Practical Applications of Bernoulli's Equation

Flow Rate Measurement

An effective way to measure the **flowrate** through a pipe is to place some type of restriction within the pipe and to measure the pressure difference between the high-pressure and low-pressure sections. Several measuring devices use Bernoulli principle to measure fluid **velocities and flowrates**.

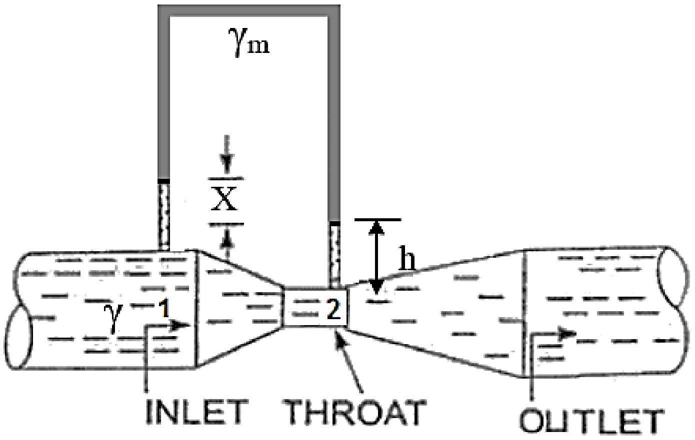
$$\frac{P_1}{\rho\ g}+\frac{V_1^2}{2g}=\frac{P_2}{\rho\ g}+\frac{V_2^2}{2g}$$

$A_1\ V_1 = A_2\ V_2$ Combining the two equations, we get

$$Q = A_2\sqrt{\frac{2(p_1-p_2)}{\rho[1-(A_2/A_1)^2]}}$$

1. ***Venturimeter:*** is a device used for measuring **the rate of a flow** of a fluid flowing through a pipe. It consists of three parts:

   (i) Short converging part
   (ii) Throat
   (iii) Diverging part



γ_m: Specific weight of the Manometer's liquid.

γ: Specific weight of the liquid flowing in the pipe.

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+Z_2$$

The pipe is horizontal, thus $Z_1 = Z_2$

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}\rightarrow\frac{V_2^2}{2g}=\frac{P_1-P_2}{\gamma}+\frac{V_1^2}{2g} \qquad 1.1$$

To find $P_1 - P_2$, we apply the hydrostatic pressure law:

$$P_1-\gamma(h+x)+\gamma_m\,x+\gamma h= P_2$$

$P_1 - P_2 = x\,(\gamma-\gamma_m)$ *, Apply in 1.1:*

$$\frac{V_2^2}{2g}=\frac{x\,(\gamma-\gamma_m)}{\gamma}+\frac{V_1^2}{2g}\rightarrow V_2^2= 2gx\left(1-\frac{\gamma_m}{\gamma}\right)+V_1^2 \qquad 1.2$$

*To find V_1**,** we apply the continuity equation:*

$A_1 V_1 = A_2 V_2$

$V_1 = \frac{A_2}{A_1} V_2$*, Apply in 1.2:*

$$V_2^2= 2gx\left(1-\frac{\gamma_m}{\gamma}\right)+\left(\frac{A_2}{A_1}\right)^2 V_2^2$$

$$\left(1-\left(\frac{A_2}{A_1}\right)^2\right)V_2^2=2gx\left(1-\frac{\gamma_m}{\gamma}\right)\rightarrow V_2^2=2gx\left(1-\frac{\gamma_m}{\gamma}\right)\frac{A_1^2}{A_1^2-A_2^2}$$

$$V_2=\sqrt{2gx\left(1-\frac{\gamma_m}{\gamma}\right)\frac{A_1^2}{A_1^2-A_2^2}}\rightarrow V_2=\frac{A_1}{\sqrt{A_1^2-A_2^2}}\sqrt{2gx\left(1-\frac{\gamma_m}{\gamma}\right)}$$

$$Q = A_2V_2=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gx\left(1-\frac{\gamma_m}{\gamma}\right)}$$

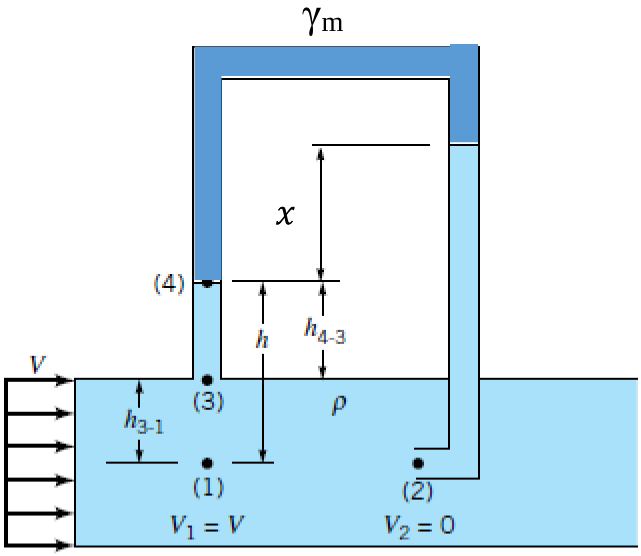
The above formula is correct when the Manometer liquid is lighter than the liquid flowing in the pipe $(\gamma_m<\gamma)$. Therefore, the term $\left(1-\frac{\gamma_m}{\gamma}\right)$ is positive. If the Manometer liquid is heavier than the liquid flowing through the pipe, the formula can be written as

$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gx\left(\frac{\gamma_m}{\gamma} - 1\right)}$$

Ex: An oil of SG = 0.8 is flowing through a Venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of x = 15 cm. Calculate the theoretical discharge of oil through the horizontal Venturimeter given that SG of mercury is 13.6.

Ex) What would be the reading of the oil-mercury differential manometer if the flowrate was increased to 60 $^{ltrs}/_{S}$.

2. ***Static Pitot tube:*** It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that **the velocity** of flow at the **stagnation point** becomes **zero**.



Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{P_1}{\rho\ g} + \frac{V_1^2}{2g} + Z1 = \frac{P_2}{\rho\ g} + \cancel{\frac{V_2^2}{2g}} + Z2$$

The pipe is horizontal, hence $Z_1 = Z_2$

$$\frac{V_1^2}{2g} = \frac{P_2 - P_1}{\rho\ g} \rightarrow V_1 = \sqrt{2g\ (\frac{P_2 - P_1}{\gamma})}$$

To find $P_2 - P_1$, we apply the field pressure law:

$$P_2 - \gamma(h + x) + \gamma_m\, x + \gamma h = P_1$$

$$P_2 - P_1 = x\ (\gamma - \gamma_m)$$

$$V_1 = \sqrt{2gx\ \frac{(\gamma - \gamma_m)}{\gamma}}$$

$$V_1 = \sqrt{2gx\ \left(1 - \frac{\gamma_m}{\gamma}\right)} \quad , \qquad Q = A_1 V_1 = A_1\sqrt{2gx\ \left(1 - \frac{\gamma_m}{\gamma}\right)}$$

Ex) A pitot tube is inserted in a pipe of 10 cm diameter. The pressure difference measured by a mercury (SG = 13.6) differential manometer gives a reading of 18 cm of mercury. Find the velocity of the flow of an oil of SG = 0.7.

Ex) The same pipe is used to transfer different liquid at the same flow rate. Find the density of that oil knowing that the oil-mercury differential manometer reading is 25 cm.