Flow Rate

It is a measure at which fluid is flowing per unit time.

Flow Rate

Mass Flow Rate $\dot{\boldsymbol{m}}$ is the mass of fluid flowing per unit time

$$\dot{\boldsymbol{m}} = \frac{mass}{time}\ (Kg/s)$$

Volume Flow Rate $\boldsymbol{Q}$ is the volume of fluid flowing per unit time

$$\boldsymbol{Q} = \frac{volume}{time} (m^3/s)$$

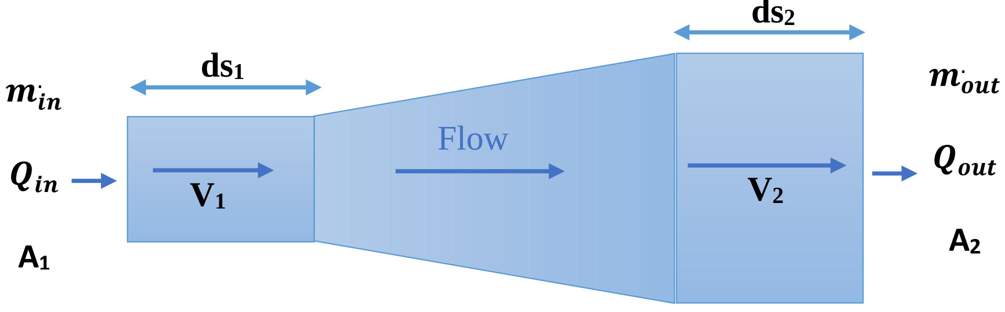
For example, an empty bucket weighs 2.0 kg. After 7 seconds of collecting water the bucket weighs 8.0 kg, then

$$\text{mass flow rate} = \dot{m} = \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}}$$
$$= \frac{8.0 - 2.0}{7}$$
$$= 0.857 kg/s \quad (kg\, s^{-1})$$

Continuity Equation

According to the ***conservation of mass***, matter cannot be created or destroyed but it can change to different forms. Thus:

Mass entering per unit time = Mass leaving per unit time



$$\dot{m}_{in} = \dot{m}_{out}$$

$$\rho_1 A_1 \frac{ds1}{dt} = \rho_2 A_2 \frac{ds2}{dt}$$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

*For **incompressible** fluid*, $\rho_1 = \rho_2$

$\boxed{A_1 V_1 = A_2 V_2 = Q}$ *volume flow rate*

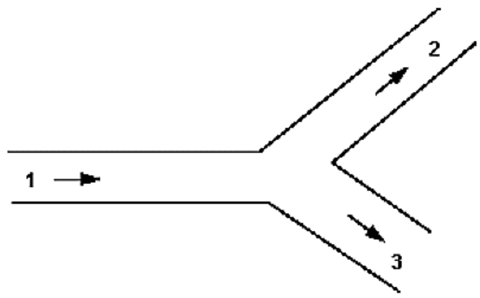
$\boxed{Q_{in} = Q_{out}}$

$\boxed{\dot{m} = \rho Q}$

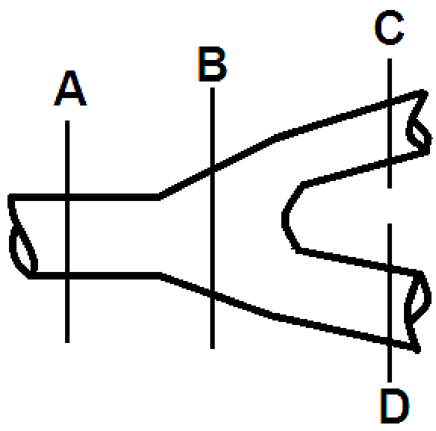
Similarly, in the case of junctions or flow dividers, as shown below:

$$Q_1 = Q_2 + Q_3$$

$$A_1 V_1 = A_2 V_2 + A_3 V_3$$



Ex: The pipe has a diameter of (1.2 m) at A, (1.5 m) at B and (0.8 m) at C. The discharge at C is (Q_A / 3) and the velocities at A & D are (3.5 m / s) and (2.5 m / s) respectively. If the flow is steady incompressible, Determine the:

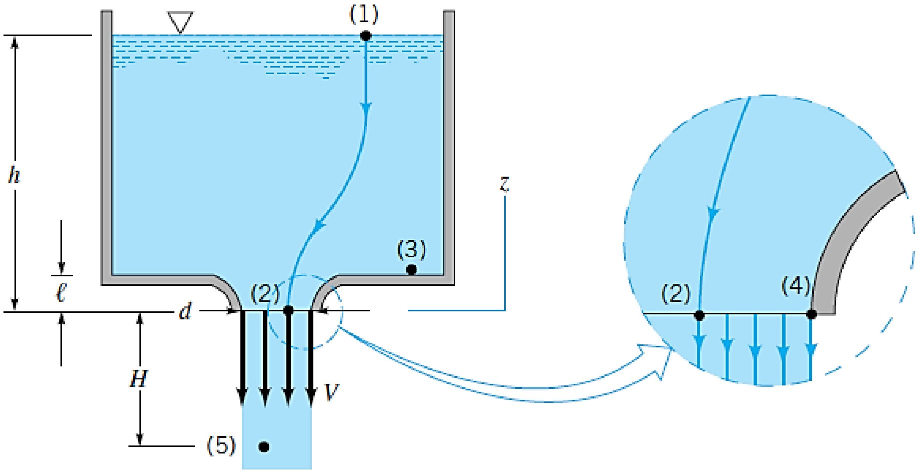


1. Discharge at A.
2. The velocity at B&C.
3. The diameter at D

Application on Bernoulli Equation

a. Free Jets

Bernoulli's equation can be applied to the flow of a liquid from large tanks, as is shown:



Applying Bernoulli's equation between point 1&2

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + Z_2$$

Both streamlines (1&2) in contact with the atmosphere, P_1=P_2= 0. Also, Z_1 = h and Z_2 = 0.

The reservoir is large, V_1 **is approximately** equal to zero. Thus,

$$h = \frac{V_2^2}{2g} \rightarrow V_2 = \sqrt{2gh}$$

The stream continues to fall as a free jet with zero pressure

Applying Bernoulli's equation between point 1&5

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_5}{\rho\, g} + \frac{V_5^2}{2g} + Z_5$$

$$h + H = \frac{V_5^2}{2g}$$

$The\ Speed\ increases\ according\ to\ V_5 = \sqrt{2g(h + H)}$

The pressure at point 3 can be obtained by:

- Applying Bernoulli's equation between 1&3

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_3}{\rho\, g} + \frac{V_3^2}{2g} + Z_3$$

$P_1 = 0$. Also, $Z_1 = h - \ell$, and $Z_3 = 0$.

The reservoir is large, V_1 and V_3 is approximately equal to zero. Thus,

$$\frac{\boldsymbol{P_3}}{\boldsymbol{\rho\, g}} = h - \ell$$

$$P_3 = \gamma\,(h - \ell)$$

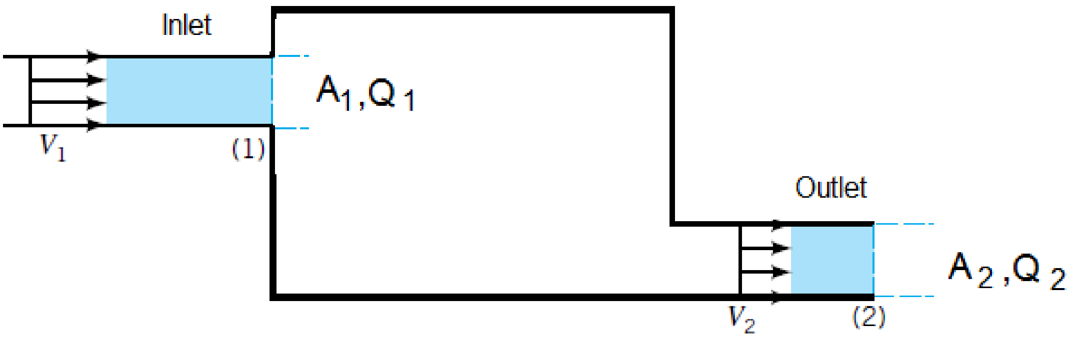
- Or by simply applying the hydrostatic field pressure law

$$P_3 - \gamma\,(h - \ell) = 0$$

$$P_3 = \gamma\,(h - \ell)$$

b. Flow in pipes

As the fluid flows within a pipe of **variable dimeter**, the velocity **changes** because the flow area is different from one section to another. In these situations, we use the **continuity equation** along with the **Bernoulli equation**.



$Q_1 = Q_2$

$A_1 V_1 = A_2 V_2$

Ex) The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 liters/s. If the pressure at section 1 is 39.24 N/cm^2, find the intensity of the pressure at section two.

Ex) A stream of water of diameter (0.1m) flows steadily from a tank of diameter (D = 1m) as shown. Determine the flowrate Q needed from the inflow pipe so that the depth remains constant, h = 2.0 m.