Lecture 1

Introduction

- The term *microwave frequencies* is generally used for those wavelengths measured in centimeters, roughly from 30 cm to 1 mm (1 to 300 GHz).
- Outlines
- Course materials
- Waveguide
- Passive elements
- Active elements

TABLE IEEE MICROWAVE FREQUENCY BANDS

Designation	Frequency range in gigahertz
HF	0.003– 0.030
VHF	0.030– 0.300
UHF	0.300– 1.000
L band	1.000– 2.000
S band	2.000– 4.000
C band	4.000– 8.000
X band	8.000– 12.000
Ku band	12.000– 18.000
K band	18.000– 27.000
Ka band	27.000– 40.000
Millimeter	40.000–300.000
Submillimeter	>300.000

MICROWAVE SYSTEMS

- A microwave system normally consists of a transmitter subsystem, including a microwave oscillator, waveguides, and a transmitting antenna, and a receiver subsystem that includes a receiving antenna, transmission line or waveguide, a microwave amplifier, and a receiver.

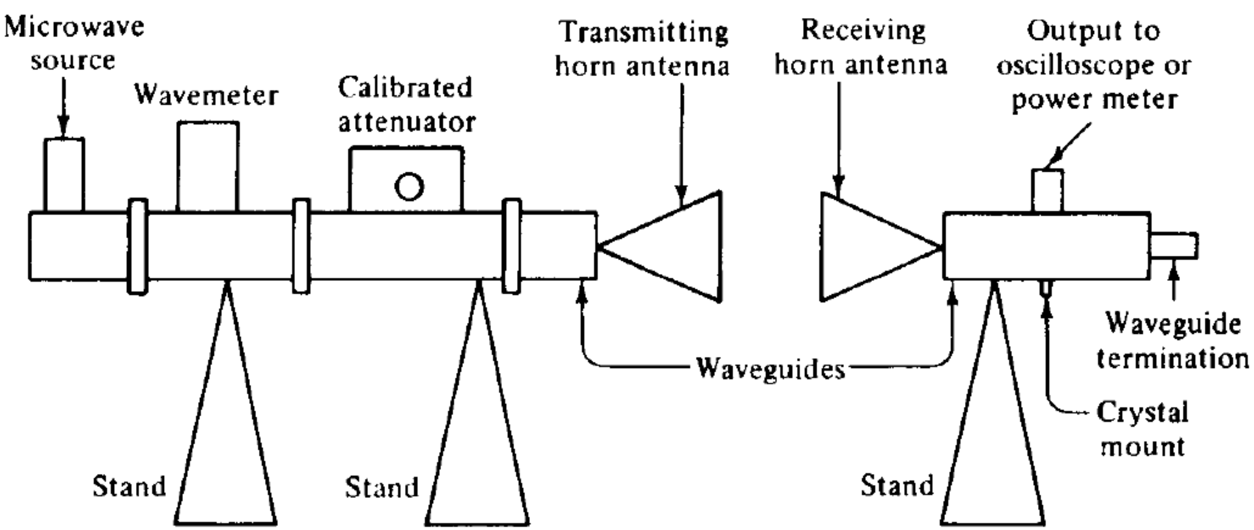


Figure Microwave system.

Applications of Microwave Engineering

- Antenna gain is proportional to the electrical size of the antenna. At higher frequencies,more antenna gain can be obtained for a given physical antenna size, and this has important consequences when implementing microwave systems.
- More bandwidth (directly related to data rate) can be realized at higher frequencies. A 1% bandwidth at 600 MHz is 6 MHz, which (with binary phase shift keying

modulation) can provide a data rate of about 6 Mbps (megabits per second), while

- at 60 GHz a 1% bandwidth is 600 MHz, allowing a 600 Mbps data rate.

- Microwave signals travel by line of sight and are not bent by the ionosphere as are lower frequency signals. Satellite and terrestrial communication links with very high capacities are therefore possible, with frequency reuse at minimally distant locations.
- The effective reflection area (radar cross section) of a radar target is usually proportional to the target's electrical size. This fact, coupled with the frequency characteristics of antenna gain, generally makes microwave frequencies preferred for radar systems.

MICROWAVE UNITS OF MEASURE

- MKS (meter-kilogram-second)

TABLE MKS UNITS

Quantity	Unit	Symbol
Capacitance	farad = coulomb per volt	F
Charge	coulomb: $A - s$	Q
Conductance	mhos	℧
Current	ampere = coulomb per second	A
Energy	joule	J
Field	volt per meter	E
Flux linkage	weber = volt · second	ψ
Frequency	cycle per second	Hz
Inductance	henry = $(V - s)/A$	H
Length	meter	m
Power	watt = joule per second	W
Resistance	ohm	Ω
Time	second	s
Velocity	meter per second	v
Voltage	volt	V

Note: 1 tesla = 1 weber/m^2 = 10^4 gausses = 3×10^{-6} ESU
1 Å (angstrom) = 10^{-10} m
1 μm (micron) = 10^{-6} m

TABLE PREFIXES

Prefix	Factor	Symbol
exa	10^{18}	E
peta	10^{15}	P
tera	10^{12}	T
giga	10^{9}	G
mega	10^{6}	M
kilo	10^{3}	k
hecto	10^{2}	h
deka	10	da
deci	10^{-1}	d
centi	10^{-2}	c
milli	10^{-3}	m
micro	10^{-6}	μ
nano	10^{-9}	n
pico	10^{-12}	p
femto	10^{-15}	f
atto	10^{-18}	a

TABLE PHYSICAL CONSTANTS

Constant	Symbol	Value
Boltzmann constant	k	1.381×10^{-23} J/°K
Electronvolt	eV	1.602×10^{-19} J
Electron charge	q	1.602×10^{-19} C
Electron mass	m	9.109×10^{-31} kg
Ratio of charge to mass of an electron	e/m	1.759×10^{11} C/kg
Permeability of free space	μ_0	1.257×10^{-6} H/m or $4\pi \times 10^{-7}$ H/m
Permittivity of free space	ϵ_0	8.854×10^{-12} F/m
Planck's constant	h	6.626×10^{-34} J·s
Velocity of light in vacuum	c	2.998×10^{8} m/s

Introduction to Electromagnetic plane wave

- Since Maxwell's fundamental concepts of electromagnetic wave theory have been established, the electric and magnetic wave equations can readily be derived from
- Faraday's electromotive force law, Ampere's circuital law, and Gauss's law for the electric and magnetic fields.

- The principles of electromagnetic plane waves are based on the relationship between the electricity and magnetism.
- A changing magnetic will induce an electric field and a changing electric will induce a magnetic field .
- the induced fields are not confined but ordinarily extend outward into space.
- The sinusoidal form of wave causes energy to be interchanged between magnetic and electric field in its direction of propagation.

- A plane wave has a plane front, a cylindrical wave has a cylindrical front, and a spherical wave has a spherical front.
- In far field free space the electric and magnetic waves are always perpendicular to each other and both normal to the direction of propagation .
- This type of wave is known as the *transverse electromagnetic* (TEM) wave.

- If only the transverse electric wave exists, the wave is called *TE-mode* wave. That means there is no component of the electric wave in the direction of propagation.
- In *TM modes* only the transverse magnetic wave exists.
- Electric and magnetic equation can be basically derived

from Maxwell's equations which are :

$$\nabla \times \vec{\mathrm{E}} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla . D = \rho_v$$

$$\nabla \times \vec{\mathrm{H}} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \nabla . B = 0$$

- where the vector operator Δ is expressed by

$$\nabla = \frac{\partial}{\partial x}\mathbf{u}_x + \frac{\partial}{\partial y}\mathbf{u}_y + \frac{\partial}{\partial z}\mathbf{u}_z \qquad \text{(cartesian)}$$

$$\nabla = \frac{\partial}{\partial r}\mathbf{u}_r + \frac{\partial}{r\,\partial \phi}\mathbf{u}_\phi + \frac{\partial}{\partial z}\mathbf{u}_z \qquad \text{(cylindrical)}$$

$$\nabla = \frac{\partial}{\partial r}\mathbf{u}_r + \frac{\partial}{r\,\partial \theta}\mathbf{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\mathbf{u}_\phi \qquad \text{(spherical)}$$

- **E** = *electric field intensity* in volts per meter
- **H** = *magnetic field intensity* in amperes per meter
- **D** = *electric flux density* in coulombs per square meter
- **B** = *magnetic flux density* in webers per square meter or in tesla
- **J** = *electric current density* in amperes per square meter
- *Pv* = *electric charge density* in coulombs per cubic meter

- In addition to Maxwell's four equations, the characteristics of the medium in which the fields exist are needed to specify the flux in terms of the fields in a specific medium. These constitutive relationships are
- **D** = ε E
- **B = μH**
- *Jc* = σ E
- $\varepsilon = \varepsilon_r \varepsilon_o$
- $\boldsymbol{\mu = \mu_r \mu_o}$

- ε = dielectric permittivity or capacitivity of the medium in farads per meter
- ε_r = relative dielectric constant (dimensionless)
- $\varepsilon_o = 8.854 \times 10^{-12}$ F/m is the dielectric permittivity of vacuum or free space
- μ = magnetic permeability or inductivity of the medium in henrys per meter
- μ_r = the relative permeability or relative inductivity (dimensionless)
- $\mu_o = 4\pi \times 10^{-7}$ H/m is the permeability of vacuum or free space
- σ = conductivity of the medium in mhos per meter

- If a sinusoidal time function in the form of e^{jwt} is assumed, ${}^{d}/_{dt}$ can be replaced by jw. Then Maxwell's equations in frequency domain are given by

- $\nabla \times \vec{E} = -jw\mu\vec{H}$
- $\nabla \times \vec{H} = (\sigma + jw\epsilon)\vec{E}$
- $\nabla . D = \rho_v$
- $\nabla . B = 0$

- In free space charge density is zero and in perfect conductor time varying or static fields do not exist
- These yields the electric and magnetic field equations as :

- $\nabla^2\vec{E} = \gamma^2\vec{E}$
- $\nabla^2\vec{H} = \gamma^2\vec{H}$
- $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Taking the curl of Eq. (2-1-11) on both sides yields

$$\nabla \times \nabla \times \mathbf{E} = -j\omega\mu\nabla \times \mathbf{H} \tag{2-1-15}$$

Substitution of Eq. (2-1-12) for the right-hand side of Eq. (2-1-15) gives

$$\nabla \times \nabla \times \mathbf{E} = -j\omega\mu(\sigma + j\omega\epsilon)\mathbf{E} \tag{2-1-16}$$

The vector identity for the curl of the curl of a vector quantity **E** is expressed as

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2\mathbf{E} + \nabla(\nabla \cdot \mathbf{E}) \tag{2-1-17}$$

In free space the space-charge density is zero, and in a perfect conductor time-varying or static fields do not exist. So

$$\nabla \cdot \mathbf{D} = \rho_v = 0 \tag{2-1-18}$$

$$\nabla \cdot \mathbf{E} = 0 \tag{2-1-19}$$

Substitution of Eq. (2-1-17) for the left-hand side of Eq. (2-1-16) and replacement of Eq. (2-1-19) yield the electric wave equation as

$$\nabla^2\mathbf{E} = \gamma^2\mathbf{E} \tag{2-1-20}$$

Lecture 2

Microwave Waveguides

- In general, a waveguide consists of a hollow metallic tube of a rectangular or circular shape used to guide an electromagnetic wave.
- Waveguides are used principally at frequencies in the microwave range
- At frequency range X band from 8.00 to 12.0 GHz, for example, the U.S. standard rectangular waveguide WR-90 has an inner width of 2.286 cm (0.9 in.) and an inner height of 1.016 cm (0.4 in.); but its outside dimensions are 2.54 cm (1 in.) wide and 1.27 cm (0.5 in.) high

- In waveguides the electric and magnetic fields are confined to the space within the guides. Thus no power is lost through radiation, and even the dielectric loss is negligible, since the guides are normally air-filled.
- there is some power loss as heat in the walls of the guides, but the loss is very small.
- It is possible to propagate several modes of electromagnetic waves within a waveguide. These modes correspond to solutions of Maxwell's equations for particular waveguides.

- A given waveguide has a definite cutoff frequency for each allowed mode. If the frequency of the impressed signal is above the cutoff frequency for a given mode, the electromagnetic energy can be transmitted through the guide for that particular mode without attenuation.

- Otherwise the electromagnetic energy with a frequency below the cutoff frequency for that particular mode will be attenuated to a negligible value in a relatively short distance.

- *The dominant mode in a particular guide is the mode having the lowest cutoff frequency.* **It is advisable to choose the dimensions of a guide in such a way that, for a given input signal, only the energy of the dominant mode can be transmitted through the guide.**

RECTANGULAR WAVEGUIDES

- **A rectangular waveguide is a hollow metallic tube with a rectangular cross section.**
- **The conducting walls of the guide confine the electromagnetic fields and thereby guide the electromagnetic wave.**
- **When the waves travel longitudinally down the guide, the plane waves are reflected from wall to wall.**

- Figure 4-1-1 shows that any uniform plane wave in a lossless guide may be resolved into TE and TM waves.

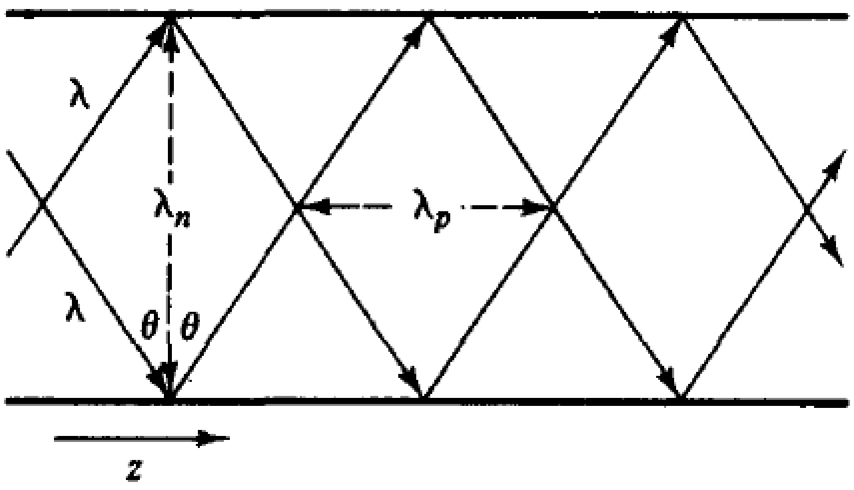
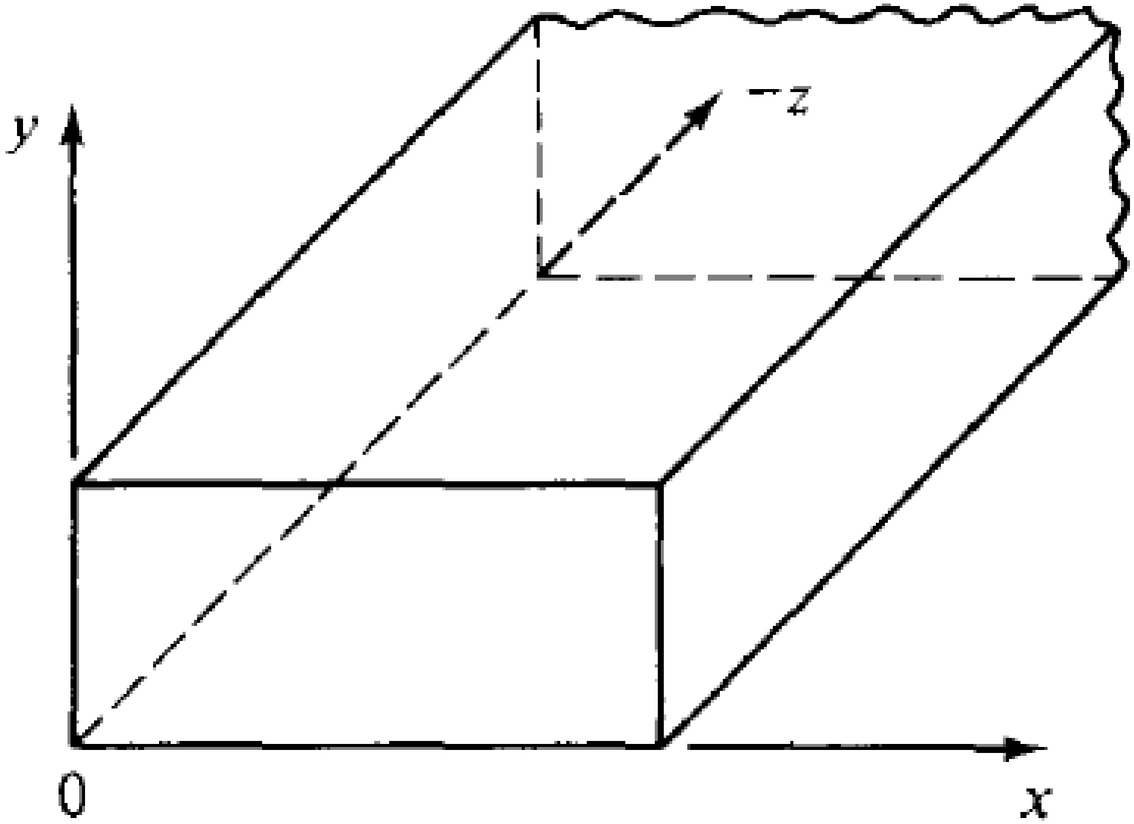


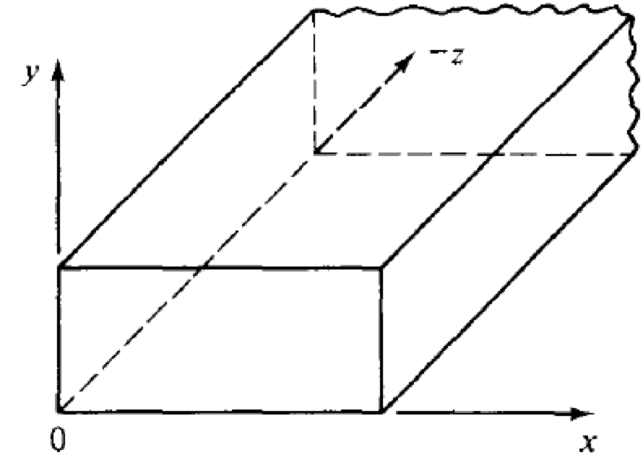
Figure 4-1-1 Plane wave reflected in a waveguide.

- In lossless waveguides the modes may be classified as either *transverse electric* (TE) mode or *transverse magnetic* (TM) mode. In rectangular guides the modes are designated TE_{mn} or TM_{mn}.
- The integer *m* denotes the number of half waves of electric or magnetic intensity in the *x* direction, and *n* is the number of half waves in the *y* direction if the propagation of the wave is assumed in the positive *z* direction.

Solutions of Wave Equations in Rectangular Coordinates

- rectangular coordinate system is shown in Figure.





$$\nabla^2 \mathbf{E} = \gamma^2 \mathbf{E}$$

$$\nabla^2 \mathbf{H} = \gamma^2 \mathbf{H}$$

where $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$. These are called the *vector wave equations*.

Rectangular coordinates are the usual right-hand system. The rectangular components of **E** or **H** satisfy the complex scalar wave equation or Helmholtz equation

$$\nabla^2 \psi = \gamma^2 \psi$$

The Helmholtz equation in rectangular coordinates is

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi$$

- By separation we assume

$\psi = X_{\chi}\, Y_y\, Z_z$..........B

Sub B in A

$$\frac{\partial \Psi}{\partial x} = YZ\frac{dX}{dx} \quad \Rightarrow \quad \frac{\partial^2 \Psi}{\partial x^2} = YZ\frac{d^2X}{dx^2}$$

same goes for $\frac{\partial \Psi}{\partial y}$ and $\frac{\partial \Psi}{\partial z}$

$$\left[YZ\frac{d^2X}{dx^2} + XZ\frac{d^2Y}{dy^2} + YX\frac{d^2Z}{dz^2} = \Upsilon^2 YXZ\right] \div XYZ$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} = \Upsilon^2$$

- Since there is three items are independent of each other and summation is γ^2 (constant) $\Longrightarrow$ each term is constant .

$$-K_x{}^2 - K_y{}^2 - K_z{}^2 = \Upsilon^2$$

$$\frac{1}{\mathrm{X}}\frac{d^2\mathrm{X}}{dx^2} = -K_x{}^2 \Longrightarrow \frac{d^2\mathrm{X}}{dx^2} + K_x{}^2 = 0$$

$$r^2 + K_x{}^2 = 0 \quad r^2 = -K_x{}^2 \Longrightarrow r = \pm jK_x$$

$$\mathrm{X} = A\sin(K_x x) + \mathrm{B}\cos(K_x x)$$

$$\mathrm{Y} = C\sin(K_y y) + \mathrm{D}\cos(K_y y)$$

$$\mathrm{Z} = E\sin(K_z z) + \mathrm{F}\cos(K_z z)$$

- So the solution :

$$\Psi = [A\sin(K_x x) + B\cos(K_x x)]\,[C\sin(K_y y) + D\cos(K_y y)]\,[E\sin(K_z z) + F\cos(K_z z)]$$

- The propagation is assumed to be in Z- direction
- The propagation constant Υ_g in guide differs from Υ

Let

$$\Upsilon g^2 = \Upsilon^2 + Kx^2 + Ky^2 \Longrightarrow \Upsilon g^2 = \Upsilon^2 + Kc^2$$

- $K_c = \sqrt{Kx^2 + Ky^2}$
- For the lossless dielectric
- $\Upsilon^2 = -\omega^2\mu\epsilon \quad then \quad \Upsilon g = \pm\sqrt{\omega^2\mu\epsilon - Kc^2}$

There are three cases for the propagation in the wave guide :

- Case 1: if $\omega^2\mu\epsilon = \mathrm{K}c^2$ and $\Upsilon g = 0,$ this is critical condition and there will be no propagation ,

$$\omega^2\mu\epsilon = \mathrm{K}c^2 \longrightarrow fc = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{Kx^2 + Ky^2}$$

Fc is the critical frequency

Case 2: if $\omega^2\mu\epsilon > \mathrm{K}c^2$ the wave will be propagating in the guide and $\Upsilon g = \pm\, jBg$

$$\Upsilon g = \pm jw\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{fc}{f}\right)^2}$$

This means that the propagating frequency must be above fc in order for a wave to propagate in guide .

Case 3:

if $\omega^2\mu\epsilon < \mathrm{K}c^2$

$$\Upsilon g = \pm\,\omega\,\sqrt{\mu\epsilon}\sqrt{\left(\frac{fc}{f}\right)^2 - 1}$$

The wave will be attenuated ,

So the final solution

$$\Psi = [A\sin(K_x x) + \mathrm{B}\cos(K_x x)]\ [C\sin(K_y y) + \mathrm{D}\cos(K_y y)]e^{-j\beta g Z}$$

the wave will propagate in the wave guide either in TE mode or in TM mode

TE mode in rectangular waveguide

lecture3

- The *TEmn* modes in a rectangular guide are characterized by *Ez* = 0.
- In other words, the *z* component of the magnetic field, *Hz,* must exist in order to have energy
- transmission in the guide. Consequently, from a given Helmholtz equation,
- $\nabla^2 H_z = \gamma^2 H_z$

$$H_z = \left[A_m \sin\left(\frac{m\pi x}{a}\right) + B_m \cos\left(\frac{m\pi x}{a}\right)\right] \times \left[C_n \sin\left(\frac{n\pi y}{b}\right) + D_n \cos\left(\frac{n\pi y}{b}\right)\right] e^{-j\beta_g z}$$

- will be determined in accordance with the given boundary conditions, where $K_x = {}^{m\pi}/_{a}$ and $K_y = {}^{n\pi}/_{b}$ are replaced. For a lossless dielectric, Maxwell's curl equations in frequency domain are

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

- In rectangular coordinates, their components are

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \qquad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z$$

- With the substitution $d/d_z = - -j\beta_g$ and $Ez = 0$, the foregoing equations are simplified to

$$\beta_g E_y = -\omega\mu H_x$$

$$\beta_g E_x = \omega\mu H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} = -j\omega\mu H_z$$

$$\frac{\partial H_z}{\partial y} + j\beta_g H_y = j\omega\epsilon E_x$$

$$-j\beta_g H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = 0$$

- Solving these six equations for *Ex, Ey, Hx,* and *Hy* in terms of *Hz* will give the TE mode field equations in rectangular waveguides as

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$$

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$E_z = 0$$

$$H_x = \frac{-j\beta_g}{k_c^2}\frac{\partial H_z}{\partial x}$$

$$H_y = \frac{-j\beta_g}{k_c^2}\frac{\partial H_z}{\partial y}$$

- where $K_c{}^2 = w^2\mu\epsilon - \beta_g{}^2$has been replaced.
- The boundary conditions are applied to the newly found field equations in such a manner that either the tangent E field or the normal **H** field vanishes at the surface of the conductor.
- Since *Ex*= 0, then ${}^{dH_z}/_{dy} = 0$ at *y* = 0, *b.* Hence *Cn* = 0. Since *Ey* = 0, then
- $\frac{\partial H_z}{\partial x} = 0$ at x=0 henca Am=0
- It is generally concluded that the normal derivative of *Hz* must vanish at the conducting surfaces-that is,

- $\frac{\partial H_z}{\partial n} = 0$
- at the guide walls. Therefore the magnetic field in the positive *z* direction is given by

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta_g z}$$

- So the TE_{mn} field equations in rectangular wave guide:

❖ $E_x = E_{\circ x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $E_y = E_{\circ y}\, sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ Ez

❖ $H_x = H_{\circ x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $H_y = H_{\circ y}\, cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $Hz = H_{\circ z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

Where m=0,1,2,3…. and n=0,1,3,2…….

$m = n = 0$ this case is impossible because thus there will be no transmission of a signal in the guide

- The cutoff wave number *kc.* as defined for the *TEmn* modes, is given by

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} = \omega_c\sqrt{\mu\epsilon}$$

- where *a* and *b* are in meters. The cutoff frequency for the *TEmn* modes, is

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\frac{m^2}{a^2} + \frac{n^2}{b^2}}$$

- The propagation constant (or the phase constant here) β_g, as defined is expressed by

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

- The phase velocity in the positive *z* direction for the *TEmn* modes is shown as

$$v_g = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

- where $V_p = {}^{1}/_{\sqrt{\mu\epsilon}}$ is the phase velocity in an unbounded dielectric.
- The characteristic wave impedance of *TEmn* modes in the guide can be given

$$Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta_g} = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

- where $\boldsymbol{\eta} = \sqrt{\frac{\mu}{\epsilon}}$ is the intrinsic impedance in an unbounded dielectric.

The wavelength $\boldsymbol{\lambda}_g$ in the guide for the *TEmn* modes is given by

$$\lambda_g = \frac{\lambda}{\sqrt{1-(f_c/f)^2}}$$

- where $\boldsymbol{\lambda} = {}^{V_p}/_{f}$ is the wavelength in an unbounded dielectric.
- Since the cutoff frequency shown in Equation above is a function of the modes and guide dimensions, the physical size of the waveguide will determine the propagation of the modes.
- Whenever two or more modes have the same cutoff frequency, they are said to be *degenerate modes.*
- In a rectangular guide the corresponding *TEmn* and *TMmn* modes are always degenerate.
- In a square guide the *TEmn,* TEnm, *TMmn,* and *TMnm* modes form a foursome of degeneracy. Rectangular guides ordinarily have dimensions of *a* = *2b* ratio.
- The mode with the lowest cutoff frequency in a particular guide is called the *dominant mode.*

- The dominant mode in a rectangular *guide with a* > *b is* the TE10 mode.
- Each mode has a specific mode pattern (or field pattern).
- It is normal for all modes to exist simultaneously in a given waveguide. The situation is not very serious, however. Actually, only the dominant mode propagates, and the higher modes near the sources or discontinuities decay very fast.

Ex:

Example:- An air filled rectangular wave guide with dimensions 7*3.5 cm operates in the dominant mode TE_{10}

a) **Find the cut off frequency**

b) **Determine the phase velocity at f=3.5 GHz**

c) **Determine λ_g at same frequency**

Solution

$$a)\ \ f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$$

$$= \frac{C}{2a}$$

$$c = \frac{1}{\sqrt{\mu_0\ \epsilon}}$$

$$= \frac{3*10^8}{2*7*10^{-2}}$$

=2.14 GHz

b) $V_g = \frac{C}{\sqrt{(1-(\frac{fc}{f})^2)}}$

$= 3.78 * 10^8 \ m/sec$

$V_p = C$

$= \frac{3*10^8}{\sqrt{(1-\left(\frac{2.14}{3.5}\right)^2)}}$

c) $\lambda_g = \frac{\lambda}{\sqrt{(1-(\frac{fc}{f})^2)}}$

$= \frac{3*10^8 / 3.5*10^9}{\sqrt{(1-(\frac{2.14}{3.5})^2)}}$

$= 10.8\ cm$

Ex2:

Example : In air filled rectangular wave guide the cutoff frequency of TE_{01} is 2GHz calculate:-

a) **The dimension of the guide**

b) **The cutoff frequency of three higher TE modes**

c) **The cutoff frequency for TE_{11} mode if guide is filled with a lossless material having $\epsilon_r = 2.25$ and $\mu_r = 1$**

Ex3:

: A rectangular waveguide dimension of a =2 cm and b= 1 cm with dielectric $\mu_r = 1$, $\epsilon_r = 81$ operates at 3 GHZ , determine all propagating modes assuming TE modes

Solution:

We apply the formula

$$f_c = \frac{1}{2 * \sqrt{\mu\epsilon}} \sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}$$

To calculate the cut off frequencies at all the propagating modes and then compare it with the operating frequency , and any frequency above 3 GHZ then the mode is considered as a non propagating mode at this wave guide

TM mode for rectangular waveguide

Lecture4

- The *TMmn* modes in a rectangular guide are characterized by *Hz* = 0. In other words, the *z* component of an electric field *E* must exist in order to have energy transmission in the guide. Consequently, the Helmholtz equation for *E* in the rectangular coordinates is given by

- $\nabla^2 E_z = \Upsilon^2 E_z$
- $E_z = \left[A_m \sin\frac{m\pi x}{a} + B_m \cos\frac{m\pi x}{a}\right].\left[C_n \sin\frac{n\pi y}{b} + D_n \cos\frac{n\pi y}{b}\right] e^{-j\beta_g \mathrm{Z}}$

According to boundary conditions:

$$Ez = E_{\circ z} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$$

Where m=1,2,3…. and n=1,2,3…….

n=m≠0

- If either $m = 0$ or $n = 0$, the field intensities all vanish. So there is no TM01 or TM10 mode in a rectangular waveguide, which means that TE10 is the dominant mode in a rectangular waveguide for $a > b$.

- So the TM_{mn} field equations in rectangular wave guide:

❖ $E_x = E_{\circ x} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $E_y = E_{\circ y}\, sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $Ez = E_{\circ z} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $H_x = H_{\circ x} \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $H_y = H_{\circ y}\, cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-j\beta g Z}$

❖ $Hz = 0$

For TM modes:

- The cut off frequency for TM_{mn} is

$$fc = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

- Phase constant

$$\beta_g = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{fc}{f}\right)^2}$$

- The wave length λ_g

$$\lambda_g = \frac{\lambda}{\sqrt{1-\left(\frac{fc}{f}\right)^2}} \qquad \lambda = \frac{V_p}{f}$$

- Phase velocity in Z- direction

$$V_g = \frac{\omega}{\beta_g} = \frac{Vp}{\sqrt{1-\left(\frac{fc}{f}\right)^2}} \qquad V_p = \frac{1}{\sqrt{\mu\epsilon}}$$

- The characteristic wave impedance

$$Z_g = \frac{\beta g}{\omega\epsilon}$$

$$Z_g = \eta\sqrt{1-\left(\frac{fc}{f}\right)^2} \qquad \eta = \sqrt{\mu/_{\epsilon}}$$

Ex/ A rectangular wave guide of (20/3*20/4 cm) is filled with a dielectric of ($\epsilon_r = 4$) what is the free space wave length that can propagate in TE_{11} mode (a=20/3 , b=20/4)

solution

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$$

=1.875 GHz

$$\lambda_c = \frac{V_p}{f_c} = \frac{V_p}{1.875 * 10^9}$$

$$V_p = \frac{1}{\sqrt{\epsilon_0\epsilon_r\mu_0\mu_r}} = 1.5 * 10^8$$

$$\lambda_c = 8\ cm$$

Ex/A rectangular wave guide 2.42*1.12 cm supporting TE_{10} at 6 GHz is filled with a dielectric of ϵ_r . What are the limits on ϵ_r if only the dominant mode propagate

Solution :

$$f_c = \frac{V_p}{2a} \rightarrow 6 * 10^9 = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} . \frac{1}{2 * 2.42 * 10^{-2}}$$

$$\epsilon_r = 1.06$$

$$f_{c2} = \frac{V_p}{2b} \rightarrow 6 * 10^9 = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}} . \frac{1}{2 * 1.12 * 10^{-2}}$$

$$\epsilon_r = 4.9824$$

$$1.0672 < \epsilon_r < 4.9824$$

Power transmission

Lecture5

- The power transmitted through a waveguide and the power loss in the guide walls can be calculated by means of the complex Poynting theorem.
- It is assumed that the guide is terminated in such a way that there is no reflection from the receiving end or that the guide is infinitely long compared with the wavelength.
- the power transmitted through a guide is given by

$$P_{tr} = \oint \mathbf{p} \cdot d\mathbf{s} = \oint \frac{1}{2}(\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s}$$

- For a lossless dielectric, the time-average power flow through a rectangular guide is given by

$$P_{tr} = \frac{1}{2Z_g} \int_a |E|^2 \, da = \frac{Z_g}{2} \int_a |H|^2 \, da$$

$$\text{where } Z_g = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$|E|^2 = |E_x|^2 + |E_y|^2$$

$$|H|^2 = |H_x|^2 + |H_y|^2$$

- For TEmn modes, the average power transmitted through a rectangular waveguide is given by

$$P_{tr} = \frac{\sqrt{1 - (f_c/f)^2}}{2\eta} \int_0^b \int_0^a (|E_x|^2 + |E_y|^2)\, dx\, dy$$

- For TM_{mn} modes

$$P_{tr} = \frac{1}{2\eta\sqrt{1-\left({}^{fc}/_{f}\right)^2}} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dxdy$$

Where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ impedance in unbounded dielectric

Ex: an air filled wave guide with 2x1 cm transports energy in TE_{10} mode ate rate of 373w .the impressed frequency is 30 GHZ. What is the peak value of electric field occurring in the guide?

Solution:

For TE_{mn} modes

$$P_{tr} = \frac{\sqrt{1-\left({}^{fc}/_{f}\right)^2}}{2\eta} \int_0^b \int_0^a \left(|E_x|^2 + |E_y|^2\right) dxdy$$

- The field components of the dominant mode TE10

$$E_x = 0 \qquad H_x = \frac{E_{0y}}{Z_g} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_g z}$$

$$E_y = E_{0y} \sin\left(\frac{\pi x}{a}\right) e^{-j\beta_g z} \qquad H_y = 0$$

$$E_z = 0 \qquad H_z = H_{0z} \cos\left(\frac{\pi x}{a}\right) e^{-j\beta_g z}$$

- $Z_g = \frac{\omega\mu_\circ}{\beta_g}$

$$\beta_g = \sqrt{\omega^2\mu_\circ\epsilon_\circ - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$$= \sqrt{\omega^2\mu_\circ\epsilon_\circ - \frac{\pi^2}{a^2}}$$

$$= \sqrt{(2\pi f)^2\frac{1}{c^2} - \frac{\pi^2}{a}} = 193.5\pi = 608.81 rad/m$$

For $\boldsymbol{TE_{10}}$

$$E_x = 0$$

$$E_y = E_{\circ y} \sin\frac{\pi x}{a} e^{-j\beta g Z}$$

$$P_{tr} = \frac{\beta g}{2\omega\mu_\circ}\int_0^b\int_0^a \left|E_{\circ y}\sin\left(\frac{\pi x}{a}\right)\right|^2 dx\, dy$$

$$= \frac{1}{2} E_{\circ y}{}^{2} \frac{\beta g}{\omega \mu_\circ} \int_0^b \int_0^a \frac{1 - \cos\frac{2\pi x}{a}}{2} dxdy$$

$$= \frac{1}{4} E_{\circ y}{}^{2} \frac{\beta g}{\omega \mu_\circ} \int_0^b x - \frac{a}{2\pi} \sin\frac{2\pi x}{a} \Big|_0^a dy$$

$$= P_{tr} = \frac{1}{4} E_{\circ y}{}^{2} \frac{\beta g}{\omega \mu_\circ} ab$$

$$373 = \frac{1}{4} E_{\circ y}{}^{2} \frac{193.5\pi \times 10^{-2} \times 2 \times 10^{2}}{2\pi \times 30 \times 10^{9} \times 4\pi \times 10^{-7}}$$

$$E_{\circ y} = 53.87 Kv/m$$

Excitations of Modes in Rectangular Waveguides

- In general, the field intensities of the desired mode in a waveguide can be established by means of a probe or loop-coupling device.
- The probe may be called a monopole antenna; the coupling loop, the loop antenna.
- A probe should be located so as to excite the electric field intensity of the mode,
- A coupling loop in such a way as to generate the magnetic field intensity for the desired mode.

- If two or more probes or loops are to be used, care must be taken to ensure the proper phase relationship between the currents in the various antennas. This factor can be achieved by inserting additional lengths of transmission line in one or more of the antenna feeders.
- Impedance matching can be accomplished by varying the position and depth of the antenna in the guide or by using impedance-matching stubs on the coaxial line feeding the waveguide.

- A device that excites a given mode in the guide can also serve reciprocally as a receiver or collector of energy for that mode.

- The methods of excitation for various modes in rectangular waveguides are shown in Fig(1).

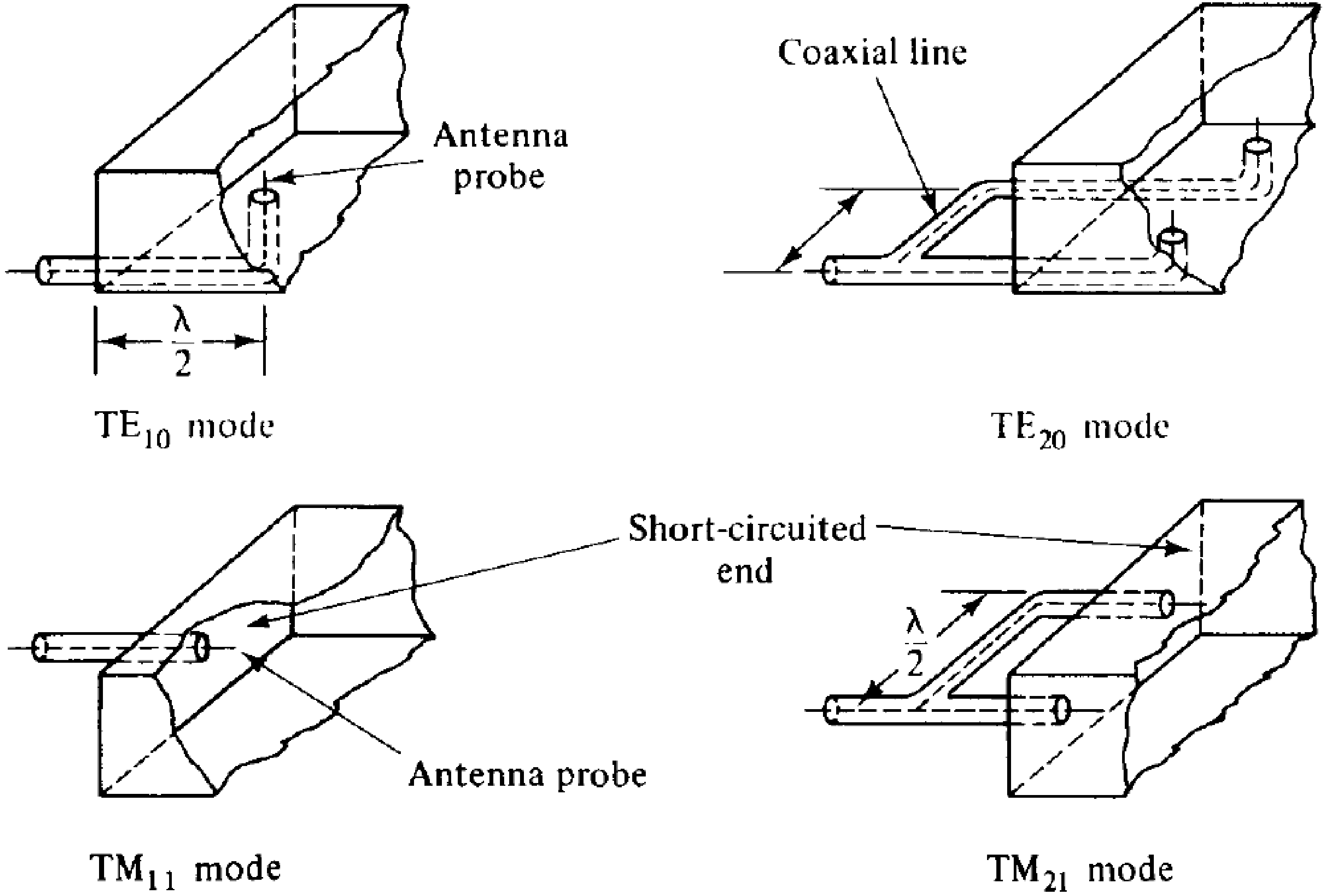


Figure 1 Methods of exciting various modes in rectangular waveguides.

- In order to excite a TE10 mode in one direction of the guide, the two exciting antennas should be arranged in such a way that the field intensities cancel each other in one direction and reinforce in the other.
- Figure 2 shows an arrangement for launching a TE10 mode in one direction only. The two antennas are placed a quarter wavelength apart and their phases are in time quadrature. Phasing is compensated by use of an additional quarter-wavelength section of line connected to the antenna feeders.

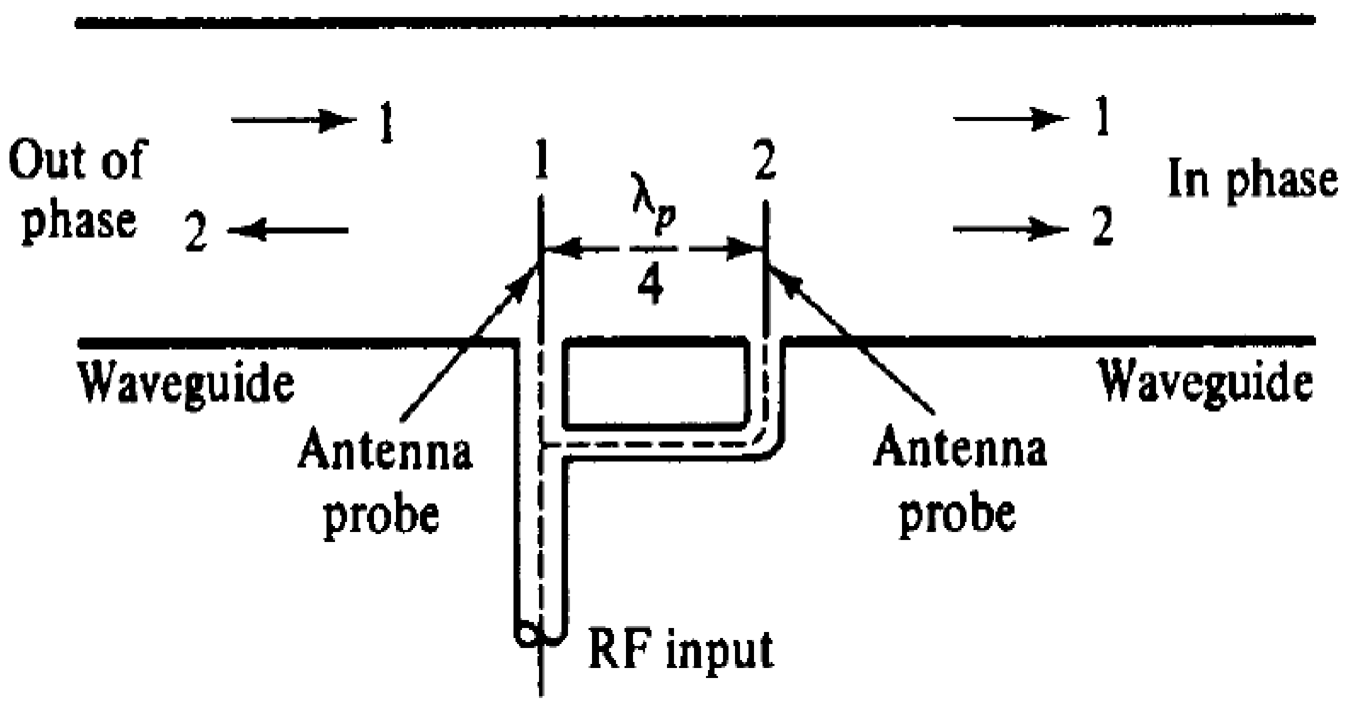


Figure 2 A method of launching a TE_{10} mode in one direction only.

- The field intensities radiated by the two antennas are in phase opposition to the left of the antennas and cancel each other, whereas in the region to the right of the antennas the field intensities are in time phase and reinforce each other. The resulting wave thus propagates to the right in the guide.

Characteristics of Standard Rectangular Waveguides

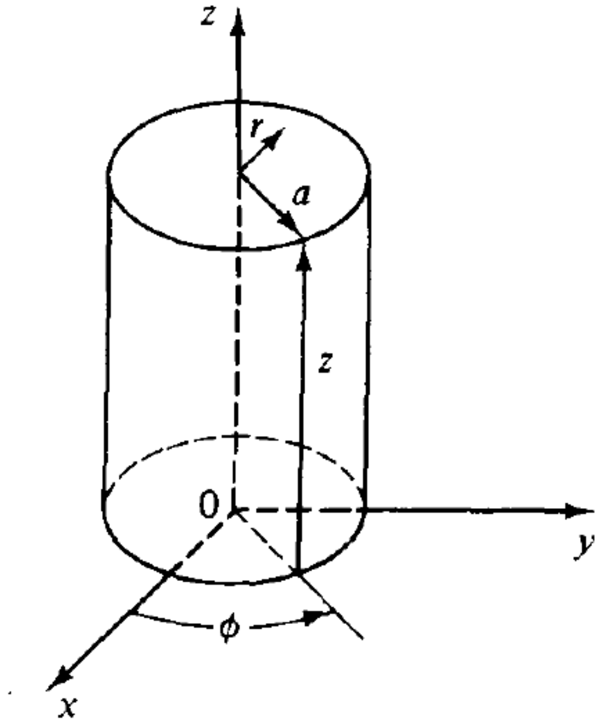
- Rectangular waveguides are commonly used for power transmission at microwave frequencies.
- Their physical dimensions are regulated by the frequency of the signal

being transmitted. For example, at X-band frequencies from 8 to 12 GHz, the outside dimensions of a rectangular waveguid are 2.54 cm (1.0 in.) wide and 1.27 cm (0.5 in

CIRCULAR WAVEGUIDES

Introduction

- A circular waveguide is a tubular, circular conductor. A plane wave propagating through a circular waveguide results in a transverse electric (TE) or transerse magnetic (TM) mode.



Solutions of Wave Equations in Cylindrical Coordinates

- The scalar Helmholtz equation in cylindrical coordinates is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi$$

- Using the method of separation of variables, the solution is assumed in the form of

$$\Psi = R(r)\Phi(\phi)Z(z)$$

- where *R* (r) = a function of the *r* coordinate only
- Ø(Ø)= a function of the Ø coordinate only
- Z (z) = a function of the *z* coordinate only
- Substitution of Eq. (4-2-2) in (4-2-1) and division of the resultant by (4-2-2) yield

$$\frac{1}{rR}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+\frac{1}{r^2\Phi}\frac{d^2\Phi}{d\phi^2}+\frac{1}{Z}\frac{d^2Z}{dz^2}=\gamma^2$$

- Since the sum of the three independent terms is a constant, each of the three terms must be a constant. The third term may be set equal to a constant γ_g.

$$\frac{d^2Z}{dz^2} = \gamma_g^2 Z$$

- The solutions of this equation are given by

$$Z = Ae^{-\gamma_g z} + Be^{\gamma_g z}$$

- where γ_g= propagation constant of the wave in the guide.

- Inserting γ_g for the third term in the left-hand side of Eq. (4-2-3) and multiplying the resultant by r^2yield

$$\frac{r}{R}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} - (\gamma^2 - \gamma_g^2)\,r^2 = 0$$

- The second term is a function of Ø only; hence equating the second term to a constant $(-n^2)$

$$\frac{d^2\Phi}{d\phi^2} = -n^2\Phi \qquad (4\text{-}2\text{-}7)$$

- The solution of this equation is also a harmonic function:

$$\Phi = A_n \sin(n\phi) + B_n \cos(n\phi) \qquad (4\text{-}2\text{-}8)$$

- Replacing the Ø term by $(-n^2)$ in Eq. (4-2-6) and multiplying through by *R,* we have

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) + [(k_c r)^2 - n^2]R = 0$$

- This is Bessel's equation of order *n* in which

$$k_c^2 + \gamma^2 = \gamma_g^2$$

- This equation is called the *characteristic equation* of Bessel's equation. For a lossless guide, the characteristic equation reduces to

$$\beta_g = \pm\sqrt{\omega^2 \mu \epsilon - k_c^2}$$

- The solutions of Bessel's equation are

$$R = C_n J_n(k_c r) + D_n N_n(k_c r)$$

- Therefore the total solution of the Helmholtz equation in cylindrical coordinates is given by

$$\Psi = [C_n J_n(k_c r) + D_n N_n(k_c r)][A_n \sin(n\phi) + B_n \cos(n\phi)]e^{\pm j\beta_g z} \quad (4\text{-}2\text{-}13)$$

- By applying boundary conditions Finally, the solution of the Helmholtz equation is reduced to

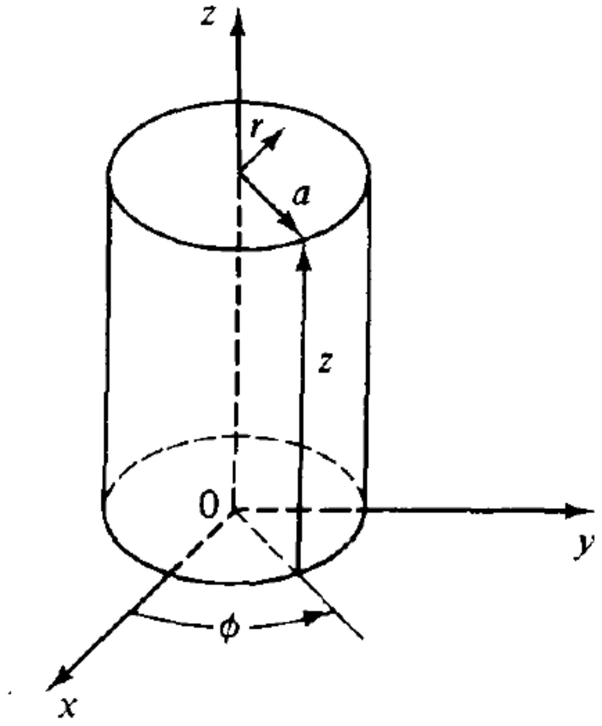
$$\Psi = \Psi_0 J_n(k_c r) \cos(n\phi) e^{-j\beta_g z}$$

TE Modes in Circular Waveguides

- It is commonly assumed that the waves in a circular waveguide are propagating in the positive z direction.
- The TEnp modes in the circular guide are characterized by *Ez* = 0.
- This means that the *z* component of the magnetic

field Hz must exist in the guide in order to have

Electromagnetic energy transmission.



- A Helmholtz equation for *Hz* in a circular guide is given by

$$\nabla^2 H_z = \gamma^2 H_z \qquad (4\text{-}2\text{-}16)$$

$$H_z = H_{0z} J_n(k_c r) \cos(n\phi) e^{-j\beta_g z}$$

- For a lossless dielectric, Maxwell's curl equations in frequency domain are given by

$$\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$$

$$\nabla \times \mathbf{H} = j\omega\epsilon\mathbf{E}$$

$$E_r = E_{0r} J_n\left(\frac{X'_{np} r}{a}\right) \sin (n\phi) e^{-j\beta_g Z}$$

$$E_\phi = E_{0\phi} J'_n\left(\frac{X'_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

$$E_z = 0$$

$$H_r = -\frac{E_{0\phi}}{Z_g} J_n\left(\frac{X'_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J_n\left(\frac{X'_{np} r}{a}\right) \sin (n\phi) e^{-j\beta_g Z}$$

$$H_z = H_{0z} J_n\left(\frac{X'_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

- Where $Z_g = {}^{E_r}/_{H_\emptyset} = -{}^{E_\emptyset}/_{H_r}$ has been replaced for the wave impedance in the guide and where *n* = 0, 1, 2, 3, ... And p = 1, 2, 3, 4, ...
- The first subscript *n* represents the number of full cycles of field variation in one revolution through *2π* rad of ∅.
- The second subscript *p* indicates the number of zeros of $E_\emptyset$.

- The mode propagation constant is determined by Eq

$$\beta_g = \sqrt{\omega^2\mu\epsilon - \left(\frac{X'_{np}}{a}\right)^2}$$

- The cutoff wave number of a mode is that for which the mode propagation constant vanishes. Hence

$$k_c = \frac{X'_{np}}{a} = \omega_c\sqrt{\mu\epsilon}$$

- The cutoff frequency for TE modes in a circular guide is then given by

$$f_c = \frac{X'_{np}}{2\pi a \sqrt{\mu\epsilon}}$$

- and the phase velocity for TE modes is

$$v_g = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

- The wavelength and wave impedance for TE modes in a circular guide are given, respectively, by

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$$

- and

$$Z_g = \frac{\omega\mu}{\beta_g} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}$$

where $\lambda = \frac{v_p}{f}$ = wavelength in an unbounded dielectric

$\eta = \sqrt{\frac{\mu}{\epsilon}}$ = intrinsic impedance in an unbounded dielectric

TABLE 4-2-1 *p*th ZEROS OF $J'_n(K_c a)$ FOR TE_{np} MODES

p	$n =$ 0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	6.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987
4	13.324	11.706	13.170			

Ex:

- A TE11 mode is propagating through a circular waveguide. The radius of the guide is 5 cm, and the guide contains an air dielectric
- a) Determine the cutoff frequency.
- b) Determine the wavelength in the guide for an operating frequency of 3 GHz.
- c) Determine the wave impedance in the guide.

- Sol.
- From the table we can get X'_{11}= 1.841=$K_c \times a.$ The cutoff wave number is

$$k_c = \frac{1.841}{a} = \frac{1.841}{5 \times 10^{-2}} = 36.82$$

- The cutoff frequency is

$$f_c = \frac{k_c}{2\pi\sqrt{\mu_0\epsilon_0}} = \frac{(36.82)(3 \times 10^8)}{2\pi} = 1.758 \times 10^9 \text{ Hz}$$

- The phase constant in the guide is

$$\beta_g = \sqrt{\omega^2 \mu_0 \epsilon_0 - k_c^2}$$

$$= \sqrt{(2\pi \times 3 \times 10^9)^2 (4\pi \times 10^{-7} \times 8.85 \times 10^{-12}) - (36.82)^2}$$

$$= 50.9 \text{ rads/m}$$

- The wavelength in the guide is

$$\lambda_g = \frac{2\pi}{\beta_g} = \frac{6.28}{50.9} = 12.3 \text{ cm}$$

- The wave impedance in the guide

$$Z_g = \frac{\omega \mu_0}{\beta_g} = \frac{(2\pi \times 3 \times 10^9)(4\pi \times 10^{-7})}{50.9} = 465\ \Omega$$

Ex2:

- An air-filled circular waveguide is to be operated at a frequency of 6 GHz and is to have dimensions such that fc = 0.8f for the dominant mode. Determine:
- **a.** The diameter of the guide
- **b.** The wavelengthλg, and the phase velocity *Vg* in the guide

- Sol:

$$f_c = \frac{X'_{np}}{2\pi a\sqrt{\mu\epsilon}}$$

- $f_c = \frac{X_{np}}{2\pi a\sqrt{\mu\epsilon}}$

- $4.8 \times 10^9 = \frac{1.841}{2\pi \times a \times \sqrt{\mu\epsilon}}$

- $4.8 \times 10^9 = \frac{1.841 \times 3 \times 10^8}{2\pi \times a}$

- a= 18mm

- $\boldsymbol{\lambda} g = {}^{\boldsymbol{\lambda}}\!\big/_{\sqrt{1-(0.8)^\wedge 2}} = {}^{2\pi}\!\big/_{B_g}$

- $B_g = \sqrt{w^2 \epsilon \mu - K_c^2}$

- $B_g = \sqrt{4\pi^2 f^2 \frac{1}{c^2} - 102^2}$
- $B_g = 73.4$

- $\boldsymbol{\lambda} g = {}^{2\pi}\!\big/_{B_g} = 0.085m$

- $V_g = {}^{w}/_{B_g} = {}^{2\pi f}/_{B_g} = 5.1 \times 10^8$

Ex3:

- An air-filled circular waveguide has a diameter of 4 cm and is to carry energy at a frequency of 10 GHz. Determine all TEnp modes for which transmission is possible.

- $f_c = \frac{X_{np}}{2\pi a\sqrt{\mu\epsilon}}$
- $f_c = \frac{X_{np}\times 3\times 10^8}{2\pi\times 2\times 10^{-2}}$
- $f_c = X_{np} \times 2.38 \times 10^9$

TABLE 4-2-1 *p*th ZEROS OF $J'_n(K_c a)$ FOR TE_{np} MODES

p	*n* = 0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	6.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987
4	13.324	11.706	13.170			

- $f_c = X_{np} \times 2.3 \times 10$
- $f_{cTE01} = 3.832 \times 2.3 \times 10^9 = 8.8GHz$
- $f_{cTE11} = 1.841 \times 2.3 \times 10^6 = 4.23GHz$
- $f_{cTE21} = 3.054 \times 2.3 \times 10^6$ =7GHz
- $f_{cTE12} = 5.331 \times 2.3 \times 10^6$ =12.26GHz
- $f_{cTE31} = 4.201 \times 2.3 \times 10^6 =$ 9.6GHz

H.W.

- A circular waveguide has a cutoff frequency of 9 GHz in dominant mode.

a. Find the inside diameter of the guide if it is air-filled.

b. Determine the inside diameter of the guide if the guide is dielectric-filled. The relative dielectric constant is $Er = 4$.

TM Mode in Circular Waveguide

- The *TMnp* modes in a circular guide are characterized by $Hz = 0$. However, the *z* component of the electric field *Ez* must exist in order to have energy transmission in the guide. Consequently, the Helmholtz equation for *Ez* in a circular waveguide is given by

$$\nabla^2 E_z = \gamma^2 E_z$$

- Its solution is given in Eq.

$$E_z = E_{0z} J_n(k_c r) \cos(n\phi) e^{-j\beta_g Z}$$

- Since *Jn(kcr)* are oscillatory functions there are infinite numbers of roots of *Jn(kcr).* Table 4-2-2 tabulates a few of them for some lower order *n.*

TABLE 4-2-2 pth ZEROS OF $J_n(K_c a)$ FOR TM_{np} MODES

p	$n =$ 0	1	2	3	4	5
1	2.405	3.832	5.136	6.380	7.588	8.771
2	5.520	7.106	8.417	9.761	11.065	12.339
3	8.645	10.173	11.620	13.015	14.372	
4	11.792	13.324	14.796			

$$E_r = E_{0r} J'_n\left(\frac{X_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

$$E_\phi = E_{0\phi} J_n\left(\frac{X_{np} r}{a}\right) \sin (n\phi) e^{-j\beta_g Z}$$

$$E_z = E_{0z} J_n\left(\frac{X_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

$$H_r = \frac{E_{0\phi}}{Z_g} J_n\left(\frac{X_{np} r}{a}\right) \sin (n\phi) e^{-j\beta_g Z}$$

$$H_\phi = \frac{E_{0r}}{Z_g} J'_n\left(\frac{X_{np} r}{a}\right) \cos (n\phi) e^{-j\beta_g Z}$$

$$H_z = 0$$

- Some of the TM-mode characteristic equations in the circular guide are identical to those of the TE mode, but some are different. For convenience, all are shown here:

$$\beta_g = \sqrt{\omega^2 \mu \epsilon - \left(\frac{X_{np}}{a}\right)^2}$$

$$k_c = \frac{X_{np}}{a} = \omega_c \sqrt{\mu \epsilon}$$

$$f_c = \frac{X_{np}}{2\pi a \sqrt{\mu \epsilon}}$$

$$v_g = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$$

$$Z_g = \frac{B_g}{\omega \epsilon} = \eta \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

- It should be noted that the dominant mode, or the mode of lowest cutoff frequency in a circular waveguide, is the mode of TE_{11} that has the smallest value of the product, $k_c a = 1.841$

Example :

- An air-filled circular waveguide has a radius of 2 cm and is to carry energy at a frequency of 10 GHz. Find all the TEnp and *TMnp* modes for which energy transmission is possible.

Power Transmission in Circular Waveguides or Coaxial Lines

- In general, the power transmitted through circular waveguides and coaxial lines can be calculated by means of the complex Poynting theorem
- For a lossless dielectric, the time-average power transmitted through a circular guide can be given by

$$P_{tr} = \frac{1}{2Z_g} \int_0^{2\pi} \int_0^a [|E_r|^2 + |E_\phi|^2] r\, dr\, d\phi$$

$$P_{tr} = \frac{Z_g}{2} \int_0^{2\pi} \int_0^a [|H_r|^2 + |H_\phi|^2] r\, dr\, d\phi$$

where $Z_g = \frac{E_r}{H_\phi} = -\frac{E_\phi}{H_r}$ = wave impedance in the guide

a = radius of the circular guide

- For *TEnp* modes, the average power transmitted through a circular guide is given by

$$P_{tr} = \frac{\sqrt{1 - (f_c/f)^2}}{2\eta} \int_0^{2\pi} \int_0^a [|E_r|^2 + |E_\phi|^2] r \, dr \, d\phi$$

Excitations of *Modes in Circular Waveguides*

- TE modes have no *z* component of an electric field, and TM modes have no *z* component of magnetic intensity. If a device is inserted in a circular guide in such a way that it excites only a *z* component of electric intensity, the wave propagating through the guide will be the TM mode; on the other hand,
- if a device is placed in a circular guide in such a manner that only the *z* component of magnetic intensity exists, the traveling wave will be the TE mode. The methods of excitation for various modes in circular waveguides are shown in Fig. 4-2-7.

- D

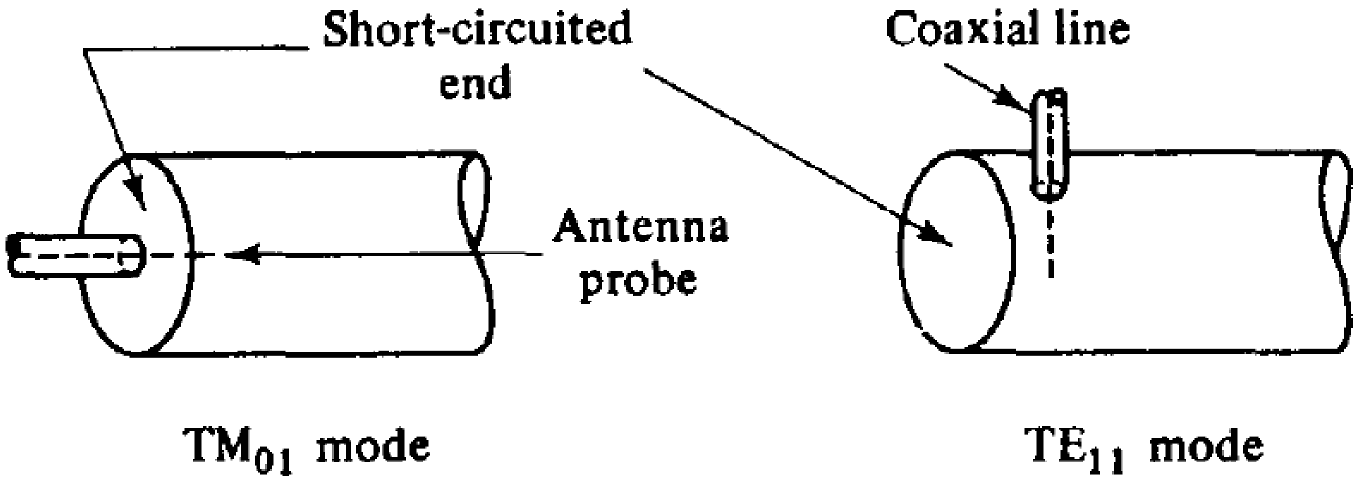


Figure 4-2-7 Methods of exciting various modes in circular waveguides.

- A common way to excite TM modes in a circular guide is by a coaxial line as shown in Fig. 4-2-8. At the end of the coaxial line a large magnetic intensity exists in the Ø direction of wave propagation. The magnetic field from the coaxial line will excite the TM modes in the guide. However, when the guide is connected to the source by a coaxial line, a discontinuity problem at the junction will increase the standing-wave ratio on the line and eventually decrease the power transmission.
- It is often necessary to place a turning device around the junction in order to suppress the reflection.

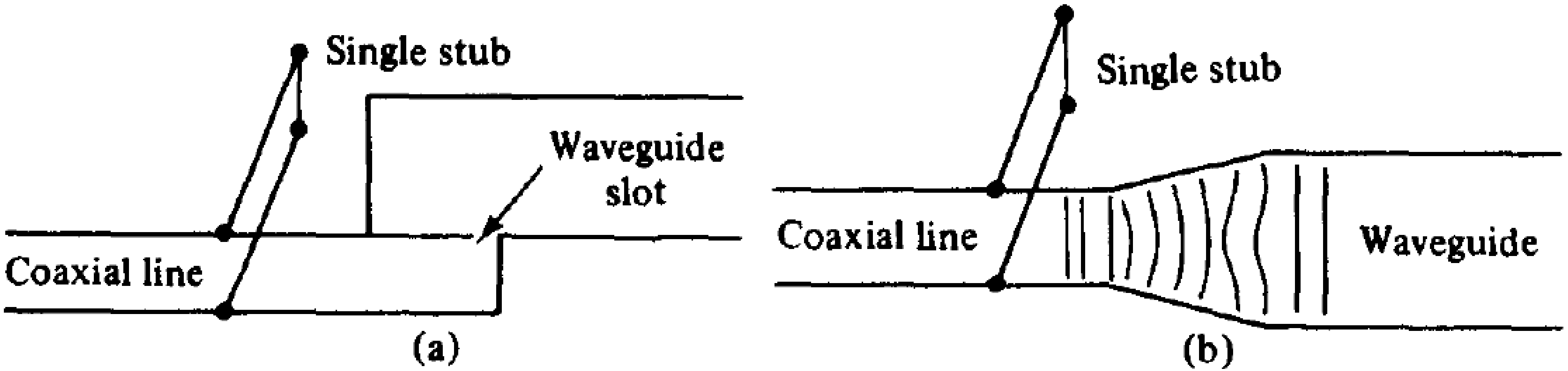


Figure 4-2-8 Methods of exciting TM modes in a circular waveguide. (a) Coaxial line with a slotted waveguide. (b) Coaxial line in series with a circular waveguide.

TABLE 4-2-8 CHARACTERISTICS OF STANDARD CIRCULAR WAVEGUIDES

EIA[a] designation WC[b]( )	Inside diameter in cm (in.)	Cutoff frequency for air-filled waveguide in GHz	Recommended frequency range for TE_{11} mode in GHz
992	25.184 (9.915)	0.698	0.80–1.10
847	21.514 (8.470)	0.817	0.94–1.29
724	18.377 (7.235)	0.957	1.10–1.51
618	15.700 (6.181)	1.120	1.29–1.76
528	13.411 (5.280)	1.311	1.51–2.07
451	11.458 (4.511)	1.534	1.76–2.42
385	9.787 (3.853)	1.796	2.07–2.83
329	8.362 (3.292)	2.102	2.42–3.31
281	7.142 (2.812)	2.461	2.83–3.88
240	6.104 (2.403)	2.880	3.31–4.54
205	5.199 (2.047)	3.381	3.89–5.33
175	4.445 (1.750)	3.955	4.54–6.23
150	3.810 (1.500)	4.614	5.30–7.27
128	3.254 (1.281)	5.402	6.21–8.51
109	2.779 (1.094)	6.326	7.27–9.97
94	2.383 (0.938)	7.377	8.49–11.60
80	2.024 (0.797)	8.685	9.97–13.70
69	1.748 (0.688)	10.057	11.60–15.90
59	1.509 (0.594)	11.649	13.40–18.40
50	1.270 (0.500)	13.842	15.90–21.80
44	1.113 (0.438)	15.794	18.20–24.90
38	0.953 (0.375)	18.446	21.20–29.10
33	0.833 (0.328)	21.103	24.30–33.20
28	0.714 (0.281)	24.620	28.30–38.80
25	0.635 (0.250)	27.683	31.80–43.60
22	0.556 (0.219)	31.617	36.40–49.80
19	0.478 (0.188)	36.776	42.40–58.10
17	0.437 (0.172)	40.227	46.30–63.50
14	0.358 (0.141)	49.103	56.60–77.50
13	0.318 (0.125)	55.280	63.50–87.20
11	0.277 (0.109)	63.462	72.70–99.70
9	0.239 (0.094)	73.552	84.80–116.00

[a] Electronic Industry Association
[b] Circular Waveguide

H.W. solution

- A circular waveguide has a cutoff frequency of 9 GHz in dominant mode.

a. Find the inside diameter of the guide if it is air-filled.

b. Determine the inside diameter of the guide if the guide is dielectric-filled. The relative dielectric constant is *Er* = 4.

- $f_c = \frac{X_{np}}{2\pi a\sqrt{\mu\epsilon}}$

- $9 \times 10^9 = \frac{1.841}{2\pi \times a \times \sqrt{\mu\epsilon}}$
- $9 \times 10^9 = \frac{1.841 \times 3 \times 10^8}{2\pi \times a}$

- $a = 0.0097m$

- $9 \times 10^9 = \frac{1.841 \times 3 \times 10^8}{2\pi \times a \times \sqrt{\epsilon_r}}$
- $9 \times 10^9 = \frac{1.841 \times 3 \times 10^8}{2\pi \times a \times \sqrt{4}}$
- a=4.85mm

Microwave Cavities and Waveguide TEE

Lecture 9

Introduction

- In general, a cavity resonator is a metallic enclosure that confines the electromagnetic energy. The stored electric and magnetic energies inside the cavity determine its equivalent inductance and capacitance.
- The energy dissipated by the finite conductivity of the cavity walls determines its equivalent resistance. In practice, the rectangular-cavity resonator and circular-cavity resonator are commonly used in many microwave applications.

- Theoretically a given resonator has an infinite number of resonant modes, and each mode corresponds to a definite resonant frequency.

- *When the frequency of an impressed signal is equal to a resonant frequency, a maximum amplitude of the standing wave occurs,* and the peak energies stored in the electric and magnetic fields are equal.
- The mode having the lowest resonant frequency is known as the *dominant mode.*

Rectangular-Cavity Resonator

- The electromagnetic field inside the cavity should satisfy Maxwell's equations, subject to the boundary conditions that the electric field tangential to and the magnetic field normal to the metal walls must vanish. The geometry of a rectangular cavity is illustrated in Fig. 4-3-1.

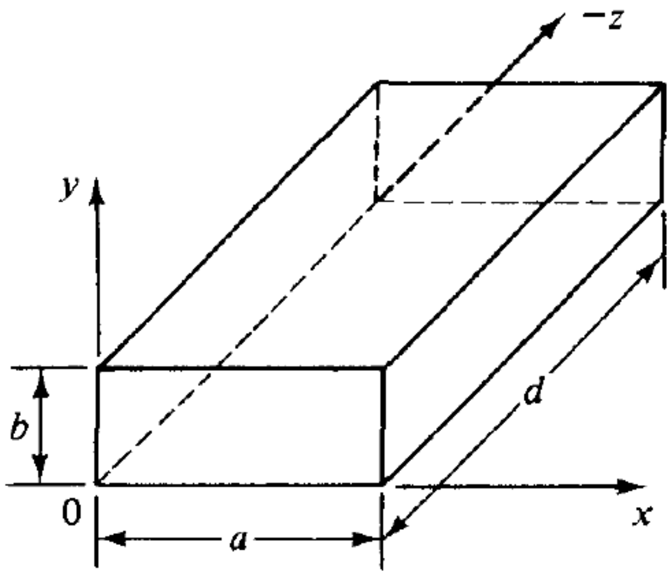


Figure 4-3-1 Coordinates of a rectangular cavity.

$$H_z = H_{0z} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) \sin\left(\frac{p\pi z}{d}\right) \qquad (\mathrm{TE}_{mnp}) \qquad (4\text{-}3\text{-}1)$$

- where $m = 0, 1, 2, 3, \ldots$ represents the number of the half-wave periodicity in the x direction and $n = 0, 1, 2, 3, \ldots$ represents the number of the half-wave periodicity in the y direction $p = 1, 2, 3, 4, \ldots$ represents the number of the half-wave periodicity in the z direction

$$E_z = E_{0z} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi x}{b}\right) \cos\left(\frac{p\pi z}{d}\right) \qquad (\mathrm{TM}_{mnp}) \qquad (4\text{-}3\text{-}2)$$

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2} \qquad (\text{TE}_{mnp}, \text{TM}_{mnp}) \qquad (4\text{-}3\text{-}4)$$

- For *a* > *b* < *d,* the dominant mode is the TE_{101} mode.

Excitation

- In general, a straight-wire probe inserted at the position of maximum electric intensity is used to excite a desired mode, and the loop coupling placed at the position of maximum magnetic intensity is utilized to launch a specific mode.

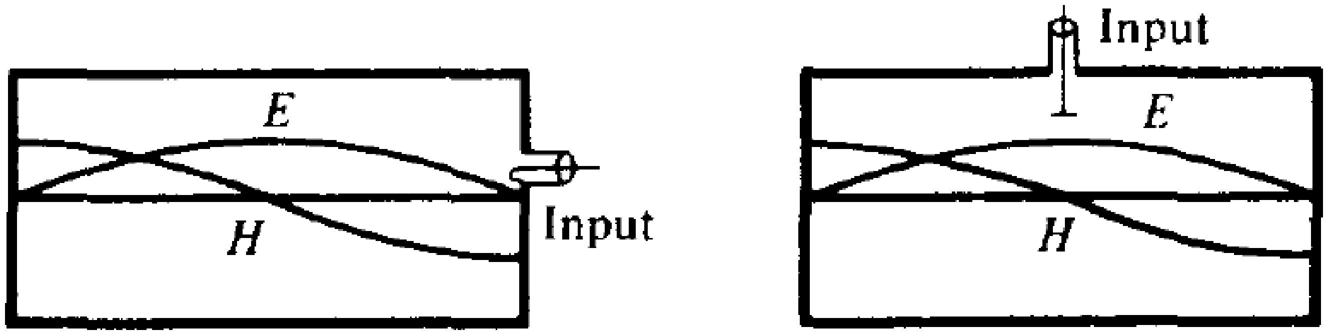


Figure 4-3-2 Methods of exciting wave modes in a resonator.

- The maximum amplitude of the standing wave occurs when the frequency of the impressed signal is equal to the resonant frequency.

Q *Factor of a Cavity Resonator*

- The quality factor *Q* is a measure of the frequency selectivity of a resonant or antiresonant circuit, and it is defined as

$$Q \equiv 2\pi \frac{\text{maximum energy stored}}{\text{energy dissipated per cycle}} = \frac{\omega W}{P} \qquad (4\text{-}3\text{-}15)$$

- where W is the maximum stored energy and *P is* the average power loss.

- At resonant frequency, the electric and magnetic energies are equal and in time quadrature. When the electric energy is maximum, the magnetic energy is zero and vice versa.
- The total energy stored in the resonator is obtained by integrating the energy density over the volume of the resonator:

$$W_e = \int_v \frac{\epsilon}{2} |E|^2 \, dv = W_m = \int_v \frac{\mu}{2} |H|^2 \, dv = W \qquad (4\text{-}3\text{-}16)$$

- where IE I and IHI are the peak values of the field intensities.

Ex:

- An air-filled circular waveguide has a radius of 3 cm and is used as a resonator for TE01 mode at 10 GHz by placing two perfectly conducting plates at its two ends. Determine the minimum distance between the two end plates.
- a=3cm
- F=10GHz

TABLE 4-2-1 pth ZEROS OF $J'_n(K_c a)$ FOR TE_{np} MODES

p	$n =$ 0	1	2	3	4	5
1	3.832	1.841	3.054	4.201	5.317	6.416
2	7.016	5.331	6.706	8.015	9.282	10.520
3	10.173	8.536	9.969	11.346	12.682	13.987
4	13.324	11.706	13.170			

Solution

$$f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{X'_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2} \qquad \text{(TE)}$$

- $f_r = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{X_{np}}{a}\right)^2 + \left(\frac{q\pi}{d}\right)^2}$
- $10 \times 10^9 = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{3.832}{3\times10^{-2}}\right)^2 + \left(\frac{\pi}{d}\right)^2}$
- $10 \times 10^9 = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{3.832}{3\times10^{-2}}\right)^2 + \left(\frac{\pi}{d}\right)^2}$

- $10 \times 10^9 = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{3.832}{3\times10^{-2}}\right)^2 + \left(\frac{\pi}{d}\right)^2}$

- d=0.0189m
- d=1.89cm

Ex:

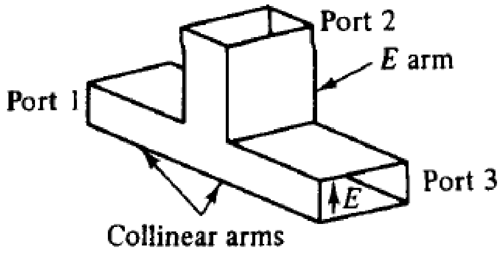
- A rectangular-cavity resonator has dimensions of a = 5 cm, b = 2 cm, and d = 15 cm. Compute:

a. The resonant frequency of the dominant mode for an air-filled cavity

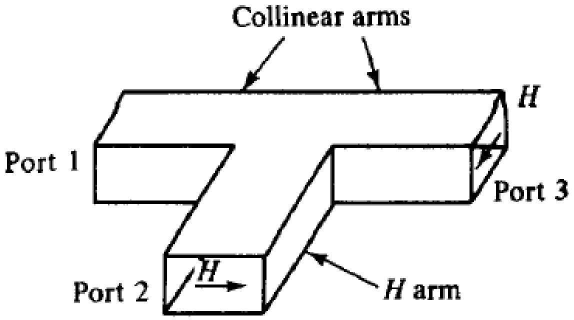
b. The resonant frequency of the dominant mode for a dielectric-filled cavity of $\epsilon_r = 2.56$

MICROWAVE HYBRID CIRCUITS

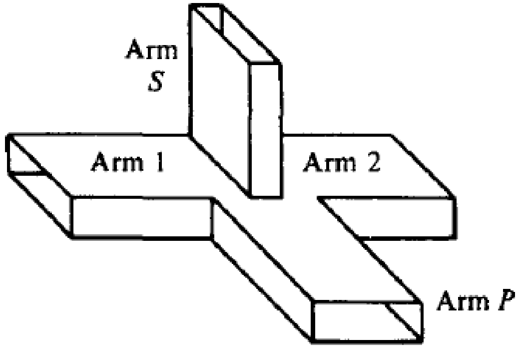
- A microwave circuit ordinarily consists of several microwave devices connected in some way to achieve the desired transmission of a microwave signal.
- The interconnection of two or more microwave devices may be regarded as a microwave junction.
- Commonly used microwave junctions include such waveguide tees as the *E* plane tee, *H* -plane tee, magic tee, hybrid ring (rat-race circuit), directional coupler, and the circulator. This section describes these microwave hybrids, which are shown in Fig. 4-4-1.



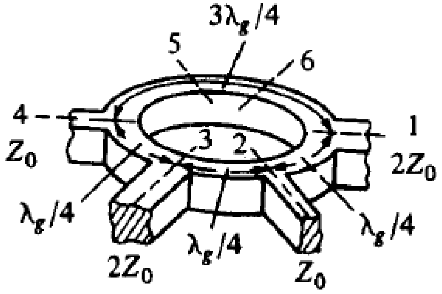
(a) *E*-plane tee



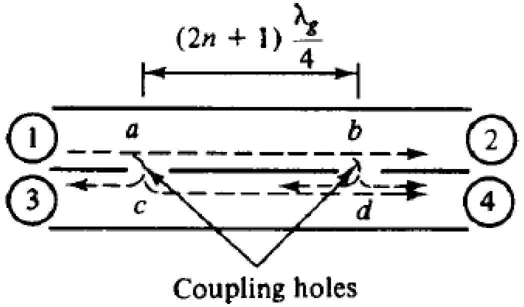
(b) *H*-plane tee



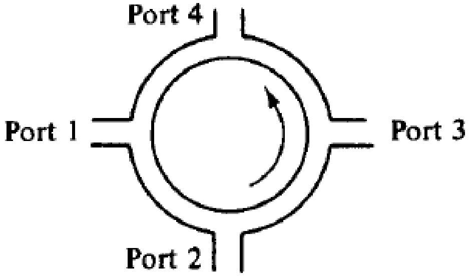
(c) Magic tee



(d) Hybrid ring



(e) Directional coupler



(f) Circulator

Two port network

- A two-port network is shown in Figure below. From network theory a two-port device can be described by a number of parameter sets, such as the *H, Y,* Z, and *ABCD* parameters.

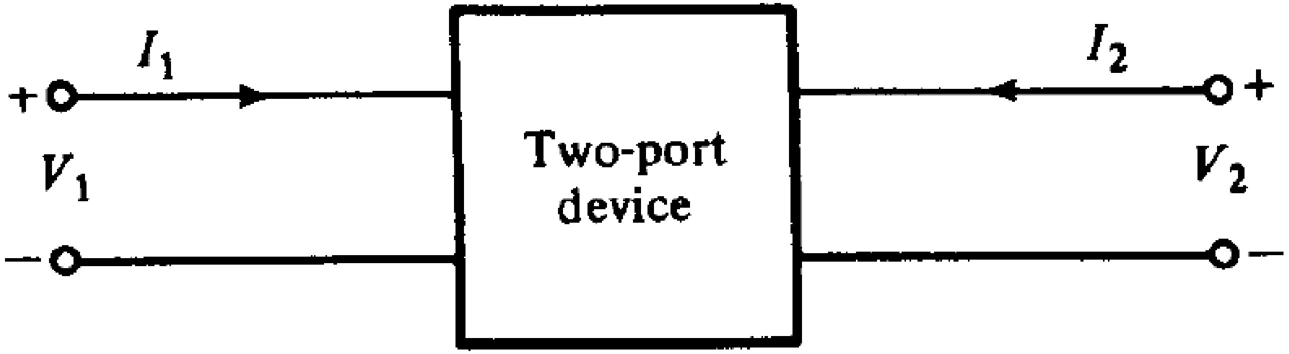


Figure 4-4-2 Two-port network.

- H parameters: $\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

$$V_1 = h_{11} I_1 + h_{12} V_2 \tag{4-4-1}$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \tag{4-4-2}$$

Y parameters: $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$$I_1 = y_{11} V_1 + y_{12} V_2 \tag{4-4-3}$$
$$I_2 = y_{21} V_1 + y_{22} V_1 \tag{4-4-4}$$

Z parameters: $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$V_1 = z_{11} I_1 + z_{12} I_2 \tag{4-4-5}$$
$$V_2 = z_{21} I_1 + z_{22} I_2 \tag{4-4-6}$$

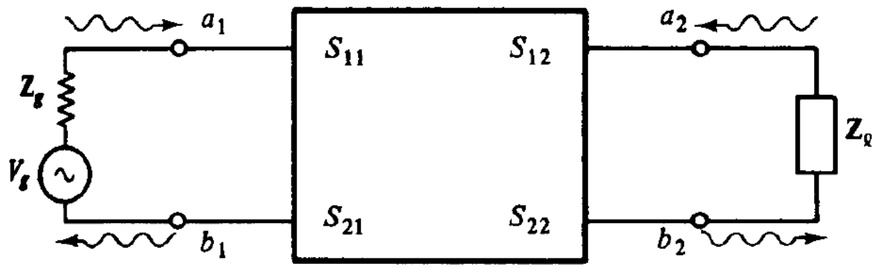
$ABCD$ parameters: $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \begin{matrix} = \\ = \end{matrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$

$$V_1 = AV_2 - BI_2 \tag{4-4-7}$$
$$I_1 = CV_2 - DI_2 \tag{4-4-8}$$

- All these network parameters relate total voltages and total currents at each of the two ports.
- If the frequencies are in the microwave range, however, the *H, Y,* and Z parameters cannot be measured for the following reasons:

1. Equipment is not readily available to measure total voltage and total current at the ports of the network.
2. Short and open circuits are difficult to achieve over a broad band of frequencies.
3. Active devices, such as power transistors and tunnel diodes, frequently will not have stability for a short or open circuit.

S parameters



- Consequently, some new method of characterization is needed to overcome these problems. The logical variables to use at the microwave frequencies are traveling waves rather than total voltages and total currents. These are the *S* parameters, which are expressed as

$$b_1 = S_{11}a_1 + S_{12}a_2 \qquad (4\text{-}4\text{-}11a)$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \qquad (4\text{-}4\text{-}11b)$$

- ff

$$b_1 = S_{11}a_1 + S_{12}a_2 \tag{4-4-11a}$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \tag{4-4-11b}$$

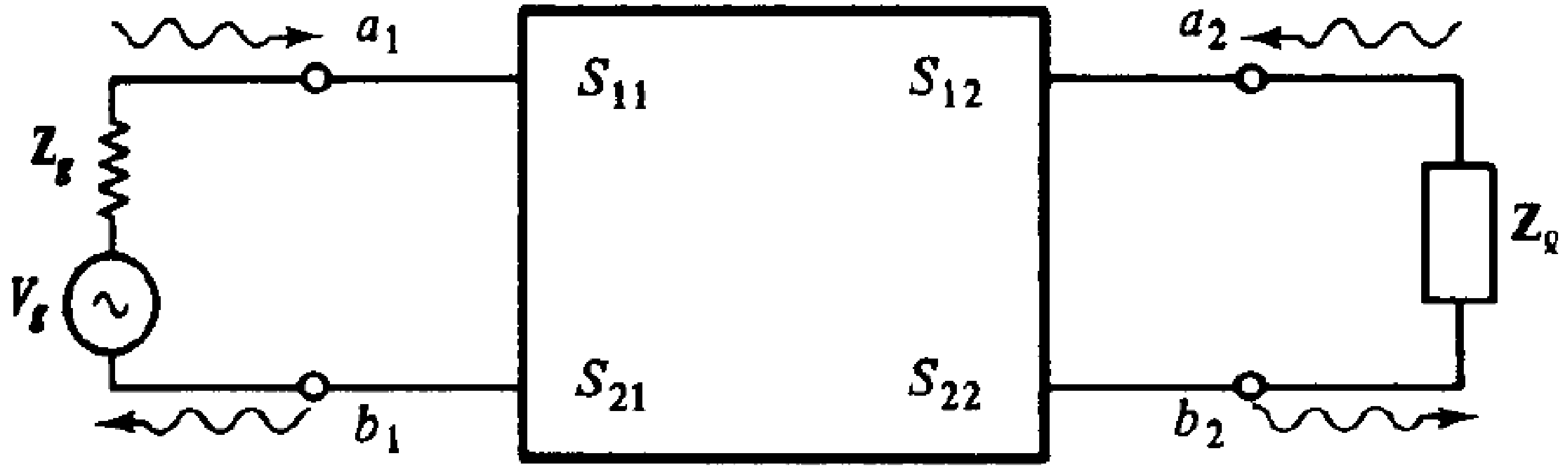
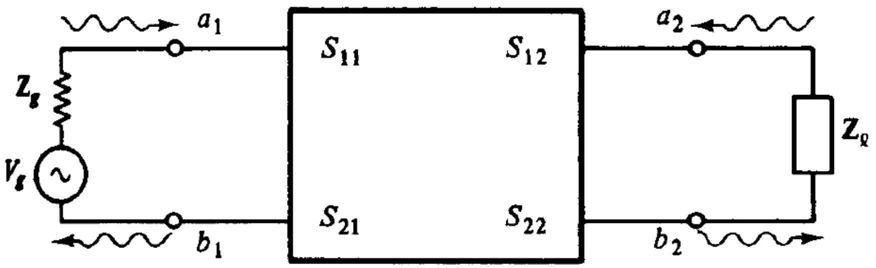


Figure 4-4-3 Two-port network.

S parameter properties

- [S] is always square matrix of order n*n
- [S] is a symmetric matrix
- $S_{ij} = S_{ji}$ where I not equal to j
- [S] matrix is a unitary matrix
- $[S] \times [S]^{*} = [I]$
- Zero diagonal matrix for perfectly matched network
- $[S]_{ij} = 0 \qquad if\ i = j$

- $$\begin{matrix} b_1 \\ b_2 \end{matrix} = \begin{matrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{matrix} \times \begin{matrix} a_1 \\ a_2 \end{matrix}$$



Waveguide Tees

- **Tee junctions.** In microwave circuits a waveguide or coaxial-line junction with three independent ports is commonly referred to as a *tee junction.*
- From the *S* parameter theory of a microwave junction it is evident that a tee junction should be characterized by a matrix of third order containing nine elements.
- six of which should be independent.
- The characteristics of a three-port junction can be explained by three theorems of the tee junction. These theorems are derived from the equiva*lent- circuit representation of the tee junction. Their* statements follow

1. A short circuit may always be placed in one of the arms of a three-port junction in such a way that no power can be transferred through the other two arms.
2. If the junction is symmetric about one of its arms, a short circuit can always be placed in that arm so that no reflections occur in power transmission between the other two arms. (That is, the arms present matched impedances.)
3. It is impossible for a general three-port junction of arbitrary symmetry to present matched impedances at all three arms.

E-plane tee {series tee).

- An E-plane tee is a waveguide tee in which the axis of its side arm is parallel to the *E* field of the main guide (see Fig. 4-4-4).

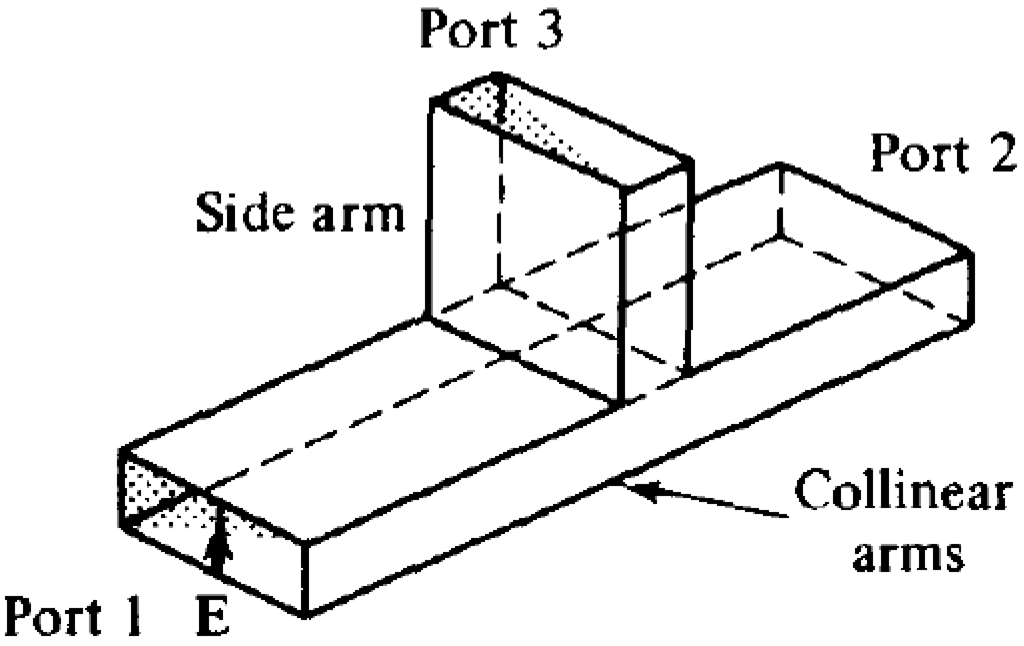


Figure 4-4-4 *E*-plane tee

- If the collinear arms are symmetric about the side arm, there are two different transmission characteristics (see Fig. 4-4-5). It can be seen from Fig. 4-4-4 that if the

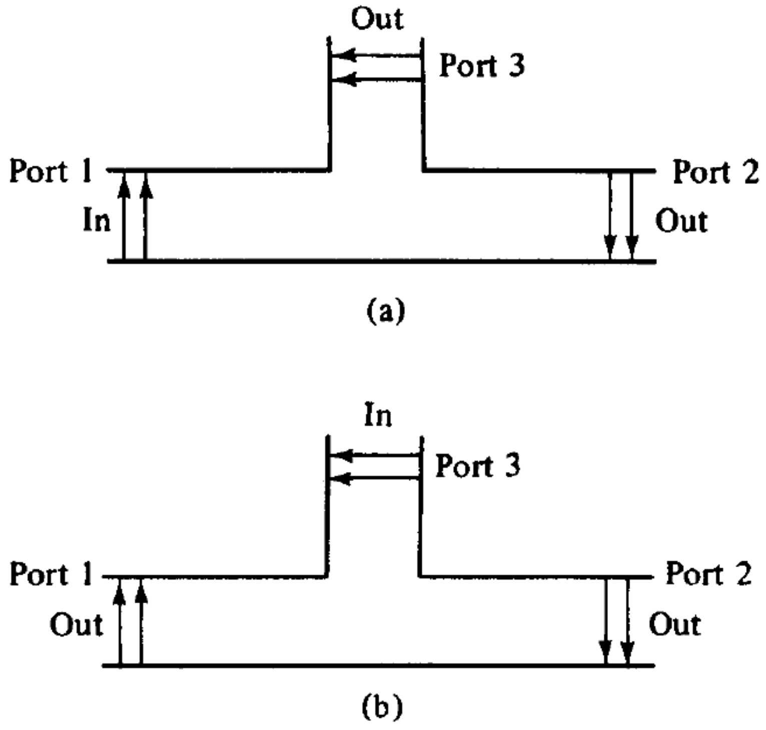


Figure 4-4-5 Two-way transmission of *E*-plane tee. (a) Input through main arm. (b) Input from side arm.

- When the waves are fed into the side arm (port 3), the waves appearing at port 1 and port 2 of the collinear arm will be in opposite phase and in the same magnitude.

$$S_{13} = -S_{23}$$

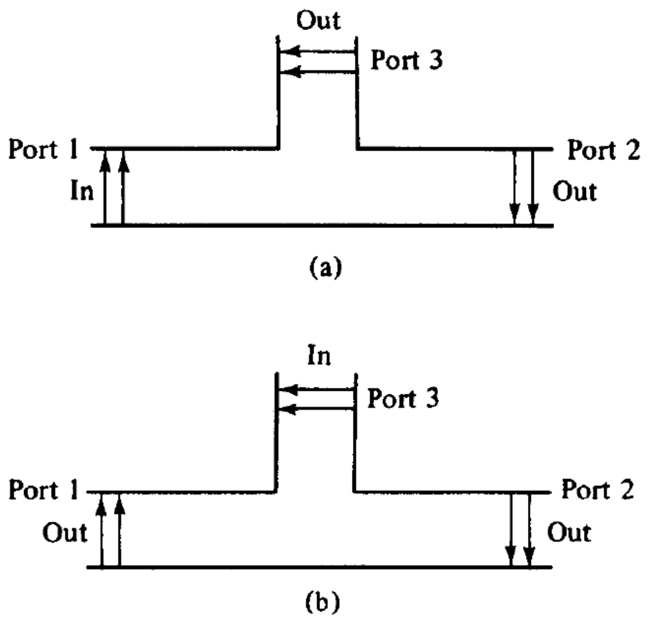


Figure 4-4-5 Two-way transmission of E-plane tee. (a) Input through main arm. (b) Input from side arm.

- The negative sign merely means that S13 and S23 have opposite signs.
- the S matrix is given by

$$\begin{matrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{matrix} \qquad \begin{matrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & S_{33} \end{matrix}$$

- From the symmetry property of S matrix, the symmetric terms are equal and they are

$$S_{12} = S_{21} \qquad S_{13} = S_{31} \qquad S_{23} = S_{32}$$

- From the zero property of S matrix, the sum of the products of each term of any column (or row) multiplied by the complex conjugate of the corresponding terms of any other column (or row) is zero and it is

$$S_{11} S_{12}^* + S_{21} S_{22}^* + S_{31} S_{32}^* = 0 \qquad (4\text{-}4\text{-}15)$$

Hence

$$S_{13} S_{23}^* = 0 \qquad (4\text{-}4\text{-}16)$$

- $$\begin{matrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{matrix} \quad * \quad \begin{matrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{matrix} \quad = \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

- From multiplying the first row by first column and row2 by column2 we find that S11 =S22

$$\begin{matrix} \boldsymbol{S_{11}} & \boldsymbol{S_{12}} & \boldsymbol{S_{13}} \\ \boldsymbol{S_{12}} & \boldsymbol{S_{11}} & \boldsymbol{-S_{13}} \\ \boldsymbol{S_{13}} & \boldsymbol{-S_{13}} & \boldsymbol{0} \end{matrix} \qquad \begin{matrix} \boldsymbol{S_{11}^*} & \boldsymbol{S_{12}^*} & \boldsymbol{S_{13}^*} \\ \boldsymbol{S_{12}^*} & \boldsymbol{S_{22}^*} & \boldsymbol{-S_{13}^*} \\ \boldsymbol{S_{13}^*} & \boldsymbol{-S_{13}^*} & \boldsymbol{0} \end{matrix} \qquad \begin{matrix} \boldsymbol{1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} \end{matrix}$$

- Multiply row3 by column3 equal 1
- $S_{13} = {}^{1}/_{\sqrt{2}}$

- Multiply row3 by column1
- S11 =S22=S12 from the previous matrix

$$\begin{matrix} \boldsymbol{S_{11}} & \boldsymbol{S_{12}} & \boldsymbol{{}^{1}/_{\sqrt{2}}} \\ \boldsymbol{S_{12}} & \boldsymbol{S_{11}} & \boldsymbol{-{}^{1}/_{\sqrt{2}}} \\ \boldsymbol{{}^{1}/_{\sqrt{2}}} & \boldsymbol{-{}^{1}/_{\sqrt{2}}} & \boldsymbol{0} \end{matrix}$$

- S11=S22=S12=1/2

H-plane tee (shunt tee).

- An H-plane tee is a waveguide tee in which the axis of its side arm is "shunting" the E field or parallel to the H field of the main guide as shown in Fig. 4-4-6.

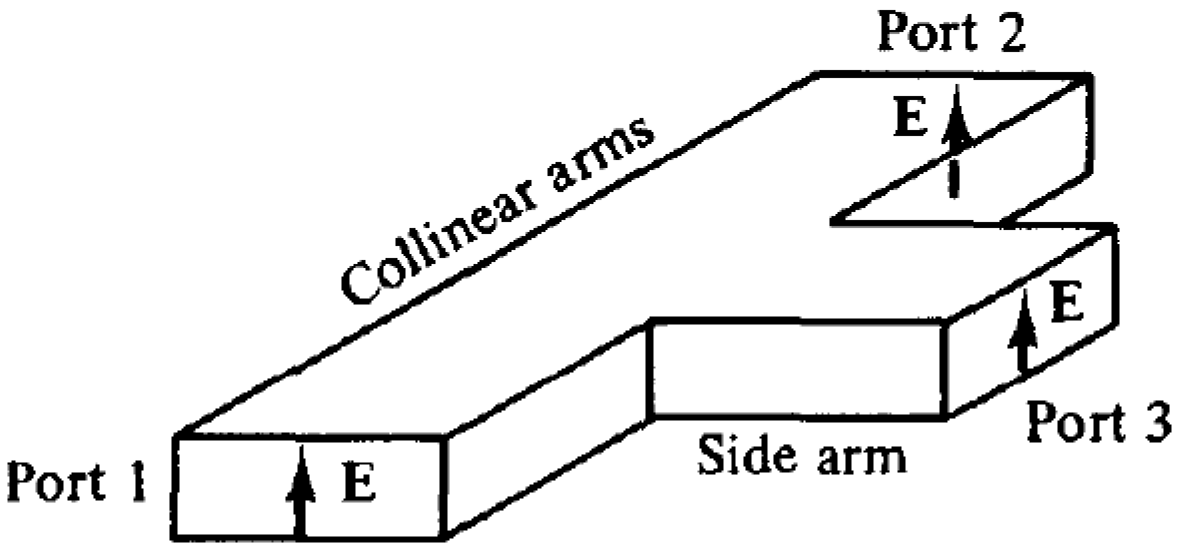
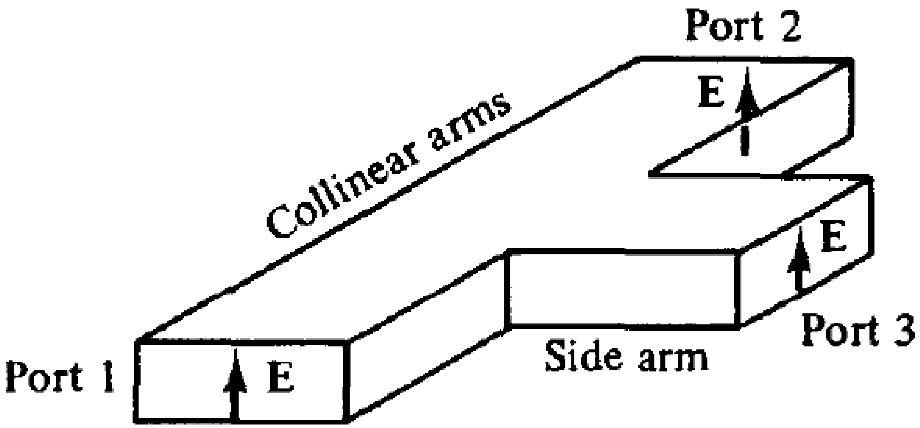


Figure 4-4-6 *H*-plane tee.

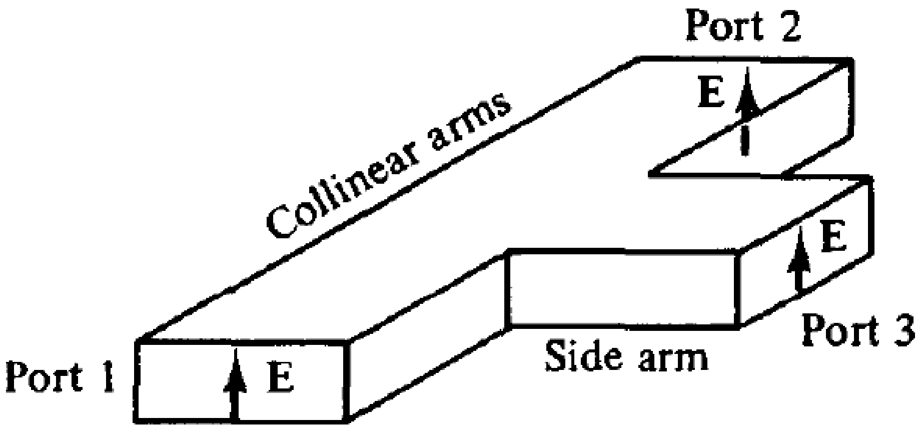
- It can be seen that if two input waves are fed into port 1 and port 2 of the collinear arm, the output wave at port 3 will be in phase and additive.
- On the other hand, if the input is fed into port 3, the wave will split equally into port 1 and port 2 in phase and in the same magnitude.



- S13=S23

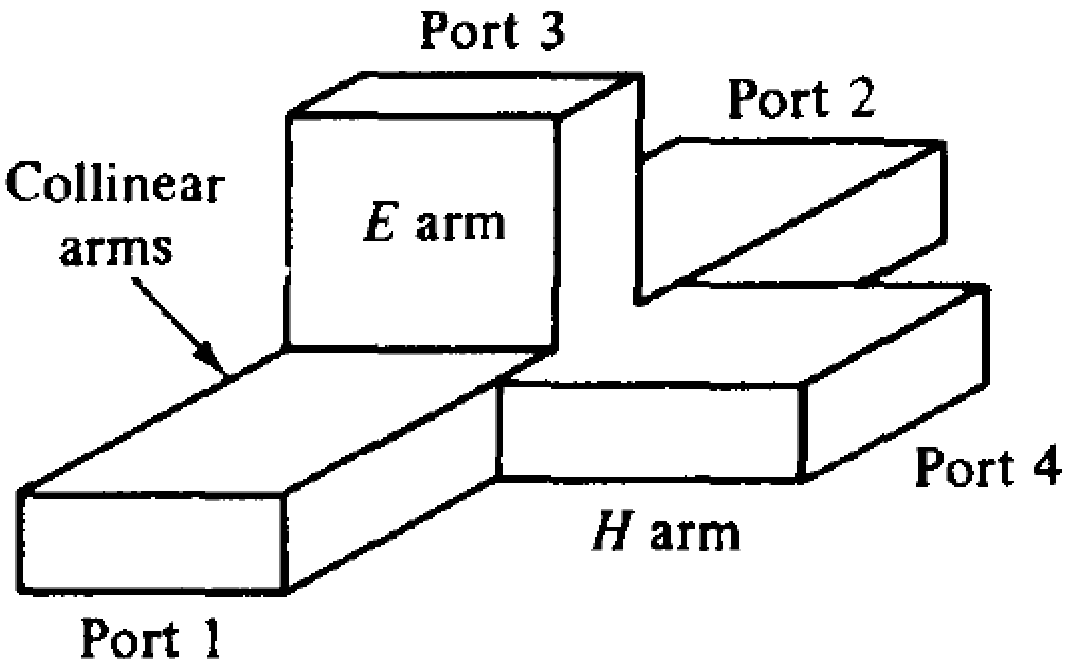
$$\begin{matrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & 0 \end{matrix}$$

$$\begin{matrix} 1/2 & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{matrix}$$



Magic T and Hybrid ring

- A magic tee is a combination of the E-plane tee and *H*-plane tee



Magic tee characteristics:

1. If two waves of equal magnitude and the same phase are fed into port 1 and port 2, the output will be zero at port 3 and additive at port 4.
2. If a wave is fed into port 4 (the Harm), it will be divided equally between port 1 and port 2 of the collinear arms and will not appear at port 3 (the *E* arm).
3. If a wave is fed into port 3 (the *E* arm), it will produce an output of equal magnitude and opposite phase at port 1 and port 2. The output at port 4 is zero.

- If a wave is fed into one of the collinear arms at port 1 or port 2, it will not appear in the other collinear arm at port 2 or port 1 because the *E* arm causes a phase delay while the Harm causes a phase advance. That is, S12 = S21 = 0.
- Therefore the S matrix of a magic tee can be expressed as

$$\mathbf{S} = \begin{bmatrix} 0 & 0 & S_{13} & S_{14} \\ 0 & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & 0 \\ S_{41} & S_{42} & 0 & 0 \end{bmatrix}$$

Magic Tee applications

- The magic tee is commonly used for mixing, duplexing, and impedance measurements.

- Suppose, for example, there are two identical radar transmitters in equipment stock. A particular application requires twice more input power to an antenna than either transmitter can deliver. A magic tee may be used to couple the two transmitters to the antenna in such a way that the transmitters do not load each other.

- The two transmitters should be connected to ports 3 and 4, respectively, as shown in Fig. 4-4-8. Transmitter 1, connected to port 3, causes a wave to originate from port 1 and another to emanate from port 2; these waves are equal in magnitude but opposite in phase. Similarly, transmitter 2, connected to port 4, gives rise to a wave at port 1 and another at port 2, both equal in magnitude and in phase. At port 1 the two opposite waves cancel each other. At port 2 the two in-phase waves add together; so double output power at port 2 is obtained for the antenna as shown in Fig. 4-4-8.

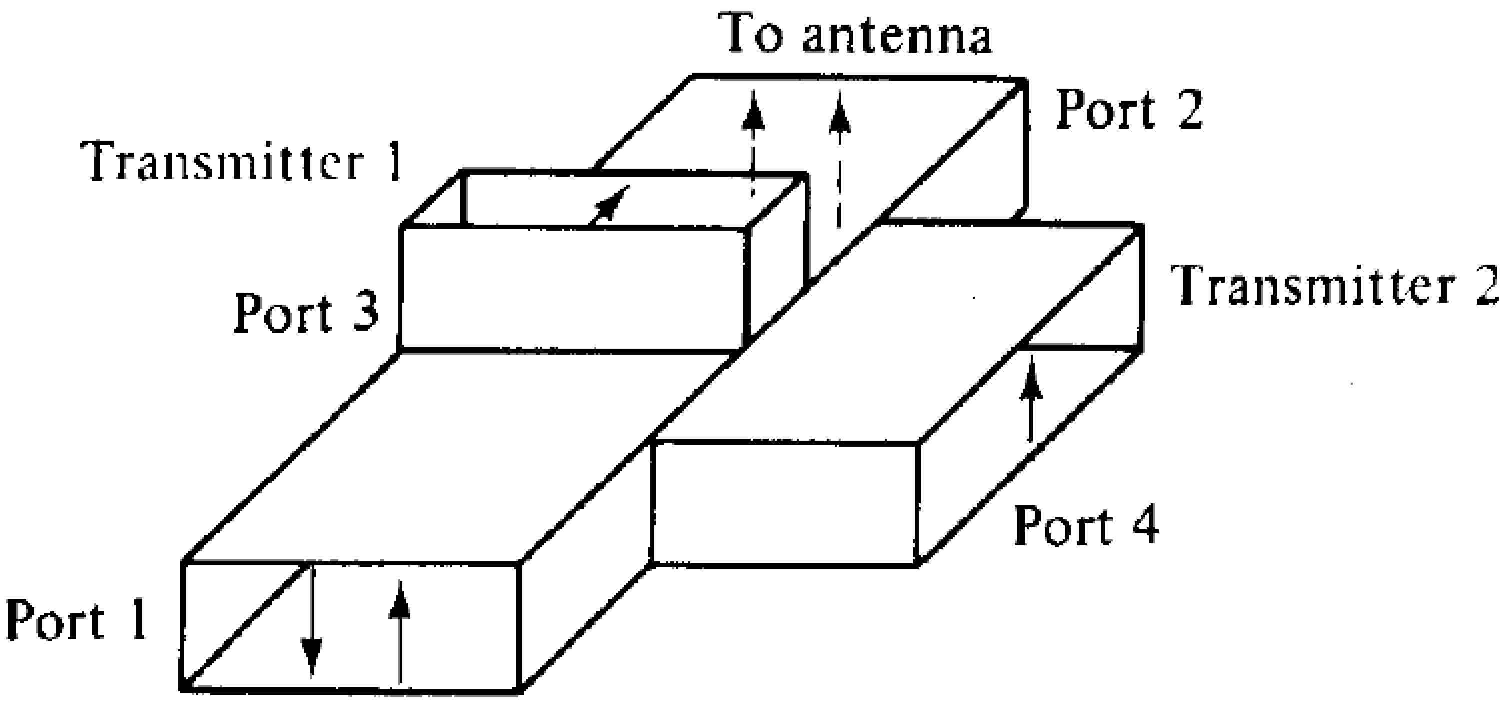


Figure 4-4-8 Magic tee-coupled transmitters to antenna.

Ex:

If a signal of $2V_{pp}$ is applied to port 1 of magic Tee determine the power at each other port assuming all ports are matched

Sol:

The power will split between 3&4

And nothing will reach port 2.

P=$\frac{(Vrms)^{2}}{R}$, lets R=1;

$$V_p = {}^{V_{pp}}/_{2} = {}^{2}/_{2} = 1$$

Vrms=$1/\sqrt{2}$ =0.707

P=0.5 watt

Power at port 1 =0.5 watt

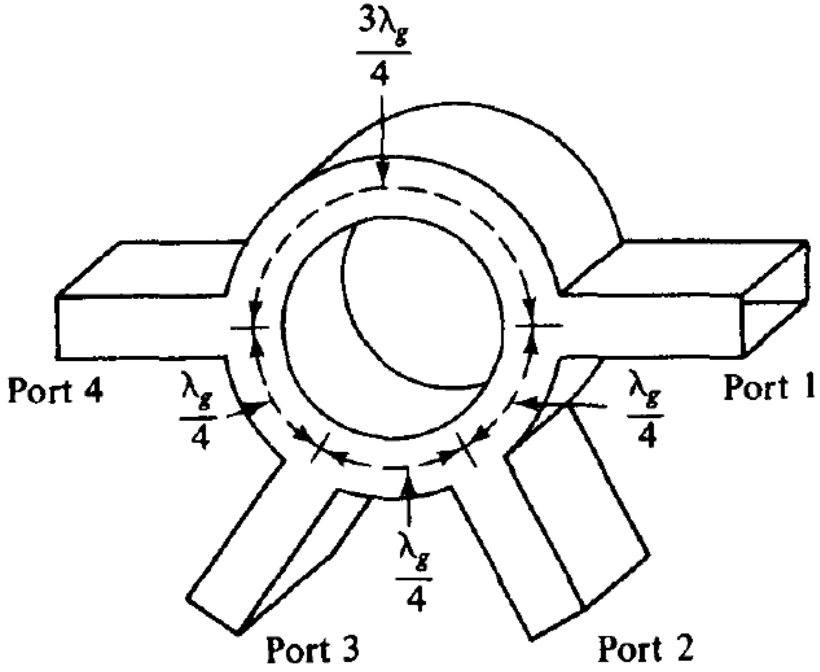
Power at port 2=0 watt

Power at port 3=0.25 watt

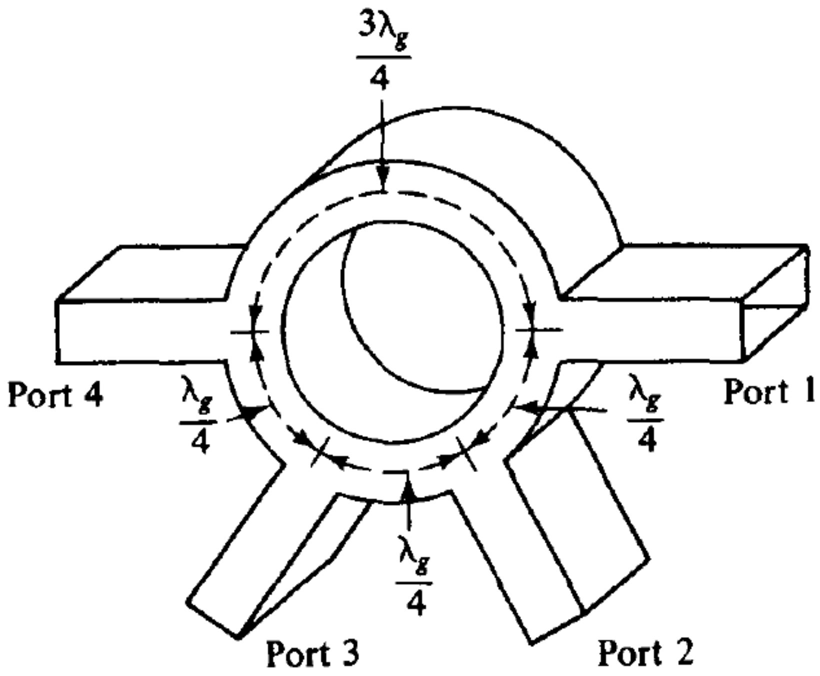
Power at port 4=0.25 watt

Hybrid ring

- A hybrid ring consists of an annular line of proper electrical length to sustain standing waves, to which four arms are connected at proper intervals by means of series junctions.



- The hybrid ring has characteristics similar to those of the hybrid tee. When a wave is fed into port 1, it will not appear at port 3 because the difference of phase shifts for the waves traveling in the clockwise and counterclockwise directions is180°.
- Thus the waves are canceled at port 3. For the same reason, the waves fed into port 2 will not emerge at port 4 and so on.



- It should be noted that the phase cancellation ocurs only at a designated frequency for an ideal hybrid ring. In actual hybrid rings there are small leakage couplings, and therefore the zero elements in the matrix are not quite equal to zero.

$$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Waveguide bends

- The waveguide corner, bend, and twist are shown in Fig. 4-4-10. These waveguide components are normally used to change the direction of the guide through

an arbitrary angle.

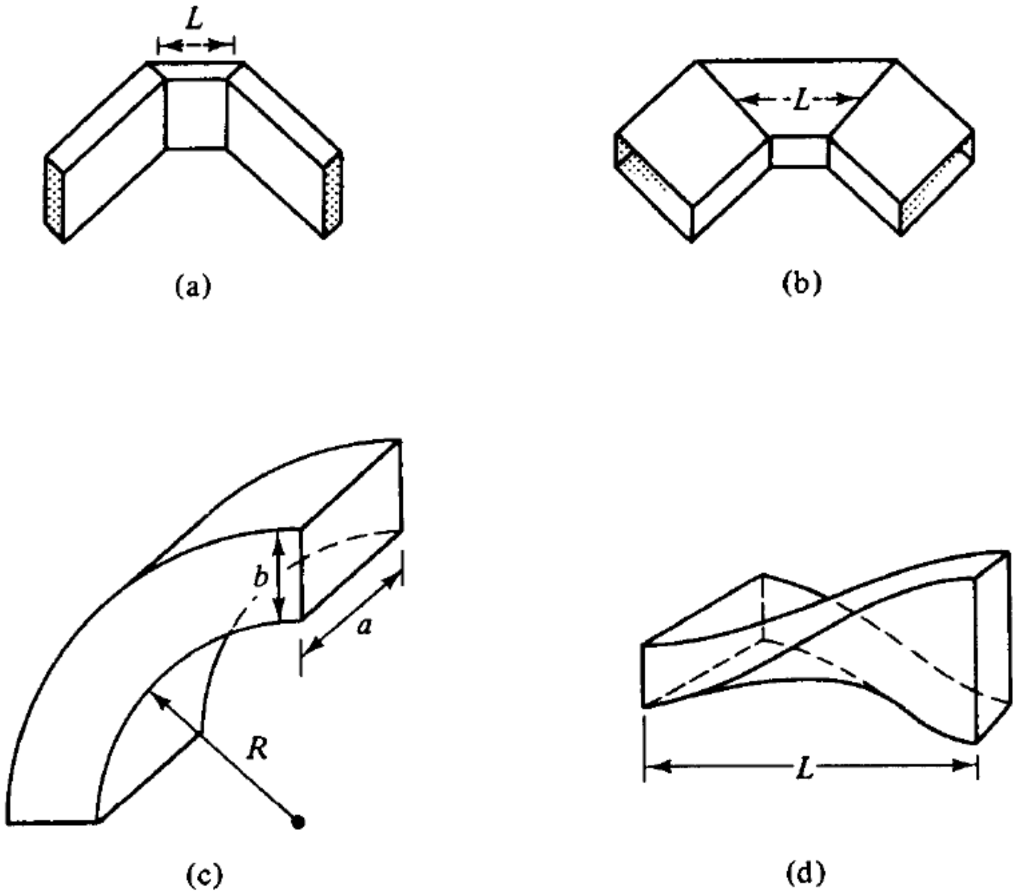


Figure 4-4-10 Waveguide corner, bend, and twist. (a) *E*-plane corner. (b) *H*-plane corner. (c) Bend. (d) Continuous twist.

- In order to minimize reflections from the discontinuities, it is desirable to have the mean length *L* between continuities equal to an odd number of quarter-wavelengths. That is,

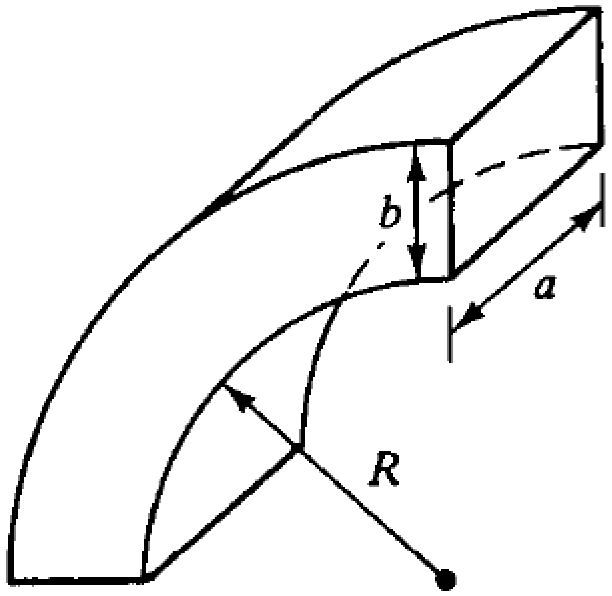
$$L = (2n + 1)\frac{\lambda_g}{4}$$

- where *n* = 0, 1, 2, 3, ... , and A8 is the wavelength in the waveguide. If the mean length *L* is an odd number of quarter wavelengths, the reflected waves from both ends of the waveguide section are completely canceled.

- For the waveguide bend, the minimum radius of curvature for a small reflection is given by

$$R = 1.5b \qquad \text{for an } E \text{ bend}$$

$$R = 1.5a \qquad \text{for an } H \text{ bend}$$



dB, dBm, dBw, dBv

Directional Couplers

lecture11

- A *directional coupler* is a four-port waveguide junction as shown in Fig. 4-5-1. It consists of a primary waveguide 1-2 and a secondary waveguide 3-4.

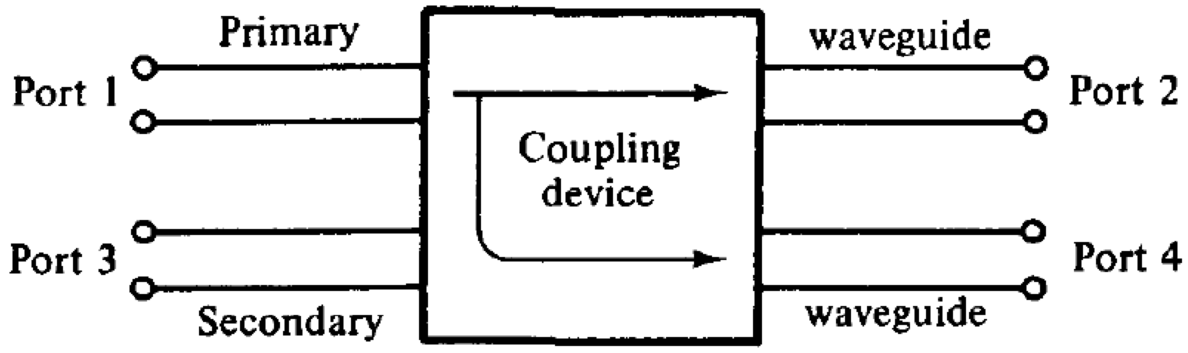


Figure 4-5-1 Directional coupler.

- When all ports are terminated in their characteristic impedances, there is free transmission of power, without reflection, between port 1 and port 2, and there is no transmission of power between port 1 and port 3 or between port 2 and port 4 because no coupling exists between these two pairs of ports.

- The degree of coupling between port 1 and port 4 and between port 2 and port 3 depends on the structure of the coupler.
- The characteristics of a directional coupler can be expressed in terms of its coupling factor and its directivity.
- Assuming that the wave is propagating from port 1 to port 2 in the primary line, the coupling factor and the directivity are defined, respectively, by

$$\text{Coupling factor (dB)} = 10 \log_{10} \frac{P_1}{P_4}$$

$$\text{Directivity (dB)} = 10 \log_{10} \frac{P_4}{P_3}$$

where P_1 = power input to port 1
P_3 = power output from port 3
P_4 = power output from port 4

- It should be noted that port 2, port 3, and port 4 are terminated in their characteristic impedances.
- *The coupling factor is* a measure *of the ratio of* power levels in the primary and secondary lines.
- Hence if the coupling factor is known, a fraction of power measured at port 4 may be used to determine the power input at port 1.
- This significance is desirable for microwave power measurements because no disturbance, which may be caused by the power measurements, occurs in the primary line.

- The directivity is a measure of how well the forward traveling wave in the primary waveguide couples only to a specific port of the secondary waveguide.
- An ideal directional coupler should have infinite directivity. In other words, the power at port 3 must be zero because port 2 and port 4 are perfectly matched. Actually, well-designed directional couplers have a directivity of only 30 to 35 dB.

Types of directional couplers

- Two-hole directional coupler
- Four-hole directional coupler,
- Reverse-coupling directional coupler (Schwinger coupler)
- Bethe-hole directional coupler
- Only the very commonly used two-hole directional coupler is described here.

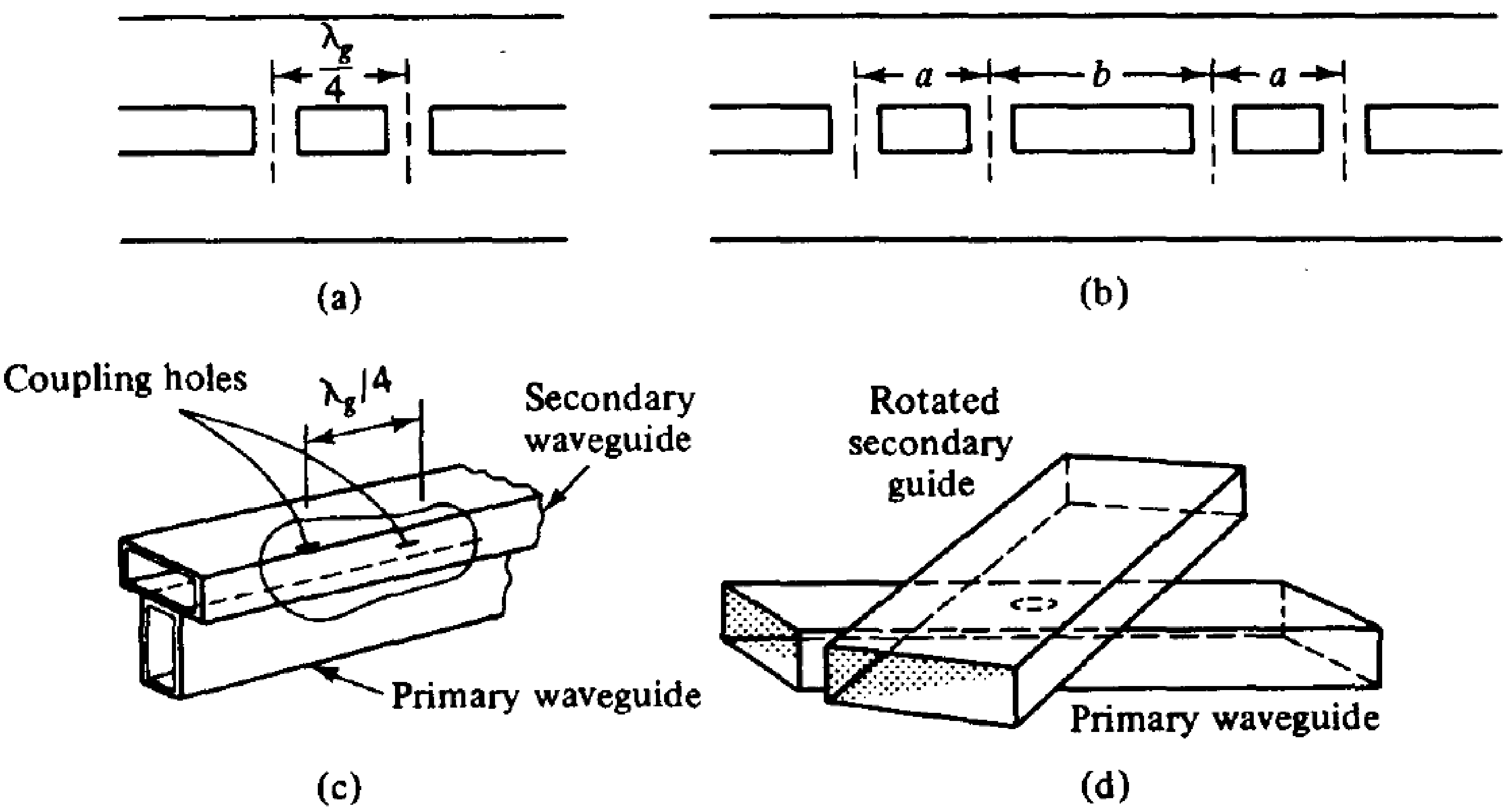


Figure 4-5-2 Different directional couplers. (a) Two-hole directional coupler. (b) Four-hole directional coupler. (c) Schwinger coupler. (d) Bethe-hole directional coupler.

Two-hole Directional Couplers

- A two-hole directional coupler with traveling waves propagating in it is illustrated in Fig. 4-5-3. The spacing between the centers of two holes must be

$$L = (2n + 1)\frac{\lambda_g}{4} \qquad (4\text{-}5\text{-}3)$$

- where n is any positive integer.

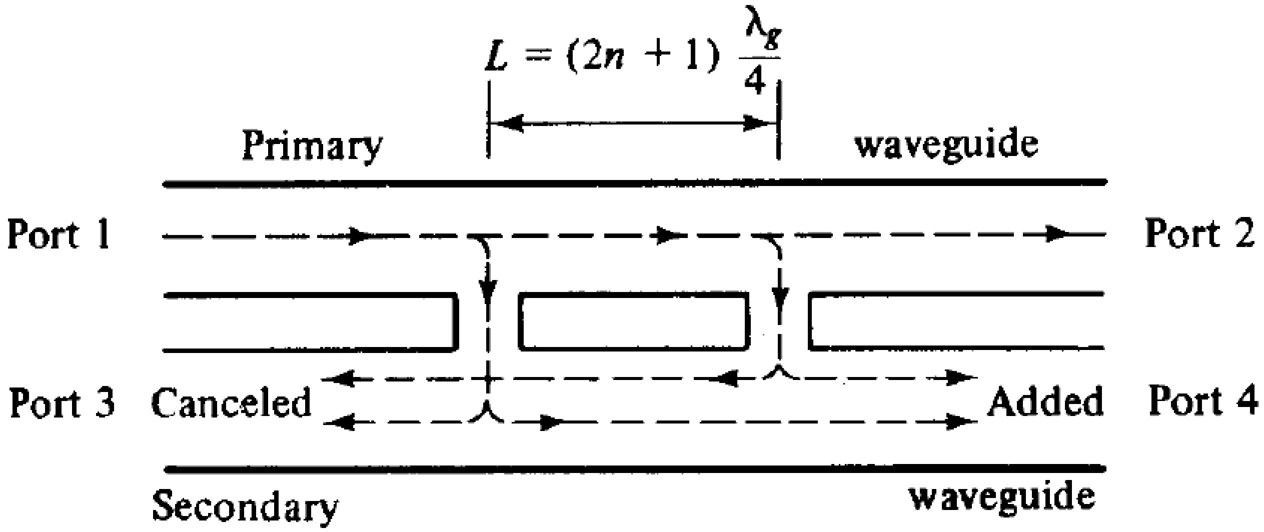


Figure 4-5-3 Two-hole directional coupler.

- A fraction of the wave energy entered into port 1 passes through the holes and is radiated into the secondary guide as the holes act as slot antennas. The forward waves in the secondary guide are in the same phase, regardless of the hole space, and are added at port 4. The backward waves in the secondary guide (waves are progressing from right to left) are out of phase by $(2L/\lambda_g)2\pi$ rad and are canceled at port 3.

S *Matrix of a Directional Coupler*

- In a directional coupler all four ports are completely matched. Thus the diagonal elements of the S matrix are zeros and

$$S_{11} = S_{22} = S_{33} = S_{44} = 0$$

- As noted, there is no coupling between port 1 and port 3 and between port 2 and port 4. Thus

$$S_{13} = S_{31} = S_{24} = S_{42} = 0$$

- Consequently, the S matrix of a directional coupler becomes
- $$\mathbf{S} = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix} \qquad (4\text{-}5\text{-}6)$$
- Equation (4-5-6) can be further reduced by means of the zero property of the S matrix,
- so we have

$$S_{12}S_{14}^* + S_{32}S_{34}^* = 0 \quad (4\text{-}5\text{-}7)$$

$$S_{21}S_{23}^* + S_{41}S_{43}^* = 0 \quad (4\text{-}5\text{-}8)$$

- Also from the unity property of the S matrix, we can write

$$S_{12}S_{12}^* + S_{14}S_{14}^* = 1 \quad (4\text{-}5\text{-}9)$$

Equations (4-5-7) and (4-5-8) can also be written

$$|S_{12}||S_{14}| = |S_{32}||S_{34}| \tag{4-5-10}$$

$$|S_{21}||S_{23}| = |S_{41}||S_{43}| \tag{4-5-11}$$

Since $S_{12} = S_{21}$, $S_{14} = S_{41}$, $S_{23} = S_{32}$, and $S_{34} = S_{43}$, then

$$|S_{12}| = |S_{34}| \tag{4-5-12}$$

$$|S_{14}| = |S_{23}| \tag{4-5-13}$$

$$S_{12} = S_{34} = p \tag{4-5-14}$$

where p is positive and real. Then from Eq. (4-5-8)

$$p(S_{23}^* + S_{41}) = 0 \tag{4-5-15}$$

Let

$$S_{23} = S_{41} = jq \tag{4-5-16}$$

where q is positive and real. Then from Eq. (4-5-9)

$$p^2 + q^2 = 1 \tag{4-5-17}$$

The **S** matrix of a directional coupler is reduced to

$$\mathbf{S} = \begin{bmatrix} 0 & p & 0 & jq \\ p & 0 & jq & 0 \\ 0 & jq & 0 & p \\ jq & 0 & p & 0 \end{bmatrix} \tag{4-5-18}$$

Example 4-5-1: Directional Coupler

- A symmetric directional coupler with infinite directivity and a forward attenuation of 20 dB is used to monitor the power delivered to a load *Zl.* Bolometer1 introduces a VSWR of 2.0 on arm 4; bolometer 2 is matched to arm 3. If

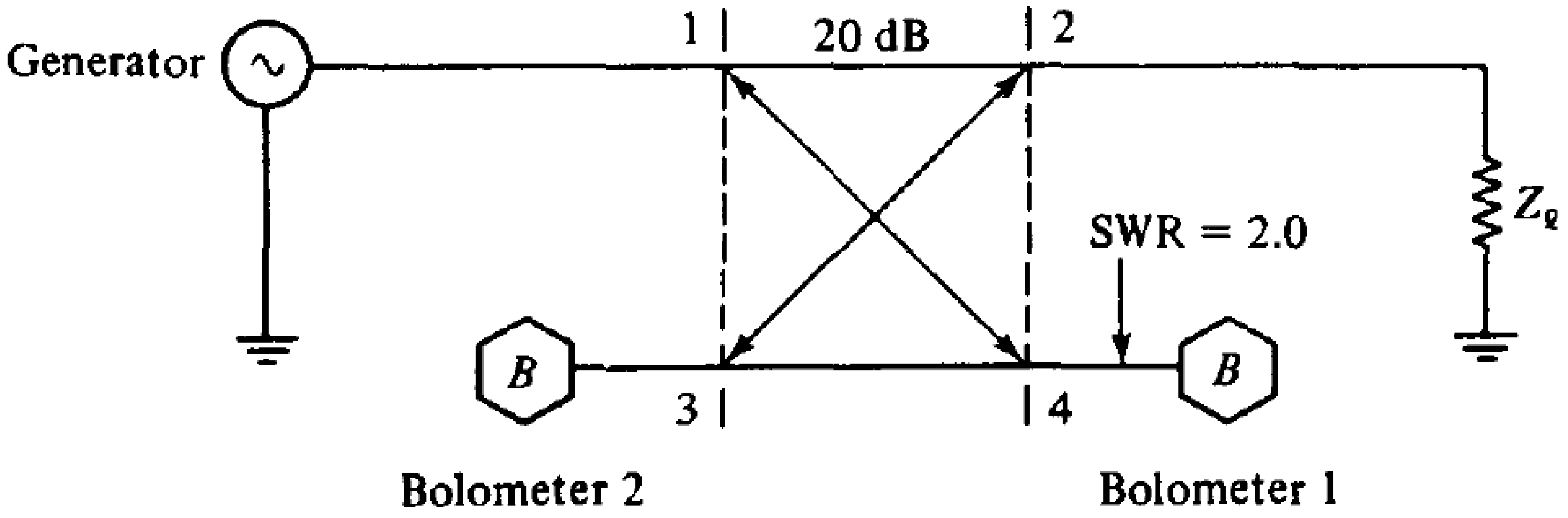


Figure 4-5-4 Power measurements by directional coupler.

- bolometer 1 reads 8 mW and bolometer 2 reads 2 mW, find: (a) the amount of power dissipated in the load *Zl;* **(b)** the VSWR on arm 2.

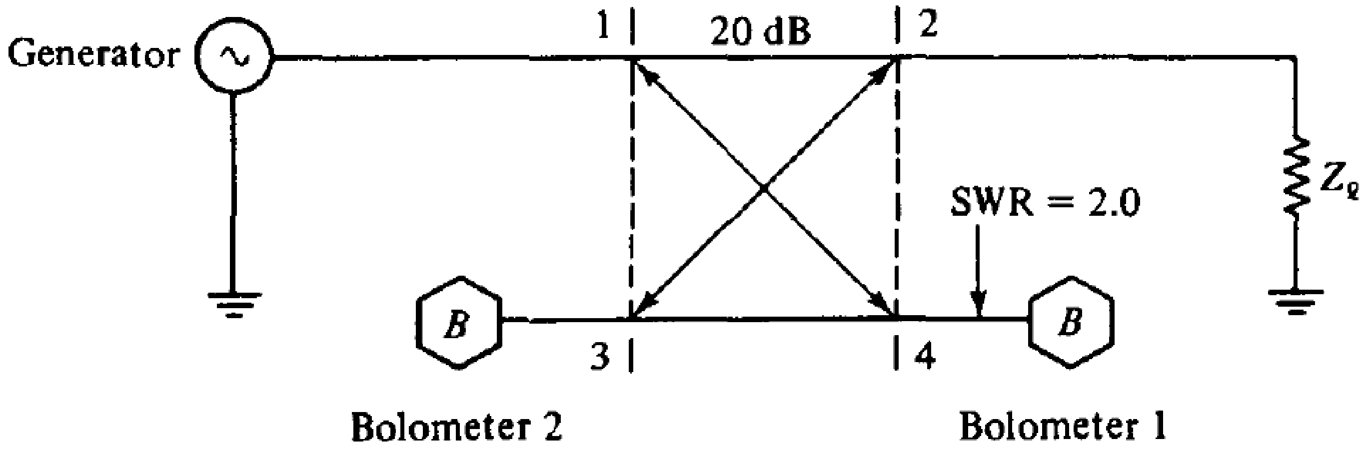
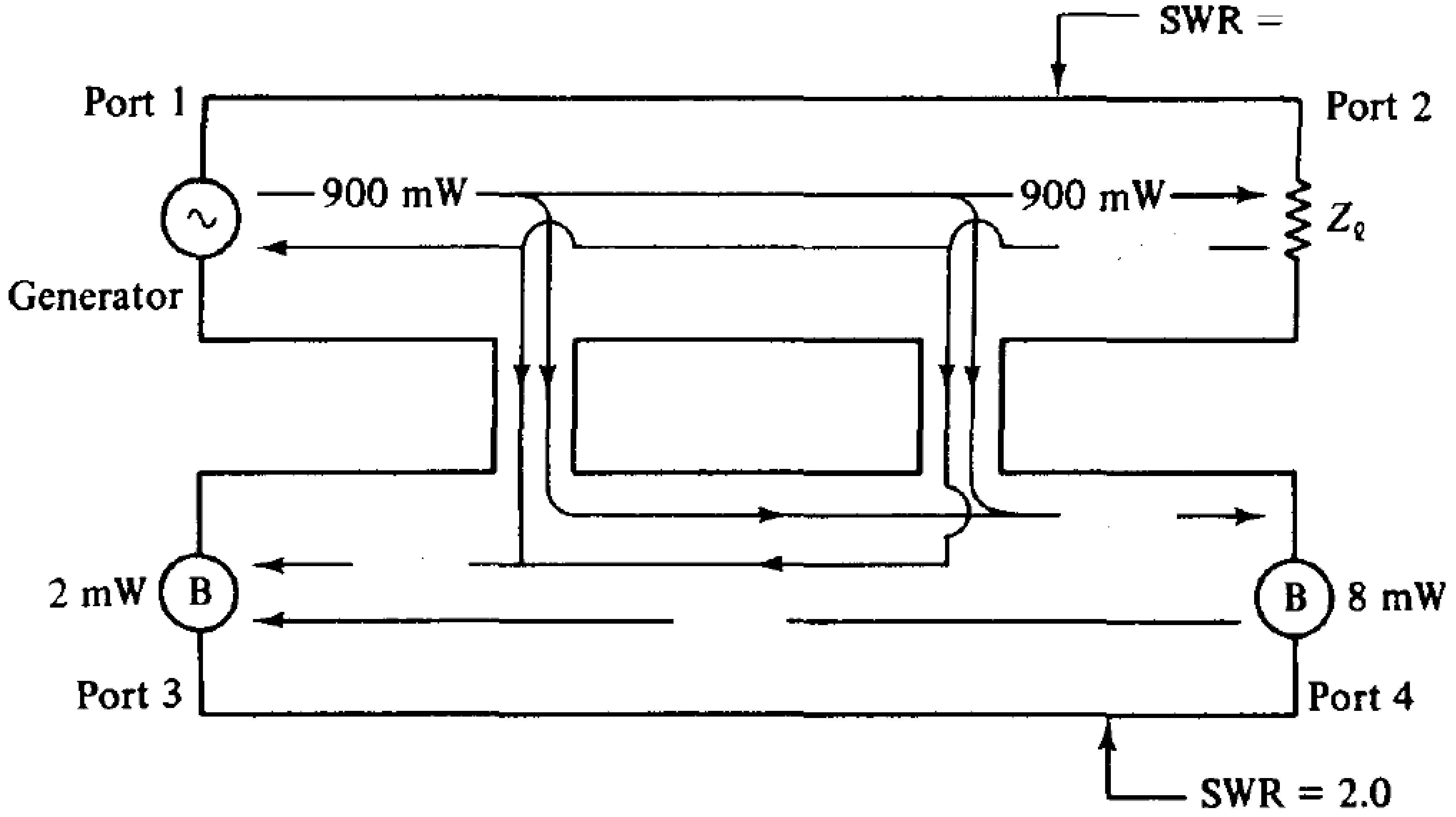


Figure 4-5-4 Power measurements by directional coupler.


SWR =
Port 1
Port 2
900 mW
900 mW
Z_ℓ
Generator
2 mW
B
B
8 mW
Port 3
Port 4
SWR = 2.0

- t

The reflection coefficient at port 4 is

$$|\Gamma| = \frac{\rho - 1}{\rho + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

Since the incident power and reflected power are related by

$$P^- = P^+ |\Gamma|^2$$

where P^+ = incident power and P^- = reflected power, then

$$|\Gamma| = \frac{1}{3} = \sqrt{\frac{P^-}{P^+}} = \sqrt{\frac{P^-}{8 + P^-}}$$

The incident power to port 4 is $P_4^+ = 9$ mW, and the reflected power from port 4 is $P_4^- = 1$ mW.

Since port 3 is matched and the bolometer at port 3 reads 2 mW, then 1 mW must be radiated through the holes.

Since 20 dB is equivalent to a power ratio of 100:1, the power intput at port 1 is given by

$$P_1 = 100P_4^+ = 900 \text{ mW}$$

and the power reflected from the load is

$$P_2^- = 100 \times (1 \text{ mW}) = 100 \text{ mW}$$

The power dissipated in the load is

$$P_\ell = P_2^+ - P_2^- = 900 - 100 = 800 \text{ mW}$$

The reflection coefficient is calculated as

$$|\Gamma| = \sqrt{\frac{P^-}{P^+}} = \sqrt{\frac{100}{900}} = \frac{1}{3}$$

Then the VSWR on arm 2 is

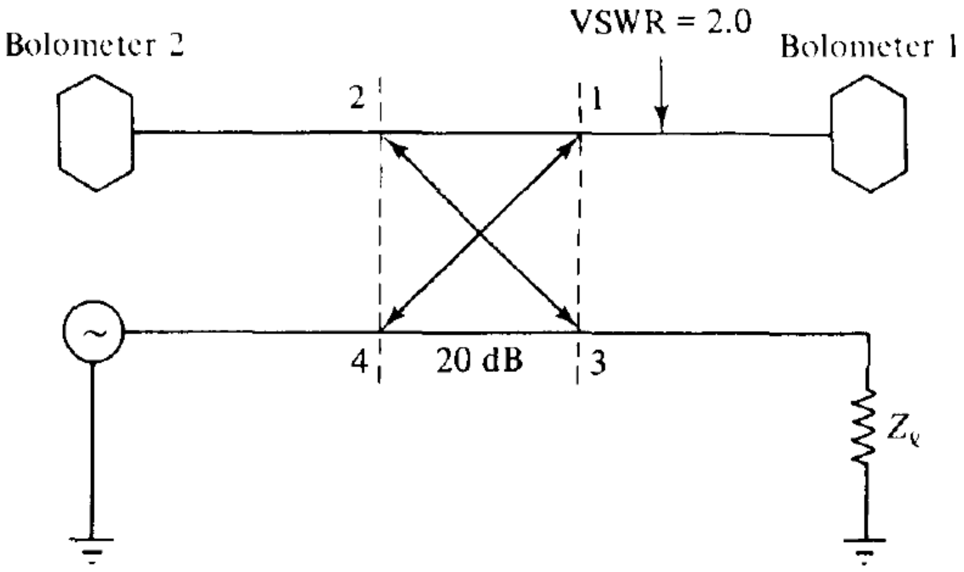
$$\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = 2.0$$

Ex2:

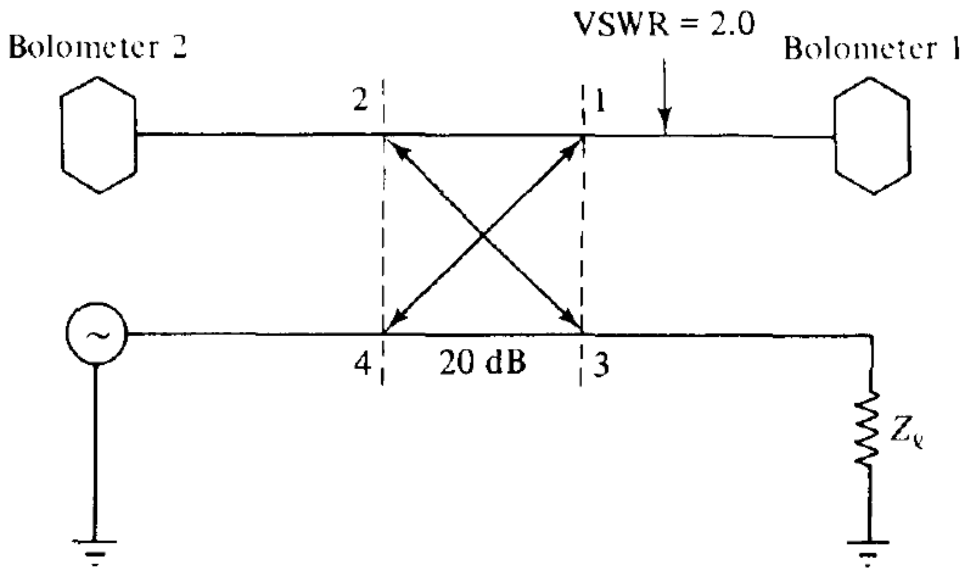
- A symmetric directional coupler has an infinite directivity and a forward attenuation of 20 dB. The coupler is used to monitor the power delivered to a load *Zl* as shown in Fig. P4-26. Bolometer 1 introduces a VSWR of 2.0 on arm 1; bolometer 2 is matched to arm 2. If bolometer 1 reads 9 mW and bolometer 2 reads 3 mW:
- a. Find the amount of power dissipated in the load *Zl* .
- **b.** Determine the VSWR on arm 3.

- The reflection coefficient at port 1 equal

$$|\Gamma| = \frac{\rho - 1}{\rho + 1} = \frac{2 - 1}{2 + 1} = \frac{1}{3}$$



P4-26



P4-26

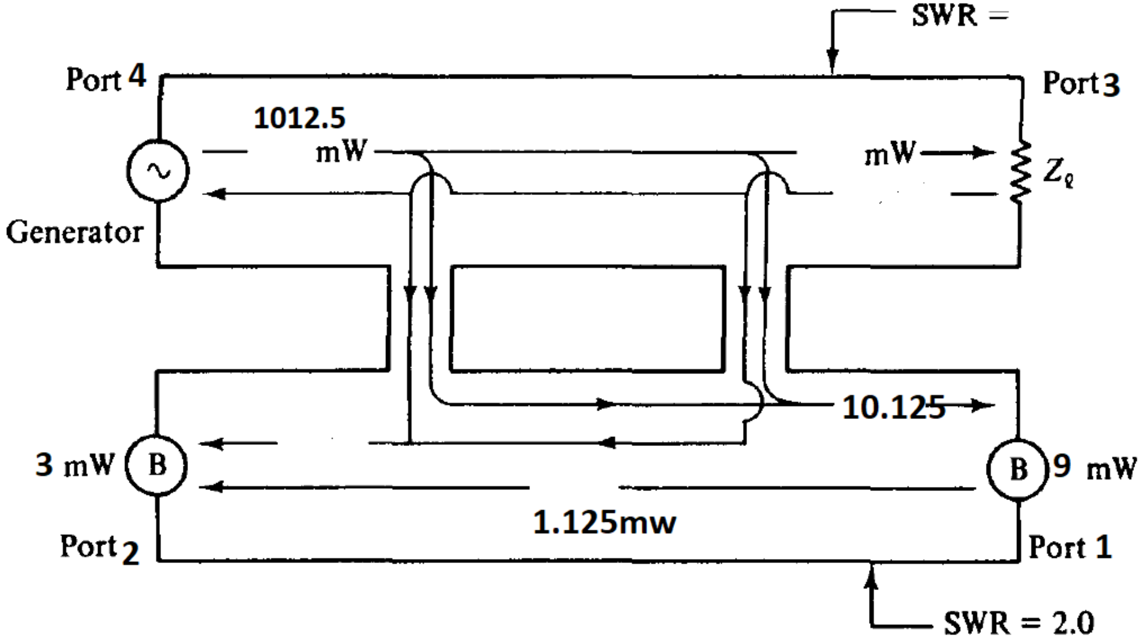
Since the incident power and reflected power are related by

$$P^- = P^+ |\Gamma|^2$$

where P^+ = incident power and P^- = reflected power, then

$$|\Gamma| = \frac{1}{3} = \sqrt{\frac{P^-}{P^+}} = \sqrt{\frac{P^-}{9 + P^-}}$$

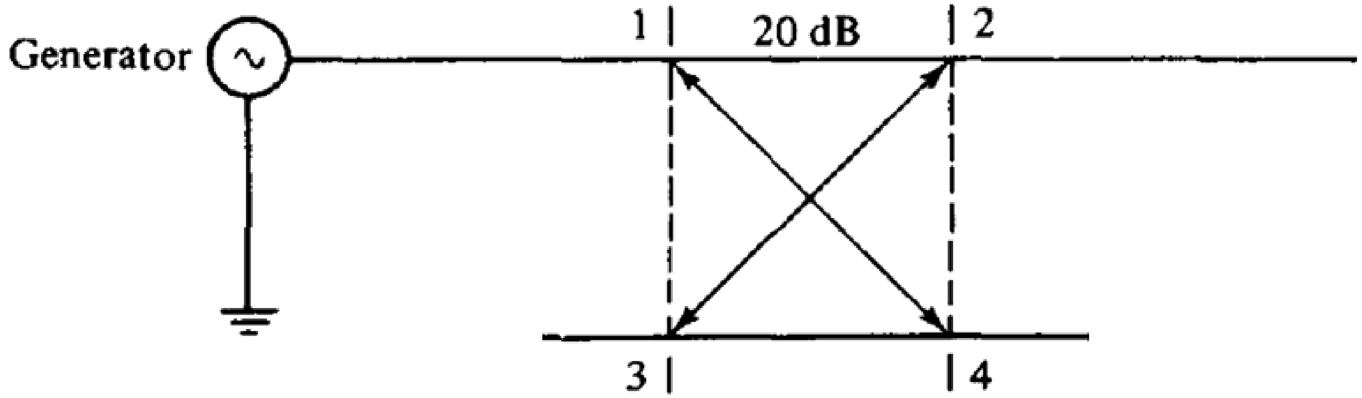
- The incident power at port 1 equal$P_i^+ = 10.125$ and the reflected power from port 1 equal $P_i^- = 1.125$
- Since port two is matched and the bolemeter read 3 mw then 1.875mw will be radiated throw hole.



- Since 20 dB is equivalent to a power ratio of 100: 1, the power intput at port 4 is given by
- $P_4 = P_1 \times 100$
- $P_4 = 10.125 \times 100$
- The power dissipated in the load is
- $P_3 = 1012.5 - 187.5mw = 825mw$

Q)

- A 20 dBm power source is connected to the input of a directional coupler having a coupling factor of 20 dB, a directivity of 35 dB, and an insertion loss of 0.5 dB. If all ports are matched, find the output powers (in dBm) at the p2, p3, and isolated ports.



- $C = 10\log {}^{P_1}/_{P_3}$ $P_2 = P_1 - I_L = 20 - 0.5 = 19.5dB$
- $D = 10\log {}^{P_4}/_{P_3}$ $P_4 = P_1 - C = 20 - 20 = 0dB$
- $I_L = 10\log {}^{P_1}/_{P_2}$ $P_3 = P_4 - D = 0 - 35 = -35dB$
- $I = 10\log {}^{P_1}/_{P_4}$

Circulator and Isolators

Introduction

- Both microwave circulators and microwave isolators are nonreciprocal transmission devices that use the property of Faraday rotation in the ferrite material.

- In order to understand the operating principles of circulators and isolators, let us describe the behavior of ferrites in the nonreciprocal phase shifter.

- A *nonreciprocal phase shifter* consists of a thin slab of ferrite placed in a rectangular waveguide at a point where the dc magnetic field of the incident wave mode is circularly polarized.

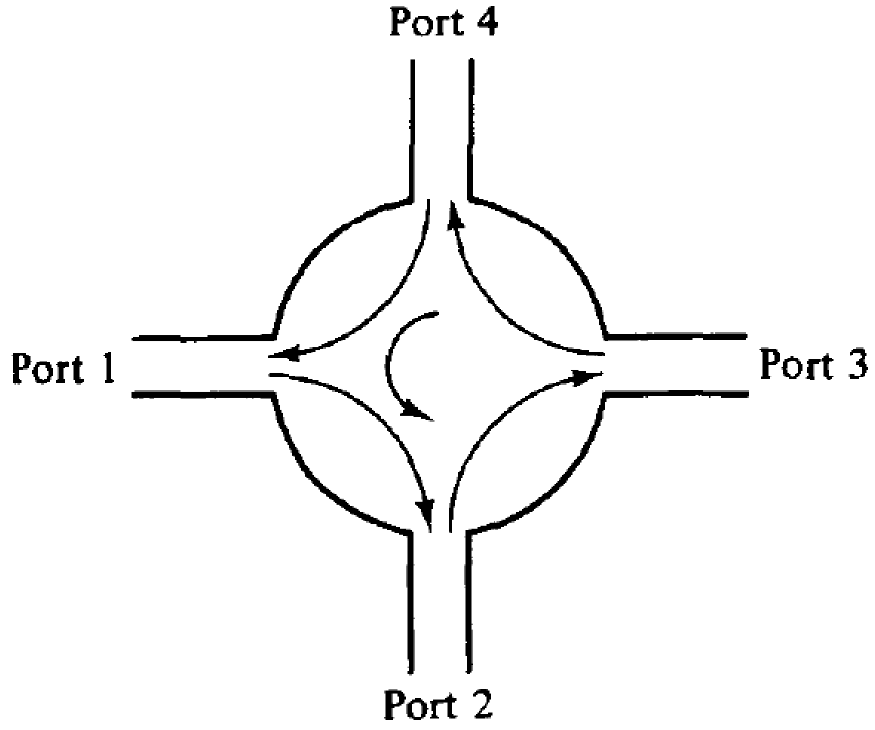
- Ferrite is a family of MeO · Fe2 03, where Me is a divalent iron metal. When a piece of ferrite is affected by a dc magnetic field, the ferrite exhibits Faraday rotation.

- It does so because the ferrite is nonlinear material and its permeability is an asymmetric tensor, as expressed by
- $B = [\mu][H]$
- Resistivity 10^{14} greater than metals it’s a dielectric materials
- $\epsilon_r \; from \; 10 - 15$
- μ_r high depends on the applied magnetic field.

- Since the ferrite materials are nonconductive then e.m. waves can easily propagate inside the ferrite
- The ferrite is usually subjected to d.c. magnetizing field this field produced by either permanent magnet or an electric magnet this ferrite characteristic can be controlled by this magnetization.

Circulator

- A *microwave circulator* is a multiport waveguide junction in which the wave can flow only from the nth port to the *(n* +1)th port in one direction



- Although there is no restriction on the number of ports, the four-port microwave circulator is the most common.

- One type of four-port microwave circulator is a combination of two 3-dB side-hole directional couplers and a rectangular waveguide with two nonreciprocal phase shifters as shown in Fig. 4-6-3.

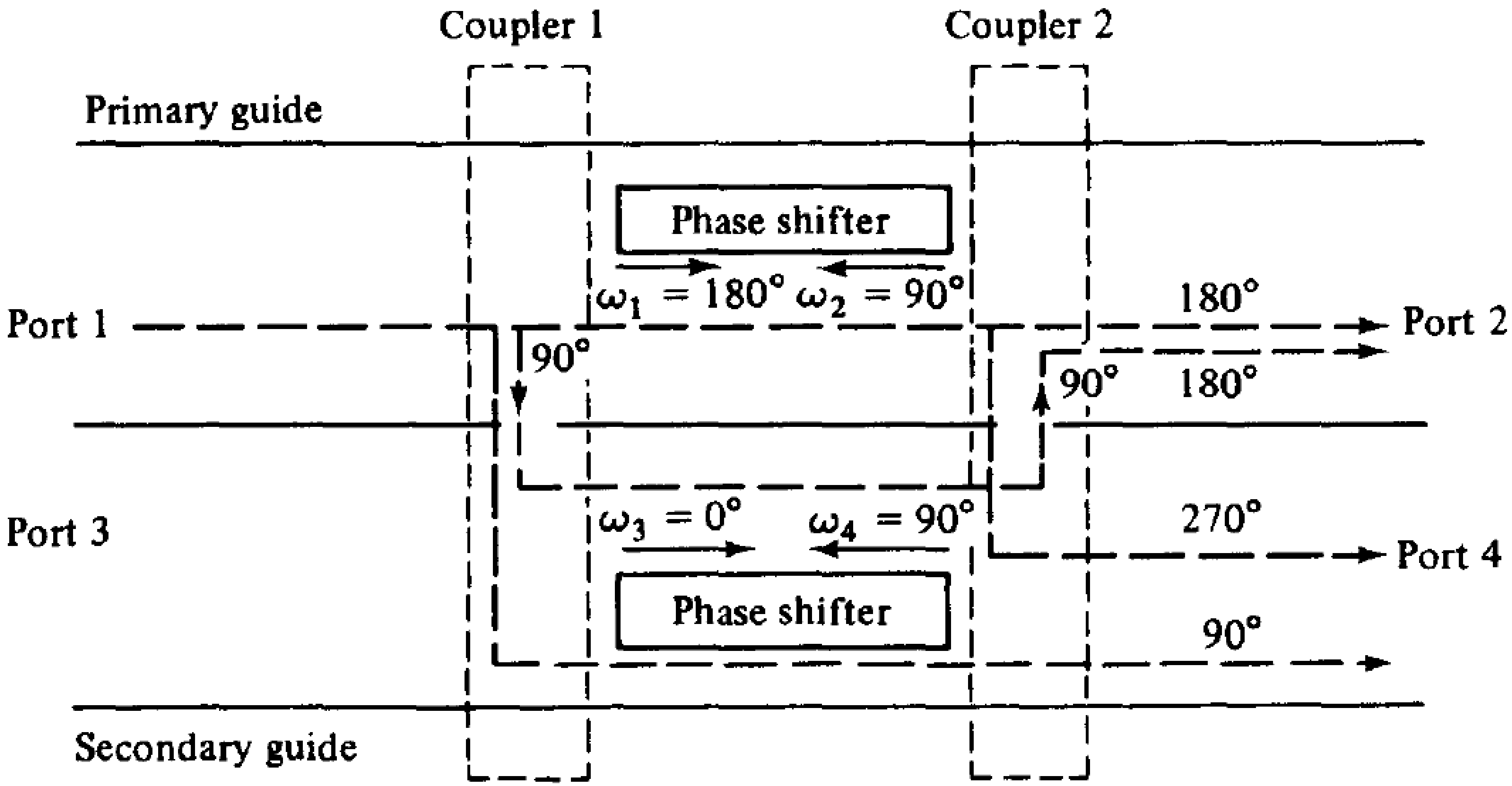


Figure 4-6-3 Schematic diagram of four-port circulator.

- The operating principle of a typical microwave circulator can be analyzed with the aid of Fig. 4-6-3. Each of the two 3-dB couplers in the circulator introduces a phase shift of 90°, and each of the two phase shifters produces a certain amount of phase change in a certain direction as indicated. When a wave is incident to port 1, the wave is split into two components by coupler **1.** The wave in the primary guide arrives at port 2 with a relative phase change of 180°. The second wave propagates through the two couplers and the secondary guide and arrives at port 2 with a relative phase shift of 180°. Since the two waves reaching port 2 are in phase, the power transmission is obtained from port 1 to port 2.

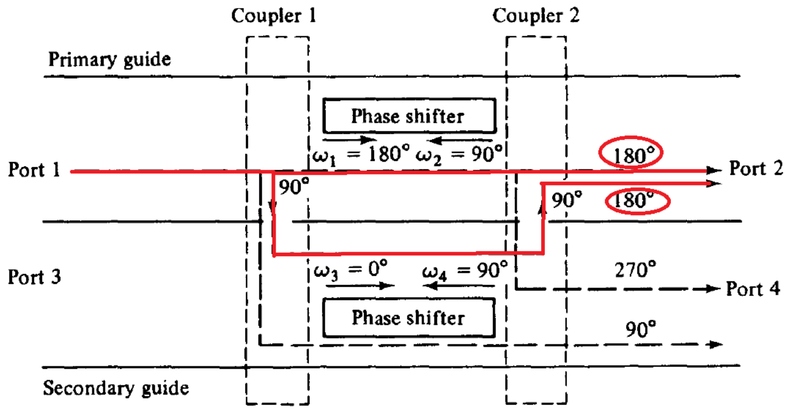


Figure 4-6-3 Schematic diagram of four-port circulator.

- However, the wave propagates through the primary guide, phase shifter, and coupler 2 and arrives at port 4 with a phase change of 270°. The wave travels through coupler 1 and the secondary guide, and it arrives at port 4 with a phase shift of 90°. Since the two waves reaching port 4 are out of phase by 180°, the power transmission from port 1 to port 4 is zero.

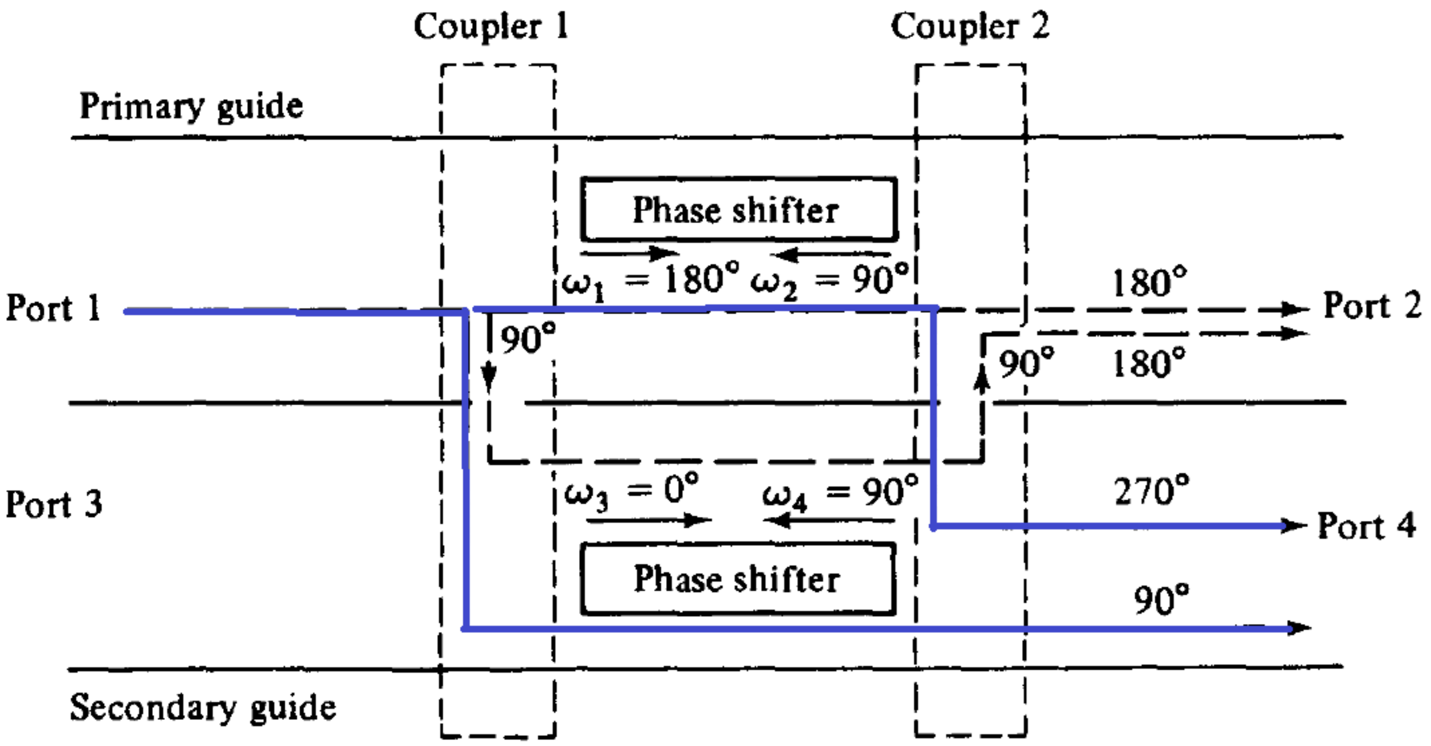


Figure 4-6-3 Schematic diagram of four-port circulator.

- In general, the differential propagation constants in the two directions of propagation in a waveguide containing ferrite phase shifters should be

$$\omega_1 - \omega_3 = (2m + 1)\pi \qquad \text{rad/s}$$

$$\omega_2 - \omega_4 = 2n\pi \qquad \text{rad/s}$$

- where *m* and *n* are any integers, including zeros. A similar analysis shows that a wave incident to port 2 emerges at port 3 and so on.

- As a result, the sequence of power flow is designated as 1 ~ 2 ~ 3 ~ 4 ~ 1.

- Many types of microwave circulators are in use today. However, their principles of operation remain the same. Figure 4-6-4 shows a four-port circulator constructed of two magic tees and a phase shifter. The phase shifter produces a phase shift of 180°.

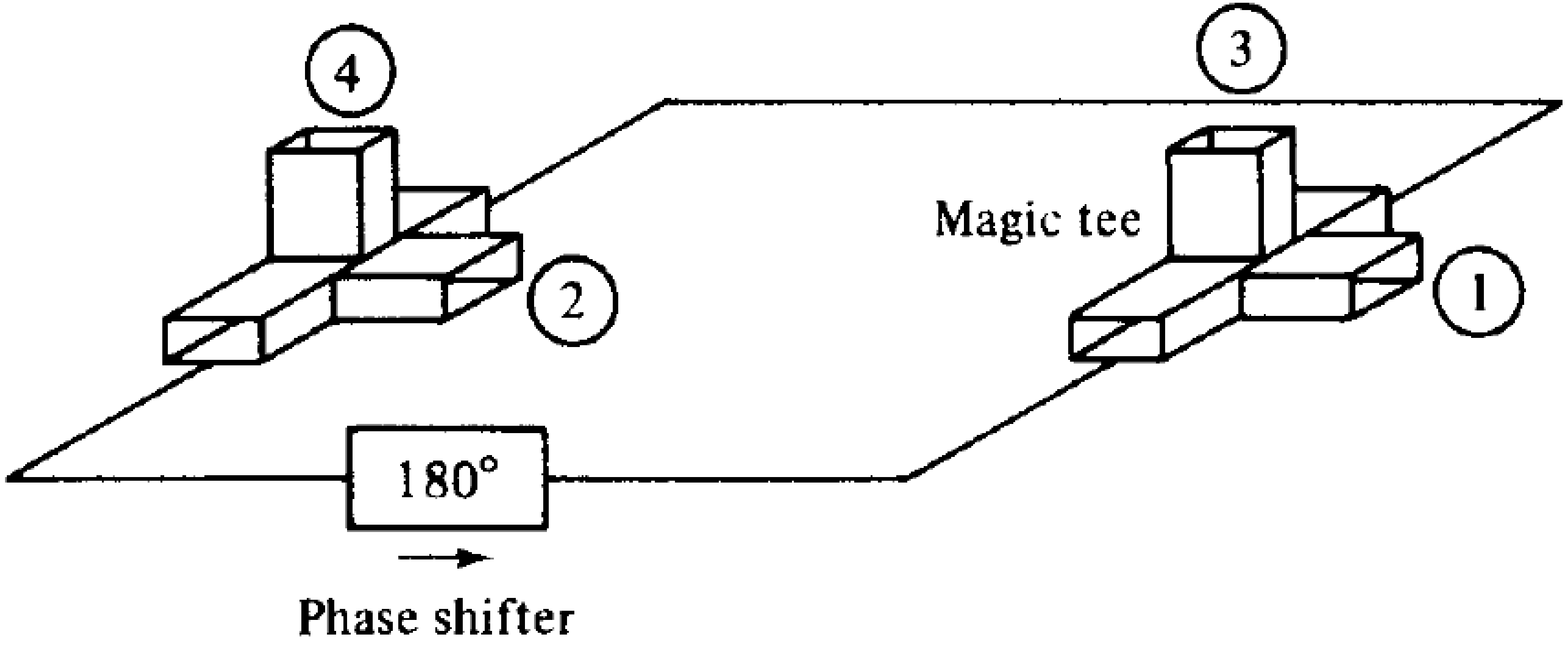


Figure 4-6-4 A four-port circulator.

Microwave Isolators

- An *isolator* is a nonreciprocal transmission device that is used to isolate one component from reflections of other components in the transmission line.
- An ideal isolator completely absorbs the power for propagation in one direction and provides lossless transmission in the opposite direction.
- Thus the isolator is usually called *uniline.* Isolators are generally used to improve the frequency stability of microwave generators such as klystrons and magnetrons, in which the reflection from the load affects the generating frequency.

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},$$

- common application uses an isolator between a high-power source and a load to prevent possible reflections from damaging the source. An isolator can be used in place of a matching or tuning network, but it should be realized that any power reflected from the load will be absorbed by the isolator, as opposed to being reflected back to the load, which is the case when a matching network is used.

- The isolator placed between the generator and load prevents the reflected power from the unmatched load from returning to the generator. As a result, the isolator maintains the frequency stability of the generator.

- Isolators can be constructed in many ways.

1. They can be made by terminating ports 3 and 4 of a four-port circulator with matched loads.
2. Isolators can be made by inserting a ferrite rod along the axis of a rectangular waveguideas shown in Fig. 4-6-5.

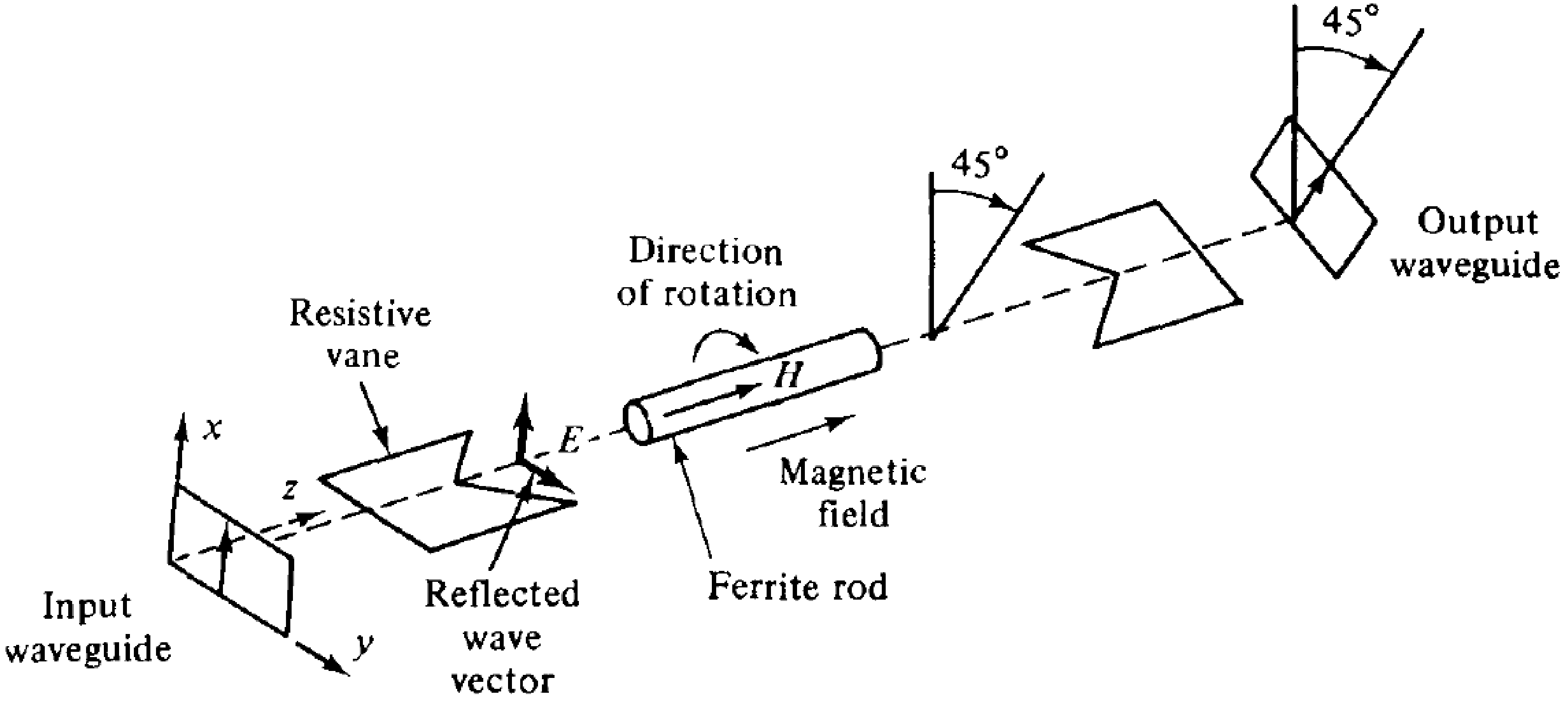


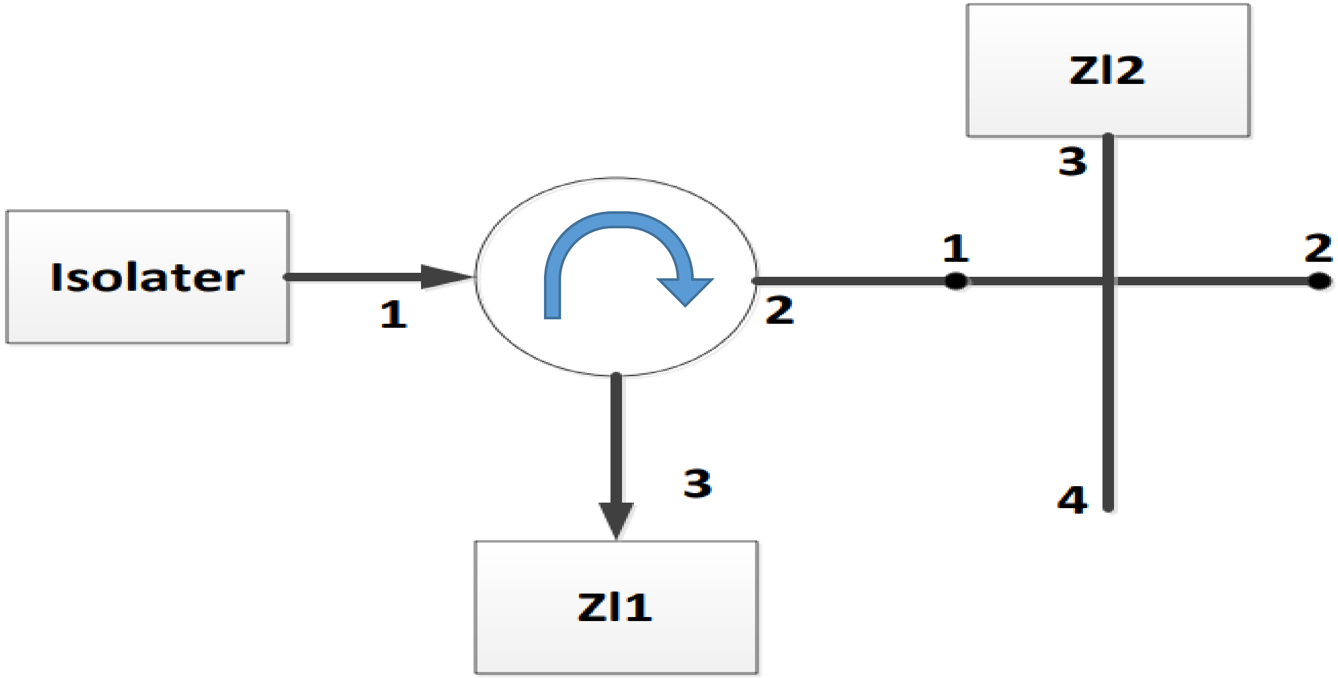
Figure 4-6-5 Faraday-rotation isolator.

- The input resistive card is in the *y-z* plane, and the output resistive card is displaced 45° with respect to the input card. The dc magnetic field, which is applied longitudinally to the ferrite rod, rotates the wave plane of polarization by 45°.
- The degrees of rotation depend on the length and diameter of the rod and on the applied dc magnetic field. An input TE10 dominant mode is incident to the left end of the isolator.
- Since the TE_{10} mode wave is perpendicular to the input resistive card, the wave passes through the ferrite rod without attenuation.
- The wave in the ferrite rod section is rotated clockwise by 45° and is normal to the output resistive card.

- As a result of rotation, the wave arrives at the output end without attenuation at all.
- On the contrary, a reflected wave from the output end is similarly rotated clockwise 45° by the ferrite rod.
- However, since the reflected wave is parallel to the input resistive card, the wave is thereby absorbed by the input card.
- The typical performance of these isolators is about 1-dB insertion loss in forward transmission and about 20 to 30 dB isolation in reverse attenuation.

Find power dissipated in ZL_1 & ZL_2 ,all ports are matched .

Insertion loss =1 dB for isolator and circulator, isolation =30 dB ,input power to the isolator is 3mw.



Sol:

$1\ dB$ ⟶ $10^{(1/10)} = 1.25$

$30\ dB$ ⟶ $10^{\left(\frac{30}{10}\right)} = 1000$

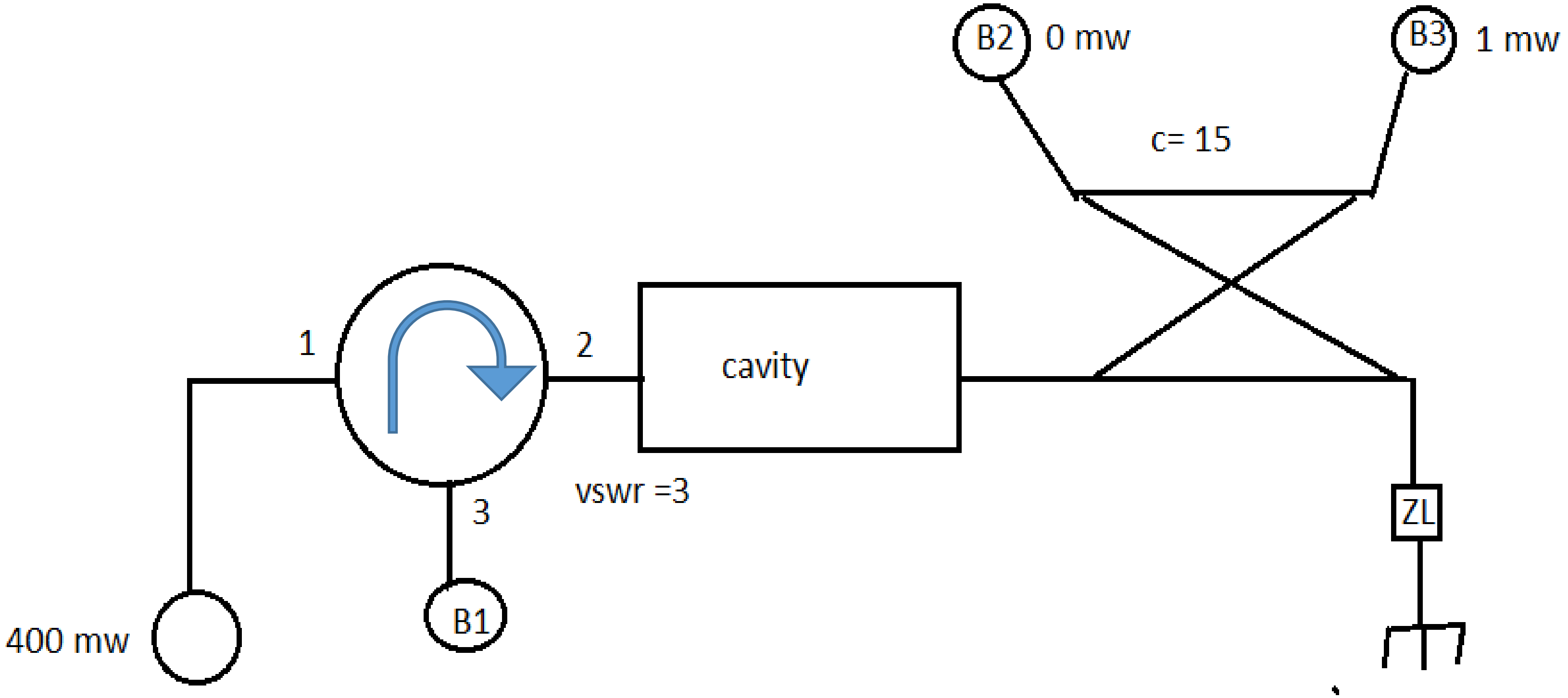
At Port 1 of circulator $\frac{3mw}{1.25} = 2.4\ mw$

At port 2 of circulator $\frac{2.4\ mw}{1.25} = 1.92\ mw$

At port 3 of circulator = $power\ \ dissipated\ at\ ZL_1$

Power dissipated at $ZL_2 = \frac{1.92mw}{2} = 0.96\ mw$

H.W for the network shown ,

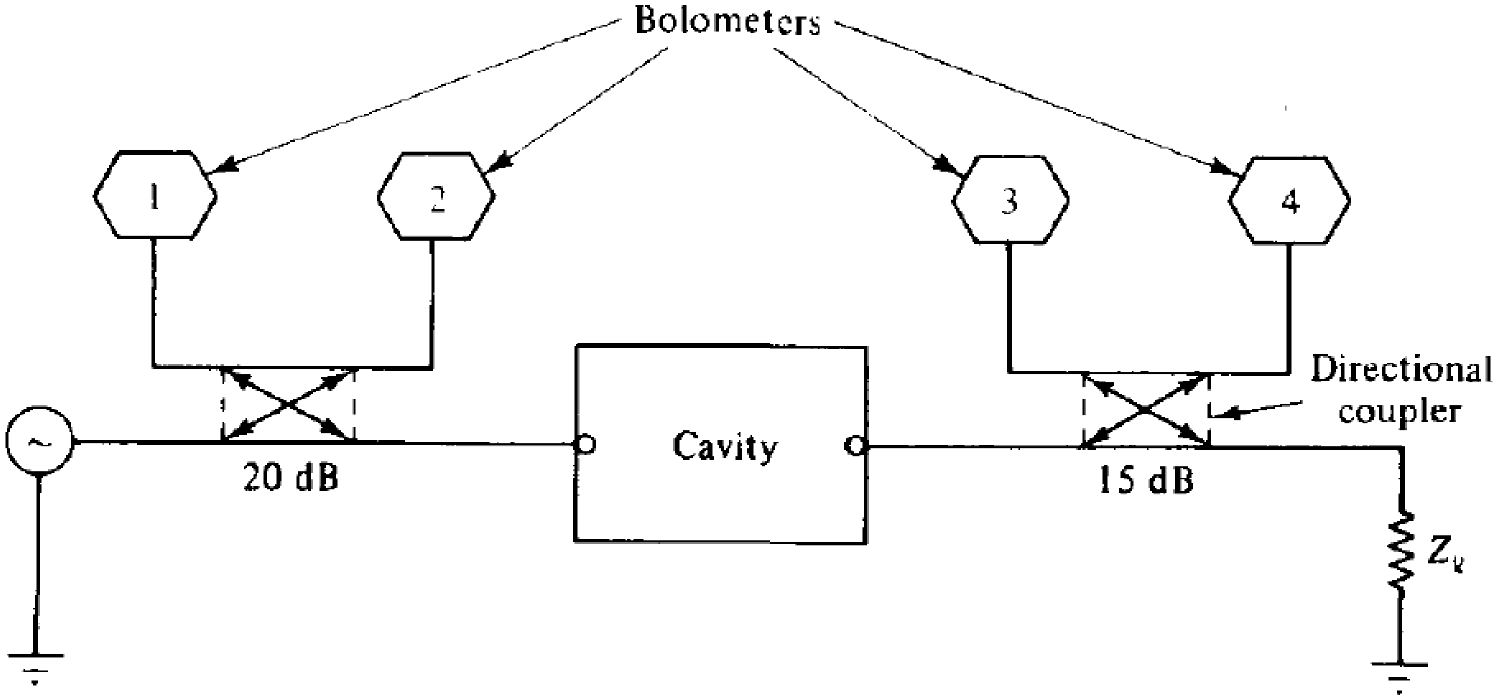


Take the isolation=$infinite$ & insertion = 1.5dB, c=15 dB .

a) **Find ZL in terms of Zo**
b) **Calculate power dissipated by ZL**
c) **What is the reading of bolometer 1**
d) **Compute the power dissipated in the cavity**

A microwave transmission system consists of a generator, an overcoupled cavity, two ideal but not identical dual directional couplers with matched bolometers, and a load Z_ℓ. The lossless transmission line has a characteristic impedance Z_0. The readings of the four bolometers (1, 2, 3, and 4) are 2 mW, 4 mW, 0 and 1 mW, respectively. The system is shown in Fig. P4-25.

a. Find the load impedance Z_ℓ in terms of Z_0.
b. Calculate the power dissipated by Z_ℓ.
c. Compute the power dissipated in the cavity.
d. Determine the VSWR on the input transmission line.
e. Find the ratio of Q_ℓ/Q_0 for the cavity.



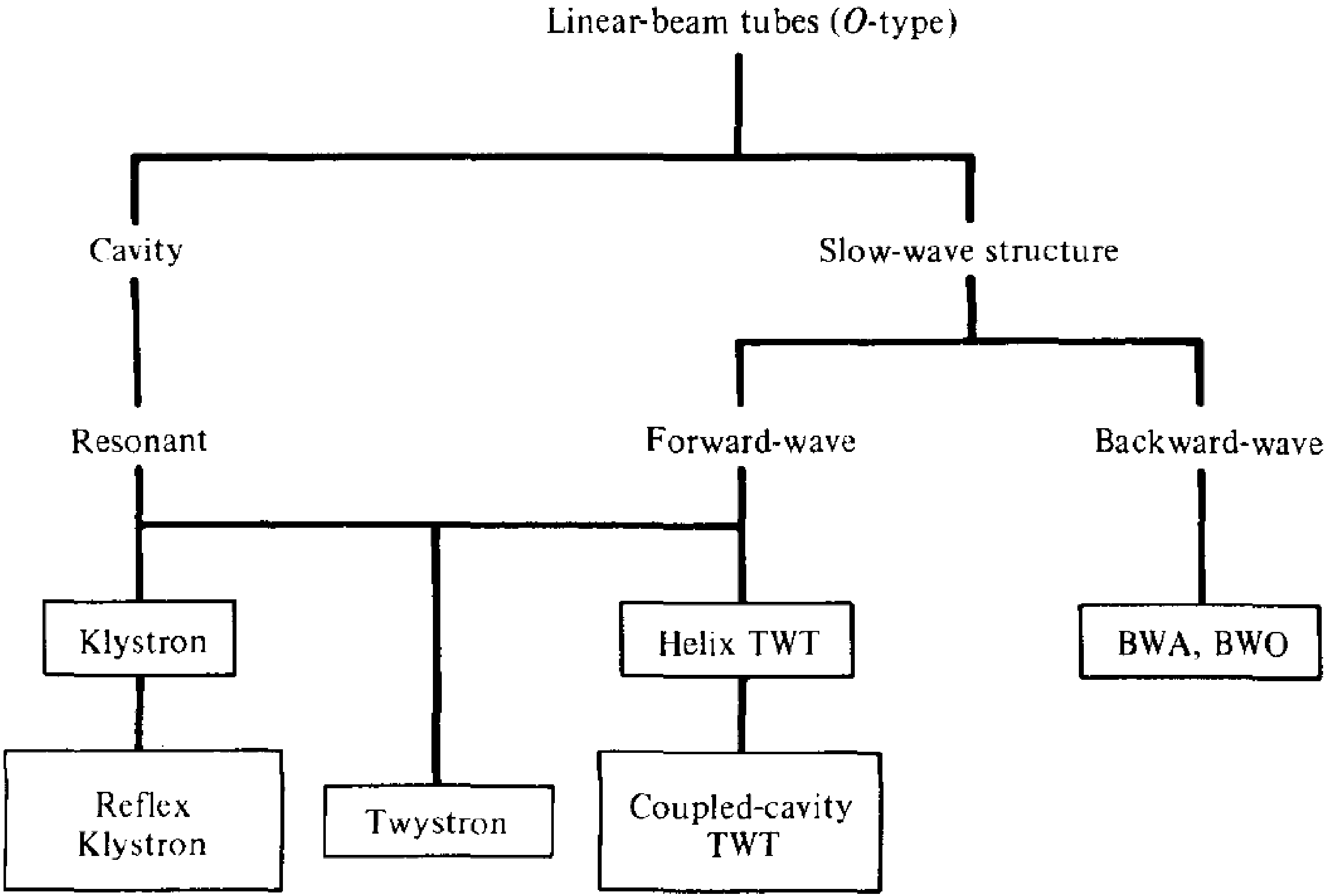
Microwave Linear-Beam Tubes (O-Type)

Lecture13 and 14

INTRODUCTION

- The most important microwave tubes at present are the linear-beam tubes (O type)
- The dominant O-type tube is the two-cavity klystron, and it is followed by the reflex klystron. The helix traveling-wave tube (TWT), the coupled-cavity TWT, the forward-wave amplifier (FWA), and the backward-wave amplifier and oscillator (BWA and BWO) are also O-type tubes,
- The advent of linear-beam tubes began with the Heil oscillators in 1935 and the Varian brothers' klystron amplifier in 1939.

- The work was advanced by the space-charge-wave propagation theory of Hahn and Ramo in 1939 and continued with the invention of the helix-type traveling-wave tube (TWT) by R. Kompfner in 1944.



Linear-beam tubes (O-type)
Cavity
Slow-wave structure
Resonant
Forward-wave
Backward-wave
Klystron
Helix TWT
BWA, BWO
Reflex Klystron
Twystron
Coupled-cavity TWT

- In a linear-beam tube a magnetic field whose axis coincides with that of the electron beam is used to hold the beam together as it travels the length of the tube.
- *O*-type tubes derive their name from the word *original* (meaning the original type of tube).
- In these tubes electrons receive potential energy from the dc beam voltage before they arrive in the microwave interaction region, and this energy is converted into their kinetic energy.

- In the microwave interaction region the electrons are either accelerated or decelerated by the microwave field and then bunched as they drift down the tube.
- The bunched electrons, in turn, induce current in the output structure. The electrons then give up their kinetic energy to the microwave fields and are collected by the collector.
- O-type traveling-wave tubes are suitable for amplification.
- At present, klystron and TWT amplifiers can deliver (Pulse) a peak power output up to 30 MW (megawatts) with a beam voltage on the order of 100 kV at the frequency of 10 GHz.

Specifications

- The average power outputs are up to 700 kW.
- The gain of these tubes is on the order of 30 to 70 dB.
- The efficiency is from 15 to 60%.
- The bandwidth is from 1 to 8% for klystrons and 10 to 15% for TWTs.

- Since the early 1960s, predictions have continued that microwave tubes will be displaced by microwave solid-state devices.
- This displacement has occurred only at the low-power and receiving circuit level of equipment.

KLYSTRONS

- The two-cavity klystron is a widely used microwave amplifier operated by the principles of velocity and current modulation. All electrons injected from the cathode arrive at the first cavity with uniform velocity.

- Those electrons passing the first cavity gap at zeros of the gap voltage (or signal voltage) pass through with unchanged velocity.
- Those passing through the positive half cycles of the gap voltage undergo an increase in velocity.
- those passing through the negative swings of the gap voltage undergo a decrease in velocity.

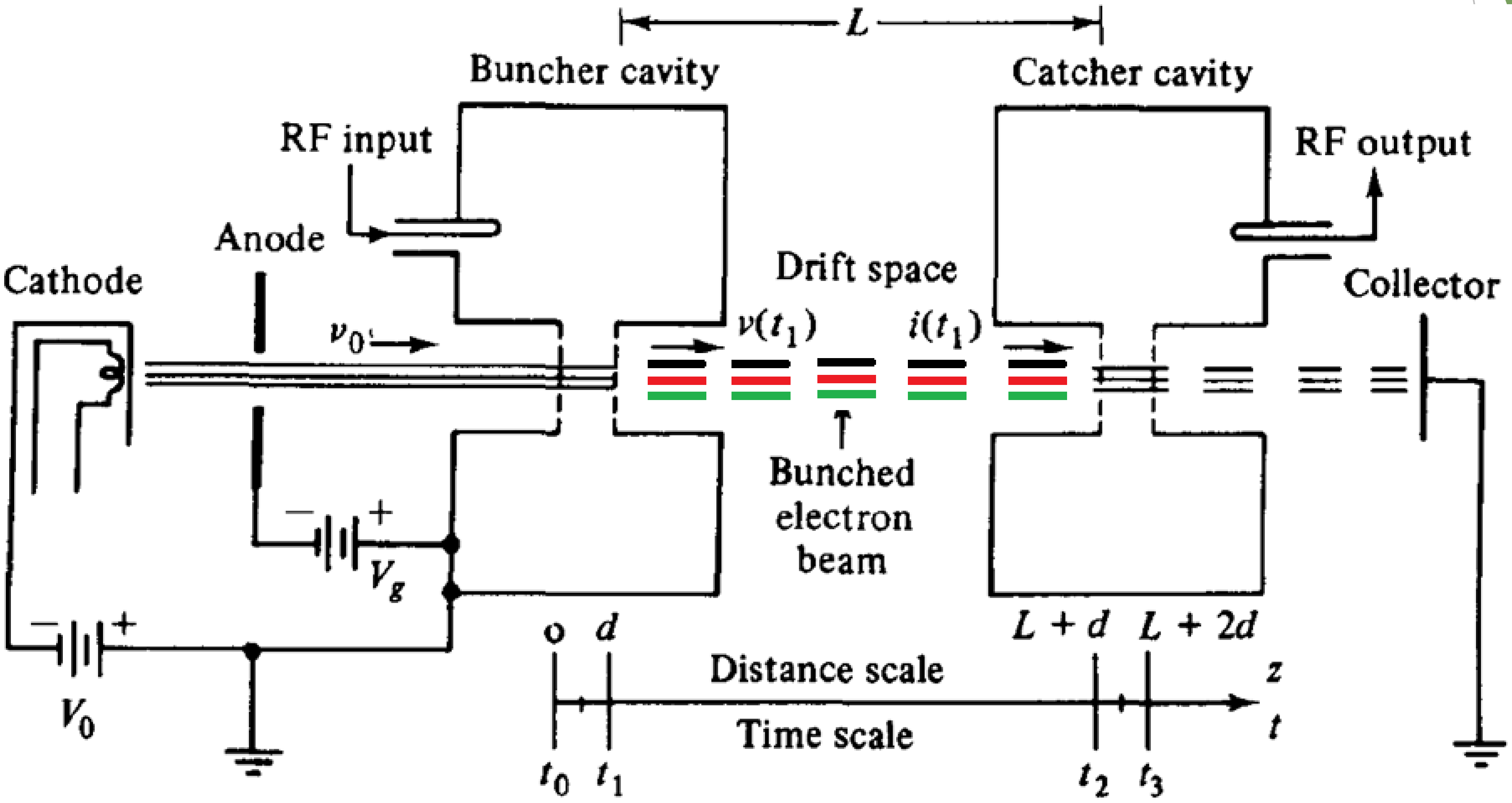


Figure 9-2-2 Two-cavity klystron amplifier.


MAGNET
CAUTION

- As a result of these actions, the electrons gradually bunch together as they travel down the drift space. *The variation in electron velocity in the drift space is known as velocity modulation.*
- The density of the electrons in the second cavity gap varies cyclically with time. The electron beam contains an ac component and is said to be current-modulated.
- The maximum bunching should occur approximately midway between the second cavity grids during its retarding phase; thus the kinetic energy is transferred from the electrons to the field of the second cavity.
- The electrons then emerge from the second cavity with reduced velocity and finally terminate at the collector.

Klystron characteristics

- The characteristics of a two-cavity klystron amplifier are as follows:
- Efficiency: about 40%.
- Power output: average power (CW power) is up to 500 kW and pulsed power is up to 30 MW at 10 GHz.
- Power gain: about 30 dB.

- The cavity close to the cathode is known as the *buncher cavity* or input cavity, which velocity- modulates the electron beam.

- The other cavity is called the *catcher cavity* or output cavity; it catches energy from the bunched electron beam. The beam then passes through the catcher cavity and is terminated at the collector.

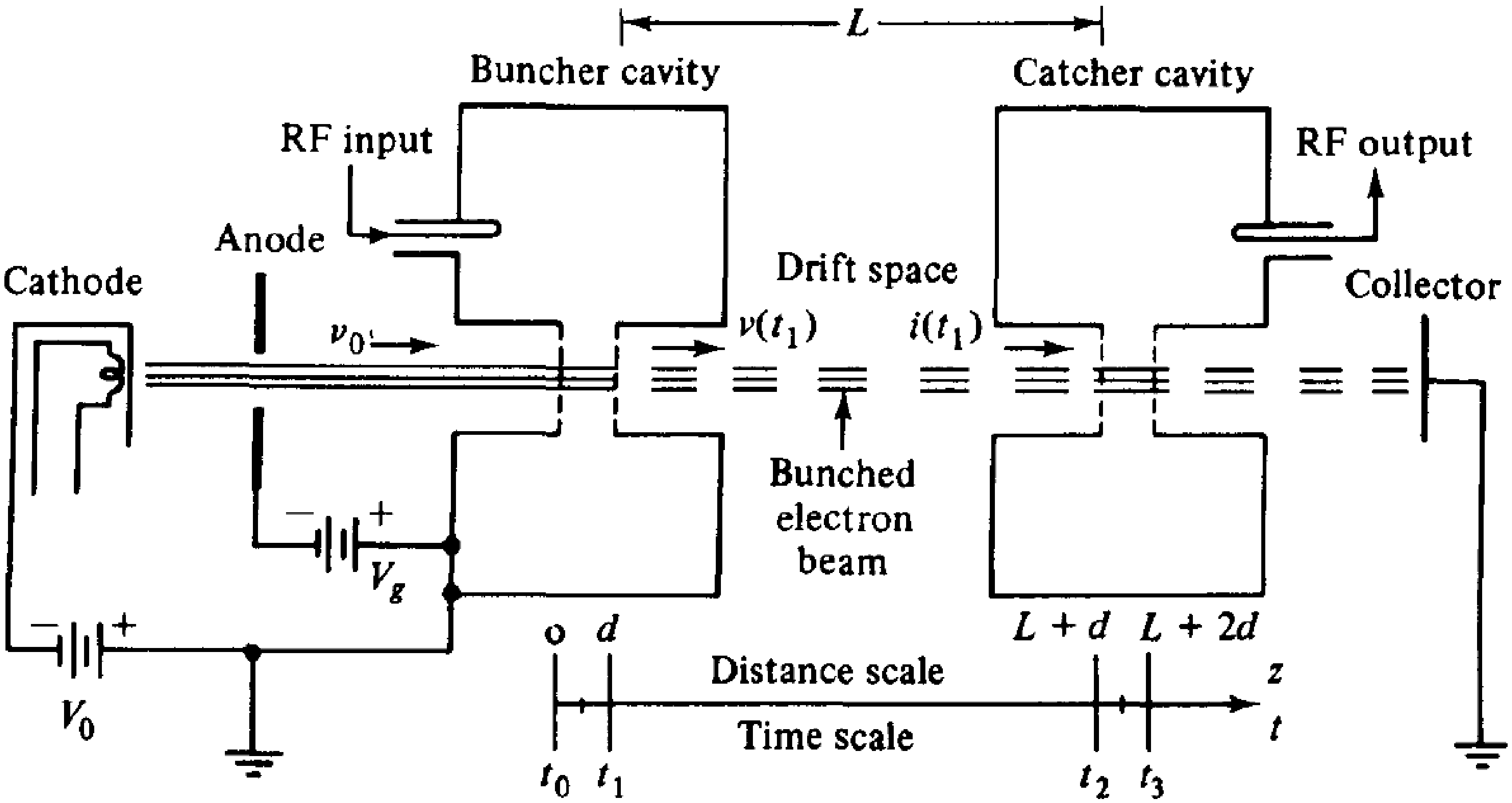


Figure 9-2-2 Two-cavity klystron amplifier.

Reentrant Cavities

- At a frequency well below the microwave range, the cavity resonator can be represented by a lumped-constant resonant circuit. When the operating frequency is increased to several tens of megahertz, both the inductance and the capacitance must be reduced to a minimum in order to maintain resonance at the operating frequency.

- Ultimately the inductance is reduced to a minimum by short wire. Therefore the reentrant cavities are designed for use in klystrons and microwave triodes. A reentrant cavity is one in which the metallic boundaries extend into the interior of the cavity. Several types of reentrant cavities are shown in Fig. 9-2-3.

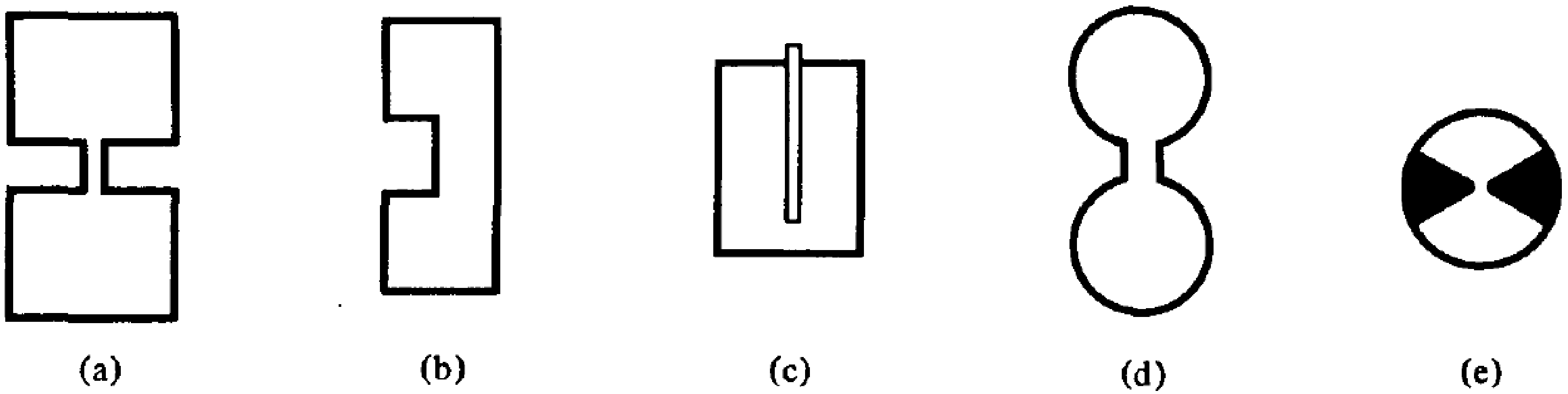


Figure 9-2-3 Reentrant cavities. (a) Coaxial cavity. (b) Radial cavity. (c) Tunable cavity. (d) Toroidal cavity. (c) Butterfly cavity.

Velocity-Modulation Process

- When electrons (emitted by cathode) are first accelerated by the high dc voltage *Vo* before entering the buncher grids, their velocity is uniform:

$$v_0 = \sqrt{\frac{2eV_0}{m}} = 0.593 \times 10^6 \sqrt{V_0} \quad \text{m/s} \qquad (9\text{-}2\text{-}10)$$

- In Eq. (9-2-10) it is assumed that electrons leave the cathode with zero velocity. When a microwave signal is applied to the input terminal, the gap voltage between the buncher grids appears as

$$V_s = V_1 \sin(\omega t)$$

where V1 is the amplitude of the signal and $V_1 \ll V_o$ is assumed.

- In order to find the **modulated velocity** in the buncher cavity in terms of either the entering time (*to*) or the exiting time ($t1$) and the **gap transit angle** θ_g as shown in Fig. 9-2-2

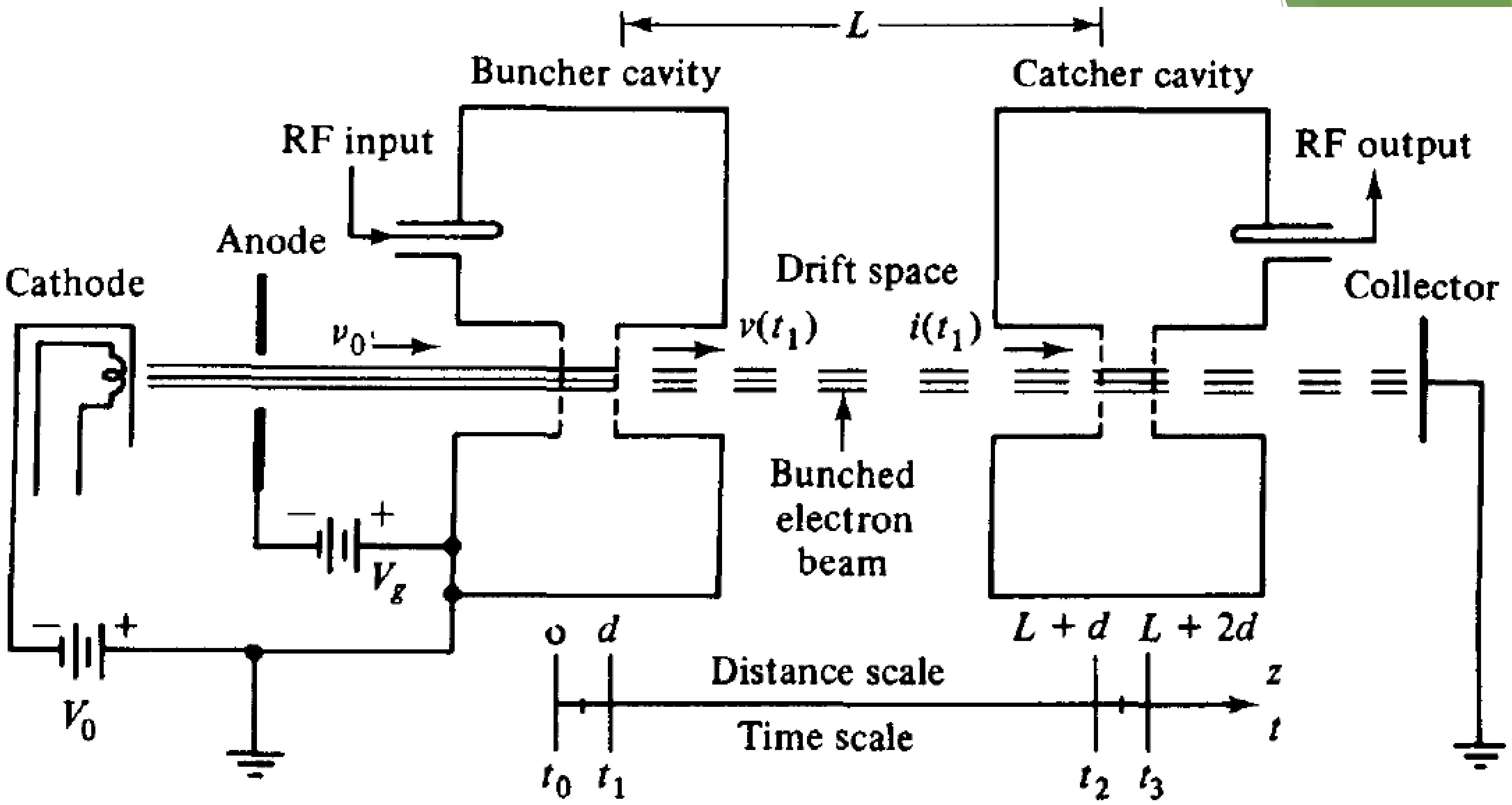


Figure 9-2-2 Two-cavity klystron amplifier.

- it is necessary to determine the average microwave voltage in the buncher gap as indicated in Fig. 9-2-6.

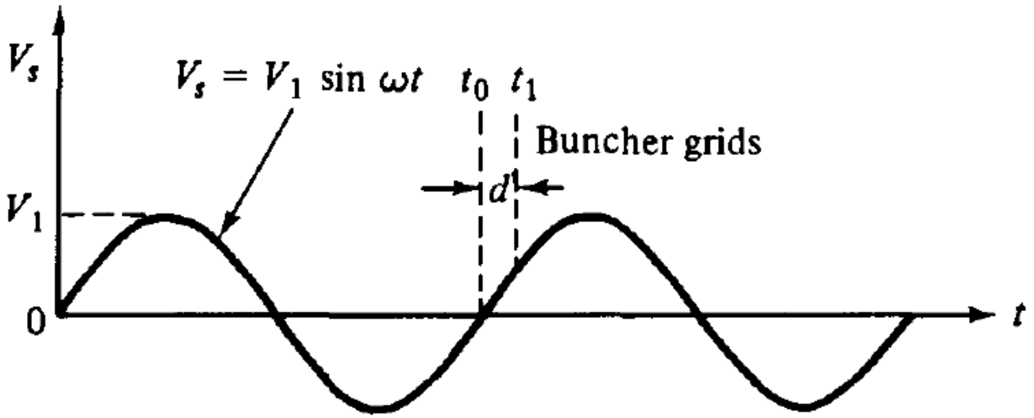


Figure 9-2-6 Signal voltage in the buncher gap.

- Since $V_1 \ll V_o$, the average transit time through the buncher gap distance *d* is

$$\tau \approx \frac{d}{v_0} = t_1 - t_0$$

- The average gap transit angle can be expressed as

$$\theta_g = \omega\tau = \omega(t_1 - t_0) = \frac{\omega d}{v_0}$$

- The average microwave voltage in the buncher gap can be found in the following way:

$$\langle V_s \rangle = \frac{1}{\tau} \int_{t_0}^{t_1} V_1 \sin(\omega t) dt = -\frac{V_1}{\omega \tau} [\cos(\omega t_1) - \cos(\omega t_0)]$$
$$= \frac{V_1}{\omega \tau} \left[\cos(\omega t_0) - \cos\left(\omega_0 + \frac{\omega d}{v_0} \right) \right] \qquad (9\text{-}2\text{-}14)$$

Let

$$\omega t_0 + \frac{\omega d}{2v_0} = \omega t_0 + \frac{\theta_g}{2} = A$$

and

$$\frac{\omega d}{2v_0} = \frac{\theta_g}{2} = B$$

Then using the trigonometric identity that cos $(A - B) - \cos(A + B) = 2 \sin A \sin B$, Eq. (9-2-14) becomes

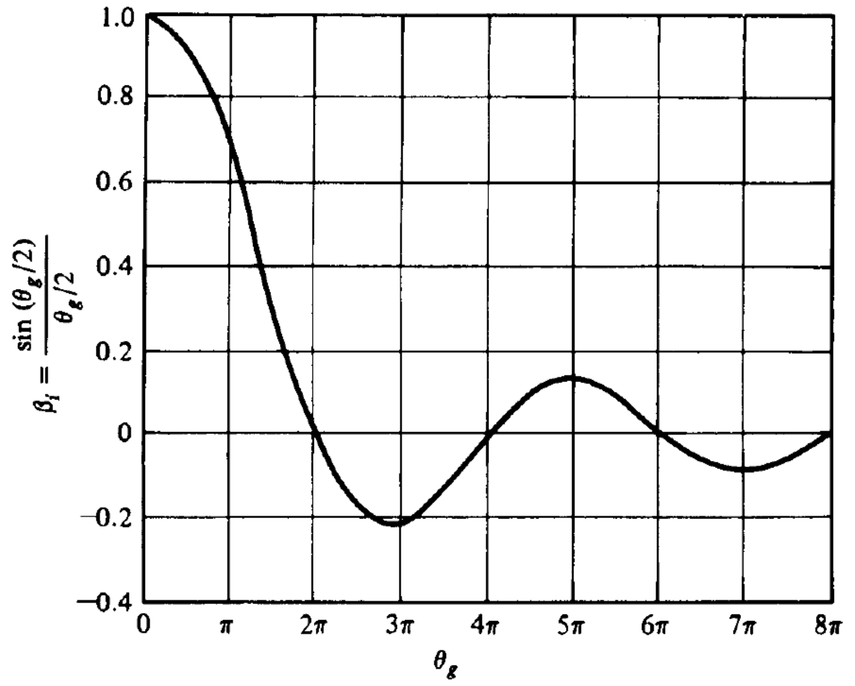
$$\langle V_s \rangle = V_1 \frac{\sin[\omega d/2v_0)]}{\omega d/(2v_0)} \sin\left(\omega t_0 + \frac{\omega d}{2v_0}\right) = V_1 \frac{\sin(\theta_g/2)}{\theta_g/2} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right) \qquad (9\text{-}2\text{-}15)$$

It is defined as

$$\beta_i \equiv \frac{\sin[\omega d/(2v_0)]}{\omega d/(2v_0)} = \frac{\sin(\theta_g/2)}{\theta_g/2} \qquad (9\text{-}2\text{-}16)$$

Note that β_i is known as the *beam-coupling coefficient* of the input cavity gap (see Fig. 9-2-7).

- It can be seen that increasing the gap transit angle *(Jg* decreases the coupling between the electron beam and the buncher cavity; that is, the velocity modulation of the beam for a given microwave signal is decreased.



- Immediately after velocity modulation, the exit velocity from the buncher gap is given by

$$v(t_1) = \sqrt{\frac{2e}{m}\left[V_0 + \beta_i V_1 \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]}$$

$$= \sqrt{\frac{2e}{m} V_0 \left[1 + \frac{\beta_i V_1}{V_0} \sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right]} \qquad (9\text{-}2\text{-}17)$$

- where the factor $\beta_i \, {}^{V_1}/_{V_o}$ is called the ***depth of velocity modulation***.

- Ff

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_0 + \frac{\theta_g}{2}\right)\right] \tag{9-2-19}$$

Equation (9-2-19) is the equation of velocity modulation. Alternatively, the equation of velocity modulation can be given by

$$v(t_1) = v_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right] \tag{9-2-20}$$

Bunching Process

- Once the electrons leave the buncher cavity, they drift with a velocity given by Eq. (9-2-19) or (9-2-20) along in the field-free space between the two cavities. The effect of velocity modulation produces bunching of the electron beam-or current modulation.
- The electrons that pass the buncher at *Vs* = 0 travel through with unchanged velocity v_o and become the bunching center. Those electrons that pass the buncher cavity during the positive half cycles of the microwave input voltage *Vs* travel faster than the electrons that passed the gap when *Vs* = 0.

- Those electrons that pass the buncher cavity during the negative half cycles of the voltage *Vs* travel slower than the electrons that passed the gap when *Vs* = 0.

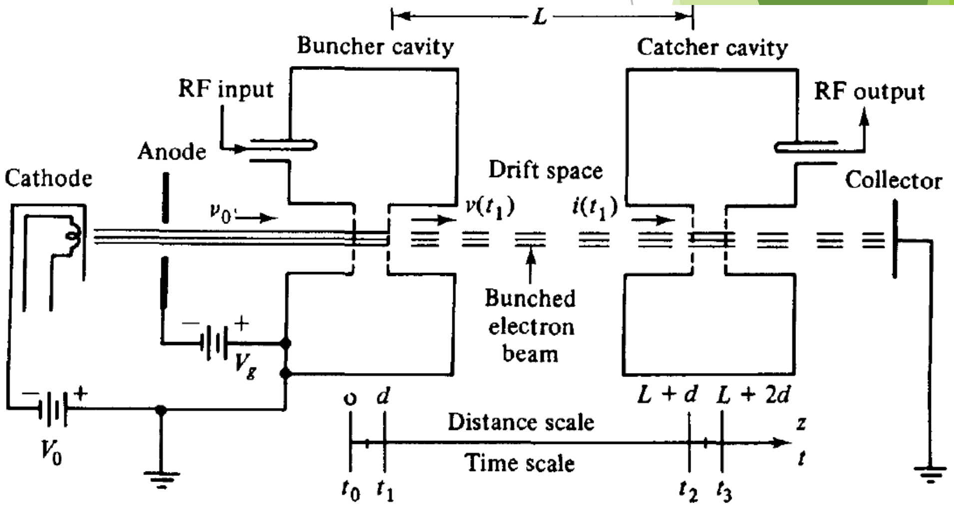


Figure 9-2-2 Two-cavity klystron amplifier.

- At a distance of ΔL along the beam from the buncher cavity, the beam electrons have drifted into dense clusters. Figure 9-2-8 shows the trajectories of minimum, zero, and maximum electron acceleration.

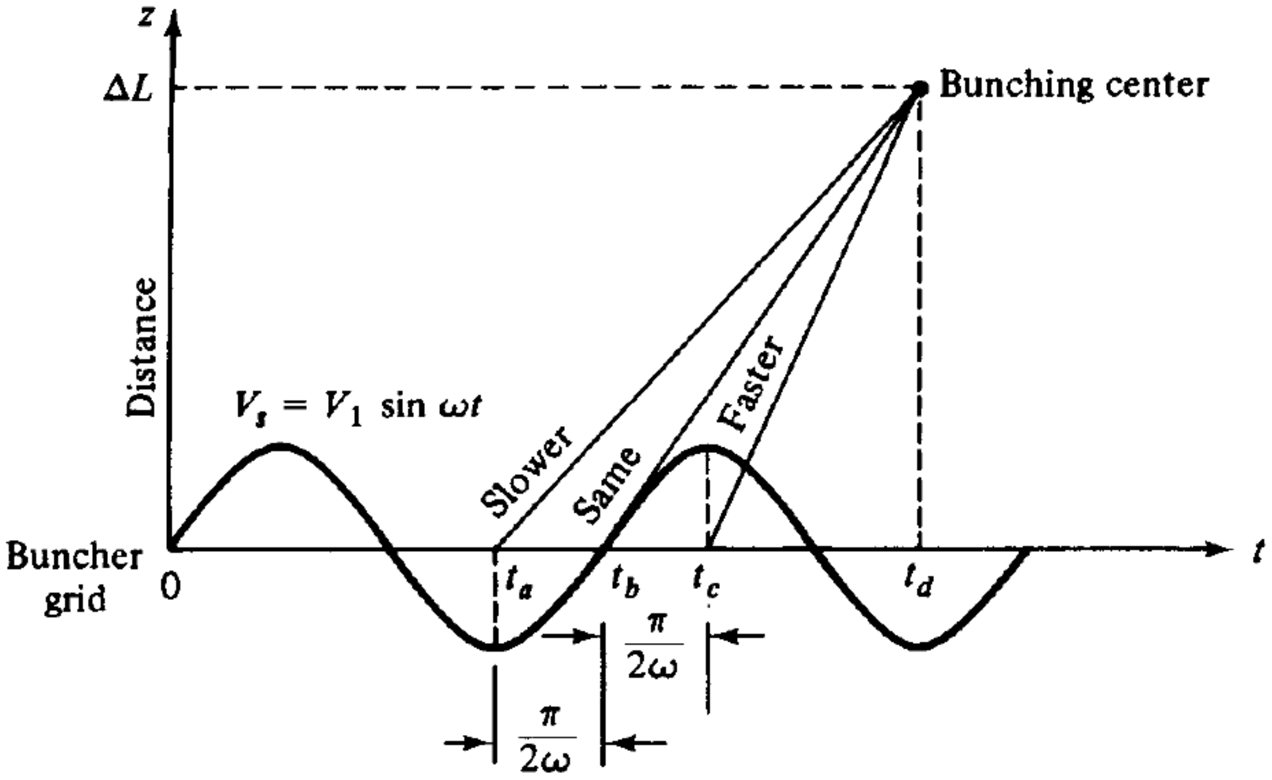


Figure 9-2-8 Bunching distance.

- The distance from the buncher grid to the location of dense electron bunching for the electron at *tb* is

$$\Delta L = v_0(t_d - t_b)$$

- Similarly, the distances for the electrons at *ta* and *tc* are

$$\Delta L = v_{\min}(t_d - t_a) = v_{\min}\left(t_d - t_b + \frac{\pi}{2\omega}\right)$$

$$\Delta L = v_{\max}(t_d - t_c) = v_{\max}\left(t_d - t_b - \frac{\pi}{2\omega}\right)$$

- The distance between bunching grids and the position of the bunch. The catching cavity grids must be placed at this position.

$$\Delta L = v_0 \frac{\pi V_0}{\omega \beta_i V_1}$$

- the *bunching parameter* of a klystron is

$$X \equiv \frac{\beta_i V_1}{2V_0} \theta_0$$

At the buncher gap a charge dQ_0 passing through at a time interval dt_0 is given by

$$dQ_0 = I_0 dt_0 \tag{9-2-36}$$

where I_0 is the dc current. From the principle of conservation of charges this same amount of charge dQ_0 also passes the catcher at a later time interval dt_2. Hence

$$I_0 |dt_0| = i_2 |dt_2| \tag{9-2-37}$$

- Current *i2* is the current at the catcher gap.

- The current arriving at the catcher cavity is then given as

$$i_2(t_0) = \frac{I_0}{1 - X\cos(\omega t_0 + \theta_g/2)}$$

$$i_2 = I_0 + \sum_{n=1}^{\infty} 2I_0 J_n(nX)\cos[n\omega(t_2 - \tau - T_0)]$$

- The fundamental component of the beam current at the catcher cavity has a magnitude

- For signal amplification we must maximize the 1st harmonic

$$I_f = 2I_0 J_1(X)$$

- This fundamental component h has its maximum amplitude at

$$X = 1.841$$

- The optimum distance *L* at which the maximum fundamental component of current occurs is computed

$$\boxed{L_{\text{optimum}} = \frac{3.682 v_0 V_0}{\omega \beta_i V_1}}$$

The induced current in the catcher cavity.

- Since the current induced by the electron beam in the walls of the catcher cavity is directly proportional to the amplitude of the microwave input voltage *V1* , the fundamental component of the induced microwave current in the catcher is given by

$$I_{2ind} = \beta_0 I_2 = \beta_0 2 I_0 J_1(X)$$

Equivalent circuit

- Figure 9-2-13 shows an output equivalent circuit in which Rsho represents the wall resistance of catcher cavity, *Rb* the beam loading resistance, *RL* the external load resistance, and Rsh the effective shunt resistance. Rsh over all shunt resistance.

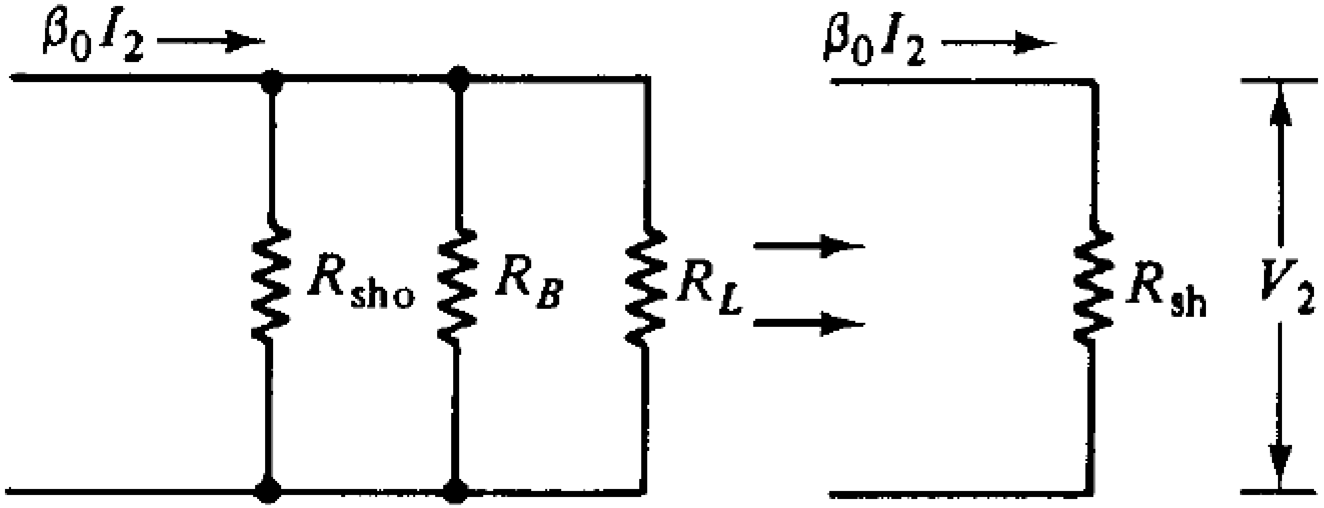
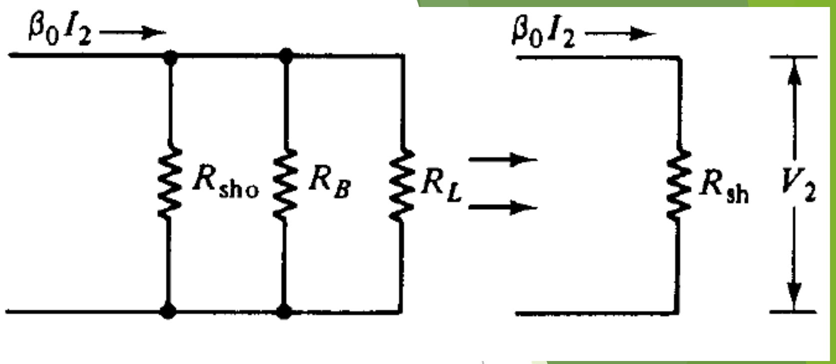


Figure 9-2-13 Output equivalent circuit.



- The output power delivered to the catcher cavity and the load is given as

$$P_{out} = \frac{(\beta_0 I_2)^2}{2} R_{sh} = \frac{\beta_0 I_2 V_2}{2}$$

- where *Rsh* is the total equivalent shunt resistance of the catcher circuit, including the load, and *V2* is the fundamental component of the catcher gap voltage.

Efficiency of klystron.

- The electronic efficiency of the klystron amplifier is defined as the ratio of the output power to the input power:

$$\text{Efficiency} \equiv \frac{P_{out}}{P_{in}} = \frac{\beta_0 I_2 V_2}{2 I_0 V_0}$$

- in which the power losses to the beam loading and cavity walls are included.
- If the coupling is perfect, *Bo*= 1, the maximum beam current approaches $I_2 \text{max} = 2 \times Io(0.582)$, and the voltage *Vi* is equal to *Vo*. Then the maximum electronic efficiency is about 58%.

Voltage gain

- The voltage gain of a klystron amplifier is defined as

$$A_v \equiv \frac{|V_2|}{|V_1|} = \frac{\beta_0 I_2 R_{sh}}{V_1} = \frac{\beta_0^2 \theta_0}{R_0} \frac{J_1(X)}{X} R_{sh}$$

- Where Ro is the beam d.c resistance
- $R_o = {}^{V_o}/_{I_0}$

Example 1: Klystron Amplifier

- A two-cavity klystron amplifier has the following parameters:

$$V_0 = 1000 \text{ V} \qquad R_0 = 40 \text{ k}\Omega$$

$$I_0 = 25 \text{ mA} \qquad f = 3 \text{ GHz}$$

Gap spacing in either cavity: $d = 1$ mm

Spacing between the two cavities: $L = 4$ cm

Effective shunt impedance, excluding beam loading: $R_{sh} = 30 \text{ k}\Omega$

a. Find the input gap voltage to give maximum voltage V_2.

b. Find the voltage gain, neglecting the beam loading in the output cavity.

c. Find the efficiency of the amplifier, neglecting beam loading.

Solution:

a. For maximum V_2, $J_1(X)$ must be maximum. This means $J_1(X) = 0.582$ at $X = 1.841$. The electron velocity just leaving the cathode is

$$v_0 = (0.593 \times 10^6)\sqrt{V_0} = (0.593 \times 10^6)\sqrt{10^3} = 1.88 \times 10^7 \text{ m/s}$$

The gap transit angle is

The beam-coupling coefficient is

$$\beta_i = \beta_0 = \frac{\sin(\theta_g/2)}{\theta_g/2} = \frac{\sin(1/2)}{1/2} = 0.952$$

The dc transit angle between the cavities is

$$\theta_0 = \omega T_0 = \omega \frac{L}{v_0} = 2\pi (3 \times 10^9) \frac{4 \times 10^{-2}}{1.88 \times 10^7} = 40 \text{ rad}$$

The maximum input voltage V_1 is then given by

$$V_{1\,max} = \frac{2V_0 X}{\beta_i \theta_0} = \frac{2(10^3)(1.841)}{(0.952)(40)} = 96.5 \text{ V}$$

b. The voltage gain is found as

$$A_v = \frac{\beta_0^2 \theta_0}{R_0} \frac{J_1(X)}{X} R_{sh} = \frac{(0.952)^2(40)(0.582)(30 \times 10^3)}{4 \times 10^4 \times 1.841} = 8.595$$

c. The efficiency can be found as follows:

$$I_2 = 2I_0 J_1(X) = 2 \times 25 \times 10^{-3} \times 0.582 = 29.1 \times 10^{-3} \text{ A}$$

$$V_2 = \beta_0 I_2 R_{sh} = (0.952)(29.1 \times 10^{-3})(30 \times 10^3) = 831 \text{ V}$$

$$\text{Efficiency} = \frac{\beta_0 I_2 V_2}{2I_0 V_0} = \frac{(0.952)(29.1 \times 10^{-3})(831)}{2(25 \times 10^{-3})(10^3)} = 46.2\%$$

MULTI CAVITY KLYSTRON AMPLIFIERS

- The typical power gain of a two-cavity klystron amplifier is about 30 dB. In order to achieve higher overall gain, one way is to connect several two-cavity tubes in cascade, feeding the output of each of the tubes to the input of the following one.

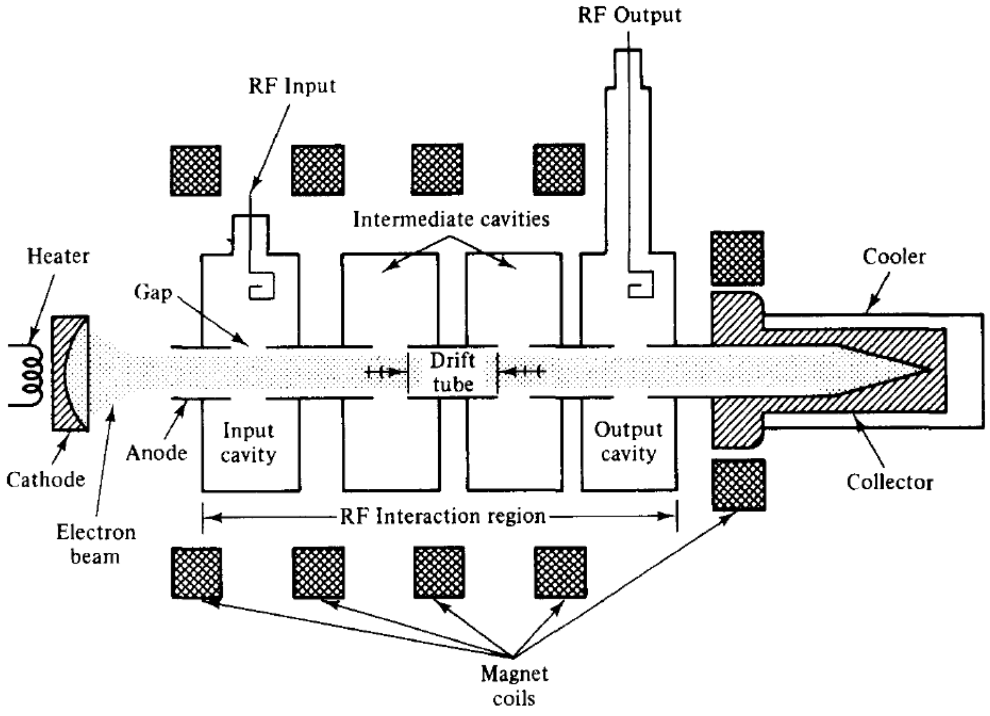


Figure 9-3-1 Schematic diagram of a four-cavity klystron amplifier. (*Courtesy of Varian Associates, Inc.*)

- besides using the multistage techniques, the tube manufacturers have designed and produced multicavity klystron to serve the high-gain requirement.
- In a multicavity klystron each of the intermediate cavities, placed at a distance of the bunching parameter X of 1.841 away from the previous cavity, acts as a buncher with the passing electron beam inducing a more enhanced RF voltage than the previous cavity, which in turn sets up an increased velocity modulation.

Ex : A two cavity klystron has accelerating voltage 1000V frequency 5GHz , and cavity gap 1mm .It is required to amplify 100V input find the gap transit angle and optimum length of drift space

Sol:

Transit angle θg (between two cavities)

$$\theta g = \frac{wd}{vo}$$

$$vo = 0.593 \times 10^{-6}\sqrt{Vo} = 1.88 \times 10^{7} \text{ m/s}$$

$$\theta g = \frac{2\pi \times 5 \times 10^{9} \times 1 \times 10^{-3}}{1.88 \times 10^{7}} = 1.67\, rad$$

$$L\ opt = \frac{3.682\ vo\ Vo}{w\ \beta i\ V1}$$

$$\beta i = \beta o = \frac{\sin(\theta g/2)}{\theta g/2} = 0.89$$

$$V1 = 100V$$

$$Lopt = \frac{3.682 \times 1.88 \times 10^{7} \times 10^{3}}{2\pi \times 5 \times 10^{9} \times 0.89 \times 100} = 2.47\, cm$$

Ex : a two cavity klystron has the following characteristics ,voltage gain 15 dB ,input power 5mw ,total shunt impedance of input cavity 30KΩ,total shunt impedance of output cavity 40k Ω

Determine

a)Input impedance (r.m.s)

b)Output voltage

c)Power delivered to load

Sol:

$$input\ power = \frac{V1^2}{Rshunt}$$

$$5 \times 10^{-3} = \frac{V1^2}{30 \times 10^3}$$

$150 = V1^2 \quad \rightarrow V1 = 12.24\ V$

$$\frac{15}{20} = \frac{20}{20} \log x$$

$0.75 = \log x \ \rightarrow 10^{0.75} = x$

Voltage gain Av = 5.6

$$Av = \frac{V2}{V1} \rightarrow 5.6 = \frac{V2}{12.24}$$

V2=68.8 V

$$Pl = \frac{V2^2}{Rshunt\ output}$$

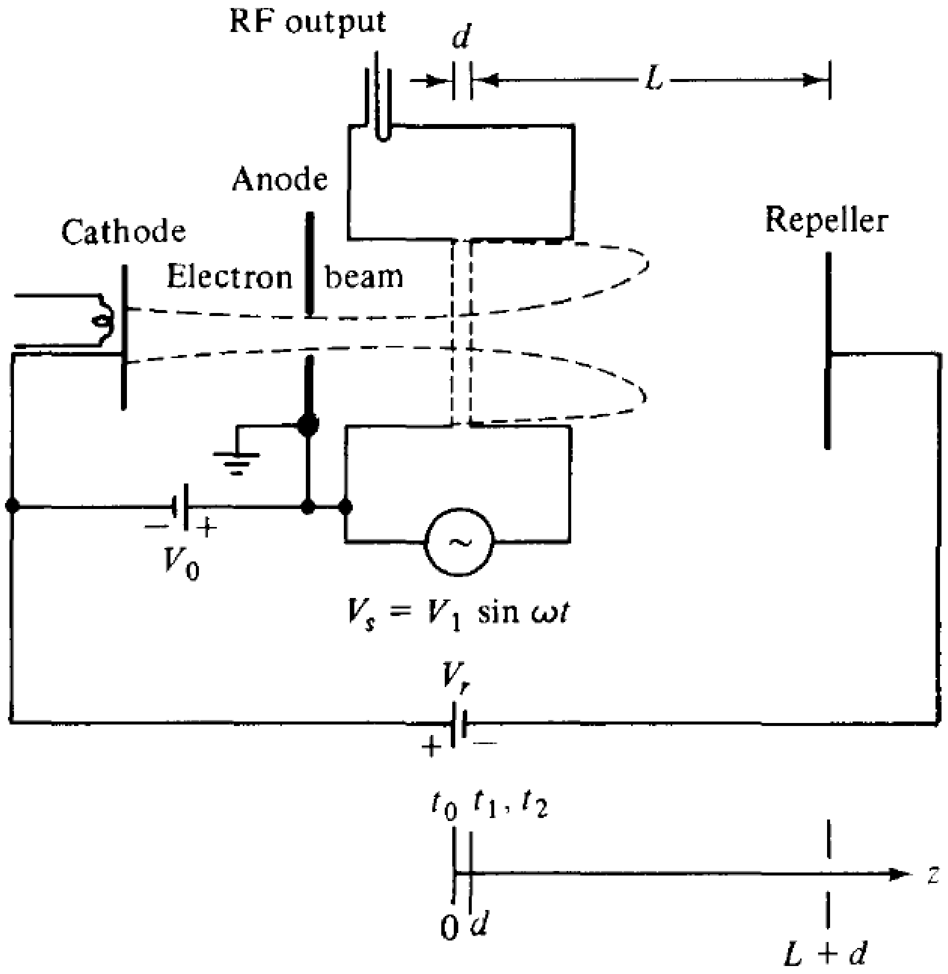
$$Pl = \frac{(68.8)^2}{40 \times 10^3} = 118\ mw$$

REFLEX KLYSTRONS

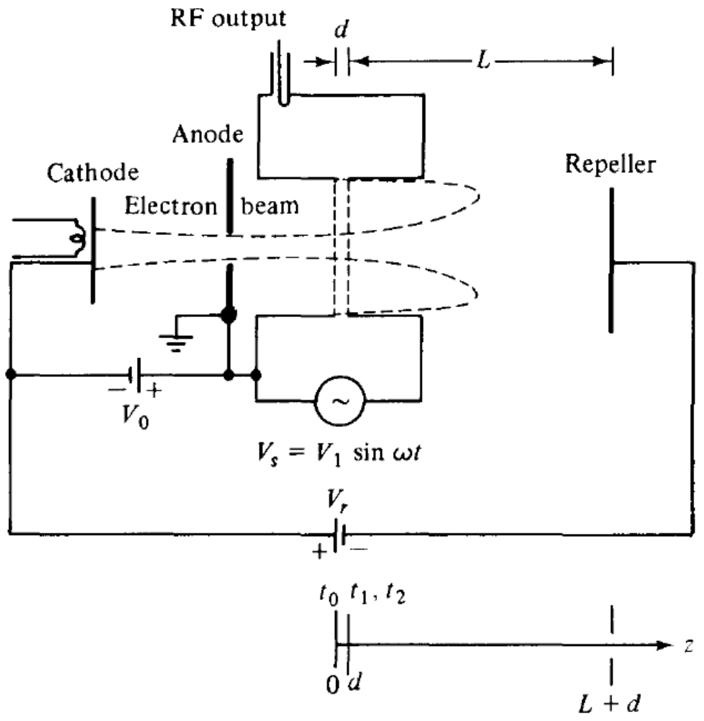
- If a fraction of the output power is fed back to the input cavity and if the loop gain has a magnitude of unity with a phase shift of multiple 2π, the klystron will oscillate.
- The reflex klystron is a single-cavity klystron that overcomes the disadvantages of the two cavity klystron oscillator.
- It is a low-power generator of 10 to 500-mW output at a frequency range of 1 to 25 GHz. The efficiency is about 20 to 30%.
- This type is widely used in the laboratory for microwave measurements and in microwave receivers as local oscillators in commercial, military, and airborne Doppler radars as well as missiles.

Schematic Diagram

- A schematic diagram of the reflex klystron is shown in Fig. 9-4-1.



- The electron beam injected from the cathode is first velocity-modulated by the cavity-gap voltage. Some electrons accelerated by the accelerating field enter the repeller space with greater velocity than those with unchanged velocity. Some electrons decelerated by the retarding field enter the repeller region with less velocity.

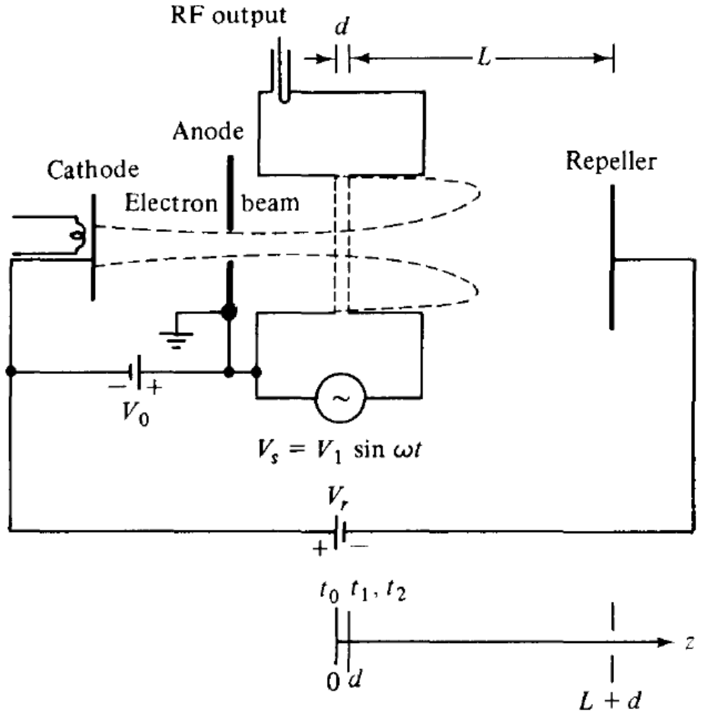


- All electrons turned around by the repeller voltage then pass through the cavity gap in bunches that occur once per cycle.

- On their return journey the bunched electrons pass through the gap during the retarding phase of the alternating field and give up their kinetic energy to the electromagnetic energy of the field in the cavity.

- Oscillator output energy is then taken from the cavity.

- The electrons are finally collected by the walls of the cavity or other grounded metal parts of the tube.



Velocity Modulation

- The analysis of a reflex klystron is similar to that of a two-cavity klystron. For simplicity, the effect of space-charge forces on the electron motion will again be neglected. The electron entering the cavity gap from the cathode at *z* = 0 and time *to* is assumed to have uniform velocity

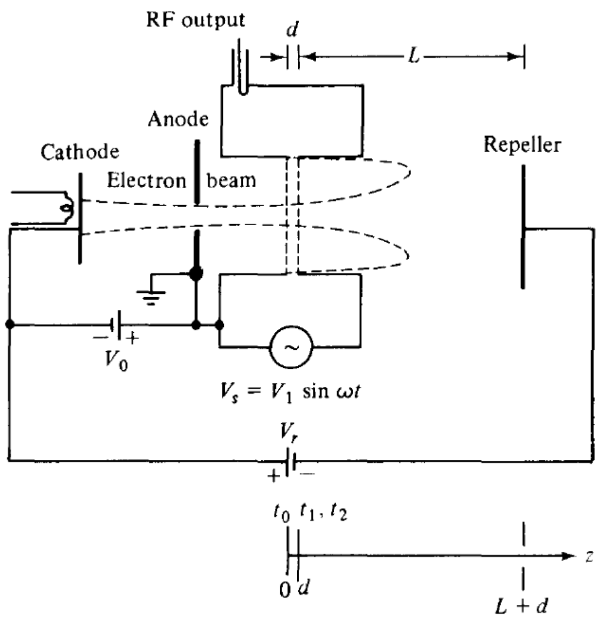
$$v_0 = 0.593 \times 10^6 \sqrt{V_0} \tag{9-4-1}$$

The same electron leaves the cavity gap at $z = d$ at time t_1 with velocity

$$v(t_1) = v_0 \left[1 + \frac{\beta_1 V_1}{2V_0} \sin \left(\omega t_1 - \frac{\theta_g}{2} \right) \right] \tag{9-4-2}$$

- for the problems up to this point are identical to those of a two-cavity klystron amplifier. The same electron is forced back to the cavity $z = d$ and time $t2$ by the retarding electric field E which is given by

$$E = \frac{V_r + V_0 + V_1 \sin(\omega t)}{L}$$



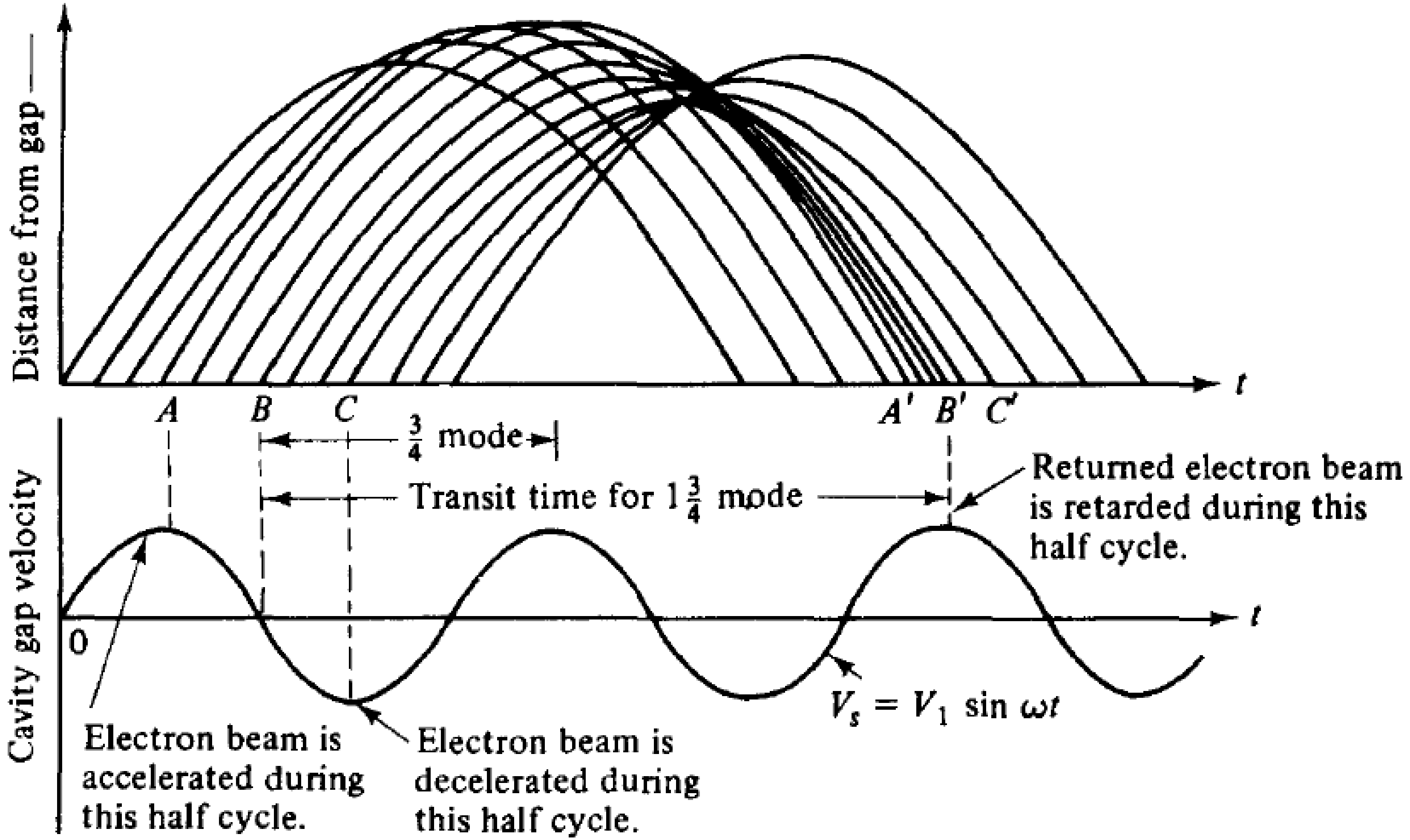


Figure 9-4-2 Applegate diagram with gap voltage for a reflex klystron.

- The round-trip transit time in the repeller region is given by

$$T' = t_2 - t_1 = \frac{2mL}{e(V_r + V_0)} v(t_1) = T'_0\left[1 + \frac{\beta_i V_1}{2V_0}\sin\left(\omega t_1 - \frac{\theta_g}{2}\right)\right]$$

- Where

$$T'_0 = \frac{2mLv_0}{e(V_r + V_0)}$$

- is the round-trip dc transit time of the center-of-the-bunch electron.

$$\theta_0' = \omega T_0'$$

is the round-trip dc transit angle of the center-of-the-bunch electron and

$$X' \equiv \frac{\beta_i V_1}{2V_0} \theta_0'$$

is the bunching parameter of the reflex klystron oscillator.

Power Output and Efficiency

- In order for the electron beam to generate a maximum amount of energy to the oscillation, the returning electron beam must cross the cavity gap when the gap field is maximum retarding. In this way, a maximum amount of kinetic energy can be transferred from the returning electrons to the cavity walls.

- It can be seen that for a maximum energy transfer, the round-trip transit angle, referring to the center of the bunch, must be given by

$$\omega(t_2 - t_1) = \omega T_0' = \left(n - \frac{1}{4}\right)2\pi = N\,2\pi = 2\pi n - \frac{\pi}{2}$$

- where $V_1 \ll V_o$ is assumed, *n* = any positive integer for cycle number, and $N = n - {}^1/_4$ is the number of modes.

- The dc power supplied by the beam voltage *Vo* is

$$P_{dc} = V_0 I_0$$

$$I_2 = 2 I_0 \beta_i J_1(X')$$

- and the ac power delivered to the load is given by

$$P_{ac} = \frac{V_1 I_2}{2} = V_1 I_0 \beta_i J_1(X')$$

$$\text{Efficiency} \equiv \frac{P_{ac}}{P_{dc}} = \frac{2X' J_1(X')}{2\pi n - \pi/2}$$

- The factor $X' J_1(X')$ reaches a maximum value of 1.25 at X'= 2.408 and $J1(X')$ =0.52.
- In practice n=2 then efficiency equal
- 22.7%

- The maximum theoretical efficiency of a reflex klystron oscillator ranges from 20 to 30%.

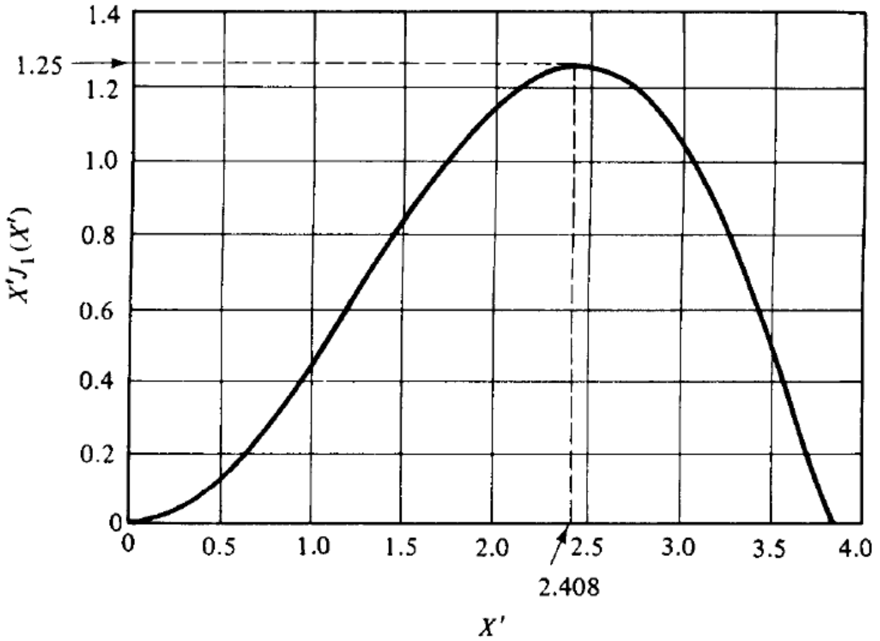


Figure 9-4-3 $X'J_1(X')$ versus X'.

Equivalent circuit

- The equivalent ciruit of a reflex klystron is shown in Fig. 9-4-6. In this circuit *L* and C are the energy storage elements of the cavity; *Ge* represents the copper losses of the cavity, *Gb* the beam loading conductance, and *Ge* the load conductance.

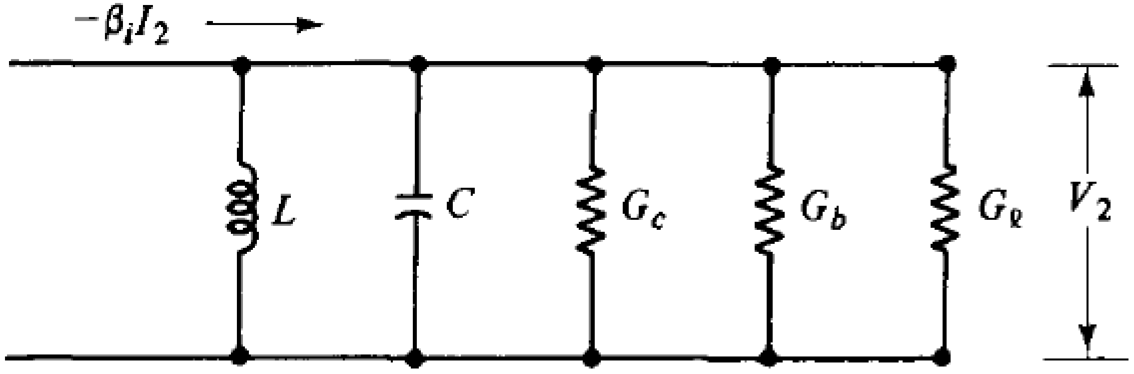


Figure 9-4-6 Equivalent circuit of a reflex klystron.

- $$output\ power\ Pac = \frac{V1\ I2}{2} = V1Io\ \beta i\ J1(\grave{x})$$

$$Pac = \frac{V1Io\ \beta i\ J1(\grave{x})}{2\pi n - \frac{\pi}{2}}$$

Where n is an integer

$$Pdc = Vo\ Io$$

$$effeciency = \frac{Pac}{Pdc} = \frac{2\ \grave{x}\ J1(\grave{x})}{2\pi n - \frac{\pi}{2}}$$

For a given beam voltage Vo the relationship between the repeller voltage and cycle number (n) required for oscillation is :

$$\frac{Vo}{(Vr+Vo)^2} = \frac{(2\pi n - \frac{\pi}{2})^2}{8w^2L^2}\frac{e}{m}$$

The output power can be expressed in terms of Vr

$$Pac = \frac{Vo\ Io\ \grave{x}\ J1(\grave{x})(Vr+Vo)}{w\ L}\sqrt{\frac{e}{2mVo}}$$

$$Pac = \frac{2VoIo\ \grave{x}\ J1(\grave{x})}{2\pi n - \frac{\pi}{2}}\ in\ general$$

$$Pacmax = \frac{0.398\,Vo\ Io}{mode}\ for\ maximum$$

Ex: a reflex klystron operates at 8 GHz with dc beam voltage 300 v ,repeller space 1 mm for $1\frac{3}{4}$ mode calculate Prfmax and corresponding repeller voltage for a beam current of l8 mA

Sol:

$$Pacmax = \frac{o.398\, VoIo}{mode}$$

$$= \frac{0.398 \times 300 \times 18 \times 10^{-3}}{1.75} = 1.228\, w$$

$$\frac{e}{m} = 1.759 \times 10^{11}$$

$$\frac{Vo}{(Vr + Vo)^2} = \frac{e}{m}\frac{(2\pi n - \frac{\pi}{2})^2}{8w^2L^2}$$

- $$\frac{Vo}{(Vr+Vo)^2} = 1.759 \times 10^{11} \frac{(4\pi - \frac{\pi}{2})^2}{8(2\pi \times 8 \times 10^9 \times 10^{-3})^2}$$

$$Vr = 233\ V$$

Ex: a reflex klystron operates under the condition

Vo=600V , L=1 mm , Rsh= 15KΩ , n=2 or ($1\frac{3}{4}$ *mode*)

$\frac{e}{m} = 1.759 \times 10^{11}$ fr=9 GHz , neglect βi

a) **Find (Vr) value**

b) **Find the direct current necessary to give gap voltage 200V**

c) **Calculate the efficiency**

a)

$$\frac{Vo}{(Vr+Vo)^2} = \frac{e}{m}\frac{(2\pi n - \frac{\pi}{2})^2}{8w^2L^2}$$

$$\frac{Vo}{(Vr+Vo)^2} = 1.759 \times 10^{11} \frac{(4\pi - \frac{\pi}{2})^2}{8(2\pi \times 8 \times 10^9 \times 10^{-3})^2}$$

$$Vr = 250\ V$$

b)

$$V2 = I2\ Rsh$$

$$I2 = 2Io\ \beta i\ J1(\grave{x})$$

$$V2 = 2Io\ J1\ (\grave{x}) \qquad we\ take\ \beta i = 1$$

$$\dot{I}o = \frac{V2}{2\,J1(\grave{x})\ \ Rsh} = \frac{200}{2 \times 0.582 \times 15 \times 10^3} = 11.45\ mA$$

c)

$$effeciency = \frac{Pac}{Pdc} = \frac{2\,\grave{x}\,J1(\grave{x})}{2\pi n - \frac{\pi}{2}}$$

$$\eta\ = \frac{2\times1.841\times0.582}{2\pi(2)-\frac{\pi}{2}}$$

$$= 19.49\ \%$$

Microwave crossed Field Tubes(M type)

Introduction

- In crossed-field devices, however, the dc magnetic field and the dc electric field are perpendicular to each other.
- In all crossed-field tubes, the dc magnetic field plays a direct role in the RF interaction process.
- Crossed-field tubes derive their name from the fact that the dc electric field and the dc magnetic field are perpendicular to each other. They are also called *M* –type tubes for propagation of waves in a magnetic field.

- In a crossed-field tube, the electrons emitted by the cathode are accelerated by the electric field and gain velocity, but the greater their velocity, the more their path is bent by the magnetic field.

- If an RF field is applied to the anode circuit, those electrons entering the circuit during the retarding field are decelerated and give up some of their energy to the RF field.
- Consequently, their velocity is decreased, and these slower electrons will then travel the de electric field far enough to regain essentially the same velocity as before.

- Because of the crossed-field interactions, only those electrons that have given up sufficient energy to the RF field can travel all the way to the anode.
- This phenomenon would make the *M-type* devices relatively efficient.

- Those electrons entering the circuit during the accelerating field are accelerated by means of receiving enough energy from the RF field and are returned back toward the cathode. This back-bombardment of the cathode produces heat in the cathode and decreases the operational efficiency.

Magnetron

- A commonly used crossed-field tube is a magnetron
- Cylindrical magnetron: The cylindrical magnetron was developed by Boot and Randall in early 1940.
- Coaxial magnetron: The coaxial magnetron introduced the principle of integrating a stabilizing cavity into the magnetron geometry.
- Voltage-tunable magnetron: The voltage-tunable magnetron has the cathode anode geometry of the conventional magnetron, but its anode can be tuned easily.
- Inverted magnetron: The inverted magnetron has the inverted geometry of the conventional magnetron with the cathode placed on the outside surrounding the anode and microwave circuit.

MAGNETRON OSCILLATORS

- Hull invented the magnetron in 1921, but it was only an interesting laboratory device until about 1940. During World War II, an urgent need for high-power microwave generators for radar transmitters led to the rapid development of the magnetron to its present state.

- All magnetrons consist of some form of anode and cathode operated in a dc magnetic field normal to a dc electric field between the cathode and anode.

- Because of the crossed field between the cathode and anode, the electrons emitted from the cathode are influenced by the crossed field to move in curved paths. If the dc magnetic field is strong enough, the electrons will not arrive in the anode but return instead to the cathode. Consequently, the anode current is cut off.

Cylindrical Magnetron

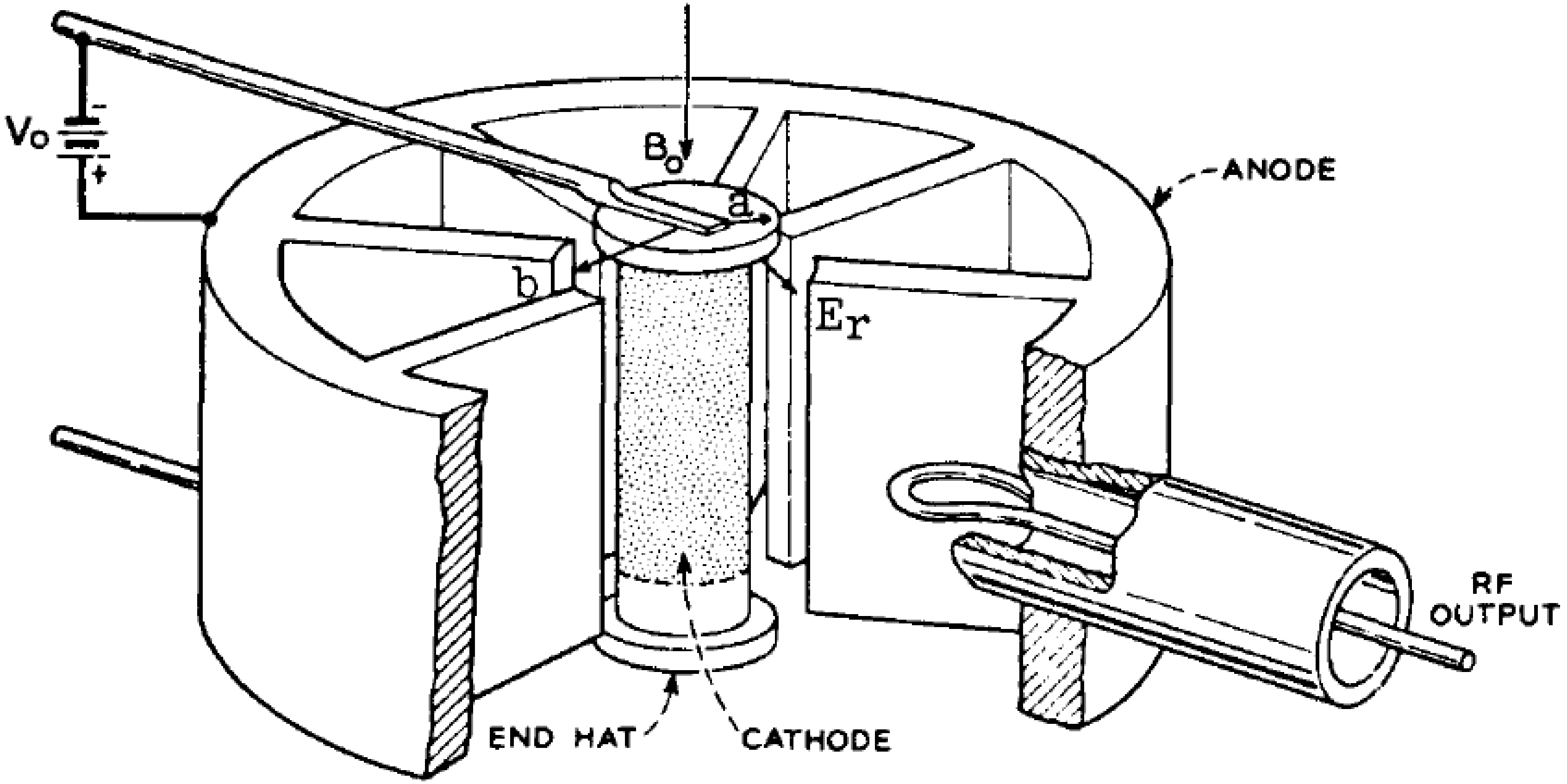


Figure 10-1-1 Schematic diagram of a cylindrical magnetron.

- In a cylindrical magnetron, several reentrant cavities are connected to the gaps.
- The dc voltage *Vo* is applied between the cathode and the anode.

- The magnetic flux density *Bo* is in the positive *z* direction.

- When the dc voltage and the magnetic flux are adjusted properly, the electrons will follow cycloidal paths in the cathode anode space under the combined force of both electric and magnetic fields as shown in Fig. 10-1-2.

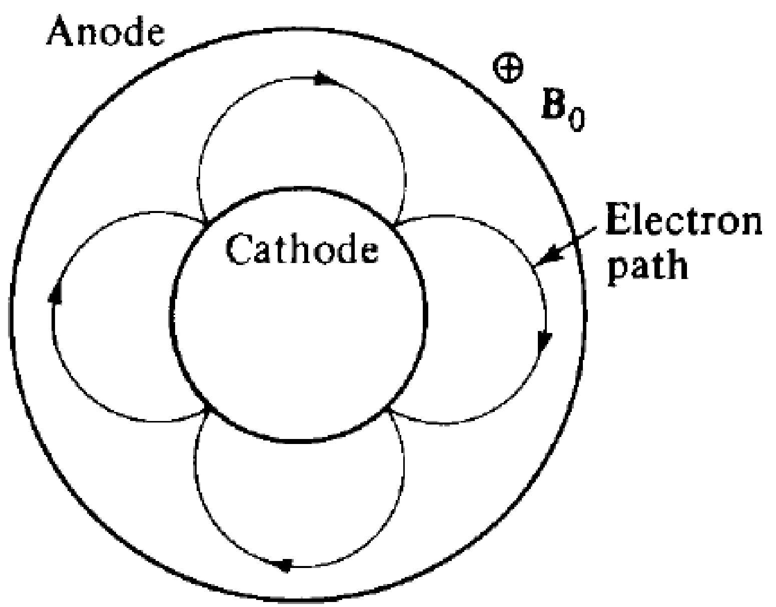


Figure 10-1-2 Electron path in a cylindrical magnetron.

- The electrons emitted by the cathode are accelerated towards the anode by the anode voltage Vo. The electrons moving at velocity v will be influenced by the magnetic field which is perpendicular to its direction of motion and the E-field.
- the force due to magnetic field
- $F_m = e \times v \times B$
- The force tries to curve the path of electron
- $F_e = e \times E$

- Total force equal to
- $F_m + F_e = m \times {}^{dv}/_{dt}$

- Force =mass*acceleration

$$\frac{d^2r}{dt^2} - r\left(\frac{d\phi}{dt}\right)^2 = \frac{e}{m}E_r - \frac{e}{m}rB_z\frac{d\phi}{dt} \qquad (10\text{-}1\text{-}1)$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2\frac{d\phi}{dt}\right) = \frac{e}{m}B_z\frac{dr}{dt} \qquad (10\text{-}1\text{-}2)$$

- where e/m = 1.759 x 1011 C/kg is the charge-to-mass ratio of the electron and Bo = Bz is assumed in the positive z direction.
- Rearrangement of Eq. (10-1-2) results in the following form

$$\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \frac{e}{m} B_z r \frac{dr}{dt} = \frac{1}{2}\omega_c \frac{d}{dt}(r^2) \qquad (10\text{-}1\text{-}3)$$

- Where $wc = {}^{e}/_{m} \times B_z$ is the cyclotron angular frequency.
- at r = a, where a is the radius of the cathode cylinder,

- If H is large compared to E then electrons return back to the cathode

- The cut off magnetic flux density Boc

$$Boc = \frac{(8\,Vo\,\frac{m}{e})^{1/2}}{b(1-\frac{a^2}{b^2})}$$

$a\ is\ radius\ of\ cathode\ cylinder$
$b\ is\ radius\ of\ anode$

- This means that Bo > Boc for a given Vo, the electrons will not reach anode
- Cut off voltage is

$$Voc = \frac{e}{8m} Bo^2 b^2 (1 - \frac{a^2}{b^2})^2$$

This means that if Vo<Voc for a given Bo, the electrons will not reach the anode

PROPERTIES OF MAGNETRON

- Operating frequency up to about 20 GHZ
- Peak O/P power up to 40Mw
- Average O/P power up to 800Kw
- Efficiency 40% _ 70%
- The efficiency and power output of magnetron depend on resonant structure and dc supply

- Ex cylindrical magnetron has the following parameters $beam\ current\ Io = 27A, a = 5cm$, b = 10cm $magnetic\ flux\ density\ Bo = 0.336\ wb/m^2$, $Vo = 27KV$

Compute :

a) The cut off voltage at fixed Bo

b) The cut off magnetic flux density

Sol:

a) $\frac{e}{m} = 1.759 \times 10^{11}$

$$Voc = \frac{e}{8m} Bo^2 b^2 (1 - \frac{a^2}{b^2})^2$$

$$Voc = \frac{1}{8} \times 1.759 \times 10^{11} \times (0.336)^2 \times (10 \times 10^{-2})^2 \times (1 - \frac{5^2}{10^2})^2$$

$$Voc = 139.5\ KV$$

b) $$Boc = \frac{(8Vo\frac{m}{e})^{1/2}}{b(1 - \frac{a^2}{b^2})}$$

$$Boc = \frac{(8 \times 26 \times 10^3 \times \frac{1}{1.759 \times 10^{11}})^{1/2}}{10 \times 10^{-2}(1 - \frac{5^2}{10^2})}$$

$$Boc = 14.495\ m\ wb/m^2$$

Linear Magnetron

- The schematic diagram of a linear magnetron is shown in Fig. 10-1-7. In the linear magnetron as shown in Fig. 10-1-7, the electric field *Ex* is assumed in the positive *x* direction and the magnetic flux density *Bz* in the positive *z* direction. The differential equations of motion of electrons in the crossed-electric and magnetic fields can be written as

$$\frac{d^2x}{dt^2} = -\frac{e}{m}\left(E_x + B_z\frac{dy}{dt}\right)$$

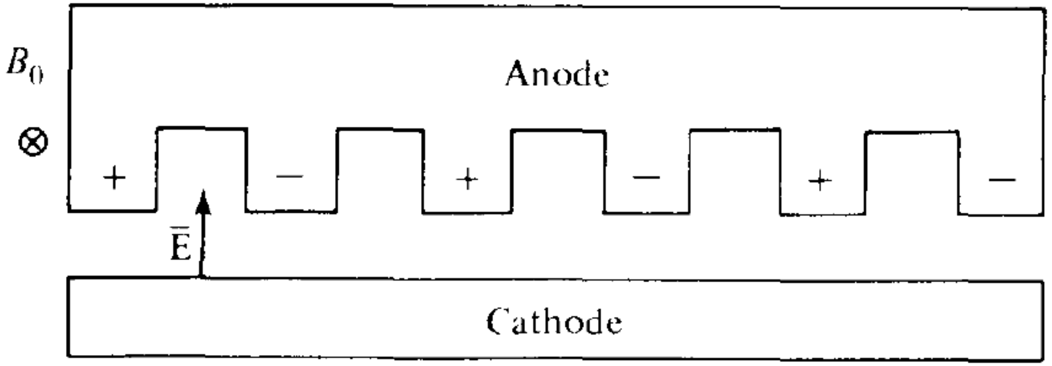


Figure 10-1-7 Schematic diagram of a linear magnetron.

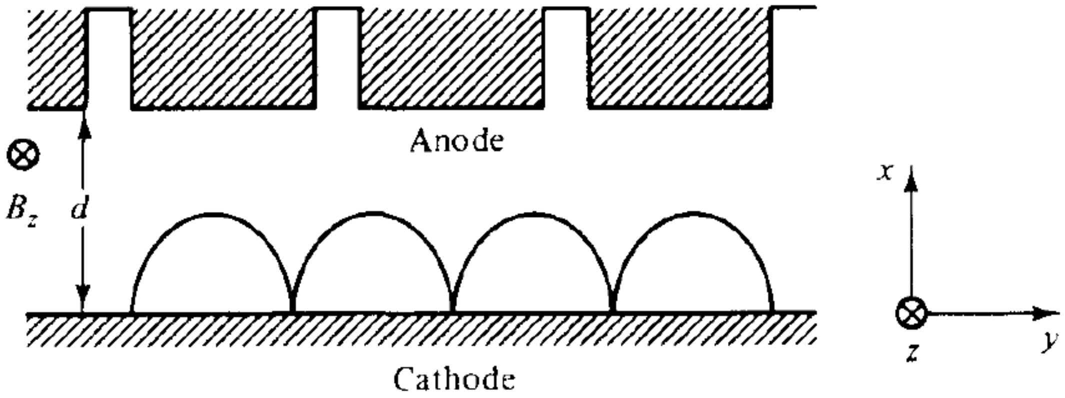


Figure 10-1-8 Electron path in a linear magnetron.