

قوانين  
الرياضيات

(نظم و سيطرة) - 1 -

## Basic Algebra Formulas

$$\textcircled{1} * a(b+c) = ab + ac$$

$$* \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$* \frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$$

$$* \frac{a/b}{c/d} = \frac{a}{b} \cdot \frac{d}{c}$$

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$$\textcircled{2} -(-a) = a, \quad \frac{-a}{b} = -\frac{a}{b} = \frac{a}{-b}$$

$$\textcircled{3} \text{ If } a \neq 0: \frac{0}{a} = 0, \quad a^0 = 1, \quad 0^a = 0$$

For any number  $a$ :

$$a \cdot 0 = 0 \cdot a = 0$$

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$$\textcircled{4} a^m a^n = a^{m+n}$$

$$(ab)^m = a^m b^m$$

$$(a^m)^n = a^{mn}$$

$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$$\text{If } a \neq 0, \quad \frac{a^m}{a^n} = a^{m-n}, \quad a^0 = 1, \quad a^{-m} = \frac{1}{a^m}$$

(A)

(4) For every positive integer n,

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{1 \cdot 2} a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} a^{n-3}b^3 + \dots + nab^{n-1} + b^n$$

For instance,

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

n=3

$$(5) a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + ab^{n-2} + b^{n-1})$$

For instance,

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^4 - b^4 = (a-b)(a^3 + a^2b + ab^2 + b^3)$$

(6) If  $a \neq 0$ ,

$$ax^2 + bx + c = a(u^2 + C)$$

$u = x + (b/2a)$
$C = c - \frac{b^2}{4a}$

⑦ If  $a \neq 0$  and  $ax^2 + bx + c = 0$ , then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Geometry Formula

A = area

B = area of base

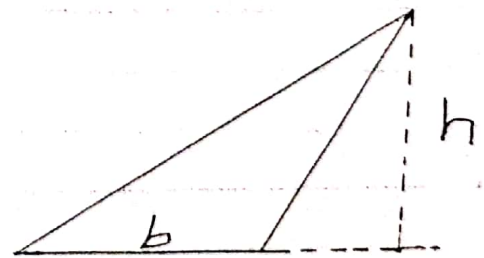
C = circumference

S = lateral area or surface area

V = volume

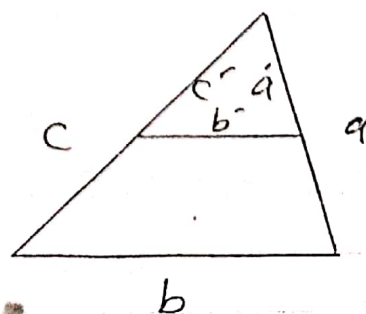
① Triangle : @ Area of Triangle

$$A = \frac{1}{2} bh$$



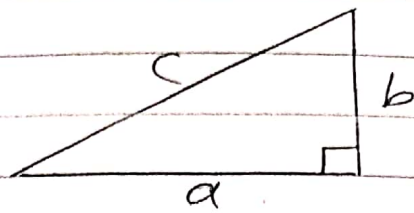
② Similar Triangles

$$\frac{a'}{a} = \frac{b'}{b} = \frac{c'}{c}$$



① Pythagorean Theorem

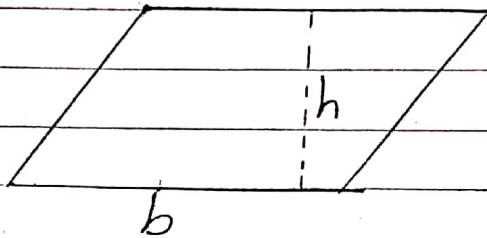
$$a^2 + b^2 = c^2$$



② Parallelogram :

Area of parallelogram

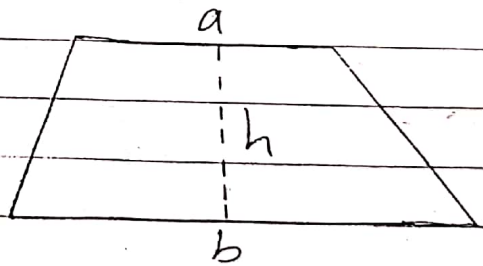
$$A = bh$$



③ Trapezoid :

Area of trapezoid

$$A = \frac{1}{2}(a+b)h$$



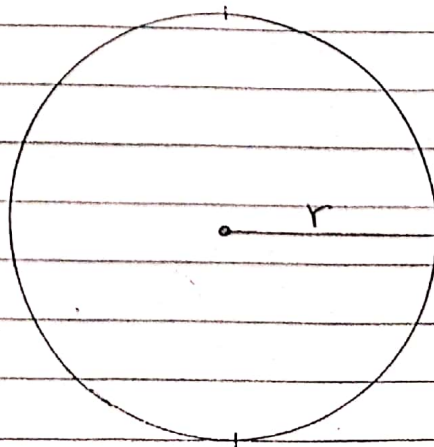
④ Circle :

Area of circle

$$\text{Area} = \pi r^2$$

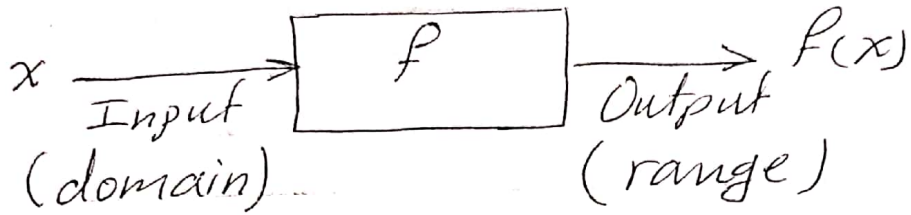
Circumference of circle

$$C = 2\pi r$$

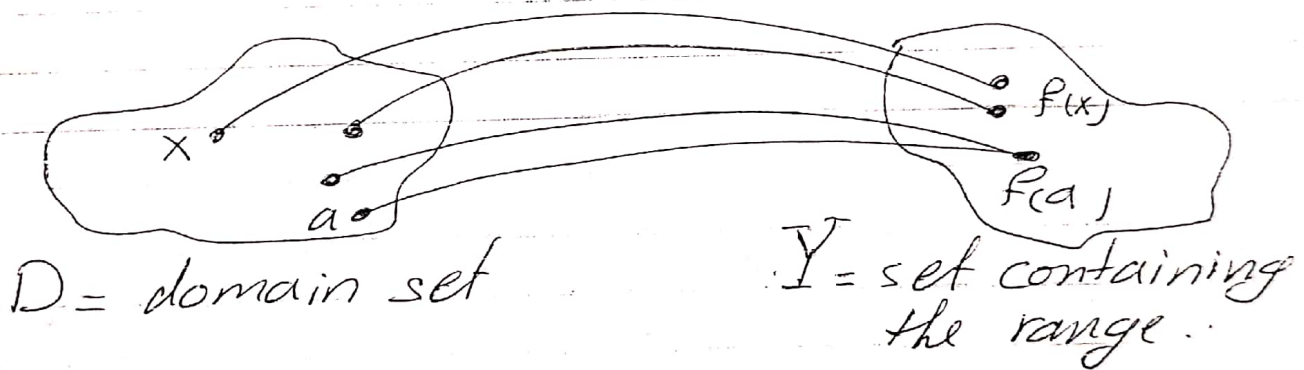


# Functions

A function  $f$  from a set  $D$  to a set  $Y$  is a rule that assigns a unique (single) element  $f(x) \in Y$  to each element  $x \in D$ .



"A diagram showing a function as a kind of machine".



A function from a set  $D$  to a set  $Y$  assigns a unique element of  $Y$  to each element in  $D$ .

Example (1): Find the domain and range of each function.

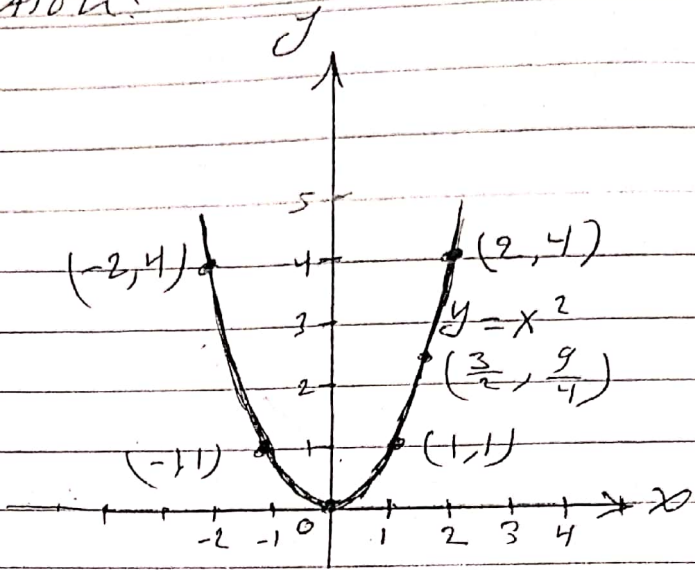
①  $y = x^2$

Domain ( $x$ ):  $-\infty < x < \infty$ ,  $(-\infty, \infty)$

Range ( $y$ ):  $0 \leq x < \infty$ ,  $[0, \infty)$

Graph of Function:

$x$	$y = x^2$
-2	4
-1	1
0	0
1	1
$\frac{3}{2}$	$\frac{9}{4}$
2	4
$\vdots$	$\vdots$



	Function	Domain (x)	Range (y)
2	$y = \sqrt{x}$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
3	$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
4	$y = \sqrt{4-x}$	$(-\infty, 4]$	$[0, \infty)$
5	$y = \sqrt{1-x^2}$	$[-1, 1]$	$[0, 1]$

H.W. Find the domain and range of each function.

①  $f(x) = 1 + x^2$

④  $y = 1 - \sqrt{x}$

②  $f(x) = \sqrt{5x+10}$

⑤  $y = \sqrt{x^2 - 4x}$

③  $f(x) = \frac{4}{3-x}$

⑥  $y = \frac{2}{x^2 - 16}$

## Even Functions and Odd Functions

A function  $y = f(x)$  is an  
- even function of  $x$  if

$$f(-x) = f(x) ,$$

- odd function of  $x$  if

$$f(-x) = -f(x) ,$$

for every  $x$  in the function's domain.

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## « Trigonometric Functions »

The Six Basic Trigonometric Functions:

$$\textcircled{1} \sin \theta = \frac{\text{opp}}{\text{hyp}}$$

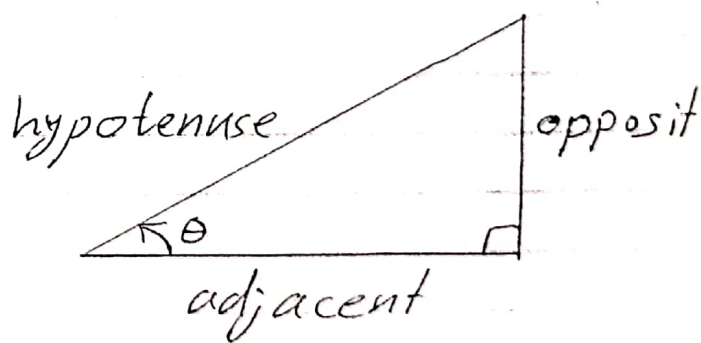
$$\textcircled{2} \cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\textcircled{3} \tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\textcircled{4} \cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\textcircled{5} \csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\textcircled{6} \sec \theta = \frac{\text{hyp}}{\text{adj}}$$



① sine:  $\sin \theta = \frac{y}{r}$

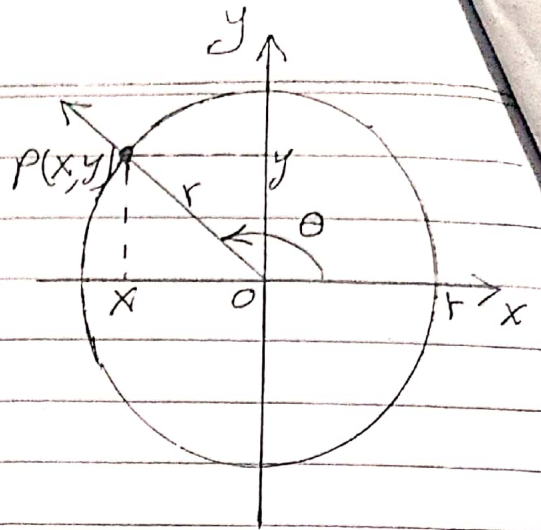
② cosine:  $\cos \theta = \frac{x}{r}$

③ tangent:  $\tan \theta = \frac{y}{x}$

④ cotangent:  $\cot \theta = \frac{x}{y}$

⑤ secant:  $\sec \theta = \frac{r}{x}$

⑥ cosecant:  $\csc \theta = \frac{r}{y}$



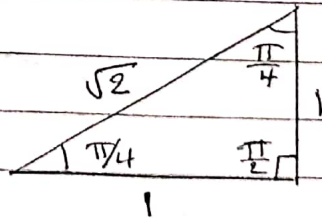
$\tan \theta = \frac{\sin \theta}{\cos \theta}$	$\cot \theta = \frac{1}{\tan \theta}$
	$= \frac{\cos \theta}{\sin \theta}$
$\sec \theta = \frac{1}{\cos \theta}$	$\csc \theta = \frac{1}{\sin \theta}$

For some angles

$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan \frac{\pi}{4} = 1$

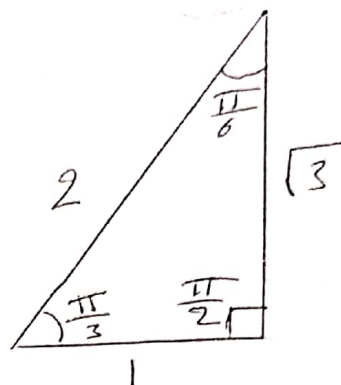




$$\sin \frac{\pi}{6} = \frac{1}{2}$$

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$



$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

## Periods of Trigonometric Functions

Period  $\pi$  :  $\tan(x + \pi) = \tan x$   
 $\cot(x + \pi) = \cot x$

Period  $2\pi$  :  $\sin(x + 2\pi) = \sin x$   
 $\cos(x + 2\pi) = \cos x$   
 $\sec(x + 2\pi) = \sec x$   
 $\csc(x + 2\pi) = \csc x$

Even
$\cos(-x) = \cos x$
$\sec(-x) = \sec x$

Odd
$\sin(-x) = -\sin x$
$\tan(-x) = -\tan x$
$\csc(-x) = -\csc x$
$\cot(-x) = -\cot x$

# Graphs of the Trigonometric Functions

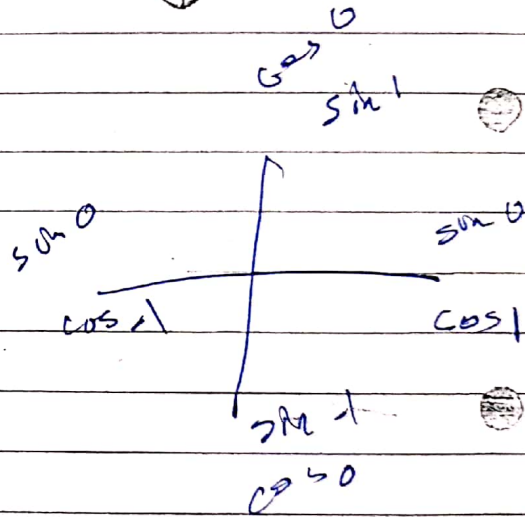
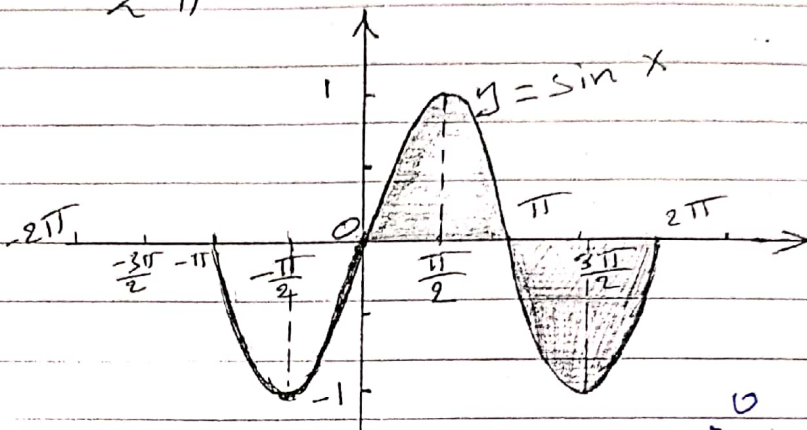
①  $y = \sin x$

Domain :  $-\infty < x < \infty$

Range :  $-1 \leq y \leq 1$

Period :  $2\pi$

$x$	$y = \sin x$
$-\pi$	0
$-\pi/2$	-1
0	0
$\pi/2$	1
$\pi$	0
$3\pi/2$	-1
$\vdots$	$\vdots$



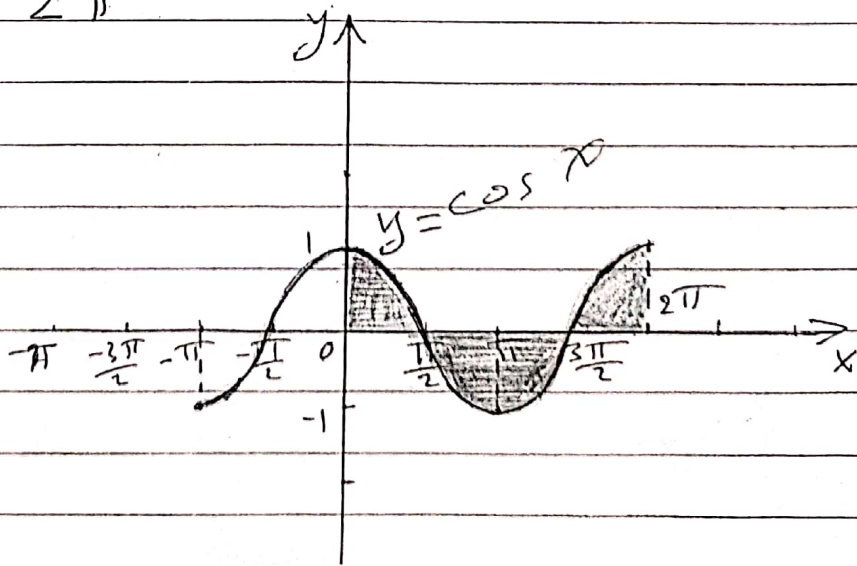
②  $y = \cos x$

Domain :  $-\infty < x < \infty$

Range :  $-1 \leq y \leq 1$

Period :  $2\pi$

$x$	$y = \cos x$
$-\pi$	-1
$-\pi/2$	0
0	1
$\pi/2$	0
$\pi$	-1
$3\pi/2$	0
$2\pi$	1
$\vdots$	$\vdots$



Trigonometric

# Trigonometric Identities

$$\cos^2 \theta + \sin^2 \theta = 1$$

①

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

②

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

③

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

④

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

⑤