Deriving Boolean Expression from a Truth Table

Logical functions are generally expressed in terms of different combinations of logical variables such as

I. Sum of the Products SOP (Minterm)

Sum of Products (SOP) is basically an **OR operation** on AND operated variables. For example, Y = AB + BC + AC is sum of products expressions.

II. Product of the Sums POS (Maxterm)

Product of Sums (POS) is an **AND operation** on OR operated variables. For example, Y = (A + B + C) (A + B` + C) (A`+ B + C`) is product of sums expressions.

Minterm

A product term (**AND** operation) containing all *the* variables of the function in either true or complemented form.

A	B	C	Minterm
0	0	0	$A'B'C'$
0	0	1	$A'B'C$
0	1	0	$A'BC'$
0	1	1	$A'BC$
1	0	0	$AB'C'$
1	0	1	$AB'C$
1	1	0	ABC'
1	1	1	ABC

Ex) Obtain the standard sum of product form of the following function:

$$\begin{aligned} F(A, B, C) &= A + BC \\ &= A(B + B')(C + C') + BC(A + A') \\ &= (AB + AB')(C + C') + ABC + A'BC \\ &= ABC + AB'C + ABC' + AB'C' + ABC + A'BC \\ &= ABC + AB'C + ABC' + AB'C' + A'BC \end{aligned}$$

Ex) Obtain the standard sum of product form of the following function:

$$\mathbf{F(A, B, C, D) = AB + ACD}$$

Maxterm

A sum term (**OR** operation) containing all *the* variables of the function in either true or complemented form.

A	B	C	Maxterm
0	0	0	$A + B + C$
0	0	1	$A + B + C'$
0	1	0	$A + B' + C$
0	1	1	$A + B' + C'$
1	0	0	$A' + B + C$
1	0	1	$A' + B + C'$
1	1	0	$A' + B' + C$
1	1	1	$A' + B' + C'$

Ex) Obtain the canonical product of the sum form of the following function:

$$F\ (A,\ B,\ C) = (A + B')\ (B + C)\ (A + C')$$

$$\begin{aligned}
F\ (A,\ B,\ C) &= (A + B')\ (B + C)\ (A + C') \\
&= (A + B' + 0)\ (B + C + 0)\ (A + C' + 0) \\
&= (A + B' + CC')\ (B + C + AA')\ (A + C' + BB') \\
&= (A + B' + C)\ (A + B' + C')\ (A + B + C)\ (A' + B + C)\ (A + B + C') \\
&\quad\ (A + B' + C') \\
&= (A + B' + C)\ (A + B' + C')\ (A + B + C)\ (A' + B + C)\ (A + B + C')
\end{aligned}$$

Boolean function is also expressed as the logical **sum** (Minterm) or **product** (Maxterm) of the minterms from the rows of a truth table:

$$\begin{aligned}
F\ (A,B,C) &= \Sigma\ (3,5,6) \\
&= m_3 + m_5 + m_6 \\
&= A'BC + AB'C + ABC'
\end{aligned}$$

$$\begin{aligned}
F\ (A,B,C\ &= \Pi\ (0,2,5) \\
&= M_0\ M_2\ M_5 \\
&= (A + B + C)\ (A + B' + C)\ (A' + B + C')
\end{aligned}$$

Deriving SOP&POS Expressions from a Truth Table

The **SOP** expression of a Boolean function can be obtained from its truth table performing **OR** of the **Minterms** having a function value of 1.

Consider the following truth table, the Boolean function is derived as follow:

Inputs			*Output*	*Product terms*
A	*B*	*C*	*Y*	
0	0	0	0	
0	0	1	0	
0	1	0	1	A′BC′
0	1	1	0	
1	0	0	1	AB′C′
1	0	1	1	AB′C
1	1	0	1	ABC′
1	1	1	0	

Thus, the output function will be:

$$Y = A'BC' + AB'C' + AB'C + ABC'$$

The **POS** expression of a Boolean function can be obtained from its truth table performing **AND** of the **Maxterm** having a function value of 0.

Inputs			*Output*	*Product terms*
A	*B*	*C*	*Y*	
0	0	0	0	
0	0	1	0	
0	1	0	1	
0	1	1	0	
1	0	0	1	
1	0	1	1	
1	1	0	1	
1	1	1	0	

Conversion between Canonical Forms

The complement of a function expressed in SOP form equals to sum of the minterms which are missing from the original function. This is because the original function is expressed minterms that make the function equal to 1, while its complement is those minterms whose values are 0.

Ex) Convert the following function from SOP form to POS form:

$$F\ (A,B,C) = \Sigma\ (2,4,5,6)$$

Solution:

$$\begin{aligned} F\ (A,B,C) &= \Sigma\ (2,4,5,6) \\ &= m_2 + m_4 + m_5 + m_6 \\ &= A'BC' + AB'C' + AB'C + ABC'. \end{aligned}$$

To convert to POS form

This has the complement that can be expressed as

$$\begin{aligned} F'\ (A,B,C) &= \ (0,1,3,7) \\ &= m_0 + m_1 + m_3 + m_7 \end{aligned}$$

Now, if we take complement of F′ by DeMorgan's theorem, we obtain F as

$$\begin{aligned} F\ (A,B,C) &= (m_0 + m_1 + m_3 + m_7)' \\ &= m_0'm_1'm_3'm'_7 \\ &= (A + B + C)(A + B + C')\ (A + B' + C')\ (A' + B' + C'). \end{aligned}$$