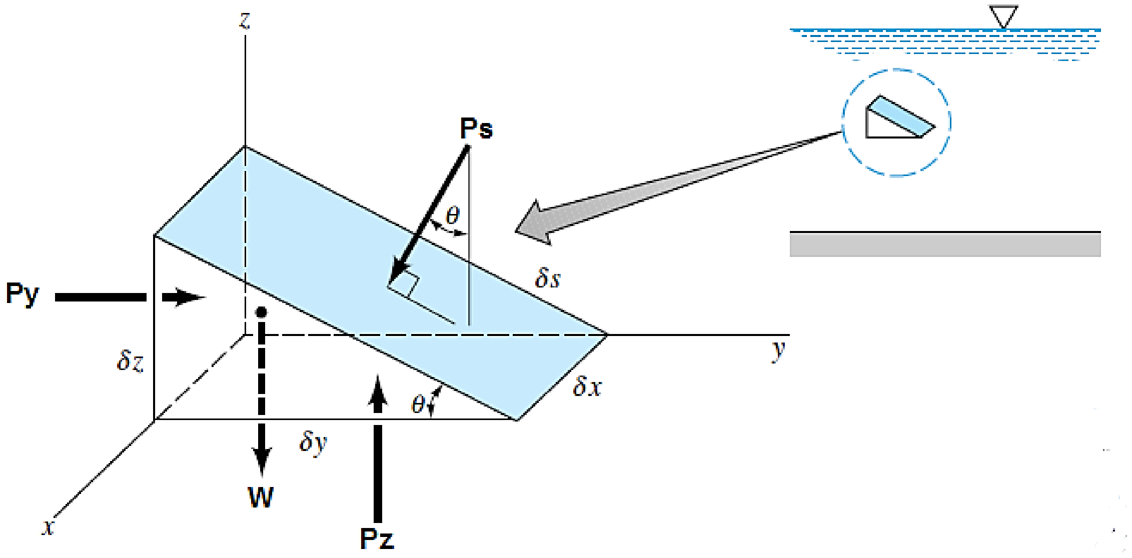
# *Fluid Static*

The fluid is either at rest or moving in such a manner so that there is no relative motion between adjacent (متجاورة) particles. In both cases, there will be no shearing stress in the fluid and the only forces that develop on the surfaces of the particles will be due to the pressure.

***Pascal's Law (Pressure at a Point)***

'The pressure acting on a point in a fluid at rest is the same in all directions'. To prove that, consider a **small triangular slice** of fluid within a fluid mass.




Since the fluid at rest (there are no shearing stresses), the only external forces acting on the slice are due to the pressure (Py, Pz and Ps) and the weight (W).

Applying Newton's 2nd law of motion (F = m.a) in the *y* and *z* directions respectively:

$$\sum Fy = m\,.\,a_y \rightarrow Py\delta x\delta z - Ps\delta x\delta s\,\sin\Theta = \rho v a_y$$

$$Py\delta x\delta z - Ps\delta x\delta s\,\sin\Theta = \rho\frac{\delta z\delta y\delta x}{2}a_y \qquad 1.1$$

$$\sum Fz = m\,.\,a_z \rightarrow Pz\delta x\delta y - Ps\delta x\delta s\,\cos\Theta - \gamma v = \rho v a_z$$

$$Pz\delta x\delta y - Ps\delta x\delta s\,\cos\Theta = (\rho a_z + \gamma)\frac{\delta z\delta y\delta x}{2} \qquad 1.2$$

$\delta y = \delta s\,\cos\Theta\,,\ \delta z = \delta s\,\sin\Theta$ *Substitute in 1.1 and 1.2 yield*

$$Py\ \delta x\delta s\ sin\Theta - Ps\ \delta x\delta s\ sin\Theta = \rho \frac{\delta s\ \delta y\ \delta x\ sin\Theta}{2} a_y$$

$$Py - Ps = \rho \frac{\delta y}{2} a_y \qquad 1.3$$

$$Pz\ \delta x\delta s\ cos\Theta - Ps\ \delta x\delta s\ cos\Theta = (\rho a_z + \gamma) \frac{\delta z\ \delta x\delta s\ cos\Theta}{2}$$

$$Pz - Ps = (\rho a_z + \gamma) \frac{\delta z}{2} \qquad 1.4$$

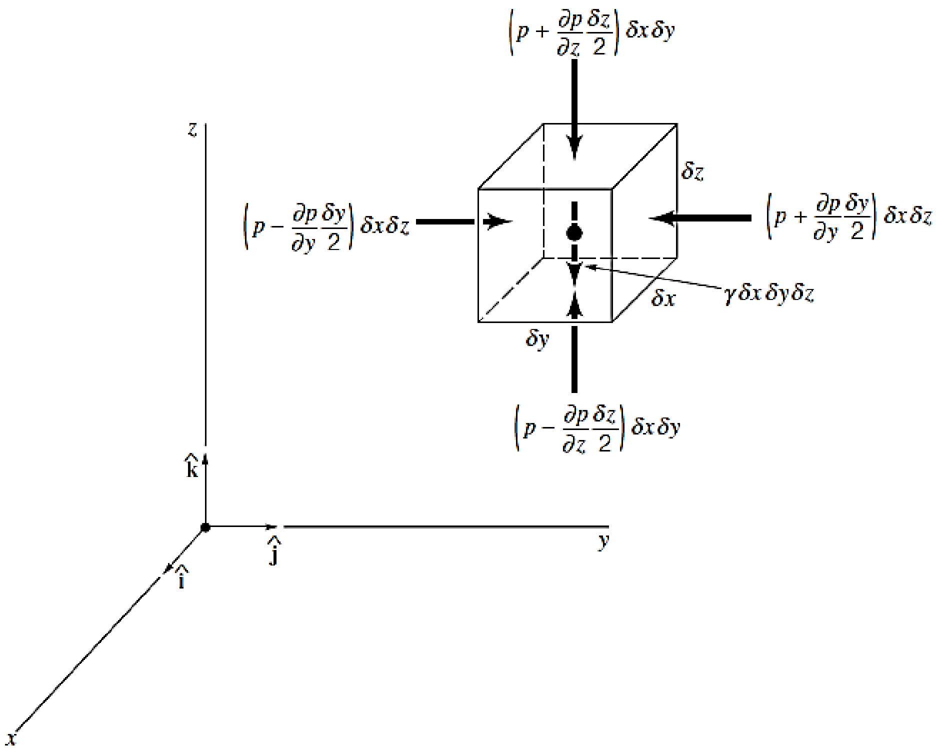
Since we are really interested in what is happening at a point, therefore $\delta x\delta y\delta z$ are **really small** and can be considered **zero** (1.3&1.4) which gives:

$$\boxed{Py = Pz = Ps}$$

The angle was arbitrarily chosen so we can conclude that the pressure at a point in a fluid at rest is **independent of direction** as long as there are no shearing stresses present.

***Hydrostatic Pressure Law***

To answer the question of how the pressure changes from point to point, consider a rectangular element of fluid as follow:



The pressure at the center of the element is *P*, then the average pressure on the various faces can be expressed in Taylor series expansion as shown above.

The resultant surface force in the *y* direction is:

$$F_y = \left(p - \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z - \left(p + \frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\,\delta z$$

$$F_y = -\frac{\partial p}{\partial y}\,\delta x\,\delta y\,\delta z$$

Similarly, for the *x* and *z* directions the resultant surface forces are:

$$F_x = -\frac{\partial p}{\partial x}\,\delta x\,\delta y\,\delta z \qquad F_z = -\frac{\partial p}{\partial z}\,\delta x\,\delta y\,\delta z$$

The resultant surface force acting on the element can be expressed in vector form as

$\mathbf{F_R} = F_X\mathbf{i} + F_Y\mathbf{j} + F_Z\mathbf{k}$

$$\mathbf{F_R} = -\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right)\delta x\,\delta y\,\delta z$$

Also, the weight of the element can be expressed in vector form as:

$\mathbf{W} = -\gamma\,\delta x\,\delta y\,\delta z\,\mathbf{k}$

Newton's second law, applied to the fluid element, can be expressed as:

$$\sum \mathbf{F} = m\,\mathbf{a}$$

$$-\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right)\delta x\,\delta y\,\delta z - \gamma\,\delta x\,\delta y\,\delta z\,\mathbf{k} = \rho\,\delta x\,\delta y\,\delta z\,\mathbf{a}$$

$$-\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right) - \gamma\mathbf{k} = \rho\,\mathbf{a}$$

*For a fluid at rest, a = 0*

$$-\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right) = \gamma\mathbf{k}$$

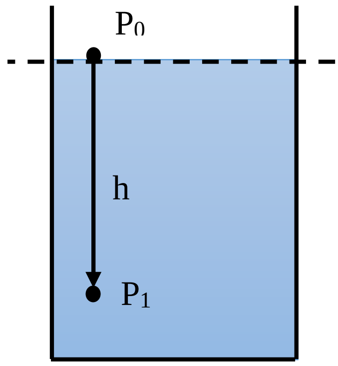
$$\frac{\partial p}{\partial x} = 0 \qquad \frac{\partial p}{\partial y} = 0 \qquad \frac{\partial p}{\partial z} = -\gamma$$

These equations show that the pressure **does not** depend on **x or y directions**. Thus, when we move from point to point in a horizontal plane, the pressure does not change and it **depends** only on **z direction.**

Since $p$ depends only on **Z direction**, the equation can be written as an ordinary differential equation.

$$\boxed{\frac{dp}{dz} = -\gamma}$$

This is the fundamental equation for fluids at rest that can be used to determine how pressure changes with elevation:




$$\int_{P0}^{P1} dp = -\gamma \int_{Z0}^{Z1} dz$$

$$P_1 - P_0 = -\gamma\,(Z_1 - Z_0)$$

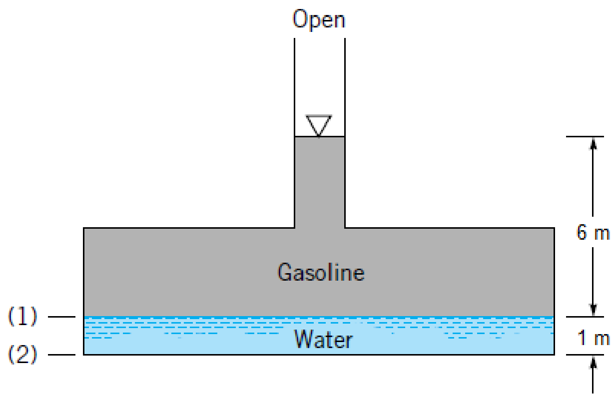
$$P_1 = P_0 - \gamma\,(-h - 0)$$

$$P_1 = P_0 + \gamma\, h$$

$$P_0 \; can\ be\ taken\ as\ zero\ (open\ to\ atmosphere)$$

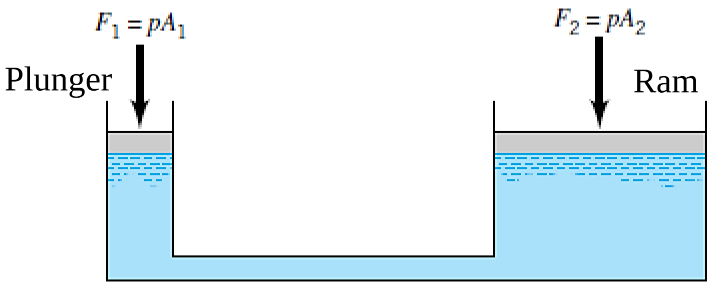
$$P_1 = \gamma\, h$$

**Ex)** Because of a leak in a buried gasoline storage tank, water has seeped in to the depth. If the specific gravity of the gasoline is 0.68. Determine the gauge pressure at the gasoline-water interface (1) and at the bottom of the tank (2).



## Application on Hydrostatic Pressure Law

The equality of pressures at same elevations throughout a system is important for the operation of hydraulic jacks, lifts and presses.




The pressure *p* acting on the faces of both pistons is the same. Therefore, the Ram area ($A_2$) can be made much larger than Plunger area ($A_1$) and hence a large mechanical advantage can be developed.

$$P_1 = P_1$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\boxed{F_2 = \frac{A_2}{A_1} F_1}$$

That is, a small force applied at the smaller piston can be used to develop a large force at the larger piston.

**Ex)** A hydraulic press has a ram of 20 cm diameter and a plunger of 3 cm diameter. It is used for lifting a weight of 10 KN. Find the force required at the plunger.

**H.W)** Can we create a hydraulic hand jack that can be used to lift heavy weights (say 1000 KN) by simply making A2 way much bigger than A1?