The principles of Mechanical engineering

* statics
* Dynamics
* thermodynamics

Engineering Mechanics by J.L. Meriam & L.G. kraige

Mechanics: is that branch of physical science which deals with the state of rest or motion of bodies under the action of forces.

Bacis concepts:-

- space: is the geometric region occupied by bodies whose position are described by Linear and angular measurments relative to a coordinate system.

- Time :- is the measure of the succession of events and is abasic quantity in dynamics

- Mass: is a measure of the inertia of a body, which is its resistance to a change of velocity.

Force: is the action of one body on an other. The action of a force is characterized by its magnitude, by the direction of its action.

Particle: is the body of negligible dimensions.

Newton's Laws:-

Law I: A particle remains at start or continues to move in a straight Line with a uniform velocity if there is no unbalanced force acting on it.

Law II: The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

Law III: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction and collinear.

Units:-

Mechanics deal with four fundametal quantities :-
Length , mass , force and time .

Quantity	Dimensional symbol	unit	symbol
mass	M	kilogram	Kg
Length	L	meter	m
Time	T	second	S
Force	F	Newton	N

(S.I. units)

بعض إصطلاحات اللاتينية
المستخدمة للتعبير عن الزوايا

θ	ثيتا
β	بيتا
γ	كاما
α	الفا
σ	سيكما
η	ايتا

The Forces

Sumation of forces (Resultant):-

1- The forces in the same direction:-

$\overrightarrow{F_1}\ ,\ \overrightarrow{F_2}$ $\quad \therefore \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$ as diretion

$R = F_1 + F_2$ magnitude

2- The forces in the opposite direction

$\overrightarrow{F_1}\ ,\ \overleftarrow{F_2}$ $\quad \therefore \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

$R = F_1 + (-F_2)$

3- Angle between direction of the forces:-

$\overrightarrow{F_1}\ ,\ F_2\ (\theta)$ $\quad \overrightarrow{R} = \overrightarrow{F_1} + \overrightarrow{F_2}$

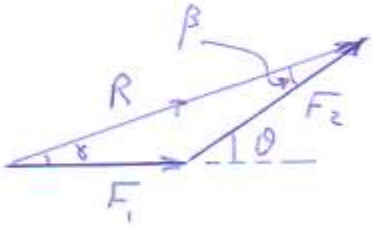
cos Law $\longrightarrow$ $R = \sqrt{F_1^2 + F_2^2 \mp 2F_1F_2\cos\theta}$

if $(0 < \theta < 90)$, take positive sign

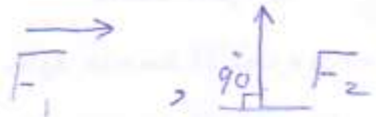
if $(90 < \theta < 180)$, take negative sign

<u>or</u>: $\dfrac{F_1}{\sin\beta} = \dfrac{F_2}{\sin\gamma} = \dfrac{R}{\sin(180-\theta)}$

this is sin Law



4- perpendicular forces



$$\vec{R} = \vec{F_1} + \vec{F_2}$$

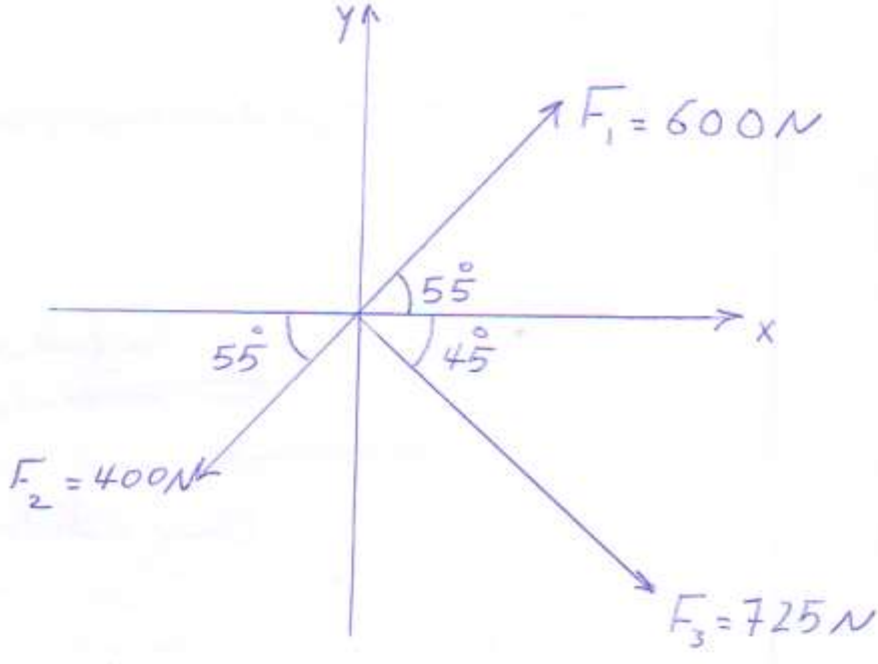
$$R^2 = F_1^2 + F_2^2$$

or $$R = \sqrt{F_1^2 + F_2^2}$$

Examples:-

Ex 1:- Find the resultant of these forces

solution



$$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

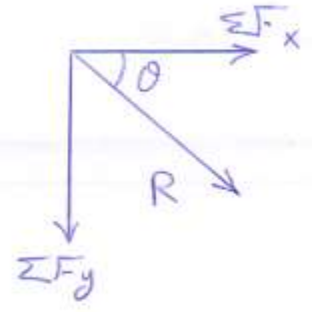
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$$

$$\Sigma F_x = 600 \cos 55 + 725 \cos 45 - 400 \cos 55$$

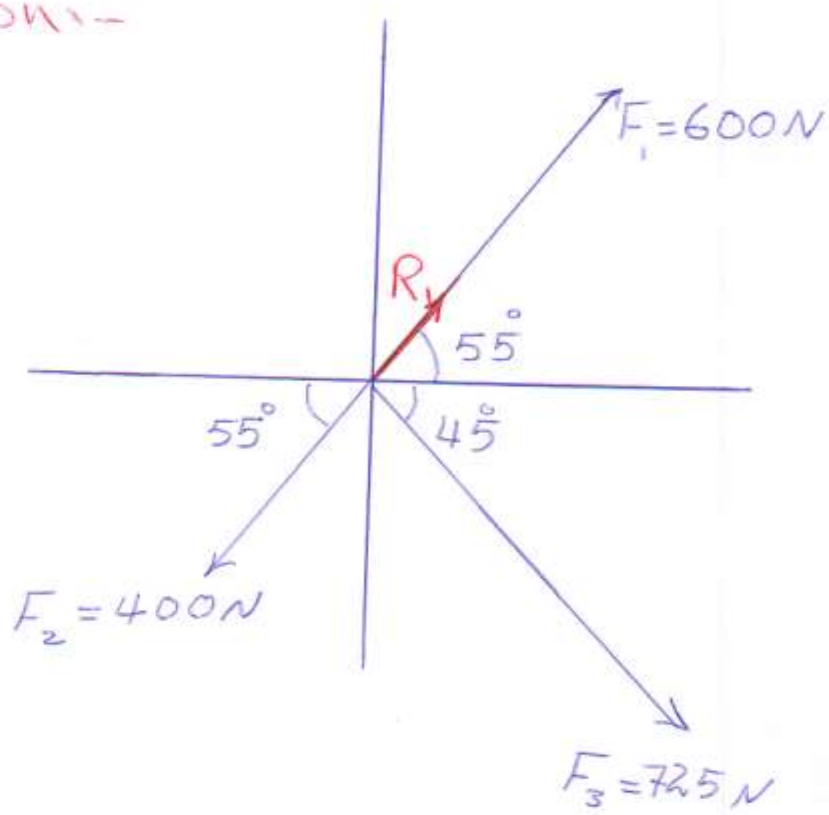
$$= 627.37\,N$$

$$\therefore R = \sqrt{(627.37)^2 + (-348.82)^2} = 717.82\ N$$

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-348.82}{627.37} \Rightarrow \theta = 29^\circ$$



Another solution:-



$$R_1 = F_1 + (-F_2)$$

$$= 600 - 400$$

$$= 200\ N$$

$$\Sigma F_x = R_{1x} + F_{3x}$$

$$= 200\cos 55 + 725\cos 45$$

$$= 627.37\ N$$

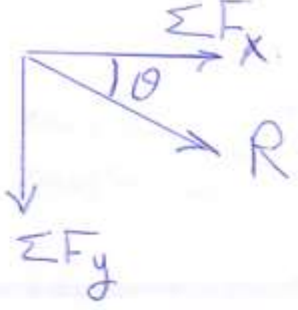
$$\Sigma F_y = R_{1y} + F_{3y}$$

$$= 200\sin 55 - 725\sin 45 \Rightarrow \Sigma F_y = -348.82\ N$$

so $R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

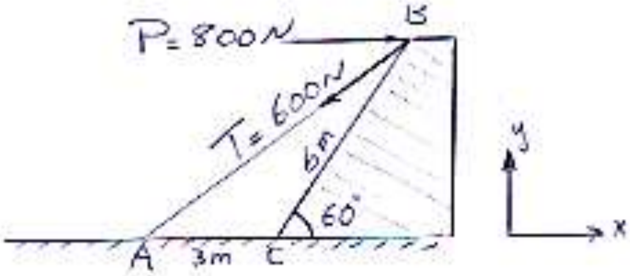
$= 717.82\ N$

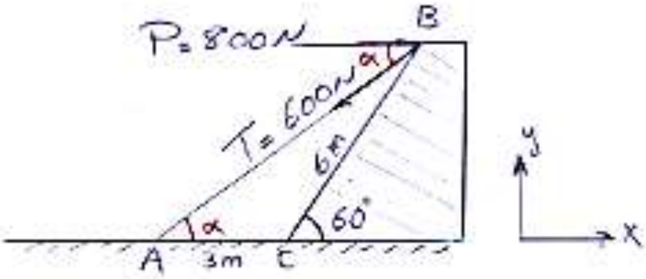
$\theta = \tan^{-1} \dfrac{\Sigma F_y}{\Sigma F_x} \Rightarrow \theta = 2.9^\circ$

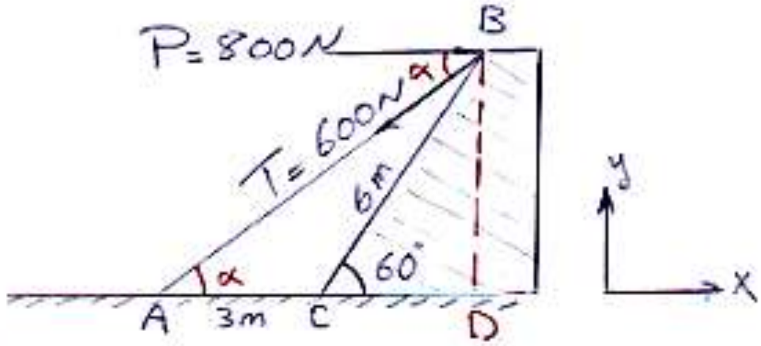


Example 2:

Combine the two forces P & T, which act on the fixed structure at B, into a single equivalent force (R).







$\sin 60 = \frac{BD}{6} \Rightarrow BD = 5.2\,m$

$\cos 60 = \frac{DC}{6} \Rightarrow DC = 3\,m$

$\therefore \tan\alpha = \frac{BD}{DC+3} = \frac{5.2}{3+3} \Rightarrow \tan\alpha = 0.867$

$\therefore \alpha = 40.9^\circ$

$$\Sigma F_x = 800 - 600 \cos\alpha$$

$$= 800 - 600 \cos 40.9 \Rightarrow \Sigma F_x = 346.5\ N$$

$$\Sigma F_y = -600 \sin\alpha = -600 \sin 40.9$$

$$\Rightarrow \Sigma F_y = -392.8\ N$$

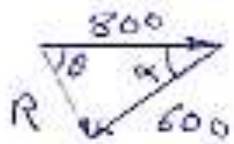
another solution :-

$$R = \sqrt{F_1^2 + F_2^2 \mp 2F_1 F_2 \cos\alpha}$$

$$\therefore R = \sqrt{800^2 + 600^2 - 2(800)(600)\cos 40.9}$$

$$\Rightarrow R = 524\ N$$

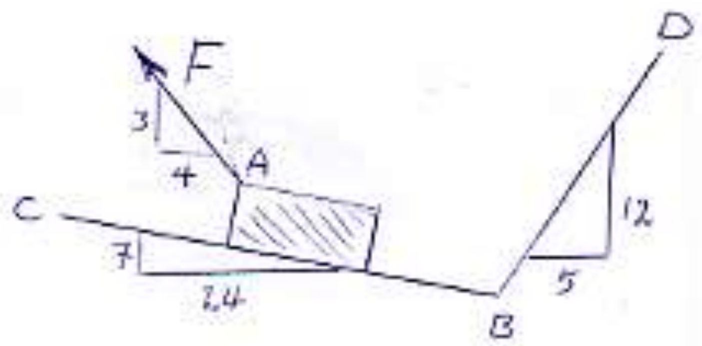
cos Law



sin Law

Example 3 :-

The force F in fig. as shown is the resultant of a 100N force along BC and a force acting parallel to BD. Determine the component parllel to BD.

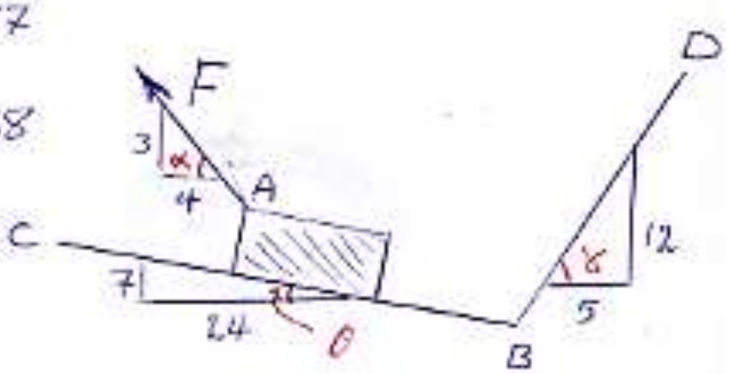


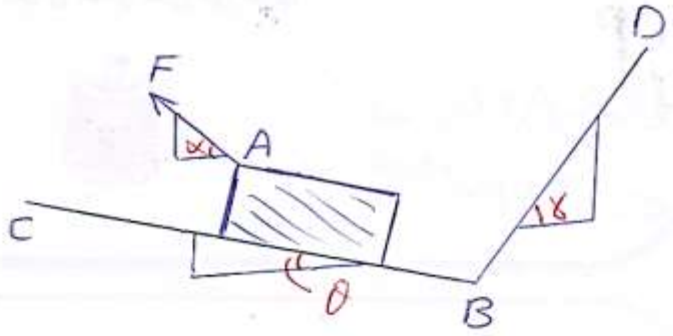
solution:-

$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.87^\circ$

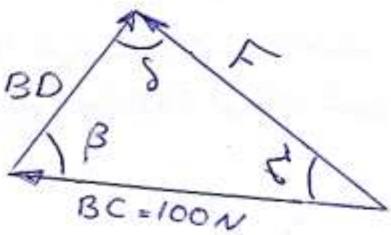
$\tan\gamma = \frac{12}{5} \Rightarrow \gamma = 67.38$

$\tan\theta = \frac{7}{24} \Rightarrow \theta = 16.26^\circ$





$\zeta = \alpha - \theta = 20.61^\circ$

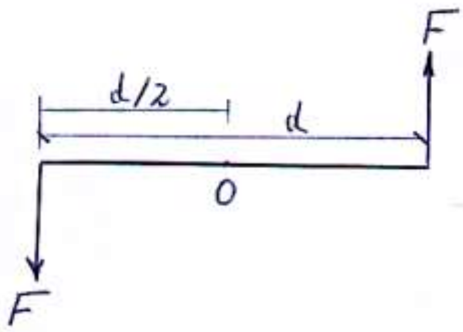


$\beta = \gamma + \theta = 83.64^\circ$

$\delta = 180 - \zeta - \beta = 75.75^\circ$

$\frac{100}{\sin \delta} = \frac{BD}{\sin \zeta} \Rightarrow BD = 36.32\ N$

The Couple



The moment produced by two equal, opposite, and noncollinear forces is called a couple.

$$\overset{\curvearrowright -}{M_o} = F \cdot \frac{d}{2} + F \cdot \frac{d}{2}$$

$$\Rightarrow \overset{\curvearrowright -}{M_o} = F \cdot d$$

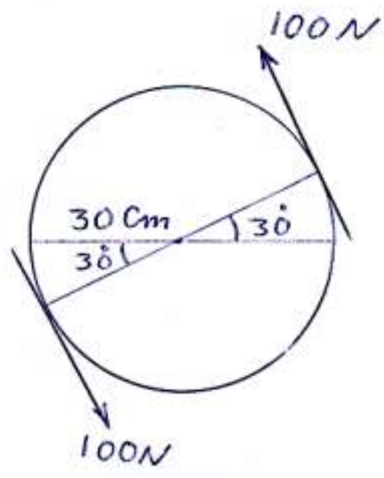
$$= C_o \text{ or } C$$

Since the sum of the forces of a couple in any direction is zero.

Couple has no tendency to translate abody in any direction but tends only to rotate the body on which it acts.

a couple is that it has the same moment with respect to every point in space. In other world, the moment of a couple is independent of its moment center.

Example: Find the couple about point O.

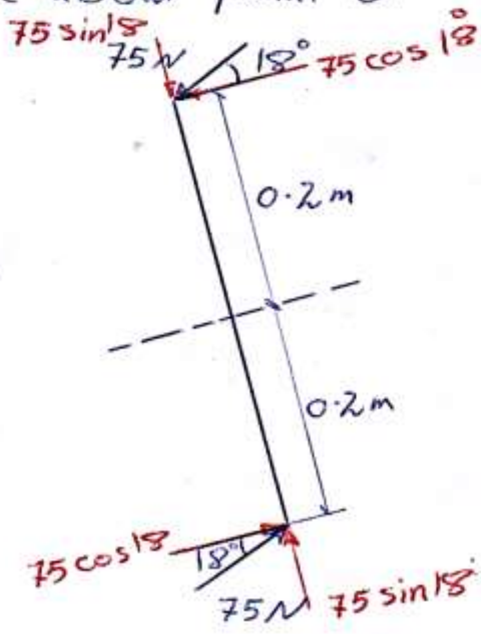


solution:-

$$C_o = 100\,(0.6)$$

$$= 60\,N.m \quad C.C.W$$

Example 2:- Determine the couple about point O.



solution:-

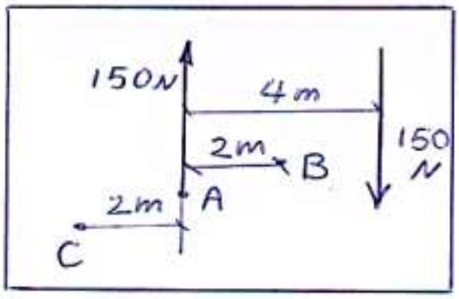
$$C_o = 75\cos 18\,(0.4) + 75\sin 18\,(0)$$

$$= 28.53\,N.m$$

Example 3:- Determine the moment of the couple in fig. as shown with respect to a) point A

b) point B

c) point C



solution:-

a) $M_A = ?$

$$\overset{\curvearrowright -}{M_A} = -150 * 4 = -600 \ N.m \quad C.W$$

b) $M_B = ?$

$$\overset{\curvearrowright -}{M_B} = -150 * 2 - 150 * 2$$

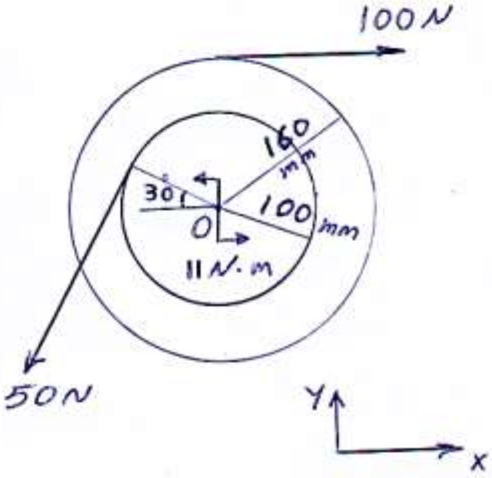
$$= -150 * 4 = -600 \ N.m \quad C.W$$

c) $M_C = ?$

$$\overset{\curvearrowright -}{M_C} = -150 * 6 + 150 * 2$$

$$= -600 \ N.m \quad C.W$$

Example 4:- Find the resultant of these forces and moment.



solution:-

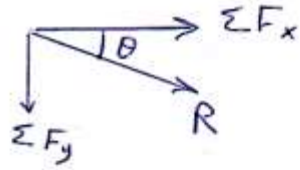
$$\Sigma F_x = 100 - 50 \sin 30 = 75 N$$

$$\Sigma F_y = -50 \cos 30 = -43.3 N$$

$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 86.6 N$$

$$\tan \theta = \frac{\Sigma F_y}{\Sigma F_x}$$

$$\theta = \tan^{-1} \frac{43.3}{75} \Rightarrow \theta \approx 30^\circ$$



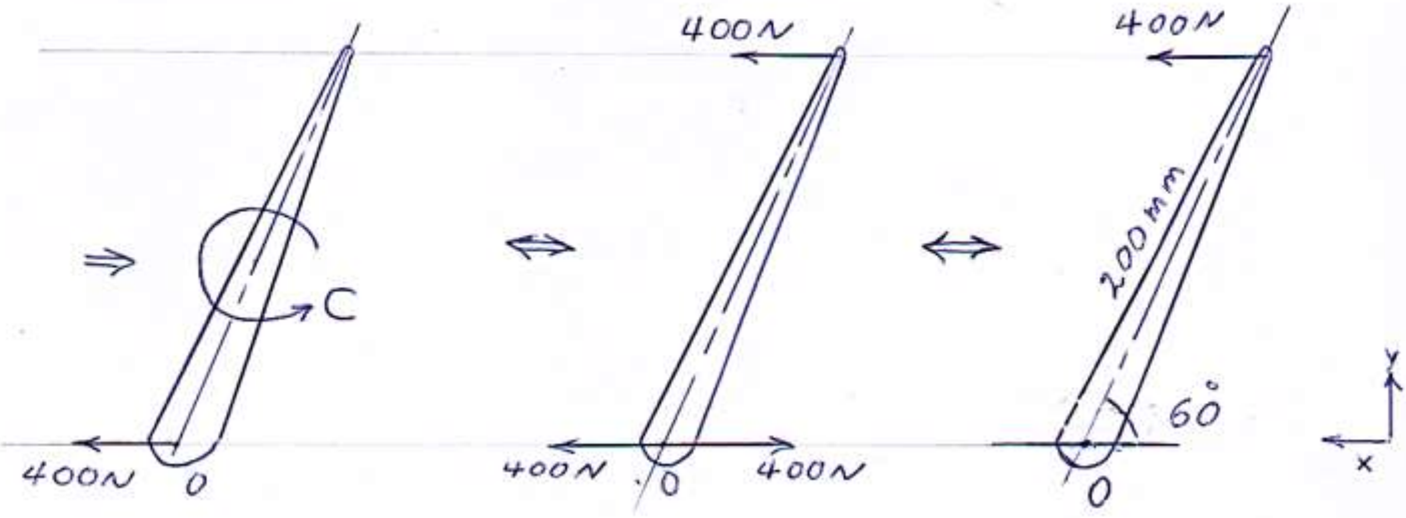
$$\overset{\curvearrowright -}{M_o} = 11 + 50(0.1) - 100(0.16)$$

$$= 0$$

Transformation of the forces

Example: Replace the horizontal 400 N force acting on the Lever by an equivalent system consisting of a force at O and a couple

solution:-



$\Sigma F_x = 400N$

$\overset{\curvearrowright}{M_o}{}^{-} = C$

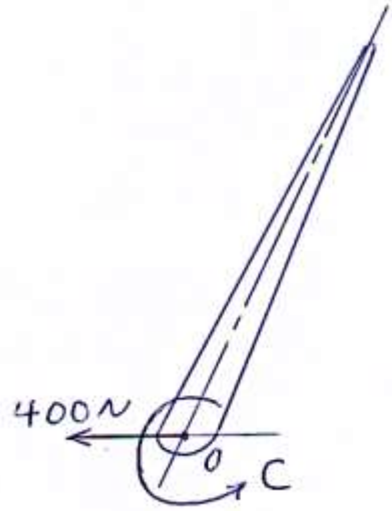
$= 400(0.2\sin 60)$

$\Sigma F_x = 400N$

$\overset{\curvearrowright}{M_o}{}^{-} = 400(0.2\sin 60)$

$\Sigma F_x = 400N$

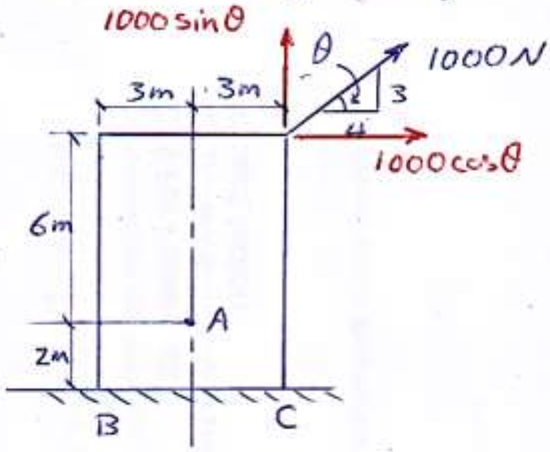
$\overset{\curvearrowright}{M_o}{}^{-} = 400(0.2\sin 60)$



$\Sigma F_x = 400N$

$\overset{\curvearrowright}{M_o}{}^{-} = C = 400(0.2\sin 60)$

Example:- By means of the transformation of a couple replace the 1000 N force of fig. by a force through A and a couple whose forces act vertically through B and C.



solution

$\tan\theta = \frac{3}{4}$

$\Rightarrow \theta = \tan^{-1}\frac{3}{4}$

$\Rightarrow \theta = 36.87°$

$\overset{\curvearrowright -}{M_A} = -1000\cos 36.87\,(6) + 1000\sin 36.87\,(3)$

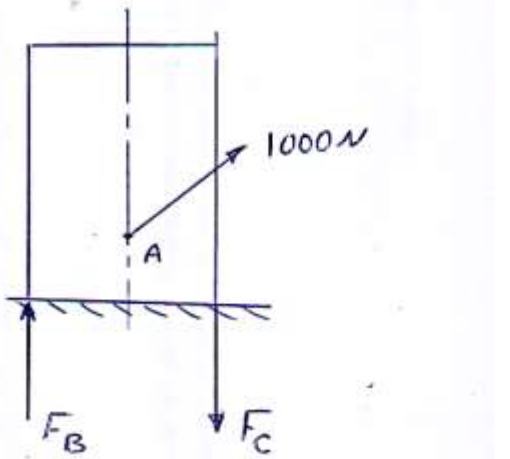
$\overset{\curvearrowright -}{M_A} = -3000\ N.m \quad C.W \quad = C_{trans.}$

$= C_{BC}$

$\therefore -3000 = -F_B * 6$

$\Rightarrow F_B = 500 N \uparrow$

$\therefore F_C = 500 N \downarrow$



after transformat

$\overset{\curvearrowright -}{M_{C\,after}} = \overset{\curvearrowright -}{M_{C\,befor}}$

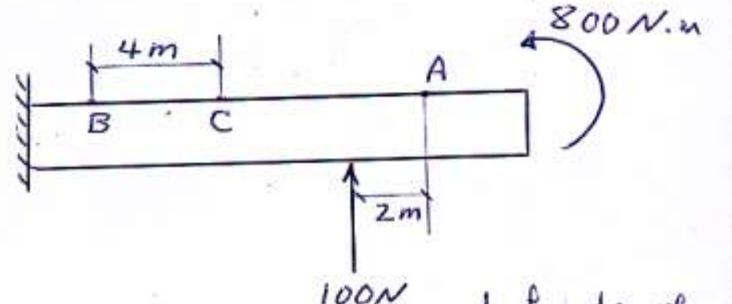
$-500(6) + F_{A_x}(2) + F_{A_y}(3) = -1000\cos 36.87\,(8)$

$F_A\cos 36.87\,(2) + F_A\sin 36.87(3) = -3400$

$\Rightarrow F_A = 1000 N$

Example: Replace the force and couple shown with a vertical force at A and a couple whose forces act vertically at B and C.

solution:-



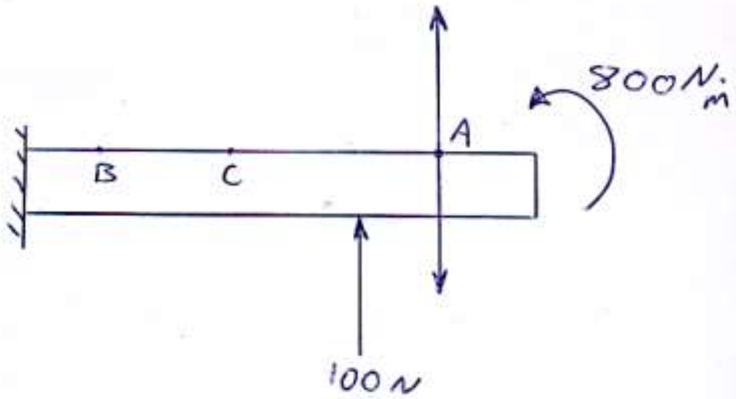
befor transforme

$$\overset{\curvearrowright -}{M_A} = 800 - 100(2)$$

$$= 600 \text{ N.m} \quad \text{C.C.W}$$

$$= C_{trans.}$$

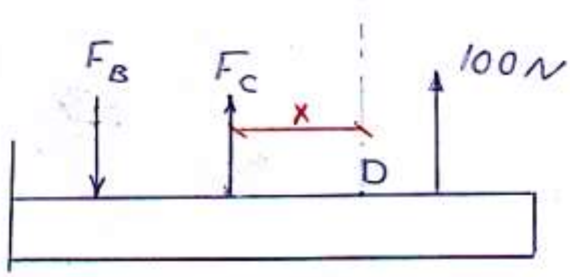
$$= C_{BC}$$



$$C_{BC} = F_B(4)$$

$$\Rightarrow F_B = \frac{C_{BC}}{4} = \frac{600}{4}$$

$$\Rightarrow F_B = 150 N = F_C$$



after transformation

$$F_B \downarrow \; , \; F_C \uparrow$$

$$M_{D_{befor}} = M_{D_{after}}$$

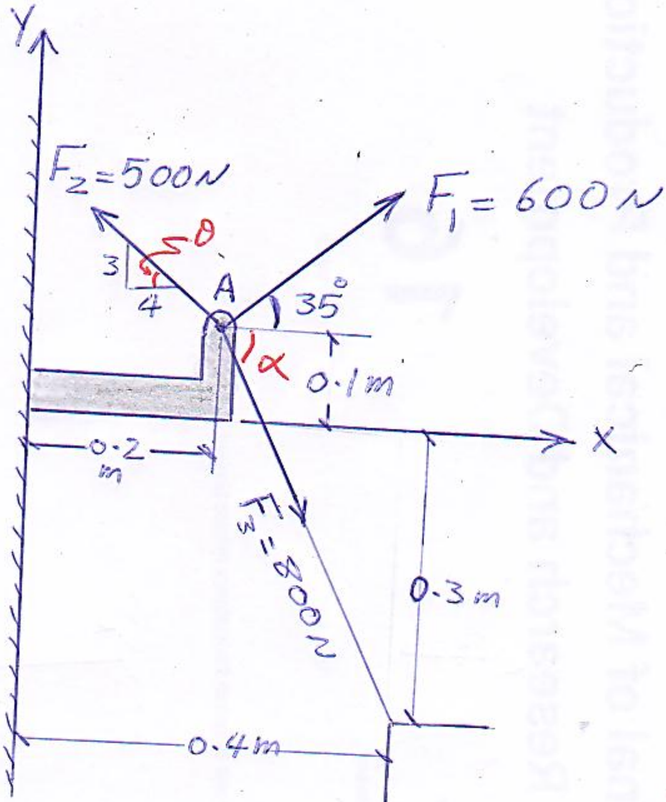
$$800(\cancel{100 \times 0}) = F_B(4+x) - F_C(x) + F_A(2)$$

$$800 = 150(4) + \cancel{150(x)} - \cancel{150(x)} + F_A(2)$$

$$\Rightarrow F_A = \frac{800 - 600}{2} = 100 \; N$$

Examples for the forces, moment, couple and the transformation of the forces

Example 1:- Determine the resultant for these forces.



solution:-

$\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.87$

$\tan\alpha = \frac{0.1+0.3}{0.4-0.2} \Rightarrow \alpha = 63.43^\circ$

$F_{1x} = +600\cos 35$, $F_{1y} = +600\sin 35$

$F_{2x} = -500\cos 36.87$, $F_{2y} = +500\sin 36.87$

$F_{3x} = +800\cos 63.43$, $F_{3y} = -800\sin 63.43$

$$\therefore \sum F_x = F_{1x} + F_{2x} + F_{3x}$$

$$= 600\cos 35 - 500\cos 36.87 + 800\cos 63.43$$

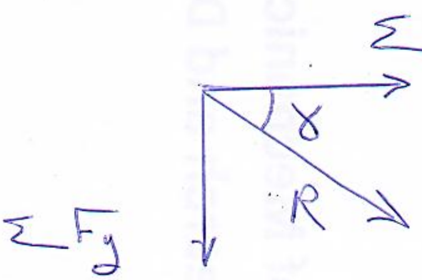
$$= 449.3\ N$$

$$\sum F_y = 600\sin 35 + 500\sin 36.87 - 800\sin 63.43$$

$$= -71.36\ N$$

$$R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$= 455\ N$$



$$\tan\gamma = \frac{\sum F_y}{\sum F_x}$$

$$= \frac{71.36}{449.3}$$

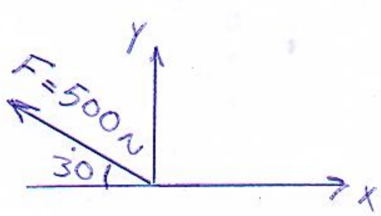
$$\Rightarrow \gamma = 9^\circ$$

or:-

$$R = 449.3\,\vec{i} - 71.36\,\vec{j}$$

Example 2:- The magnitude of force F = 500N, Express F as a vector in terms of $\vec{i}, \vec{j}$

solution:-



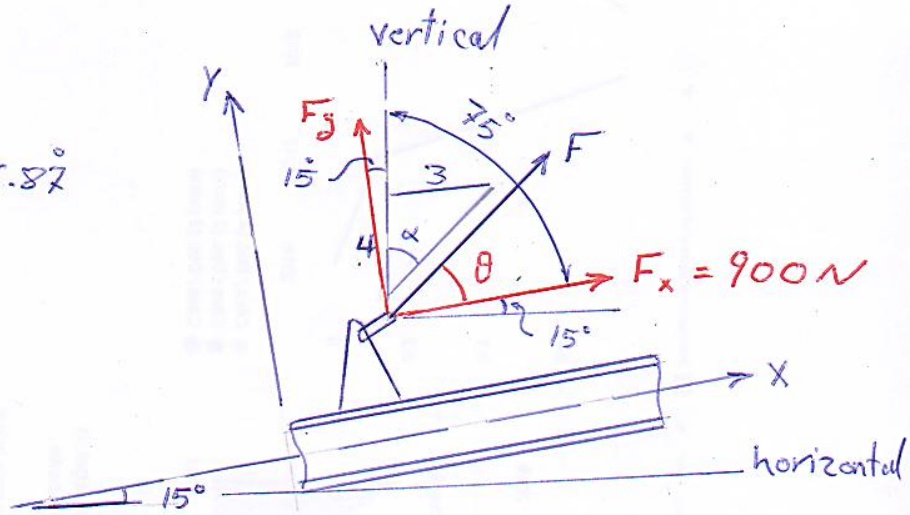
$$F_x = -500\cos 30 = -433.01\ N$$

$$F_y = 500\sin 30 = 250\ N$$

$$\therefore F = -433.01\,\vec{i} + 250\,\vec{j}\ \ N$$

Example 3:- A cable exerts a force F on the branchlet of the structural member to which it is attached. If the magnitude of x-component of F is 900 N, calculate the y-component and the magnitude of F.

solution:-



$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.87^\circ$

to find θ :-

$\theta = 90 - 15 - \alpha$

$= 90 - 15 - 36.87$

$= 38.13^\circ$

$\therefore F_x = F\cos 38.13^\circ$

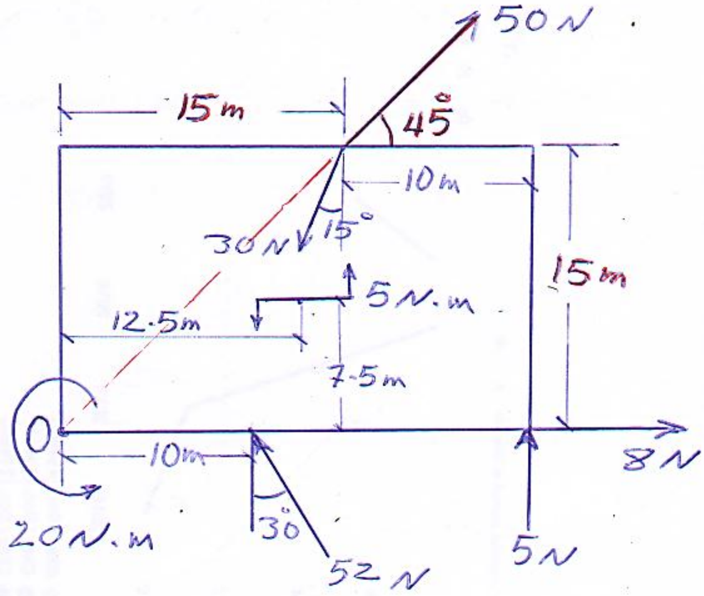
$\Rightarrow F = \frac{900}{\cos 38.13} \Rightarrow F = 1144.3\ N$

$F_y = F\sin 38.13$

$= 1144.3 \sin 38.13 \Rightarrow F_y = 706.7\ N$

Example 4: Find the resultant moment about point O.

solution:-



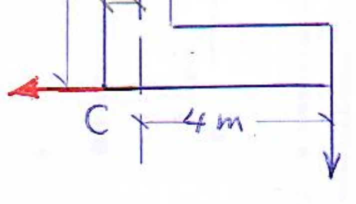
$$\sum \overset{\curvearrowright -}{M_0} = \sum (F \cdot d)_0 + C_1 + C_2$$

$$= 5 \times 25 + 52 \cos 30 (10) + 30 \sin 15 (15) - 30 \cos 15 * (15) + 20 + 5$$

$$= \quad N.m$$

Example 5:- Replace the 90 N force of fig. by a force through A and a couple whose forces act horizontally through B and C.

solution:-



$$\overset{\curvearrowright -}{M_A} = -90 (4) = -360 \ N.m \ C.W$$

$$\overset{\curvearrowright -}{M_A} = C_{CB} = -360 \ N.m$$

$$C_{CB} = F_B (6) = F_C (6)$$

$$\therefore -360 = -F_B (6) \Rightarrow F_B = 60 N = F_C$$

befor

after

between them

$$M_{C_{befor}} = M_{C_{after}}$$

$$-90(4+x) = F_A x + 60 * 6 \Rightarrow$$

$\cancel{360} - 90(x) - \cancel{360} = F_A \cdot x$

$-90(\cancel{x}) = F_A \cdot \cancel{x} \Rightarrow F_A = -90N$

or $F_A = 90N \downarrow$

The resultant of a system of forces

The resultant of asystem of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

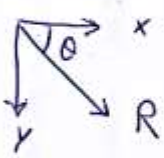
Equilibrium of abody is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. when the resultant of all forces on abody is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

$$\therefore\ R = F_1 + F_2 + F_3 + \cdots = \Sigma F$$

$$R_x = \Sigma F_x = F_{1x} + F_{2x} + \cdots$$

$$R_y = \Sigma F_y = F_{1y} + F_{2y} + \cdots$$

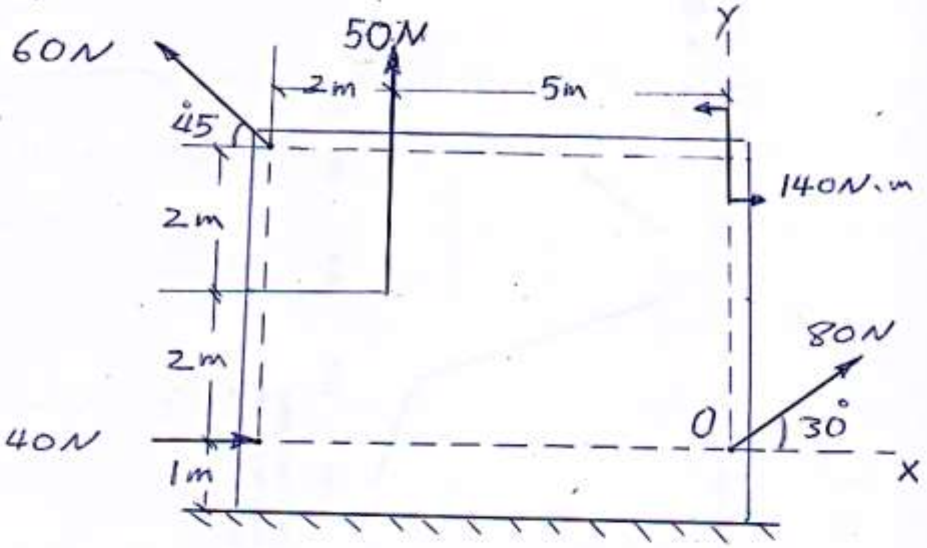
$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$



$$\Sigma M_{\text{any point}} = \Sigma (F.d)_{\text{same point}}$$

$$= R \cdot d_{\text{same point}}$$

Example:- Determine the resultant of the four forces and one couple which act on the plate shown



solution:-

$$R_x = \Sigma F_x = 40 + 80\cos 30 - 60\cos 45$$

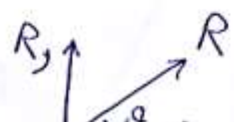
$$= 66.9 N$$

$$R_y = \Sigma F_y = 50 + 60\sin 45 + 80\sin 30$$

$$= 132.4 N$$

$$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{66.9^2 + 132.4^2}$$

$$= 148.3 N$$



$$\tan\theta = \frac{R_y}{R_x}$$

$$\Rightarrow \theta = 63.2^\circ$$

$$\overset{\curvearrowright}{M_o} = \Sigma (F.d)_o$$

$$= 140 - 50(5) + 60\cos 45(4) - 60\sin 45(7)$$

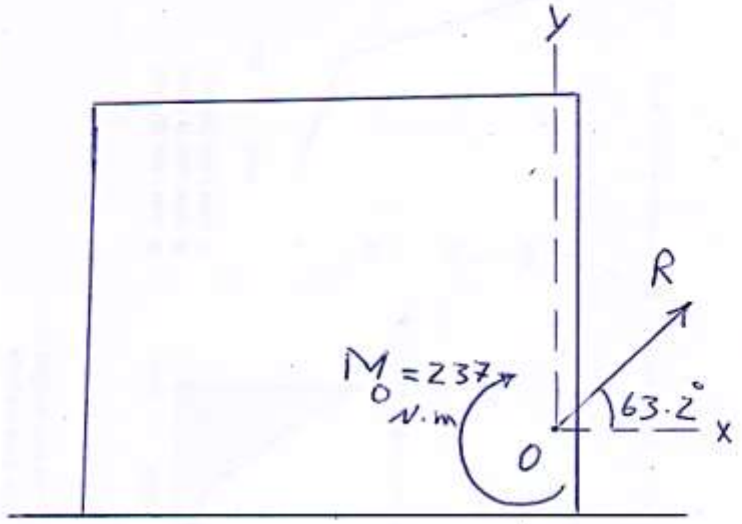
$$= -237 \ N.m$$

Now:-

$$\Sigma \overset{\curvearrowright -}{M_0} = \Sigma (F.d)_0 = R\, d_{R0}$$

$$\Rightarrow \quad 237 = 148.3\, d_{R0}$$

$$\Rightarrow d_{R0} = 1.6\ m$$

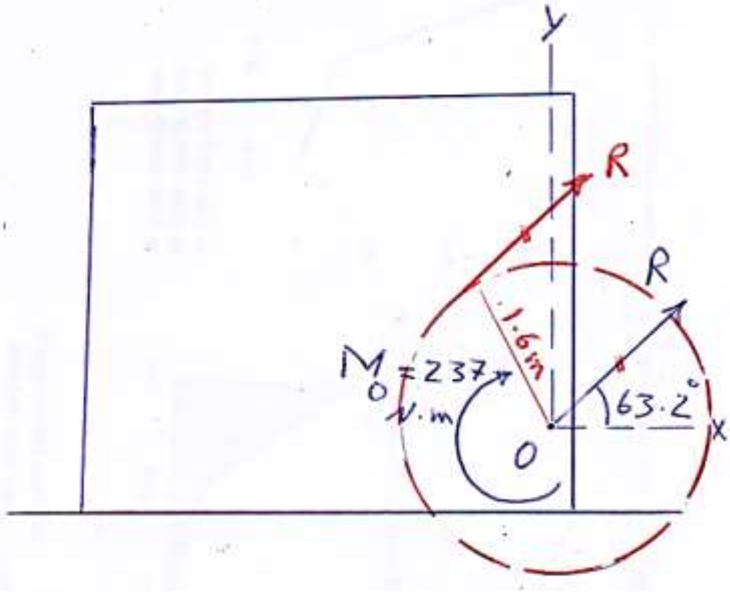


Now:-

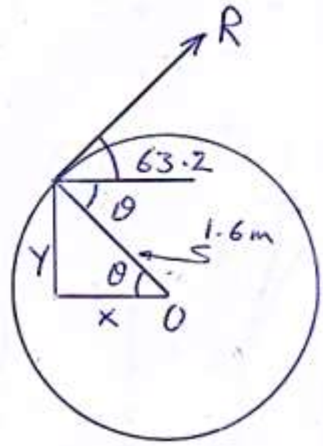
$$\Sigma \overset{\curvearrowright -}{M_0} = \Sigma (F.d)_0 = R\, d_{R0}$$

$$\Rightarrow \quad 237 = 148.3\, d_{R0}$$

$$\Rightarrow d_{R0} = 1.6\ m$$



$\theta = 90 - 63.2 = 26.8$

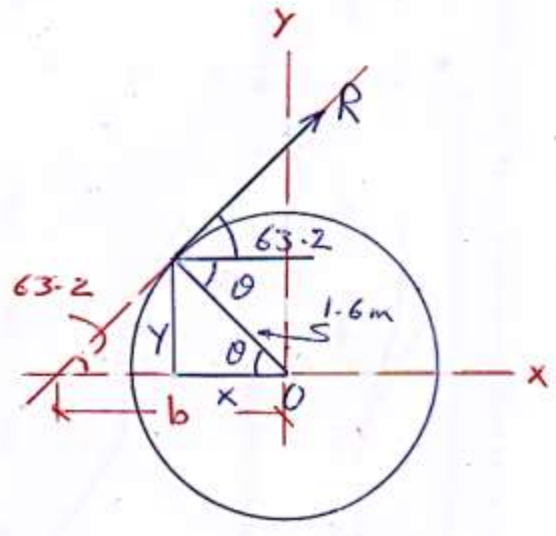


$\therefore x = 1.6\cos\theta$

$= 1.428\ m$

$\therefore y = 1.6 \sin 26.8$

$= 0.722\ m$



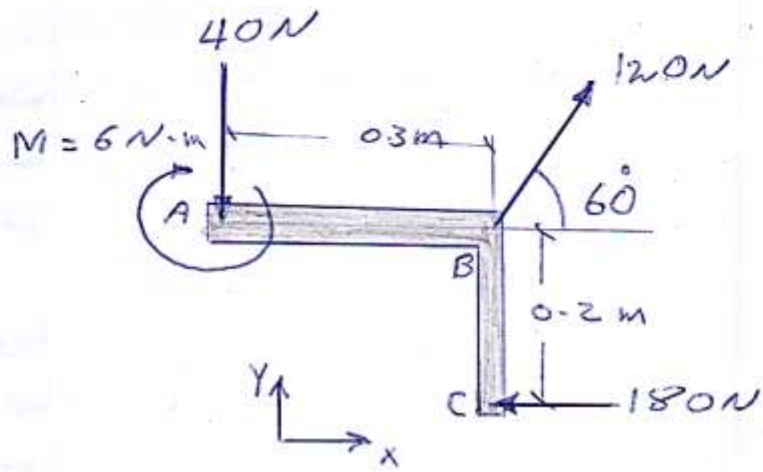
or:-

$$\frac{1.6}{\sin 63.2} = \frac{b}{\sin 90} \Rightarrow b = 1.792\ m$$

Example 2:- The three forces shown and a couple of moment $M = 6 N \cdot m$ are applied to an angle bracket.

a) find the resultant of the forces.

b) Locate the points where the Line of action of resultant intersects AB and Line BC.



solution

a)

$$\Sigma F_x = R_x = 120 \cos 60 - 180$$

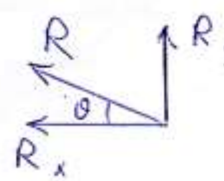
$$= -120 N$$

$$\Sigma F_y = R_y = 120 \sin 60 - 40$$

$$= 64 N$$

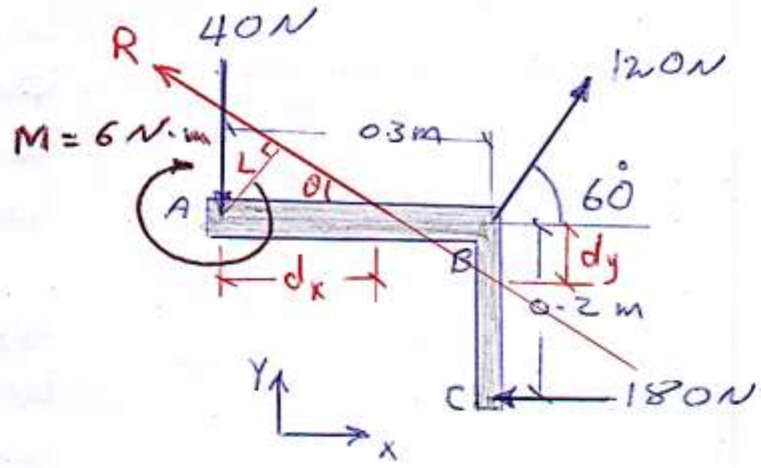
$$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-120)^2 + 64^2}$$

$$= 136 N$$



$$\tan \theta = \frac{R_y}{R_x} \Rightarrow \theta = \tan^{-1} \frac{64}{120}$$

$$\Rightarrow \theta = 28^\circ$$



b) Line of action for the resultant R intersects Line AB :-

$$\Sigma \overset{\curvearrowright}{M_A}^{-} = \Sigma (F.d)_A + M$$

$$= 120\sin 60 (0.3) - 180 (0.2) - 6$$

$$= -4.823 \text{ N.m} \quad C.W$$

Now : for the resultant intersects on AB :-

$$\Sigma \overset{\curvearrowright}{M_A}^{-} = R.L + M$$

or

$$\Sigma \overset{\curvearrowright}{M_A}^{-} = R_x(d_y) + R_y d_x + M$$

$$-4.823 = \cancel{120(0)} + 64.d_x - 6$$

$$\Rightarrow d_x = 0.0184 \text{ m} \quad \text{right to point A}$$

if the resultant intersects on BC :-

$$\Sigma \overset{\curvearrowright}{M_A}^{-} = R_x . d_y + R_y . d_x + M$$

$$-4.823 = -120 . d_y + \cancel{64(0)} - 6$$

$$\Rightarrow d_y = 0.01314\ m \quad \text{below point } A$$

or

$$\overset{\curvearrowright -}{M_A} = R.L + M$$

$$-4.823 = 136 . L - 6 \Rightarrow L = 0.00865 m$$

f the resultant of all system (i.e. the forces & couples) :-

$$\Sigma \overset{\curvearrowright -}{M_A} = R.L \quad \text{only}$$

or

$$\Sigma \overset{\curvearrowright -}{M_A} = R_x . d_y + R_y . d_x \quad \text{only}$$

i.e.-

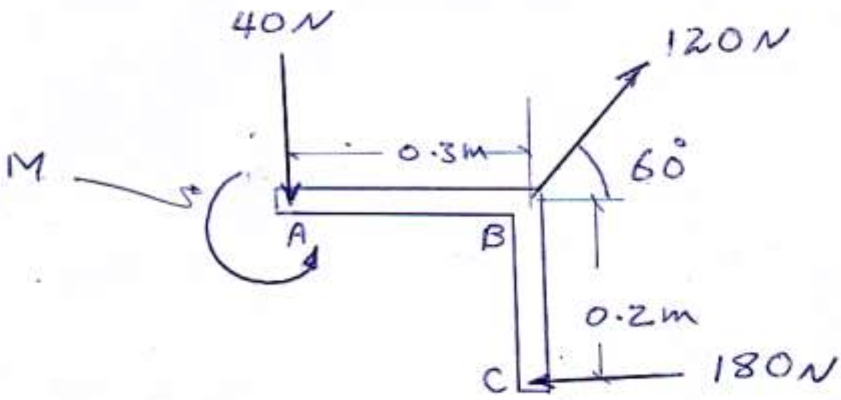
$$-4.823 = \cancel{120}(0) + 64 . d_x$$

$$\Rightarrow d_x = -0.0754\ m$$

if the sign is positive, (right position)
if the sign is negative, (the position of the resultant at another side of the point)

Example 3:- The three forces and Couple M are applied to an angle bracket. Find the moment of the couple, if the Line of action of the resultant of the force system is to pass through point

a) A , b) B , c) C .



solution:-

a) The resultant pass through point A

From the previous example:-

$R = 136 N$

$\theta = 28^\circ$

now:-

$$\Sigma M_A = \Sigma (F.d)_A + M$$

$$= 120 \sin 60 (0.3) - 180(0.2) + M$$

$$\Sigma M_A = -4.823 + M \quad \text{---} (1)$$

$\Sigma M_A = R . L = 0$ (L=0 becouse R pass through A)

$\therefore -4.823 + M = 0 \implies M = 4.823 \ N.m$ C.C.W

We can resolve (a) by take the moment about any point for example; M_B or M_c and we get same result.

now:-

b) The resultant pass through point B

$$\Sigma \overset{\curvearrowright -}{M_A} = \Sigma (F.d)_A + M = (R_x \cdot d_y)_B + (R_y \cdot d_x)_B$$

$$\Rightarrow \cancel{40(0)} + 120\sin 60(0.3) + \cancel{120\cos 60(0)} - 180(0.2)$$

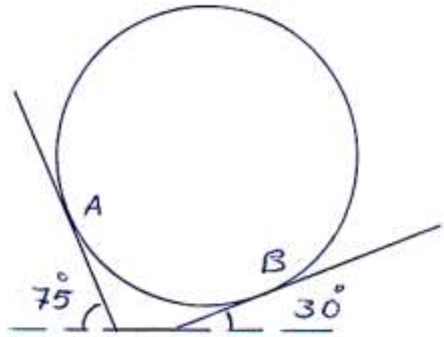
$$+ M = \cancel{136\cos 28(0)} + 136\sin 28(0.3)$$

$$\Rightarrow M = 24\ N.m$$

c) to find M if the resultant pass through point C :-

same method above.

Example:- The 20 kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at A & B.



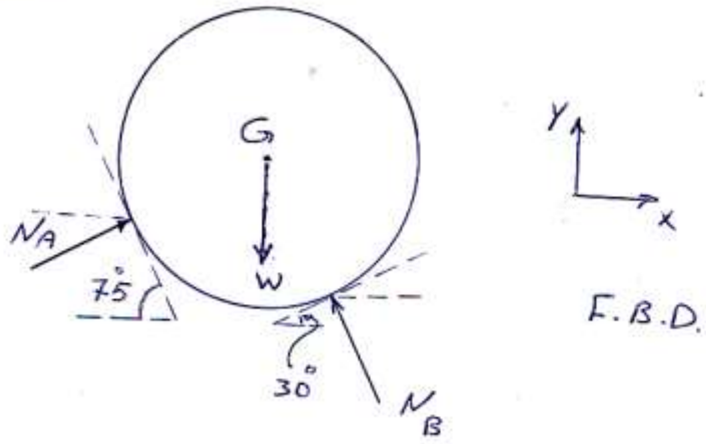
solution:-

$\Sigma F_x = 0$

$N_A \cos 15 - N_B \cos 60 = 0$ ---(1)

$\Sigma F_y = 0$

$N_A \sin 15 + N_B \sin 60 - W = 0$ ---(2)



$W = mg = 20(9.81)$
$= 196.2 \text{ N}$

sub W in (2) we get:-

$N_A \sin 15 + N_B \sin 60 - 196.2 = 0$

$\Rightarrow N_A = \dfrac{196.2 - N_B \sin 60}{\sin 15}$

sub N_A in (1) we get:-

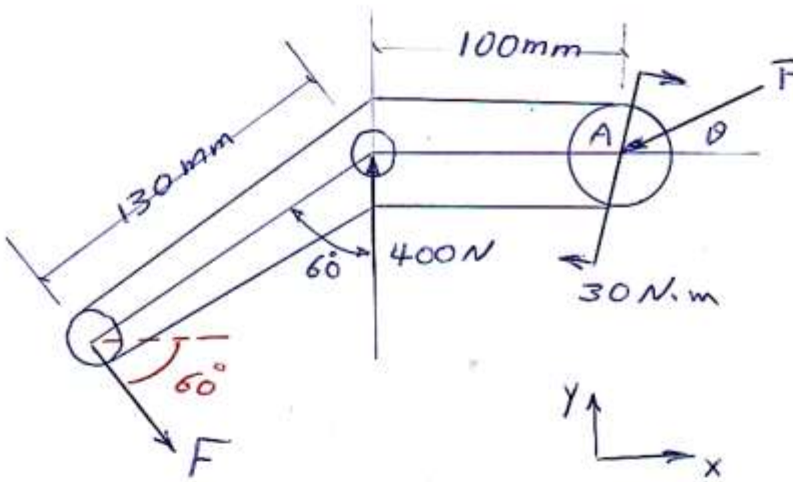
$N_B = 196.2\ N$

sub N_B in (3) we get

$N_A = 101.6N$

Example:- The Lever is in equilibrium under the action of the three forces and one couple as shown in its free-body diagram. Determine F, P, and θ.

solution:-



$\Sigma F_x = 0$

$F\cos 60 - P\cos\theta = 0$ ------ (1)

$\Sigma F_y = 0$

$400 - F\sin 60 - P\sin\theta = 0$ --- (2)

$\Sigma M_A = 0$

$-30 - 400(0.1) + F\cos 60\,(0.13\cos 60) + F\sin 60\,(0.1 + 0.13\sin 60) = 0$

$\Rightarrow F = 32318\ N$

sub F in (1) & (2) we get:-

$$323.18 \cos 60 - P \cos\theta = 0$$

$$\Rightarrow P\cos\theta = 161.588. \text{ --- } (3)$$

&

$$400 - 323.18 \sin 60 = P \sin\theta$$

$$\Rightarrow P\sin\theta = 120.12 \text{ --- } (4)$$

Now

$$\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} = \frac{120.12}{161.588} \Rightarrow \tan\theta = 0.743$$

$$\Rightarrow \theta = 36.6^{\circ}$$

sub θ in 3 or 4 we get

$$P = 201.47 \text{ N}$$

The Friction

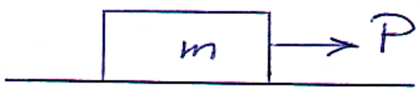
Friction is a force that resists the movement of two contacting surfaces that slide relative to one another. This force always acts tangent to the surface at the points of contact and is directed so as to oppose the possible or existing motion between the surfaces.

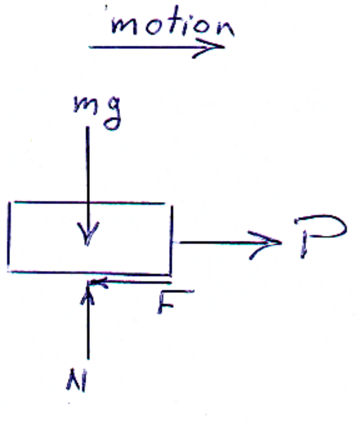
Types of friction:-

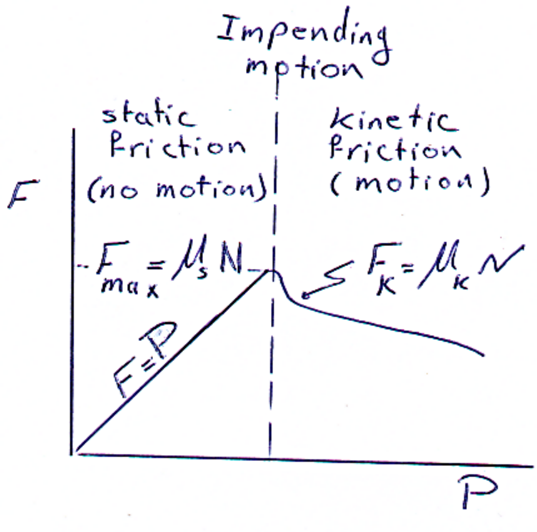
1- Dry friction
2- Fluid firction
3- internal friction

Types of surfaces:-

1- Smooth surfaces : (no friction)
2- Coarse surfaces : (friction)







note:- $(\mu_s \ \& \ \mu_k) < 1$

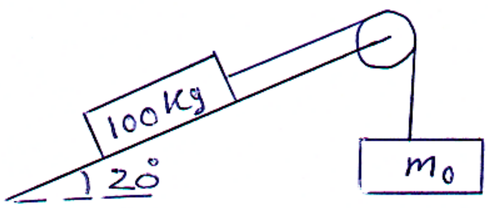
The region up to the point of slippage or impending motion is called the range of static friction, and in this range the value of friction force is determined by the equations of equilibrium. This friction force may have any value from zero up to and including the maximum value.

The maximum value of static friction F_{max} is proportional to the normal force N. Thus, we may write

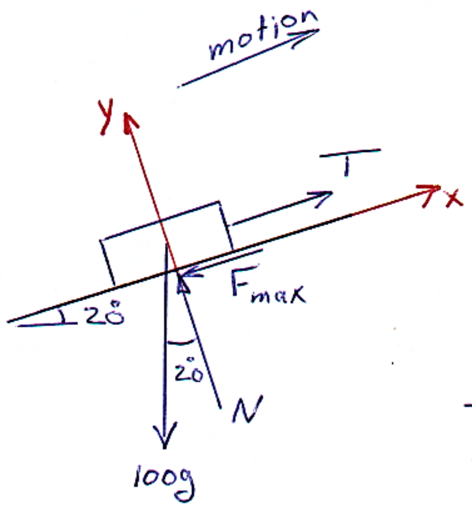
$$F_{max} = \mu_s N$$

where μ_s is the proportionality constant, called the coefficient of static friction.

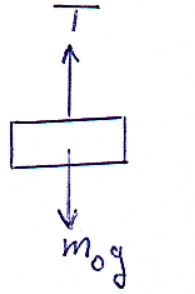
Example 1:- Determine the range of values which the mass m_o may have so that the 100 kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.3



case 1 : if m_o move or impending motion to down



case 1:



$T - m_o g = 0$

$\Rightarrow T = m_o g$

$\Sigma F_y = 0$

$N - 981 \cos 20 = 0$

$\Rightarrow N = 922\,N$

$F_{max} = \mu_s N$

$= 0.3(922)$

$= 277\,N$

$\Sigma F_x = 0$

$m_o(9.81) - 277 - 981 \sin 20 = 0$

$\Rightarrow m_o = 62.4\,kg$

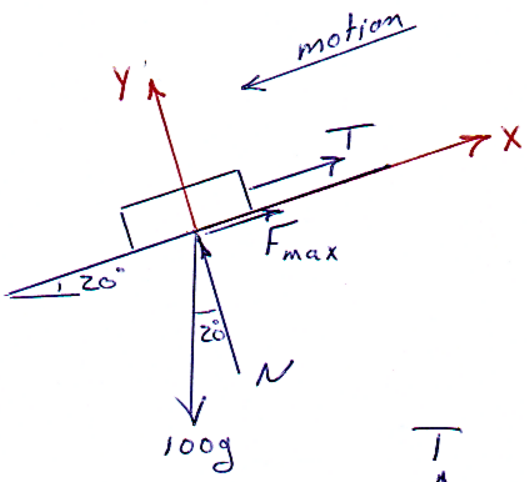
now:

Case 2: if m_o impending move to up:-

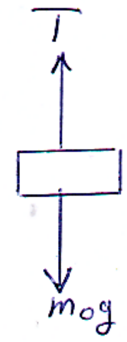
$\Sigma F_x = 0$

$m_o(9.81) + 277 - 981 \sin 20 = 0$

$\Rightarrow m_o = 6.01\,kg$

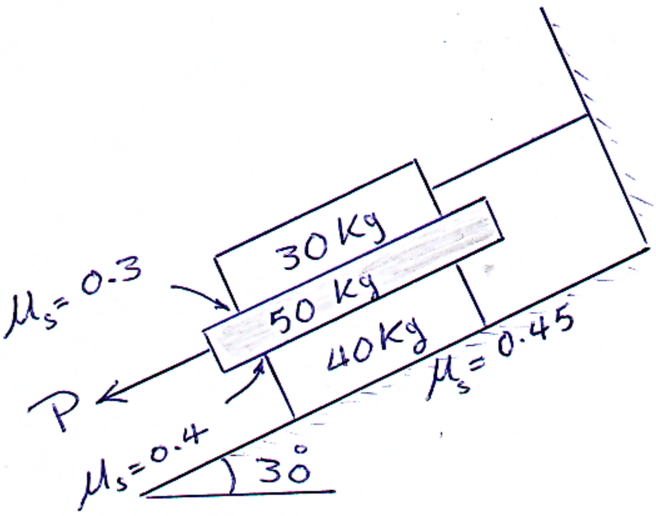


Thus, m_o may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

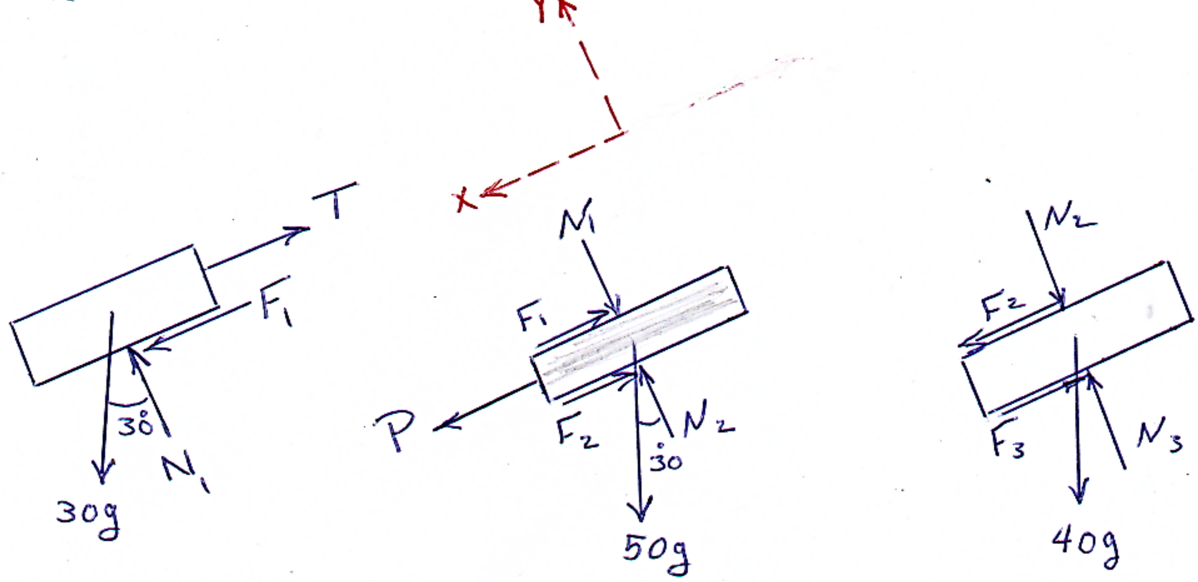


$T = m_o g$

Example: The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown. Determine the maximum value which P may have before any slipping taken place.



solution:-



$\Sigma F_y = 0$

For 30kg: $N_1 - 30(9.81)\cos 30 = 0$

$\Rightarrow N_1 = 255\ N$

For 50 kg: $N_2 - 50(9.81)\cos 30 - 255 = 0$

$\Rightarrow N_2 = 680\ N$

For 40kg: $N_3 - 40(9.81)\cos 30 - 680 = 0$

$\Rightarrow N_3 = 1019\ N$

$F_{max} = \mu_s N$

$\therefore F_1 = 0.3(255) = 76.5\ N$

$F_2 = 0.4(680) = 272\ N$

The assumed equilibrium of forces at impending motion for the 50kg block gives

$\Sigma F_x = 0$: $P - 76.5 - 272 + 50(9.81)\sin 30 = 0$

$\Rightarrow P = 103.1\ N$

we now check on the validity of our initial assum-ption. For the 40 kg block with $F_2 = 272\ N$ the friction force F_3 would be given by

$\Sigma F_x = 0$: $272 + 40(9.81)\sin 30 - F_3 = 0$

$\Rightarrow F_3 = 468\ N$

But the maximum possible value of F_3 is

$F_3 = \mu_s N_3 = 0.45(1019) = 459\,N$

Thus, 468 N can not be supported and our initial assumption was wrong. we conclude, therefore, that slipping occurs first between the 40 kg block and the incline. with corrected value $F_3 = 459\,N$, equilibrim of the 40 kg block for its impending motion requires.

$$\Sigma F_x = 0 : \quad F_2 + 40(9.81)\sin 30 - 459 = 0$$

$$\Rightarrow F_2 = 268$$

Equilibrium of the 50 kg gives, finally

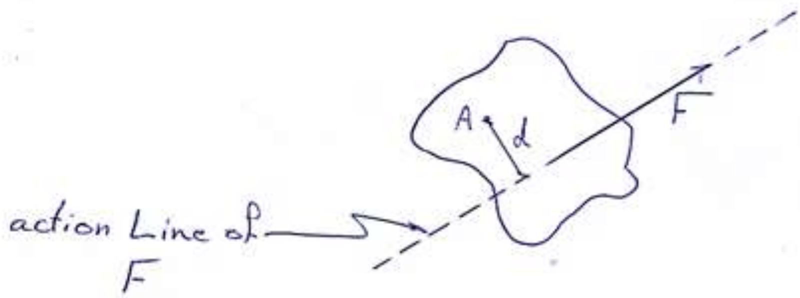
$$\Sigma F_x = 0 : \quad P + 50(9.81)\sin 30 - 263 - 76.5 = 0$$

$$\Rightarrow P = 93.8\,N$$

Thus, with $P = 93.8\,N$, motion impends for the 50 kg and 40 kg blocks as a unit

The moment of aforce

The moment of aforce is a measure of its tendency to turn or rotate abody about the moment axis. defined as the product of the magnitude F of the force and of the perpendicular distance (d) from A to the Line of action of F.



$$M_A = +F \cdot d$$

The moment of a force

Example 1 :-

Calculate the magnitude of the moment about the base point O of the 600 N force in four differen[t] ways.

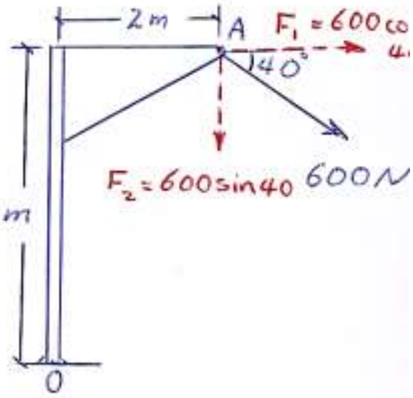
solution:-

① $M_o = -F_1(4) - F_2(2)$

$= -600\cos40 \times 4 - 600\sin40(2)$

$= -2610$ N.m

or $= 2610$ N.m C.W



② $M_o = -F.d$

$d = BC + CO$

$AD = 2\sin 40 = 1.285$ m

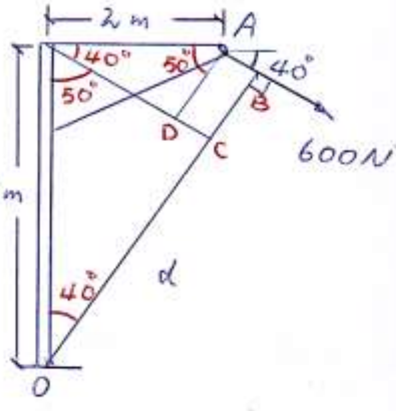
$= BC$

$CO = 4\cos40 = 3.1$ m

$\therefore d = 4.35$ m

$\therefore M_o = -600(4.35)$

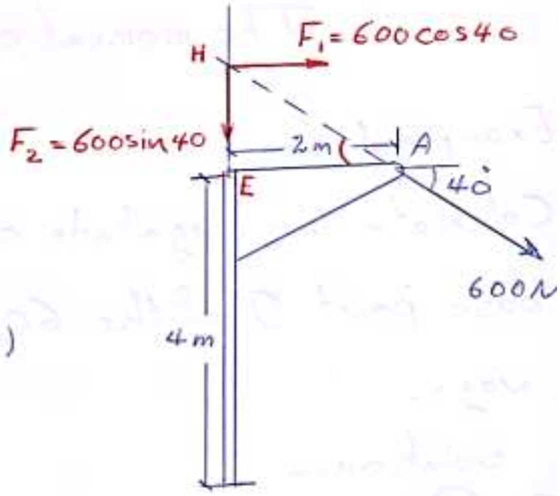
$= -2610.1$ N.m



③ $\overset{\curvearrowright -}{M_o} = -F_1(HE+4) - F_2(0)$

$= -F_1(HE+4)$

$HE = 2\tan 40 = 1.678\ m$



$\therefore \overset{\curvearrowright -}{M_o} = -600\cos 40(1.678+4)$

$= -2610\ N.m$

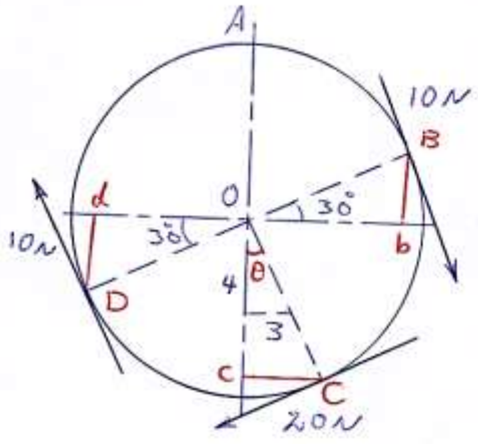
④ same ③ way but in right of point O.

Example 2:-

Determine the moment of the force system of fig. as shown

① with respect of O.

② with respect of A.



solution:-

① $\overset{\curvearrowright -}{M_o} = -10\times5 - 20\times5 - 10\times5$

$= -200 \ N.m \quad C.W$

② $\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.87°$

$Cc = 5\sin 36.87 = 3 \ m$

$CO = 4 \ m$

$Bb = Dd = 5\sin 30 = 2.5 \ m$

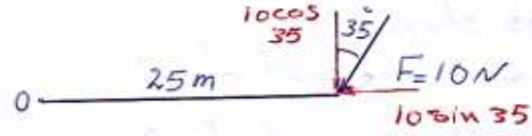
$Ob = Od = 5\cos 30 = 4.33 \ m$

$\overset{\curvearrowright -}{M_A} = -20\cos 36.87\ (4+5) - 20\sin 36.87\ (3) - (10\cos 30 * 4.33) * 2 + 10\sin 30\,(5-2.5) - 10\sin 30\,(5+2.5)$

$= -280 \ N.m$

Example 3:-

Find the moment of this force about point O.

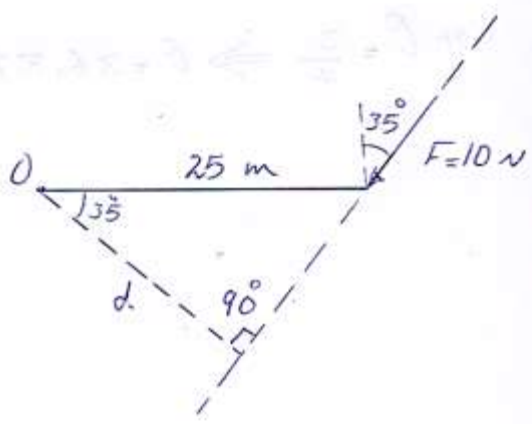


solution:-

$$\overset{\curvearrowright -}{M_o} = -10\cos 35 * 25 + 10\sin 35 * 0$$

$$= 204.8\ N.m\ C.w$$

Another solution



$$d = 25\cos 35$$

$$= 20.48\ m$$

$$\therefore \overset{\curvearrowright -}{M_o} = -10 * 25\cos 35$$

$$= 204.8\ N.m$$

The Equilibrium

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero.

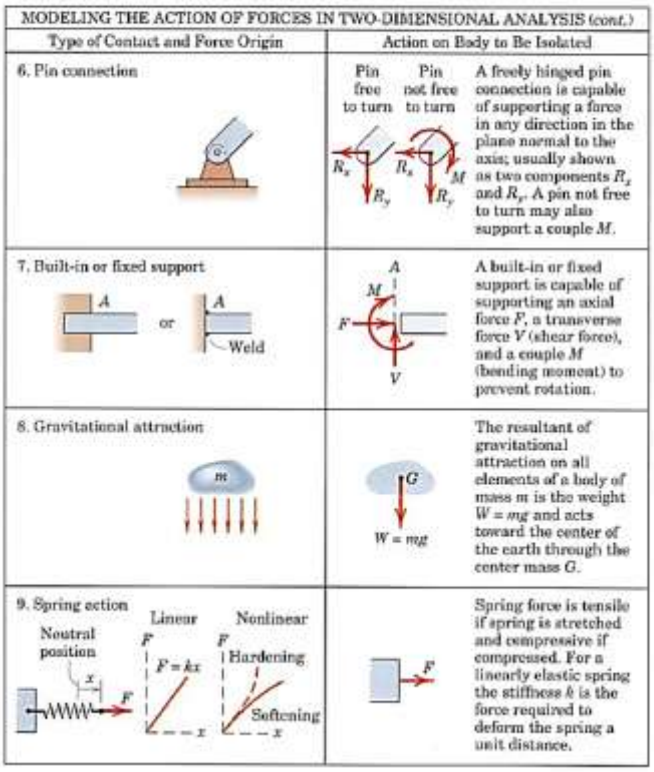
i.e.:

$$\left.\begin{aligned}\Sigma F_x &= 0 \\ \Sigma F_y &= 0\end{aligned}\right\} \Rightarrow R = 0$$

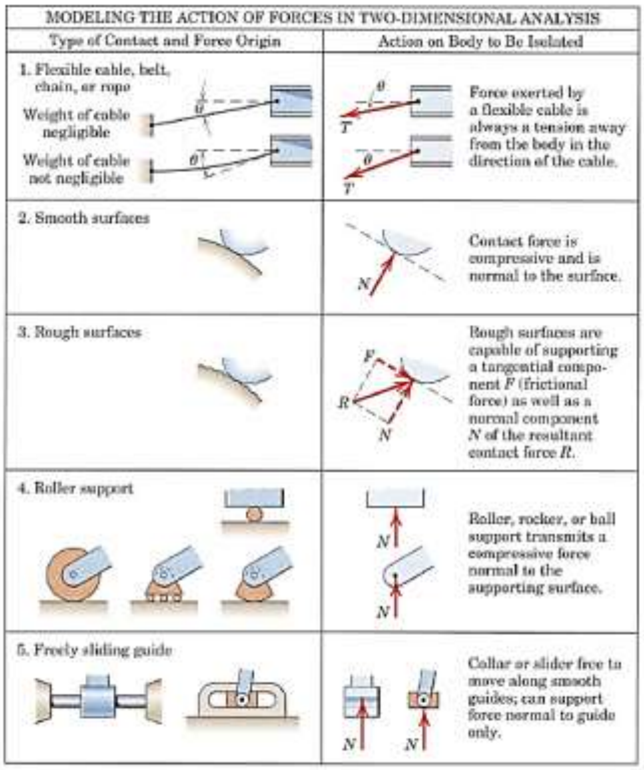
$$\& \; \Sigma M = 0$$

The free-body diagram (F.B.D):-

is the most important single step in the solution of problems in mechanics.

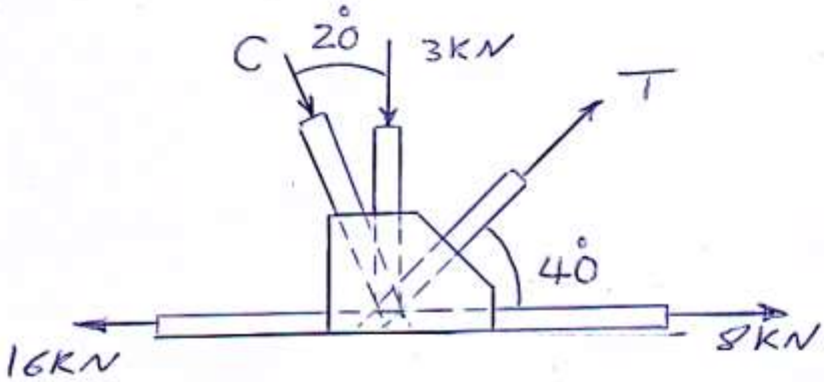


MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (cont.)	
Type of Contact and Force Origin	Action on Body to Be Isolated
6. Pin connection	Pin free to turn; Pin not free to turn; R_x, R_y, M. A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support: A or A (Weld)	A, M, F, V. A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction: m	G, $W = mg$. The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action: Neutral position, x, F; Linear: F, $F = kx$, x; Nonlinear: F, Hardening, Softening, x	F. Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

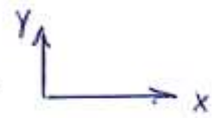


MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	Action on Body to Be Isolated
1. Flexible cable, belt, chain, or rope: Weight of cable negligible (θ); Weight of cable not negligible (θ)	θ, T. Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces	N. Contact force is compressive and is normal to the surface.
3. Rough surfaces	F, R, N. Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support	N, N. Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide	N, N. Collar or slider free to move along smooth guides; can support force normal to guide only.

Example:- Determine the magnitudes of the C and T, which along with the other three forces shown, act on the bridge-truss joint.



solution:-



$$\Sigma F_x = 0$$

$$8 - 16 + T\cos 40 + C\sin 20 = 0$$

$$\Rightarrow 0.766\,T + 0.342\,C = 8 \quad \text{------}(1)$$

$$\Sigma F_y = 0$$

$$T\sin 40 - C\cos 20 - 3 = 0$$

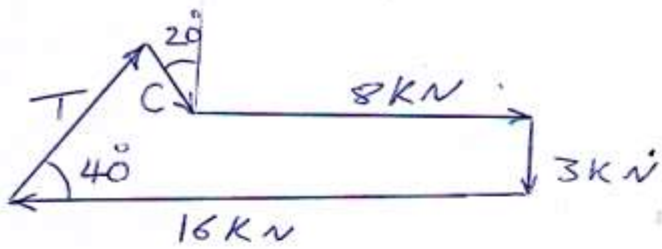
$$\Rightarrow 0.643\,T - 0.94\,C = 3 \quad \text{------}(2)$$

from (1) & (2) we get

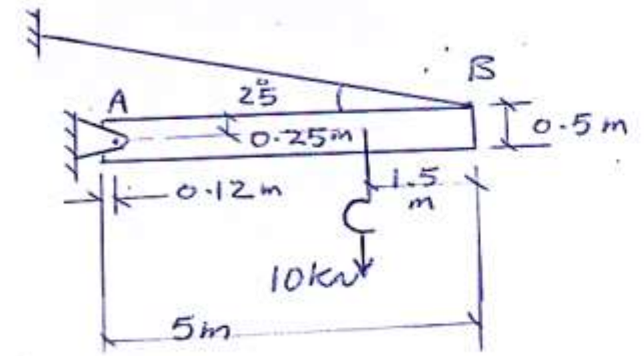
$T = 9.09\ KN$, $C = 3.03\ KN$

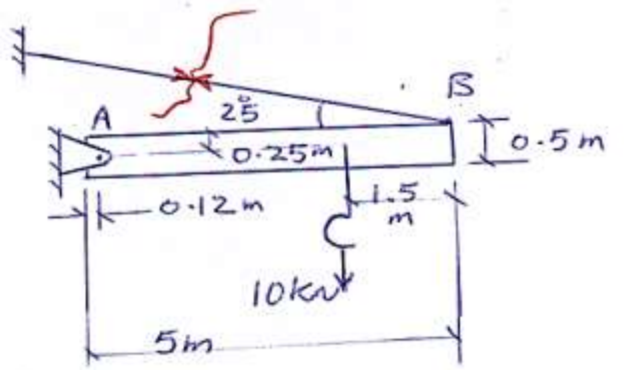
another solution:-

by scaler

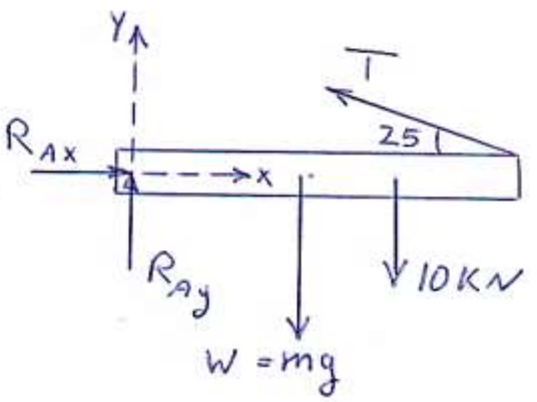


Example 2:- Determine the magnitude T of the tension in the supportion cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5m I-beam with a mass of 95 kg per meter of Length.





solution:-



$$W = mg = 95 \times 5 \times 9.81$$
$$= 4.66 \text{ kN}$$

$$\Sigma F_x = 0$$
$$-T\cos 25 + R_{Ax} = 0 \quad \text{-----} (1)$$

$$\Sigma F_y = 0$$
$$R_{Ay} - 4.66 - 10 + T\sin 25 = 0 \quad \text{-----} (2)$$

now

$$\Sigma \overset{\curvearrowright -}{M_A} = 0$$

$$T\sin 25\,(5-0.12) + T\cos 25\,(0.25) - 10\,(5-1.5-0.12) - 4.66(2.5-0.12) = 0$$

$$\Rightarrow T = 19.61 \text{ kN}$$

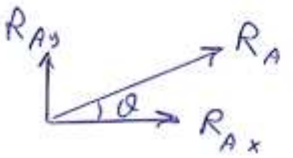
sub. T in (1) & (2) we get

$$R_{Ax} = 17.77 \text{ kN}$$

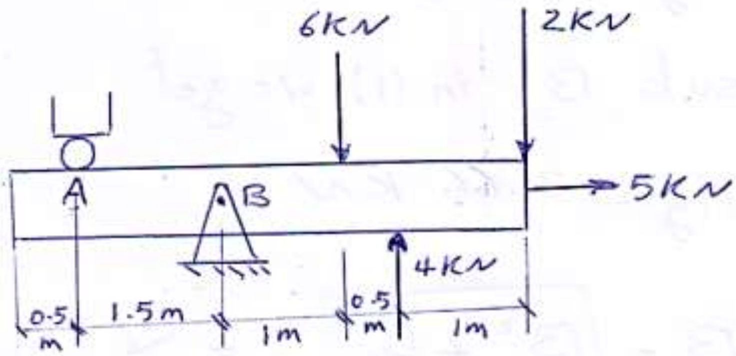
$$R_{Ay} = 6.37 \text{ kN}$$

$$\therefore R_A = \sqrt{R_{Ax}^2 + R_{Ay}^2} \Rightarrow R_A = 18.88 \text{ kN}$$

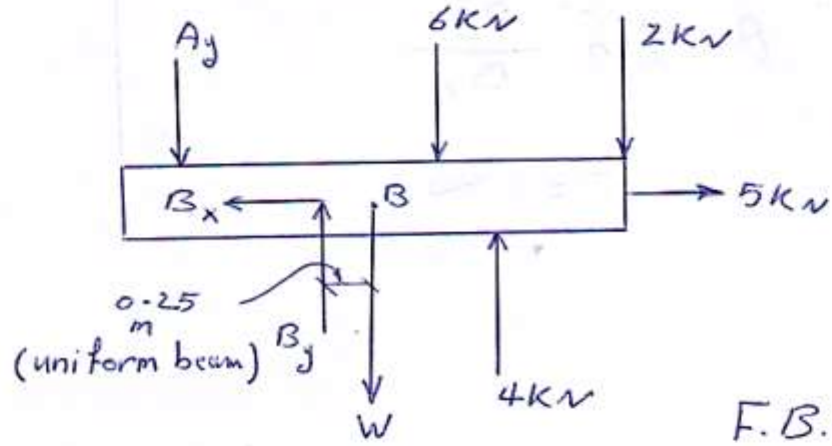
$$\tan\theta = \frac{R_{Ay}}{R_{Ax}}$$
$$\Rightarrow \theta = 19.72^\circ$$



Example 3:- The uniform 4.5 m beam has a mass of 200 kg & Loaded by the parallel force shown. Calculate the reaction at the support points A & B.



solution



F.B.D.

$\Sigma F_x = 0$

$5 - B_x = 0$

$\Rightarrow B_x = 5 KN$

$\Sigma F_y = 0$

$B_y - A_y - 6 - 2 + 4 - W = 0$

$B_y - A_y - 4 - \frac{200}{1000}(9.81) = 0 \quad ---(1)$

$B_y - A_y = 5.962 \quad ---(1)$

$\sum M_A = 0$

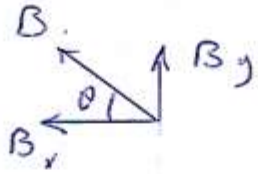
$+B_y(1.5) + 4 \times 3 - \frac{200}{1000}(9.81) * (1.75) - 2(4) - 6(2.5) = 0$

$B_y = 9.623 \text{ kN}$

sub B_y in (1) we get

$A_y = 3.66 \text{ kN}$

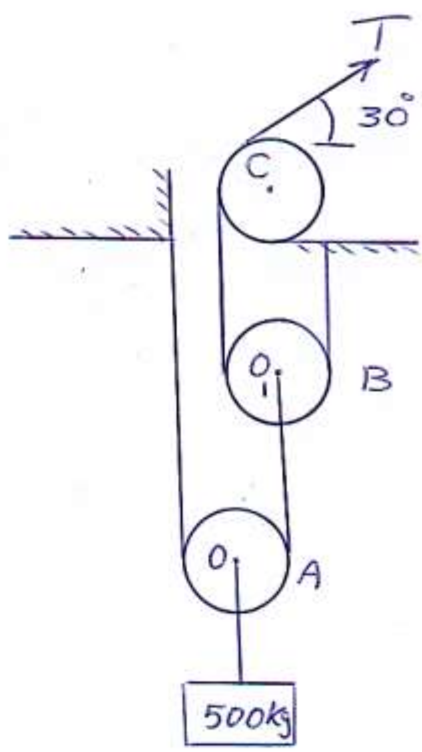
$B = \sqrt{B_x^2 + B_y^2} =$



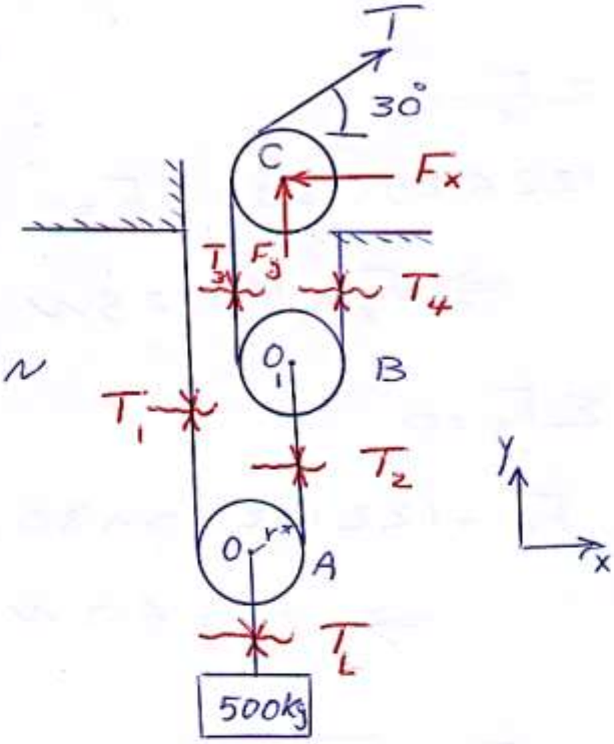
$\theta = \tan^{-1} \frac{B_y}{B_x}$

$\theta =$

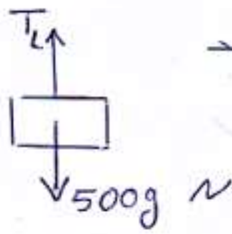
Example 4:- Calculate the tension T in the cable which supports the 500 kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the Load. Find the magnitude of the total force on the bearing of pulley C.



Example 4:- Calculate the tension T in the cable which supports the 500 kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the Load. Find the magnitude of the total force on the bearing of pulley C.



solution:-
for the Load

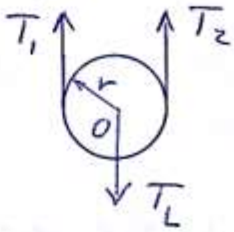


$\Sigma F_y = 0$

$T_L - 500(9.81) = 0$

$\Rightarrow T_L = 4905\ N$

for pulley A:-



$\Sigma \overset{\curvearrowright -}{M_O} = 0$

$T_1 r - T_2 r = 0$

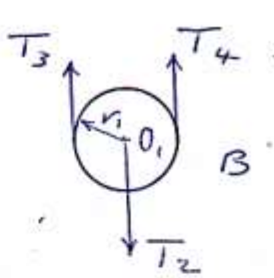
$\Rightarrow T_1 = T_2 \quad \text{----}(1)$

$\Sigma F_y = 0$

$T_1 + T_2 - 4905 = 0$

$\Rightarrow 2T_1 = 4905 \Rightarrow T_1 = T_2 = 2452.5\,N = \frac{1}{2} T_L$

for pulley B:-



$T_3 + T_4 - T_2 = 0$

$T_3 + T_4 = 2452.5 \quad \text{---}(2)$

$\Sigma \overset{\curvearrowright -}{M_O} = 0$

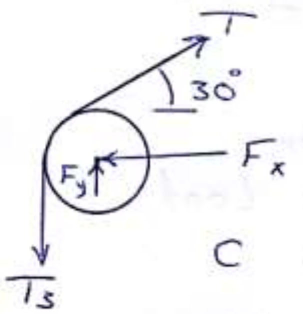
$\Rightarrow T_3 r_1 = T_4 r_1 \Rightarrow T_3 = T_4$

sub T_4 in (2) we get

$2T_3 = 2452.5 \quad \Rightarrow \quad T_3 = T_4 = 1226.25\ N = \frac{1}{4} T_L$

pulley C :-

$T_3 = T = 1226.25\ N$



$\Sigma F_x = 0$

$1226.25 \cos 30 - F_x = 0$

$\Rightarrow F_x = 163.125\ N$

$\Sigma F_y = 0$

$F_y + 1226.25 \sin 30 - 1226.25 = 0$

$\Rightarrow F_y = 613\ N$

$\therefore F_c = \sqrt{F_x^2 + F_y^2} \Rightarrow F_c = 1226\ N$

$\theta = \tan^{-1} \frac{F_y}{F_x}$

$\Rightarrow \theta = 30^\circ$