Lectures in
Physical Electrons

University of Mosul

Department of Electronics and Communication

in College of Electronic Engineering

by

Dr. Qais Thanon

2012-2013

Greek Letters

alpha	α	beta	β	chi	χ	delta	δ
epsilon	ϵ	phi	ϕ	varphi	φ	gamma	γ
eta	η	iota	ι	kappa	κ	lambda	λ
mu	μ	nu	ν	pi	π	varpi	ϖ
theta	θ	vartheta	ϑ	rho	ρ	sigma	σ
varsigma	ς	tau	τ	upsilon	υ	omega	ω
xi	ξ	psi	ψ	zeta	ζ		
Delta	Δ	Phi	Φ	Gamma	Γ	Lambda	Λ
Pi	Π	Theta	Θ	Sigma	Σ	Omega	Ω
Xi	Ξ	Psi	Ψ				

List of Constants

Charge of Electron : 1.6×10^{-19} C	Mass of the Electron : 9.1×10^{-31} kg
Permittivity of Space : 8.85×10^{-12} $F\ m^{-1}$	Speed of the Light : 3×10^{8} m s^{-1}
Plank's constant : 6.62×10^{-34} $J\ s$	Boltzmann's constant : 1.38×10^{-23} $J\ K^{-1}$

Chapter 1

Energy bands in Solids

All matters are made of atoms; and all atoms consist of electrons, protons, and neutrons. In this chapter, you will learn about the structure of the atom, electron orbits and shells, valence electrons, ions, and the semiconductive materials. Semiconductive material is important because the configuration of certain electrons in an atom is the key factor in determining how a given material conducts electrical current.

1.1 Atomic Structure

An atom is the smallest particle of an element that retains the characteristics of that element. Each atom consists of central nucleus surrounded by orbiting electrons. The nucleus consists of positively charge particles called *protons* and uncharged particles called *neutrons*. Electrons are the basic particles of negative charge.

CHARGED PARTICLES

Electric charges, positive or negative, occur in multiples of the electronic charge. The electron is one of the fundamental particles constituting the atom. The charge

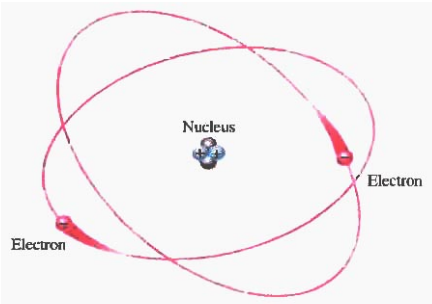

Figure 1.1: Atomic structure

of an electron is negative and is denoted by *(e)*. The magnitude of e is 1.6×10^{-19} Coulomb and the mass of the electron is 9.1×10^{31} *kg*. The proton have the same charge of electron but with opposite sign and the mass of proton equal to 1.672×10^{-27} *kg*. The charge of a positive ion is an integral multiple of the charge of the electron, although it is of opposite sign. For the case of singly ionised particles, the charge is equal to that of the electron. For the case of doubly ionised particles, the ionic charge is twice that of the electron.

The electron in obit moving under the influence of the balance between the central force and the electrostatic force between the electron and the positive nucleus.

$$\frac{e^2}{4\pi\epsilon_o r^2} = \frac{mv^2}{r}$$

The electron velocity in orbit can be calculated by:

$$v = \sqrt{\frac{e^2}{4\pi\epsilon_o mr}} \tag{1.1}$$

Where m is the mass of electron, e is charge of electron and v is electron velocity and r is the radius of orbit and ϵ_o is the permittivity of space $= 8.85 \times 10^{-12}$ F/m.

According to equation (1.1) Bohr developed a model which contains three postulates:

1. Instead of infinity of orbits which would be possible in classical mechanics is only possible for an electron to move in an orbit for which its orbital angular momentum is an integer multiple by Plank's constant divided by 2π.

$$P = mvr = n\frac{h}{2\pi} = n\hbar \qquad (1.2)$$

where

$$\hbar = \frac{h}{2\pi}$$

h is Plank's constant $= 6.62 \times 10^{-34}\ J\ s.$

Equation (1.2) shows that the angular momentum of electron is *quantized*. From equations (1.1) and (1.2) it can be written:

$$r_n = \frac{n^2 h^2 \epsilon_o}{\pi m e^2} \qquad (1.3)$$

When n=1 equation (1.3) will gives $r_1 = 5.29 \times 10^{-11} m$. $r_1 \equiv$ Bohr radius $\equiv$ Hydrogen radius (a_o). Equation (1.3) can be written as:

$$r_n = a_o n^2 \qquad (1.4)$$

where,

$$a_o = \frac{h^2 \epsilon_o}{\pi m e^2}$$

2. Electromagnetic radiation is emitted if an electron initially moves from an orbit of total energy (E_i) to lower orbit of energy (E_f). The frequency of the emitted radiation is equal to:

$$f = \frac{E_i - E_f}{h} \qquad (1.5)$$

3. An atom has a finite state of energy. These states are separated and the electron in which state is stationary and non-radiating.

1.2 Electron Energy in Orbit

The total energy of electron in orbit around the nucleus is the sum of kinetic energy and electrostatic energy, that means:

E = electrostatic (work)+ kinetic energy

The potential energy of the electron at a distance r from the nucleus is $\frac{-q^2}{4\pi\epsilon_o r}$, and its kinetic energy is $\frac{mv^2}{2}$. Then, according to the conservation of energy and using equation (1.1) we can write:

$$E = \frac{e^2}{8\pi\epsilon_o r} + \frac{-e^2}{4\pi\epsilon_o r} \tag{1.6}$$

The first term in the above equation represents the kinetic energy of the electron in the orbit whilst the second term represents the work, i.e., the electro-static energy. The electrostatic energy can be define as the work which required to move an electron from infinite to distance r from the positive nucleus. As a result the total energy of the orbital electron can be written as:

$$E = \frac{-e^2}{8\pi\epsilon_o r} \tag{1.7}$$

It is necessary to observe that equation (1.7) shows that the electron energy is **negative** when it bounded to atom. **If the total energy of electron is greater than zero, then the electron has enough energy to separate from atom**. Substituting the value of r from (1.3) in the equation (1.7), the last equation would be as:

$$E = \frac{-me^4}{8h^2\epsilon_o^2} \times \frac{1}{n^2} \tag{1.8}$$

The magnitude $\frac{-me^4}{8h^2\epsilon_o^2}$ is constant and represents the energy of electron in the ground state of the hydrogen atom. Then it can be re-write equation (1.8) as:

$$E = E_r \times \frac{1}{n^2} \tag{1.9}$$

Where $E_r = \frac{-me^4}{8h^2\epsilon_o^2}$ and the magnitude of this constant is equal to *-13.6 eV*. **This value is the energy which required to ionize the hydrogen atom**.

It can be observed that it is used the *electron − volt* as a unit of the energy instead of the 'Joule'. For energies involved in electron devices, 'Joule' is too large a unit. Such small energies are conveniently measured in *electron − volt*, abbreviated eV. The *electron − volt* is defined as the kinetic energy gained by an electron, initially at rest, in moving through a potential difference of 1 volt. Since $e = 1.6 \times 10^{-19}$ Coulomb, then each $1eV$ will equal to 1.6×10^{-19} Joule.

1$eV = 1.6 \times 10^{-19}$ **Joule**.

1.3 Atomic Energy Level

According to equation (1.9) the energy of second state is -3.4ev. Then it is required to 10.2ev to raise the electron from ground state to second state. Therefore there is only discrete value of electron energy exist within atomic structures. This energy can be calculate using the following equation:

$$\Delta E = E_R \times \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right) \tag{1.10}$$

For each integral value of n in equation (1.9) a horizontal line is drawn. These lines are arranged vertically in accordance with the numerical values calculated from equation (1.9). Such graph is called an energy-level diagram and is indicated in Fig. 1.2 for hydrogen. The number to the left of each line gives the energy of this level in electron volts. The number immediately to the right of a line is the value of n. Theoretically, an infinite number of levels exist for each atom, but only the first five and the level for $n = \infty$ are indicated in Fig. 1-2.

As the electron is given more and more energy, it moves into stationary states which

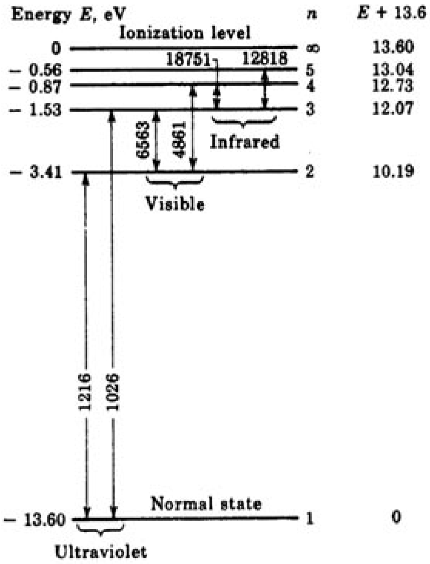

Figure 1.2: The lowest five energy levels and the ionization level of hydrogen. The spectral lines are in angstrom units.

are farther and farther away from the nucleus. When its energy is large enough to move it completely out of the field of influence of the ion, it becomes *'detached'* from it. The energy required for this process to occur is called the *ionization potential* and is represented as the highest state in the energy-level diagram.

Solved problems

1. `Calculate the electron velocity in the second orbit of the hydrogen atom?`

Solution

According to equation (1.1) the electron velocity in the orbit is given by:

$v = \sqrt{\frac{e^2}{4\pi\epsilon_o mr}}$

Since $r = a_o \times n^2$

$r = 5.29 \times 10^{-11} \times 2^2 = 2.11 \times 10^{-10} m.$

$v = \sqrt{\frac{(1.6\times10^{-19})^2}{4\pi\times8.85\times10^{-12}\times9.1\times10^{-31}\times2.11\times10^{-10}}}$

$v = 1.96 \times 10^6 m.s^{-1}$

....

2. `The total energy of electron in the second orbit is (-3.4ev). Find the kinetic energy and work?`

Solution

As it is shown before in equation (1.6) that the total energy of the electron in the orbit is the sum of kinetic energy and work. Then, E=$K.E$ + W

but $K.E = \frac{e^2}{8\pi\epsilon_o r}$ and $W = \frac{-e^2}{4\pi\epsilon_o r}$

then $E = \frac{e^2}{8\pi\epsilon_o r} + \frac{-e^2}{4\pi\epsilon_o r} = \frac{-e^2}{8\pi\epsilon_o r}$=-K.E=$\frac{W}{2}$

$K.E = -E = -(-3.4) = 3.4$ eV

$W = 2 \times E = 2 \times -3.4 = -6.8$ eV

....

3. `A photon with energy of 5eV is emitted for electron transition from orbit to another. Find the wavelength of the emitted electromagnetic wave?`

Solution

$E = h \times \nu = \frac{h\times c}{\lambda}$

$\lambda = \frac{h\times c}{E}$=$\frac{6.62\times10^{-34}\times3\times10^8}{5\times1.6\times10^{-19}}$=0.24 μ m.

....

4. `What is the frequency of the electromagnetic wave which emitted from the electron transition from the second orbit to the first orbit?`

   Solution

   $\Delta E = E_R \times \left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = -13.6 \times \left(\frac{1}{2^2} - \frac{1}{1^2}\right) = 10.2eV$

1.4 De Broglie Hypothesis

Since a photon is absorbed by only one atom, the photon acts as if it were concentrated in a very small volume of space, in contradiction to the concept of a wave associated with radiation. De Broglie postulated that the dual character of wave and particle is not limited to radiation, but is also exhibited by particles such as electrons, atoms, or macroscopic masses. He postulated that a particle of momentum $p = mv$ has a wavelength λ associated with it given by:

$$\lambda = \frac{h}{P} \tag{1.11}$$

We can make use of the wave properties of a moving electron to establish Bohr's postulate that a stationary state is determined by the condition that the angular momentum must be an integral multiple of $(h/2\pi)$. It seems reasonable to assume that an orbit of radius r will correspond to a stationary state if it contains a standing-wave pattern. In other words, a stable orbit is one whose circumference is exactly equal to the electronic wavelength λ, or to $(n\lambda)$, where n is an integer (but not zero). Thus:

$$2\pi r = n\lambda = \frac{nh}{mv} \tag{1.12}$$

`Prove equation 1.12?`

1.5 The energy-band theory of crystals

As we have seen, all the electrons of a given atom having the same value of n belong to the same prescribed bands (electronic shell). Each shell around the nucleus corresponds to a certain energy band and is separated from adjacent shells by energy gaps, in which no electron can exist.

A crystal is a solid consisting of a regular and repetitive arrangement of atoms or molecules (strictly speaking, ions) in space. If the positions of the atoms in the crystal are represented by points, called lattice points, we get a crystal lattice. The distance between the atoms in a crystal is fixed and is termed the lattice constant of the crystal. To discuss the behaviour of electrons in a crystal, we consider an isolated atom of the crystal. If Z is the atomic number, the atomic nucleus has a positive charge Ze. At a distance r from the nucleus, the electrostatic potential due to the nuclear charge is (in SI units).

$$V(r) = \frac{Ze}{4\pi\epsilon_o r} \tag{1.13}$$

Since an electron carries a negative charge, the potential energy of an electron at a distance r from the nucleus is:

$$E_p(r) = -eV = \frac{-Ze^2}{4\pi\epsilon_o r} \tag{1.14}$$

$V(r)$ is positive while $E_p(r)$ is negative. Both $V(r)$ and $E_p(r)$ are zero at an infinite distance from the nucleus. Figs. 1.3(a) and (b) show the variation of $V(r)$ and $E_p(r)$, respectively with r.

Now, consider two identical atoms placed close together. The net potential energy of an electron is obtained as the sum of the potential energies due to the two individual nuclei. In the region between the two nuclei, the net potential energy is clearly smaller than the potential energy for an isolated nucleus (see Fig. 1.3).

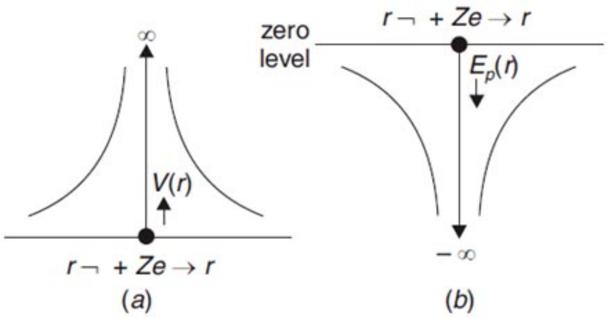

Figure 1.3: Variation of (a) Potential in the field of a nucleus with distance, (b) Potential energy of an electron with its distance from the nucleus.

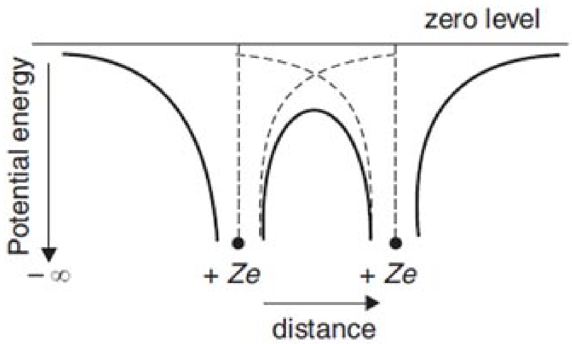

Figure 1.4: Potential energy variation of an electron with distance between two identical nuclei.

The potential energy along a line through a row of equispaced atomic nuclei, as in a crystal, is diagrammatically shown in figure 1.5. The potential energy between the nuclei is found to consist of a series of humps. The separation between the split-off energy levels is very small. This large number of discrete and closely spaced energy levels forms an *energy band*. Energy bands are represented schematically by shaded regions in figure 1.5(b).

The width of an energy band is determined by the parent energy level of the isolated atom and the atomic spacing in the crystal. The lower energy levels are not greatly affected by the interaction among the neighbouring atoms, and hence form narrow bands. The higher energy levels are greatly affected by the interatomic interactions and produce

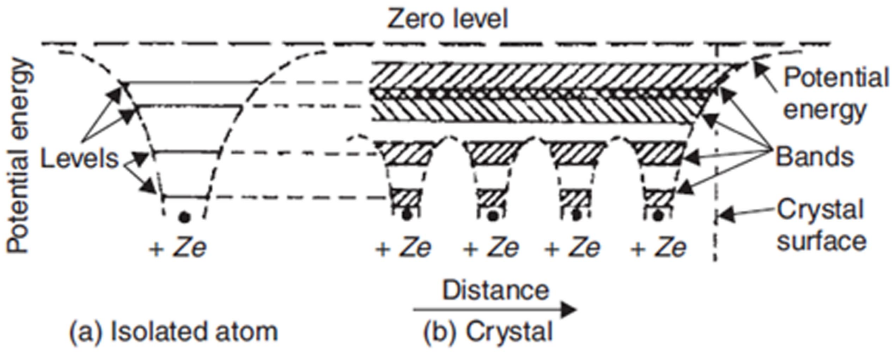

Figure 1.5: Splitting of energy levels of isolated atoms into energy bands as these atoms are brought close together to produce a crystal.

wide bands.

The lower energy bands are normally completely filled by the electrons since the electrons always tend to occupy the lowest available energy states. The higher energy bands may be completely empty or may be partly filled by the electrons.

The interatomic spacing, although fixed for a given crystal, is different for different crystals. The width of an energy band thus depends on the type of the **crystal**, and is larger for a crystal with a small interatomic spacing. **The lower energy bands are normally completely filled by the electrons since the electrons always tend to occupy the lowest available energy states. The higher energy bands may be completely empty or may be partly filled by the electrons.** The lower energy band calls the valence band and the first, unfilled or partially filled, band above the valence is called conduction band. The energy gap between the valence and conduction can be calculated as:

$$E_g = E_c - E_v \tag{1.15}$$

On the basis of the band structure, crystals can be classified into metals, insulators, and semiconductors.

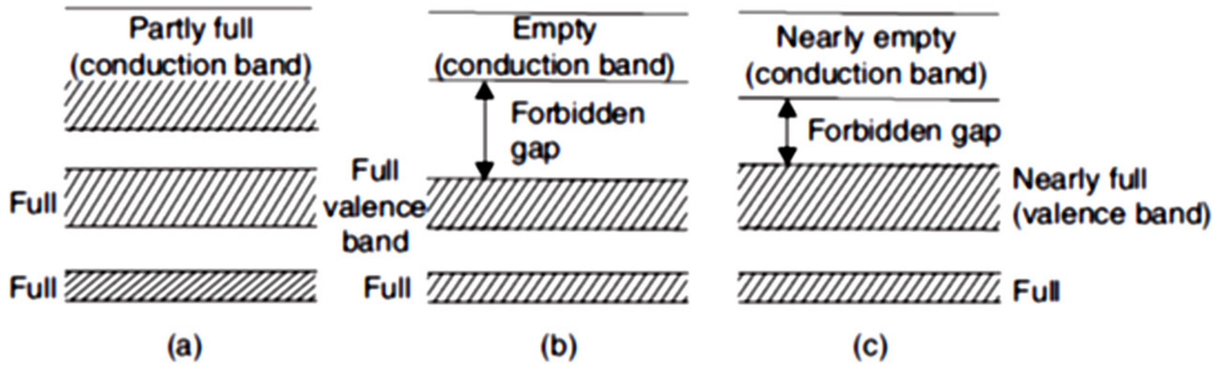

Figure 1.6: Energy band structure of (a) metal, (b) insulator, and (c) semiconductor.

1.5.1 Metals

A crystalline solid is called a metal if the uppermost energy band is partly filled [figure 1.6(a)] or the uppermost filled band and the next unoccupied band overlap in energy. The electrons in the uppermost band find neighbouring vacant states to move in, and thus behave as free particles. In the presence of an applied electric field, these electrons gain energy from the field and produce an electric current, so that a metal is a good conductor of electricity. The partly filled band is called the conduction band. The electrons in the conduction band are known as free electrons or conduction electrons.

1.5.2 Insulators

When the forbidden energy gap between the valence band and the conduction band, is very large, only a few electrons can acquire enough thermal energy to move from the valence band into the conduction band. Such solids are known as insulators. Since only a few free electrons are available in the conduction band, an insulator is a bad conductor of electricity. The energy band structure of an insulator is schematically shown in figure 1.6(b).

1.5.3 Semiconductors

A material for which the width of the forbidden energy gap between the valence and the conduction band is relatively small (1 eV) is referred to as a semiconductor. As the forbidden gap is not very wide, some of the valence electrons acquire enough thermal energy to go into the conduction band. These electrons then become free and can move about under the action of an applied electric field. The absence of an electron in the valence band is referred to as a hole. The holes also serve as carriers of electricity. The electrical conductivity of a semiconductor is less than that of a metal but greater than that of an insulator. The band diagram of a semiconductor is given in figure 1.6(c).
At 0 ^oK the semiconductor becomes insulator because the electrons do not has energy to jump to conduction band.

1.6 Fermi–Dirac distribution function

Fermi Dirac distribution function describes the energies of single particles in a system comprising many identical particles that obey the Pauli exclusion principle. Electrons are Fermions, and thus follow Fermi Dirac distribution function.
At room temperature, the thermal energy of the atoms may allow a small number of electrons to participate in the conduction process in semiconductor. The probability for filling the band by electrons depends on temperature.

$$f(E) = \frac{1}{1 + exp\left(\frac{E-E_f}{kT}\right)} \tag{1.16}$$

where $f(E)$ is the probability of occupancy of the state with energy E, E_F is a char-

acteristic energy for a particular solid and is referred to as the Fermi level, T is the absolute temperature in ^oK and k is Boltzmann's constant ($k = 1.38 \times 10^{-23}\ J\ K^{-1} = 8.614 \times 10^{-5}\ eV\ K^{-1}$). Fermi energy can be defined as the energy at which there would be a fifty percent chance of finding an electron.
Figure 1.7 shows the probability $f(E)$ against $\frac{E}{E_F}$ for $T = 0$, 300 and 2000 K.

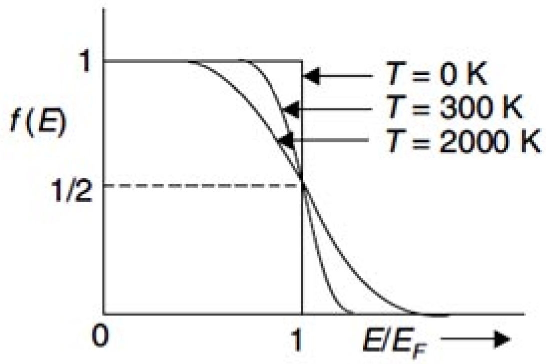

Figure 1.7: $f(E)$ as a function of E/E_F for different values of T

From Fermi Dirac function, equation 1.16 and figure 1.7, it can be conclude that:

1. At the absolute zero of temperature, i.e. at T = 0 K, equation 1.16 shows that $f(E) = 1$ for $E < E_F$ and $f(E) = 0$ for $E > E_F$. Thus all the energy states below E_F are occupied by the electrons and all the energy states above E_F are completely empty. (The probability of finding electron above Fermi level at zero ^oK is zero).

2. At temperatures greater than the absolute zero, $f(E) > 0$ for $E > E_F$, as shown in figure 1.7. This means that at a finite temperature, some of the electrons in the quantum states below EF acquire thermal energy to move into states above E_F. The probability of electron above Fermi level at T>0 ^oK is given by:

$$f(E) = exp\left(\frac{-(E - E_f)}{kT}\right) \tag{1.17}$$

3. The probability of electron to fill a state below Fermi level at $T>0\ ^oK$ is given by:

$$f\left(E\right)=1-exp\left(\frac{-\left(\left|E-E_f\right|\right)}{kT}\right) \tag{1.18}$$

Solved problems

1. `Find the probability of an electron to occupy a level (0.1 eV) above Fermi level at 27` ^oC?

   Solution

   $f\left(E\right)=exp\left(\frac{-\left(E-E_f\right)}{kT}\right)=exp\left(\frac{-(0.1)}{8.614\times10^{-5}\times300}\right)=0.0209$

2. `The probability for an electron to occupy a level at 120` ^oC `is` (2×10^{-6}). `Find the location of this level with respect to Fermi level?`

   Solution

   $f\left(E\right)=\frac{1}{1+exp\left(\frac{E-E_f}{kT}\right)}=\frac{1}{1+exp\left(\frac{E-E_f}{8.614\times10^{-5}\times300}\right)}=2\times10^{-6}$

   $E-E_f=0.444eV$ above Fermi level.

Review Questions

1. What is the orbital angular momentum of the electron in the third orbit? (Ans. $3\times\hbar=1.97\times10^{-15}eV$).

2. What is the energy of the electron in the Hydrogen atom after absorption an electromagnetic wave with frequency of $3.284\times10^{15}Hz$?(Ans. 13.59 eV).

3. An electron, initially at rest, gains a speed of 10^7 m/s after being accelerated through a potential difference of V volt. Determine V. What is the final kinetic energy of the electron in J and eV? (Ans. 284.7 volt, $4.555\times10^{-17}J$, 284.7 eV)

4. A particle carries a positive charge numerically equal to the electronic charge. It acquires a velocity of 200 km/s after moving through a potential difference of 210 V. Determine the mass of the particle relative to the electronic rest mass. (Ans. 1844 m_o).

5. A particle of charge $1.2 \times 10^{-8} C$ and mass $5g$ travels a distance of $3m$ under the action of a potential difference of $500V$. Calculate the final velocity and the acceleration of the particle if it starts from rest. (Ans. $4.898 \times 10^{-2} m/s$, $4 \times 10^{-4} m/s^2$).

6. A potential difference of 400 volt is applied between two parallel metal plates 4 cm apart. An electron starts from rest from the negative plate. Obtain (i) the kinetic energy of the electron when it reaches the positive plate and (ii) the time required by the electron to reach the positive plate. (Ans. 400 eV, $6.745 \times 10^{-9} s$).

7. Find the probability for an electron to occupy a level at 50 ^oC if this level is below Fermi level by (0.2 eV)?

Chapter 2

Transport Phenomena in Semiconductor

The current is defined as the flow of charge particles. In metal the current results from the flow of negative charges (electrons),whereas the current in a semiconductor results from the motion of both electrons and positive charges (holes). A pure semiconductor may be doped with impurity atoms so that the current is due predominantly either to electrons or to holes. The transport of the charges, i.e. conductivity, in a crystal under the influence of an electric field (a drift current), and also as a result of a nonuniform concentration gradient (a diffusion current), is investigated in this chapter.

2.1 Mobility and Conductivity

As it is observed in the preceding chapter, according to the energy band theory, the materials can be classified into three types: insulators, conductors and semiconductors. A conductor is a solid in which an electric current flows under the influence of the electric field. By contrast, application of an electric filed produces no current in an insulator. The energy gap for an insulator is so wide that hardly any electrons acquired enough to jump to conduction band. If a constant electric field is applied to the metal, the electrons will move with acceleration equal:

$$a = \frac{e\,E}{m} \tag{2.1}$$

where, E is the electric field in unit $(V\,m^{-1})$

But the electron suffering from collisions with other particles in metal and it speed between two successive collisions is $(a\,t)$. t : is the relaxation time. The distance between two successive collisions is called the ***mean free path*** and equal to:

$$l = v_d\,t \tag{2.2}$$

Electrical mobility is the ability of charged particles (i.e. electrons) to move through a medium in response to an electric field that is pulling them. The external electrical field gives electron drift velocity and acceleration, therefore the drift velocity is equal to:

$$v_d = \mu\,E \tag{2.3}$$

where μ is the electron mobility in unit $(m^2V^{-1}s^{-1})$ and it is equal to (et/m). The minus sign means the drift velocity is in the direction opposite to that of the external electric field.

Electrical Conductivity: Electrical conductivity is a measure of a material's ability to conduct an electric current. It is commonly represented by Greek letter σ. The following figure shows a box of metal with length L and cross section area A. The voltage V was applied on the ends of the box. According to Ohm's law: $V = I\,R$ and;

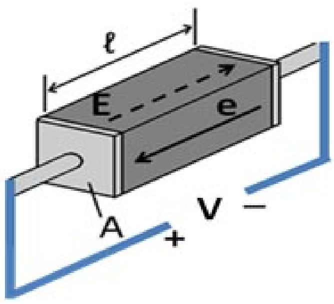

Figure 2.1: Box of metal

$$R = \rho \frac{L}{A} \tag{2.4}$$

where ρ is the resistivity in unit (Ωm).

Since $R = V/I$

Then 2.4 becomes:

$(I/A) = (1/\rho) \times (V/L) = J$ (Current density)

but $(V/L) = E$ and $(1/\rho) = \sigma$, then:

$$J = \sigma\, E \tag{2.5}$$

Where J represents the drift current density.

Now, consider that the metal contains (n) of free electrons per unit volume, then the total free electrons inside the metal is:

$$q = -n\, e\, A\, L \tag{2.6}$$

but $I = (q/t) = q \times (v_d/L)$, where $v_d = (L/t)$

Since $I = J \times A$ and from 2.5 we have $J = \sigma\, E$, then it can be write:

$$\sigma E = -q \frac{v_d}{L \times A} = -n\, e\, v_d \tag{2.7}$$

Substituting 2.7 in equation 2.3, we can obtain:

$$\sigma = -n\, e\, \mu \tag{2.8}$$

This equation shows that the conductivity depends on the density of the free electrons and the mobility of these electrons. Equation 2.8 can be written in new form as shown below:

$$\sigma = \frac{n\, e^2\, l}{m\, v_d} \tag{2.9}$$

2.2 Diffusion Current

In addition to a conduction current, the transport of charges in a semiconductor may be accounted for by a mechanism called diffusion, not ordinarily encountered in metals. The essential features of diffusion are now discussed. It is possible to have a nonuniform concentration of particles in a semiconductor. As indicated in figure 2.2, the concentration n of electrons varies with distance x in the semiconductor, and there exists a concentration gradient, dn/dx, in the density of carriers. The existence of a gradient implies the density of electrons immediately on one side of the surface is larger than the density on the other side. The electrons are in a random motion as a result of their thermal energy. Accordingly, electrons will continue to move back. The net transport of electrons across the surface constitutes a current in the positive X direction.

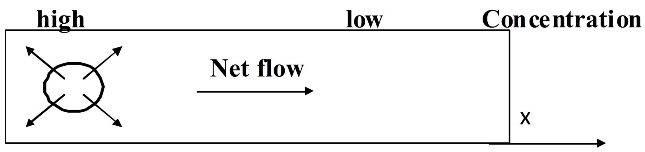

Figure 2.2: A nonuniform concentration n(x) results in a diffusion current

It should be noted that this net transport of charge is not the result of mutual repulsion among charges of like sign, but is simply the result of a statistical phenomenon. This diffusion is exactly analogous to th at which occurs in a neutral gas if a concentration gradient exists in the gaseous container. The diffusion electron-current density J_n (amperes per square meter) is proportional to the concentration gradient, and is given by: $J_{diff} \propto (dn/dx)$
also, $J_{diff} \propto D$
where D is diffusion constant $(m^2 sec^{-1})$

$$J_{diff} = e\,D\,\frac{dn}{dx} \tag{2.10}$$

D and μ are related by Einstein relation:

$$D = \frac{kT}{e}\mu \tag{2.11}$$

where T is the temperature in ^oK

Total Current: It is possible for both a potential gradient and a concentration gradient to exist simultaneously within a semiconductor. In such a situation the total hole current is the sum of the drift current 2.5 and the diffusion current 2.10;

$$J_{tot} = eD\frac{dn}{dx} + \sigma E \tag{2.12}$$

2.3 Work Function

Free electron moves in metal by random motion in absent external operator or in additional to drift and /or diffusion motion. The kinetic energy makes electrons reach Fermi level. Then the energy required rising electron to state outside the metal is E_s (surface energy), therefore the work function (ϕ) given as:

$$\phi = E_s - E_f \tag{2.13}$$

The **work function** is the minimum energy (usually measured in electron volts) needed to remove an electron from a solid to a point immediately outside the solid surface (or energy needed to move an electron from the Fermi level into vacuum). Here "immediately" means that the final electron position is far from the surface on the atomic scale but still close to the solid on the macroscopic scale. The work function is a characteristic property for any solid phase of a substance with a conduction band (whether empty or partly filled). For a metal, the Fermi level is inside the conduction band, indicating that the band is partly filled. For an insulator, the Fermi level lies within the band gap, indicating an empty conduction band; in this case, the minimum energy to remove an electron is about the sum of half the band gap and the electron affinity. When the electron absorbs energy E then the $K.E.$ for this electron outside the metal will be:

$$\frac{1}{2}\, m\, v^2 = E - \phi \tag{2.14}$$

This is called **electronic emission**. There is four type of electronic emission: *Thermionic Emission, Photo Emission, Field Emission and Secondary Emission.* If a thermal energy is supplied to the electrons in the metal, then the energy distribution of the electrons changes, because of the increasing in the temperature. The thermal energy given to the charge carrier overcomes the binding potential (work function) and can release it from the metal surface. This is called **Thermionic emission.** According to the *Richardson-Dushman* equation the emitted electron current density, $J(A.m^{-2})$, is related to the absolute temperature T by the equation:

$$J = A_o T^2 exp(\frac{-\phi}{kT}) \tag{2.15}$$

where (A_o) is Richardson-Dushman constant. As mentioned before, the work function is the minimum energy that must be given to an electron to liberate it from the surface of a particular substance. In the photoelectric effect, electron excitation is achieved by absorption of a photon. If the photon's energy is greater than the substance's work function, **photoelectric emission** occurs and the electron is liberated from the surface. Excess photon energy results in a liberated electron with non-zero kinetic energy. The photo-electric work function is:

$$\phi = hf_o$$

f_o is the minimum (threshold) frequency of the photon required to produce photoelectric emission.

Field emission (F_E) (also known as **field electron emission** and **electron field emission**) is emission of electrons induced by an electrostatic field. Field emission in pure metals occurs in high electric fields and strongly dependent upon the work function. The emission current density is given as:

$$J \propto \ exp\,(\frac{-\phi}{e\,E\,x_o}) \tag{2.16}$$

x_o is the gap thickness.

Secondary electron emission is a phenomenon where primary incident electrons of sufficient energy, when hitting a surface of material, induce the emission of secondary electrons. It was found experimentally the number of secondary electrons depend on the following the number and the energy of primary electrons, the angle of incidence of the particles on the material, the type of the material, and the physical condition of the surface. The secondary emission ratio (δ) defined as:

$$\delta = \frac{no.\,of\,the\,secondary\,emitted\,electrons}{no.\,of\,the\,primary\,incident\,electrons} \tag{2.17}$$

Solved problems

1. A silicon crystal having across section area of ($0.001cm^2$) and a length of ($10^{-3}cm$) is connected at its ends to ($10V$) battery at temperature (300^oK). Find the resistivity and the conductivity of the silicon crystal if the current passing through the crystal is ($100mA$).

   Solution

$$J = \sigma E \Rightarrow \sigma = \frac{J}{E} = \frac{I/A}{V/L}$$

$$\sigma = \frac{100 \times 10^{-2}/0.001 \times 10^{-4}}{10/(10^{-3} \times 10^{-2})} = 10\,(\Omega m)^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{10} = 0.1\,\Omega\,m$$

....

2. Calculate the average drift velocity of hole in a bar of silicon with across sectional area ($10^{-4}cm^2$), containing holes concentration of ($4.5\times 10^{15}cm^{-3}$) and carrying a current of ($45mA$)?

   Solution

$$v_d = \mu\,E.....(a)$$

$$\sigma = pe\,\mu.....(b)$$

$$J = \sigma E \Rightarrow \sigma = \frac{J}{E}.....(c)$$

Therefore equation (b) becomes:

$$\frac{J}{E} = pe\mu \Rightarrow \mu = \frac{J}{pe\mu}.....(d)$$

From equations (a and d),

$$v_d = \left(\frac{J}{peE}\right) \times E = \frac{J}{pe} = \frac{I/A}{pe} = \frac{45 \times 10^{-3}/10^{-4} \times 10^{-4}}{4.5 \times 10^{15} \times 10^6 \times 1.6 \times 10^{-19}} = 6250\,ms^{-1}$$

3. A current of $1\mu A$ passing through an intrinsic silicon bar has $3mm$ length and $50{\times}100\mu m^2$ cross-section. The resistivity of the bar is $2.3{\times}10^5\Omega cm$ at 300^oK. Find the voltage across the bar?

Solution

$$J_d = \frac{I}{A} = \sigma\, E$$
$$E = \frac{J_d}{\sigma} = \frac{I}{A} \times \frac{1}{\sigma} = \frac{I}{A} \times \rho$$
$$E = \frac{10^{-6}}{50 \times 10^{-6} \times 100 \times 10^{-6}} \times 2.3 \times 10^5 \times 10^{-2} = 4.6 \times 10^5 V.m^{-1}$$
$$V_{bar} = E \times L = 4.6 \times 10^5 \times 3 \times 10^{-3} = 1380\, V$$

4. The electron density variation along the x-axis is given as $[10^{28}\, exp(-10^{-6}\, x)]$. Find the diffusion current at $(x = 0)$ and $(x = 10^{-5}m)$ if the mobility of electron is $(4 \times 10^{-3}m^2V^{-1}s^{-1})$ at $T = 300^oK$?

Solution

$$J_{diff} = eD\frac{dn}{dx}$$
$$D = \frac{kT}{e}\mu = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 4 \times 10^{-3} = 1.035 \times 10^{-4} m^2 sec^{-1}$$
$$\frac{dn}{dx} = -10^{28} \times 10^{-6} \times exp\left(-10^{-6}x\right)$$
$$J_{diff} = 1.6 \times 10^{-19} \times 1.035 \times 10^{-4} \times \left[-10^{28} \times 10^{-6} \times exp\left(-10^{-6}x\right)\right]$$
$$1. J_{diff(at\, x=0)} = -1.6 \times 10^{-11}\; Am^{-2}$$
$$2. J_{diff(at\, x=10^{-5})} = 1.035{\times}10^{-4}{\times}1.6{\times}10^{-19}{\times}\left[-10^{28} \times 10^{-6} \times exp\left(-10^{-6} \times 10^{-5}\right)\right]$$
$$= 1.76 \times 10^{11}\; Am^{-2}$$

5. A bar of copper of $(2cm)$ length and resistively of $(1.8 \times 10^{-8}\Omega m)$ is connected to power supply of $(10V)$. Find the mobility and drift velocity of the electrons if electron density in copper is $(8.5 \times 10^{28}m^{-3})$?

Solution

$$\sigma = n\, e\, \mu \;\Rightarrow\; \mu = \frac{1}{n\, e\, \rho}$$

$$\mu = \frac{1}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.8 \times 10^{-8}} = 4.08 \times 10^{-3}\, m^2V^{-1}s^{-1}$$

2.4 Generation and Recombination of Charges

Generation = break up of covalent bond to form electron and hole pairs. A pure silicon crystal at room temperature derives heat (thermal) energy from the surrounding environment, causing some valence electrons to gain sufficient energy to jump the gap from valence band into the conduction band, becoming free electrons. When an electron jumps to C.B., a vacancy is left in the valence band. This vacancy is called a hole. If

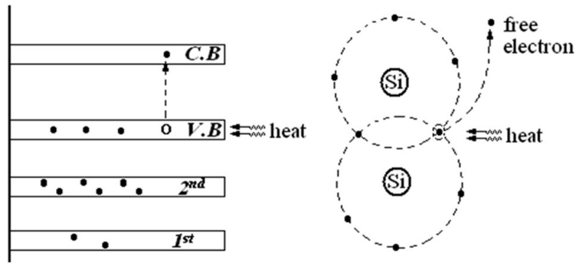

Figure 2.3: Free charge carrier generation in semiconductor

n and p is the free electron and hole concentration, respectively, per volume unit, at equilibrium status n=p=n_i. Where n_i is the carrier concentration. Recombination occurs when a conduction-band electron loses energy and falls back into a hole in the valence band.

To summarise, a piece of intrinsic semiconductor at room temperature has, at any instant, a number of conduction-band (free) electrons that are unattached to any atom and are essentially drifting randomly throughout the material. There is also an equal number of holes in the valence band created when these electrons jump into the conduction band.

Electron and Hole Current

When a voltage is applied across a piece of intrinsic silicon the thermally generated free electrons in the conduction band, which are free to move randomly in the crystal structure, are now easily attracted toward the positive end. This movement of free electrons is one type of current in a semiconductor material and is called *electron current.*

Another type of current occurs in the valence band, where the holes created by the free electrons exist. Electrons remaining in the valence band are still attached to their atoms and are not free to move randomly in the crystal structure as are the free electrons. However, a valence electron can move into a nearby hole with little change in its energy level. thus leaving another hole where it came from. Effectively the hole has moved from one place to another in the crystal structure. as illustrated in Figure 2.4 This is called *hole current*.

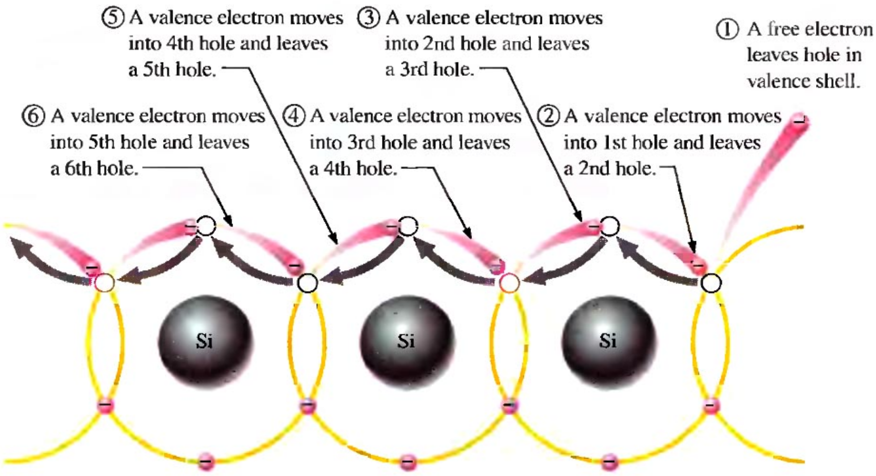

Figure 2.4: Hole current in intrinsic silicon

2.4.1 Electrons and Holes Density in an Intrinsic Semiconductor

In a pure (*intrinsic*) semiconductor the number of holes is equal to the number of free electrons and the electrical properties determined by host martial. In intrinsic semiconductor the carrier concentration can be determined from Fermi-Dirac function distribution:

$$n = N_c \, exp\left(-\frac{E_c - E_f}{kT}\right) \tag{2.18}$$

$$p = N_v \, exp\left(-\frac{E_f - E_v}{kT}\right) \tag{2.19}$$

Where n and p are the electron and hole concentration, respectively. N_c is the active level density at C.B. and N_v is the active level density at V.B. and given by:

$$N_c = \left(\frac{2\,\pi\,m_n^*\,kT}{h^2}\right)^{3/2}$$

and

$$N_v = \left(\frac{2\,\pi\,m_p^*\,kT}{h^2}\right)^{3/2}$$

m_n^*: effective mass of electron. m_p^*: effective mass of hole.

(*Effective mass*: When we apply an external force to an electron in a crystal, it may not respond as if it were a free electron. This is because of the interaction with the crystal lattice). Then the number of carriers is:

$$n_i = \sqrt{n\,p} = \sqrt{N_c\,N_v}\;exp\left(-\frac{E_c - E_v}{2kT}\right) \tag{2.20}$$

but $E_g = E_c - E_v$, then;

$$n_i = \sqrt{n\,p} = \sqrt{N_c\,N_v}\;exp\left(-\frac{E_g}{2kT}\right) \tag{2.21}$$

It can be observed that the concentration of electrons and holes in pure semiconductor independent on the location of the Fermi level but it is depending on the temperature.

2.4.2 Electrons and Holes Density in an Extrinsic Semiconductor

Semiconductor materials do not conduct current well and are of limited value in their intrinsic state. This is because of the limited number of free electrons in the conduction band and holes in the valence band. Intrinsic silicon (or germanium) must be modified by increasing the number of free electrons or holes to increase its conductivity and make it useful in electronic devices. This is done by adding impurities to the intrinsic material as you will learn in this section. Two types of extrinsic (impure) semiconductor materials, n-type and p-type, are the key building blocks for most types of electronic devices. An extrinsic semiconductor can be formed from adding impurity atoms to the intrinsic semiconductor in process known as doping. The electrical properties of extrinsic semiconductor are determined by chemical impurities. For example, silicon has four valence electrons. Doping silicon with Aluminum (**Al**) will produce a hole. The dopant atoms have not enough number of electrons to share bonds with surrounding silicon atoms. One of the silicon atoms has a vacancy for an electron. It creates hole that contribute in conduction process and the semiconductor is called p-type as shown in figure 2.5(a). The dopant atoms are called **acceptors**. While if the silicon is doped with Phosphor (**P**) or Arsenide (**As**), which they have extra electron in valence bands, the dopant atoms contributes an additional electron to the crystal and the semiconductor is called n-type as shown in figure 2.5(b). The dopant atoms are called **donors**.

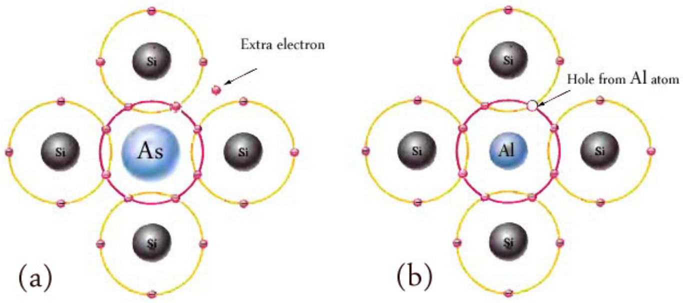

Figure 2.5: Extrinsic s.c. (a) n-type s.c. (b) p-type s.c.

Determination of electrons density for n-type semiconductor

If, to intrinsic silicon, there is added a small percentage of phosphor (P) atoms, a doped, impure, or extrinsic, semiconductor is formed. The fifth electron of the phosphor (P) will be released by energy $0.05eV$, which is the smallest energy required an electron of silicon atom by 20 times (E_g= $1.1eV$).

Then the density of electron in host semiconductor which doped by N_D atoms is:

$$n = N_D$$

N_D is the *concentration of donor atoms.*

The increasing of the electron density in conduction band case shifting in Fermi-level up word to C.B, then the difference in energy between old and new position of Fermi-level is:

$$\Delta E_n = E_{fn} - E_{fi}$$

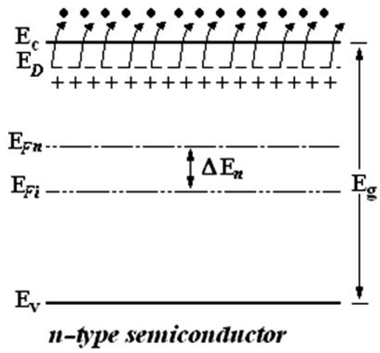

Figure 2.6: Energy band structure in n-type s.c.

The concentration of the electrons in conduction band is:

$$n = N_D = N_c\, exp\left(-\frac{E_c - E_{fn}}{kT}\right) \tag{2.22}$$

but, $E_{fn} = \Delta E_n + E_{fi}$
then 2.22 becomes;

$$n = N_D = N_c\, exp\left(-\tfrac{E_c - \Delta E_n - E_{fi}}{kT}\right)$$

$$n = N_D = N_c\, exp\left(-\frac{E_c - E_{fi}}{kT}\right) \times exp\left(\frac{\Delta E_n}{kT}\right)$$

Since;

$$n_i = N_c\, exp\left(-\frac{E_c - E_{fi}}{kT}\right)$$

then,

$$n = N_D = n_i\, exp\left(\frac{\Delta E_n}{kT}\right) \tag{2.23}$$

From the above equation the shift in Fermi level can be calculated as:

$$\Delta E_n = kT\, ln\left(\frac{n}{n_i}\right) = kT\, ln\left(\frac{N_D}{n_i}\right) \tag{2.24}$$

Determination of holes density of n-type semiconductor The shifting of Fermi level by ΔE_n up word in the n-type semiconductor, as shown in figure 2.6, means the new Fermi level shift away from valance band, and this will cause a new concentration of holes. Since the hole density given as:

$$p = N_v \, exp\left(-\frac{E_{fn} - E_v}{kT}\right) \tag{2.25}$$

but, $E_{fn} = \Delta E_n + E_{fi}$ then 2.25 becomes;

$$p = N_v \, exp\left(-\frac{E_{fi} - E_v + \Delta E_n}{kT}\right)$$

$$p = N_v \, exp\left(-\frac{E_{fi} - E_v}{kT}\right) \times exp\left(-\frac{\Delta E_n}{kT}\right)$$

Since;

$$n_i = N_v \, exp\left(-\frac{E_{fi} - E_v}{kT}\right)$$

then,

$$p = n_i \, exp\left(-\frac{\Delta E_n}{kT}\right) \tag{2.26}$$

It can be observe that the hole density decrease with Fermi level shifting upward:

$$n\,p = n_i^2$$

Since $n = N_D$, then;

$$p = n_i^2/N_D$$

Determination of holes density for p-type semiconductor The doping solid is the aluminum (**Al**) or boron (**B**), which have 3 valence electrons only, so the doping atom need one additional electron to bonded with the silicon structure, in this case it's can be filled from the nearest bond electrons and this will cause break up the bond near vacancy. The energy which required is (0.05 eV) for the boron (**B**) atoms, the number of charge carriers is equal to the number of holes (doping atoms). The impurity atoms in this case called the acceptors atoms and it's density is:

$$p \; = \; N_A$$

where N_A is the *concentration of acceptor atoms*.

The concentration of holes can be determined from Fermi-Dirac function as below:

$$p = N_v \, exp\left(-\frac{E_{fp} - E_v}{kT}\right) \tag{2.27}$$

new position of Fermi-level is:

$$\Delta E_p = E_{fi} - E_{fp}$$

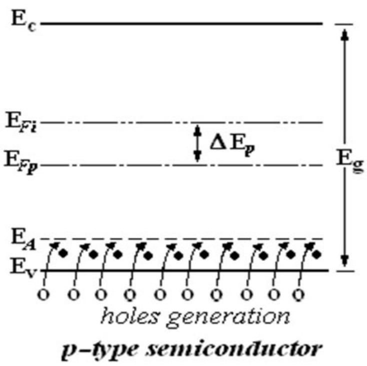

Figure 2.7: Energy band structure in p-type s.c.

$$p = N_A = N_v \, exp\left(-\frac{E_{fi} - \Delta E_p - E_v}{kT}\right)$$

$$p = N_v \, exp\left(-\frac{E_{fi} - E_v}{kT}\right) \times exp\left(\frac{\Delta E_p}{kT}\right)$$

Since;

$$n_i = N_v \, exp\left(-\frac{E_{fi} - E_v}{kT}\right)$$

then,

$$p = n_i \, exp\left(\frac{\Delta E_p}{kT}\right) \tag{2.28}$$

From the above equation the shift in Fermi level can be calculated as:

$$\Delta E_p = kT \, ln\left(\frac{p}{n_i}\right) = kT \, ln\left(\frac{N_A}{n_i}\right) \tag{2.29}$$

and the number of electrons is equal to:

$$n = \frac{n_i^2}{N_A}$$

.....℘℘℘℘℘℘℘...℘℘℘℘℘℘℘.....

Solved problems

....

1. The electron density in pure silicon is $1.45\times10^{16}m^{-3}$ at 300^oK. Find the electron density when the temperature change to 350^oK, take $E_g = 1.1eV$?

<u>*Solution*</u>

$$n_{i1} = \sqrt{N_c N_v}\, exp\left(-\frac{E_g}{2kT_1}\right) .....(1$$

$$n_{i2} = \sqrt{N_c N_v}\, exp\left(-\frac{E_g}{2kT_2}\right) .....(2$$

$$\frac{n_{i1}}{n_{i2}} = \frac{exp\left(-\frac{E_g}{2kT_1}\right)}{exp\left(-\frac{E_g}{2kT_2}\right)}$$

$$n_{i2} = n_{i1}\frac{exp\left(\frac{E_g}{2kT_1}\right)}{exp\left(\frac{E_g}{2kT_2}\right)}$$

$$n_{i2} = n_{i1} exp\left(\frac{E_g}{2k}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]\right) = 1.45\times10^{16}\, exp\left(\frac{1.1}{2\times 8.62\times 10^{-5}} \times \left[\frac{1}{300} - \frac{1}{350}\right]\right)$$

$$n_{i2} = 3.03\times 10^{17}\, m^{-3}$$

---

2. Pure semiconductor with energy gap of $1.42\, eV$ and charge carrier density of $1.79\times10^{12}\, m^{-3}$ at $300\, ^oK$. Determine the position of the Fermi level with respect of the mid of gap if $N_c = 4.7\times 10^{23}\ m{-}3$. What is the value of N_v?

<u>*Solution*</u>

Since the charge concentration in pure semiconductor is equal to electron concentration, so:

$$n = N_c\, exp\left(-\frac{E_c - E_f}{kT}\right)$$

$$E_c - E_f = kT\; ln\frac{N_c}{n} = \frac{1.38\times10^{-23}}{1.6\times10^{-19}}\times300\;\times ln\left(\frac{4.7\times10^{23}}{1.79\times10^{12}}\right) = 0.68\,eV$$

The Fermi level located at 0.68 eV under the E_c, but the mid of the energy gap at 0.71 eV under the E_c. Therefore the position of Fermi level would be 0.03 eV above the mid of the gap.

** **Home work ⇒ Find** N_v **

---

3. Pure silicon has electron concentration $1.45\times10^{16}\,m^{-3}$ at $300\,^oK$ was doped with $10^{22}m^{-3}$ phosphor (P) atoms. Find the electron and hole densities at $300\,^oK$ and $500\,^oK$?

Solution

$n_i = 1.45\times10^{16}\,m^{-3}$ before doping at $500\,^oK$ the doping solid is phosphor (P) which is donor atoms, then; $n = N_D = 10^{22}m^{-3}$ the density of the solid after doping at $300\,^oK$

$$\frac{n_{i1}}{n_{i2}} = \frac{exp\left(-\frac{E_g}{2kT_1}\right)}{exp\left(-\frac{E_g}{2kT_2}\right)}$$

$$n_{i2} = n_{i1}\frac{exp\left(\frac{E_g}{2kT_1}\right)}{exp\left(\frac{E_g}{2kT_2}\right)}$$

$$n_{i2} = n_{i1}\times exp\left(\frac{E_g}{2k}\left[\frac{1}{T_1}-\frac{1}{T_2}\right]\right) = 1.45\times10^{16}\times exp\left(\frac{1.1}{2\times8.62\times10^{-5}}\times\left[\frac{1}{300}-\frac{1}{500}\right]\right)$$

$$n_{i2} = 7.2\times10^{19}\,m^{-3}$$

$$p_2 = \frac{n_{i2}^2}{N_D} = 5.2\times10^{17}\,m^{-3}$$

---

4. The electron concentration in pure silicon is $1.5\times10^{16}\,m^{-3}$ at $300\,^oK$. The silicon was doped with $10^{22}m^{-3}$ donor atoms. Find the electron and hole densities after doping and calculate the position of the new Fermi level with to the initial position?

Solution

After doping the density of electron is $n = N_D = 10^{22}\ m^{-3}$, while the hole density is given as;

$$p = \frac{n_i^2}{N_D} = \frac{1.5 \times 10^{16^2}}{10^{22}} = 2.25 \times 10^{10}\ m^{-3}$$

$$\Delta E_n = kT\ ln \left(\frac{N_D}{n_i}\right)$$

$$\Delta E_n = \frac{1.38 \times 10^{-23}}{1.6 \times 10^{-19}} \times 300\ \times ln \left(\frac{10^{22}}{1.5 \times 10^{16}}\right) = 0.347\, eV$$

The new position of Fermi level is above the initial position by $0.347\, eV$

---

5. `If the position of Fermi level in impure semiconductor at` $0.3\, eV$ `above the mid of the energy gap at` $300\ ^oK$, `where the` $E_g = 1.1\ eV$ `and` $n_i = 1.45 \times 10^{16}\ m^{-3}$.

   `What is type of the impurities and what are its concentration?`

   Solution

   Since the Fermi level location is above the mid of the energy gap, therefore the semiconductor would be from n-type. So the impurities are donor atoms.

$$N_D = n_i \times exp \left(\frac{\Delta E_n}{kT}\right)$$

$$N_D = 1.45 \times 10^{16} \times exp \left(\frac{0.3}{8.614 \times 10^5 \times 300}\right) = 1.58 \times 10^{21}\ m^{-3}$$

2.5 Electrical conduction in semiconductor

As it was seen that the electron motion and the electrical conduction in metal depend on several parameters which describe the electrical motion in metal, this description for the electron motion and the electrical conduction is same to these in semiconductor but with take care the ratio of doping.

2.5.1 Electrical conduction in intrinsic semiconductor

The electrical conduction in intrinsic semiconductor is same the general formula of conductivity in metal:

$$\sigma = n\, e\, \mu$$

Applied this formula on an intrinsic semiconductor, the electrons and holes contribute in electrical conduction then:

$$\sigma_i = n\, e\, \mu_n + p\, e\, \mu_p \tag{2.30}$$

where;
n=concentration of electrons (m^{-3})
μ_n=electron mobility ($m^2\, V^{-1}\, s^{-1}$)
p=concentration of holes (m^{-3})
μ_p=hole mobility ($m^2\, V^{-1}\, s^{-1}$)

In pure semiconductor the concentration of electrons equal to concentration of holes; i.e., p=n=n_i, then equation 2.30 can be written as;

$$\sigma_i = n_i\, e\, (\mu_n + \mu_n) \tag{2.31}$$

and n_i is;

$$n_i = \sqrt{N_c\, N_v}\, exp\left(-\frac{E_g}{2kT}\right)$$

$$n_i = \sqrt{\left[\frac{2\pi\, m_n^*\, k\, T}{h^2}\right]^{3/2} \times \left[\frac{2\pi\, m_p^*\, k\, T}{h^2}\right]^{3/2} \times \left[exp\left(-\frac{E_g}{2kT}\right)\right]}$$

$$n_i = \left[\frac{2\pi\, k}{h^2}\right]^{3/2} \times \left[m_n^*\, m_p^*\right]^{3/4} \times T^{3/2} \times \left[exp\left(-\frac{E_g}{2kT}\right)\right]$$

$$n_i \propto T^{3/2} \times \left[exp\left(-\frac{E_g}{2kT}\right)\right]$$

Since it is known that;

$$\mu_n = \frac{e\, t_n}{m_n^*}$$

and

$$\mu_p = \frac{e\, t_p}{m_p^*}$$

where, t_n and t_p are the relaxation time of electrons and holes respectively.

The mobility depends on the relation time and effective mass of moving charges.

Heating the semiconductor causes vibration of atoms and this will effect on electron motion inside the crystal structure and hence the electrons collision with atoms will increase due to vibration of atoms and therefore the *mobility will decreases.*

$$\mu \propto T^{-3/2}$$

Then it can be conclude that:

$$\sigma \propto n_i \Longrightarrow \sigma \propto T^{3/2} \times \left[exp \left(-\frac{E_g}{2kT} \right) \right]$$

and;

$$\sigma \propto \mu \Longrightarrow \sigma \propto T^{3/2}$$

As a result of that the semiconductor conductivity affected by temperature as:

$$\sigma_i = \sigma_o\, exp \left(-\frac{E_g}{2kT} \right) \tag{2.32}$$

where, σ_o is a constant and **independent to temperature**. Equation 2.32 can also be written as:

$$ln\,(\sigma_i) = ln\,(\sigma_o) - \frac{E_g}{2kT}$$

The semiconductor conductivity changes strongly with temperature variation.

$$\frac{1}{\sigma_i}\frac{d\sigma_i}{dT} = -\frac{E_g}{2kT} \tag{2.33}$$

2.5.2 Electrical conduction in extrinsic semiconductor

When the semiconductor is doped by impurities has N_D concentration ($n >> p$):

$$\sigma_{(n)} = n\,e\,\mu_n + p\,e\,\mu_p \tag{2.34}$$

In other words equation 2.34 can be written as;

$$\sigma_{(n)} = \sigma_n + \sigma_p$$

But $n \times p = n_i^2 \Longrightarrow N_D \times p = n_i^2 \Longrightarrow p = (n_i^2/N_D)$, then equation 2.34 can also be written as;

$$\sigma_{(n)} = N_D\,e\,\mu_n + \frac{n_i^2}{N_D}\,e\,\mu_p \tag{2.35}$$

$\because N_D = n >> n_i$ that is meaning the p concentration has no effect and ($\sigma_n >> \sigma_p$), so: $\sigma_{(n)} = N_D\,e\,\mu_n$ In the same manner if the semiconductor is doped by impurities have N_A acceptor atoms concentration ($N_A = p >> n_i$): $\sigma_{(p)} = N_A\,e\,\mu_p$

2.6 Diffusion and Drift currents density in semiconductor

There are two mechanisms by which holes and electrons move through a silicon crystal diffusion and drift.

Diffusion current density: As aforementioned the diffusion current density is given as:

$$J_{diff} = -e\,D\,\frac{dn}{dx} \tag{2.36}$$

Since the diffusion current density in semiconductor is due to electrons and holes motion, then:

$$J_{diff} = J_{diff(n)} + J_{diff(p)} = -e\left(D_n\,\frac{dn}{dx} + D_p\,\frac{dp}{dx}\right) \tag{2.37}$$

Drift current density: As aforementioned the drift current density is given as:

$$J_d = -e\,n\,\mu_n \tag{2.38}$$

The free electrons will drift in the direction opposite to that of E. **The total drift current density** is obtained by combining the two charge carriers:

$$J_{drift} = J_d = e\,E\,(n\,\mu_n + p\,\mu_p) \tag{2.39}$$

2.7 Photo-conductivity

When the semiconductor exposure to electromagnetic wave has energy (hf) then this energy will cause a generation of a new charge carriers contribute in electrical conduction process, this called the *Photo-conductivity*: If the energy of exposure photon is: $hf \geq E_g$. In other words the minimum wavelength of the absorbed electromagnetic radiation which can produce a new charge carriers will given as:

$$\lambda \leq \frac{1.24}{E_g}\ (\mu m) \tag{2.40}$$

The ability of the semiconductor to absorb photons depend on its nature and frequency. If the semiconductor surface exposure to the ray of the photons $\left(n_{ph(o)}\right)$, so the number of the photons will decrease with penetration depth (x) of the surface and the number of the photons which will arrive to depth (x) would be:

$$n_{ph(x)} = n_{ph(o)}.exp\,(-\alpha x) \tag{2.41}$$

α : is the absorption constant. α proportion to the absorption of solid ability to photons, so if α is large the solid has good ability to absorb.

Solved problems

1. Pure germanium has (4×10^{22}) $atom.\,cm^{-3}$ doped by indium atoms, the impurity is add to the extent of 1 part in (10^{8}) germanium atoms, if the intrinsic concentration of germanium 2.5×10^{13} cm^{-3}, note that $\mu_n = 3800\,cm^2(V\,s)^{-1}$ and $\mu_p = 1800\,cm^2(V\,s)^{-1}$.

   (a) Find the conductivity and the resistivity before the doping?

   (b) Find the conductivity and the resistivity after the doping?

   (c) What can you conclude from 1 and 2?

Solution

1. The conductivity of pure semiconductor (before doping) is given by:

$$\sigma = ne\mu_n + pe\mu_p$$

since the semiconductor is intrinsic then, $n = p = n_i$

$$\sigma = n_i e\left(\mu_n + \mu_p\right)$$

$$\sigma = 2.5 \times 10^{13} \times 1.6 \times 10^{-19} \times (3800 + 1800) = 0.0224\,S\,cm^{-1}$$

$$\rho = \frac{1}{\sigma}$$

$$\rho = \frac{1}{0.0224} = 44.64\,\Omega\,cm$$

2. Doping pure germanium with indium will produce increasing in hole density, so:

$$N_A = \frac{4 \times 10^{22}}{10^{8}} = 4 \times 10^{14} cm^{-3}$$

$$n \times p = n_i^2 \Rightarrow n \times N_A = n_i^2 \Rightarrow n = \frac{n_i^2}{N_A}$$

$$n = \frac{\left(2.5 \times 10^{13}\right)^2}{4 \times 10^{14}} = 1.56 \times 10^{12} cm^{-3}$$

$$\frac{(2.5 \times 10^3)^2}{10^{14}}$$

$$\sigma = ne\mu_n + pe\mu_p$$

$$\sigma = 1.6 \times 10^{-19} \times \left(1.56 \times 10^{12} \times 3800 + 4 \times 10^{14} \times 1800\right) = 0.116 Scm^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{0.116} = 8.62 \Omega cm$$

3. The ratio between the conductivity after and before doping is given as:

$$\frac{\sigma_{(after\, doping)}}{\sigma_{(before\, doping)}} = \frac{0.116}{0.0224} = 5.18$$

The conductivity of the germanium increased more than 5 time after doping with indium.

2. `Pure silicon doped by antimony has concentration equal to` $2 \times 10^{15}\, atom.\, cm^{-3}$ `, until` $N_D - N_A \gg 2n_i$`, note that they represent replacement of less than` $10^{-5}\%$ `of the atoms in the silicon. Find the conductivities` $\left(\sigma_{(n)},\, \sigma_{(p)}\; and\; \sigma\right)$ `and the resistivity` ρ `of the silicon? note that` $\mu_n = 1260\, cm^2\, (V\, s)^{-1}$ `and` $\mu_p = 460\, cm^2\, (V\, s)^{-1}$`.`

Solution

When the pure silicon doped with antimony atoms mean doping by donor atoms:

$$N_D = 2 \times 10^{15} atom\; cm^{-3}$$

$$\therefore 10^{-5} \times 2 \times 10^{15} - 0 \gg 2n_i$$

$$n_i = 10^{10} atom\; cm^{-3}$$

$$n.p = n_i^2$$

$$p\, N_D = n_i^2 \Longrightarrow p = (n_i^2)/N_D$$

$$p = (10^{10})^2/(2 \times 10^{15}) = 5 \times 10^4\; atoms\; cm^{-3}$$

$$\sigma_n = N_D\, e\, \mu_n = 2 \times 10^{15}\; \times 1.6 \times 10^{-19} \times 1260 = 0.403\; S\; cm^{-1}$$

$$\sigma_p = p\, e\, \mu_p = 5 \times 10^4\; \times 1.6 \times 10^{-19} \times 460 = 368 \times 10^{-14}\; S\; cm^{-1}$$

$$\sigma = \sigma_n + \sigma_p$$

$$\because \sigma_n \gg \sigma_p$$

$$\therefore \sigma = \sigma_n$$

$$\sigma = 0.403\, S\, cm^{-1}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{0.403} = 2.48\,\Omega\, cm$$

2.8 Hall Effect

Suppose that an electric current J_x is flow in a semiconductor in the x-direction, and a magnetic field B_z is applied normal to the s.c. in the z-direction. The current J_x will cause from the motion of holes (if the s.c. from p-type) with speed v_{Dx} under the influence the electric field E_x. These holes will effect to force F_L known as Lorentz Force and in $(-y)$ direction as shown in Figure below.

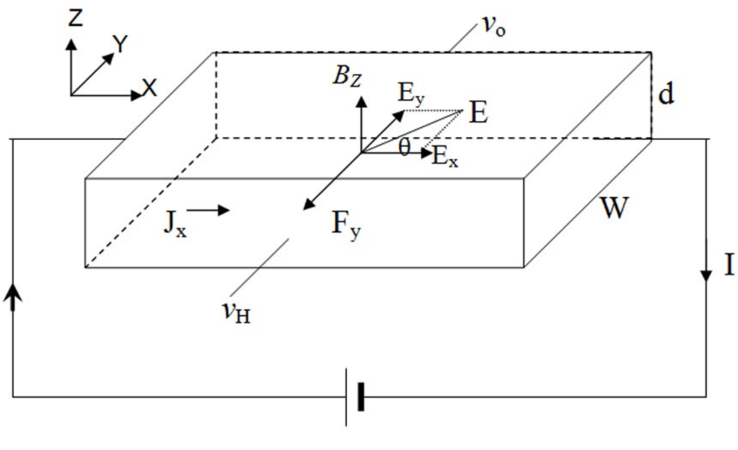

Figure 2.8: Hall effect setup

$$F_L = e \times v_{Dx} \times B_z \tag{2.42}$$

This force will push the holes in the front surface direction; this will cause a high density of holes on this surface while the back surface would be empty from the holes. Since the flow of hole current in y-direction (E_y). This field cause from the distribution of holes, and the field will cause a force on holes equal to Lorentz force, i.e.:

$$e\,E_y = e \times v_{Dx} \times B_z \tag{2.43}$$

but

$$J_x = e \times v_{Dx} \times p$$

$$E_y = \frac{J_x\,B_z}{p\,e} \tag{2.44}$$

or,

$$\frac{1}{p\,e} = \frac{E_y}{J_x\,B_z}$$

The quantity

$$\frac{1}{p\,e}$$

called Hall Coefficient (R_H), then;

$$R_H = \frac{1}{p\,e} = \frac{E_y}{J_x\,B_z} \tag{2.45}$$

The induced potential between front and back surfaces can be measured and then;

$$V_H = E \times W$$

If the thickness of the slide is d then,

$$I = J_x \times W \times d$$

Substituting the value of J_x in equation 2.45, then:

$$R_H = \frac{V_H\,d}{I\,B_z} = \frac{1}{p\,e} \tag{2.46}$$

Since V_H, $d\,I$, and B_Z are a measurable values, then it can be calculated the Hall coefficient experimentally and hence calculated the hole concentration from equation 2.46. Similar analysis can be done for a semiconductor of n-type and find:

$$R_H = \frac{1}{n\,e} \tag{2.47}$$

It is seen in last figure that there is a result of electrical field (E) in (θ) direction about the direction of (E_x), then,

$$tan\theta = \frac{E_y}{E_x} \tag{2.48}$$

Substituting the value of (E_y) from equation 2.46 in equation 2.48, then it can find,

$$tan\theta = \frac{J_x\, B_z}{p\, e} \times \frac{\sigma}{J_x} = \mu_p\, B_z \tag{2.49}$$

or,

$$\mu_p = R_H\, \sigma \tag{2.50}$$

According to this simple analysis it can measured the mobility of hole from Hall coefficient and conductivity.

Solved problems

1. A current of $(0.12\,A)$ pass through n-type semiconductor have a width of $(w = 2\,mm)$ and thickness of $(d = 1\,mm)$. If the voltage along the width of this sample of semiconductor is $(3.4\,mV)$ and a normal magnetic field of $(500\,Gauss)$ applied on this piece of semiconductor, find the Hall coefficient and electron density?

   Solution

$$R_H = \frac{V_H \times d}{I \times B_z}$$

$$R_H = \frac{3.4 \times 10^{-3} \times 1 \times 10^{-3}}{0.12 \times 500 \times 10^{-4}} = 5.6 \times 10^{-4}\, m^3\, C^{-1}$$

$$R_H = \frac{1}{n\, e}$$

$$5.6 \times 10^{-4} = \frac{1}{n \times 1.6 \times 10^{-19}} \Rightarrow n = 2 \times 10^{22}\, m^{-3}$$

2. Find the Hall coefficient, electron density and the angle between the field components for n-type semiconductor wire having thickness of ($d = 2\,mm$)? The normal applied magnetic field on this semiconductor is ($B = 0.1\,T$) and the current which passing through it is ($10\,mA$), and ($V_H = 1\,mV$), ($\mu_n = 0.36\,m^2\,s^{-1}$).

Solution

$$R_H = \frac{V_H \times d}{I \times B_z}$$

$$R_H = \frac{1 \times 10^{-3} \times 2 \times 10^{-3}}{10 \times 10^{-3} \times 0.1} = 2 \times 10^{-3}\,m^3\,C^{-1}$$

$$R_H = \frac{1}{n\,e}$$

$$2 \times 10^{-3} = \frac{1}{n \times 1.6 \times 10^{-19}} \Rightarrow n = 3.1 \times 10^{21}\,m^{-3}$$

$$tan\theta = \mu_p\,B_z = 0.36 \times 0.1 = 0.036 \Rightarrow \theta = tan^{-1}\,(0.036) = 2^o$$

Review Questions

1. Define mobility? Give its dimensions?
2. Indicate pictorially how a hole contributes to conduction?
3. (a) Define intrinsic concentration of holes. (b) What is the relationship between this density and the intrinsic concentration for electrons? (c) What do these equal at 0^oK?
4. Given an intrinsic semiconductor specimen, state two physical processes for increasing its conductivity? Explain briefly.
5. Explain physically the meaning of the following statement: An electron and a hole recombine and disappear?
6. Define (a) donor, (b) acceptor impurities?
7. What properties of a semiconductor are determined from a Hall effect experiment?
8. A pure silicon contains 5×10^{28} atom per cubic meter and the ratio of broken bonds are one bond per 10^{12} silicon atom at 38^oC. What is the ratio of broken of covalent bonds if the temperature raised to 75^oC, where $E_g = 1.1\ eV$?
9. A piece of pure semiconductor contains 5×10^{18} donor atoms at $27\ ^oC$. How far the Fermi level will move and in which direction if an additional donor atoms of concentration of 10^{22} will put in?
10. What is the wavelength of the electromagnetic waves which can release an electron from Germanium and Silicon? where $E_g(Ge) = 0.66\ eV$ and $E_g(Si) = 1.1\ eV$.
11. The charge carriers concentration in pure silicon is $4.5 \times 10^{16}\ m^{-3}$ at $300\,^oK$. Where the Fermi level would be if the silicon doped by 1×10^{21} donor atoms? Find the location of Fermi level at $200\,^oK$ and $900\,^oK$. Use $E_g(Si) = 1.1\ eV$.
12. If the conductivity of the pure Germanium change with temperature according the following relation, so what is the energy gap of the Germanium?

$$exp\left(\frac{-4350}{T}\right)$$

13. A resistor of pure Silicon with a resistance of 2500 Ω at 20 ^oC, if the resistance of this resistor increased by 1% of its initial vaule when the temperature increased to 100 ^oC, what is the energy gap of Silicon?

14. The resistivity of pure Silicon doped with impurities is $9.27 \times 10^{-7}\ \Omega\, m$ and the Hall coefficient is $3.84 \times 10^{-4}\ m^3\, C^{-1}$, what is the density and the mobility of impurities atoms?

Chapter 3

P-N Junction (Diode)

If one side of a piece of silicon dope with a trivalent impurity and the other side with a pentavalent impurity, a $(p-n)$ junction will formed between the resulting p-type and n-type portions and a basic diode will created. A diode is a device that conducts current in only one direction. In this chapter we demonstrate the characteristics of the $(p-n)$ junction region. The volt-ampere characteristics of the $(p-n)$ junction is studied. The capacitance across the junction is calculated.

3.1 The structure of p-n junction

If a donor impurities are doped into one side and acceptors into the other side of a single crystal of a pure semiconductor, a $(p-n)$ junction will create. This two-terminal device called a junction diode. 3.1 Fig.(1) shows the schematic symbol of the diode. The key feature of this device that conducts current in only one direction. When the ***n-type***

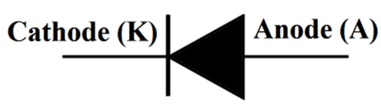

Figure 3.1: Diode schematic symbol.

semiconductor connect to ***p-type*** semiconductor the ***n*** region loses free electrons as they diffuse across the junction. This creates a layer of positive ions near the junction. The ***p*** region loses holes as electrons and holes combine. This creates a layer of negative ions near the junction. These two layers of positive and negative form the depletion region, as shown in figure3.2, with a built-in potential which is called the *contact or barrier potential (V). The depletion region is completely free from the charges.*

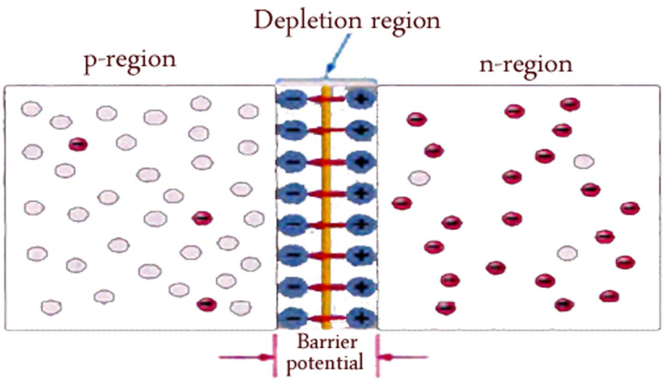

Figure 3.2: Formation of the depletion region.

From the basic conception of the semiconductor, it is easy to determine the value of the ***barrier*** or ***contact potential***. From the energy levels in figure 3.3 it can be observed that:

$$E_{cp} - E_{cn} = e\,V_o$$

In p-side, the electron concentration as a minor charge carrier is given as:

$$n_p = N_c \times exp\left(-\frac{E_{cp} - E_{fp}}{kT}\right)$$

$$E_{cp} = E_{fp} - kT \times ln\left(\frac{n_p}{N_c}\right)$$

whilst the electron concentration as a major charge carrier in n-side is given as:

$$n_n = N_c \times exp\left(-\frac{E_{cn} - E_{fn}}{kT}\right)$$

$$E_{cn} = E_{fn} - kT \times ln\left(\frac{n_n}{N_c}\right)$$

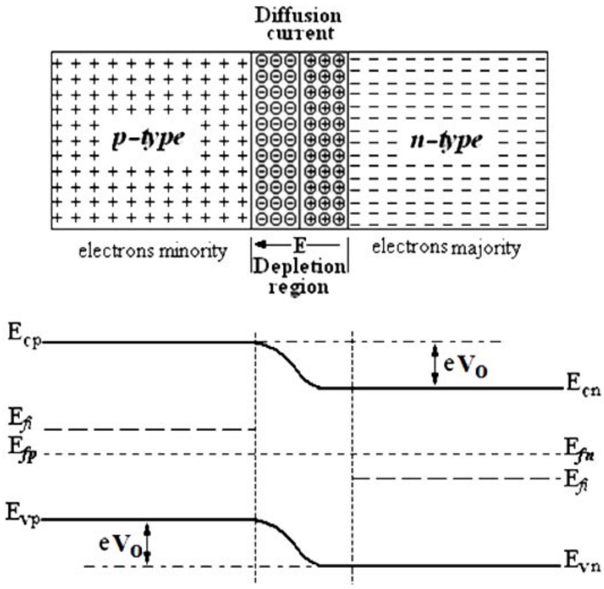

Figure 3.3: The P-N junction and its energy levels after contacted.

and

$$eV_o = E_{cp} - E_{cn}$$

$$\therefore\ eV_o = \left[E_{fp} - kT \times ln\left(\frac{n_p}{N_c}\right)\right] - \left[E_{fn} - kT \times ln\left(\frac{n_n}{N_c}\right)\right)]$$

Since the junction is in *equilibrium* status, then:$\Rightarrow$ $E_{fp} = E_{fn}$, then it can be write:

$$V_o = \frac{kT}{e} ln\left(\frac{n_n}{n_p}\right) \tag{3.1}$$

Equation 3.1 can also be written as;

$$n_n = n_p \times exp\left(\frac{eV_o}{kT}\right)$$

Since $n_n = N_D$ and $n_p = n_i^2/N_A$, then the above equation becomes:

$$V_o = \frac{kT}{e} ln\left(\frac{N_D \times N_A}{n_i^2}\right) \tag{3.2}$$

In same way, from n-side it could be found;

$$V_o = \frac{kT}{e} ln\left(\frac{p_p}{p_n}\right)$$

$$or\ \ p_p = p_n \times exp\left(\frac{eV_o}{kT}\right)$$

Solved problems

1. A pn junction was formed from two pieces of silicon contain $N_D = 10^{24}\,m^{-3}$ and $N_A = 10^{20}\,m^{-3}$ at $300\,^oK$. Find the barrier potential? The carrier concentration for pure silicon is: $n_i = 1.45 \times 10^{16}\,m^{-3}$.

Solution

$$V_o = \frac{kT}{e} \times ln\left(\frac{N_D \times N_A}{n_i^2}\right) = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times ln\left[\frac{10^{24} \times 10^{20}}{(1.45 \times 10^{16})^2}\right] = 0.7\,volt$$

2. A pure silicon has $n_i = 1.45{\times}10^{16}\,m^{-3}$, doped in its two ends by phosphor (P) and Boron (B) with carrier densities $N_D = 10^{22}\,m^{-3}$ and $N_A = 10^{20}\,m^{-3}$, respectively to form a p-n junction at $300\,^oK$. Find:

   (a) The location of Fermi-level in n and p parts?

   (b) Determine the barrier potential?

Solution

(a) For n-side

$$N_D = n_i \times exp\left(\frac{E_{fn} - E_{fi}}{kT}\right)$$

$$\therefore\ E_{fn} - E_{fi} = kT \times ln\frac{N_D}{n_i} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times ln\left[\frac{10^{22}}{(1.45 \times 10^{16})}\right]$$

$$= 0.35\,eV\ above\ Fermi\ level$$

(b) For p-side

$$N_A = n_i \times exp\left(\frac{E_{fi} - E_{fp}}{kT}\right)$$

$$\therefore\ E_{fi} - E_{fp} = kT \times ln\frac{N_A}{n_i} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times ln\left[\frac{10^{20}}{(1.45 \times 10^{16})}\right]$$

$$= 0.27\,eV\ below\ Fermi\ level$$

$$V_o = \frac{kT}{e} \times ln\left(\frac{N_D \times N_A}{n_i^2}\right) = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times ln\left[\frac{10^{22} \times 10^{20}}{(1.45 \times 10^{16})^2}\right] = 0.62\,volt$$

3.2 PN junction characteristics

If the external potential of V volt is applied across the P-N junction this will bias the diode. There are two type of diode bias:

3.2.1 Forward bias

Connecting an external voltage across the junction will cause a disturbance in the junction equilibrium status and a net current will flow through the P-N junction. Connecting the positive terminal of the external voltage source to the p-side and the negative terminal to the n-type will cause a forward bias for the junction as shown in figure 3.4.

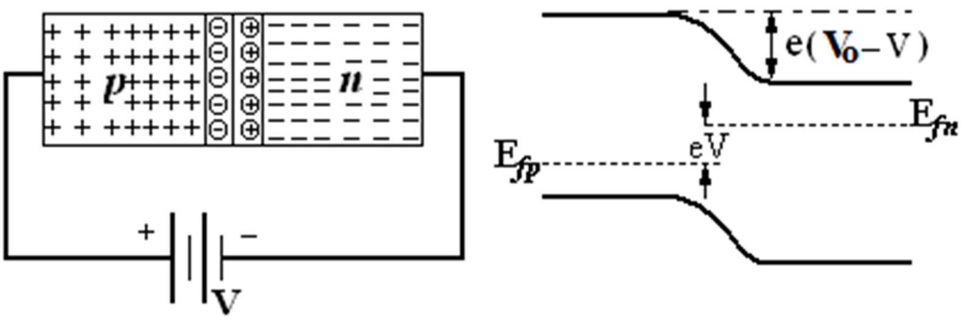

Figure 3.4: Forward bias voltage (V) applied to p-n junction and the result energy band structure.

The application of forward bias potential V will cause an injection of electrons from n-side and hole from p-side in opposite direction across the junction region and some of these carriers will recombine with the ions near the boundary region and reduce the width of depletion region. On being injected across the junction, these carriers immediately become minority carriers and the density of the minority carriers near the junction rise to new values n_{po} and p_{no}. as shown in figure 3.4.

$$n_{po} = n_n \times exp\left[-\frac{e\,(V_o - V)}{kT}\right] \tag{3.3}$$

$$n_{po} = n_n \times exp\left[-\frac{e\,(V_o)}{kT}\right] \times exp\left[\frac{eV}{kT}\right]$$

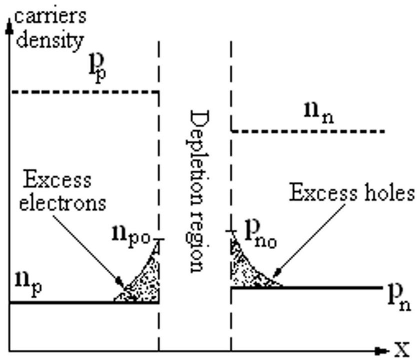

Figure 3.5: The excess carriers in the terminals of junction.

$$\because\ n_n = n_p \times exp\left[\frac{e\,V_o}{kT}\right]$$

$$So\ n_p = n_n \times exp\left[\frac{-e\,V_o}{kT}\right]$$

$$\therefore\ n_{po} = n_p \times exp\left[\frac{e\,V}{kT}\right] \tag{3.4}$$

where n_{po} is the electrons excess at the edge of the depletion region in p side.

In same manner it can be prove that:

$$\therefore\ p_{no} = p_n \times exp\left[\frac{e\,V}{kT}\right] \tag{3.5}$$

where p_{no} is the electrons excess at the edge of the depletion region in n side.

As shown in figure 3.5, the density of carrier decreases far from the depletion region to exist the forward bias.

$$n(x) = n_p - (n_p - n_{po})\,exp\left(-\frac{x}{L_n}\right) \tag{3.6}$$

L_n :the electron diffusion length in p-side.

$$p(x) = p_n - (p_n - p_{no})\,exp\left(-\frac{x}{L_p}\right) \tag{3.7}$$

L_p :the electron diffusion length in n-side.

The total current densities are due to holes and electrons diffusion motion is defined as:

$$J_D = J_n + J_p = e\,D_n\,\frac{dn}{dx} + e\,D_p\,\frac{dp}{dx}$$

with caring to the direction of (x), then the above equation can be written as:

$$J_D = \left[\frac{e\,D_n\,n_p}{L_n} + \frac{e\,D_p\,p_n}{L_p} \right] \times \left[exp\left(\frac{eV}{kT}\right) - 1 \right] \tag{3.8}$$

Equation 3.8 can be written as;

$$J_F = J_s \times \left[exp\left(\frac{eV}{kT}\right) - 1 \right] \tag{3.9}$$

where;

$$J_s = \left[\frac{e\,D_n\,n_p}{L_n} + \frac{e\,D_p\,p_n}{L_p} \right]$$

The value J_s represents the saturation current density, whilst J_F the forward current density. Saturation current flows through the junction in equilibrium case between the diffusion currents under the effect of (V_o).

3.2.2 Reverse bias

If the positive terminal of the applied voltage connect to the $n-type$ and the negative terminal to the $p-type$, as illustrated in figure 3.6, the junction will bias in reverse direction. Because unlike charges attract, the positive side of the bias-voltage source "*pulls*" the free electrons, which are the majority carriers in the n region, away from the pn junction. As the electrons flow toward the positive side of the voltage source, additional positive ions are created. This results in a widening of the depletion region and a depletion of majority carriers. In the p region, electrons from the negative side

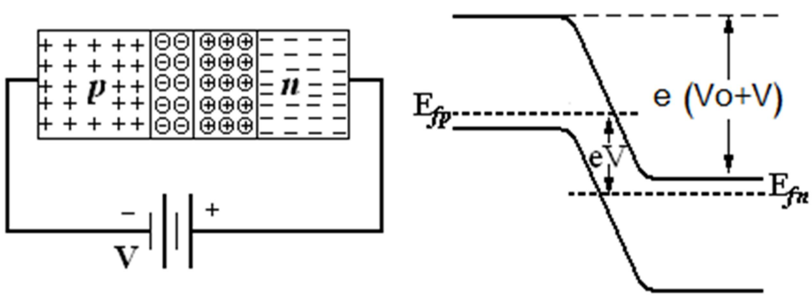

Figure 3.6: A (p-n) junction under the reverse bias and the equivalent energy band structure.

of the voltage source enter as valence electrons and move from hole to hole toward the depletion region where they create additional negative ions. This results in a widening of the depletion region and a depletion of majority carriers. The flow of valence electrons can be viewed as holes being *"pulled"* toward the positive side. The initial flow of charge carriers is transitional and lasts for only a very short time after the reverse-bias voltage is applied. As the depletion region widens, the availability of majority carriers decreases as shown in figure 3.7. As more of the n and p regions become depleted of majority carriers, the electric field between the positive and negative ions increases in strength until the potential across the depletion region equals the bias voltage, V_{bias}. At this point. The transition current essentially ceases (dies) except for a very small reverse current that can usually be neglected. Reverse Current: The extremely small current

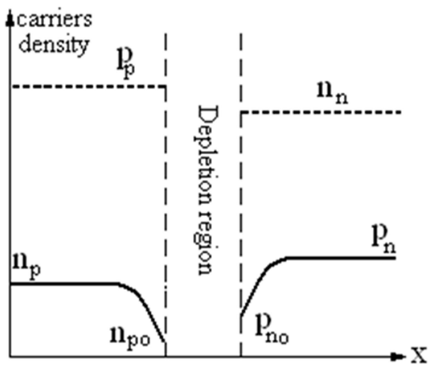

Figure 3.7: The carrier densities in bulk of reverse bias p-n junction.

that exists in reverse bias after the transition current dies out is caused by the minority carriers in the n and p regions that are produced by thermally generated electron-hole pairs. The small number of free minority electrons in the p region are *"pushed"* toward the *pn* junction by the negative bias voltage. When these electrons reach the wide depletion region, they *"fall down the energy hill"* and combine with the minority holes in the n region as valence electrons and flow toward the positive bias voltage, creating a small hole current. The conduction band in the p region is at a higher energy level than the conduction band in the n region. Therefore, the minority electrons easily pass through the depletion region because they require no additional energy. The current in reverse-bias condition called **Reverse Saturation Current (I_s)**.

The applied voltage lead to increase the height of barrier potential to a new value equal to $(V_o + V)$, thereby reducing the diffusion current through the junction. The current density at reverse bias is given as:

$$J_R = J_s \times \left[1 - exp\left(\frac{-eV}{kT}\right)\right] \tag{3.10}$$

where J_R is the reverse current density.

The general formula of the current density in the p-n junction can be form as:

$$J = \pm\, J_s \times \left[exp\left(\frac{\pm eV}{kT}\right) - 1\right] \tag{3.11}$$

where (+):forward and (−):reverse. If the junction cross-section area is equal to (A) then the current is:

$$I = A.J \; and \; I_s = A.J_s$$

$$\therefore I = \pm\, I_s \times \left[exp\left(\frac{\pm eV}{kT}\right) - 1\right] \tag{3.12}$$

Figure 3.8 shows the relation between the current and voltage in both forward and

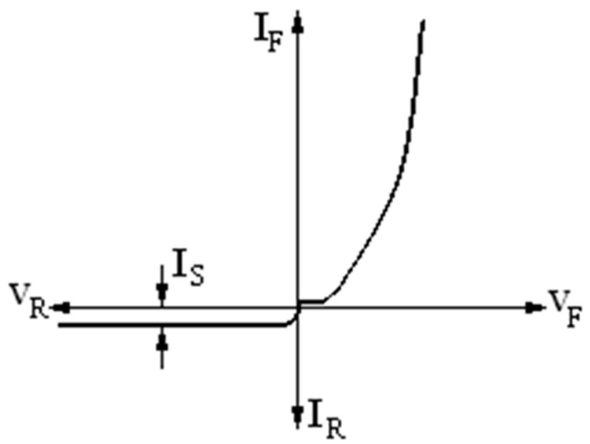

Figure 3.8: $I - V$ characteristics of the $p - n$ junction.

reverse biases. For the large values of V_F, where $(V_F > 4kT)$ the forward current is:

$$I_F = I_s\, exp\left(\frac{eV_F}{kT}\right)$$

and for long time the reverse bias $(I_R = I_S)$.

3.3 Depletion region and depletion capacitances

The two space charge density layers at the junction vary in width with the applied voltage, and therefore in the amount of charges they contain as the bias voltage changes. If (w) is the width of the depletion region then:

$$w = d_n + d_p \tag{3.13}$$

where (d_n) with in the n-side and (d_p) with in the p-side.

It can be related between (d_n and d_p) from the charge density in these regions:

$$N_A^- \, A \, d_p = N_D^+ \, A \, d_n \quad or \quad \frac{d_p}{d_n} = \frac{N_D^+}{N_A^-}$$

Since the width of the depletion region effect by biasing voltage then:

$$w = \left[\frac{2\epsilon}{e} \left(\frac{1}{N_A^-} + \frac{1}{N_D^+} \right) (V_o \pm V) \right]^{\frac{1}{2}} \tag{3.14}$$

The $-ve$ sign for the forward bias and $+ve$ sign for the reverse bias. Where $\epsilon = \epsilon_o \, \epsilon_r$.

If the reverse bias at $|V_R| > V_o$ and $N_A \gg N_D$ or $N_D \gg N_A$;

$$w = \left[\frac{2\epsilon \, V_R}{e \, N} \right]^{\frac{1}{2}} \tag{3.15}$$

N: the impurities concentration for **much less** type. The depletion region is a free of charges (carriers), therefore it can be considered as insulator with capacitance.

$$C_j = \frac{\epsilon}{w} = \sqrt{\frac{e \, \epsilon \, N}{2 \, V_R}} \; in \; unit \; \left(\frac{F}{m^2} \right) \tag{3.16}$$

It can see that the capacitance of depletion region dependent on applied reverse voltage.

$$C_j \propto V_R^{-1/2} \; (variable \, capacitor \, for \, abrupt \, junction)$$

$$C_j \propto V_R^{-1/2} \; (variable \, capacitor \, for \, graded \, junction)$$

The capacitance property for a pn junction in reverse bias is very useful for IC but its value variable depend on applied voltage. **variable capacitor→Tuning ≡ called (varactor)**

3.4 Diffusion capacitors

It is very important than the junction capacitor, where it is depend on the density of minority carriers on the edge of the depletion region. These carriers effect with external applied voltage (V_F), then the diffusion capacity is given by:

$$C_D = \frac{e^2}{2\,k\,T}\,[p_n\,L_p\,+ n_p\,L_n]\,exp\left(\frac{e\,V_F}{k\,T}\right) \tag{3.17}$$

Note that;

$$C_D\,\propto\,exp\left(\frac{e\,V_F}{k\,T}\right)$$

$$I_F = I_S\,exp\left(\frac{e\,V_F}{k\,T}\right)$$

$$\frac{dI_F}{dV_F} = \frac{e}{k\,T}\times I_S\,exp\left(\frac{e\,V_F}{k\,T}\right)$$

$$\therefore\,\frac{dI_F}{dV_F} = \frac{e\,I_F}{k\,T} = \frac{1}{r_d}$$

Multiply equation 3.17 by $\frac{I_F}{I_F}$

$$C_D = \frac{e\,I_F}{k\,T}\times\frac{e}{2\,I_F}\,[p_n\,L_p\,+ n_p\,L_n]\,exp\left(\frac{e\,V_F}{k\,T}\right)$$

$$C_D = \frac{1}{r_d}\times\frac{e}{2\,I_F}\,[p_n\,L_p\,+\,n_p\,L_n]\,exp\left(\frac{e\,V_F}{k\,T}\right) \tag{3.18}$$

$$I_S = e\left[\frac{D_n\,n_p}{L_n}+\frac{D_p\,p_n}{L_p}\right] = saturation\ current$$

$$L = \sqrt{D\,t}\,\Rightarrow\,D_n = \frac{L_n^2}{t_n}\ and\ D_p = \frac{L_p^2}{t_p}$$

t_n and t_p are the life times for the minority carriers which effect in I_s.

$$I_S = e\left[\frac{L_n^2\,n_p}{t_n\,L_n}+\frac{L_p^2\,p_n}{t_p\,L_p}\right] = e\left[\frac{L_n\,n_p}{t_n}+\frac{L_p\,p_n}{t_p}\right]$$

$$I_F = e\left[\frac{L_n\,n_p}{t_n}+\frac{L_p\,p_n}{t_p}\right]\,exp\left(\frac{e\,V_F}{k\,T}\right) \tag{3.19}$$

Substituting equation 3.19 in equation 3.18

$$C_D = \frac{1}{r_d} \times \frac{e}{2e\left[\frac{L_n\, n_p}{t_n} + \frac{L_p\, p_n}{t_p}\right] exp\left(\frac{e\,V_F}{k\,T}\right)} \times [p_n\, L_p\, +\, n_p\, L_n]\, exp\left(\frac{e\,V_F}{k\,T}\right)$$

$$C_D = \frac{1}{2r_d} \times \left[\frac{p_n\, L_p\, +\, n_p\, L_n}{\frac{L_n\, n_p}{t_n} + \frac{L_p\, p_n}{t_p}}\right] \tag{3.20}$$

Since the $p-n$ junction current would be form from either(n_p or p_n) then the equation 3.20 becomes:

$$C_D\, r_d = \frac{1}{2} t \tag{3.21}$$

t:the life times of the minority carriers that have large effect in I_s.
r_d:the dynamic resistance for the $p-n$ junction.

C_D is very important in digital circuit because it is limited the speed of the switching **(on/off)** which is given by:

$$t_{off} = C_D \frac{k\,T}{e\,I_R}\ or\ t_{off} = \frac{I_F}{I_R} \times \frac{L_p^2}{2\,D_n} \tag{3.22}$$

Solved problems

1. A pn junction has a hole density in p-side $10^{24}\, m^{-3}$ and electron density in n-side $10^{22}\, m^{-3}$, the cross-section area for the pn junction is $10^{-6}\, m^2$, the mobility of the holes is $0.2\, m^2\, (V\, s)^{-1}$ and the mobility of the electrons is $0.4\, m^2\, (V\, s)^{-1}$. The diffusion length of the minorities are ($L_n = 300\, \mu m$ and $L_p = 200\, \mu m$). If $\epsilon_r = 16$ and $n_i = 10^{19} m^{-3}$ at room temperature. Determine;

   (a) The density of majority and minority carriers and the conductivity?

   (b) The barrier potential?

   (c) The diffusion constant for the both types of the carriers?

   (d) Saturation current?

   (e) The junction current when $V_F = 0.25V$?

   (f) The junction current for the reverse bias, at high reverse voltage?

(g) Width of the depletion region at $V_R = 10V$?

(h) Depletion capacity at $V_R = 10V$?

(i) Ratio of holes current to electrons current across the junction?

Solution:

(a) At p-side:

$$n_p = \frac{n_i^2}{p_p} = \frac{(10^{19})^2}{10_{24}} = 10^{14}\, m{-}3\, electrons\, minority$$

$$N_A = 10^{24}\, m^{-3}\, holes\, majority$$

$$\sigma_p = e\, p_p\, \mu_p = 1.6 \times 10^{-19} \times 10^{24} \times 0.2 = 3.2 \times 10^4\, S\, m^{-1} \quad (p - side)$$

At n-side:

$$p_n = \frac{n_i^2}{n_n} = \frac{(10^{19})^2}{10_{22}} = 10^{16}\, m{-}3\, holes\, minority$$

$$N_D = 10^{22}\, m^{-3}\, electrons\, majority$$

$$\sigma_n = e\, n_n\, \mu_n = 1.6 \times 10^{-19} \times 10^{22} \times 0.4 = 0.64 \times 10^4\, S\, m^{-1} \quad (n - side)$$

(b) The barrier potential:

$$V_o = \frac{kT}{e} \times ln\frac{N_D \times N_A}{(n_i)^2} = \frac{1}{40} \ln\left(\frac{10^{22} \times 10^{24}}{(10^{19})^2}\right) = 0.46\, Volt$$

(c) The diffusion constant given as:

$$D_n = \frac{kT}{e}\, \mu_n = \frac{1}{40} \times 0.4 = 0.01\, m^2\, s^{-1}$$

$$D_p = \frac{kT}{e}\, \mu_p = \frac{1}{40} \times 0.2 = 0.005\, m^2\, s^{-1}$$

(d) The saturation current given as:

$$I_s = J_s \times A = A \times e\left[\frac{D_n\, n_p}{L_n} + \frac{D_p\, p_n}{L_p}\right]$$

$$I_s = 10^{-6} \times 1.6 \times 10^{-19} \times \left[\frac{0.01 \times 10^{14}}{300 \times 10^{-6}} + \frac{0.005 \times 10^{16}}{200 \times 10^{-6}}\right] = 0.04\, \mu A$$

(e) The forward current:

$$I_F = I_S \left[exp\left(\frac{e\,V_F}{k\,T}\right) - 1\right] = 0.04{\times}10^{-6}\left[exp\left(\frac{1.6\times 10^{-19}\times 0.25}{8.614\times 10^{-5}\times 290}\right) - 1\right] = 0.88\,mA$$

(f) At high reverse voltage $I_R = I_s = 0.04\,\mu A$

(g) The depletion region width is:

$$w = \left[\frac{2\epsilon}{e}\left(\frac{1}{N_A^-} + \frac{1}{N_D^+}\right)(V_o + V_R)\right]^{\frac{1}{2}}$$

since $\epsilon = \epsilon_o \epsilon_r$

$$w = \left[\frac{2\times 16\times 8.85\times 10^{-12}}{1.6\times 10^{-19}}\left(\frac{1}{10^{24}} + \frac{1}{10^{22}}\right)(0.46 + 10)\right]^{\frac{1}{2}} = 1.34\,\mu m$$

(h) The junction capacitor:

$$C_j = \frac{\epsilon}{w}\times A = \frac{\epsilon_o\,\epsilon_r}{w}\times A = \frac{16\times 8.85\times 10^{-12}}{1.34\times 10{-}6}\times 10^{-6} = 100\,pF$$

(i) The ratio of I_p to I_n is:

$$\frac{I_p}{I_n} = \frac{\frac{e\,D_p\,p_n}{L_p}}{\frac{e\,D_n\,n_p}{L_n}} = \frac{\frac{0.005\times 10^{16}}{200\times 10^{-6}}}{\frac{0.01\times 10^{14}}{300\times 10^{-6}}} = 75$$

2. The conductivity of n-side in the Ge $p - n$ junction is $10^4\,S\,m^{-1}$ and for the p-side is $10^2\,S\,m^{-1}$. Find the barrier potential for the junction at $300\,^oK$? where $n_i = 2.5\times 10^{19} m^{-3}$, $\mu_n = 0.36\,m^2\,V\,s$ and $\mu_p = 0.16\,m^2\,V\,s$.

Solution:

At n-side:

$$\sigma_{(n)} = n_n\,e\,\mu_n + p_n\,e\,\mu_p = N_D\,e\,\mu_n + \frac{n_i^2}{N_D}\,e\,\mu_p$$

$$10^4 = 1.6\times 10^{-19}\times\left(0.36\,N_D + \frac{(2.5\times 10^{19})^2}{N_D}\times 0.16\right) \Rightarrow N_D = 1.7\times 10^{23}\,m^{-3}$$

At p-side:

$$\sigma_{(p)} = p_p\,e\,\mu_p + n_p\,e\,\mu_n = N_A\,e\,\mu_p + \frac{n_i^2}{N_A}\,e\,\mu_n$$

$$10^2 = 1.6 \times 10^{-19} \times \left(0.16\, N_A + \frac{(2.5 \times 10^{19})^2}{N_A} \times 0.36\right) \Rightarrow N_A = 3.9 \times 10^{21}\, m^{-3}$$

Then the barrier potential is:

$$V_o = \frac{kT}{e} \times ln \frac{N_D \times N_A}{(n_i)^2} = \frac{8.614 \times 10^{-5} \times 300}{1.6 \times 10^{-19}} \times ln \frac{3.9 \times 10^{21} \times 1.7 \times 10^{23}}{(2.5 \times 10^{19})^2} = 0.36\, Volt$$

3. A forward $p - n$ junction is connected to 100Ω resistance and to power supply of $10\, volt$. If the applied voltage was reversed and $t_{off} = 0.1\, \mu s$. Find?

   (a) The average reverse bias current during the period of inverting if $D_n = 0.0031\, m^2 s$ and $L_p = 0.5\, \mu m$

   (b) The diffusion capacitor C_D.

Solution:

(a) The reverse current:

$$I_F = \frac{V}{R} = \frac{10}{100} = 0.1\, A$$

$$t_{off} = \frac{I_F}{I_R} \times \frac{L_p^2}{2\, D_n} \Rightarrow I_R = \frac{I_F}{t_{off}} \times \frac{L_p^2}{2\, D_n}$$

$$I_R = \frac{0.1}{0.1 \times 10^{-6}} \times \frac{(0.5 \times 10^{-6})^2}{2 \times 0.0031} = 40\, \mu A$$

(b) The diffusion capacitor C_D:

$$C_D = t_{off}\, I_R\, \frac{e}{kT} = 0.1 \times 10^{-6} \times 40 \times 10^{-6} \times 40 = 160\, pF$$

Review Questions

1. What is a ***pn*** junction?
2. Describe the depletion region?
3. What is the effect of forward bias on the depletion region?
4. Which bias condition produces majority carrier current and how?
5. A cylindrical ***pn*** junction with 200μ m diameter and 10μ m length for each side. In n-side $N_D = 10^{22}\,m^{-3}$, $\mu_n = 0.13\,m^2\,(V.s)^{-1}$ and in p-side $N_A = 10^{22}m^{-3}$, $\mu_p = 0.05\,m^2\,(V.s)^{-1}$ at 20 ^oC, if $n_i = 1.4 \times 10^{16}\,m^{-3}$. Find:
   (a) The barrier potential?
   (b) The bulk resistor?
   (c) The voltage which required to allow a current of 100mA passing through the junction, if the saturation current is 1nA?
6. An abrupt Si ***pn*** junction ($A = 10^{-4}\,cm^2$) has the following properties at 300 ^oK:
   p-side $N_A = 10^{17}\,cm^{-3}$, $t_n = 0.1\,\mu s$, $\mu_p = 200\,cm^2\ (V.s)^{-1}$ and $\mu_n = 700\,cm^2\ (V.s)^{-1}$
   n-side $N_D = 5\times10^{22}\,cm^{-3}$, $t_p = 10\,\mu s$, $\mu_p = 450\,cm^2\ (V.s)^{-1}$ and $\mu_n = 1300\,cm^2\ (V.s)^{-1}$
   Find:
   (a) The depletion capacitance for V_R=100V?
   (b) The total excess stored electric charge and the electric field far from the depletion region on the p-side when the current =20mA?

Chapter 4

Diodes and their applications

If one side of a piece of silicon dope with a trivalent impurity and the other side with a pentavalent impurity, a $(p-n)$ junction will formed between the resulting p-type and n-type portions and a basic diode will created. A diode is a device that conducts current in only one direction. In this chapter we demonstrate the characteristics of the $(p-n)$ junction region. The volt-ampere characteristics of the $(p-n)$ junction is studied. The capacitance across the junction is calculated.

4.1 Introduction

Several common physical configurations of diodes are illustrated in Figure 4.1. The anode and cathode are indicated on a diode in several ways, depending on the type of package. The cathode is usually marked by a band, a tab, or some other feature. On those packages where one lead is connected to the case, the case is the cathode.

Summary of diode biasing:

Forward bias:

- Bias voltage connections: positive to (p) region: negative to (n) region.
- The bias voltage must be greater than the barrier potential.

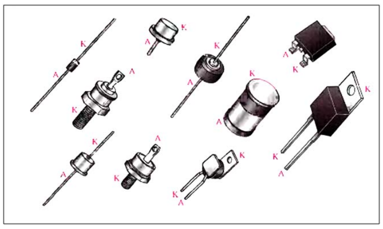

Figure 4.1: Typical diode packages with terminal identification.

- Majority carriers flow toward the (pn) junction.
- Majority carriers provide the forward Current.
- The depletion region narrows.

Reverse bias:

- Bias voltage connections: positive to (n) region; negative to (p) region.
- The bias voltage must be less than the breakdown voltage.
- Majority carriers flow away from the (pn) junction during short transition time.
- Minority carriers provide the extremely small reverse current.
- There is no majority carrier current after transition time.
- The depletion region widens.

4.2 The diode model

There are three models of the diode:

4.2.1 The ideal model

The ideal model of a diode is a simple switch. When the diode is forward biased, it acts like closed (on) switch, as shown in figure 4.2a. When the diode is reversed biased. It acts like an open (off) switch, as shown figure 4.2b. The barrier potential, the forward dynamic resistance, and the reverse current are all neglected.

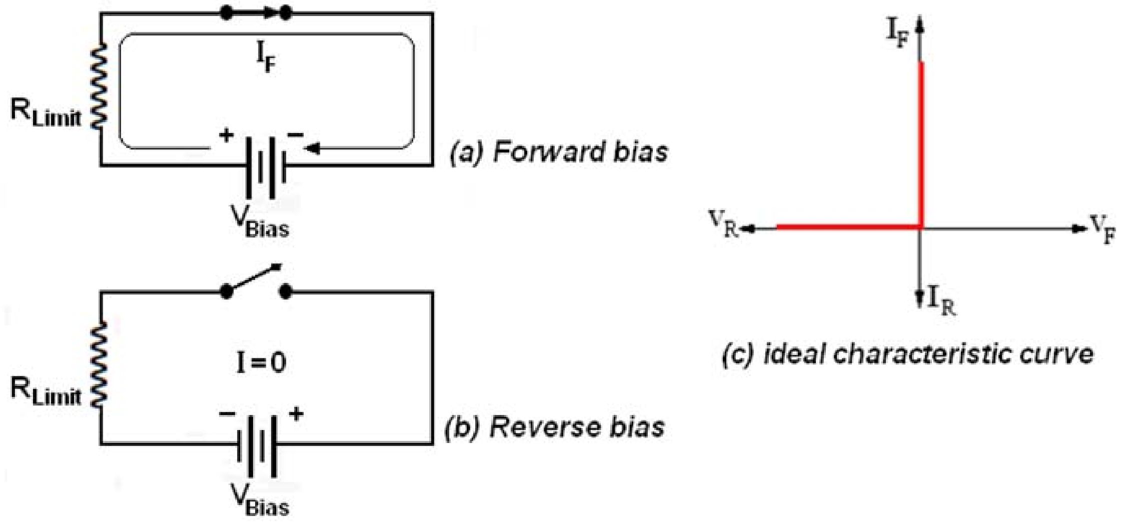

Figure 4.2: The ideal model of the diode (a) forward bias, (b) reverse bias and (c) ideal characteristic curve.

In figure 4.2c, the ideal $V-I$ characteristic curve graphically depicts the ideal diode operation.

In the ideal diode model: $V_F = 0$, $I_R = 0$ and $V_R = V_{bias}$.

4.2.2 The practical model

The practical model adds the barrier potential to the ideal switch model. When the diode is forward biased, it is equivalent to a closed switch in series with a small equivalent voltage source equal to the barrier potential with the positive side toward the anode, as indicated in figure3a. This equivalent voltage source represents the fixed voltage drop (V_F) produced across the forward biased ($p-n$) junction of the diode and is not an active source of voltage. This voltage (V_F) consists of the barrier potential voltage (V_o) plus the small voltage drop across dynamic resistance of the diode (r_d), as indicated by the portion of the curve to the right of the origin. The curve slopes because the voltage drops due to dynamic (r_d) as the current increases.

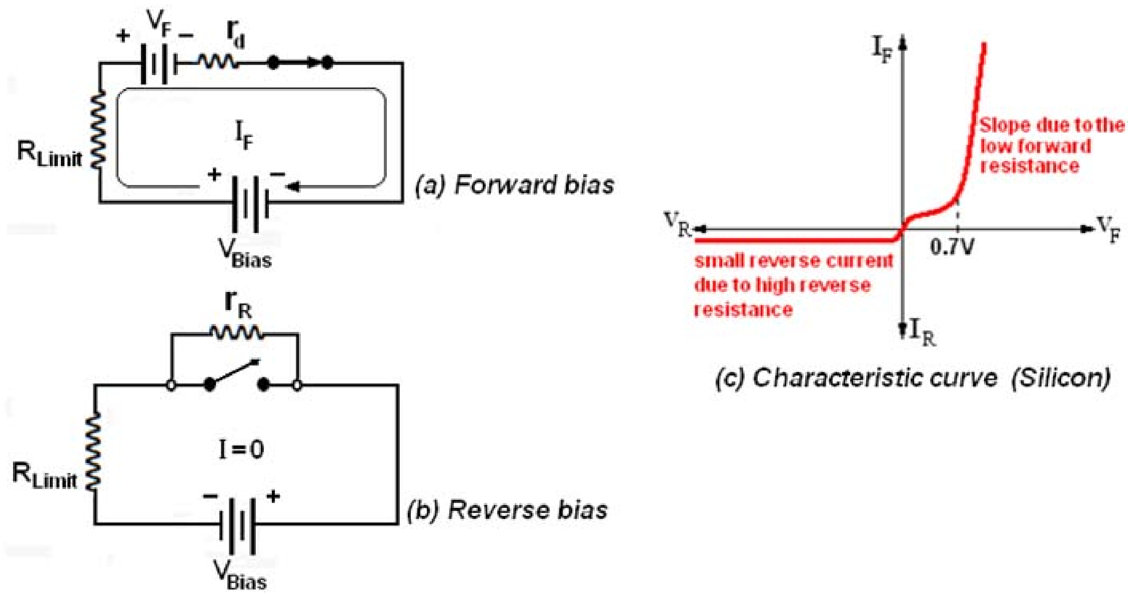

Figure 4.3: The complete model of the diode (a) forward bias, (b) reverse bias and (c) ideal characteristic curve (silicon).

4.2.3 The complete model

For the complete model of a silicon diode, the following formulas apply:

$$V_F = V_o + I_F \times r_d$$

$$I_F = \frac{(V_{Bias} - V_o)}{(R_{Limit} + r_d)}$$

The reverse current is taken into account with the parallel resistance and is indicated by the portion of the curve to the left of the origin.

Solved problem

a) Determine the forward voltage and forward current for the diode in figure (a) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume $r_d = 10\Omega$ at the determined value of forward current.

b) Determine the reverse voltage and reverse current for the diode in figure (b) for each of the diode models. Also find the voltage across the limiting resistor in each case. Assume I_R=1μA.

Solution:

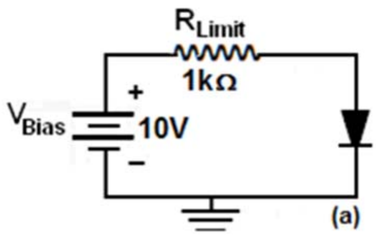

Figure 4.4:

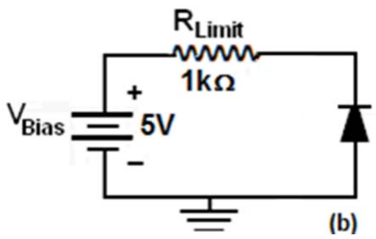

Figure 4.5:

$a)$ Ideal model:

$$V_F = 0\,volt$$
$$I_F = \frac{V_{bias}}{R_{Limit}} = \frac{10}{1 \times 10^3} = 10\,mA$$
$$V_{R_{limit}} = I_F \times R_{Limit} = 10\,mA \times 1k\Omega = 10\,volt$$

Practical model:

$$V_F = 0.7\,volt$$
$$I_F = \frac{V_{bias} - V_F}{R_{Limit} + r_d} = \frac{10 - 0.7}{1 \times 10^3 + 10} = 9.21\,mA$$
$$V_d = 0.7 + I_F \times r_d = 0.7 + 9.21mA \times 10 = 792mV$$
$$V_{R_{limit}} = I_F \times R_{Limit} = 9.21\,mA \times 1k\Omega = 9.21\,volt$$

If we neglected r_d then;

$$I_F = \frac{V_{bias} - V_F}{R_{Limit}} = \frac{10 - 0.7}{1 \times 10^3} = 9.3\,mA$$
$$V_{R_{limit}} = I_F \times R_{Limit} = 9.3\,mA \times 1k\Omega = 9.3\,volt$$

$b)$ Ideal model:

$$I_R = 0\,A$$

$$V_R = VBias = 5\,V$$

$$V_{R_{limit}} = 0\,volt$$

Practical model

$$I_R = 1\,\mu A$$

$$V_{R(limit)} = I_R \times R_(Limit) = 1\mu A \times 1k\Omega = 1mV$$

$$V_R = V_(Bias) - V_{R_(limit)=5V-1mV=4.999\,volt}$$

4.3 Diode Applications

Because of their ability to conduct current in one direction and block current in the other direction, diodes are used in circuits called rectifiers that convert *ac* voltage into *dc* voltage. Rectifiers are found in all *dc* power supplies that operate from an *ac* voltage source. A power supply is an essential part of each electronic system from the simplest to the most complex. In this section, you will study the most basic type of rectifiers, the half-wave rectifier, full-wave rectifiers and power supply filters and regulators, and the diode limiting and clamping circuits, and voltage multipliers.

4.3.1 Half-wave rectifier

Figure 4.6, illustrates the process called half wave rectification. The diode connection to an ac source and to a load resistor R_L will form a half-wave rectifier. When the input

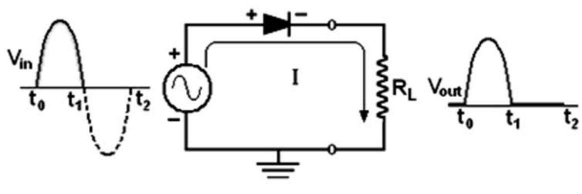

Figure 4.6: Half-wave rectifier circuit with the input and output voltage waveform.

voltage (V_in) goes positive during the time duration between t_0 to t_1, as shown in Figure 4.6, the diode will biased forward and conducts current through the load resistor. The current produces an output voltage across the load R_L which has the same shape as the positive half cycle of the input voltage. As the input voltage goes negative, during the second half of the input voltage cycle (t_1 to t_2), the diode will biased reverse. As a result there is no current will pass through the load R_L and the voltage across the load resistor is $0\,V$. The net result is that only the positive half cycles of the ac input voltage appear across the load as shown in Figure 5. Since the output does not change polarity, it is a **pulse dc voltage**. When the practical diode model is used with the

Figure 4.7: Half-wave output voltage for three input cycles.

barrier potential of (V_o) taken into account. During the positive half cycle, the input voltage must overcome the barrier potential before the diode becomes forward biased. This result in a half wave output with a peak value that is less than the peak value of the input by (V_o), as shown in Figure 4.8.

$$V_{p(out)} = V_{p(in)} - V_o$$

The mean value of the output voltage (V_{avg} or V_{dc}) can be calculated mathematically

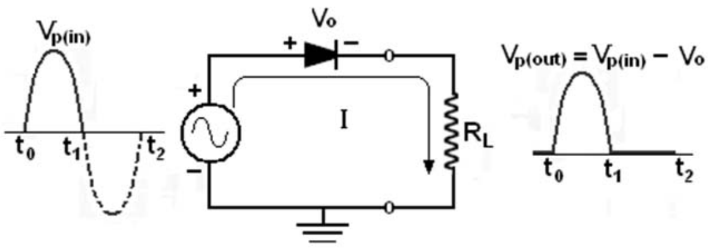

Figure 4.8: The effect of the barrier potential on the half wave rectified output voltage is to reduce the peak value of the input by V_o.

by the area under the curve over a full cycle, as shown in Figure 4.9 dividing by 2π, the number of radians in a full cycle, where V_p is the peak value of the voltage.

$$V_{avg} = \frac{V_p}{\pi}$$

The root mean square value of the output voltage (V_{rms}) is given as:

$$V_{rms} = \frac{V_p}{\sqrt{2}}$$

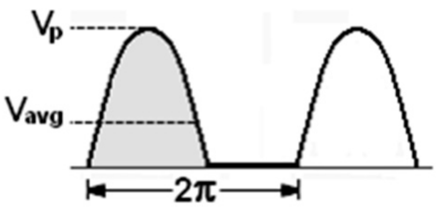

Figure 4.9: Average value of the half wave rectified signal.

4.3.5 Half wave rectifier with transformer coupled input voltage

A transformer is often used to couple the ac input voltage from the source to the rectifier, as shown in figure 4.10. Transformer coupling provides two advantages: 1. It allows the source voltage to be stepped up or stepped down as needed. 2. The ac source is electrically isolated from the rectifier, thus preventing a shock hazard in the secondary circuit. The turn's ratio (n) is equal to the ratio of secondary turns ($N_{secondary}$) to the primary turns ($N_{primary}$):

$$n = \frac{N_{secondary}}{N_{primary}}$$

$$N_{secondary} = n \times N_{primary}$$

If $n > 1$, the secondary voltage is greater than the primary voltage. If $n < 1$, the secondary voltage is less than the primary voltage. The peak secondary voltage, $V_{p(secondary)}$ in a transformer coupled half wave rectifier is equal to $Vp(in)$. Therefore:

$$V_{p(out)} = V_{p(secondary)} - V_o$$

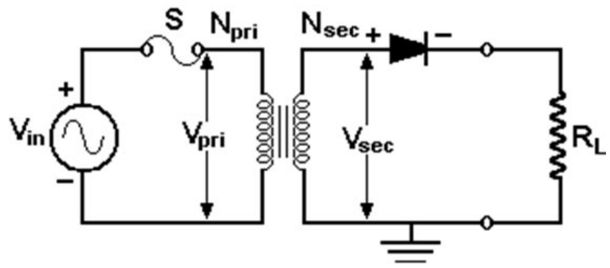

Figure 4.10: Half wave rectifier with transformer coupled input voltage.

4.4 Full Wave Rectifier

A full-wave rectifier allows unidirectional (one-way) current through the load during the entire 360^o of the input cycle, whereas a half-wave rectifier allow current through the load only during one-half of the cycle. The result of full-wave rectification is an output voltage with a frequency twice the input frequency that pulsate every half-cycle of the input, as shown in Figure 4.11. The average or dc value for a full wave rectified sinusoidal voltage is twice that of the half wave, as shown in the following formula:

$$V_{dc} = V_{avg} = \frac{2 \times V_p}{\pi}$$

V_avg is approximately 63.7% of V_P for a full wave rectifiered voltage.

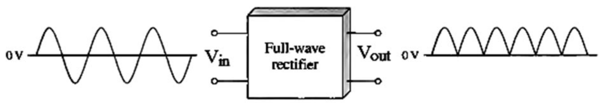

Figure 4.11: Full wave rectifier.

4.6.1 The centre tapped full wave rectifier

A centre tapped rectifier use two diodes connected to the secondary of a center tapped transformer as shown in figure 4.12. The input voltage is coupled through the transformer to the centre tapped secondary. Half of the total secondary voltage appears between the center tap and each end of the secondary winding as shown. For a positive half cycle of the input voltage, the polarities of the secondary voltages are as shown in figure 4.13(a). This condition forward biases diode D_1 and reverse biases diode D_2 . The current path is through D_1 and the load resistor R_L as indicated.

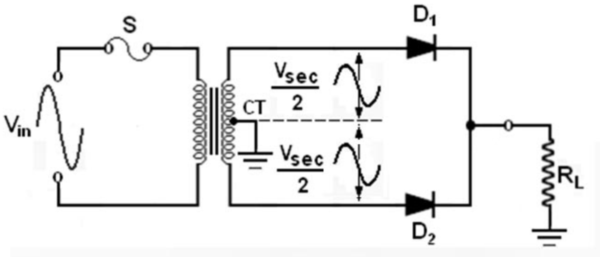

Figure 4.12: A centre-tapped full-wave rectifier.

For a negative half cycle of the input voltage, the voltage polarities on the secondary are as shown in figure 4.13(b). This condition reverses biases D_1 and forward biases D_2. The current path is through D_2 and R_L as indicated. Because the output current during through the load, the output voltage developed across the load resistor is a full wave rectified dc voltage as shown. The output voltage of a center tapped full wave rectifier

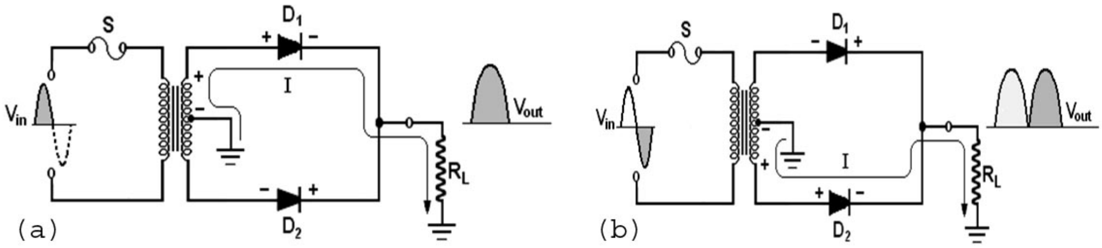

Figure 4.13: Basic operation of a centre tapped full wave rectifier. a) During positive half cycles, D_1 is forward biased and D_2 is reverse biased. b) During negative half cycles. D_2 is forward-biased and D_1 is reverse biased.

is always one half of the **total secondary voltage** less the diode drop, no matter what is the turn's ratio.

$$V_{out} = \frac{V_{sec}}{2} - V_o$$

4.6.8 The Bridge Full wave Rectifier

The bridge rectifier uses four diodes connected as shown in figure 4.14. When the input cycle is positive, as shown in figure 4.14a, diodes D_1 and D_2 are forward biased and conduct current in the direction shown by arrow. A voltage is developed across R_L that looks like the positive half of the input cycle. During this time, diodes D_3 and D_4 are reverse biased. When the input cycle is negative as in figure 4.15b, diodes D_3 and D_4 are forward biased and conduct current in the same direction through R_L as during the positive half cycle. During the negative half cycle, D_1 and D_2 are reverse biased. A full wave rectified output voltage appears across R_L as a result of this action. ***Bridge***

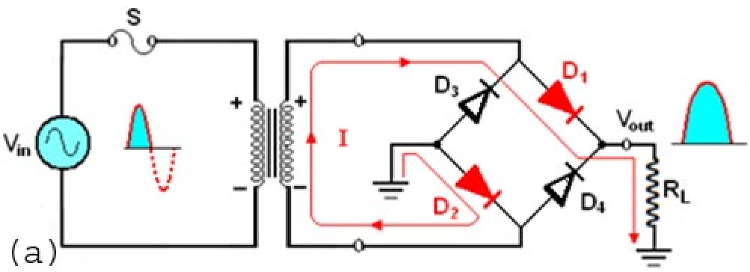

Figure 4.14: Figure 12(a) During the positive half cycle of the input. D_1 and D_2 are forward biased and conduct current D_3 and D_4 are reverse biased.

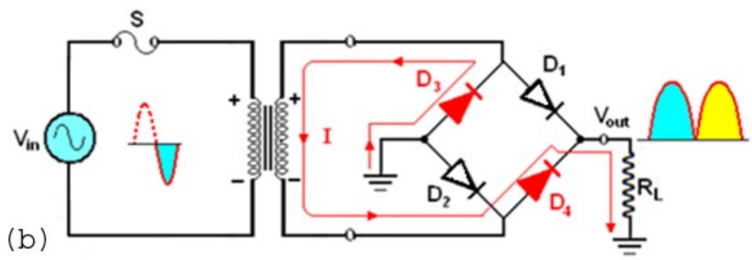

Figure 4.15: Figure 12(b) During the negative half cycle of the input. D_3 and D_4 are forward biased and conduct current while D_1 and D_2 are reverse biased.

output voltage:

During the positive half cycle of the total secondary voltage, diodes D_1 and D_2 are forward biased. Neglecting the diode drops, the secondary voltage appears across the load resistor. The same is true when D_3 and D_4 are forward biased during the negative half cycle.

$$V_{p(out)} = V_{p(sec)}$$

It can be seen in figure 4.16. Two diodes are always in series with the load resistor during both the positive and negative half cycles. If these diode drops are taken into account, the output voltage is:

$$V_{p(out)} = V_{p(sec)} - V_o$$

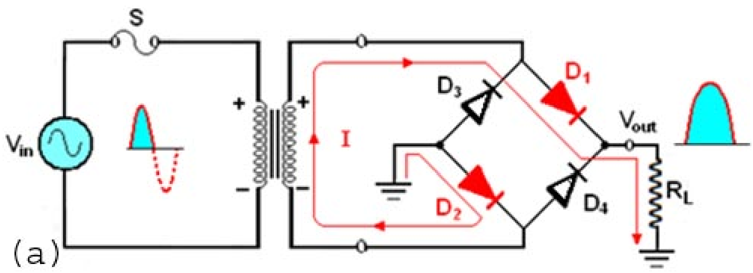

Figure 4.16: Bridge operation during a positive half cycle of the primary and secondary voltages.

4.7 Power Supply Filters and Regulators

In the most power supply applications, the standard 50 Hz AC power line voltage should be converted to an approximately constant DC voltage. The pulsating DC output of a rectifier must be filtered to reduce the large voltage variations.

4.9.1 Power Supply Filters and Regulators

The filter is simply a capacitor connected from the rectifier output to ground. R_L represents the equivalent resistance of a load.

1. During the positive first quarter cycle of the input, the diode is forward biased, allowing the capacitor to charge to within V_o of the input peak, as shown in figure 4.17.

2. When the input begins to decrease below its peak. As shown in figure 4.18, the capacitor retains its charge and the diode becomes reverse biased because the cathode is more positive than the anode. During the remaining part of the cycle,

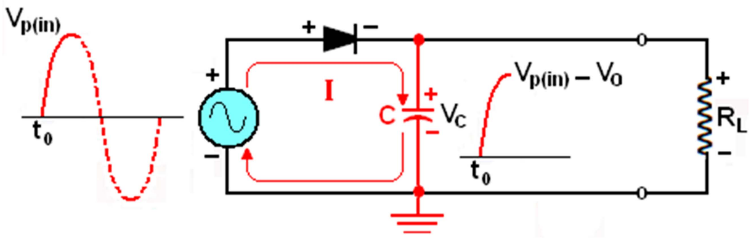

Figure 4.17: Initial charging of the capacitor (diode is forward biased) happens only once when power is turned on.

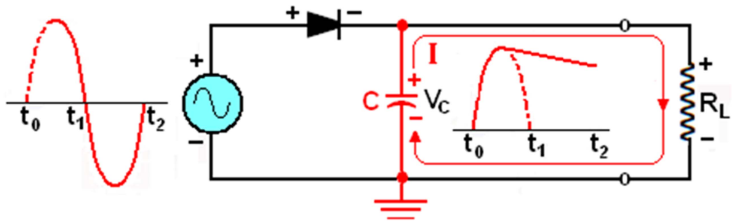

Figure 4.18: The capacitor discharges through R_L after peak of positive alternation when the diode is reverse biased.

the capacitor can discharge only through the load resistance at a rate determined by the $R_L C$ time constant.

3. During the first quarter of the next cycle, as shown in figure 4.19, the diode will again become forward biased when the input voltage exceeds the capacitor voltage by approximately V_o.

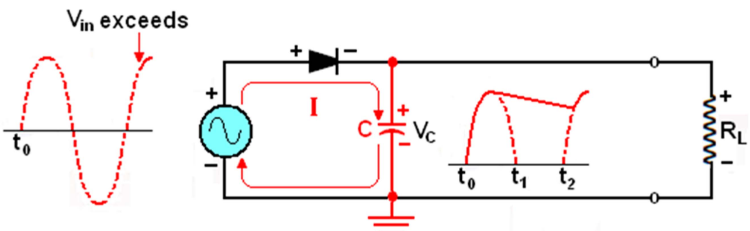

Figure 4.19: The capacitor charges back to peak of input when the diode becomes forward-biased.

4.9.2 Ripple Factor

The variation in the capacitor voltage due to the charging and discharging is called the ripple voltage. Generally, ripple is undesirable; thus, the smaller the ripple, the better the filtering action, as shown in figure 4.20. When filtered, the full wave rectified voltage has a smaller ripple than does a half wave voltage for the same load resistance and capacitor values. The capacitor discharges less during the shorter interval between full-wave pulses, as shown in figure 4.20a and b. The ripple factor is an indication of

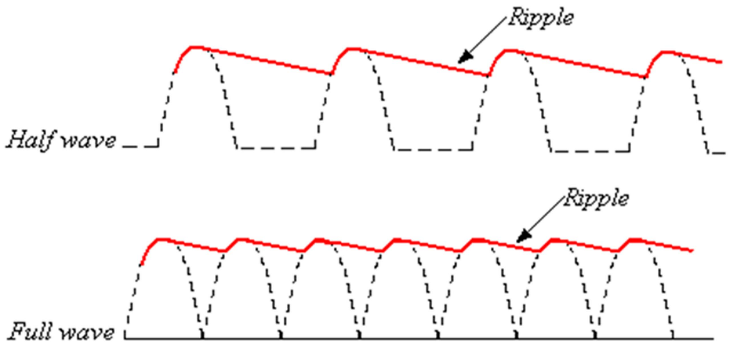

Figure 4.20: Comparison of ripple voltages for (a) half wave and (b) full wave rectified voltages with the same filter capacitor and load.

the effectiveness of the filter and is defined as:

$$r = \frac{V_{rms}}{V_{dc}}$$

As shown in figure 4.21. The ripple factor can be lowered by increasing the value of the filter capacitor or increasing the load resistance. For a full wave rectifier with a capacitor input filter, approximations for the peak to peak ripple voltage, V_{rms} and the DC value of the filter output voltage is $V_{d.c}$, are given in the following expressions. The variable $V_{p(rect)}$ is the unfiltered peak rectified voltage.

$$V_{dc} = \left(1 - \frac{0.00417}{R_L C}\right) V_{p(rect)}$$

$$V_{rms} = \left(\frac{0.0024}{R_L C}\right) V_{p(rect)}$$

The last formulas are for ripple filtered signal.

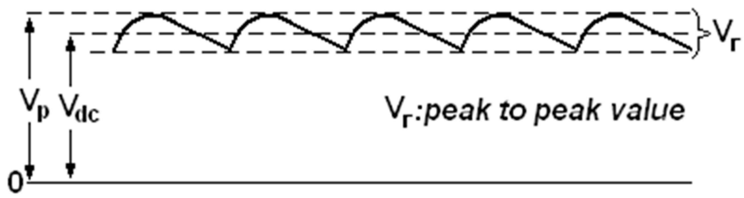

Figure 4.21: V_{rms} and V_{dc} determination for the ripple factor calculation

4.10.3 Inductor Input Filter

When a choke is add to the filter input, as in figure 4.22, a reduction in the ripple voltage V_{rms} is achieved. The choke has a high reactance at the ripple frequency. The capacitive reactance is low compared to both R_L and X_L (10 time at least). The magnitude of the out ripple voltage of the filter is determined with voltage divider equation:

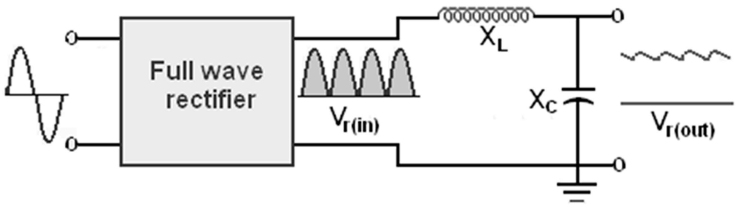

Figure 4.22: The LC filter as it look to the AC component.

$$V_{rms(out)} = \frac{X_C}{|X_L - X_C|} V_{rms(in)}$$

Since the choke presents a winding resistance R_w in series with load resistance R_L. This resistance produce undesirable reduction of DC value, therefore R_w must be small compared of R_L.

$$V_{dc(out)} = \frac{R_L}{R_L - R_w} V_{dc(in)}$$

4.12 Diode Clipping Circuits

Diode circuits, called limiters or clippers, are sometimes used to clip off portions of signal voltages above or below certain levels. Another type of diode circuit, called a clamper, is used to add or restore a DC level to an electrical signal. In the present lecture we will discuss the clipping circuits only.

4.12.1 Positive Clipping Circuit

Figure 4.23 shows a diode clipper that clips the positive part of the input voltage. As the input voltage goes positive, the diode becomes forward biased and conducts the current. So point A is limited to $(+\ V_o)$, when the input go back below V_o, the diode is reverse biased and appears as an open the output voltage is look like the negative part of the input voltage, but with magnitude determined by the voltage divider formed by R_1 and the load resistance R_L, as follow:

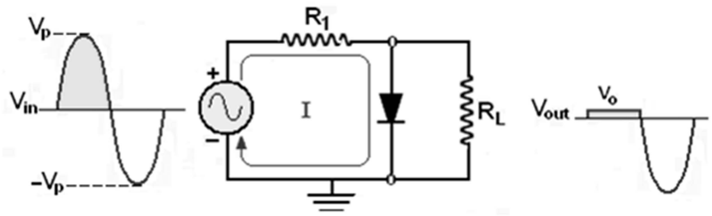

Figure 4.23: Circuit for the positive clipper.

$$V_{out} = \frac{R_L}{R_L + R_1} V_{in}$$

if R_1 is much less than $R_L \Rightarrow V_{out} \approx V_{in}$

4.13.2 Negative Clipping Circuit

To obtain negative biased clipper circuit, the diode and bias voltage must connect as shown in figure 4.24. In this case, the voltage at point A must go below $(-V_B - V_o)$ to forward bias the diode an initiate limiting action as shown.

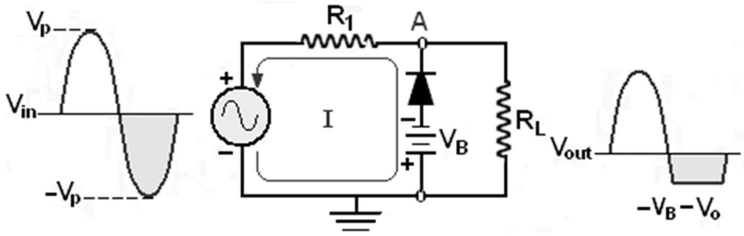

Figure 4.24: Circuit for the negative clipper.