هندسة الاتصالات الرقمية

قسم هندسة الاتصالات

المرحلة الثالثة

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DIGITAL COMMUNICATIONS

- **BASE BAND SIGNAL**
- **DIGITAL CARRIEAR MODULATION**
- **INFORMATION THEORY**

Digital Communications Syllabus

Base Band Signals

1 – Introduction to Digital Communications
2 – Sampling theory
 Ideal sampling
 Natural sampling
3 – Pulse Amplitude Modulation (PAM)
 Equations and waveform in time domain
 Generation and Detection Circuits
 Required Channel Bandwidth
4 – Pulse Time Modulation (PTM)
 Pulse Width Modulation (PWM) and Pulse Position Modulation (PPM)
 Equations and waveform in time domain
 Generation and Detection Circuits
 Required Channel Bandwidth
5 – Time Division Multiplexing for sampled Signals (TDM)
6 – Pulse Code Modulation (PCM)
 Equations and waveform in time domain
 Generation and Detection Circuits
 Signal to Noise Ratio (S/N)
 Required Channel Bandwidth
7 – Time Division Multiplexing for PCM Signals (TDM-PCM)
8 – Compression – Expansion technique
9 – Delta Modulation (DM)
 Equations and waveform in time domain
 Generation and Detection Circuits
 Signal to Noise Ratio (S/N)
 Required Channel Bandwidth
10 – Line Coding for base band signals
11 – Error Detection and Correction technique
12 – Inter-symbol interference problem in Base band signal
13 – Power Spectral Density (PSD) for base band signals
14 – Probability of error for base band signals

Digital Carrier Modulation

1 – Introduction to Modulation process

2- Amplitude Shift Keying (ASK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

3 – Matched Filter

4 – M-ary ASK

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

5- Frequency Shift Keying (FSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

6- M-ary FSK

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

7 – Phase Shift Keying (PSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

8 – Carrier Recovery Circuits

9 Deferential Phase Shift Keying (DPSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

10– M-ary PSK (QPSK, 8-PSK, OQPSK, MSK, GMSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

12 – Amplitude-Phase Shift Keying (APK) (16_QAM)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

13 – Comparison and applications for the digital carrier modulation

Information Theory

1 – Introduction to information theory
Historical review, Shannon para diagram and applications
2 – Information Measurements and Entropy
3 – Source Coding (Shannon and Huffman)
Algorithm, Code length, Average code word and the source code efficiency
4 – Channel Coding
Forward error correction
Error Detection
5 – Information Channel
Channel modeling
Channel types
Channel capacity

References

1 - "Digital and analog communication" 2001ByL.W.Couch (sixth edition)

2 – "Introduction to Communication Systems" 1992 ByF.Stremler

3 – "Digital Communications" 2004 By Abba Kattoush

4 – "ELEMENTS OF INFORMATION THEORY" 2006 By THOMAS M. COVER and JOY A. THOMAS

5. Communication Systems by Simon Haykin 4th edition
6. Digital Communications by Proakis 5th edition

DIGITAL COMMUNICATIONS

Lecture (1-1): INTRODUCTION TO DIGITAL COMMUNICATIONS

- **Introduction**
- **Historical Reviw**
- **Digital and Analog Source and System**
- **Advantages and Disadvantages of Digital Communications**
- **Introduction to Signal Types**
- **Sampling Theory**

Introduction:

Digital communication is rapidly advancing applications area. The increase in demand for voice, data and internet connections, is the principal driving force behind the growth in telecommunications. The design and manufacturing of hardware and software for digital communication networks are among the fastest growing engineering areas. In order to adapt and contribute effectively to these changes electrical engineers need to acquire a solid foundation and understanding of digital communications.

Historical Reviw:

In the following table contains the important events in the history of data and digital communication

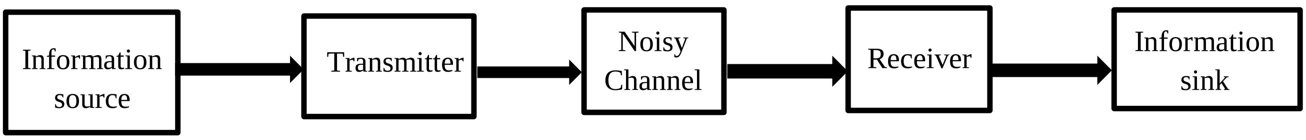
1837	Line telegraphy invented
1866	First permanent transatlantic telegraph

1901	First transatlantic radio message by Marconi, UK to Canada
1928	Gaussian thermal noise paper of Johnson and Nyquist
1937	PCM proposed by Reeves
1943	Matched filtering proposed by North
1945	Early computers constructed
1946	ARQ system developed by Duuren
1948	Proofs of sampling theorem Mathematical theory of communications by Shannon
1950	Signal space theory applied to communication Beginnings of computer software
1958	Matched filter applied to communications
1960	Error-correcting codes begin rapid development
1962	T1-carrier system by Bell Laboratories
1966	Optical fiber proposed by Kao & Hockman Packet switching
1967	Forney proposed the trellis Viterbi proposed his algorithm
1970	Medium scale data networks by ARPA/TYMNET LANs, MANs and WANs Microprocessors appear Large-scale integrated circuits appear
1971	The term ISDN coined by CCITT
1974	Internet concept by Cerf &Kahn
1975	Speech and image digitization begins rapid development
1976	Bandwidth-efficient coded modulations begin to appear Digital telephone trunks first installed
1978	Navistar GPS launched by Global

1980	Digital optical fiber telephone trunks begin to be installed OSI 7 layer reference model adopted by ISO
1981	HDTV demonstrated by NHK in Japan IBM PC is introduced
1985	ISDN basic rate access in UK
1986	SONET/SDH introduced in USA
1990	Use of the Internet accelerators
1991	Beginning of digital signal processing with microprocessor
1992	GSM cellular system in Europe
1993	PCN concept launched
1994	IS-95 CDMA specification by Qualcom

Digital and Analog Source and System:

The purpose of communication system is to transmit information from the source to the sink. A typical communication system as shown in the following figure which consist of the main component:

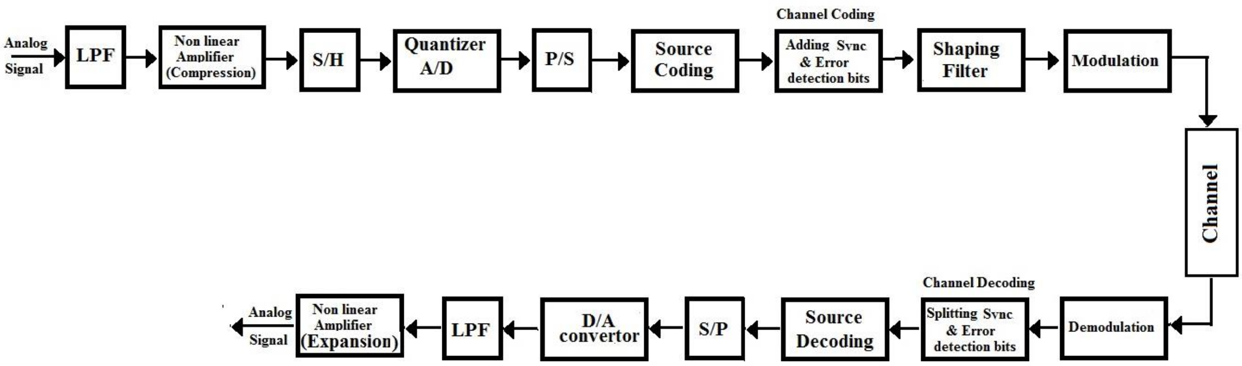


1. The Information source: such as television picture, human voice, data, etc. Information usually converted to base-band electrical waveform called information signal by an input transducer.
2. The transmitter: usually convert the base-band signal to band pass signal appropriate to a practical transmission medium for efficient transmission.
3. The channel: it the medium through which the transmitter output is sent such as wire, coaxial cable, optical fiber, etc.

4. The receiver: it processes the signal received from the channel by eliminating the signal modifications made at the transmitter and the channel.
5. The destination (sink): which is the consumer of information.

Depending on the type of source and destination, the communication system can be a digital communication system or an analog communication system.

a. **Analog communication system** is defined as system that transfers information from an analog source to the destination, where an analog source producing infinite (continued) possible massages. For example, the temperature of certain location can vary over a continues range and can assume an infinite number of possible messages (values).
b. **Digital communication system** is defined as system that transfers information from a digital source to the destination, where a digital source producing a finite set of possible massages. For example, printed language consists of 26 letters, 10 numbers, space and several punctuation marks. Thus, a typewriter is a digital source with finite number of massages that can be emitted.



Block diagram of the Digital Communication system

Advantages and Disadvantages of Digital Communications:

Digital communication systems usually represent an increase in complexity over the equivalent analog systems. However, most of communications have become digital because of advantages of digital communications over analog communication. Some of the reasons are:

1) Error can be corrected. possibility of channel coding to minimize effects of noise and interference. Digital communication is rugged in the sense that it is more immune to channel noise and distortion: it is inherently more efficient than analog in realizing the exchange of SNR for bandwidth. Also regenerative repeaters along the transmission path can prevent the accumulation of noise along the path. Digital signals can be coded to yield extremely low error rates and high fidelity.
2) Digital hardware implementation is flexible and permits the use of microprocessors, minicomputer, and digital switching and large-scale integrated circuits. It is easier and more efficient to multiplex several digital signals than analog
3) Signal manipulation (e.g. encryption) is simple. In digital dealing is with numbers rather than waveforms. Numbers can manipulate by simple logic circuits or microprocessors. Complex operations can easily be performed in order to accomplish signal processing, security in transmission and encryption or channel encoding functions.
4) New services. E-mail, multimedia, digital voice and voice broadcasting, airline booking systems, computer modems, electronic banking, electronic marketing and electronic government are one of the new services in these days, where these services are difficult to imagine in an analog word.

5) Standardization of signals, irrespective of their type, origin, or the services they support, leading to integrated services digital networks (ISDN).
6) Compatibility and flexibility. Digital signals are possible to multiplex and transmit in similar shared medium. Network functions such as switching and multiplexing are easier to implement to digital signals. Control and servicing information can be combined with information signals. All sorts of features such as disc-player music search and telephone voice mail become economic.
7) Cheap hardware. Digital hardware has become very cheap which makes all other advantages cheap to buy.
8) Message security. Along encryption is fundamentally difficult, but the encryption of symbols is just fundamentally not difficult.
9) Easy with which bandwidth, power and time can be treated off in order to optimized the use of these limited resources.
10) Facility for source code for data compression.
11) Increased demand for data transmission.
12) Increased scale of integration, sophistication and reliability of digital electronics for signal processing, combined with decrease cost.
13) Greater dynamic range (differences between largest and smallest values) is often possible.
14) Cost of transmission.

Some disadvantages of digital communication system are listed below:

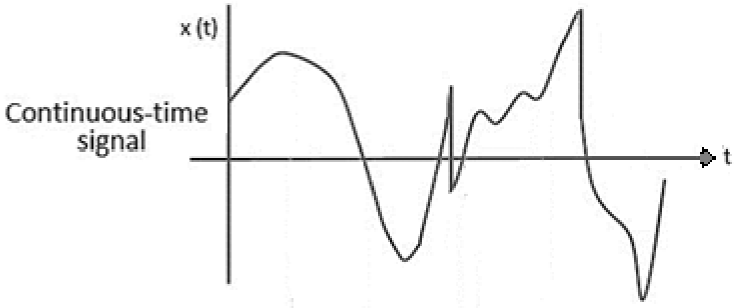
1. Usually represent an increase in complexity over the equivalent analog system

2. Generally, requires more bandwidth than analog.
3. Synchronization is required.

Introduction to Signal Types:

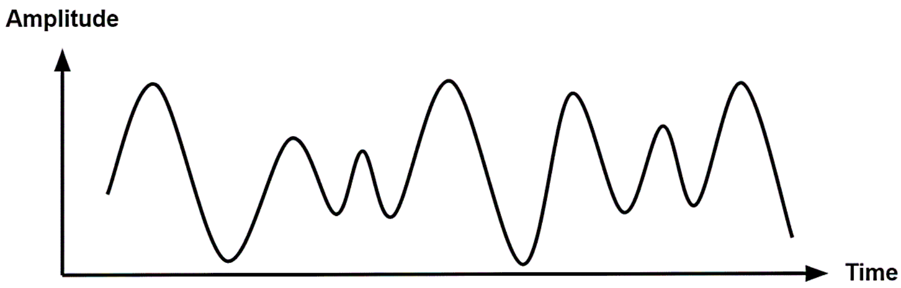
In general, the signals divide into two types:

1. **Continuous signal:** It is a signal that has an amplitude over all time, regardless of its value.



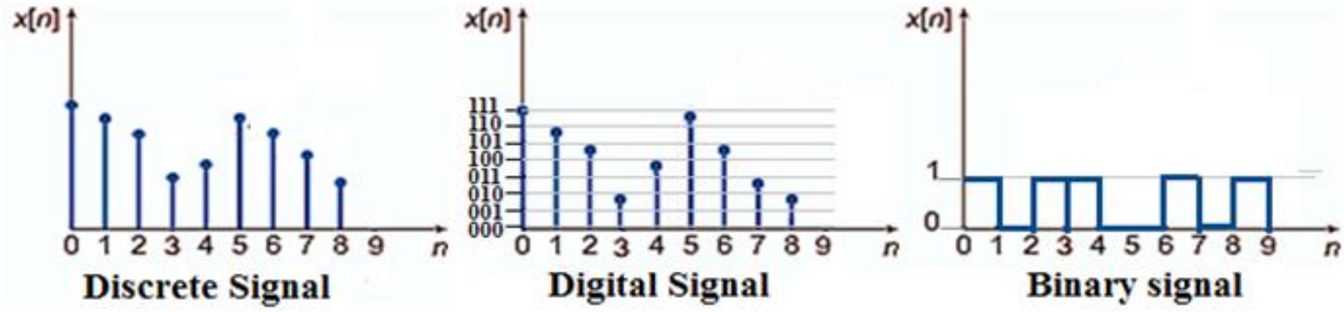
A continuous signal has a special case when the amplitude of the signal changed smoothly with time it's called an **Analog signal** and any points must achieve the following equation:

$$x(t) = \frac{x(t-1)+x(t+1)}{2}$$



Discrete signal: it's a signal has an amplitude in specific parts of the time, where the amplitude values may be integer or decimal value.

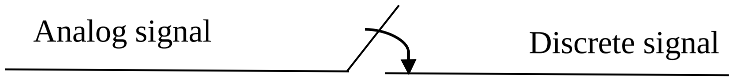
Digital signal it's a discrete signal but the amplitude values was quantized into limited value (if two value only (1 or 0) only represent binary signal or base band signal).



Sampling Theory:

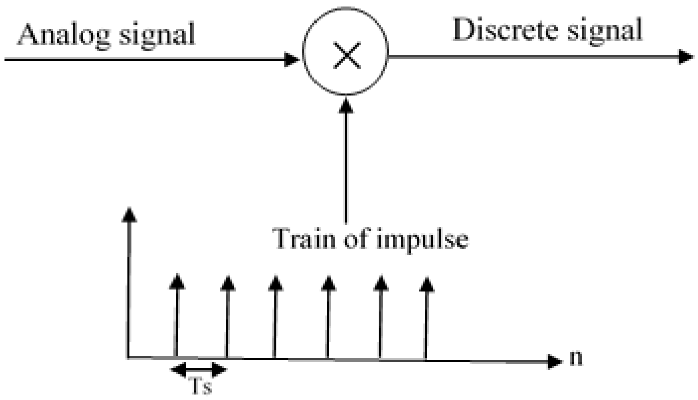
Sampling: is a signal converted with conditions necessary to convert an analog signal into discrete signal without loss of information.

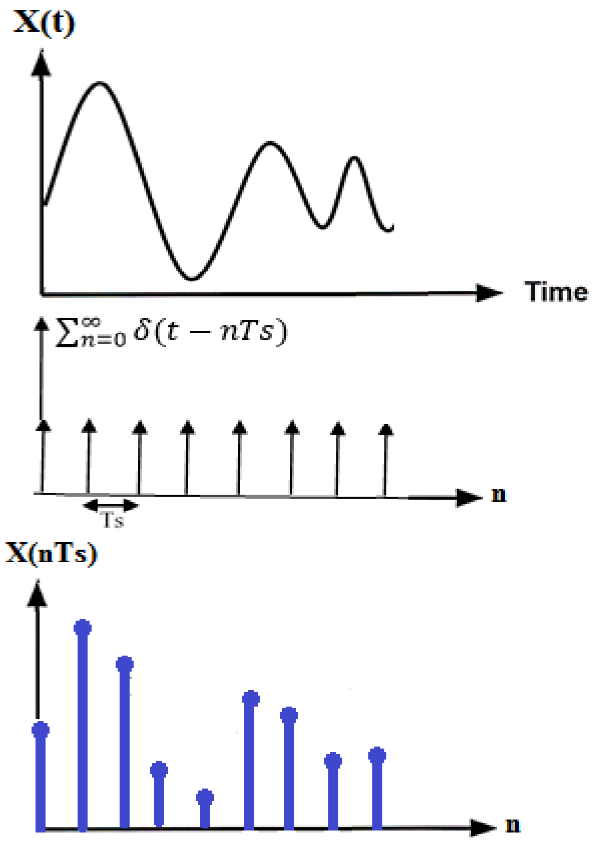
To understand the sampling technique, suppose that there is a key working on forming the analog signal.



a) Ideal Sampling

Ideal sampling consists of multiplying the analog signal by train of impulses as shown in the following figure.





$Train\ of\ impulses\ (S(t) = \sum_{n=0}^{\infty}\delta(t-nTs)$

$y(nTS) = x(t) * \sum_{n=0}^{\infty}\delta(t-nTs)$ in Time domain

Using Fourier Series: for the term ($\sum_{n=0}^{\infty}\delta(t-nTs)$)

$A(t) = \sum_{n=-\infty}^{\infty} C_n e^{-jn\omega st}$

$\int_{-\infty}^{\infty} F(t)\,\delta(t)\,dt = f(0)$

Where: $C_n = \frac{1}{T}\int_{-T/2}^{T/2} A(t)e^{-jn\omega st}\,dt$

$e^{-jn\omega st} = A\cos nwst + jB\sin nwst$

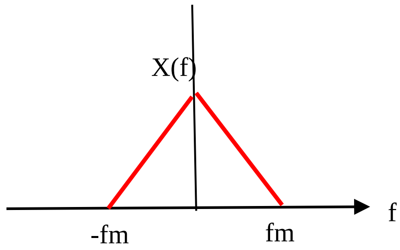
$C_n = \frac{1}{Ts}\int_{-\frac{T}{2}}^{\frac{T}{2}}\sum_{n=0}^{\infty}\delta(t-nTs)e^{-jn\omega st}\ dt$

$C_n = \frac{1}{Ts}$

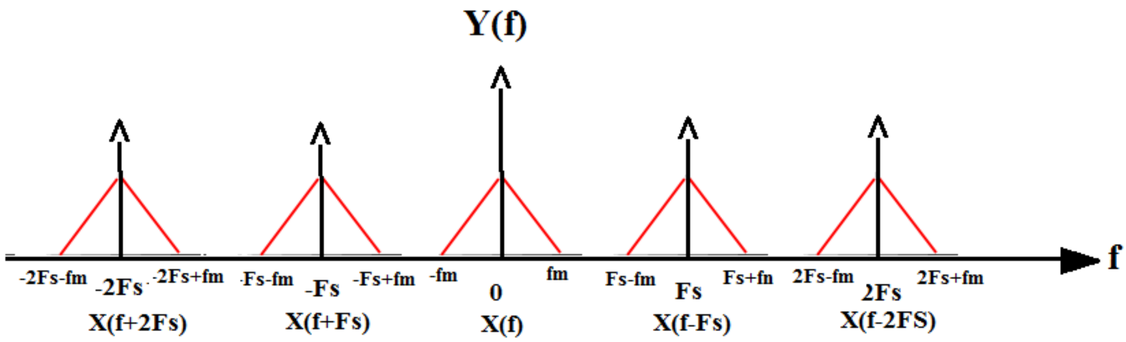
$y(nTs) = x(t) * \sum_{n=-\infty}^{\infty}\frac{1}{Ts}e^{-jn\omega st}$ using Fourier Transform

$Y(f) = \frac{1}{Ts}\sum_{n=-\infty}^{\infty} X(f-nfs)$

If the spectrum of the analog input signal are;



The spectrum of the output (Sampled Signal can be as shown :-



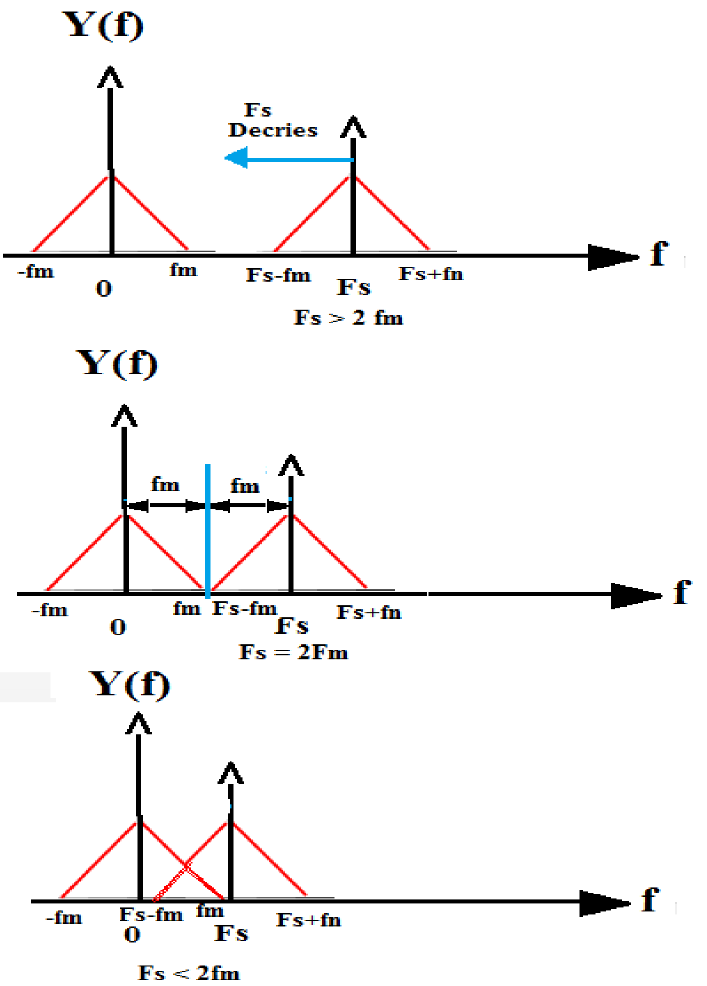
Sampling Conditions

1 – The signal that be sampled must be an **analog** Signal

2 – The analog signal must be band limited.

3 - The sampling time interval must be equal

4 –The sampling frequency must be equal or grater than 2* Fmax (How?)



Note :-

Practically $\boxed{fs = 2\ f_{max} + 20\%\ f_{max}}$

As the modulation is loading the signal on carrier to increase the frequency so increase the speed of transmitting data, also the sampling theory is considered as modulation because the frequency of the signal increase rather than double of maximum frequency of the signal.

DIGITAL COMMUNICATIONS

Lecture (2-1): PULSE AMPLITUDE MODULATION (PAM)

- **Equations and Waveform in Time Domain**
- **Generation and Detection Circuits**
- **Required Channel Bandwidth**

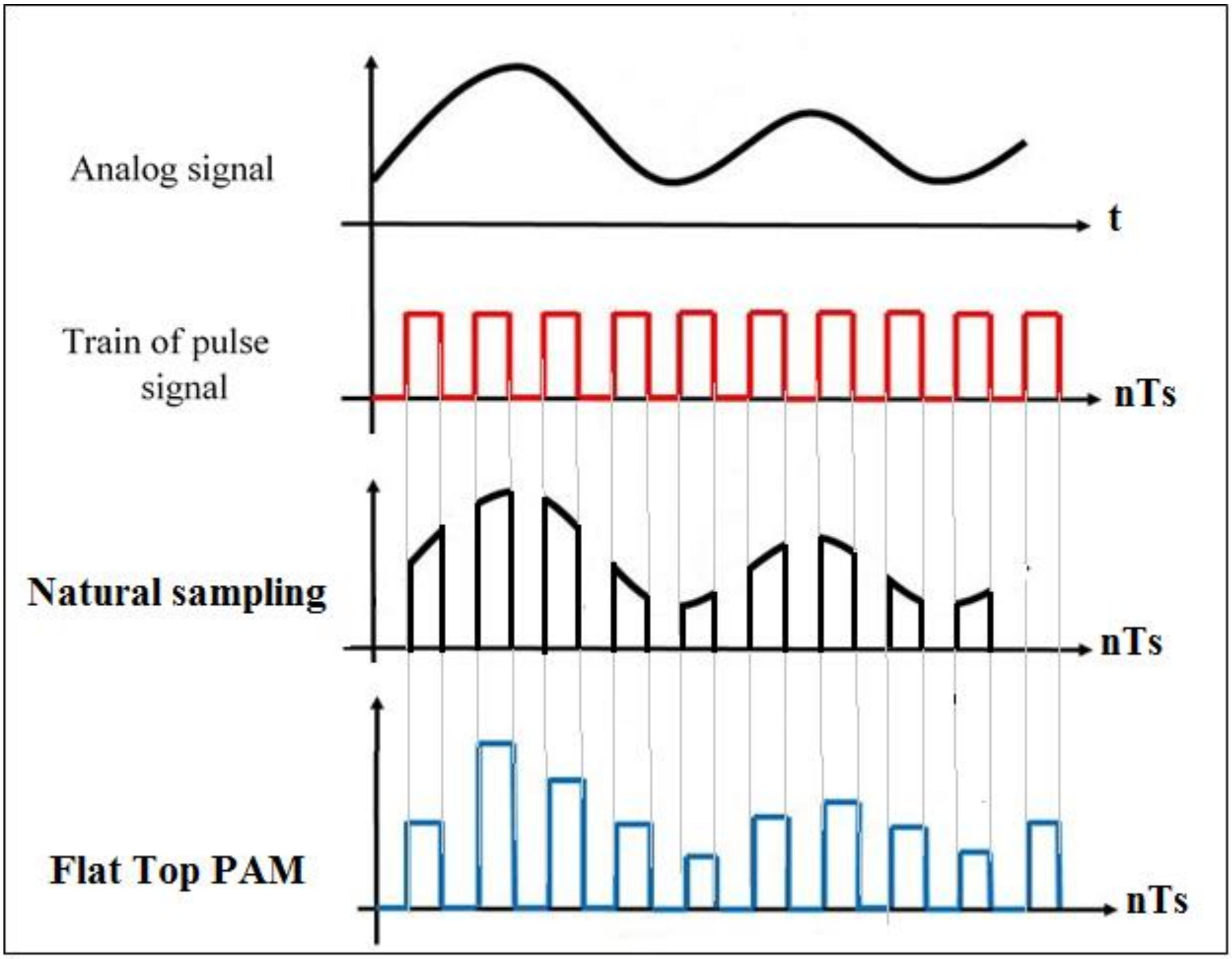
PULSE TIME MODULATION (PTM)

PULSE WIDTH MODULATION (PWM) AND PULSE POSITION MODULATION (PPM)

- **Equations and waveform in time domain**
- **Generation and Detection Circuits**
- **Required Channel Bandwidth**

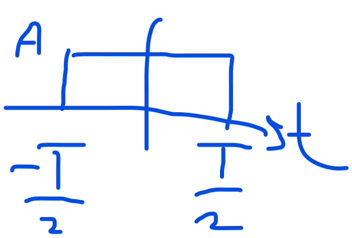
Equations and Waveform in Time Domain:

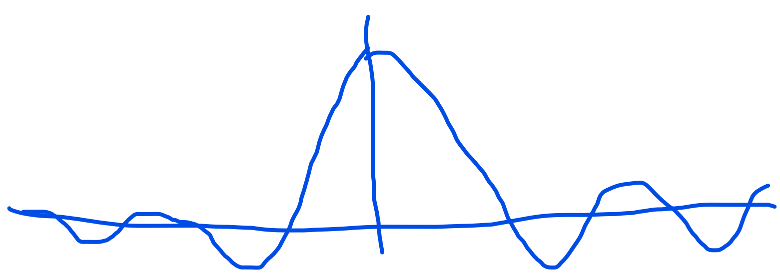
Pulse amplitude modulation (PAM) in engineering term that is used to describe the conversion of the analog signal to a pulse-type signal in which the amplitude of the pulse denotes analog information. PAM is studied first because the analog-to-PAM conversion process is the first step in the converting an analog wave form to a PCM (digital) signal. The waveform of the PAM signal can be shown in the figure bellow.



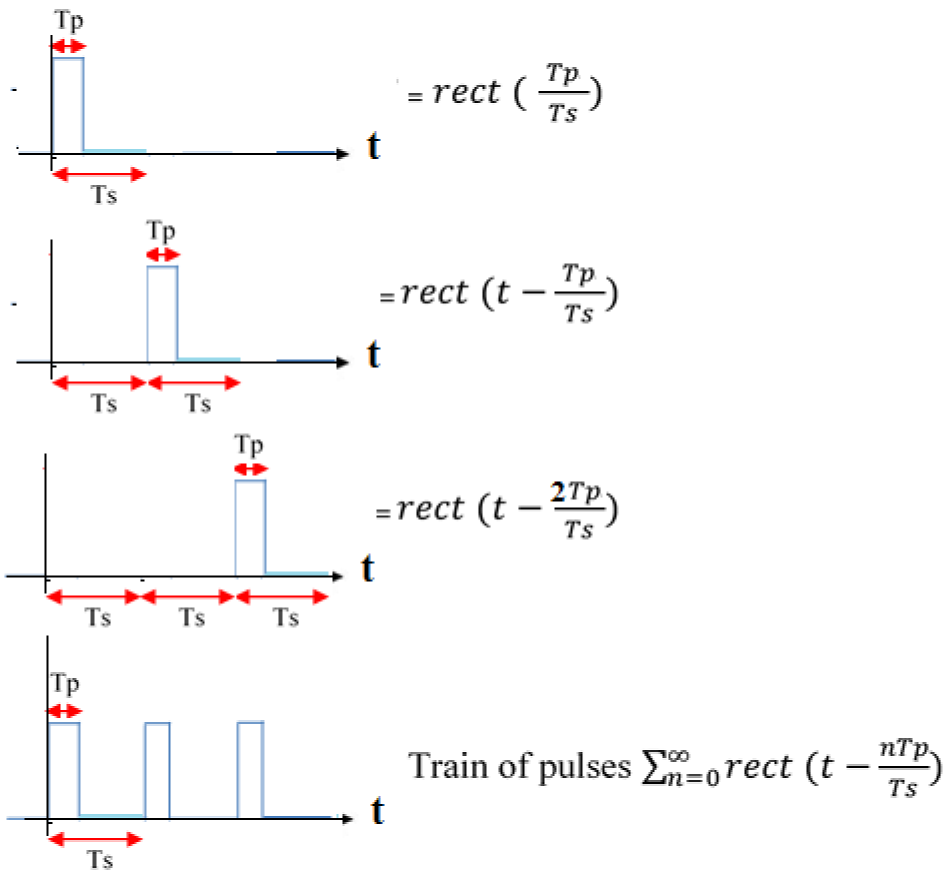
In the sampling theory the impulses is not real signal but it supposed to facilitated the derivation, so it would replace by a train of pulses with width equal

τ Tp and period Ts. To evaluate the main equation for PAM, the first we must find the formula of the train of pulses from the mathematical form of pulse signal (rectangle signal) as shown:-





$$A\,\mathrm{rect}\left(\frac{t}{T}\right) \xleftrightarrow{F.T} AT\,\mathrm{sinc}(fT)$$



If we substitute the train of pulse formula instead of the train of impulses formula in the main equation of the sampling process we can get

$\tau = Tp$

$y(nTs) = x(t) \otimes \sum_{n=0}^{\infty} \delta(t - nTs)$ train of impulse ideal sampling eq.

$y(nTs) = x(t) \otimes \sum_{n=0}^{\infty} rect(t - \frac{nTp}{Ts})$ train of pulses PAM equation

To specify the spectrum of the PAM signal we must applying the same steps that we approach in the ideal sampling derivation.

Convert the rect part to sin and cos using fourier series

$$C_n = \frac{1}{Ts} \int_{\frac{-Ts}{2}}^{\frac{Ts}{2}} \sum_{n=0}^{\infty} rect\left(t - \frac{nTp}{Ts}\right) * e^{-jn\omega st}\, dt$$

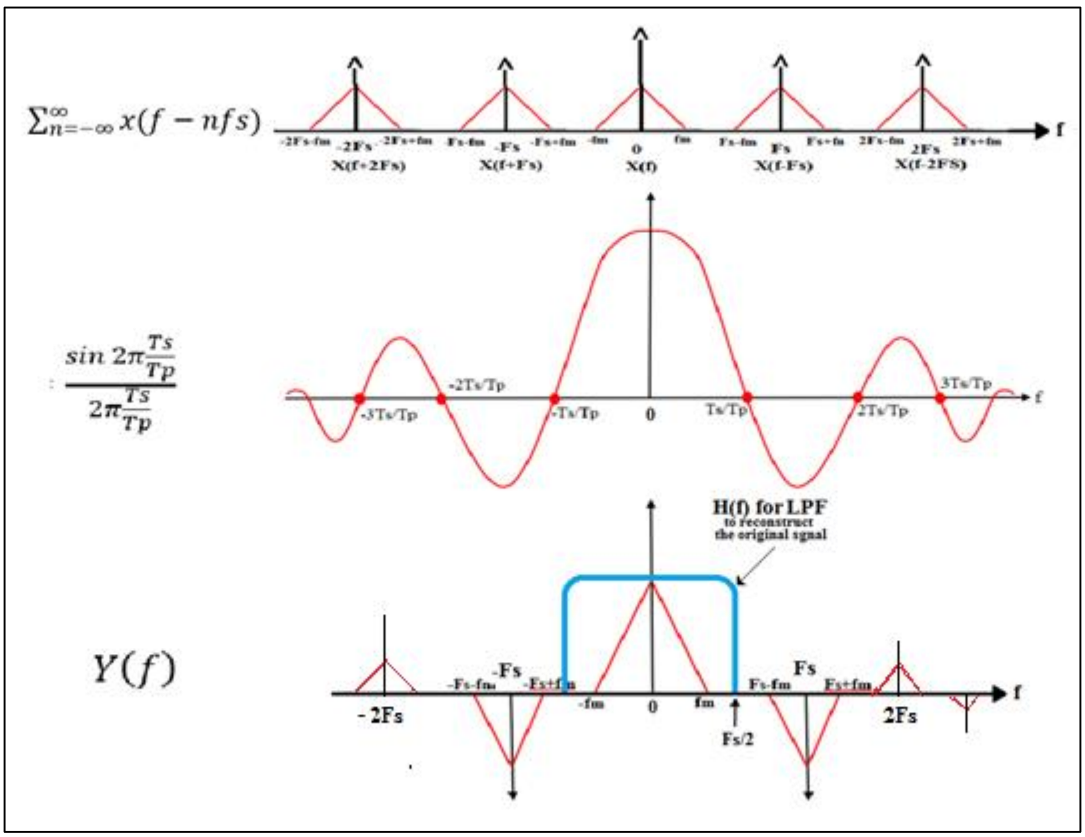
$$C_n = \frac{1}{Ts} \cdot \frac{\sin 2\pi\frac{Ts}{Tp}}{2\pi\frac{Ts}{Tp}}$$

$$\therefore\ y(nTs) = x(t) \otimes \frac{1}{Ts}\sum_{n=-\infty}^{\infty} \frac{\sin 2\pi\frac{Ts}{Tp}}{2\pi\frac{Ts}{Tp}} * e^{-jn\omega st}$$

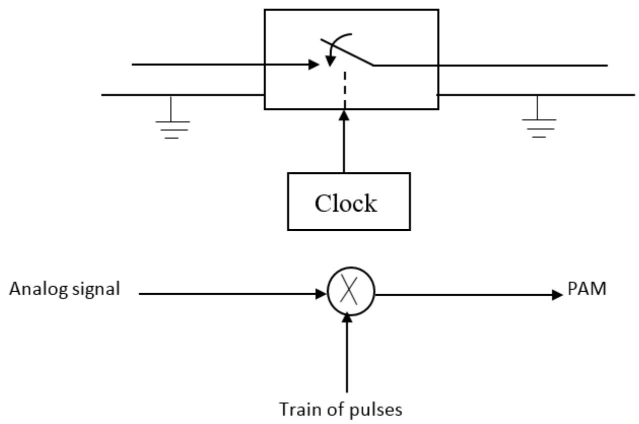
By using fourier transform, the signal equation in frequency domain:

$$Y(f) = \frac{1}{Ts}\sum_{n=-\infty}^{\infty} x(f - nfs) * \frac{\sin 2\pi\frac{Ts}{Tp}}{2\pi\frac{Ts}{Tp}}$$

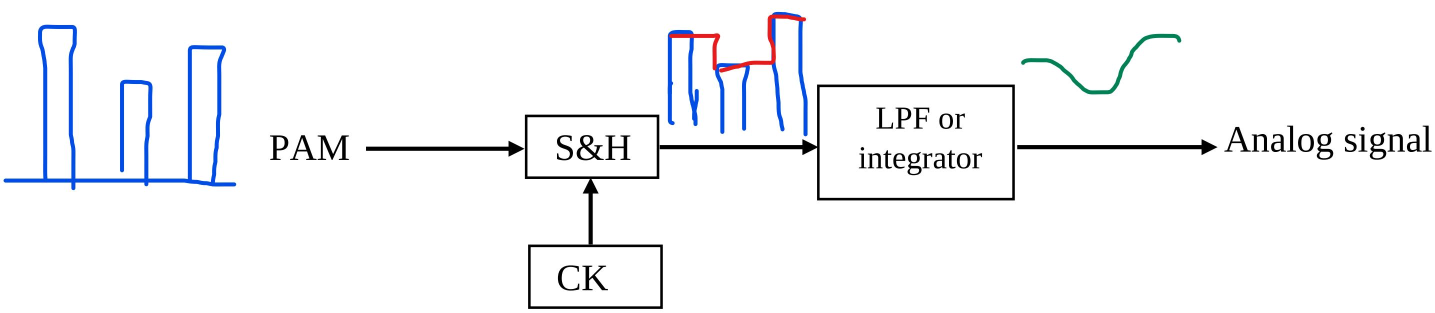
The above equation can be considered in the following figure.



Generation and Detection Circuits:



Generation circuits for PAM



Detection circuit for PAM

Required Channel Bandwidth:

By taking the first component only because it has the half power, so we try to make the top of the sinc signal be flat which is achieved by reducing the Tp.

$T_s \gg T_p$

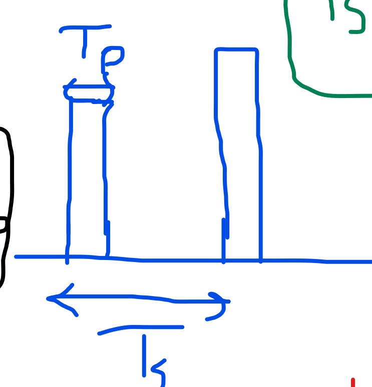
$T_p : \tau$

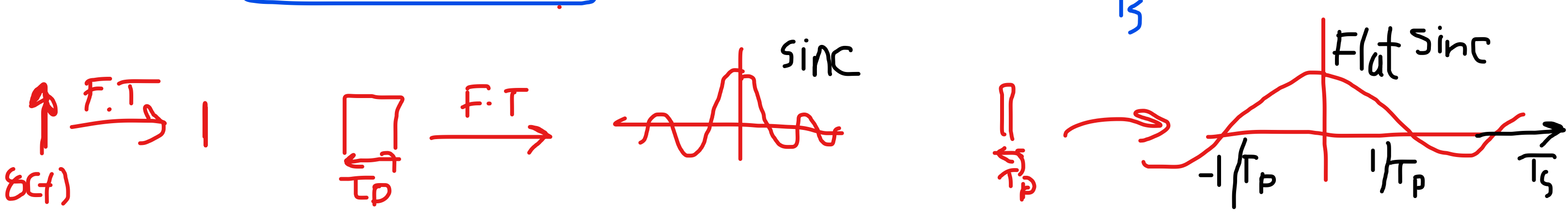
Where Tp/Ts ≤ 0.1

$\therefore$ *requied channel BW* $= \frac{1}{2} fs$

$\boxed{BW_{PAM} = \frac{1}{2T_p}}$

$\boxed{T_p = 0.1\, T_s}$





PULSE TIME MODULATION (PTM)

In pulse time modulation (PTM) the sample values of analog signal are encoded on to the time axis of a digital signal and they are analogous to angle modulation. Two types of PTM exist pulse width modulation and pulse position modulation. PTM has a great immunity to additive noise when compared with PAM. In general, PTM and PAM are easier to generate and detect than pulse code modulation PCM, since PCM requires analog to digital converter (ADC).

PTM signaling is not widely used in communication across channels because relatively wide channel band width is needed. However, PTM signal may be found internally in digital communication terminal equipment.

PULSE WIDTH MODULATION (PWM)

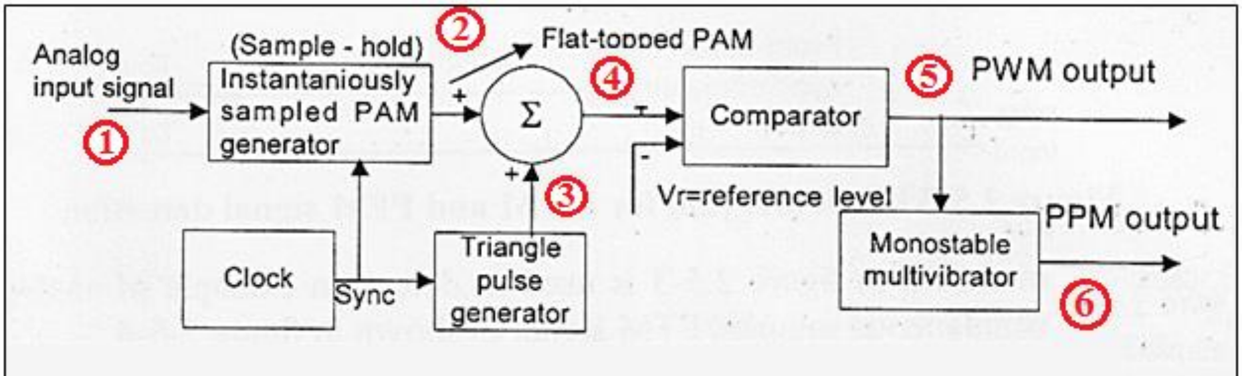
In pulse width modulation the width of each pulse the varies in accordance with the instantaneous sample value of the information signal. The larger sample value, the wider is the corresponding pulse.

Since the pulse width is not constant, the power of the wave form is also not constant. Thus, as the signal amplitudes increase, transmitted power also increases, while bearing no additional information.

PULSE POSITION MODULATION (PPM)

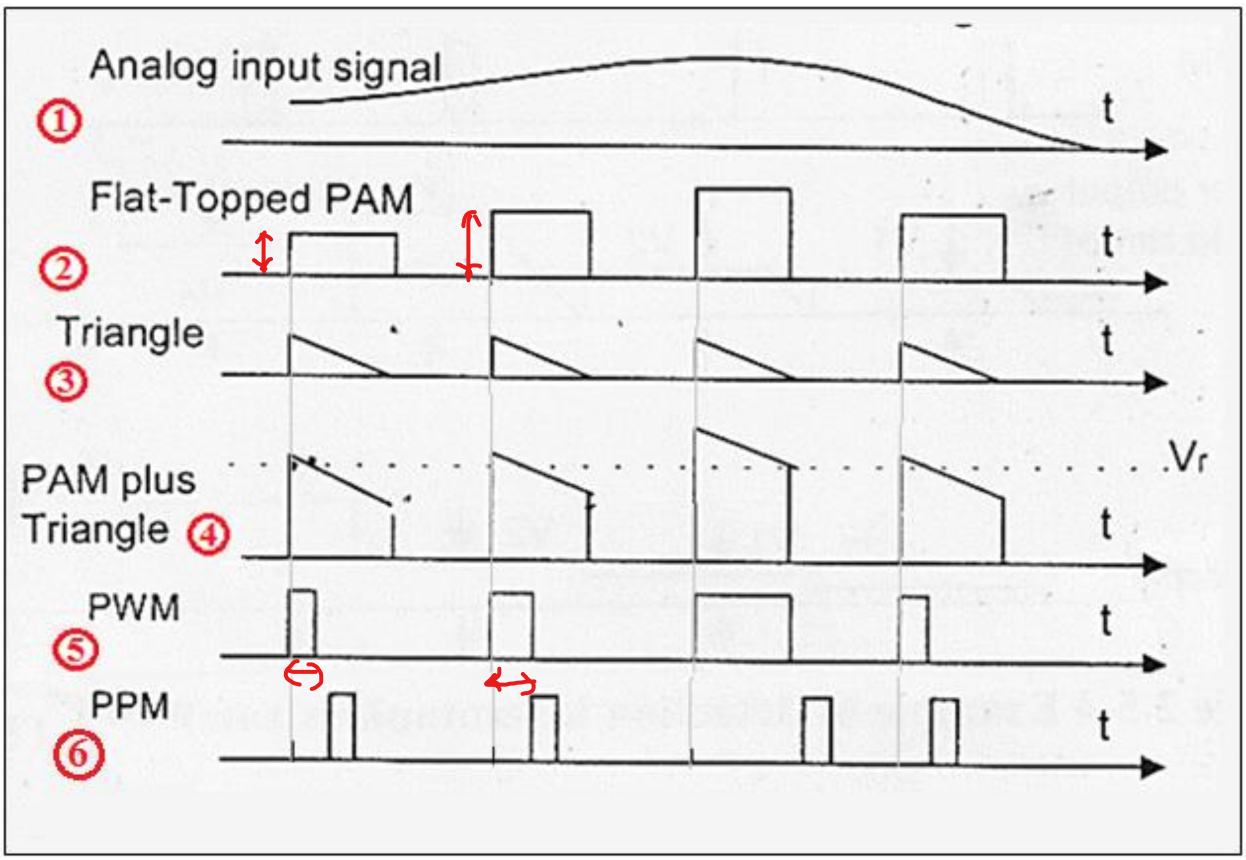
In PPM the position of a pulse relative to its not modulated time of occurrence is varied in accordance with message signal. PPM has the advantage of PWM without the problem of a variable power that is a function of signal amplitude. the two types of modulation (PWM and PPM) are usually called PTM.

Generation and Detection Circuits:

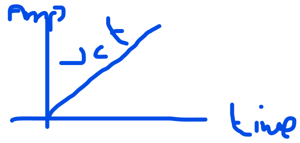


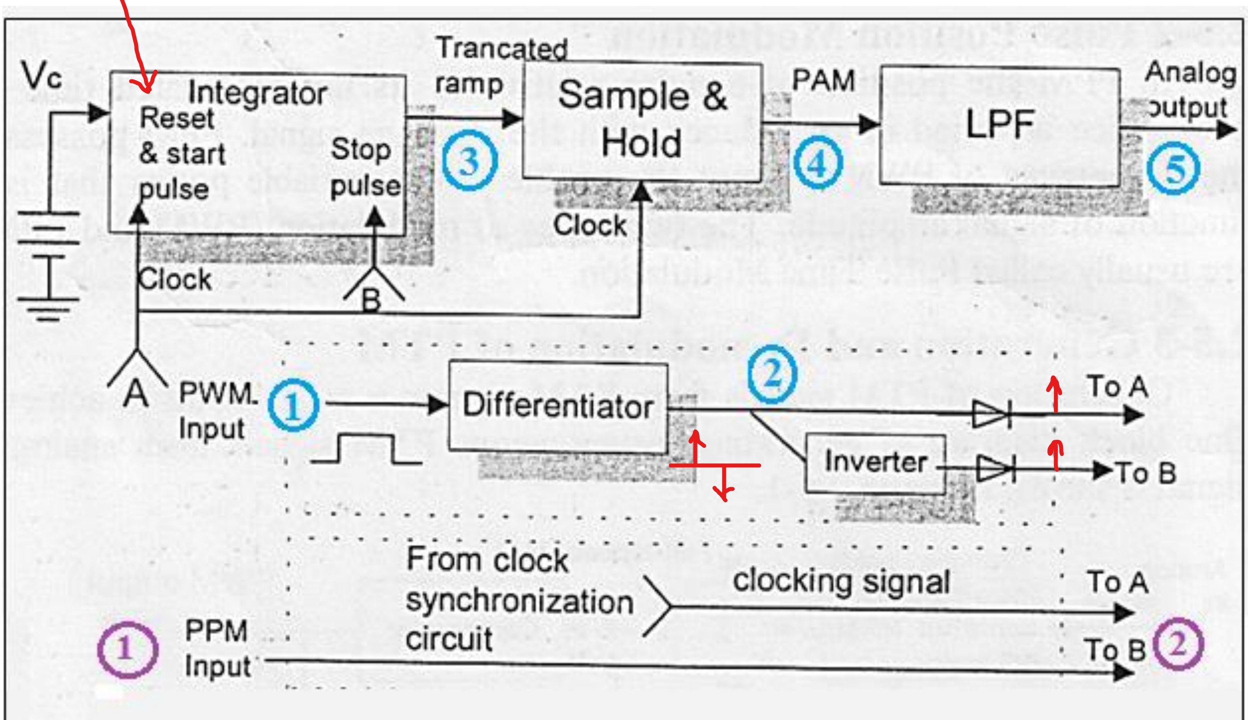
Generation Circuits for PTM, PWM and PPM

The waveform in time domain:

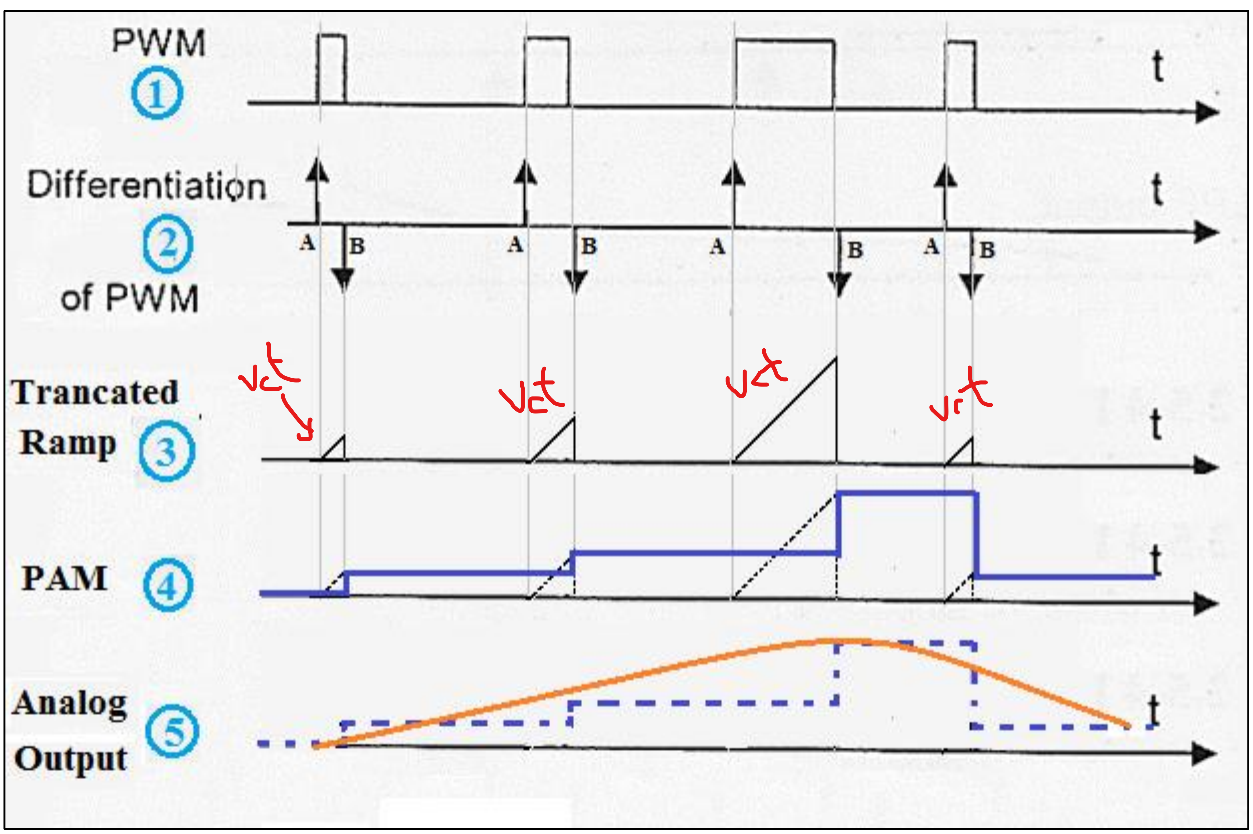


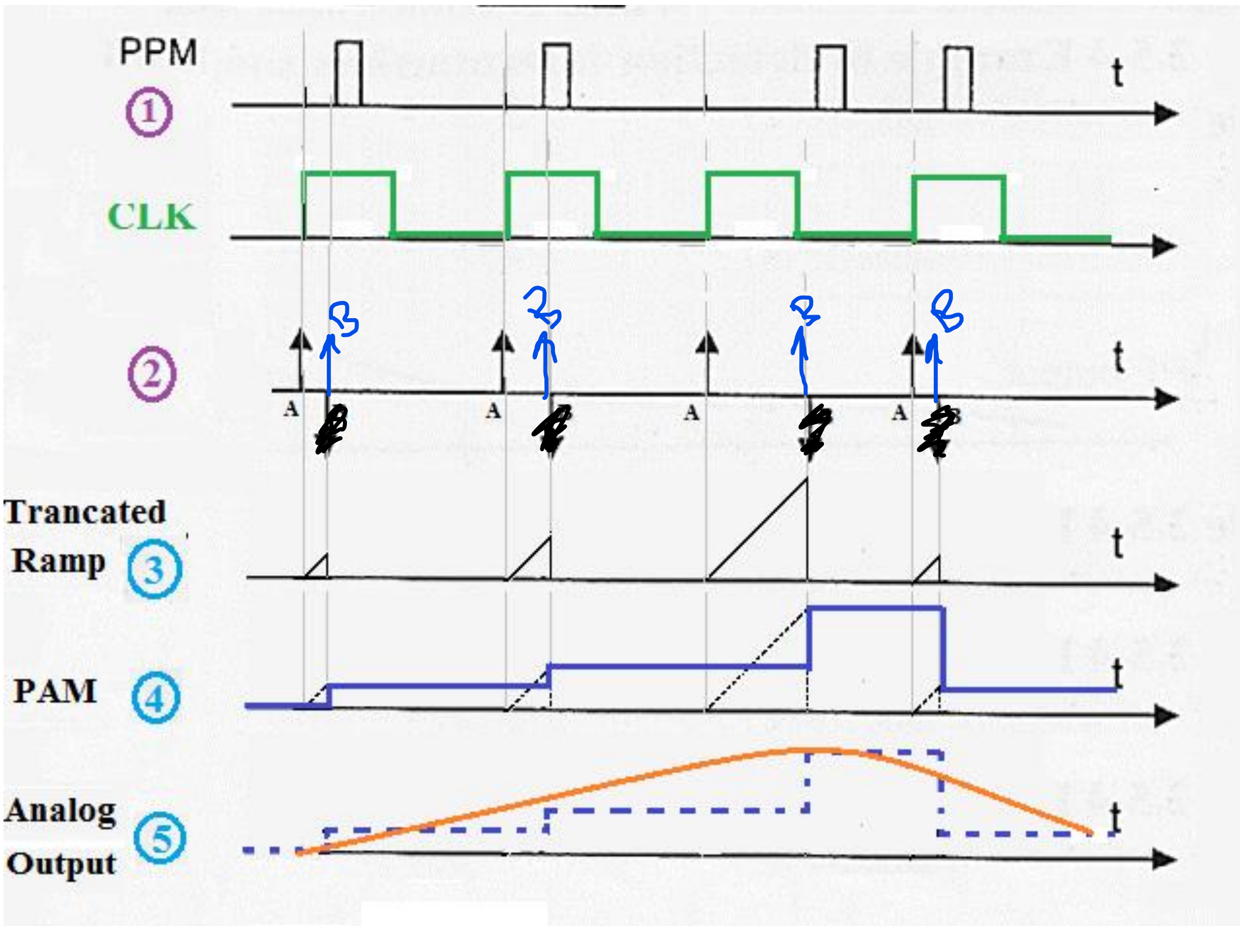
$$\int V_c \, dt = V_c t \Big|_A^B$$



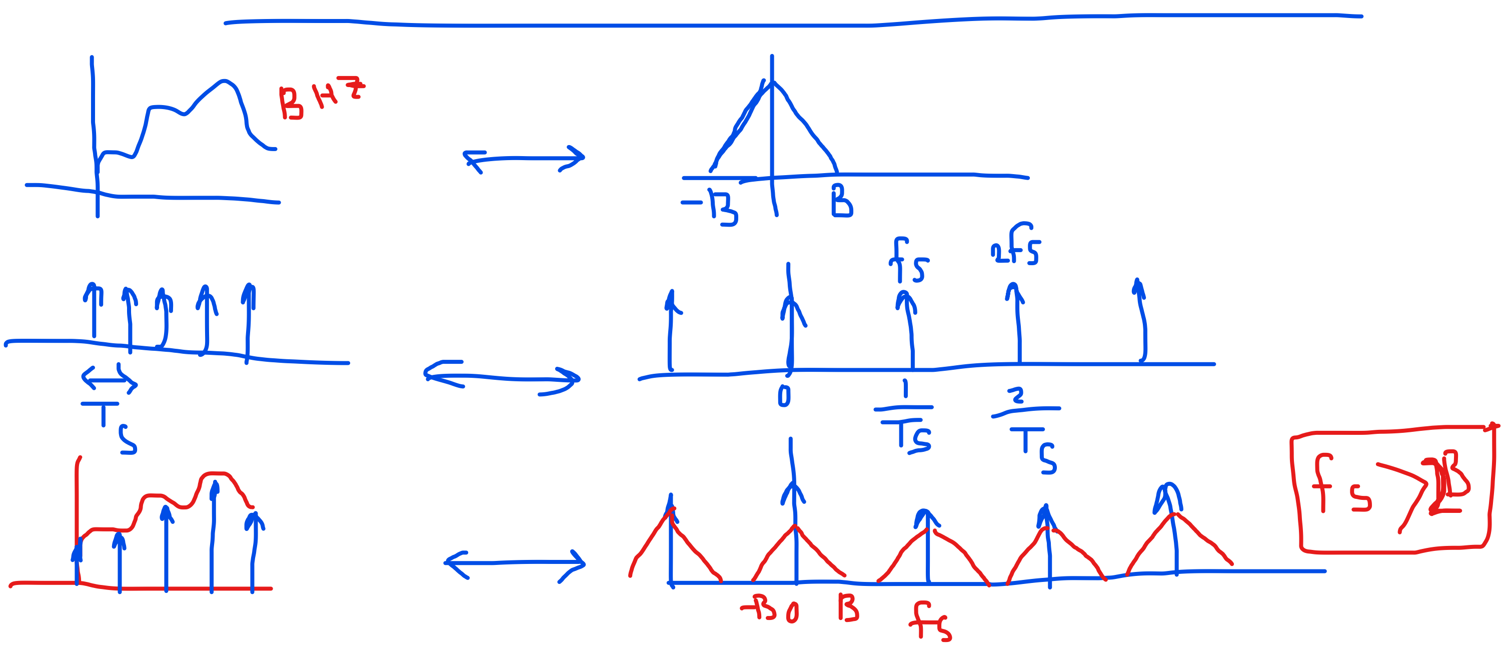


Detection Circuits for PTM, PWM and PPM





PPM
1
CLK
2
A
B
Trancated Ramp
3
PAM
4
Analog Output
5
t



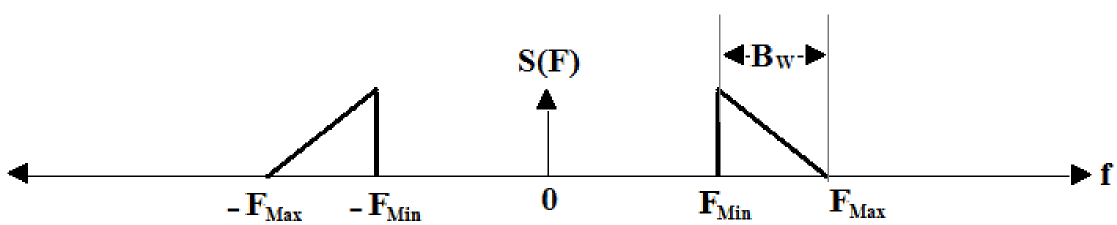
B Hz
-B
B
T_s
f_s
$2f_s$
0
$\frac{1}{T_s}$
$\frac{2}{T_s}$
-B 0 B
f_s
$f_s > 2B$

LPF → $f_s \geq 2f_{max}$

DIGITAL COMMUNICATIONS

Lecture (1-3): Tutorial

Band-pass Sampling Theorem: If a bandpass signal s(t) has a spectrum of bandwidth BW=B and started at lower frequency f_{Min} an stopped at upper frequency f_{Max} then s(t) can be recovered from its sampled version S (t) by band pass filtering if $\boxed{fs=2f_{Max}/K}$ where $\boxed{K}$ is the largest integer not exceeding f_{Max}/B. All higher sampling rates are not necessarily usable unless they exceed $2f_u$.



$$B_W = F_{Max} - F_{Min}$$

$$K = INT\left(\frac{F_{Max}}{B_W}\right)$$ min int i.e.

1.9 → k=1

2.9 → k=2

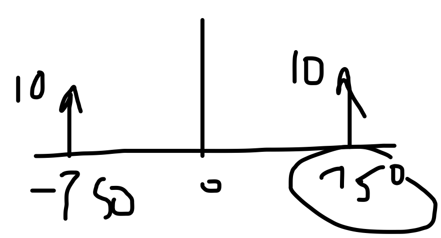
1.6 → k=1

$$Fs = 2 \times \frac{F_{Max}}{K}$$

Example: find the Nyquist rate (fs) and the Nyquist interval for each of the following signals;

$T_s = \frac{1}{f_s}$

1. S1(t)= 20 cos (1500πt) → fm=750Hz
2. S2(t)= 20 cos (1500πt) * cos (4000πt)
3. S3(t)= sin (500πt) / (πt)
4. S4(t)= [sin (500πt) / (πt)]^2
5. S5(t)= [sin (500πt) / (πt)]^2 *sin (6000 πt)
6. S6(t)= sin (5000 πt) + cos (8000 πt)
7. S7(t)= sin (500πt) / (πt) * sin (6000 πt)
8. S8(t)= 5 + 5cos (4000 πt)



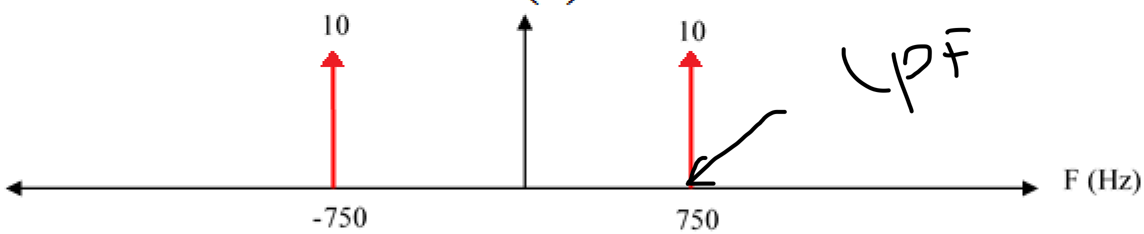
$f_s \geq 2f_{max}$

Solution:

$2\pi 750t$

1. **S1(t) = 20 cos (1500πt)**

S1(F)



LPF

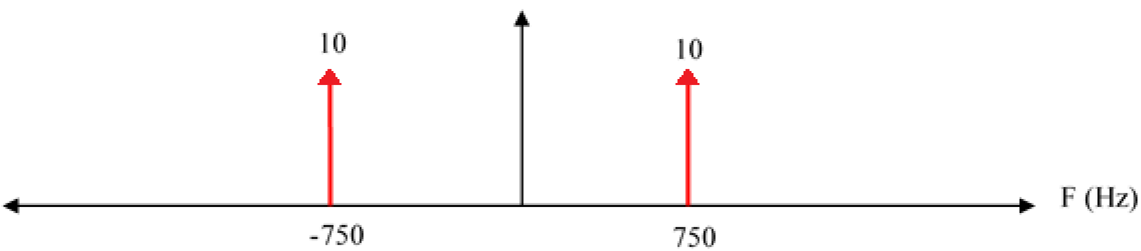
Fmax = 750 Hz

Fs ≥ 2*Fmax = 2* 750 = 1500 S/Sec

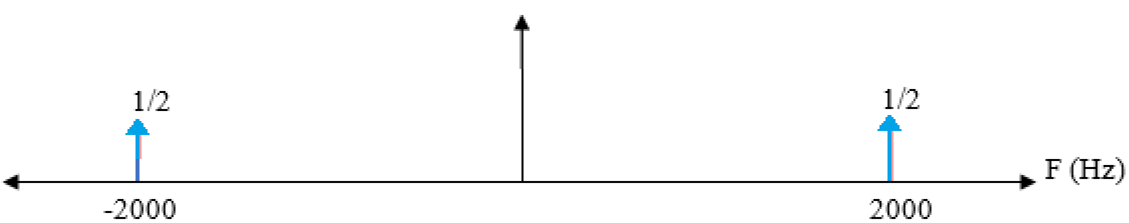
2. **S2(t)= 20 cos (1500πt) * cos (4000πt)**

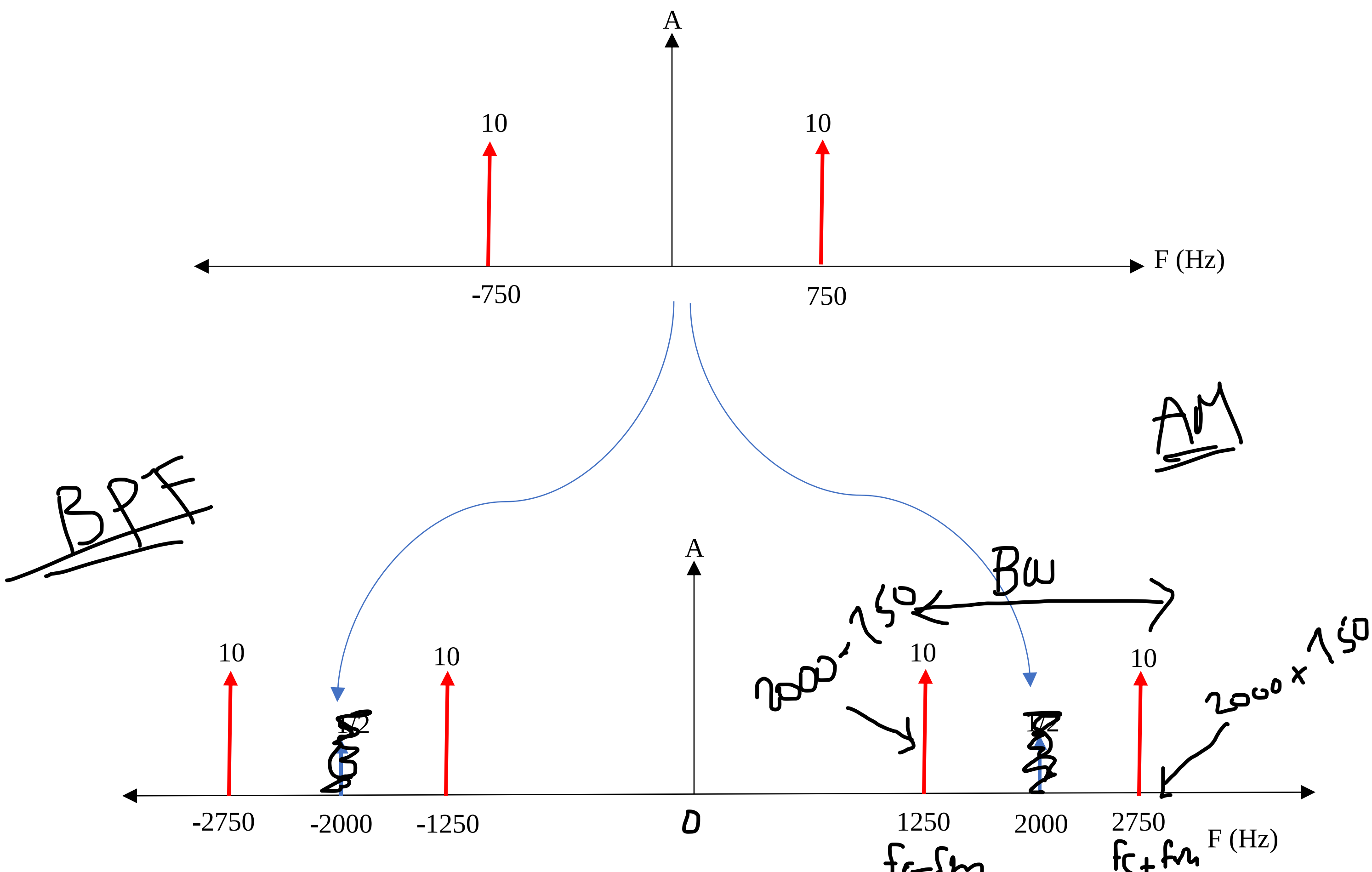
$\cos A \times \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

F [20 cos (1500πt)]



F [cos (4000πt)]





To find the Fs of the signal we must choose the filter type LPF or BPF, by looking to the final signal spectrum, the suitable filter type is BPF (Band-pass Sampling Theorem).

fmin

fmax

Fs= 2fmax/k

K= fmax/BW

BW= 2750-1250= 1500 , fmax= 2750

K= 2750/1500 = 1.8333 ≃ 1

K = 1

LPF ⊂ BPF

❖ Note: in K factor we take the integer number only

Fs= (2*2750) / 1 = 5500 s/sec samples/sec

Nyquist interval (Ts)= 1/ fs = 1/5500

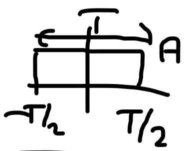
$\text{Sinc } kf = \dfrac{\sin(\pi k f)}{\pi k f}$

$\boxed{k \text{ Sinc } kt \xleftrightarrow{F.T} \text{Rect } \dfrac{f}{k}}$ Π

3. S3(t) = sin (500πt) / (πt)

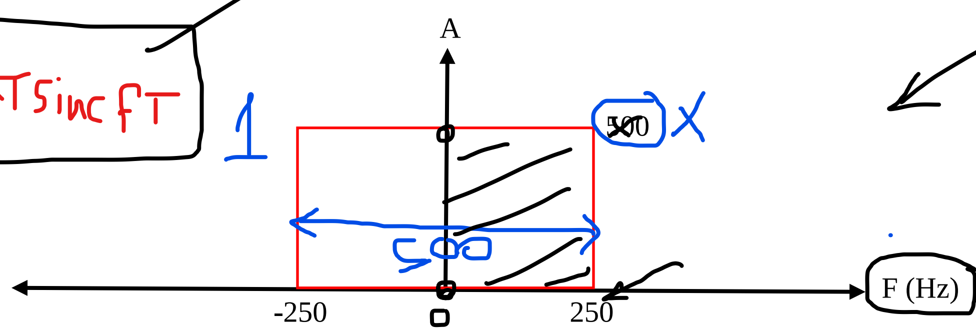
S2(t) = sin (500πt) / (πt)] *500/500

= 500 [sin (500πt) / (500πt)] $= 500 \text{ Sinc } 500t \xleftrightarrow{F.T} \text{Rect } \dfrac{f}{500}$



$\boxed{A \text{rect } \dfrac{t}{T} \xrightarrow{F.T} AT \text{Sinc } fT}$

$\boxed{-\dfrac{T}{2} \rightarrow \dfrac{T}{2}}$



freq. dom.

By looking to the final spectrum, we find that LPF is will used

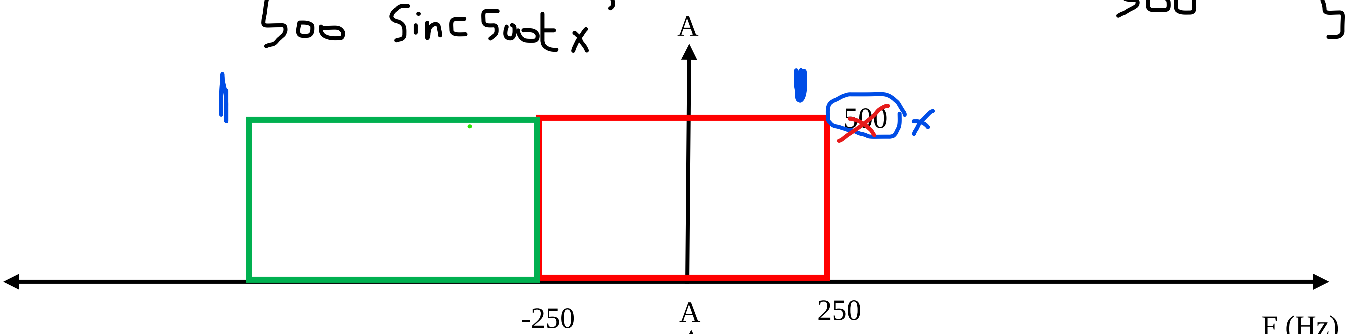
Fs ≥ 2*fmax= 2*250=500 s/sec

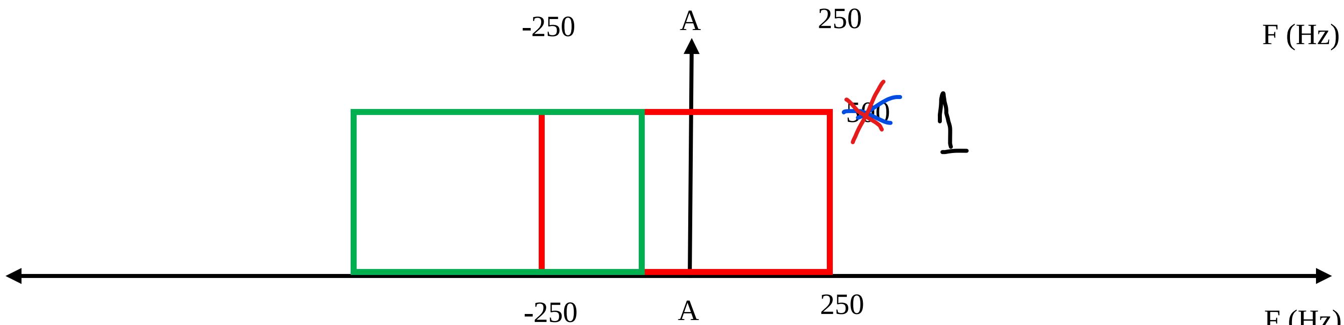
4. S4(t)= [sin (500πt) / (πt)]^2

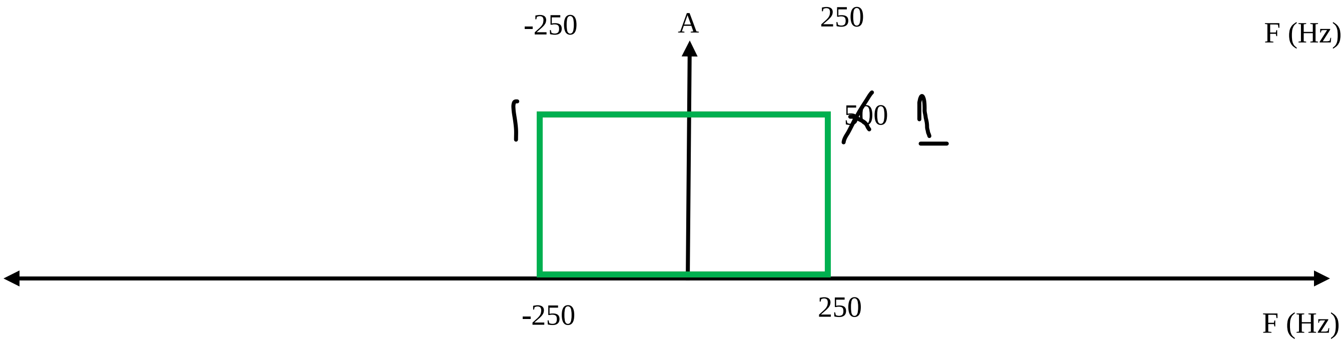
mult. Conv.

= 500 sin (500πt) / (500πt) * 500 sin (500πt) / (500πt) $\xrightarrow{F.T} \Pi \dfrac{f}{500} \circledast \Pi \dfrac{f}{500}$

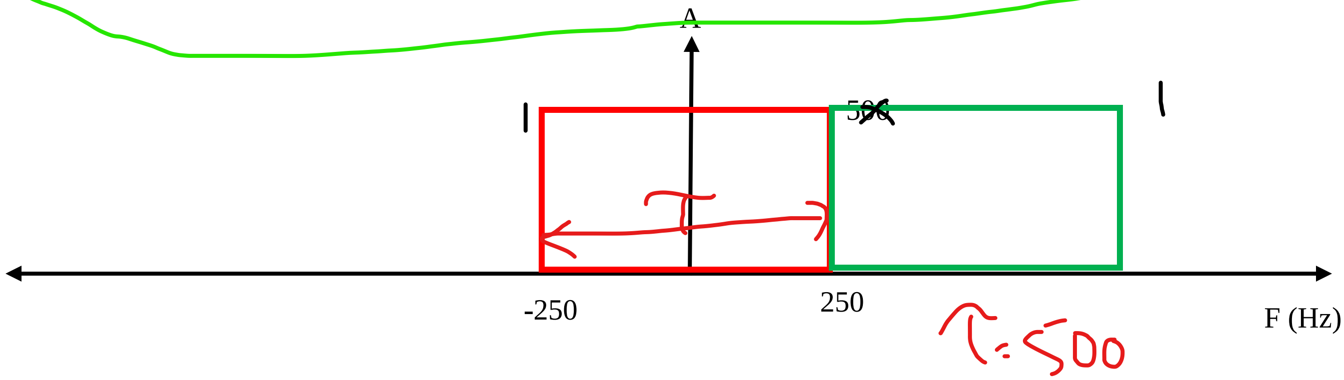
500 Sinc 500t x

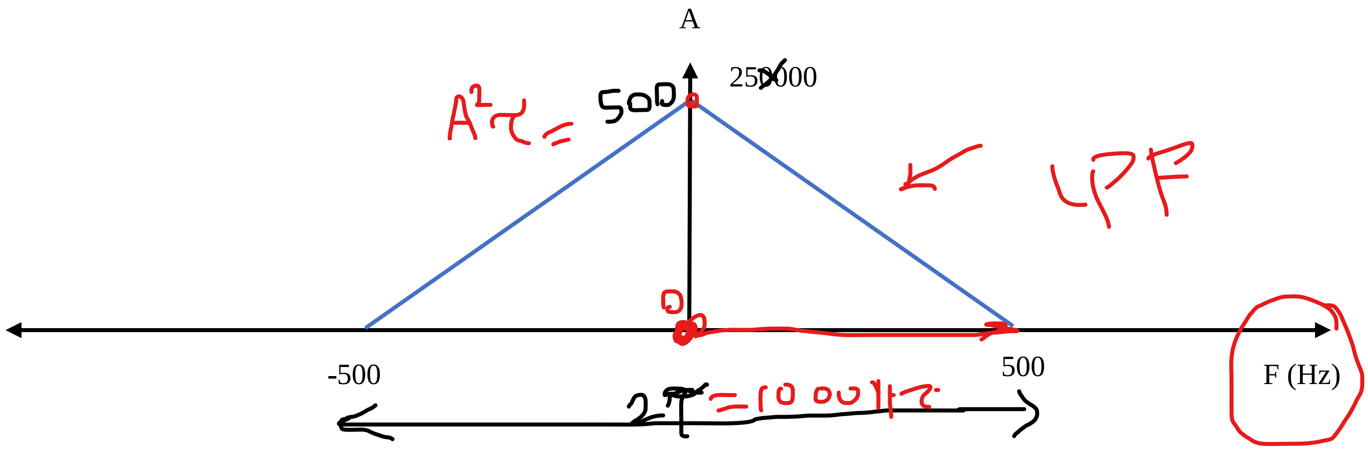






$$A\,\mathrm{rect}\frac{f}{\tau} \otimes A\,\mathrm{rect}\frac{f}{\tau} = A^2\tau\,\mathrm{tri}\frac{f}{2\tau}$$





By looking to the final spectrum, we find that LPF is will used

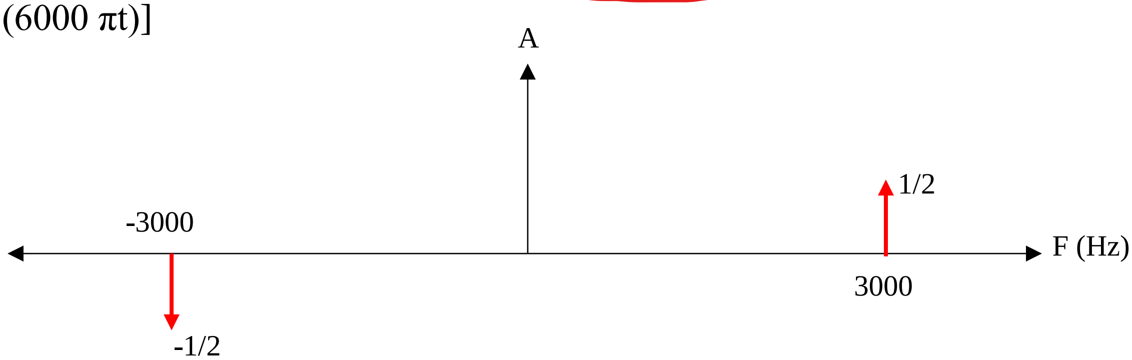
Fs $\geq$ 2*fmax = 2*500 = 1000 s/sec

$\frac{e^{j\theta} - e^{-j\theta}}{2j}$

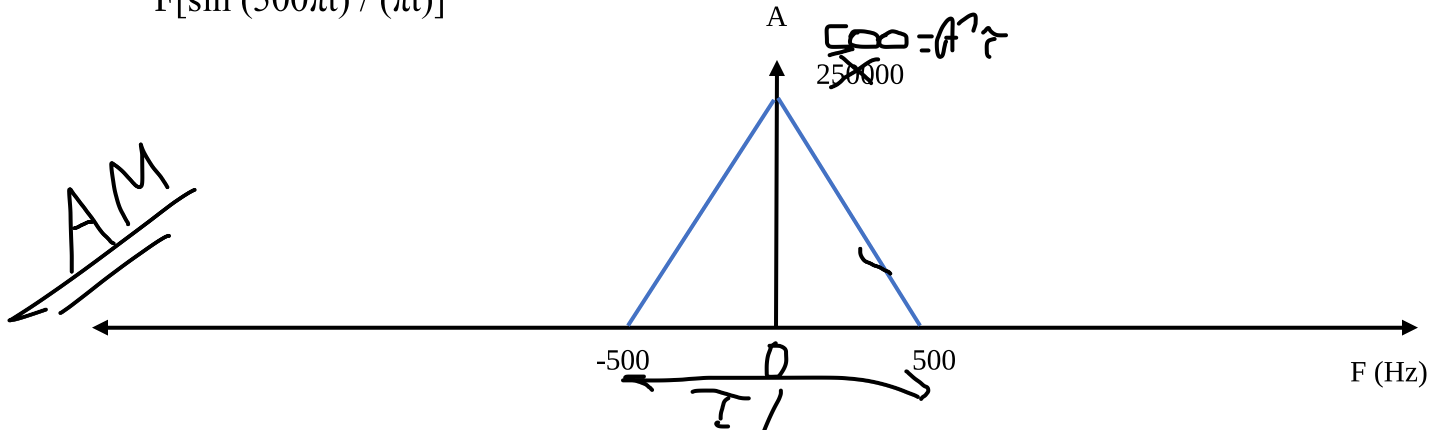
$2 \times \pi\, 3000\, t$

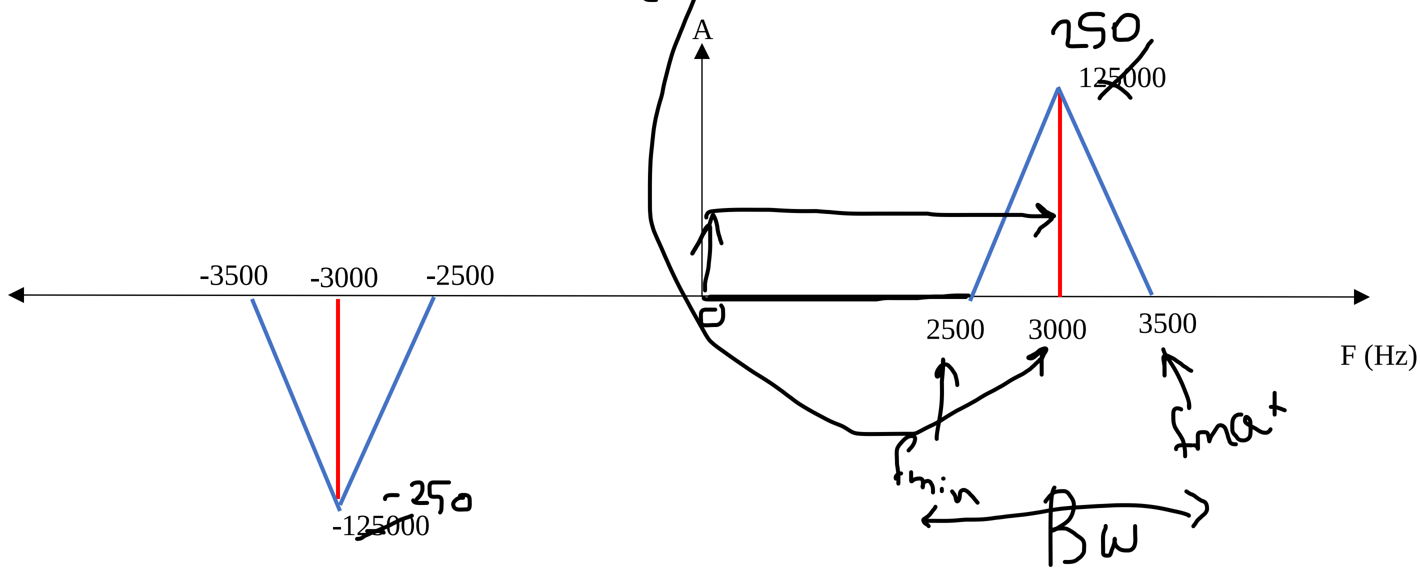
5. S5(t)= [sin (500πt) / (πt)]^2 *sin (6000 πt)

F[sin (6000 πt)]



$F[\sin(500\pi t) / (\pi t)]^2$





By looking to the final spectrum, we find that BPF is will used

Fmax= 3500 Hz , BW= 3500-2500=1000 Hz

K=fmax/BW= 3500/1000=3.5 ∴ k=3

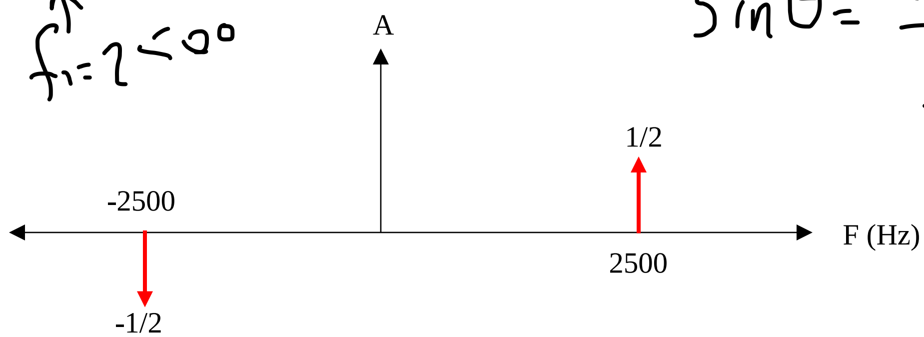
Fs=2*fmax/k= 2*3500/3 = 2.333 k s/sec

$f_2 = 4000$

6. S6(t)= sin (5000 πt) + cos (8000 πt)

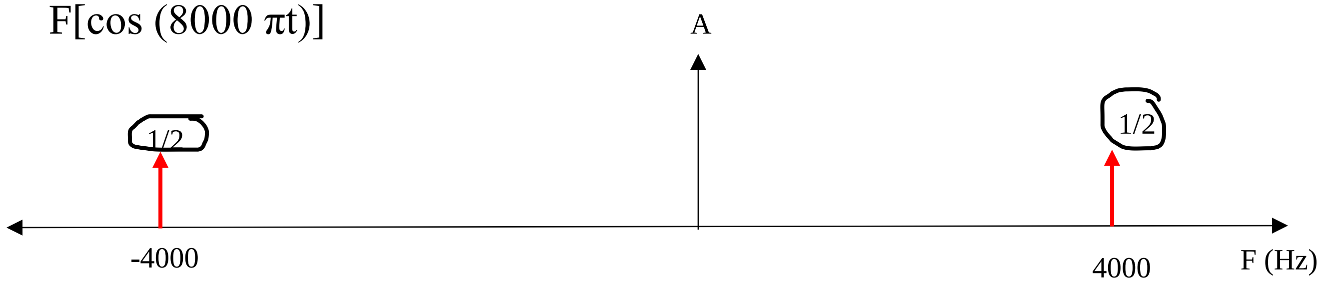
$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{j2}$

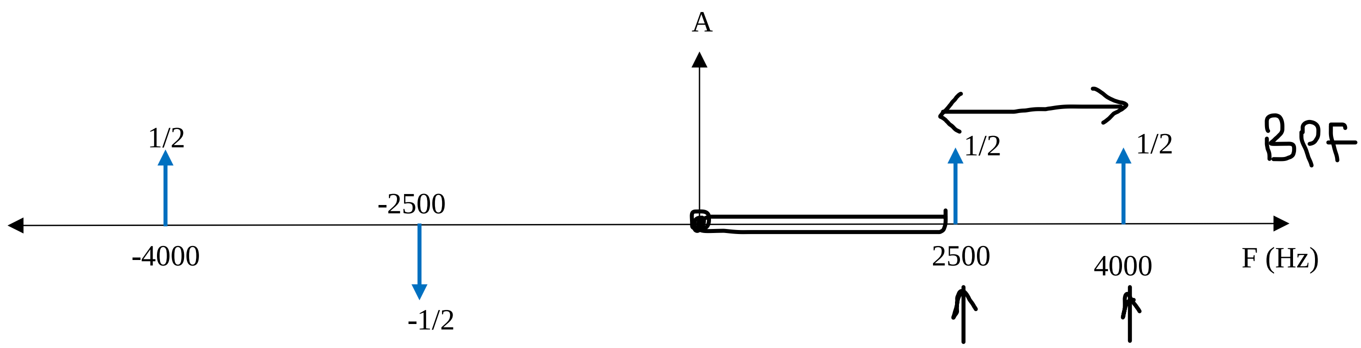
F[sin(5000 πt)] $f_1 = 2500$



$$\cos\theta = \frac{e^{j\theta}+e^{-j\theta}}{2}$$

F[cos (8000 πt)]





By looking to the final spectrum, we find that BPF is will used

Fmax=4000 Hz , BW= 4000-2500=1500 Hz

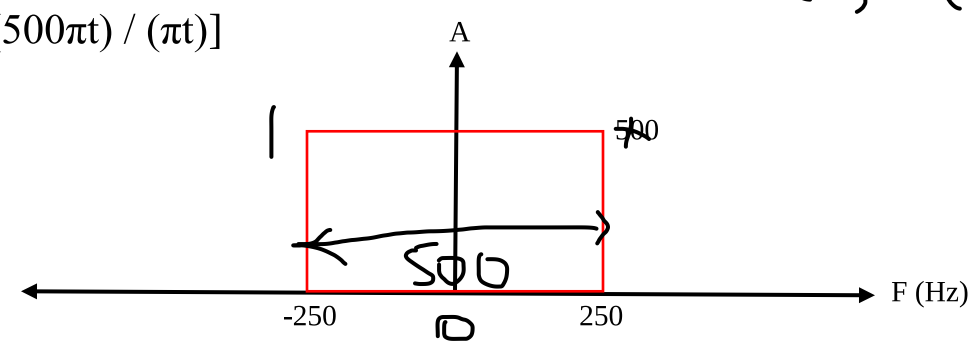
K =fmax/BW= 4000/1500=2.6667 .: k=2

Fs=2*fmax/k= 2*4000/2 = 4000 s/sec

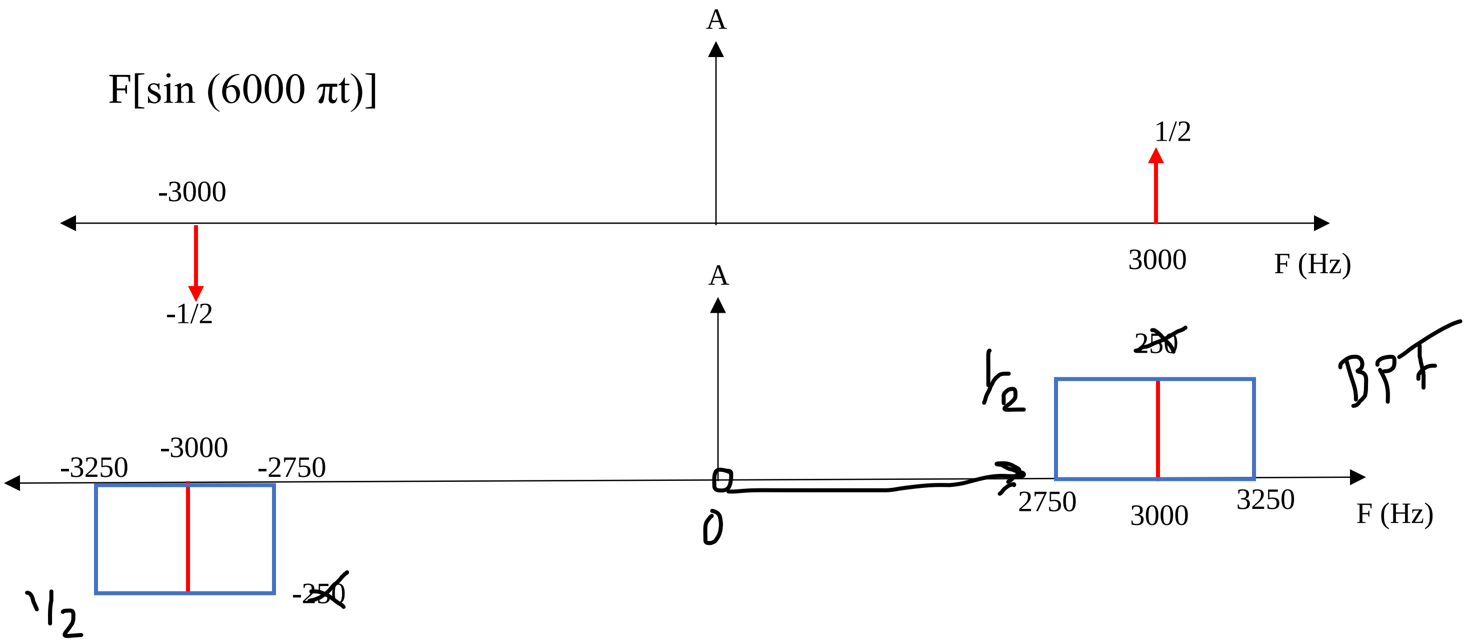
7. **S7(t)= sin (500πt) / (πt)* sin (6000 πt)**

$500\,\text{sinc}\,500t \leftrightarrow \text{Rect}\,\frac{f}{500}$

F[sin (500πt) / (πt)]



F[sin (6000 πt)]



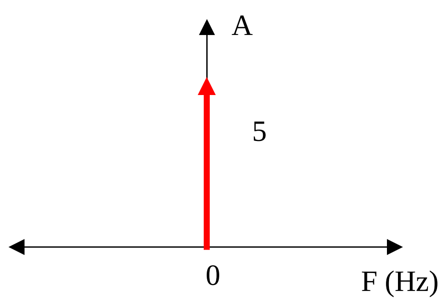
By looking to the final spectrum, we find that BPF is will used

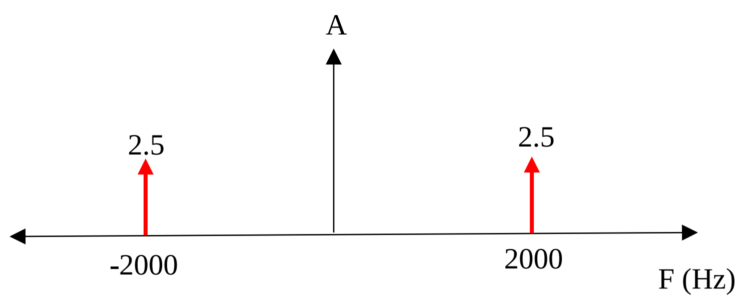
Fmax=3250 Hz , BW= 3250-2750=500 Hz

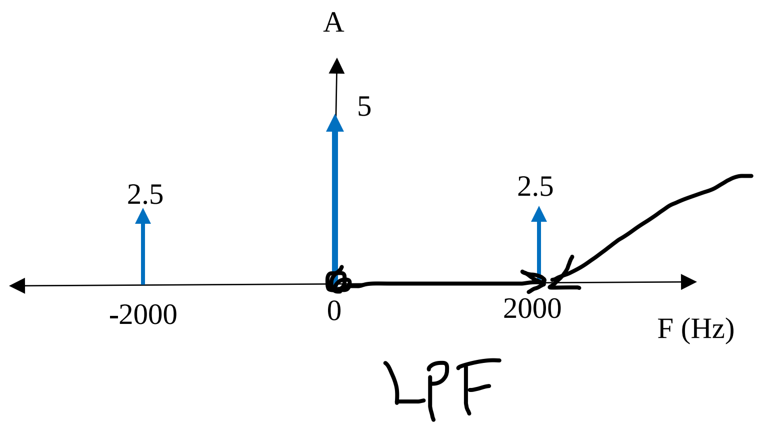
K =fmax/BW= 3250/500 = 6.5 ∴ k=6

Fs=2*fmax/k= 2*3250/6 = 1083.33 s/sec

8. S8(t)= 5 + 5cos (4000 πt)







By looking to the final spectrum, we find that LPF is will used

Fs≥ 2*fmax= 2*2000≥4000 s/sec

DIGITAL COMMUNICATIONS

Lecture (1-4): TIME DIVISION MULTIPLEXING FOR SAMPLED SIGNALS (TDM)

Introduction:

Multiplexing is a technique, which allows many users to share a single communication channel (wire, cable, radio link, fiber optic cable, satellite) simultaneously. So, a number of information sources share the same communication channel. Information transmitted can be either voice (telephone signals) or data (computer signal, images, video etc). Well-known application of multiplexing is telephone communication system.

There are many different types of multiplexing that has been used in communication and you'd better know of at least some name of these techniques. There are some communication systems that is using only one of these techniques and there are some other communication systems that are using multiples of these techniques in combination.

- TDM (Time Division Multiplexing): This would be the most straight forward method. We split the data exchange time into multiple small slots and transmit/receive different data onto different slot. GSM is one of example of communication system that is extensively using this technology)
- FDM (Frequency Division Multiplexing): In this technique, we split a communication channel (physical resource) into different frequency blocks and transmit/receive different stream of data through different frequency blocks. OFDM (Orthogonal Frequency Division Multiplexing) is one typical example.

- CDM (Code Division Multiplexing): In this technique, we split a communication channel (physical resource) into different code (orthogonal code) and allocate each stream of data onto different code. CDMA and WCDMA is the most widely used example.

The sampled signals can be separated from each other if they are not overlapping in either time or frequency, this propriety exploit by adding signals together and transmit them on one channel, so time division multiplexing (TDM) can be defined:

TDM is the time interleaving of samples from several sources so that the information from these sources can be transmitted serially over a single communication channel. There are conditions to transmit the signals in TDM system:

1) Preserving the same sampling frequency for each signal.
2) Keep the time between each two samples constant.
3) Need the synchronization between the transmitter and receiver to know which signal was received.

Note: the needed BW in this system was higher than any BW for all the signals.

The basic steps to design the TDM system:

1) Choose the smallest common factor from the sampling frequencies of all signals.
2) Divided sampling frequency of each signal on this factor and records the result.
3) Sum the above result and see if the summation results divisible without remainder or not.

Example:

Design a TDM system to transmit these two signals in to one channel:

S1(t)= band limited signal to 2KHz

S2(t)= band limited signal to 2KHz

Solution:

As we mention before in the advantage of sampling theory that we can use the same channel to transmit more than one signal at the same time, so to transmit these two signals must know the sampling frequency for them

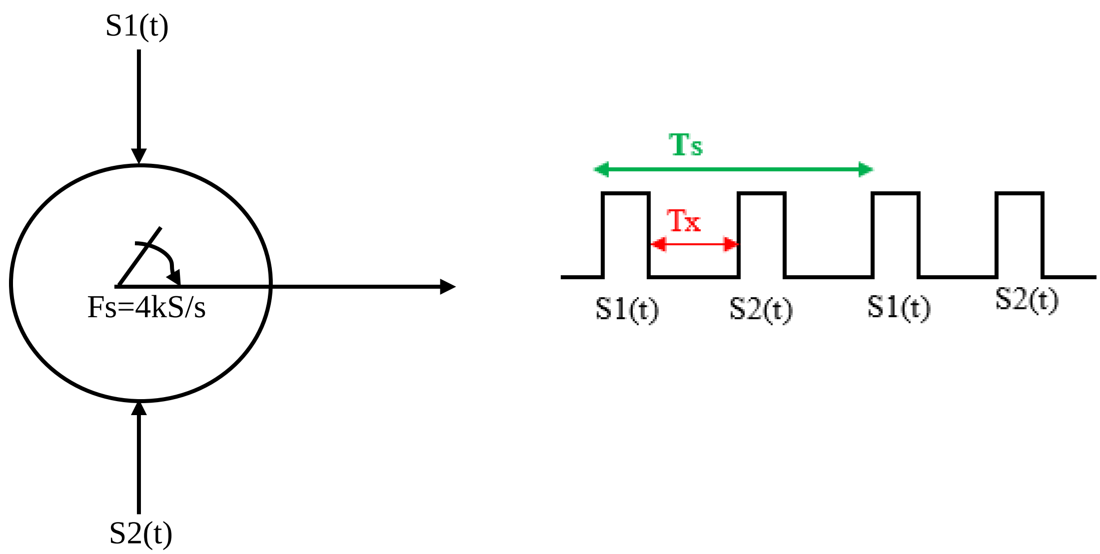
Fs for S1(t)= 2*f_{max} =2*2=4KS/sec

Fs for S2(t)= 2*f_{max} =2*2=4KS/sec

By applicate the above steps to design TDM system

1) The smallest common factor from the sampling frequencies= 4KS/sec
2) Fs1 4/4 = 1 (number of pieces for each signal)

   Fs2 4/4 = 1 2/1 = 2
3) Sum the number of pieces = 2 , 2/1 =2 2/1 = 2



Ts= 1/4 msec

Tx= Ts/number of signals= Ts/2 = (1/4)/2 = 1/8 msec

(where TX is the time between any two samples)

Fx=1/Tx = 8 KS/sec

Required channel BW= ½ fx= ½ * 8 = 4KHz

Example:

Design a TDM system to transmit these three signals in to one channel:

S1(t)= band limited signal to 2KHz

S2(t)= band limited signal to 2KHz

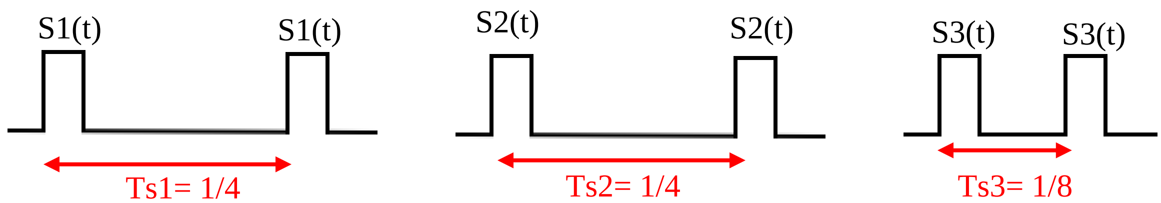
S3(t)= band limited signal to 4KHz

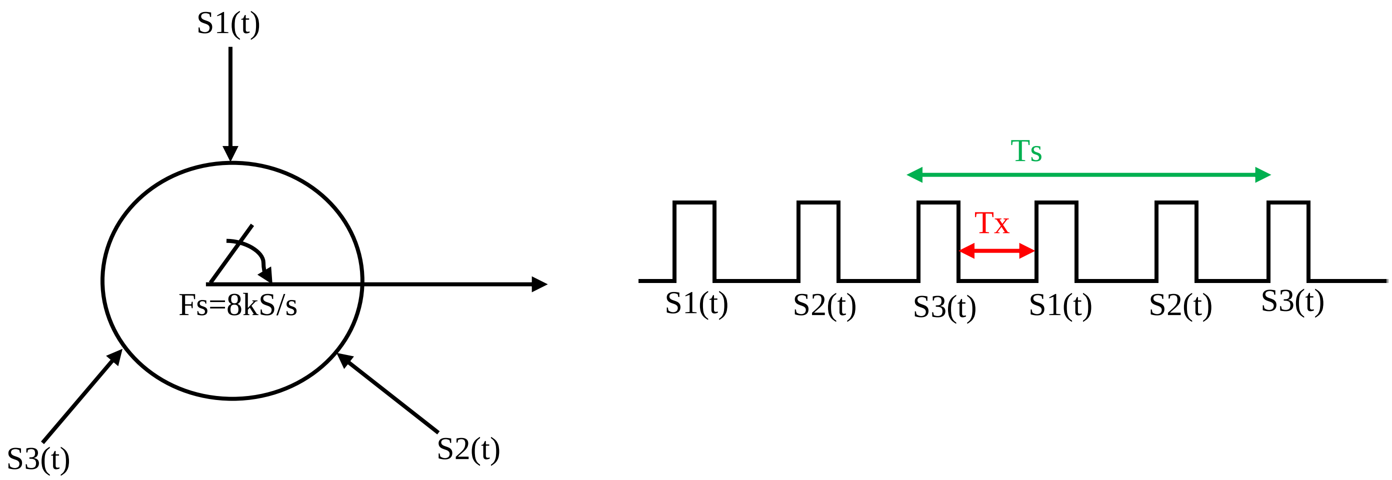
Solution:

Fs for S1(t)= 2*f_{max} =2*2=4KS/sec

Fs for S2(t)= 2*f_{max} =2*2=4KS/sec

Fs for S3(t)= 2*f_{max} =2*4=8KS/sec





Ts= 1/8 msec

Tx= Ts/3 = 1/24 msec

Fx= 1/Tx = 24 K S/sec

Required channel bandwidth= ½ Fx= 12 KHz

choose the smalest fs and divide all fs' on it

<u>The right solution:</u> <u>number of pieces</u>

Fs for S1(t)= 2*f_{max} =2*2=4KS/sec /4K $\boxed{1}$ 4

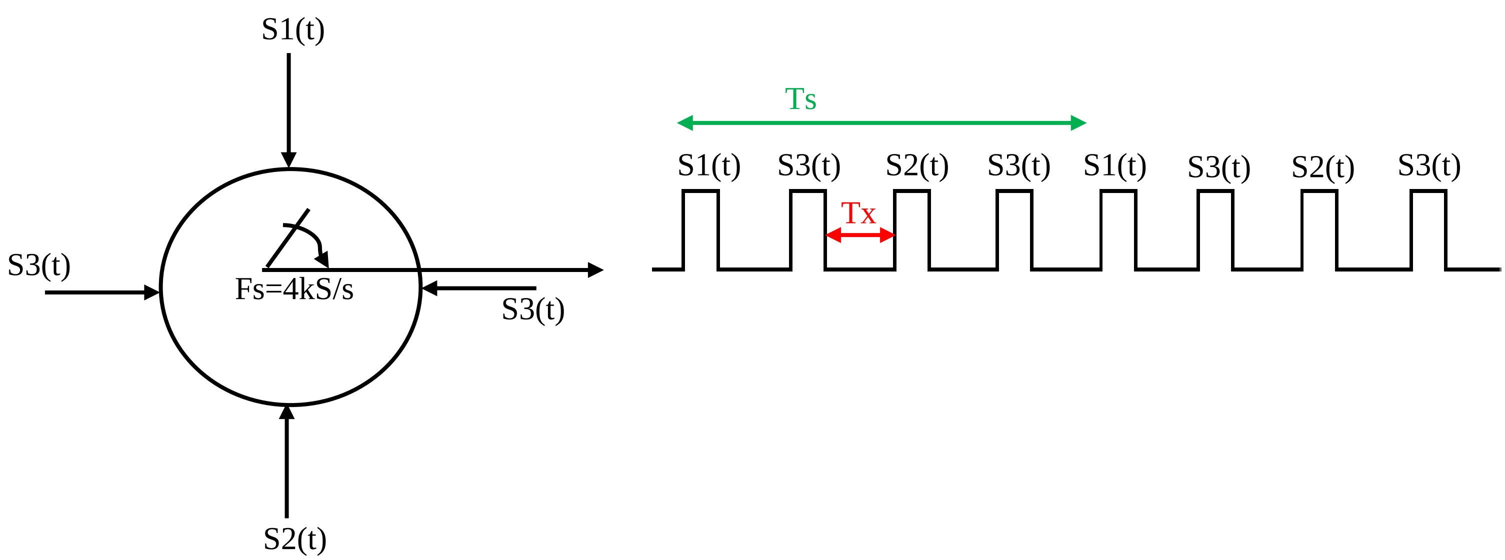
Fs for S2(t)= 2*f_{max} =2*2=4KS/sec /4K $\boxed{1}$ 4

Fs for S3(t)= 2*f_{max} =2*4=8KS/sec /4K $\boxed{2}$ 2

The sum result= 4

4/1 =4 , 4/2=2

Just to check if there is a reminder or not!



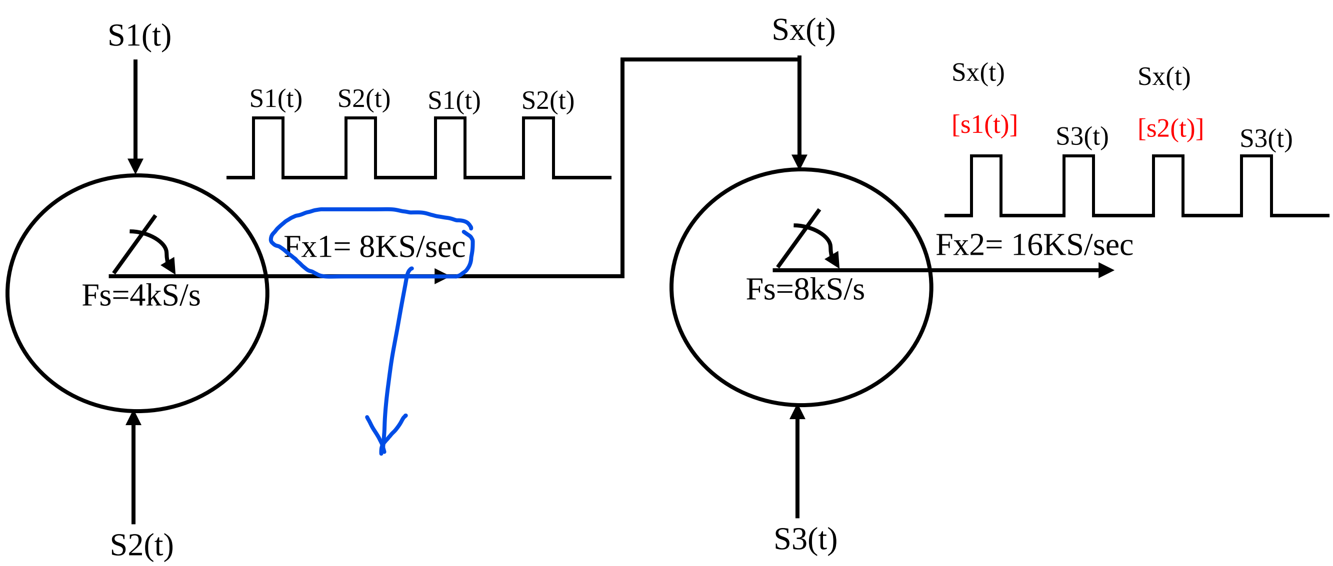
Ts=1/4

Tx= (¼)/4 = 1/16 msec

Fx= 16 K S/sec

BW= ½ fx = ½ * 16 = 8KHz

Other solution:



T_{s1}= 1/4 msec

T_{x1}= T_{s1}/number of signals= T_{s1}/2 = (1/4)/2 = 1/8 msec

F_{x1}=1/T_{x1} = 8 KS/sec

T_{s2}= 1/8 msec

T_{x2}= T_{s2}/number of signals= T_s/2 = (1/8)/2 = 1/16 msec

F_{x2}=1/T_{x2} = 16 KS/sec

Required channel bandwidth= ½ F_{x2}= 8 KHz

DIGITAL COMMUNICATIONS

Lecture (5-1): TDM - Tutorial

Example 1:

The following signals are sampled each at its Nyquist rate and multiplexed and transmitted by using TDM:

S1(t) bandlimited to 1.8 KHz

S2(t) bandlimited to 3.6 KHz

S3(t) bandlimited to 0.6 KHz

S4(t) bandlimited to 0.6 KHz

S5(t) bandlimited to 0.6 KHz

Solution:

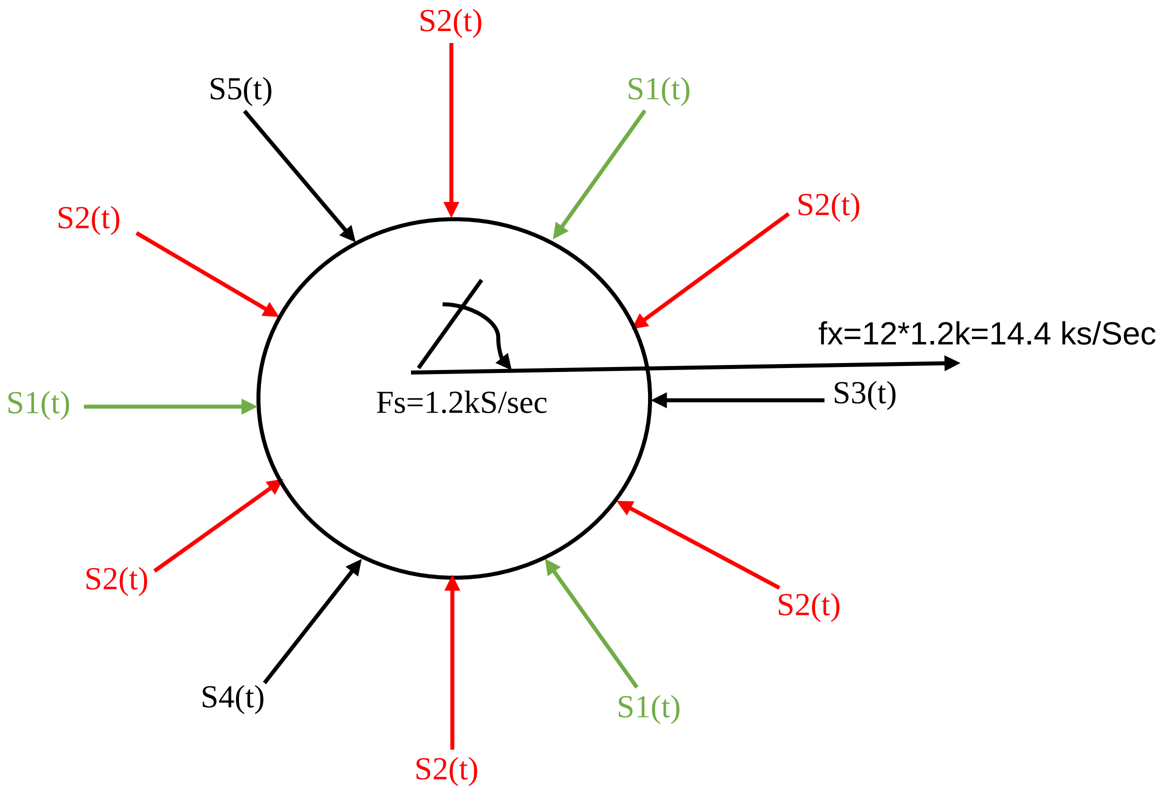
	number of pieces
Fs for S1(t)= 2*f_{max} =2*1.8=3.6KS/sec /1.2k	3
Fs for S2(t)= 2*f_{max} =2*3.6=7.2KS/sec	6
Fs for S3(t)= 2*f_{max} =2*0.6=1.2KS/sec	1
Fs for S4(t)= 2*f_{max} =2*0.6=1.2KS/sec	1
Fs for S5(t)= 2*f_{max} =2*0.6=1.2KS/sec	1

= 12 pieces

The sum of pieces= 12

12/3=4, 12/6=2, 12/1=12

All pieces are divisible



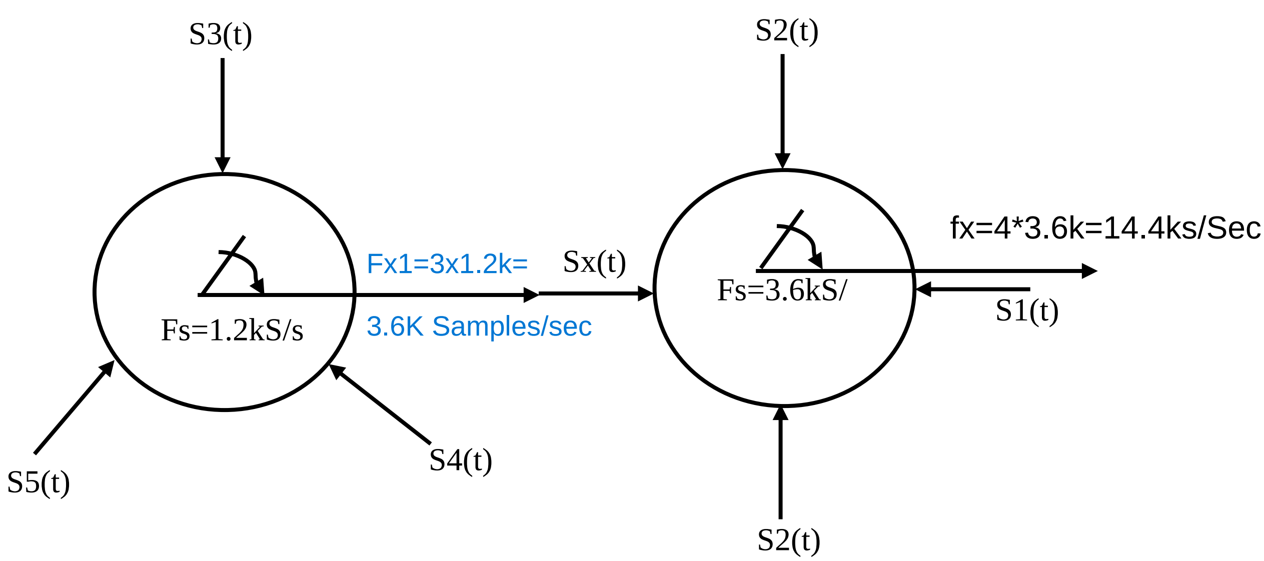
Ts=1/1.2

Tx= (1/1.2)/12 = 1/14.4msec

Fx= 14.4 K S/sec

BW= ½ fx = ½ * 14.4 = 7.2KHz

<u>Other solution:</u>



Ts=1/3.6

Tx= (1/3.6)/4 = 1/14.4msec

Fx= 14.4 K S/sec (**Or** Fx= Fs* number of pieces= 3.6*4 = 14.4 KS/sec)

BW= ½ fx = ½ * 14.4 = 7.2KHz

Example 2:

The following signals are sampled each at its Nyquist rate and multiplexed and transmitted by using TDM:

S1(t) bandlimited to 4.8 KHz

S2(t) bandlimited to 2.4 KHz

S3(t) bandlimited to 0.6 KHz

S4(t) bandlimited to 0.6 KHz

S5(t) bandlimited to 0.6 KHz

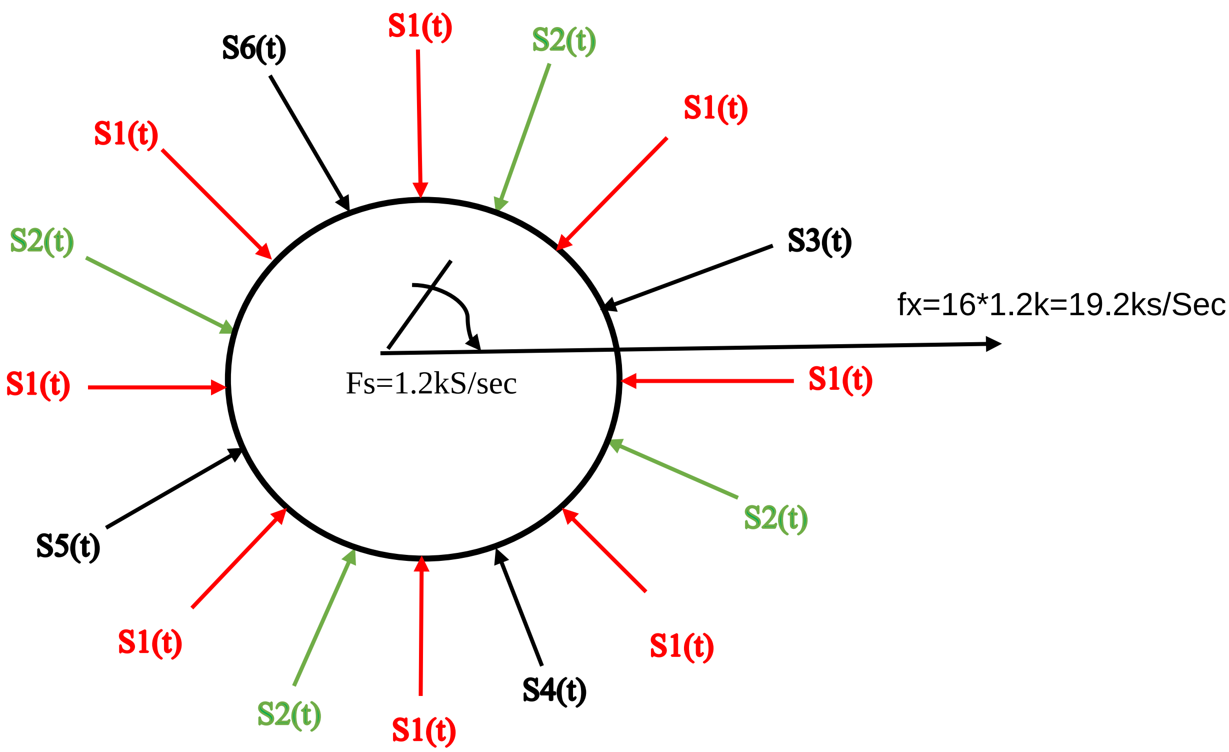
S6(t) bandlimited to 0.6 KHz

Solution:

		number of pieces
Fs for S1(t)= $2*f_{max}$ =2*4.8=9.6KS/sec	/1.2KS/s =	8
Fs for S2(t)= $2*f_{max}$ =2*2.4=4.8KS/sec	〃	4
Fs for S3(t)= $2*f_{max}$ =2*0.6=1.2KS/sec	〃	1
Fs for S4(t)= $2*f_{max}$ =2*0.6=1.2KS/sec	〃	1
Fs for S5(t)= $2*f_{max}$ =2*0.6=1.2KS/sec	〃	1
Fs for S6(t)= $2*f_{max}$ =2*0.6=1.2KS/sec	〃	1

The sum of pieces= 16

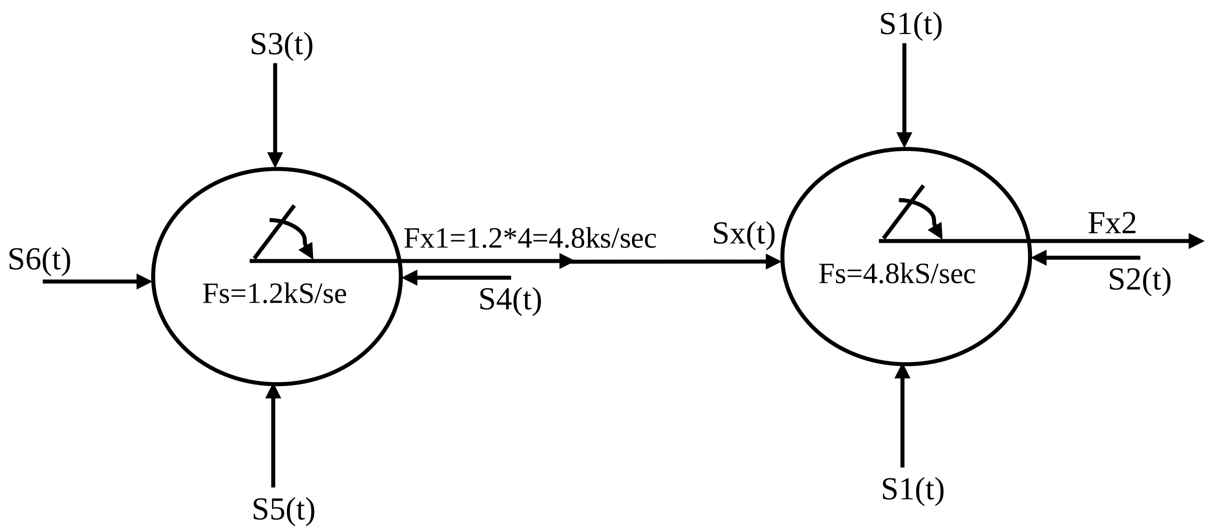
16/8=2,   16/4=4,   16/1=16



Fx= Fs* number of pieces= 1.2*16 = 19.2 KS/sec

BW= ½ fx = ½ * 19.2= 9.6 KHz

Other solution



Fx2= 4.8*4=19.2KS/sec

BW= ½ fx2 = ½ * 19.2= 9.6 KHz

Example 3:

The following 36 channels are to be multiplexed using TDM: Using

1 channel sampled at 8KHz

1 channel sampled at 4KHz

18 channel sampled at 1KHz

16 channel sampled at 0.125KHz

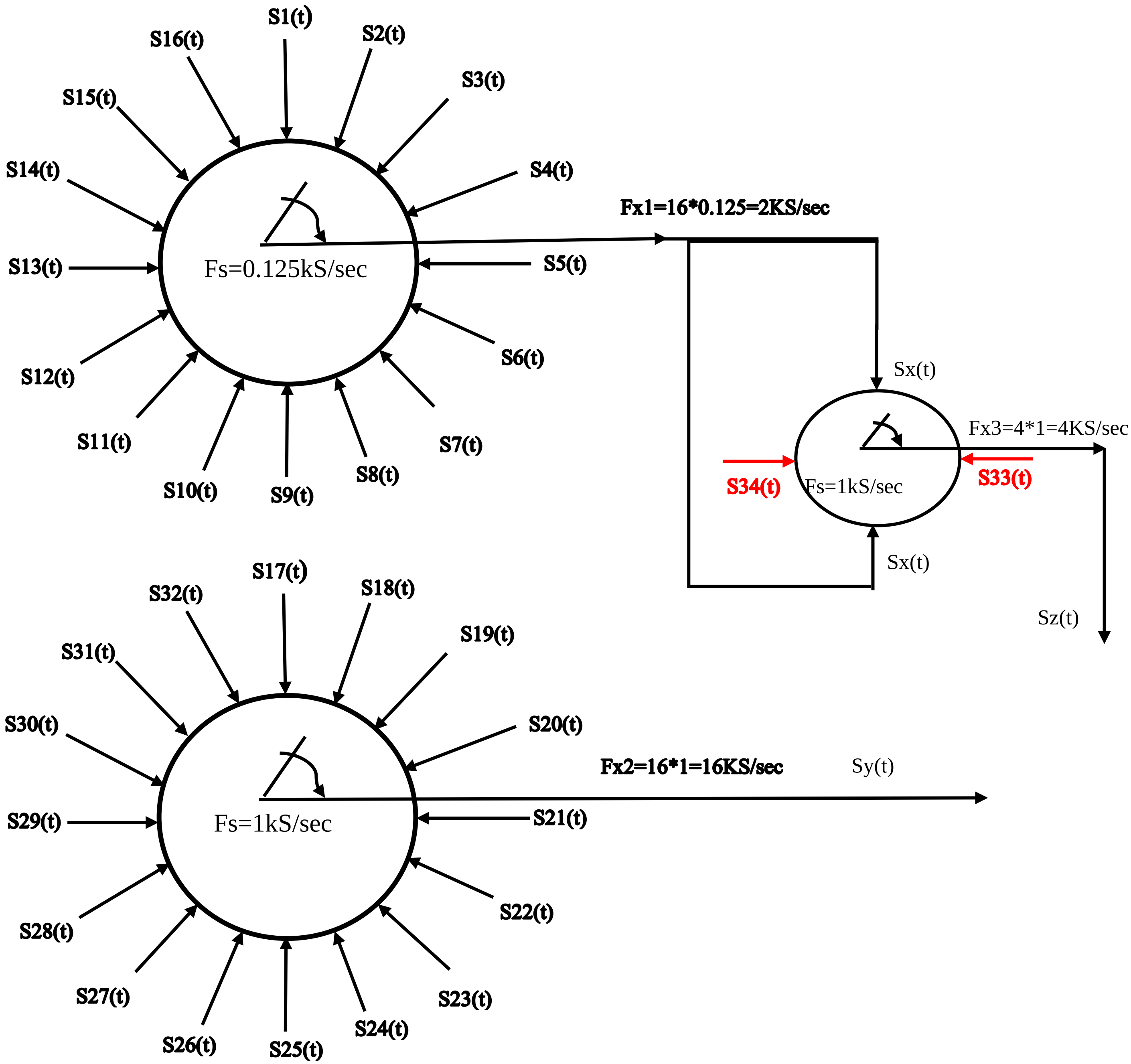
* 16 pieces commutator of 0.125 ks/s
* 16 " " of 1 ks/s

Solution:

First, we notice that every channel was sampled which mean that the above frequencies given are Fs.

To solve this example, we have 16 channels sampled at 0.125KHz will sub-commutation firstly.

* 4 " of 1 ks/s
* 8 " of 4 ks/s

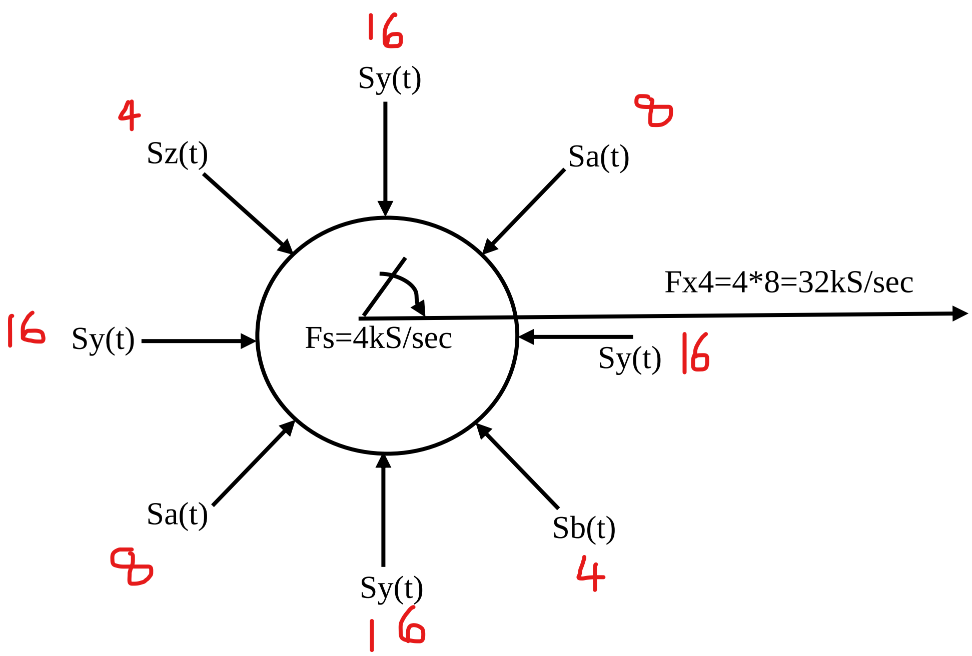

S1(t)
S2(t)
S3(t)
S4(t)
S5(t)
S6(t)
S7(t)
S8(t)
S9(t)
S10(t)
S11(t)
S12(t)
S13(t)
S14(t)
S15(t)
S16(t)
Fs=0.125kS/sec
Fx1=16*0.125=2KS/sec
Sx(t)
Fx3=4*1=4KS/sec
S34(t)
Fs=1kS/sec
S33(t)
Sx(t)
Sz(t)
S17(t)
S18(t)
S19(t)
S20(t)
S21(t)
S22(t)
S23(t)
S24(t)
S25(t)
S26(t)
S27(t)
S28(t)
S29(t)
S30(t)
S31(t)
S32(t)
Fs=1kS/sec
Fx2=16*1=16KS/sec
Sy(t)

If we looking at the remaining signals and the produced signals from the above TDM system we have:

		number of pieces
1 channel sampled at 8KHz	[Sa(t)]	2
1 channel sampled at 4KHz	[Sb(t)]	1
1 channel sampled at 16KHz	[Sy(t)]	4
1 channel sampled at 4KHz	[Sz(t)]	1

The sum of pieces = 8

8/2=4 , 8/1=8 , 8/4=2



BW= ½ fx4 = ½ * 32= 16 KHz

If the above example was modified to be, Design TDM system to transmit these channels:

1 channel sampled at 8KHz

18 channel sampled at 1KHz

16 channel sampled at 0.125KHz

The solution would be the as the first part of the previous example, the difference would be in the second part:

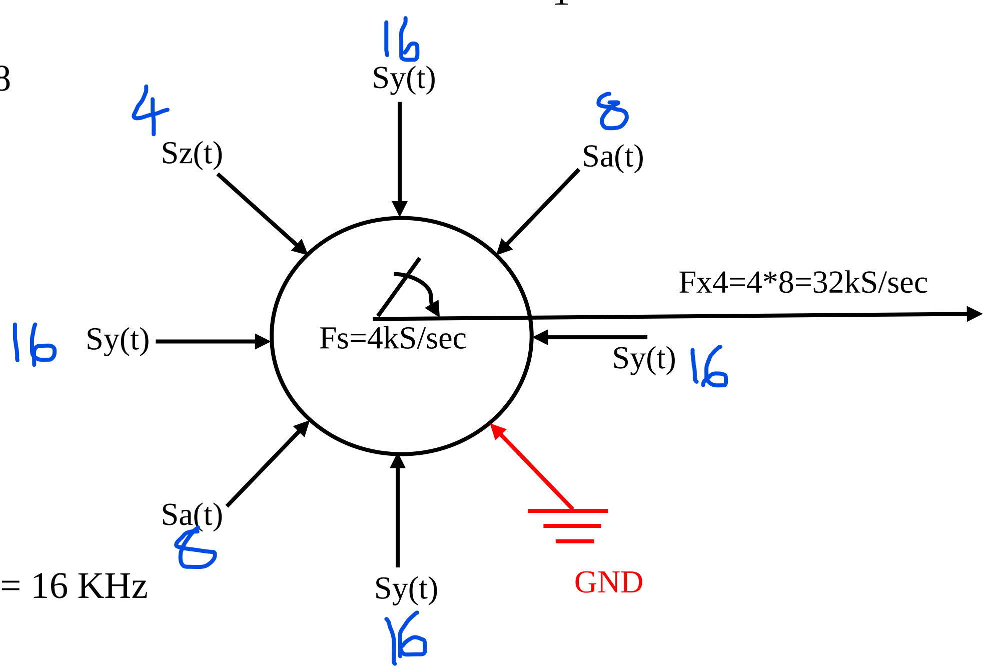
			number of pieces
1	channel sampled at 8KHz	[Sa(t)]	2
1	channel sampled at 16KHz	[Sy(t)]	4
1	channel sampled at 4KHz	[Sz(t)]	1

The sum of pieces = 7

We notice that the summation of pieces divisible with remainder. To solve this problem, we add an additional channel which called ground (GND) to the other channels so the sum of pieces =8

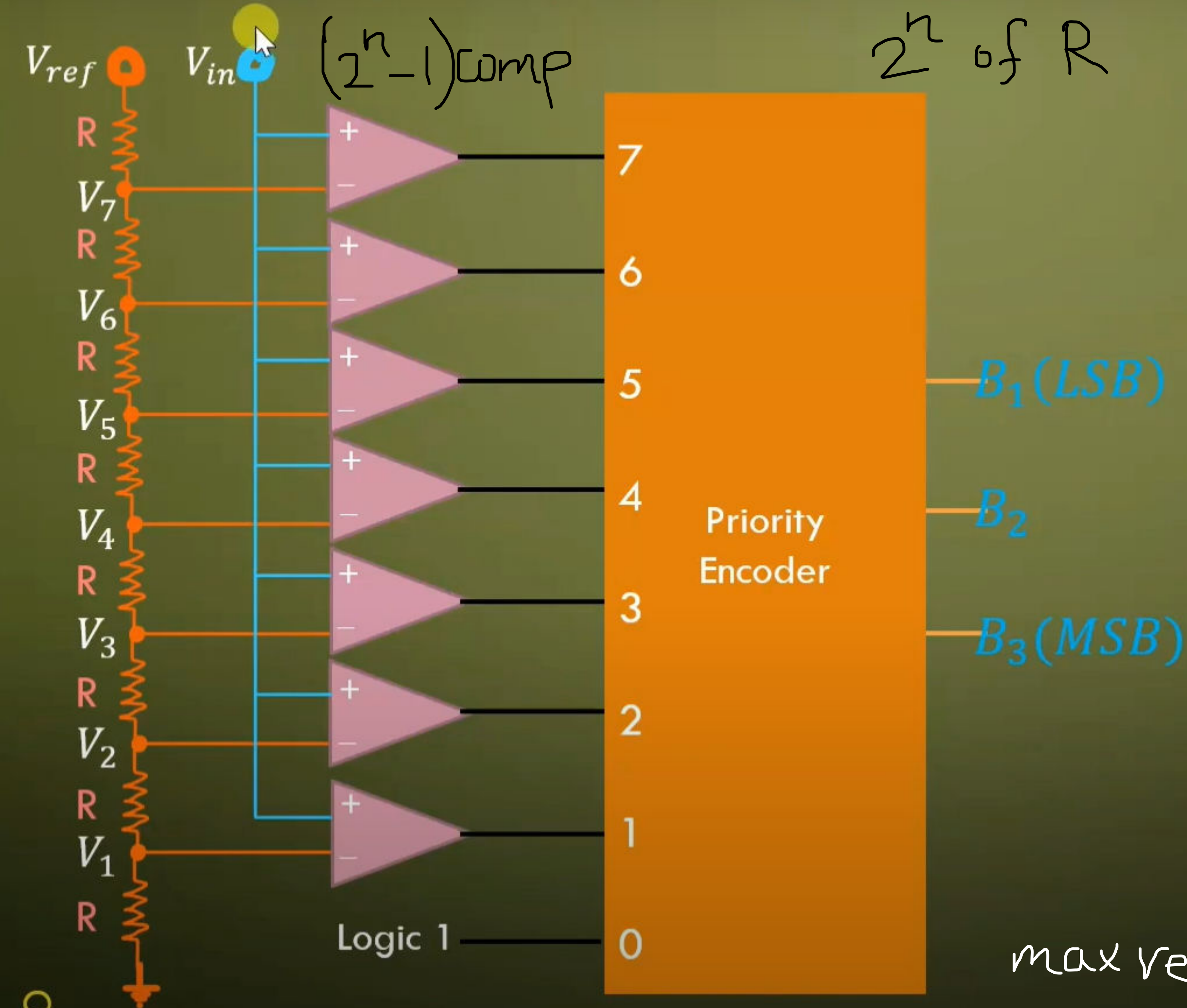
			number of pieces
1	channel sampled at 8KHz	[Sa(t)]	2
1	channel sampled at 16KHz	[Sy(t)]	4
1	channel sampled at 4KHz	[Sz(t)]	1
1	channel GND		1

The sum of pieces = 8



BW= ½ fx4 = ½ * 32= 16 KHz

Structure of Flash ADC



$$V_1 = V_{ref}\left[\frac{R}{8R}\right] = \frac{V_{ref}}{8}$$

$$V_2 = V_{ref}\left[\frac{2R}{8R}\right] = \frac{2V_{ref}}{8}$$

$$\cdots$$

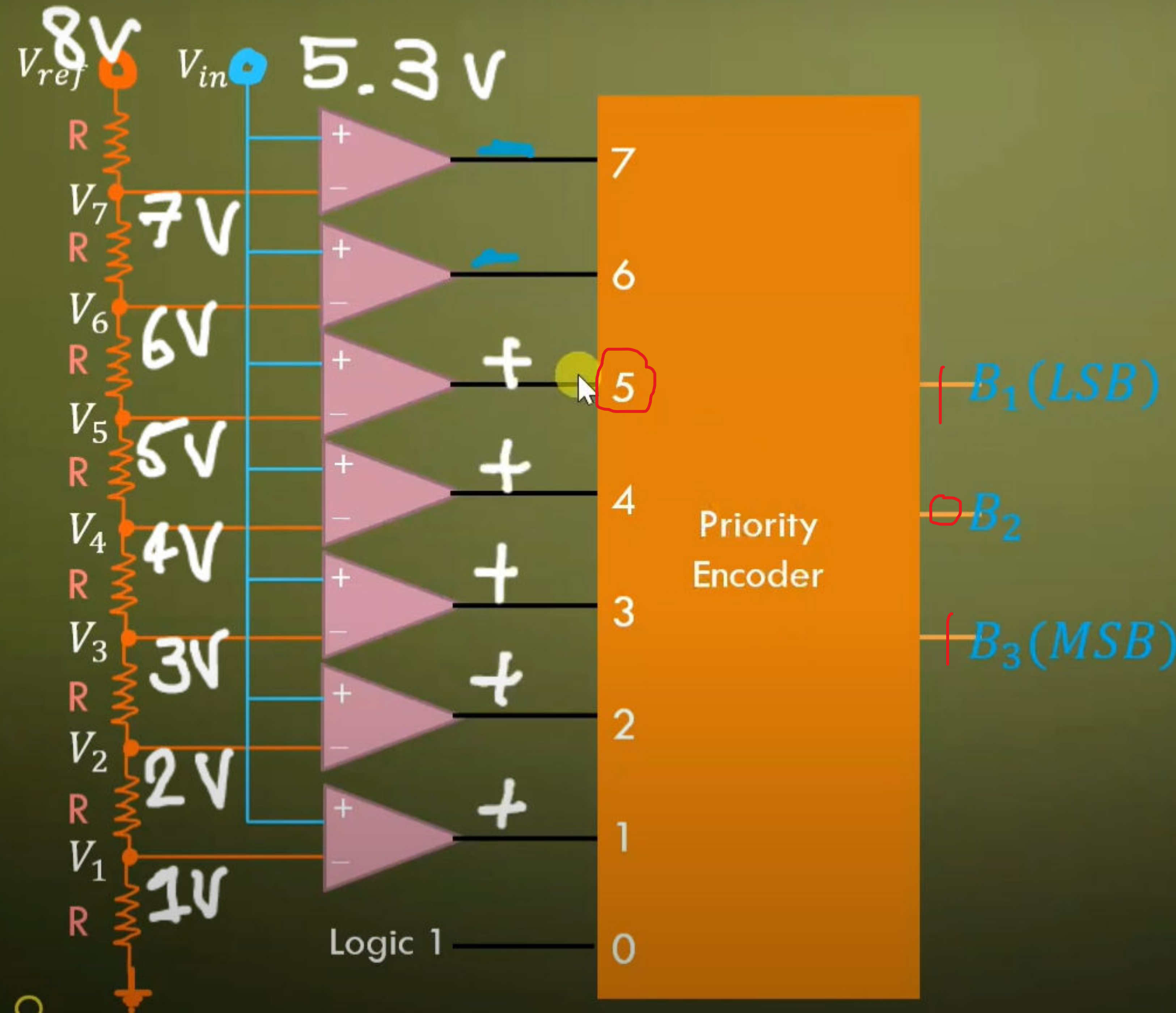
$$V_7 = V_{ref}\left[\frac{7R}{8R}\right] = \frac{7V_{ref}}{8}$$

In Priority encoder, here priority is given in descending order. So, highest (7) and lowest (0) priority is given.

During Conversion, input amplitude should be constant, so we need to connect sample and hold circuit at input signal.

max resolusion is 8bits

Structure of Flash ADC



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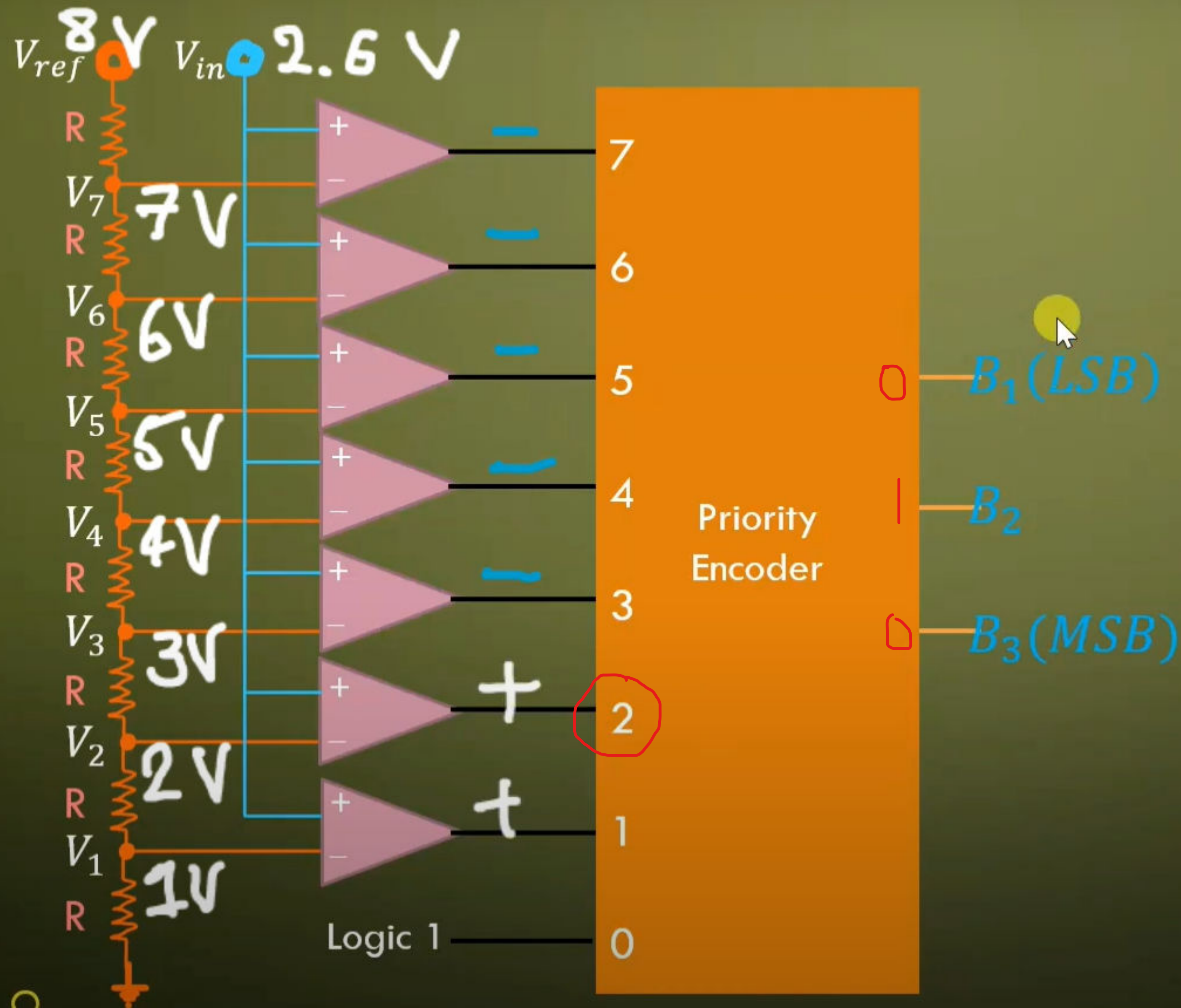
Structure of Flash ADC



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Advantages and Disadvantages of Flash ADC

- It is fastest ADC (Giga Samples per second)
- Useful with large bandwidth input

- High Power consumption
- Limited resolution (Up to 8 bits)
- Large size due to comparator (for 8 bits 255 comparator)
- With more comparator need of higher accurate matching network

Applications of Flash ADC

- Satellite Communication
- RADAR processing
- Oscilloscope

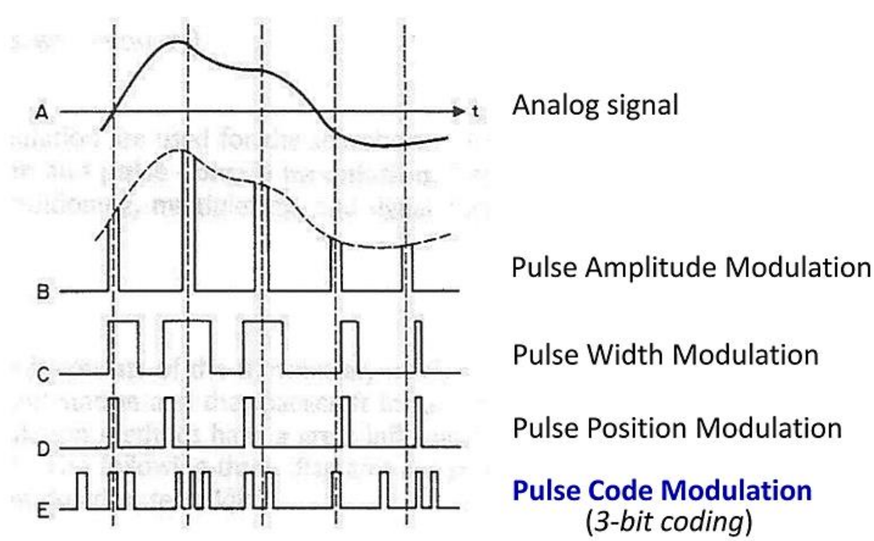
DIGITAL COMMUNICATIONS

Lecture (6-1): Pulse Code Modulation (PCM)

- **Equations and waveform in time domain**
- **Generation and Detection Circuits**
- **Signal to Noise Ratio (S/N)**
- **Required Channel Bandwidth**

Introduction:

Pulse Code Modulation (PCM) is essentially an analog to digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in serial bit stream. In other words, the information that contained in the amplitude (PAM) will converted to a code (using binary code) transmitted in the same sample time



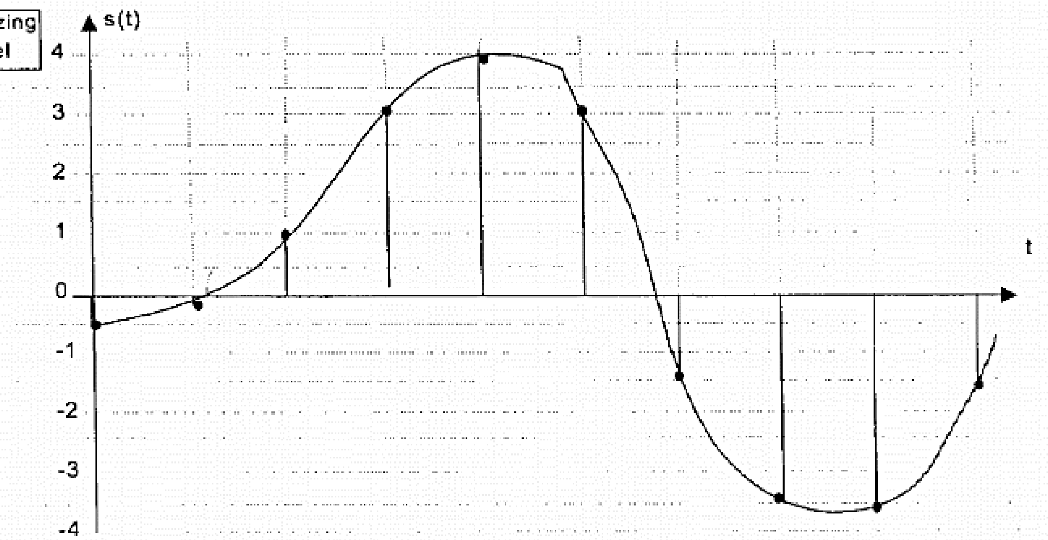
There are two method to get PCM the first is **uniform quantization** and the second is **non uniform quantization**.

<u>Uniform quantization</u>: If we have a message signal S(t) with amplitude varies from (sp , -sp),which called dynamic range of S(t), So to understand the transfer mechanism in uniform quantization the dynamic range is divided into L uniformly spaced interval each with step ΔV

$\Delta V = 2\ sp/L$

A sample amplitude value is approximated by the midpoint of the interval in which it lies as shown in the following figure.

Code Number	Quantizing Level
7	3.5
6	2.5
5	1.5
4	0.5
3	-0.5
2	-1.5
1	-2.5
0	-3.5



Sample number	0	1	2	3	4	5	6	7	8	9
Sample value	-0.45	-0.18	0.95	3.1	3.95	3.0	-1.45	-3.4	-3.6	-1.6
Quantized value	-0.5	-0.5	0.5	3.5	3.5	2.5	-1.5	-3.5	-3.5	-1.5
Code number	3	3	4	7	7	6	2	0	0	2
Binary number	011	011	100	111	111	110	010	000	000	010

The PCM properties:

1. The transmitted code consists of bits which have a constant amplitude and width, thus the bit information confined by two probability either “1” or “0” represented by VCC for “1” and 0 for “0”.
2. Easy to encode the information because it is a digital.
3. It can be storage in a memory

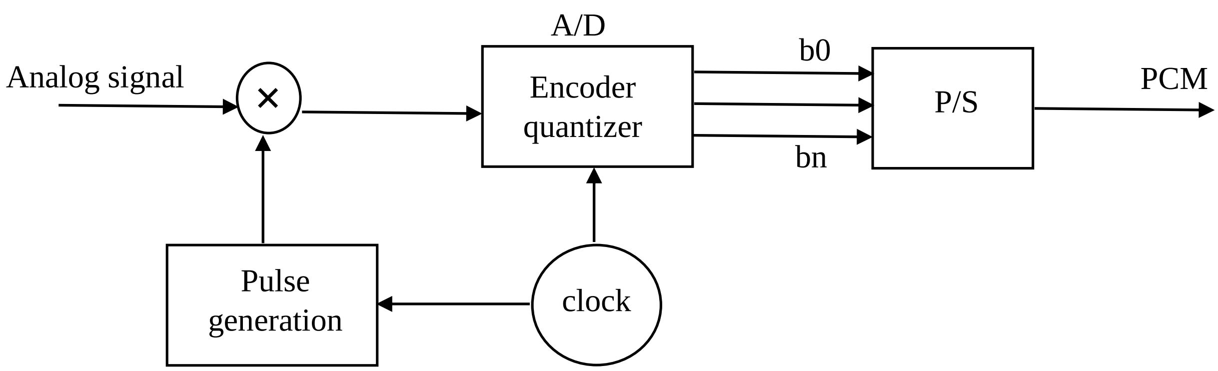
The PCM disadvantage:

1. The quantization processes generate an error cause by approximation of the sampled value.
2. If there is loss in the first bit for the first sample, the receiver will start from the second bit and consider as the first one so will shift the bits.
3. The noise can change the value of the bit.
4. Using the PCM increase the required channel bandwidth.

And to solve these problems there are three solutions:

1. Increase the number of levels so the number of bits that represent the sample value would increase which mean that the quantization error will decrease.
2. Send a synchronization signal with different amplitude and shape to in the begging of the transmitted signal to recognize the first bit in the received signal.
3. Adding a parity bit to the transmitted signal to check if the received signal expose to the noise and make any change in it.

Generation and Detection Circuits:

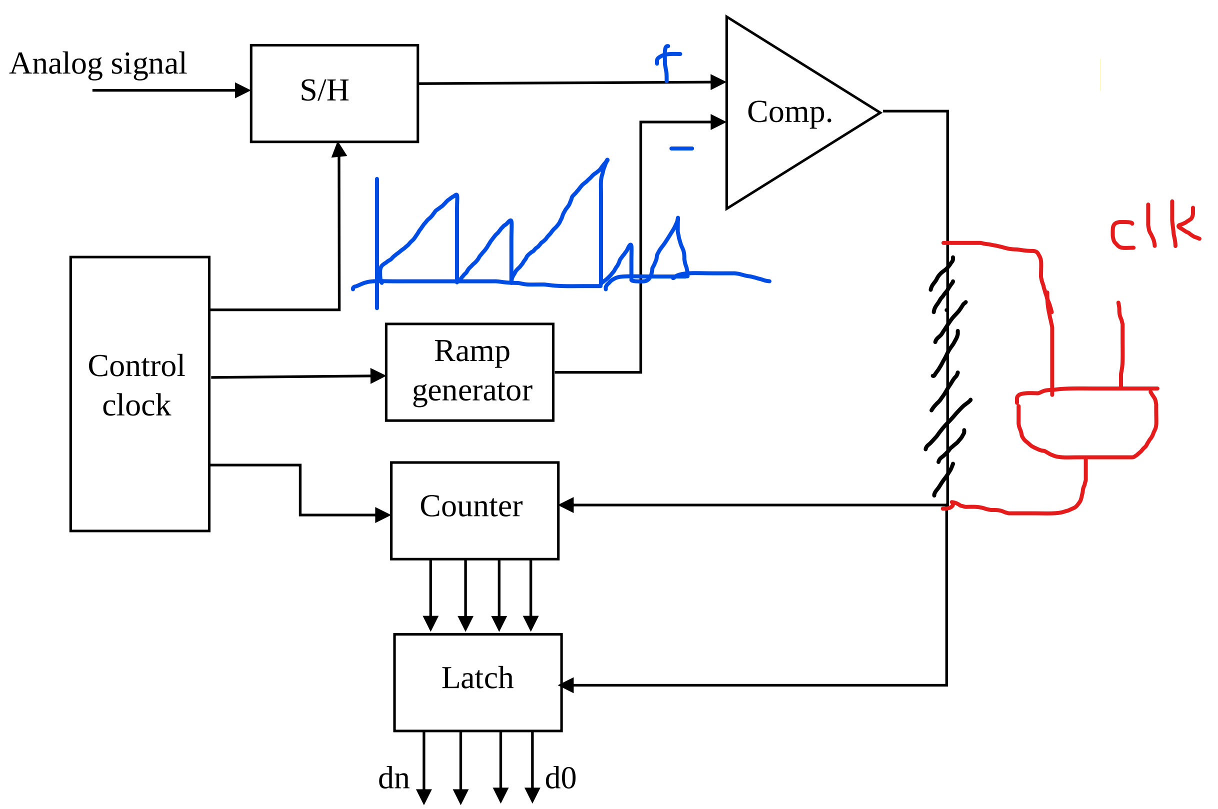


Generation circuit for PCM

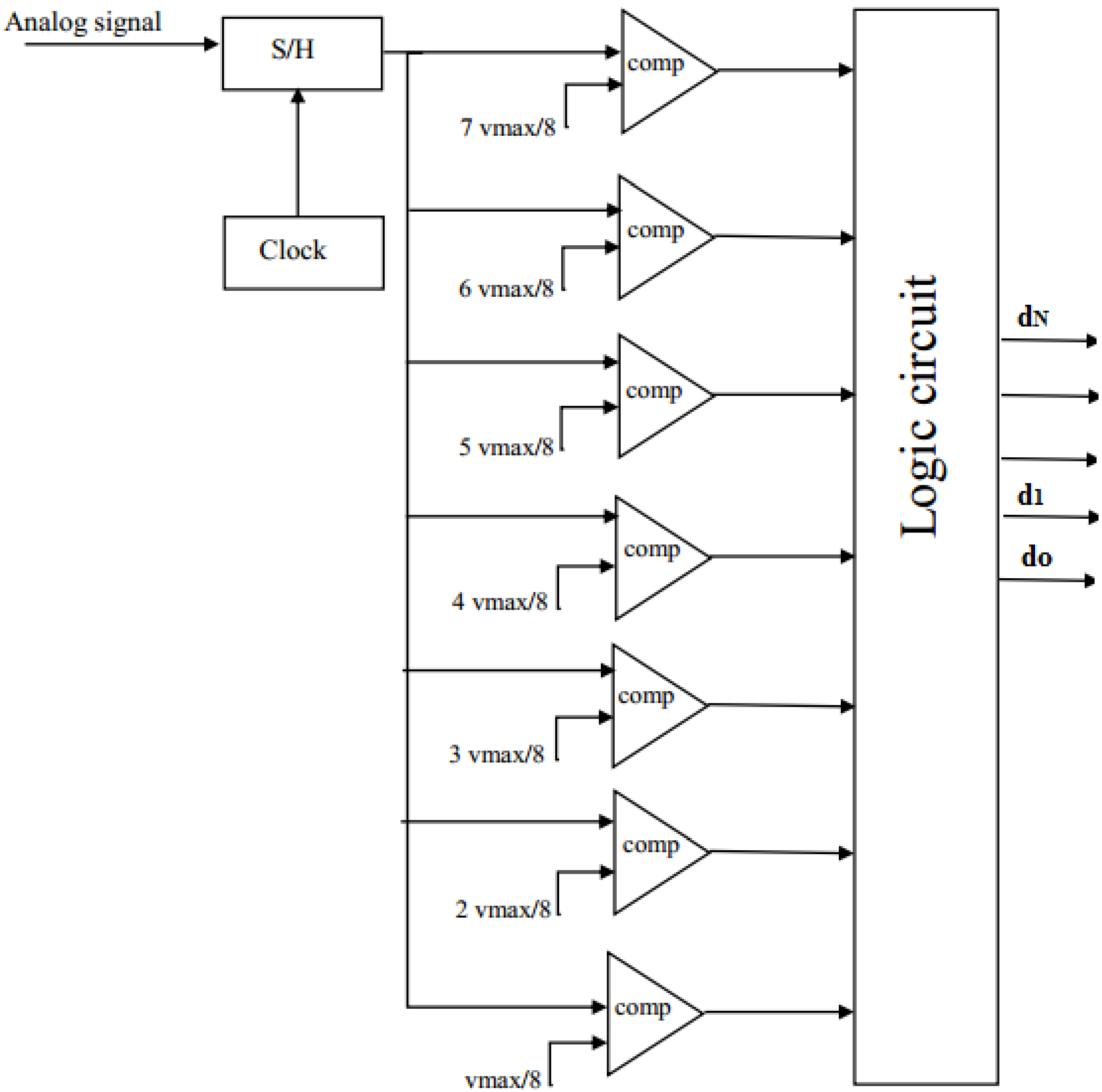
There are two type of A/D convertor:

1) Ramp A/D convertor
2) Flash A/D convertor

Ramp A/D convertor



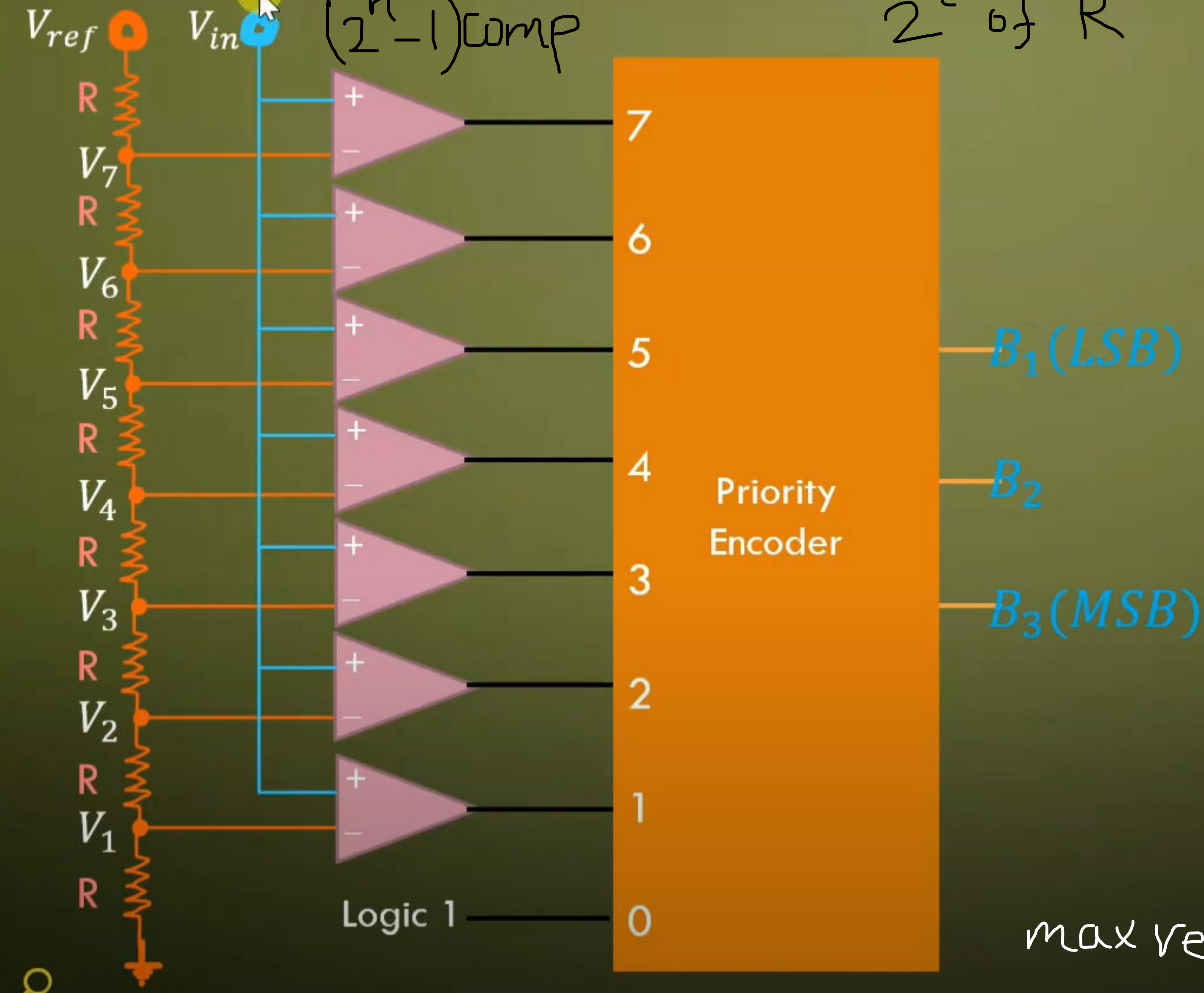
Flash A/D convertor



Structure of Flash ADC

(2^n-1) comp

2^n of R



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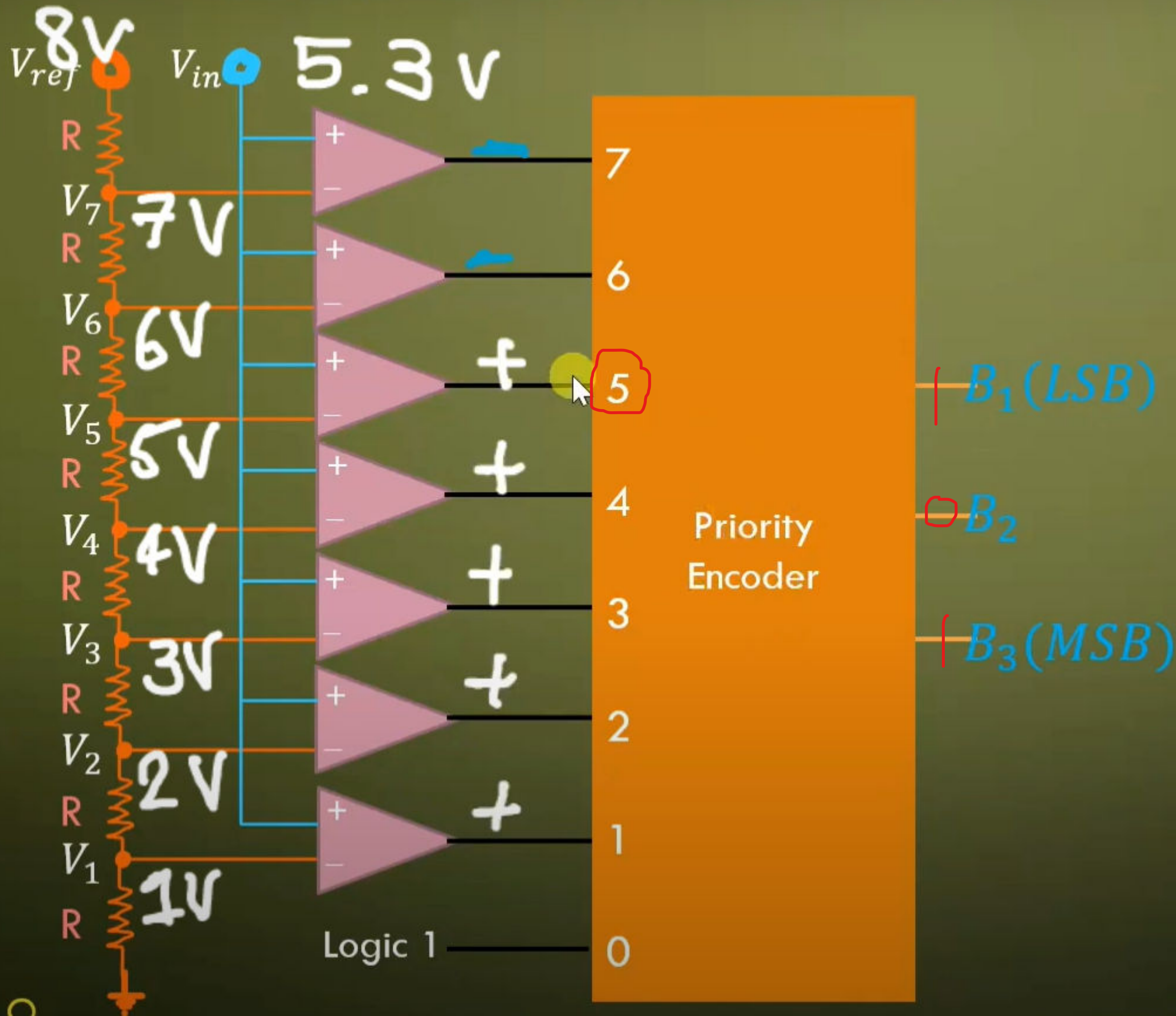
Structure of Flash ADC



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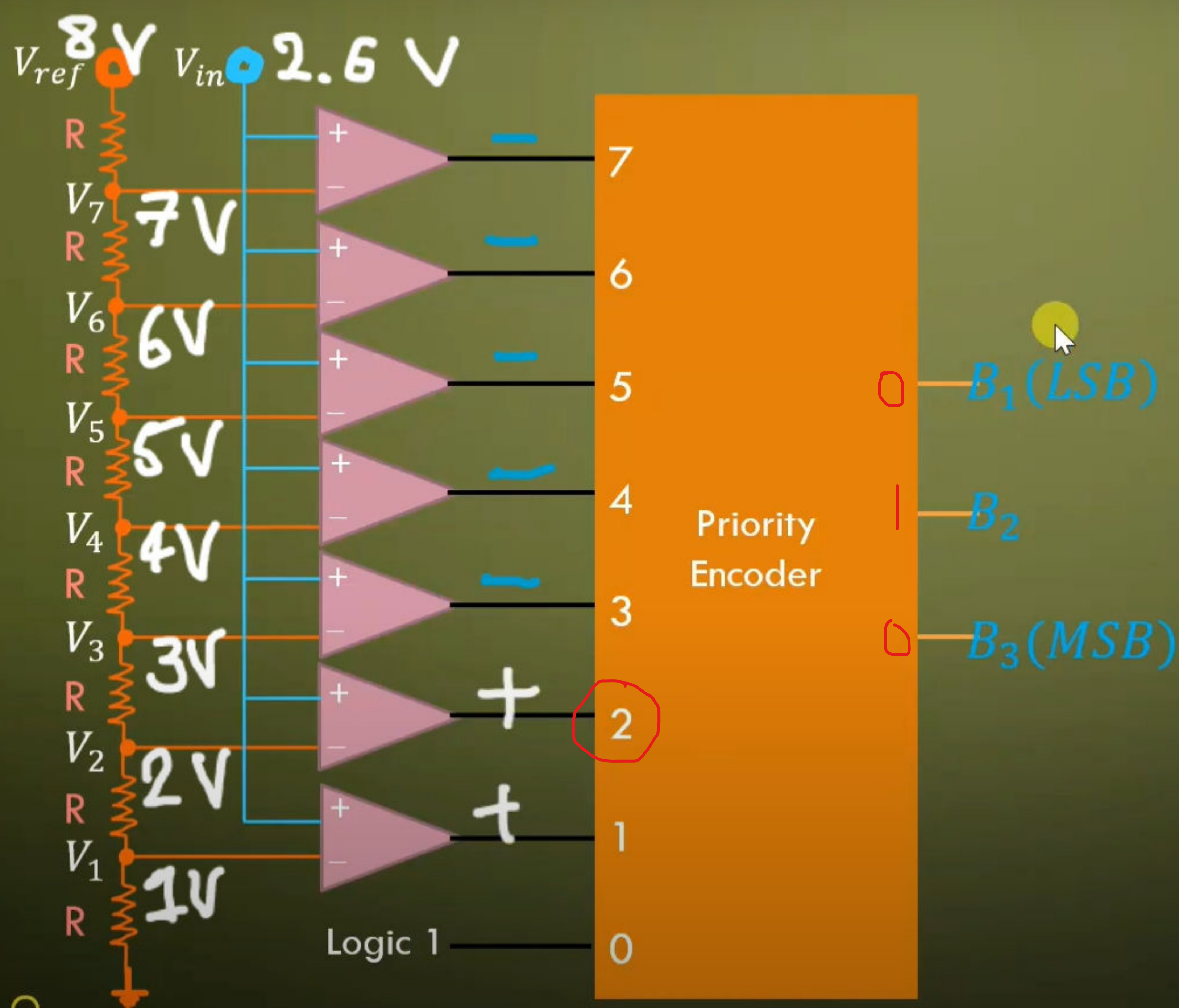

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- Useful with large bandwidth input

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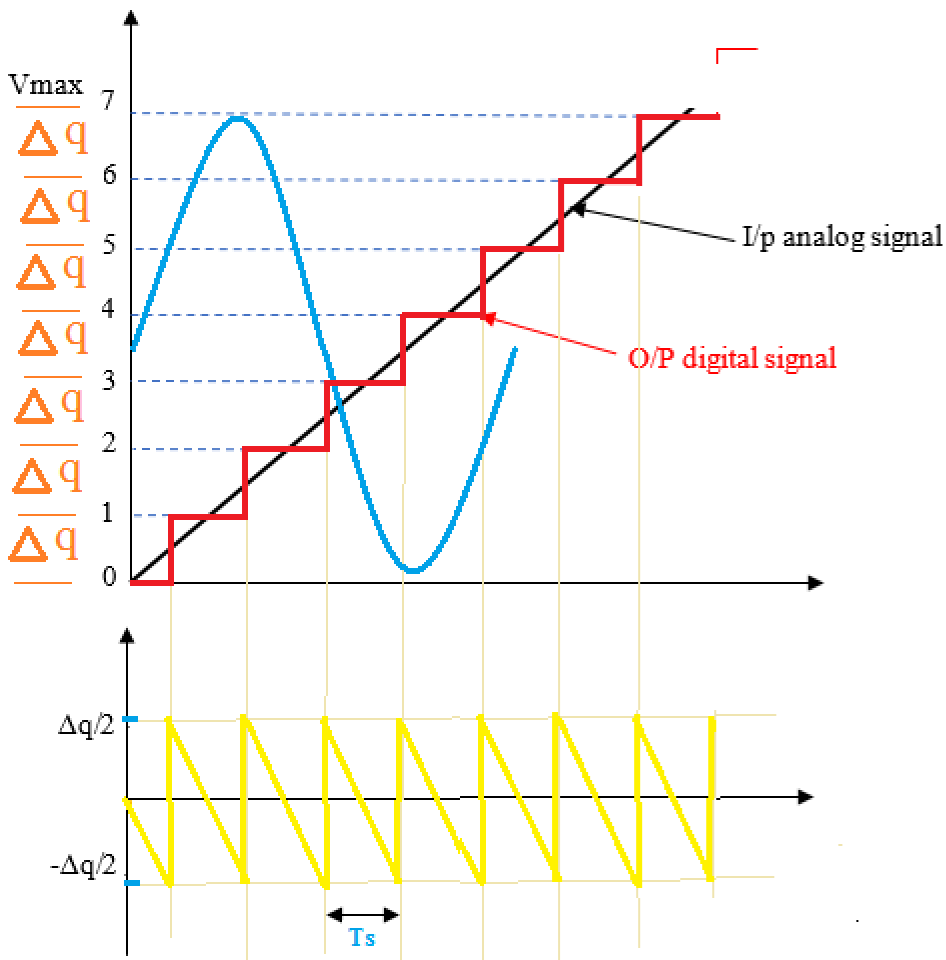
- Limited resolution (Up to 8 bits)
- Large size due to comparator (for 8 bits 255 comparator)
- With more comparator need of higher accurate matching network

Applications of Flash ADC

- Satellite Communication
- RADAR processing
- Oscilloscope

Signal to Noise Ratio (S/N):



$\frac{S}{N} = signal\ power/noise\ power$

$P = Vrms^2/R$

$Vrms(for\ quantization\ error) = \Delta q/2\sqrt{3}$

$\therefore noise\ power = \Delta q^2/12$

The input signal is sin wave

For the input signal: = A cos(wct)

$V_{p-p} = ql * \Delta q$ where ql is the number of levels ql=L =2^n

$Vrms = \frac{V_{p-p}}{2\sqrt{2}} = \frac{ql*\Delta q}{2\sqrt{2}}$

$signal\ power = ql^2 \Delta q^2/8$

$$\therefore \frac{S}{N} = \frac{ql^2 \Delta q^2/8}{\Delta q^2/12} = \frac{3}{2}\, ql^2 = \frac{3}{2}\,(2^N)^2$$

Where N is the number of bits used in PCM

$ql = 2^N$ this equation used to find the number of levels

Also, the S/N can be calculated in dB by using the following equation:

$$\boxed{\frac{S}{N} = 1.76 + 6.02N}$$

rms

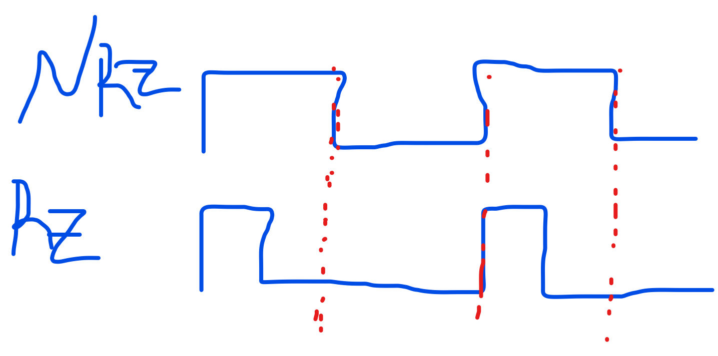
peak SNR=4.77+6.02n

Peak SNR= 3L^2 = 4.77+6.02n
Average SNR= L^2 =6.02n
RMS SNR= 3L^2/2=1.77+6.02n

Required Channel Bandwidth:

To find the required channel BW we must calculate the signaling rate (Sr), where

$Sr = \frac{1}{Tb}$



$Tb = Ts/N$

□ $Ts = 1/Fs$

$\therefore Tb = 1/NFs$

$\therefore Sr$= N*Fs

$BW = Sr/\eta$ where η is the efficiency of the line coding maybe (1 or 2)

η =1 for RZ and =2 for NRZ

DIGITAL COMMUNICATIONS

Lecture (7-1): Compression – Expansion technique (Companding)

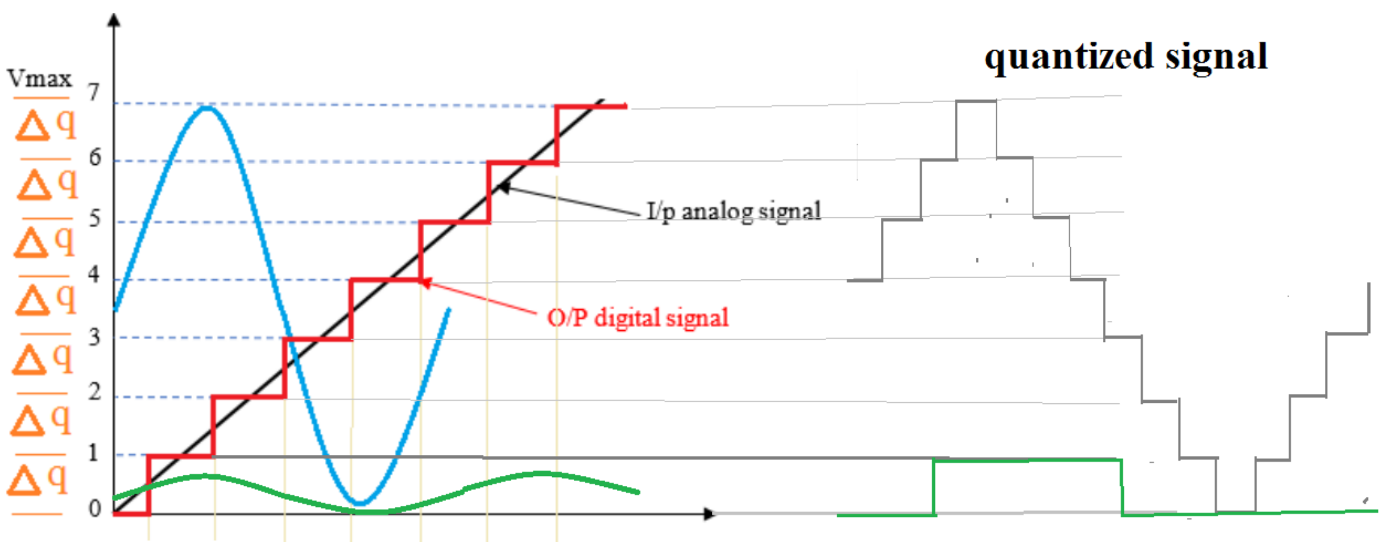
Line Coding for base band signals

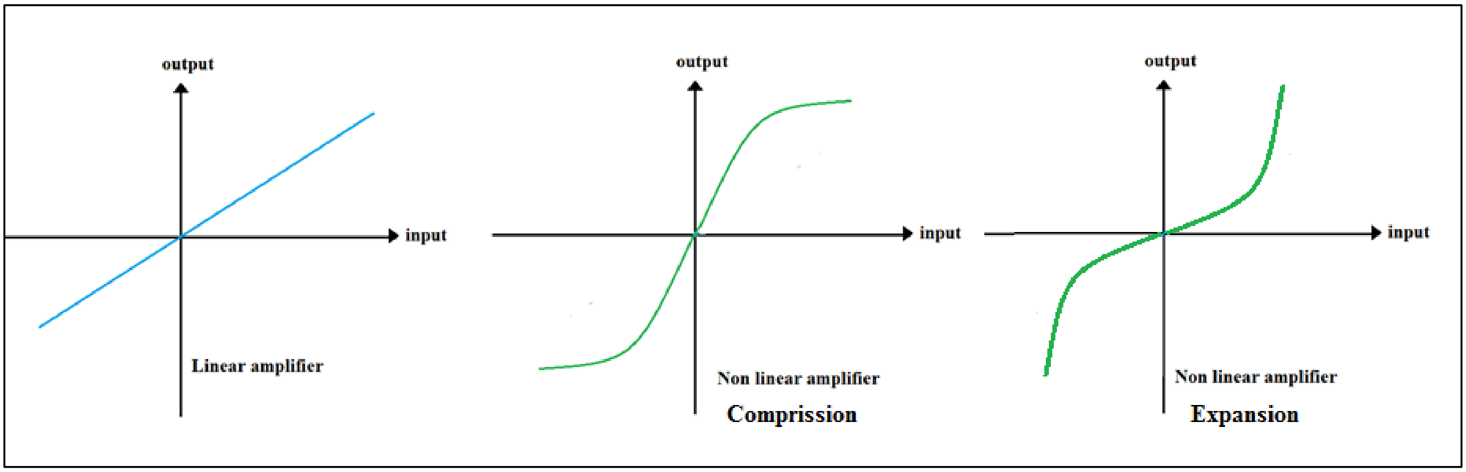
Time Division Multiplexing for PCM Signals (TDM-PCM)

Introduction:

The scientists found thatin speech signal, the smaller amplitudes dominate and the larger amplitudes are much less frequent which mean that the S/N will be low most of the time. This problem belongs to the fixed value of the step size (Δq), where the step size in uniform quantization (Δq=2Sp/L) and the noise power is directly proportional to the square of the step size ($noise\ power = \Delta q^2/12$).

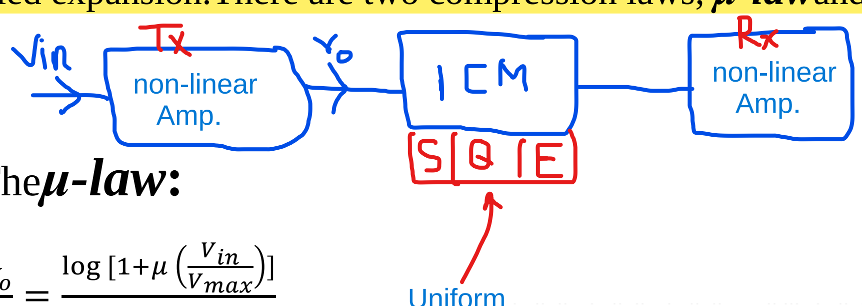
$\epsilon = \frac{\Delta q}{2\sqrt{3}}$





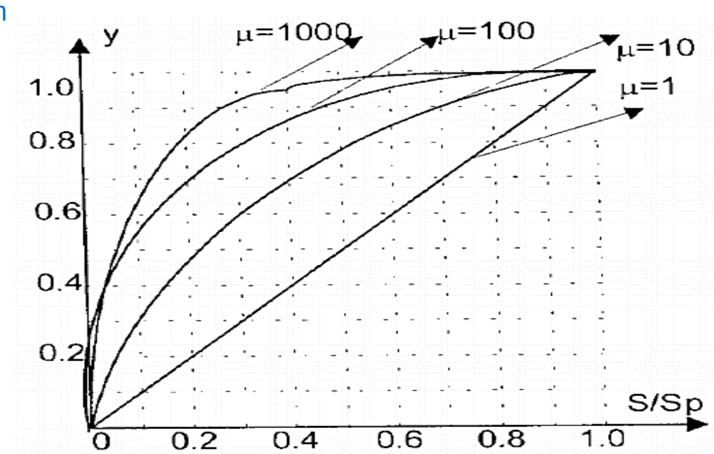
<u>Compression – Expansion technique:</u>

This technique solved the problem first by passing the analog speech signal through a compression(nonlinear) amplifier and then into a PCM generation circuit that uses a uniform quantizer, where the nonlinear amplifier working on amplify the small amplitudes and compress the large amplitudes. This process was in transmitter called compression but in the receiver the amplifier work vice versa in transmitter and called expansion.There are two compression laws, ***μ-law***and ***A-law***:



1. The***μ-law*:**

$$\frac{V_o}{V_i} = \frac{\log\left[1+\mu\left(\frac{V_{in}}{V_{max}}\right)\right]}{\log(1+\mu)}$$

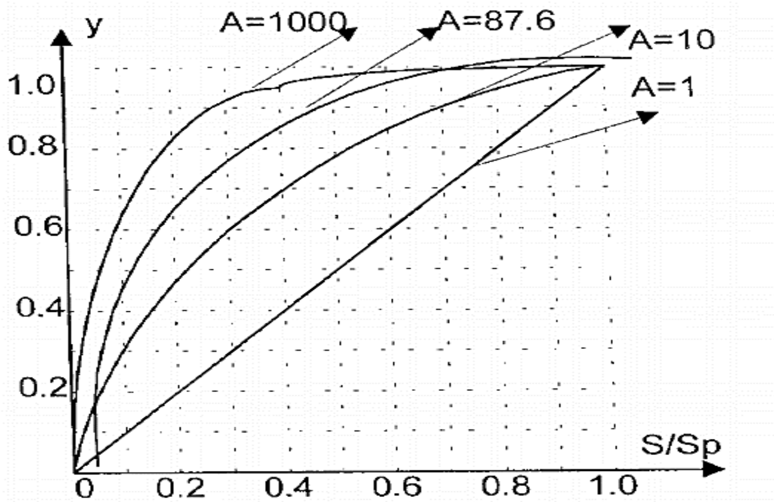


μ-law characteristic

2. The ***A-law:***

$$\frac{V_o}{V_i} = \frac{A * \frac{V_{in}}{V_{max}}}{1+\log A} \qquad 0 \leq V_{in}/V_{max} \leq 1/A$$

$$\frac{V_o}{V_i} = \frac{1+\log(A|\frac{V_{in}}{V_{max}}|)}{1+logA} \qquad 1/A \leq V_{in}/V_{max} \leq 1$$



A-law characteristic

The output SNR in PCM:

bit rate * $R = nf_s$

bandwidth * $B_T = nf_s / \eta$

* $f_s \geq 2f_m$

$B_T = 2nf_m / \eta \Rightarrow \boxed{n = \frac{\eta B_T}{2f_m}}$

$L = 2^n$

$L^2 = (2^n)^2 = 2^{2n}$

$$SNR_o \cong \frac{3L^2}{[\ln(1+\mu)]^2} \rightarrow for \quad \mu - law$$

$$SNR_o \cong \frac{3L^2}{[1+\ln A]^2} \rightarrow for \quad A - law$$

$$SNR_o \cong 3L^2 \frac{\overline{m^2(t)}}{S_p^2} \rightarrow for \quad uniform$$

These three upper equations can be expressed as:

$$SNR_o = C(2)^{2n} = C(2)^{\frac{\eta B_T}{f_m}} \rightarrow$$

$\boxed{n = \frac{\eta B_T}{2f_m}}$

$\eta = 1$ for bipolar linecoding

where

$$C = 3\frac{\overline{m^2(t)}}{S_p^2} \rightarrow for \quad uniform$$

$$C = \frac{3}{[\ln(1+\mu)]^2} \rightarrow for \quad \mu - law$$

$$C = \frac{3}{[1+\ln A]^2} \rightarrow for \quad A - law$$

$10\log_{10} 2^{2n} = 20n\log 2$

$= 6.02$

In dB's

$$(SNR_o)_{dB} = 10\log(SNR_o) = \alpha + 6.02n\, dB$$

$$\alpha = 10\log C$$

where

$$\alpha = 4.77 + 20\log\frac{S_{rms}}{S_p} \rightarrow for \quad uniform$$

$$\boxed{\begin{aligned} \alpha &= 4.77 - 20\log[\ln(1+\mu)] \rightarrow for \quad \mu - law \\ \alpha &= 4.77 - 20\log[1+\ln(A)] \rightarrow for \quad A - law \end{aligned}}$$

$3L^2$ (P)

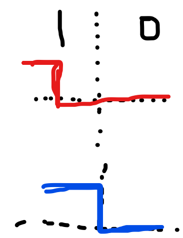
L^2 (A)

$\frac{3L^2}{2}$ (R)

Line Coding for base band signals:

There are two major categories of signaling:

1) Return-to-Zero (RZ): in which the wave form returns to a zero-volt level portion of the bit interval (usually one half).
2) Non-Return-to-Zero (NRZ): in which the wave form does not returns to a zero-volt level before the next bit interval.



Note: NRZrequired more power than RZ but its power spectral density (PSD) bandwidth is less than that of RZ.

Polar signaling:

In polar signaling, 1 is transmitted by a pulse with V volt and 0 is transmitted by a minus pulse with -V volt.

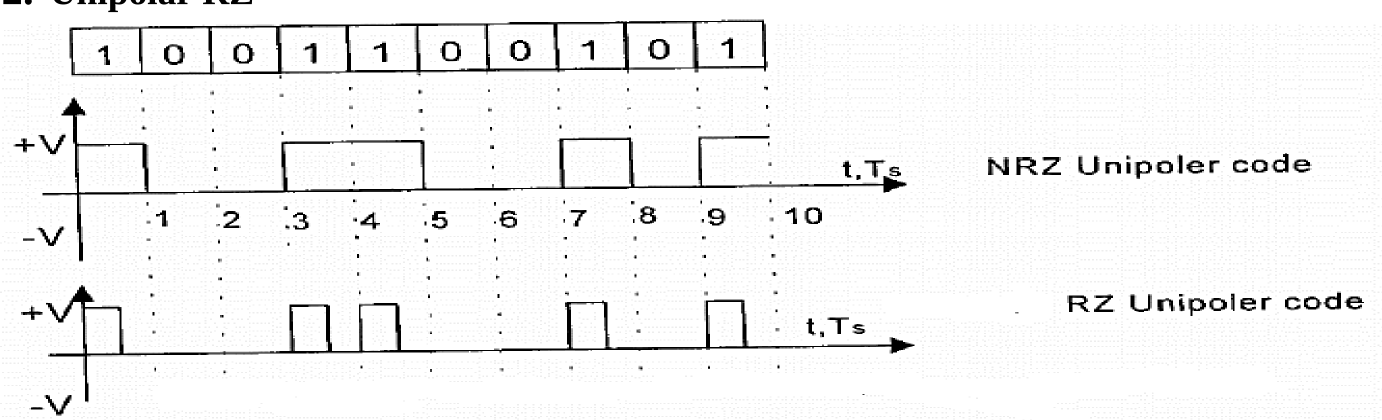
Unipolar or On-Off Signaling:

In unipolar signaling, 1 is transmitted by a pulse with V volt and 0 there is no pulse transmitted.

Line Coding Types:

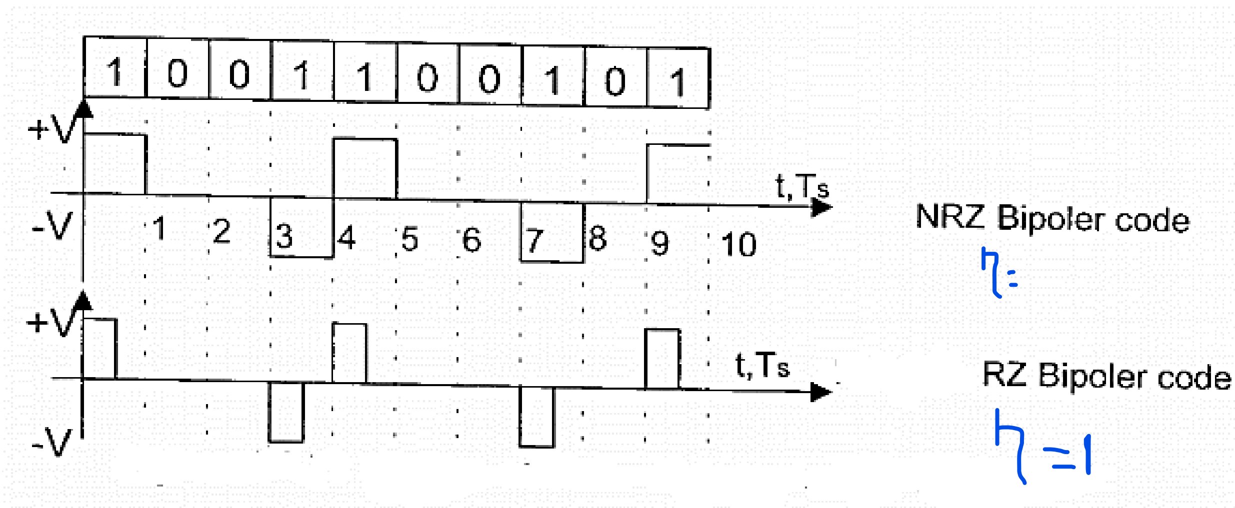
1. Unipolar NRZ

2. Unipolar RZ

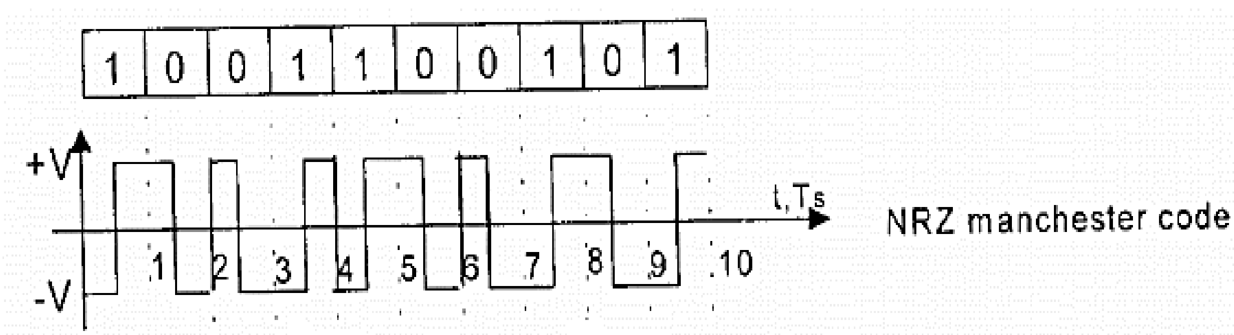


3. **Bipolar NRZ**

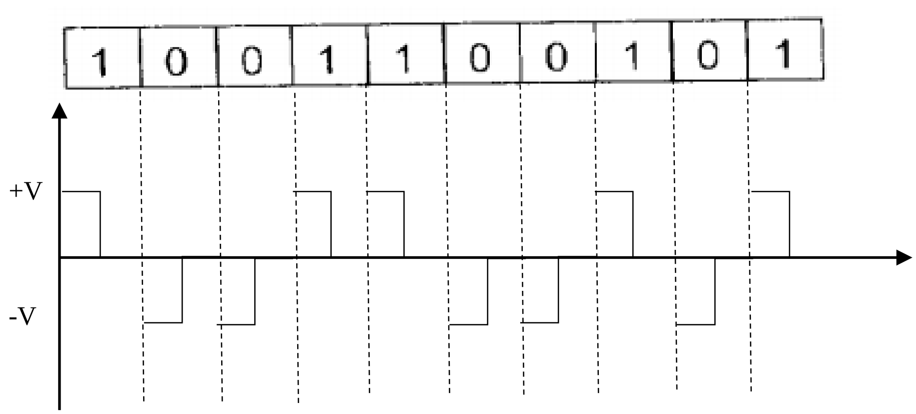
4. **Bipolar RZ**



5. **Manchester code**



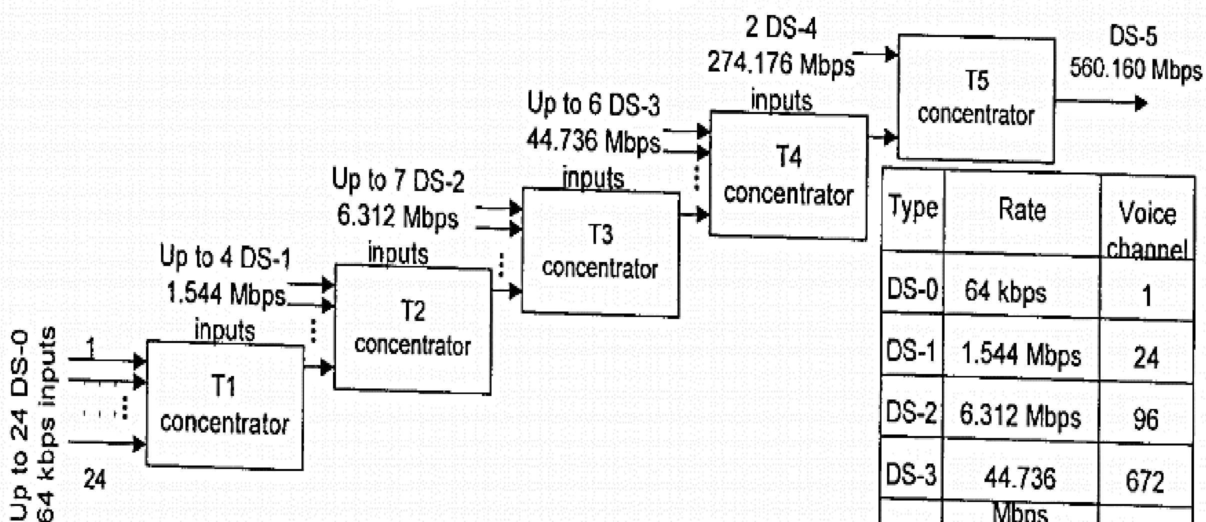
6. **Return-to-Bias (RB)**



Time Division Multiplexing for PCM Signals (TDM-PCM):

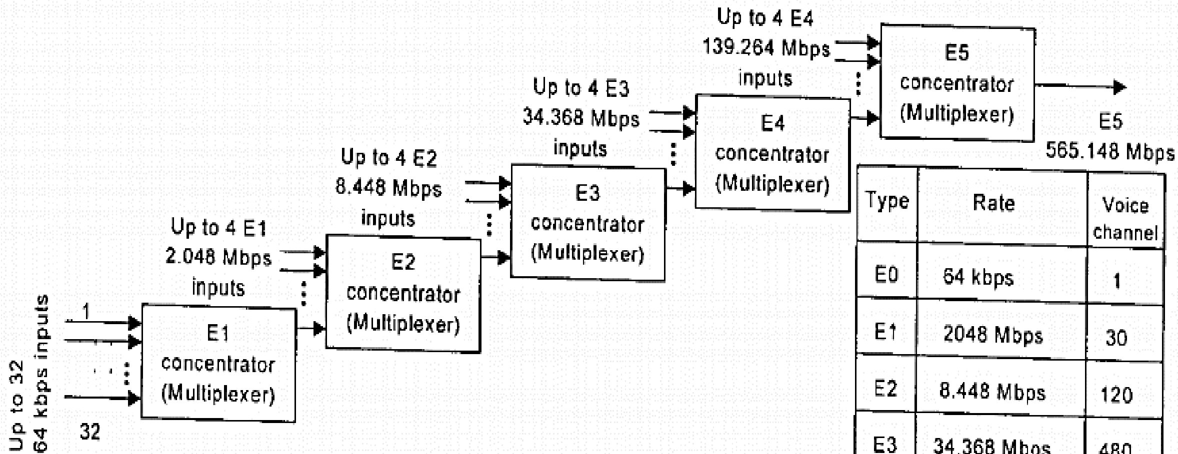
There are two categories of TDM systems:

1) Digital computer TDM system: in this category digital signals from several sources are merged for TDM transmission over a high-speed line to a digital computer. The output rates of these multiplexers have been standardized to 1200, 2400, 3600, 4800, 7200, 9600 and 19200 bps.
2) Common carrier TDM system: this category of TDM system is used by the common carrier such as AT&T, to combine different sources into high- speed digital TDM signal for transmission over toll network. The standards adopted by North America and Japan are different from those that have been adopted in Europe and other parts of the world.



Type	Rate	Voice channel
DS-0	64 kbps	1
DS-1	1.544 Mbps	24
DS-2	6.312 Mbps	96
DS-3	44.736 Mbps	672
DS-4	274.176 Mbps	4032
DS-5	560.160 Mbps	8064

North American digital TDM hierarchy



Type	Rate	Voice channel
E0	64 kbps	1
E1	2048 Mbps	30
E2	8.448 Mbps	120
E3	34.368 Mbps	480
E4	139.264 Mbps	1920
E5	565.148 Mbps	7680

International Telecommunications Union (ITU) digital TDM hierarchy

DIGITAL COMMUNICATIONS

Lecture (8-1): Delta Modulation (DM)

Equations and waveform in time domain

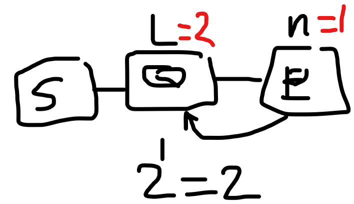
Generation and Detection Circuits

Signal to Noise Ratio (S/N)

Required Channel Bandwidth

Adaptive Delta Modulation (ADM)

$\boxed{L=2^n}$



$+\Delta q \quad -\Delta q$

Introduction:

The PCM system has many advantages mention in the previews lectures, but also this system has disadvantage where if the user wants to decrease the error, the S/N must increase. The S/N raised by increasing the number of bits that represent the amplitude of the sample as shown in the following equation:

$$\frac{S}{N} = \frac{3}{2}L^2 = \frac{3}{2}\left(2^N\right)^2 \qquad L=2^n$$

r.m.S
$$\frac{S}{N} = 1.76 + 6.02N$$

This increasing lead to increase in the required bandwidth which considered as an important factor to evaluate the operating system. One of the must popular solution to this problem is the delta modulation system.

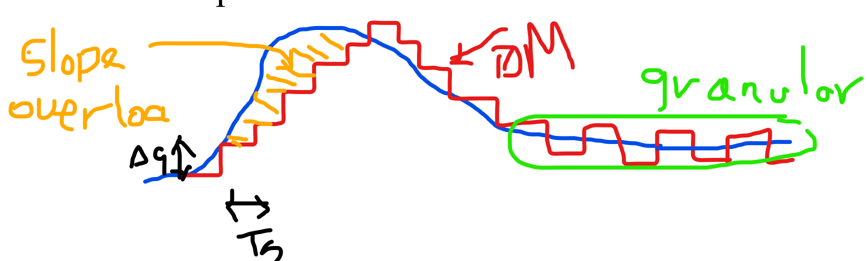
Delta Modulation (DM):

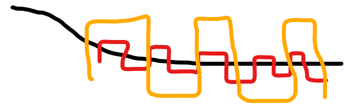
DM is a simple technique based on sending the difference between the amplitude of a sample and previews sample instead of sending the amplitude value of each one by using **one bit**, where if the difference is positive "**1**" is send, if the difference is negative "**0**" is send. These two possibilities known as either +Δ or -Δ.

$n=1$

$L=2$

$+\Delta q \quad -\Delta q$





At every sample point the quantized waveform can only either increase or decrease by Δ.

Equations and waveform in time domain:

The basic principle of delta modulation may be formalized in the following set of discrete time relations: $+\Delta q$ $-\Delta q$

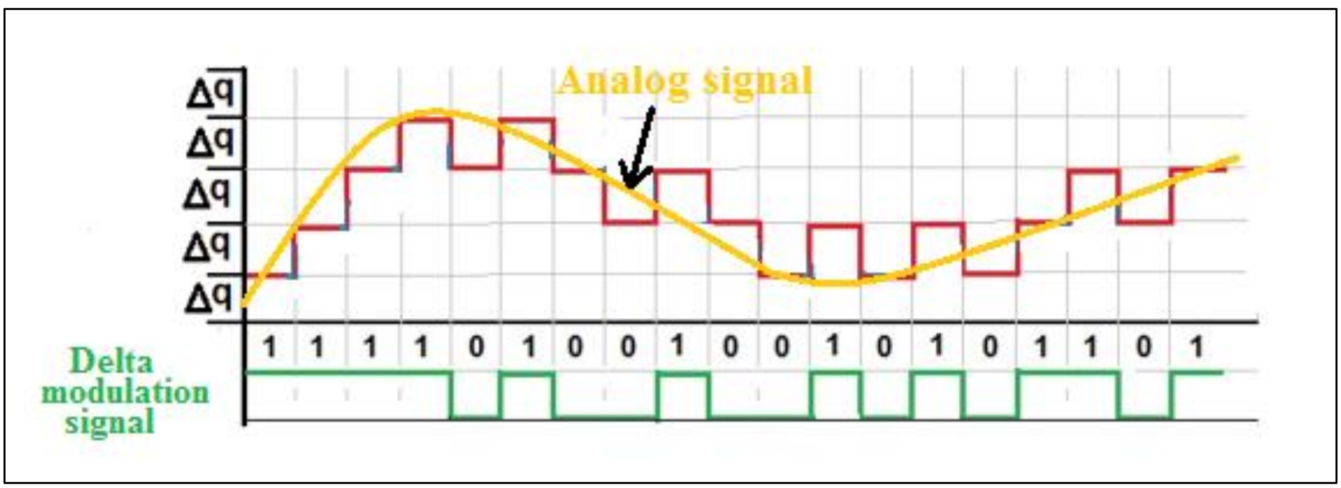
$$e(nTs) = S(Ts) - Sq(nTs - Ts)$$

$$eq(nTs) = \Delta\, sgn\, [e(nTs)] \quad +\Delta \quad -\Delta$$

$$Sq(nTs) = Sq(nTs - Ts) + e_q(nTs)$$

Where: Ts: the sampling period

S(t): the input signal



Sq(nTs): the staircase approximation of S(t)

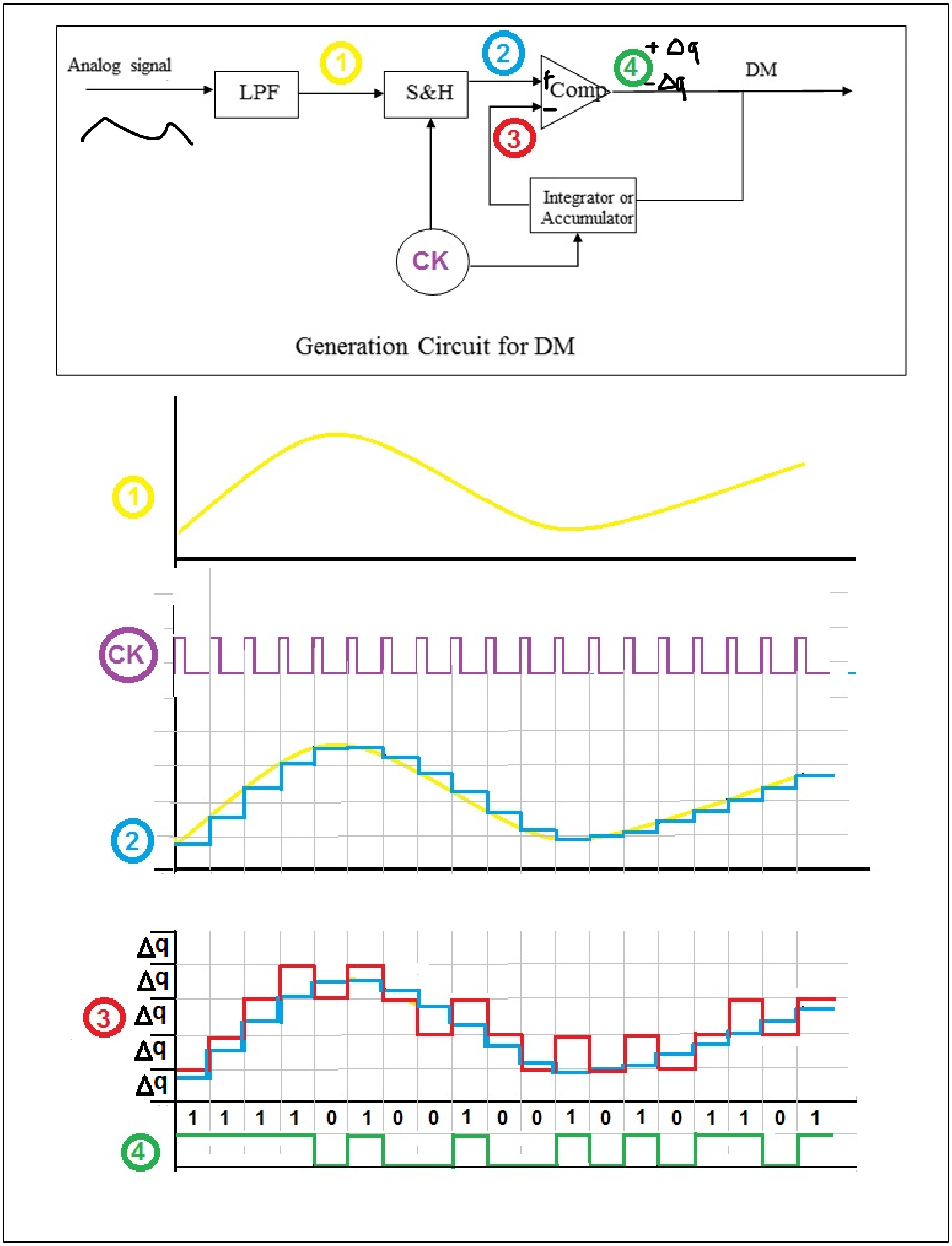
e(nTs): an error signal

S(nTs): the present sample

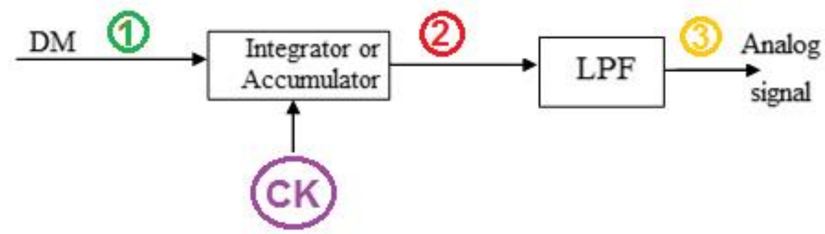
Sq(nTs-Ts): the latest approximation to it

eq(nTs): the quantized version of e(nTs)

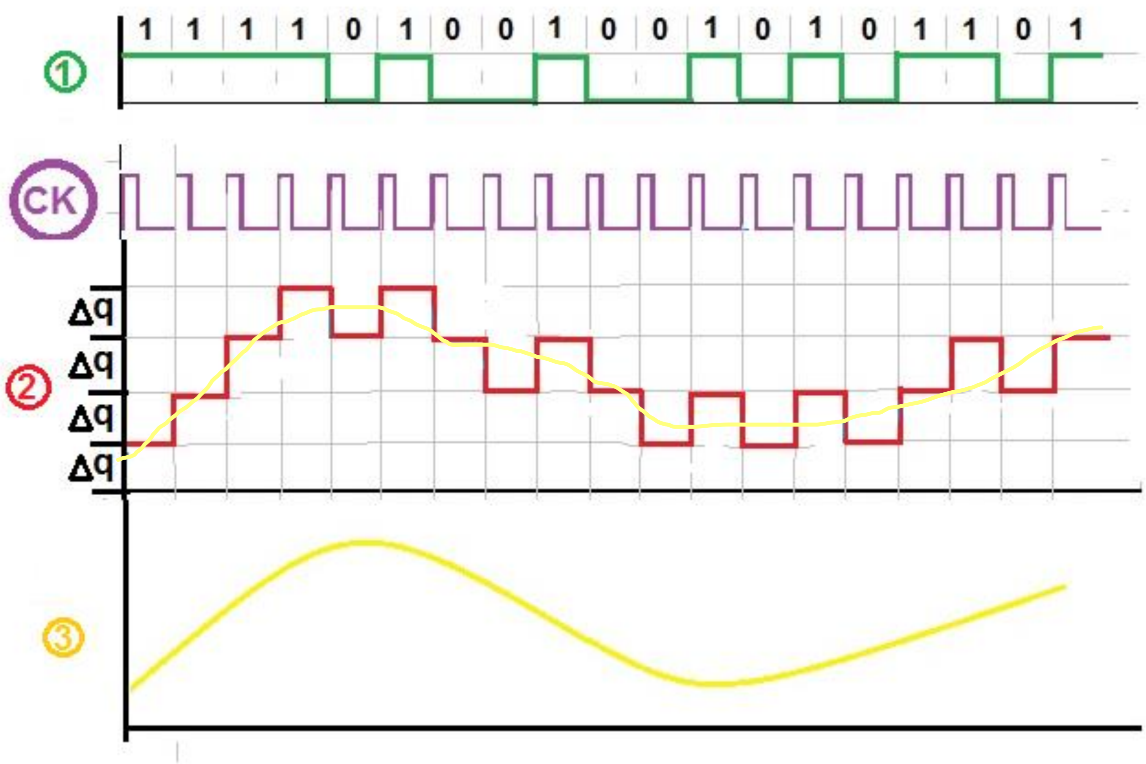
Generation and Detection Circuits:



Generation Circuit for DM



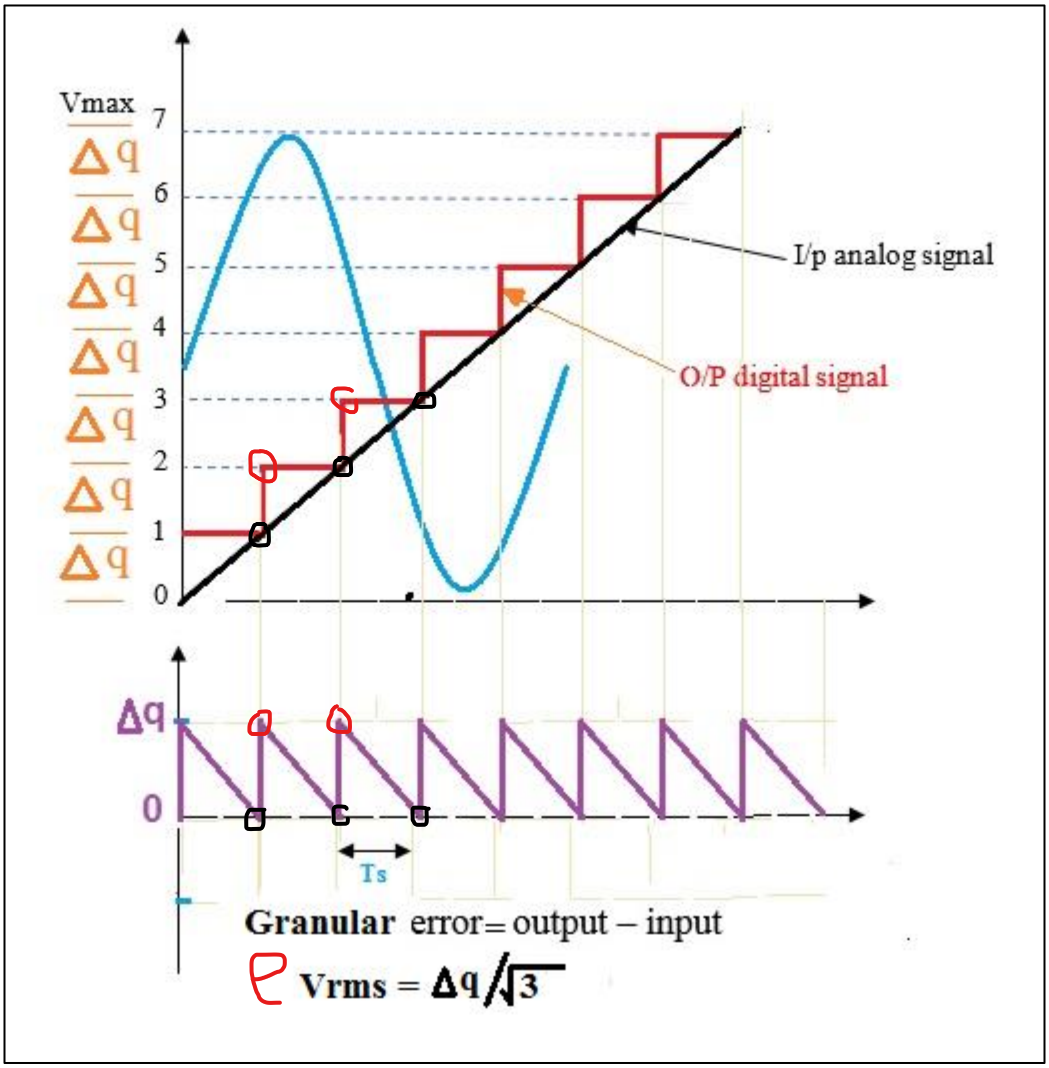
Detection Circuit for DM



Granular (Threshold) Noise and Slope Overload Noise:

To define the Granular noise and slope overload noise let us consider a signal shown in the following figure having different slopes and see the effect of high slope and low slope on the quality of the output stare-case and DM signal.

Granular noise and threshold of coding happened when variation in s(t) smaller than the step value and these variations are lost in DM.

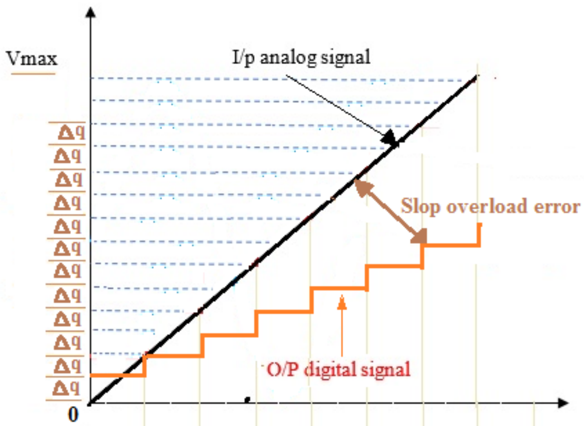


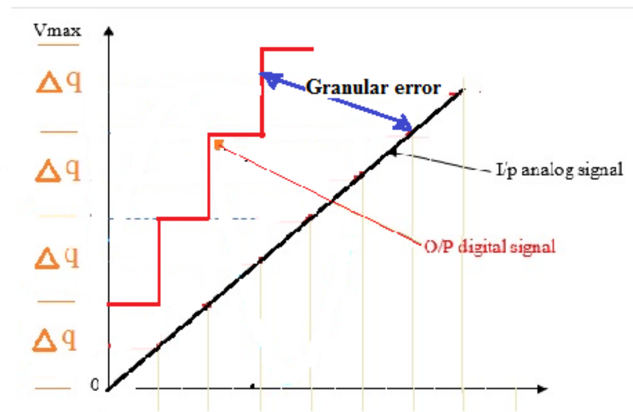
$$e^2_{v.rms} = \frac{1}{2\Delta q}\int_{-\Delta q}^{\Delta q} \epsilon^2 \, d\epsilon = \frac{\Delta q^2}{3} \Rightarrow \boxed{e_{v.m.s} = \frac{\Delta q}{\sqrt{3}}}$$

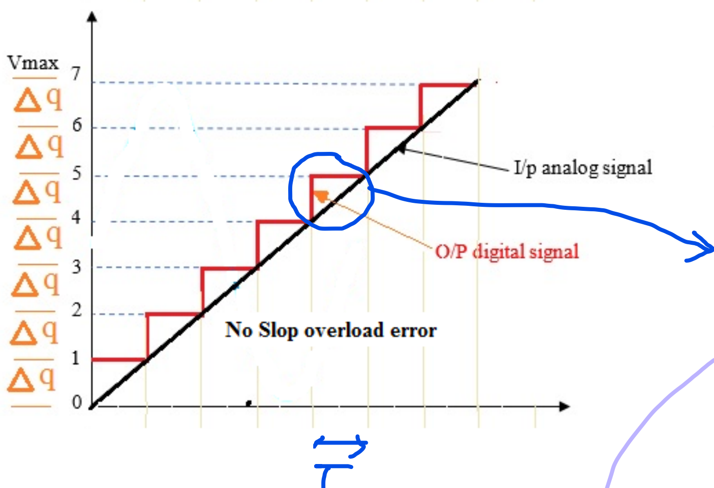
$\left(\frac{1}{\sqrt{2}}\right)^2$ for r.m.s

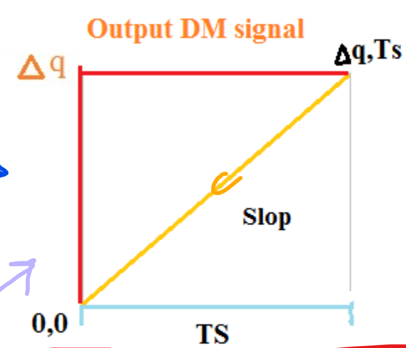
Slop over load error :-

The slope overload noise happened when the output signal cannot follow the input signal during Ts.

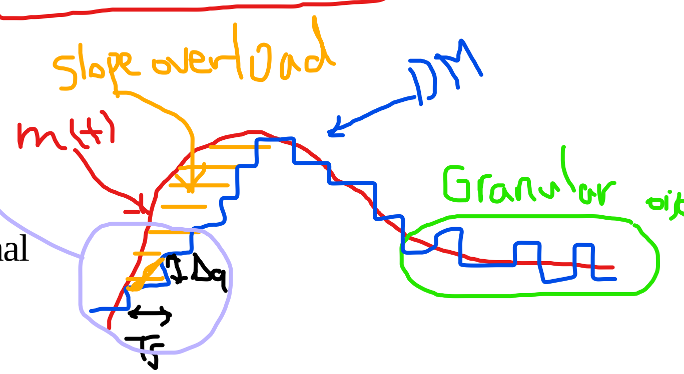








$\boxed{\text{Slop} = \Delta q / Ts \;=\; \Delta q * Fs}$



Slop=Δq/Ts

- For no slop overload error

Slop for o/p = the differentiation of the I/P signal

$$\frac{\Delta q}{Ts} = \frac{d}{dt} A \sin wat \Big|_{to}$$

$$\Delta q * fs = A\, wa \cos wat$$

$$\Delta q * fs = A 2\pi fa \qquad \boxed{\Delta q = \frac{A 2\pi fa}{fs}} \quad (\text{ for no slop overload error})$$

max of cos

$\Delta q\, f_s = A_m\, 2\pi f_m$

$$\boxed{f_s \geq \frac{2\pi A f_a}{\Delta q}} \quad \text{or} \quad \boxed{f_s \Delta q \geq 2\pi A f_a}$$

Example: $m(t) = 6\sin(20\pi t) + 4\cos(40\pi t)$. Determine the minimum sampling frequency to avoid slope overload in DM, if $\Delta_q = 0.1\pi$.

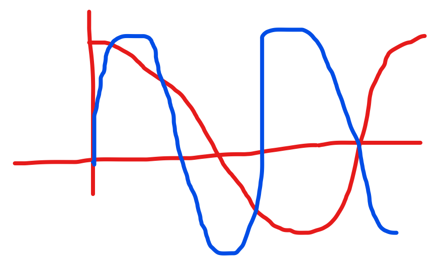
$$m(t) = 6\sin(20\pi t) + 4\cos(40\pi t), \text{ then}$$

$$\frac{dy}{dt} m(t) = 6\cos(20\pi t) \times 20\pi + 4(-\sin(40\pi t)) \times 40\pi$$
$$= 120\pi\cos(20\pi t) - 160\pi\sin(40\pi t).$$

Now taking the peak magnitude of the above equation gives

$$\left|\frac{dy}{dt} m(t)\right|_{max} = 120\pi + 160\pi = 280\pi.$$

Thus,



$$f_s \Delta \geq 280\pi$$
$$f_s \geq \frac{280\pi}{\Delta}$$
$$f_s \geq \frac{280\pi}{0.1\pi}$$
$$f_s \geq 2800 \text{ samples/sec.}$$

Example: A DM modulator is to be designed to transmit information of an analogue waveform that has a peak-to-peak level of 1V and a bandwidth of 3.4 kHz. If DM system is to be used to transmit the information of voice (analogue) signal, select the appropriate step size when the sampling rate is 30 kHz.

Sol.: Recall

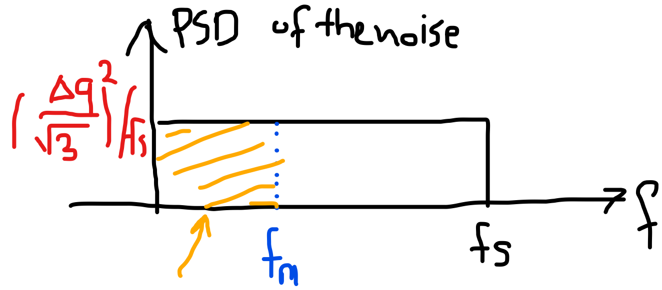
$$f_s \Delta \geq 2\pi A_m f_m.$$

Hence, the minimum step size can be given as

$$\Delta_{min} = \frac{A_m 2\pi f_m}{f_s} = \frac{0.5 \times 2\pi \times 3400}{30 \times 10^3} = 0.4273.$$

<u>Signal to Noise Ratio (S/N):</u>

To drive S/N equation the fooling condition must be realized to get no slope overload error:



$$\boxed{slope\ output\ signal \geq slope\ input\ signal}$$

$$noise\ Vrms = \frac{\Delta q}{\sqrt{3}}$$

$$P_N = noise\ power = \frac{\Delta q^2}{3}$$

$$P.N.S.D = power\ noise\ spectrum\ density$$

$$\boxed{N} \longrightarrow P.N.S.D = \frac{PN}{fs} * BW = \frac{\frac{\Delta q^2}{3}}{fs} * BW = \frac{\Delta q^2}{3fs} * \cancel{BW}\ f_a$$

$$\boxed{N_q = \frac{\Delta_q^2}{3f_s} \cdot f_a = \text{quantization noise power}}$$

Let $s(t) = A \sin wat$

$$\boxed{S} \longrightarrow signal\ power = \frac{Vrms^2}{R} = \left(\frac{A}{\sqrt{2}}\right)^2 = \frac{A^2}{2}$$

$$\boxed{\frac{S}{N} = \frac{signal\ power}{noise\ power}} = \frac{A^2/2}{\frac{\Delta q^2}{3fs} * \cancel{BW}\ f_a}$$

$$\Delta q = \frac{2\pi A f_a}{f_s}$$

$$\boxed{Bw = f_a}$$

By substituting the Δq for no slop overload error in the last equation

$$\frac{\boldsymbol{S}}{\boldsymbol{N}} = \frac{\boldsymbol{3fs^3}}{\boldsymbol{8\pi^2 * fa^2 * BW}} \Longrightarrow \boxed{\frac{S}{N} = \frac{3f_s^3}{8\pi^2 f_a^3}}$$

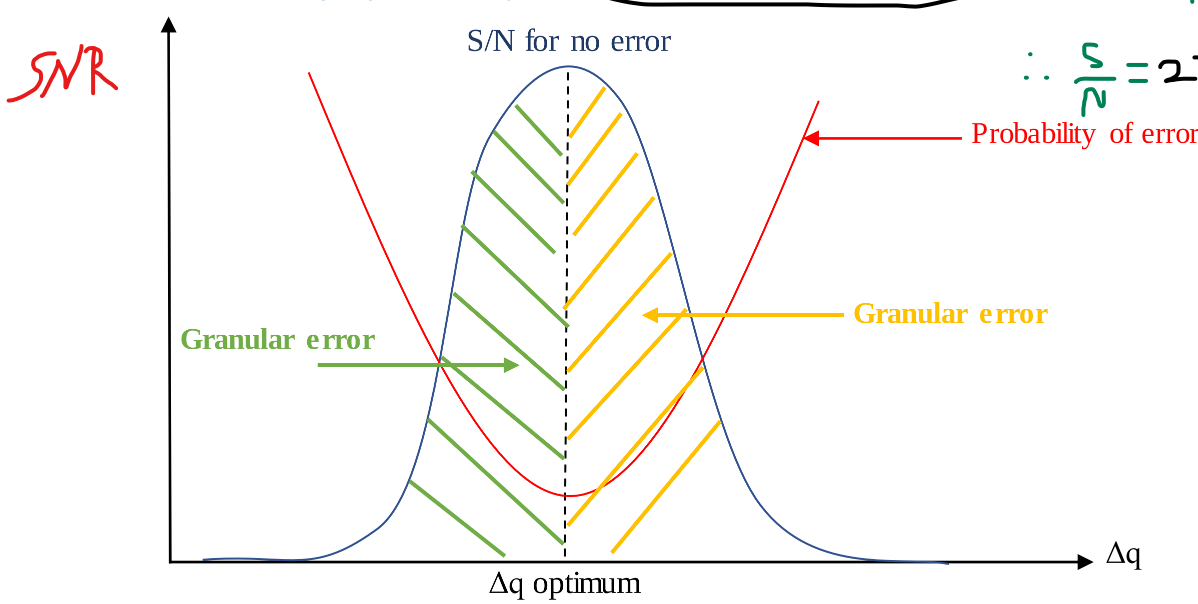
Ex: let $f_s = 64\ kHz$

$f_a = 3.5\ kHz$

$\therefore \frac{S}{N} = 23.66\ dB$

H.W

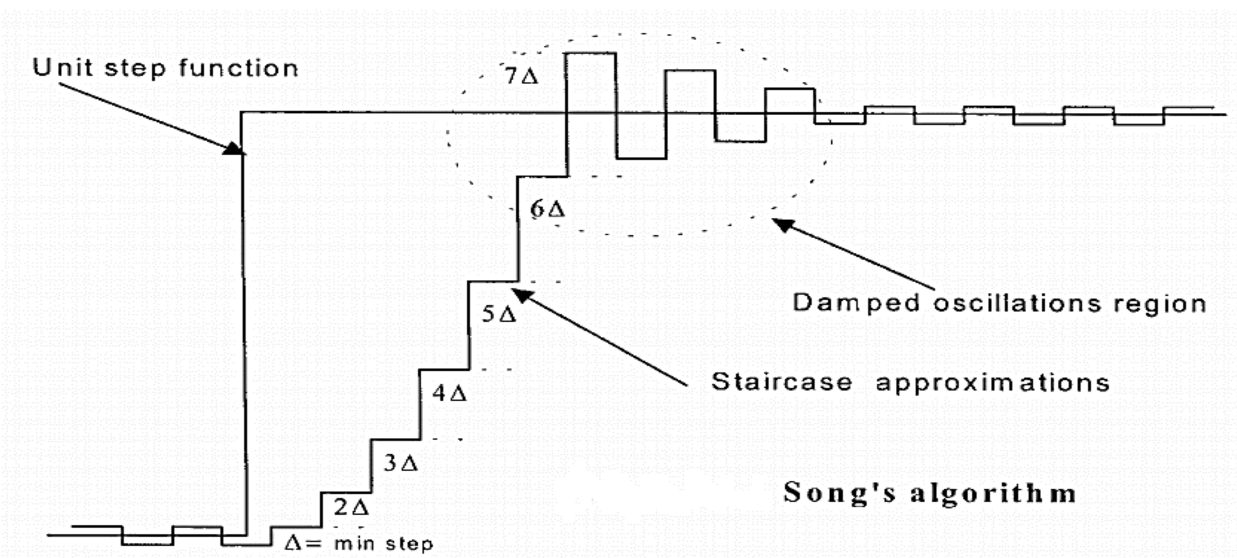
SNR dB= 10log10(SNR linear)



Adaptive Delta Modulation (ADM):

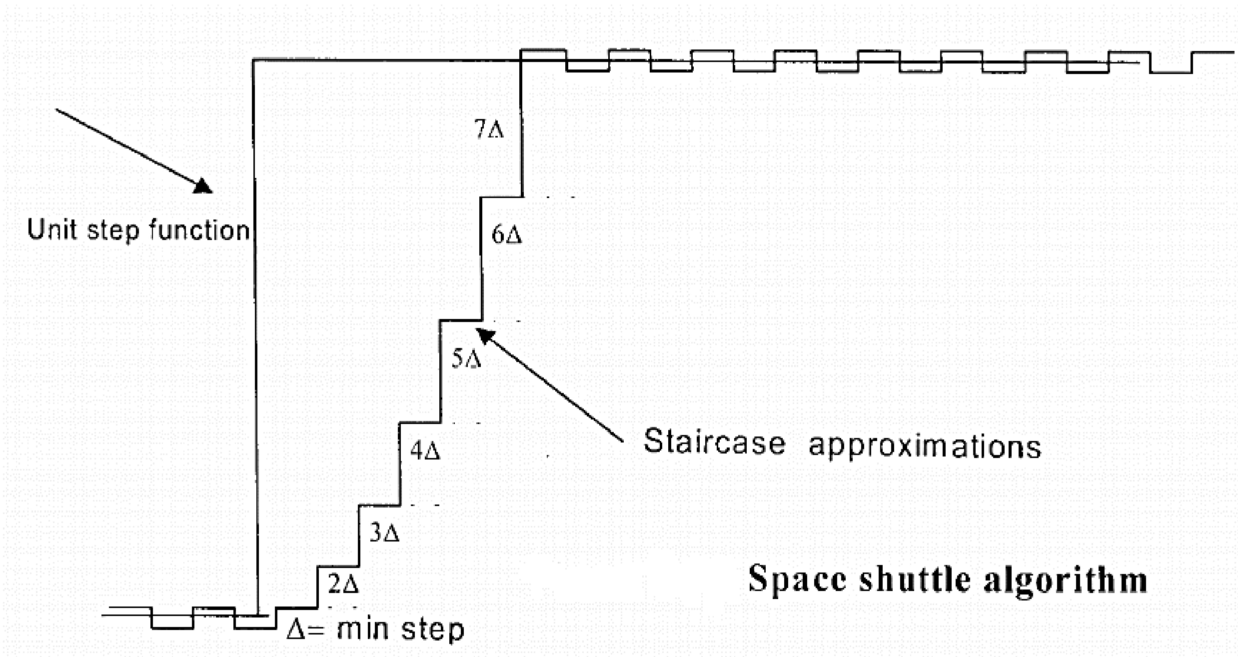
Adaptive Delta Modulation (ADM) is scheme that permits adjustment of the step size depending on the characteristics of the analog signal. There are two type of ADM:

1) **Song Algorithm:** It compare the transmitted bit with the previous bit, if the two are the same the step size is increased by a fixed amount Δ. if the two bits are different, the step size is reduced by the fixed amount Δ, thus the step size always changes, and it can get larger and larger without limit, if necessary.



2) **Space shuttle Algorithm:** It is a modification of Song's algorithm to eliminate the damped oscillation. When the present bit is the same as the

pervious one, the step size increased by a fixed amount Δ. However, when the bits disagree, the step size reverts immediately to its minimum size Δ.



DIGITAL COMMUNICATIONS

Lecture (9-1): Tutorial

Example: If the signal Sp=20V and L=256 and μ=255 in μ-law compressor, what is the voltage between levels when there is no compression? For μ=255, what is the smallest and what is the largest effective separation between levels?

Solution:

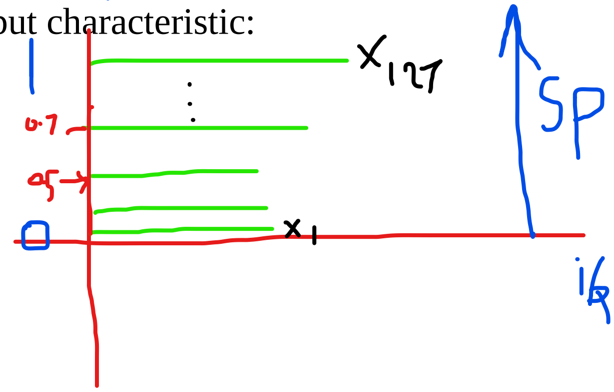
With uniform quantizing the step size Δ is define as:

$\Delta = 2S_p / L = (2*20) / 256 = 0.156$ V = constant

With compression the smallest effective separation between levels will be the one closest to the origin, and the largest effective separation between levels will be the one closest to |x|=1.

We have 127 level positive and 127 level negative values for (y) [because we have L=256, or steps=255]

The μ-law compressor has the following input-output characteristic:



$$y = \pm \frac{\ln(1+\mu|x|)}{\ln(\mu+1)} \qquad |x| < 1$$

$$y = \pm \frac{\ln(1+255|x|)}{\ln 256} \qquad |x| < 1$$

The first value of y (y_1= 1/127)

Let (x_1) be the value of (x) corresponding to (y_1), that is:

$$\frac{1}{127} = \pm \frac{\ln(1+255|x_1|)}{\ln 256}$$ by solving this equation to find $|x_1|$: (exp(1/127*log(256))-1)/255

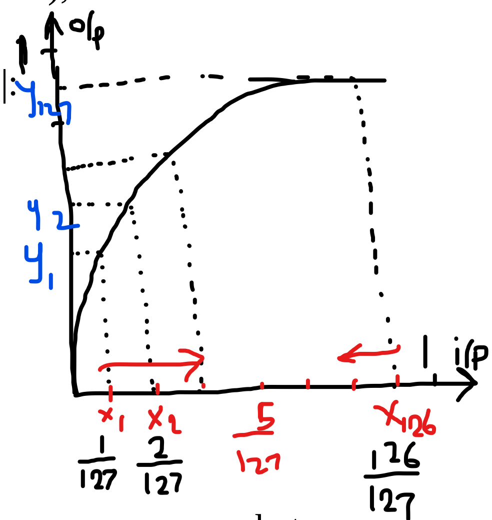
$|x_1|=1.75*10^{-4}$

The smallest effective separation between levels is given by:

Δ min= S_p * $|x_1|$ = 20* $1.75*10^{-4}$ = $3.5*10^{-3}$ V = 3.5 mV

Let (x_{127}) be the value of (x) corresponding to (y_{127}= 1-y_1=126/127), that is:

$\frac{126}{127} = \pm \frac{\ln(1+255|x_{127}|)}{\ln\ 256}$ by solving this equation to find $|x_{127}|$:



$|x_{127}|=0.957$

The largest effective separation between levels is given by:

Δ max= S_p * (1-$|x_{127}|$) = 20* (1- 0.957) = 0.86 V

<u>Example:</u> If the signal Sp=20V and L=256 and A=87.6 in A-law compressor, what is the smallest and what is the largest effective separation between levels?

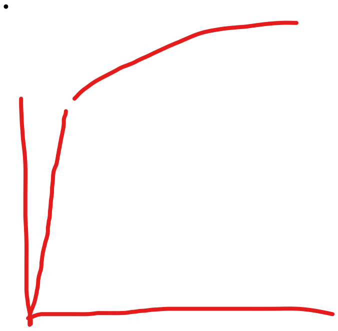
<u>Solution:</u>

With compression the smallest effective separation between levels will be the one closest to the origin, and the largest effective separation between levels will be the one closest to |x|=1.

We have 127 level positive and 127 level negative values for (y) [because we have L=256, or steps=255]

The A-law compressor has the following input-output characteristic:

$$y = \begin{cases} \frac{A}{1+\ln A}\, x & 1/A \geq |x| \\ \frac{sgn(x)}{1+\ln A}\,[1+\ln A|x|] & 1/A \leq |x| \leq 1 \end{cases}$$



$$y = \begin{cases} \frac{87.6}{1+\ln 87.6}\, x & 1/87.6 \ge |x| \\ \frac{sgn(x)}{1+\ln 87.6}\,[1+\ln 87.6|x|] & 1/87.6 \le |x| \le 1 \end{cases}$$

The first value of (x) corresponding to (y_1=1/127), that is:

$$\frac{1}{127} = \begin{cases} \frac{87.6}{1+\ln 87.6}\, x_1 & 1/87.6 \ge |x| \end{cases}$$

Solving to find $|x_1|$:

$|x_1|=4.9*10^{-4}$

Thus, the smallest effective separation between levels is given by:

Δ min= S_p * $|x_1|$ = 20* 4.9*10^{-4} = 9.8*10^{-3} V = 9.8 mV

Let (x_{127}) be the value of (x) corresponding to (y_{127}= 1-y_1=126/127), that is:

$$\frac{126}{127} = \frac{\cancelto{1}{sgn(x_{127})}}{1+\ln 87.6}\,[1+\ln 87.6|x_{127}|] \qquad 1/87.6 \le |x| \le 1$$ by solving this equation to find $|x_{127}|$:

(exp(((126/127)*(1+log(87.6)))-1))/87.6 = X_{127}

log=ln

$|x_{127}|=0.958$

The largest effective separation between levels is given by:

Δ max= S_p * (1-$|x_{127}|$) = 20* (1- 0.958) = 0.84 V

Example: find the channel transmission bandwidth required to transmit 24-voice signal using TDM system bipolar line coding, each with:

RZ

- Bandwidth=4KHz ← 0.3 → 3.4K
- Fs=8K sample/sec
- Number of quantization levels 256

Solution:

The number of bits in each code word:

L=256 = 2^N

N= 8

DS0

Sr=N*fs=8*8000 =64 Kpulse/second for each voice channel

F_{TDM} = 24 voice * Sr= 24*64000=1.536 Mbit/sec

For the bipolar the η=1 RZ

∴ BW= F_{TDM} / η =1.536 KHz

Example: audio signal band-limited to (300-3300 Hz) is sampled at a sampling rate 8K sample/sec and the required (SNR)=30dB.

uniform
a) What is the number of levels L needed and what is N_{min} needed?
b) Calculate the minimum system bandwidth required.
c) Repeat parts (a) and (b) when μ=255 and μ-law is used.

Solution:

(a) SNR=1.76+6.02N

N= 4.7 $\therefore$L= 26 level

But the number of bits must be integer number $\therefore$ N=5 bit/sample

(b) BW= N*fs= 5*8000=40KHz

(c) $\alpha = 10\log\{\frac{3}{[\ln(1+\mu)]^2}\} = -10.$

$$\boxed{\begin{aligned}\alpha &= 4.77 - 20\log[\ln(1+\mu)] \\ \alpha &= -10.1\end{aligned}}$$

SNR=20 log L-10.1

Log L= 40.1/20 =2.005

L=101.2

$\therefore L_{min}$= 102

N=$\log_2$ L = 6.67

$SNR = \alpha + 6.02n$

$30 = -10.1 + 6.02n$

$\boxed{n \simeq 7}$

$\therefore N_{min}$ =7 bit

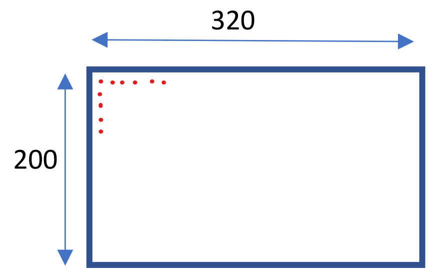
BW=N*fs=7*8000=56KHz

Example: A lookout system consists of two digital cameras each of them has the resolution (200*~~230~~ 320 pixel with 256 gray levels) and four microphone each of them followed by a low pass filter with cutoff frequency equal to 4KHz, if there is a need to transmit one frame of picture per second the sound will be transmitted to the digital form with S/N ratio of not less than 98.08 dB.

- Design TDM system to transmit these signals into one channel
- Calculate the transmitted band width if the digital signal represented by
  a) Return to zero b) Non return to zero
- If only one of the two cameras were used (repeat 1 & 2)

Solution:

1) To design the TDM system we must determine what are the signals will be transmitted. First, we have a two digital camera which mean we have two pictures, to calculate the signals of them:



Number of pixels for each frame = 200*320

= 64000 pixels for each frame

These pixels used to transmit 256 gray level, so

we must know the number of bits which used to Represent these 256 levels

$2^N = 256$

$\therefore N = 8$ bit

To understand the way of sending the digital camera signal, each pixel similar a point of the whole picture and each pixel has its own value represented by a one gray level, so each pixel represented by 8 bits.

overall Sr x 8

$\therefore$ No. of bit/frame = 64000*8 = 512 Kbit Sr for 1 sec this represents sr

$\because$ the transmit time for each frame = 1 second (form the question)

T_b=1 /512000

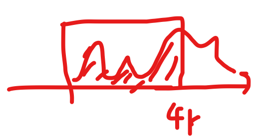
$Sr = n\,fs = \frac{1}{T_b}$

$\boxed{S_r = 1/\ T_b}$ = 512 kbit/sec for one digital camera signal and let us called it S1

$\therefore$ for the second digital camera Sr = 512 Kbit/sec and let us called it S2

Second, we have 4 microphones followed by LPF, and $\boxed{Fc=4KHz}$ which mean that fc=fmax.

$\therefore fs \geq 2*fmax = 2*4=8KHz$ ~~/sec~~



These voice signal converted to a digital signal with S/N =98.08, using S/N equation:

S/N for PCM= 1.76 + 6.02N

98.08 = 1.76 + 6.02N

N =16 bit

∴ Sr = N*fs= 16*8=128Kbit/sec

Now, we have 6 digital signal (2 from camera and 4 from microphone)

S1= 512 Kbit/sec 4

S2= 512 Kbit/sec 4

S3= 128 Kbit/sec 1

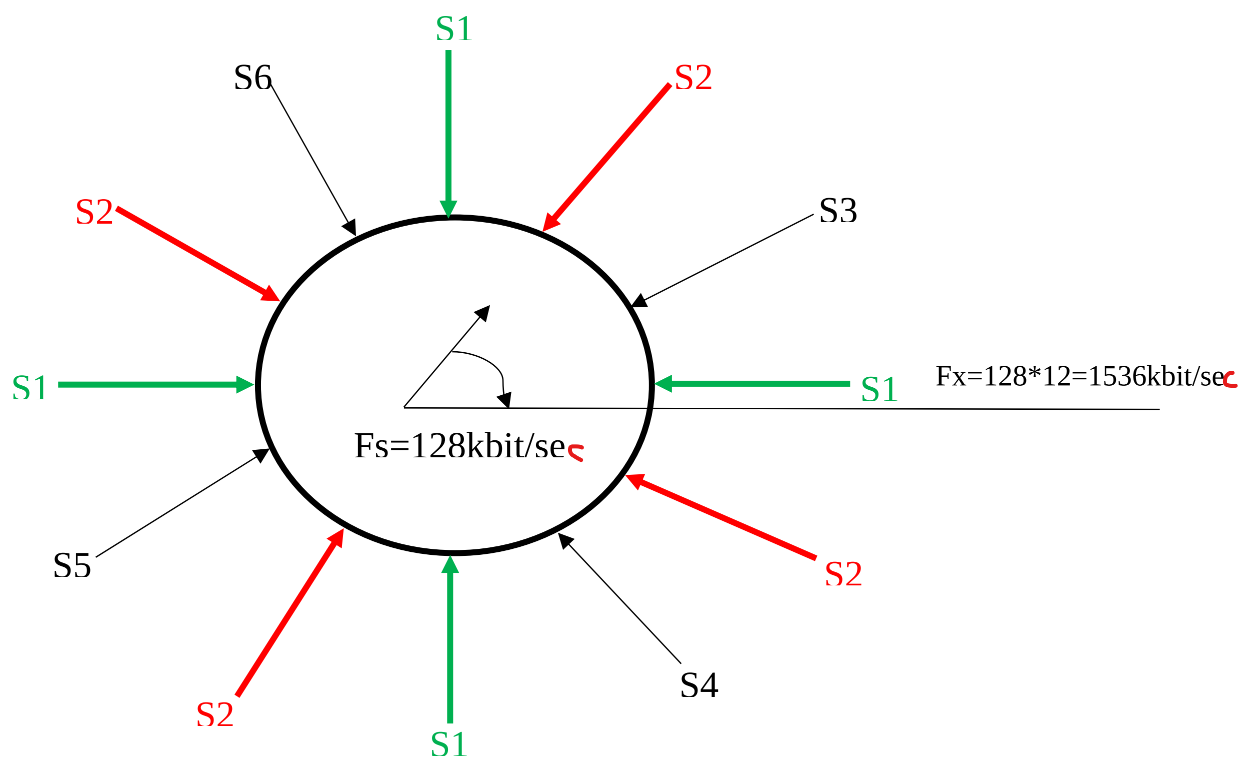
S4= 128 Kbit/sec 1

S5= 128 Kbit/sec 1

S6= 128 Kbit/sec 1

No. of pieces=12

By using the same steps of designing TDM system, we get:



2) To find the bandwidth:

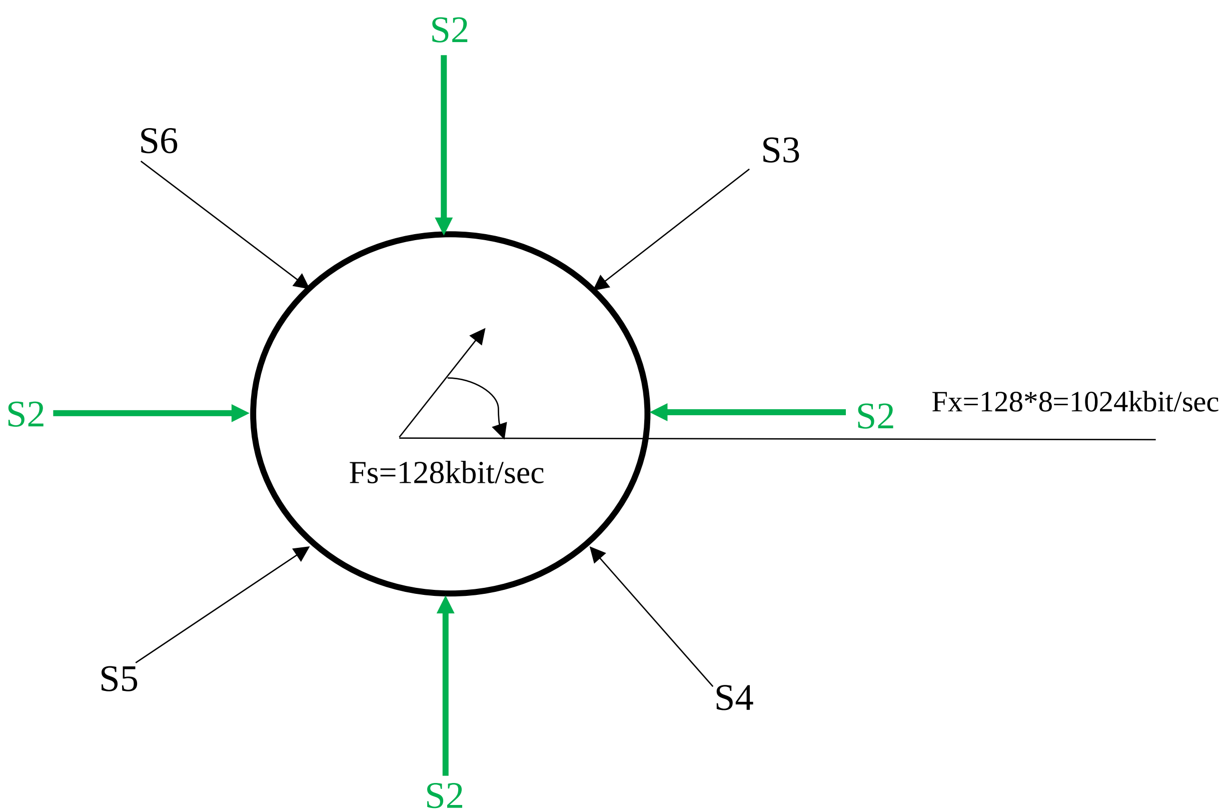
Required BW for return to zero= Sr/η = 1536/1 = 1536 KHz

Required BW for nonreturn to zero= Sr/η = 1536/2 = 768 KHz

3) By canceling the one digital camera signal (S1), the reminded signals transmitted in the TDM system are:

S2= 512 Kbit/sec	4
S3= 128 Kbit/sec	1
S4= 128 Kbit/sec	1
S5= 128 Kbit/sec	1
S6= 128 Kbit/sec	1

No. of pieces=8



To find the bandwidth:

Required BW for return to zero= Sr/η = 1024/1 = 1024 KHz

Required BW for nonreturn to zero= Sr/η = 1024/2 = 512 KHz

DIGITAL COMMUNICATIONS

Lecture (10-1): Error Detection and Correction Technique

Introduction:

The transmitted digital signal through the channel may exposed to noise and make an error (**Single error**) or errors (**Burst error**) in the transmitted data by changing one bit or more, to ensure that the received data is correct the transmitted one must be documenting to know if there are error or not. There are some techniques working on detect the errors only and other detect and correct errors.

Error detection technique:

1) Vertical Redundancy Check (VRC)
2) Longitudinal Redundancy Check (LRC)
3) Vertical Redundancy Check and Longitudinal Redundancy Check (VRC & LRC)
4) Check Sum
5) Cyclic Redundancy Check (CRC)

Error detection and correction technique is **Hamming Code Method**.

1) Vertical Redundancy Check (VRC):

This technique is a parity check error were it working on adding bits to the samples of data, as we know there are two types of parity check (even parity check error and odd parity check error) and by choosing one of them to use in the transmitter on condition that the receiver know which the technique used in the transmitter side, to understand the idea of VRC we take an example.

Example: Used VRC technique to transmit this data

[11001100 01001100 11110000 10101010 10000001 11100000

00000001 11000011]

Solution:

By choosing one type of parity (even for example) so the transmitted data will be:

Transmitted data: [11001100**0** 01001100**1** 11110000**0** 10101010**0** 10000001**0** 11100000**1** 00000001**1** 11000011**0**]

In the receiver, if there is no error in the transmitted data after made checking to the received data, we notice that the parity is right:

Received data: [11001100**00** 01001100**10** 11110000**00** 10101010**00** 10000001**00** 11100000**10** 00000001**10** 11000011**00**]

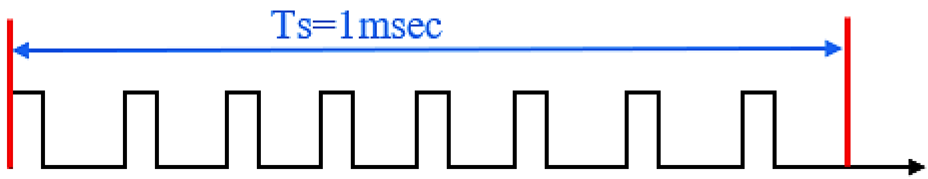
But if there is burst error in the samples, the receiver knows there are errors in the sample if the number of errors is odd, and did not know the number of errors in one sample:

Received data: [11001100**11** 01001100**10** 11110000**00** 1010101**101** 10000001**00** 11100000**10** 00000001**10** 11**1**00011**01**]

Received data: [11**1**0**0**100**11** 01001100**10** 11110000**00** 1010101**101** 10000001**00** 11100000**10** 00000001**10** 11**1**00011**01**]

Received data: [1100010010 0100110010 1111000000 1010101101 1000000100 1110000010 0000000110 1110001101]

Assume that the Ts=1msec before using VRC technique, the required channel BW:

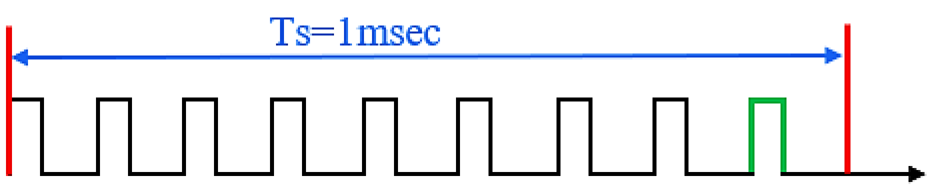


Tb=Ts/number of bit = 1msec/8

Sr= 1/Tb =8KS/sec

BW= Sr/η = 8KHz

To calculate the required channel bandwidth after using VRC technique:



We notice that Ts does not change, only Tb will change to be:

Tb= 1mesc / 9

Sr= 1/Tb = 9KS/sec

BW = Sr/η = 9KHz

2) Longitudinal Redundancy Check (LRC):

This technique is a parity check error were it working on adding a complete one sample to the frame of data, as we mention before, there are two types of parity check and by choosing one of them to use in the transmitter on condition that the receiver know which the technique used in the transmitter side. To understand the idea of LRC we take an example.

Example: Used LRC technique to transmit this data

[11001100 01001100 11110000 10101010 10000001 11100000 00000001 11000011]

Solution:

By choosing one type of parity (even for example) so the transmitted data will be:

1	1	0	0	1	1	0	0	
0	1	0	0	1	1	0	0	
1	1	1	1	0	0	0	0	
1	0	1	0	1	0	1	0	
1	0	0	0	0	0	0	1	
1	1	1	0	0	0	0	0	
0	0	0	0	0	0	0	1	
1	1	0	0	0	0	1	1	
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	⇐ LRC parity sample

Transmitted data: [11001100 01001100 11110000 10101010 10000001 11100000 00000001 11000011 **01111001**]

if two bits at the same positions in different column are flipped, LRC will not be able to detect them

In the receiver, if there is no error in the transmitted data after made checking to the received data, where the receiver made check to the data and the parity sample as shown below:

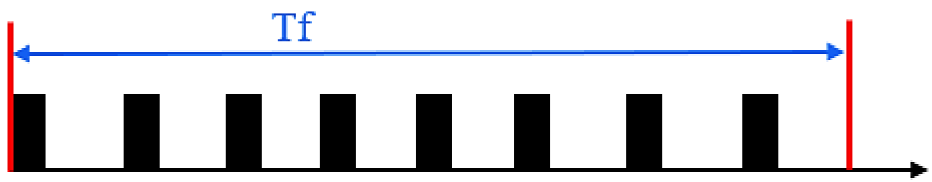
1	1	0	0	1	1	0	0
0	1	0	0	1	1	0	0
1	1	1	1	0	0	0	0
1	0	1	0	1	0	1	0
1	0	0	0	0	0	0	1
1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	1
1	1	0	0	0	0	1	1
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**

⇐ Checking LRC parity in the receiver

This technique could recognize one error or an odd number of errors for one column, but did not know the number of errors:

1	1	0	0	1	1	0	0
0	1	0	0	1	1	**1**	0
1	**0**	1	1	0	0	0	0
1	0	1	0	1	0	**0**	0
1	0	0	**1**	0	0	0	1
1	1	1	0	0	0	**1**	0
0	0	0	0	0	**1**	0	1
1	1	0	0	0	0	1	1
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**
0	**1**	**0**	**1**	**0**	**1**	**1**	**0**

Assume that the Ts=1msec before using LRC technique, the required channel BW:



Tf mean time of the frame, where the frame consist of number of samples and each sample has number of bits.

∴ Tf = Ts * number of samples

Tf = 1msec * 8 = 8 msec

To find Tb we must divide Tf on the number of bits

Number of bits = number of bits for one sample * number of samples

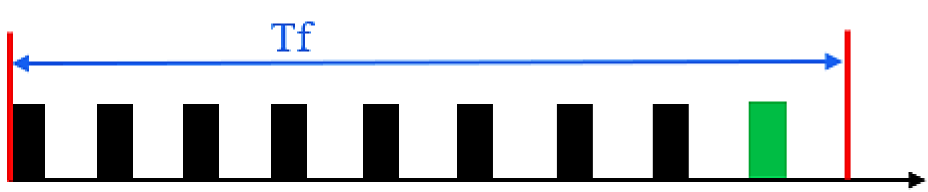
= 8bit * 8sample = 64 bit for the frame

∴Tb = Tf / number of bits for the frame = 8msec/ 64 bit

Sr= 1 / Tb = 64 /8 = 8 Kb/sec

BW= Sr / η = 8 /1 = 8 KHz

To calculate the required channel bandwidth after using LRC technique:



Tf = Ts * number of samples

Tf = 1msec * 8 = 8 msec

Number of bits = number of bits for one sample * number of samples

= 8bit * 9sample = 72 bit for the frame

∴Tb = Tf / number of bits for the frame = 8msec/ 72 bit

Sr= 1 / Tb = 72 /8 = 9 Kb/sec

BW= Sr / η = 9 /1 = 9 KHz

3) Vertical Redundancy Check and Longitudinal Redundancy Check (VRC & LRC):

This technique is a parity check error merge the two above techniques where it adding bit to each sample in the frame of data and adding a complete one sample to the frame of data. This technique could recognize the number of errors even when it is even or odd, also if there is one error bit in the sample then it could find which bit is error (find the error location). To understand the idea of VRC & LRC we take an example.

Example: Used VRC & LRC technique to transmit this data

[11001100 01001100 11110000 10101010 10000001 11100000

00000001 11000011]

Solution:

By choosing one type of parity (even for example) so the transmitted data will be:

1	1	0	0	1	1	0	0	**0**
0	1	0	0	1	1	0	0	**1**
1	1	1	1	0	0	0	0	**0**
1	0	1	0	1	0	1	0	**0**
1	0	0	0	0	0	0	1	**0**
1	1	1	0	0	0	0	0	**1**
0	0	0	0	0	0	0	1	**1**
1	1	0	0	0	0	1	1	**0**
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**

VRC parity bits

LRC parity sample

Transmitted data: [11001100**0** 01001100**1** 11110000**0** 10101010**0** 10000001**0** 11100000**1** 00000001**1** 11000011**0** **011110011**]

In the receiver, if there is no error in the transmitted data after made checking to the received data, where the receiver made check to the data and the parity sample as shown below:

1	1	0	0	1	1	0	0	**0**	**0**
0	1	0	0	1	1	0	0	**1**	**0**
1	1	1	1	0	0	0	0	**0**	**0**
1	0	1	0	1	0	1	0	**0**	**0**
1	0	0	0	0	0	0	1	**0**	**0**
1	1	1	0	0	0	0	0	**1**	**0**
0	0	0	0	0	0	0	1	**1**	**0**
1	1	0	0	0	0	1	1	**0**	**0**
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

If there is one error in the sample then it finds the error and its location:

1	1	0	0	1	1	0	0	**0**	**0**
0	1	0	0	**0**	1	0	0	**1**	**1**
1	1	1	1	0	0	0	0	**0**	**0**
0	0	1	0	1	0	1	0	**0**	**1**
1	0	0	0	0	0	0	1	**0**	**0**
1	1	1	0	0	0	0	0	**1**	**0**
0	0	0	0	0	0	0	**0**	**1**	**1**
1	1	0	0	0	0	1	1	**0**	**0**
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**	**0**
1	**0**	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**0**

If there are two error in the sample then it finds there is error in the data but it could not find their locations:

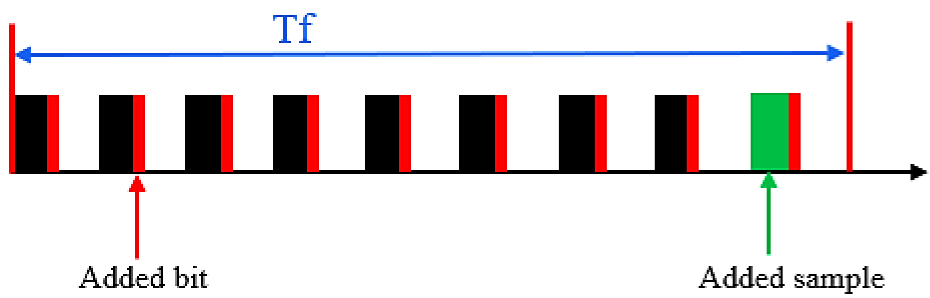
1	1	0	0	1	1	0	0	**0**	**0**
0	1	0	0	1	1	0	0	**1**	**0**
1	1	1	1	0	0	0	0	**0**	**0**
1	0	1	0	1	0	1	0	**0**	**0**
0	0	0	0	0	0	**1**	1	**0**	**0**
1	1	1	0	0	0	0	0	**1**	**0**
0	0	0	0	0	0	0	1	**1**	**0**
1	1	0	0	0	0	1	1	**0**	**0**
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**	**0**
1	**0**	**0**	**0**	**0**	**0**	**1**	**0**	**0**	**0**

It knows that there are errors at the 1st and 6th columns, but it could not know the exact locations of the rows.

1	**0**	0	0	1	1	0	0	**0**	**1**
0	1	0	0	1	1	0	0	**1**	**0**
1	1	1	1	0	0	0	0	**0**	**0**
1	0	1	0	1	0	1	0	**0**	**0**
1	0	0	0	0	0	0	1	**0**	**0**
1	1	1	0	0	0	0	0	**1**	**0**
0	**1**	0	0	0	0	0	1	**1**	**1**
1	1	0	0	0	0	1	1	**0**	**0**
0	**1**	**1**	**1**	**1**	**0**	**0**	**1**	**1**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

It knows that there are errors at the 1st and 7th rows, but it could not know the exact locations of the rows.

To calculate the required channel bandwidth after using VRC & LRC technique:



Tf = Ts * number of samples

Tf = 1msec * 8 = 8 msec

Number of bits = number of bits for one sample * number of samples

= 9bit * 9sample = 81 bit for the frame

∴Tb = Tf / number of bits for the frame = 8msec/ 81 bit

Sr= 1 / Tb = 81 /8 = 10.125 Kb/sec

BW= Sr / η = 10.125 /1 = 10.125 KHz

DIGITAL COMMUNICATIONS

Lecture (11-1): Error Detection and Correction Technique

4) Check Sum:

In this technique the transmitter depending on the summation of the samples in the frame, where the result of the summation added as a sample to the data on condition that the number of the bits in the added sample did not exceed the number of bits in the one origin data sample. In the receiver, to check the received data it working on sum them and compared the sum result with the transmitted sum result if they are same it means there is no error if it not that means there is error or errors. To understand this technique, we take an example.

Example: Used Check Sum technique to transmit this data

[10011011 11001110 11110000 10001000]

Solution:

		1	0	0	1	1	0	1	1
		1	1	0	0	1	1	1	0
		1	1	1	1	0	0	0	0
		1	0	0	0	1	0	0	0
1	**0**	**1**	**1**	**1**	**0**	**0**	**0**	**0**	**1**

← Check sum result sample

$\underbrace{1\ 0}_{\text{Neglected}}$

Neglected

Transmitted data [10011011 11001110 11110000 10001000 **1010001**]

In the receiver, if there is no error in the transmitted data after made checking to the received data, where the receiver made check to the data and compare the sum result sample as shown below:

1	0	0	1	1	0	1	1
1	1	0	0	1	1	1	0
1	1	1	1	0	0	0	0
1	0	0	0	1	0	0	0
1	**1**	**1**	**0**	**0**	**0**	**0**	**1**

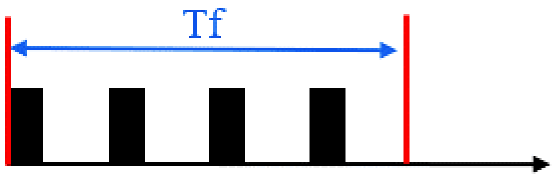
11100001 = **11100001**

If there is error in the data, the sum results are not equal:

1	0	0	1	1	0	1	1
1	1	0	0	1	1	1	0
1	1	1	1	0	0	0	**1**
1	0	0	0	1	0	0	0
1	**1**	**1**	**0**	**0**	**0**	**1**	**0**

11100010 ≠ **11100001**

Assume that the Ts=1msec before using Check Sum technique, the required channel BW:



□ Tf = Ts * number of samples

Tf = 1msec * 4 = 4 msec

To find Tb we must divide Tf on the number of bits

Number of bits = number of bits for one sample * number of samples

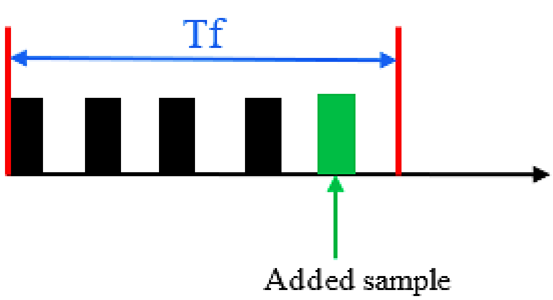
= 8bit * 4sample = 32 bit for the frame

□Tb = Tf / number of bits for the frame = 4msec/ 32 bit

Sr= 1 / Tb = 32 /4 = 8 Kb/sec

BW= Sr / η = 8 /1 = 8 KHz

To calculate the required channel bandwidth after using LRC technique:



Tf = 4msec

Number of bits = number of bits for one sample * number of samples

= 8bit * 5sample = 40 bit for the frame

$\Rightarrow$ Tb = Tf / number of bits for the frame = 4msec/ 40 bit

Sr= 1 / Tb = 40 /4 = 10 Kb/sec

BW= Sr / η = 10 /1 = 10 KHz

5) <u>Cyclic Redundancy Check (CRC):</u>

This technique is used a different way of adding bits to the transmitted data where it depending on diving the origin data on the polynomial and the reminded result was sent with the origin data.

CRC can detect number of error bit $\leq$ number of polynomial bits in percent (100%)

CRC can detect number of error bit > number of polynomial bits in percent (99.98%)

The polynomial chose in this technique conditionally based on:

1) The polynomial must be divided by (x+1) without reminder
2) The polynomial must not divide by (x) without reminder

The polynomial given in the following form:

X^6+X^2+X+1 here we notice that this form = $X^6+X^5+X^4+X^3+X^2+X^1+X^0$

But X^5, X^4 & X^3 are not exist which mean their values = zero, by substituting each one with a bit either "1" or "0":

$X^6= 1$, $X^5=0$, $X^4=0$, $X^3=0$, $X^2=1$, $X^1= 1$, $X^0 = 1$

$\Rightarrow$ The polynomial = 1000111

To understand CRC we take example.

Example: for the following stream of data use the polynomial (X^3+ X^2+1) to send the data using CRC technique.

Data stream [100100]

Solution:

To solve this question, we follow the next steps:

1) Find the polynomial

   X^3+ X^2+1 $\longrightarrow$ 1101

   □the polynomial = **1101**

2) Adding No. of zero bits to the original data = No. of polynomial bits -1`

   No. of polynomial bits = 4 bits

   □ No. of zero bits to the original data = 3 zero bits

   □ the divided data = 100100**000**

3) Divide the stream data by the polynomial

   ❖ Division process depending on (XOR gate) XOR=module-2 add

4) After the division process, we take the reminder result with number of bits = No. of polynomial bits -1`

5) Stat the transmitted data + reminder

XOR=module2

x	y	o/p(Add)
0	0	0
1	0	1
0	1	1
1	1	0

$\boxed{Tx}$

```
           1 1 1 1 0 1
1101 | 1 0 0 1 0 0 0 0 0
       1 1 0 1
       0 1 0 0 0
         1 1 0 1
         0 1 0 1 0
           1 1 0 1
           0 1 1 1 0
             1 1 0 1
             0 0 1 1 0
               0 0 0 0
               0 1 1 0 0
                 1 1 0 1
                 0 0 0 1
```

$\underbrace{001}_{\text{reminder}}$

$\boxed{Rx}$

```
           1 1 1 1 0 1
1101 | 1 0 0 1 0 0 0 0 1
       1 1 0 1
       0 1 0 0 0
         1 1 0 1
         0 1 0 1 0
           1 1 0 1
           0 1 1 1 0
             1 1 0 1
             0 0 1 1 0 1
                 1 1 0 1
                 0 0 0 0
```

- The transmitted data [1 0 0 1 0 0 0 0 1]

In the receiver side:

- If the received data divided on the same polynomial if the reminder = zero this mean there is no error in the data
- If the received data divided on the same polynomial if the reminder ≠ zero this mean there is error in the data

H.W: If Ts before CRC is 1msec, calculate the BW before and after adding CRC redundant bits for error detection for the above example?

DIGITAL COMMUNICATIONS

Lecture (12-1): Error Detection and Correction Technique

Hamming Code Error Detection and Correction:

The five techniques we take before just detect if there is error or not, but in this technique could detect and correct error. As like other method of error detection there are added bits to the stream of original data when it transmitted, the way of adding bits and their location can be explained in the following steps:

1) **Calculating the number of the added bits**

There is a condition used to calculate the added bits to the original data, if (d) the number of the data bits and (r) the number of the added bits:

$$2^r \geq d + r + 1$$

Ex: if the data [11010111]

d= 8bits , r =we choose values until the condition become true

if we take r = 1

$2^1 \geq 8 + 1 + 1 \Longrightarrow 2 \geq 10$ faluse

If r =2

$2^2 \geq 8 + 2 + 1 \Longrightarrow 4 \geq 11$ faluse

If r=3

$2^3 \geq 8 + 3 + 1 \Longrightarrow 8 \geq 12$ faluse

If r =4

$2^4 \geq 8 + 4 + 1 \Longrightarrow 16 \geq 13$ true

□ the added bits to the data = 4 bits

2) The added bits distribution

To understand the method of distributing the added bits, we should know the sequence of the bits:

Data	1	1	0	1	0	1	1	1
sequence	d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0

Added bits	r_3	r_2	r_1	r_0
sequence	8	4	2	1

Where the sequence of the added bits = 2^r

3) Finding the value of the added bits

To find the value of each added bit we follow the next way:

r_3	r_2	r_1	r_0	Sequence Of data
0	0	0	1	r_0
0	0	1	0	r_1
0	0	**1**	**1**	d_0
0	1	0	0	r_2
0	**1**	0	**1**	d_1
0	**1**	**1**	0	d_2
0	**1**	**1**	**1**	d_3
1	0	0	0	r_3
1	0	0	**1**	d_4
1	0	**1**	0	d_5
1	0	**1**	**1**	d_6
1	**1**	0	0	d_7
1	1	0	1	Neglected
1	1	1	0	Neglected
1	1	1	1	Neglected

□ the transmitted data sequence will be:

d_7	d_6	d_5	d_4	r_3	d_3	d_2	d_1	r_2	d_0	r_1	r_0

d_7	d_6	d_5	d_4	d_3	d_2	d_1	d_0
1	1	0	1	0	1	1	1

First, we find the equation parity of each added bit by taking the bit sequence at each "1" for one column. So, the value of the added bits:

r_0 = parity of {d_0 , d_1 , d_3 , d_4 , d_6 }

r_0 = parity of {1 , 1 , 0 , 1 , 1 } = 1 0 $\boxed{0}$

$r_0 = 0$

r_1 = parity of {d_0 , d_2 , d_3 , d_5 , d_6 }

r_1 = parity of {1 , 1, 0 , 0 , 1 } = 1 $\boxed{1}$

$r_1 = 1$

r_2 = parity of {d_1 , d_2 , d_3 , d_7}

r_2 = parity of {1 , 1 , 0 , 1} = 1 $\boxed{1}$

$r_2 = 1$

r_3 = parity of { d_4 , d_5 , d_6 , d_7}

r_3 = parity of { 1 , 0 , 1 , 1 } = 1 $\boxed{1}$

$r_3 = 1$

- Notice that each data bit repeats twice in the equations or more

The transmitted data: [d_7 d_6 d_5 d_4 r_3 d_3 d_2 d_1 r_2 d_0 r_1 r_0]

The transmitted data: [1 1 0 1 1 0 1 1 1 1 1 0]

In the receiver to check the data if there is error or not, we use the same r equations but with little difference, where r will be in parity equations:

Er_0 = parity of { r_0 ,d_0 , d_1 , d_3 , d_4 , d_6 }

Er_1 = parity of { r_1 ,d_0 , d_2 , d_3 , d_5 , d_6 }

Er_2 = parity of { r_2 ,d_1 , d_2 , d_3 , d_7}

Er_3 = parity of { r_3 ,d_4 , d_5 , d_6 , d_7}

Where be using these parity equations, we find the location of the error if there is error if not the result will be zero.

- ❖ If there is no error:

  Er_0 = parity of { r_0 ,d_0 , d_1 , d_3 , d_4 , d_6 }

  Er_0 = parity of {0 , 1 , 1 , 0 , 1 , 1 } $= 1\ 0\ \boxed{0}$

  Er_0 = 0

  Er_1 = parity of { r_1 ,d_0 , d_2 , d_3 , d_5 , d_6 }

  Er_1 = parity of {1 , 1 , 1, 0 , 0 , 1 } $= 1\ 0\ \boxed{0}$

  Er_1 = 0

  Er_2 = parity of { r_2 ,d_1 , d_2 , d_3 , d_7}

  Er_2 = parity of {1 , 1 , 1 , 0 , 1} $= 1\ 0\ \boxed{0}$

  Er_2 = 0

  Er_3 = parity of { r_3 ,d_4 , d_5 , d_6 , d_7}

  Er_3 = parity of {1 , 1 , 0 , 1 , 1 } $= 1\ 0\ \boxed{0}$

  Er_3 = 0

  The error location: [Er_3 Er_2 Er_1 Er_0]

  The error location:[0 0 0 0]

- ❖ If there is error in the bit **d_3**:

  The received data: [1 1 0 1 1 1 1 1 1 1 1 0]

  Be using the check method at the receiver:

  Er_0 = parity of { r_0 ,d_0 , d_1 , d_3 , d_4 , d_6 }

  Er_0 = parity of {0 , 1 , 1 , 1 , 1 , 1 } $= 1\ 0\ 1$

  Er_0 = 1

Er_1 = parity of { r_1 ,d_0 , d_2 , d_3 , d_5 , d_6 }

Er_1 = parity of {1 , 1 , 1, 0 , 1 , 1 } = 101

Er_1 = 1

Er_2 = parity of { r_2 ,d_1 , d_2 , d_3 , d_7}

Er_2 = parity of {1 , 1 , 1 , 1 , 1} = 101

Er_2 = 1

Er_3 = parity of { r_3 ,d_4 , d_5 , d_6 , d_7}

Er_3 = parity of {1 , 1 , 0 , 1 , 1 } = 100

Er_3 = 0

The error location: [Er_3 Er_2 Er_1 Er_0]

The error location: [0 1 1 1] this mean the bit with sequence 7 is error

From the received bits

The received data: [1 1 0 1 1 1 1 1 1 1 1 0]

which exactly point to **d3**

Thin the correct data stream are :-

The correct data: [1 1 0 1 1 0 1 1 1 1 1 0]

- **Notice that this technique can correct only one bit error.**

DIGITAL COMMUNICATIONS

Lecture (13-1): Tutorial (Error Detection and Correction Technique)

<u>Example (1):</u> The following data stream [1100101 , 1010010 , 1111111] are received through the 140 KHz channel bandwidth, if you know that the data have been treated with (Hamming) error detection and correction technique before transmitted, and one of these streams attack with one bit error, answer the following:

A) Detect which stream contain the error and correct the error

B) State the original data stream (without the error detection and correction bits).

C) Calculate the required channel BW before adding error detection bits.

D) Depending on the original data calculated in (B), find the transmitted data using VRC parity and calculate the transmitted channel BW.

<u>Solution:</u>

A) From the question the received data treated by using Hamming code, so to know which stream has a wrong bit we must know the equation of parity which refer to the wrong bit.

$2^r \geq d + r + 1$

$2^r \geq 7$

$2^3 \geq 7$ $\quad r=3 \quad r_0, r_1, r_2$

d+r+1=7

d+r =6

d=3

r_2	r_1	r_0
4	2	1
2^2	2^1	2^0

r_2	r_1	r_0	Sequence Of data	
0	0	1	r_0	
0	1	0	r_1	2
0	**1**	**1**	d_0	
1	0	0	r_2	4
1	0	**1**	d_1	
1	**1**	0	d_2	
1	**1**	**1**	d_3	

r_0 = parity of $\{d_0\ ,\ d_1\,,\ d_3\}$

r_1 = parity of $\{d_0\ ,\ d_2\,,\ d_3\}$

r_2 = parity of $\{d_1\ ,\ d_2\,,\ d_3\}$

Er_0 = parity of $\{\ r_0\,,\ d_0\ ,\ d_1\,,\ d_3\}$

Er_1 = parity of $\{\ r_1\,,\ d_0\ ,\ d_2\,,\ d_3\}$

Er_2 = parity of $\{\ r_2\,,\ d_1\ ,\ d_2\,,\ d_3\}$

We check the first stream: [1100101]

Data	1	1	0	0	1	0	1
sequence	d_3	d_2	d_1	r_2	d_0	r_1	r_0

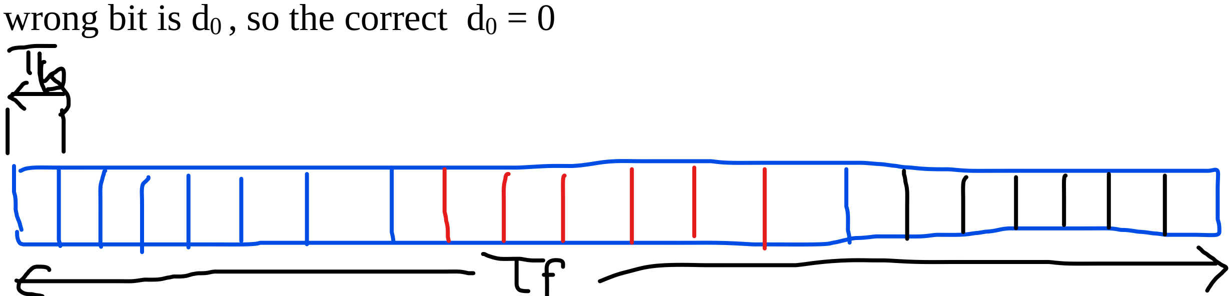
Er_0 = parity of $\{1\ ,\ 1\ ,\ 0\ ,\ 1\}= 1$

Er_1 = parity of $\{0\ ,\ 1\ ,\ 1\ ,\ 1\ \}= 1$

Er_2 = parity of $\{0\ ,\ 0\ ,\ 1\ ,\ 1\ \}= 0$

The error location: [Er_2 Er_1 Er_0] = [0 1 1] = the bit with number 3 = d_0

$\therefore$ the wrong bit is d_0, so the correct $d_0 = 0$



B) The original data stream: [1100 , 1010 , 1111]

C) To calculate the transmitted BW, first we find the Tb: let $\boxed{\eta = 1}$

after hamming

$Sr = 1/Tb$ ➡ $Tb_{(old)} = 1/(140*10^3)$

$Tb_{(old)} = Tf / N$ ➡ $Tf = Tb_{(old)} * N = [1/(140*10^3)] * 21 = \boxed{21/(140*10^3)}$ constant

before

$Tb_{(new)} = Tf / N = [21/(140*10^3)] / 12 = 1/80$ msec

$Sr = 1/ Tb_{(new)} = 1/ [1/80 \text{ msec}] = 80$ Kbit/sec

$BW = Sr / \eta = 80KHz$

Tf is constant with and without error detection and correction

D) The transmitted data using VRC: [1100**0** , 1010**0** , 1111**0**]

From (C) the $Tf = 21/(140*10^3)$

$Tb_{(new)} = Tf / N = [21/(140*10^3)] / 15 = 1/100$ msec

$Sr = 1/ Tb_{(new)} = 1/ [1/100 \text{ msec}] = 100$ Kbit/sec

$BW = Sr / \eta = 100KHz$

<u>Example (2):</u> The following data stream [11101010 11101] are received through the 130 KHz channel bandwidth, if you know that the data ~~will be~~ have been treated with CRC error detection technique with the polynomial ($p = X^5+X^3+X+1$), answer the following:

A) Detect if this stream contains the error or not.

B) State the original data stream (before adding error detection bits).

C) Calculate the required channel BW before adding error detection bits.

Solution:

A) Polynomial = 101011

```
             110 10 10 0
        ______________________________
101011 | 1 1 1 0 1 0 1 0 1 1 1 0 1
         1 0 1 0 1 1
         0 1 0 0 0 1 1
           1 0 1 0 1 1
           0 0 1 0 0 0 0
               0 0 0 0 0 0
               0 1 0 0 0 01
                 1 0 1 0 1 1
                 0 0 1 0 1 0 1
                   0 0 0 0 0 0
                   0 1 0 1 0 1 1
                     1 0 1 0 1 1
                     0 0 0 0 0 0 0
                       0 0 0 0 0 0
                       0 0 0 0 0 0 1
                         0 0 0 0 0 0
                         0 0 0 0 0 1
```

There is reminder at the receiver side which mean the stream contains error

B) The original data stream can be found by removing the number of bits that added to the data which could be know through the polynomial. Because at the transmitter side we added bits of the reminder equal to the No. of polynomial bits -1, as No. of polynomial bits = 6

$\therefore$ the added bits at the transmitter = 5 bits

$\therefore$ The original data stream: [11101010]

C) To calculate the transmitted BW, first we find the Tb:

$Sr = 1/Tb \longrightarrow Tb_{(old)} = 1/(130*10^3)$

$Tb_{(old)} = Tf / N \longrightarrow Tf = Tb_{(old)} * N = [1/(130*10^3)] * 13 = 0.1$ msec

$Tb_{(new)} = Tf/ N = 0.1$ msec$/ 8 = 1/80$ msec

$Sr = 1/ Tb_{(new)} = 1/ [1/80$ msec$] = 80$ Kbit/sec

$BW = Sr / \eta = 80$KHz

<u>Example (3):</u> The following data stream [100010010101101 , 110110111101100 , 101101111011011 , 110110111101100] are received through the 150 KHz channel bandwidth, if you know that the data ~~will be~~ have been treated with (Hamming) error detection and correction technique before transmitted, and one of these streams attack with one bit error, answer the following:

A) Find the wrong received stream and state the correct stream.

B) State the original data stream (without the error detection and correction bits).

C) Depending on the original data calculated in (B), find the transmitted data using VRC&LRC parity and calculate the transmitted channel BW.

D) Depending on the original data calculated in (B) and using the following polynomial calculate the data transmitted of the (last stream only) (p= X^5+X^3+X+1), calculate the required transmitted channel BW.

Solution:

A) From the question the received data treated by using Hamming code, so to know which stream has a wrong bit we must know the equation of parity which refer to the wrong bit.

$2^r \geq d + r + 1$

$2^r \geq 15$ ⟶ $2^4 \geq 15$

d+r+1=15 ⟶ d+r =14 ⟶ d=10

r_3	r_2	r_1	r_0	Sequence Of data
0	0	0	1	r_0
0	0	1	0	r_1
0	0	**1**	**1**	d_0
0	1	0	0	r_2
0	**1**	0	**1**	d_1
0	**1**	**1**	0	d_2
0	**1**	**1**	**1**	d_3
1	0	0	0	r_3
1	0	0	**1**	d_4
1	0	**1**	0	d_5
1	0	**1**	**1**	d_6
1	**1**	0	0	d_7
1	**1**	0	**1**	d_8
1	**1**	**1**	0	d_9
1	**1**	**1**	**1**	d_{10}

r_0 = parity of $\{d_0 , d_1 , d_3 , d_4 , d_6 , d_8 , d_{10}\}$

r_1 = parity of $\{d_0 , d_2 , d_3 , d_5 , d_6 , d_9 , d_{10}\}$

r_2 = parity of $\{d_1 , d_2 , d_3 , d_7 , d_8 , d_9 , d_{10}\}$

r_3 = parity of $\{d_4 , d_5 , d_6 , d_7 , d_8 , d_9 , d_{10}\}$

Er_0 = parity of $\{ r_0 , d_0 , d_1 , d_3 , d_4 , d_6 , d_8 , d_{10}\}$

Er_1 = parity of $\{ r_1 , d_0 , d_2 , d_3 , d_5 , d_6 , d_9 , d_{10}\}$

Er_2 = parity of $\{ r_2 , d_1 , d_2 , d_3 , d_7 , d_8 , d_9 , d_{10}\}$

Er_3 = parity of $\{ r_3 , d_4 , d_5 , d_6 , d_7 , d_8 , d_9 , d_{10}\}$

We check the first stream: [100010010101101]

Data	1	0	0	0	1	0	0	1	0	1	0	1	1	0	1
sequence	d_{10}	d_9	d_8	d_7	d_6	d_5	d_4	r_3	d_3	d_2	d_1	r_2	d_0	r_1	r_0

Er_0 = parity of { 1 , 1 , 0 , 0 , 0 , 1 , 0 , 1}= 0

Er_1 = parity of { 0 , 1 , 1 , 0 , 0 , 1 , 0 , 1 }= 0

Er_2 = parity of { 1 , 0 , 1 , 0 , 0 ,0 , 0 , 1 }= 1

Er_3 = parity of { 1 , 0 , 0 , 1 , 0 ,0 , 0 , 1 }= 1

The error location: [Er_3 Er_2 Er_1 Er_0] = [1 1 0 0] = the bit with number 12 = d_7

$\therefore$ the wrong bit is d_7 , so the correct $d_7 = 1$

B) The original data stream: [100**1**1000101 , 11011011101 , 10110111010 , 11011011101]

C)

1	0	0	1	1	0	0	0	1	0	1	**1**
1	1	0	1	1	0	1	1	1	0	1	**0**
1	0	1	1	0	1	1	1	0	1	0	**1**
1	1	0	1	1	0	1	1	1	0	1	**0**
0	**0**	**1**	**0**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**0**

The transmitted data: [10011000101**1** , 11011011101**0** , 10110111010**1** , 11011011101**0** , **001011111110**]

To calculate the transmitted BW, first we find the Tb:

Sr= 1/Tb ⟶ $Tb_{(old)} = 1/(150*10^3)$

$Tb_{(old)} = Tf / N$ ⟶ $Tf = Tb_{(old)} * N = [1/(150*10^3)] * 60 = 0.4$ msec

$Tb_{(new)} = Tf/ N = (0.4*10^{-3}) / 60$

$Sr = 1/ Tb_{(new)} = 60/(0.4*10^{-3}) = 150$ Kbit/sec

BW= Sr / η = 150KHz

D) $p= X^5+X^3+X+1 =101011$ data=11011011101

1 1 1 0 1 0 0 0 0 0 1

101011 | 1 1 0 1 1 0 1 1 1 0 1 **0 0 0 0 0**

1 0 1 0 1 1

0 1 1 1 0 1 1

1 0 1 0 1 1

0 1 0 0 0 0 1

1 0 1 0 1 1

0 0 1 0 1 0 1

0 0 0 0 0 0

0 1 0 1 0 1 0

1 0 1 0 1 1

0 0 0 0 0 1 1

0 0 0 0 0 0

0 0 0 0 1 1 0

0 0 0 0 0 0

0 0 0 1 1 0 0

0 0 0 0 0 0

0 0 1 1 0 0 0

0 0 0 0 0 0

0 1 1 0 0 0 0

1 0 1 0 1 1

0 1 1 0 1 1 0

1 0 1 0 1 1

0 1 1 1 0 1

The transmitted data: [1 1 0 1 1 0 1 1 1 0 1 **1 1 1 0 1**]

To calculate the transmitted BW, first we find the Tb:

$Sr = 1/Tb \longrightarrow Tb_{(old)} = 1/(150*10^{3})$

$Tb_{(old)} = Tf / N \longrightarrow Tf = Tb_{(old)} * N = [1/(150*10^{3})] * 60 = 0.4$ msec

$Tb_{(new)} = Tf / N = (0.4*10^{-3}) / 64$

$Sr = 1/ Tb_{(new)} = 64/(0.4*10^{-3}) = 160$ Kbit/sec

$BW = Sr / \eta = 160$KHz

DIGITAL COMMUNICATIONS

Lecture (14-1): Tutorial (Error Detection and Correction Technique)

Example (1): the following data stream (0110) are the PCM for one sample from an analog signal which bandlimited to 10KHz, answer the following:

A) Draw this stream using bipolar RZ and bipolar NRZ

B) Find the S/N and the required channel BW to transmit this PCM signal

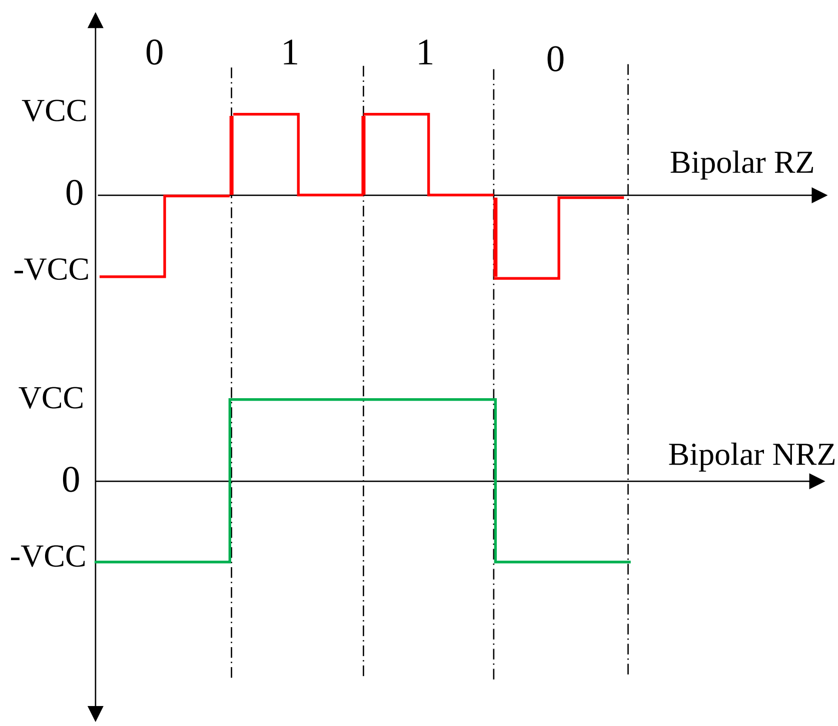
C) State the transmitted data if error detection and correction are used

D) Calculate the required channel BW after using CRC technique with polynomial $=x^5+x^3+x+1$

E) Repeat (D) if VRC error detection technique is used

Solution:

A)



B) The number of bits for data = 4 bits =N

S/N = 1.76+6.02N

= 25.84 dB

Bandlimited mean (f_{max}) = 10KHz

Fs= 2* f_{max} = 2*10 = 20KHz

BW = Sr / η

Sr = N*fs= 4 * 20 = 80Kbit/sec

BW = 80/1 =80KHz

C)

$$2^r \geq d + r + 1$$

$$2^3 \geq 4 + 3 + 1$$

$\therefore$ r = 3

r_2	r_1	r_0	Sequence Of data
0	0	1	r_0
0	1	0	r_1
0	**1**	**1**	d_0
1	0	0	r_2
1	0	**1**	d_1
1	**1**	0	d_2
1	**1**	**1**	d_3

r_0 = parity of {d_0 , d_1 , d_3} = 1

r_1 = parity of {d_0 , d_2 , d_3}= 1

r_2 = parity of {d_1 , d_2 , d_3}= 0

The transmitted data = [0110011]

D)

P= 101011 from the No. of polynomial bits we can know the No. of the added bits to the transmitted data = No. of polynomial bits – 1 = 5 bit

Sr = N*fs= (4+5) * 20 = 9 * 20 = 180Kbit/sec

BW = 180/1 = 180KHz

E) The No. of added bits = 1 bit

Sr = N*fs= (4+1) * 20 = 5 * 20 = 100Kbit/sec

BW = 100/1 =100KHz

Example (2): the following data stream [11010 10001 10101 10010 11100 10111] are the PCM for 6 samples from an analog signal band limited to 3KHz, answer the following:

A) Find the transmitted data if VRC&LRC error detection bits technique used

B) Calculate the required channel BW before and after adding the error detection bits.

C) Design TDM system to transmit the previous signal (after adding error detection bits) **with the following digital signals**

[(S1 to S7) Sr = 2Kbit/sec] , [(S8 to S10) Sr = 14 Kbit/sec] ,

[(S11 to S18) Sr = 1.75 Kbit/sec]

D) Calculate the final signaling rate (Sr) and the required channel BW for your designed system

Solution:

A)

1	1	0	1	0	**1**
1	0	0	0	1	**0**
1	0	1	0	1	**1**
1	0	0	1	0	**0**
1	1	1	0	0	**1**
1	0	1	1	1	**0**
0	**0**	**1**	**1**	**1**	**1**

The transmitted data: [110101 100010 101011 100100 111001 101110 001111]

B) Bandlimited (f_{max}) = 3KHz

$Fs = 2^* f_{max} = 2^*3 = 6KHz$

$BW_{(before\ adding)} = Sr / \eta$

Sr = N*fs= 5 * 6 = 30Kbit/sec

BW = 30/1 =30KHz

After adding error detection bits:

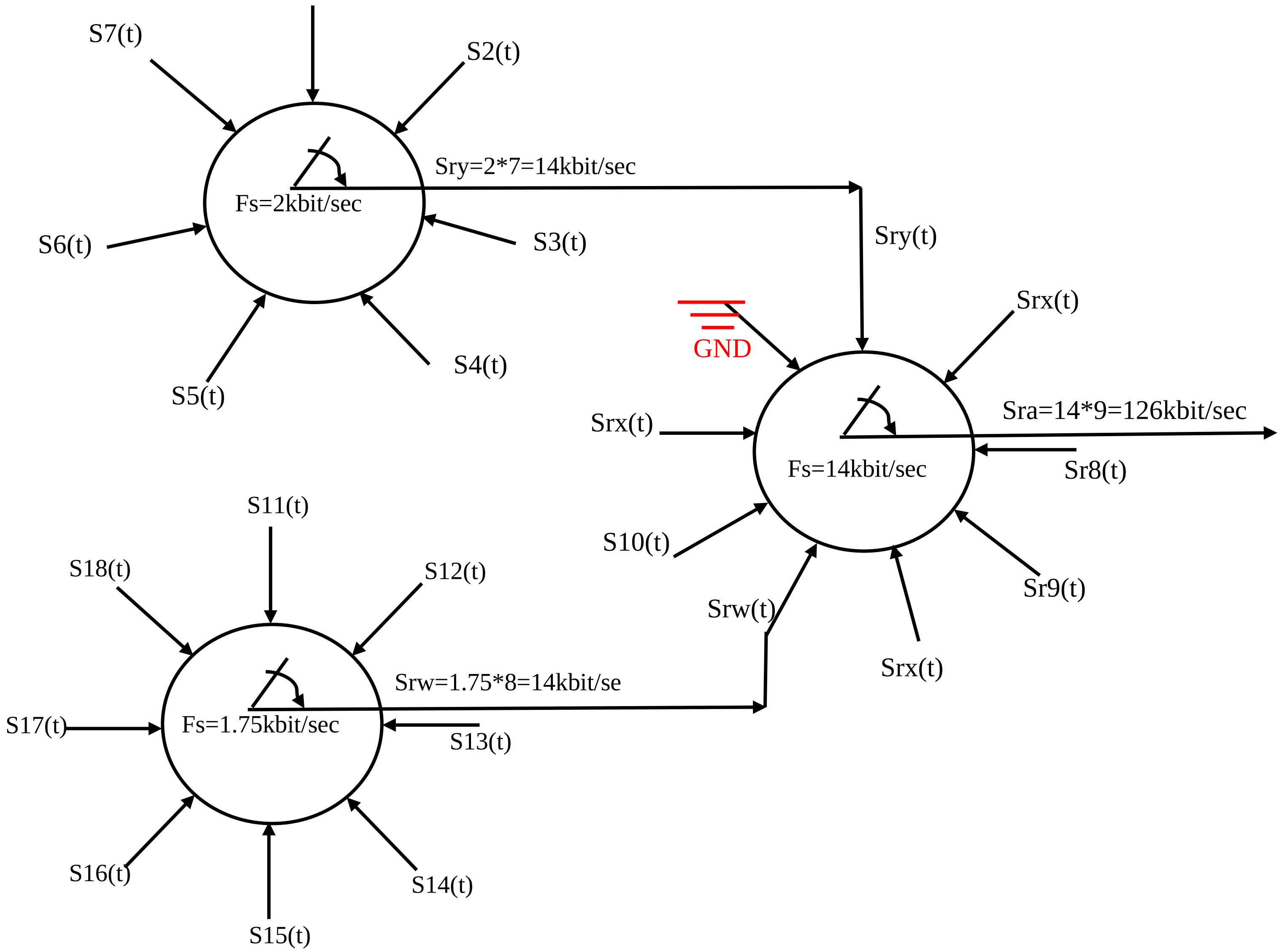
Tf =N*Tb = 30 * 1/(30 Kbit/sec) = 1msec

$Tb_{new} = Tf / N = 1/42$ msec

Sr = 1/Tb = 42 Kbit/sec [let's called it Srx]

$BW_{(after\ adding)} = Sr / \eta = 42$ KHz

C)



Srx=42 Kbit /sec	3
Sry = 14 Kbit/sec	1
Srw = 14 Kbit/sec	1
Sr8 = 14Kbit /sec	1
Sr9 = 14Kbit /sec	1
Sr10 = 14Kbit /sec	1

Sum of pieces = 8 (we add ground) so it will be =9

D) The final signaling rate =126 Kbit/sec

BW = Sr / η = 126 KHz

H.W:

A delta modulation (DM) system is tested with 10KHz sinusoidal signal, 10Vp-p. At the input, the signal is sampled 10 times the Nyquist rate. fs=200KHZ

a) Draw sampled signals for 50 micro second duration time
b) Draw the PAM for the sampled signal in (a) to get BW = fs/2
c) Draw the pulse width modulation for PAM in (b) Return to bias line coding
d) State and draw the DM transmitted data stream using RB for the PAM in(b)
e) Calculate the step size required to prevent slop overload error and minimize granular error
f) Calculate the required channel BW for the DM system
g) If the receiver input is bandlimited to 200KHz, what is the average S/N ratio
h) If the PAM in (b) converted to PCM with 50mV uniform step size, calculate the S/N ratio
i) Calculate the required channel BW for the PCM system in (h)
j) Discuss the results for (f,g,h,i), how can be raised the S/N with reduce the required channel BW?

DIGITAL COMMUNICATIONS

Lecture (14-1): Tutorial (Error Detection and Correction Technique)

<u>Example (1):</u> the following data stream (0110) are the PCM for one sample from an analog signal which bandlimited to 10KHz, answer the following:

A) Draw this stream using bipolar RZ and bipolar NRZ

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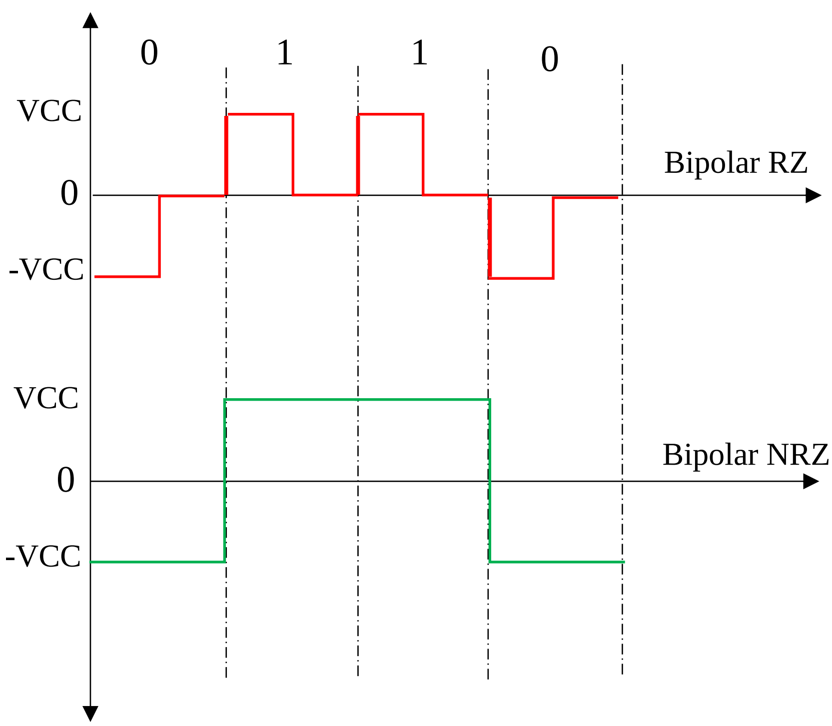
C) State the transmitted data if Hammimg error detection and correction are used

D) Calculate the required channel BW after using CRC technique with polynomial $=x^5+x^3+x+1$

E) Repeat (D) if VRC error detection technique is used

<u>Solution:</u>

A)



B) The number of bits for data = 4 bits =N

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= 25.84 dB

Bandlimited mean (f_{max}) = 10KHz

Fs= 2* f_{max} = 2*10 = 20KHz

BW = Sr / η

Sr = N*fs= 4 * 20 = 80Kbit/sec

BW = 80/1 =80KHz

C)

$$2^r \geq d + r + 1$$

$$2^3 \geq 4 + \boxed{3} + 1$$

∴ r = 3

r_2	r_1	r_0	Sequence Of data
0	0	1	r_0
0	1	0	r_1
0	**1**	**1**	d_0
1	0	0	r_2
1	0	**1**	d_1
1	**1**	0	d_2
1	**1**	**1**	d_3

r_0 = parity of $\{d_0 , d_1 , d_3\}$ = 1

r_1 = parity of $\{d_0 , d_2 , d_3\}$= 1

r_2 = parity of $\{d_1 , d_2 , d_3\}$= 0

The transmitted data = [0110011]

D)

P= 101011 from the No. of polynomial bits we can know the No. of the added bits to the transmitted data = No. of polynomial bits – 1 = 5 bit

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C) Design TDM system to transmit the previous signal (after adding error detection bits) **<u>with the following digital signals</u>**

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[(S11 to S18) Sr = 1.75 Kbit/sec]

D) Calculate the final signaling rate (Sr) and the required channel BW for your designed system

Solution:

A)

1	1	0	1	0	**1**
1	0	0	0	1	**0**
1	0	1	0	1	**1**
1	0	0	1	0	**0**
1	1	1	0	0	**1**
1	0	1	1	1	**0**
0	**0**	**1**	**1**	**1**	**1**

The transmitted data: [110101 100010 101011 100100 111001 101110 001111]

B) Bandlimited (f_{max}) = 3KHz

Fs= 2* f_{max} = 2*3 = 6KHz

BW $_{\text{(before adding)}}$ = Sr / η

Sr = N*fs= 5 * 6 = 30Kbit/sec

BW = 30/1 =30KHz

After adding error detection bits:

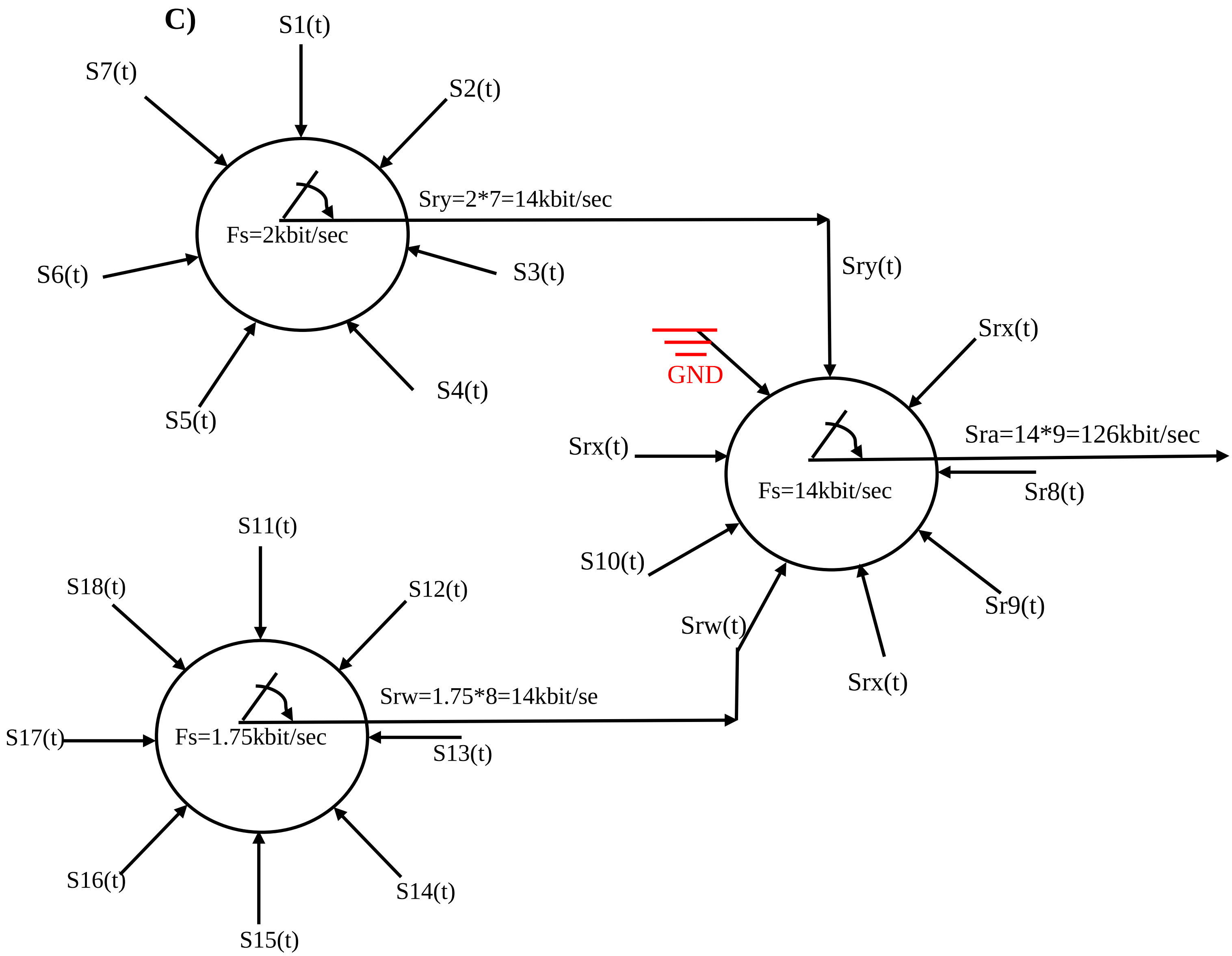
Tf =N*Tb = 30 * 1/(30 Kbit/sec) = 1msec

Tb $_{\text{new}}$ = Tf / N = 1/42 msec

Sr = 1/Tb = 42 Kbit/sec [let's called it Srx]

BW $_{\text{(after adding)}}$ = Sr / η = 42 KHz

C)



Srx=42 Kbit /sec	3
Sry = 14 Kbit/sec	1
Srw = 14 Kbit/sec	1
Sr8 = 14Kbit /sec	1
Sr9 = 14Kbit /sec	1
Sr10 = 14Kbit /sec	1

Sum of pieces = 8 (we add ground) so it will be =9

D) The final signaling rate =126 Kbit/sec

BW = Sr / η = 126 KHz

DIGITAL COMMUNICATIONS

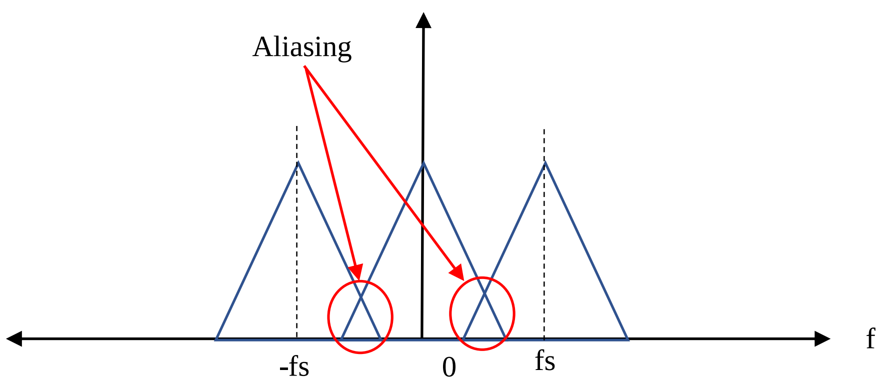
Base Band signal

Lecture (15-1): Problems in Base Band Signal

The problems appear in Transmitting the baseband signal through the channel are:

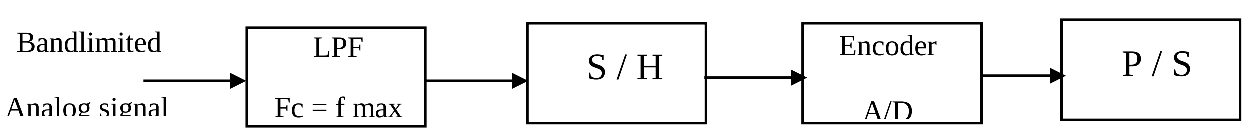
1. Aliasing
2. Inter symbol interference

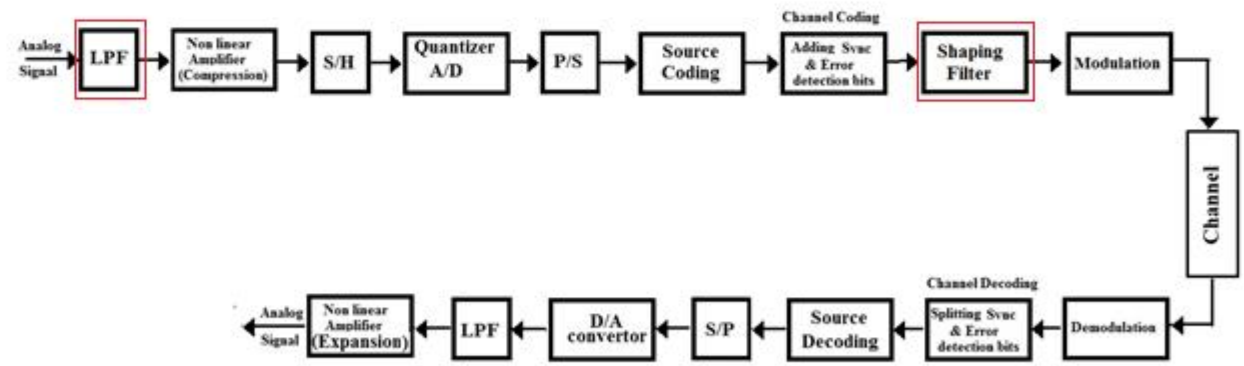
1. **<u>Aliasing:</u>** this problem is a result of overlapping in the spectrum of the samples transmitted signal, where this problem happened when **($f_s < 2f_{max}$)**



To solve this problem, we used Nyquist's rate which add 20% of Fmax as a safety region to avoid the overlapping between the transmitted bits, also adding LPF to the circuit to enforce the system not take a frequency higher than fmax.

$F_s = 2 * f_{max} + 20\% \, f_{max}$

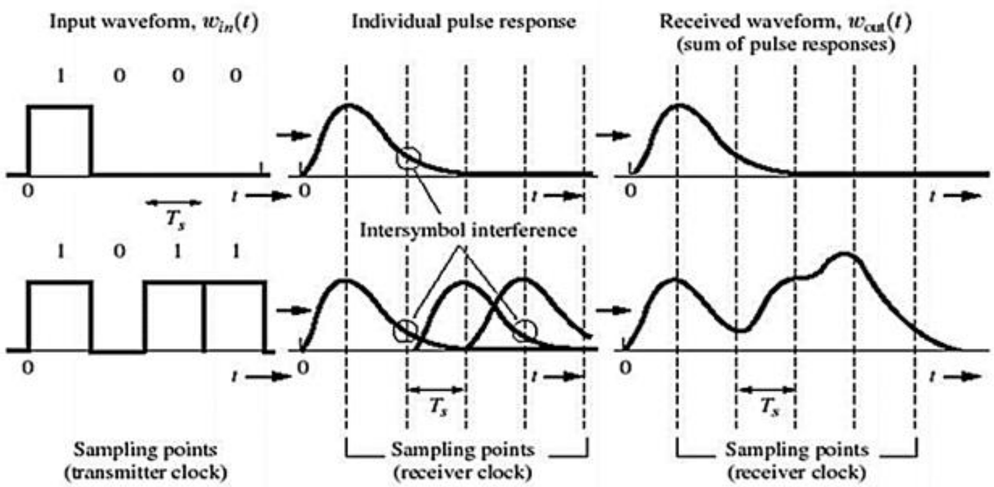


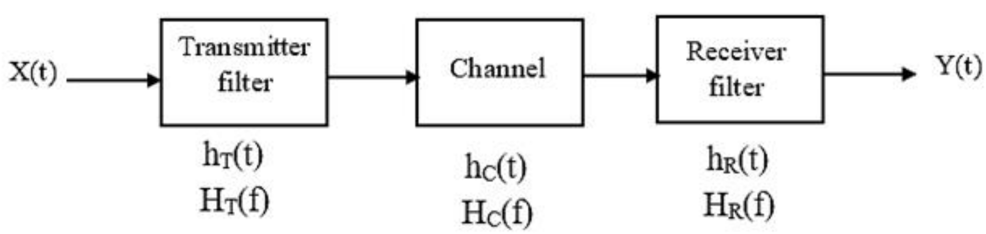


Block diagram of the Digital Communication system

2. Inter Symbol Interference:

Through sending bits in the channel which is look like LPF, the overlapping happened in the transmitted bits so in the receiver made the wrong the decision (occurs when a pulse spreads out in such a way that it interferes with adjacent at the sample instant).





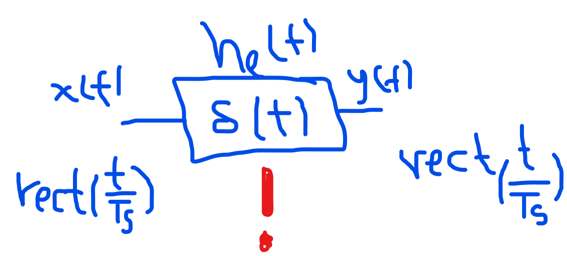
$y(t) = x(t) \otimes h_T(t) \otimes h_C(t) \otimes h_R(t)$

$Y(f) = X(f).\underline{H_T(f).H_C(f).H_R(f)}$

$He(f) = H_T(f).H_C(f).H_R(f)$

$Y(f) = X(f).He(f)$

$y(t) = x(t) \otimes h_e(t)$



From the last equation we can conclude that if the transmitted signal was rect(t/Ts)

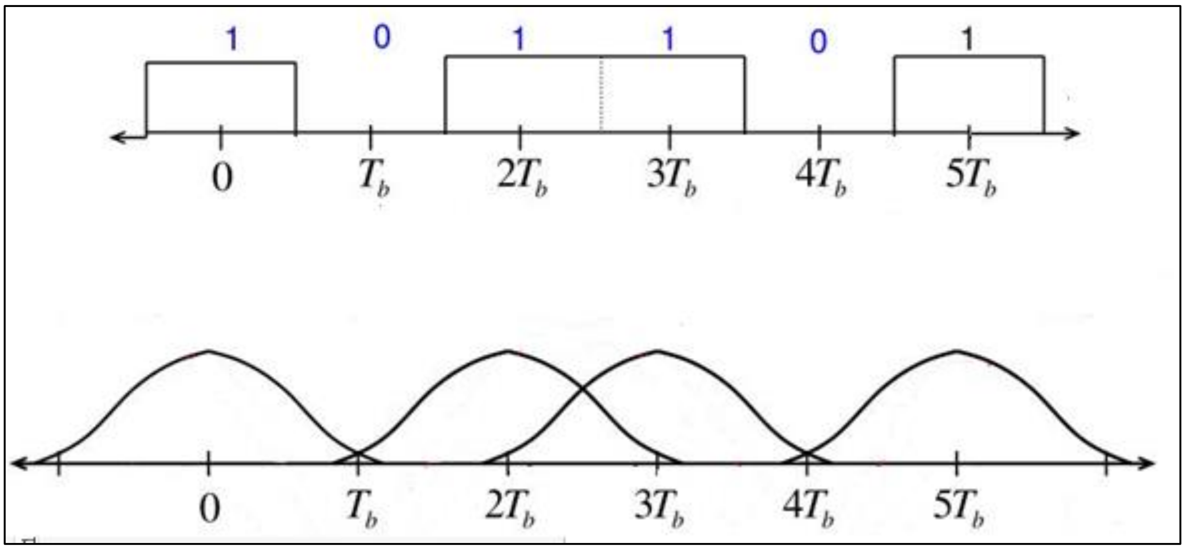
x(t)= rect (t/Ts)

if $h_e(t) = \delta(t)$

$\Rightarrow$ y(t)= rect (t/Ts)

(because the transmitted signal is a bit with rectangular shape) the received signal must also was rect (t/Ts) and this can be achieved when $h_e(t) = \delta(t)$ and this not existed, this was the first solution.

The second solution, as the received signal would arrive like sinc signal at the receiver we suppose that the first null would be at the center



$$y(t) = sinc\left(\frac{t}{Ts}\right)$$

$$Y(f) = X(f).He(f)$$

$$rect\left(\frac{f}{fs}\right) = sinc\left(\frac{f}{fs}\right).He(f)$$

x(t)=rect (t/Ts) → He (f) → y(t)=sinc (t/Ts)

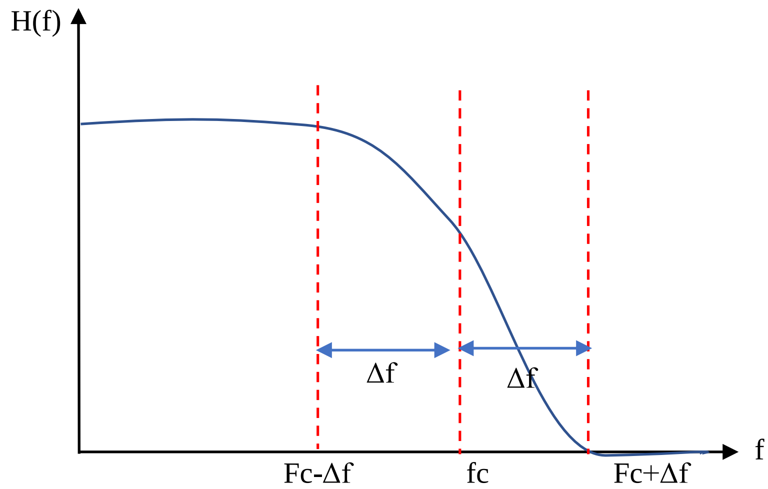
X(f)=sinc (f/fs)   Y(f)=rect (f/fs)

$$\boxed{He(f) = \frac{rect(\frac{f}{fs})}{sinc(\frac{f}{fs})}}$$

The third solution, made a shift in the transfer function by $(e^{-i\omega t})$ and multiplied it with an attenuation factor (K).

$$He(f) = \frac{rect(\frac{f}{fs})}{sinc(\frac{f}{fs})} * Ke^{-i\omega t}$$

Raised cosine filter (shaping filter):



Roll factor (r)= Δf /fc

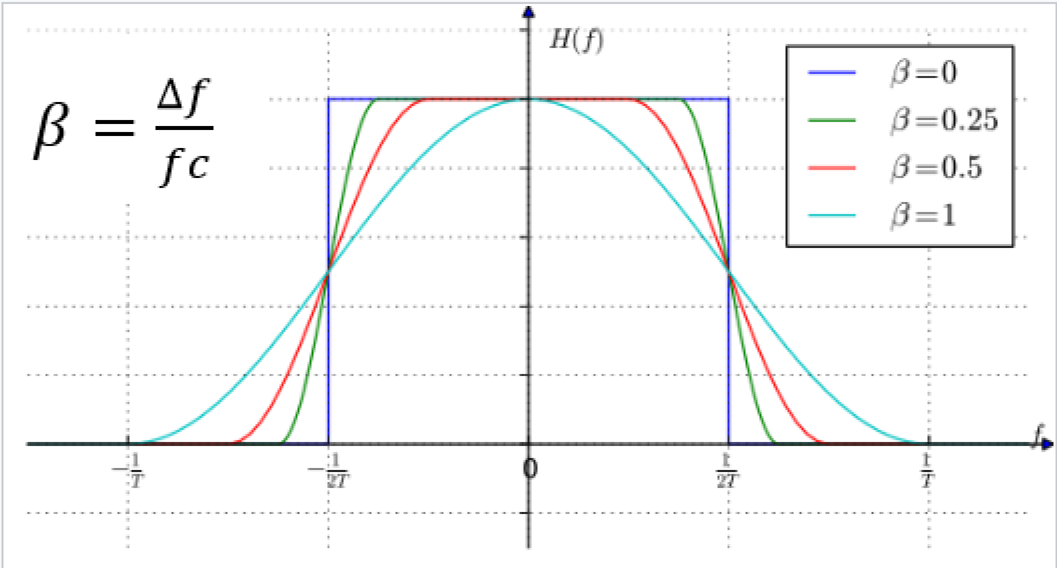
$H(f) = 1 \ \ \{ if \quad |f| \leq (fc - \Delta f)\}$

$H(f) = \frac{1}{2}\left[1 + \cos\left(\frac{\pi(|f|)-(fc-\Delta f)}{2\Delta f}\right)\right] \qquad \{if \quad (fc - \Delta f) \leq |f| \leq (fc + \Delta f)\}$

$H(f) = 0 \quad \{ if \quad |f| > (fc + \Delta f)\}$

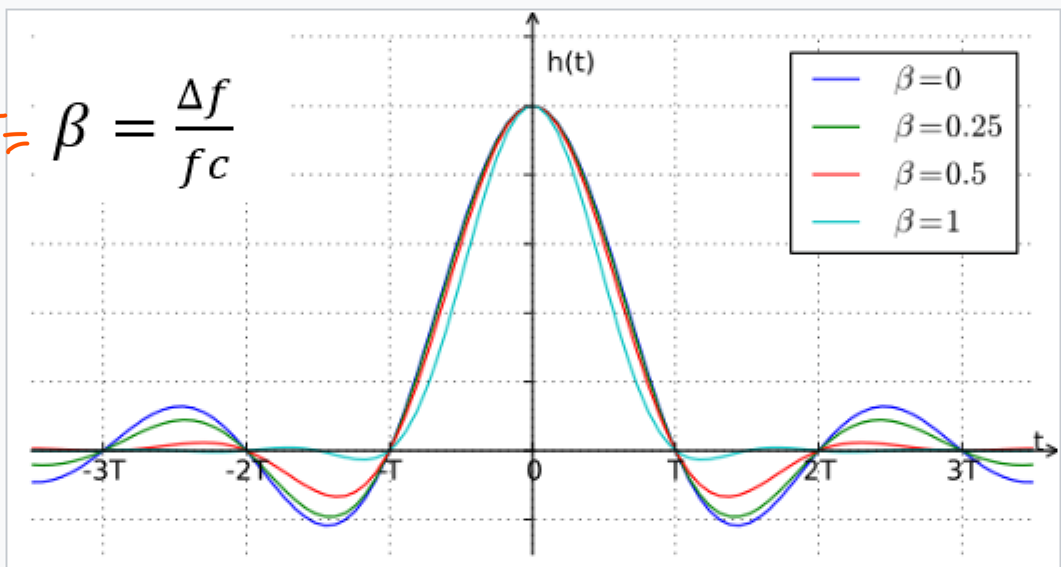
$req\ BW\ (after\ adding\ r.c.f) = BW_{old}(1 + r)$

$$\gamma = \beta = \frac{\Delta f}{fc}$$



Frequency response of raised-cosine filter with various roll-off factors

$$\gamma = \beta = \frac{\Delta f}{fc}$$



Impulse response of raised-cosine filter with various roll-off factors

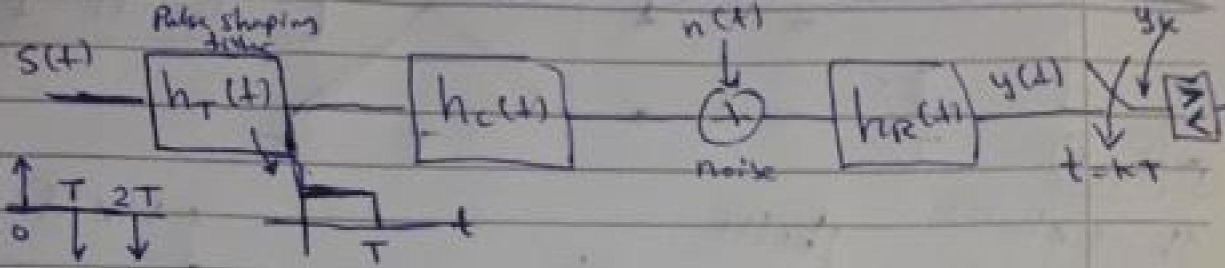
$$H(f) = \begin{cases} 1, & |f| \leq \frac{1-\beta}{2T} \\ \frac{1}{2}\left[1 + \cos\left(\frac{\pi T}{\beta}\left[|f| - \frac{1-\beta}{2T}\right]\right)\right], & \frac{1-\beta}{2T} < |f| \leq \frac{1+\beta}{2T} \\ 0, & \text{otherwise} \end{cases}$$

The impulse response of such a filter[1] is given by:

$$h(t) = \begin{cases} \frac{\pi}{4T}\operatorname{sinc}\left(\frac{1}{2\beta}\right), & t = \pm\frac{T}{2\beta} \\ \frac{1}{T}\operatorname{sinc}\left(\frac{t}{T}\right)\frac{\cos\left(\frac{\pi\beta t}{T}\right)}{1-\left(\frac{2\beta t}{T}\right)^2}, & \text{otherwise} \end{cases}$$

Were $\beta = \frac{\Delta f}{fc}$ $= r$

Pulse shaping and Raised Cosine filter

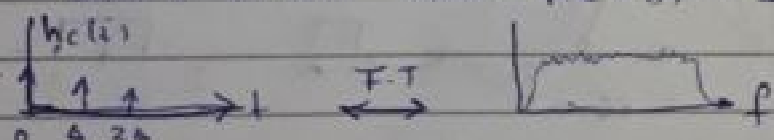


$$y(t) = \underbrace{h_R(t) \circledast h_c(t) \circledast h_T(t)}_{h_e(t)} \circledast S(t) + h_R(t) \circledast n(t)$$

where $h_e(t) = h_R(t) \circledast h_c(t) \circledast h_T(t)$ $\boxed{\text{Time}}$
= effective filter of the system

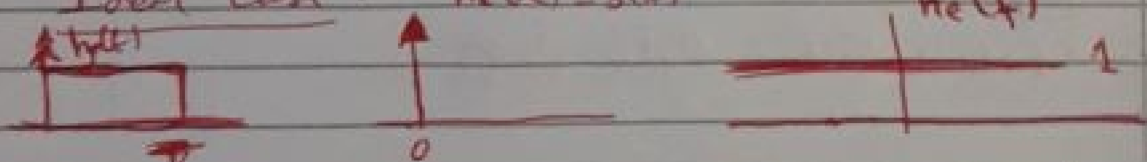
$$H_e(f) = H_R(f) \cdot H_c(f) \cdot H_T(f) \quad \boxed{\text{freq}}$$

Let the channel $H_c(f)$ is flat fading which means that $H_c(f) \simeq 1$

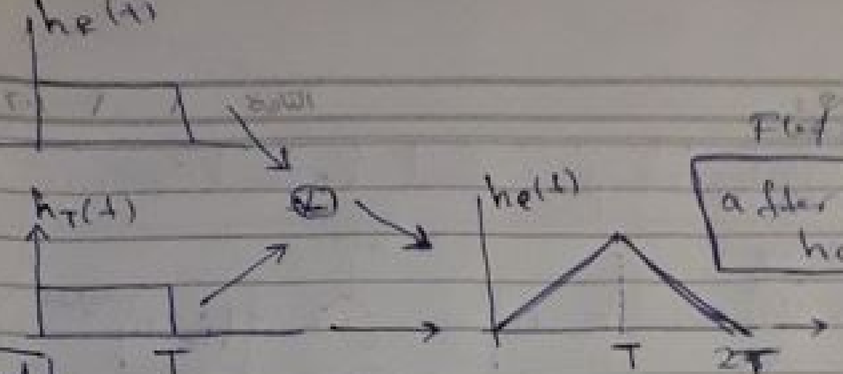


$\boxed{\text{channel}}$

① Ideal Case: $h_e(t) = \delta(t)$



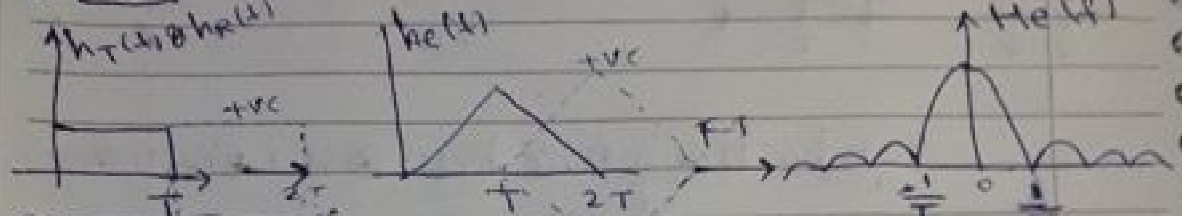
* It is assumed that $h_T(t) = \text{Rect}\,\frac{t}{T_s}$ and $h_e(t) = \delta(t)$, which not existed in the real-world



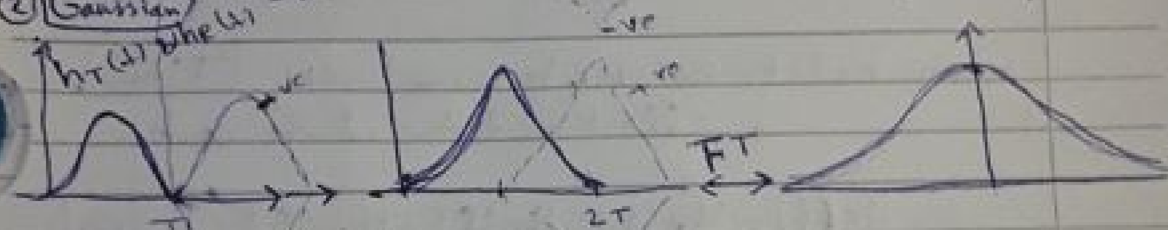
Flat fading

a filter assuming $h_c(t) = 1$

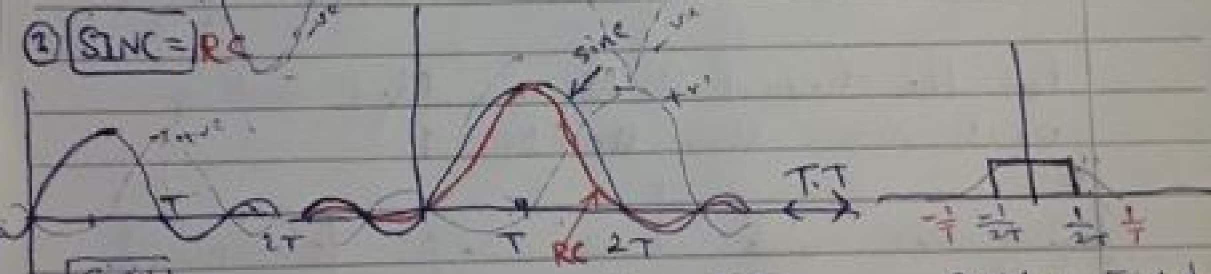
① Rect



② Gaussian



③ SINC = RC



$\boxed{\frac{\sin \pi x}{x}}$

Problem in time ISI

Problem Fixed in freq. domain

RC

④ Raised Cosine Filter Pulse shapping

DIGITAL COMMUNICATIONS

Lecture (16-1): Power Spectral Density (PSD) for Base Band Signals

Introduction:

Power spectral density (PSD) is the average of power distribution in frequency domain. The PSD consist of two part, the absolute value and the function distribution (which mean that the power distributed in frequency domain with a function and amplitude) like $\delta(t)$ has absolute value equal 1 at frequencies (from 0 to ∞).

$$power = \frac{V^2}{R} \quad or \quad power = energy/time$$

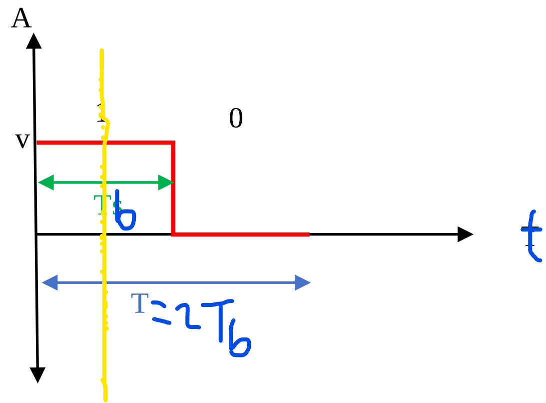
The energy according to Parseval theorem: the summation of energy in time domain equal the summation of energy in frequency domain or the average power in in time domain equal the average power in frequency domain.

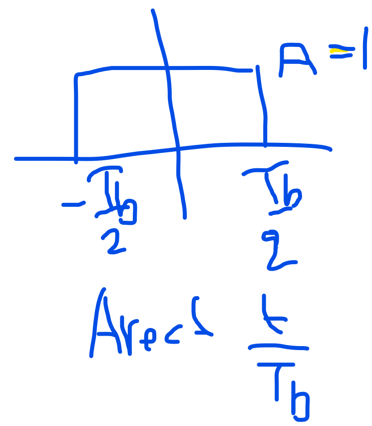
$$\frac{1}{T}\int_{-T/2}^{T/2}|x(t)dt|^2 = \frac{1}{T}\int_{-\infty}^{\infty}|x(f)df|^2$$

In this study we take the PSD for these types of base band signal:

1) PSD for NRZ unipolar
2) PSD for NRZ bipolar
3) PSD for RZ unipolar
4) PSD for RZ bipolar

1) PSD for NRZ unipolar:





To represent the above signal for bit "1" :

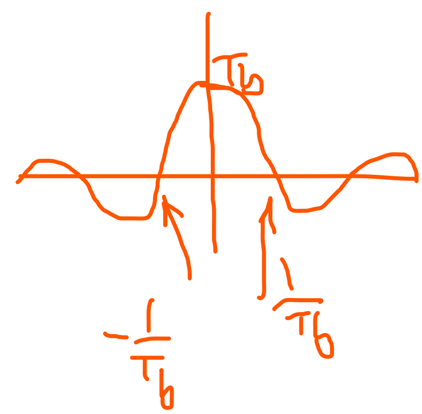
$x(t) = rect\,\frac{t}{T_b}$ $\qquad X(f) = T_b\left(\frac{\sin 2\pi f T_b}{2\pi f T_b}\right)$

$T_b\,\mathrm{sinc}(f T_b)$

To represent the above signal for bit "0" :

$x(t) = 0$

To find the average voltage for the NRZ unipolar

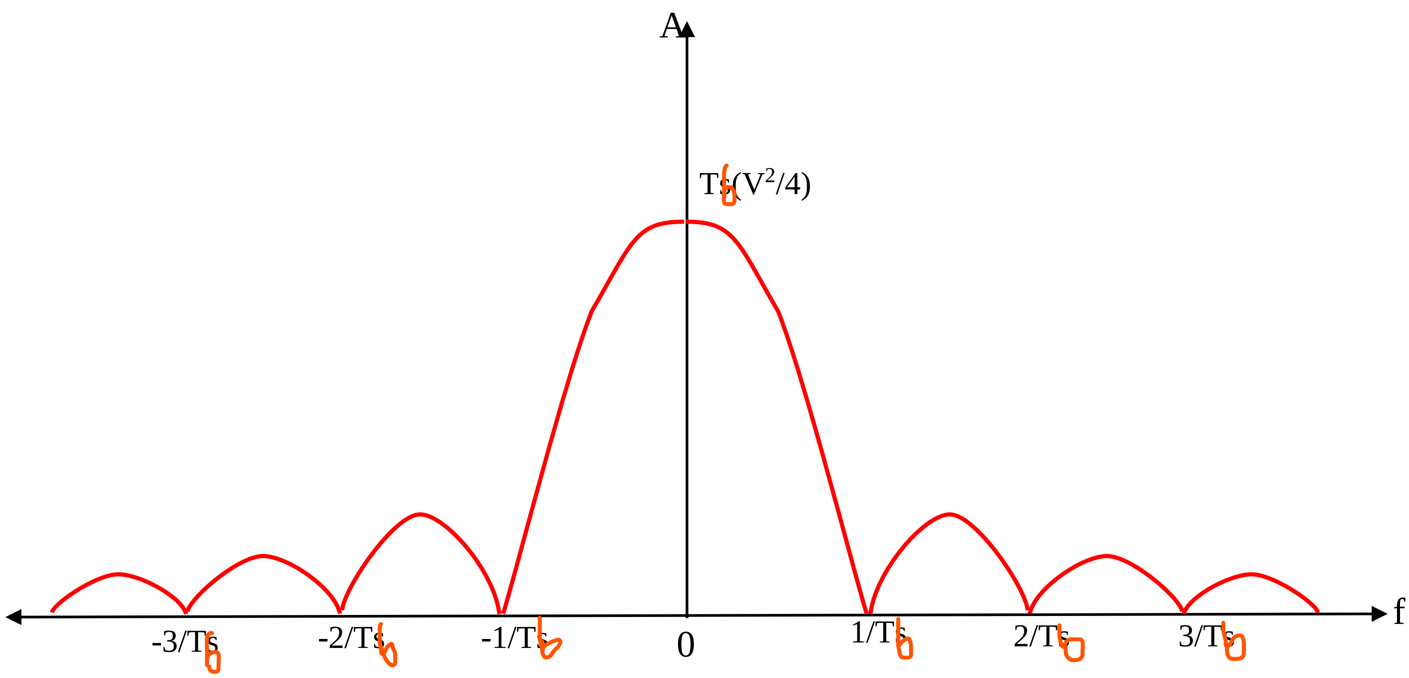


$$average\ voltage = \frac{v+0}{2} = \frac{v}{2}$$

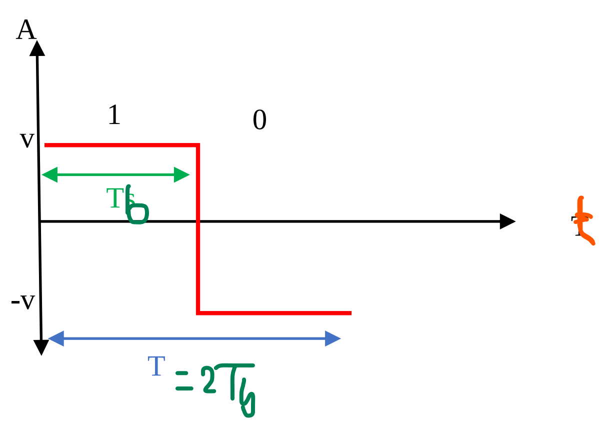
$$energy = \frac{v^2}{4}$$

$$power = \frac{energy}{T_b} = \frac{v^2}{4T_b}$$ amplitude as power

$$PSD = \frac{v^2}{4T_b} * T_b^2\left(\frac{\sin 2\pi f T_b}{2\pi f T_b}\right)^2 = \frac{v^2 T_b}{4}\left(\frac{\sin 2\pi f T_b}{2\pi f T_b}\right)^2$$



2) PSD for NRZ bipolar:



To represent the above signal for bit "1" :

$x(t) = rect\frac{t}{T_b}$ $\qquad X(f) = T_b(\frac{\sin 2\pi f T_b}{2\pi f T_b}) = T_b \, sinc(f T_b)$

To represent the above signal for bit "0" :

$x(t) = -rect\frac{-t}{T_b} \longrightarrow X(f) = \left| -T_b \, sinc(f T_b) \right|$

$= T_b \, sinc(f T_b)$

To find the average voltage for the NRZ bipolar

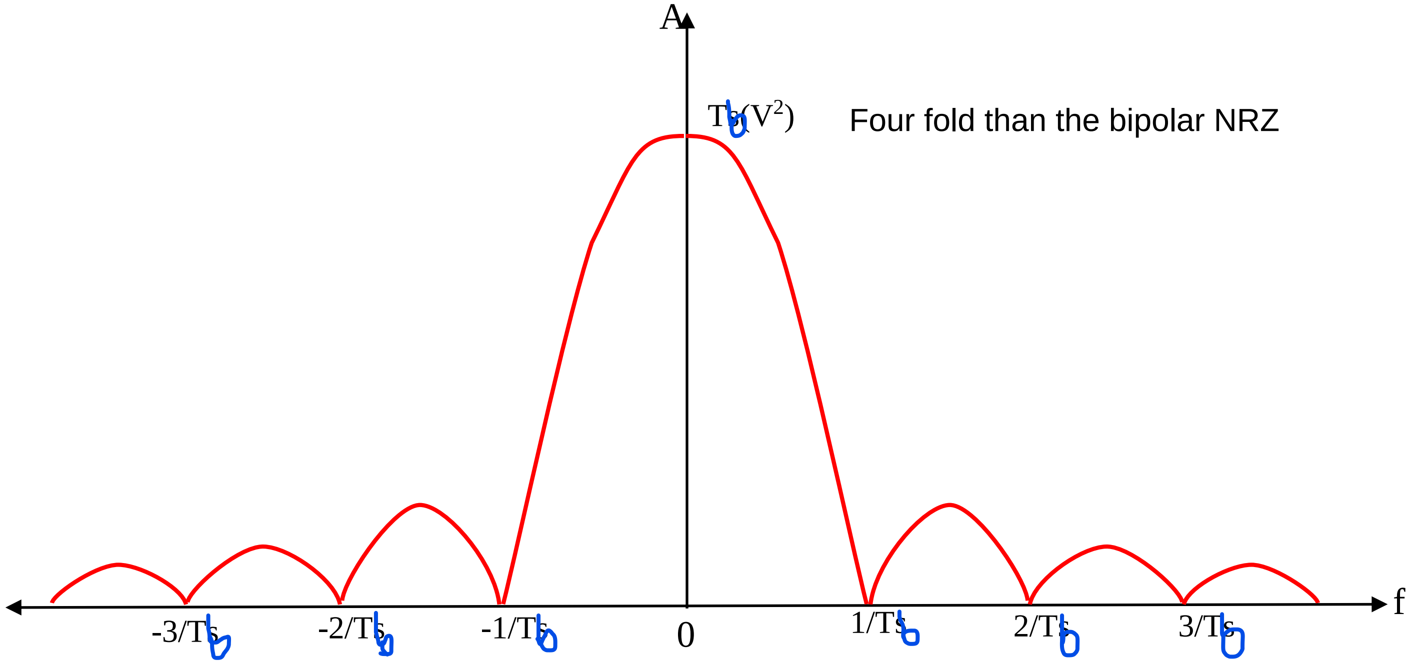
$$average\ voltage = \frac{v+v}{2} = v$$

The average sum of the absolute voltage (|v|+|-v|)/2 over T=2Tb

$$energy = v^2$$

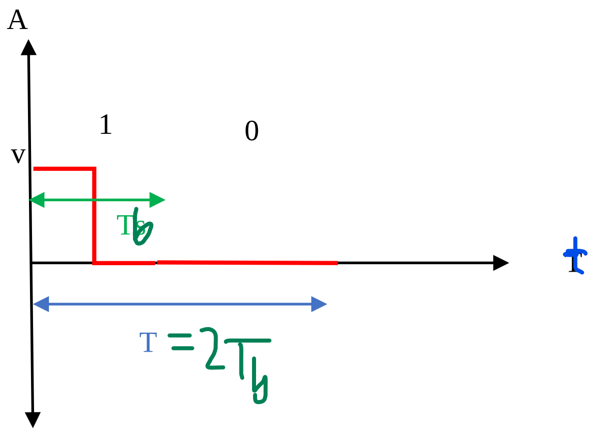
$$power = \frac{energy}{T_b} = \frac{v^2}{T_b}$$ amplitude as power

$$PSD = \frac{v^2}{T_b} * T_b^2 \left(\frac{\sin 2\pi f T_b}{2\pi f T_b}\right)^2 = v^2 T_b \left(\frac{\sin 2\pi f T_b}{2\pi f T_b}\right)^2$$



The same required bandwidth

3) PSD for RZ unipolar:



To represent the above signal for bit "1" :

$$x(t) = rect\,\frac{t}{T_b/2} \qquad X(f) = T_b\left(\frac{\sin 2\pi f T_b/2}{2\pi f T_b/2}\right)$$

To represent the above signal for bit "0" :

$$x(t) = 0$$

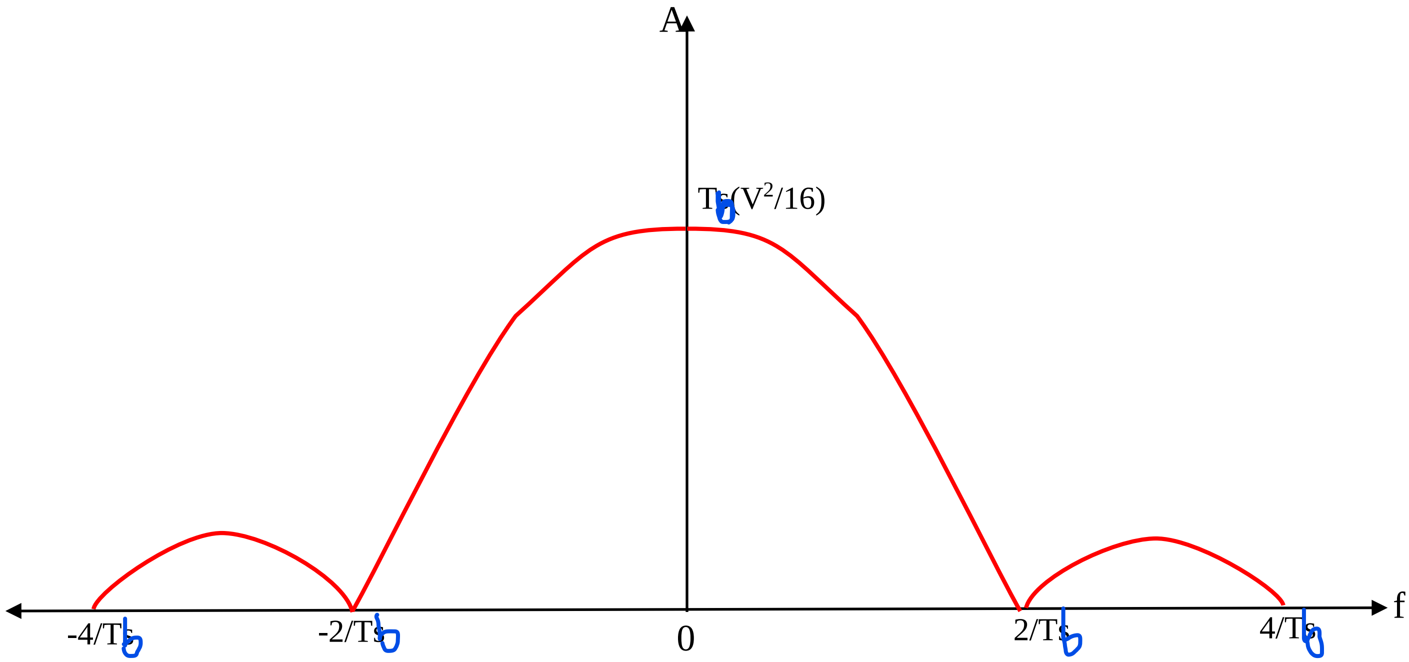
To find the average voltage for the NRZ bipolar

$$average\ voltage = \frac{v+0+0+0}{4} = \frac{v}{4}$$

$$energy = v^2\ /16$$

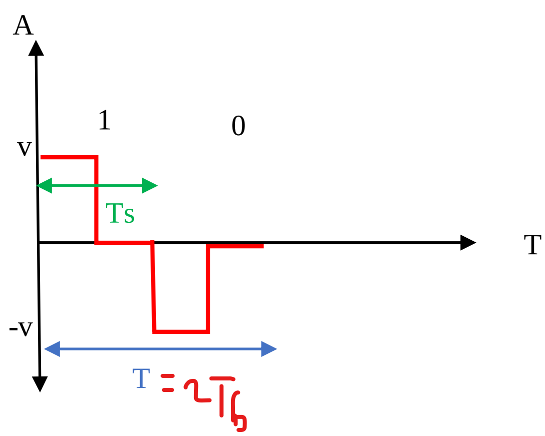
$$power = \frac{energy}{T_b} = \frac{v^2}{16T_b}$$ amplitude as power

$$PSD = \frac{v^2}{16T_b} * T_b^2\left(\frac{\sin 2\pi f T_b/2}{2\pi f T_b/2}\right)^2 = \frac{v^2}{16}Ts\left(\frac{\sin 2\pi f T_b/2}{2\pi f T_b/2}\right)^2$$



Twice the BW required for NRZ

4) PSD for RZ bipolar:



To represent the above signal for bit "1" :

$$x(t) = rect\,\frac{t}{T_b/2} \qquad X(f) = T_b\left(\frac{\sin 2\pi f T_b/2}{2\pi f T_b/2}\right)$$

To represent the above signal for bit "0" :

$$x(t) = -rect\,\frac{t}{T_b/2}$$

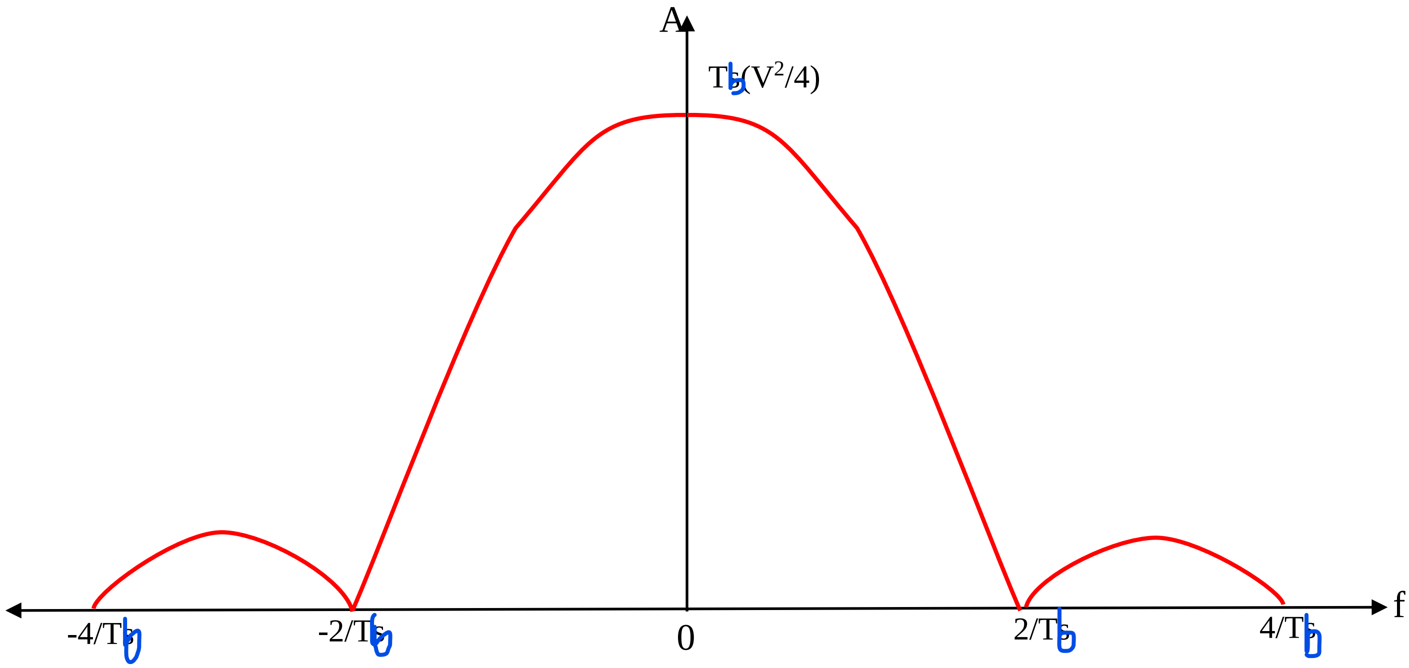
To find the average voltage for the NRZ bipolar

$$average\ voltage = \frac{v+0+v+0}{4} = \frac{v}{2}$$

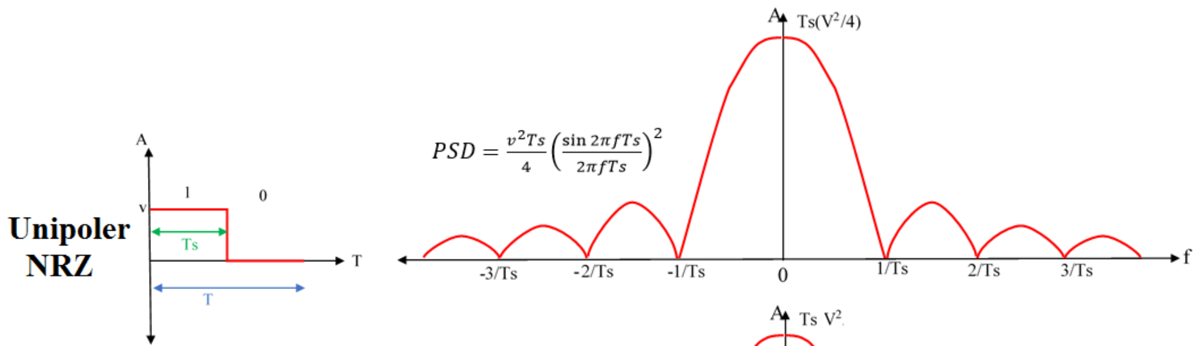
$$energy = \frac{v^2}{4}$$

$$power = \frac{energy}{Ts} = \frac{v^2}{4Ts}$$ amplitude as power

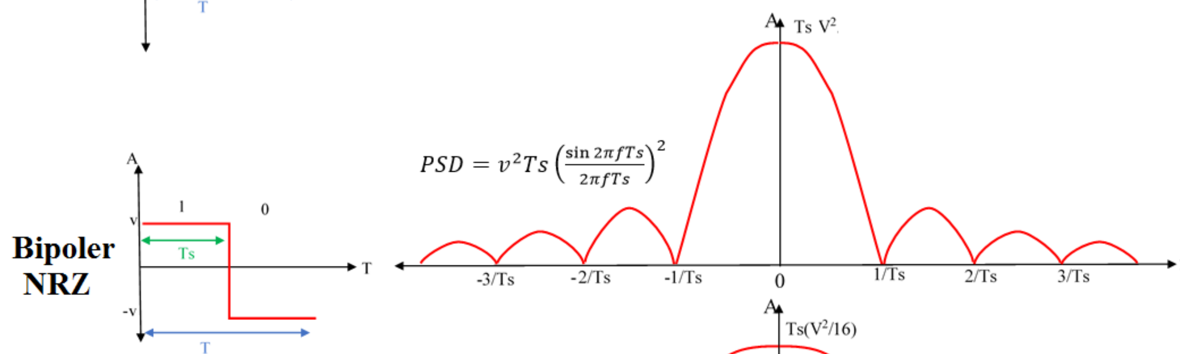
$$PSD = \frac{v^2}{4Ts} * Ts^2 \left(\frac{\sin 2\pi f Ts/2}{2\pi f Ts/2}\right)^2 = \frac{v^2}{4} Ts \left(\frac{\sin 2\pi f Ts/2}{2\pi f Ts/2}\right)^2$$



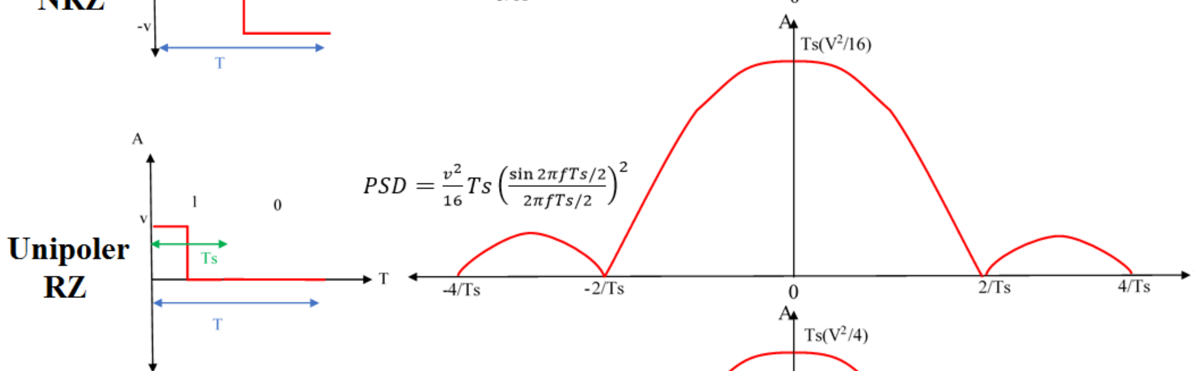
Replace all Ts with Tb for all the SINCs below



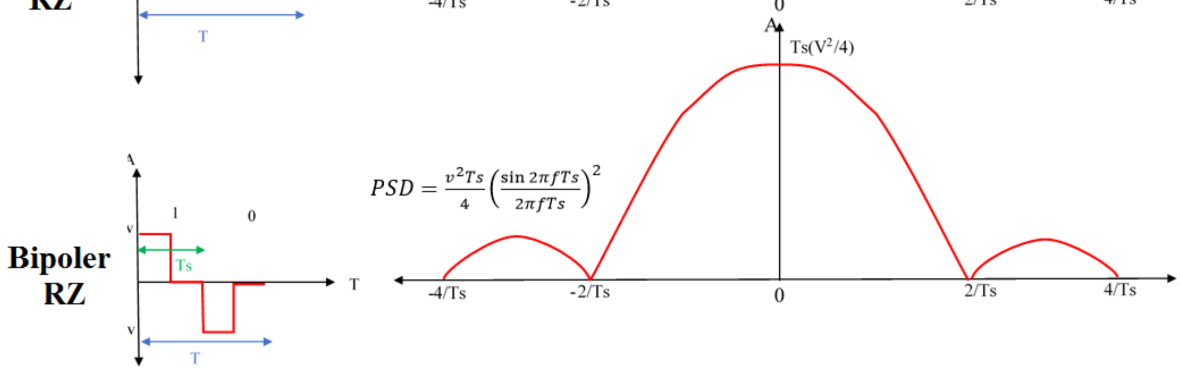
$$PSD = \frac{v^2 Ts}{4}\left(\frac{\sin 2\pi f Ts}{2\pi f Ts}\right)^2$$



$$PSD = v^2 Ts\left(\frac{\sin 2\pi f Ts}{2\pi f Ts}\right)^2$$



$$PSD = \frac{v^2}{16} Ts\left(\frac{\sin 2\pi f Ts/2}{2\pi f Ts/2}\right)^2$$



$$PSD = \frac{v^2 Ts}{4}\left(\frac{\sin 2\pi f Ts}{2\pi f Ts}\right)^2$$

$$\boxed{\eta = \begin{cases} 2 & \text{for NRZ} \\ 1 & \text{for RZ} \end{cases}}$$

$$BW = \frac{Sr}{\eta} \Longrightarrow$$

BW(NRZ)=Sr/2

BW(RZ)=Sr/1

DIGITAL COMMUNICATIONS

Lecture (17-1): Probability of Error for Base Band Signals

Introduction:

The variables can be classified into two types the deterministic and non-deterministic (random) variable. Deterministic is the opposite of a random event. It tells us that some future event can be calculated exactly, without the involvement of randomness. For example, the conversion between Celsius and Kelvin is deterministic, because the formula is *not* random…it is an exact formula that will always give you the correct answer (assuming you perform the calculations correctly):

$$\text{Kelvin} = \text{Celsius} + 273.15.$$

If something is deterministic, you have all of the data necessary to predict (determine) the outcome with 100% certainty. The process of calculating the output (in this example, inputting the Celsius and adding 273.15) is called a deterministic process or procedure. A few more examples:

- Rolling a fair die: each number on a six-sided die has the same odds (1/6) of coming up.
- Calculating what your savings account balance will be in a month (add up your deposits and the prevailing interest rate).
- The relationship between a circumference and radius of a circle, or the area and radius of a circle.

On the other hand, a random event or process can't be determined with an exact formula. You can ballpark it, or "hazard a good guess," but you can't assign probabilities to it. For example, the odds of seeing a black cat on your way to work tomorrow cannot be calculated, as the process is completely random, or stochastic. this event can be represented by the probability of occurrence.

Probability: it is a limiting value of the relative frequency of occurrence is called the probability of the out come (A) and written as P(A).

Example:
Coin is tossed with Head and Tail faces.
P(H)=0.5 and P(T)=0.5

We can repeat this experiment 10, 100, or 1000 times and calculate the average of the results

$$\boldsymbol{P(A) = \lim_{N\to\infty} \frac{NA}{N}} \qquad \boldsymbol{0 \le P(A) \le 1}$$

Where:

A: is the occurrence , NA: number of the occurrence of the event A

N: the total number of experiment frequency

P(A) maximum=1 , P(A) minimum =0 , $\sum_{A=0}^{N} P(A) = 1$

Usually, the events in the nature are two types: deterministic and nondeterministic. The deterministic events like variables subject to law or constant equation to find the value while the nondeterministic events not subject to law like rainfall rate.

If two events, A and B, are mutually exclusive (they cannot occur simultaneously),

Relations for Variables Under Probability:

MEE

1) **Mutually Exclusive Events:** two or more events are defined mutually exclusive if they cannot possibly occur together simultaneously (the occurrence of one outcome precludes the occurrence of the other).

$$\boldsymbol{P(A) * P(B) = 0 = P(A * B) = P(A \cap B)}$$
$$\boldsymbol{P(A + B) = P(A \cup B) = P(A) + P(B)}$$

intersection = AND= (.)=(*) $\cap$

union= OR = (+) $\cup$

Example: A sample subset S={1, 2, 3, 4, 5, 6}, we have Three groups A={1, 2, 3}, B={5, 6} and C={3,4,5}

$A \cap B = \{\ \} = \phi$

P(A and B)=P(A.B)=0 MEE

$A \cap C = \{3\}$

P(A.C)=1/6 not MEE

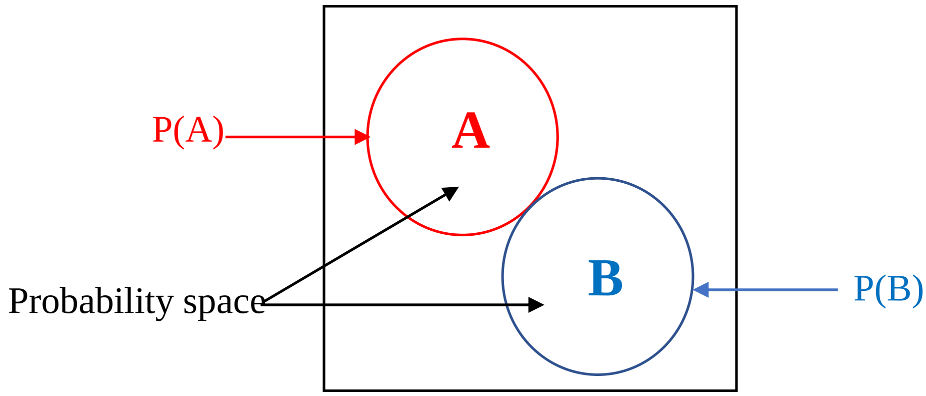
$\neq 0$

$B \cap C = \{5\}$

P(B.C)=1/6 not MEE

$\neq 0$

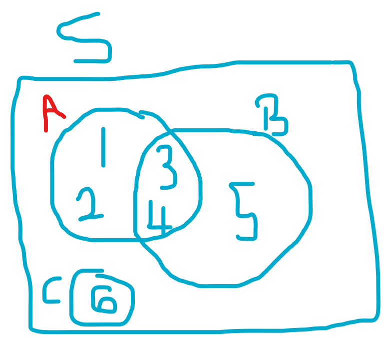
The mutually events in digital communication are when sending "1" or "0", where sending bit "1" prevent sending bit "0" at the same time. If sending "1" is event A and sending "0" is event B, the following figure illustrate the probability.



If the events are mutually exclusive and equally to occur

$$\boldsymbol{P_{(i)} = \frac{1}{N} \qquad 0 \le P(A) \le 1}$$

$$\boldsymbol{\sum_{i=0}^{N} P(i) = 1}$$



$$\boldsymbol{P(A) * P(B) = 0 = P(A * B) = P(A \cap B)}$$

$$\boldsymbol{P(A + B) = P(A \cup B) = P(A) + P(B)}$$

2) Joints Events: two or more events are defined joint if they may can possible occur together.

Example:

S={1,2,3,4,5,6} with A={1,2,3,4} and B={3,4,5}

$P(A \cap B) = \frac{2}{6} = P(A) \cdot P(B) = \frac{4}{6} \cdot \frac{3}{6} = \frac{12}{36} = \frac{2}{6}$

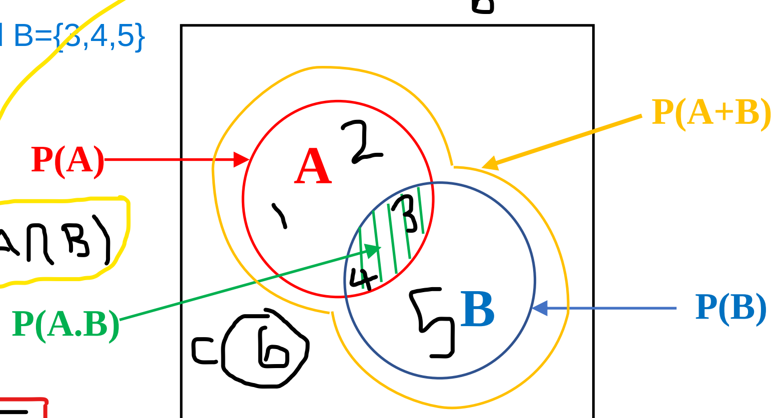
$P(A \cup B) = P(A \text{ or } B)$

$= P(A) + P(B) - P(A \cap B)$

$= \boxed{\frac{4}{6} + \frac{3}{6}} - \frac{2}{6}$

$= \boxed{\frac{5}{6}} < 1$

$\frac{7}{6} > 1$ Wrong



2nd Method: $A \cup B = \{1, 2, 3, 4, 5\} \Rightarrow P(A \cup B) = \boxed{\frac{5}{6}}$ ✓

Let C={6} then P(B + C)= P(B) + P(C) - P(B.C)=
= 3/6 + 1/6 - 0 = 4/6 because it is MEE case

$$\boxed{\boldsymbol{P(A+B) = P(A \cup B) = P(A) + P(B) - P(A.B)}}$$ ✓

$$\boldsymbol{P(A.B) = P(A \cap B) = P(A).P(B)}$$ ✓

In general, the joints events can be understanded if we have two coins and of throw them at the same time, the probability will be:

(A)	(B)
T	T
T	H
H	T
H	H

PA(T)=2/4, PB(T)=2/4 P(A.B)=PA(T).PB(T)=1/4

PA(T)=2/4, PB(H)=2/4 P(A.B)=PA(T).PB(H)=1/4

PA(H)=2/4, PB(T)=2/4 P(A.B)=PA(H).PB(T)=1/4

PA(H)=2/4, PB(H)=2/4 P(A.B)=PA(H).PB(H)=1/4

=1=100%

PA(T) = 2/4 , PB(T) = 2/4 , P(A.B)= PA(T).PB(T) = 2/4 *2/4 = 4/16 =1/4

The joint events in digital communication are when:

- occur bit ”1” only
- occur bit “1” with noise
- occur bit ”0” only
- occur bit “0” with noise

P(B|A)

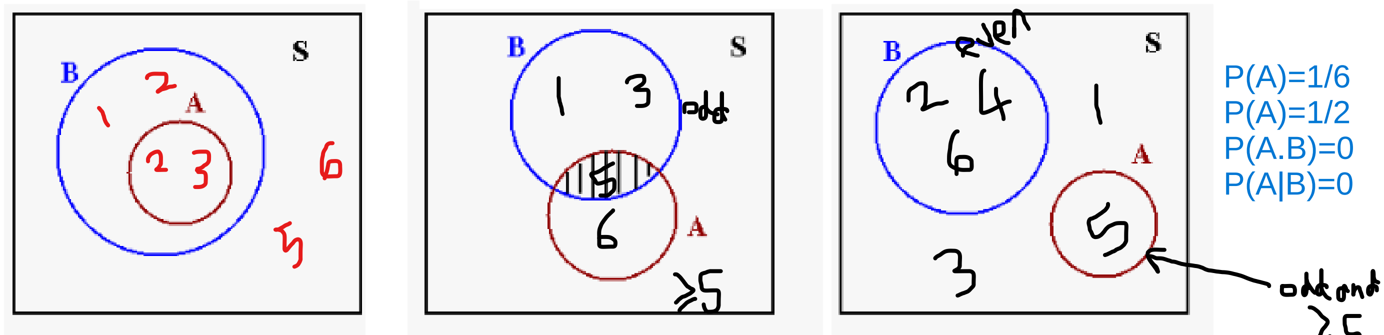
Conditional Events: The ***conditional probability*** of an event *B* is the probability that the event will occur given the knowledge that an event *A* has already occurred. This probability is written *P(B|A)*, notation for the *probability of B given A*

1)if its raining then it should be already cloudy.
2)if you get 70% in an exam, then you should be examined already.
3) etc..

S={1, 2, 3,4, 5,6} with A={2,3} and B={1,2,3,4}

P(A|B)=P(A)+P(B)= 2/6+4/6=6/6=1

S={1,2,3,4,5,6} with A={2,3,5} and B={1,2,3,4}
P(A|B)=P(A.B)/P(B)=(2/6)/(4/6)=2/4=1/2
P(B|A)=P(A.B)/P(A)=(2/6)/(3/6)=2/3



Possibility-1 : A subset of B
P(A|B) = 1

Possibility 3 : Sets A and B intersect
P(A|B) = P(A ∩ and B)/P(B)

Possibility 2 : A and B are independent
P(A|B) = 0

P(A)=1/6
P(A)=1/2
P(A.B)=0
P(A|B)=0

If the events independent   P(A/B) = P(A)   P(A|B)=P(A.B)/P(B)= P(A).P($\cancel{B}$)/P($\cancel{B}$)=P(A)

P(B/A) = P(B)   P(B|A)=P(B.A)/P(A)= P(B).P($\cancel{A}$)/P($\cancel{A}$)=P(B)

If the events dependent   P(A/B) = P(A.B)/P(B)

P(B/A) = P(A.B)/P(A)

The conditional events in digital communication are when sending bit "1" and may receive it "0", or sending bit "0" and may receive it "1" .

Examples:
If P(A)=.5, P(B)=.4, and P(A.B)=.2, then P(A|B)=.2/.4=.5 = P(A) and A and B are independent.
If P(A)=.6, P(B)=.4, and P(A.B)=.2, then P(A|B)=.2/.4=.5 which is not equal to P(A)=0.6, and A and B are not independent   (so they are dependent).

Random Variable: is the outcome from any random experiment or/and formula between the outcome for any random experiment

Probability Density Function (P.D.F): is the function probability distribution of the variables space or axis. $f_X(x)$

Commutative Probability Distribution Function (CDF):

$$CDF = \int_{-\infty}^{\infty} P.D.F\, dv = F_X(x)$$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = F_X(x)$$

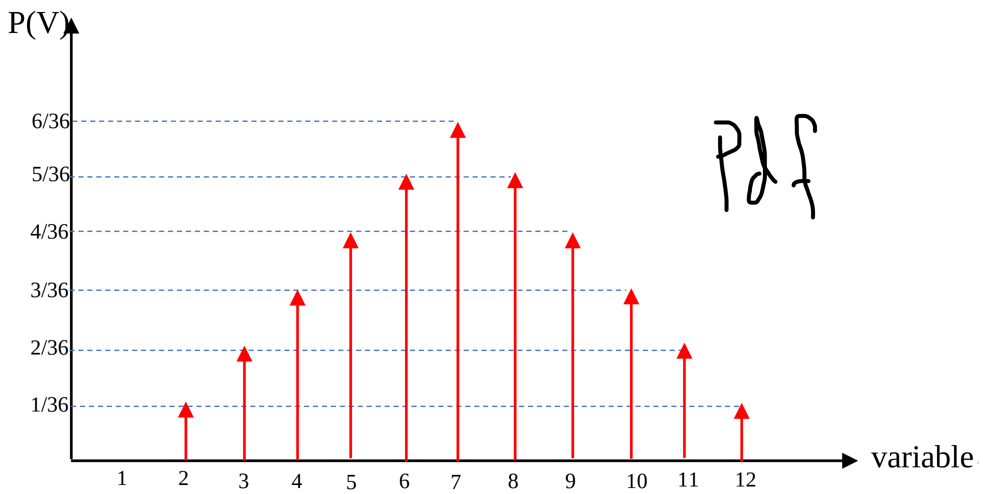
Example: find PDF for two dice.

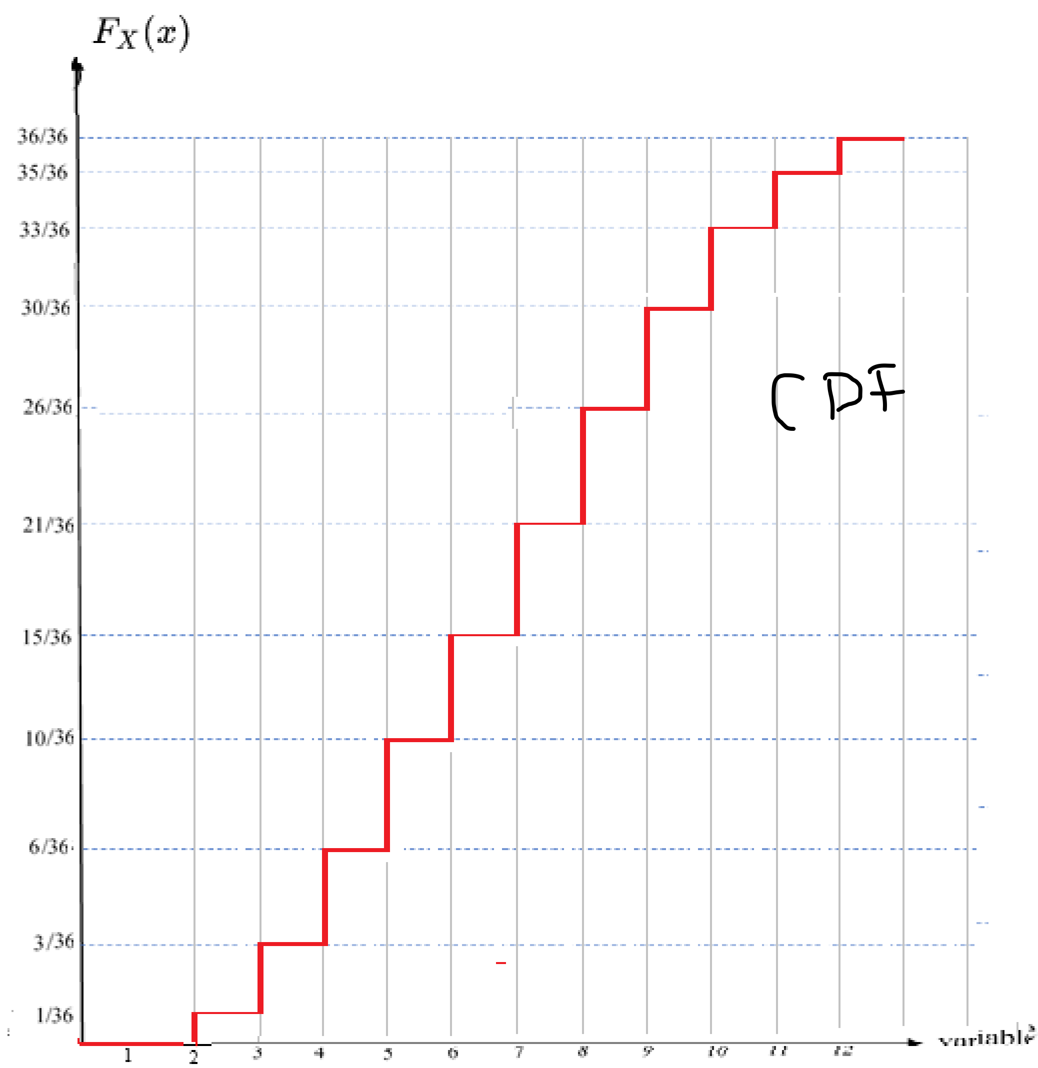
A	B	Random variable(xi)=A+B
1	1	2
1	2	3
1	3	4
1	4	5
1	5	6
1	6	7
2	1	3
2	2	4
2	3	5
2	4	6
2	5	7
2	6	8
3	1	4
3	2	5
3	3	6
3	4	7
3	5	8
3	6	9
4	1	5
4	2	6
4	3	7
4	4	8
4	5	9
4	6	10
5	1	6
5	2	7
5	3	8
5	4	9
5	5	10
5	6	11
6	1	7
6	2	8
6	3	9
6	4	10
6	5	11
6	6	12

Ni= the number of occurance

The probability continues for all other cases

Xi	Ni	P(xi)
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36


P(V)
6/36
5/36
4/36
3/36
2/36
1/36
1
2
3
4
5
6
7
8
9
10
11
12
variable
Pdf


$F_X(x)$
36/36
35/36
33/36
30/36
26/36
21/36
15/36
10/36
6/36
3/36
1/36
1
2
CDF

$$F_X(x) = \mathrm{P}(X \le x) = \sum_{x_i \le x} \mathrm{P}(X = x_i) = \sum_{x_i \le x} p(x_i).$$

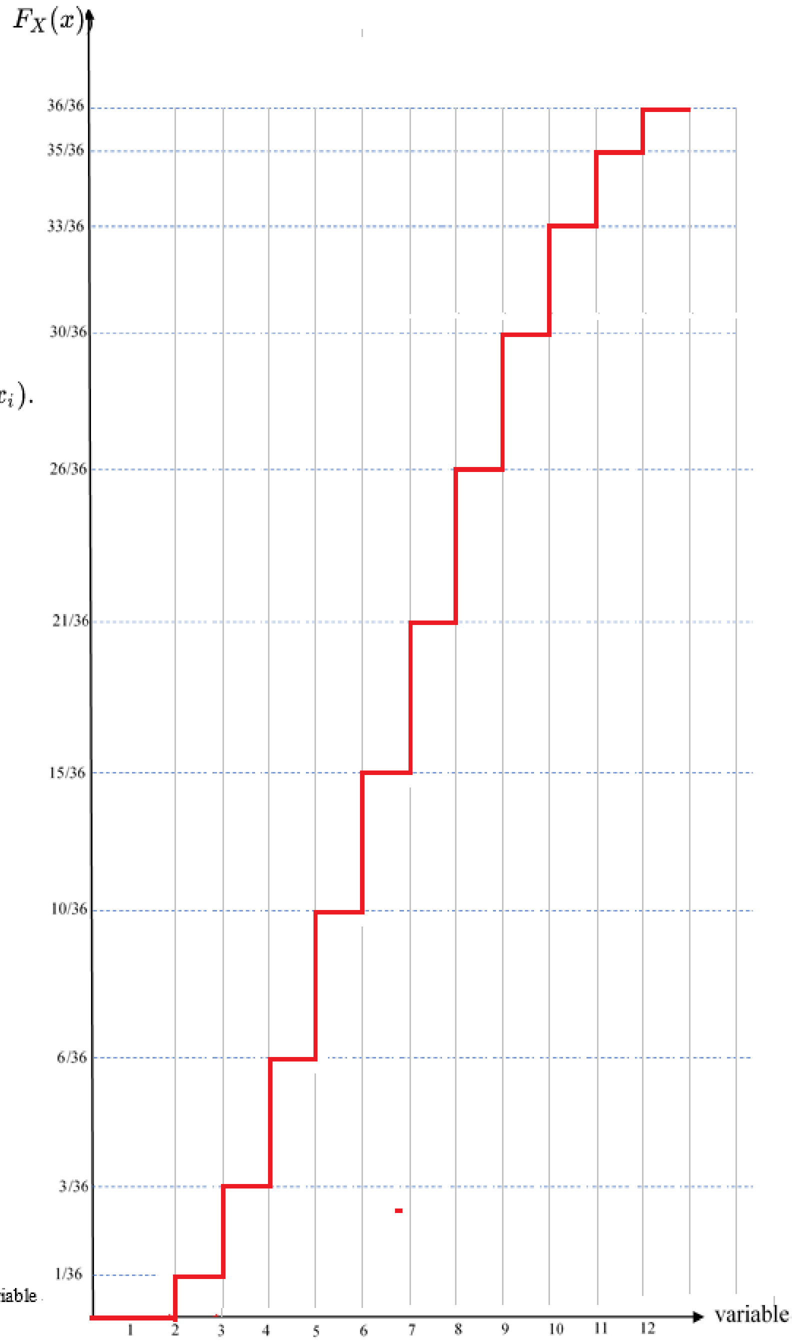
$$\lim_{x \to -\infty} F_X(x) = 0, \quad \lim_{x \to +\infty} F_X(x) = 1.$$

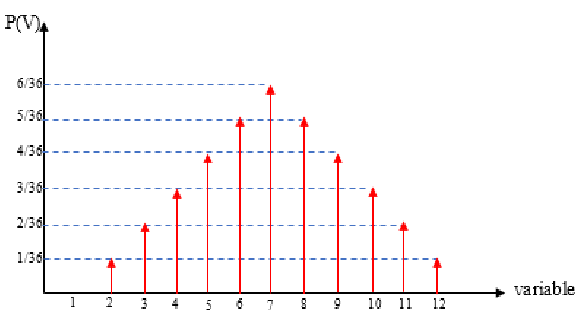
$$F_X(b) - F_X(a) = \mathrm{P}(a < X \le b) = \int_a^b \mathrm{P}_X(x)\, dx$$

$$\mathrm{P}(x) = \frac{dF(x)}{dx}$$

$$F_X(x) = \int_{-\infty}^{x} \mathrm{P}_X(t)\, dt.$$

$$\mathrm{P}(X = b) = F_X(b) - \lim_{x \to b^-} F_X(x).$$





Bayes rule:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(A|B) = \frac{P(A.B)}{P(B)} \text{ — ①} \qquad P(B|A) = \frac{P(B.A)}{P(A)} \text{ — ②}$$

$$P(A.B) = P(B.A) \text{ — ③}$$

$$P(A.B) = P(B|A)\,P(A) \leftarrow \text{from ②} \hookrightarrow \text{in ①}$$

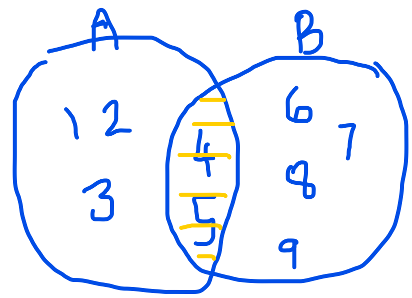
$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$

Ex:

$A = \{1,2,3,4,5\}$ $B = \{4,5,6,7,8,9\}$

Find P(A|B) using conditional probability and confirm your result using Bayesian rule?

① $P(A|B) = \dfrac{P(A \cdot B)}{P(B)}$



$S = 1,2,3,4,5,6,7,8,9\}$

$P(A \cdot B) = P(A \text{ and } B) = P(A \cap B) = \dfrac{2}{9}$

$P(A) = \dfrac{5}{9}$; $P(B) = \dfrac{6}{9}$

$P(A|B) = \dfrac{2/9}{6/9} = \dfrac{2}{6} = \dfrac{1}{3}$

② $P(A|B) = \dfrac{P(B|A)\,P(A)}{P(B)}$

$= \dfrac{2/5 \cdot 5/9}{6/9} = \dfrac{2}{6} = \dfrac{1}{3}$

$P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{P(A \cap B)}{P(A)}$

$= \dfrac{2/9}{5/9} = \dfrac{2}{5}$

Conclusions:

① $P(A \cap B) = P(A) \cdot P(B)$

② $P(A \cup B) = P(A) + P(B) - P(A \cdot B)$ $\quad \cap$

③ $P(A|B) = \dfrac{P(A \cdot B)}{P(B)} = \dfrac{P(B|A)\,P(A)}{P(B)}$

④ If P(A|B)=P(A), then the two events A and B are independent

DIGITAL COMMUNICATIONS

Lecture (18-1): Probability of Error for Base Band Signals

Statistical Variable:

1) Mean Value (M) or Average Value:

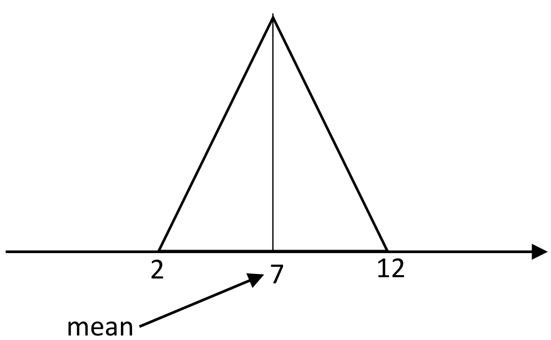
$M = \int_{-\infty}^{\infty} x . P(x)\, dx$ for continuous

$M = \sum_{i=0}^{N} x_i \, P(x_i)$ for discrete

Example: find mean value for the two dice example

$M = \sum_{i=0}^{N} x_i \, P(x_i)$

M= 2*1/36 + 3*2/36 + 4*3/36 + 5*4/36 + 6*5/36 + 7*6/36 + 8*5/36 + 9*4/36 + 10*3/36 + 11*2/36 + 12*1/36 = 7



2) Mean Square Value (M^2):

$M^2 = \int_{-\infty}^{\infty} x^2 . P(x)\, dx$ for continuous

$M^2 = \sum_{i=0}^{N} {x_i}^2 \, P(x_i)$ for discrete

Example: find mean value for the two dice example

$$M^2 = \sum_{i=0}^{N} x_i^{\,2}\, P(x_i)$$

M^2= 4*1/36 + 9*2/36 + 16*3/36 + 25*4/36 + 36*5/36 + 49*6/36 + 64*5/36 + 81*4/36 + 100*3/36 + 121*2/36 + 144*1/36 — 49

3) Mean Square Root of Variance or Standard of Deviation ($\boldsymbol{\sigma}$) :

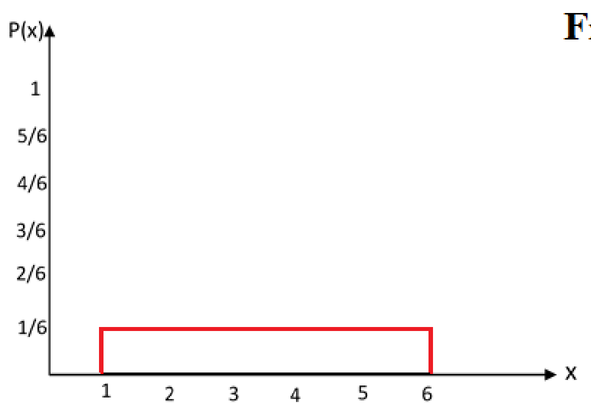
$\sigma = \sqrt{\sigma^2}$ where σ^2 = variance

$\sigma^2 = \int_{-\infty}^{\infty} (x - M)^2 . P(x)dx$ for continuous

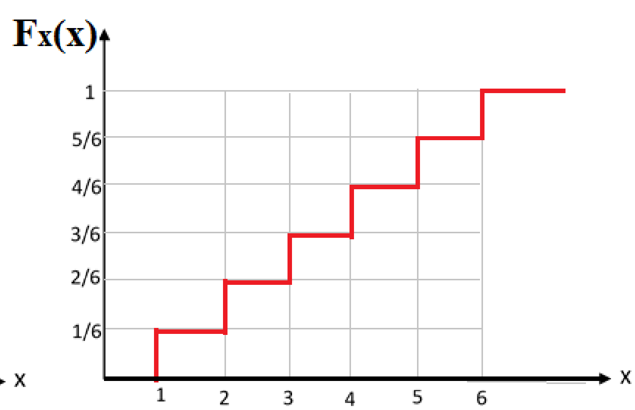
$\sigma^2 = \sum_{i=0}^{N} (x_i - M)^2 . P(x_i)$ for discrete

Some of Probability Distribution Function:

1) **Uniform Distribution:** uniform PDF for mutually exclusive variables and likely equally events, like sending bit “1” or bit “0”. The following figure is uniform distribution for throwing one dice.



probability distribution function (P.D.F)



Commutative Distribution Function (CDF)

2) **Binomial Distribution:** if we have two events only and mutually exclusive, also the condition must be realized:

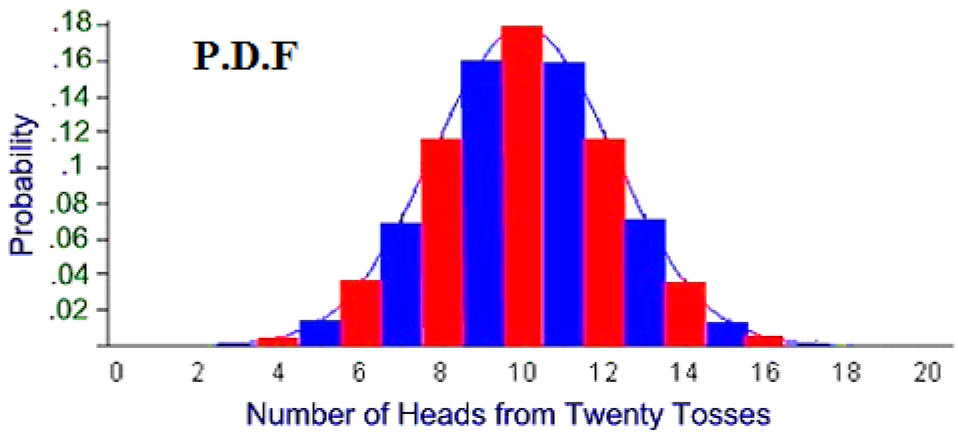
P(A) + P(B) = 1

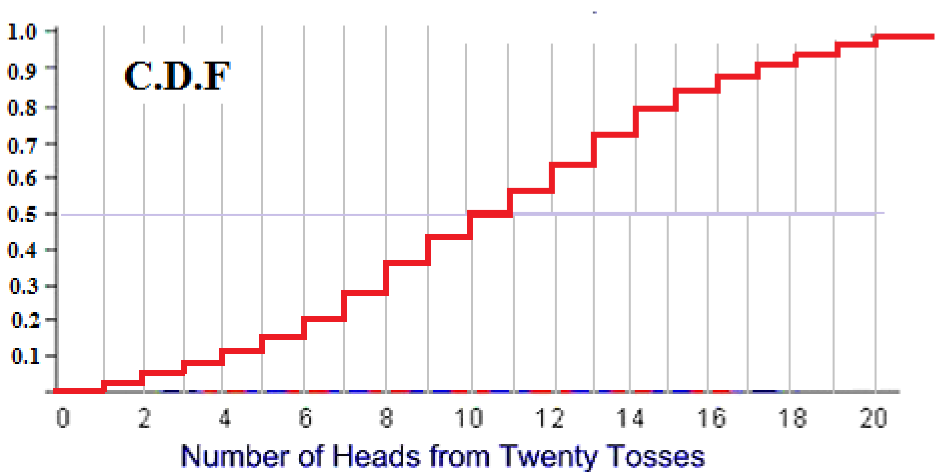
P(A) = 1- P(B)

So, if we have two events (like throw coin) and made an experiment repeated with N times (like N= 20), the probability of occurring Head (10 time) in this experiment.

$P_i = (i^n) e^i \, q^{n-i}$ where q=1-P(A)

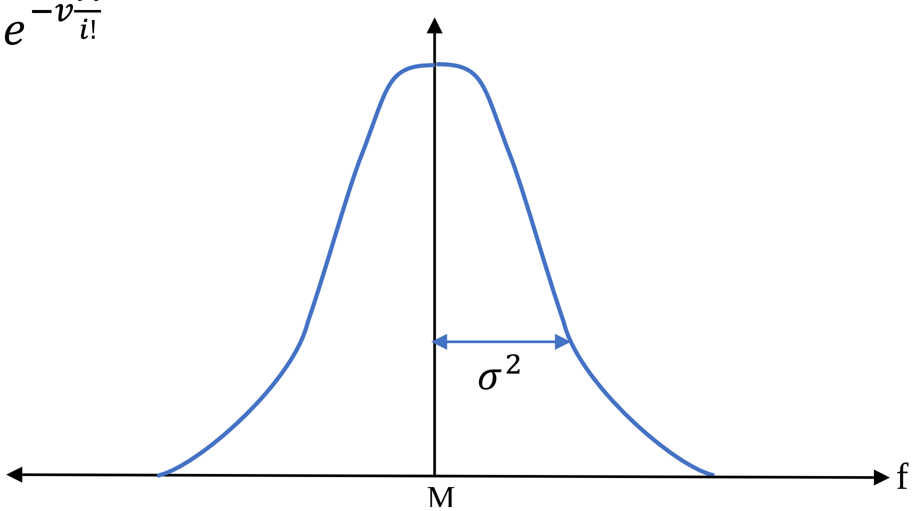
$$(i^n) = \frac{n!}{i!(n-1)!}$$

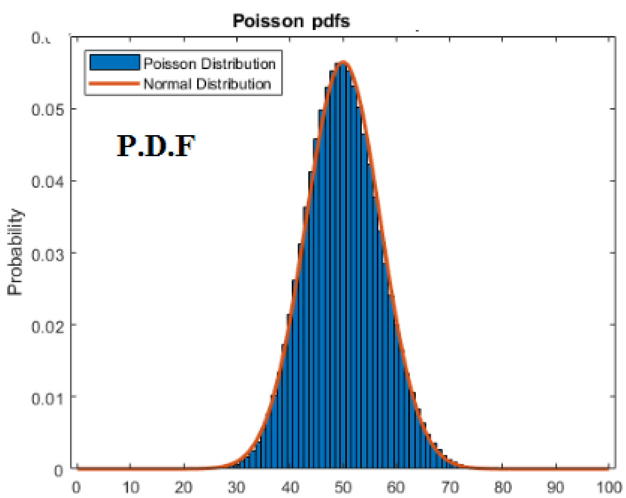


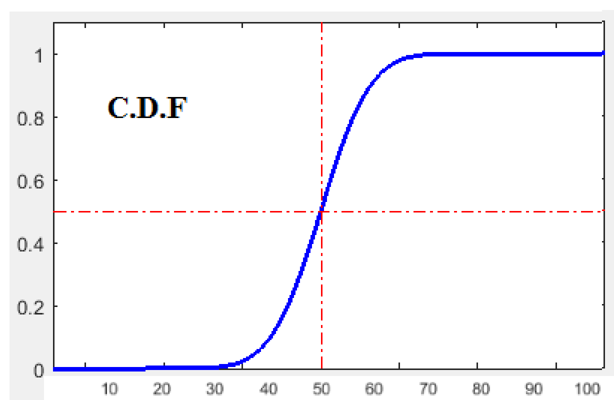


3) **Poisson Distribution:** if a certain type represents the count of the number of occurrences of same event during intervals of time.

$$P_i = e^{-v\frac{vi}{i!}}$$

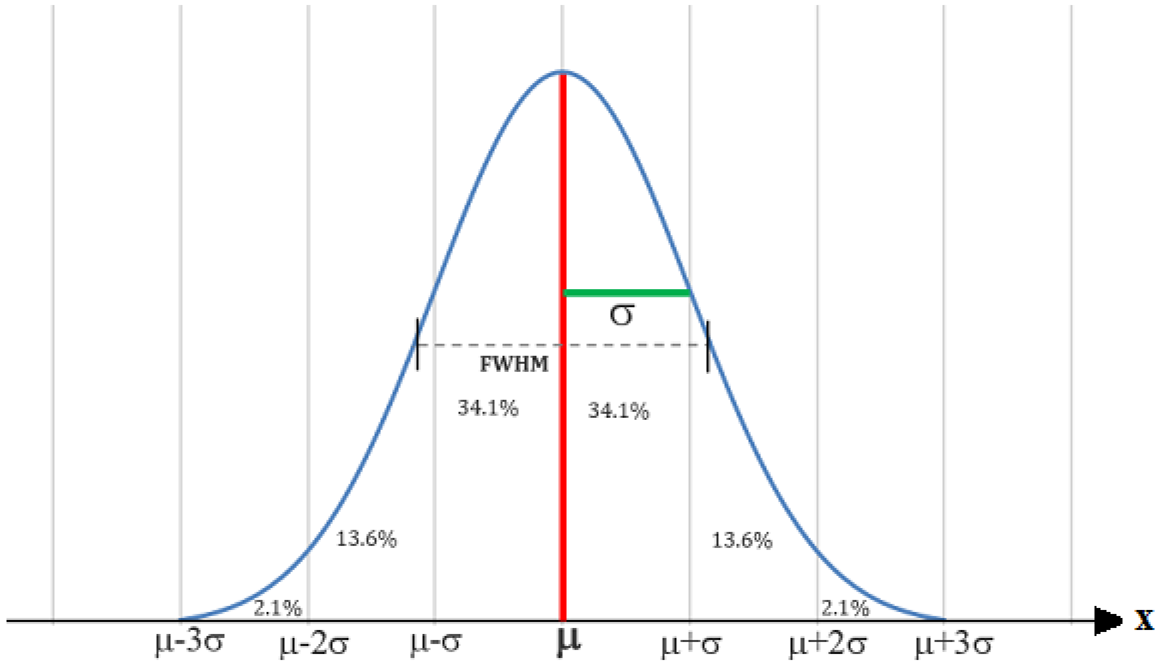


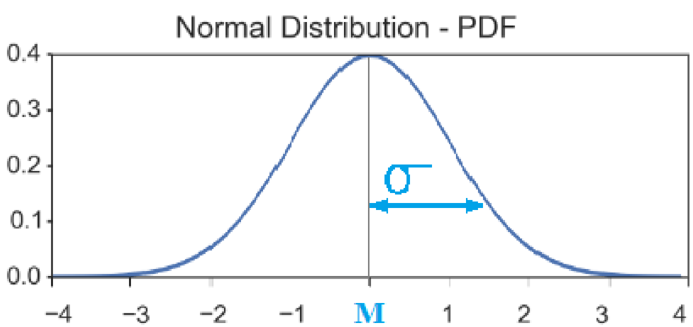


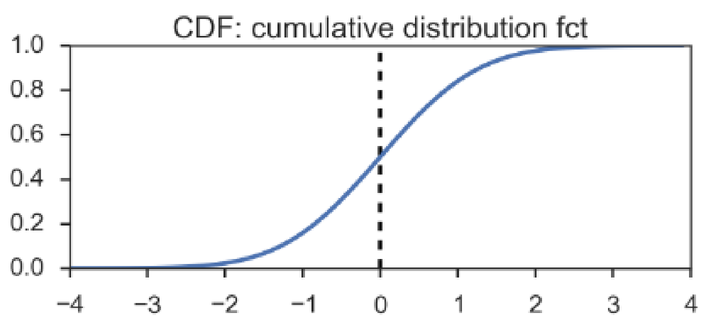


4) **Gaussian Distribution (normally distribution):** it is a probability distribution that is symmetric about the mean, showing that data near the mean are more frequent in occurrence than data far from the mean.

$$\boldsymbol{P(x) = \frac{1}{\sqrt{2\pi\sigma}}\, e^{\frac{-(x-M)^2}{2\sigma^2}}}$$







Gaussian distribution properties:-

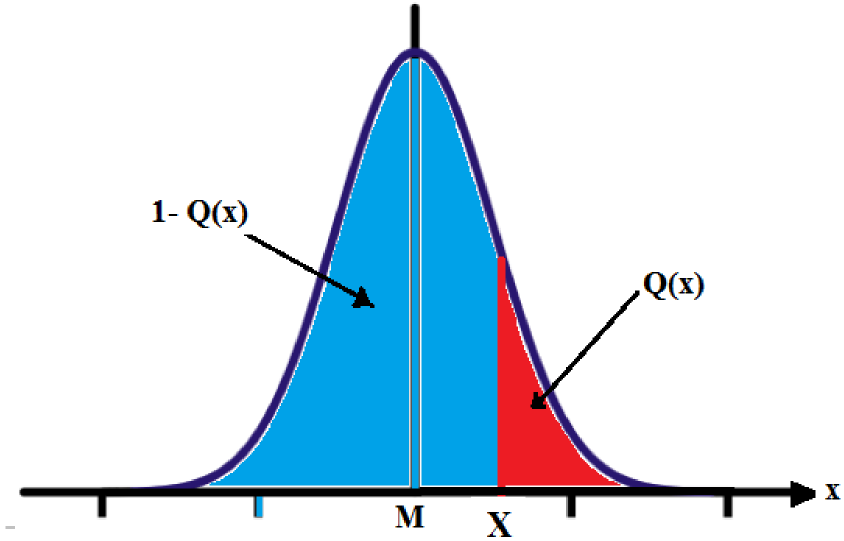
1 – The area under the Gaussian function = 1

$$Fx(x) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx = 1$$

2 – The two sides around the main value are exactly identical.

$$\int_{-\infty}^{M} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx = \int_{M}^{\infty} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx = \ 1/2$$

3 - The Commutative probability function at x value =



$$\boldsymbol{Fx(X) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx}$$

$$\boldsymbol{Fx(X) = 1 - \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx}$$

If M = 0

$$\boldsymbol{\int_{x}^{\infty} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx \; = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-x^2}{2\sigma^2}} dx \; = Q(x)\; (error\ function)}$$

$$\boldsymbol{\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-(x-M)^2}{2\sigma^2}} dx \; = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma}\, e^{\frac{-x^2}{2\sigma^2}} dx \; = (1 - Q(x))}$$ erfc

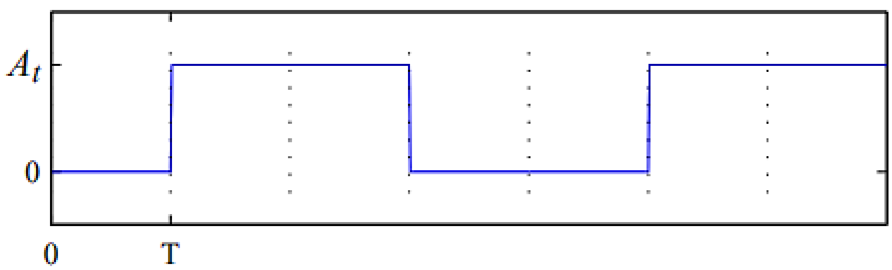
$$\boldsymbol{(error\ function\ complementary)}$$

DIGITAL COMMUNICATIONS

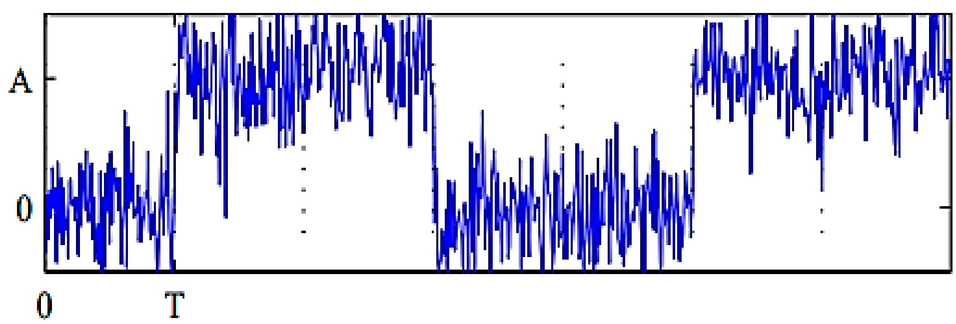
Lecture (19-1):Probability of Error for Base Band Signals

In a binary PCM system, binary digits may be represented by two pulse levels. If these levels are chosen to be 0 and A, the signal is termed unipolar binary signal. If the level switches between −A/2 and A/2 it is called bipolar binary signal.

Suppose we are transmitting digital information, and decide to do this using two-level pulses each with period T:



The binary digit 0 is represented by a signal of level 0 for the duration T of the transmission, and the digit 1 is represented by the signal level At. In the event of a noisy Gaussian channel (with high bandwidth) the signal at the receiver may look as follows:



A very simple detector could be obtained by sampling the received signal at some time instant Ts in the range (0, T), and using the value to make a decision. The value obtained would be one of the following:

y(Ts) = n(Ts) signal absent

y(Ts) = A + n(Ts) signal present.

A reasonable rule for the decision of whether a 0 or a 1 was received is the following:

y(T) ≤ Threshold signal absent — 0 received

y(T) >Threshold signal present — 1 received

The threshold which we would usually choose somewhere between 0 and A. Suppose now that n(Ts) has a Gaussian distribution with a mean of zero and a variance of σ^2. Under the assumption that a zero was received the probability is:

$$P_0 = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}}$$

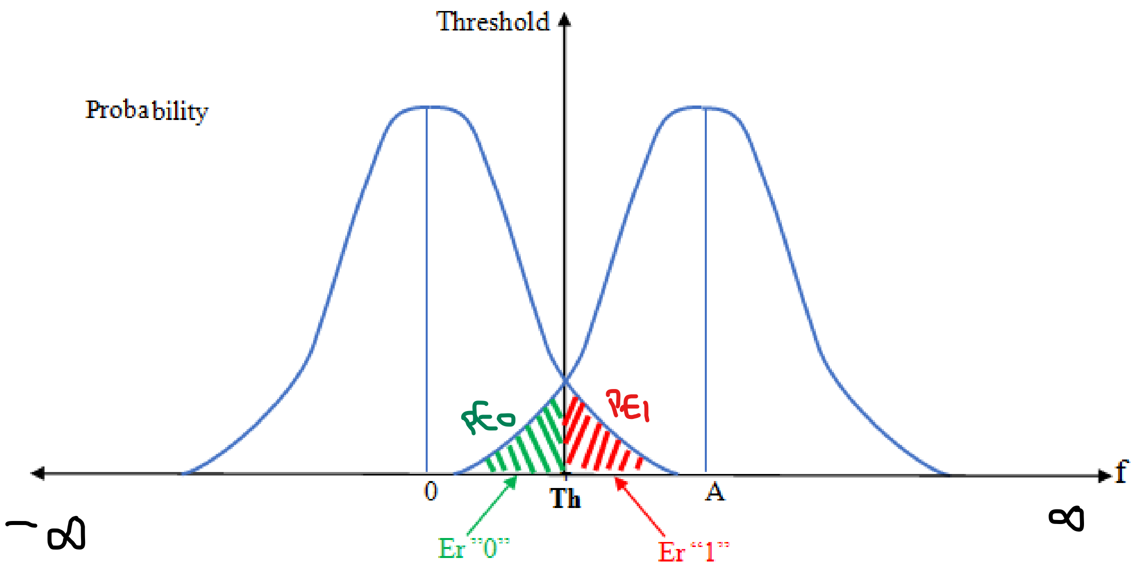
M = 0 for '0'

Similarly, when a signal is present, the probability is:

M = A for "1"

$$P_1 = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-M)^2}{2\sigma^2}}$$

These probabilities are shown below.



Using the decision rule described, it is evident that we sometimes decide that a signal is present even when it is in fact absent. The probability of such a false alarm occurring (mistaking a zero for a one) is:

Receiving 1 instead of 0

$$PE_0 = \int_{-\infty}^{Th} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-M)^2}{2\sigma^2}} \, dx = \int_{-\infty}^{Th} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-A)^2}{2\sigma^2}} \, dx$$

not easy to solve so we use tables instead.

Similarly, the probability of a missed detection (mistaking a one for a zero) is:

Receiving 0 instead of 1

$$PE_1 = \int_{Th}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-(x-M)^2}{2\sigma^2}} \, dx = \int_{Th}^{\infty} \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{-x^2}{2\sigma^2}} \, dx$$

Letting PT_0 and PT_1 be the source digit probabilities of zeros and ones, respectively, we can define the overall probability of error to be:

The probability of sending "0" = $PE_1 . PT_0$

The probability of sending "1" = $PE_0 \cdot PT_1$

$PE = PE_1 \cdot PT_0 + PE_0 \cdot PT_1$

- PT_0 & PT_1 are mutually exclusive and likely equally
- $PT_0 = PT_1 = ½$

$PE = ½\, PE_1 + ½\, PE_0$

One from the characteristic of the Gaussian distribution that the right side analog the left side.

- $Er_0 = Er_1$
- $PE = 2(½\, PE_1)$ or $= 2(½\, PE_0)$
- $PE = PE_1 = PE_0 = \int_{Th}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}}\, dx$ by solving it numerically

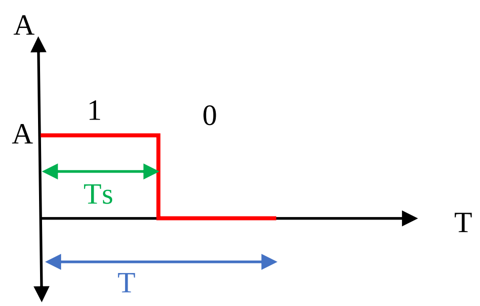
If Th = A/2

$PE = PE_1 = PE_0 = \int_{A/2}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-x^2}{2\sigma^2}}\, dx$

$$\boldsymbol{PE = Q\left(\frac{A}{2\sigma}\right)}$$

$$\boldsymbol{Q\left(\frac{A}{2\sigma}\right) = 1 - erf\left(\frac{A}{2\sigma}\right)}$$

Probability of Error for Unipolar NRZ:



$$P_{average} = \frac{A^2 + 0}{2} = \frac{A^2}{2} = S\ (signal\ power)$$

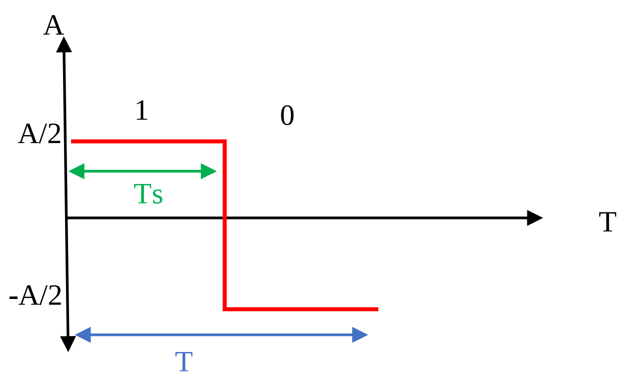
$$A = \sqrt{2 * S}$$

$\sigma^2 = \mathrm{n}_o{}^2(\mathrm{t}) = \mathrm{N}$ Where $\mathrm{n}_o{}^2(\mathrm{t})$ is the average noise power

$$\therefore\ \sigma = \sqrt{N}$$

$$\boldsymbol{PE = Q\,\frac{\sqrt{2S}}{2\sqrt{N}} = Q\sqrt{\frac{S}{2N}}}$$

<u>Probability of Error for Bipolar NRZ:</u>

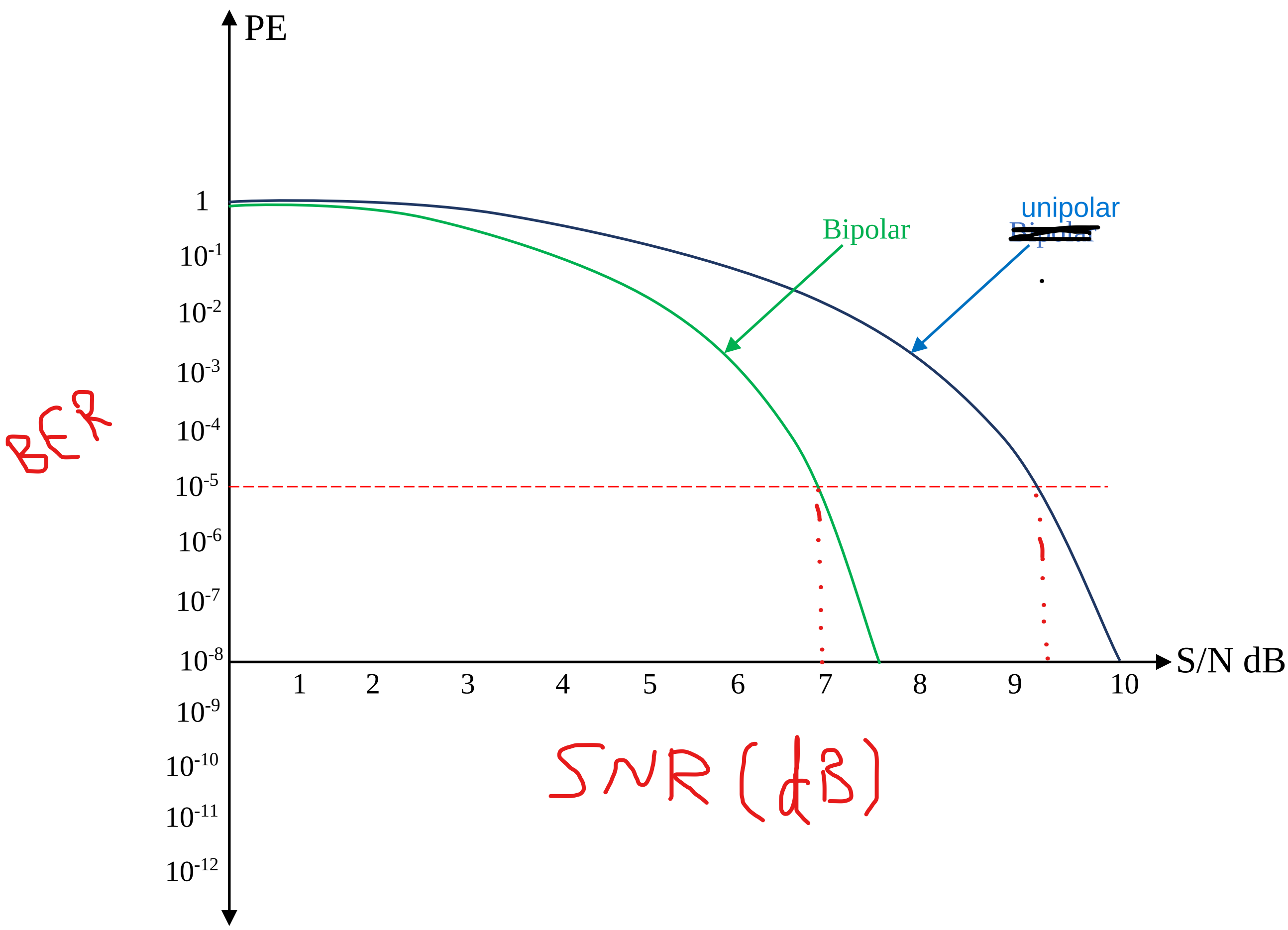


$$P_{average} = \frac{A^2}{4} = S\ (signal\ power)$$

$$A = 2\sqrt{S}$$

$$PE = Q\left(\frac{A}{2\sigma}\right) = Q\,\frac{2\sqrt{S}}{2\sqrt{N}} = Q\sqrt{\frac{S}{N}}$$

$$\therefore\ \boldsymbol{PE = Q\sqrt{\frac{S}{N}}}$$



PE
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
BER
Bipolar
unipolar
~~Bipolar~~
S/N dB
1
2
3
4
5
6
7
8
9
10
SNR (dB)

DIGITAL COMMUNICATIONS

H.W.

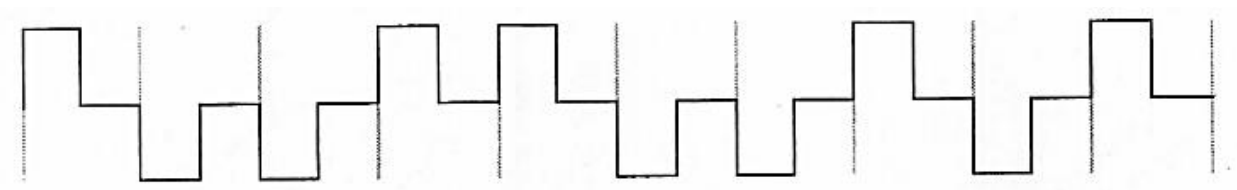
Q1/ A) Find the sampling frequency for the following signals:

1) $S1 = 20\cos(1500\pi t) * \cos(4000\pi t)$

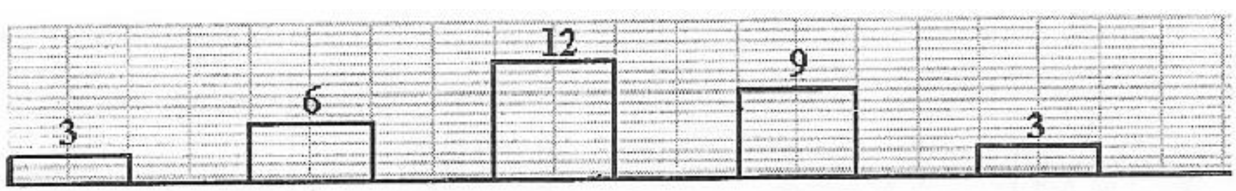
2) $S2 = \left[\sin\left(\frac{500\pi t}{\pi t}\right)\right]^2$

3) $S3 = \left[\frac{sin4000\pi t}{\pi t}\right] * 5\cos(1200\pi t)$

B) The following waveform is the RB (return to bios) representation of the serial digital data, find the digital data code and draw the NRZ (non-return to zero) and RZ (return to zero) waveforms.



C) The waveform below is the PAM for the analog signal, draw the analog signal, PWM and the PPM for this signal.



Q2/ The following data stream (10011011 11001110 11110000 10001000 11100000 11011010 11111111 01110111 10011001 11100111 01011010) will be transmitted with signaling rate equal to 80 k bit/sec.

1- Find the data transmitted using L- parity, V- parity and L&V parity.

2- Find the Hamming code of the last byte of the data only in which two error can be detected and one error can be corrected, calculate the transmitted bandwidth? how can reduce the transmitted bandwidth? Explain and calculate the reduced transmitted bandwidth?

3- By using the following polynomial calculate the data transmitted of the (first byte of the data only) (p= x5 +x3 +x+l).

Q3/ A- Design the TDM system of the following 43 signals and calculate the transmitted bandwidth.

1- S1 band limited to 40kHz
2- S2 band limited to 20kHz
3- 18 signal (S3 to S20) each band limited to 5kHz
4- 14 signal (S21 to S34) each band limited to 312.5 Hz
5- 9 signal (S35 to S43) each band limited to 625 Hz

B- If these analog signals converted to digital signals with signal to Noise ratio not less than 61.96 dB find the signaling rate in each join of the designed TDM system.

Q4/ If figure (1) is the spectrum of the periodic signal answer the following for one period of this signal in time domain: -

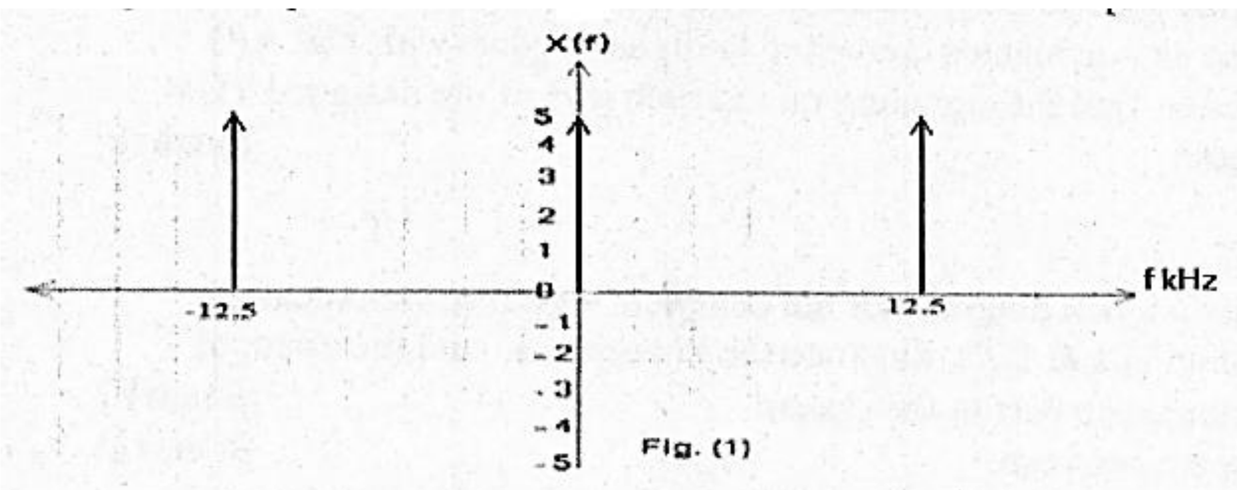


Fig. (1)

1- Find the rated sampling frequency and draw the sampled signal.
2- Draw the sampled signal if it sampled at four time of its sampling rate.
3- If this signal modulated by PAM within bandwidth needed not greater than (0.5 Fs), draw the (flat top) output signal.
4- Draw the P WM output for this signal.
5- Draw the PPM output for this signal.
6- If the PCM are used with S/N ratio not less than (25.84dB), calculate the required bandwidth and draw the output signal for the PCM system with (RZ) line coding.
7- If the L (even parity) error detection system are used, calculate the transmitted data and the required bandwidth.
8- If the V (even parity) error detection system are used, calculate the transmitted data and the required bandwidth.
9- If the L&V (even parity) error detection system are used, calculate the transmitted data and the required bandwidth.
10- If CRC error detection system are used which detect 8 error with 100% percent, state the polynomial and calculate the required bandwidth.

Q4/ An analog signal ($20 \cos 2\pi 2000t$) has been converted to digital signal using 8 bit A/D converter.

1- Calculate the step size if uniform quantization.
2- Calculate the minimum step size and maximum step size if low compression mode is used with (μ=255).
3- Calculate the minimum step size and maximum step size if A low compression mode is used with (A=87.6).

Q5/ The following data (101000110101110) is arrived in the receiver side, if you know that the data was coded using CRC with the polynomial ($X^5+X^4+X^2+1$).

A- Check if the received data is without error or not?

B- If the required bandwidth before adding the CRC error detection bits were 150kHz, what is the required bandwidth after adding the CRC code, find also the maximum frequency of the original analog signal?

C- Find the transmitted data if the suitable Hamming code will be used. Find the required transmitted bandwidth.

Q6/ There is a need to transmit the following signal through one channel

S1 Analog signal band limited to 24kHz

S2 Analog signal band limited to 12kHz

(S3 to S20) Analog signals each band limited to 3kHz

(S21 to S36) Analog signals each band limited to 375Hz

A- Design a T DM system to transmit these signals and calculate the transmitted bandwidth channel?

B- If the above signal converted to the digital signal (before multiplexed) with S/N not less than 25.84 dB, calculate the transmitted bandwidth and the signaling rate of each node for the TDM system you design it in (A)

Digital Carrier Modulation

1 – Introduction to Modulation process

2- Amplitude Shift Keying (ASK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

3 – Matched Filter

4 – M-ary ASK

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

5- Frequency Shift Keying (FSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

6- M-ary FSK

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

7 – Phase Shift Keying (PSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

8 – Carrier Recovery Circuits

9 Deferential Phase Shift Keying (DPSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

10– M-ary PSK (QPSK, 8-PSK, OQPSK, MSK, GMSK)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

12 – Amplitude-Phase Shift Keying (APK) (16_QAM)

Equations and waveform in time domain, Generation and Detection Circuits, Constellation Diagram and Power Spectral Density, Required Channel Bandwidth and Probability of error

13 – Comparison and applications for the digital carrier modulation

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (1 - 2) (INTRODUCTION)

A) Why need Modulation
B) Modulation process consideration.
C) The specification of Carrier Signal
D) Modulation types for Base Band Signal

A)- Why need Modulation:-

Because of the base band signal have low frequency so they are suitable for transmission over a pair of wire or coaxial cable only, base band signal cannot transmitted over the radio link because this would required very large antennas size, hence modulation techniques are used to rise the frequency for baseband signal into the high frequency for the carrier signal. Also if there are a need to send more than one signal into one channel in the same time, the modulation technique must be used to convert each information signal to specific frequency band that differ from the frequency band for the other signals and multiplex all the signal using Frequency Division Multiplexing (FDM).

Because of the carrier signal are clearly sinusoidal signal its more suitable for transmitted over the almost communication channels in reverse of the baseband signal which have sharp edge that contained high frequency component which cause some errors.

From the studies of digital modulation technique we can discover that by using some type of modulation we can send data with signaling rate greater than the channel bandwidth. From short illustration above we can conclude why we use the modulation by

1 – To reduce the antenna size for radio transmissions.

2 – To multiplex many information signal (FDM) and transmit them into one channel.

3 – To avoid the baseband signal problem in the channel.

4 – To increase the signaling rate that can be transmitted through a limited channel.

*B)- **Modulation process Consideration**:-*

The word modulation refer to consideration of multiplication between two signals one of them is the information signal and the other is the carrier (sinusoidal) signal, in this process the carrier signal take the job of carrying the information signal in different forms and transmit it to the receiver side through the channel. In fact any multiplication process between any two signals can be considered as modulation process ***(why)***

C) - The specification of Carrier Signal

From (B) we can deduce that the carrier signal is a sinusoidal signal and can be write the carrier signal formula as

$S(t) = A \, Cos \, (2\pi ft + \emptyset)$

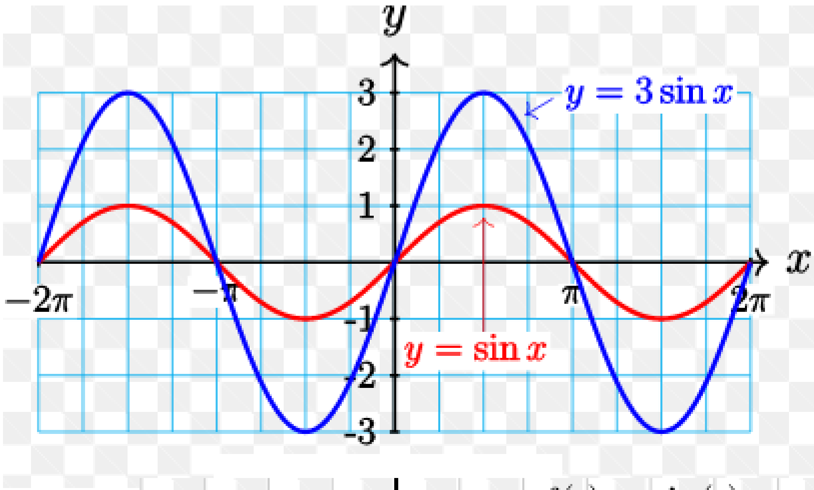
The instantaneous value of the carrier signal can be changed by the following three variables

A :- The amplitude of the carrier signal

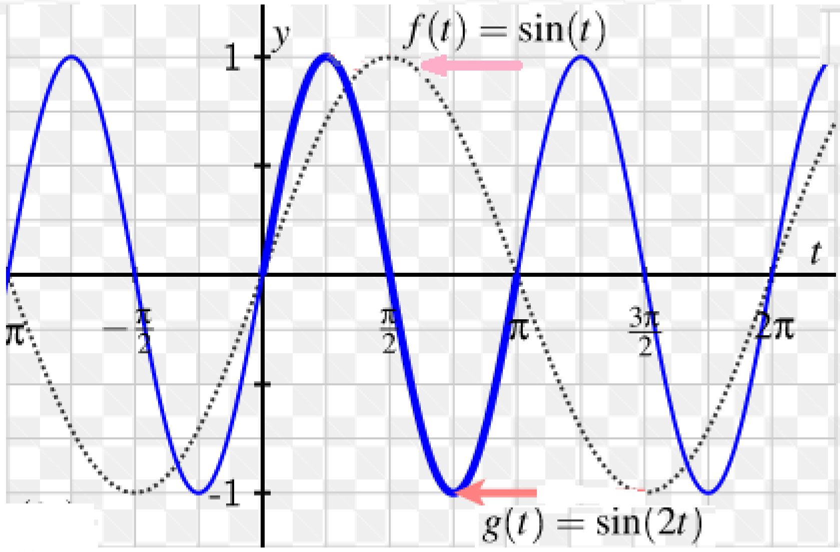
F :- The frequency of the carrier signal

Φ :- The phase of the carrier signal

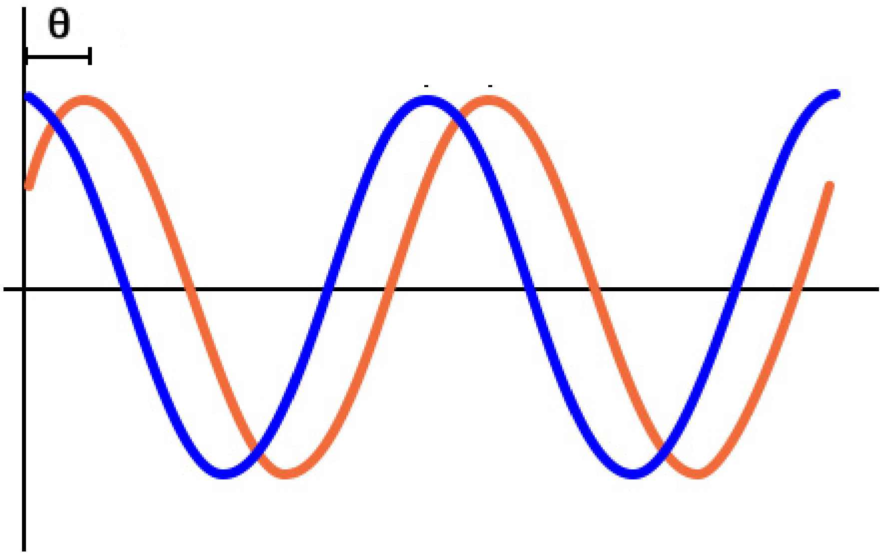
so the carrier signal can carry the information signal into three forms Amplitude , Frequency, and Phase.



y
$y = 3\sin x$
3
2
1
x
-2π
$-\pi$
π
2π
-1
-2
-3
$y = \sin x$



y
$f(t) = \sin(t)$
1
t
π
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
-1
$g(t) = \sin(2t)$



θ

D)- Modulation types for Base Band Signal

According to the specifications of the carrier signal and the specifications of the base band signal (digital signal) three main types of modulations can be used in digital communications systems :-

1 – Amplitude Shift Keying **ASK**

2 – Frequency Shift keying FSK

3 – Phase Shift Keying **PSK**

Note that other types of modulations can be created from the three main types of modulations ASK,FSK and PSK.

QPSK, OQPSK, CPM
Probability Of Error for AWGN and Flat Fading Channels [4]

16:332:546 Wireless Communication Technologies Spring 2005*
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Abstract

This article discusses QPSK, OQPSK, $\pi/4$ DQPSK and the trade offs involved. Later we discuss various CPM schemes and their relative bandwidth efficiencies. Probability of Bit error is discussed for various schemes assuming an AWGN channel. A model is devised, ignoring any changes in phase, to determine the error probabilities for flat fading channels. The article ends with a brief discussion of non-coherent detection.

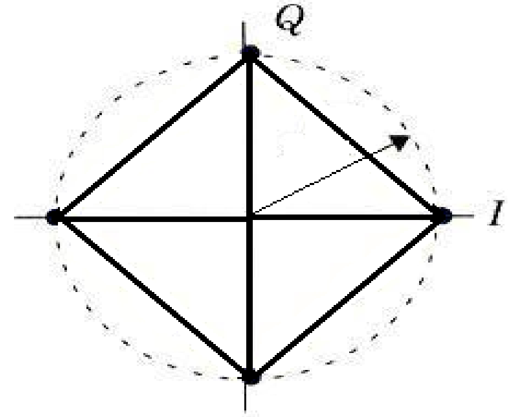


Figure 1: The figure shows a QPSK constellation. The dark black lines show all possible phase changes.

1 QPSK and Offset QPSK Signaling

QPSK or Quadrature Phase Shift Keying, involves the splitting of a data stream $m_k(t) = m_0, m_1, m_2, \ldots$, into an in-phase stream $m_I(t) = m_0, m_2, m_4, \ldots$ and a quadrature stream $m_Q(t) = m_1, m_3, m_5, \ldots$. Both the streams have half the bit rate of the data stream $m_k(t)$, and modulate the cosine and sine functions of a carrier wave simultaneously. As a result, phase changes across intervals of $2T_b$, where T_b is the time interval of a single bit (the $m_k(t)$s). The phase transitions can be as large as $\pm\pi$ as shown in Figure 1.

Sudden phase reversals of $\pm\pi$ can throw the amplifiers into saturation. As shown in Figure 2 [1], the phase reversals of $\pm\pi$ cause the envelope to go to zero momentarily. This may make us susceptible to *non-linearities* in amplifier circuitry. The above may be prevented using linear amplifiers but they are more *expensive and power consuming*. A solution to the above mentioned problem is the use of OQPSK.

*Taught by Dr. Narayan Mandayam, Rutgers University.

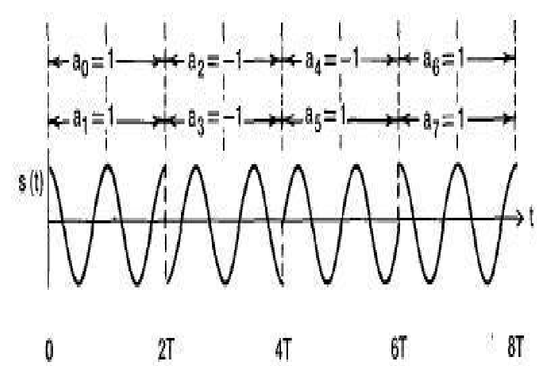


Figure 2: The figure shows a QPSK waveform. As is seen across the dotted line corresponding to a phase shift of π, the envelope reduces to zero temporarily.

OQPSK modulation is such that phase transitions about the origin are avoided. The scheme is used in IS-95 handsets. In OQPSK the pulse streams $m_I(t) = m_0, m_2, m_4, \ldots$ and $m_Q(t) = m_1, m_3, m_5, \ldots$ are *offset in alignment*, in other words are staggered, by one bit period (half a symbol period). Figure 3 [2], shows the staggering of the data streams in time. Figure 4 [1], shows the OQPSK waveform undergoing a phase shift of $\pm\pi/2$. The result of *limiting* the phase shifts to $\pm\pi/2$ is that the envelope will not go to zero as it does with QPSK.

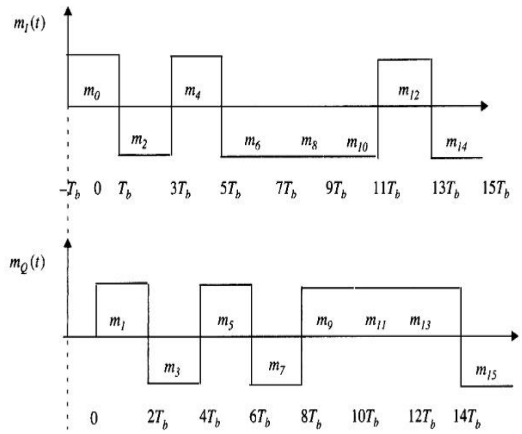


Figure 3: The figure shows the staggering of the in phase and quadrature modulated data streams in OQPSK. The staggering restricts the phase changes to ± 90 as shown in Figure 4.

In OQPSK, the phase transitions take place every T_b seconds. In QPSK the transitions take place every $2T_b$ seconds.

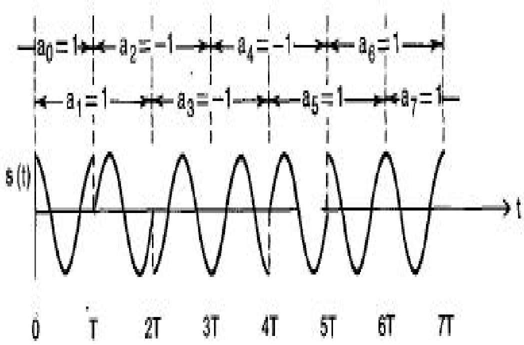


Figure 4: The figure shows a QPSK waveform. As is seen across the dotted lines the phase changes are of $\pm\pi/2$.

The OQPSK constellation is as shown in Figure 5.

2 $\pi/4$ DQPSK Signaling

The signaling is a compromise between QPSK and OQPSK in that the maximum transitions are allowed to be $\pm 3\pi/4$. The scheme is used in North American TDMA (IS-136). Figure 6 shows an example set of phase values that we may choose.

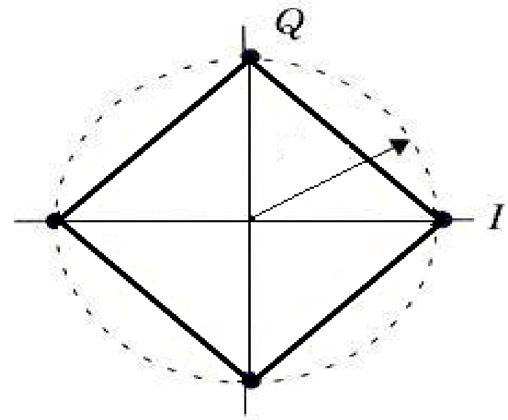


Figure 5: The figure shows a OQPSK constellation. The dark black lines show all possible phase changes. The signal space is the same as in the case of QPSK, though phase changes are restricted to ± 90.

Information bits m_{Ik}, m_{Qk}	Phase shift ϕ_k
1 1	$\pi/4$
0 1	$3\pi/4$
0 0	$-3\pi/4$
1 0	$-\pi/4$

Figure 6: The figure shows an allowable table of phase transitions in $\pi/4$ DQPSK. The maximum phase translation allowed is ± 135.

Figure 7 [2] gives two possible constellations and their all possible phase transitions.

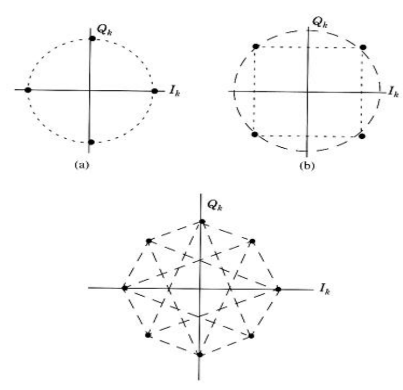


Figure 7: $\pi/4$ DQPSK constellations and all possible phase phase translations.

3 CPM

Constant envelope and very good spectral characteristics make CPM, Continuous Phase Modulation, a preferred choice in wireless communications. The complex baseband equivalent is given by

$$v(t) = A\ exp\Big\{ j2\pi k_f \int_{-\infty}^{t} \sum_n x_n h_f(\tau - nT)\ d\tau \Big\} \\ = A\ exp\{j\phi(t)\} \tag{1}$$

where A is the amplitude, k_f is the peak frequency deviation, $h_f(t)$ is the frequency shaping pulse and T is the symbol duration. The symbol source sequence is $\{x_n\} = \{\pm 1, \pm 3, \pm 5, \ldots, \pm(M-1)\}$, where M is the alphabet size.

$$\begin{aligned} \phi(t) = \ & 2\pi k_f \int_{-\infty}^{kT} \sum_{n=-\infty}^{k-1} x_n h_f(\tau - nT)\ d\tau \\ & + 2\pi k_f \int_{kT}^{t} h_f(\tau - kT)\ d\tau \\ & \text{where, } kT \le t \le (k+1)T \end{aligned} \tag{2}$$

If $h_f(t) = 0$ for $t > T$, the CPM signal is called *full response* CPM. If $h_f(t) \neq 0$ for $t > T$, the CPM signal is called *partial response* CPM. Using the *standard form* of representation of a baseband signal,

$$v(t) = A \sum_k b(t - kT, \underline{x}_k) \tag{3}$$

$$b(t, \underline{x}_k) = u_T(t) \exp\Big\{ j\Big[\beta(\tau) \sum_{-\infty}^{k-1} x_n + x_k \beta(t)\Big]\Big\} \tag{4}$$

In the above equation, $\beta(\tau)\sum_{-\infty}^{k-1} x_n$ is the accumulated excess phase (memory) and $x_k\beta(t)$ is the excess phase for the current symbol.

$$\beta(t) = \begin{cases} 0 & t < 0 \\ 2\pi k_f \int_0^t h_f(\tau)\ d\tau & 0 \le t \le T \\ \beta(T) & t \ge T \end{cases} \tag{5}$$

Two terms that characterize CPM are the *Average Frequency Deviation* $\bar{k}_f$ and the *Modulation Index* h.

$$\bar{k}_f = (k_f/T) \int_0^T h_f(\tau)\ d\tau \tag{6}$$

$$h = \beta(\tau)/\pi = 2\bar{k}_f T \tag{7}$$

3.1 CPFSK

A conventional FSK signal is generated by shifting the carrier by an amount $f_n = 1/2\ \Delta f I_n, I_n = \pm 1, \pm 3, \ldots, \pm(M-1)$, to reflect the digital information that is being transmitted. The type of FSK signal is memoryless. Further, the switching from one frequency to another may be accomplished by having $M = 2^k$ separate oscillators tuned to the desired frequencies and selecting one of the M frequencies according to the k-bit symbol that is to be transmitted in a signal interval of duration $T = k/R$ seconds. However, such abrupt switching from one oscillator output to another in successive signaling intervals results in relatively large spectral side lobes outside of the main spectral band of the signal and, consequently, this method requires a large frequency band for transmission of the signal. To avoid the use of signals having large spectral side lobes, the information bearing signal frequency modulates a single carrier whose frequency is changed continuously. The resulting frequency modulated signal is phase continuous and, hence, it is called *continuous-phase* FSK.

$$h_f(t) = u_T(t) \tag{8}$$

$$\bar{k}_f = k_f, h = 2k_f T \tag{9}$$

$$\beta(t) = \begin{cases} 0 & t < 0 \\ 2\pi k_f t = \pi h t/T & 0 \le t \le T \\ \pi h & t \ge T \end{cases} \tag{10}$$

CPM signals are usually described by sketching the excess phase $\phi(t)$.

$$\phi(t) = \beta(T) \sum_{n=-\infty}^{k-1} x_n + x_k\beta(t - kT) \text{ for all } \{x_n\} \tag{11}$$

$\phi(t)$ is plotted for Binary CPFSK, $\{x_n\} = \{-1, 1\}$ in Figure 8 [3].

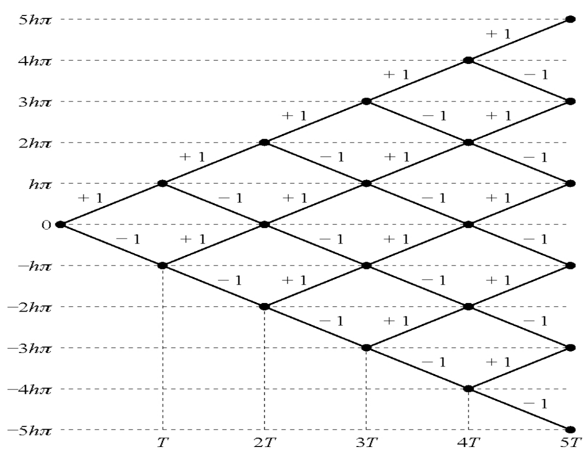


Figure 8: Phase trajectory for binary CPFSK.

The figure corresponds to a rectangular pulse representing a bit. Hence, the phase changes at a constant rate (straight line). Figure 9 compares the phase changes between a raised cosine pulse and a rectangular pulse shape.

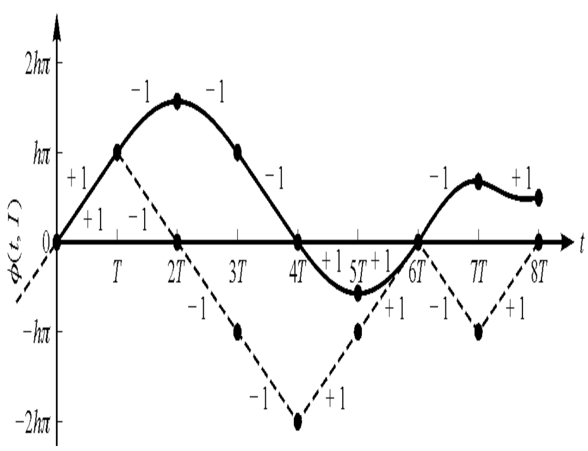


Figure 9: Phase trajectory for binary CPFSK using a rectangular pulse (dotted) and a raised cosine pulse. The I is the $\{x_n\}$

3.1.1 MSK

MSK, Minimum shift keying is a special case of binary CPFSK with $h = 0.5$.

$$\beta(t) = \begin{cases} 0 & t < 0 \\ 2\pi k_f t = \pi t/2T & 0 \leq t \leq T \\ 0.5\pi & t \geq T \end{cases} \tag{12}$$

Therefore, the carrier phase during the interval $kT \leq t \leq (k+1)T$ is given by,

$$\begin{aligned} \phi(t) &= 2\pi f_c t + \pi/2 \sum_{n=-\infty}^{k-1} x_n + 0.5\pi\ x_k\ ((t - kT)/T) \\ &= \Big(2\pi f_c + \pi x_k/2T\Big)t + \pi/2 \sum_{n=-\infty}^{k} x_n - \pi/2\ x_k \end{aligned} \tag{13}$$

The MSK bandpass waveform is then given as

$$\begin{aligned} s(t) &= A\cos\Big[(2\pi f_c + \pi x_k/2T)t + \\ &\quad \pi/2 \sum_{n=-\infty}^{k-1} x_n - \pi k/2\ x_k\Big] \\ &\quad \text{where } kT \leq t \leq (k+1)T \\ &= A\cos\Big[2\pi(f_c + x_k/4T)t + \\ &\quad \pi/2 \sum_{n=-\infty}^{k-1} x_n - \pi k/2\ x_k\Big] \\ &\quad \text{where } x_k \epsilon (-1, 1) \end{aligned} \tag{14}$$

Since, x_k can be ± 1, two different frequencies are modulated for ± 1. The frequencies are $f_c \pm 1/4T$. Therefore, the difference between the frequencies is $1/2T$ which is the minimum frequency separation required to ensure orthogonality between two sinusoids of duration T, assuming coherent demodulation. The above is the reason why the scheme is called *minimum* shift keying.
The Power Spectral Density of MSK is shown in the Figure 10 for different pulse shapes. Figure 11 compares the spectra of MSK and OQPSK. Note that the main lobe of MSK is 50% wider than that for OQPSK. However, the side lobes in MSK fall off considerably faster, making MSK more bandwidth efficient. Even greater efficiency than MSK can be achieved by further reducing h. However, the FSK signals will no longer be orthogonal and there will be an increase in the error probability.

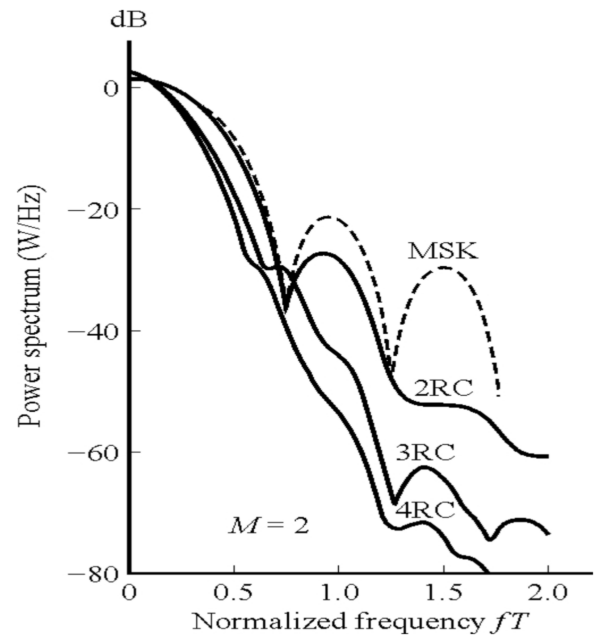


Figure 10: PSD for MSK. The dotted curve corresponds to MSK. The other curves correspond to a raised cosine pulse, partial response CPFSK with $h = 0.5$, lasting for $2T$,$3T$,$4T$. It is clearly seen that as the spreading in time is increased, the bandwidth efficiency increases.

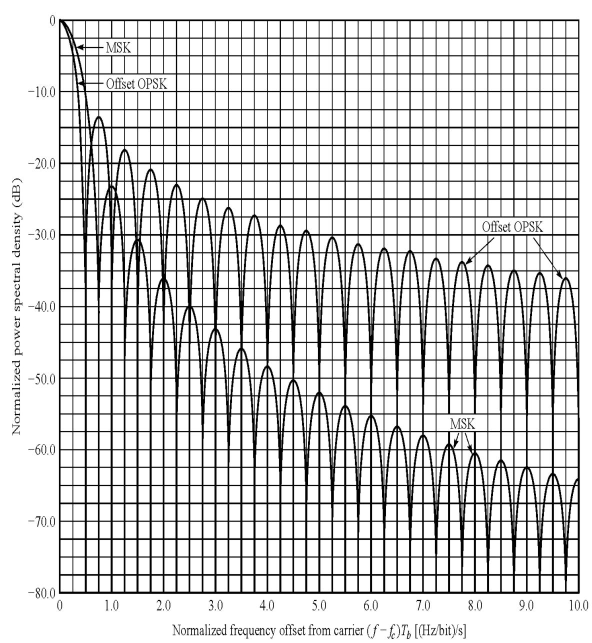


Figure 11: PSD comparison of MSK and OQPSK.

3.2 Partial Response CPM

The idea is to make $h_f(t)$ of duration greater than T.

$$h_f(t) = h_f(t)\; u_{kT}(t)$$
$$\text{where } u_{kT} = \sum_{k=0}^{K-1} u_T(t - kT) \tag{15}$$

3.2.1 GMSK

Pass the rectangular pulse $h_f(t)$ through a premodulation filter given as

$$H(f) = exp\Big\{ - (f/B)^2 \ln 2/2 \Big\} \tag{16}$$

B is the bandwidth of the filter. H(f) is bell shaped about $f = 0$. Therefore, the name Gaussian MSK. BT is used to parameterize GMSK schemes. B is the bandwidth of the premodulation filter defined above and T is the symbol duration. The next two figures make it amply clear in both the time and frequency domain, that decreasing BT improves spectral occupancy.

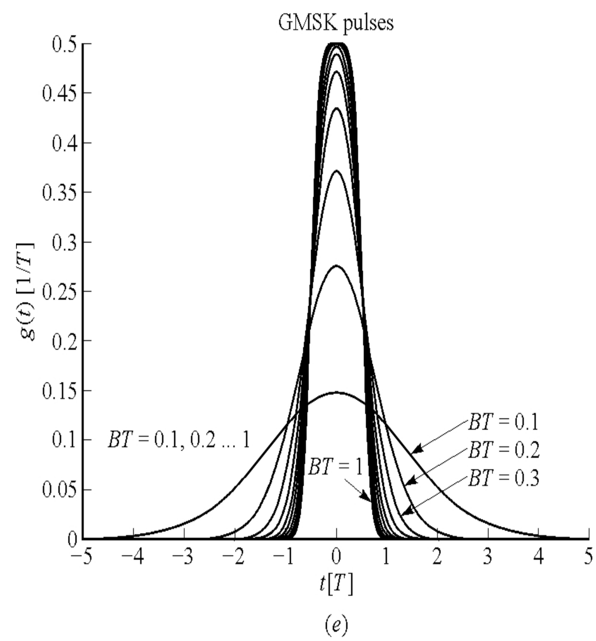


Figure 12: GMSK pulses for different BT.

From Figure 12, we observe that when $BT = 0.3$, the GMSK pulse may be truncated at $|t| = 1.5T$ with a relatively small error incurred. GMSK with $BT = 0.3$ is used in GSM.

The decrease in spectral occupancy is accompanied by increase in ISI, as we no longer adhere to the Nyquist Criterion. To counter the ISI, GMSK requires equalization. Thus, we can say that GMSK is a bandwidth efficient scheme but not a power efficient one. The table below shows occupied RF bandwidth for GMSK and MSK as a fraction of R_b, the bit rate, containing a given percentage of power. Notice that GMSK is spectrally tighter than MSK.

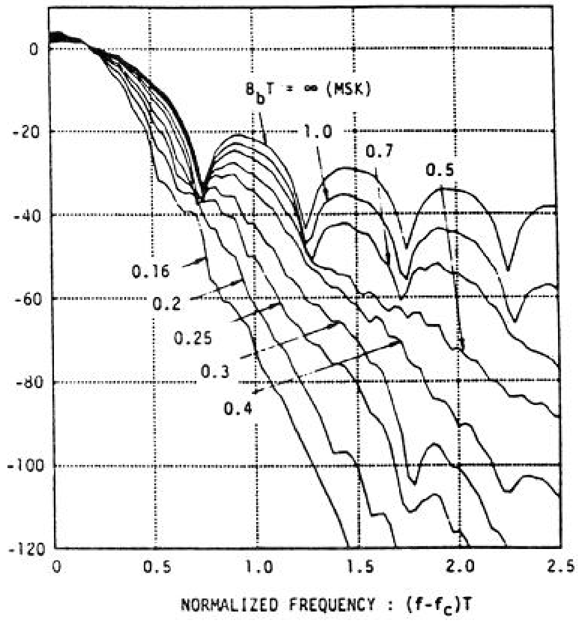


Figure 13: Power spectral density for a GMSK signal.

BT	90%	99%	99.9%	99.99%
0.2 GMSK	0.52	0.79	0.99	1.22
0.25 GMSK	0.57	0.86	1.09	1.37
0.5 GMSK	0.69	1.04	1.33	2.08
MSK	0.78	1.20	2.76	2

4 M-PSK Bandwidth/Power

The MPSK waveform is given by,

$s_i(t) = (2E_s/T_s)^{0.5}\cos(2\pi f_c(t) + 2\pi/M\ (i-1))$
where $0 \le t \le T_s$ and $T_s = (\log_2 M)T_b$
$E_s = E_b \log_2 M\ and T_b$ is the energy per symbol. (17)

The table below highlights the bandwidth and power efficiency of M-PSK signals. Power efficiency η_p is defined as E_b/N_o required for $P_e = 10^{-6}$. The bandwidth is the first null bandwidth.

M	2	4	8	16	32	64
η_b	0.5	1	1.5	2	2.5	3
E_b/N_o	10.5	10.5	14	18.5	23.4	28.5

5 Optimum Receivers AWGN

For a signal transmitted over an AWGN channel [3], either a correlation demodulator or a matched filter demodulator produces the vector $\underline{r} = [r_1 r_2 \ldots r_N]$, which contains all the relevant information in the received signal waveform. Once the vector $\underline{r}$ has been received, an optimum decision needs to be made regarding which signal $\underline{s}_m$ was transmitted, given that $\underline{r}$ has been received where $1 \le m \le M$. The decision criterion is based on selecting the signal corresponding to the maximum of the set of a posteriori probabilities $\{P(\underline{s}_m|\underline{r})\}$. The decision criterion is called the MAP (maximum a posteriori probability) criterion. It can be proved that the criterion minimizes the probability of error and therefore a detector that implements it is known as the optimum detector.

Using Bayes' rule, the a posteriori probabilities can be expressed as $P(\underline{s}_m|\underline{r}) = p(\underline{r}|\underline{s}_m)P(\underline{s}_m)/p(\underline{r})$, where $p(\underline{r}|\underline{s}_m)$ is the conditional PDF of the observed vector given that $\underline{s}_m$ was transmitted and $P(\underline{s}_m)$ is the a priori probability of the mth signal being transmitted.

If we further assume that all M signals are equally probable a priori, the optimum detection rule reduces to finding the transmitted signal that maximizes $p(\underline{r}|s_m)$. The criterion is also known as the ML (maximum likelihood) criterion.

For an AWGN channel,

$$p(\underline{r}|\underline{s}_m) = (\pi N_0)^{-N/2} exp\Big[(-1/N_0)\ \sum_{k=1}^{N}(r_k - s_{mk})^2\Big] \quad 0 \le m \le M \tag{18}$$

Therefore,

$$\ln p(\underline{r}|\underline{s}_m) = (-N/2)\ \ln(\pi N_0) - 1/N_0 \sum_{k=1}^{N}(r_k - s_{mk})^2 \tag{19}$$

The maximum of $\ln p(\underline{r}|\underline{s}_m)$ over s_m is equivalent to finding the signal $\underline{s}_m$ that minimizes $\sum_{k=1}^{N}(r_k - s_{mk})^2$, which is the same as the Euclidean distance between the received vector and the vector $\underline{s}_m$.

Further, if we assume that all the $\underline{s}_m$ have the same energy, the criteria becomes selecting the vector $\underline{s}_m$ that has the maximum correlation with the received vector $\underline{r}$. Remember that the vector was received at the output of the matched filter/ correlation demodulator.

All the observations made above are assuming memoryless modulation. However, for signals that have memory, the maximum likelihood sequence detection algorithm must be used. The algorithm searches for the minimum Euclidean distance path through the trellis that characterizes the memory in the transmitted signal.

5.1 Probability Of Error

5.1.1 Binary FSK

It is given by $Q(\sqrt{E_b/N_o})$.

5.1.2 MSK

MSK has one of two possible frequencies over any symbol interval.

$$
\begin{aligned}
s(t) = A\ \cos\Big[&(2\pi f_c + \pi x_k/2T)t + \\
&\pi/2 \sum_{n=-\infty}^{k-1} x_n - \pi/2\ kx_k\Big] \\
\text{where } kT \leq t \leq (k+1)T&
\end{aligned} \tag{20}
$$

Consider interval $0 \leq t \leq T_b$ and denote $A = \sqrt{2E_b/T_b}$. Then

$$
\beta(t) = \begin{cases} \sqrt{2E_b/T_b}\cos(2\pi f_1 t + \theta(o)) & \text{symbol 1} \\ \sqrt{2E_b/T_b}\cos(2\pi f_2 t + \theta(o)) & \text{symbol 0} \end{cases} \tag{21}
$$

On first guess one may conclude that MSK has the same error probability as BFSK, but since it has memory of phase it does better. Using phase trellis it can be shown that $P_e \approx Q(\sqrt{2E_b/T_b})$, the approximation is valid for high SNR values. Therefore, MSK is approximately same in BER performance as BPSK.

5.1.3 GMSK

The probability of error of GMSK can be shown to be $P_e \approx Q(\sqrt{2\alpha E_b/T_b})$ where α is a constant for a given BT_b. For example, for $BT_b = 0.25$, $\alpha \approx 0.68$. Similarly, for $BT_b = \infty$, $\alpha \approx 0.85$ which is the case of MSK.

We had earlier noted that GMSK improved bandwidth efficiency over MSK since it had a narrower main lobe and a faster roll-off of side bands. But from above it is clear that $P_e(GMSK) > P_e(MSK)$

6 B/W vs Power efficiency trade off

As BT_b decreases, bandwidth efficiency increases, but power efficiency decreases (because P_e increases).

7 Error Probabilities for Flat Fading Channels

Consider the transmitted waveform,

$$
s_i(t) = \sqrt{2E_s/T_s}\cos(2\pi f_c t + 2\pi/M\ (i-1)), \quad 0 \leq t \leq T_s \tag{22}
$$

7.1 Flat fading received model

$x(t) = g(t)s_i(t) + w(t)$, where $w(t)$ is AWGN. $g(t)$ is the attenuation in the amplitude of the signal due to fading. Assume that the channel is *flat and slow*. Therefore $T_s >> \sigma_\tau$ and $T_s << T_c$. Therefore $g(t)$ is effectively a constant over the symbol duration. Let $g(t) = \alpha$, then $x(t) = \alpha s_i(t) + w(t)$ for $0 \leq t \leq T_s$.

For a constant α the maximum likelihood decoding rule for optimum detection (assuming inputs are equiprobable) holds true. Therefore, the receiver structure remains the same. In general it can be shown that,

$$
P_e \leq \sum_{\substack{k=1 \\ k\neq i}} Q(\alpha d_{ik}/\sqrt{2N_0}) \tag{23}
$$

Typically, α is Rayleigh or Rician distributed for non LOS and LOS situations respectively. Therefore the probability of error may be written as,

$$
P_e \leq \sum_{\substack{k=1 \\ k\neq i}} \int_0^\infty Q(\alpha d_{ik}/\sqrt{2N_0}) f_\alpha(\alpha)\ d\alpha \tag{24}
$$

Consider $M = 2$, the SNR is given as

$\gamma_b = \alpha^2 E_b/N_0$. Let $\beta = \alpha^2$. If α is Rayleigh, β is exponential.

$$\bar{P}_e = \int_0^\infty Q(\sqrt{2\beta E_b/N_0}) f_\beta(\beta)\ d\beta \quad (25)$$

Rewrite, $\gamma_b = \beta E_b/N_0$. Then

$$E[\gamma_b] = \bar{\gamma}_b = E_b/N_0\ E[\beta] \quad (26)$$

$$f(\gamma_b) = (\bar{\gamma}_b)^{-1}\ exp(-\gamma_b/\bar{\gamma}_b)\ ,\ \gamma_b \geq 0 \quad (27)$$

Therefore,

$$\bar{P}_e = \int_0^\infty Q(\sqrt{2\gamma_b})(\gamma_b)^{-1} exp(-\gamma_b/\bar{\gamma}_b)\ d\gamma_b \quad (28)$$

Integrating by parts we get,

$$\bar{P}_e = 0.5 - (1/2\sqrt{\pi}) \int_0^\infty exp(-\gamma_b(1+\bar{\gamma}_b)/\bar{\gamma}_b)\ \gamma_b^{-0.5} d\gamma_b \quad (29)$$

Substituting,

$$z = \gamma_b(1+\bar{\gamma}_b)/\bar{\gamma}_b) \quad (30)$$

$$\bar{P}_e = 1/2 - 1/2\ (\pi(1+\bar{\gamma}_b)/\bar{\gamma}_b)^{-1} \int_0^\infty e^{-z}\ z^{-0.5}\ dz \quad (31)$$

$$\bar{P}_e = 1/2 - 1/2\ (\pi(1+\bar{\gamma}_b)/\bar{\gamma}_b)^{-1}\Gamma(1/2) \quad (32)$$

$$\bar{P}_e = 1/2 - 1/2\ ((1+\bar{\gamma}_b)/\bar{\gamma}_b)^{-1} \quad (33)$$

For high SNR ($\bar{\gamma}_b$) we can say that $P_e \propto (SNR)^{-1}$, unlike AWGN where they were exponentially related. Consider the probability of error, if using Binary FSK on a flat fading channel as modeled above.

$$P_e = 1/2 - 1/2\ ((2+\bar{\gamma}_b)/\bar{\gamma}_b)^{-1} \quad (34)$$

Therefore, coherent PSK is 3dB better than coherent FSK.

It is important to note that in the above model, we assumed that coherent detection was possible. That is why the phase was completely ignored in the model. For coherent detection to be possible in a fading channel, we need pilot signals.

7.2 Detection of signals with unknown phase

If we assume that the phase of the signal is not known at the receiver we will have to use *Non-Coherent* detection. Let the transmitted signal be,

$$s_i(t) = \sqrt{2E/T}\cos(2\pi f_i t),\ 0 \leq t \leq T \quad (35)$$

The received signal may be written as,

$$x(t) = \sqrt{2E/T}\cos(2\pi f_i t + \theta) + w(t) \quad (36)$$

where $w(t)$ is the AWGN and θ is the unknown phase. So we can assume that θ is a random variable uniformly distributed in the interval $[0, 2\pi]$. Rewriting, $x(t)$ as

$$x(t) = \sqrt{2E/T}\Big\{\cos(2\pi f_i t)\cos(\theta) - \sin(2\pi f_i t)\sin(\theta)\Big\} + w(t) \quad (37)$$

The signal may be received using a *Quadrature Receiver.*

7.3 Non Coherent Orthogonal Modulation

Assume that bit 1 is transmitted as $s_1(t)$ and bit 0 is transmitted as $s_2(t)$. As the modulation is orthogonal, $s_1(t)$ and $s_2(t)$ are orthogonal. We further assume that the received signal is $x(t)$.

$$x(t) = \begin{cases} g_1(t) + w(t) & \text{,for } 0 \leq t \leq T, \text{ if 1 is Tx} \\ g_2(t) + w(t) & \text{,for } 0 \leq t \leq T, \text{ if 0 is Tx} \end{cases} \quad (38)$$

We can further assume that $g_1(t)$ and $g_2(t)$ are orthogonal. The receiver is shown in Figure 14.

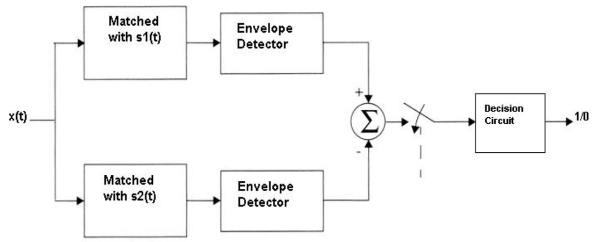


Figure 14: Non-Coherent detection. Each ARM of the receiver is a Quadrature Receiver.

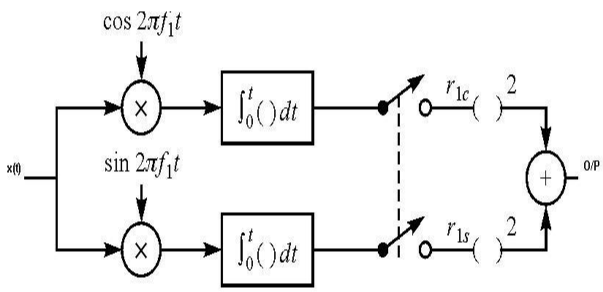


Figure 15: A Quadrature Receiver.

References

[1] Digital Communications Fundamentals and Applications, Dr. Bernard Sklar.

[2] Digital Communications, Dr. Proakis.

[3] Wireless Communications Principles and Practice, Dr. Rappaport.

[4] Wireless communication technologies, lecture notes, Spring 2005, Dr. Narayan Mandayam, Rutgers University.

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (2 - 2) (Amplitude Shift Keying ASK)

In this lecture and beyond we shall study each type of digital modulation systems in the following main parts:-

A) - The idea of the modulation system.
B) – The equations of the modulated signal
C) - The waveform in time domain
D) – Constellation diagram of the modulation type.
E) – The power spectral density (PSD) of the modulated signal.
F) –The required channel bandwidth for the modulation system.
G) – The generation circuits for the modulation system.
H) – The detection circuits for the modulation system.
I) – The probability of error for the modulation system.

A) - The idea of the ASK modulation system.

We can simply consider the idea of the ASK modulation system by send the logic one (1") of the base band signal by sinusoidal signal with amplitude value A1 and send the logic zero (0") of the base band signal by the sinusoidal signal with other value of amplitude A2 .

B) – The equations of the ASK modulated signal .

The equation of the ASK modulated signal can be written as:-

$S(t) = A/2[(1 + mdi(t)] \cos wct$ ……………………………………………………(2)

.m is the modulation index while 0<m<=1

$di(t)$ = 1 for (1") and = -1 for (0")

Then $S1(t) = A/2[(1 + m)] \cos wct$ ……………………………………………………(3)

$S0(t) = A/2[(1 - m)] \cos wct$ ……………………………………………………(4)

if m = 1 $S1(t) = A \cos wct$ $S0(t) = 0$ (O.O.K)

(O.O.K) on . off . keying it’s a special case from the ASK ***when m=1***

$\neq$

If m != 1 let m= 0.5

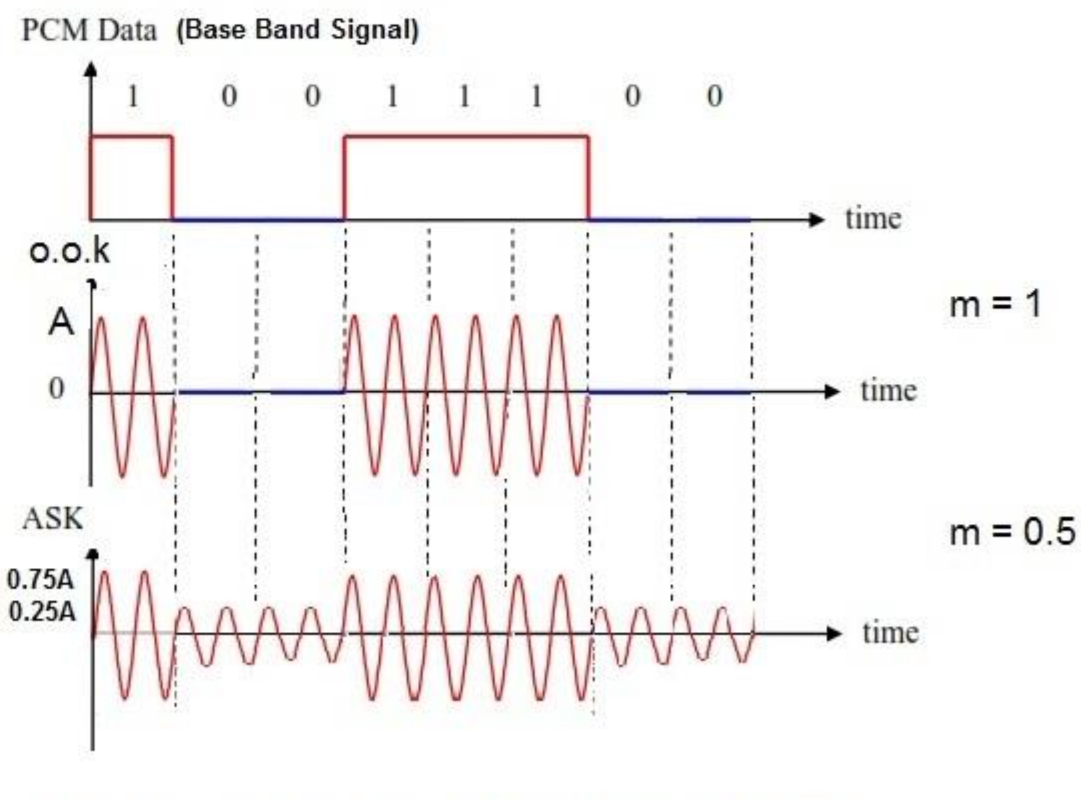
$S1(t) = A/2[(1 + 0.5)] \cos wct$ = $0.75\ A \cos wct$

$S0(t) = A/2[(1 - 0.5)] \cos wct$ = $0.25A \cos wct$

<u>Notes :- because of the wide use of the O.O.K they take sign of the ASK in the digital communications.</u>

C) <u>– The waveform in time domain</u>

The waveform in time domain for the ASK modulated signal is shown in Figure (1) below:-



Wave form in time domain for ASK and O.O.K

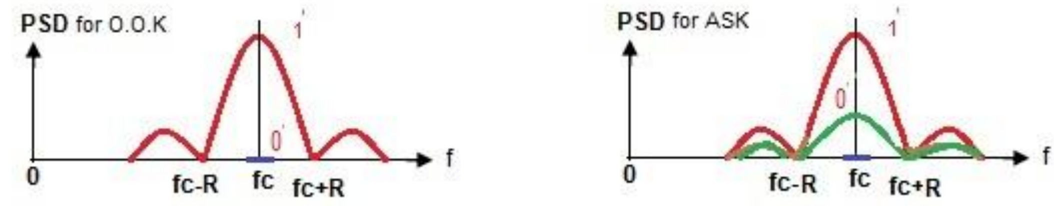
<u>*D – Constellation diagram of the ASK modulation type.*</u>

<u>Constellation diagram:-</u> is very similar to vector diagram. It's a method of represented symbol states in carrier modulated waveforms in terms of their amplitude and phase. The X axis is taken as a reference for symbols that are in phase *(I)* with carrier ***A Coswct***, and The Y axis is taken as a reference for symbols that are in phase *(Q)* with carrier ***A Sinwct***. So figure (2) show the constellation diagram for ASK and O.O.K

Constellation Diagram For ASK and O.O.K

E) -The power spectral density (PSD) of the ASK modulated signal.

The power spectral density for the ASK modulated signal is shown in figure (3)



PSD for ASK and O.O.K

F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the ask and the O.O.K above we can extract the required channel bandwidth which represent the frequency band between the two nulls (fc-R) and (Fc+R) then

The ***reqBW for ASK = 2R*** …………………………………………………………………….(5)

For more correct representation for the required channel bandwidth for the ASK we must write the low as :-

reqBW for ASK = 2* BW for B.B.S before the modulation process.

$\boxed{R = S_r}$

$S_r = n f_s = \frac{1}{T_b}$

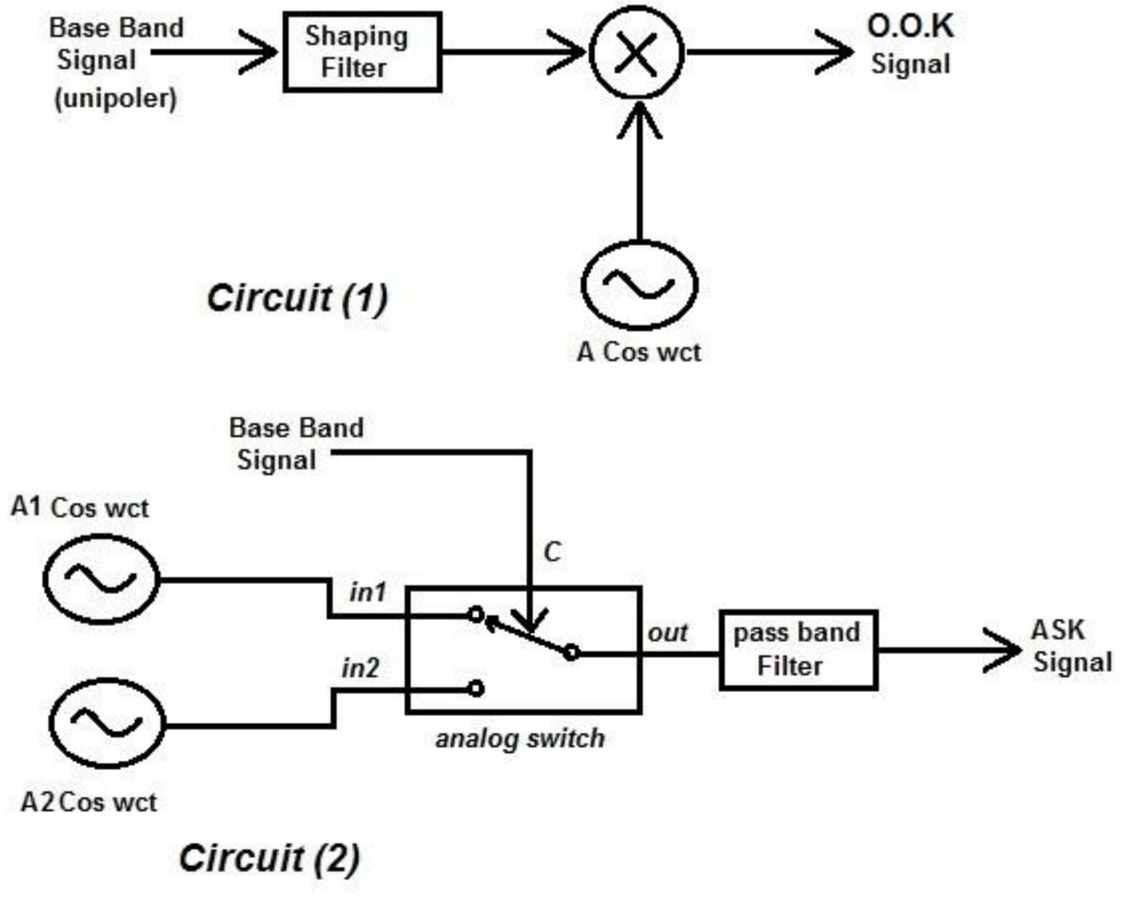
$BW = \frac{S_r}{\eta}$

$\eta = 1$ RZ

$\eta = 2$ NRZ

G – The generation circuits for the ASK modulation system.

The generation circuit for the ASK and O.O.K are shown in figure (4) below :-



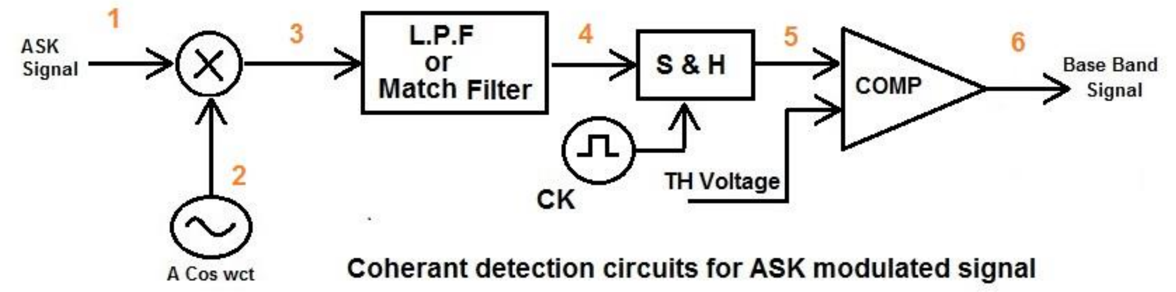
Generation Circuits For ASK System

H)– The detection circuits for the ASK modulation system.

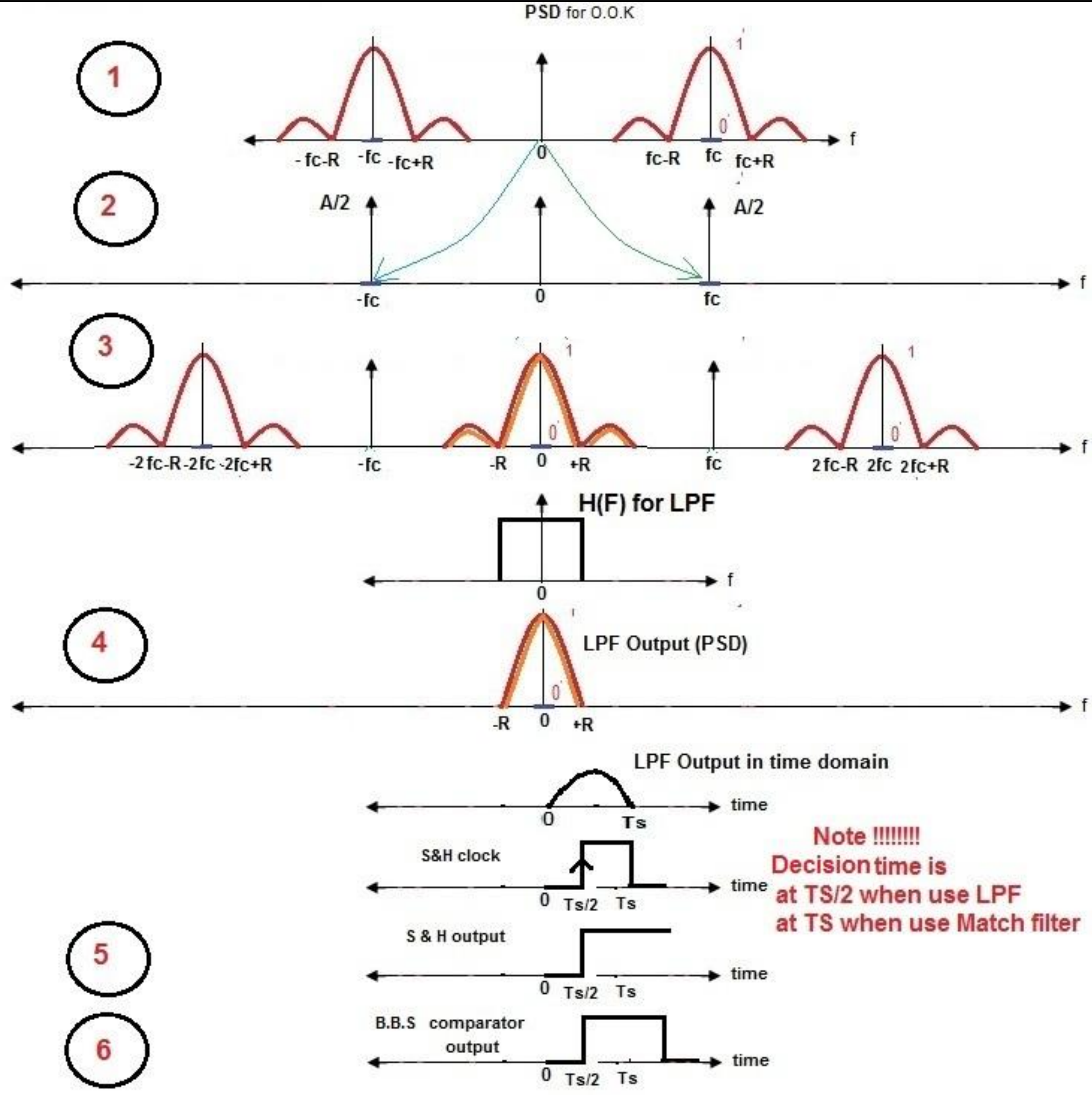
There are two types of detection circuits for the ASK modulated signals

1 - The first type is the ***coherent detection*** circuit which required sinusoidal source in the receiver side that has quietly the same specifications for the carrier signals in the generation circuits of ASK modulator circuit .

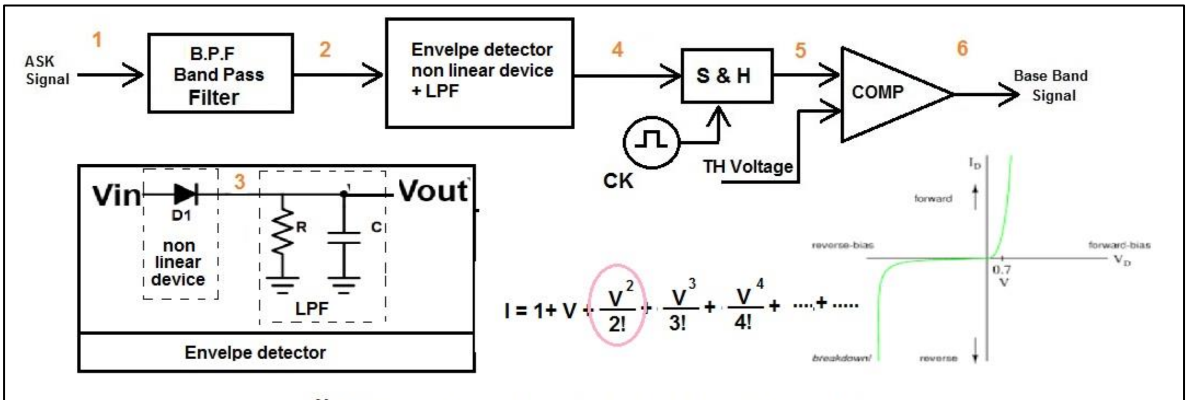
2 – The second type is the ***non coherent detection*** circuit which not need of the sinusoidal source in the receiver side, its replace this source by no linear device and low pass filter (envelope detector) , these two types of detection circuits are shown in figure (5) and (6) respectively.



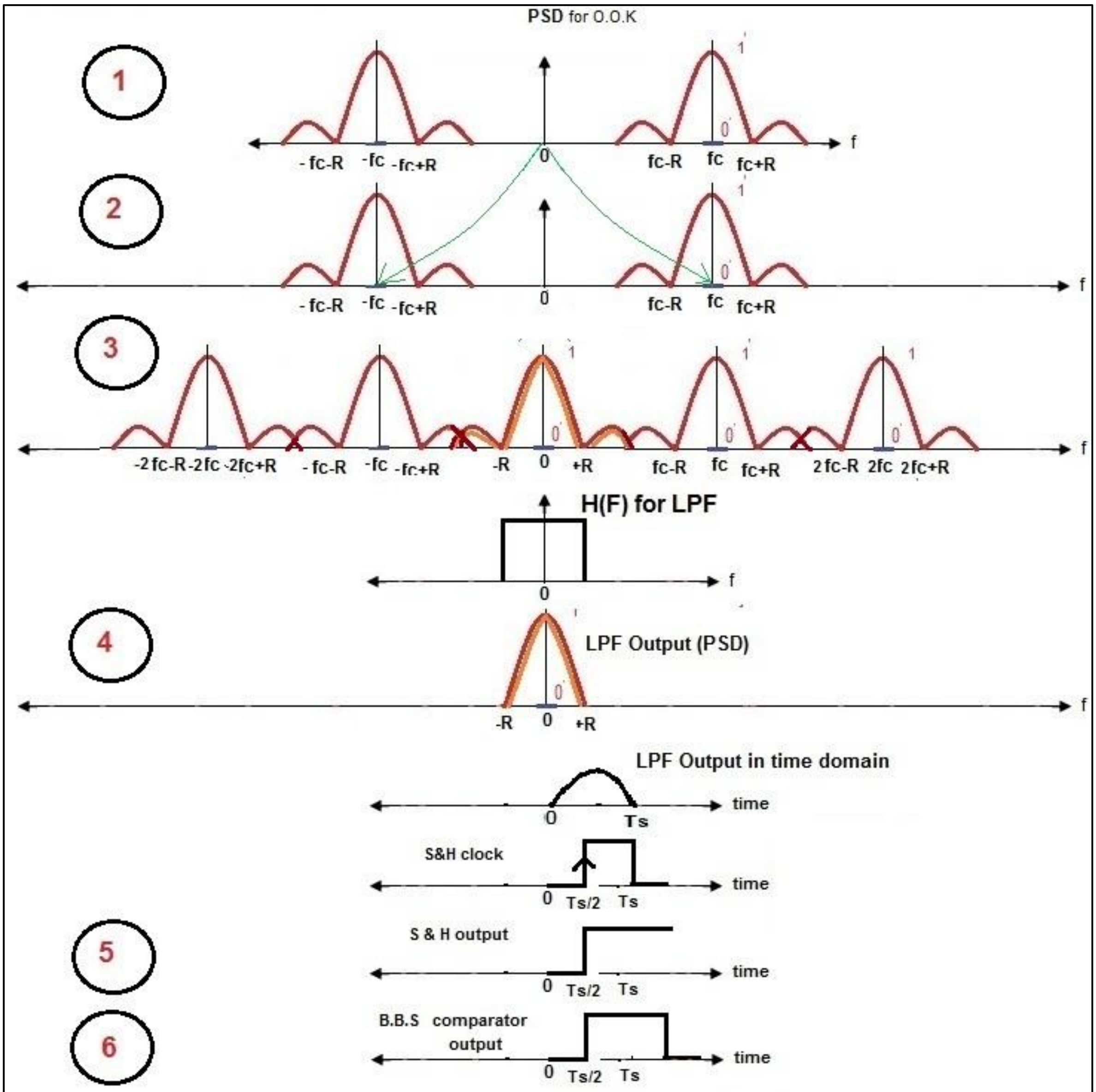
Coherant detection circuits for ASK modulated signal



Signal analysis for the ASK coherant detection circuit



Non Coherant detection circuits for ASK modulated signal



Signal analysis for the ASK Non coherant detection circuit

I)– The probability of error for the ASK modulation system.

From the waveform of the ASK signal in time domain we can see clearly the similarity between the unipoler NRZ and the ASK signal then we can deduce that the probability of error for the ASK system is equally equal the probability of error for unipoler NRZ baseband signal

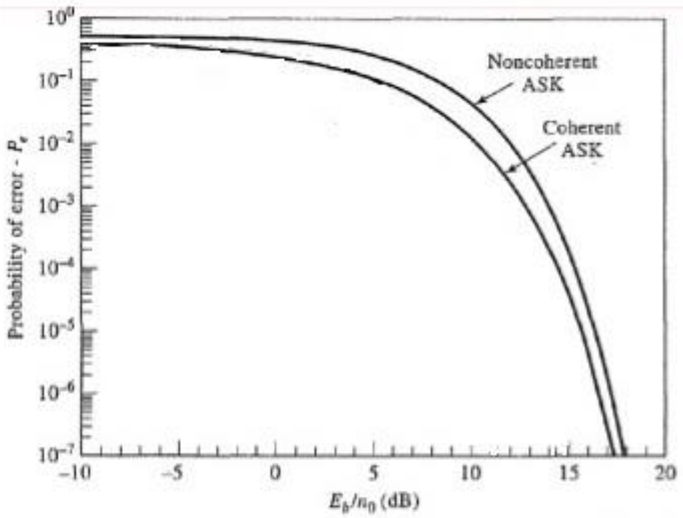
$$Pe_{ASK} = Pe_{NRZ} = Q\left(\sqrt{\frac{S}{2N}}\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Note that equation (6) represent the probability of error for the ASK system when using the coherent detection circuits in the receiver side otherwise the probability of error for ASK system when using non coherent detection circuits is **<u>worst than the system that use the coherent detection circuits by one dB .</u>**

The formula of the probability of error for Non coherent ASK system is

$$Pe_{ASK\ non\ coh} = \frac{1}{2}\ e^{\frac{1}{2}\left(\frac{Eo}{No}\right)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Figure (7) show the probability of error for Coherent and non coherent detection ASK system.

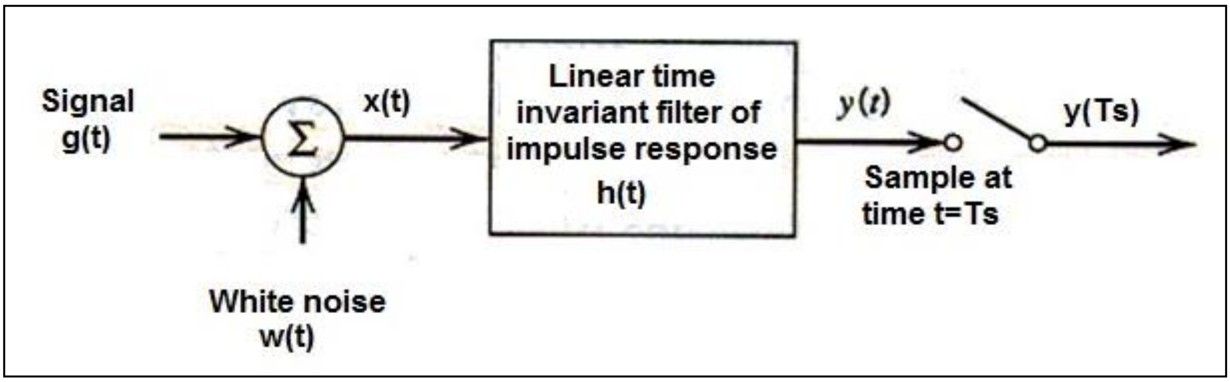


Probability of error for ASK System

Digital Communications
Digital Carrier Modulation
Lecture (3 – 2)
MATCHED FILTER

– A basic problem that often arises in the study of communication systems is that of detecting a pulse transmitted over a channel that is corrupted by channel noise

– A matched filter is a linear filter designed to provide the maximum signal-to-noise power ratio at its output. This is very often used at the receiver.



– Consider that the filter input $x(t)$ consists of a pulse signal $g(t)$ corrupted by additive noise $w(t)$. It is assumed that the receiver has knowledge of the waveform of the pulse signal $g(t)$.

The function of receiver is to detect the pulse signal $g(t)$ in an optimum manner, given the received signal $x(t)$.

$\boldsymbol{x(t) = g(t) + w(t)}$ $\qquad\qquad$ $\boldsymbol{y(t)=x(t)*h(t)}$

$\boldsymbol{y(t)= [g(t)+ w(t)]*h(t)}$

y(t)=g(t)*h(t)+w(t)*h(t) ***..... linear proparty***

y(t)=go(t)+No(t)

where go(t)=g(t)*h(t) ....(1)

and No(t) = w(t)*h(t) ....(2)

in frequency domain we can write (1) and (2) as :-

Go(f) = G(f).H(f)..;...(3)

No(f) = W(f).H(f) ....(4)

– The purpose of the circuit is to design an impulse response $h(t)$ of the filter such that the output signal-to-noise ratio is maximized.

Signal Power

Let $G(f)$ and $H(f)$ denoted the Fourier transform of $g(t)$ and $h(t)$.

we can write go(t) from eq (3) in other form using inverse Fourier transform of Go(f) as:-

$$g_0(t) = \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df$$

$$\text{The signal power} = |\ g_0(t)\ |^2 = |\ \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi ft)df\ |^2$$

Noise Power

Since $w(t)$ is white with a power spectral density $\frac{No}{2}$,the spectral density function of Noise is

$$S_N(f) = \frac{N_0}{2}|H(f)\ |^2$$

– The noise power = $E[n^2(t)] = \dfrac{N_0}{2}\int_{-\infty}^{\infty}|\ H(f)\ |^2 df$

S/N Ratio

– Thus the signal to noise ratio become

$$\text{S/N} = \frac{\left| \int_{-\infty}^{\infty} H(f)G(f)\exp(j2\pi fT)df \right|^2}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df} \quad \text{.............} (5)$$

- (the output is observed at $t = Ts$)

– Our problem is to find, for a given $G(f)$, the particular form of the transfer function $H(f)$ of the filter that makes S/N at maximum.

– To simplified Eq.(5) Scwarzs theory can help us to do this simplification

Schwarz's inequality:

If $\int_{-\infty}^{\infty} |\mathbf{A}(x)|^2 dx < \infty$ and $\int_{-\infty}^{\infty} |\mathbf{B}(x)|^2 dx < \infty$,

$$\left| \int_{-\infty}^{\infty} \mathbf{A}(x)\, \mathbf{B}(x) dx \right|^2 \le \int_{-\infty}^{\infty} |\mathbf{A}(x)|^2 dx \int_{-\infty}^{\infty} |\mathbf{B}(x)|^2 dx$$

This equation holds, if and only if, we have A(x) = KB*(x), where k is an arbitrary constant, and * denotes complex conjugation. Applying the Schwarz's inequality to the numerator of equation (1), we have

$$\left| \int_{-\infty}^{\infty} H(f) G(f) \exp(j2\pi f T) df \right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |G(f)|^2 df \quad \text{...(6)}$$

Note that : $|e^{j2\pi fT}| = 1$, Substituting (6) into (5) , The S/N ratio be

$$S/N \le \frac{2}{N_0} \int_{-\infty}^{\infty} |G(f)|^2 df \quad \text{or} \quad S/N \le \frac{2E}{N_0} \quad \text{......(7)}$$

where the energy E= $\int_{-\infty}^{\infty} |G(f)|^2 df$ is the input signal energy

Notice that the S/N ratio does not depend on the transfer function H(f) of the filter but only on the signal energy. The optimum value of H(f) is then obtained as

$$H(f) = kG^*(f)\exp(-j2\pi fT)$$

Taking the inverse Fourier transform of $H(f)$ we have

$$h(t) = k\int_{-\infty}^{\infty} G^*(f)\exp[-j2\pi f(T-t)]df$$

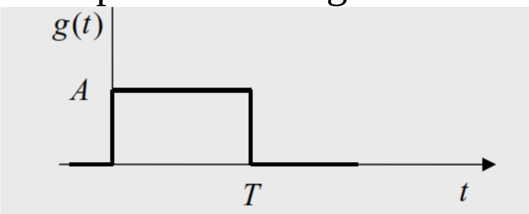
and $G^*(f) = G(-f)$ for real signal $g(t)$

$$h(t) = k\int_{-\infty}^{\infty} G(-f)\exp[-j2\pi f(T-t)]df$$

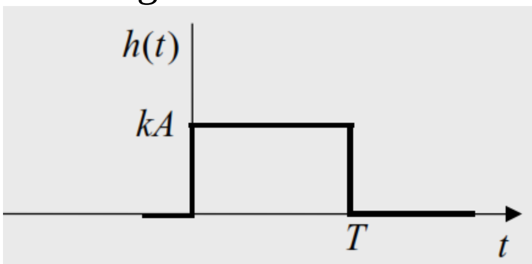
$$h(t) = kg(T-t) \qquad \ldots\ldots(8)$$

<u>Equation (4) shown that the impulse response of the filter is the time-reversed and delayed version of the input signal g(t) . "Matched with the input signal"</u>

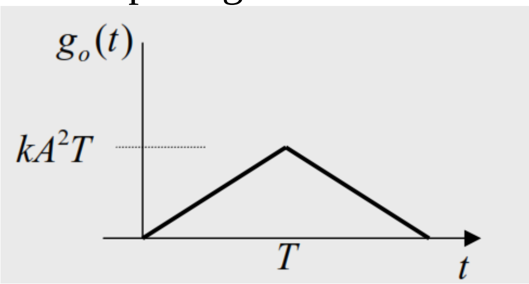
Example: let the signal is a rectangular pulse.



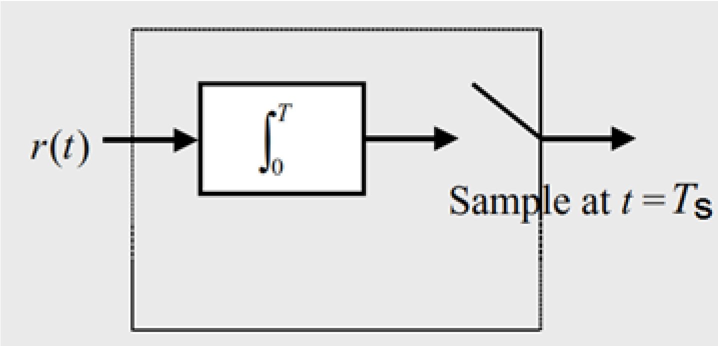
The impulse response of the matched filter has exactly the same waveform as the signal.



The output signal of the matched filter has a triangular waveform.



In this special case, the matched filter may be implemented using a circuit known as integrate-and- dump circuit.



While r(t) is the input signal , y(t) is the output signal , h(t) is the impulse response for the block system.

Assuming the output of $y(t) = r(t) \otimes h(t)$

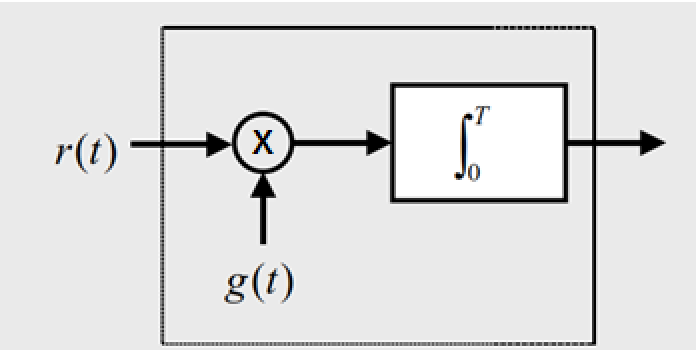
$$y(t) = \int_0^t r(\tau)h(t-\tau)d\tau \text{ ----------- } (9)$$

Substitute (8) into (9) we have $y(t) = \int_0^t r(\tau)g[T_S - (t-\tau)]d\tau$

Substitute $t = T_S$

$$y(T_S) = \int_0^t r(\tau)g(\tau)d\tau \quad \ldots\ldots (10) \quad \textbf{(Correlation Process)}$$

Equation (10) represent Correlation Process between r(t) and g(t)
And can be considered as shown below



Correlator

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (4-2)

(M-Ary ASK)

A) - The idea of the M-ary ASK modulation system.

After we study the ASK digital modulation system we conclude that the required bandwidth for this system is twice the bandwidth of the base band signal (before the modulation process).

As we know that the bandwidth is one of the important parameters which effects on the cost of the system, therefore the researchers and the communication engineers worked toward reduce the required bandwidth of the system, only one way to reduce this bandwidth, that is reduce the bandwidth of the base band signal before the modulation process, this can be done by representing more than one bit into one symbol and that lead produce more than two symbols according to the number of bits that combines together into one symbol. in this lecture we shall study 4_ary ASK system which combine two bits into one symbol.

B) – The equations of the 4-Ary ASK modulated signal .

The equation of the 4-Ary ASK modulated signal can be written as:-

$S(t) = A/2[(1 + mdi(t)] \cos wct$ ……………………………………………………(2)

.m is the modulation index while 0<m<=1

$di(t)$ = 1 for (11") $di(t)$ = -1 for (00")

$di(t)$ = 0.5 for (10") $di(t)$ = -0.5 for (01")

Then $S11(t) = A/2[(1 + m)] \cos wct$ ……………………………………………………(3)

$S00(t) = A/2[(1 - m)] \cos wct$ … ………………………………………(4)

$S01(t) = A/2[(1 - 0.5m)] \cos wct$ ……………………………………………(5)

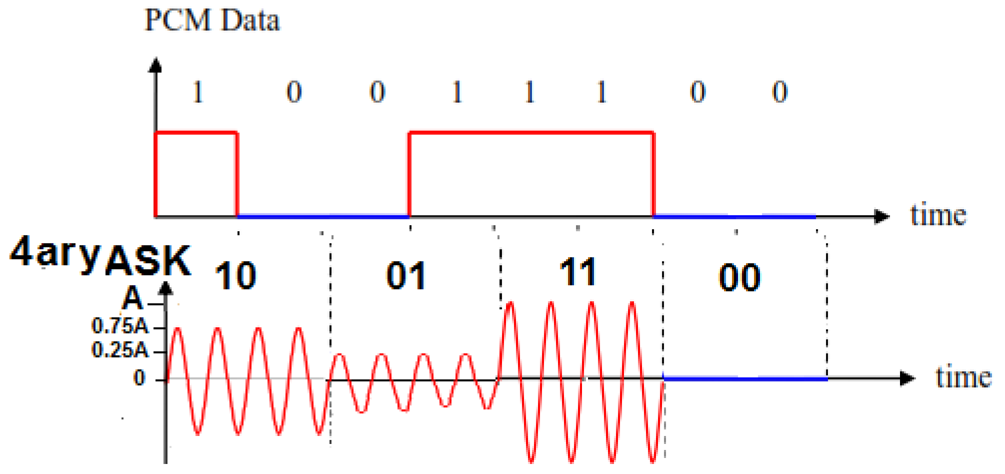
$S10(t) = A/2[(1 + 0.5m)] \cos wct$ ………………………………………………(6)

<u>if m = 1</u> $S11(t) = A \cos wct$ $S00(t) = 0$

$S10(t) = 0.75A \cos wct$ $S01(t) = 0.25A \cos wct$

C) – The waveform in time domain

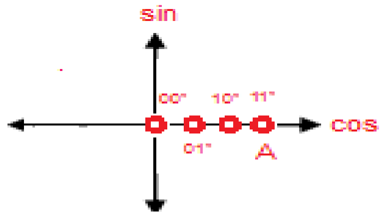
The waveform in time domain for the 4-Ary ASK modulated signal is shown in Figure (1) below:-



Waveform for 4-Ary ASK modulated signal

D – Constellation diagram of the ASK modulation type.

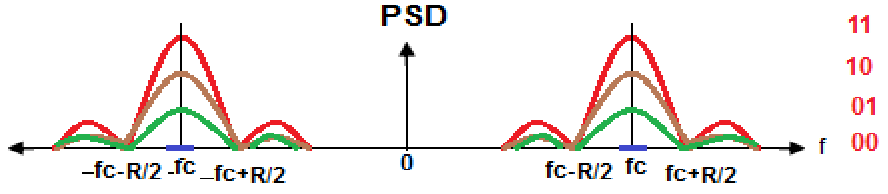
figure (2) show the constellation diagram for the 4-Ary ASK



4-Ary ASK
Constellation diagram

E) -The power spectral density (PSD) of the ASK modulated signal.

The power spectral density for the 4-Ary ASK modulated signal is shown in figure (3) below



PSD For 4-Ary ASK

F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the 4-Ary ASK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (fc-R/2) and (Fc+R)/2 then

The ***reqBW for 4-Ary ASK = 2R/2 = R*** ………………………………………………….(7)

For more correct representation for the required channel bandwidth for the 4-Ary ASK we must write the low as :-

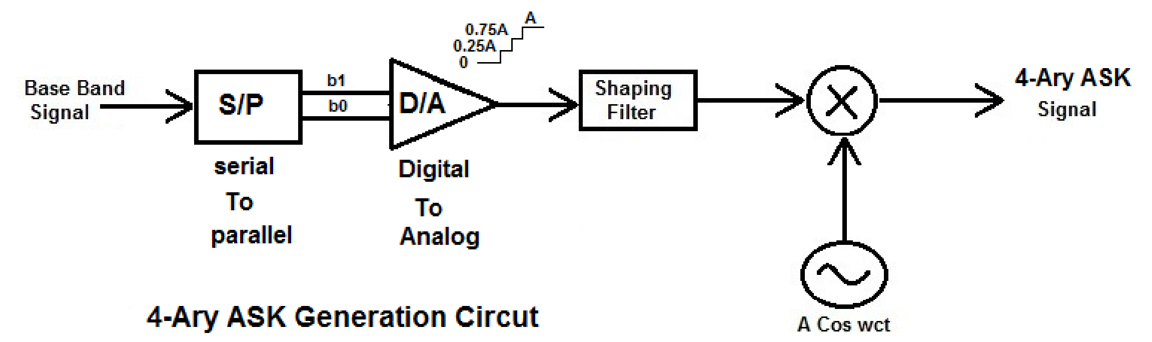
req BW for 4-Ary ASK = BW for B.B.S before the modulation process.

In general we can write the equation of the required bandwidth for the

M-Ary ASK is reqBw M-Ary ASK = $\frac{2 * BW\ for\ B.B.S}{Log_2(M)}$

G – The generation circuits for the ASK modulation system.

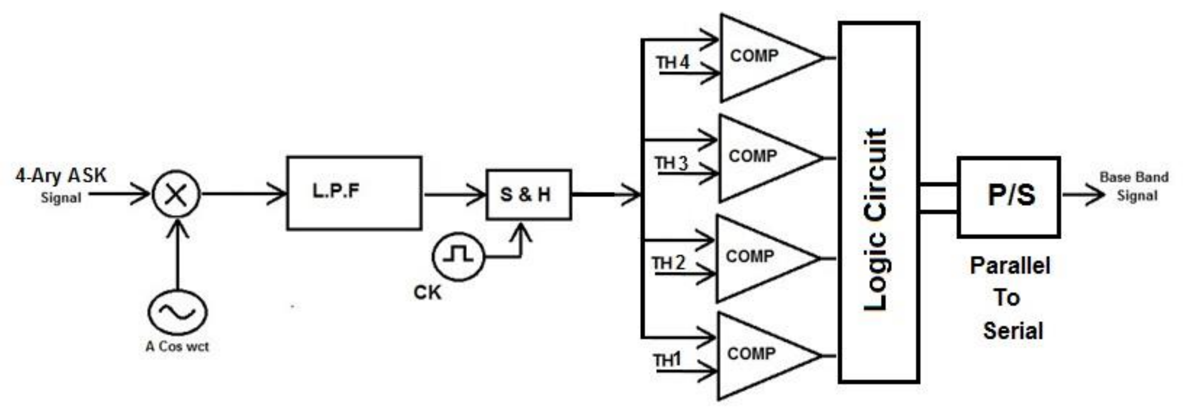
The generation circuit for the 4-Ary ASK are shown in figure (4) below :-



4-Ary ASK Generation Circut

H)– The detection circuits for the ASK modulation system.

The Coherent Detection for the 4-Ary ASK modulated signals are Shown in figure(5) below.



Coherent Detection Circuit for 4-Ary ASK Modulation Signal

I)– The probability of error for the ASK modulation system.

The probability of error for the M-Ary ASK system can be written as

$$P_e \approx \frac{P_s}{\log M} \qquad \text{...(8)}$$

Where P_s is the symbol probability of error which equal

$$P_s \cong 2\frac{M-1}{M}Q\left[\sqrt{\frac{3\log M}{M^2-1}\times\frac{2E_b}{N_o}}\right] \qquad \text{................(9)}$$

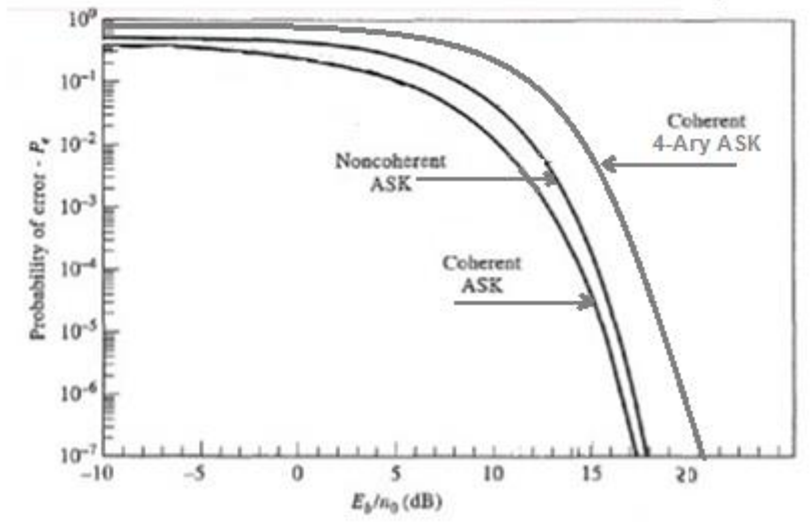
Then the probability of error is :-

$$P_e \cong \frac{2}{\log M}Q\left[\sqrt{\frac{2E_b}{N_o}\times\frac{3\log M}{M^2-1}}\right] \qquad \text{............ (10)}$$

P_e for 4-Ary ASK =$Q\left[\sqrt{\frac{2Eb}{No} * \frac{2}{5}}\right]$ ....(11)

Note that equation (11) represent the probability of error for the Coherent 4-Ary ASK system which is **<u>worst than the probability of error for Coherent detection binary ASK by three dB .</u>**

Figure (6) show the probability of error for Coherent and non coherent detection ASK system.



Probability of error for Binary and 4-Ary ASK System

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (5 - 2) (Frequency Shift Keying FSK)

In this lecture and beyond we shall study the FSK modulation systems in the following main parts:-

A) - The idea of the FSK modulation system.
B) – The equations of the FSK modulated signal
C) - The waveform of the FSK in time domain
D) – Constellation diagram of the FSK modulation type.
E) – The power spectral density (PSD) of the FSK modulated signal.
F) –The required channel bandwidth for the FSK modulation system.
G) – The generation circuits for the FSK modulation system.
H) – The detection circuits for the FSK modulation system.
I) – The probability of error for the FSK modulation system.

A) - The idea of the FSK modulation system.

We can simply consider the idea of the FSK modulation system by send the logic one (1") of the base band signal by sinusoidal signal with frequency (f1) and send the logic zero (0") of the base band signal by the sinusoidal signal with other frequency (f2).

B) – The equations of the FSK modulated signal .

The equation of the FSK modulated signal can be written as:-

$S(t) = Acos[(wct + 2\pi kf \int Sb(t)dt]$ ……………………………………………(1)

Sb(t) = +V or –V

Δf = KfV the maximum frequency diviation

$S(t) = Acos[(wct + 2\pi\Delta f di(t)]$ ……………………………………………………(2)

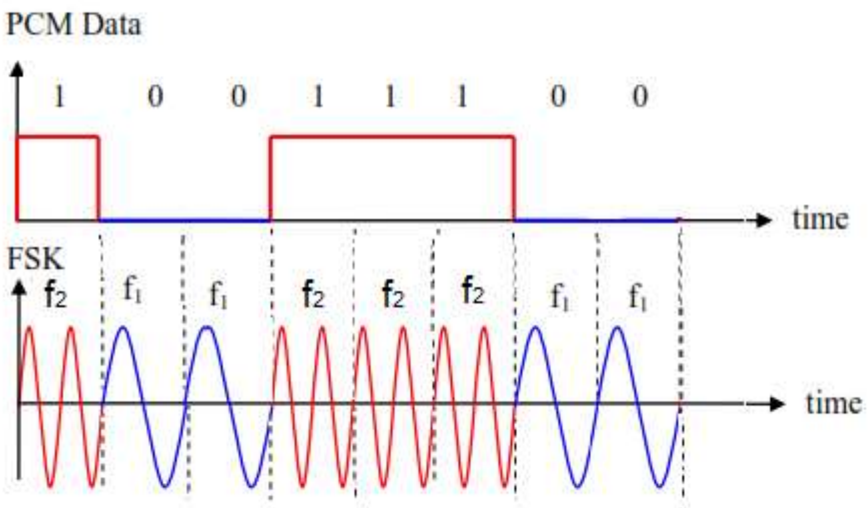
$di(t)$ = 1 for (1") and = -1 for (0")

Then $S1(t) = Acos[(wct + \Delta f\ ]$ ....(3)

$S2(t) = Acos[(wct - \Delta f\ ]$ ....(4)

C) – The waveform of the FSK signal in time domain

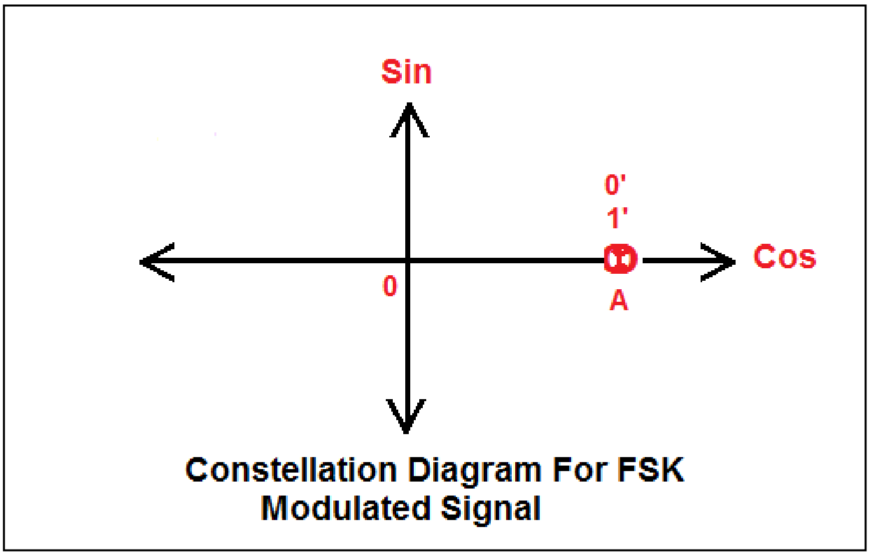
The waveform in time domain for the FSK modulated signal is shown in Figure (1) below:-



Waveform for FSK modulation signal in time domain

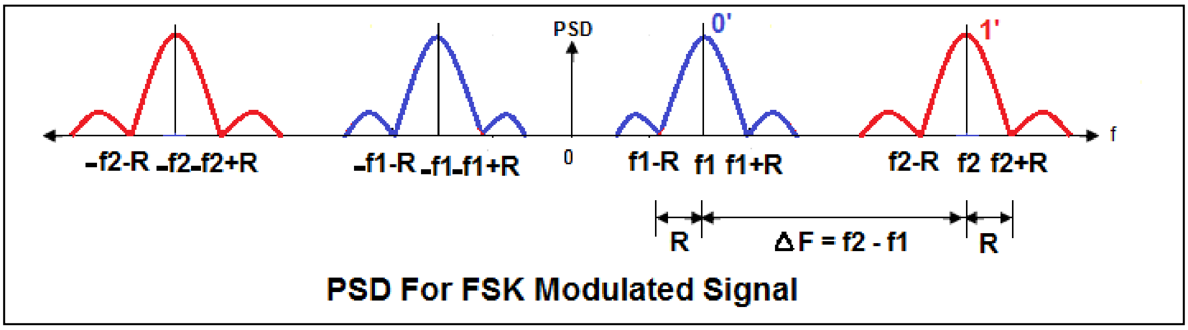
D – Constellation diagram of the FSK modulation type.

Constellation diagram:- Figure (2) show the constellation diagram for FSK signal



E) -The power spectral density (PSD) of the FSK modulated signal.

The power spectral density for the FSK modulated signal is shown in figure (3)



F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the FSK modulated signal above we can extract the required channel bandwidth which represent the frequency band between the two nulls (f1-R) and (F2+R) then

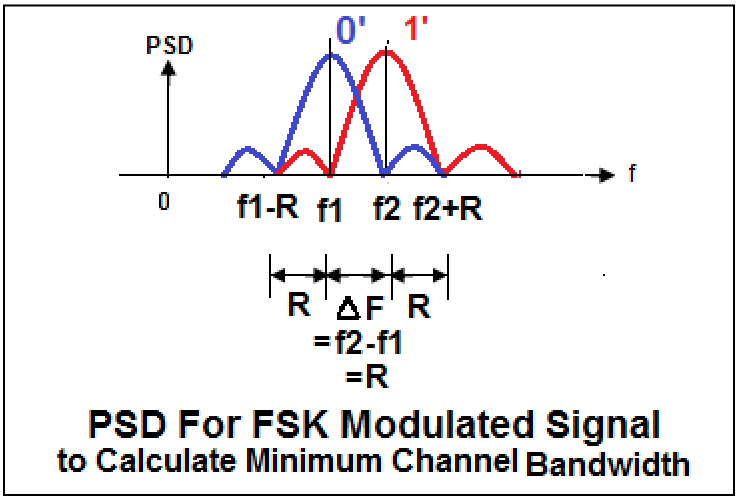
The ***reqBW for FSK = R+ΔF+R*** ……………………………………………………………….(5)

reqBW for FSK = 2R+Δ F

For more correct representation for the required channel bandwidth for the FSK we must write the low as :-

reqBW for FSK = 2* BW for B.B.S + ΔF

The minimum channel bandwidth to transmit the FSK modulated signal can be extract from figure (4) below .



From figure (4) above one can see that minimum require bandwidth for FSK modulated signal done with minimum ΔF (f2-f1) which equal to (R) then

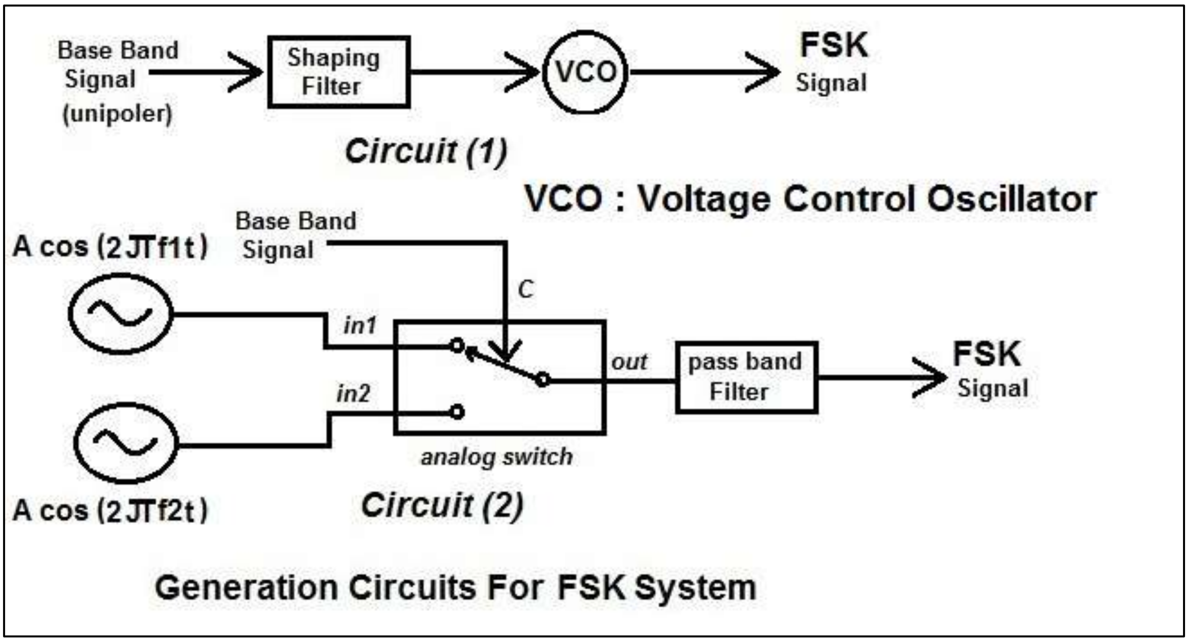
Minimum reqBw for FSK = 3R

For more accurate **minimum reqBw for FSK = 3* Bw B.B.S before modulation process.**

Also one can be see that minimum f1 = R (how ????)

G – The generation circuits for the ASK modulation system.

Two types of generation circuits for the FSK are shown in figure (5) below :-

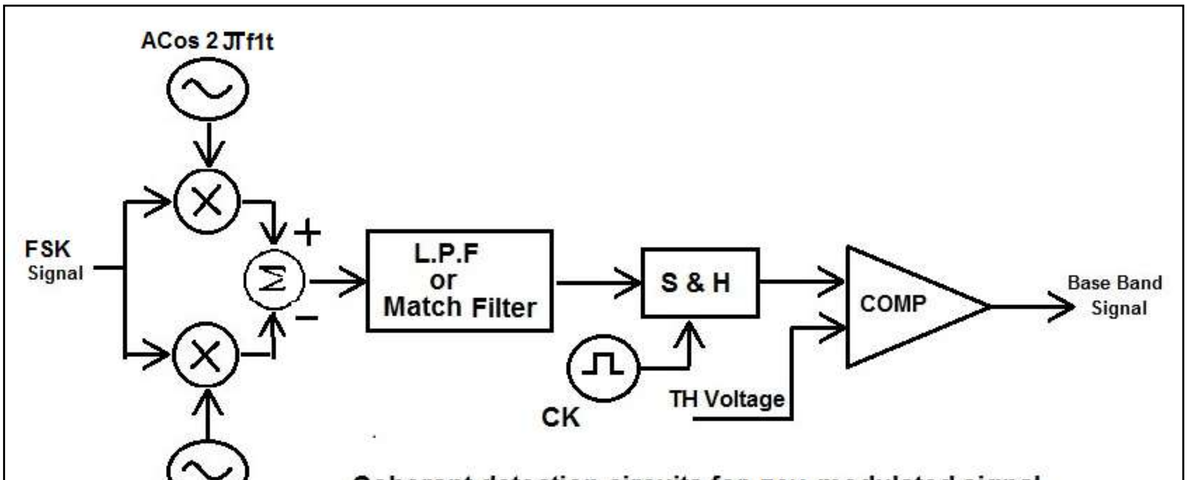


H)– The detection circuits for the FSK modulation system.

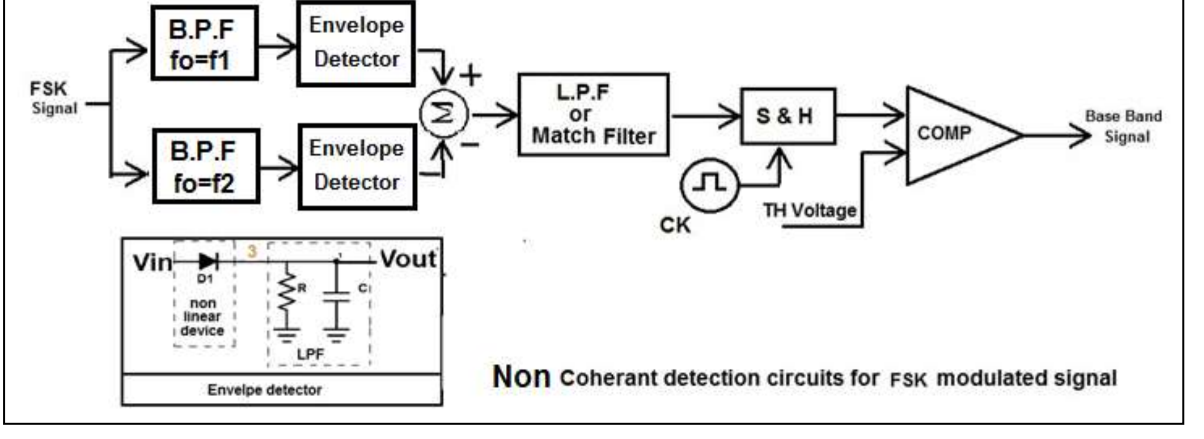
There are two types of detection circuits for the FSK modulated signals

1 - The first type is the ***coherent detection*** circuit which required two sinusoidal sources in the receiver side that has quietly the same specifications for the carrier signals in the generation circuits of FSK modulator circuit .

2 – The second type is the ***non coherent detection*** circuit which not need of the sinusoidal source in the receiver side, its replace this source by two set of non linear device and low pass filter (two set of envelope detector) , these two types of detection circuits are shown in figure (6) and (7) respectively.



Coherant detection circuits for FSK modulated signal



Non Coherant detection circuits for FSK modulated signal

I)– The probability of error for the FSK modulation system.

When comparing the waveform of the FSK signal in time domain with the waveform for the unipolar and bipolar NRZ base band signals we can see clearly that the FSK not similar to none of them but it lies in between of the unipolar and bipolar NRZ ,then we can deduce that the probability of error for the FSK system

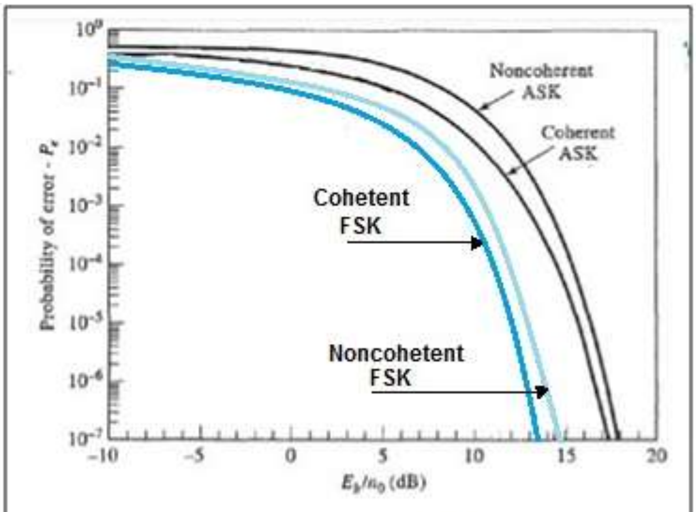
$$Pe_{FSK} = Pe_{NRZ} = Q\left(\sqrt{\frac{S}{N}}\right) \qquad \text{...(6)}$$

Note that equation (6) represent the probability of error for the FSK system when using the coherent detection circuits in the receiver side otherwise the probability of error for FSK system when using non coherent detection circuits is **<u>worst than the system that use the coherent detection circuits by one dB .</u>**

The formula of the probability of error for Non coherent FSK system is

$$Pe_{ASK\ non\ coh} = \frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{e}^{\left(\frac{\boldsymbol{Eo}}{\boldsymbol{No}}\right)} \qquad \text{..(7)}$$

Figure (7) show the probability of error for Coherent and non coherent detection FSK system with the probability 0f error for ASK.



Probability of error for FSK System

From figure (7) above, one can be see that ***<u>the probability of error for coherent FSK are better than this in the coherent ASK by 3dB</u>*** and the probability of error for none coherent FSK are worst than the coherent FSK by 1dB.

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (6-2)

(M-Ary FSK)

A) <u>- The idea of the M-ary FSK modulation system.</u>

From the past lecture we deduced that the gain which we get from the FSK system is the main enhancement of the probability of error over the ASK system, while the FSK system suffer from the large required bandwidth (3*R)

therefore the researchers and the communication engineers thought about using M-Ary FSK to reduce the required bandwidth of the system. in this lecture we shall study 4_Ary FSK system which combine two bits into one symbol, each symbol have same amplitude and different frequency from the other.

B) – <u>The equations of the 4-Ary FSK modulated signal</u> .

The equation of the 4-Ary FSK modulated signal can be written as:-

$$S(t) = A[\cos(2\pi fct + \frac{2\pi Nt}{2TB}) \quad \text{...}(1)$$

$$S(t) = A[\cos(2\pi fct + \frac{2\pi NRt}{2}) \quad \text{...}(2)$$

R=1/Tb

N =1,2,3,……M

$$S00(t) = A\cos(2\pi(fc + \frac{R}{2})t$$

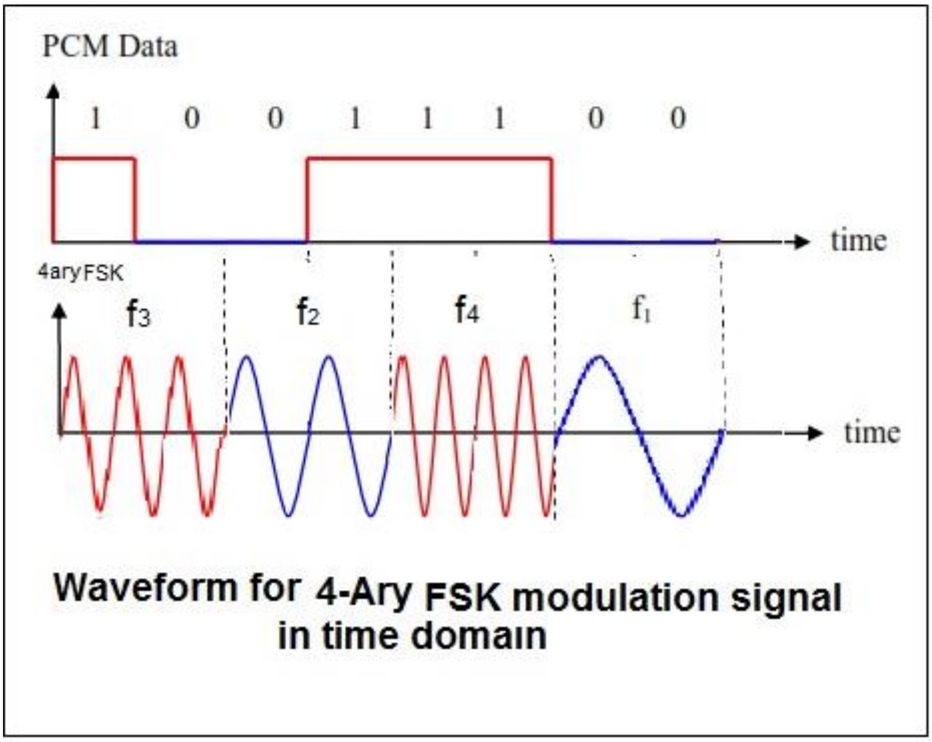
$$S01(t) = A\cos(2\pi(fc + R)t$$

$$S10(t) = A\cos(2\pi(fc + \frac{3R}{2})t$$

$$S11(t) = A\cos(2\pi(fc + 2R)t$$

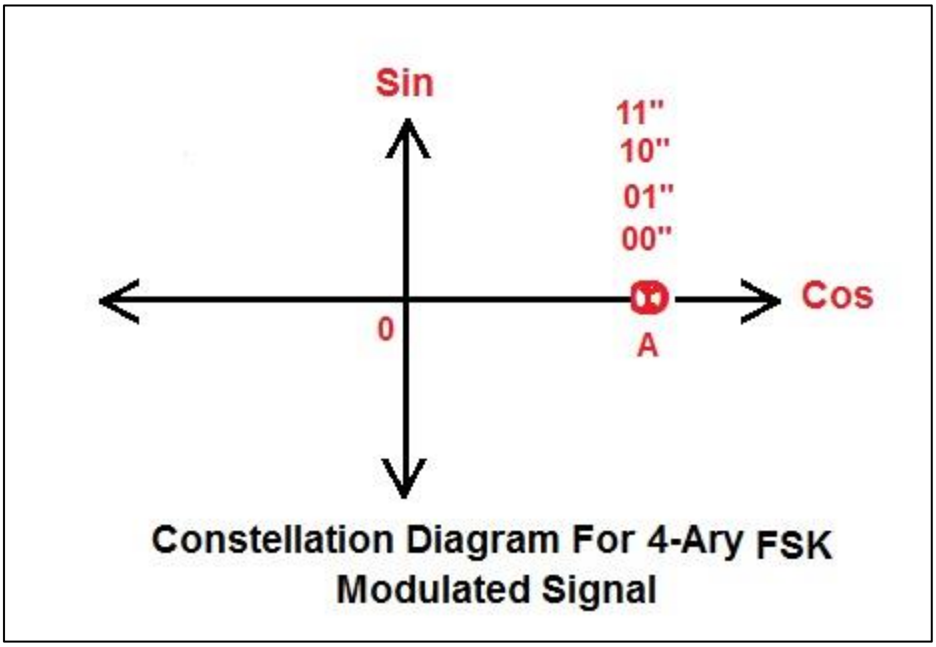
C) <u>– The waveform in time domain</u>

The waveform in time domain for the 4-Ary FSK modulated signal is shown in Figure (1) below:-



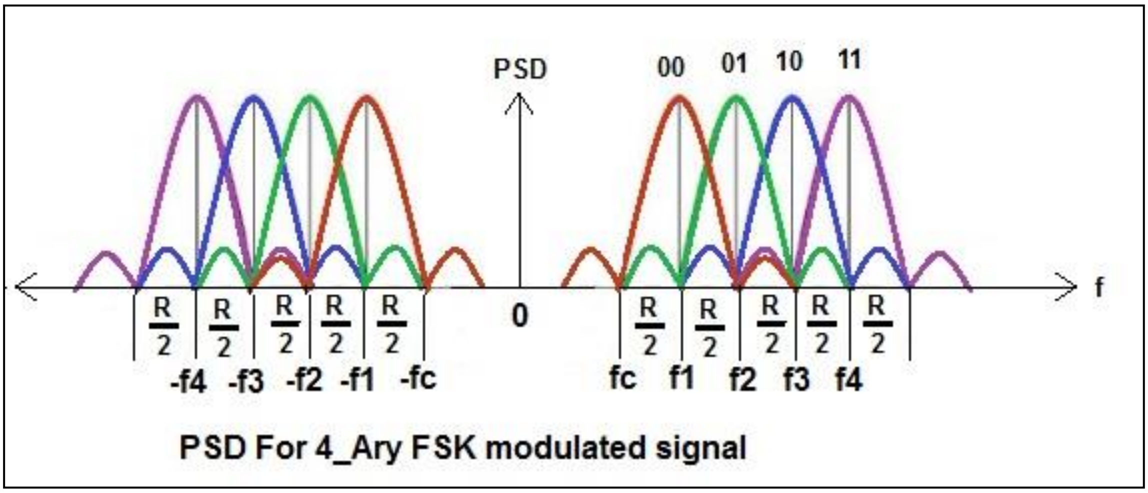
D – Constellation diagram of the 4-Ary FSK modulation type.

figure (2) show the constellation diagram for the 4-Ary FSK



E) -The power spectral density (PSD) of the 4-Ary FSK modulated signal.

The power spectral density for the 4-Ary FSK modulated signal is shown in figure (3)



F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the 4-Ary FSK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (f1-R/2) and (F4+R/2) then

The ***reqBW for 4-Ary FSK = 5R/2 =*** ……………………………………………………….(7)

For more correct representation for the required channel bandwidth for the 4-Ary FSK we must write the low as :-

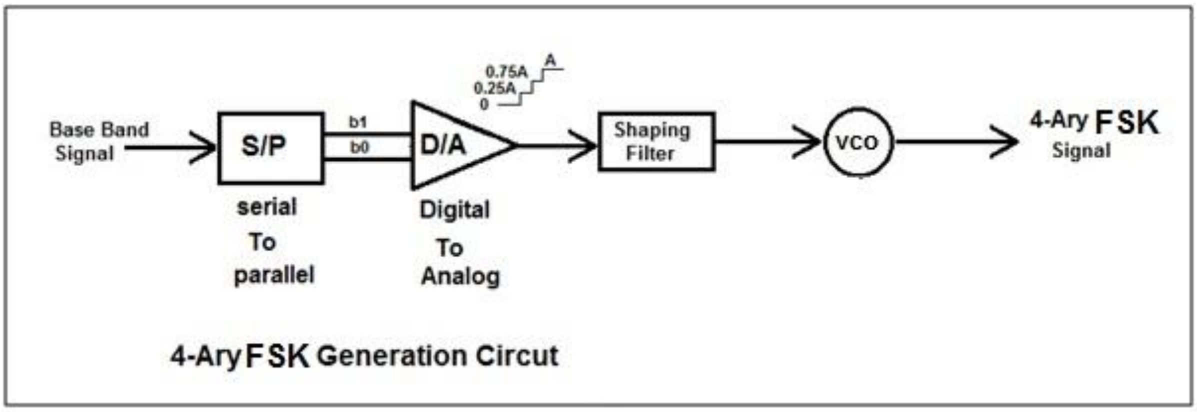
req BW for 4-Ary FSK = 5/2 *BW for B.B.S before the modulation process.

In general we can write the equation of the required bandwidth for the

M-Ary FSK is reqBw M-Ary ASK = $\frac{[M+1]*BW\ for\ B.B.S}{Log_{2}(M)}$

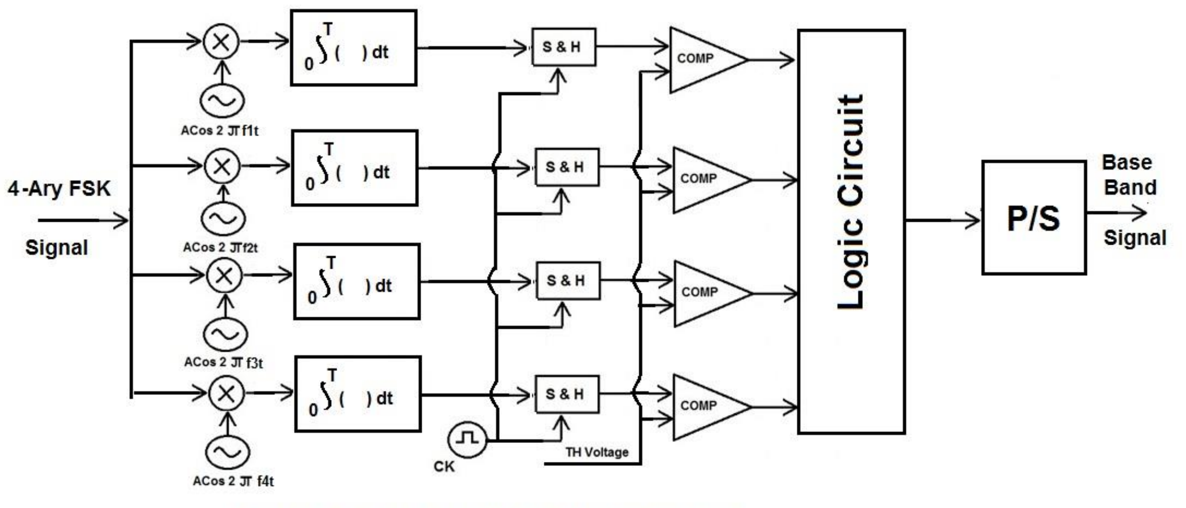
G – The generation circuits for the 4-Ary FSK modulation system.

The generation circuit for the 4-Ary FSK are shown in figure (4) below :-

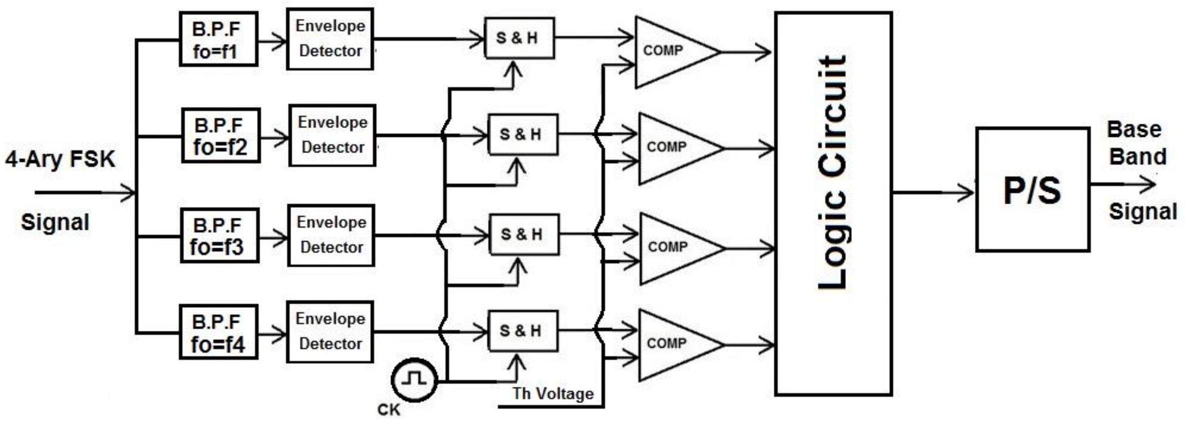


H)– The detection circuits for the M-Ary FSK modulation system.

The Coherent Detection and non coherent detection circuits for the 4-Ary FSK modulated signals are Shown in figure(5) and figure(6) below.



Coherent Detection Circuit For 4-Ary FSK System



Non Coherent Detection Circuit For 4-Ary FSK System

I)– The probability of error for the M-Ary FSK modulation system.

The probability of error for the M-Ary FSK system can be written as

$$Pe = \frac{Ps}{Log_{2}M} \qquad \text{..(8)}$$

Where P_s is the symbol probability of error which equal for coherent detection are :-

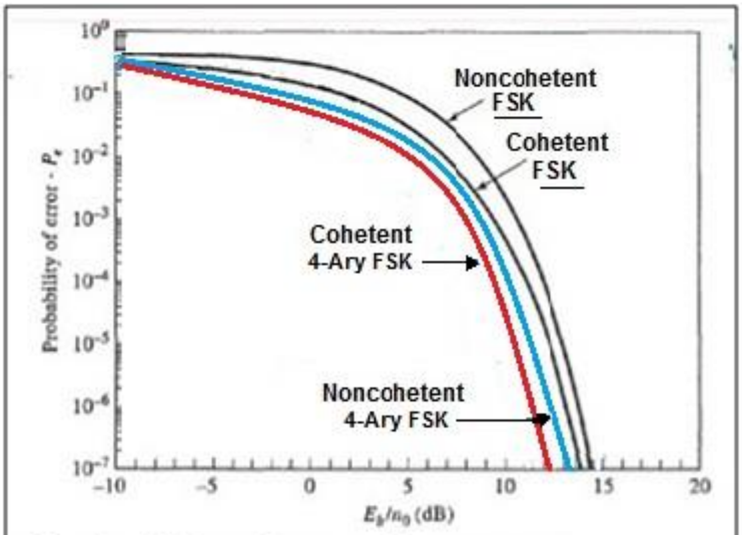
$$Ps = \frac{(M-1)}{Log_{2M}} \; Q\left(\sqrt{\frac{Eo}{No}}\right) \quad \text{..(9)}$$

And Ps for non coherent detection are ;-

$$Ps = \frac{M-1}{2} \; e^{-(\frac{Eo}{2No})} \quad \text{..(10)}$$

Note that equation (9 & 8) represent the probability of error for the Coherent 4-Ary FSK system which is **<u>better than the probability of error for Coherent detection binary FSK by about 1.6 dB .while the probability of error for non coherent detection 4-Ary FSK is worst than that in coherent detection 4_Ary FSK by one dB.</u>**

Figure (7) show the probability of error for Coherent and non coherent detection 4-Ary FSK system.



Probability of error for FSK And 4-Ary FSK System

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (7 - 2) (Phase Shift Keying PSK)

In this lecture we shall study the PSK modulation systems in the following main parts:-

A) - The idea of the PSK modulation system.
B) – The equations of the PSK modulated signal
C) - The waveform of the PSK in time domain
D) – Constellation diagram of the PSK modulation type.
E) – The power spectral density (PSD) of the PSK modulated signal.
F) –The required channel bandwidth for the PSK modulation system.
G) – The generation circuits for the PSK modulation system.
H) – The detection circuits for the PSK modulation system.
I) – The probability of error for the PSK modulation system.

A) - The idea of the PSK modulation system.

We can simply consider the idea of the PSK modulation system by send the logic one (1") of the base band signal by sinusoidal signal with frequency (fc) and amplitude (A) and phase (ϕ1)and send the logic zero (0") of the base band signal by the sinusoidal signal with seam frequency (fc) and amplitude (A) but with other phase (ϕ2). Note that the phase of the carrier signal must be synchronized with the starting (rise edge) of the Base band signal bit.

B) – The equations of the PSK modulated signal .

The equation of the PSK modulated signal can be written as:-

$S(t) = Acos[(wct + \Delta\emptyset di(t)]$ ……..(1)

$di(t)$ = 1 for (1") and = -1 for (0")

Then $S1(t) = Acos[(wct + \Delta\emptyset\ ]$ ……..(2)

$S0(t) = Acos[(wct - \Delta\emptyset\ ]$ ……..(3)

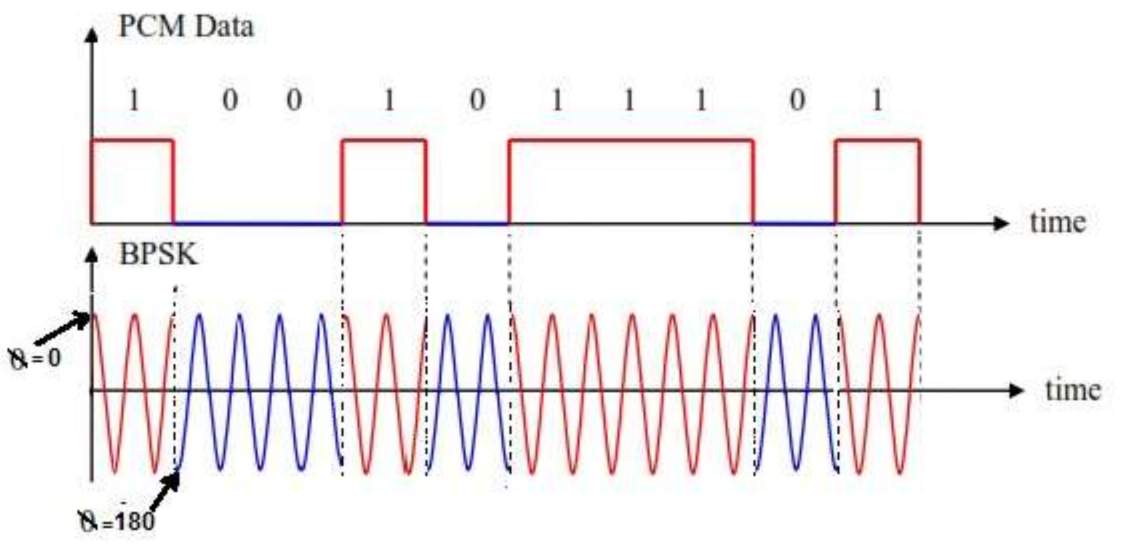
$$S1(t) = A\cos[(wct + \emptyset 1)\ ] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

$$S0(t) = A\cos[(wct + \emptyset\ 0)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

Usually $(\emptyset 1 = 0\ )$ and $((\emptyset 0 = \ \pi)$

C) – The waveform of the PSK signal in time domain

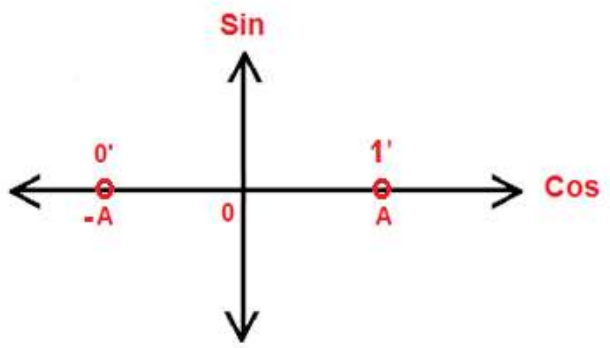
The waveform in time domain for the PSK modulated signal is shown in Figure (1) below:-



Waveform for PSK modulated signal in time domain

D – Constellation diagram of the PSK modulation type.

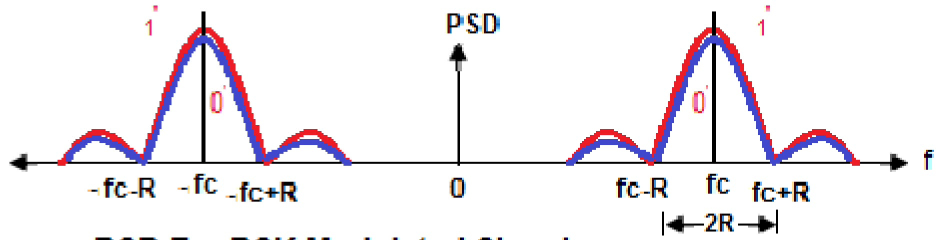
Constellation diagram:- Figure (2) show the constellation diagram for PSK signal



Constellation Diagram For PSK Modulated Signal

E) -The power spectral density (PSD) of the PSK modulated signal.

The power spectral density for the PSK modulated signal is shown in figure (3)



PSD For PSK Modulated Signal

F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the PSK modulated signal above we can extract the required channel bandwidth which represent the frequency band between the two nulls (Fc-R) and (Fc+R) then

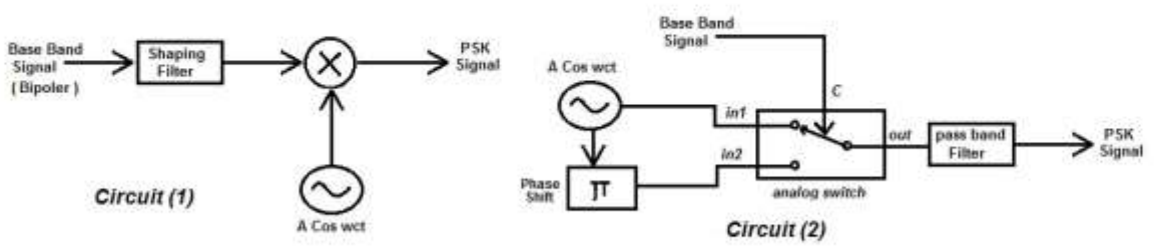
The ***reqBW for PSK = 2R*** ……………………………………………………………….(6)

For more correct representation for the required channel bandwidth for the PSK we must write the low as :-

reqBW for PSK = 2* BW for B.B.S

G – The generation circuits for the PSK modulation system.

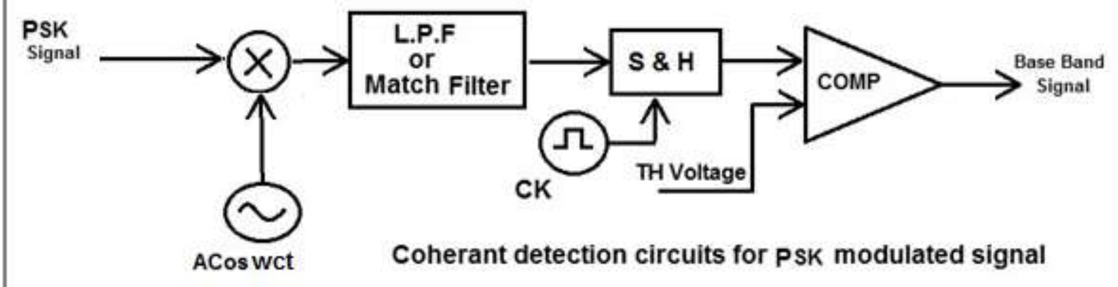
Two types of generation circuits for the PSK are shown in figure (4) below :-



Generation Circuits For PSK System

H)– The detection circuits for the PSK modulation system.

There is only one type of detection circuits for the PSK modulated signals Which is the ***coherent detection*** circuit which required one sinusoidal sources in the receiver side that has quietly the same specifications for the carrier signal in the generation circuits of PSK modulator circuit. This detection circuit is shown in figure (6) below.



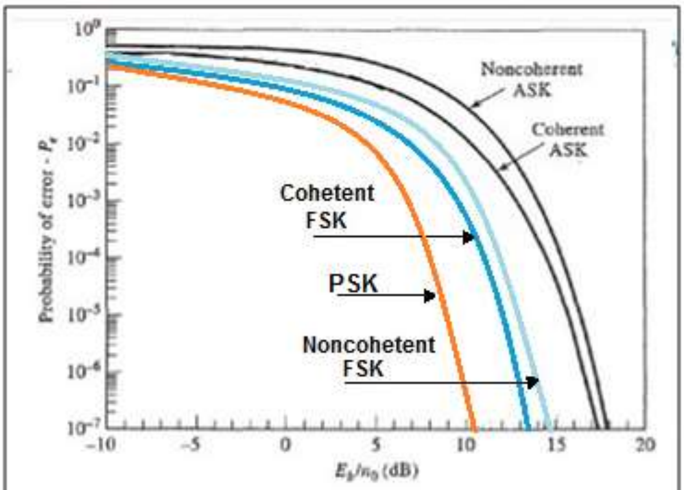
Coherant detection circuits for PSK modulated signal

I)– The probability of error for the PSK modulation system.

When comparing the waveform of the PSK signal in time domain with the waveform for the unipolar and bipolar NRZ base band signals we can see clearly that the PSK are similar to bipolar NRZ ,then we can deduce that the probability of error for the PSK system

$$Pe_{PSK} = Pe_{NRZ} = Q\left(\sqrt{\frac{2S}{N}}\right) \quad \text{...(7)}$$

Figure (7) show the probability of error for Coherent detection PSK system with the probability 0f error for FSK and ASK.



Probability of error for PSK System

From figure (7) above, one can be see that ***<u>the probability of error for coherent PSK are better than this in the coherent FSK by 3dB</u>*** and better than the probability of error for none coherent ASK by 6 dB.

- ***- Carrier Recovery :-***

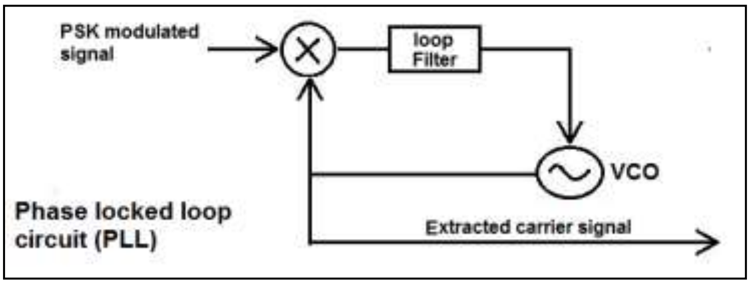
In the transmitter of a digital communications system, a carrier wave is modulated by a baseband signal. At the receiver the baseband information is extracted from the incoming modulated waveform. In an the PSK modulation system, the carrier signal oscillators of the transmitter and receiver must be perfectly matched in frequency and phase thereby permitting perfect coherent demodulation of the modulated baseband signal. However, transmitters and receivers rarely have the same carrier oscillator specification.

Communications receiver systems are usually far away from the transmitting systems and contain their own oscillators with frequency and phase offsets and instabilities. Doppler shift may also contribute to frequency differences in mobile radio frequency communications systems.

All these frequency and phase variations must be estimated from the modulated received signal to reproduce or recover the carrier signal at the receiver and permit coherent demodulation.

Carrier signal extracting methods :-

1 - For PSK signals with phase shift between the two symbols **less than 180 degree**, the phase locked loop circuit can extract the carrier signal from the modulated PSK signal as shown in figure (8) below :-



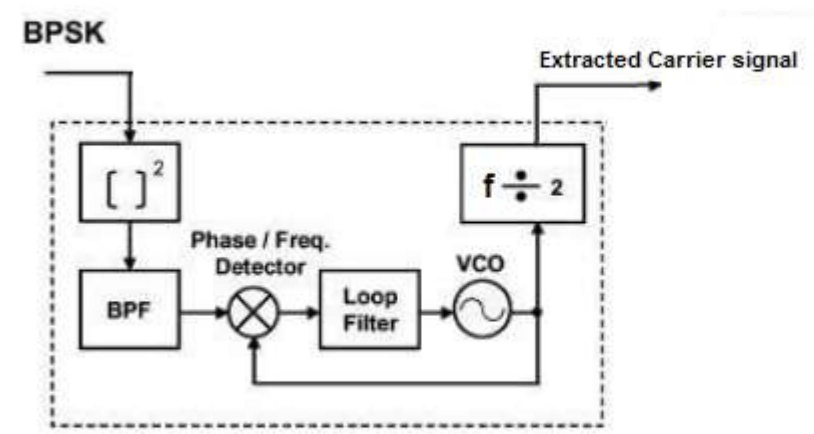
A **phase-locked loop** or **phase lock loop** (**PLL**) is a control system that generates an output signal whose phase is related to the phase of an input signal. There are several different types; the simplest is an electronic circuit consisting of a variable frequency oscillator (VCO) and a phase detector in a feedback loop. The oscillator generates a periodic signal, and the phase detector compares the phase of that signal with the phase of the input periodic signal, adjusting the oscillator to keep the phases matched.

2 – For the BPSK with phase shift equal 180 degree between the two symbols there are two methods for extracting the carrier signal from the PSK modulated signal,

A- The squaring method

B- The Costas loop method

- -The squaring method :- in this method the BPSK signal is squared and band pass filtered before inter the PLL circuit, the squared process cause doubles the frequency of the carrier signal, this twice frequency term requires filtering (within the PLL) to remove the channel noise, and then the frequency must be halved to get the required coherent carrier frequency.



Squaring loop carrier recovery

Let m(t) be the message signal transmitted.

The BPSK modulated signal is given by A m(t) cos(ωc t + θc).

The squared output is given by A2m 2 (t) cos2 (ωc t + θc).

The BPF output is given by A0cos2(ωc t + θc).

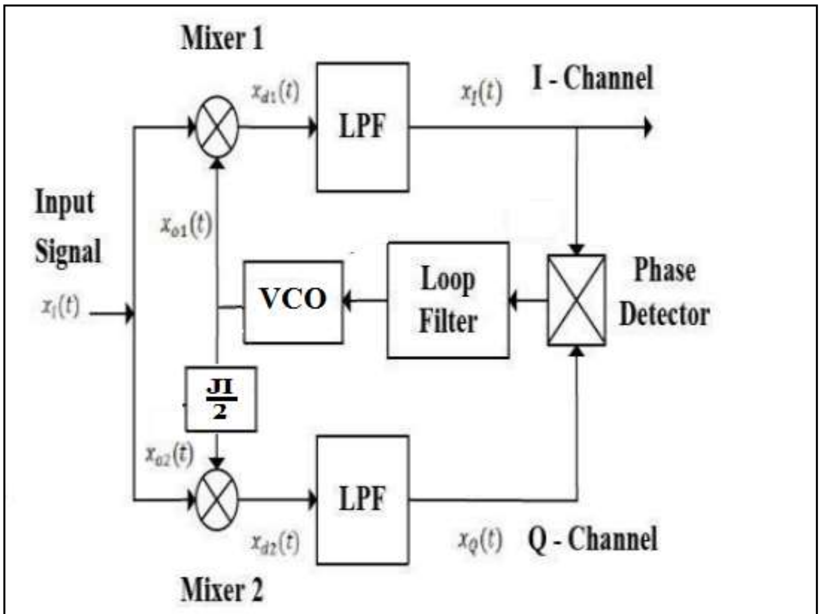
PLL output is given by A0cos2(ωc t + θe).

Frequency divider output is given by A0cos(ωc t + θe)

The recovered carrier output is given by cos(ωc t + θe)

The recovered carrier has phase error of θe

- - The Costas loop recover method is an optimal method to attain data and carrier recovery for BPSK signal. It comprises of mixer, low pass filter, phase detector, loop filter and voltage controlled oscillator (VCO) and phase shifter by (Л /2). The arm connected to in phase signal is called I channel and the output of the (Л/2) phase shift is connected to quadrature phase signal is called Q channel. The BPSK modulated signal is multiplied with in phase and quadrature phase carrier signal. They are then passed through LPF where high frequency component are filtered out. The phase detector estimates the phase difference between the two arms of the signals. The error signal is given to loop filter where it removes the unnecessary spikes. The loop filter controls the phase and frequency of VCO output signal which gives the carrier signal.



Costas loop recover method

Let BPSK modulated output is given by m(t) cos(ωc t + θc)

In phase LPF output is given by m(t) cos(θc- θv)

Q- phase LPF output is given by m(t) sin(θc- θv)

PLL is used to lock the carrier phase of modulated signal with the recovered carrier. The recovered carrier will be cos (ωc t)

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (8 - 2) (differential Phase Shift Keying DPSK)

In this lecture we shall study the DPSK modulation systems in the following main parts:-

A) - The idea of the DPSK modulation system.
B) – The detection circuits for the DPSK modulation system.
C) – The generation circuits for the DPSK modulation system.
D) – The equations of the DPSK modulated signal
E) - The waveform of the DPSK in time domain
F) – Constellation diagram of the DPSK modulation type.
G) – The power spectral density (PSD) of the DPSK modulated signal.
H) –The required channel bandwidth for the DPSK modulation system.
I) – The probability of error for the DPSK modulation system.

A) <u>- The idea of the DPSK modulation system.</u>

As we see in the previous lecture (7-2) the PSK modulation system has the best probability of error (lower than the ASK and FSK) and they has required channel bandwidth lower than the FSK modulation system and equal to the required channel bandwidth of the ASK modulated system, but one can see also that the PSK modulated signal cannot detected by using the non coherent demodulation circuit. This cause that the PSK demodulation circuits be more complexity of the detection circuits than the ASK and FSK systems. The researchers and the communication engineers exerted their utmost efforts to solve this problem, and they came up with the idea of depending on the past received signal in order to be a reference to the just received signal.

By correlated the just received signal with the previous signal we can conclude two cases

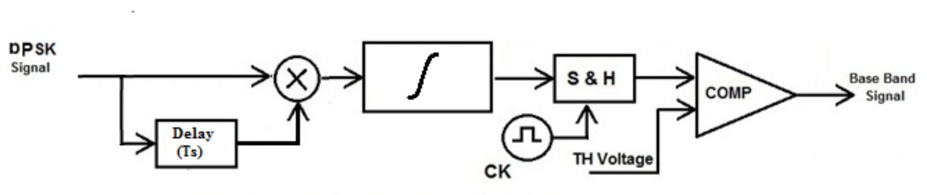
* - If the output of the correlator give the maximum power at t = Ts, then the just received signal is identical to the previous received signal,

* - If the correlator output not give the maximum power at t = Ts, then the just received signal is opposite to the previous received signal.

in this way the coherent sinusoidal signal generator can be cancelled in the receiver side. But note that we shall get the data which represent the difference between the previous received signal and the just received signal not the origin transmitted base band signal, to solve this problem we must made re arranging of the base band signal in the receiver side to avoid this problem. These ideas can be understands by explaining detection and generation circuits for the DPSK modulation system.

B)– The detection circuits for the DPSK modulation system.

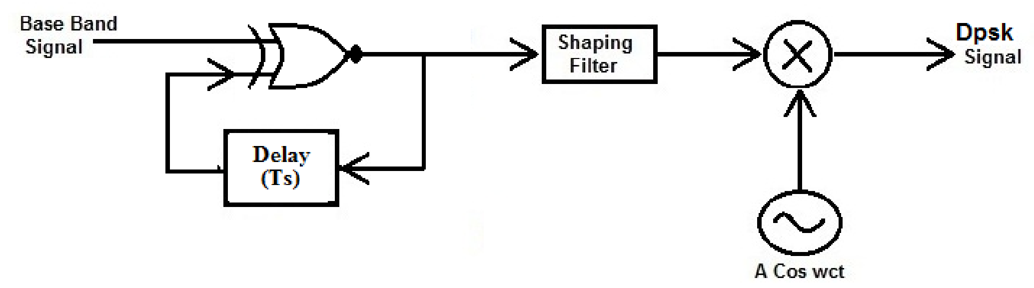
As illustrated in the above paragraph the detection circuit for DPSK contain delay unit by t= Ts (symbol time), correlator, and decision circuit which include S/H and comparator, the detection circuit for DPSK are shown in figure (1) below.



Detection circuits for DPSK modulated signal

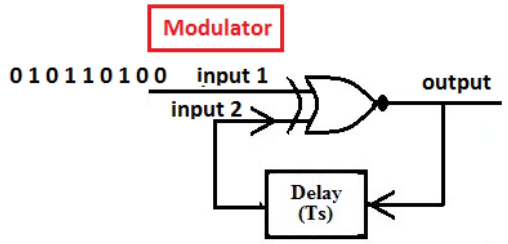
C – The generation circuits for the DPSK modulation system.

In order to avoid the problems in the detected data in the receiver side which illustrated in paragraph (A), we must made in the modulated circuit the opposite action on the data that done in the receiver, if we consider the correlator in the receiver represent the (XNOR) function (if the two inputs are identical, the output be 1"), then in the modulation circuit we must add the (XNOR) that has two inputs (the present data and the previous output for the XNOR gate), the generation circuit for DPSK are shown in figure (2) below

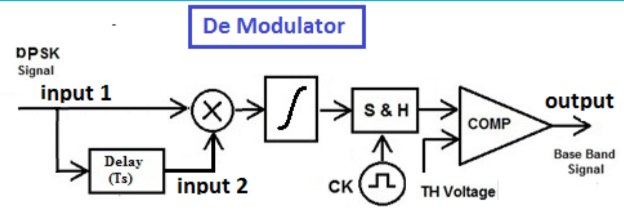


Generation Circuits For DPSK System

In order insure that the generation and the detection circuits are work correctly let us exam the generation and detection circuits by the following data stream (001011010).



Input 1	Input 2	output
0	0	1
0	1	0
1	0	0
0	0	1
1	1	1
1	1	1
0	1	0
1	0	0
0	0	1



Input 1	Input 2	output
1	0	0
0	1	0
0	0	1
1	0	0
1	1	1
1	1	1
0	1	0
0	0	1
1	0	0

D) – <u>The equations of the DPSK modulated signal</u> .

The equation of the DPSK are same in PSK modulated signal but the input base band signal that specified the ($di(t)$ = -1 or +1) is not the real input base band signal but its be the output of the XNOR gate (Y xnor) in the receiver as we illustrate in previous paragraphs, then the equations of the DPSK modulated signal can be written as :-

$S(t) = Acos[(wct + \Delta\emptyset di(t)]$ ………………………………………………(1)

Y xnor = (Y xnor (i-1)) XNOR b(i)

Where
Y xnor is the present value of the XNOR output
b(i) is the present input bit from data stream
y xnor (i-1) is the previous value of the XNOR output

$di(t)$ = 1 (if Y xnor =(1")) $di(t)$ = -1 (if Y xnor =(0"))

Then $S1(t) = Acos[(wct + \Delta\emptyset\ ]$ ……………………………………………………(2)

$S0(t) = Acos[(wct - \Delta\emptyset\ ]$ ……………………………………………………(3)

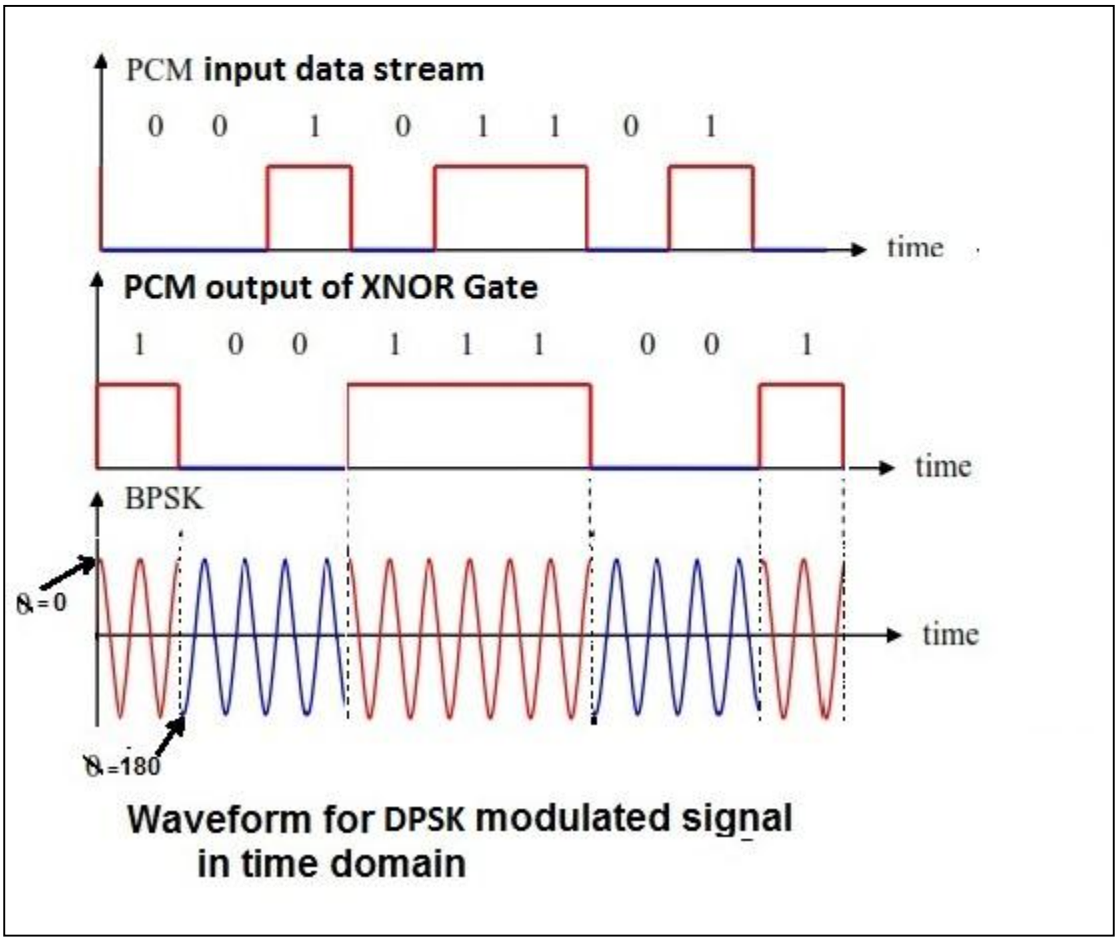
$S1(t) = Acos[(wct + \emptyset 1)\ ]$ ……………………………………………………(4)

$S0(t) = Acos[(wct + \emptyset\ 0)]$ …………………………………………………(5)

Usually ($\emptyset 1 = 0$) and (($\emptyset 0 = \ \pi$)

E) – The waveform of the DPSK signal in time domain

The waveform in time domain for the DPSK modulated signal is shown in Figure (3) below:-

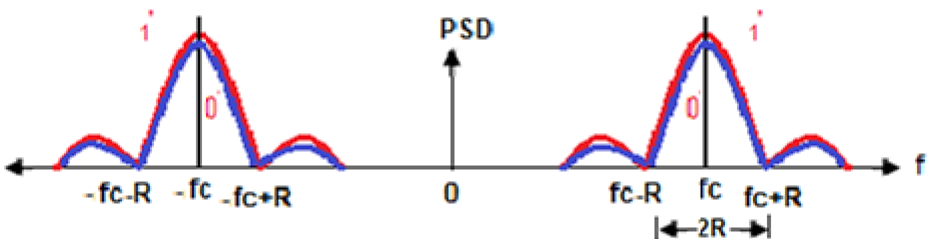


F – Constellation diagram of the DPSK modulation type.

As shown Figure (3) above for the waveform of the DPSK modulated signal, there are no specific symbol for all input 0" but the (0") may be represented by sinusoidal signal in 0 started with 0 degree phase or with 180 degree phase angle, and this also happened for (1"), so that there is no constellation diagram can be specify the DPSK system.

E) -The power spectral density (PSD) of the DPSK modulated signal.

The power spectral density for the DPSK modulated signal is same in PSK modulated signal, this can be shown in figure (4) below



PSD For DPSK Modulated Signal

F) –The required channel bandwidth for the DPSK modulation system.

From the PSD diagram for the DPSK modulated signal above we can conclude that it's identical to the PSD for PSK system and so the required channel bandwidth for DPSK is equal to required channel bandwidth for PSK

The ***reqBW for DPSK = 2R***

For more correct representation for the required channel bandwidth for the DPSK we must write the low as:-

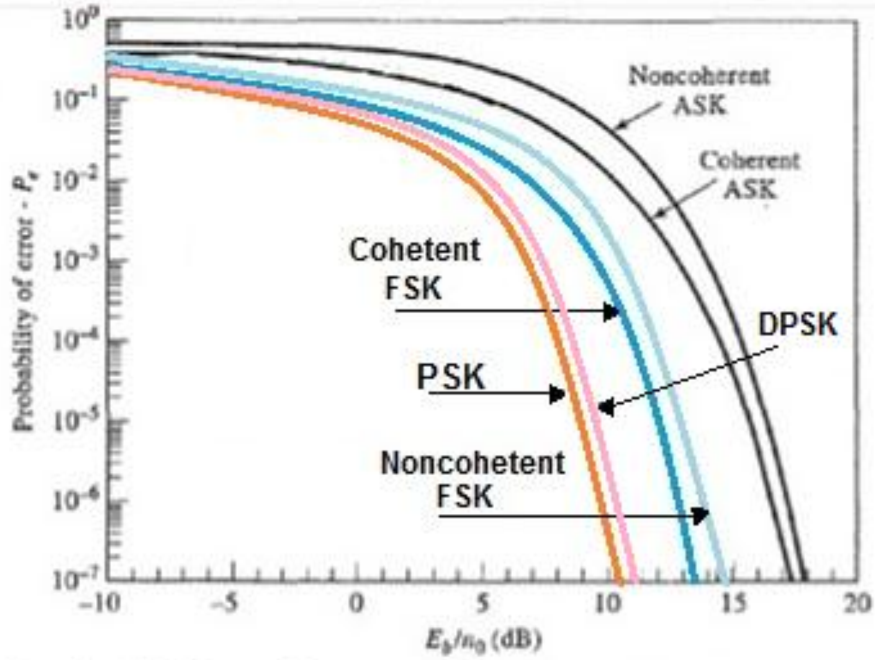
reqBW for DPSK = 2* BW for B.B.S

I)– The probability of error for the DPSK modulation system.

The equation of the probability of error for the DPSK can be written as :- em

$$Pe_{DPSK} = \frac{1}{2}e^{\frac{-Eb}{No}}$$

Figure (5) show the probability of error for DPSK and PSK system with the probability 0f error for FSK and ASK.



Probability of error for DPSK,PSK, FSK and ASK Systems

From figure (5) above, one can be see that ***the probability of error for coherent DPSK are better than this in the coherent FSK by 2dB*** and better than the probability of error for coherent ASK by 5 dB but it is worse than the probability of error for PSK by one dB.

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (9-2) (M-Ary PSK) (QPSK)

In order to reduced the required channel bandwidth, the researchers and the communication engineers thought about using M-Ary PSK as will as using the m-ary ASK and Mary FSK, the M-ary PSk system are largely used in digital communication systems, specially the 4-ary PSK which called QPSK (Quadrature Phase Sift Keying) and 8-PSK which is used in the satellite communication channels, in this lecture we shall study the QPSK modulation systems while the 8-psk will be study in the next lectures**.**

A) - The idea of the 4-aryPSK modulation system.

The idea of QPSK is combine each two bits from B.B.S and send them in 4 types of symbols, these symbols have the same amplitude and frequency, but each of them has specific Phase at the starting of B.B.S bit, as the whole phase can be represent the sinusoidal signal are equal 360 degree (2π) then the phase difference between each neighboring symbols must be 90 degree, for this reason the expression (quadrature) called for 4-ary PSK.

B) – The equations of the QPSK modulated signal.

The equation of the QPSK modulated signal can be written as:-

$S_{QPSK}(t) = A\cos(wct + \phi)$ ………………………………………………………..(1)

$\phi = (2i - 1) * \frac{\pi}{4}$ ………………………………………………………..(2)

$S_{QPSK}(t) = A\cos(wct + [(2i - 1) * \frac{\pi}{4}])$ ……………………………….(3)

$S11(t) = A\cos(wct + \frac{\pi}{4})$ for (i = 1)

$S01(t) = A\cos(wct + 3\frac{\pi}{4})$ for (i = 2)

$S00(t) = A\cos(wct + 5\frac{\pi}{4})$ for (i = 3)

$S10(t) = A\cos(wct + 7\frac{\pi}{4})$ for (i = 4)

Using the trigonometric expansion equation (3) can be written as

$$S_{QPSK}(t) = A\cos\left[(2i-1)\,\frac{\pi}{4}\right] cos(wct) + A\sin\left[(2i-1)\,\frac{\pi}{4}\right] sin(wct)\ ) \;...(4)$$

Eq (4) above can be written as

$$S_{QPSK}(t) = a\cos(wct) + b\sin(wct) \quad ....(5)$$

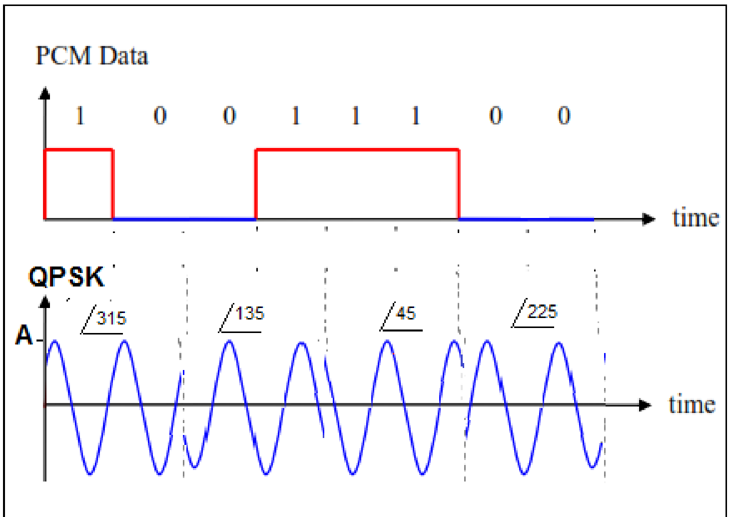
Where $a = A\cos\left[(2i-1)\,\frac{\pi}{4}\right]$ and $b = A\sin\left[(2i-1)\,\frac{\pi}{4}\right]$

From Eq. (5) we can calculate the four cases as shown in table (1) below

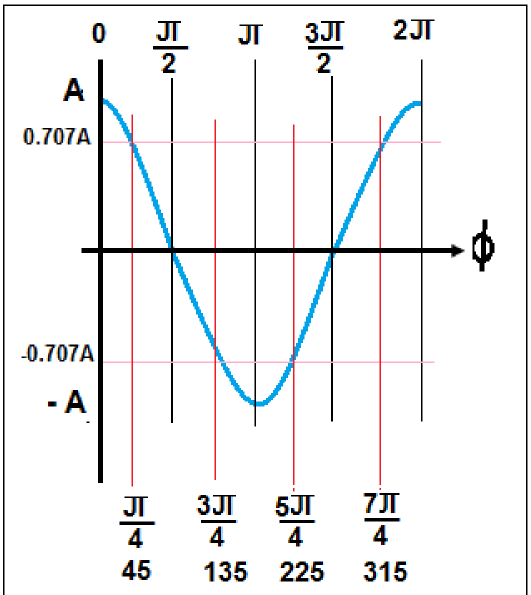
Input data	(i)	$(2i-1)\,\frac{\pi}{4}$	(a)	(b)	QPSK phase
11	1	$\frac{\pi}{4}$	$\frac{A}{\sqrt{2}}$	$\frac{A}{\sqrt{2}}$	$\frac{\pi}{4}$
01	2	$\frac{3\pi}{4}$	$-\frac{A}{\sqrt{2}}$	$\frac{A}{\sqrt{2}}$	$\frac{3\pi}{4}$
00	3	$\frac{5\pi}{4}$	$-\frac{A}{\sqrt{2}}$	$-\frac{A}{\sqrt{2}}$	$\frac{5\pi}{4}$
10	4	$\frac{7\pi}{4}$	$\frac{A}{\sqrt{2}}$	$-\frac{A}{\sqrt{2}}$	$\frac{7\pi}{4}$

C) – The waveform in time domain

The waveform in time domain for the QPSK modulated signal is shown in Figure (1) below:-



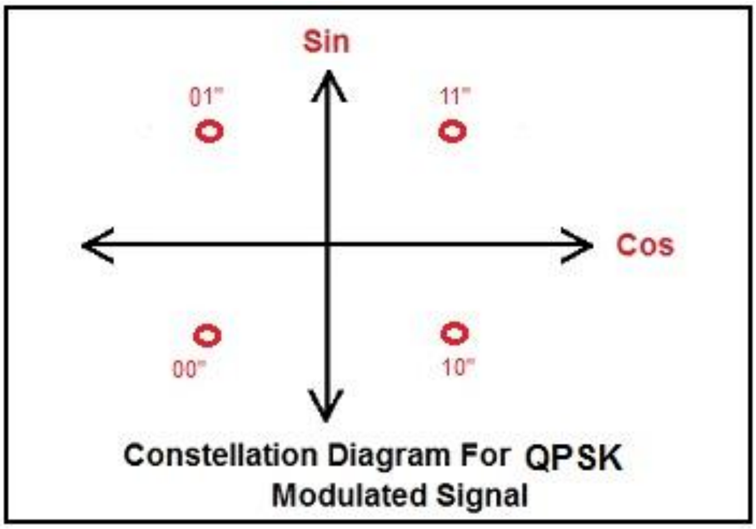
Waveform For QPSK



one cycle from cosine signal

D – Constellation diagram of the QPSK modulation System.

As illustrated in the previous paragraph the QPSK signal contain the cosine component and the sine component, the results of summation the two component produce sinusoidal signal with the phases (45,135,225,315) degree. Figure (2) show the constellation diagram for the QPSK

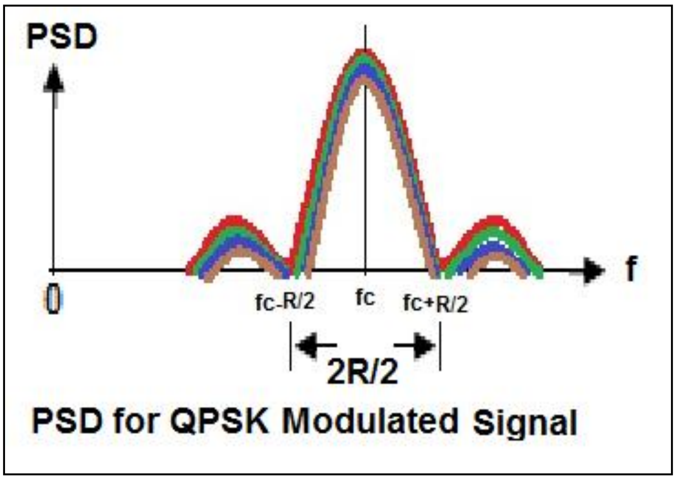


Note :- The constellation diagram shown in figure (2) above is corresponding to the QPSK generation circuit that shown in figure (4) in next paragraphs.

E) -The power spectral density (PSD) of the QPSK modulated signal.

The four symbols of the QPSK signal have the same amplitude and the same frequency, so they appear together in the same place in the PSD.

The power spectral density for the QPSK modulated signal is shown in figure (3)



F) –The required channel bandwidth for the modulation system.

From the PSD diagram for the QPSK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (FC-R/2) and (FC+R/2) then

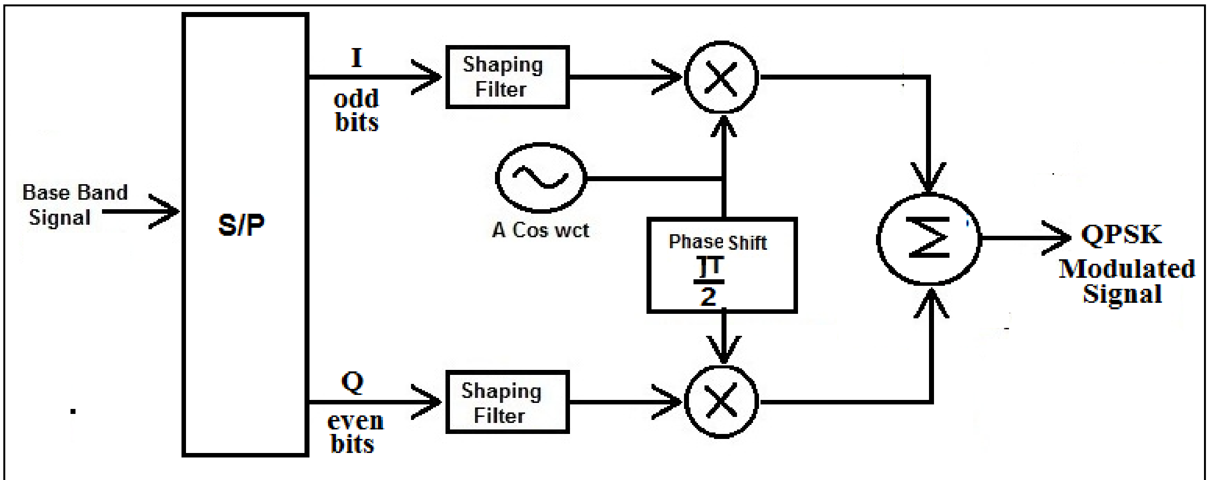
The ***reqBW for QPSK = 2*R/2 = R*** ……………………………………………………(6)

For more correct representation for the required channel bandwidth for the QPSK we must write the low as :-

reqBW for QPSK = BW for B.B.S before the modulation process.

G – The generation circuits for the QPSK modulation system.

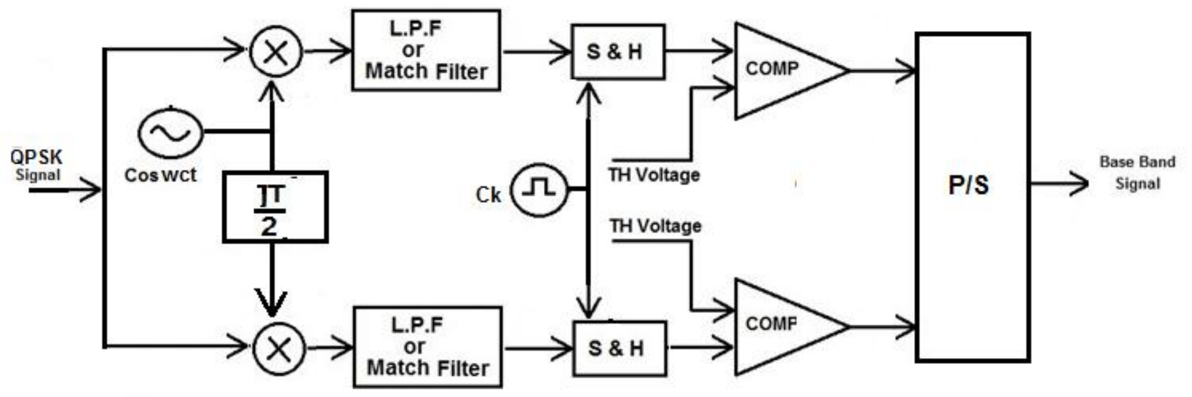
The generation circuit for the QPSK can be considered as two generation circuit for the PSK system, the first to generate the cosine component of the QPSK modulated signal, and the second is for the sine component for the QPSK signal generator are shown in figure (4) below :-



Generation Circuits For QPSK System

H)– The detection circuits for the QPSK modulation system.

The detection circuits for the QPSK modulated signal can be considered as a two detection circuits of the PSK modulation signal, one for the cosine component of the signal and the second for the sine component of the QPSK modulated signal, the carrier recovery will be used in detection circuit to re generate the carrier (cosine wave signal) while the sine wave signal can be extracted from the cosine signal by shifting it by 90 degree (Л/2) using phase shifter circuit, finally each branch of detection circuit extract one bit from the original B.B.S, that extracted in parallel and converted to serial form using (P/S) unit. The detection circuit for QPSK modulated signal shown in figure(5) below.



detection circuits for QPSK modulated signal

I)– The probability of error for the QPSK modulation system.

The probability of error for the QPSK system is the same as that of coherent BPSK especially at S/N large or equal 10dB, and it can be written as

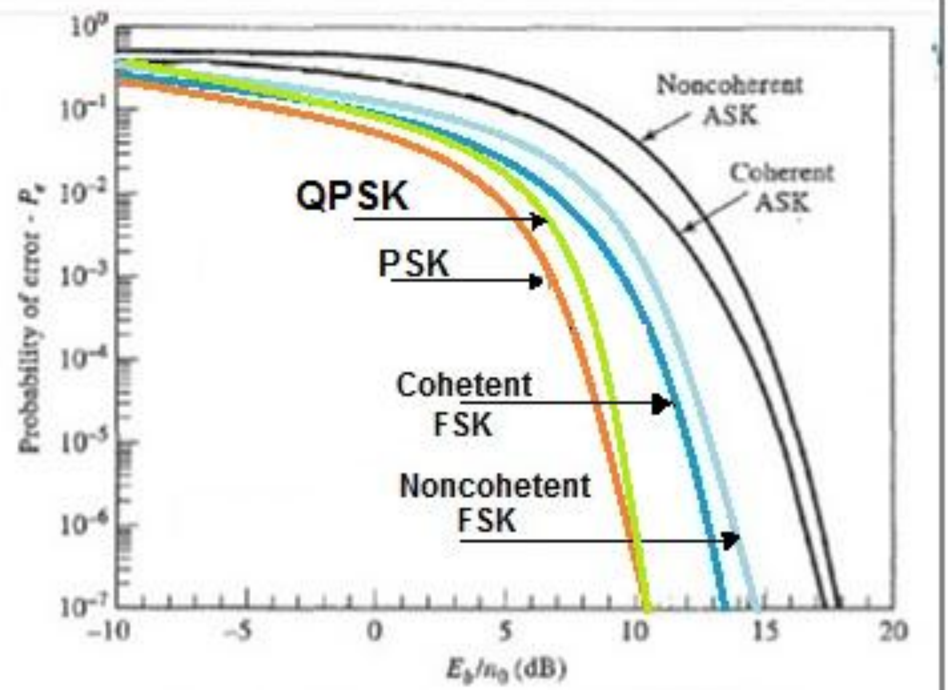
$$Pe_{\ QPSK} = Pe_{\ BPSK} = Q\left(\sqrt{\frac{2S}{N}}\right) = Q\left(\sqrt{\frac{2E}{no}}\right)$$

And the symbol error probability for it is twice that of bit error probability under the same noise and given by:

$$PS_{\ QPSK} = 2\ \ Pe_{\ QPSK} = 2\ Q\left(\sqrt{\frac{2S}{N}}\right) = 2\ Q\left(\sqrt{\frac{2E}{no}}\right)$$

Where P_s is a symbol probability of error.

Figure (6) show the probability of error for QPSK and other systems.



Probability of error for QPSK, PSK, FSK, and ASK Systems

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE(10-2) (Differential Quadrature Phase Shift Keying) (DQPSK)

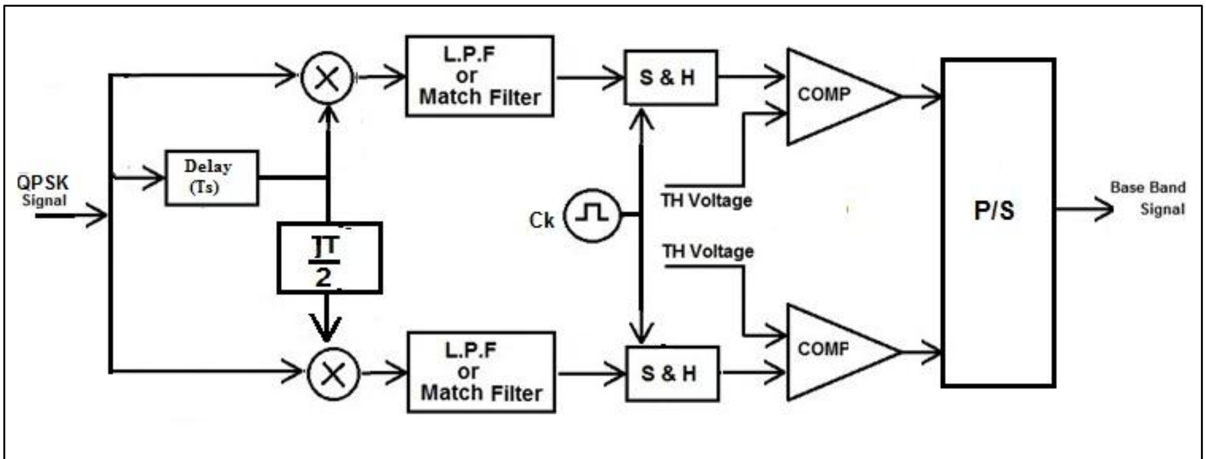
As illustrated in previous lecture, the QPSK system can be considered as an optimum modulation system in both the probability of error and the required channel bandwidth, but the weak point of this system is the complexity in detection circuits. To decrease the complexity of the detection circuit the communication engineers thought about using Differential Quadrature Phase Shift Keying (DQPSK) as well as using the DPSK to decrease the complexity of the detection circuit for PSK system**.**

A) - The idea of the DQPSK modulation system.

The decreasing of QPSK detection circuit complexity, can be achieved by cancelling the carrier recovery circuit in the receiver side, this can be done only by depending on the previous received modulated signal as a reference to the present received signal, as in DPSK demodulator circuits but with duplicate the DPSK system units, because that the QPSK modulated signal consist of two sinusoidal signals (sine and cosine). This duplication in demodulation circuit must be done in the transmitting circuits also as shown in the next paragraphs.

B)– The detection circuits for the DQPSK modulation system.

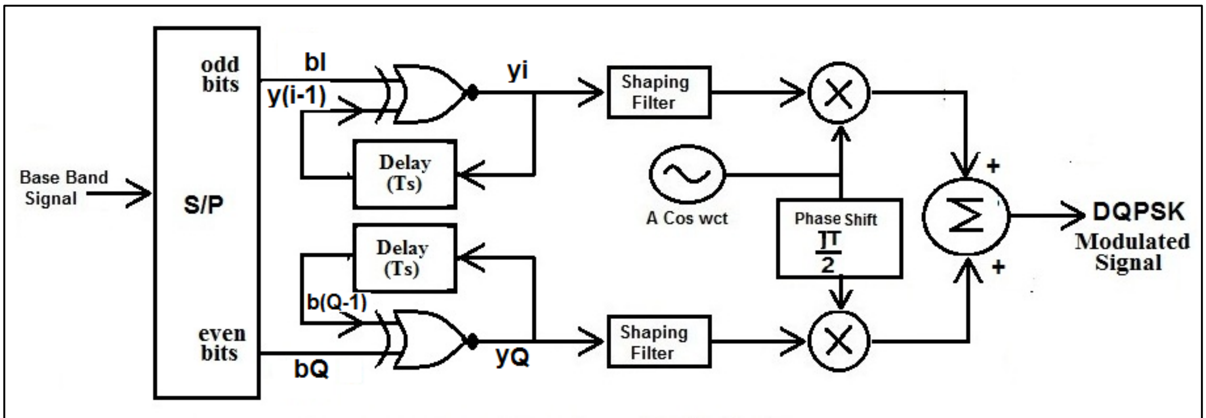
The detection circuits for the DQPSK modulated signal can be implemented by remove the carrier recovery unit from the QPSK detection circuit and adding delay by (TS) unit and phase shifter by 90 degree (Л/2) unit to this circuit. The detection circuit for DQPSK modulated signal shown in figure (1) below.



Detection Circuits for DQPSK Signal

C – The generation circuits for the DQPSK modulation system.

The generation circuit for the DQPSK can be considered as two generation circuit for the DPSK system, the first to generate the cosine component of the DQPSK modulated signal, and the second is for the sine component for the DQPSK signal generator are shown in figure (2) below :-



Generation Circuits For DQPSK System

D) – The equations of the DQPSK modulated signal.

The equation of the DQPSK modulated signal is same for the QPSK modulated signal but the input B.B.S (odd and even bits) are rearranged as:-

yi = bI XNOR y(i-1)

yQ = bQ XNOR y(Q-1)

$S_{DQPSK}(t) = A\cos(wct + \phi)$ ……………………………………………………………(1)

$$\phi = (2i - 1) * \frac{\pi}{4} \quad \text{...(2)}$$

$$S_{DQPSK}(t) = A\cos(wct + [(2i - 1) * \frac{\pi}{4}]) \quad \text{......................................(3)}$$

$S11(t) = A\cos(wct + \frac{\pi}{4})$ for (i = 1) and (yi = 1" , yQ =1")

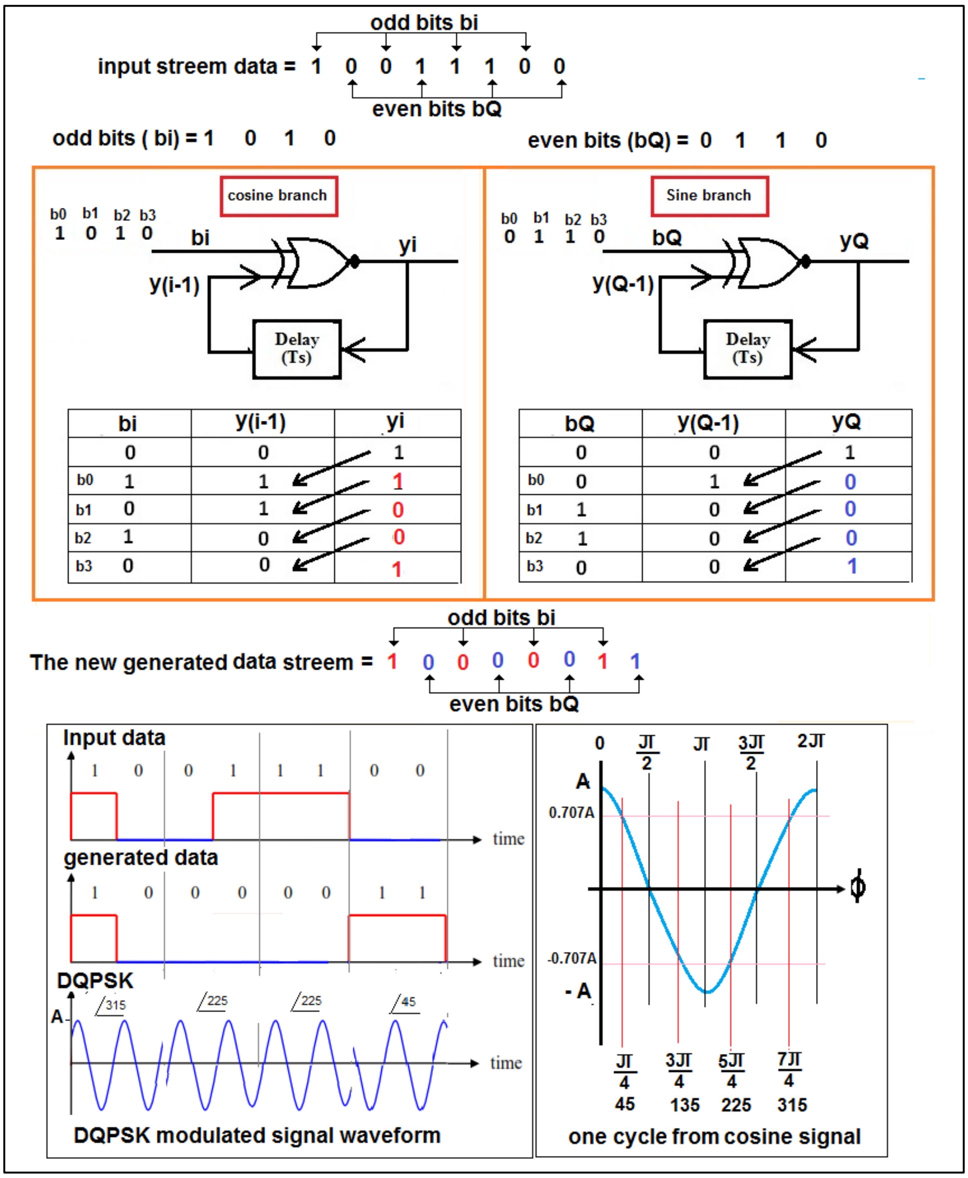
$S01(t) = A\cos(wct + 3\frac{\pi}{4})$ for (i = 2) and (yi =1" , yQ = 0 ")

$S00(t) = A\cos(wct + 5\frac{\pi}{4})$ for (i = 3) and (yi = 0" , yQ = 0")

$S10(t) = A\cos(wct + 7\frac{\pi}{4})$ for (i = 4) and (yi = 0" , yQ = 1")

E) – The waveform in time domain

The waveform in time domain for the DQPSK modulated signal for the input data stream (1 0 0 1 1 1 0 0) can be evaluated after calculate the new generated data stream as shown in Figure (3) below:-

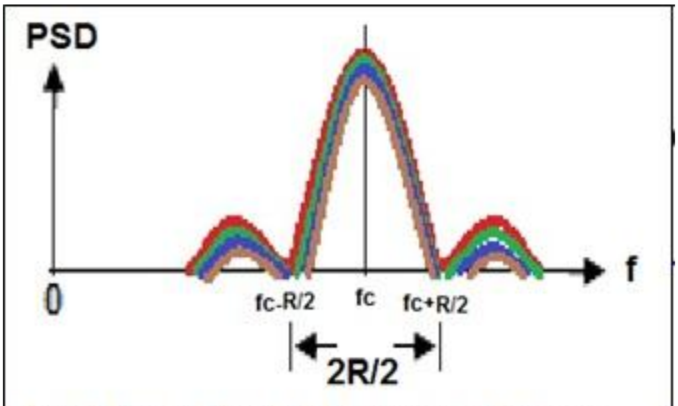


F – Constellation diagram of the DQPSK modulation System.

As shown in figure (3) above, there are no specific wave form for any two input bits because the waveform not depend on the input data but it depend on the output data of (XNOR and delay)circuits, so we can say that there is no constellation diagram can be specify the DQPSK modulated system

G) -The power spectral density (PSD) of the DQPSK modulated signal.

Similar to the PSD for the QPSK, the four symbols of the DQPSK signal have the same amplitude and the same frequency, so they appear together in the same place in the PSD. The power spectral density for the DQPSK modulated signal is shown in figure (4) below



PSD for DQPSK Modulated Signal

H)–The required channel bandwidth for the DQPSK modulation system.

From the PSD diagram for the DQPSK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (FC-R/2) and (FC+R/2) then

The ***reqBW for QPSK = 2*R/2 = R*** ……………………………………………………(6)

For more correct representation for the required channel bandwidth for the DQPSK we must write the low as :-

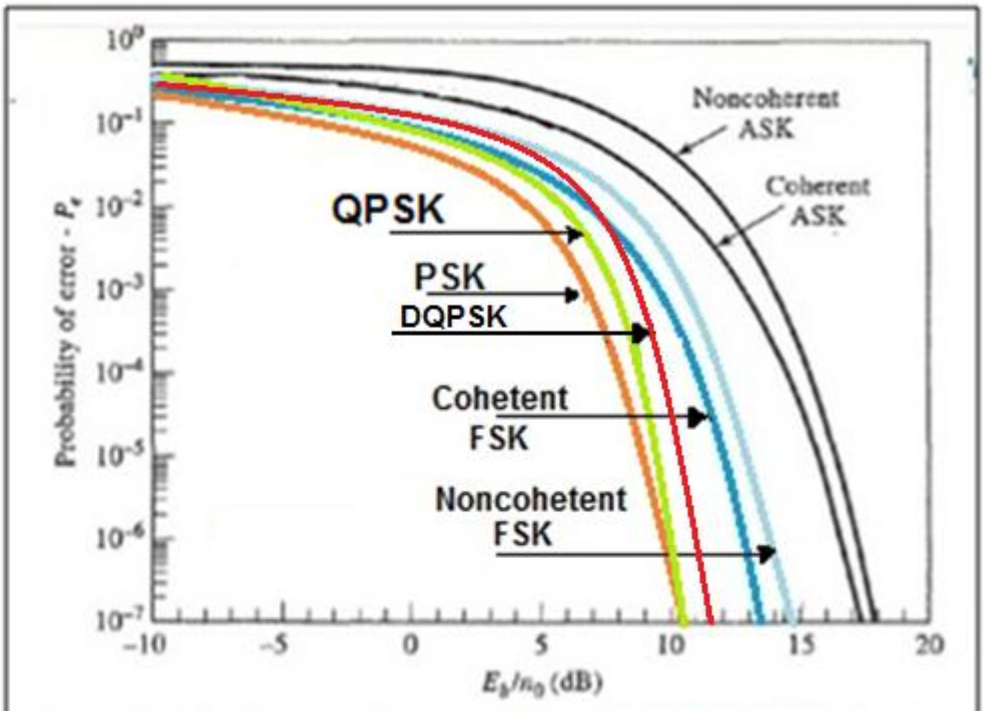
reqBW for DQPSK = BW for B.B.S before the modulation process.

I)– The probability of error for the DQPSK modulation system.

The equation of the probability of error for the DQPSK can be written as

$$Pe_{\,DQPSK} = Q\left(\sqrt{1.716(\frac{S}{N})}\right)$$

From the above expression for the DQPSK probability of error, one can conclude that the probability of error for DQPSK worse than the probability of error for QPSK by one dB. Figure (5) show the probability of error for DQPSK system with other modulation systems.



Probability of error for DQPSK, QPSK, PSK, FSK, and ASK Systems

DIGITAL COMMUNICATIONS

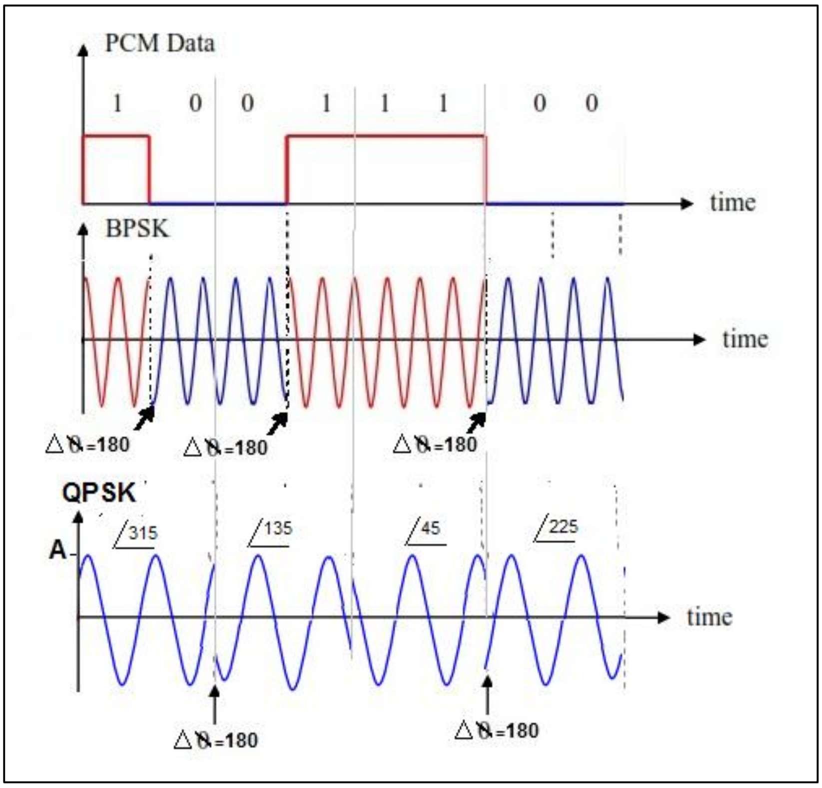
Digital Carrier Modulation

LECTURE (11-2) Offset quadrature phase shift keying (OQPSK)

. Minimum shift keying (MSK)

. Gaussian minimum shift keying (GMSK)

Although the good specification that produced by PSK, DPSK, QPSK, and DQPSK systems in the required channel bandwidth and the probability of error, there is a very effected problem appear when using these systems, this problem is the sudden transient edge (180 degree) between the end of one symbol and the start of the next symbol as shown in figure (1) below:-

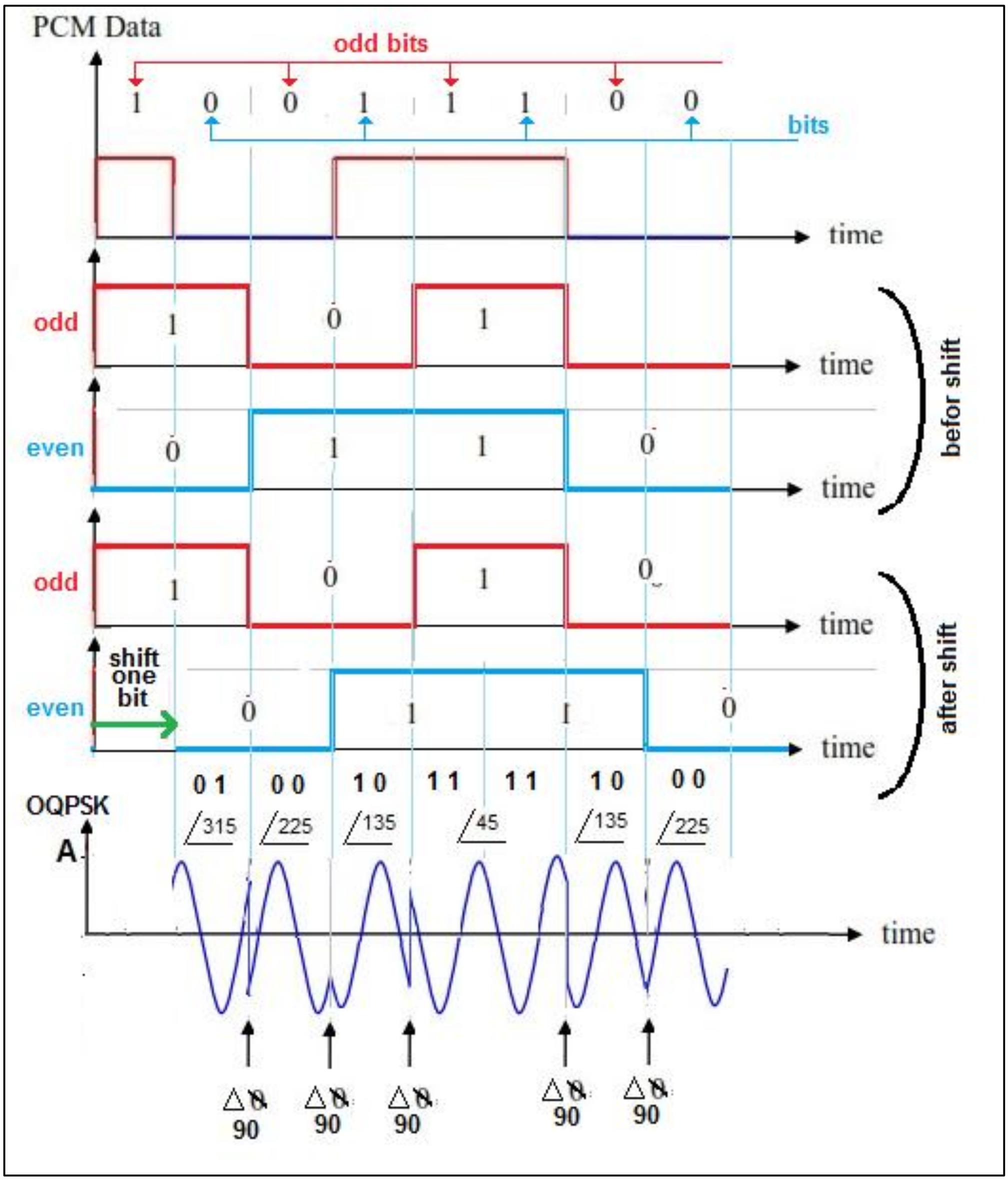


To solve this problem the OQPSK system was Introduced to reduce the sudden transient edge from 180 degree to 90 degree at maximum, while the MSK and GMSK are used to cancel this sudden transient edge.

1 – The offset quadrature phase shift keying (OQPSK)

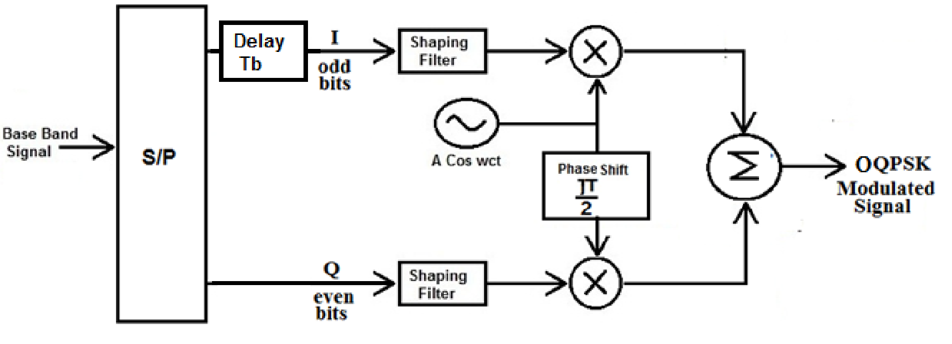
A) - The idea of the OQPSK modulation system.

The idea of OQPSK is as in QPSK system but with shifting one branches of data (odd data or even data) by one duration bit before entered them to the QPSK modulator, this process achieve decreasing in the transient edge from 180 degree to 90 degree at maximum as illustrated below :-



B – The generation circuits for the OQPSK modulation system.

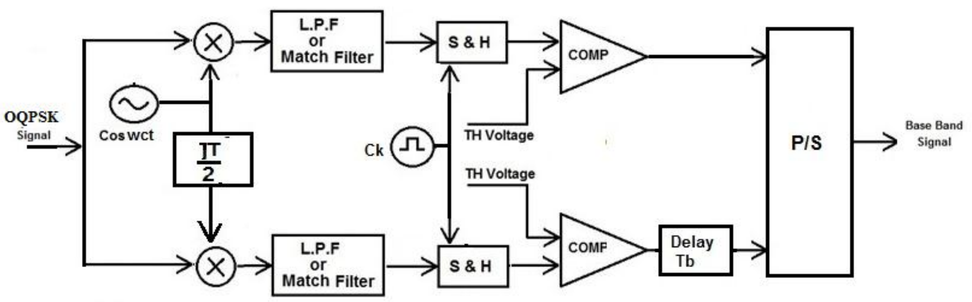
The generation circuit for the OQPSK is exactly similar to the QPSK system but with adding delay unit by Tb on one of the two branches of the QPSK generation circuit. The OQPSK signal generator are shown in figure (3) below:-



Generation Circuits For OQPSK System

<u>C – The detection circuits for the OQPSK modulation system.</u>

The detection circuits for the OQPSK modulated signal is also similar to the detection circuit of the QPSK system but with adding delay unit by Tb duration time in opposite branch which delayed in generation circuit (if we add delay unit at odd data branch in generation circuit we must add the delay unit at even data branch in detection circuit and verse versa. The detection circuit for OQPSK modulated signal are shown in figure (4) below:-



detection circuits for OQPSK modulated signal

D) - The required channel bandwidth for OQPSK system.

The required channel bandwidth cannot be stated in an expression because the symbol time once occupied one bit duration time and other occupied two bit duration time, but exactly its larger than the required channel bandwidth for QPSK and less than the required channel bandwidth for PSK system.

E) – The probability of error for the OQPSK modulation system.

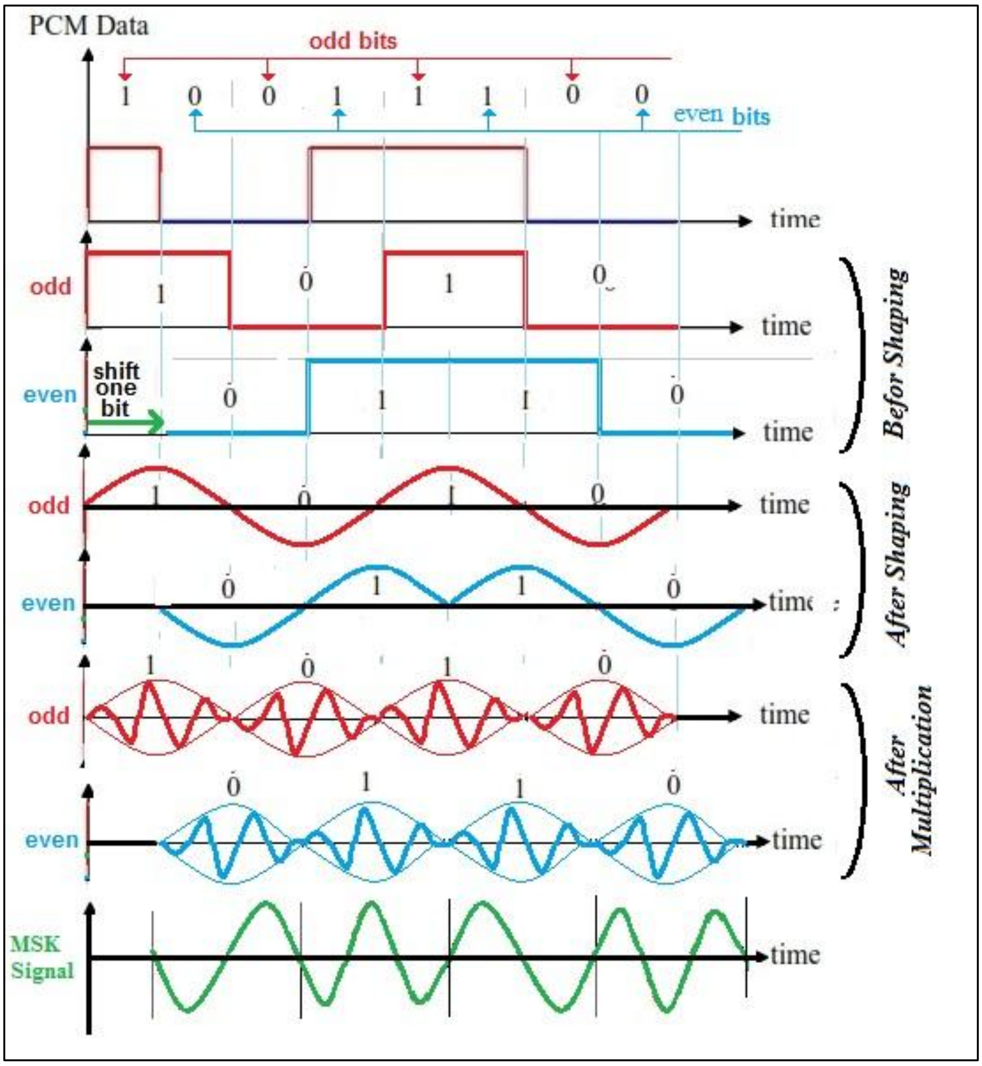
The probability of error for the OQPSK system is the same as that for QPSK system.

2- The Minimum Shift Keying (MSK)

Although the OQPSK system has reduced the sudden transfer edge between the end of one symbol and the beginning of the other symbol, this system has not been able to completely cancel this sudden transition edge while the MSK system has successfully accomplished that task.

A) - The idea of the MSK modulation system.

The MSK system can be considered as an extension of the OQPSK system with changing the shape of the two base-band signals (I & Q) from square wave to upper part of the form of sinusoidal signal before inserting them into the two multipliers with the corresponding carriers' signals.



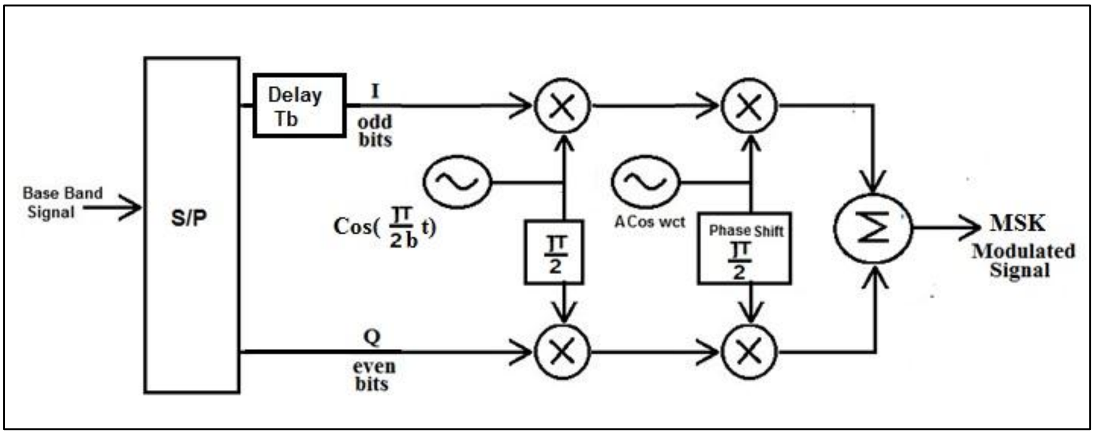
B) - The equations of the MSK modulation system.

The formula of the MSK modulated signal can written as

$$s(t) = A_I(t)\cos\left(\frac{\pi t}{2T_b}\right)\cos(2\pi f_c t) + A_Q(t)\sin\left(\frac{\pi t}{2T_b}\right)\sin(2\pi f_c t)$$

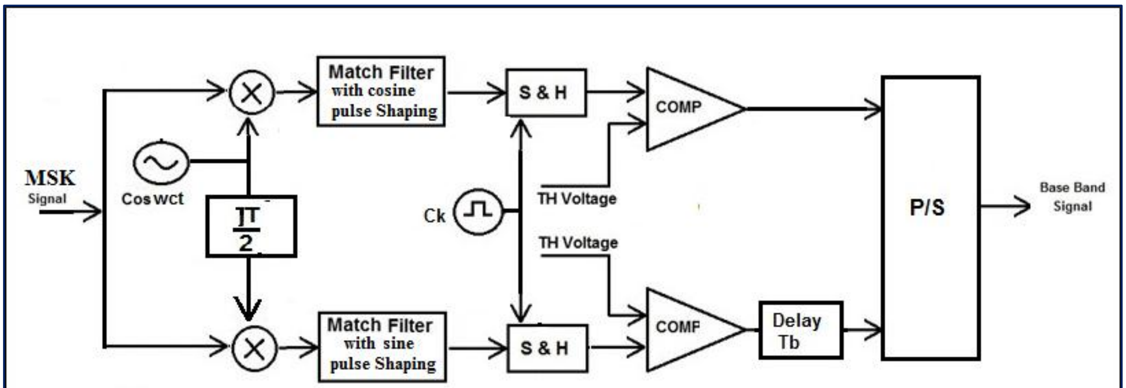
C – The generation circuits for the MSK modulation system.

The generation circuit for the MSK is modified version from the OQPSK system but by adding the sinusoidal shaping stage to the OQPSK generation circuit as shown in figure (6) below :-



D – The detection circuits for the MSK modulation system.

The detection circuits for the MSK modulated signal is similar to the detection circuit of the OQPSK system but the matched filter will be matched the carrier signal that shaped by sinusoidal signal as shown in figure (7) below.



detection circuits for MSK modulated signal

F) - The required channel bandwidth for MSK system.

The required channel bandwidth exactly its larger than the required channel bandwidth for QPSK and less than the required channel bandwidth for PSK system, the formula of the required channel bandwidth can approximately written as:-

reqBW for MSK = **3R/2 = 1.5 R** **or** ***= 1.5* BW for B.B.S***

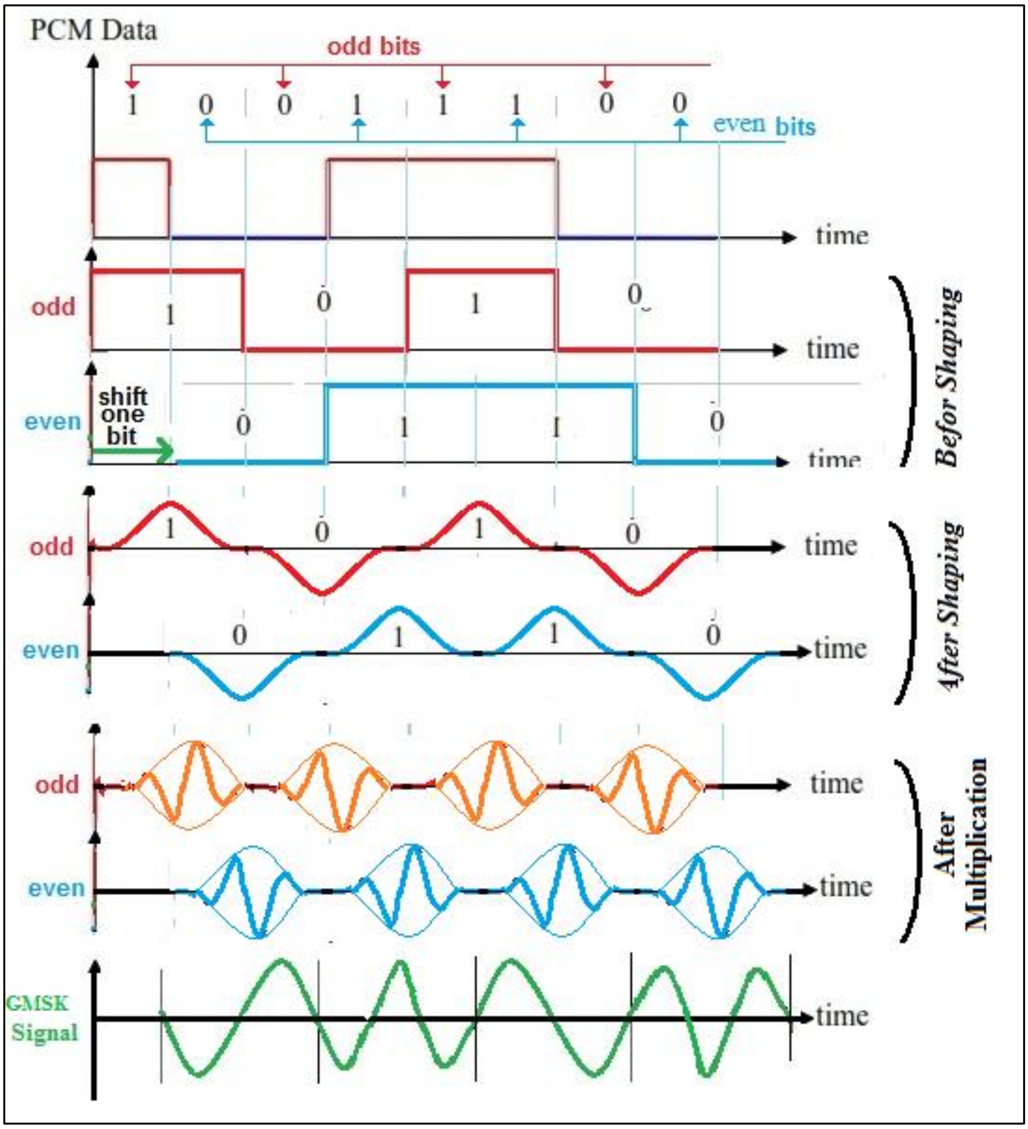
G) – The probability of error for the MSK modulation system.

The probability of error for the MSK system is the same as that for non-coherent FSK and its formula can be written as:- $Pe_{MSK} = \frac{1}{2}\, e^{\left(\frac{Eo}{No}\right)}$

3- The Gaussian Minimum Shift Keying (GMSK)

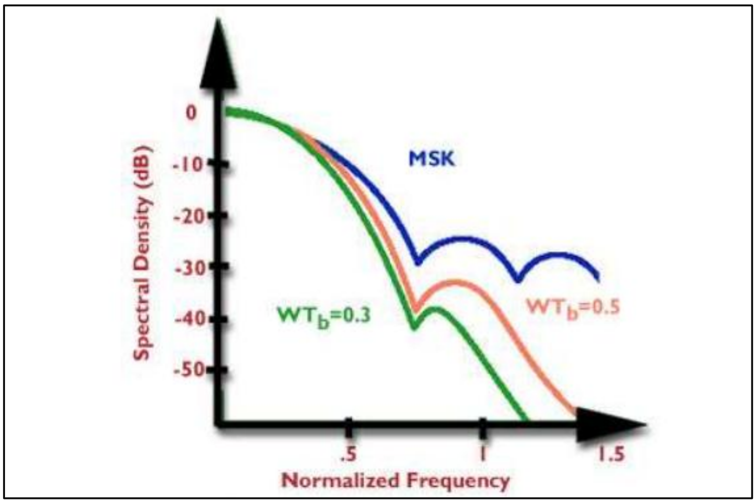
GMSK is derivative of MSK where the bandwidth required is further reduced by passing the modulating waveform through a Gaussian filter.

The Gaussian filter minimizes the instantaneous frequency variations over time. The phase of MSK signal is continuous but its frequency is discontinuous, by reducing this discontinuity a smoother modulated signal can be generated which called GMSK modulated signal.



A) - The idea of the GMSK modulation system.

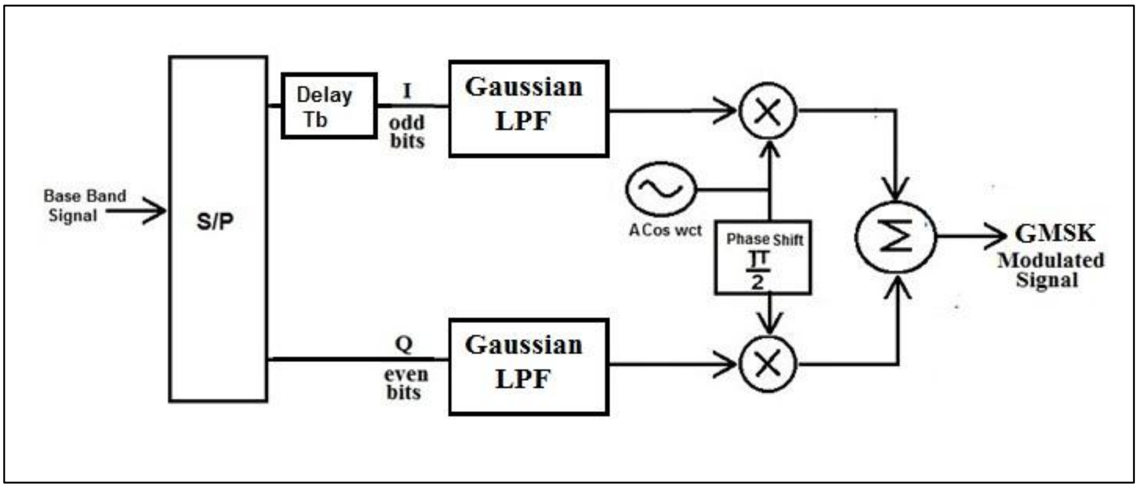
The principle parameter in designing an appropriate Gaussian filter is the time-bandwidth product (WTb). As shown in the following figure (9) for the frequency response of different Gaussian filters. Note that MSK has a time-bandwidth product of infinity.



As can be seen from above, GMSKs power spectrum drops much quicker than MSK's. Furthermore, as time-bandwidth product is decreased, the roll-off is much quicker. Even though MSK's power spectrum density falls quite fast, it does not fall fast enough so that interference between adjacent signals in the frequency band can be avoided. To take care of the problem, the original binary signal is passed through a Gaussian shaped filter before it is modulated with MSK.

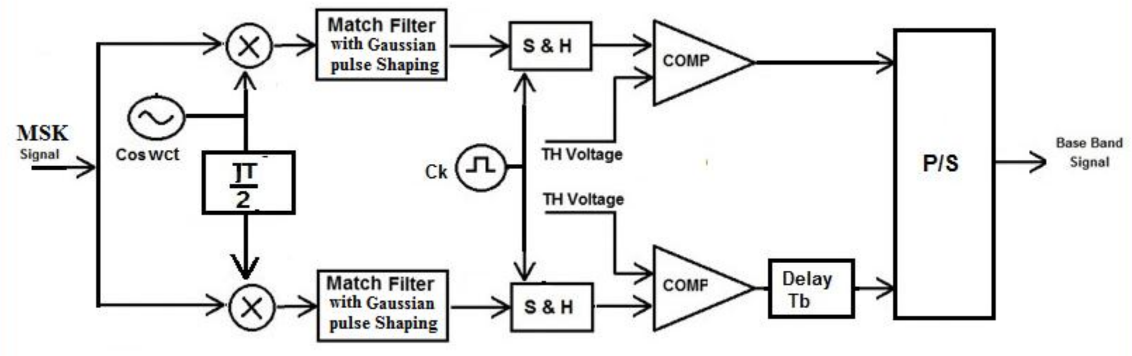
B – The generation circuits for the GMSK modulation system.

The generation circuit for GMSK is shown in figure (10) below



C – The detection circuits for the GMSK modulated signal.

The detection circuits for the GMSK modulated signal is similar to the detection circuit of the MSK system but the matched filter will be matched the carrier signal that shaped by Gaussian signal instead of sinusoidal signal as shown in figure (11) below.



detection circuits for MSK modulated signal

D) - The required channel bandwidth for GMSK system.

The required channel badwidth for GMSK is depend on the specification of the Gaussian filter (time-bandwidth product) as in example, In the GSM standard, Gaussian Minimum Shift Keying with a time-bandwidth product of 0.3 was chosen as a compromise between spectral efficiency and intersymbol interference. With this value of time-bandwidth product (WTb), 99% of the power spectrum is within a bandwidth of 250 kHz, and since GSM spectrum is divided into 200 kHz channels for multiple access, there is very little interference between the channels. The speed at which GSM can transmit at, with time-bandwidth product (WTb) =0.3, is 271 kb/s. (It cannot go faster, since that would cause intersymbol interference).

DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (12-2) (M-Ary PSK) (8PSK)

After good specifications that produced by the PSK, QPSK and the derivative system from it, and the In order to more reducing in the required channel bandwidth with keeping the acceptant specification that we get, the 8-PSK modulated system present as a suitable method to achieve this task.

A) - The idea of the 8-aryPSK modulation system.

Eight Phase Shift Keying (8PSK) is a method to transmit digital information on a carrier by changing the phase of the carrier. In 8PSK there are 8 different phase changes defined, each phase change represents the transmission of 3 bits. 8PSK is used in e.g. EDGE. This phase changes are either

0, 45, 90, 135, 180, 225, 270 and 315 degrees.

Or 22.5, 67.5, 112.5, 152.5, 202.5, 247.5, 292.5, 337.5 degrees

B) – The equations of the 8PSK modulated signal.

The equation of the 8PSK modulated signal can be written as:-

$S_{8PSK}(t) = A\cos(wct + \phi)$ ……………………………………………………………(1)

$\phi = (2i - 1) * \frac{\pi}{8}$ ……………………………………………………………(2)

$S_{8PSK}(t) = A\cos(wct + [(2i - 1) * \frac{\pi}{8}])$ ……………………………………(3)

Substituting i = 1,2,3,4,5,6,7,8

$S111(t) = A\cos(wct + \frac{\pi}{8})$ $\phi = 22.5$ for (i = 1)

$S110(t) = A\cos(wct + 3\frac{\pi}{8})$ $\phi = 67.5$ for (i = 2)

$S100(t) = A\cos(wct + 5\frac{\pi}{8})$ $\phi = 112.5$ for (i = 3)

$S101(t) = A\cos(wct + 7\frac{\pi}{8})$ $\phi = 157.5$ for (i = 4)

$S001(t) = A\cos(wct + 9\frac{\pi}{8})$ $\phi = 202.5$ for (i = 5)

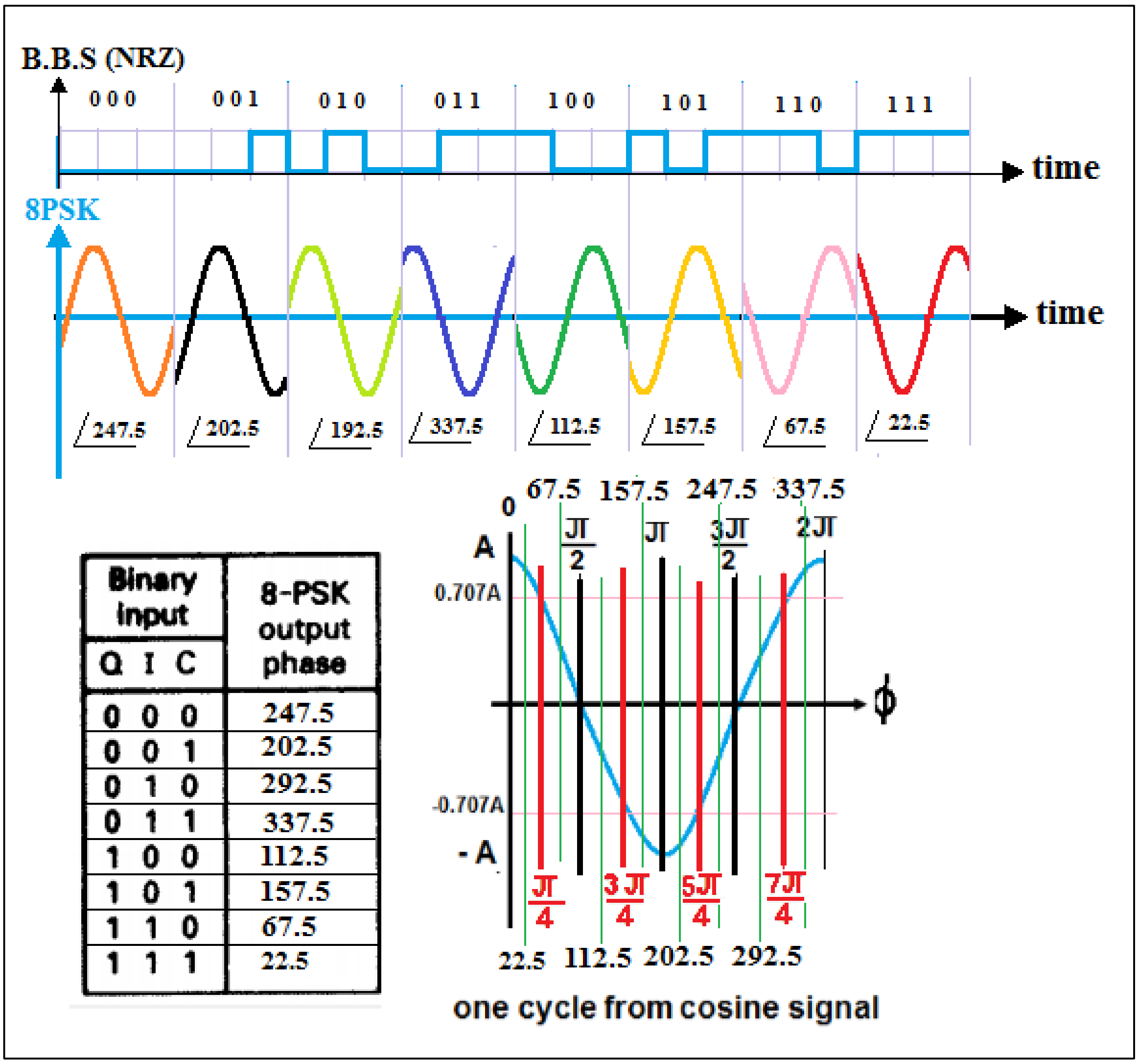
$S000(t) = A\cos(wct + 11\frac{\pi}{8})$ $\phi = 247.5$ for (i = 6)

$S010(t) = A\cos(wct + 13\frac{\pi}{8})$ $\phi = 292.5$ for (i = 7)

$S011(t) = A\cos(wct + 15\frac{\pi}{8})$ $\phi = 337.5$ for (i = 8)

C) – The waveform for the 8psk modulated signal in time domain

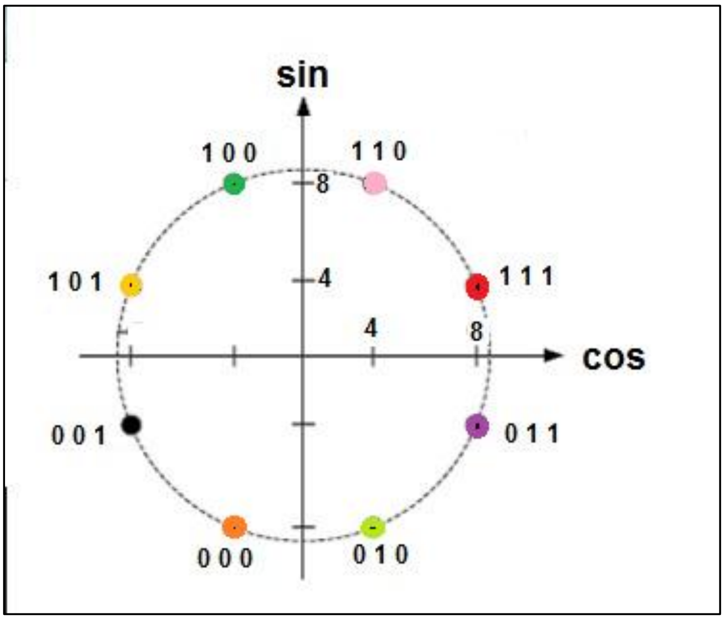
The waveform in time domain for the 8PSK modulated signal is shown in Figure (1) below:-



Binary input Q I C	8-PSK output phase
0 0 0	247.5
0 0 1	202.5
0 1 0	292.5
0 1 1	337.5
1 0 0	112.5
1 0 1	157.5
1 1 0	67.5
1 1 1	22.5

D – Constellation diagram of the 8PSK modulation System.

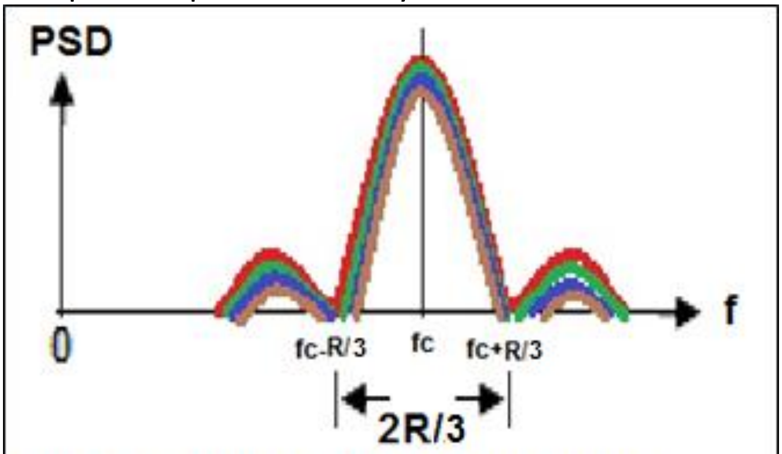
As illustrated in the previous paragraph the 8PSK signal contain the cosine component and the sine component, the results of summation the two component produce sinusoidal signal with the phases (22.5, 67.5, 112.5, 157.5, 202.5, 247.5, 292.5, and 337.5) degree. Figure (2) show the constellation diagram for the 8PSK



E) -The power spectral density (PSD) of the 8PSK modulated signal.

The eight symbols of the 8PSK signal have the same amplitude and the same frequency, so they appear together in the same place in the PSD.

The power spectral density for the 8PSK modulated signal is shown in figure (3)



PSD for 8PSK Modulated Signal

F) –The required channel bandwidth for the 8PSK modulation system.

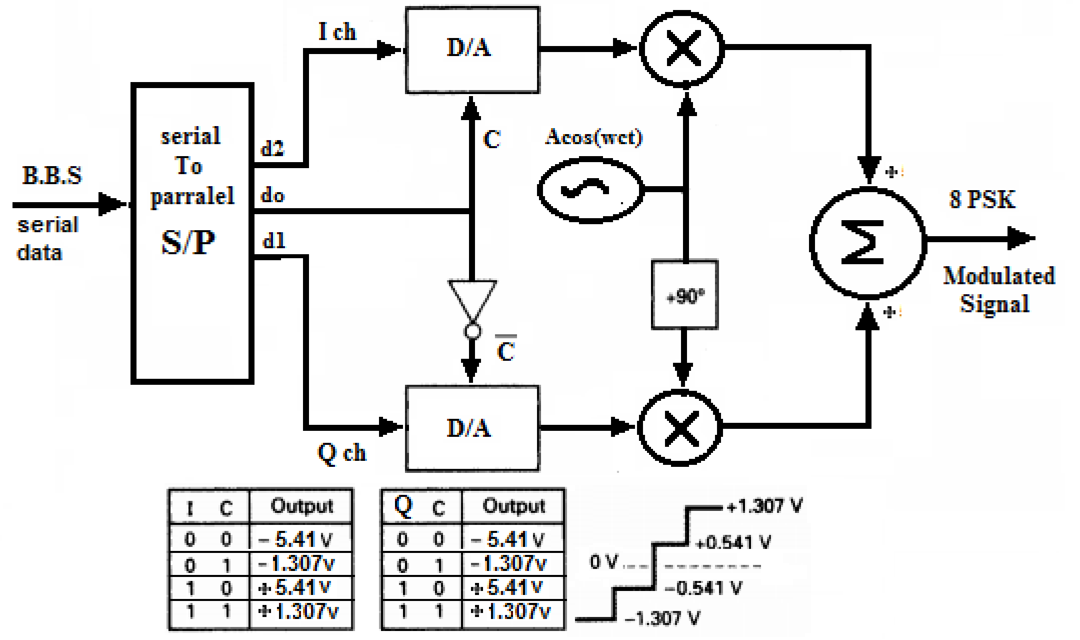
From the PSD diagram for the 8PSK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (FC-R/3) and (FC+R/3) then

The ***reqBW for 8PSK = 2*R/3 = 0.666 R*** ……………………………………………….(6)

Or ***reqBW for 8PSK = 0.666* BW for B.B.S*** before the modulation process.

G – The generation circuits for the 8PSK modulation system.

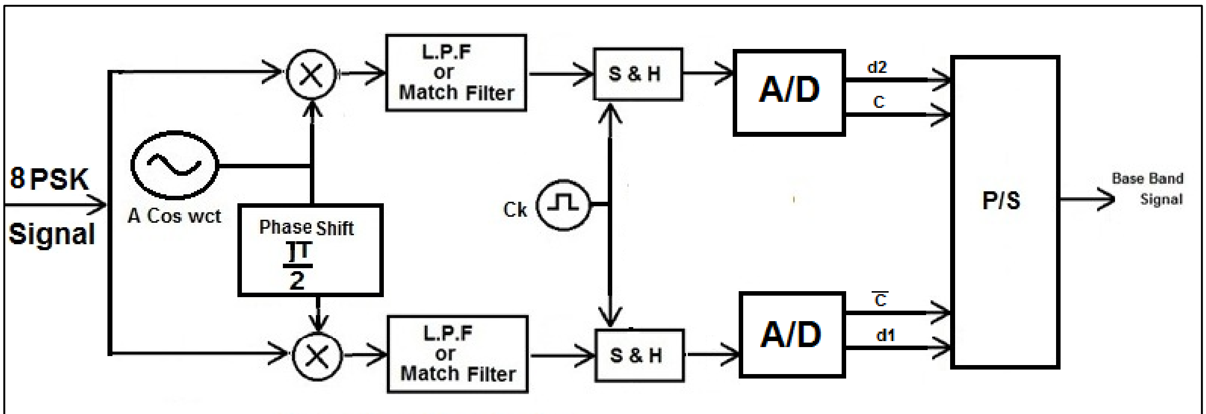
The generation circuit for the 8PSK are shown in figure (4) below:-



Data in	I	Q	C	$\overline{C}$	In I	V i	inQ	V Q	phase
000	0	0	0	1	0 0	-0.541	0 1	-1.307	247.5
001	0	0	1	0	0 1	-1.307	0 0	-0.541	202.5
010	0	1	0	1	0 0	-0.541	1 1	+1.307	112.5
011	0	1	1	0	0 1	-1.307	1 0	+0.541	292.5
100	1	0	0	1	1 0	+0.541	0 1	-1.307	157.5
101	1	0	1	0	1 1	+1.307	0 0	-0.541	337.5
110	1	1	0	1	1 0	+0.541	1 1	+1.307	67.5
111	1	1	1	0	1 1	+1.307	1 0	+0.541	22.5

H)– The detection circuits for the 8PSK modulation system.

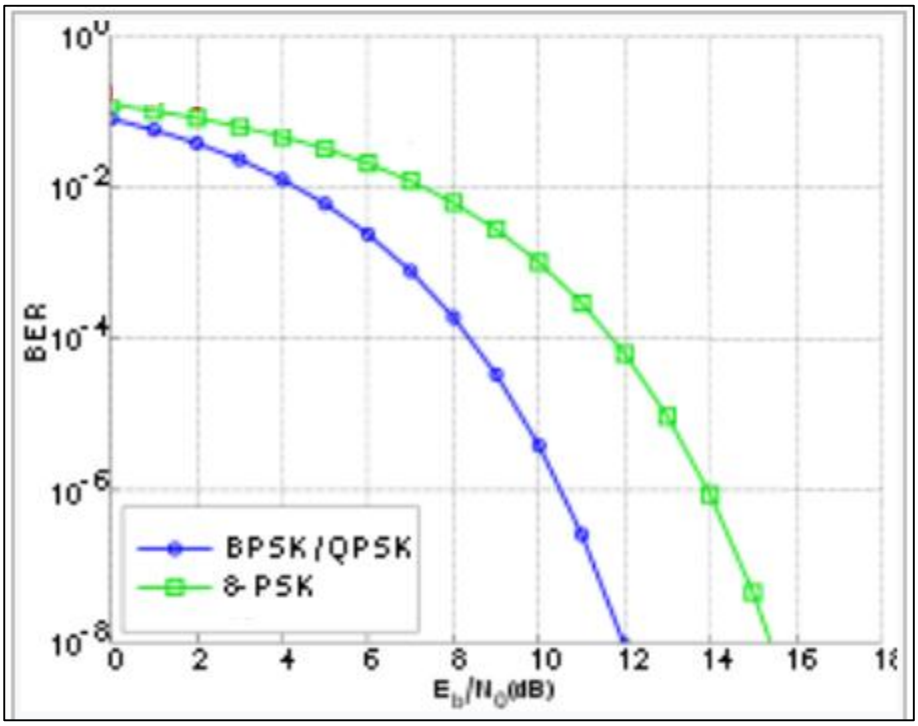
The detection circuits for the 8PSK modulated signal are shown in figure (5) below.



Detection Circuits for 8 PSK Signal

I)– The probability of error for the 8PSK modulation systems.

The probability of error for the 8PSK system as shown in figure (6) below is worse than the probability of error for QPSK and PSK by 3dB .



DIGITAL COMMUNICATIONS

Digital Carrier Modulation

LECTURE (13-2) Amplitude and phase-shift Keying (APK) Comparison and application for different types of digital modulation systems

After we study the three types of digital carrier modulation system, the question that can one ask it what will happen if we combine two types of digital carrier modulation system, in fact , a modulation method combining two or more symbol types could give improved performance in the inevitable trade-off between bandwidth efficiency and noise performance. When the spectral efficiency is the most important factor and the channel has good amplitude linearity.

The digital modulation that conveys data by changing, or modulating, both the amplitude and the phase of a reference signal (the carrier wave). In other words, it combines both amplitude-shift keying (ASK) and phase-shift keying (PSK) to increase the symbol-set. It called amplitude and phase shift keying (APK). The First M-ary amplitude phase keying (M-ary ApK) proposed systems introduced a pair of multi-level ASK signals modulated by two carrier waves of the same frequency but out of phase with each other by 90°(sine and cosine), a condition known as orthogonally or quadrature amplitude modulation (QAM). The QAM is used extensively as a modulation scheme for digital communication systems, such as in 802.11 Wi-Fi standards. Arbitrarily high spectral efficiencies can be achieved with QAM by setting a suitable constellation size, they limited only by the noise level and linearity of the communications channel. 16-QAM and 64-QAM is being used in optical fiber systems and in many practical systems as bit rates increase. In this lecture we shall study the 16-QAM modulation system

A) - The idea of the 16-QAM modulation system.

The 16QAM is a method to transmit digital Base band signal on a carrier by changing the phase of the carrier and amplitude. In 16-QAM there are 16 different symbol that represent 4 base band bits, each tow bits from the B.B.S are converted to 4 levels and then the both converted signals are modulated by two carrier signals that are in the same amplitude and frequency but they different in phase by 90 degree, finally these two modulated signals are added together using linear summation unit to represent the 16-QAM modulated signal.

B) – The equations of the 16-QAM modulated signal.

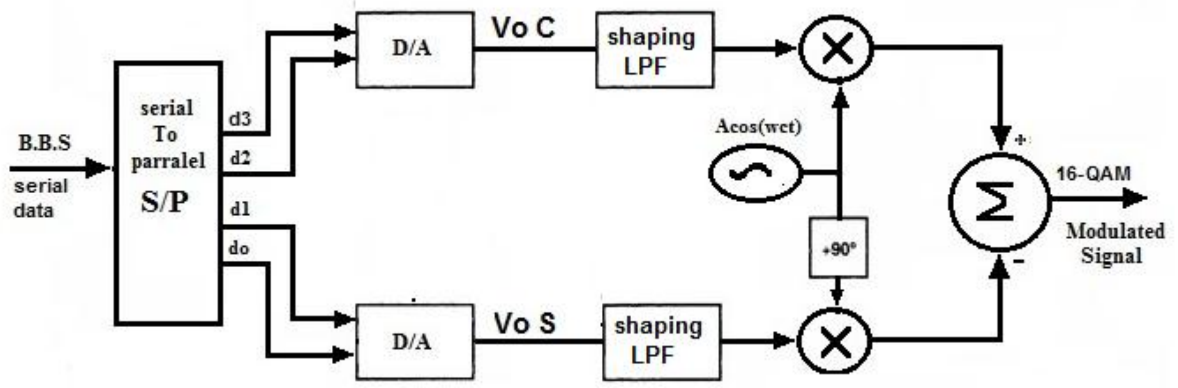
The equation of the 16-QAM modulated signal can be written as:-

$$\boxed{\begin{array}{l} s_i(t) = A_i g(t)\cos\left[2\pi f_c t + \theta_c + \varphi_j\right], \\ i = 1,2,\cdots,M_1, j = 1,2,\cdots,M_2, 0 \le t \le T_s \end{array}}$$

- As both amplitude and phase are used to carry symbol information, it is very bandwidth efficient
- Signal set size $M=M_1M_2$: $2^1 \times 2^1 = 4$, $2^2 \times 2^2 = 16$, $2^3 \times 2^3 = 64$, etc → 4QAM, 16QAM, 64QAM

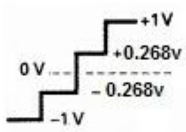
C – The generation circuits for the 16-QAM modulation system.

As illustrated in paragraph (A) from this lecture the generation circuit for 16QAM are shown in figure (1) below



d3 d2	Output
0 0	-1 v
0 1	- 0.268v
1 0	+1 v
1 1	+ 0.268v

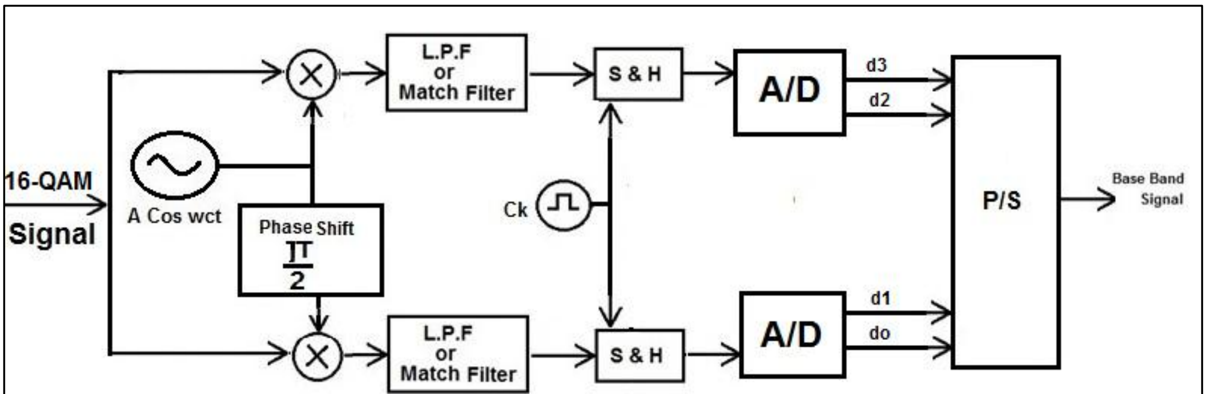
d1 d0	Output
0 0	-1 v
0 1	- 0.268v
1 0	+1 v
1 1	+ 0.268v



Data in	d3 d2	d1 do	Vo C	Vo S	AMPL	phase
0000	0 0	0 0	- 1	-1	1.414A	135
0001	0 0	0 1	- 1	-0.268	1.035A	165
0010	0 0	1 0	- 1	+1	1.414A	225
0011	0 0	1 1	- 1	+0.268	1.035A	195
0100	0 1	0 0	-0.268	-1	1.035A	105
0101	0 1	0 1	-0.268	-0.268	0.367A	135
0110	0 1	1 0	-0.268	+1	1.035A	255
0111	0 1	1 1	-0.268	+0.268	0.367A	225
1000	1 0	0 0	+1	-1	1.414A	45
1001	1 0	0 1	+1	-0.268	1.035A	15
1010	1 0	1 0	+1	+1	1.414A	315
1011	1 0	1 1	+1	+0.268	1.035A	345
1100	1 1	0 0	+0.268	-1	1.035A	75
1101	1 1	0 1	+0.268	-0.268	0.367A	45
1110	1 1	1 0	+0.268	+1	1.035A	195
1111	1 1	1 1	+0.268	+0.268	0.367A	315

D)– The detection circuits for the 16-QAM modulation system.

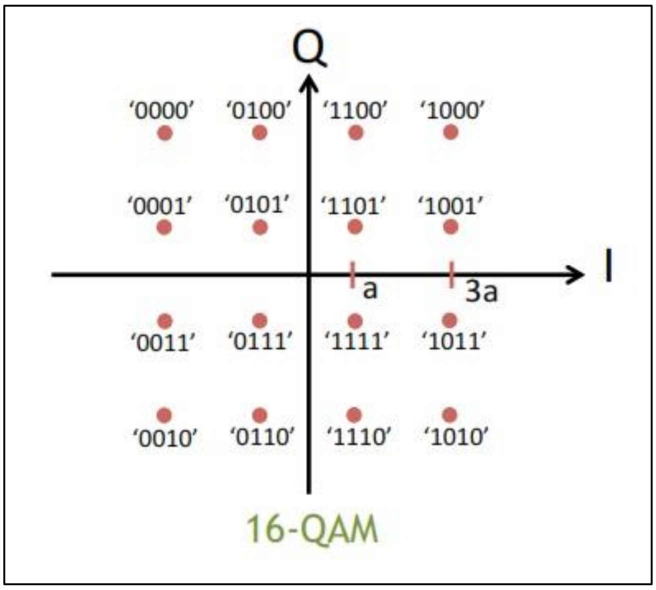
The detection circuits for the 16-QAM modulated signal are shown in figure (2) below.



Detection Circuits for 16-QAM Signal

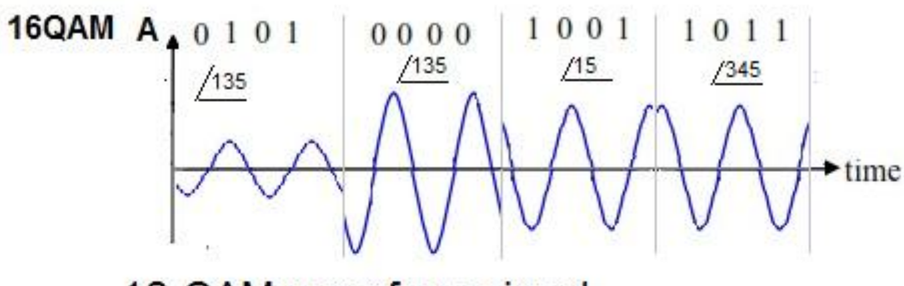
E – Constellation diagram of the 16-QAM modulation System.

The amplitude that may be the 16-QAM symbols appear are (0.367A, 1.035A, and 1.414A) while these symbols appear in the following phases (15, 45, 75, 105, 135, 165, 195, 225, 255, 285, 315, and 345)   Figure (3) show the constellation diagram for the 16-QAM



F) – The waveform for the 16-QAM modulated signal in time domain

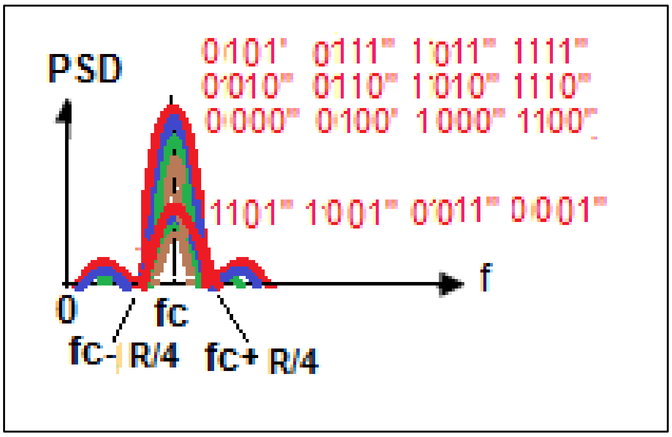
The waveform for the stream (0101, 0000,1001, and 1011) as an example in time domain for the 16-QAM modulated signal according to the constellation diagram and the generation circuit in previous paragraph is shown in Figure (4) below:-



16-QAM waveform signal

G) -The power spectral density (PSD) of the 16-QAM signal.

The eight symbols of the 16-QAM modulated signal have three amplitude and the same frequency, so they appear together in the same place in the PSD. The power spectral density for the 16-QAM modulated signal is shown in figure (5)



H)–The required channel bandwidth for the 16-QAM system.

From the PSD diagram for the 8PSK above we can extract the required channel bandwidth which represent the frequency band between the two nulls (FC-R/4) and (FC+R/4) then

The ***reqBW for 16-QAM = 2*R/4 = 0.5 R*** ……………………………………………….(6)

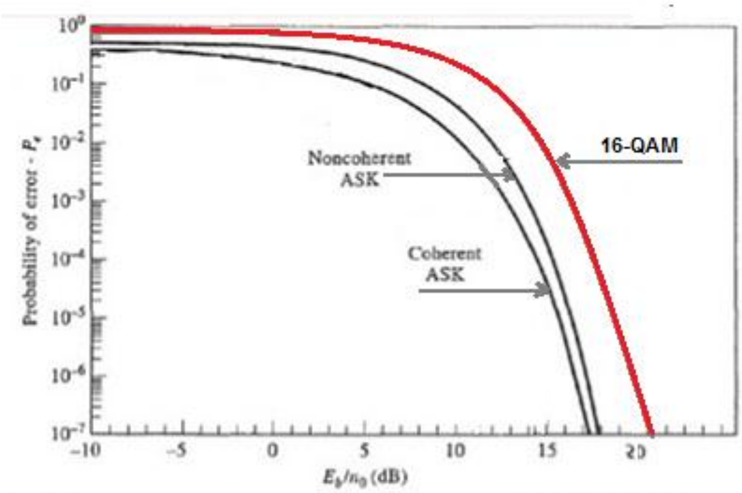
Or ***reqBW for 16-QAM = 0.5* BW for B.B.S*** before the modulation process.

I)– The probability of error for the 16-QAM modulation systems.

The probability of error formela for the 16-QAM can be written as :-

$$Pe = \frac{4}{\log_2 M}\left(1-\frac{1}{\sqrt{M}}\right)Q\left[\sqrt{\frac{3\log_2 M * Eb}{No(M-1)}}\right]$$ Where Eb is the average energy per bit

The probability of error for the 16-QAM system as shown in figure (6) below is worse than the probability of error for ASK by 3dB .



Probability of error for ASK and 16-QAM

- ***Bandwidth efficiency*** describes the ability of a modulation scheme to accommodate data within a limited bandwidth, In general, it is defined as *the ratio of the data bit rate R to the required RF bandwidth B*:

$$\eta_B = \frac{R}{B} \text{(bps/Hz)} \qquad \text{Large } \eta_B \text{ is preferred}$$

Comparison between different types of digital modulation systems

	Mod. Type	Min. Req. Bandwidth	bandwidth efficiency	Probability of error
1	ASK_{coh}	$2R$	1 bit/sec/Hz	$Q(\sqrt{Eb/2No})$
2	ASK_{coh}^{non}	$2R$	1 bit/sec/Hz	$\frac{1}{2}e^{-Eb/2No}$
3	FSK_{coh}	$3R$	2/3 bit/sec/Hz	$Q(\sqrt{Eb/No})$
4	FSK_{coh}^{non}	$3R$	2/3 bit/sec/Hz	$\frac{1}{2}e^{-Eb/2No}$
5	BPSK	$2R$	1 bit/sec/Hz	$Q(\sqrt{2Eb/No})$
6	DPSK	$2R$	1 bit/sec/Hz	$\frac{1}{2}e^{-Eb/No}$
7	M-ASK	$2R/(\log_2 M)$	$\log_2 M$	$\frac{2}{\log_2 M} Q\left[\sqrt{\frac{\log_2 M}{M^2-1} \cdot \frac{2Eb}{No}}\right]$
8	M-FSK Coh	$\frac{(M+1)R}{\log_2 M}$	$\frac{2\log_2 M}{M+1}$	$\frac{M-1}{\log_2 M} Q\left[\sqrt{\frac{Es}{No}}\right]$
9	M-FSK Non-coh	$\frac{(M+1)R}{\log_2 M}$	$\frac{2\log_2 M}{M+1}$	$\frac{M-1}{2\log_2 M} e^{-Es/2No}$
10	QPSK, OQPSK	R	2 bit/sec/Hz	$Q(\sqrt{2E/No})$
11	DQPSK	R	2 bit/sec/Hz	$Q(\sqrt{1.1716E/No})$
12	MSK	$1.5R$	1 bit/sec/Hz	$Q(\sqrt{2Eb/No})$
13	M-PSK	$2R/(\log_2 M)$	$\log_2 M$	$\frac{2}{\log_2 M} Q\left[\sqrt{\log_2 M \frac{2Eb}{No}} \operatorname{Sin} \frac{\pi}{M}\right]$
14	QAM	$2R/(\log_2 M)$	$\log_2 M$	$\frac{4}{\log_2 M}(1-\frac{1}{\sqrt{M}}) Q\left[\sqrt{\frac{3\log_2 M Eb}{No(M-1)}}\right]$

Applications for different modulation types in communication systems

	Modulation format	Application
1	MSK, GMSK	Global System for Mobile Communication (GSM), Cellular Digital Packet Data (CDPD).
2	BPSK	Deep space telemetry, cable modems
3	QPSK, $\pi/4$-DQPSK	Satellite, Code Division Multiple Access (CDMA), North American Digital Cellular (NADC), Trans European Trunked Radio (TETRA), Personal Handy-phone System (PHS), Pacific Digital Cellular System (PDC), Local Multi-point Distribution System (LMDS), Digital Video Broadcast—Satellite (DVB-S), cable (return path), cable modems, Terrestrial Flight Telephone System (TFTS)
4	OQPSK	Code Division Multiple Access (CDMA), satellite
5	FSK, Gaussian FSK (GFSK)	Digital Enhanced Cordless Telephone (DECT), paging, RAM (Wireless data network) mobile data, Advanced Mobile Phone System (AMPS), Cordless Telephone 2(CT2), European Radio Message System (ERMES), land mobile, public safety
6	8, 16 VSB	North American digital TV (ATV), broadcast, cable
7	8PSK	Satellite, aircraft, telemetry pilots for monitoring broadband video systems
8	16 QAM	Microwave digital radio, modems, Digital Video Broadcast Cable (DVB-C), Digital Video Broadcast Terrestrial (DVB-T)
9	32 QAM	Terrestrial microwave, Digital Video Broadcast Terrestrial (DVB-T)
10	64 QAM	Digital Video Broadcast Cable DVB-C, modems, broadband set top boxes, Multi-channel Multi-point Distribution System (MMDS)
11	256 QAM	Modems, Digital Video Broadcast Cable (DVB-C) (Europe), Digital Video (US)
12	512-QAM, 1024-QAM	Modems

BB= baseband (without carrier)
BP= band Pass (after carrier)

[Q1] Figure (1) show the constellation diagram for Specific type of digital modulation system

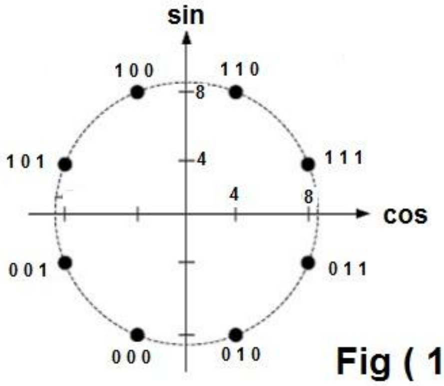
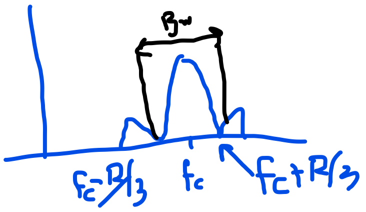


Fig (1)

A – For which type of digital modulation system belong this diagram?, Draw the generation and detection circuits for this type of modulation?

B – Draw the power spectral density (PSD) and calculate the required bandwidth if the signaling rate of the base band signal equal 60 KHz R followed by raised cosine filter (with r= 0.5)

BB BW=1/2(1+r)R
BP BW= (1+r)R



C – Draw the wave form for the following stream in time domain (suppose the carrier signal is sine wave with frequency (60 kHz))
[110010011000]

D – Repeat (C) using QPSK digital modulation system

E – Draw the constellation diagram, PSD and calculate the required bandwidth When using QPSK

F – How can be **reduce** the suddenly change between ending and staring symbols that clearly appear in (D), draw the generation and detection circuit for the modulation that reduce this suddenly change . OQPSK,GMSK, MSK

G – How can be avoid the **complexity in the detection circuit** of the QPSK de modulation due to using coherent detection only ? draw the generation and detection circuit for this type of modulation that solve this problem. DQPSK

H – Calculate and draw the transmitted sequence in (NRZ) form in the **generation circuit** for the modulation type in (G) Table for I and Q branches with XNOR

I - Calculate and draw the base band sequence that generated in the final stage in **the detection circuit** in (NRZ) form for the modulation type in (G)

J – Compare between the digital modulation types that are stated or you suggested to solve the above questions according to (probability of error, required channel bandwidth , the stability of the system, the circuit complexity)

Squared Method, Costas loop, PLL

Q[2] Draw and explain briefly one of the carrier recovery circuits ?

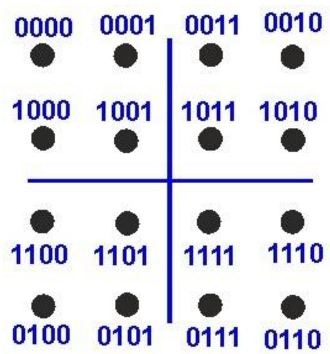
Q[3] Derive the impulse response and the transfer function for the matched filter, and prove that we can get max (E/ ŋ) when using matched filter ?

[Q4] A- prove that H(f) for the matched filter = KG*(f) e $^{-jwTs}$ and h(t) = kg(T_s - t)

B- Draw the MSK generation and detection circuits.

C-Draw the complete block diagram of the digital communication system, and illustrate briefly each part of them.

[Q5] Figure (2) show the constellation diagram for a digital modulation system.



A – For which type of digital modulation system belong this diagram?, Draw the generation and detection circuits for this type of modulation?

B – Draw the power spectral density (PSD) and calculate the required bandwidth if the signaling rate of the base band signal equal 80 Ks/sec

C – Draw the wave form for the following stream in time domain (suppose the carrier signal is sine wave with frequency (80 kHz)) [100111001000]

D – Repeat (C) using QPSK digital modulation system

E – Draw the constellation diagram, PSD and calculate the required bandwidth When using QPSK.

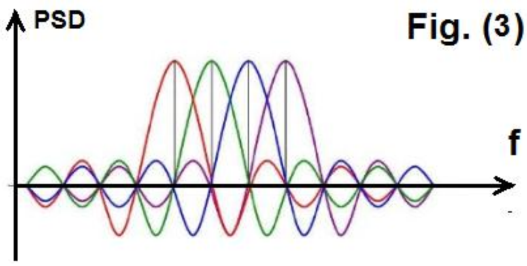


Fig. (3)

F – Figure (3) show the PSD diagram for digital modulation system for which type of digital modulation system belong this diagram?, Draw the generation and detection circuits for this type of modulation?

G - Draw the constellation diagram, and calculate Minimum frequency and maximum frequency if the required bandwidth is 200khz and signaling rate (80 Ks/sec).

H – Repeat (C) using this type of modulation digital modulation system

I – Compare between the digital modulation types that are stated or you suggested to solve the above questions according to (probability of error, required channel bandwidth , the stability of the system, the circuit complexity).

[Q6] Figure (5) Shows the waveform output of the data sequence using one of the digital modulation system, using the probability of errors curves shown in figure (6) and (7) answer the following :-

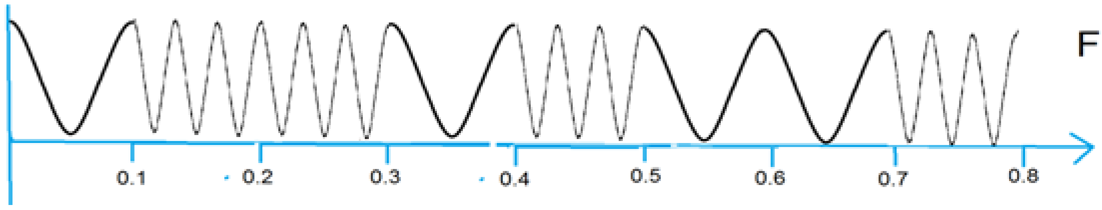


Fig. **(5)**

1) – What is the modulation type?, draw the PSD and constellation diagram of it.

2) – Calculate the required transmitted bandwidth to transmit this type of modulation?

3) – State the minimum (S/N) that needed to transmit this data within the accepter probability of errors and state the original data stream .

4) – According to BW and (S/N) that calculated in (2 and 3) state all types of the digital modulation systems that can be used instead of the digital modulation system used in figure (1)?, draw the constellation diagram of two type of them.

5) – State the probability of errors of the digital modulation systems stated in (4) (depending on the (S/N) calculated in (3).

6) – Draw the waveform output of the one of digital modulation system (stated in (4)), and state maximum signaling rate of it.

7) – Using (S/N) = 17 dB what is the digital modulation systems that can be used (and not stated in (4)), draw the waveform output of the same data sequence in figure (5).

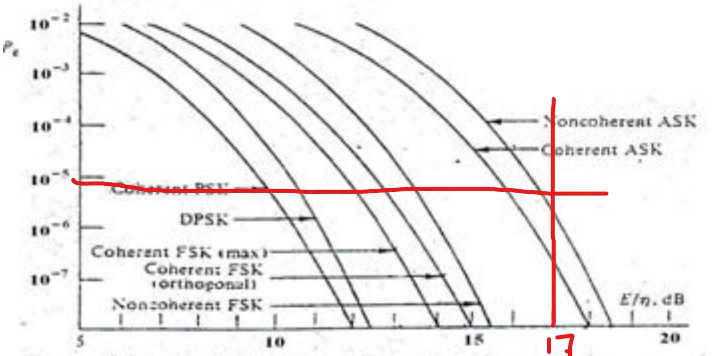
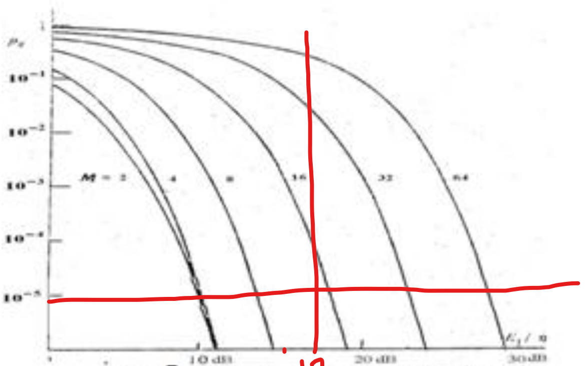


Figure 6 Error probabilities for binary digital modulation systems



Figur 7 Probability of error of M-ary PSK.

fmax=6khz, Bw=2Khz, K=fmax/Bw=6/2=3, fs=2*fmax/k=2*6000/3=4KHz, Sr=R=nfs=4*4000=16Khz

[Q7] An analog signal will be passed through aband-pass filter (Fc1 = 4KHz, Fc2 = 6kHz) and transmitted in the Digital communication system with FSK modulation (after rated sampling, 16 bit quintazation level, and raised cosine low pass filter with r=0.4).

n=4 bits

1 – Draw the generation and detection circuits of the (FSK) system.

Base band B=1/2(1+r)R
after carrier BT=(1+r)R

2 – Calculate the minimum required transmitted channel bandwidth?.

fmax

3 – What is the maximum frequency of the band limited analog signal if the following type of the modulations are used with that channel bandwidth calculated in (2).

R=67.2/2 R=67.2 R=67.2*2

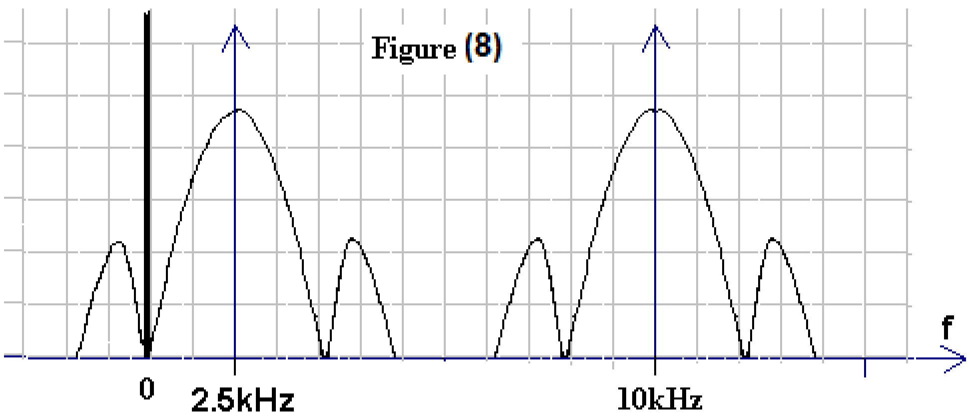
a- ASK b- DQPSK c- 16QAM .

R=nfs fs=2fmax fmax=R/2n

4 – Draw the constellation diagram and calculate the minimum $\frac{E}{\eta}$ equired for accepted probability of error of the following types of the modulation?.

a- ASK b- FSK c- PSK d- QPSK

[Q8] Figure (8) Shows the PSD of the digital modulated signal, depending on this figure and the curves of the probability of error shown in figure (6,7) answer the following questions?



A – What is the type of the modulation ? draw the time domain of the modulated signal for the following stream (10011011)

B – Calculate the required transmitted bandwidth. What is the minimum transmitted bandwidth for this type of modulation?

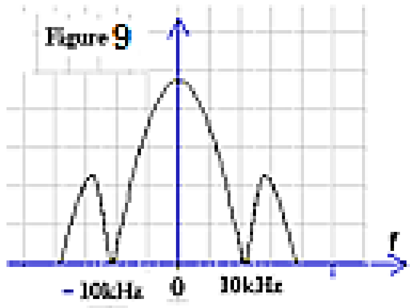
C – Draw the block diagrams of two types for the modulation and the demodulation circuits for this modulation .

D – Calculate the required (E / ŋ) within the accepted bit error rate ? , state and draw the constellation diagram for all types of the digital modulation that can be used to transmit the same data within the same (E / ŋ).

E- Calculate the maximum signaling rate can be transmitted using the digital modulation types stated in (D) within the minimum transmitted bandwidth calculated in (C).

[Q9] If figure (9) is the PSD of the NRZ baseband signal answer the following for the following data stream **(011010110010)**

A) Draw the time domain waveform , PSD , constellation diagram , calculate the required transmitted bandwidth and extract the required (E / η) for acceptable probability of error if the following digital modulation system are used :-



B) – ASK

C) – FSK

D) - PSK

E) – QPSK

F) – QAM-16

1) - State the transmitted data, draw the time domain waveform if DPSK modulation system are used to transmit the data stream above.

2) – State the transmitted data , and draw the generation and detection circuits if DQPSK modulation system are used to transmit the data stream above.

	ASK	FSK	PSK	DPSK	QPSK
1200	✓	✓	✓	✓	✓
2400	✓	✗	✓	✓	✓
3600	-	✗	✗	✓	✓

[Q10] There is a need to transmit a serial digital data in the

A- 1200 bit/sec B – 2400 bit/sec C- 3600 bit/sec

Through the communication channel which is limited to 5kHz, if the probability of error accepted was not greater than 10^{-5} and the transmitter have $\frac{E}{\eta}$ less than 17dB. Benefiting from the probability of error curves fig(6,7)

2R 3R 2R 2R R R

1 – Discuss the ability of using (ASK, FSK, PSK, DPSK, QPSK, DQPSK) modulation.

5K/2 5K/3 5K/2 5K/2 5k 5K

2 – Draw the constellation diagram of the digital modulation stated in (1).

3 – What is the maximum bit-rate of the digital data can be transmitted through this channel, using the digital modulation stated in (1).

4 – If there is a more than one type of the digital modulation can be used in each case, decide what is the best type you favorite (as a communication engineer) to used and why ?

5 – Draw the transmitter and receiver block diagram for the digital modulation type you decide to used.

through

[Q11] An analog signal will be transmitted ~~throw~~ Digital communication system with

BW=R=BT here n=4 bits

DQPSK modulation (after rated sampling, 12 bit quintazation, and raised cosine low pass filter with r=0.5) .

B=1/2(1+r)R base band
BT=(1+r)R band pass
BT=1.5R here

2 – If the channel bandwidth offered is 360 kHz, what is the max frequency of the analog input signal?.

BT=360=1.5R, R=360/1.5=240Kbps=nfs=2nfmax,
fmax=240Kbps/2*4=30Khz

3 – what is the required bandwidth to transmit this analog signal through another digital communication system with the following type of the modulations

2R 3R 0.5R

a- ASK b- FSK c- 16QAM .

4 – Draw the constellation diagram of the

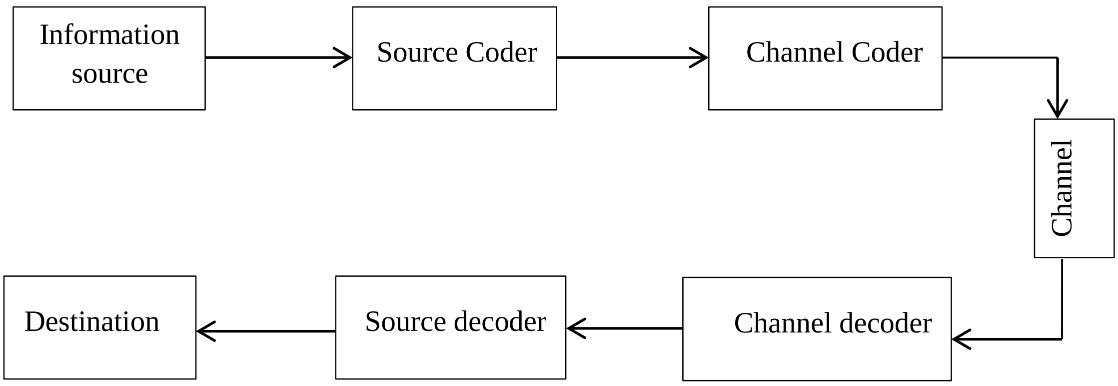
a- ASK b- FSK c- 16QAM

5 – what is the best type of the modulation (DQPSK, ASK, FSK, or 16QAM) in your opinion? Why?.

DIGITAL COMMUNICATIONS
Information Theory
Lecture (1-3) INTRODUCTION

- Information theory started when Cloud-Shannon (1916-2001) published his paper "Mathematical Theory of Communication" in 1948.
- Shannon benefit from "Ralph Hartly" (1889- 1976) whose has first attempt to science definition of quantities measure of information (1928).
- Shannon also benefit from "Hary Nyquist" (1881-1970) whose find the relationship between the information and the frequency, and between the information and the time of arrival of information.
- The information theory was found the mean of measuring the content of information for messages and the best ways to send them, and give the answers of:
    1) What is " Informations" and how to measure it?
    2) What are the fundamental limit of the storage and transmission of information?
    3) Also the information theory illustrate the information cycle within the communication system. Which called **Shannon-para-diagram**.

Paradigm



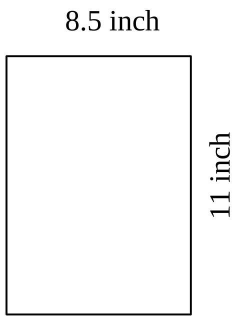
a) Information Source:- is a device which randomly delivers symbols from alphabet as example , PC which connected to internet.
b) Source Coder:- always one to represent the data source more compactly by eliminating redundancy (its aims to reduce data rate).

c) Channel Coder:- adds redundancy to protect transmitted signal agents transmission errors (increase data rate).
d) Channel:- is a system which links a transmitter to receiver include signaling equipment, pair of cooper wire, coaxial cable ….. etc.
e) Source decoder and Channel decoder are converse to source and channel coders.

Information Theory applied on:

1) Communication theory.
2) Pure mathematic.
3) Computer science.
4) Translation machines.
5) Genetics science.
6) Psychological science.
7) Illnesses diagnoses.

Example(1): To illustrate the result of using information theory methods; the transmission of facsimile



1" block pixel or dot

0" white pixel or dot

Resolution is 200 pixel or dot per inch= $4*10^4$ dot/inch^2

No. of pixel per page= $8.5* 11* 4* 10^4$= 3.74 Mbits

3.74*10^6/14.4*10^3/60=4 min and 20 sec

Which modem (14.4 kb/sec) the transmission of one page take 4 minutes and 20 seconds, but by using "Huffman coding" one of source coding technique. The time of transmission reduced to 17 second.

Example(2): The musical stereo analog signal left & right channel are sampled at 44.1 kHz (for cd quality), each sample quantized to 16 bit then one second contain $44.1*10^3*16*2$= 1.411 Mbits, by using MP3 coding Algorithm the value reduced to 128 kbits without distinguish between the original sound and the coded sound.
a-CD which has capacity of 650Mbits can store more than 10 hours of MP3 stereo music.

DIGITAL COMMUNICATIONS
Information Theory
Lecture (2-3)   INFORMATION MEASUREMENT

Shannon pursued the idea of the information contain of the event (E) is depend solely on the probability of this event occurrence P(E) and the following facts must be gets:

1) If the probability of ~~a certain~~ an event happening is certain, there is no information held by the event

   If $P(E) = 1$ $h(E) = 0$

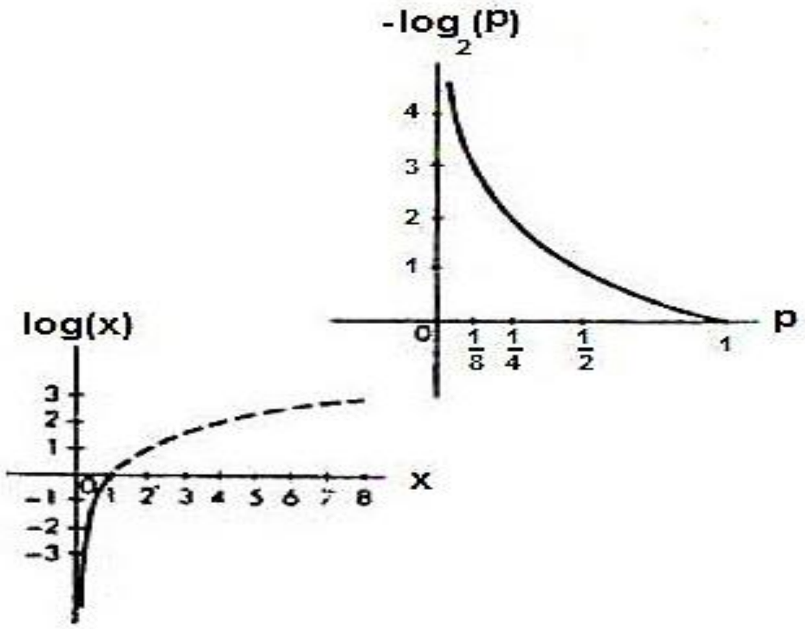
   $h(E)$ = information contain

2) If the chance of the event is minimal, the information contained in the event is large as possible

   $P(E) \lim_{\rightarrow 0}$ $h(E) \approx$ Maximum

   Then only algorithmic function satisfy the previous axiom

$$h(E) = \log \frac{1}{P(E)} = -\log P(E)$$



As h(E) represent information measure of the event (E) it is express in different units according to the Base of algorithm.

Log Base	Unite
2	bit or Sh (Shannon)
e	natural unit (Nepers)
3	trit
10	decimal (digit)

Example(3): find the information contain of the drawn one of 52 pack card if E = the card drawn is the king of hearts.

Each card has the same probability $P(E) = \frac{1}{52}$

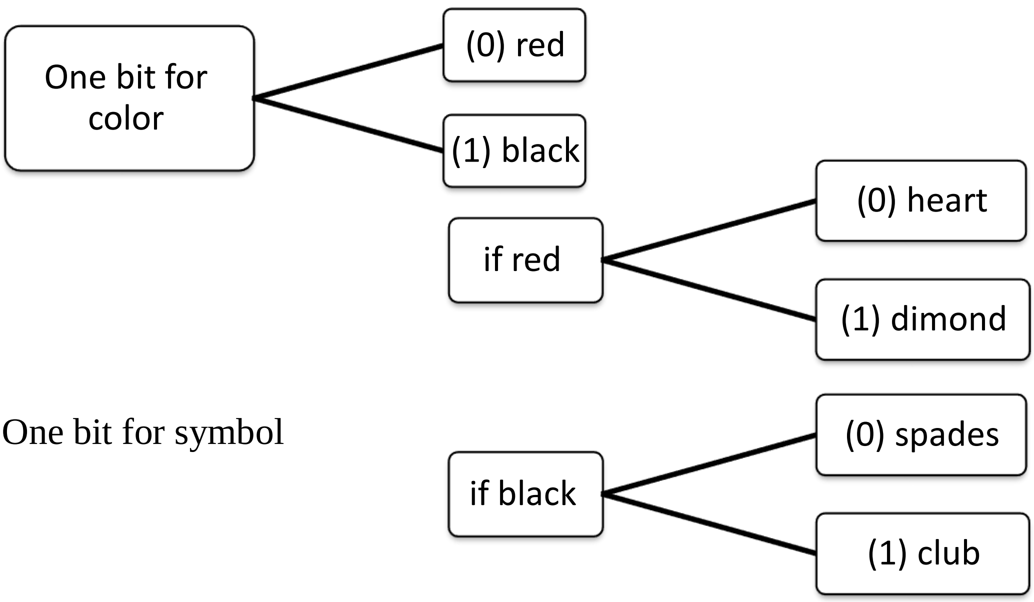
$h(E) = \log_2 \frac{1}{1/52} = -\log_2 \frac{1}{52} = \log_2 52$

h(E) = 5.703 bit 5.703 not integer

If the pack card are 32 card find h(E).

$h(E) = \log_2 \frac{1}{1/32} = -\log_2 \frac{1}{32} = \log_2 32 = 5$ bit

as h(E) = 5 an integer No. then 5 bit are required to satisfy the playing card among 32 card.



The last 3 bit represent the card No. within the heart red card at each stage, we divide the set of left card into two subset having the same No. of elements.

We can interpret that if N event have equally probability then the h(x) = log N ……………………… (1)

If the event (E) depend on the event (F) the uncertainty " information contain" of the event (E) can be found by using the condition probability P(E/F).

$$h\left(\frac{E}{F}\right) = -\log P\ (E/F)$$

while $P\left(\frac{E}{F}\right) = \frac{P(E.F)}{P(F)} = \frac{P(\ E\cap F)}{P(F)}$

$P(E.F) = P(E).P(F)$ if the events independent

P(E.F) = 0 if the events are Mutually Exclusive

$$\left.\begin{aligned} \therefore\ P(E/F) &= P(E) \\ P(F/E) &= P(F) \end{aligned}\right\} \text{If the events independent}$$

Example (4): From 32 pack card what is the uncertainty of (E) the card drawn is king of heart known F the card is heart?

$$P\left(\frac{E}{F}\right) = \frac{P(E\cap F)}{P(F)}$$

$$P(E\cap F) = P(E).P(F)$$

$P(F) = \frac{1}{4}$ one from 4 symbol

$$P(E\cap F) = P(E).P(F) = \frac{1}{8}\times\frac{1}{4} = \frac{1}{32}$$

$$P\left(\frac{E}{F}\right) = \frac{1/32}{1/4} = \frac{4}{32} = \frac{1}{8}$$

Note that P(E/F) = P(E) = 1/8

Because the events are independent

$$\therefore h\left(\frac{E}{F}\right) = -\log P\left(\frac{E}{F}\right) = -\log \frac{1}{8} = 3\ bits$$

$$h(E.F) \quad or \quad h(E \cap F) = ?$$

$$P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)}$$

$$P\left(\frac{E}{F}\right).P(F) = P(E \cap F)$$

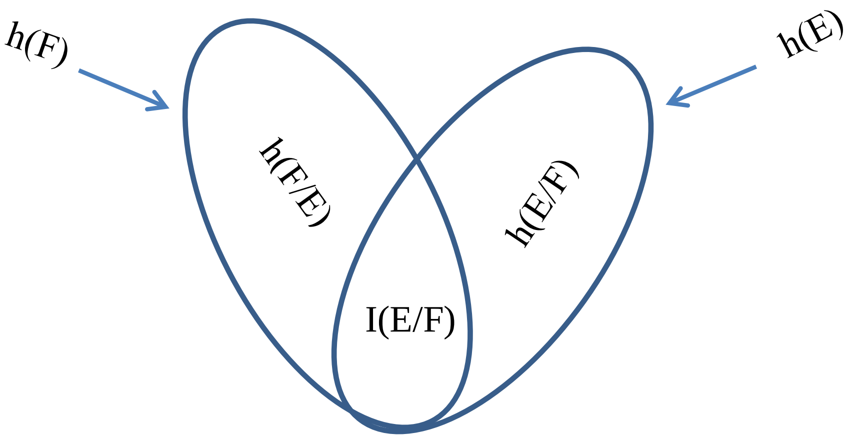
$$-\log P(E \cap F) = -\log P\left(\frac{E}{F}\right) - \log P(F)$$

$$\therefore h(E \cap F) = h\left(\frac{E}{F}\right) + h(F)$$

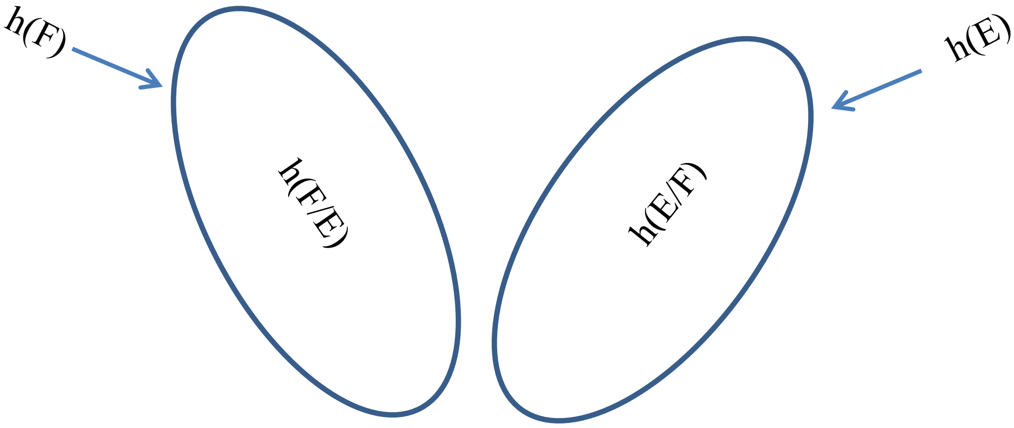
$$\left.\begin{aligned} \text{As } P\left(\frac{E}{F}\right) &= P(E) \\ P\left(\frac{F}{E}\right) &= P(F) \end{aligned}\right\} \text{If the events are independent}$$

$$\therefore h(E \cap F) = h(E) + h(F)$$

Dependent



Independent



For dependent event

$h(E.F) = h(E \cap F) = h(E) + h\left(\frac{F}{E}\right) = h(F) + h(\frac{E}{F})$

So $h\left(\frac{F}{E}\right) = h(E \cap F) - h(E)$

$h\left(\frac{E}{F}\right) = h(E \cap F) - h(F)$

$I(E;F) = h(E \cap F) - h\left(\frac{F}{E}\right) - h(\frac{E}{F})$

Mutual information between (E&F)

$I(E;F) = h(F) - h(\frac{F}{E})$

$I(E;F) = h(E) - h(\frac{E}{F})$

$I(E;F) = h(E) + h(F) - h(E \cap F)$

For independent events

$h(E \cap F) = h(E) + h(F)$

$h\left(\frac{E}{F}\right) = h(E)$

$h\left(\frac{F}{E}\right) = h(F)$

$I(E;F) = 0$

Example (5): for previous example(4) find the mutual information between E and F.

$I(E;F) = h(E) + h(F) - h(E \cap F)$

Or $\quad = h(E) - h(\frac{E}{F})$

$h(E) = -\log P(E) = -\log \frac{1}{32} = 5bit$

$h(E) - h\left(\frac{E}{F}\right) = 5 - 3 = 2\ bit$

.:. the mutual information between E&F are 2 bit

Example (6): two playing cards are simultaneously drawn from pack 32 card let E respectively F be the event at least one of the two card drawn is red respectively the king of spades is one of the two drawn cards, what is the amount of mutual information between E and F ?

R $I(E;F) + h(E) - h(\frac{E}{F})$

T $h(E) = -\log P(E)$ probability of one of card is red

$= P(E) = \frac{16}{32} \times \frac{15}{31} + \frac{16}{32} \times \frac{16}{31} + \frac{16}{32} \times \frac{16}{31}$

$P(E) = \frac{47}{62}$

$P\left(\frac{E}{F}\right) = \frac{16}{31}$

$I(E;F) = \log_2 \frac{62}{47} - \log_2 \frac{31}{16} = -0.5546\ bit$

DIGITAL COMMUNICATIONS
Information Theory
Lecture (3-3) SOURCE ENTROPY

If the h(E) is the information measurement of the event (E).

(H) the entropy is the measurement of the average information contain of the whole source of information

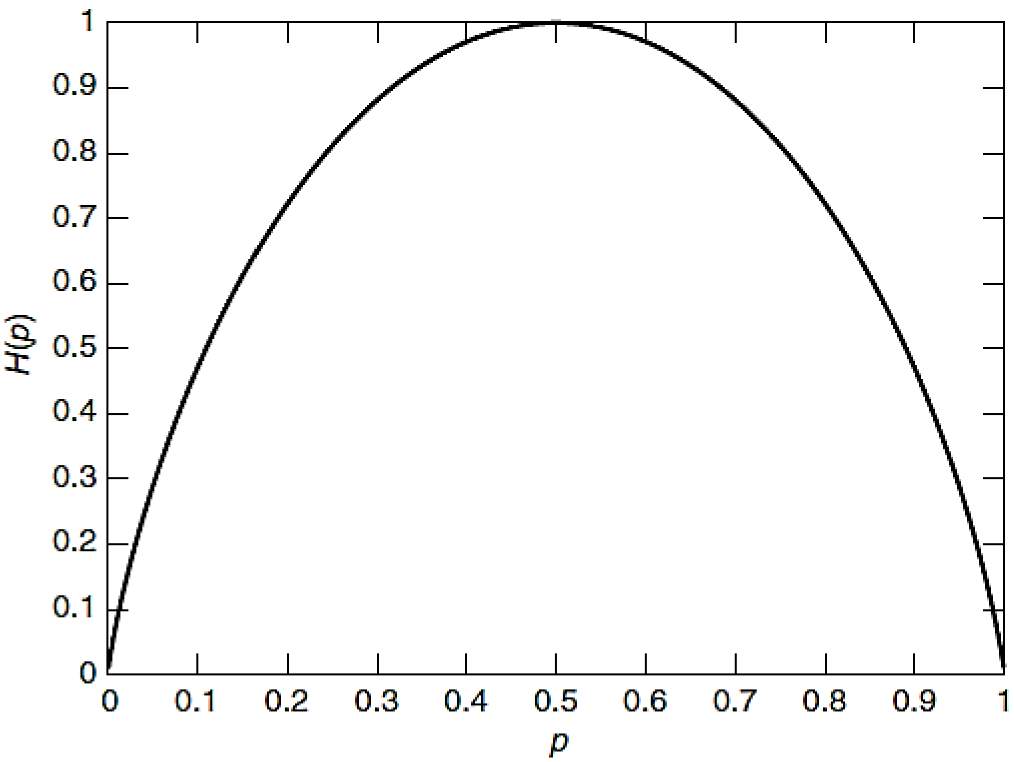
$H = P1 \log P1 - P2\ \log P2 - P3 \log P3 - \ldots \ldots \ldots \ldots$ For binary source

$H = \ -P1\ \log_2 P1 - P2\ \log_2 P2$

If P1= P2 = 1/2 uniform distribution

$H = \ -\frac{1}{2}\ \log_2 0.5 - \ \frac{1}{2}\ \log_2 0.5$

$= \ \frac{1}{2} + \ \frac{1}{2} = 1$



$H = \sum_{i=1}^{N} - Pi\ \log_2(Pi)$

Example (7): a random experiment consist of drawning one card from pack of 32 playing card. Let x be:

$x = \sqrt{3}$

$x = 7$

$x = \log \pi$

find the entropy of the information source.

$P(x = \sqrt{3})$ (red) = 1/2 or 16/32

$P(x = 7)$ (spade) = 8/32 = 1/4

$P(x = \log \pi)$ (diamond) = 8/32 = 1/4

$h(x = \sqrt{3}) = -\log\frac{1}{2} = 1\ bit$

$h(x = 7) = -\log\frac{1}{4} = 2\ bit$

$h(x = \log \pi) = -\log\frac{1}{4} = 2\ bit$

the average uncertainty (H) became

$H=(-\frac{1}{2}\log_2\frac{1}{2} - \frac{1}{4}\log_2\frac{1}{4} - \frac{1}{4}\log_2\frac{1}{4})$

=1/2 + 1/2 +1/2 = 1.5 bit

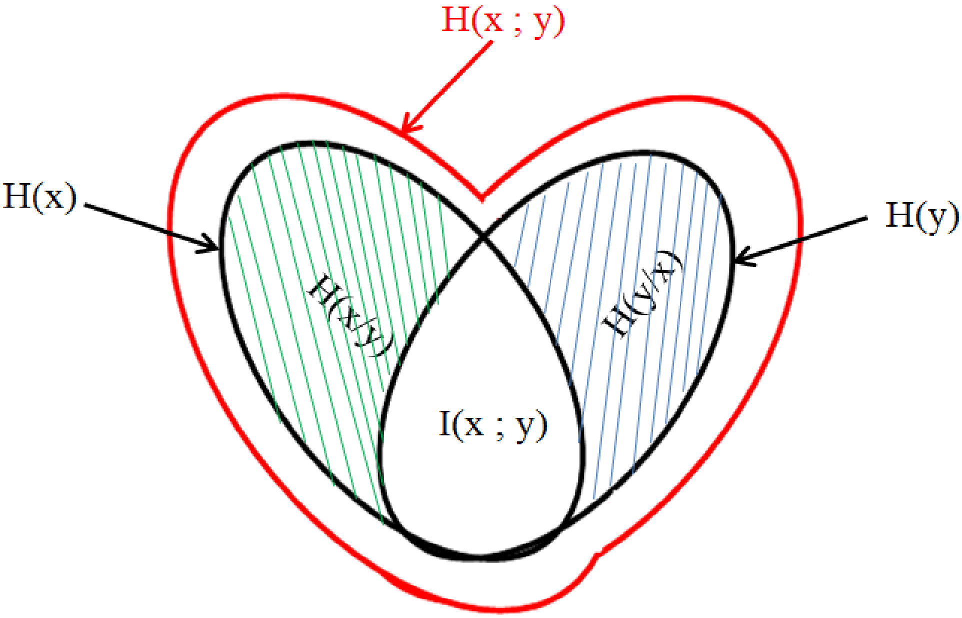
This means that the average number of bits required to represent value of x is 1.5 bits.

Note that:

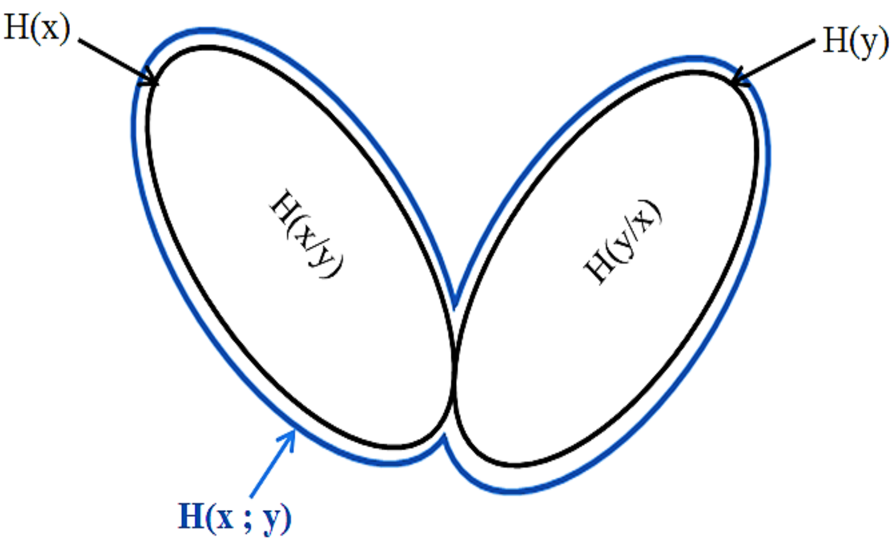
1) H(x) depend only on probability of x not on the actual values taken by x.
2) If n is finite the maximum H(x) if and only if x is uniform distribution over the value P=1/N
   H(x)= log N

Join and Conditional Entropy

Dependent



Independent



$$H(x,y) = H(x) + H(\frac{y}{x})$$

$$H(x,y) = H(y) + H(\frac{x}{y})$$

$$H(x,y) = H(x) + H(y) - I(x;y)$$

$$H\left(\frac{x}{y}\right) < H(x)$$

$$H(\frac{y}{x}) < H(y)$$

$$H\left(\frac{x}{y}\right) = H(x,y) - H(y)$$

$$H(\frac{y}{x}) = H(x,y) - H(x)$$

If x & y are independent

$$H\left(\frac{x}{y}\right) = H(x)$$

$$H\left(\frac{y}{x}\right) = H(y)$$

$$H(x;y) = H(x) - H(y)$$

$$I(x;y) = 0$$

Let us consider (x,y) be discrete variables taking an variable values

x=[x_1, x_2, x_3, ………. , x_n] and y=[y_1, y_2, y_3, ………. ,y_m] with

pi = p(x = xi) , p_J = p(y = y_J) , pi_J = p(x = xi ∩ y = y_J)

The joint entropy of pair (x,y)

$$H(x,y) = -\sum_{i=1}^{N}\sum_{J=1}^{m} P_{iJ} \log P_{iJ}$$

$$H\left(\frac{x}{y}\right) = \sum_{J=1}^{m} P_J \ H(\frac{X}{Y} = Y_J)$$

Where

$$H\left(\frac{x}{y_{=y_J}}\right) = -\sum_{i=1}^{N} p(x = x_i/y_J) \log P(x = x_i/y_{=yJ})$$

$$I(x;y) = H(x) - H\left(\frac{x}{y}\right)$$

$$= H(y) - H\left(\frac{y}{x}\right)$$

$$I(x;y) = I = \log\frac{P(x,y)}{P(x).P(y)}$$

Example (9): If the joint probability of 2 group x=(x_1 , x_2 , x_3) and y=(y_1, y_2) are

$$P(x,y) = \begin{array}{c c} & \begin{array}{cc} y1 & y2 \end{array} \\ \begin{array}{c} x1 \\ x2 \\ x3 \end{array} & \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.125 \\ 0.0625 & 0.0625 \end{bmatrix} \end{array}$$

Find P(x) , P(y) , H(x) , H(y) , I(x1, y2) , H(x/y) , H(y/x).

Solution:

$$P(x_i) = \sum_{J=1}^{m} P(x_i , y_J)$$

$$P(x_1) = \sum_{J=1}^{m} P(x_1, y_J)$$ sum the first row

$$P(x_2) = \sum_{J=1}^{m} P(x_2 , y_J)$$ sum the second row

$$P(x_3) = \sum_{J=1}^{m} P(x_3, y_J)$$ sum the third row

$$P(x) = \begin{bmatrix} 0.5 & + & 0.25 \\ 0 & + & 0.125 \\ 0.0625 & + & 0.0625 \end{bmatrix} = \begin{bmatrix} 0.75 \\ 0.125 \\ 0.125 \end{bmatrix}$$

$$P(y_J) = \sum_{i=1}^{N} P(x_i, y_J)$$

$$P(y_1) = \sum_{i=1}^{N} P(x_i, y_1)$$ sum the first column

$$P(y_2) = \sum_{i=1}^{N} P(x_i, y_2)$$ sum the second column

$$P(y) = [0.5 + 0 + 0.0625 \qquad 0.25 + 0.125 + 0.0625]$$

$$P(y) = [0.5625 \qquad 0.4375]$$

$$H(x) = -\sum_{i=1}^{N} P(x_i)\ \log P(x_i)$$

$$H(x) = -0.75\ \log(0.75) - 0.125\ \log(0.125) - 0.125\ \log(0.125)$$

$H(x) = 1.0627$ bit/symbol

$$H(y) = -\sum_{J=1}^{m} P(y_J)\ \log P(y_J)$$

$$= -0.5625\ \log(0.5625) - 0.4375\ \log(0.4375)$$

$H(y) = 0.9887$ bit/symbol

The join entropy=?

$$H(x,y) = -\sum_{J=1}^{m}\sum_{i=1}^{N} P(x_i, y_J)\ \log P(x_i, y_J)$$

From the matrix probability P(x,y)

$$H(x,y) = -0.5\ \log 0.5 - 0.25\ \log 0.25 - 0.125\ \log 0.125 - 0.0625 \log 0.0625 - 0.0625\ \log 0.0625$$

$H(x,y) = 1.875$ bit/symbol

$$I(x_i, y_J) = \log \frac{P(x_i, y_J)}{P(x_i).P(y_J)}$$

$P(x_1, y_2)$ can gets from the probability

Matrix = 0.25

$P(x_1)$ = can gets from P(x) matrix

= 0.75

$P(y_2)$ = can get from P(y) matrix

= 0.4375

$$I(x_1, y_2) = \log(\frac{0.25}{0.75*0.4375})$$

H(x/y) the loss entropy

$$H\left(\frac{x}{y}\right) = H(x,y) - H(x)$$

= 1.875 - 1.0627= 0.8123

H(y/x) the noise entropy

$$H\left(\frac{y}{x}\right) = H(x,y) - H(y) = 1.875 - 0.9887$$

DIGITAL COMMUNICATIONS
Information Theory
Lecture (4-3) SOURCE CODING

There are two types of coding for information:

1) Fixed length coding: which have no reduction of No. of bits to represent all variables:
   $N = \log_2 m$ where m= number of variable
   N= bit length code
2) Variable length code: the usual code are:
   a) Shannon-Fano code
   b) Huffmen code
   c) Extended source
   d) Compression code

Also variable length code can be classified into:

a) Lossless coding
b) Lossy coding

Lossless_variable length source coding the major job of this source coding are:

- ❖ Transforming the source information into standardized source almost without redundancy so that the bit rate is reduced.
- ❖ The main idea to reduce redundancy and so the bit rate is that the data compression can be achieved by assigning short code_word to commonly occurring "more probability" message and longer code_word to less frequency message.
- ❖ The amount of information of all messages to be transmitted is retained after coding without loss of information.

Notes, some of source coder due to loss a few an amount of information so that called lossy compression source coding. There are many types of lossless source coding algorithm as Shannon-Fano, Huffmen, LZ, LZW, ….. ets.

Shannon-Fano Algorithm

The Shannon-Fano encoding scheme is based on the principle of that each code bit which can be described by random variable, must have maximum entropy.

1. Arrange the source massage such as the probabilities are in the decrease order.
2. Divide the list of massages into two subset as balanced as possible, in the sense of sum of elementary probability.
3. Assign respectively the symbol "0" to the up element and "1"to the farther ones.
4. Repeat the step (2&3) with each subset until the operation becomes impossible.

Example(10): Let the text [AAAA BBB CDC] find the minimum code word using Shanonn-Fano, find the average code word LC and the source code efficiency, also find the probability of sending zero and one.

Solution:

symbol	N	P(x)	Code word	l_i	0_i	1_i
A	4	0.4	0	1	1	0
B	3	0.3	1 0	2	1	2
C	2	0.2	1 1 0	3	1	2
D	1	0.1	1 1 1	3	0	3

LC=average code word

$$LC = \sum_{i=1}^{n} l_i * P(x_i)$$

$$= 1 * 0.4 + 2 * 0.3 + 3 * 0.2 + 3 * 0.1 = 1.9\ bit$$

Source efficiency= $\frac{H(x)}{LC} * 100\%$

$$H(x) = -\sum_{i=1}^{n} P(x) \log_2 P(x)$$

$$= 0.4\ \log_2 0.4 - 0.3\ \log_2 0.3 - 0.2 \log_2 0.2 - 0.1 \log_2 0.1$$

= 0.528 + 0.521 + 0.46 + 0.333

H(x) = 1.839 bits

Source code efficiency = $\frac{1.839}{1.9} * 100\%$ = 96.78%

$$P(0) = \frac{\sum_{i=1}^{n} 0_i P(x)}{LC} = \frac{0.4+0.3+0.2}{1.9} = 0.473$$

$$P(1) = \frac{\sum_{i=1}^{n} l_i\, P(x)}{LC} = \frac{0.3+2*0.2+3*0.1}{1.9} = \frac{1}{1.9} = 0.527$$

$$P(0) = \frac{\sum_{i=1}^{n} i\; 0\; P(x)}{LC}$$

$$P(0) = \frac{2*0.4+1*0.19+1*0.16+1*0.15}{2.25} = \frac{0.8+1.9+0.16+0.15}{2.25} = \frac{1.3}{2.25} = 0.577$$

$$P(1) = \frac{\sum_{i=1}^{n} i1\; P(x)}{LC}$$

$$P(1) = \frac{0.19*1+0.16*1+0.15*2+0.1*3}{2.25} = \frac{0.19+0.16+0.3+0.3}{2.25} = \frac{0.95}{2.25} = 0.423$$

Or P(1) = 1- P(0) = 1- 0.577 = 0.423

Example(11): Source with 5 messages with respective probability (0.4 , 0.19 , 0.16 , 0.15 , 0.1), find the Shannon-Fano code, the average code word length, the source efficiency and the probability of "1" and "0".

Solution:

	P(MSi)	**Code word**			**Li**	**L 0**	**L 1**
MS1	0.4	0 1st step	0 2nd step	3rd step	2	2	0
MS2	0.19	0	1		2	1	1
MS3	0.16	1	0		2	1	1
MS4	0.15	1	1	0	3	1	2
MS5	0.1	1	1	1	3	0	3

The code are (00 , 01 , 10 , 110 , 111)

$$Lc = \sum_{i=1}^{m} Li * P(x_i)$$

$$= 2*0.4 + 2*0.19 + 2*0.16 + 3*0.15 + 3*0.1$$

$$= 0.8 + 0.38 + 0.32 + 0.45 + 0.3$$

$$Lc = 2.25 \;\; bits$$

$$H(x) = -\sum_{j=1}^{n} P(i) \log P(i)$$

$$= -0.4\log 0.4 - 0.19\log 0.19 - 0.16\log 0.16 - 0.15\log 0.15 - 0.1\log 0.1$$

= 2.15 bit

$$\eta = \frac{H(x)}{Lc} * 100\% = \frac{2.15}{2.25} * 100\% = 95.6\%$$

Huffman Code

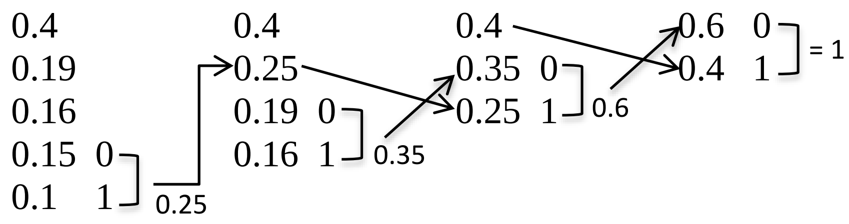
The Huffman code is optimum in the sense that no other instantaneous code for the same probability distribution can have a better efficiency, practically the code efficiency of Huffman algorithm is greater than or equal to the code efficiency obtained by Shannon-Fano code algorithm.

Huffman algorithm based on the following steps:

1- Arrange the outputs(message symbol) in decreasing order of their probability.
2- Combine the two least probable message together into single new output, and replace the two previous ones and whose probability is the sum of corresponding probabilities.
3- If the number of remaining outputs is (1) one output, then go to the next step; otherwise go to step (1), with a new list to be arranged with number of "outputs" reduced.
4- Assign arbitrary "0" and "1" (up to "0-1") as first symbol of the 2 words (Nodes) corresponding to the 2 remaining outputs. "last iteration node with sum=1".
5- If an output is the result of the merger of 2 outputs in preceding iteration append the current word with 0" and 1" to obtain the word for preceding outputs and repeat(5) if no output is preceded by another output in an iteration, then, stop.

Example (12): Repeat example(11) using Huffman algorithm.

Solution:



0.4 ≡ 1 , 0.19 ≡ 000 , 0.16 ≡ 001 , 0.15 ≡ 010 , 0.1 ≡011

P(x)	Word-code	Li	L0	L1
0.4	1	1	0	1
0.19	000	3	3	0
0.16	001	3	2	1
0.15	010	3	2	1
0.1	011	3	1	2

The average word code is $Lc = \sum_{i=1}^{m} L_i * P(x_i)$

$= 0.4 * 1 + 0.6 * 3 = 2.2\ bits$

$H(x) = -\sum_{i=1}^{n} P(x_i) \log P(x_i)$

$= -0.4 \log 0.4 - 0.19 \log 0.19 - 0.16 \log 0.16 - 0.15 \log 0.15 - 0.1 \log 0.1 = 2.15$

$\eta = \frac{H(x)}{Lc} * 100\% = \frac{2.15}{2.2} * 100\% = 97.7\%$

$P(0) = \frac{\sum_{j=1}^{n} L0 * P(xi)}{Lc} = \frac{3*0.19+2*0.16+2*0.15+0.1}{2.2} = \frac{1.29}{2.2} = 0.586$

$P(1) = \frac{\sum_{j=1}^{n} L1 * P(xi)}{Lc} = \frac{0.4+0.16+0.15+0.2}{2.2} = \frac{0.91}{2.2} = 0.414$

Example(13): Let the source taking values in (A, B, C, D, E, F, G) with probabilities {0.4, 0.2, 0.15, 0.1, 0.05, 0.05, 0.05} respectively, find the average code word and the source code efficiency if using

A) Shannon-Fano algorithm   B) Huffman algorithm

Solution:

Shannon-Fano algorithm

symbol	P					Code word	Li
A	0.4	1	1			11	2
B	0.2	1	0			10	2
C	0.15	0	1	1		011	3
D	0.1	0	1	0		010	3
E	0.05	0	0	1	1	0011	4
F	0.05	0	0	1	0	0010	4
G	0.05	0	0	0		000	3

$$Lc = \sum_i^m L_i * P(x_i)$$

$$Lc = 2 * 0.4 + 2 * 0.2 + 3 * 0.15 + 3 * 0.1 + 4 * 0.05 + 4 * 0.05 + 3 * 0.05$$

Lc= 2.5 bit the average word code

$$H(x) = -\sum_{i=1}^n P(x_i) \log P(x_i)$$

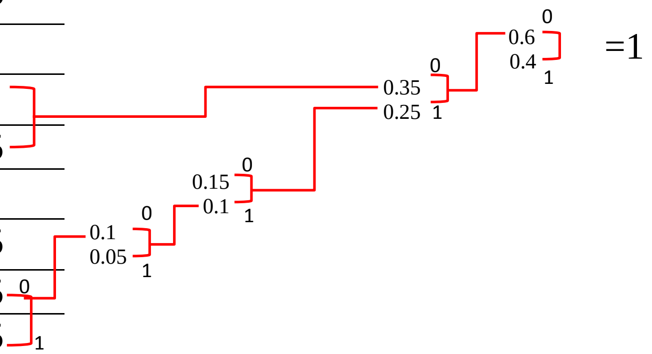
$$= -0.4 \log 0.4 - 0.2 \log 0.2 - 0.1 \log 0.1 - 0.3 * 0.05 \log 0.05$$

H(x)= 0.5287 + 0.4642 + 0.3321 + 3*0.216 = 2.1893 bit

$$\eta = \frac{H(x)}{Lc} * 100\% = \frac{2.1893}{2.5} * 100\% = 87.57\%$$

A) Huffman algorithm

symbol	P
A	0.4
B	0.2
C	0.15
D	0.1
E	0.05
F	0.05
G	0.05



symbol	P	Word code	Li
A	0.4	1	1
B	0.2	000	3
C	0.15	100	3
D	0.1	110	3
E	0.05	1010	4
F	0.05	00010	5
G	0.05	10010	5

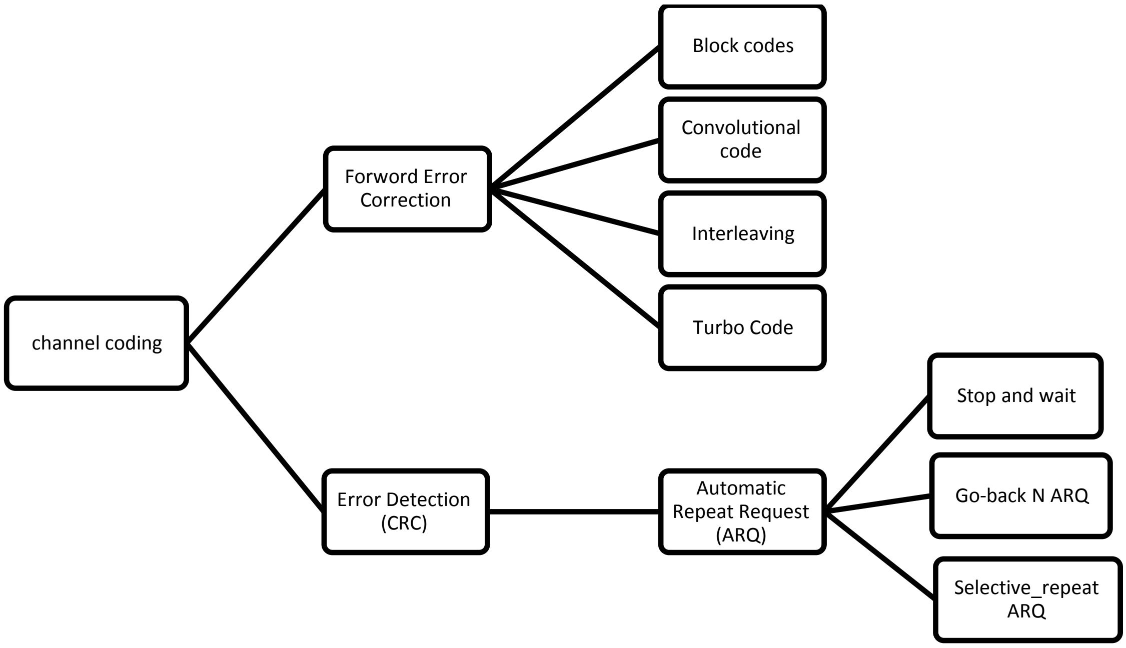
$$Lc = \sum_i^n L_i * P(x_i)$$

$$Lc = 0.4 + 3 * 0.45 + 4 * 0.05 + 5 * 0.1 = 0.4 + 1.35 + 0.2 + 0.5$$

Lc= 2.45 bits

$$\eta = \frac{H(x)}{Lc} * 100\% = \frac{2.1893}{2.45} * 100\% = 89.36\%$$

DIGITAL COMMUNICATIONS
Information Theory
Lecture (5-3)   CHANNEL CODING

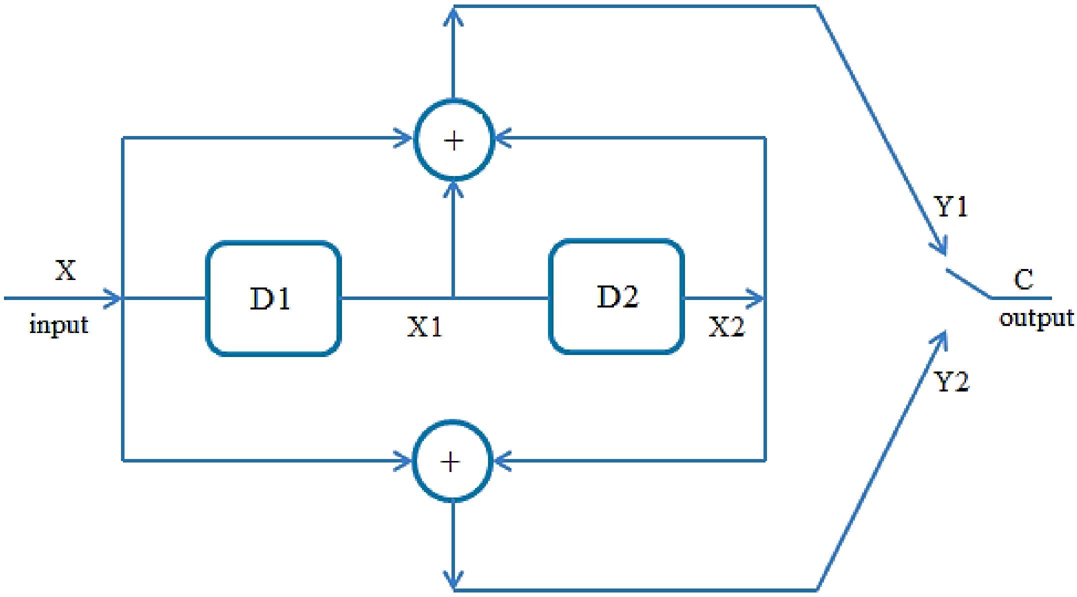


- **Block Code:** information is divided in to blocks of length k, r parity bits are added to each block.

  Total length n = k + r

  Code rate = R = k / n

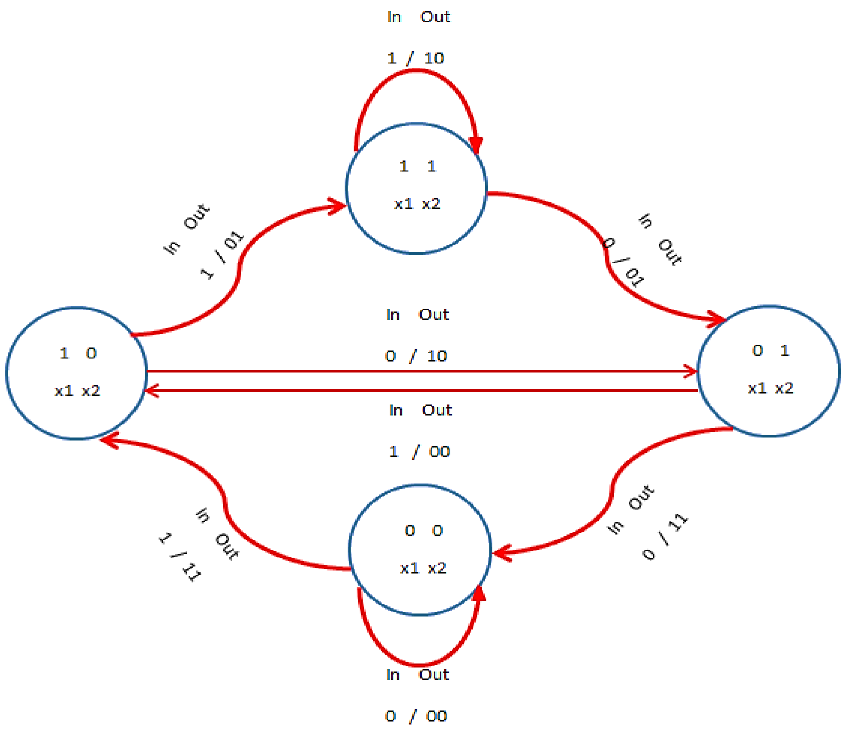
- **Convolution Codes:** encoding of information stream not blocks, value of certain information symbols. Easy implementation using shift register , Decoding is mostly performed by the vibration algorithm.



Input	1	1	1	0	0	0
output	1 1 y1 y2	0 1 y1 y2	1 0 y1 y2	0 1 y1 y2	1 1 y1 y2	0 0 y1 y2

$$y_1 = x + x_1 + x_2$$

$$y_2 = x + x_2$$



❖ **Interleaving:**

Input data a_1 , a_2 , a_3 , ………, a_{16}

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 \\ a_5 & a_6 & a_7 & a_8 \\ a_9 & a_{10} & a_{11} & a_{12} \\ a_{13} & a_{14} & a_{15} & a_{16} \end{bmatrix}$$

Interleaving

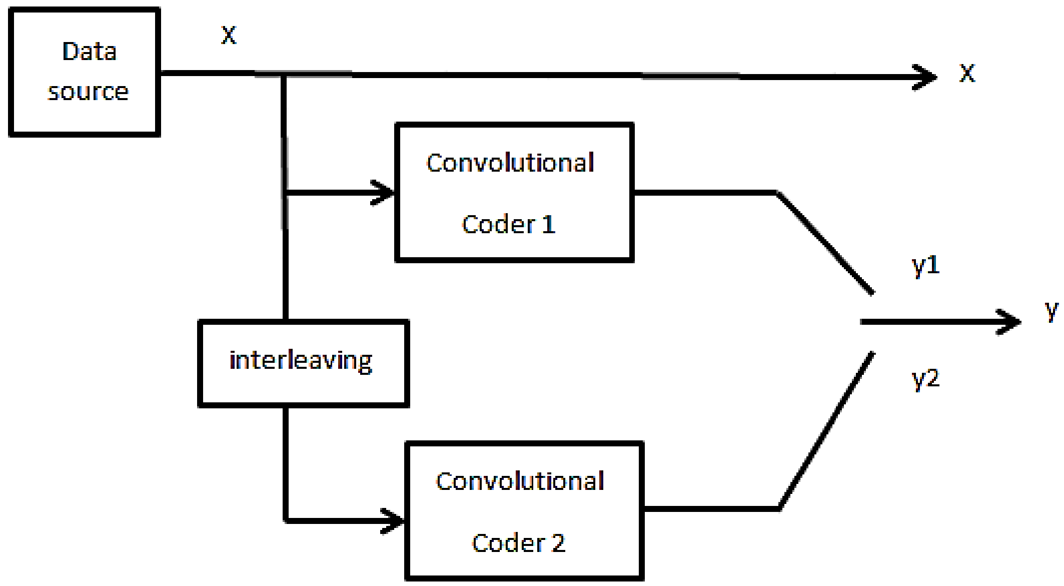
$$\begin{bmatrix} a_1 & a_5 & a_9 & a_{13} \\ a_2 & a_6 & a_{10} & a_{14} \\ a_3 & a_7 & a_{11} & a_{15} \\ a_4 & a_8 & a_{12} & a_{16} \end{bmatrix}$$

a_1 , a_5 , a_9 , a_{13} , a_2 , a_6 , a_{10} , a_{14} , a_3 , a_7 , a_{11} , a_{15} , a_4 , a_8 , a_{12} , a_{16}

to avoid burst error

❖ **Turbo coders**

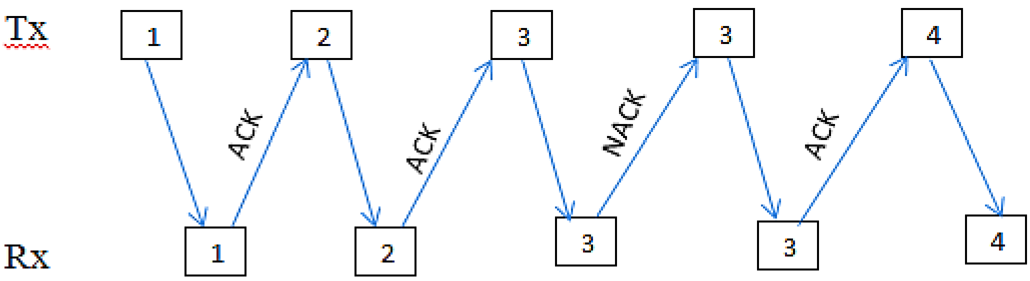
Turbo codes have been proposed for low-power applications such as third generation of mobile, personal communication service, adhoc, sensor network and satellite communications.



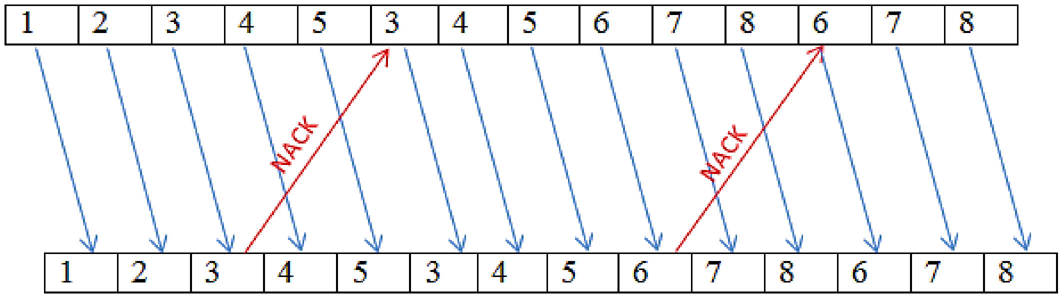
- **Error detections**

1) CRC
2) Automatic Repeat Request (ARQ)

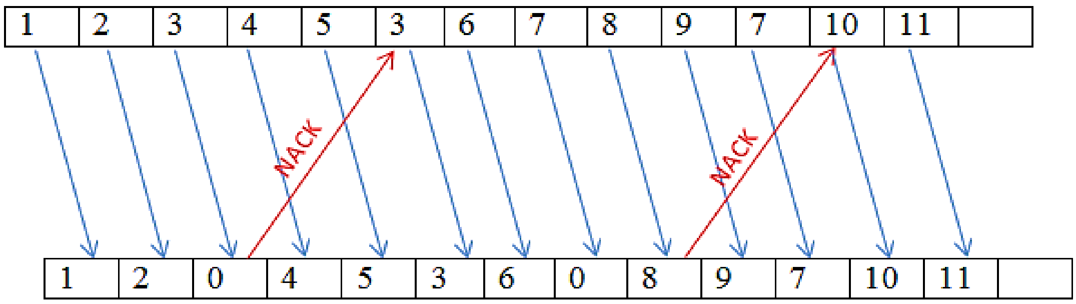
A) Stop - and wait ARQ (SAW-ARQ)



B) Go - Back - N ARQ (GBN ARQ)



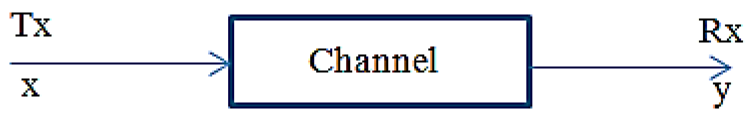
C) Selective-Repeat ARQ (SRARQ)

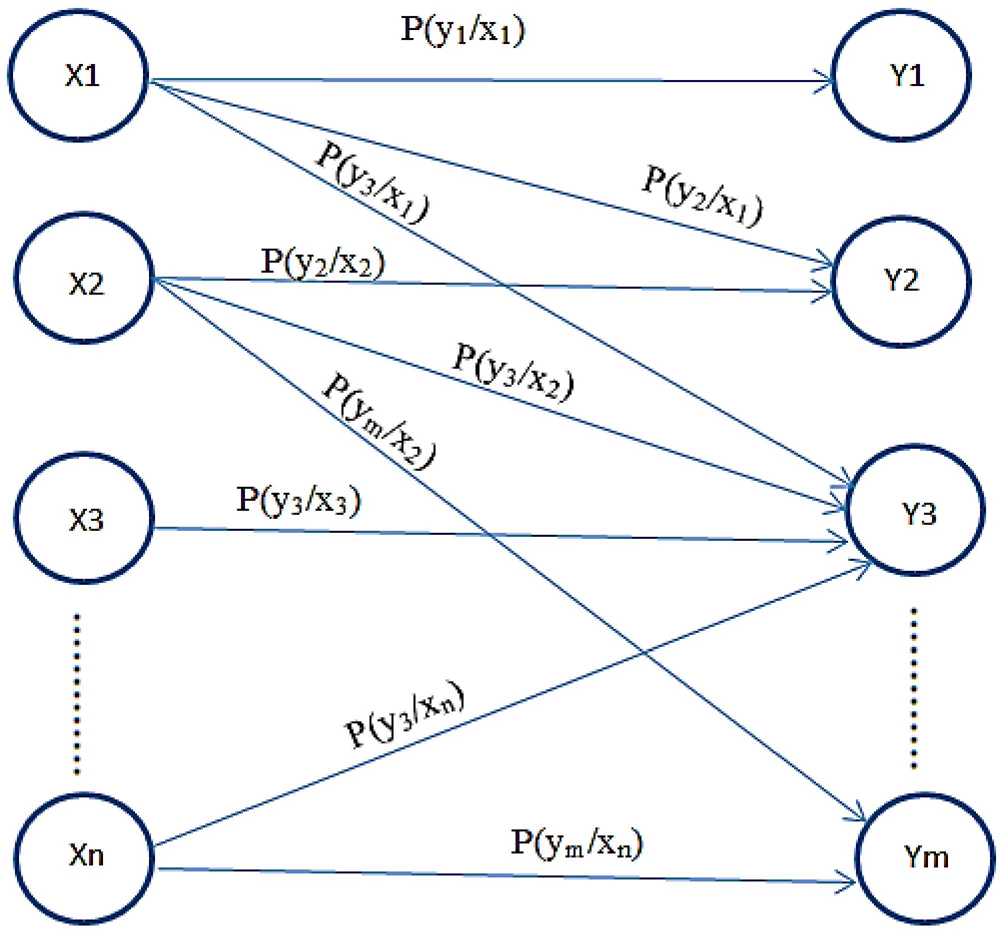


DIGITAL COMMUNICATIONS
Information Theory
Lecture (6-3)   INFORMATION CHANNEL

- ✓ **Channel modeling**
- ✓ **Channel types**
- ✓ **Channel capacity**

❖Channel Modeling





$$P(y/x) = \begin{bmatrix} P(\frac{y_1}{x_1}) & P(\frac{y_2}{x_1}) & P(\frac{y_3}{x_1}) & \cdots & P(\frac{y_m}{x_1}) \\ P(\frac{y_1}{x_2}) & P(\frac{y_2}{x_2}) & P(\frac{y_3}{x_2}) & \cdots & P(\frac{y_m}{x_2}) \\ P(\frac{y_1}{x_3}) & P(\frac{y_2}{x_3}) & P(\frac{y_3}{x_3}) & \cdots & P(\frac{y_m}{x_3}) \\ \vdots & \vdots & \vdots & & \vdots \\ P(\frac{y_1}{x_n}) & P(\frac{y_2}{x_n}) & P(\frac{y_3}{x_n}) & \cdots & P(\frac{y_m}{x_n}) \end{bmatrix}$$

Notes: $\sum_{i=1}^{n} P\left(\frac{y_j}{x_i}\right) = 1$ (The sum of each row =1)

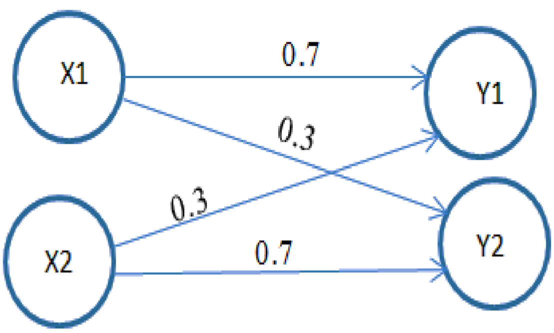
❖Channel Types

1) Symmetric channels

2) A-Symmetric channels

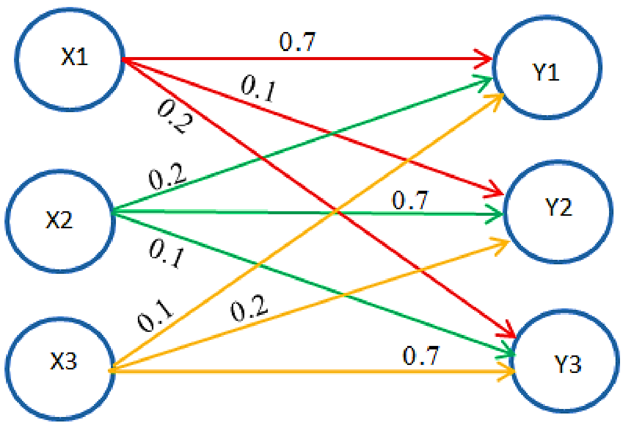
1) Symmetric channels: where **n = m** and the rotation of every row result the next row from p(y/x). There are three types of symmetric channel:

A) Binary symmetric channel



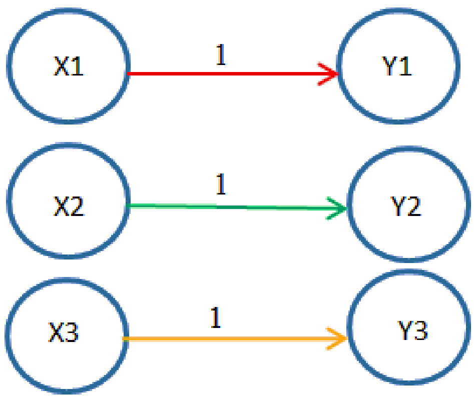
$$P\left(\frac{y}{x}\right) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

B) Tirrary symmetric channel



$$P\left(\frac{y}{x}\right) = \begin{bmatrix} 0.7 & 0.1 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}$$

C) Noiseless channel



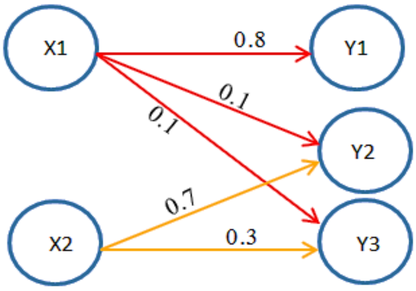
$$P\left(\frac{y}{x}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$H\left(\frac{x}{y}\right) = H\left(\frac{y}{x}\right) = 0$$

2) A-Symmetric channels $n \neq m$

Example(13): complete drawing the following channel model and find H(x), H(y), H(x,y), Noise intropy, losses intropy , if $p(x_1) = 0.6$.

Solution:



$$P\left(\frac{y}{x}\right) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

$$P(x, y) = P\left(\frac{y}{x}\right) . P(x)$$

$P(x) = [P(x_1) \;\; P(x_2)]$ where $P(x_1) = 0.6$

$\therefore P(x_2) = 1 - 0.6 = 0.4$

$P(x) = [\ 0.6 \quad 0.4\ ]$

$$P(x, y) = \begin{bmatrix} 0.8 * 0.6 & 0.1 * 0.6 & 0.1 * 0.6 \\ 0 & 0.7 * 0.4 & 0.3 * 0.4 \end{bmatrix}$$

$$P(x, y) = \begin{bmatrix} 0.48 & 0.06 & 0.06 \\ 0 & 0.28 & 0.12 \end{bmatrix}$$

$$P(y) = [0.48 + 0 \qquad 0.06 + 0.28 \qquad 0.06 + 0.12]$$

$P(y) = [0.48 \qquad 0.34 \qquad 0.18]$ to check the result ,the sum=1

$$H(x) = -\sum_{i=1}^{n} P(x_i) \log P(x_i)$$

$$H(x) = -0.6 \log 0.6 - 0.4 \log 0.4$$

$$H(x) = 0.442 + 0.528 = 0.97\ bits$$

$$H(y) = -\sum_{i=1}^{j} P(y_j) \log P(y_j)$$

$$H(y) = -0.48 \log 0.48 - 0.34 \log 0.34 - 0.18 \log 0.18$$

$$H(x) = -0.508 + 0.529 + 0.445 = 1.486\ bits$$

$$H(x, y) = -\sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i, y_j) \log P(x_i, y_j)$$

$$= -0.48 \log 0.48 - 2 * 0.06 \log 0.06 - 0.28 \log 0.28 - 0.12 \log 0.12$$

$$= 0.508 + 0.487 + 0.514 + 0.367 = 1.876$$

$$H\left(\frac{y}{x}\right) (the\ noise\ intropy) = H(x,y) - H(x)$$

$$= 1.876 - 0.97 = 0.906 \quad bit/s$$

$$H\left(\frac{x}{y}\right) (the\ losses\ intropy) = H(x,y) - H(y)$$

$$= 1.876 - 0.97 = 0.906 \quad bit/s$$

❖Channel Capacity

Shannon theorem

$C = B \log_2(S/N + 1)$, Maximum rate (Max R) = C

Example(14): find the maximum rate if BW = 3.3KHz and S/N = 15dB

Solution:

BW= $3.3*10^3$ Hz S/N=15 dB= 10 log S/N

S/N= $10^{15/10}$ = 31.5

$$C = 3.3 * 10^3 \log_2(31.5 + 1) = 16.574\ KHz$$

Max R = C = 16.574 bit/sec

Lecture (7-3) Tutorial

Example:

The five symbols which have the following frequency and probabilities, design suitable Shannon-Fano binary code. Calculate average code length, source entropy and efficiency.

Symbol	Count	Probabilities	Binary codes	Length
A	15	0.385	00	2
B	7	0.1795	01	2
C	6	0.154	10	2
D	6	0.154	110	3
E	5	0.128	111	3

The average code word length:

$$L = \sum_{j=1}^{m} P(x_j) l_j$$

$$L = 2 \times 0.385 + 2 \times 0.1793 + 2 \times 0.154 + 3 \times 0.154 + 3 \times 0.128$$
$$= 2.28\ bits/symbol$$

The source entropy is:

$$H(Y) = -\sum_{j=1}^{m} P(y_j)\log_2 P(y_j)$$

$$H(Y) = -[0.385ln0.385 + 0.1793ln0.1793 + 2 \times 0.154ln0.154 + 0.128l0.128]/ln2$$

$$H(Y) = 2.18567\ bits/symbol$$

The code efficiency:

$$\eta = \frac{H(Y)}{\text{L}} \times 100 = \frac{2.18567}{2.28} \times 100 = 95.86\%$$

Example

Develop the Shannon - Fano code for the following set of messages, $p(x) = [0.35\ \ 0.2\ \ 0.15\ \ 0.12\ \ 0.1\ \ 0.08]$ then find the code efficiency.

Solution

x_i	$p(x_i)$	Code			l_i
x_1	0.35	0	0		2
x_2	0.2	0	1		2
x_3	0.15	1	0	0	3
x_4	0.12	1	0	1	3
x_5	0.10	1	1	0	3
x_6	0.08	1	1	1	3

$$L_C = \sum_{i=1}^{6} l_i p(x_i) = 2.45 \text{ bits/symbol}$$

$$H(X) = -\sum_{i=1}^{6} p(x_i)\log_2 p(x_i) = 2.396 \text{ bits/symbol}$$

$$\eta = \frac{H(X)}{L_C} \times 100\% = 97.796\%$$

Example: Design Huffman codes for $A = \{a_1, a_2, \dots \dots . a_5\}$, having the probabilities $\{0.2, 0.4, 0.2, 0.1, 0.1\}$.

Symbol	Step 1	Step 2	Step 3	Step 4	Codeword
a_2	0.4	0.4	0.4	0.6 0	1
a_1	0.2	0.2	0.4 0	0.4 1	01
a_3	0.2	0.2 0	0.2 1		000
a_4	0.1 0	0.2 1			0010
a_5	0.1 1				0011

Letter	Probability	Codeword
a_2	0.4	1
a_1	0.2	01
a_3	0.2	000
a_4	0.1	0010
a_5	0.1	0011

The average code word length:

$$L = 0.4 \times 1 + 0.2 \times 2 + 0.2 \times 3 + 0.1 \times 4 + 0.1 \times 4 = 2.2\ bits/symbol$$

The source entropy:

$$H(Y) = -[0.4ln0.4 + 2 \times 0.2ln0.2 + 2 \times 0.1ln0.1]/ln2 = 2.12193 \text{ bits/symbol}$$

The code efficiency:

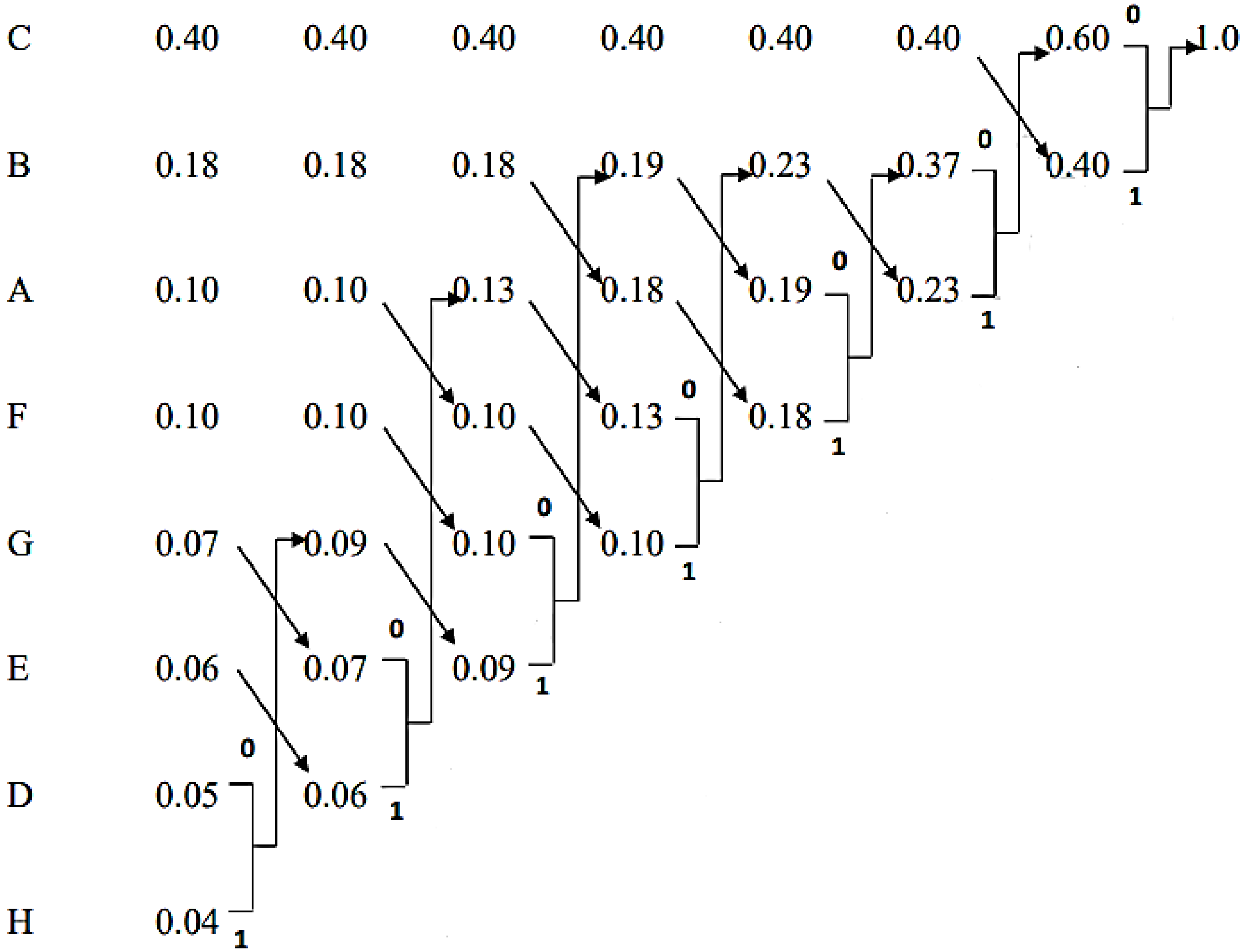
$$\eta = \frac{2.12193}{2.2} \times 100 = 96.45\%$$

Example

Develop the Huffman code for the following set of symbols

Symbol	A	B	C	D	E	F	G	H
Probability	0.1	0.18	0.4	0.05	0.06	0.1	0.07	0.04

Solution



So we obtain the following codes

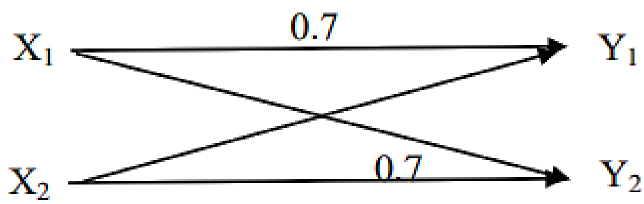
Symbol	A	B	C	D	E	F	G	H
Probability	0.1	0.18	0.4	0.05	0.06	0.1	0.07	0.04
Codeword	011	001	1	00010	0101	0000	0100	00011
l_i	3	3	1	5	4	4	4	5

$$H(X) = -\sum_{i=1}^{8} p(x_i)\log_2 p(x_i) = 2.552 \text{ bits/symbol}$$

$$L_C = \sum_{i=1}^{8} l_i p(x_i) = 2.61 \text{ bits/symbol}$$

Example 9:

For the BSC shown:



Find the channel capacity and efficiency if $I(x_1) = 2bits$

Solution:

$$P(Y \mid X) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$$

Since the channel is symmetric then

$C = log_2 m + K$ and $n = m$

$where\ n\ and\ m\ are\ number\ row\ and\ column\ repestively$

$$K = 0.7log_2 0.7 + 0.3log_2 0.3 = -0.88129$$

$$C = 1 - 0.88129 = 0.1187 \quad bits/symbol$$

The channel efficiency $\eta = \frac{I(X,Y)}{C}$

$$I(x_1) = -log_2 P(x_1) = 2$$

$$P(x_1) = 2^{-2} = 0.25 \quad then\ P(X) = [0.25 \quad 0.75]^T$$

And we have $P(x_i, y_j) = P(x_i)P(y_j \mid x_i)$ so that

$$P(X,Y) = \begin{bmatrix} 0.7 \times 0.25 & 0.3 \times 0.25 \\ 0.3 \times 0.75 & 0.7 \times 0.75 \end{bmatrix} = \begin{bmatrix} 0.175 & 0.075 \\ 0.225 & 0.525 \end{bmatrix}$$

$$P(Y) = [0.4 \quad 0.6] \quad \rightarrow H(Y) = 0.97095\ bits/symbol$$

$$I(X,Y) = H(Y) + K = 0.97095 - 0.88129 = 0.0896\ bits/symbol$$

Then $\eta = \frac{0.0896}{0.1187} = 75.6\%$

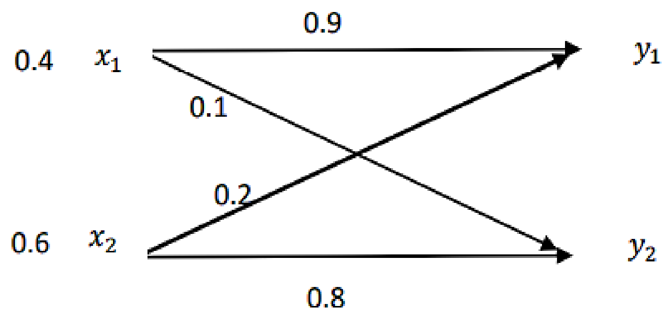
Review questions:

A binary source sending x_1 with a probability of 0.4 and x_2 with 0.6 probability through a channel with a probabilities of errors of 0.1 for x_1 and 0.2 for x_2.Determine:

1- Source entropy.
2- Marginal entropy.
3- Joint entropy.
4- Conditional entropy $H(Y \mid X)$.
5- Losses entropy $H(X \mid Y)$.

Solution:

1- The channel diagram:



Or $P(Y|X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$

$$H(X) = -\sum_{i=1}^{n} p(x_i)\, log_2 p(x_i)$$

$$H(X) = -\frac{[0.4\ln(0.4) + 0.6\ln(0.6)]}{ln2} = 0.971\frac{bits}{symbol}$$

2- $P(X,Y) = P(Y|X) \times P(X)$

$$\therefore P(X,Y) = \begin{bmatrix} 0.9 \times 0.4 & 0.1 \times 0.4 \\ 0.2 \times 0.6 & 0.8 \times 0.6 \end{bmatrix} = \begin{bmatrix} 0.36 & 0.04 \\ 0.12 & 0.48 \end{bmatrix}$$

$$\therefore P(Y) = [0.48 \quad 0.52]$$

$$H(Y) = -\sum_{j=1}^{m} p(y_j)\, log_2 p(y_j)$$

$$H(Y) = -\frac{[0.48\ln(0.48) + 0.52\ln(0.52)]}{\ln(2)} = 0.999\ bits/symbol$$

3- $H(X,Y)$

$$H(X,Y) = -\sum_{j=1}^{m}\sum_{i=1}^{n} P(x_i, y_j)\log_2 P(x_i, y_j)$$

$$H(X,Y) = -\frac{[0.36\ln(0.36) + 0.04\ln(0.04) + 0.12\ln(0.12) + 0.48\ln(0.48)]}{\ln(2)}$$

$$= 1.592\ bits/symbol$$

4- $H(Y|\mathrm{X})$

$$H(Y|\mathrm{X}) == -\sum_{j=1}^{m}\sum_{i=1}^{n} P(x_i, y_j)\log_2 P(y_j|x_i)$$

$$H(Y|\mathrm{X}) = -\frac{[0.36\ln(0.9) + 0.12\ln(0.2) + 0.04\ln(0.1) + 0.48\ln(0.8)]}{\ln(2)}$$

$$= 0.621\frac{bits}{symbol}$$

Or $H(Y|\mathrm{X}) = H(X,Y) - H(X) = 1.592 - 0.971 = 0.621\frac{bits}{symbol}$

5- $H(X|\mathrm{Y}) = H(X,Y) - H(Y) = 1.592 - 0.999 = 0.593\ bits/symbol$