Elementary Fluid Dynamics

As was discussed earlier, Fluid Static is the situations where the fluid is considered to be ***stationary (Rest)***. In contrast, Fluid Dynamics studies and analyses fluid ***in motion (Flow)*** along a streamline.

Flow Classification

The flow of the fluid particles can be classified as follow:

Flow
- ***Viscous***: is a flow with a considerable amount of viscosity (Real Flow $\mu \neq 0$).
- ***Non-viscous***: is a flow of inviscid fluid (Ideal Flow $\mu = 0$).

Flow
- ***Laminar Flow***: each particle of the fluid follows a smooth path, usually at low velocities, which never interfere with one another.
- ***Turbulent Flow***: irregular flow where each particle of the fluid flows, usually at high velocities, through whirlpool paths (دوامات) .

Flow
- ***Uniform***: at a given instant, the fluid particles have the same velocity at **every point** along the streamline.
- ***Non-uniform***: at a given instant, velocity **is not** the same at every point along the streamline.

Flow
- ***Steady***: at a given point, the velocity may differ from other points along the streamline but ***DO NOT*** change with time.
- ***Unsteady***: at a given point along the streamline, the conditions change with time.

Combining the last two types above, we can classify any flow as:

- *Steady uniform flow.*
- *Steady non-uniform flow.*
- *Unsteady uniform flow.*

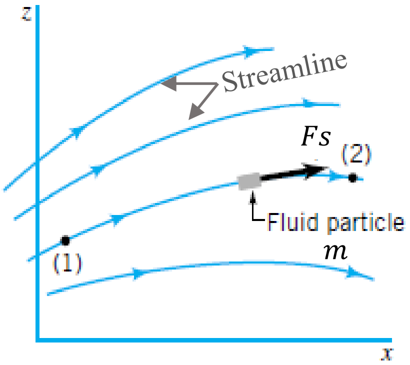
- *Unsteady non-uniform flow.*

Equations of motion

As a fluid particle moves from one location to another along the streamline, it usually experiences acceleration and deceleration. This **dynamic behaviour** of the fluid flow is analysed using several equations of motion such as:

1. ***Newton 2^nd^ law of motion***

According to Newton's second law of motion, the net force F_S acting on a fluid element in the streamline direction (***S***) is equal to mass ***m*** of the fluid element multiplied by the acceleration in the S-direction. Mathematically:



$$F_S = m \, . \, a_S$$

In fluid flow, the following ***forces*** are present:

- F_G, gravity force.
- F_P, pressure force.
- F_V, force due to viscosity.
- F_C, force due to compressibility.
- F_R, force due to turbulence.

Thus, $F_S = F_G + F_P + F_V + F_C + F_R$

2. ***Euler equation of motion***

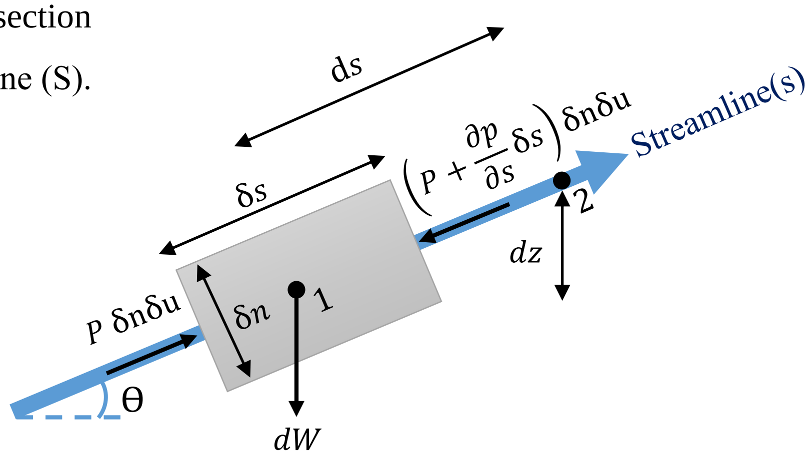
The following assumptions are made in the derivation of Euler's equation:

- The flow is ideal, *i.e.*, viscosity is zero ($F_V = 0$)
- The flow is steady, *i.e.*, velocity change is zero
- The flow is incompressible ($F_C = 0$)
- The flow is non-rotational ($F_R = 0$)

Therefore, only forces due to **gravity** and **pressure** are taken into consideration.

$$F_S = F_G + F_P$$

Consider a fluid element of cross-section δnδu and length δS along the streamline (S).



The forces acting on the element are:

1. Pressure force *p* δnδu in the direction of flow.
2. Pressure force $\left(P+\frac{\partial p}{\partial s}\delta s\right)$ δnδu opposite to the direction of flow.
3. Weight of element *dW* = *specific weight x volume* = *ρg δnδuδs*.

$$\sum Fs = m \,.\, a_s$$

$$P\delta n\delta u-\left(P+\frac{\partial p}{\partial s}\delta s\right)\delta n\delta u - \rho g\ \delta n\delta u\delta s\ Sin\Theta = \rho\ \delta n\delta u\delta s\,.\,a_s$$

The acceleration (a_s) is the rate change of velocity V(s) in the (S) direction, *Therefore*

$$a_s=\frac{dv(s)}{dt}$$

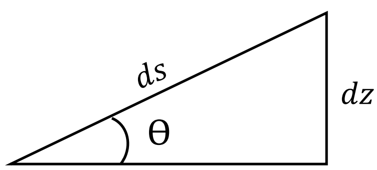
The velocity may change from point to point, *Thus*

$$a_s=\frac{\partial v}{\partial s}\frac{ds}{dt},\ \ where\ \frac{ds}{dt}=v$$

$$a_s=v\frac{\partial v}{\partial s}$$

$$-\frac{\partial p}{\partial s}\delta n\delta u\delta s - \rho g\ \delta n\delta u\delta s\ Sin\Theta = \rho\,\delta n\delta u\delta s\,.\,v\frac{\partial v}{\partial s} \qquad dividing\ by\ \rho\ \delta n\delta u\delta s$$

$$-\frac{\partial p}{\rho\,\partial s}-g\ Sin\Theta = v\frac{\partial v}{\partial s} \qquad sin\Theta=\frac{dz}{ds}$$



$$\frac{\partial p}{\rho\,\partial s}+g\ \frac{dz}{ds}+v\frac{\partial v}{\partial s}=0 \qquad dividing\ by\ g$$

$$\frac{1}{\rho\, g}\frac{\partial p}{\partial s}+\frac{V}{g}\frac{\partial v}{\partial s}+\frac{dz}{ds}=0$$ *Euler equation of motion.*

3. *Bernoulli equation*

Bernoulli equations can be obtained by rearranging and integrating Euler equation derived above and as follow:

$$\frac{1}{\rho\, g}\frac{dp}{ds}+\frac{V}{g}\frac{dv}{ds}+\frac{dz}{ds}=0$$ *this is simplified into:*

$$\frac{dp}{\rho\, g}+\frac{V\ dv}{g}+dz=0$$ *which can be integrated to:*

$$\frac{p}{\rho\, g}+\frac{V^2}{2g}+Z=constant\ along\ the\ streamline$$

Therefore, for **any two points (1&2)** on a streamline in steady, inviscid and incompressible flow the Bernoulli equation can be applied in the form:

$$\boxed{\frac{P_1}{\rho\, g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho\, g}+\frac{V_2^2}{2g}+Z_2}$$ *Bernoulli equation of motion where:*

$\boldsymbol{\frac{p}{\rho\, g}}$ *is the pressure energy per unit weight of the fluid or pressure head*

$\boldsymbol{\frac{V^2}{2g}}$ *is the kinetic energy per unit weight of the fluid or kinetic head*

$\boldsymbol{Z}$ *is the potential energy of the fluid or potential head*

Ex: A large tank, opened to the atmosphere, is filled with water to a height of (5m). A tab near to the bottom is opened and water flows from the smooth and rounded outlet. Determine the water velocity at the outlet.