Ninevah University
College of Electronics Engineering
Systems & Control Engineering

Four-Variable Karnaugh Maps

Similar to the method used for two-variable and three-variable Karnaugh maps, four-variable Karnaugh maps may be constructed with 16 squares consisting of 16 minterms as shown

$F\ (w, x, y, z)$

$wx \backslash yz$	00	01	11	10
00	m_0 $w'x'y'z'$	m_1 $w'x'y'z$	m_3 $w'x'yz$	m_2 $w'x'yz'$
01	m_4 $w'xy'z'$	m_5 $w'xy'z$	m_7 $w'xyz$	m_6 $w'xyz'$
11	m_{12} $wxy'z'$	m_{13} $wxy'z$	m_{15} $wxyz$	m_{14} $wxyz'$
10	m_8 $wx'y'z'$	m_9 $wx'y'z$	m_{11} $wx'yz$	m_{10} $wx'yz'$

(y: columns 11, 10; x: rows 01, 11; w: rows 11, 10; z: columns 01, 11)

Ex) Simplify the Boolean function: $F\ (w, x, y, z)\ = (0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

$$F\ =\ \bar{y}\ +\ \bar{w}\bar{z}\ +\ x\bar{z}$$

$wx \backslash yz$	00	01	11	10
00	m_0 1	m_1 1	m_3	m_2 1
01	m_4 1	m_5 1	m_7	m_6 1
11	m_{12} 1	m_{13} 1	m_{15}	m_{14} 1
10	m_8 1	m_9 1	m_{11}	m_{10}

Ex) Simplify the expression: $F\ (W, X, Y, Z)\ =\ \sum(3, 4, 5, 7, 9, 13, 14, 15)$

F = W′XY′ + W′YZ + WY′Z + WXY.

	Y′Z′	Y′Z	YZ	YZ′
W′X′			1	
W′X	1	1	1	
WX		1	1	1
WX′		1		

Ex) Simplify the Boolean function $F\ (W, X, Y, Z)\ =\ \sum(0, 1, 2, 5, 8, 9, 10)$

Ex) Simplify the Boolean function $F(A, B, C, D) = \bar{A}\bar{B}\bar{C} + \bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}$

Solution:

First of all, we need to write the function in the standard form (4 variables for each minterms)

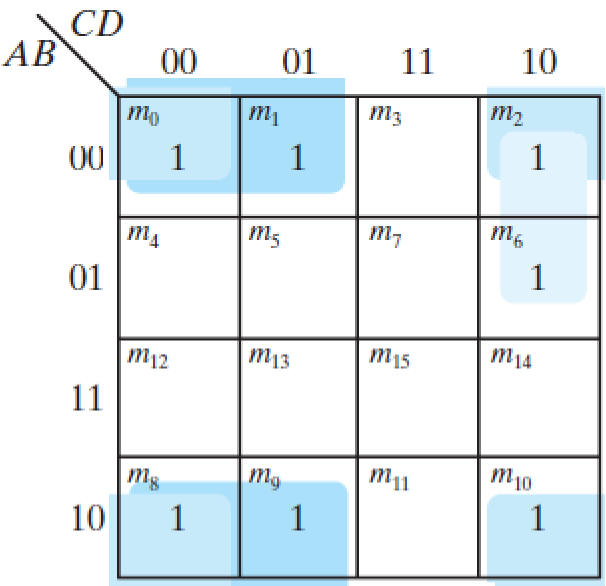
$F\ =\ \bar{A}\bar{B}\bar{C} + \bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}$

$F\ =\ \bar{A}\bar{B}\bar{C}\ (D + \bar{D}) + \bar{B}C\bar{D}(A + \bar{A}) + \bar{A}BC\bar{D} + A\bar{B}\bar{C}(D + \bar{D})$

$F\ =\ \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}\ + A\bar{B}C\bar{D}\ + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D}$

$F\ =\ m_1 + m_0\ + m_{10} + m_2 + m_6 + m_9 + m_8$

$F\ =\ \bar{B}\bar{C} + \bar{A}C\bar{D}\ + \bar{B}\bar{D}$



Ex) Simplify the Boolean function $F\ (W,X,Y,Z) = wxy\ +\ xz\ +\ wx'z\ +\ w'x$

Multiple Solutions

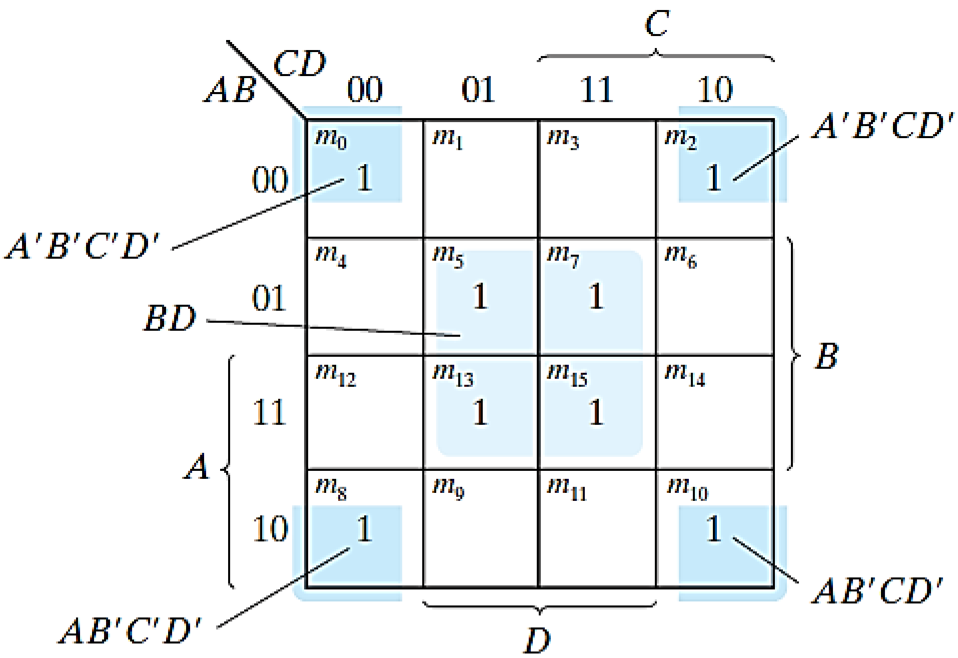
Sometimes, there are more than one solution to the same function. All possible solutions can be found as follow

Ex) Simplify the following function $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$

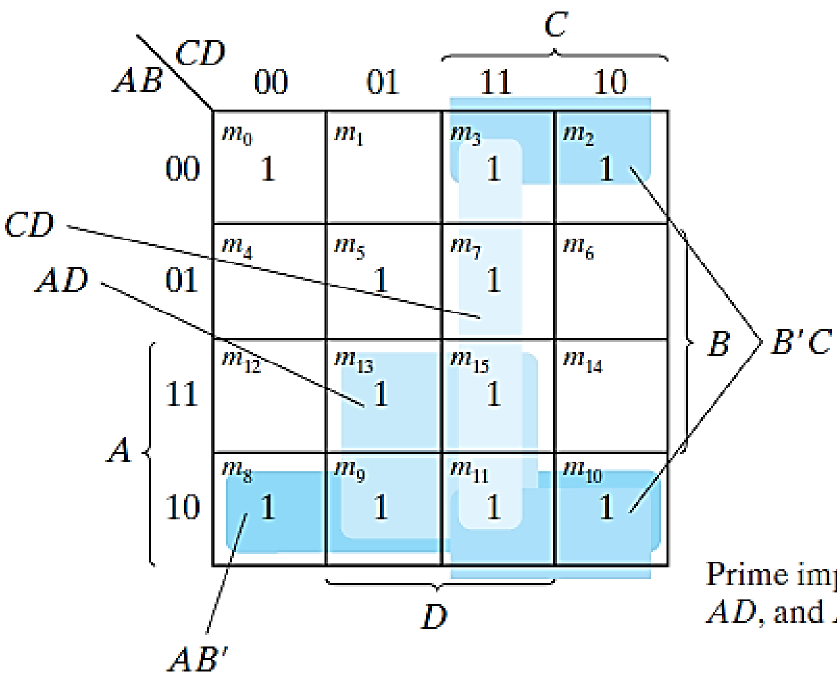
$AB \backslash CD$	00	01	11	10
00	m_0 1	m_1	m_3 1	m_2 1
01	m_4	m_5 1	m_7 1	m_6
11	m_{12}	m_{13} 1	m_{15} 1	m_{14}
10	m_8 1	m_9 1	m_{11} 1	m_{10} 1

First we identify the essential minterms, a minterm is essential if there is only one way to include minterm within four adjacent squares.

Essential groups are $BD + \bar{B}\bar{D}$



Then, we find the expression for the remaining minterms (m_3, m_9, m_{11}).



Prime implicants CD, $B'C$, AD, and AB'

Minterm m_3 can be covered with either group CD or B'C. Similarly, minterm m_9 can be covered with either AD or AB'. The simplified expression is obtained from the logical sum of the two essential prime groups and any two prime groups that cover all the minterms. Thus the function could be:

$$\begin{aligned} F &= BD + B'D' + CD + AD \\ &= BD + B'D' + CD + AB' \\ &= BD + B'D' + B'C + AD \\ &= BD + B'D' + B'C + AB' \end{aligned}$$

Don’t Care Conditions

The logical sum of the minterms specifies the states under which the function is equal to 1. The function is equal to 0 for the rest of the minterms. However, in some applications the function is not specified for certain group of minterms. For example, four-bit BCD range from (0-9), thus it has six combinations from (10- 15) that are not used and considered to be unspecified.

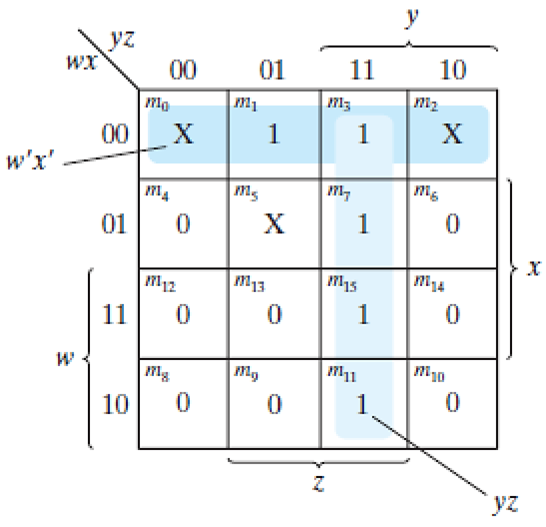
The unspecified minterms of a function is called don’t-care conditions. These don’t-care conditions can be used on a map to provide further simplification of the Boolean expression. Such a minterm cannot be marked with a 1 or 0 on which combination gives the simplest expression.

Ex) Simplify the Boolean function

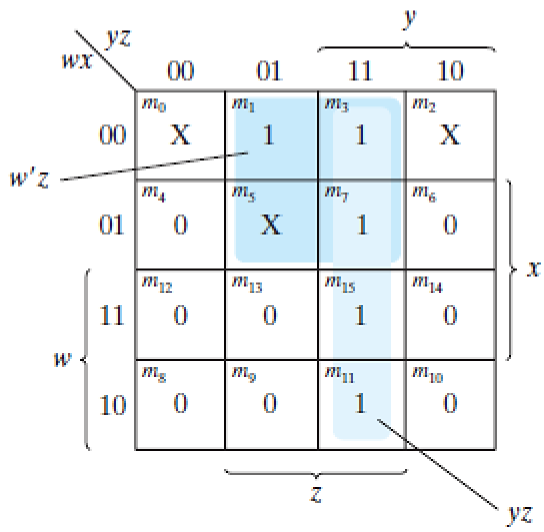
$$F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

which has the don’t-care conditions

$$d(w, x, y, z) = \Sigma(0, 2, 5)$$



(a) $F = yz + w'x'$



(b) $F = yz + w'z$

In Map a:

m_0 & $m_2 = 1$

$m_5 = 0$

In Map b:

m_0 & $m_2 = 0$

$m_5 = 1$