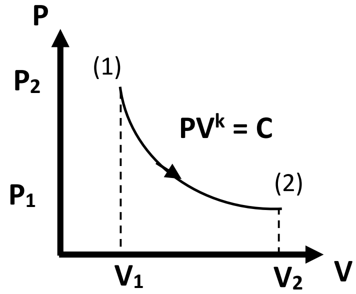
Polytropic Process

During an actual expansion and compression processes of gases, pressure and volume are often related by $PV^k = C$



In Polytropic process, all the state variables (P, T and V) change which is different from the three previous processes in which one of the state variable were held constant.

$$\Delta U = Q - W$$

$$W = \int_{V_1}^{V_2} P\, dV = \int_{V_1}^{V_2} C\, V^{-k}\, dV$$

$$W = C\,\frac{V_2^{-k+1} - V_1^{-k+1}}{-k+1} = \frac{C\,V_2^{-k+1} - C\,V_1^{-k+1}}{-k+1}$$

$$W = \frac{P_2\, V_2^k\, V_2^{-k+1} - P_1\, V_1^k\, V_1^{-k+1}}{-k+1} = \frac{P_2 V_2 - P_1 V_1}{-k+1}$$

$$W = \frac{mR(T_2 - T_1)}{-k+1}\ for\ k \neq 1$$

$$P\,V^k = C \rightarrow P_1\, V_1^k = P_2\, V_2^k$$

(H.W.) Use the ideal gas equation along with the equation ($PV^k = C$) to derive the following relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1}$$

The exponent (***k***) can take on any value from $-\infty$ *to* ∞ depending on the particular process.

- When $k = 0$, the process is an isobaric (constant-pressure) process.
- When $k = \infty$ the process is an isometric (constant-volume) process.
- When $k = 1$ the process is an isothermal (constant-temperature) process.
- When $k = \gamma$ the process is an adiabatic process

Where $\gamma = \frac{C_p}{C_v}$, specific heat ratio.

An adiabatic process is a process during which there is no heat transfer (Q = 0)

There are two ways a process can be adiabatic:

- Either the system is well insulated so that only a negligible amount of heat can pass through the boundary.
- Or both the system and the surroundings are at the same temperature.

An adiabatic process should not be confused with an isothermal process. Even though there is no heat transfer during an adiabatic process, the temperature of a system can still be changed by other means such as work.

$$\Delta U = \cancel{Q} - W$$

$$W = -\Delta U$$

$$\Delta U = m\, C_v\, \Delta T$$

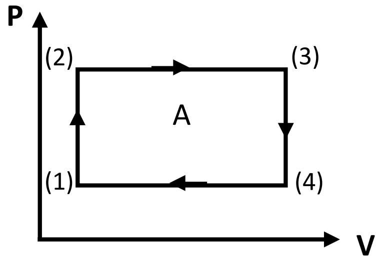
$$W = \int_{V_1}^{V_2} P\, dV = \frac{P_2V_2 - P_1V_1}{-\gamma+1}$$

(H.W.): Starting from the 1st law of thermodynamic, Prove that $T_1\, V_1^{\gamma-1} = T_2\, V_2^{\gamma-1}$ for an adiabatic process.

Ex$_1$) Air undergoes a Polytropic compression in a piston–cylinder assembly from P_1 = 1 bar, T1 = 22 ^{o} C to P_2 = 5 bars. determine the work and heat transfer per unit mass, if K = 1.3.

Thermodynamics Cycles

A system is said to have completed a cycle when it starts at a certain state then goes through a series of thermodynamics processes and finally end up at the same state. That is, for a cycle the initial and final states are identical.



$\Delta U = Q - W$ 1st law of thermodynamic

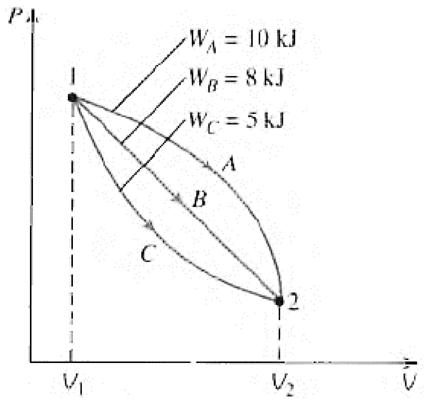
The work done during a cycle is simply equal to the area inside the curve (A). Also, it is simply the sum of the individual work done of the processes that make up the cycle.

$$W = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

The cycle (1-4) shown above produces a positive net work output because the work done by the system during the expansion process (2-3) is greater than the work done on the system during the compression part of the cycle (4-1***)*** and the difference between these two is the net work done during the cycle (A). that is

$$W = W_{2-3} - W_{4-1}$$

A gas can follow several different paths (A or B or C) as it expands from state 1 to state 2. In general, each path will have a different area underneath it, and since this area represents the magnitude of the work, the work done will be different for each process. This is because work is a path function which means it depends on the path followed as well as the end states.



For a complete cycle, there is no change in the internal energy (ΔU = 0) because the system returns to the same initial state that has the same volume, pressure and hence the same temperature (ΔT = 0).

In conclusion, during a cycle ΔU is always zero because the internal energy is a state function and during a cycle the initial and final states are identical, thus all the state variable remains constant. While W cannot be zero because it is path function.

Therefore, the net work output during a cycle is equal to net heat input.

$$W = Q_{net} = Q_{\ in} - Q_{out}$$

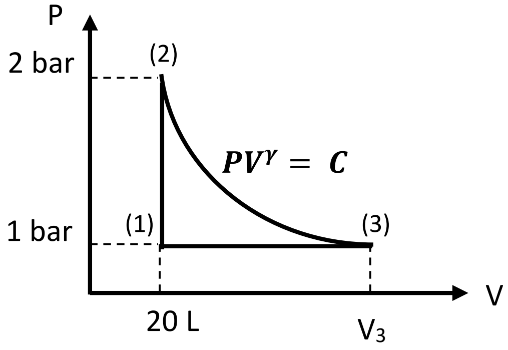
Ex$_2$) 0.5 kg of air undergoes a thermodynamic cycle consisting of three processes.

1. Process 1–2: constant volume
2. Process 2–3: constant temperature expansion
3. Process 3–1: constant pressure compression

At state 1, the temperature is 300 K, and the pressure is 1 bar. At state 2, the pressure is 2 bars. By employing the 1st law of thermodynamic and ideal gas equation of state:

a) Sketch the cycle on *p*–*v* coordinates.
b) Determine the temperature at state 2 in Kelvin.
c) Determine the specific volume at state 3 in m^3/kg.
d) Work done during the entire cycle
e) Internal energy during the compression process.

Ex₃) 1 mole of a gas initially at 245 K undergoes the following thermodynamic cycle



Calculate the work done by the gas. Take Cv = 3/2 Ru