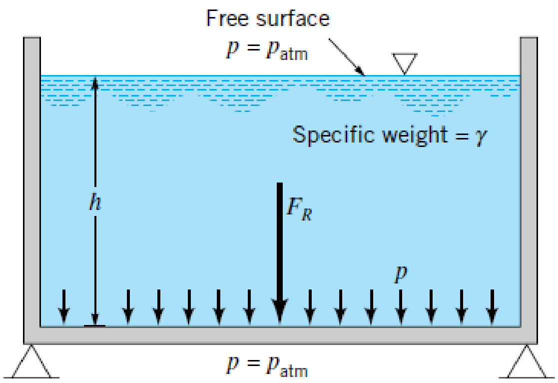
Hydrostatic Force on Submerged Surfaces

When a surface is submerged (مغمور) in a fluid, forces develop perpendicularly to the surface of contact. The calculations of these forces are important in the design of **storage tanks**, **dams, gates, etc.**

Force on Horizontal Surfaces

In horizontal surfaces, the pressure is constant and uniformly distributed. Therefore, the magnitude of the resultant force is simply equal to



$$FR = P \,.\, A = \gamma h \,.\, A$$

Location: it acts through the centroid of the

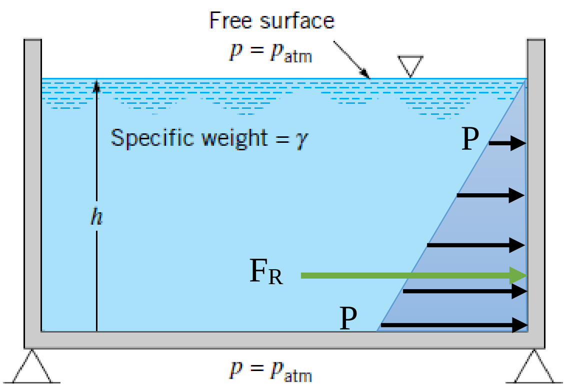
Surface area.

Force on Vertical Surfaces

In horizontal surfaces, the pressure will vary **linearly** with depth Therefore, the magnitude of the resultant force can be calculated from the average pressure:

$$FR = P_{av} \,.\, A = \frac{P_{Max} - P_{Min}}{2} \,.\, A$$

$$FR = \frac{P_{Max}}{2} \,.\, A = \frac{\gamma h}{2} \,.\, A$$

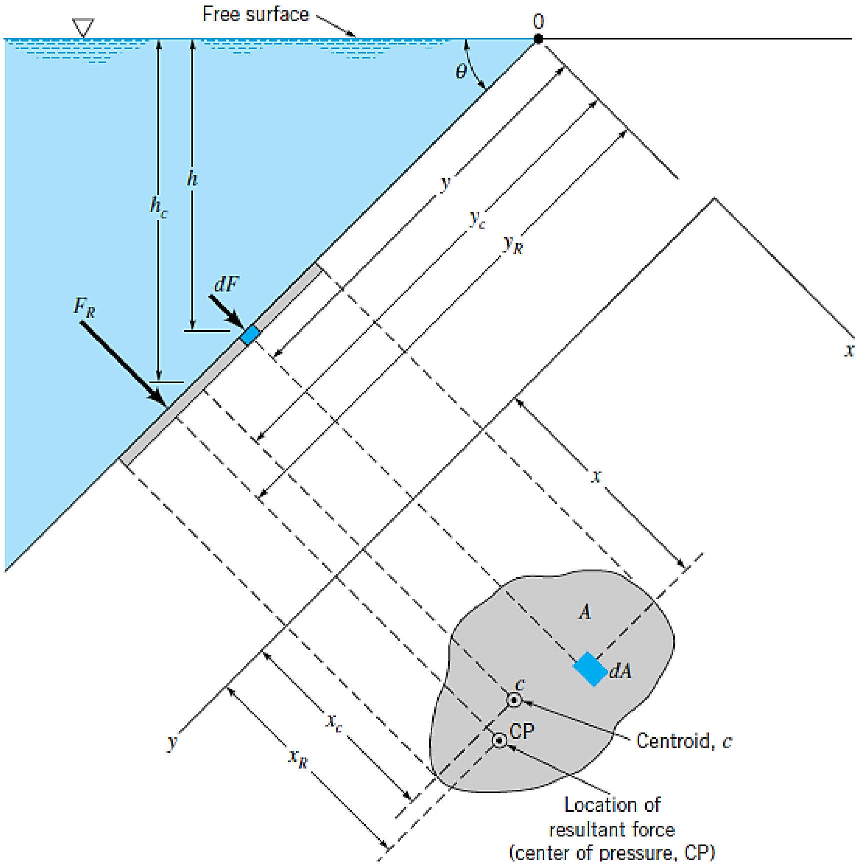


Location: it acts through the centroid of the area

Formed by the pressure on the surface (Triangle)

Force on Inclined Surfaces

In more general case where the surface is inclined, the pressure **will also vary linearly** with depth. To calculate the **magnitude** and **location** of the resultant force, we follow the following procedure:



- The *x–y* coordinate system is defined in the origin (0) and *y* is directed along the surface passing through the centroid.
- The entire surface is divided into a number of small parallel strips (dA). The force on small strip (dF) is calculated as follow: dF = $\gamma h\ dA$
- The magnitude of the resultant force can be found by summing these small forces over the entire surface:

$$F_R = \int_A \gamma h\ dA = \int_A \gamma y \sin\theta\ dA$$

where $h = y \sin\theta$. For constant γ and θ

$$F_R = \gamma \sin\theta \int_A y\ dA$$

- The integral Part is the ***first moment of the area*** *and it is equal to* $A.y_C$

  Thus, the resultant force can be written as:

$$F_R = \gamma A y_c \sin\theta \qquad h_c = y_c \sin\Theta$$

or more simply as

$$\boxed{F_R = \gamma h_c A}$$

h_C: the vertical distance from the **fluid surface** to the **centroid of the area**.

The ***location (Cp)*** of the resultant force can be determined by taking moments around the origin (0) in the X and Y directions:

$$F_R y_R = \int_A y\, dF \qquad dF = \gamma\, h\, dA$$

$$h = y \sin\Theta$$

$$\gamma A y_c \sin\theta\, y_R = \int_A \gamma \sin\theta\, y^2\, dA \qquad dF = \gamma .\, y \sin\Theta\, dA$$

$$y_R = \frac{\int_A y^2\, dA}{y_c A}$$

The integral part is the second moment of area (I_X) with respect to the free surface *(X) axis.*

$$y_R = \frac{I_x}{y_c A}$$

The parallel axis theorem can be used to determine I_X

$$I_x = I_{xc} + A y_c^2$$

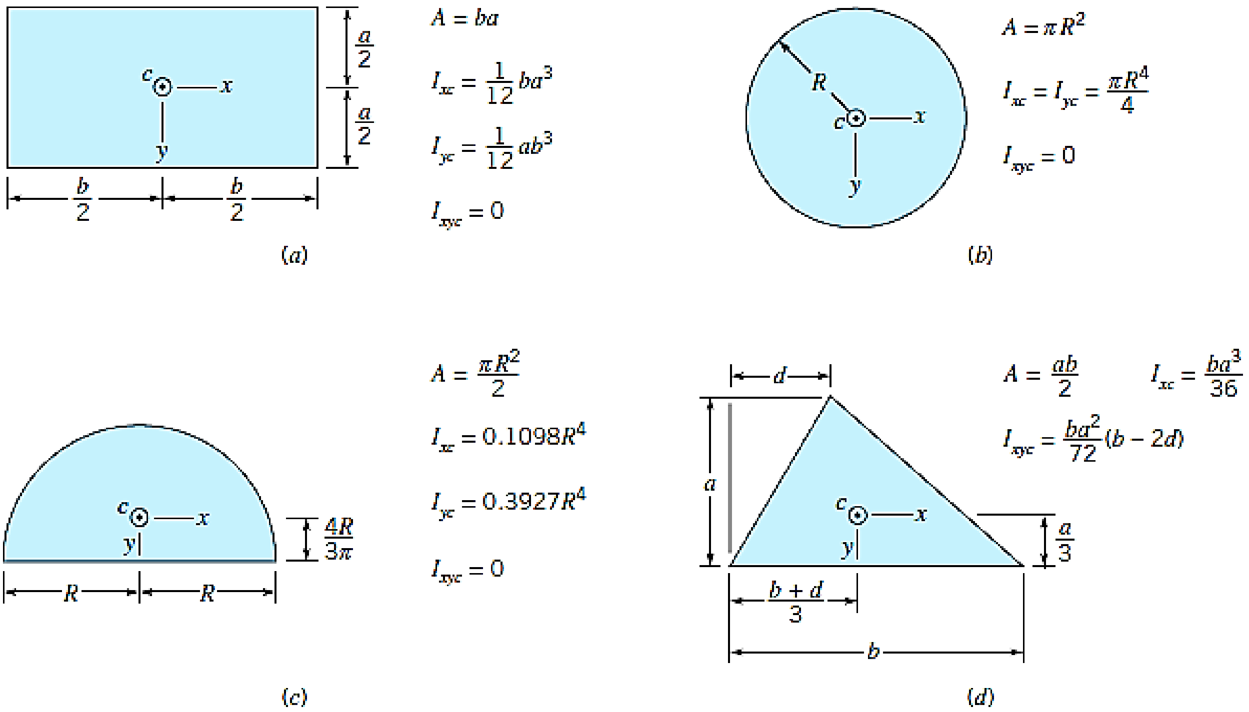
Where I_{XC} is the second moment of area with respect to the *centroid axis*.

$$y_R = \frac{I_{xc}}{y_c A} + y_c$$

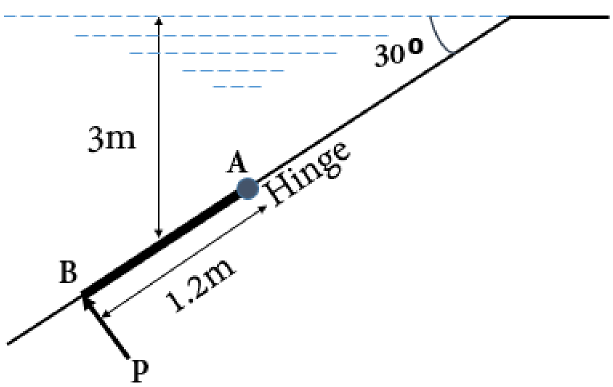
The *x* coordinate (X_R) can be determined in a similar manner by summing moments about the *y* axis.

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

The geometric properties (area, Centroid & moment of area) of some common shapes are given below:



Ex1) An inclined rectangular gate AB (1.2 m height x 3.5 m width) is installed to control the discharge of water. The end A is hinged. Determine the force P normal to the gate applied at B to open it.



Ex2) A 4-m-diameter circular gate is located in the inclined (60^0) wall of a large reservoir containing water. The gate is mounted on a shaft along its horizontal diameter. For a water depth of 10 m above the shaft determine:

1. The **magnitude and location** of the resultant force exerted on the gate by the water.
2. The **moment** that would have to be applied to the shaft to open the gate.