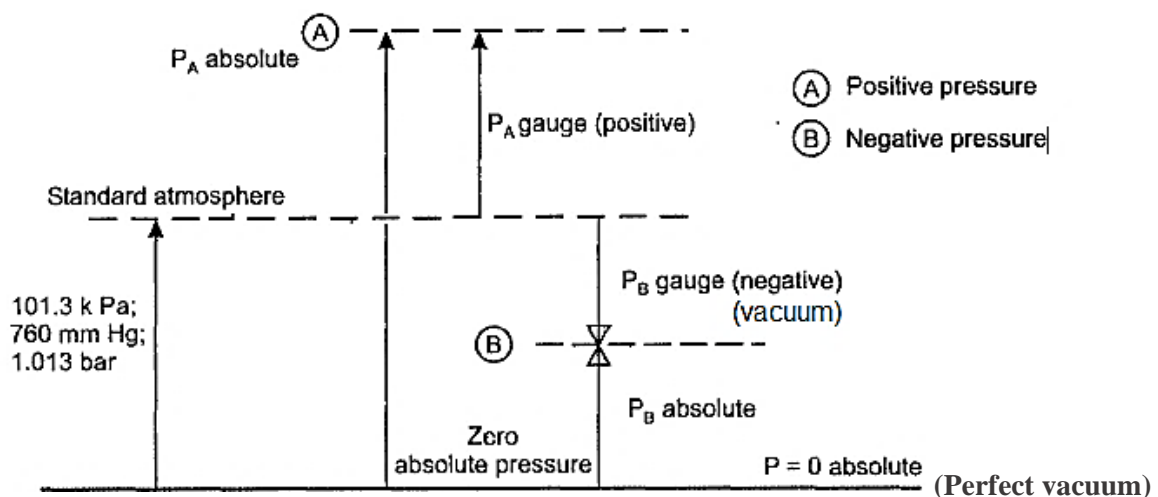
Pressure is defined as the normal force per unit area. The SI unit is (N/m²) or Pascal (Pa).

$$P = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

The **gauge pressure** is the pressure measured by a manometer relative to the atmospheric pressure.

The **atmospheric pressure** is the pressure caused by the weight of the atmosphere (air).

The **absolute pressure** is the pressure obtained relative to a perfect vacuum condition.



$$(P_A)_{\text{absolute}} = P_{\text{atm}} + (P_A)_{\text{gauge}}$$

$$(P_B)_{\text{absolute}} = P_{\text{atm}} - (P_B)_{\text{vacuum}}$$

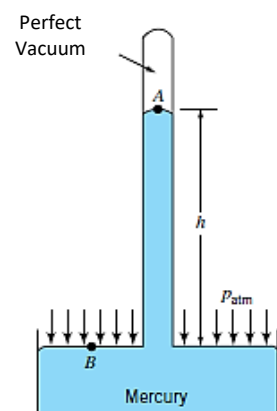
The **measurement of the Atmospheric Pressure** is usually accomplished with a **mercury barometer**. A tube is initially filled with mercury and then turned upside down in a container of mercury. The column of mercury will come to an **equilibrium position** (@760mm) where the weight of the fluid **balances** the force due to the atmospheric pressure. Thus,

$$P_{\text{atm}} = \gamma_{\text{mercury}} \times h$$

$$P_{\text{atm}} = S.G_{\text{mercury}} \times \gamma_{\text{Water}} \times h$$

$$P_{\text{atm}} = 13.6 \times 9810 \times 0.76 = 101.3 \text{ kpa}$$

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Measurement of Pressure

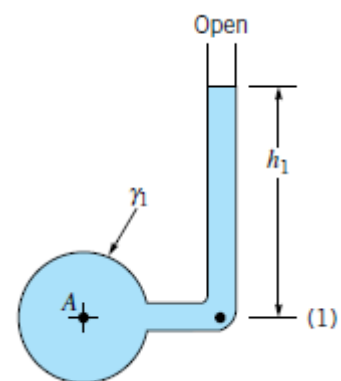
Manometers is a standard technique for measuring pressure which involves the use of liquid columns in vertical or inclined tubes. The common types of manometers are:

- Piezometer tube.
- U-tube manometer.
- Differential manometer.
- Inclined-tube manometer.

Piezometer Tube

The simplest type of manometer. It consists of a vertical tube opened at the top and attached to the container in which the pressure is desired to be measured.

$$P_A = P_1 = \gamma_1 h_1$$



Advantages and Disadvantages

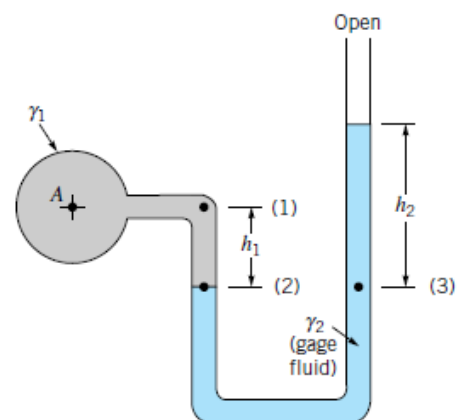
1. Very simple and accurate.
2. Measure **liquid pressure** only.
3. Only suitable if the pressure in the container is **greater** than atmospheric pressure.
4. Pressure must be relatively small so that the **height** of the column is reasonable.

U-Tube Manometer

To overcome the difficulties in piezometer tube, U tube manometer is used which includes an additional liquid (gauge fluid).

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 = 0 \quad (\text{See the note below})$$

$$P_A = \gamma_2 h_2 - \gamma_1 h_1$$



Note: Pressure *increases* (+) as we move *downward* and *decreases* (-) as we move *upward*.

Advantages

- Measure **gas** and **liquid** pressures.
- Adjustable column height for different pressure ranges.

How do we adjust the measurement range?

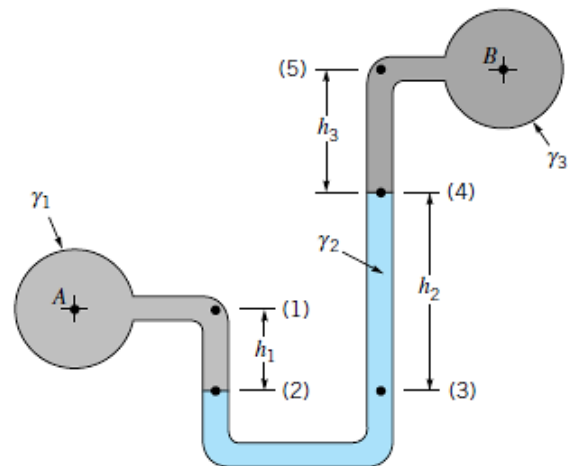
If the pressure P_A is **large**, then a **heavy gauge fluid** (such as mercury) can be used and a reasonable column height (not too long) can still be maintained. Alternatively, if the pressure P_A is **small**, a **lighter gage fluid** (such as water) can be used so that a relatively large column height (readable) can be achieved.

Differential manometer

It is basically a U type manometer connected from both sides with a gauge liquid in between.

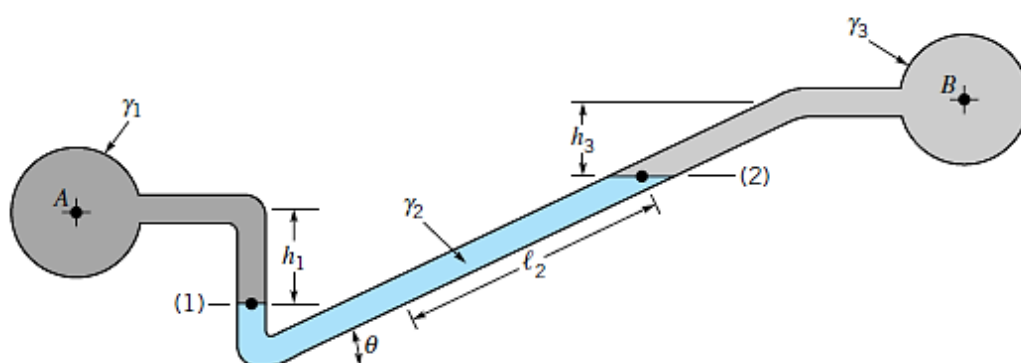
$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$



Inclined-tube manometer

The inclined-tube manometer is often used to measure **small differences in gas pressures**.



$$P_A + \gamma_1 h_1 - \gamma_2 L_2 \sin \theta - \gamma_3 h_3 = P_B$$

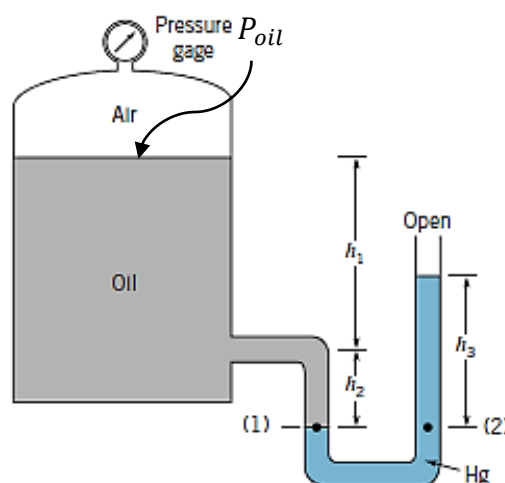
$$P_A - P_B = \gamma_2 L_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

The contributions of the gas columns ($\gamma_1 h_1$ & $\gamma_3 h_3$) can be neglected which gives:

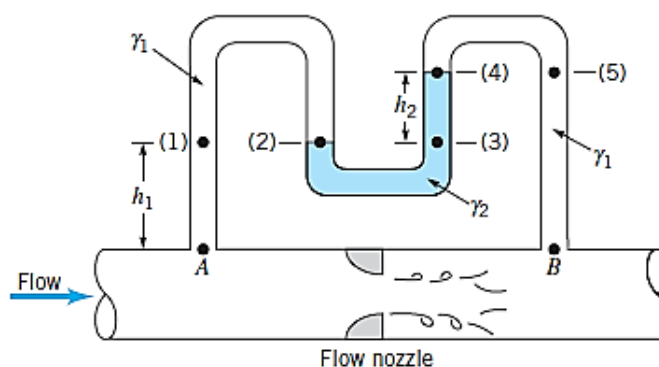
$$L_2 = \frac{P_A - P_B}{\gamma_2 \sin\theta}$$

For a given pressure difference, the **sensitivity** can be changed by **adjusting** the manometer's angle. Thus, for relatively small angles the differential reading along the inclined tube (L_2) can be large even for small pressure differences.

Ex) A closed tank contains compressed air and oil (SG = 0.9). A U-tube manometer using mercury (SG = 13.6) is connected to the tank. For column heights $h_1 = 140$ cm, $h_2 = 24$ cm and $h_3 = 36$ cm, determine the pressure reading of the gage.



Ex) Determine the value of the pressure drop (ΔP) created by the nozzle shown. For $\gamma_1 = 9.8$ KN/m³, $\gamma_2 = 15.6$ KN/m³, $h_1 = 1$ m and $h_2 = 0.5$ m.

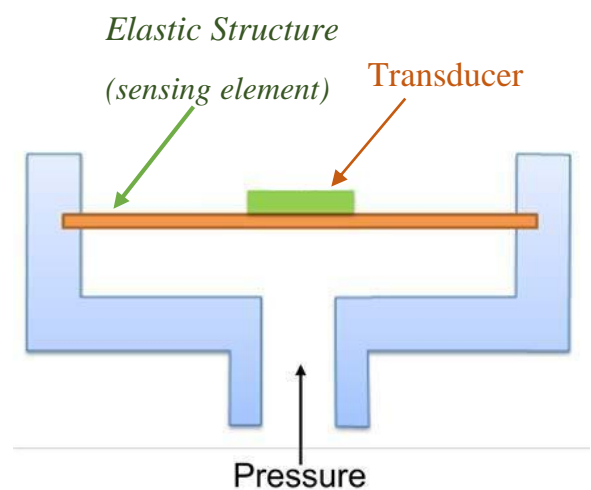


Pressure Sensor

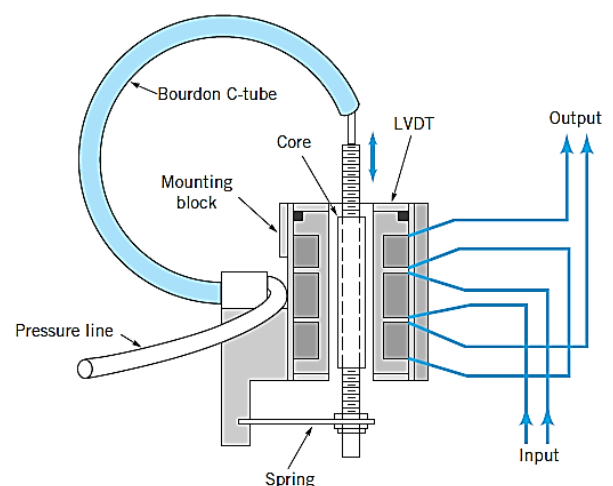
Although manometers are widely used, they have some disadvantages:

- They are not well suited for measuring very high pressures.
- Not suitable for pressures that are changing rapidly with time.
- They require the measurement of one or more column heights which can be time consuming.

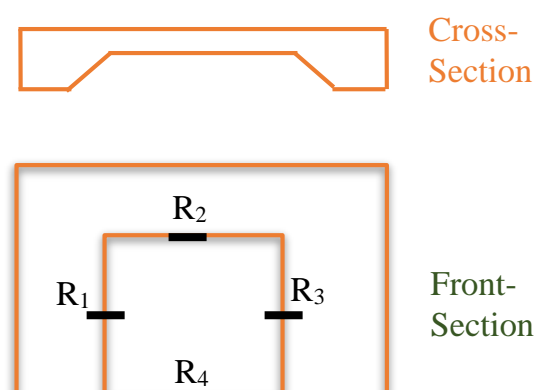
To overcome these problems, other types of pressure measuring instruments have been developed. Most of these instruments make use of the idea that when a pressure acts on an elastic structure, the structure will deform and this deformation can be related to the magnitude of the pressure.



The most familiar device is the Bourdon Tube Pressure Sensor. The essential mechanical element in this device is the hollow, elastic curved tube Bourdon tube which is connected to the pressure source. As the pressure within the tube increases the tube tends to straighten. This deformation can be translated into an electrical output using linear variable differential transformer (LVDT).



Because of the relatively high stiffness of the Bourdon tube, **it cannot respond to rapid changes in pressure**. To overcome this difficulty, the sensing element is replaced by a thin, elastic diaphragm. Also, a strain gauge is attached on the surface of the diaphragm. As the pressure changes, the diaphragm deflects and this deflection can be sensed by strain gage and



Hydrostatic Force on Submerged Surfaces

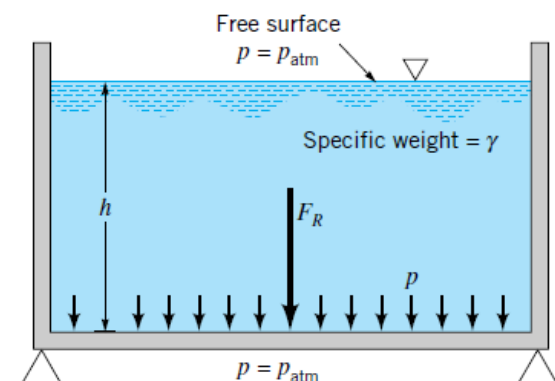
When a surface is submerged (مغمور) in a fluid, forces develop perpendicularly to the surface of contact. The calculations of these forces are important in the design of **storage tanks, dams, gates, etc.**

Force on Horizontal Surfaces

In horizontal surfaces, the pressure is constant and uniformly distributed. Therefore, the magnitude of the resultant force is simply equal to

$$FR = P \cdot A = \gamma h \cdot A$$

Location: it acts through the centroid of the Surface area.



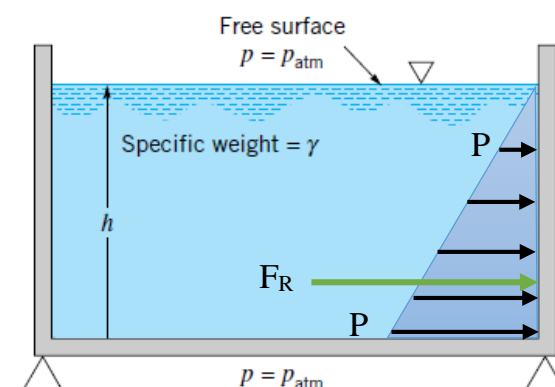
Force on Vertical Surfaces

In horizontal surfaces, the pressure will vary **linearly** with depth. Therefore, the magnitude of the resultant force can be calculated from the average pressure:

$$FR = P_{av} \cdot A = \frac{P_{Max} - P_{Min}}{2} \cdot A$$

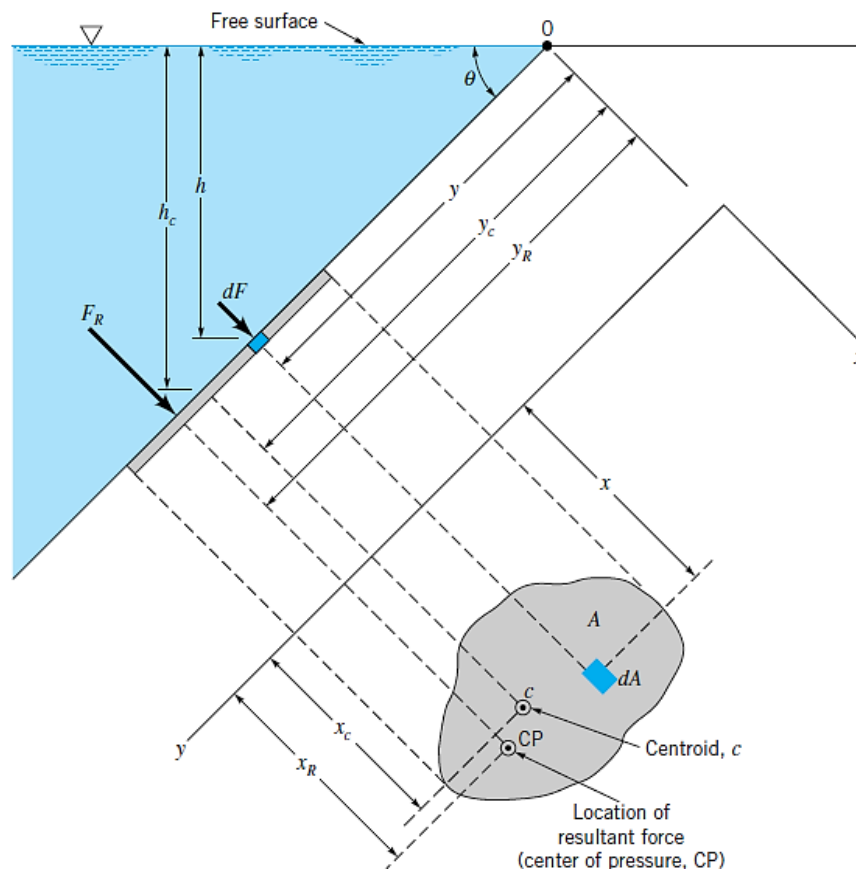
$$FR = \frac{P_{Max}}{2} \cdot A = \frac{\gamma h}{2} \cdot A$$

Location: it acts through the centroid of the area Formed by the pressure on the surface (Triangle)



Force on Inclined Surfaces

In more general case where the surface is inclined, the pressure **will also vary linearly** with depth. To calculate the **magnitude** and **location** of the resultant force, we follow the following procedure:



- The x – y coordinate system is defined in the origin (0) and y is directed along the surface passing through the centroid.
- The entire surface is divided into a number of small parallel strips (dA). The force on small strip (dF) is calculated as follow: $dF = \gamma h dA$
- The magnitude of the resultant force can be found by summing these small forces over the entire surface:

$$F_R = \int_A \gamma h dA = \int_A \gamma y \sin \theta dA$$

where $h = y \sin \theta$. For constant γ and θ

$$F_R = \gamma \sin \theta \int_A y dA$$

- The integral Part is the **first moment of the area** and it is equal to $A \cdot y_c$

Thus, the resultant force can be written as:

$$F_R = \gamma A y_c \sin \theta \qquad h_c = y_c \sin \theta$$

or more simply as

$$F_R = \gamma h_c A$$

h_c : the vertical distance from the **fluid surface** to the **centroid of the area**.

The **location** (C_p) of the resultant force can be determined by taking moments around the origin (0) in the X and Y directions:

$$\begin{aligned}
 F_R y_R &= \int_A y \, dF : & dF &= \gamma h \, dA \\
 \gamma A y_c \sin \theta y_R &= \int_A \gamma \sin \theta y^2 \, dA & h &= y \sin \theta \\
 & & dF &= \gamma \cdot y \sin \theta \, dA \\
 y_R &= \frac{\int_A y^2 \, dA}{y_c A}
 \end{aligned}$$

The integral part is the second moment of area (I_x) with respect to the free surface (X) axis.

$$y_R = \frac{I_x}{y_c A}$$

The parallel axis theorem can be used to determine I_x

$$I_x = I_{xc} + A y_c^2$$

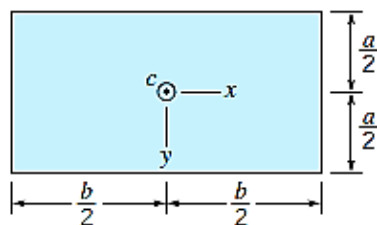
Where I_{xc} is the second moment of area with respect to the **centroid axis**.

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

The x coordinate (X_R) can be determined in a similar manner by summing moments about the y axis.

$$x_R = \frac{I_{yc}}{y_c A} + x_c$$

Properties (area, Centroid & moment of area) of some common shapes are given below:



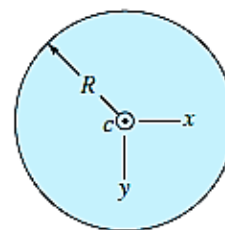
(a)

$$A = ba$$

$$I_{xx} = \frac{1}{12} ba^3$$

$$I_{yy} = \frac{1}{12} ab^3$$

$$I_{xyc} = 0$$

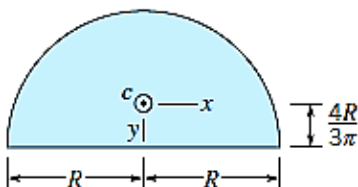


(b)

$$A = \pi R^2$$

$$I_{xx} = I_{yy} = \frac{\pi R^4}{4}$$

$$I_{xyc} = 0$$



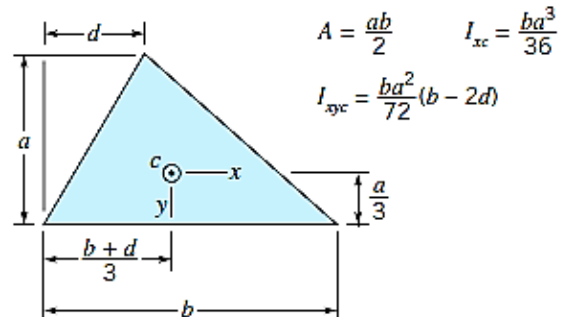
(c)

$$A = \frac{\pi R^2}{2}$$

$$I_{xx} = 0.1098 R^4$$

$$I_{yy} = 0.3927 R^4$$

$$I_{xyc} = 0$$

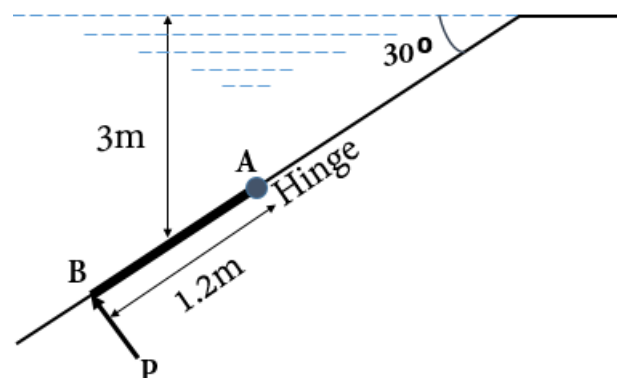


(d)

$$A = \frac{ab}{2} \quad I_{xx} = \frac{ba^3}{36}$$

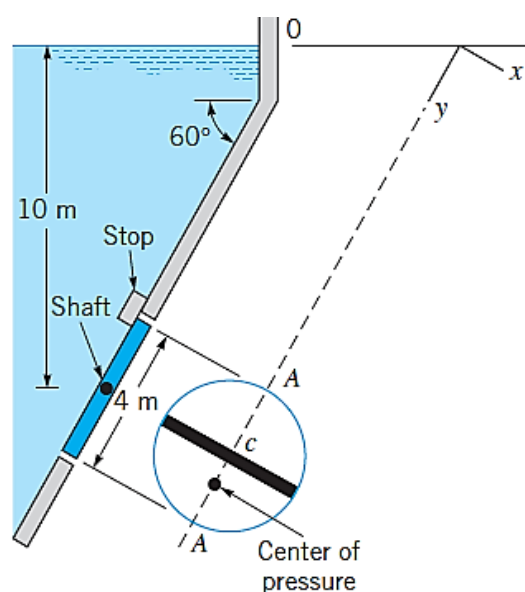
$$I_{xyc} = \frac{ba^2}{72} (b - 2d)$$

Ex1) An inclined rectangular gate AB (1.2 m height x 3.5 m width) is installed to control the discharge of water. The end A is hinged. Determine the force P normal to the gate applied at B to open it.



Ex2) A 4-m-diameter circular gate is located in the inclined (60°) wall of a large reservoir containing water. The gate is mounted on a shaft along its horizontal diameter. For a water depth of 10 m above the shaft determine:

1. The **magnitude and location** of the resultant force exerted on the gate by the water.
2. The **moment** that would have to be applied to the shaft to open the gate.

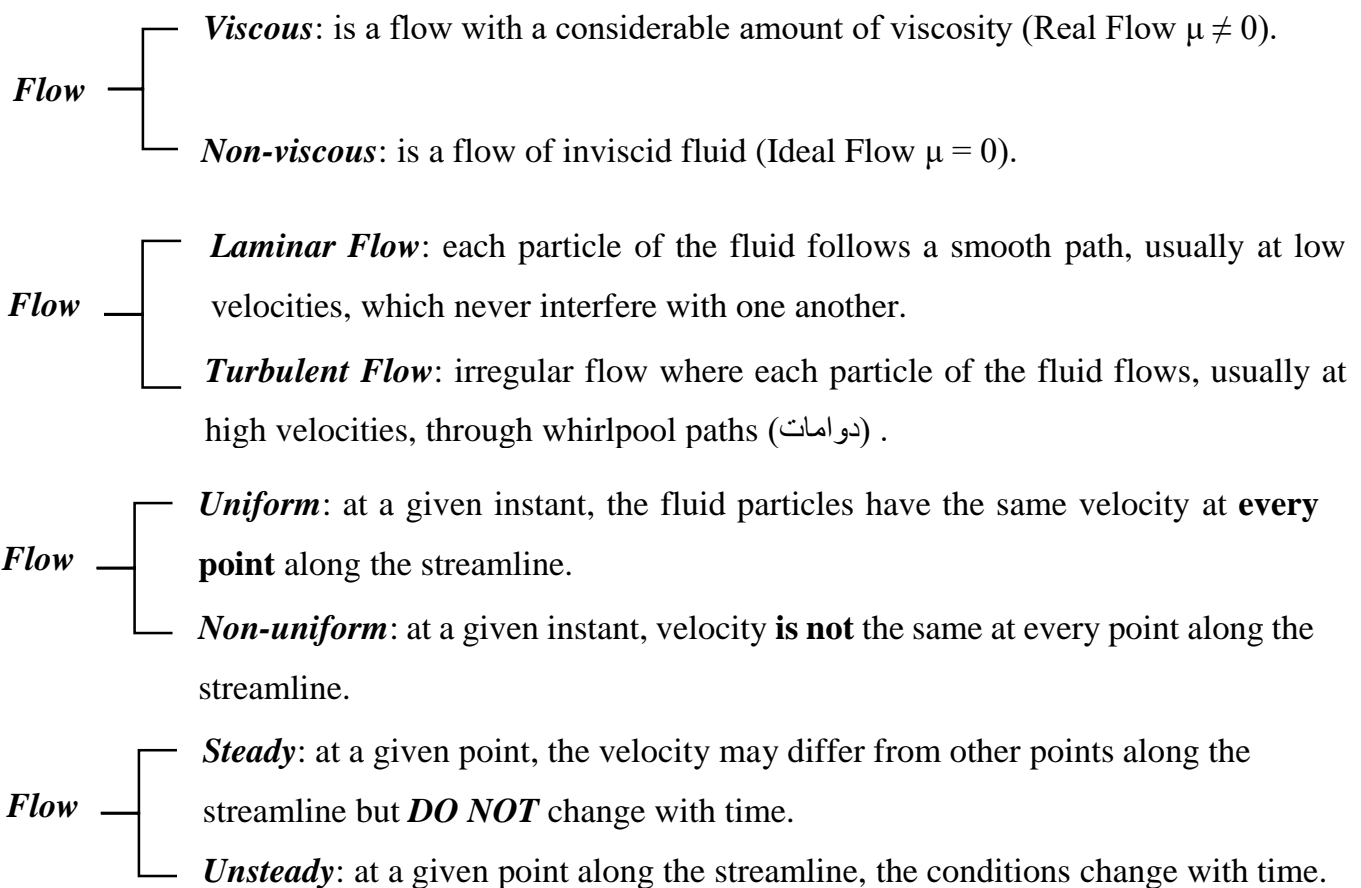


Elementary Fluid Dynamics

As was discussed earlier, Fluid Static is the situations where the fluid is considered to be *stationary (Rest)*. In contrast, Fluid Dynamics studies and analyses fluid *in motion (Flow)* along a streamline.

Flow Classification

The flow of the fluid particles can be classified as follow:



Combining the last two types above, we can classify any flow as:

- *Steady uniform flow.*
- *Steady non-uniform flow.*
- *Unsteady uniform flow.*

- *Unsteady non-uniform flow.*

Equations of motion

As a fluid particle moves from one location to another along the streamline, it usually experiences acceleration and deceleration. This **dynamic behaviour** of the fluid flow is analysed using several equations of motion such as:

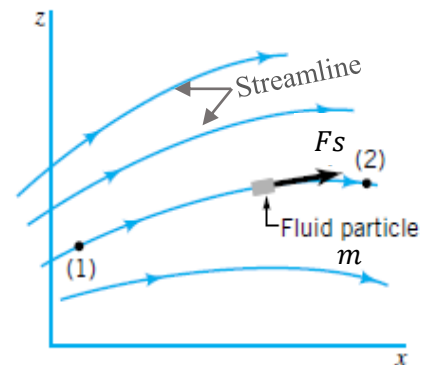
1. Newton 2nd law of motion

According to Newton's second law of motion, the net force F_S acting on a fluid element in the streamline direction (S) is equal to mass m of the fluid element multiplied by the acceleration in the S -direction. Mathematically:

$$F_S = m \cdot a_S$$

In fluid flow, the following **forces** are present:

- F_G , gravity force.
- F_P , pressure force.
- F_V , force due to viscosity.
- F_C , force due to compressibility.
- F_R , force due to turbulence.



Thus, $F_S = F_G + F_P + F_V + F_C + F_R$

2. Euler equation of motion

The following assumptions are made in the derivation of Euler's equation:

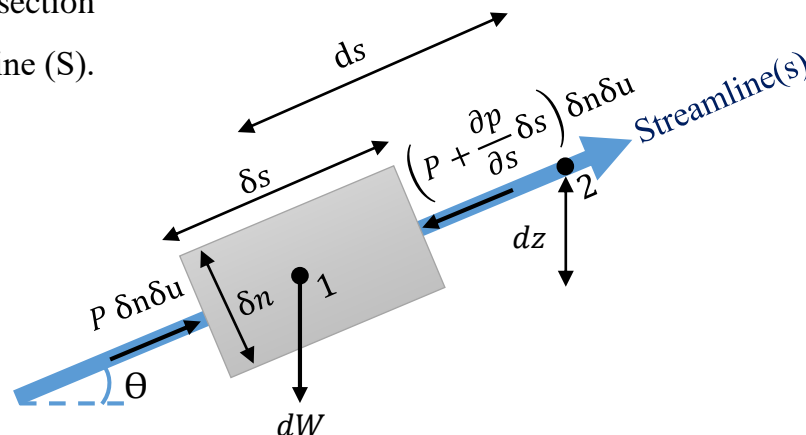
- The flow is ideal, *i.e.*, viscosity is zero ($F_V = 0$)
- The flow is steady, *i.e.*, velocity change is zero
- The flow is incompressible ($F_C = 0$)
- The flow is non-rotational ($F_R = 0$)

Therefore, only forces due to **gravity** and **pressure** are taken into consideration.

$$F_S = F_G + F_P$$

Prepared by: Mohammed N. Y. Almagid

Consider a fluid element of cross-section $\delta n \delta u$ and length δs along the streamline (S).



The forces acting on the element are:

1. Pressure force $p \delta n \delta u$ in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} \delta s\right) \delta n \delta u$ opposite to the direction of flow.
3. Weight of element $dW = \text{specific weight} \times \text{volume} = \rho g \delta n \delta u \delta s$.

$$\sum F_s = m \cdot a_s$$

$$P \delta n \delta u - \left(P + \frac{\partial p}{\partial s} \delta s\right) \delta n \delta u - \rho g \delta n \delta u \delta s \sin \theta = \rho \delta n \delta u \delta s \cdot a_s$$

The acceleration (a_s) is the rate change of velocity $V(s)$ in the (S) direction, *Therefore*

$$a_s = \frac{dv(s)}{dt}$$

The velocity may change from point to point, *Thus*

$$a_s = \frac{\partial v}{\partial s} \frac{ds}{dt}, \text{ where } \frac{ds}{dt} = v$$

$$a_s = v \frac{\partial v}{\partial s}$$

$$-\frac{\partial p}{\partial s} \delta n \delta u \delta s - \rho g \delta n \delta u \delta s \sin \theta = \rho \delta n \delta u \delta s \cdot v \frac{\partial v}{\partial s}$$

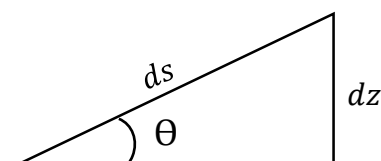
$$-\frac{\partial p}{\rho \partial s} - g \sin \theta = v \frac{\partial v}{\partial s}$$

$$\frac{\partial p}{\rho \partial s} + g \frac{dz}{ds} + v \frac{\partial v}{\partial s} = 0$$

$$\sin \theta = \frac{dz}{ds}$$

dividing by g

dividing by $\rho \delta n \delta u \delta s$





$$\frac{1}{\rho g} \frac{\partial p}{\partial s} + \frac{V}{g} \frac{\partial v}{\partial s} + \frac{dz}{ds} = 0 \quad \text{Euler equation of motion.}$$

3. Bernoulli equation

Bernoulli equations can be obtained by rearranging and integrating Euler equation derived above and as follow:

$$\frac{1}{\rho g} \frac{dp}{ds} + \frac{V}{g} \frac{dv}{ds} + \frac{dz}{ds} = 0 \quad \text{this is simplified into:}$$

$$\frac{dp}{\rho g} + \frac{V dv}{g} + dz = 0 \quad \text{which can be integrated to:}$$

$$\frac{p}{\rho g} + \frac{V^2}{2g} + Z = \text{constant along the streamline}$$

Therefore, for **any two points (1&2)** on a streamline in steady, inviscid and incompressible flow the Bernoulli equation can be applied in the form:

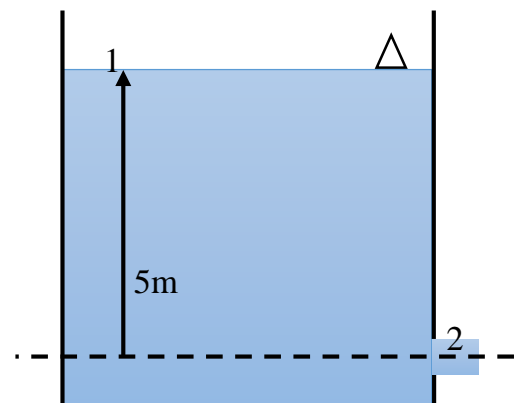
$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 \quad \text{Bernoulli equation of motion where:}$$

$\frac{p}{\rho g}$ is the pressure energy per unit weight of the fluid or pressure head

$\frac{V^2}{2g}$ is the kinetic energy per unit weight of the fluid or kinetic head

Z is the potential energy of the fluid or potential head

Ex: A large tank, opened to the atmosphere, is filled with water to a height of (5m). A tab near to the bottom is opened and water flows from the smooth and rounded outlet. Determine the water velocity at the outlet.



Flow Rate

It is a measure at which fluid is flowing per unit time.

Flow Rate

Mass Flow Rate \dot{m} is the mass of fluid flowing per unit time

$$\dot{m} = \frac{\text{mass}}{\text{time}} \text{ (Kg/s)}$$

Volume Flow Rate Q is the volume of fluid flowing per unit time

$$Q = \frac{\text{volume}}{\text{time}} \text{ (m}^3\text{/s)}$$

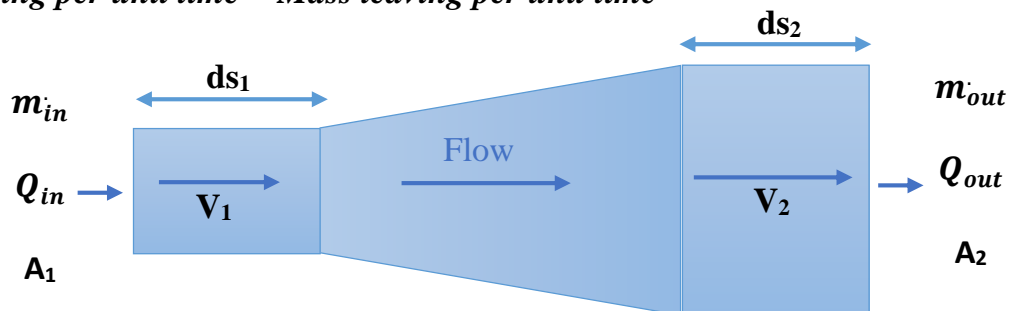
For example, an empty bucket weighs 2.0 kg. After 7 seconds of collecting water the bucket weighs 8.0 kg, then

$$\begin{aligned} \text{mass flow rate} = \dot{m} &= \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}} \\ &= \frac{8.0 - 2.0}{7} \\ &= 0.857 \text{ kg/s} \quad (\text{kg s}^{-1}) \end{aligned}$$

Continuity Equation

According to the **conservation of mass**, matter cannot be created or destroyed but it can change to different forms. Thus:

Mass entering per unit time = Mass leaving per unit time



$$\dot{m}_{in} = \dot{m}_{out}$$

$$\rho_1 A_1 \frac{ds_1}{dt} = \rho_2 A_2 \frac{ds_2}{dt}$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

For **incompressible** fluid, $\rho_1 = \rho_2$

$$A_1 V_1 = A_2 V_2 = Q \quad \text{volume flow rate}$$

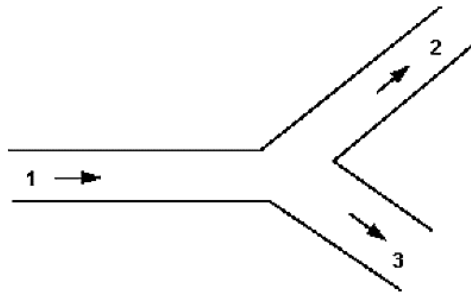
$$Q_{in} = Q_{out}$$

$$\dot{m} = \rho Q$$

Similarly, in the case of junctions or flow dividers, as shown below:

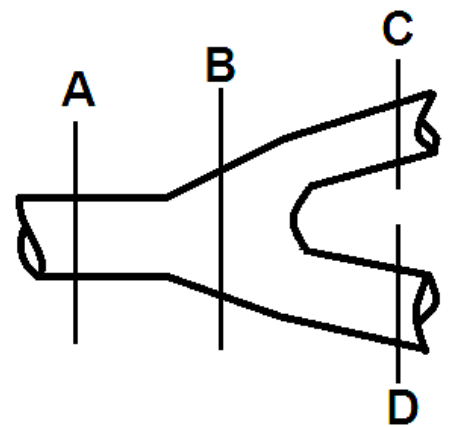
$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$



Ex: The pipe has a diameter of (1.2 m) at A, (1.5 m) at B and (0.8 m) at C. The discharge at C is ($Q_A / 3$) and the velocities at A & D are (3.5 m / s) and (2.5 m / s) respectively. If the flow is steady incompressible, Determine the:

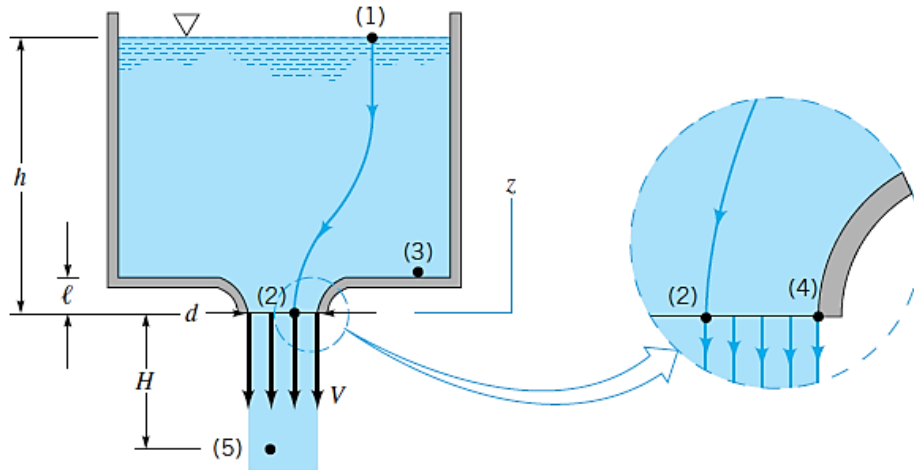
1. Discharge at A.
2. The velocity at B&C.
3. The diameter at D



Application on Bernoulli Equation

a. Free Jets

Bernoulli's equation can be applied to the flow of a liquid from large tanks, as is shown:



Applying Bernoulli's equation between point 1&2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

Both streamlines (1&2) in contact with the atmosphere, $P_1=P_2=0$. Also, $Z_1 = h$ and $Z_2 = 0$.

The reservoir is large, V_1 is **approximately** equal to zero. Thus,

$$h = \frac{V_2^2}{2g} \rightarrow V_2 = \sqrt{2gh}$$

The stream continues to fall as a free jet with zero pressure

Applying Bernoulli's equation between point 1&5

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_5}{\rho g} + \frac{V_5^2}{2g} + Z_5$$

$$h + H = \frac{V_5^2}{2g}$$

The Speed increases according to $V_5 = \sqrt{2g(h + H)}$

The pressure at point 3 can be obtained by:

- Applying Bernoulli's equation between 1&3

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\rho g} + \frac{V_3^2}{2g} + Z_3$$

$P_1 = 0$. Also, $Z_1 = h - \ell$, and $Z_3 = 0$.

The reservoir is large, V_1 and V_3 is approximately equal to zero. Thus,

$$\frac{P_3}{\rho g} = h - \ell$$

$$P_3 = \gamma (h - \ell)$$

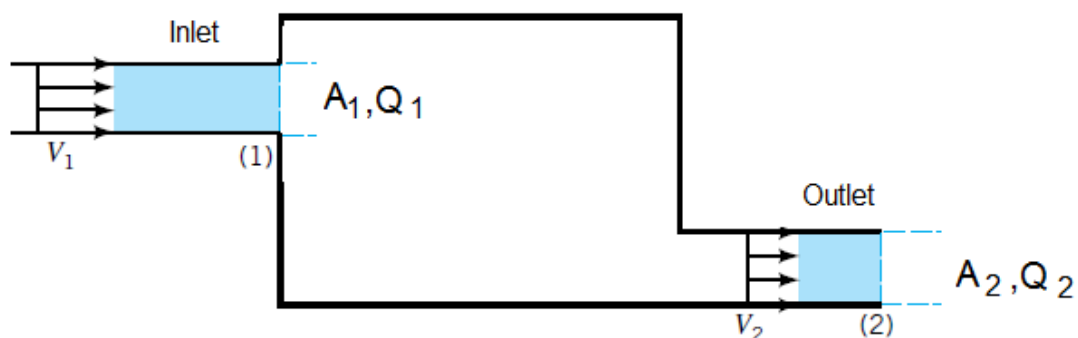
- Or by simply applying the hydrostatic field pressure law

$$P_3 - \gamma (h - \ell) = 0$$

$$P_3 = \gamma (h - \ell)$$

b. Flow in pipes

As the fluid flows within a pipe of **variable dimeter**, the velocity **changes** because the flow area is different from one section to another. In these situations, we use the **continuity equation** along with the **Bernoulli equation**.

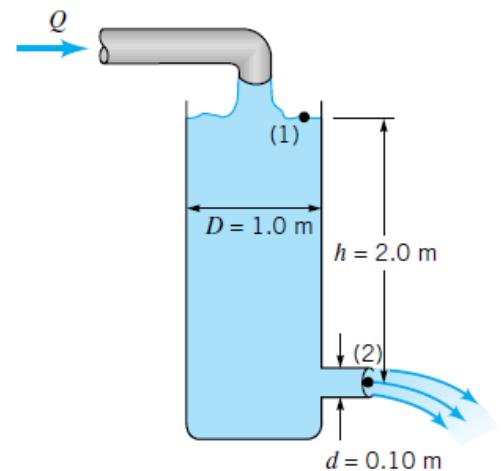


$$Q_1 = Q_2$$

$$A_1 V_1 = A_2 V_2$$

Ex) The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 liters/s. If the pressure at section 1 is 39.24 N/cm², find the intensity of the pressure at section two.

Ex) A stream of water of diameter (0.1m) flows steadily from a tank of diameter ($D = 1\text{m}$) as shown. Determine the flowrate Q needed from the inflow pipe so that the depth remains constant, $h = 2.0\text{ m}$.



Practical Applications of Bernoulli's Equation

Flow Rate Measurement

An effective way to measure the **flowrate** through a pipe is to place some type of restriction within the pipe and to measure the pressure difference between the high-pressure and low-pressure sections. Several measuring devices use Bernoulli principle to measure fluid **velocities and flowrates**.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

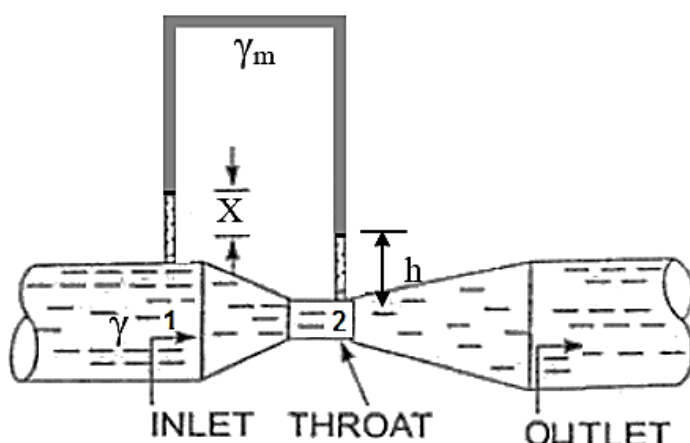
$$A_1 V_1 = A_2 V_2$$

Combining the two equations, we get

$$Q = A_2 \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

1. Venturimeter: is a device used for measuring **the rate of a flow** of a fluid flowing through a pipe. It consists of three parts:

- (i) *Short converging part*
- (ii) *Throat*
- (iii) *Diverging part*



γ_m : Specific weight of the Manometer's liquid.

γ : Specific weight of the liquid flowing in the pipe.

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

The pipe is horizontal, thus $Z_1 = Z_2$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} \rightarrow \frac{V_2^2}{2g} = \frac{P_1 - P_2}{\gamma} + \frac{V_1^2}{2g} \quad 1.1$$

To find $P_1 - P_2$, we apply the hydrostatic pressure law:

$$P_1 - \gamma(h + x) + \gamma_m x + \gamma h = P_2$$

$P_1 - P_2 = x(\gamma - \gamma_m)$, Apply in 1.1:

$$\frac{V_2^2}{2g} = \frac{x(\gamma - \gamma_m)}{\gamma} + \frac{V_1^2}{2g} \rightarrow V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma}\right) + V_1^2 \quad 1.2$$

To find V_1 , we apply the continuity equation:

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2}{A_1} V_2, \text{ Apply in 1.2:}$$

$$V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma}\right) + \left(\frac{A_2}{A_1}\right)^2 V_2^2$$

$$\left(1 - \left(\frac{A_2}{A_1}\right)^2\right) V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma}\right) \rightarrow V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma}\right) \frac{A_1^2}{A_1^2 - A_2^2}$$

$$V_2 = \sqrt{2gx \left(1 - \frac{\gamma_m}{\gamma}\right) \frac{A_1^2}{A_1^2 - A_2^2}} \rightarrow V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gx \left(1 - \frac{\gamma_m}{\gamma}\right)}$$

$$Q = A_2 V_2 = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gx \left(1 - \frac{\gamma_m}{\gamma}\right)}$$

The above formula is correct when the Manometer liquid is lighter than the liquid flowing in the pipe ($\gamma_m < \gamma$). Therefore, the term $\left(1 - \frac{\gamma_m}{\gamma}\right)$ is positive. If the Manometer liquid is heavier than the liquid flowing through the pipe, the formula can be written as

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gx \left(\frac{\gamma_m}{\gamma} - 1 \right)}$$

Ex: An oil of SG = 0.8 is flowing through a Venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of $x = 15$ cm. Calculate the theoretical discharge of oil through the horizontal Venturimeter given that SG of mercury is 13.6.

Ex) What would be the reading of the oil-mercury differential manometer if the flowrate was increased to 60 ltrs/s .

Ex) A pitot tube is inserted in a pipe of 10 cm diameter. The pressure difference measured by a mercury ($SG = 13.6$) differential manometer gives a reading of 18 cm of mercury. Find the velocity of the flow of an oil of $SG = 0.7$.

Ex) The same pipe is used to transfer different liquid at the same flow rate. Find the density of that oil knowing that the oil-mercury differential manometer reading is 25 cm.

Pneumatic & Hydraulic Systems

Fluid Power: Power generated by an effective pumped or compressed fluids to provide force and motion to mechanisms. This force may be in the form of pushing, pulling, lifting or cutting. Fluid power includes hydraulics, which involves liquids, and pneumatics which uses air.

Advantages of Fluid Power

Hydraulics and pneumatics systems have a number of favorable characteristics:

- Very high power to weight ratio.
- Self-lubricating and cooling
- Low Initial cost, especially Pneumatic System
- Motion can be transmitted via fluid without the need for complicated systems of gears, cams, and levers
- The forces generated are transmitted over large distances with small loss
- It can provide smooth, flexible and uniform action without vibration
- It provides variable motions in both rotary and straight-line
- Fluid power systems are economical to operate

Disadvantages of Fluid Power

- Leaks must be prevented. This is a serious problem with the high pressure obtained in many fluid power installations
- Movement of the fluid within the lines and components can cause friction against the containing surfaces which can lead to serious losses in efficiency
- Fluid must be kept clean, clogging can cause series damages

Pneumatic or Hydraulic?

- In general, when the application requires a low amount of power and only fairly accurate control, a pneumatic system may be used.
- If the application requires a great amount of pressure and/or extremely accurate control, a hydraulic system should be used.

Pneumatic	Hydraulic
Power to weight ratio is lower than the Hydraulic System	Very High Power to weight ratio
Relatively cheap	More expensive than pneumatic
Can exhaust to atmosphere	Mess from oil leaks
Temperature has less effect	Oil property changes with temperatures
Safe in potentially explosive environment	Danger from oil leaks

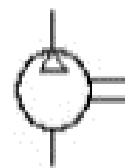
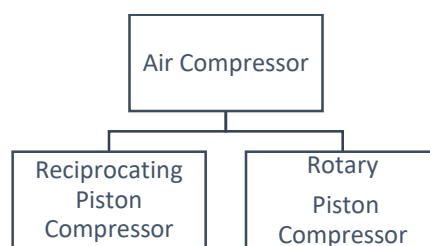
Pneumatic systems components

The Pneumatic system consists of

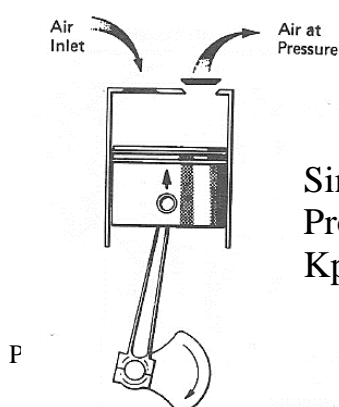
- Air compressors
- Air filter, dryer and separator
- Air reservoir (tanks)
- Regulator, relief, check and control valves
- Actuation cylinders

Air compressors

Rises the air pressure from atmospheric value to the desired level. Pneumatic components are designed for a maximum operating pressure of 800 to 1000 Kpa (8-10 bar).

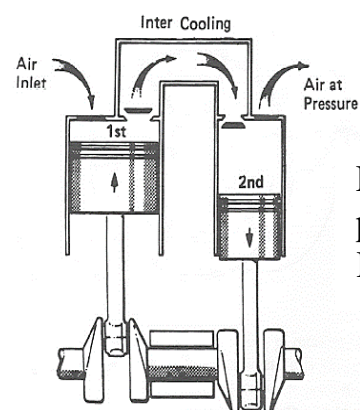


Reciprocating piston compressors



Single acting piston
Produces up to 400
Kpa (4 bar)

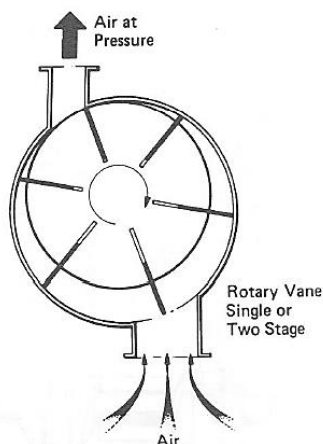
Almaged



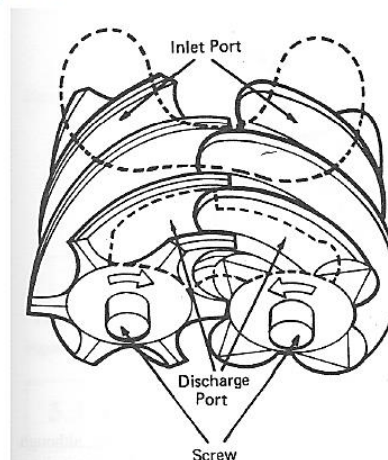
Double acting piston
produces up to 800
Kpa (8 bar)

Rotary compressors: It uses rotating elements to compress and increase the pressure of the air.

Rotary Vane compressors:

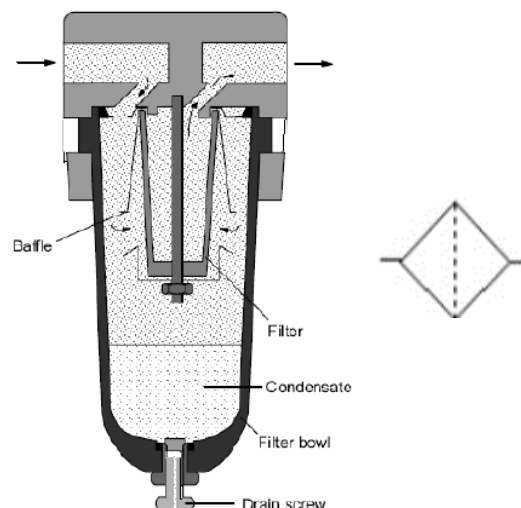


Screw compressor:



Air preparation: involves the following

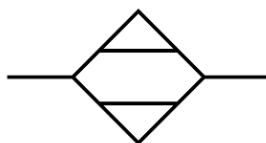
1. A pneumatic filter is a device which removes contaminants from a compressed airstream. One characteristic of compressed-air filters is the size. The size of the filter element indicates the minimum particle size which can be filtered out of the compressed air. Typical 5 microns.



2. Water separators separate out the mist of water droplets which are sometimes suspended in air by centrifugal means.



3. Pneumatic Dryer: An extra stage to dry out the compressed water using silica gel.

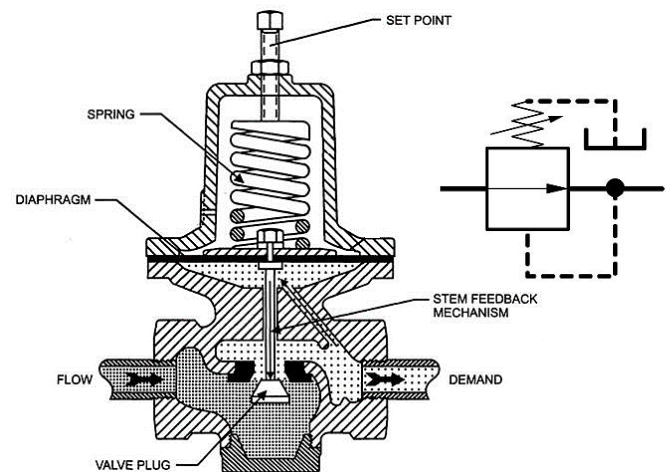


Reservoirs: store the compressed air from the compressor.

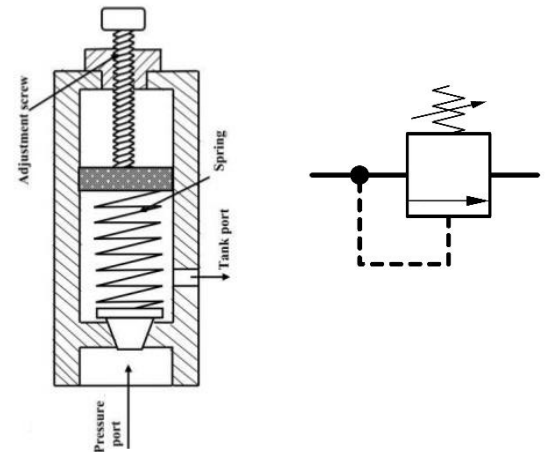


Pneumatic Valves

Pressure regulator valve: is a control valve that **reduces** the input pressure of a fluid to a desired value at its output keeping the operating pressure constant, regardless of pressure fluctuations or air consumption in the system **as long as** the input pressure at the pressure regulator is higher than the output pressure.



Pressure relief valve: When the set maximum pressure is reached, the pressure-relief valve opens and the air is exhausted to atmosphere.



Pressure check valve: on-return valve or one-way valve is a valve that normally allows fluid (liquid or gas) to flow through it in **only one** direction.

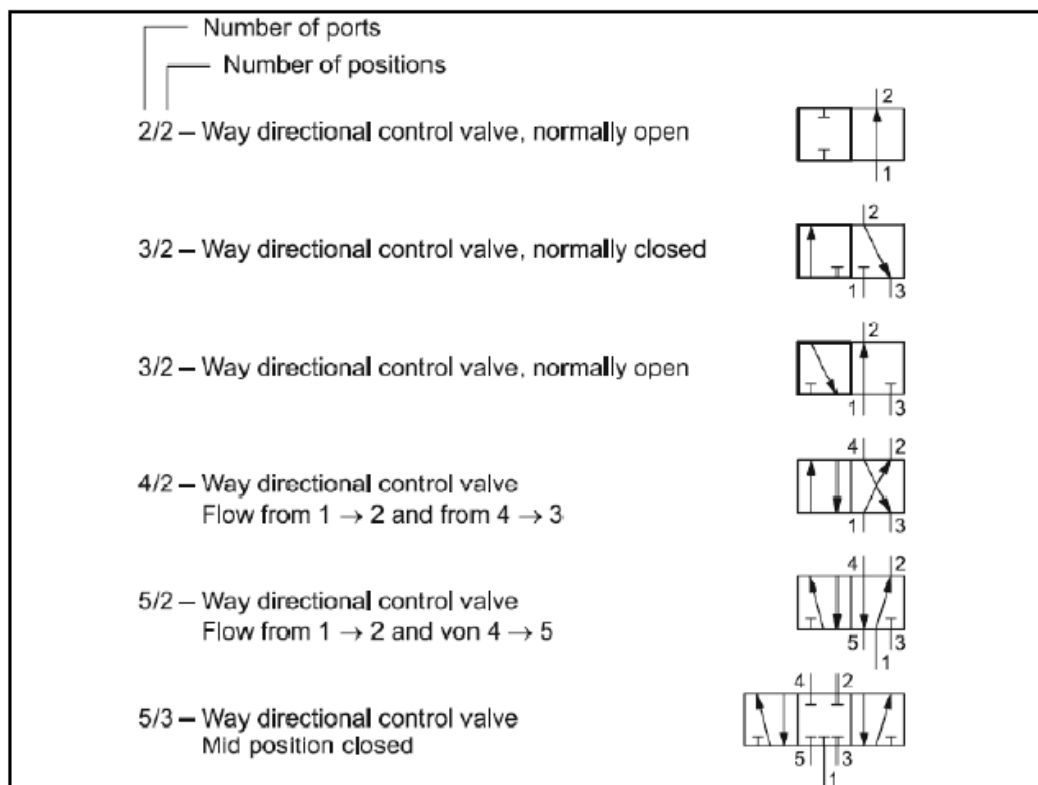


Directional control valves

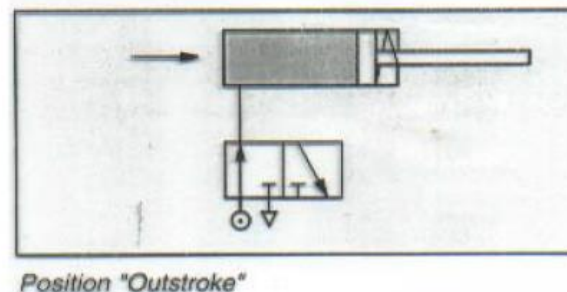
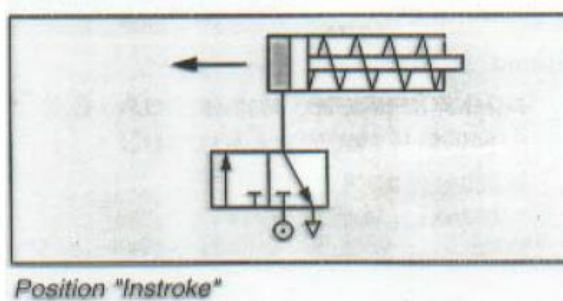
Devices that ***influence the control taken by an air*** by opening the passage of air and directing it to particular air lines or canceling it by blocking their passage or relieving the air to atmosphere via an exhaust port.

The directional control valve is characterized according to:

- Number of switching positions & Ports (2-way, 3-way, etc.)
- Method of actuation (Manual, mechanical, electrical)

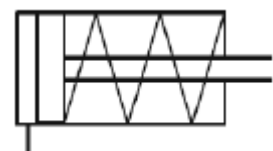


Ex) 3/2 – Way directional valve, normally closed



Pneumatic actuator: is an output device that converts the supplied fluid energy into a useful work (linear or rotary motion).

Single-acting cylinders: air is applied on **only** one side of the piston face. The other side is open to atmosphere and The return movement of the piston is produced by a built-in spring.



Double-acting cylinder: air is applied on **both** sides of the piston face. So that motion is controller in both directions.

