Applications of Derivatives

Ex(1): Skech a graph of the function

$$f(x) = x^4 - 4x^3 + 10$$

Sol:

① $f'(x) = 4x^3 - 12x^2$

Let $f'(x) = 0 \Rightarrow 4x^3 - 12x^2 = 0$

$\Rightarrow 4x^2(x-3) = 0$

$\Rightarrow$ either $4x^2 = 0 \Rightarrow x = 0$ $(0, 10)$

or $(x-3) = 0 \Rightarrow x = 3$ $(3, -17)$

② $f'(x)$: $-\ -\ -$ | $-\ -\ -$ | $+\ +\ +$

decreasing $x < 0$ | 0 | decreasing $0 < x < 3$ | min 3 | increasing $x > 3$

We see that f is decreasing on $(-\infty, 0]$ and $[0, 3)$, and increasing on $[3, \infty)$

③ A local minimum at $x = 3$ $(3, -17)$

④ $f''(x) = 12x^2 - 24x$

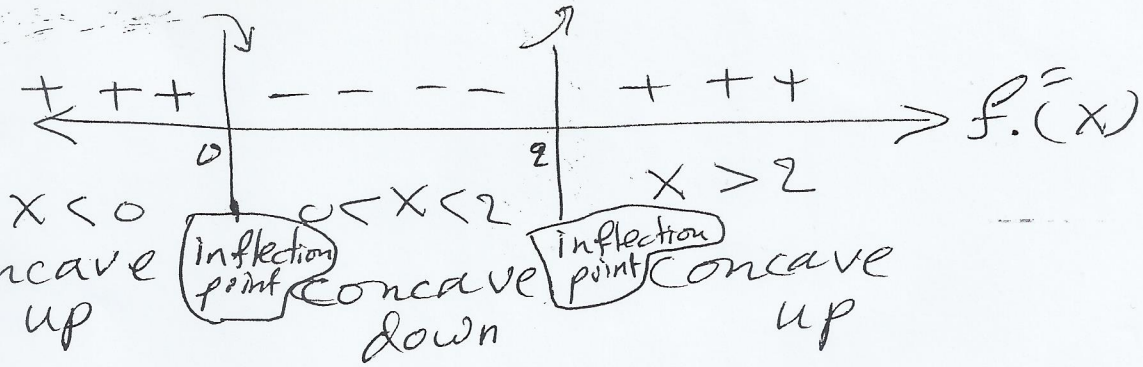
Let $f''(x) = 0 \Rightarrow 12x^2 - 24x = 0$

$\Rightarrow 12x(x-2) = 0$

either $12x = 0 \Rightarrow x = 0$ $(0, 10)$

or $x - 2 = 0 \Rightarrow x = 2$ $(2, -6)$

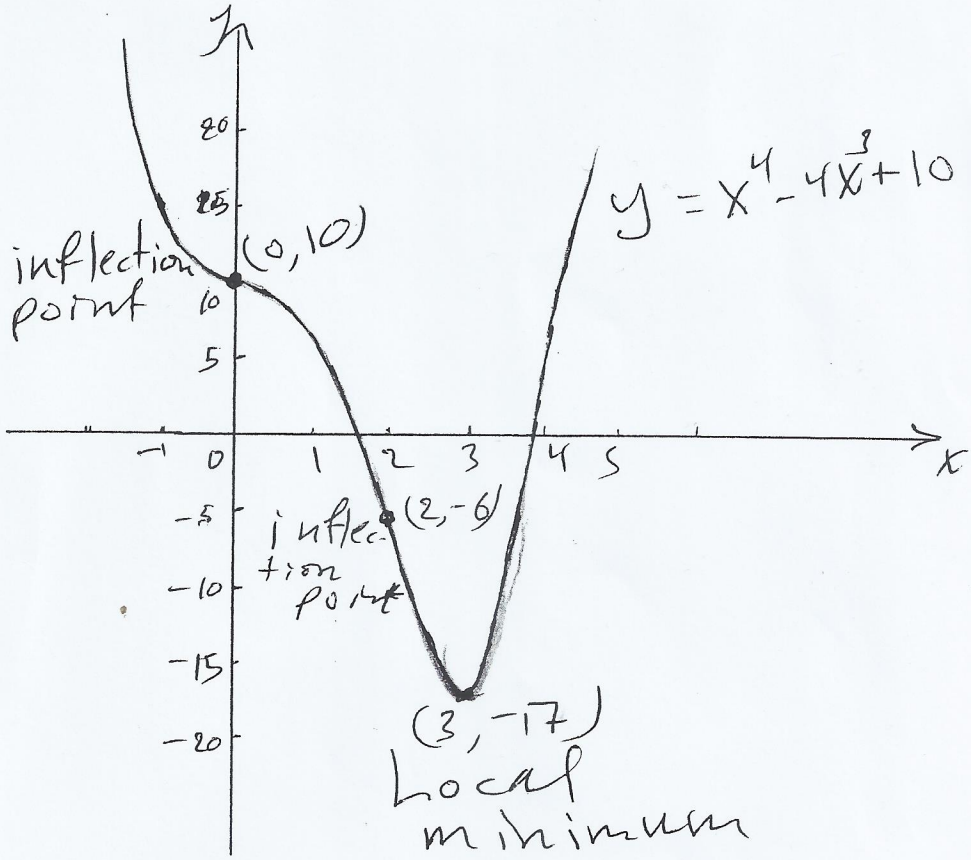
(5)



We see that f is concave up on the intervals $(-\infty, 0)$ and $(2, \infty)$, and concave down on $(0, 2)$.

(6) Plot the curve:

x	y
-2	58
-1	15
0	10
1	7
2	-6
3	-17
4	10
⋮	⋮



H.W:

Exercises (4.4)