40%   15% فا   15% ف   %10 Quiz

Date : / /   Subject: المصفوفة

Matrices

$$A = \begin{array}{c} \text{Column} \\ \begin{bmatrix} ٢ & ٣ & ٤ \\ ٥ & -٣ & -٤ \end{bmatrix}_{٢\times٣} \end{array} \xleftarrow{} \text{row}$$

$$B = \begin{bmatrix} e^{x} & 2x^{2} \\ 3x & e^{-x} \end{bmatrix}$$

إذا أردنا تحديد (-3) نقول $a_{22} = -3$

Matrices : which are (مساو) rectangular (مستطيلي) arrange of numbers of functions, It help us to express a large a mount of data and functions in an orghized and simplified from

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 2 \end{bmatrix}_{2\times3} \rightarrow \text{row}$$

Column

No. of row   No. of column

$A_{2\times3}$

$a_{22} = -1$

$a_{13} = 2$

Propreties of Matrices

1) Equaltiy

$$A = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}_{2\times2}$$

$A \neq B$

$$C = \begin{bmatrix} 3 & -2 \\ 4 & 5 \end{bmatrix}_{2\times2}$$

$$B = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix}_{2\times2}$$

$A = C$

$C \neq B$

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2) Square matrix   عدد الصفوف يساوي عدد الاعمدة

main diagonal القطر الرئيسي (الاعظم الامامي)   Secondry diagonal القطر الثانوي

$$A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & -2 & 5 \\ 6 & 7 & 8 \end{bmatrix}_{3\times 3}$$

3) unity matrix

1- لازم تكون مربعة
2- عناصر القطر الرئيسي قيمتها 1 وبقية العناصر صفر.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

وهي تمثل العدد (1) نتائجها في بعض العمليات.

4) Zero matrix

$$Z = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

كل مصفوفه عناصرها صفارة

* ملاحظة: vertical | عمودي

Horizontal. افقي / صف row

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« Addition and Subtraction of Matrices »

- to add or subtract two matrices, the two matrices must have the same size

Ex: If $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ 4 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 0 & 1 \end{bmatrix}$ Find $A+B$? $A-B$?

Sol $A+B = \begin{bmatrix} 3 & 0 \\ 5 & 3 \\ 4 & 0 \end{bmatrix}$ $A-B \begin{bmatrix} 1 & 2 \\ 1 & -3 \\ 4 & -2 \end{bmatrix}$

Ex:

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$ Find $A+B$? $A-B$?

Sol

$A+B = \begin{bmatrix} 7 & 7 & 7 \\ 7 & 7 & 7 \end{bmatrix}$ $A-B = \begin{bmatrix} -5 & -3 & -1 \\ 1 & 3 & 5 \end{bmatrix}$

* إذا كانت المصفوفتان غير متساويتان لا يمكن الجمع أو الطرح ،

ضرب المصفوفات

Multiplication of Matrices

1 Multiply a matrix with constant k, we multiply each element of the matrix with k

Ex: If $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$, $k = 2$ Find $A * k = ?$

Sol $A * k = \begin{bmatrix} 8 & -4 \\ 6 & 2 \end{bmatrix}$

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2 To multiply a matrix A by B, the number of columns in A must be equal to the number of rows in B then we multiply row in A by column in B

Ex: If $A = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & 0 \end{bmatrix}_{2\times 3}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ -1 & 3 \end{bmatrix}_{3\times 2}$ Find $A \times B = ?$ $B \times A = ?$

Sol

$$A * B = \begin{bmatrix} 1+0+1 & 0+6-3 \\ 0+0+0 & 0+2+0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}_{2\times 2}$$

$$B * A = \begin{bmatrix} 1+0 & 3+0 & -1+0 \\ 0+0 & 0+2 & 0+0 \\ -1+0 & -3+3 & 1+0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}_{3\times 3}$$

Ex:

If $A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -7 & 4 \end{bmatrix}_{2\times 3}$, $B = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 1 & 4 \\ 2 & 1 & 4 \end{bmatrix}_{3\times 3}$ Find $A * B = ?$ $B * A = ?$

Sol

$$A * B = \begin{bmatrix} 6+3+2 & 8+3+1 & 10+12+4 \\ 6-7+8 & 8-7+4 & 10-28+16 \end{bmatrix} = \begin{bmatrix} 11 & 12 & 26 \\ 7 & 5 & -2 \end{bmatrix}$$

B * A = [we can not multiply B*A because the No. of columns in does not equal to the No. of rows in]

$\boxed{B * A \neq A * B}$ *

عملية الضرب غير أبدالية *

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Ex 3 If $A = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -5 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ -5 & 0 \\ 6 & -2 \\ -1 & -3 \end{bmatrix}$ find $A*B = ?$ $B*A = ?$

Sol

$$A*B = \begin{bmatrix} 2-10-18-4 & 3+0+6-12 \\ 0+25-6-1 & 0+0+2+3 \end{bmatrix} = \begin{bmatrix} -30 & -3 \\ 18 & -1 \end{bmatrix}$$

$$B*A = \begin{bmatrix} 2+0 & 4-15 & -6-3 & 8+3 \\ -5+0 & -10+0 & 15+0 & -20+0 \\ 6+0 & 12+10 & -18+2 & 24-2 \\ -1+0 & -2+15 & 3+3 & -4-3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -11 & -9 & 11 \\ -5 & -10 & 15 & -20 \\ 6 & 22 & -16 & 22 \\ -1 & 13 & 6 & -7 \end{bmatrix}$$

Notes.

1- $A*B \neq B*A$

2- $A(BC) = (AB)*C$

3- $(kA)B = k(AB)$

4- $(A+B)*C = A*C + B*C$

where A, B and C are matrix. where as k is constant

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Ex1. show that $(A+B)C = A*C + B*C$

If $A = \begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

sol

$$A+B = \begin{bmatrix} 6 & 3 \\ -1 & 12 \end{bmatrix}$$

$$(A+B)C = \begin{bmatrix} 12+3 & 6+0 \\ -2+12 & -1+0 \end{bmatrix} = \begin{bmatrix} 15 & 6 \\ 10 & -1 \end{bmatrix}$$

$$A*C = \begin{bmatrix} 8+2 & 4+0 \\ 2+8 & 1+0 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 10 & 1 \end{bmatrix}$$

$$B*C = \begin{bmatrix} 4+1 & 2+0 \\ -4+4 & -2+0 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 0 & -2 \end{bmatrix}$$

$$A*C + B*C = \begin{bmatrix} 15 & 6 \\ 10 & -1 \end{bmatrix}$$

$\therefore (A+B)C = A*C + B*C$

$k = 2$ //واجب.

show that

$(kA).B = k(AB)$

المحددات

« Determinats »

For each square matrix, there is a number called determinats and denoted by
يرمز

det. A , |A|

* If $A = [2] \longrightarrow |A| = |2| = 2$

* If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow |A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - cb$

In general : If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$$|A| = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11} \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

Ex: If $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ find det. A ?

Sol

$$|A| = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix} = 6 - (-4) = 10$$

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Ex: If $A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 2 & 0 & 5 \\ 3 & -1 & 2 \\ 4 & -2 & 3 \end{bmatrix} \end{array} \leftarrow$ find det. A?

* نختار الصف الذي يحتوي على أصفار .

Sol

$$|A| = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 2 & 0 & 5 \\ 3 & -1 & 2 \\ 4 & -2 & 3 \end{bmatrix} \end{array} = 2\left[-3-(-4)\right] - 0\left[9-(8)\right] + 5\left[-6-(-4)\right]$$

$$= 2 - 0 - 10 = -8$$

Ex: If $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ find det. A?

Sol

$$|A| = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix} \end{array} = 1\left[12-(0)\right] - 3\left[4-(-4)\right]$$

$$= 12 - 24 = -12$$

* مصفوفة الأشارات

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

Ex: If $A = \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 3 \\ -3 & -2 & 4 \end{bmatrix}$ find det. A

Sol:

$$|A| = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & 1 & -1 \\ -2 & -1 & 3 \\ -3 & -2 & 4 \end{vmatrix} \end{array} = 2[-4-(-6)] - 1[-8-(-9)] + 1[4-3]$$

$$= 4 - 1 - 1 = 2$$

Ex:

$A = \begin{bmatrix} 5 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & -2 & -1 & 3 \\ 0 & -3 & -2 & 4 \end{bmatrix}$ find det. A?

Sol:

$$|A| = \begin{array}{c} + \\ - \\ + \\ - \end{array} \begin{vmatrix} 5 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 \\ 0 & -2 & -1 & 3 \\ 0 & -3 & -2 & 4 \end{vmatrix} = 5 \begin{vmatrix} 2 & 1 & -1 \\ -2 & -1 & 3 \\ -3 & -2 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 & 0 \\ -2 & -1 & 3 \\ -3 & -2 & 4 \end{vmatrix}$$

$$+ 0 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 1 & -1 \\ -3 & -2 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 1 & 0 \\ 2 & 1 & -1 \\ -2 & -1 & 3 \end{vmatrix}$$

$$= 5 \begin{vmatrix} 2 & 1 & -1 \\ -2 & -1 & 3 \\ -3 & -2 & 4 \end{vmatrix} = 5 * 2 = 10$$

Ex: H.w

If $A = \begin{bmatrix} 2 & -1 & 3 \\ -2 & 0 & -1 \\ -4 & -3 & 2 \end{bmatrix}$ find det. A

H.w 2 $A = \begin{bmatrix} 5 & 0 & 1 & 0 \\ 0 & 2 & 1 & -1 \\ -1 & -2 & -1 & 3 \\ 0 & -3 & -2 & 4 \end{bmatrix}$

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« Inverse of the Matrix »

To find the inverse of the square matrix A^{-1}

1- find the det. A
2- write down the minor of A (min A)
3- write down the cofactor of A (cof. A)
4- find the adjoint of A by transposing Cof A.
5-

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

* شرط كون المصفوفة مربعة ،

Ex:

If $A = \begin{bmatrix} 2 & 0 \\ 1 & -3 \end{bmatrix}$ find A^{-1} ?

Sol

$$|A| = \begin{vmatrix} 2 & 0 \\ 1 & 3 \end{vmatrix} = 6$$

$$\min A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

$$\text{cof } A = \min A \cdot \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

$$\text{cof } A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$

تحويل كل صف الى عامود

$$\text{adj } A = (\text{cof. } A)^T = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

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Ex: If $A = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ find A^{-1}

Sol

$$|A| = \begin{vmatrix} \overset{+}{1} & \overset{-}{3} & \overset{+}{0} \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{vmatrix} \leftarrow \quad = 1[12] - 3[4-(-4)] = 12 - 24 = -12$$

نجد مصفوفة ال min

$$\min A = \begin{bmatrix} 12 & 8 & 6 \\ 6 & 2 & 3 \\ 12 & 4 & 0 \end{bmatrix}$$

نجد مصفوفة ال cof

$$\text{Cof}\,A = \min.A \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} 12 & -8 & 6 \\ -6 & 2 & -3 \\ 12 & -4 & 0 \end{bmatrix}$$

نجد

$$\text{adj}\,A = (\text{cof}\,A)^T = \begin{bmatrix} 12 & -6 & 12 \\ -8 & 2 & -4 \\ 6 & -3 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}\,A}{|A|} = \frac{-1}{12} \begin{bmatrix} 12 & -6 & 12 \\ -8 & 2 & -4 \\ 6 & -3 & 0 \end{bmatrix}$$

Ex : If $A = \begin{bmatrix} 2 & 3 & -4 \\ 3 & -2 & 5 \\ 1 & 4 & -3 \end{bmatrix}$ find A^{-1}

Sol

$$|A| = \begin{vmatrix} 2 & 3 & -4 \\ 3 & -2 & 5 \\ 1 & 4 & -3 \end{vmatrix} = 2[6-20] - 3[-9-5] - 4[12-(-2)]$$

$$= -28 + 42 - 56 = -42$$

$$\min A = \begin{bmatrix} -14 & -14 & 14 \\ 7 & -2 & 5 \\ 7 & 22 & -13 \end{bmatrix}$$

$$\operatorname{cof} A = \min A \cdot * \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} = \begin{bmatrix} -14 & 14 & 14 \\ -7 & -2 & -5 \\ 7 & -22 & -13 \end{bmatrix}$$

$$= \operatorname{adj} A = (\operatorname{cof} A)^T = \begin{bmatrix} -14 & -7 & 7 \\ 14 & -2 & -22 \\ 14 & -5 & -13 \end{bmatrix}$$

$$A^{-1} = \frac{\operatorname{adj} A}{|A|} = \frac{-1}{42} \begin{bmatrix} -14 & -7 & 7 \\ 14 & -2 & -22 \\ 14 & -5 & -13 \end{bmatrix}$$

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Rank of the Matrix

اكبر مصفوفه مربعه قيمة الديتيرمنت لما لا يساوي صفر

It is the order of the heighest square matrix with det. does not equal zero. If A is a square matrix (n*n). ∴ it has rank = n if and only if ⇒ det A ≠ 0.

Ex: find the rank of the

$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -1 & 7 & 4 & 9 \\ 7 & -7 & 0 & -5 \end{bmatrix}_{3\times 4}$$

نلغي أحد الأعمدة

Sol/

$$|A_1| = \begin{vmatrix} 0 & 2 & 2 \\ 7 & 4 & 9 \\ -7 & 0 & 5 \end{vmatrix}$$

$$|A_5| = \begin{vmatrix} 3 & 0 \\ -1 & 7 \end{vmatrix} = 21 \neq 0$$

$$|A_2| = \begin{vmatrix} 3 & 2 & 2 \\ -1 & 4 & 9 \\ 7 & 0 & -5 \end{vmatrix}$$

∴ rank = 2

$$|A_3| = \begin{vmatrix} 3 & 0 & 2 \\ -1 & 7 & 9 \\ 7 & -7 & -5 \end{vmatrix}$$

$$|A_4| = \begin{vmatrix} 3 & 0 & 2 \\ -1 & 7 & 4 \\ 7 & -7 & 0 \end{vmatrix}$$

* ليش لغينا أحد الاعمدة
لان حتى تصير مصفوفه مربعة

* فيديو توضيحي

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« Solving Equation using matrices »

If we have (n) eq. with (m) unknown Variables.
for example:

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

These eq. Can be writen in matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad AX = B$$

هنالك ثلاث طرق لحل المعادلات

1- Grammer's Rule
2- Inverse of the matrix
3- Gauss Elemination

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1. Gramer's Rule

$$x = \frac{|A_x|}{|A|} \qquad y = \frac{|A_y|}{|A|} , \qquad z = \frac{|A_z|}{|A|}$$

$$|A_x| = \begin{bmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{bmatrix} , \quad |A_y| = \begin{bmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{bmatrix}$$

$$|A_z| = \begin{bmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b \end{bmatrix}$$

Ex: using Gramer's rule solve the following Eq.

$$2x - 3y + 4z = -19$$
$$4y + 6x - 2z = 8$$
$$4z + x + 5y = 23$$

ملاحظة: يجب عدد المتغيرات يساوي عدد المعادلات

١- نستخرج المصفوفة A والمصفوفة B

٢- نحسب ال {Determinats} المصفوفة A

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Sol

$$2x - 3y + 4z = -19$$
$$6x + 4y - 2z = 8$$
$$x + 5y + 4z = 23$$

$$A = \begin{bmatrix} 2 & -3 & 4 \\ 6 & 4 & -2 \\ 1 & 5 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} -19 \\ 8 \\ 23 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 2 & -3 & 4 \\ 6 & 4 & -2 \\ 1 & 5 & 4 \end{bmatrix} = 2|16+10| + 3|24+2| + 4|30-4|$$
$$= 234$$

$$|A_x| = \begin{bmatrix} -19 & -3 & 4 \\ 8 & 4 & -2 \\ 23 & 5 & 4 \end{bmatrix} = -19[16+10] + 3[32+46] + 4[40-92]$$
$$= -468$$

$$|A_y| = \begin{bmatrix} 2 & -19 & 4 \\ 6 & 8 & -2 \\ 1 & 23 & 4 \end{bmatrix} = 2[32+46] + 19[24+2] + 4[138-8]$$
$$2 * 78 + 19 * 26 + 4 * 130$$
$$156 + 494 + 520 = 1170$$

$$|A_z| = \begin{bmatrix} 2 & -3 & -19 \\ 6 & 4 & 8 \\ 1 & 5 & 23 \end{bmatrix} = 2[92-40] + 3[138-8] - 19[30-4]$$
$$2 * 52 + 3 * 130 - 19 * 26$$
$$104 + 390 - 494 = 0$$

$$x = \frac{|A_x|}{|A|} = \frac{-468}{234} = -2$$

$$y = \frac{|A_y|}{|A|} = \frac{1170}{234} = 5$$

$$z = \frac{|A_z|}{|A|} = \frac{0}{234} = 0$$

$x = -2$
$y = 5$
$z = 0$

Ex: solve the Linear system eq. using Gramer's rule.

$$x + 2y - z =$$
$$2x - y + z = 4$$
$$3x + y = 7$$

Sol/

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 3 & 1 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 4 \\ 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

$$A = 3[2-1] - 1[1+2] = 0$$

because of $|A| = 0$ there is no sol. for this system eq.

② Inverse of the matrix:

$$\frac{Ax}{A} = \frac{B}{A} \quad \div A \quad \Rightarrow \quad x = \frac{B}{A} \quad \Rightarrow \quad \boxed{x = A^{-1}B}$$

شروط أستخدام هذه الطريقة : ١- أن الـ Determinants لايساوي صفر
٢- أن تكون مربعة.

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Ex: Solve the following system by inverse matrix ;

$$x + 2y = 1$$
$$x - y = 4$$

sol/

$A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix} = -1 - 2 = -3$

$\min A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

* بما أن ال Determinats لا يساوي صفر
فـ كمل الحل أحسن لك.

$Cof.A = \begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix}$

$adj\, A = \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{adj\, A}{|A|} = -\dfrac{1}{3} \begin{bmatrix} -1 & -2 \\ -1 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}$

$X = A^{-1} B$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (\frac{1}{3} + \frac{8}{3}) \\ (\frac{1}{3} - \frac{4}{3}) \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

$x = 3$

$y = -1$

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Ex : Solve the following system by inverse of the matrix

$$x + 2y + 2z = 5$$
$$3x - 2y + z = -6$$
$$2x + y - z = -1$$

Sol

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 2 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ -6 \\ -1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & -2 & 1 \\ 2 & 1 & -1 \end{vmatrix} = 25$$

$$\min A = \begin{bmatrix} 1 & -5 & 7 \\ -4 & -5 & -3 \\ 6 & -5 & -8 \end{bmatrix}$$

$$\text{cof. } A \begin{bmatrix} 1 & 5 & 7 \\ 4 & -5 & 3 \\ 6 & 5 & -8 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 1 & 4 & 6 \\ 5 & -5 & 5 \\ 7 & 3 & -8 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{25} \begin{bmatrix} 1 & 4 & 6 \\ 5 & -5 & 5 \\ 7 & 3 & -8 \end{bmatrix}$$

$$X = A^{-1} B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{25} & \frac{4}{25} & \frac{6}{25} \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \\ \frac{7}{25} & \frac{3}{25} & -\frac{8}{25} \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{5}{25} - \frac{24}{25} - \frac{6}{25} \\ \frac{5}{5} + \frac{6}{5} - \frac{1}{5} \\ \frac{35}{25} - \frac{18}{25} + \frac{8}{25} \end{bmatrix} = \begin{bmatrix} \frac{-25}{25} \\ \frac{10}{5} \\ \frac{25}{25} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

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Ex: Solve the following system

$$x - y + z = 4$$
$$2x + y + z = 7$$
$$2z - x - 2y + 1 = 0$$

Sol

أرتبكة هذا المثال

$$x - y + z = 4$$
$$2x + y + z = 7$$
$$-x - 2y + 2z = -1$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -2 & 2 \end{bmatrix} , \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} , \quad B = \begin{bmatrix} 4 \\ 7 \\ -1 \end{bmatrix}$$

$$A = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ -1 & -2 & 2 \end{vmatrix} = 6$$

$|A| = 6$

$|Ax| = 18$

$|Ay| = 0$

$|Az| = 6$

$$\text{min } A = \begin{bmatrix} 4 & 5 & -3 \\ 0 & 3 & -3 \\ -2 & -1 & 3 \end{bmatrix}$$

$$x = \frac{|Ax|}{|A|} = \frac{18}{6} = 3$$

$$y = \frac{|Ay|}{|A|} = \frac{0}{6} = 0$$

$$\text{cof } A = \begin{bmatrix} 4 & -5 & -3 \\ 0 & 3 & 3 \\ -2 & 1 & 3 \end{bmatrix}$$

$$z = \frac{|Az|}{|A|} = \frac{6}{6} = 1$$

Date : / /   Subject:

③ Gauss Elimination

Ex: Solve the linear System.

$$x_2 - x_1 + 2x_3 = 2$$
$$x_3 + 3x_1 - x_2 = 6$$
$$3x_2 - x_1 + 4x_3 = 4$$

SOL

1- Arrange the variable.

$$-x_1 + x_2 + 2x_3 = 2$$
$$3x_1 - x_2 + x_3 = 6$$
$$-x_1 + 3x_2 + 4x_3 = 4$$

2- write down the Augmented matrix

$[A : B]$

$$\tilde{A} = \left[\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 3 & -1 & 1 & 6 \\ -1 & 3 & 4 & 4 \end{array}\right]$$

مبدأ جاوس

3- eliminate the 1st element from row 2 and row 3 depending on row 1

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 2 & 2 & 2 \end{array}\right] \begin{array}{l} \\ R_2 + 3R_1 \\ R_3 - R_1 \end{array}$$

④ Eliminate the 2nd element from Row 3 depending on row 2

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 2 \\ 0 & 2 & 7 & 12 \\ 0 & 0 & -5 & -10 \end{array}\right] \begin{array}{l} \\ \\ R_3 - R_2 \end{array}$$

$$-5x_3 = -10 \Rightarrow \boxed{x_3 = 2}$$

شرح الطريقة ٢

في البداية نرتب المعادلات . ثم نصفر العامود الاول تاوطي رقم من اليسار والثالث أيضاً

الساع الثاني

$$2x_2 + 7x_3 = 12$$

$$2x_2 + 14 = 12$$

$$\boxed{x_2 = \frac{-2}{2} = -1}$$

الساع الاول

$$-1x_1 + 1 + 4 = 2 \quad (\times -1)$$

$$\boxed{x_1 = 1}$$

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Date :   / /   Subject:

Ex: Solve the following Eq.

$$3x_1 + 2x_2 + x_3 = 3$$

$$2x_1 + x_2 + x_3 = 0$$

$$6x_1 + 2x_2 + 4x_3 = 6$$

Sol

$$\tilde{A} = \left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 6 & 2 & 4 & 6 \end{array}\right]$$

نصفرهم

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -2 \\ 0 & -2 & 2 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - \frac{2}{3}R_1 \\ R_3 - 2R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 0 & -\frac{1}{3} & \frac{1}{3} & -2 \\ 0 & 0 & 0 & 12 \end{array}\right] \begin{array}{l} \\ \\ R_3 - 6R_2 \end{array}$$

$$0 \times 3 = \frac{12}{0}$$

There is no Solution for these eqs because of $(0 = 12)$

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Date : / / Subject:

Ex: Solve the following Eqs

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$0.6x_1 + 1.5x_2 + 1.5x_3 - 5.4x_4 = 2.7$$
$$1.2x_1 - 0.3x_2 - 0.3x_3 + 2.4x_4 = 2.1$$

Sol/

$$\tilde{A} = \left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0.6 & 1.5 & 1.5 & -5.4 & 2.7 \\ 1.2 & -0.3 & -0.3 & 2.4 & 2.1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & -1.1 & -1.1 & 4.4 & -1.1 \end{array}\right] \begin{array}{l} \\ R_2 - 0.2R_1 \\ R_3 - 0.4R_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 3 & 2 & 2 & -5 & 8 \\ 0 & 1.1 & 1.1 & -4.4 & 1.1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R_3 + R_2 \\ \\ \end{array}$$

* نطلع للساف الفوق ⇑ لان {صفر = صفر}

$$1.1x_2 + 1.1x_3 - 4.4x_4 = 1.1 \quad \div 1.1$$

$$x_2 + x_3 - 4x_4 = 1 \qquad x_2 = 1 - x_3 + 4x_4 \quad \text{—} \ (1)$$

$$3x_1 + 2x_2 + 2x_3 - 5x_4 = 8$$
$$3x_1 + 2(1 - x_3 + 4x_4) + 2x_3 - 5x_4 = 8$$
$$3x_1 + 3x_4 = 6 \qquad \boxed{\text{Let } x_4 = 1 \quad x_1 = 1}$$
$$x_1 + x_4 = 2$$

{في هالحالة مالا نهاية من الحلول لكل هكذا معادلات}

There is infinity of solution Because of we let the value of x_4 to find the other variable

Date : / / Subject:

Ex: for the following Currents find the Value of currents I_1, I_2, I_3 ?

$$I_2 = I_3 + I_1$$
$$10I_2 + 20I_1 = 80$$
$$25I_3 + 10I_2 = 90$$

Sol

$$I_1 - I_2 + I_3 = 0$$
$$20I_1 + 10I_2 = 80$$
$$10I_2 + 25I_3 = 90$$

$$\tilde{A} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 20 & 10 & 0 & 80 \\ 0 & 10 & 25 & 90 \end{array}\right]$$

* نحوّل الصف الأخير الى الأعلى
كما تعتقد أنشا فيكة؟!!

$$\tilde{A} = \left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 20 & 10 & 0 & 80 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 30 & -20 & 80 \end{array}\right] R_3 - 20R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 10 & 25 & 90 \\ 0 & 0 & -95 & -190 \end{array}\right] R_3 - 3R_2$$

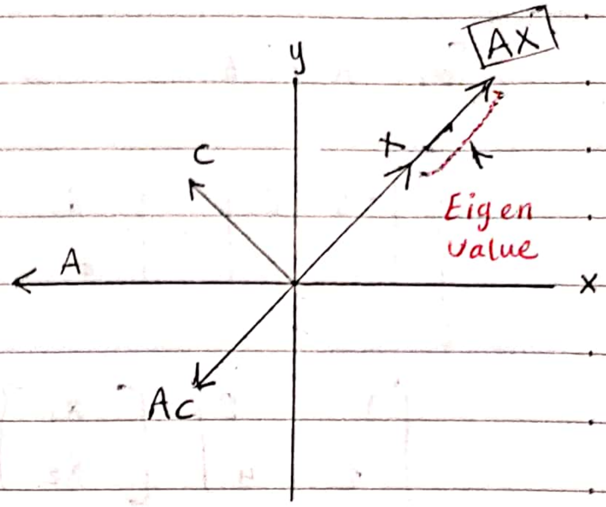
$$-95I_3 = -190$$
$$\boxed{I_3 = 2}$$

$$10I_2 + 25I_3 = 90$$
$$10I_2 = 40$$
$$\boxed{I_2 = 4}$$

$$I_1 - I_2 + I_3 = 0$$
$$I_1 - 4 + 2 = 0$$
$$\boxed{I_1 = 2}$$

قيم ذاتية Eigen Value and Eigen vector متجهات ذاتية

$$Ax = \lambda x$$



Where

A : is a Square matrix $(n \times n)$

x : is eigen vector

λ : is eigen value

$$Ax - \lambda x = 0$$

$$[A - \lambda I] = 0 \quad \text{Charactristic Eq.}$$

Ex: find the eigen values and eigen vectors of the following matrix.

$$A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

Sol/

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} -5-\lambda & 2 \\ 2 & -2-\lambda \end{bmatrix} = 0$$

$$(-5-\lambda)(-2-\lambda) - 4 = 0$$

$$10 + 5\lambda + 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 + 7\lambda + 6 = 0$$

$$(\lambda + 6)(\lambda + 1) = 0$$

$$\boxed{\lambda_1 = -6 \qquad \lambda_2 = -1}$$

eigen Value

Date : / / Subject:

$[A - \lambda I]X = 0$

for $\lambda = -6$

$$\left[\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix}\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 + 2x_2 = 0$

$2x_1 + 4x_2 = 0$

$x_1 = -2x_2$

Let $x_1 = 2$

$\therefore x_2 = -1$

$$X_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

for $\lambda = -1$

$$\begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-4x_1 + 2x_2 = 0$

$2x_1 - x_2 = 0$

$x_1 = \frac{x_2}{2} \Rightarrow$ let $x_1 = 2$

$x_2 = 4$

$$X_2 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

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Date : / / Subject:

Ex: find the eigen values and eigen vector if following matrix.

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

sol/

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix} = 0$$

$$\lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda_{1,2} = \pm\sqrt{-i} = \pm j$$

$$[A - \lambda I] X = 0$$

for $\lambda = j$

$$\left(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} j & 0 \\ 0 & j \end{bmatrix} \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} -j & 1 \\ -1 & -j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$-jx_1 + x_2 = 0$$

$$-x_1 - jx_2 = 0$$

$x_2 = jx_1$ Let. $x_1 = 1$ $x_2 = j$ $X_1 = \begin{bmatrix} 1 \\ j \end{bmatrix}$

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Date : / / Subject:

for $\lambda = -j$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} -j & 0 \\ 0 & -j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} +j & 1 \\ -1 & j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$j x_1 + x_2 = 0$$
$$- x_1 + j x_2 = 0$$

$$x_2 = -j x_1$$

Let $x_1 = 1$
$x_2 = -j$

$$X_2 = \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

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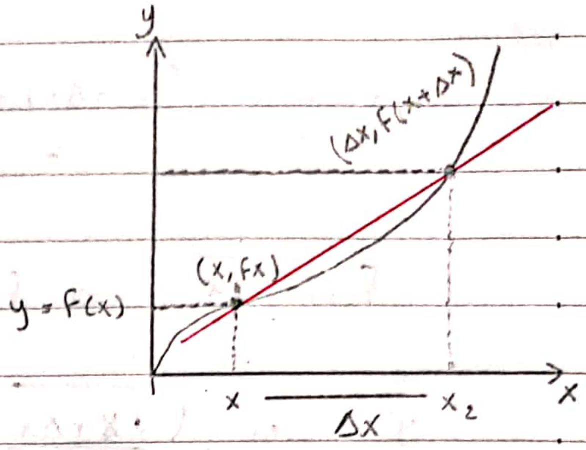
« Differentiation » مشتقة



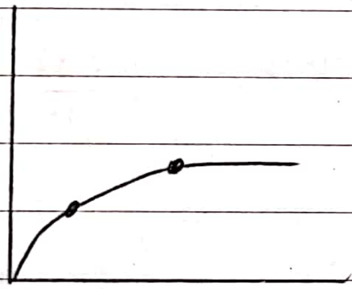
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{f(x+\Delta x) - f(x)}{x + \Delta x - x}$$

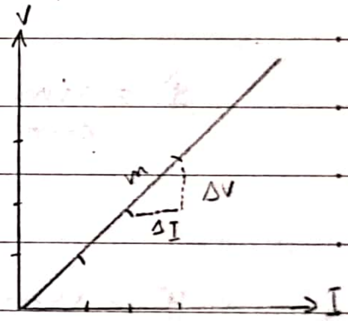
$$= \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{dy}{dx} = \frac{\Delta y}{\Delta x} = m = y'$$

في حالة جهاز متغير عند زيادة التيار



$R = \frac{V}{I}$



Ex: find $\frac{dy}{dx}$ if $y(x) = x^2$ using Limits.

Sol

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x) = y(x) = x^2$$

$$f(x+\Delta x) = (x+\Delta x)^2$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - x^2}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x (2x + \Delta x)}{\Delta x}$$

$$\frac{dy}{dx} = 2x + 0 = 2x$$

Date : / / Subject:

Ex: If $y = \sqrt{x}$ find y' using limits

$$y' = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x) = \sqrt{x} \Rightarrow f(x+\Delta x) = \sqrt{x+\Delta x}$$

$$y' = \lim_{\Delta x \to 0} \frac{\left(\sqrt{x+\Delta x} - \sqrt{x}\right)}{\Delta x} * \frac{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}{\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$y' = \lim_{\Delta x \to 0} \frac{x + \Delta x + \sqrt{x}\sqrt{x+\Delta x} - \sqrt{x}\sqrt{x+\Delta x} - x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x\left(\sqrt{x+\Delta x} + \sqrt{x}\right)}$$

$$y' = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x} + \sqrt{x}}$$

$$y' = \frac{1}{2\sqrt{x}}$$

* كل x نعوضها بـ $(x+\Delta x)$

Differentiation Rule

1- $\frac{d}{dx} c = 0$

2- $\frac{d}{dx} x^n = n x^{n-1}$

3- $\frac{d}{dx} cx^n = n \cdot c x^{n-1}$

4- $\frac{d}{dx}(g(x) \mp f(x)) = \frac{d}{dx} g(x) \mp \frac{d}{dx} f(x)$

5- $\frac{d}{dx}(f(x) \times g(x)) = f(x) \frac{d\, g(x)}{dx} + g(x) \frac{d\, f(x)}{dx}$

6- $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \frac{d\, f(x)}{dx} - f(x) \frac{d\, g(x)}{dx}}{(g(x))^2}$

7- $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} \cdot \frac{d\, f(x)}{dx}$

Ex: find y' if

1- $y(x) = 6x^3 + 5x + 10 \Rightarrow y' = 18x^2 + 5$

2- $y(x) = \frac{1}{x^4} + x^{-\frac{3}{4}} \Rightarrow y(x) = x^{-4} + x^{-\frac{3}{4}}$

$\Rightarrow y'(x) = -4x^{-5} - \frac{3}{4} x^{-\frac{7}{4}}$

3- $y(x) = (x^2+1)(x^3+3) \Rightarrow y' = (x^2+1)(3x^2) + (x^3+3)(2x)$

$\Rightarrow 3x^4 + 3x^2 + 2x^4 + 6x$

$\Rightarrow 5x^4 + 3x^2 + 6x$

$\frac{\text{Up - Down}}{\text{Down}}$

99% of people didn't know where it place

Date : / /

Subject:

Ex: find $\frac{dy}{dx}$ if $y(t) = \frac{t^2-1}{t^3+1}$

$$\frac{d}{dt} = \frac{(t^3+1)\,2t - (t^2-1)\,3t^2}{(t^3+1)^2}$$

$$= \frac{2t^4+2t-3t^4+3t^2}{(t^3+1)^2} = \frac{-t^4+3t^2+2t}{(t^3+1)^2}$$

Date : / / Subject:

Differentiation of trigonometric functions

1- $\frac{d}{d\theta} \sin u = \cos u \frac{du}{d\theta}$

2- $\frac{d}{d\theta} \cos u = -\sin u \frac{du}{d\theta}$

3- $\frac{d}{d\theta} \tan u = \sec^2 u \frac{du}{d\theta}$

4- $\frac{d}{d\theta} \cot u = -\csc^2 u \frac{du}{d\theta}$

5- $\frac{d}{d\theta} \sec u = \sec u \tan u \frac{du}{d\theta}$

6- $\frac{d}{d\theta} \csc u = -\csc u \cot u \frac{du}{d\theta}$

Ex: find y' if $y(x) = \sin^2 x^2 = (\sin x^2)^2$

Sol $y' = 2 \sin x^2 (\cos x^2) \cdot (2x)$

$= 4x \sin x^2 \cos x^2$

Ex: Proof that $\frac{d}{d\theta} \tan\theta = \sec^2\theta$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\frac{d}{d\theta} \tan\theta = \frac{(\cos\theta)^2 + (\sin\theta)^2}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta$

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Date : / /

Subject:

Ex: find $\frac{dy}{dx}$ if $y(x) = \frac{\cos x}{1-\sin x}$

Sol

$$y(x) = \frac{(1-\sin x)(-\sin x) - (\cos x)(-\cos x)}{(1-\sin x)^2}$$

$$= \frac{-\sin x + \sin^2 x + \cos^2 x}{(1-\sin x)^2} = \frac{-\sin x + 1}{(1-\sin x)^2} = \frac{1}{(1-\sin x)}$$

« The Chain Rule »

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad \left.\begin{matrix} y(u) \\ u(x) \end{matrix}\right] \frac{dy}{dx}$$

قاعدة السلسلة

Ex: find $\frac{dy}{dx}$ if $y(t) = \tan(5 - \sin 2t)$

Sol

1 ـ نجد مشتقة ما داخل القوس ثم نفرضه u

Let $u(t) = 5 - \sin 2t \rightarrow \frac{du}{dt} = -2\cos 2t$

$\therefore y(u) = \tan u \longrightarrow \frac{dy}{du} = \sec^2 u$

2 ـ نطبق قانون السلسلة

$$\frac{dy}{dt} = \frac{dy}{du} \cdot \frac{du}{dt} \rightarrow \frac{dy}{dt} = \sec^2 u \cdot (-2\cos 2t)$$

$$= \sec^2(5 - \sin 2t)(-2\cos 2t)$$

Date : / / Subject:

Ex2: If $y(u) = \cos u$ and $u(x) = x^2+1$ then find $\frac{dy}{dx}$?

Sol

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{du} = -\sin u$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = -\sin u \cdot 2x$$

$$\frac{dy}{du} = -2x \sin(x^2+1)$$

{الاشتقاق الضمني} « Implicit differentiation »

$$x^2 + y^2 + 3xy = 0 \Rightarrow x^2 = -y^2 - 3xy \Rightarrow x^2 = y(-y-3x)$$

The above eq. we can't put in form of $y = f(x)$ therefore we use Implicit differentiation.

خطوات الاشتقاق الضمني :

1 - نشتق طرفي المعادلة بالنسبة لـ x

2 - نعزل الحدود التي تحتوي $\frac{dy}{dx}$ بأحد طرفي المعادلة ومن ثم نجد قيمة $\frac{dy}{dx}$

ملاحظة مهمة جداً : عندما نشتق y نضرب في $\frac{dy}{dx}$ ، وأن x تمثل ثابت

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Date : / /   Subject:

Ex: find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$

SoL:

$$2y\frac{dy}{dx} = 2x + \cos xy \left(x \cdot \frac{dy}{dx} + y\right)$$

$$2y\frac{dy}{dx} - x\cos xy\frac{dy}{dx} = 2x + y\cos xy$$

$$\frac{dy}{dx}(2y - x\cos xy) = 2x + y\cos xy$$

$$\boxed{\frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}}$$

Ex: find $\frac{dy}{dx}$ if $y^3 + 3x^2y^2 + xy + x^2 = 2$

$$3y^2\frac{dy}{dx} + \left[3x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 6x\right] + \left[x\frac{dy}{dx} + y\right] + 2x = 0$$

التحقق

$$3y^2\frac{dy}{dx} + 6x^2y\frac{dy}{dx} + x\frac{dy}{dx} = -6xy^2 - y - 2x$$

حسب نعزل الحدود

$$\frac{dy}{dx} = \frac{-6xy^2 - y - 2x}{3y^2 + 6x^2y + x}$$

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معادلة المماس — معادلة الخط العامودي

Ex 4: find the tangent line and normal to the curve.

$$\frac{x-y}{x-2y} = 2 \rightarrow \text{at } (3,1)$$

Sol:

$$x - y = 2x - 4y$$

$$1 - \frac{dy}{dx} = 2 - 4\frac{dy}{dx}$$

$$4\frac{dy}{dx} - \frac{dy}{dx} = 2 - 1$$

$$3\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{3}$$

$$\frac{y-y_1}{x-x_1} = m$$

نقطة وميل نستخدمة

tangent line

$$\frac{y-1}{x-3} = \frac{1}{3} \Rightarrow 3(y-1) = x-3 \Rightarrow 3y = x \Rightarrow \boxed{y = \frac{1}{3}x}$$

$$m_{normal} = \frac{-1}{m_{tangent}} \Rightarrow m_n = \frac{-1}{\frac{1}{3}} = -3$$

$$\frac{y-1}{x-3} = -3 \Rightarrow -3x + 9 = y - 1 \Rightarrow \boxed{y = -3x + 10} \text{ Normal.}$$

« Partial Derivatives » — «المشتقات الجزئية»

$$z = f(x,y)$$

$\frac{dz}{dx}$ اشتقاق بالنسبة لـ x

$\frac{dz}{dy}$ اشتقاق بالنسبة لـ y

مشتقة ثانية $\frac{d^2z}{dx\,dy}$

$$z_x , \quad z_y , \quad z_{xy}$$

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Date : / /

Subject:

Ex: find $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$

If $f(x,y) = y \sin xy$

هنا المشتقة جزئية يعني
إذا x ثابت y متغير واذا
x متغير y ثابت. وشكراً

Sol:

$$\frac{\partial f}{\partial x} = y \cos xy \cdot y = y^2 \cos xy$$

$$\frac{\partial f}{\partial y} = y \cos xy (x) + \sin xy$$

$$\frac{\partial f}{\partial y} = yx \cos xy + \sin xy$$

Ex: verify that $f_{xy} = f_{yx}$, if $f(x,y) = x \sin y + y \sin x + xy$

Sol:

$$f_x = \sin y + y \cos x + y$$

$$f_{xy} = \cos y + \cos x + 1$$

$$f_y = x \cos y + \sin x + x$$

$$f_{yx} = \cos y + \cos x + 1$$

$$\therefore f_{xy} = f_{yx}$$

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نشق

Ex : A body is moving on a coordinate Line with ~~S(t)~~
$S(t) = t^2 - 3t + 2$ in meter and t in Sec where $0 < t < 2$

الإزاحة

1 - find the body displacement and avarage Velocity for the given time interval.

فترة

2 - find the speed and acceleration of the body at endpoints of the interval.

$$\text{Displacement} = \Delta S = S(0) - S(2)$$

$S(0) = 2$ $\quad S(2) = 0$ $\quad \Delta S = 2 - 0 = 2\,m$

$$\text{Speed ava.} = \frac{\Delta S}{\Delta t} = \frac{2}{2} = 1\ m/sec$$

$v(t) = 2t - 3$

$t = 0$ $\quad |v(0)| = |-3| = 3\ m/s$

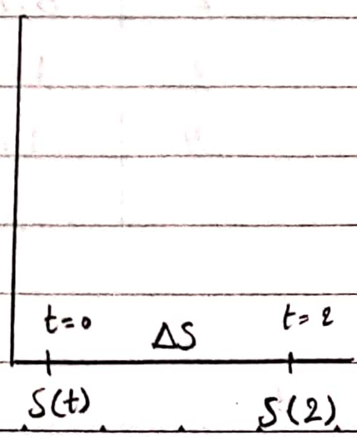
$t = 2$ $\quad |v(2)| = |4 - 3| = 1\ m/s$

نشتق

يطلع

$a(t) = 2\ m/s^2$ $\quad t = 0$, $\quad t = 2$

ولاكن لايوجد مكان للتعويض هههه.



Functions

algebraic

دوال جبريه

$y(x) = mx + c$

Transcendental

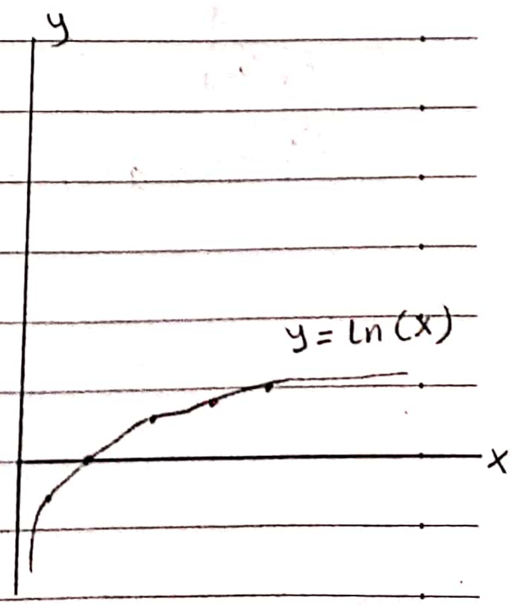
دوال فاصله

- $\ln(x)$ — مقلوبها ↳ e^x
- $\log_a(x)$ — مقلوبها ↳ a^x
- Trigonometric — دوال مثلثية — sin, cos, tan
- Hyperbolic — دالة زائدية — $\sinh(x)$, $\cosh(x)$

Ln (X) (Natural Log)

$F(x) = \ln(x) \longrightarrow x > 0$

x	Ln(x)
0	unde...
0.5	-0.7
1	0
2	0.7
3	1.0
4	1.4



Date : / /

Subject:

1- $\frac{d}{dx} \ln u(x) = \frac{1}{u(x)} \frac{du}{dx}$

2- $\int \frac{1}{u(x)} du = \ln u(x) + C$

{ Properties of Ln }

1- $\ln(ab) = \ln(a) + \ln(b)$

ملاحظة : a,b هي متغيرات أو ثوابت

2- $\ln \frac{a}{b} = \ln a - \ln b$

Ex 3- $\ln\left(\frac{1}{b}\right) = \ln 1 - \ln b = -\ln b$

4- $\ln a^n = n \ln a$

Ex: use properties of Ln to simplify the following expression.

1- $\ln \sin\theta - \ln \frac{\sin\theta}{5} = \ln\left(\frac{\sin\theta}{\frac{\sin\theta}{5}}\right) = \ln 5$

2- $\ln \sec\theta + \ln\cos\theta = \ln(\sec\theta \cdot \cos\theta) = \ln\left(\frac{1}{\cos\theta} \cdot \cos\theta\right)$

$= \ln(1) = 0$

3- $\ln(3x^2 - 9x) + \ln\left(\frac{1}{3x}\right) = \ln\left(\frac{3x^2 - 9x}{3x}\right)$

$\ln \frac{(3x(x-3))}{3x} = \ln x-3$

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4- $3 \ln \sqrt[3]{t^2-1} - \ln(t+1)$

$= 3 \ln (t^2-1)^{\frac{1}{3}} - \ln(t+1)$

$= 3 \frac{1}{3} \ln (t^2-1) - \ln(t+1)$

$= \ln \frac{t^2-1}{t+1} = \ln\left(\frac{t^2-1}{t+1}\right) = \ln\left(\frac{(t-1)(t+1)}{(t+1)}\right)$

$= \boxed{\ln t-1}$

5- $\frac{1}{2} \ln (4t^4) - \ln 2$

$= \ln \sqrt{4t^4} - \ln 2 = \ln 2t^2 - \ln 2$

$= \ln \frac{2t^2}{2} = \ln t^2$

Ex: find $\frac{dy}{dx}$

a) $y(x) = \ln 2x$ $\qquad \frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

b) $y(x) = \ln(x^2+3)$ $\qquad \frac{dy}{dx} = \frac{1}{x^2+3} \cdot 2x = \frac{2x}{x^2+3}$

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c) $y = \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1}$

Sol $\ln y = \ln \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{(x-1)}$

$\ln y = \ln (x^2+1)(x+3)^{\frac{1}{2}} - \ln (x-1)$

$\ln y = \ln (x^2+1) + \ln (x+3)^{\frac{1}{2}} - \ln (x-1)$

$\ln y = \ln (x^2+1) + \frac{1}{2} \ln (x+3) - \ln (x-1)$

$$\left[\frac{1}{y} \frac{dy}{dx} = \left[\frac{2x}{x^2+1} + \frac{1}{2}\left(\frac{1}{x+3}\right) - \frac{1}{(x-1)} \right] \right] * y$$

$$\frac{dy}{dx} = y \left[\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1} \right]$$

$$\frac{dy}{dx} = \frac{(x^2+1)(x+3)^{\frac{1}{2}}}{x-1} * \left[\frac{2x}{x^2+1} + \frac{1}{2x+6} - \frac{1}{x-1} \right]$$

« Exponential »

$y(x) = e^{x} = \exp(x) = \ln^{-1}(x)$

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Important Rules of e^x

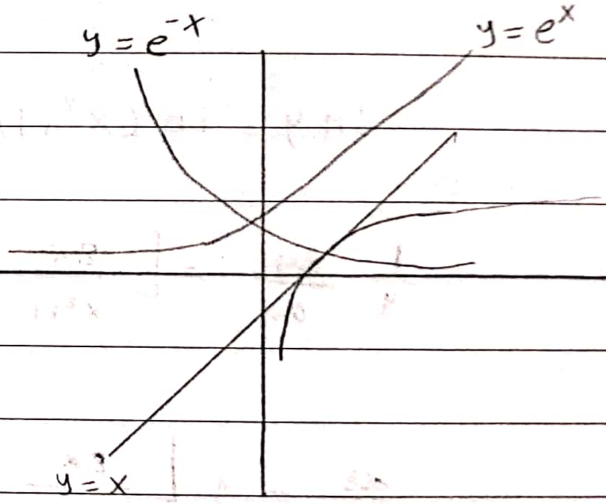
1- $e^{x_1} \cdot e^{x_2} = e^{x_1 + x_2}$

2- $e^{-x_1} = \dfrac{1}{e^{x_1}}$

3- $\dfrac{e^{x_1}}{e^{x_2}} = e^{x_1} \cdot e^{-x_2} = e^{x_1 - x_2}$

4- $(e^{x_1})^{x_2} = e^{x_1 x_2} = (e^{x_2})^{x_1}$

✓ 5- $e^{\ln x} = x$



✓ 6- $\ln e^x = x$

✓ 7- $\dfrac{d}{dx} e^u = e^u \dfrac{du}{dx}$

✓ $\int e^u \, du = e^u + c$

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Ex : put the following expressions in form of $y = f(x)$

that is mean

الهدف: بصفحة بصفحة

جماعة X = y

1- $\ln(y-1) - \ln 2 = x + \ln x$

$$\ln \frac{y-1}{2} = x + \ln x$$

$$e^{\ln\left(\frac{y-1}{2}\right)} = e^{(x+\ln x)} \Rightarrow \frac{y-1}{2} = e^{x} e^{\ln x} \Rightarrow \frac{y-1}{2} = x e^{x}$$

$$y - 1 = 2x e^{x} \Rightarrow \boxed{y = 2x e^{x} + 1}$$

2- $\ln(y^2-1) - \ln(y+1) = \ln(\sin x)$

$$\ln \frac{y^2-1}{y+1} = \ln \sin x$$

$$\ln(y-1) = \ln \sin x$$

$$e^{\ln(y-1)} = e^{\ln(\sin x)}$$

$$y - 1 = \sin x$$

$$y = \sin x + 1$$

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Ex: $y(x) = e^{\sin x}$

$$\frac{dy}{dx} = e^{\sin x} \cdot \cos x$$

2) $y(x) = (1+2x)\, e^{-2x}$

$$\frac{dy}{dx} = -2(1+2x)\, e^{-2x} + 2\, e^{-2x}$$

$$= -2e^{-2x} - 4x\, e^{-2x} + 2\, e^{-2x} = -4x\, e^{-2x}$$

General Log

$y = \log_a x$ when $a = e^1$

$y = \log_{e^1} x \Rightarrow \ln(x)$

when $a = 2$

$y = \log_2 x$

when $a = 10$

$y = \log_{10} x = \log x$

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Subject:

Inverse of general Log.

$y = a^x$

Ex: Evaluate the following expressions:

1- $\log_8 64 = x$

$8^x = 64 \Rightarrow 8^x = 8^2.$ $\boxed{x = 2}$

2- $3^x = 81 \Rightarrow 9 \times 9 = 3^2 . 3^2 = 3^4$ $\boxed{x = 4}$

3- $\log_{10} 1000 = X$

$10^x = 1000 \Rightarrow 10^x = 10^3 \Rightarrow \boxed{x = 3}$

4- $\log 0.0001 = X$

$10^x = 10^{-4} \Rightarrow \boxed{x = -4}$

5- $\log_a 1 = X$

$a^x = 1 \Rightarrow a^x = 1^0 \Rightarrow x = 0$

6- $\log_{16} 8 = X$

$16^x = 8 \Rightarrow 2^{4x} = 2^3 \Rightarrow 4X = 3$

$\Rightarrow \boxed{X = \frac{3}{4}}$

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7- $\log_8 x = \frac{-4}{3} \Rightarrow 8^{-\frac{4}{3}} = x$

$$x = \frac{1}{8^{\frac{4}{3}}} = \frac{1}{\sqrt[3]{8^4}} = \frac{1}{8\sqrt[3]{8}} = \boxed{\frac{1}{16}}$$

Ex: 2/ Solve the following Eq.

$$\log(x-1) + \log(x+8) = 2\log(x+2)$$

$$(\log(x-1) + \log(x+8)) = (2\log(x+2))$$

$$\log(x-1)(x+8) = \log(x+2)^2$$

Now take the inverse of log

$$10^{\log(x-1)(x+8)} = 10^{\log(x+2)^2}$$

$$(x-1)(x+8) = (x+2)^2$$

$$x^2 + 8x - x - 8 = x^2 + 4x + 4$$

$$3x - 12 = 0$$

$$3x = 12$$

$$\boxed{x = 4}$$

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Drivative of log

$$\frac{d}{dx}\log_a(u) = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot \frac{du}{dx}$$

where $u = F(x)$

Drivative of a^u

$$\frac{d}{dx} a^u = \ln a \cdot a^u \frac{du}{dx}$$

Ex: Find the derivative of.

1- $y = \log_{10}(3x+1)$

$$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{3}{(3x+1)}$$

2- $y = \log_3(1+Q\ln 3)$

$$\frac{dy}{dQ} = \frac{1}{\cancel{\ln 3}} \cdot \frac{\ln 3}{(1+Q\ln 3)}$$

$$\frac{dy}{dQ} = \frac{1}{1+Q\ln 3}$$

3. $y = 3^{\sin x}$

$$\frac{dy}{dx} = (\ln 3)(3^{\sin x})(\cos x)$$

4. $y = 5^{-\cos 2t}$

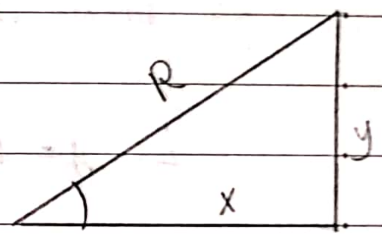
$$\frac{dy}{dt} = (\ln 5) \cdot 5^{-(\cos 2t)} \cdot (2 \sin 2t)$$

Trigonometric functions

$\sin \theta = \frac{y}{r}$, $\tan \theta = \frac{y}{x}$

$\cos \theta = \frac{x}{r}$, $\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y}$

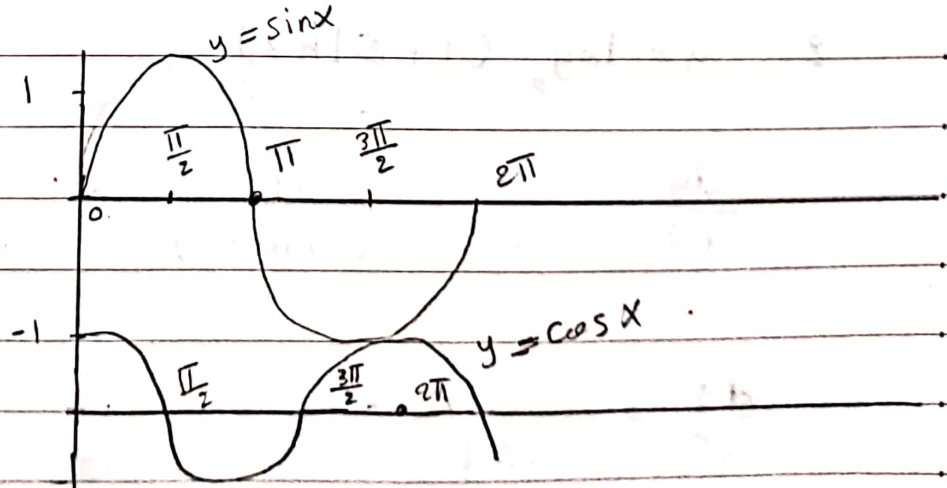
$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}$, $\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y}$



$\cos -\theta = \cos \theta$

$\sec -\theta = \sec \theta$

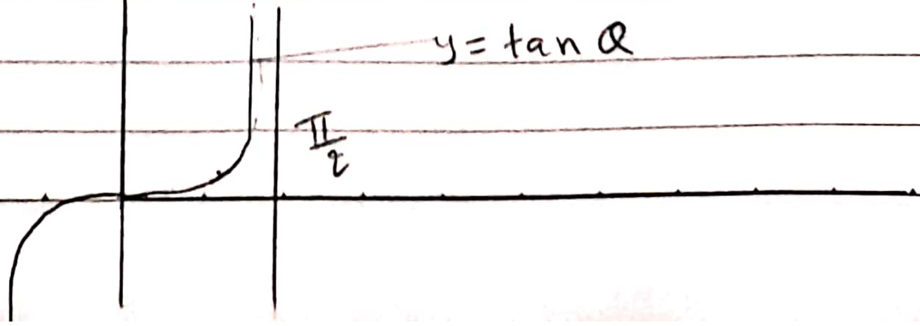
$\sin -\theta = -\sin \theta$



$\cos \pi = -1$
$\cos(-\pi) = -1$

$\sin \frac{\pi}{2} = 1$

$\sin -\frac{\pi}{2} = -1$



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Subject:

- $\sin^2 Q + \cos^2 Q = 1$
- $\sin^2 Q = \dfrac{1 - \cos 2Q}{2}$
- $\cos^2 Q = \dfrac{1 + \cos 2Q}{2}$
- $1 + \tan^2 Q = \sec^2 Q$
- $1 + \cot^2 Q = \csc^2 Q$
- $\sin 2Q = 2 \sin Q \cos Q$
- $\cos 2Q = \cos^2 Q - \sin^2 Q$

$\cos -Q = \cos Q$

$\sec -Q = \sec Q$

$\sin -Q = -\sin Q$

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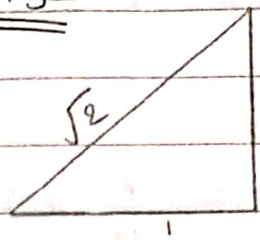
توضيحات



$$\sin\theta = \frac{1}{\sqrt{2}} \qquad \theta = \frac{\pi}{4}$$

$$\sin^{-1}(\sin\theta) = \sin^{-1}\frac{1}{\sqrt{2}}$$

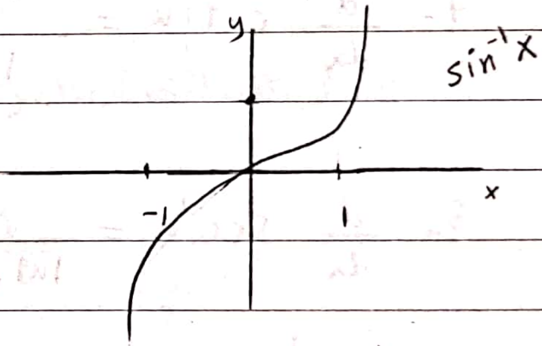
$$\theta = \sin^{-1}\frac{1}{\sqrt{2}} \quad , \quad \theta = \frac{\pi}{4}$$

$$\frac{1}{\sin\theta} = (\sin\theta)^{-1} \neq \sin^{-1}\theta$$

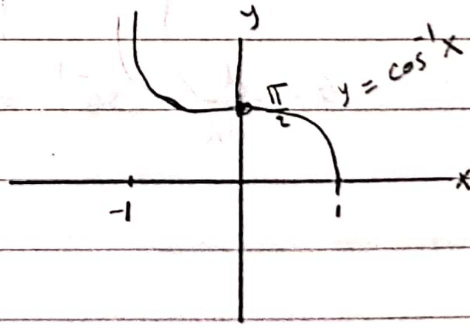
$$\theta = \frac{\pi}{4}$$

« Inverse of trigonometric functions »

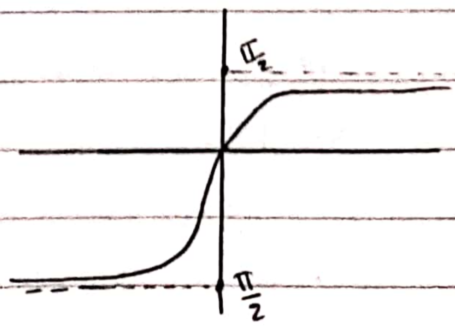
$y = \sin^{-1} x$



$y = \cos^{-1} x$



$y = \tan^{-1} x$



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Dreviative of the inverse of trigonometric functions.

1_ $\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$

2_ $\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$

3_ $\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}$

4_ $\frac{d}{dx} \cot^{-1} u = \frac{-1}{1+u^2} \frac{du}{dx}$

5_ $\frac{d}{dx} \sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}$

6_ $\frac{d}{dx} \csc^{-1} = \frac{-1}{|u|\sqrt{u^2-1}} = \frac{du}{dx}$

Ex: prove $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$

$\sin^2\theta + \cos^2\theta = 1$

$\cos\theta = \sqrt{1-\sin^2\theta}$

Sol

$y = \sin^{-1} x$

$\sin y = \sin(\sin^{-1} x) \Rightarrow \sin y = x$

$$\cos y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

Ex:2/ A partical moves a long the X-axis so that its position any time $t \geq 0$ is $x(t) = \tan^{-1}\sqrt{t}$ what is the velocity of the partical when $t = 16$?

المشتقة الأولى للمسافة هي السرعة.

Sol

$$V(t) = \frac{dx(t)}{dt} = \frac{1}{1+(\sqrt{t})^2} \cdot \frac{1}{2\sqrt{t}} = \frac{1}{2\sqrt{t}\,(1+t)}$$

When $t=16$ $\quad V(16) = \frac{1}{2\times 4\,(1+16)} = \frac{1}{136}$ m/sec

---

Ex: find $\frac{d}{dx} \sec^{-1}(5x^4)$

Sol $\frac{d}{dx} \sec^{-1}(5x^4) = \frac{20x^3}{|5x^4|\sqrt{(5x^4)^2-1}} = \frac{4}{x\sqrt{25x^8-1}}$

$\frac{1}{\sin\theta} \neq \sin^{-1}\theta$ / ملاحظة / مهمة جداً *

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Ex: find $\frac{d}{dx}$ $y = \csc^{-1}(x^2+1$

Sol/

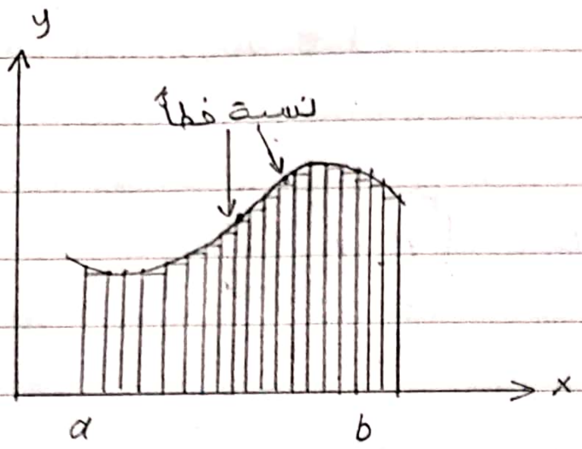
$$= \frac{-2x}{|x^2+1|\sqrt{(x^2+1)^2-1}}$$

$$= \frac{-2x}{|(x^2+1)|\sqrt{(x^2+1)^2-1}}$$

Ex: $y = \tan^{-1}(\ln(x))$

Sol $y' = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x} = \frac{1}{x+x(\ln(x))^2}$

Date : / / Subject: التكامل



Δx $\quad\quad$ $\Delta x \to 0$

شون نحسب المساحة

بالتكامل تطلع نقسمها الى
قطع مستطيلة ومتساوية

$$a_1 = f(x)\,\Delta x$$

مجموع المساحات $A = a_1 + a_2 + \ldots$

$$= f(x)\,\Delta x + f_2(x)\,\Delta x + \ldots$$

$$\sum_{n=1} f(x)\,\Delta x_n \qquad A = \lim_{\Delta x \to 0} \sum_{n=1} f(x)\,\Delta x_n = \boxed{\int_a^b f(x)\,dx}$$

{There are two types of integral}

1- finit Integral $\int_a^b f(x)\,dx = F(b) - F(a) = \text{constant (area)}$

2- In finit Integral $\int f(x)\,dx = F(x) + C \Rightarrow$ Anti-Derivative

Some properties of Integral

1- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

2- $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

3- $\int_a^a f(x)\,dx = 0$

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Date : / / Subject:

قانونين بخصوص التكامل.

$1 - \int du = u + C$ مساوي ← تكامل أي ثابت

$2 - \int u^n \, du = \frac{u^{n+1}}{n+1} + C \qquad \boxed{n \neq -1}$

Ex: find the integral of :.

$1. \int 5x - x^2 + 2 \, dx = \frac{5x^2}{2} - \frac{x^3}{3} + 2x + C$

$2. \int \sqrt{2x+1} \, dx = \int (2x+1)^{\frac{1}{2}} dx \times \frac{2}{2}$

$= \frac{1}{2} \frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{(2x+1)^{\frac{3}{2}}}{3} + C$

Ex: find $y(x)$ if $\frac{dy}{dx} = x\sqrt{1+x^2}$ at $x=0, y=-3$

Sol $\frac{dy}{dx} = x\sqrt{1+x^2} \Rightarrow \int dy = \int x\sqrt{1+x^2} \, dx$

$\Rightarrow y = \int x(1+x^2)^{\frac{1}{2}} dx \times \frac{2}{2}$

$\Rightarrow y = \frac{1}{2} \frac{(1+x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{(1+x^2)^{\frac{3}{2}}}{3} + C$

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at $x = 0$, $y = -3$

$$-3 = \frac{(1+0)^{\frac{3}{2}}}{3} + c \Rightarrow -3 - \frac{1}{3} = c \Rightarrow c = \frac{-10}{3}$$

$$y = \frac{(1+x^2)^{\frac{3}{2}}}{3} - \frac{10}{3}$$

✓ Ex: find the integral of

$$\int \frac{3r}{\sqrt{1-r^2}}\, dr$$

Sol:

$$= \int 3r(1-r^2)^{-\frac{1}{2}}\, dr$$

$$= -\frac{3}{2} \frac{(1-r^2)^{\frac{1}{2}}}{\frac{1}{2}} + c \Rightarrow -3(1-r^2)^{\frac{1}{2}} + c$$

Ex: $\int \frac{3r}{\sqrt{1-r^2}}\, dx = \frac{3r}{\sqrt{1-r^2}}\, x + c$

لانه بالنسبه لـ x

فهذا يعتبر ثابت

Date : / / Subject:

Integration of trigonometric function

1- $\int \sin u \, du = -\cos u + c$

2- $\int \cos u \, du = \sin u + c$

3- $\int \sec^2 u \, du = \tan u + c$

4- $\int \csc^2 u \, du = -\cot u + c$

5- $\int \sec u \tan u \, du = \sec u + c$

6- $\int \csc u \cot u \, du = -\csc + c$

Ex: $\int \tan u \, du = \int \frac{\sin u}{\cos u} du \quad * \frac{-1}{-1}$

$= -1 \ln|\cos u| + c = \ln|\sec u| + c$???

Ex: find the integral of

Sol $\int \cos 2t \, dt \quad * \frac{2}{2} = \frac{1}{2} \sin 2t + c$

$$\int \frac{\cos 2x}{\sin^3 2x}\,dx = \int \cos 2x \sin^{-3} 2x \, dx \quad * \frac{2}{2}$$

$$= \frac{1}{2} \frac{\sin^{-2} 2x}{-2} + c = \frac{-1}{4 \sin^2 2x} + c$$

$$\int (1+\cos\theta)^3 \sin\theta \, d\theta \quad * \frac{-1}{-1}$$

$$= \frac{-(1+\cos\theta)^4}{4} + c$$

Ex: find the Integral $\int \frac{1+\cos 2x}{\sin^2 2x}\,dx$

Sol:

$$= \int \frac{1}{\sin^2 2x}\,dx + \int \frac{\cos 2x}{\sin^2 2x}\,dx$$

$$= \int \csc^2 2x \, dx + \int \cos 2x \sin^{-2} 2x \, dx$$

$$= \frac{-\cot 2x}{2} + \frac{1}{2}\left(\frac{\sin^{-1} 2x}{-1}\right) + c$$

$$= \frac{-\cot 2x}{2} - \frac{1}{2 \sin 2x} + c$$

$$= -\frac{\cot 2x}{2} - \frac{\csc 2x}{2} + c$$

Date : / /

Subject:

Ex:

$$\int \sin 3x \cos 7x \, dx$$

Sol

for Sure

قانونا زيوو زكب القوانين

$$\boxed{\sin a \cos b = \frac{1}{2}(\sin(a+b)) + \frac{1}{2}(\sin(a-b))}$$

$$\int \frac{1}{2}(\sin 10x) + \frac{1}{2}(\sin -4x)\, dx$$

$$= \frac{-1}{20}\cos 10x + \frac{1}{8}\cos 4x + c$$

الدوال

Integration of Exponent Functions تكامل الأسية

1- $\int e^u \, du = e^u + c$

2 $\int a^u \, du = \frac{a^u}{\ln a} + c$

الدوال

Integration of log Functions تكامل اللوغارتمية

1- $\int \frac{1}{u} \, du = \ln|u| + c$

2- $\int \log_a u \, du = \int \frac{\ln u}{\ln a} \, du$

Ex: find the integral of :

1- $\displaystyle\int_0^2 \frac{2x}{x^2-5}\,dx = \left[\ln|x^2-5|\right]_0^2$

$= \ln(1) - \ln(5) = -\ln 5$

2- $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{4\cos\theta}{3+2\sin\theta}\,d\theta$

$\displaystyle = 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2\cos\theta}{3+2\sin\theta}\,d\theta$

$= 2\left[\ln|3+2\sin\theta|\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2\left[\ln(5) - \ln(1)\right] = 2\ln 5$

3- $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1+e^{\cot\theta})\csc^2\theta\,d\theta$

$\displaystyle = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \csc^2\theta\,d\theta + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} e^{\cot\theta}\csc^2\theta\,d\theta$

$= -\cot\theta\Big]_{\frac{\pi}{4}}^{\frac{\pi}{2}} + \left[-e^{\cot\theta}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= [0+1] + [-1+e^1] = e^1 \cong 2.7$

اي حتا اكل درجة كاملة

4- $\displaystyle\int_0^{\frac{\pi}{2}} 7^{\cos t}\sin t\,dt$ نضرب ونقسم على (-1)

$\displaystyle = \left[-\frac{7^{\cos t}}{\ln 7}\right]_0^{\frac{\pi}{2}} = \frac{-1}{\ln 7}\left[7^0 - 7^1\right]$

$\displaystyle = \frac{-1}{\ln 7}[-6] = \frac{6}{\ln 7}$

Date : / / Subject:

5- $\int \frac{\log_2 x}{x}\, dx = \int \frac{\ln x}{x \ln 2}\, dx$

$= \frac{1}{\ln 2} \int \frac{\ln x}{x}\, dx = \frac{1}{\ln 2} \frac{(\ln x)^2}{2} + c$

6- $\int \frac{e^x - e^{-x}}{\sqrt{e^x + e^{-x}}}\, dx = \int e^x - e^{-x} (e^x + e^{-x})^{-\frac{1}{2}}\, dx$

$= \frac{(e^x + e^x)^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{e^x + e^{-x}} + c$

« Solve the following integral »

1- $\int \cot^3 y \csc^2 y\, dy$

$= \frac{-(\cot y)^4}{4} + c$

2- $\int_0^{\sqrt{\ln 2}} 2x e^{x^2}\, dx$

$= e^{x^2} \Big]_0^{\sqrt{\ln 2}} = e^{\ln 2} - e^0 = 1$

3- $\int \frac{1 + \sin x}{\cos^2 x}\, dx$

$= \int \frac{1}{\cos^2 x}\, dx + \int \frac{\sin x}{\cos^2 x}\, dx$

$= \int \sec^2 x\, dx + \int \sin x \cos^{-2} x\, dx$ $\frac{-1}{-1}$ ×

$= \tan x - \frac{(\cos^{-1} x)}{-1} + c = \tan x + \sec x + c$

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تضرب x مرافق المقام

4- $\int \frac{1}{1+\sin x}\,dx \quad * \quad \frac{1-\sin x}{1-\sin x}$

$$\int \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx = \int \frac{1-\sin x}{1-\sin^2 x}\,dx = \int \frac{1-\sin x}{\cos^2 x}\,dx$$

5- $\int \frac{2\log(x+1)}{(x+1)}\,dx$

$$= \frac{2}{\ln 10}\int \frac{\ln(x+1)}{x+1}\,dx$$

$$= \frac{2}{\ln 10}\,\frac{(\ln(x+1))^2}{2} + c = \frac{(\ln(x+1))^2}{\ln 10} + c$$

ملاحظات :

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Some integrals produces of trigonometric function

1- $$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$$

2- $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c$$

3- $$\int \frac{du}{|u|\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c$$

where $u = f(x)$ a : is constant.

ملاحظات :

1) أولاً نطلع قيم du ، u ، u^2 ، a

ثانياً نطبق القانون

Ex: Solve the following Integral.

(A) $\int \frac{x}{\sqrt{1-x^4}}\,dx \quad * \frac{2}{2}$

$\alpha = 1$

$u^2 = x^4$

$u = x^2$

$du = 2x\,dx$

$= \int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c$

$= \frac{1}{2}\int \frac{2x\,dx}{\sqrt{1-x^4}} = \frac{1}{2}\sin^{-1}\left(\frac{x^2}{1}\right) + c$

(B) $\int \frac{1}{e^x\sqrt{1-e^{-2x}}}\,dx \quad * \frac{-1}{-1}$

$a = 1$

$u^2 = e^{-2x}$

$u = e^{-x}$

$du = -e^{-x}\,dx$

$= -\int \frac{-e^{-x}\,dx}{\sqrt{1-e^{-2x}}}$

$= -1\sin^{-1}(e^{-x}) + c = -\sin^{-1}(e^{-x}) + c$

c) $\int \frac{1}{1+e^x}\,dx \qquad \int \frac{1}{1+\frac{1}{e^{-x}}}\,dx$

$= \int \frac{1}{\frac{e^{-x}+1}{e^{-x}}}\,dx = \int \frac{e^{-x}}{e^{-x}+1}\,dx \quad * \frac{-1}{-1}$

$= -\ln|e^{-x}+1| + c$

فكرة جيدة

Date : / / Subject:

d) $\int \frac{5^x}{5^{2x}+1}\,dx \quad * \frac{\ln 5}{\ln 5}$

$a=1$

$u^2 = 5^{2x}$

$u = 5^x$

$du = 5^x \ln 5\, dx$

$= \frac{1}{\ln 5} \int \frac{5^x \ln 5\, dx}{(5^{2x}+1)}$

$= \frac{1}{\ln 5}\left[\tan^{-1}(5x) + c\right]$

« Integrated of hyperbolic functions »

1~ $\int \sinh u\, du = \cosh u + c$

2~ $\int \cosh u\, du = \sinh u + c$

3~ $\int \operatorname{sech}^2 u\, du = \tanh u + c$

4~ $\int \operatorname{csch}^2 u\, du = -\coth u + c$

5~ $\int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u + c$

6~ $\int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u + c$

Ex: Evalute the following in integrals

(A) $\int 4\cosh(3x+\ln 2)\,dx \quad * \frac{3}{3}$

$= \frac{4}{3}\int 3\cosh(3x+\ln 2)\,dx$

$= \frac{4}{3}\sinh(3x+\ln 2)+c$

(B) $\int \frac{\operatorname{sech}(\ln x)\cdot\tanh(\ln x)}{x}\,dx$

$= -\operatorname{sech}(\ln x)+c$

(C) $\int 4e^{-\theta}\sinh\theta\,d\theta$

$= \int 4e^{-\theta}\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)d\theta$

$= \int 2(e^{0}-e^{-2\theta})\,d\theta$

$= \int 2-2e^{-2\theta}\,d\theta = [2\theta+e^{-2\theta}]+c$

H.W $\int \frac{1}{e^{x}+e^{-x}}\,dx$

The key to the Solution.

$* \frac{e^{x}}{e^{x}}$

Date : / /   Subject: Chapter 2

« Power of trigonometric functions »

Ⓐ $\sin^n x$, $\cos^n x$ { When $n \to$ odd }

Ex: $\int \sin^3 x \, dx$

في حالة تجي آس فردي نسحب وحدي
منه تصير زوجي

Sol: $= \int \sin^2 x \sin x \, dx$ $\{\sin^2 x + \cos^2 x = 1\}$

$= \int (1 - \cos^2 x) \sin x \cdot dx$

$= \int (\sin x \, dx) - \int (\cos^2 x \sin x \, dx)$

$= \int \sin x \, dx - \int \cos^2 x \sin x \, dx$

$= -\cos x + \dfrac{\cos^3 x}{3} + C$

Ex: $\int \cos^5 x \, dx$

Sol: $= \int \underbrace{\cos^4 x}_{(\cos^2 x)^2} \cos x \, dx = \int (1 - \sin^2 x)^2 \cos x \, dx$

$= \int (1 - 2\sin^2 x + \sin^4 x) \cos x \, dx$

$= \int \cos x \, dx - \int 2 \sin^2 x \cos x \, dx + \int \sin^4 x \cos x \, dx$

$= \sin x - 2 \dfrac{\sin^3 x}{3} + \dfrac{\sin^5 x}{5} + C$

Date : / /

Subject:

- Ex: $\int \sin^3 x \cos^3 x \, dx$

■ عند وجود دالتين وكلاهما فردي، نأخذ وحدي والثانية تبقى.

Sol:

$$= \int \sin^3 x \cos^2 x \cos x \, dx$$

$$= \int \sin^3 x \, (1 - \sin^2 x) \cos x \, dx$$

$$= \int \sin^3 x \cos x \, dx - \int \sin^5 x \cos x \, dx$$

$$= \frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} + C$$

Ex: $\int \sin^2 x \cos^3 x \, dx$

Sol: $= \int \sin^2 x \cos^2 x \cos x \, dx$

$$= \int \sin^2 x \, (1 - \sin^2 x) \cos x \, dx$$

$$= \int \sin^2 x \cos x \, dx - \int \sin^4 x \cos x \, dx$$

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$

Date : / / Subject:

Ex: $\int \sin^7 x \, dx$ $\qquad \sin^6 x = (\sin^2 x)^3$

Sol: $= \int \sin^6 x \sin x \, dx = \int (\sin^2 x)^3 \sin x \, dx$

$= \int (1-\cos^2 x)^3 \sin x \, dx$

$= \int (1-2\cos^2 x+\cos^4 x)(1-\cos^2 x) \sin x \, dx$

$= \int (1-2\cos^2 x+\cos^4 x-\cos^2 x+2\cos^4 x-\cos^6 x) \sin x \, dx$

$= \int \sin x \, dx - \int 3\cos^2 x \sin x \, dx + \int 3\cos^4 x \sin x \, dx$

$- \int \cos^6 x \sin x \, dx$

$= -\cos x + 3\dfrac{\cos^3 x}{3} - 3\dfrac{\cos^5 x}{5} + \dfrac{\cos^7 x}{7} + C$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$
$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

when $n \longrightarrow$ even

Date : / /

Subject:

Ex: $\int \sin^2 x \, dx$

Sol: $= \int \frac{1}{2}(1-\cos 2x)\,dx = \int \frac{1}{2}\,dx - \int \frac{1}{2}\cos 2x\,dx$

$= \frac{1}{2}x - \frac{1}{2\times 2}\sin 2x + c = \frac{1}{2}x - \frac{1}{4}\sin 2x + c$

Ex: $\int \cos^4 x\,dx$

Sol: $= \int (\cos^2 x)^2\,dx = \int \left(\frac{1}{2}\cdot(1+\cos 2x)\right)^2 dx$

$= \frac{1}{4}\int (1+2\cos 2x + \cos^2 2x)\,dx$

$= \frac{1}{4}\int \left(1+2\cos 2x + \frac{1}{2}(1+\cos 4x)\right)dx$

$= \frac{1}{4}\left[\int 1\,dx + \int 2\cos 2x\,dx + \int \frac{1}{2}\,dx + \int \frac{1}{4\times 2}\cos 4x\,dx\right]$

$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right] + c$

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Date : / / Subject:

Ex: $\int \sin^2 x \cos^2 x \, dx$

إذا جاءت الدالتين زوجيتين

سنعوض القانونين معاً.

Sol: $\int \frac{1}{2}(1-\cos 2x) \cdot \frac{1}{2}(1+\cos 2x)\, dx$

$= \frac{1}{4}\int (1+\cos 2x - \cos 2x - \cos^2 2x)\, dx$

$= \frac{1}{4}\int (1-\cos^2 2x)\, dx$

$= \frac{1}{4}\int \left(1 - \frac{1}{2}(1+\cos 4x)\right) dx$

$= \frac{1}{4}\left[\int 1\, dx - \int \frac{1}{2} dx - \int \frac{1}{2}\cos 4x\, dx\right]$

$= \frac{1}{4}\left[x - \frac{1}{2}x - \frac{1}{8}\sin 4x\right] + C$

$= \frac{1}{8}x - \frac{1}{32}\sin 4x + C$

B- $\sec^n x$, $\csc^n x$

When $n \to$ odd (Coming Soon)

When $n \to$ even $\Rightarrow$

$\boxed{\begin{array}{l} \sec^2 x = 1 + \tan^2 x \\ \csc^2 x = 1 + \cot^2 x \end{array}}$

Ex: $\int \sec^2 x \, dx$

Sol: $\tan x + c$

Ex: $\int \sec^4 x \, dx$

Sol: $\int \sec^2 x \sec^2 x \, dx = \int (1 + \tan^2 x) \sec^2 x \, dx$

$= \int \sec^2 x \, dx + \int \tan^2 x \sec^2 x \, dx$

$= \tan x + \frac{\tan^3 x}{3} + c$

Ex $\int \csc^6 x \, dx$

Sol: $= \int \csc^4 x \csc^2 x \, dx = \int (\csc^2 x)^2 \csc^2 x \, dx$

$= \int (1 + \cot^2 x)^2 \csc^2 x \, dx$

$= \int (1 + 2\cot^2 x + \cot^4 x) \csc^2 x \, dx$

$= \int \csc^2 x \, dx + \int 2\cot^2 x \csc^2 x \, dx + \int \cot^4 x \csc^2 x \, dx$

$= -\cot x - 2\frac{\cot^3}{3} - \frac{\cot^5}{5} + c$

C. Power of tan, cot

$$\boxed{\begin{array}{l} \sec^2 x = \tan^2 x + 1 \\ \csc^2 x = \cot^2 x + 1 \end{array}}$$

لايوجد فرق
إذا زوجي أوفردي

- $\int \tan^2 x \, dx$

$$\int (\sec^2 x - 1) \, dx = \int \sec^2 x \, dx - \int 1 \, dx = \tan x - x + C$$

- $\int \tan^3 x \, dx$

$$\int \tan x \tan^2 x \, dx = \int \tan x (\sec^2 x - 1) \, dx$$

$$= \int \tan \sec^2 x \, dx - \int \tan x \, dx$$

$$= \frac{\tan^2 x}{2} - \int \frac{\sin x}{\cos x} \, dx = \frac{\tan^2 x}{2} + \ln |\cos x| + C$$

- $\int \cot^4 x \, dx$

$$\int \cot^2 x . \cot^2 x \, dx = \int \cot^2 x (\csc^2 x - 1) \, dx$$

$$= \int \cot^2 x \csc^2 x \, dx - \int \cot^2 x \, dx \qquad \csc^2 x - 1$$

$$= \int \cot^2 x \csc^2 x \, dx - \int \csc^2 x \, dx + \int 1 \, dx$$

$$= -\frac{\cot^3 x}{3} + \cot x + x + C$$

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دائماً نسحب تربيع

$$\int \tan^6 x \, dx = \int \tan^4 x \tan^2 x \, dx$$

$$= \int \tan^4 x (\sec^2 x - 1) \, dx = \int \tan^4 x \sec^2 x \, dx - \int \tan^4 x \, dx$$

$$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x (\sec^2 x - 1) \, dx$$

$$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x \sec^2 x \, dx + \int \tan^2 x \, dx$$

$$= \int \tan^4 x \sec^2 x \, dx - \int \tan^2 x \sec^2 x \, dx + \int (\sec^2 x - 1) \, dx$$

$$= \frac{\tan^5 x}{5} - \frac{\tan^3 x}{3} + \tan x - x + c$$

2 Integration by trigonometric Substitution.

$\sqrt{a^2 - u^2}$, $\sqrt{u^2 - a^2}$, $\sqrt{a^2 + u^2}$

نفرض نفرض

$\sqrt{a^2 - u^2} \rightarrow u = a \sin\theta$

$\sqrt{u^2 - a^2} \rightarrow u = a \sec\theta$

$\sqrt{a^2 + u^2} \rightarrow u = a \tan\theta$

where u is $f(x)$

Q/ $\int \frac{x^2}{\sqrt{9-x^2}}\,dx$

$x = 3\sin Q$

$dx = 3\cos Q\, dQ$

$$= \int \frac{9\sin^2 Q}{\sqrt{9-9\sin^2 Q}} \cdot 3\cos Q\, dQ$$

$$= \int \frac{9\sin^2 Q}{\cancel{3}\sqrt{(1-\sin^2 Q)}} \cdot \cancel{3}\cos Q\, dQ \qquad \boxed{\cos^2 Q}$$

$$= \int \frac{9\sin^2 Q\cos Q\, dQ}{\cos Q} = \int 9\sin^2 Q\, dQ$$

$$= 9\int \frac{1}{2}(1-\cos 2Q)\, dQ = \frac{9}{2}\left[Q - \frac{\sin 2Q}{2}\right] + c \qquad \boxed{\sin 2Q = 2\sin Q\cos Q}$$

الى هنا طلع جزء من

الحل فقط! الجزء الثاني نعوض عن قيمة Q

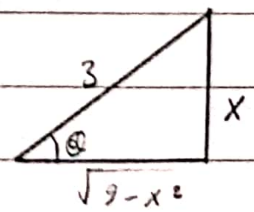
$\frac{x}{3} = \sin Q$

$$\sin^{-1}\frac{x}{3} = \sin^{-1}(\sin Q) \qquad Q = \sin^{-1}\frac{x}{3}$$

$$= \frac{9}{2}\left[\sin^{-1}\frac{x}{3} - \frac{\cancel{2}\sin Q\cos Q}{\cancel{2}}\right] + c$$

$x = 3\sin Q$

$\frac{x}{3} = \sin Q$



$$= \frac{9}{2}\left[\sin^{-1}\frac{x}{3} - \frac{x}{3}\cdot\frac{\sqrt{9-x^2}}{3}\right] + c$$

$\cos Q = \frac{\sqrt{9-x^2}}{3}$

Q/ $\int \frac{x^3}{\sqrt{x^2+16}}\,dx$

$x = 4\tan Q$

$dx = 4\sec^2 Q\,dQ$

$$= \int \frac{64\tan^3 Q}{\sqrt{16\tan^2 Q+16}} \cdot 4\sec^2 Q\,dQ$$

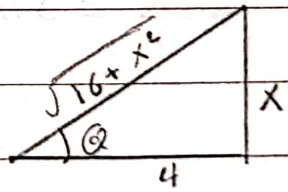
$$= \int \frac{64\tan^3 Q}{\cancel{\sec} Q} \cdot \cancel{\sec^2} Q\,dQ = \int 64\tan^3 Q\sec Q\,dQ$$

$$= \int 64\tan^2 Q\tan Q\sec Q\,dQ$$

$$= 64\int (\sec^2 Q - 1)\tan Q\sec Q\,dQ$$

$$= 64\int \sec^2 Q\sec Q\tan Q\,dQ - \int \tan Q\sec Q\,dQ$$

$$= 64\left[\frac{\sec^3 Q}{3} - \sec Q\right] + C$$



$\tan Q = \frac{x}{4}$

$\sec Q = \frac{1}{\cos Q}$

$$= 64\left[\frac{1}{3}\cdot\left[\frac{\sqrt{16+x^2}}{4}\right]\right]^3 - \left[\frac{\sqrt{16+x^2}}{4}\right] + C$$

Intgration by Parts

التكامل بطعم التجزئة

$\int \ln x \, dx$, $\int \tan^{-1} x \, dx$, $\int x \cos x \, dx$, $\int e^x \cos x \, dx$

- The above integral can not be integrated by normal form. Therefore we use integration by parts.

$$\int u \, dv = uv - \int v \, du$$

The priority of choosing $\underline{u}$

الأولوية للي ياخذ u

1- $\ln x$, $\tan^{-1} x$
2- x^n
3- trigonometric functions
4- Exp. function

- $\int \ln x \, dx$

Let $u = \ln x$

$\frac{du}{dx} = \frac{1}{x} dx$

$\int dv = \int dx$

$v = x$

$$\int u \, dv = uv - \int v \, du$$

$$\int \ln x \, dx = x \ln x - \int \cancel{x} \frac{1}{\cancel{x}} dx$$

$$\int \ln x \, dx = x \ln x - x + C$$

الى هنا نتوقف بالحل.

Date : / / Subject:

- $\int \tan^{-1}x \, dx$

Let $u = \tan^{-1}x$

$du = \frac{1}{1+x^2} dx$

$\int u \, dv = uv - \int v \, du$

$\int dv = \int dx$

$v = x$

$\int \tan^{-1}x \, dx = x \tan^{-1}x - \int x \frac{1}{1+x^2} dx * \frac{2}{2}$

$\int \tan^{-1}x \, dx = x \tan^{-1}x - \frac{1}{2} \ln |1+x^2| + c$

- $\int x e^x \, dx$

Let $u = x \longrightarrow du = dx$

$\int dv = \int e^x dx \rightarrow v = e^x$

$\int u \, dv = uv - \int v \, du$

$\int x e^x dx = x e^x - \int e^x dx$

$\int x e^x dx = x e^x - e^x + c$

Date : / / Subject:

- $\int x^3 e^{x^2} dx = \int x^2 x e^{x^2} dx$

Let $x^3 = u \rightarrow du = 3x^2$
$\int dv = \int e^{x^2} dx \rightarrow v = ?$

Let $u = x^2 \longrightarrow du = 2x\, dx$

$\int dv = \int x e^{x^2} dx * \frac{2}{2} \rightarrow v = \frac{1}{2} e^{x^2}$

we need the derivative of the exponent (2x), therefor we take it from x^3

$\int u\, dv = uv - \int v\, du$

$\int x^3 e^{x^2} dx = \frac{1}{2} x^2 e^{x^2} - \int \frac{1}{2} e^{x^2} 2x\, dx$

$\int x^3 e^{x^2} dx = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + c$

- $\int x^3 \cos x^2 dx = \int x^2 x \cos x^2 dx$

Let $u = x^2 \longrightarrow du = 2x\, dx$
$\int dv = \int x \cos x^2 dx \rightarrow v = \frac{1}{2} \sin x^2$

$\int u\, dv = uv - \int v\, du$

$\int x^2 x \cos x^2 dx = \frac{1}{2} x^2 \sin x^2 - \int \frac{1}{2} \sin x^2 \cdot 2x\, dx$

$\int x^3 \cos x^2 dx = \frac{1}{2} x^2 \sin x^2 - \int \frac{1}{2} \sin x^2 \cdot 2x\, dx$

$\int x^3 \cos x^2 dx = \frac{1}{2} x^2 \sin x^2 + \frac{1}{2} \cos x^2 + c$

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But we have a Special cases حالات خاصة

$\boxed{1}$ $\int x^n \sin x \, dx$, $\int x^n e^x \, dx$

- $\int x \cos x \, dx$

let $u = x \longrightarrow du = 1 \, dx$

$\int dv = \int \cos x \, dx \Rightarrow v = \sin x$

$$\int u \, dv = uv - \int v \, du$$

$$= \int x \cos x \, dx = x \sin x - \int \sin x \, dx$$

$$= \int x \cos x \, dx = x \sin x + \cos x + C$$

Date : / / Subject:

- $\int x^2 \sin x \, dx$

Let $u = x^2 \longrightarrow du = 2x \, dx$

$\int dv = \int \sin x \, dx \longrightarrow v = -\cos x$

$u = x \quad du = 1 dx$

$\int dv = \int \cos x \, dx$

$v = \sin x$

$$\int u \, dv = uv - \int v \, du$$

$$\int x^2 \sin x \, dx = -x^2 \cos x - \int -\cos x \cdot 2x \, dx$$

السؤال كامل

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \boxed{\int x \cos x \, dx}$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \left[x \sin x - \int \sin x \, dx \right]$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x + c$$

- $\int x^5 \cos x \, dx$

u	sign	dv
x^5	$+$	$\cos x$
$5x^4$	$-$	$\sin x$
$20x^3$	$+$	$-\cos x$
$60x^2$	$-$	$-\sin x$
$120x$	$+$	$\cos x$
120	$-$	$\sin x$
0		$-\cos x$

$= x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x$
$- 60x^2 \cos x + 120x \sin x + 120 \cos x$
$+ c$

2 $\int e^x \sin x \, dx$

- $\int e^x \sin x \, dx$

Let $u = \sin x \rightarrow du = \cos x \, dx$
$dv = e^x dx \rightarrow v = e^x$

$\int u \, dv = uv - \int v \, du$

$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$ سؤال كامل

Let $u_1 = \cos x \rightarrow du_1 = -\sin x \, dx$
$dv_1 = e^x dx \rightarrow v_1 = e^x$

$\int e^x \sin x \, dx = e^x \sin x - [e^x \cos x + \int e^x \sin x \, dx]$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$\int e^x \sin x \, dx = \frac{1}{2} [e^x \sin x - e^x \cos x] + c$

فكرة

Date : / / Subject:

- $\int \sec x \, dx \times \frac{\sec x + \tan x}{\sec x + \tan x}$ {حالة خاصة}

$$= \int \frac{\sec^2 x + \tan x \sec x}{\sec x + \tan x} \, dx$$

$$= \ln |\sec x + \tan x| + c$$

- $\int \csc x \, dx \times \frac{\csc x + \cot x}{\csc x + \cot x}$

$$= \int \frac{-(-\csc^2 x - \csc x \cot x)}{\csc x + \cot x} = -\ln |\csc x + \cot x| + c$$

- $\int \sec^3 x \, dx \rightarrow \int \sec x \sec^2 x \, dx$ integration by parts

Let $u = \sec x \rightarrow du = \sec x \tan x \, dx$

$dv = \sec^2 x \, dx \rightarrow v = \tan x$

ملاحظة:
- سهلة التكامل نفرضها dv
- سهلة الاشتقاق نفرضها u

$$\int u \, dv = uv - \int v \, du$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \tan x \sec x \tan x \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \tan^2 x \sec x \, dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \left[\int (\sec^2 - 1) \sec x \right] dx$$

$$\int \sec^3 x \, dx = \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

Date : / / Subject: ....

من المثال السابق

$$2\int \sec^3 x\,dx = \sec x \tan x + \int \sec x\,dx$$

$$2\int \sec^3 x\,dx = \sec x \tan x + \ln|\sec x + \tan x|$$

$$\int \sec^3 x\,dx = \frac{1}{2}\left[\sec x \tan x + \ln|\sec x + \tan x|\right] + c$$

- $\int x^2 e^x\,dx$

طريقة الجدول

u	sign	dv
x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
0		e^x

$$\int x^2 e^x\,dx = x^2 e^x - 2xe^x + 2e^x + c.$$

ملاحظة: عندما يأتي x^n لأي أس وجاء معها e^x أو $\sin x$ أو $\cos x$ نستخدم طريقة الجدول

- $\int x^3 \ln x\,dx$

let $u = \ln x \rightarrow du = \frac{1}{x}\,dx$

$dv = x^3\,dx \rightarrow v = \frac{x^4}{4}$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^3 \ln x\,dx = \ln x \,\frac{x^4}{4} - \int \frac{x^{\cancel{4}}}{4}\cdot\frac{1}{\cancel{x}}\,dx$$

$$\int x^3 \ln x\,dx = \frac{x^4}{4}\ln x - \frac{x^4}{16} + c$$

- $\int x^5 e^x \, dx$

u		dv
x^5	$+$	e^x
$5x^4$	$-$	e^x
$20x^3$	$+$	e^x
$60x^2$	$-$	e^x
$120x$	$+$	e^x
120	$-$	e^x
0		e^x

$$\int x^5 e^x \, dx = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120 e^x + C$$

- $\int x^5 e^{x^2} \longrightarrow \int \underbrace{x^4}_{u} \, \underbrace{x e^{x^2}}_{dv} \, dx$

u	dv
x^4	$x.e^{x^2}$
$4x^3$	
$12x^2$	
$24x$	
24	
0	

خطأ هذا السؤال لا يحل بهذه الطريقة
وإنما يحل بالطريقة العادية

Date : / / Subject:

- $\int \cos\sqrt{x}\, dx$

let $z = \sqrt{x}$

$dz = \frac{1}{2\sqrt{x}} dx$

$\int \cos z \; 2z\, dz$

$dx = 2\sqrt{x}\, dz$

$dx = 2z\, dz$

$= 2\int \underbrace{z}_{u} \underbrace{\cos z}_{dv}\, dz$

Partial Fraction

$\boxed{\int \frac{h(x)}{g(x)}\, dx}$

① If the order of $h(x) \geq g(x)$ then we divide $\frac{h(x)}{g(x)}$ and integrat

- $\int \frac{x^2+3x+5}{x+2}\, dx$

$= \int (x+1)\, dx + \int \frac{3}{x+2}\, dx$

$$\begin{array}{r|l} & x+1 \\ \hline x+2 & x^2+3x+5 \\ & \mp x^2 \mp 2x \\ \hline & 0 + x + 5 \\ & \mp x \mp 2 \\ \hline & 0 \quad 3 \end{array}$$

$= \frac{x^2}{2} + x + 3\ln|x+2| + C$

كتبناها هوني لأنه بنفس الدرس

- $$\int \frac{3x^3+5x^2-2x+4}{x^2+1}$$

$$\begin{array}{r} 3x+5 \\ x^2+1 \,\overline{)\, 3x^3+5x^2-2x+4} \\ \mp 3x^3 \mp 0 \mp 3x \\ \hline 0+5x^2-5x+4 \\ \mp 5x^2 \mp 0 \mp 5 \\ \hline -5x-1 \end{array}$$

Integration by parts

$$= \int (3x+5)\,dx + \int \frac{-5x-1}{x^2+1}\,dx$$

$$= \int (3x+5)\,dx - \int \frac{2}{2}\frac{5x}{x^2+1}\,dx - \int \frac{1}{x^2+1}\,dx$$

$$= \frac{3x^2}{2} + 5x - \frac{5}{2}\ln|x^2+1| - \tan^{-1}x + c$$

2- If order of the $h(x) < g(x)$ then we can spreate it into Partial function.

$$\int \frac{h(x)}{g(x)}\,dx = \int \frac{h(x)}{(x-r_1)(x-r_2)} \Longrightarrow \int \frac{A}{(x-r_1)}\,dx + \int \frac{B}{(x-r_2)}\,dx$$

$$\frac{h(x)}{(x^2+r_1)(x+r_2)^2} = \frac{Ax+B}{(x^2+r_1)} + \frac{C}{(x+r_2)} + \frac{D}{(x+r_2)^2}$$

$$\overline{(x^2+r_1)(x+r_2)^2}$$

إذا طلع رنفس هذا يعني أنو ع نمشي ع

- $\int \frac{5x+1}{(2x-1)(x+1)}\,dx$ $\qquad$ $\frac{5x+1}{(2x-1)(x+1)} = \frac{A}{(2x-1)} + \frac{B}{(x+1)}$

$$\frac{5x+1}{(2x-1)(x+1)} = \frac{A(x+1)+B(2x-1)}{(2x-1)(x+1)}$$

$$5x+1 = Ax + A + 2Bx - B$$

$$5x+1 = (A+2B)x + A - B$$

$$A + 2B = 5 \quad \text{——} \quad ①$$

$$A - B = 1 \quad \text{——} \quad ②$$

$$3B = 4 \longrightarrow \boxed{B = \frac{4}{3}}$$

$\boxed{A = \frac{7}{3}}$ نعوض بالمعادلة

$$\int \frac{5x+1}{(2x-1)(x+1)}\,dx = \frac{7}{3}\int \frac{1}{2x-1}\,dx + \frac{4}{3}\int \frac{1}{x+1}\,dx$$

$$= \frac{7}{3}\cdot\frac{1}{2}\ln|2x-1| + \frac{4}{3}\ln|x+1| + c$$

قيمة A $\qquad$ قيمة B

Date : / / Subject:

- $$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx$$

$$= \int \frac{x^2+2x-1}{x(2x^2+3x-2)}\,dx = \int \frac{x^2+2x-1}{x(2x-1)(x+2)}\,dx$$

$$\frac{x^2+2x-1}{x(2x-1)(x+2)} = \frac{A}{x} + \frac{B}{(2x-1)} + \frac{C}{(x+2)}$$

$$\frac{x^2+2x-1}{\cancel{x(2x-1)(x+2)}} = \frac{A(2x-1)(x+2)+Bx(x+2)+Cx(2x-1)}{\cancel{x(2x-1)(x+2)}}$$

يلغى

$$x^2+2x-1 = 2Ax^2+3Ax-2A+Bx^2+2Bx+2Cx^2-Cx$$

$$x^2+2x-1 = (2A+B+2C)x^2+(3A+2B-C)x-2A$$

$2A+B+2C=1$ —— ①

$3A+2B-C=2$ —— ②

$-2A=-1$ —— ③ $\Rightarrow \boxed{A=\frac{1}{2}}$

$B+2C=0$ —— ④

$4B-2C=1$ —— ⑤

$5B=1$ $\quad 2C=-\frac{1}{5}$

$\boxed{B=\frac{1}{5}}$ $\quad \boxed{C=-\frac{1}{10}}$

$$= \frac{1}{2}\int \frac{1}{x}\,dx + \frac{1}{5}\int \frac{1}{2x-1}\,dx - \frac{1}{10}\int \frac{1}{x+2}\,dx$$

$$= \frac{1}{2}\ln|x| + \frac{1}{10}\ln|2x-1| - \frac{1}{10}\ln|x+2| + c$$

Date : / / Subject:

$$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$$

$$\frac{-2x+4}{(x^2+1)(x-1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x-1)} + \frac{D}{(x-1)^2}$$

$A = 2$, $B = 1$, $C = -2$, $D = 1$

$$\int \frac{2x+1}{x^2+1}\,dx - 2\int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$= \int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx - 2\int \frac{1}{x-1}\,dx + \int (x-1)^{-2}\,dx$$

$$= \ln|x^2+1| + \tan^{-1}x - 2\ln|x-1| - (x-1)^{-1} + C$$

Date : / / Subject:

محاضرة الأثنين / أسئلة منوعة ♡

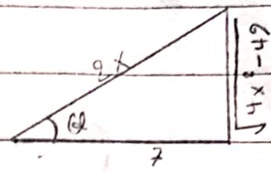
■ $\int \frac{dx}{\sqrt{4x^2-49}}$

$u = 2x$

$a = 7$

$x = \frac{7}{2}\sec\theta$

$dx = \frac{7}{2}\sec\theta\tan\theta\, d\theta$



$$\int \frac{\frac{7}{2}\sec\theta\tan\theta}{\sqrt{4\cdot\frac{49}{4}\sec^2\theta-49}}\, d\theta$$

$$\int \frac{7\sec\theta\tan\theta}{14\tan\theta}\, d\theta = \frac{1}{2}\int \sec\theta\left(\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}\right) d\theta$$

$$= \ln|\sec\theta+\tan\theta| + C = \ln\left|\frac{2x}{7}+\frac{\sqrt{x^2-49}}{7}\right| + C$$

■ $\int \frac{dx}{(4-x^2)^{3/2}}$

$a = 2, \; u = 2x$

$x = 2\sin\theta, \; dx = 2\cos\theta$

$$\int \frac{2\cos\theta}{\sqrt{(4-4\sin^2\theta)^3}}\, d\theta = \int \frac{2\cos\theta}{\sqrt{64(1-\sin^2\theta)}}$$

$$\int \frac{2\cos\theta}{8\sqrt{(\cos^2\theta)^3}}\, d\theta = \int \frac{\cos\theta}{4\cos^3\theta}\, d\theta = \int \frac{1}{4\cos^2\theta}\, d\theta$$

$$= \frac{1}{4}\int \sec^2\theta\, d\theta = \frac{1}{4}\tan\theta + C = \frac{1}{4}\left[\frac{x}{\sqrt{4-x^2}}\right] + C$$

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Date : / / Subject:

■ $\int \sqrt{1-\cos 2x}\, dx$

$\sin^2 x = \frac{1-\cos 2x}{2}$

$2\sin^2 x = 1-\cos 2x$

$\int \sqrt{2\sin^2 x}\, dx$

$= \sqrt{2}\int \sin x\, dx$

$= -\sqrt{2}\cos x + C$

■ $\int \frac{\tan^2 x}{\csc x}\, dx$

$\int \frac{\sec^2 x - 1}{\csc x}\, dx = \int (\sec^2 x - 1)\sin x\, dx$

$= \int \sec^2 x \sin x\, dx - \int \sin x\, dx = \int \frac{1}{\cos^2 x}\sin x\, dx - \int \sin x\, dx$

$= \int \cos^{-2} x \sin x\, dx - \int \sin x\, dx = \cos^{-1} x + \cos x + C$

$= \frac{1}{\cos x} + \cos x + C$

$\int \frac{9x^3 - 3x + 1}{x^3 - x^2}\, dx \Rightarrow$

$$\begin{array}{r|l} x^3 - x^2 & 9x^3 - 3x + 1 \\ & 9x^3 - 9x^2 \\ \hline & 9x^2 - 3x + 1 \end{array}$$

$= \int 9\, dx + \int \frac{9x^2 - 3x + 1}{x^3 - x^2}\, d\alpha$

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Date : / / Subject: A_s (A sub s)

slab

مساحة الشريحة

« Areas Between Curves »

14 Mar

A] When the slab is moving along the x-axis

Area of the slab $(A_s) = \Delta y \, \Delta x$

Because the slab is moving a long the x-axis than

$\Delta x = dx$

Δy = Upper curve − lower curve = $f(x) - g(x)$

Data on top − Data in Down

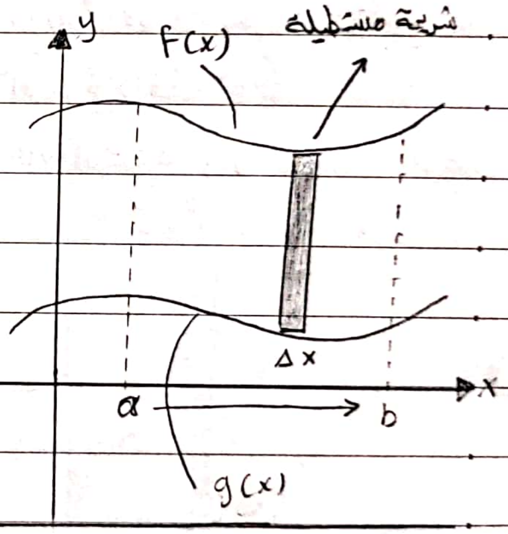
To find the Total area than

تكامل لفترة مغلقة

$A = \int A_s = \int \Delta y \, \Delta x$ طول x عرض

$= \int_a^b f(x) - g(x) \, dx$

حدود التكامل هي الفترة المعطاة بالسؤال

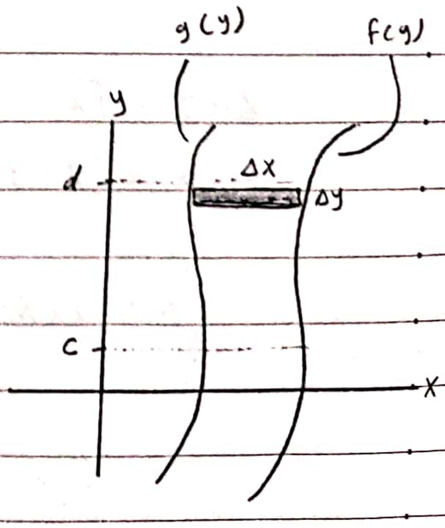


B] When the slab is moving a long the y-axis

$A_s = \Delta x \, \Delta y \Rightarrow \Delta y = dy$

Δx = Right curve − left curve = $f(y) - g(y)$

$A = \int A_s = \int \Delta x \, \Delta y = \int_c^d f(y) - g(y) \, dy$



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Date : ./ / Subject:

A: find the area of the region (منطقة) in the 1st quadrant (ربع) that is bounded (يحدد) by $\boxed{y=\sqrt{x}}$ and the line (خط) $\boxed{y=x-2}$

أولاً نرسم المنطقة المطلوب نحسب مساحتها

ثانياً نحدد الطريقة التي نشتغل بيها هل نختار شريحة عامودية أم شريحة أفقية

عملية الرسم

نفرض x ونوجد y

ناخذ قيم موجبة وسالبة والصفر من ضمنها

معادلة الخط المستقيم

x	y = x − 2
0	−2
2	0
1	−1
4	2

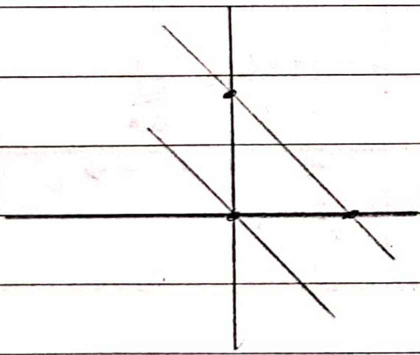
x	$y=\sqrt{x}$
−4	$\sqrt{-4}$
−1	$\sqrt{-1}$
0	0
1	1
4	2

خيالي

معادلة الخط المستقيم متى نرسمها دائماً الخط المستقيم يتقاطع مع المحورين فهذه حالة الحالة الثانية يمر بنقطة الأصل

نقطة التقاطع بين الدالتين

نفرض قيمة x صفر ونوجد y ونفرض قيمة y صفر ونوجد قيمة x



طريقة ثانية وهي بأن نساوي الدالتين

$$x-2=\sqrt{x}$$

$$x=(x-2)^2 \Rightarrow x=x^2-4x+4 \Rightarrow x^2-5x+4=0$$

$$(x-4)(x-1)=0 \qquad x_1=4 \rightarrow y_1=2$$

$$x_2=1 \rightarrow y_1=1$$

ملاحظة: عدم تساوي النقطة (1,1) في الجدول هو لان أخذنا الجزء الموجب فقط

Date : / / Subject:

ملاحظة: $y=\sqrt{x}$ $\Leftarrow$ $y^2=x$ حال Curve

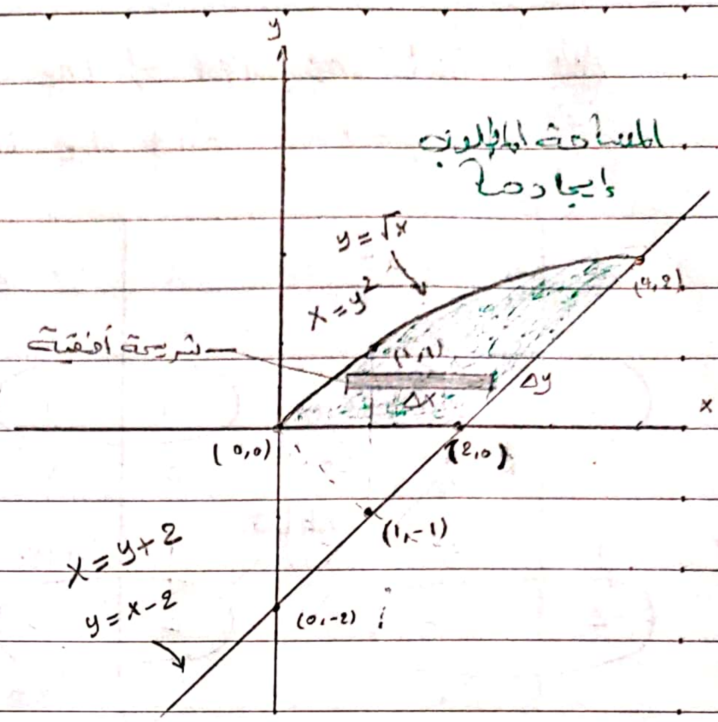
لما نجذر الطرفين $y=\mp\sqrt{x}$ ولكن

بالسؤال عطى $+\sqrt{x}$ فقط (يعني كما

الجزء الموجب فقط ولو كما $\boxed{-\sqrt{x}}$ أيضاً

كان لازم نكمل الجزء السالب في الرسم

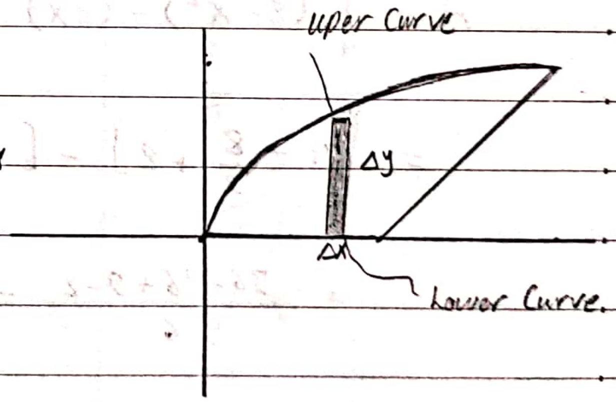
اذن سوف تهمل النقطة لان محددنا بالموجب



$$A=\oint A_s=\oint \Delta x\,\Delta y=\int f(y)-g(y)\,dy$$

$$A=\int_0^2 (y+2)-y^2\,dy=\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_0^2=\left[2+4-\frac{8}{3}\right]=\frac{10}{3}$$

في حالة أخذنا الشريحة بصورة عامودية



$$A=\oint A_s=\oint \Delta y\,\Delta x=\int_a^b f(x)-g(x)\,dx$$

$$=\int_0^2 \sqrt{x}-0\,dx+\int_2^4 \sqrt{x}-(x-2)\,dx$$

$$=\left.\frac{x^{3/2}}{3/2}\right]_0^2+\left[\frac{x^{3/2}}{3/2}-\frac{x^2}{2}+2x\right]_2^4=\frac{10}{3}$$

Date : / /

Subject: آول خطوة نرسم

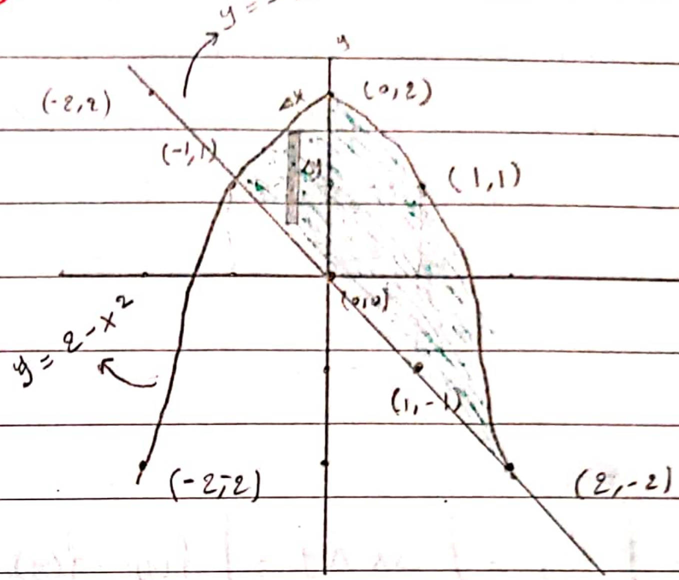
مغلق　　　　　　　　　　　　　　　　قطع المكافئ

A: find the area of the region enclosed by the Parabola $y = 2 - x^2$ and the line $y = -x$

x	$y = 2 - x^2$
-2	-2
-1	1
0	2
1	1
2	-2

نقطتين تقاطع

x	$y = -x$
-2	2
-1	1
0	0
1	-1
2	-2

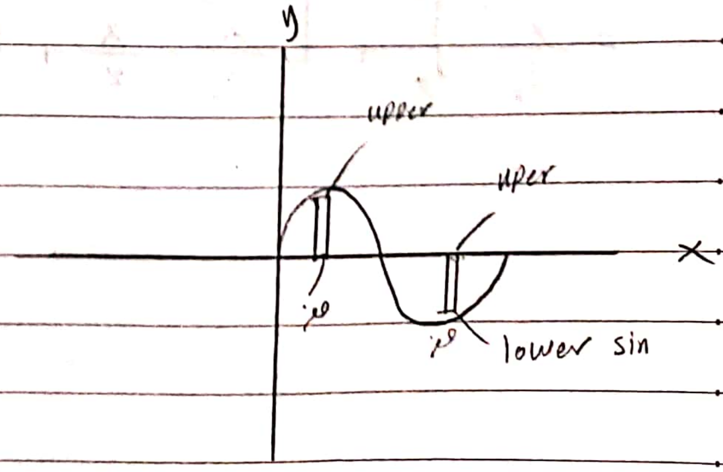


$$A = \int A_s = \int \Delta y \, \Delta x = \int_a^b f(x) - g(x) \, dx$$

$$A = \int_{-1}^{2} (2 - x^2) - (-x) \, dx = \left[2x - \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^{2}$$

$$= \left[4 - \frac{8}{3} + 2 \right] - \left[-2 + \frac{1}{3} + \frac{1}{2} \right] = 6 - \frac{8}{3} + \frac{3}{2} - \frac{1}{3}$$

$$= \frac{36 - 16 + 9 - 2}{6} = \frac{27}{6} = \frac{9}{2} \text{ unit sq}$$



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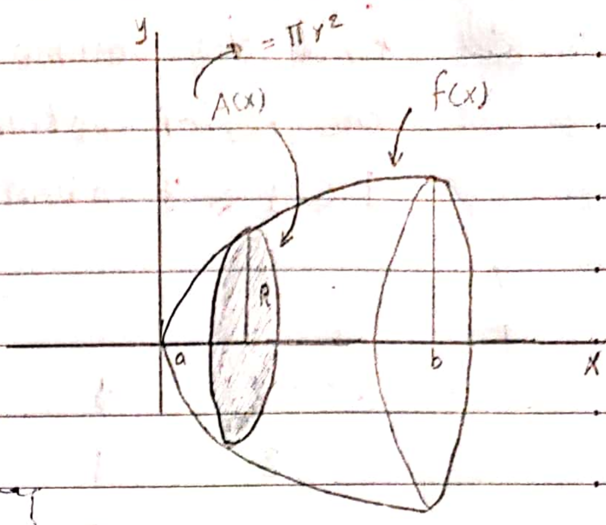
Volumes

1- Disk method



$$V = \int_a^b A(x)\, dx$$

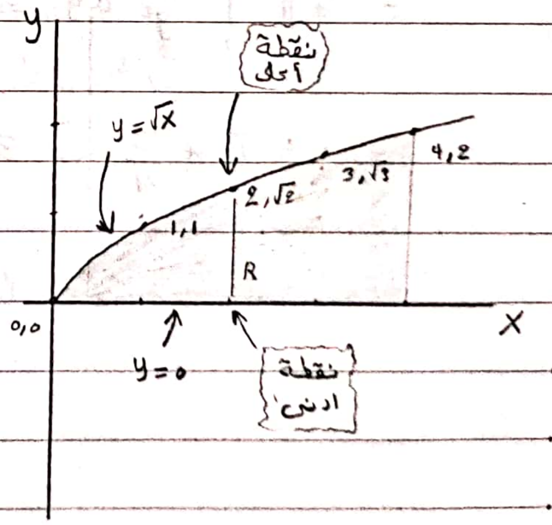
$$\boxed{\begin{array}{ll} V = \int_a^b \pi R^2[x]\, dx & \text{, about } x\text{-axis} \\ V = \int_c^d \pi R^2[y]\, dy & \text{, about } y\text{-axis} \end{array}}$$

$$R = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

قانون المسافة

① Find the volume of the solid generated revolving the region bounded by $y = \sqrt{x}$, $0 \leq x \leq 4$, and the x-axis about the x-axis

x	$y = \sqrt{x}$
0	0
1	1
2	$\sqrt{2} = 1.4$
3	$\sqrt{3} = 1.7$
4	2



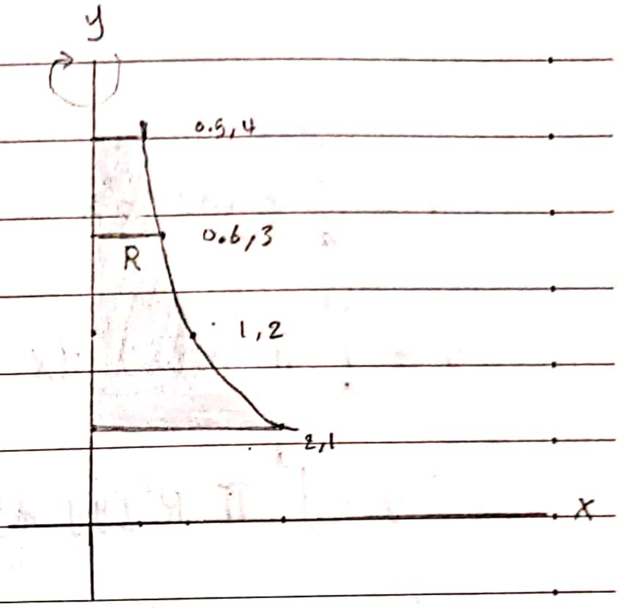
$$V = \int_a^b \pi\, [R(x)]^2\, dx = \pi \int_0^4 [\sqrt{x} - 0]^2\, dx$$

$$= \pi \left[\frac{x^2}{2}\right]_0^4 = 8\pi$$

: / /

Subject:

Find the volume of the solid generated by revolving the region between the y-axis and the curve $x = 2/y$, $1 \leq y \leq 4$ about the y-axis



$x = 2/y$	y
2	1
1	2
2/3	3
1/2	4

$$V = \int_c^d \pi \left[R(y)\right]^2 dy \Rightarrow V = \pi \int_1^4 \left[\frac{2}{y} - 0\right]^2 dy$$

$$= 4\pi \int_1^4 \frac{1}{y^2}\, dy = 4\pi \left[-\frac{1}{y}\right]_1^4 = 4\pi\left[-\frac{1}{4} - (-1)\right]$$

$$= 4\pi * \frac{3}{4} = 3\pi$$

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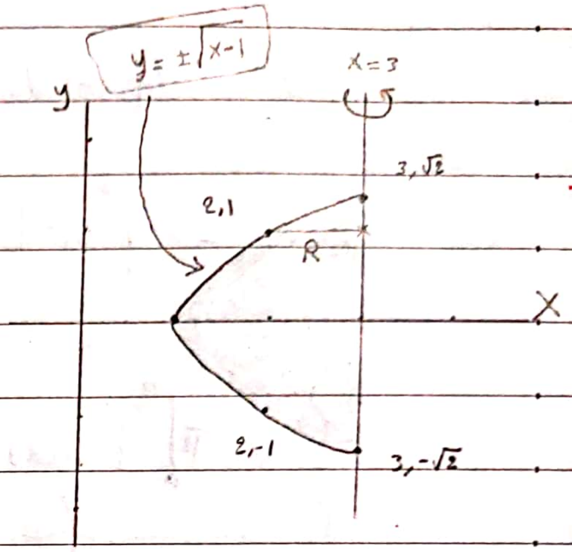
(o) Find the volume of the solid generated by revolving the region bounded by $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

x	$y = \mp\sqrt{x-1}$
-1	$\pm\sqrt{-2}$
0	$\pm\sqrt{-1}$
1	0
2	± 1
3	$\pm\sqrt{2}$

Ignore



$$\boxed{V = \int_c^d \pi \left[R(y)\right]^2 dy}$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[3 - (y^2+1)\right]^2 dy \qquad = \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[2 - y^2\right]^2 dy$$

$$= \pi \int_{-\sqrt{2}}^{\sqrt{2}} \left[4 - 4y^2 + y^4\right] dy \qquad = \pi \left[4y - \frac{4y^3}{3} + \frac{y^5}{5}\right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$= \frac{64\pi\sqrt{2}}{15}$$

2- washer method

1 Apr

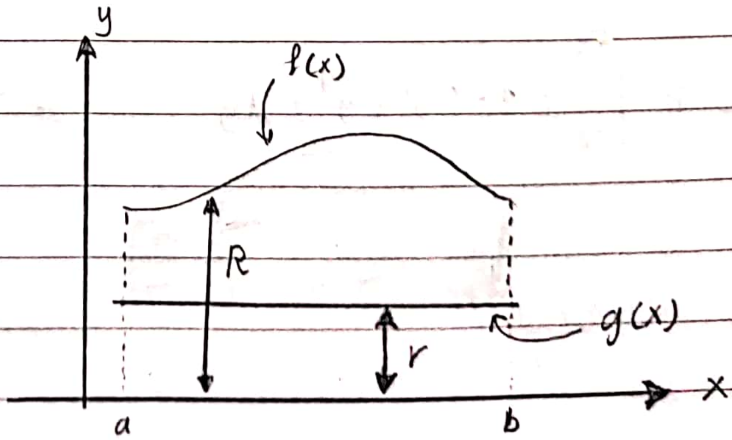
$$V = V_T - V_{space}$$ نفس الفترة

$$= \int_a^b \pi [R(x)]^2 dx - \int_a^b \pi [r(x)]^2 dx$$

$$\boxed{V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx}$$ about x-axis

outer radius بالنسبة لمحور الدوران iner raduis

$$\boxed{V = \pi \int_c^d [R(y)]^2 - [r(y)]^2 \, dy}$$ about y-axis



إذا كانت المساحة المحصورة بين منحنيين ويوجد فراغ معين مع محور الدوران

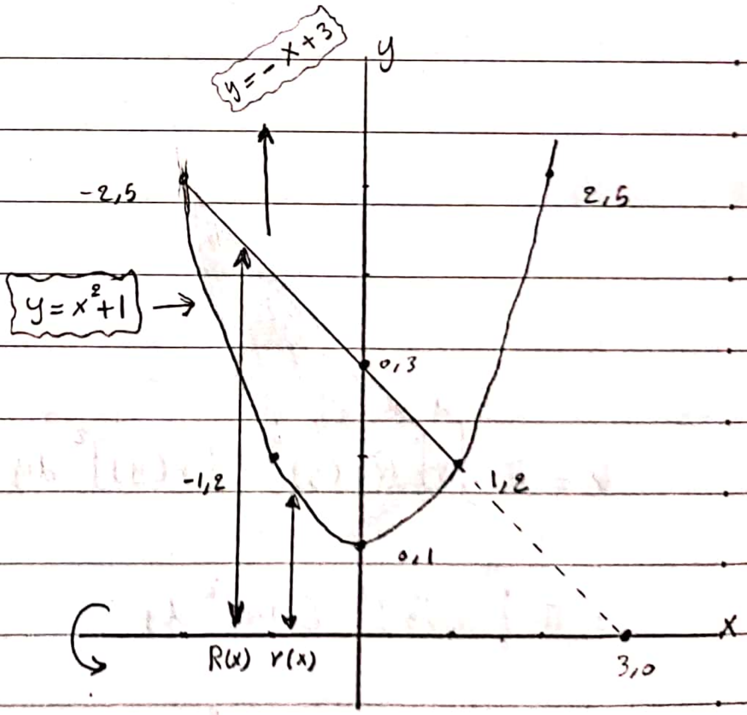
Date : / /

Subject:

1. The region bounded by the curve $y=x^2+1$ and the line $y=-x+3$ is revolved about the x-axis to generated a solid. find the volume of the solide.

x	$y=x^2+1$
-2	5
-1	2
0	1
1	2
2	5

x	$y=-x+3$
0	3
3	0
-2	5
-1	4
1	2
2	1



$$V=\pi\int_a^b [R(x)]^2-[r(x)]^2\,dx$$

$$=\pi\int_{-2}^{1}[-x+3]^2-[x^2+1]^2\,dx$$

$$=\pi\int_{-2}^{1}(x^2-6x+9-x^4-2x^2-1)\,dx$$

$$=\pi\int_{-2}^{1}(-x^2-6x+8-x^4)\,dx$$

$$=\pi\left[-\frac{x^3}{3}-\frac{6x^2}{2}+8x-\frac{x^5}{5}\right]_{-2}^{1}$$

$$=\frac{117\pi}{5}$$

(0) The region bounded by the curve $y = x^2$ and the line $y = 2x$ in the 1st quadrant is revolved about the y-axis to generate a solid, find the volume of the solid

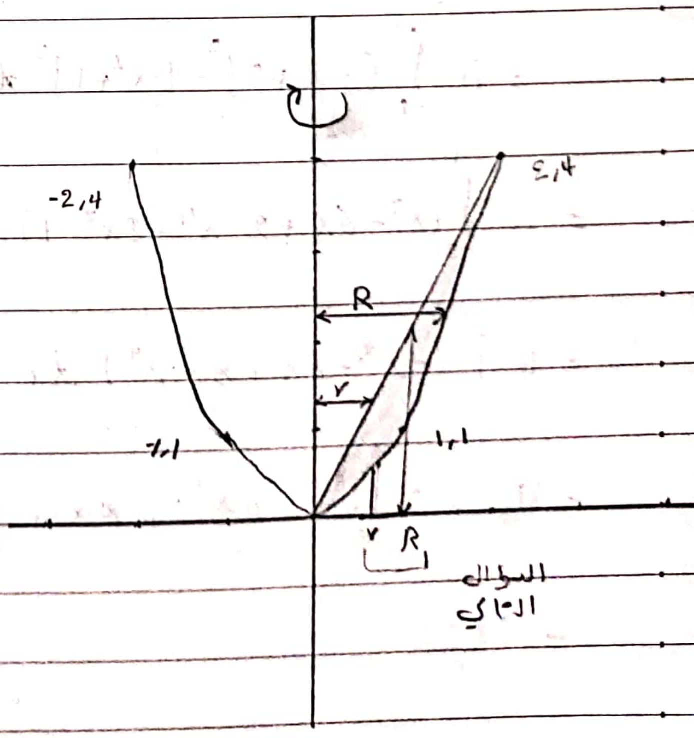
X	$y = x^2$
-2	4
-1	1
0	0
1	1
2	4

X	$y = 2x$
-2	-4
-1	-2
1	2
2	4

$$V = \pi \int_c^d [R_y(y)]^2 - [r(y)]^2 \, dy$$

$$= \pi \int_0^4 [\sqrt{y}]^2 - [y/2]^2 \, dy$$

$$= \pi \left[\frac{y^2}{2} - \frac{y^3}{12} \right]_0^4 = \frac{8}{3}\pi$$



نفس السؤال السابق لو كان الدوران حول محور x

$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx$$

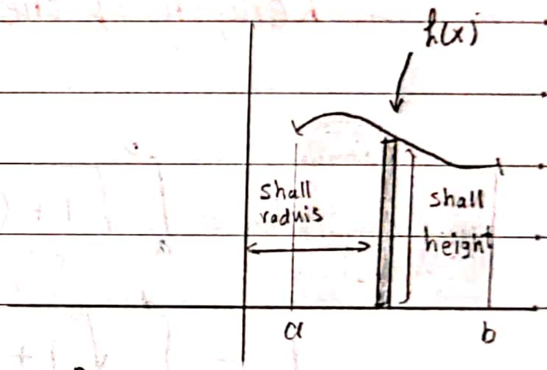
$$= \pi \int_0^2 [2x]^2 - [x^2]^2 \, dx \quad = \pi \left[\frac{4x^3}{3} - \frac{x^5}{5} \right]_0^2$$

3- Shell method

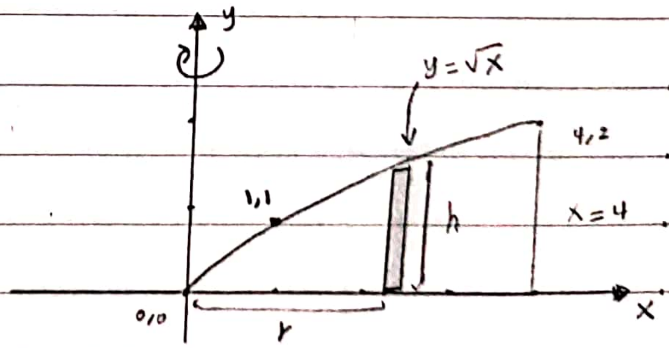
طريقة الاسطوانة

$$V = \int_a^b 2\pi \begin{bmatrix} \text{shell} \\ \text{radius} \end{bmatrix} \begin{bmatrix} \text{shell} \\ \text{height} \end{bmatrix} dx$$

القانون الرئيسي



The region bounded by the curve $y=\sqrt{x}$, the x-axis and the line $x=4$ is revolved about y-axis to generate a solid. find the volume of the solid



x	$y=\sqrt{x}$
-4	$\sqrt{-4}$
-1	$\sqrt{-1}$
0	0
1	1
4	2

تخيلي

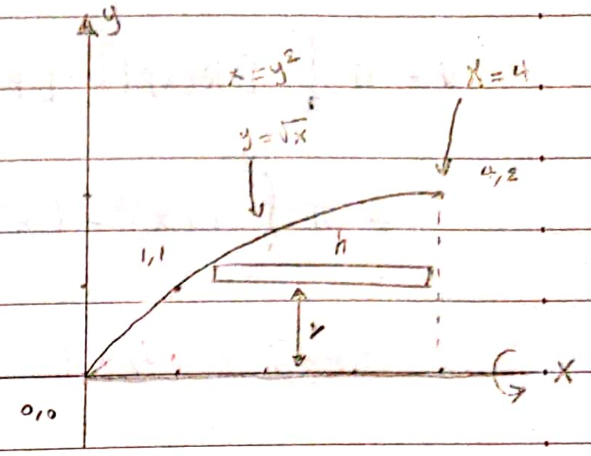
$$V = 2\pi \int_a^b h \cdot r \, dx \quad = 2\pi \int_0^4 \sqrt{x} \; x \, dx$$

$$= \frac{x^{5/2}}{\frac{5}{2}} \Big]_0^4 =$$

فكرة

Date : / / Subject: ....

If the rotation about x~axis



$$V = 2\pi \int_c^d r.h \; dy$$

$$= 2\pi \int_0^2 y(4 - y^2) \; dy$$

$$= 2\pi \left[\frac{4y^2}{2} - \frac{y^4}{4} \right]_0^2 = 8\pi$$

- Length of the curve

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \; dx$$

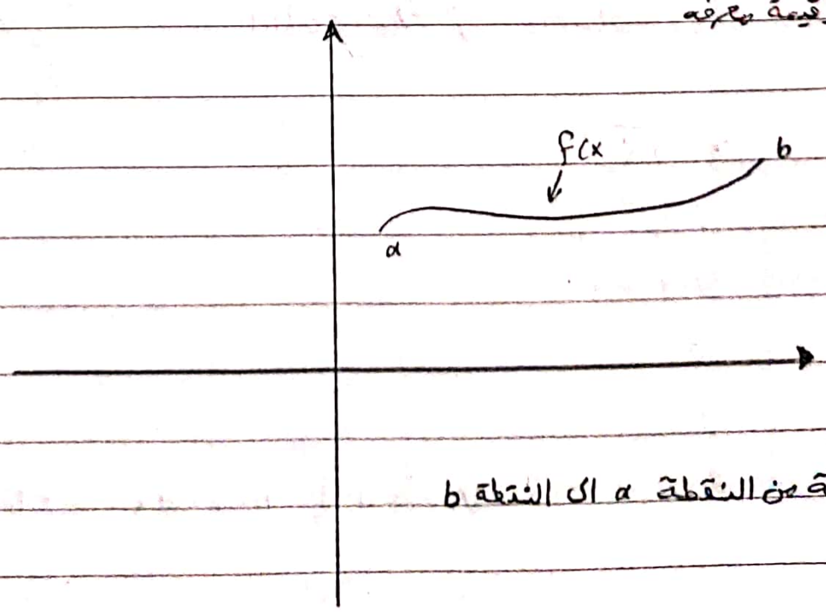
$\boxed{\frac{dy}{dx}}$ is continous for the given interval.

يطبق في حالة قيمة $\frac{dy}{dx}$ قيمة معرفه

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \; dy$$

$\boxed{\frac{dx}{dy}}$ is continuous for the given interval.

يطبق في حالة $\frac{dx}{dy}$ قيمة معرفه



إيجاد مسافة من النقطة a الى النقطة b

Date : / / Subject:

→ find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{\frac{3}{2}} - 1 \quad \underline{0 \leq x \leq 1}$

$$\frac{dy}{dx} = \frac{\cancel{4}^{2}\sqrt{2}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}} x^{\frac{1}{2}} = \boxed{2\sqrt{2}\,\sqrt{x}}$$

القيمة المعرفة أي قيمة اعتيادية

القيمة الغير معرفه هي ناتج القسمة على صفر

when $x = 0 \longrightarrow \frac{dy}{dx} = 0$, $x = 1 \longrightarrow \frac{dy}{dx} = 2\sqrt{2}$ القيم هنا معرفة

$$\left[\frac{dy}{dx}\right]^2 = 8x$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_0^1 \sqrt{1 + 8x}\, dx \qquad * \frac{8}{8}$$

$$= \frac{1}{8} \int_0^1 8(1+8x)^{\frac{1}{2}}\, dx$$

$$= \frac{1}{8} \left[\frac{(1+8x)^{\frac{3}{2}}}{3/2} \right]_0^1 = \frac{1}{12} \left[(1+8x)^{\frac{3}{2}} \right]_0^1$$

$$= \frac{1}{12} [27 - 1] = \frac{26}{12}$$

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Date : / / Subject:

Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x=0$ to $x=2$

$$\frac{dy}{dx} = \frac{\cancel{2}}{3}\left(\frac{x}{2}\right)^{-\frac{1}{3}} \cdot \frac{1}{\cancel{2}} = \frac{1}{3\left(\frac{x}{2}\right)^{1/3}} = \frac{1}{3}\left(\frac{2}{3}\right)^{1/3}$$ فكرة

when $x = 0 \longrightarrow \frac{dy}{dx} = \text{undifined}$ $\frac{dy}{dx}$ غير معرفة لهذه الفترة

$$x = 2 \longrightarrow \frac{dy}{dx} = \frac{1}{3}$$

$$y = \left(\frac{x}{2}\right)^{2/3} = \sqrt[3]{\left(\frac{x}{2}\right)^2}$$

نستخدم هذه الطريقة اي ناخذ الدالة ونجعلها $x = (y)$

$$y^3 = \left(\frac{x}{2}\right)^2 \Rightarrow \sqrt{y^3} = \frac{x}{2} \Rightarrow x = 2\sqrt{y^3}$$

نشتقها بالنسبة لـ y

$$\frac{dx}{dy} = \cancel{2} \cdot \frac{3}{\cancel{2}} y^{1/2} = 3\sqrt{y}$$

$$\left(\frac{dx}{dy}\right)^2 = 9y \qquad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$ فكرة

$y = 0$ to $y = 1$

$$= \int_0^1 \sqrt{1 + 9y}\, dy$$

$$= \frac{1}{9}\left[\frac{(1+9y)^{3/2}}{3/2}\right]_0^1 = 2.2$$

Date : / / Subject:

Find the length of the curve $x = \frac{y^3}{3} + \frac{1}{4y}$ $y=1$ to $y=3$

$$\frac{dx}{dy} = \frac{3y^2}{3} - \frac{y^{-2}}{4} = y^2 - \frac{1}{4y^2}$$

when $y=1 \longrightarrow \frac{dx}{dy} = \frac{3}{4}$, $y=3 \longrightarrow \frac{dx}{dy} = 9 - \frac{1}{36}$

$$\left(\frac{dx}{dy}\right)^2 = \left(y^2 - \frac{1}{4y^2}\right)^2$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^3 \sqrt{1 + \left(y^2 - \frac{1}{4y^2}\right)^2}\, dy$$ فكرة

$$= \int_1^3 \sqrt{1 + y^4 - \frac{2y^2}{4y^2} + \frac{1}{16y^4}}\, dy$$

$$= \int_1^3 \sqrt{1 + \underbrace{y^4 - \frac{1}{2} + \frac{1}{16y^4}}}\, dy$$

$$= \int_1^3 \sqrt{y^4 + \frac{1}{2} + \frac{1}{16y^4}}\, dy$$

$$= \int_1^3 \sqrt{\left(y^2 + \frac{1}{4y^2}\right)^2}\, dy$$ فكرة

$$= \int_1^3 \left(y^2 + \frac{1}{4y^2}\right) dy$$

$$= \int_1^3 y^2 + \frac{1}{4} y^{-2}\, dy$$

$$= \left[\frac{y^3}{3} - \frac{1}{4} y^{-1}\right]_1^3 = \frac{53}{6}$$

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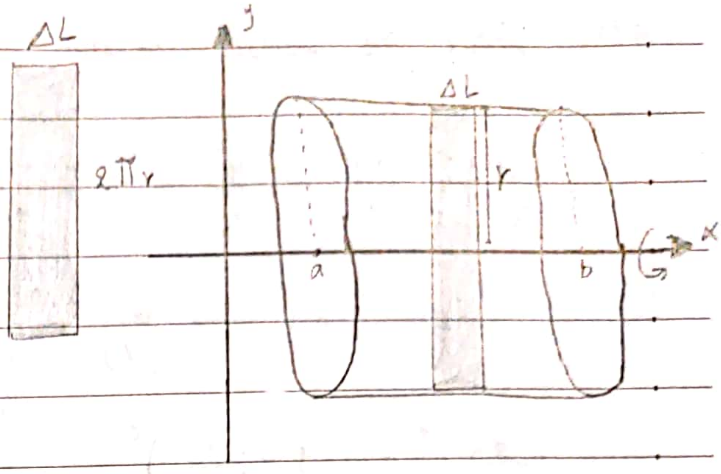
Surface area



$$S = \int A_s$$

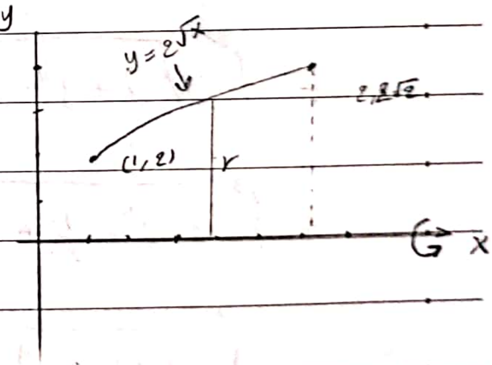
$$= \int 2\pi r \, \Delta L$$

$$S = \int_a^b 2\pi r(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx \quad \text{about } x\text{-axis}$$

$$S = \int_c^d 2\pi r(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy \quad \text{about } y\text{-axis}$$

Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$ $1 \leq x \leq 2$ about the x-axis

x	$y = 2\sqrt{x}$
1	2
2	$2\sqrt{2}$



$$S = \int_a^b 2\pi r \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$r(x) = 2\sqrt{x}$$

$$\frac{dy}{dx} = 2 \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$$

$$S = \int_1^2 2\pi \cdot 2\sqrt{x} \sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2} \, dx$$

$$= 4\pi \int_1^2 \sqrt{x\left(1 + \frac{1}{x}\right)} \, dx$$

$$= 4\pi \int_1^2 \sqrt{x + 1} \, dx$$

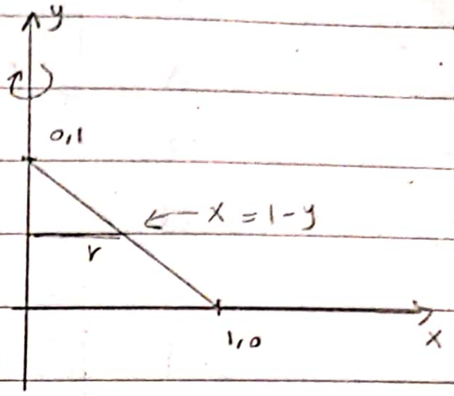
$$= 4\pi \int_1^2 (x+1)^{\frac{1}{2}} \, dx$$

$$= 4\pi \left. \frac{(x+1)^{3/2}}{3/2} \right]_1^2 =$$

Date : / /   Subject:

The Line segment $x = 1 - y$ $0 \leq y \leq 1$ is revolved about the y-axis to generate the cone (مخروط) find its lateral surface area {which excludes the base area}

$x = 1-y$	y
1	0
0	1



$r(y) = 1 - y$, $\frac{dx}{dy} = -1$

$$S = \int_c^d 2\pi \, r(y) \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$$

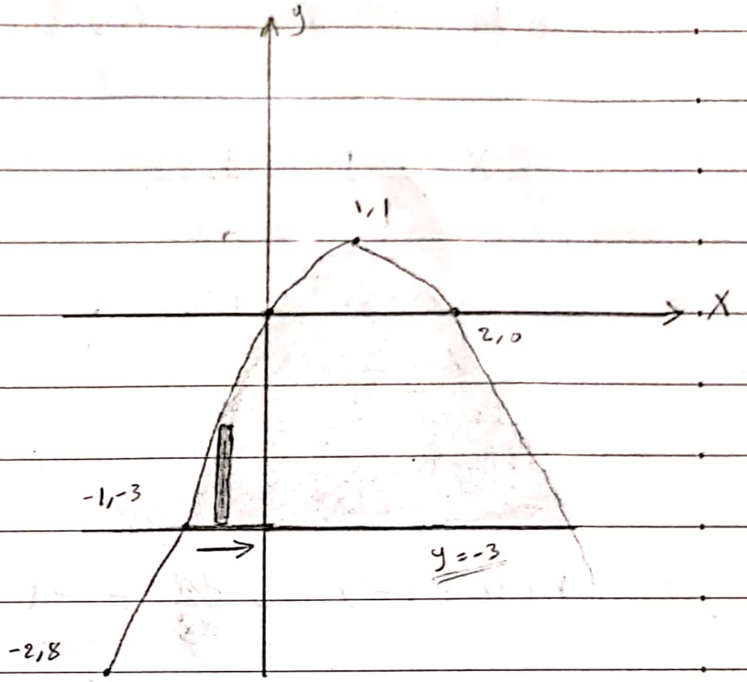
$$S = 2\pi \int_0^1 (1-y) \sqrt{1 + (-1)^2} \, dy$$

$$= 2\sqrt{2}\,\pi \int_0^1 (1-y) \, dy = 2\sqrt{2}\,\pi \left[y - \frac{y^2}{2} \right]_0^1 = \sqrt{2}\,\pi$$

Date : / / Subject:

find the area of the region enclosed by the curve $y = 2x - x^2$ and $y = -3$

x	$y = 2x - x^2$
-2	-8
-1	-3
0	0
1	1
2	0



$A = \oint As$

$= \int_a^b f(x) - g(x) \; dx$

$= \int_{-1}^{3} (2x - x^2) - (-3) \; dx$

$-3 = 2x - x^2$

$2x - x^2 + 3 = 0$

$x - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$y = 2x - x^2$

$x_1 = 3$

$x_2 = -1$

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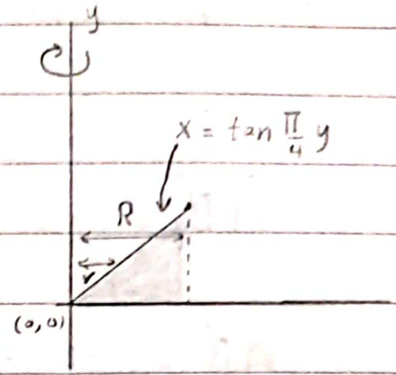
Date : / / Subject:

Find the volume of the solid generated by revolving the enclosed region $x = \tan \frac{\pi}{4} y$ and the x-axis for $0 \leq y \leq 1$ about the y-axis

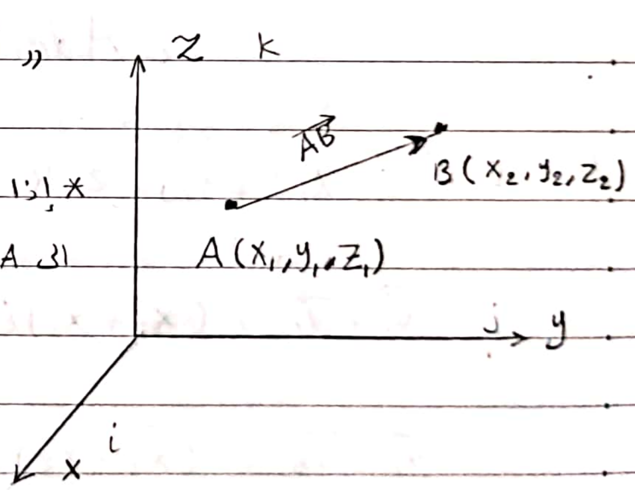


$x = \tan \frac{\pi}{4} y$	y
0	0
1	1

$$V = \pi \int_c^d [R y]^2 - [r(y)]^2 \, dy$$

$$V = \pi \int_0^1 [1]^2 - \left[\tan\left(\frac{\pi}{4} y\right)\right]^2 dy$$

« Vectors »



* إذا أنطلق من B الى A يسمى $\overrightarrow{BA}$

قيمة

$$\overrightarrow{AB} = (x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)\, k = xi + yj + zk$$

{unit vector}

طول

$$|\overrightarrow{AB}| = \sqrt{x^2 + y^2 + z^2}$$

هو متجه طوله وحده واحدة $\overrightarrow{U_{AB}} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$ {unit vector}

نحتاجه في بعض التطبيقات

نعوض بدل قيمة x 1, 2, 3,

Date : / / Subject:

Let $A(-3,4,1)$ and $B(-5,2,2)$ two points in space find:

1- The vector $\overrightarrow{AB}$ 2- Length of $\overrightarrow{AB}$ 3 unit vector of $\overrightarrow{AB}$.

$$\overrightarrow{AB} = (-5-(-3))i + (2-4)j + (2-1)K$$

$$\overrightarrow{AB} = -2i - 2j + 1K$$

$$|\overrightarrow{AB}| = \sqrt{4+4+1} = \sqrt{9} = 3$$

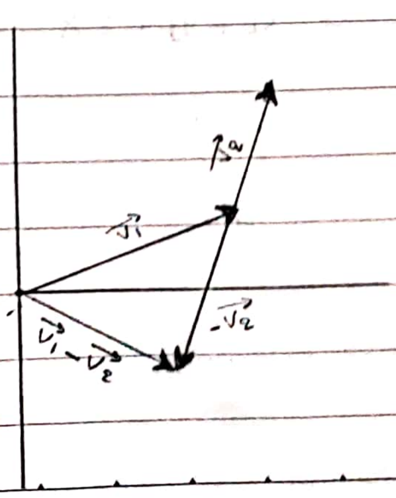
$$\overrightarrow{U_{AB}} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-2i-2j+K}{3} \Rightarrow = -\frac{2}{3}i - \frac{2}{3}j + \frac{1}{3}K$$

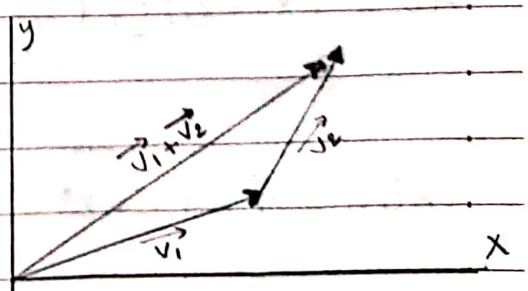
« Addition & Subtraction »

$\vec{V_1} = x_1 i + y_1 j + z_1 K$ $\vec{V_2} = x_2 i + y_2 j + z_2 K$

$$\vec{V_1} + \vec{V_2} = (x_1 + x_2)i + (y_1 + y_2)j + (z_1 + z_2)K$$

$$\vec{V_1} - \vec{V_2} = (x_1 - x_2)i + (y_1 - y_2)j + (z_1 - z_2)K$$





Date : / / Subject:

Multiply a vector by constant {Like scale}

$$C\vec{V_1} = Cx_1 i + Cy_1 j + Cz_1 K$$

Let $\vec{V_1} = -i + 3j + K$ and $\vec{V_2} = 4i + 7j$ find :

1- $\vec{V_1} + \vec{V_2}$ 2- $\vec{V_1} - \vec{V_2}$ 3 $\left|\frac{1}{2}\vec{V_1}\right|$

$\vec{V_1} + \vec{V_2} = 3i + 10j + K$ $\vec{V_1} - \vec{V_2} = -5i - 4j + K$

بالنسبة $\left|\frac{1}{2}\vec{V_1}\right| = \left|-\frac{1}{2}i + \frac{3}{2}j + \frac{1}{2}K\right| = \sqrt{\frac{1}{4} + \frac{9}{4} + \frac{1}{4}} = \sqrt{\frac{11}{4}} = \frac{1}{2}\sqrt{11}$

الغرض هو حساب الزاوية ①

1- Dot product {scaler}

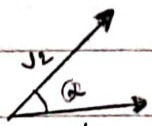
الغرض الثاني هو إيجاد مسقط متجه بأتجاه متجه أخر

$\vec{V_1} = x_1 i + y_1 j + z_1 K$ $\vec{V_2} = x_2 i + y_2 j + z_2 K$

قانون Dot product $\boxed{\vec{V_1} \cdot \vec{V_2} = |\vec{V_1}|\,|\vec{V_2}| \cos\theta}$ الزاوية المحصورة بينهما

$\vec{V_1} \cdot \vec{V_2} = x_1 x_2 + y_1 y_2 + z_1 z_2$ في حالة الزاوية غير معلومة

- ✓ * $i \cdot i = j \cdot j = K \cdot K = 1$ $\cos 0$
- ✓ * $i \cdot j = i \cdot k = j \cdot k = 0$ $\cos 90$
- ✓ * $\vec{V_1} \perp \vec{V_2} \Rightarrow \vec{V_1} \cdot \vec{V_2} = 0$
- ✓ * $\vec{V_1} \parallel \vec{V_2} \Rightarrow \vec{V_1} \cdot \vec{V_2} = |\vec{V_1}|\,|\vec{V_2}|$



$$\theta = \cos^{-1} \frac{\vec{V_1} \cdot \vec{V_2}}{|\vec{V_1}|\,|\vec{V_2}|}$$

Date : / / Subject:

3 find the angle between $\vec{A} = i - 2j - 2K$ and $\vec{B} = 6i + 3j + 2K$.

$$\vec{A} \cdot \vec{B} = 6 - 6 - 4 = -4$$

$$|\vec{A}| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$|\vec{B}| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \cos^{-1}\left(\frac{-4}{3 * 7}\right) = 100.9^\circ$$

Vector Proj

$$\vec{A} \cdot \vec{B} = |\vec{A}|\,|\vec{B}| \cos\theta$$

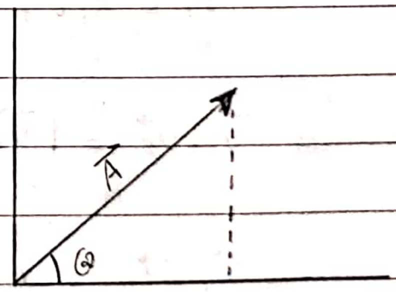
إيجاد مسقط

$$\vec{A} \cdot \vec{B} = Proj_{B}^{A} \cdot |\vec{B}|$$

مسقط باتجاه

$$\boxed{\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|} = Proj_{B}^{A}}$$

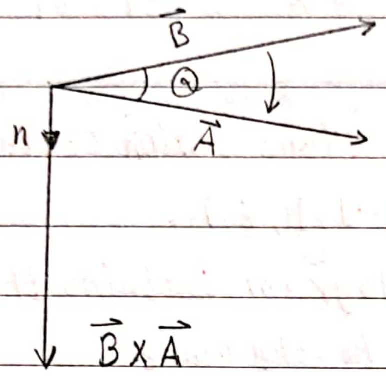
في حالة ماكو زاوية



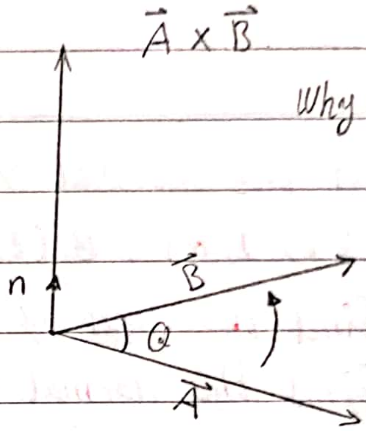
$$Proj_{B}^{A} = |A| \cos\theta$$

2- Cross Product

$$\boxed{\vec{A} \times \vec{B} = \underbrace{n}_{\text{is a normal unit vector}} |\vec{A}|\,|\vec{B}| \sin\theta}$$



Why to Down?



Why to up?

• We apply the right palm base

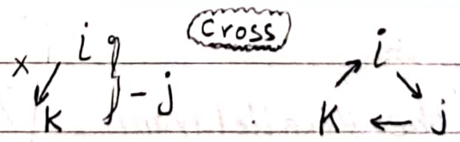
$$\vec{A} = x_1 i + y_1 j + z_1 K \quad , \quad \vec{B} = x_2 i + y_2 j + z_2 K$$

$$\vec{A} \times \vec{B} = \begin{array}{c} \\ \\ \\ \end{array}\begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{K} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} \begin{array}{l} \\ \Leftarrow A \\ \Leftarrow B \end{array}$$

فائدة ①:- نوجد متجه عامودي على متجهين
الي بحاج نعمل بينهم Cross

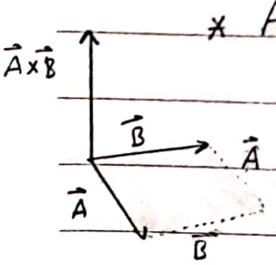
* طريقة ثانية لحساب cross من خلال Determinit

Some Comments :

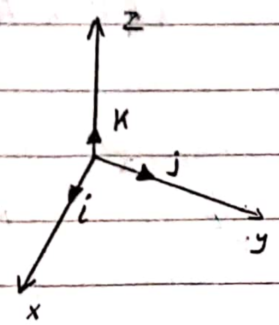


* $i \times i = j \times j = K \times K = 0$ ⟸ إذا عملنا cross للـ vector مع نفسو

* $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$



الفائدة الثانية :
إذا كملتو $|\vec{A} \times \vec{B}|$ بحاج بيعطيني
معلومات عن مساحة متوازي أضلاع



Date : / / Subject:

X find $\vec{u} \times \vec{v}$ if $\vec{u} = 2i + j + K$ and $\vec{v} = -4i + 3j + K$

$$\vec{u} \times \vec{v} = \begin{vmatrix} i & j & K \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix} = -2i - 6j + 10K$$

$\vec{v} \times \vec{u} = 2i + 6j - 10K$ فكرة

مطلوب حساب vector كامودي على هذا المستوى

X - find the normal vector to the plane which contains point $A(1,-1,0)$, $B(2,1,-1)$, $C(-1,1,2)$. مستوى يمر بالنقاط

متوازي أضلاع

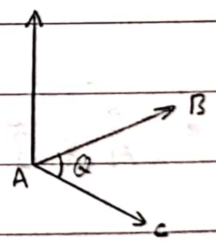
- find the area of the parallelogram contains the points.
- find the normal unit vector to the plane

①

$\overrightarrow{AB} = i + 2j - K$, $\overrightarrow{AC} = -2i + 2j + 2K$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} i & j & K \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix} \end{array}$$

الترتيب مهم



$= 6i - j0 + 6K = 6i + 6K$

② Area of the Parallelgram $= |\overrightarrow{AB} \times \overrightarrow{AC}|$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{36 + 36} = \sqrt{72}$

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③ قانون ال unit vector

$$\text{normal unit vector} = \frac{\overrightarrow{AB} \times \overrightarrow{Ac}}{|\overrightarrow{AB} \times \overrightarrow{Ac}|} = \frac{6i + 6K}{\sqrt{72}} = \frac{6i}{\sqrt{72}} + \frac{6K}{\sqrt{72}}$$

3- Parametric Eq. of the Line in space : 3D

الأسلوب مختلف

$\overrightarrow{P_0 P} = t\vec{V}$ → Scaling parameter

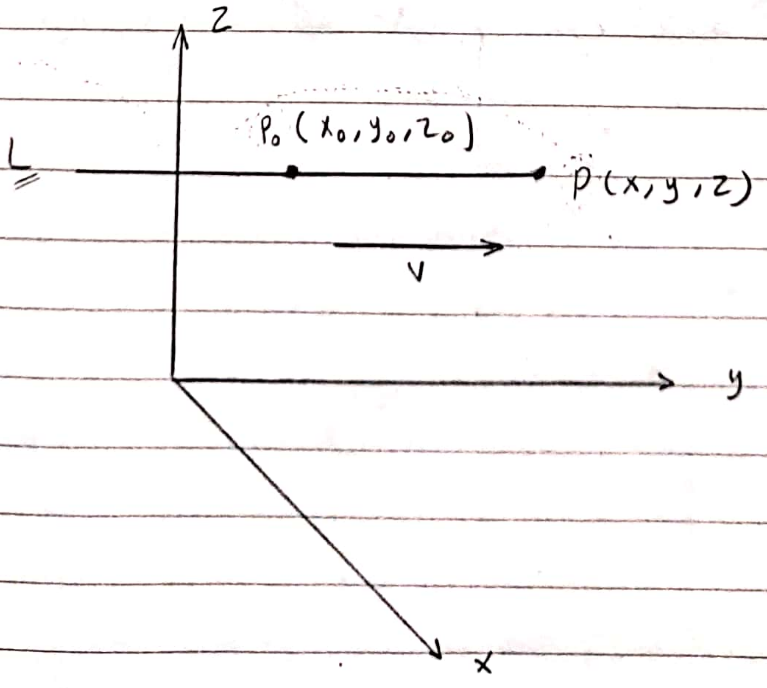
$$(x - x_0)i + (y - y_0)j + (z - z_0)K = t\vec{V}$$

$$\vec{V} = v_1 i + v_2 j + v_3 K$$

$$(x - x_0)i + (y - y_0)j + (z - z_0)K = t(v_1 i + v_2 j + v_3 K)$$

$$x - x_0 = t v_1 \qquad \boxed{x = x_0 + t v_1}$$
$$y - y_0 = t v_2 \qquad \boxed{y = v_0 + t v_2}$$
$$z - z_0 = t v_3 \qquad \boxed{z = z_0 + t v_3}$$

Parametric Eq. in the space



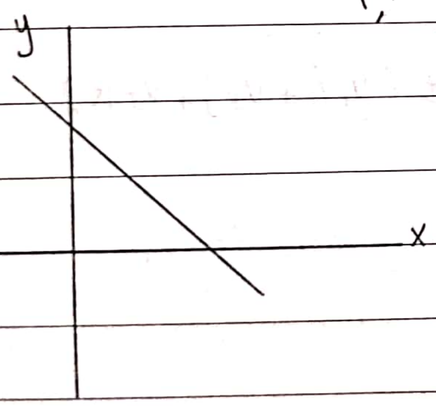
Find the Parametric eq. of the Line passes through $(-2,0,4)$ parallel to $\overline{V} = 2i + 4j - 2K$

$x = x_0 + t v_1$ $\qquad$ $x = -2 + 2t$

$y = y_0 + t v_2$ $\qquad$ $y = 0 + 4t$

$z = z_0 + t v_3$ $\qquad$ $z = 4 - 2t$

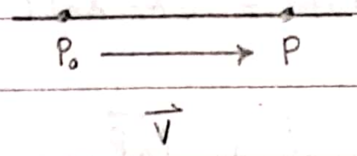
Notes



1- $\dfrac{y - y_1}{x - x_1} = m$, $\dfrac{y - y_1}{x - x_1} = \dfrac{y_2 - y_1}{x_2 - x_1}$

2- لو طلب (t) ف أكيد حاج يعطيني نقطة ثانية.

L. Find the Parametric eq. the Line through $P(-3,2,-3)$ and $Q(1,-1,4)$



$\overrightarrow{PQ} = 4i - 3j + 7K$

for $P(-3,2,-3)$

$$\left.\begin{aligned} x &= x_0 + tv_1 \\ y &= y_0 + tv_2 \\ z &= z_0 + tv_3 \end{aligned}\right\} \quad \begin{aligned} x &= -3 + 4t \\ y &= 2 - 3t \\ z &= -3 + 7t \end{aligned}$$

المعادلات المطلوبة

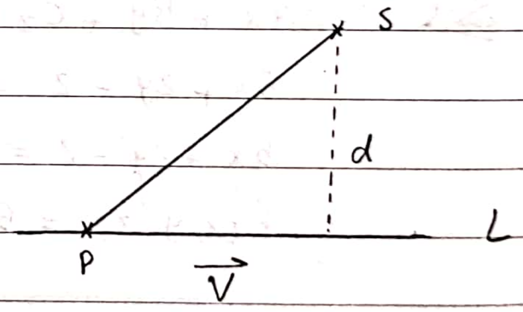
for $Q(1,-1,4)$

$$\left.\begin{aligned} x &= x_0 + tv_1 \\ y &= y_0 + tv_2 \\ z &= z_0 + tv_3 \end{aligned}\right\} \quad \begin{aligned} x &= 1 + 4t \\ y &= -1 - 3t \\ z &= 4 + 7t \end{aligned}$$

المعادلات المطلوبة

4. The distance from point to line in space

$$\boxed{d = \frac{|\overrightarrow{PS} \times \vec{v}|}{|\vec{v}|}}$$



Find the distance from the Point $S(1,1,5)$ to the Line

$L: x = 1 + t, \; y = 3 - t, \; z = 2t$ eq: of line

t_0

$\vec{v} = i - j + 2K$

$P(1,3,0)$

$S(1,1,5)$

$\overrightarrow{PS} = -2j + 5K$

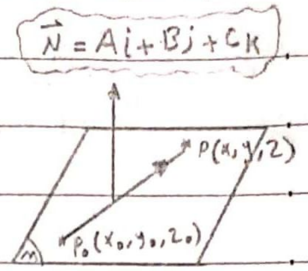
$$\overrightarrow{PS} \times \vec{v} = \begin{vmatrix} i & j & k \\ 0 & -2 & 5 \\ 1 & -1 & 2 \end{vmatrix} = i + 5j - 2K$$

$$d = \frac{\sqrt{1+25+4}}{\sqrt{1+1+4}} = \frac{\sqrt{30}}{\sqrt{6}} = \sqrt{\frac{30}{6}} = \sqrt{5}$$

بما أنه تقع على الخط المستقيم P

إذن أطبق أصبا من المعادلات الي هي اعدا المطلق

5- plane Eq. in space :



$$\overrightarrow{P_0P} = (x-x_0)i + (y-y_0)j + (z-z_0)K$$

$$\vec{N} \perp \overrightarrow{P_0P}$$

نعمل بينهم Dot product

$$\vec{N} \cdot \overrightarrow{P_0P} = |\vec{N}|\,|\overrightarrow{P_0P}| \cos\theta = 90 \quad = 0$$

ممكن تكون P_0 غير معطاة في السؤال

ونرجع نساويهم بالصفر

$$\vec{N} \cdot \overrightarrow{P_0P} = 0$$

$$(x-x_0)A + (y-y_0)B + (z-z_0)C = 0$$

نفرز الحدود x وحد و x_0 وحد وهكذا

$$\boxed{Ax + By + Cz = Ax_0 + By_0 + Cz_0 = D}$$

تمثل معادلة المستوي في الفراغ

Where D : Constant

لأن الناتج سيكون رقم ثابت

Ex: find the eq. for the plane passes through $P_0(-3,0,7)$ and perpendiculer to $\vec{N} = 5i + 2i - K$

(مستوي — الذي يمر — عامودي)

نحتاج قيم A,B,C نحصلهم من الفكتر العامودي

اما x_0 و y_0 و z_0 نحصلهم من النقطة

Sol

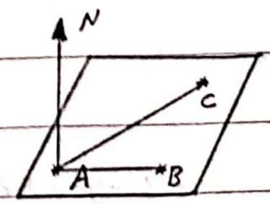
$$Ax + By + Cz = Ax_0 + By_0 + Cz_0$$

$$5x + 2y - z = -15 + (-7) = -22$$

$$5x + 2y - z = -22$$

$$f(x,y,z) = 5x + 2y - z + 22$$

Ex: find the eq for the plane passes through $A(0,0,1)$, $B(2,0,0)$, $C(0,3,0)$



Sol

$$\overrightarrow{AB} = 2i - K$$

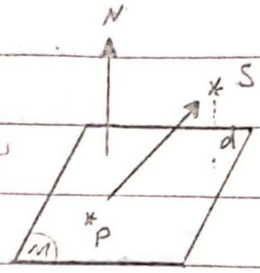
$$\overrightarrow{AC} = 3j - K$$

$$\vec{N} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & K \\ 2 & 0 & -1 \\ 0 & 3 & -1 \end{vmatrix} = 3i + 2j + 6K$$

تسمية N

$$\boxed{3x + 2y + 6z = 6}$$ plan eq.

6- The distance from the Point to a plane :



$$\boxed{d = \left| \overrightarrow{PS} \cdot \frac{\vec{N}}{|\vec{N}|} \right|}$$

مد مطلق

بعد النقطة S عن المستوى

Ex: find the distance from $S(1,1,3)$ to the plane $3x+2y+6z=6$

معطيني معادلة ال Plane

Sol

$\vec{N} = 3i + 2j + 6k$

To find P, we make $x=y=0$ then $6z=6 \Rightarrow \boxed{z=1}$

then $P(0,0,1)$ $\overrightarrow{PS} = i+j+2k$

$$d = \left| \overrightarrow{PS} \cdot \frac{\vec{N}}{|\vec{N}|} \right| \Rightarrow \left| i+j+2k \cdot \frac{3i+2j+6k}{\sqrt{9+4+36}} \right|$$

$$d = \left| \frac{3}{7} + \frac{2}{7} + \frac{12}{7} \right| = \frac{17}{7}$$

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Ex: find the parametric eq. for the line in which the planes $3x-6y-2z=15$ and $2x+y-2z=5$ intersect.

تتقاطع

Sol $3x-6y-2z=15$ and $2x+y-2z=5$.

$\vec{N}_1 = 3i-6j-2k$ $\qquad$ $\vec{N}_2 = 2i+j-2k$

$$\vec{N}_1 \times \vec{N}_2 = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -2 \end{vmatrix} = 14i + 2j + 15k$$

هذا الفكتر يوازي الخط المستقيم للتقاطع

بما أنه النقطة اي أريدها تقع كل الخط المستقيم ف تاع أصفر أحد المتغيرات

To find P_0 set the value of $z=0$ then

$$\begin{aligned} 3x-6y &= 15 \\ 2x+y &= 5 \quad \times 6 \\ \hline 15x &= 45 \end{aligned}$$

$x=3$

$y=-1$

$P_0(3,-1,0)$

$x = x_0 + tv_1$

$y = y_0 + tv_2$

$z = z_0 + tv_3$

$x = 3 + 14t$

$y = -1 + 2t$

$z = 15t$

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7~ Angles Between planes

$A x + By + Cz = D$

$A_1 x + B_1 y + C_1 z = D_1$

$\vec{N}_1 = Ai + Bj + Ck$

$\vec{N}_2 = A_1 i + Bj + C_1 K$

$$Q = \cos^{-1} \frac{\vec{N}_1 * \vec{N}_2}{|\vec{N}_1| \, |\vec{N}_2|}$$

Ex: find the angle between the planes

$3x - 6y - 2z = 0$

$2x + y - 2z = 5$

Sol

$\vec{N}_1 = 3i - 6j - 2K$

$\vec{N}_2 = 2i + j - 2K$

$$Q = \cos^{-1} \frac{\vec{N}_1 \vec{N}_2}{|\vec{N}_1| |\vec{N}_2|}$$

$$\cos^{-1}\left(\frac{6 - 6 + 4}{\sqrt{9+36+4} * \sqrt{4+1+4}} \right) = \cos^{-1}\left(\frac{4}{21}\right)$$

$Q = 79^{0}$

إذا ممنوعة الحاسبة نتفف في هوني ☺

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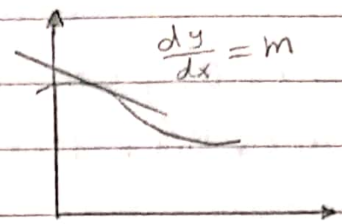
8) Gradient , Divergence and Curl of vectors :

"المشتقة الجزئية"

ديم أوبريتر

$$\nabla \equiv \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} K$$



وهو قيمة متجهة

A - gradient الميل

إذا مطلوب من عندي إيجاد الميل عند نقطة معينة تاخ آخد النقطة وأوجد الميل عند تلك النقطة.

ديم

$$grad\ f = \nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} K$$

نفسها

وإذا ما مطلوب مني إذن تاخ يكون الميل هو $\Longleftarrow$ $f(x,y,z)$ معادلة المستوى

Ex: find the gradient of $f(x,y,z) = 2y^3 + 4xz + 3x$

المشتقة الجزئية

Sol $\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j + \frac{\partial f}{\partial z} K$ نشتق بالنسبة لـ x ، و y و z ثوابت تكون ثم y ثم z

$$\nabla f = (4z+3) i + (6y^2) j + (4x) K$$

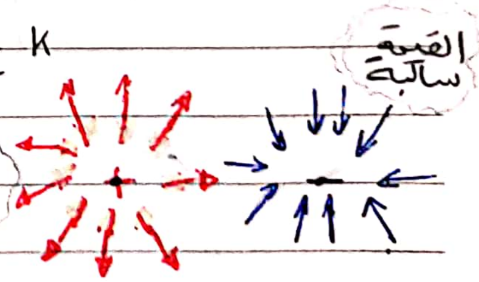
B - Divergence الانتشار

$\vec{V} = v_1 i + v_2 j + v_3 K$

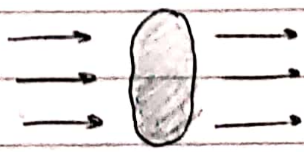
$\vec{\nabla} = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} K$

هو أنو نعمل Dot product بين

$$\boxed{\nabla \cdot \vec{V} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}}$$



الناتج هو قيمة لان ع نعمل Dot Product



إذا القيمة = 0

الأسهم الداخلة = الأسهم الخارجة

ملاحظة: إذا تم قيمة ال div كم نقطة معينة تام ينطيني نقطة مثلاً (x, y, z)

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بالنسبة لـ z بالنسبة لـ y بالنسبة لـ x

Ex: find the Div of $\vec{V} = (3xz)i + (2xy)j - (yz^2)K$

vector field

ملاحظة: هذه المركبات هي دوال

نعوضها ونجد قيمة div

$$Div. \vec{V} = 3z + 2x - 2yz \qquad (1, 0, 0)$$

إذا موجبة انتشار وإذا سالبة تجميع وإذا صفر لاهو انتشار ولا تجميع

c- Curl: الدوران أو اللّفة

Curl: ينطينا الفكتر العامودي.
من خلال الفكتر العامودي تام نعرف اتجاه دوران المجال

$$\vec{V} = v_1 i + v_2 j + v_3 K$$

$$\nabla \times \vec{V} = \begin{vmatrix} i & j & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

معاملات ال

Find the Div and the Curl of $\vec{V} = xyi + yzj + xzK$

$$\nabla \cdot \vec{V} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$$\nabla \cdot \vec{V} = y + z + x$$

$$\nabla \times \vec{V} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{K} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & yz & xz \end{vmatrix}$$

$$\nabla \times \vec{V} = \left(\frac{\partial}{\partial y}(xz) - \frac{\partial}{\partial z} yz\right)i - \left(\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial z}(xy)\right)j$$

بالنسبة لـ بالنسبة لـ

$$+ \left(\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy)\right)K$$

$$\nabla \times \vec{V} = -yi - zj - xK$$

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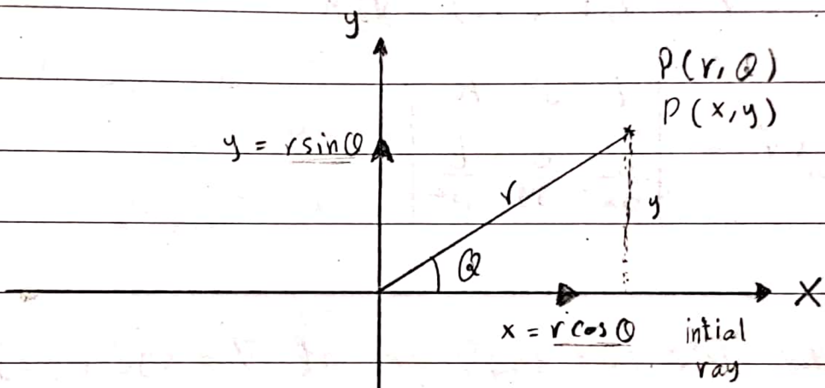
Polar

Polar to Cartisian	Cartisian to Polar
$y = r\sin Q$	$r = \sqrt{x^2+y^2}$
$x = r\cos Q$	$Q = \tan^{-1}\frac{y}{x}$



Q هي الزاوية المحصورة بين r ومحور x أو intial ray

* الزاوية Q ممكن تكون موجبة وممكن تكون سالبة ، تكون موجبة إذا كان اتجاه القياس عكس عقارب الساعة وسالبة إذا كان اتجاه الحركة مع عقارب الساعة

* ال r ممكن تكون موجبة أو سالبة {بعدين نوضحها}

* النقطة (x,y) بال Cartisian لها تمثيل واحد

* ب Polar لو تحركنا 360 ثاع نرجع على نفس المكان لو أتحرك 2π أخرى أيضاً أرجع على نفس المكان وحتى -360 أيضاً ، إذن تدي مالانهاية من الصيغ للنقطة بال Polar

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Replace the following polar eq. by equivalent cortisian eq

1. $r\cos\theta = 2 \longrightarrow x = 2$

2. $r^2\cos\theta\sin\theta = 4 \longrightarrow xy = 4$

$r\cos\theta \, r\sin\theta = 4$

3. $r = 1 + 2r\cos\theta \longrightarrow \sqrt{x^2+y^2} = 1 + 2x$

$x^2 + y^2 = (1+2x)^2$

$x^2 + y^2 = 1 + 4x + 4x^2$

$-3x^2 + y^2 - 4x - 1 = 0$

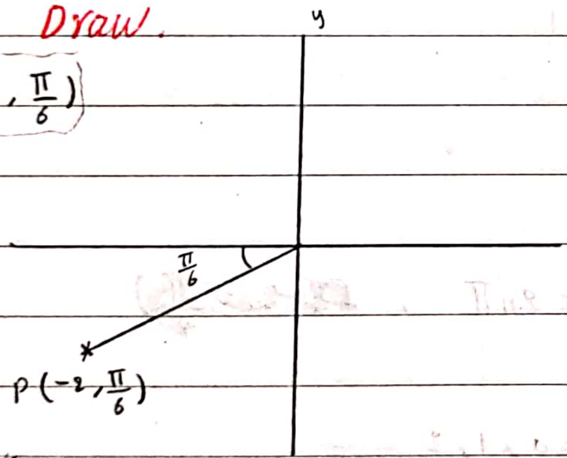
مسافة وحدة

Draw.

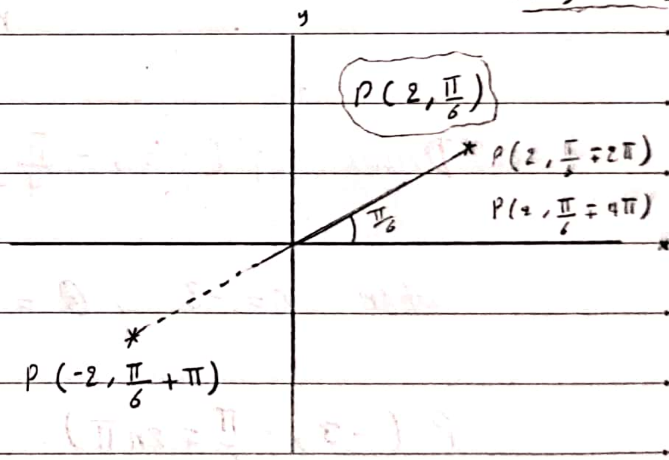
$P(-2, \frac{\pi}{6})$

r سالبة

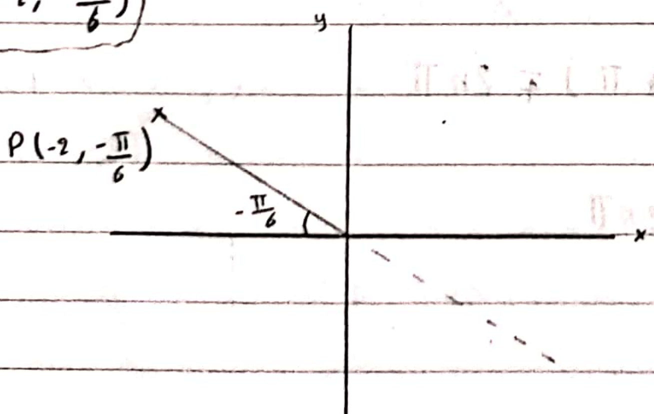
م/ الثانية



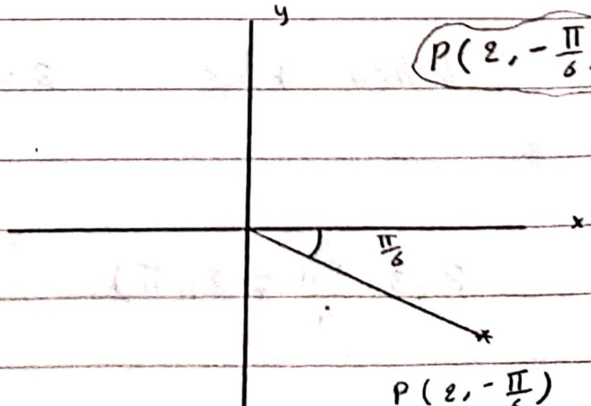
$P(2, \frac{\pi}{6})$



$P(-2, -\frac{\pi}{6})$



$P(2, -\frac{\pi}{6})$



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Find all the Polar coordinates pair of the Point $P(2, \frac{\pi}{6})$.

موجبة و بقت موجبة

① $r = +2$

معاها الزاوية
نجمع أو نطرح منها 2π

موجبة

$r = 2$, $\theta = \frac{\pi}{6} \mp 2n\pi$ $\boxed{P(2, \frac{\pi}{6} \mp 2n\pi)}$

ومعاها سالبة

where $n = 0, 1, 2, 3 \ldots$

اما ازيدها π أو أنقصها حسب السؤال

② $r = -2$

$r = -2$, $\theta = (\frac{\pi}{6} + \pi) \mp 2n\pi = \frac{7\pi}{6} \mp 2n\pi$

$P(-2, \frac{7\pi}{6} \mp 2n\pi)$

$n = 0, 1$

Draw $P(-3, -\frac{\pi}{4})$

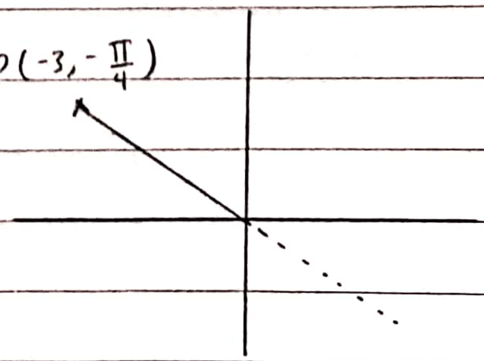
When $r = -3$, $\theta = -\frac{\pi}{4} \mp 2n\pi$, [illegible]

$P(-3, -\frac{\pi}{4} \mp 2n\pi)$ $n = 0, 1, 2$ —

When $r = 3$ $\theta = (-\frac{\pi}{4} + \pi) \mp 2n\pi$

$= \frac{3\pi}{4} \mp 2n\pi$

$P(3, \frac{3\pi}{4} \mp 2n\pi)$



$n = 0, 1, 2$ —

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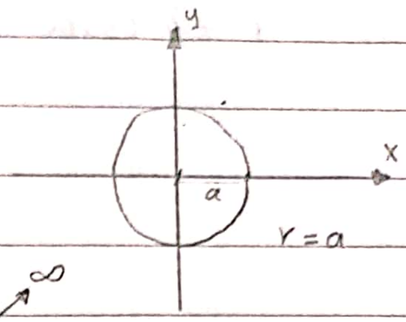
Polar Eq. رسم الدوال

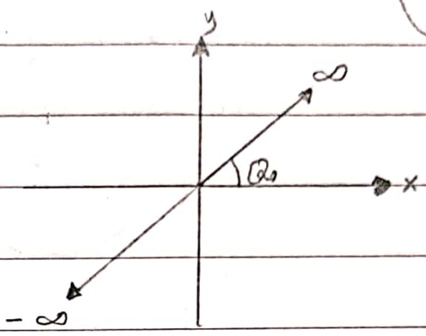
1- Circle and Lines



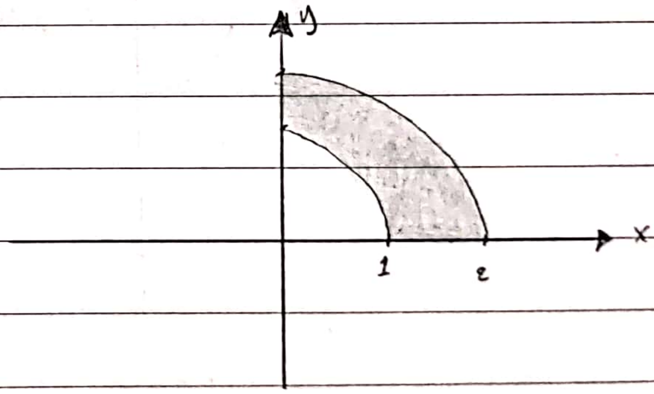
A- $r = a$ a: is constant

B- $\theta = \theta_0$

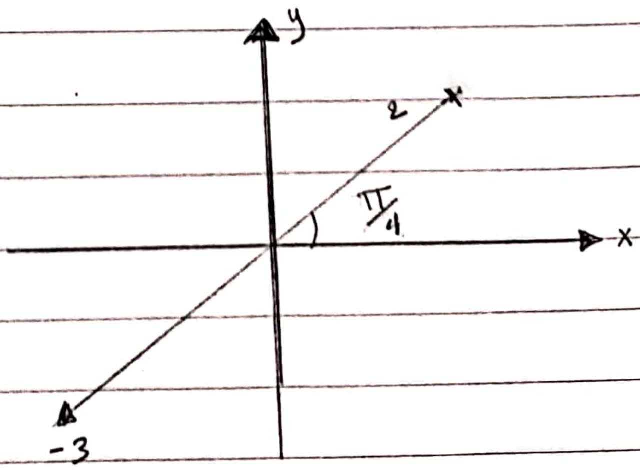
θ_0: is constant



Ex: a) $1 \leq r \leq 2$ & $0 \leq \theta \leq \frac{\pi}{2}$



b) $-3 \leq r \leq 2$ & $\theta = \frac{\pi}{4}$



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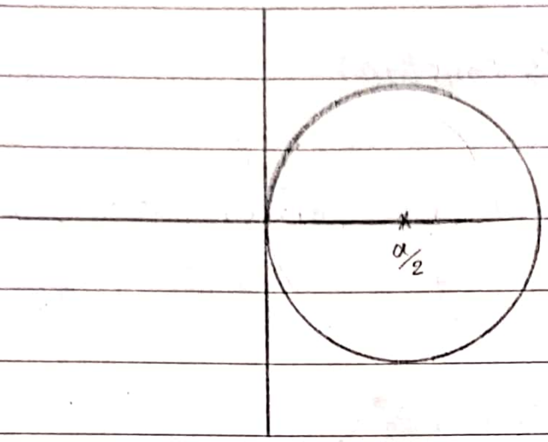
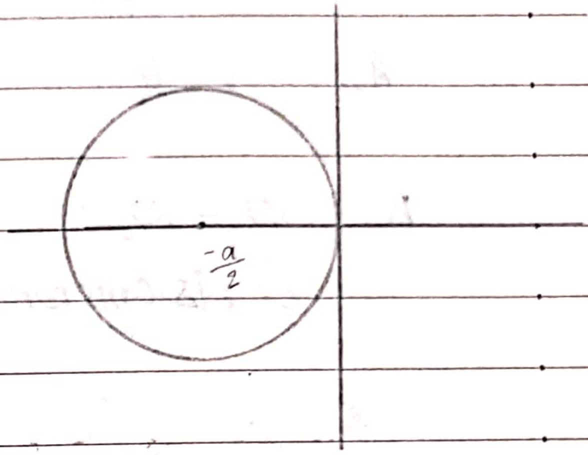
C- Circles

a: constant

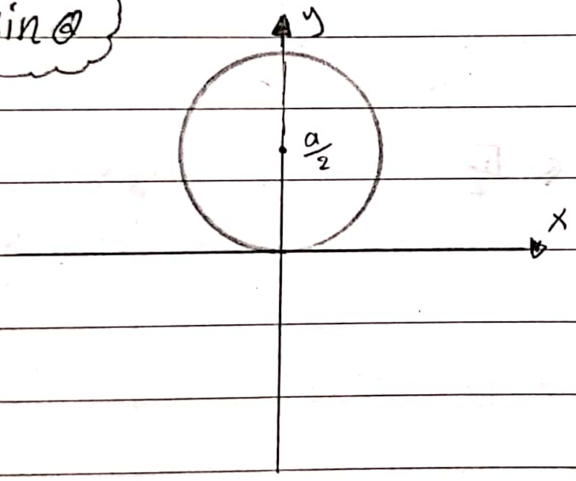
$r = a\cos\theta$

$r = -a\cos\theta$

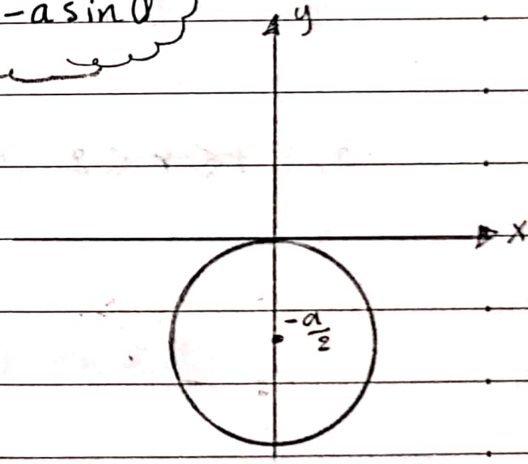




$r = a\sin\theta$



$r = -a\sin\theta$



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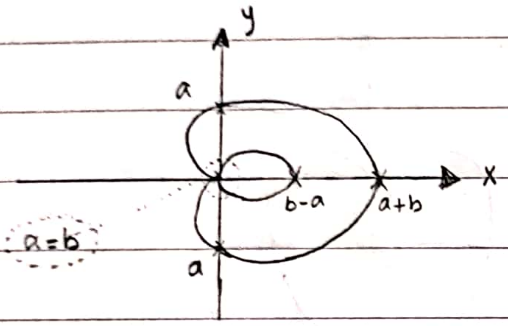
2- Limacon Eq :   الأشكال الحلزونية

$$\boxed{\begin{array}{l} r = a \pm b\cos\theta \\ r = a \pm b\sin\theta \end{array}}$$

$\boxed{a \ \& \ b : \text{is constant}}$

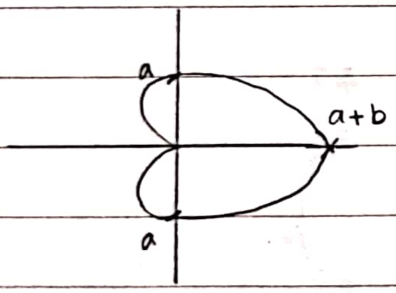
1- الشكل الأول ، إذا كانت النسبة بين a الى b أقل من 1 $\frac{a}{b} < 1$

ويسمى بـ {Inner loop limacon} اللوب الداخلي



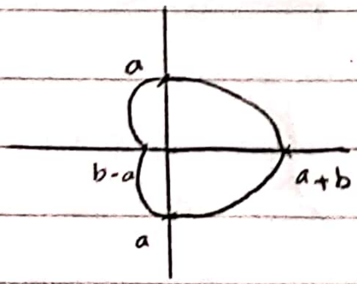
2- الشكل الثاني ، إذا كانت النسبة بين a الى b تساوي 1 $\frac{a}{b} = 1$

ويسمى بـ {Cardiod} شكل القلب



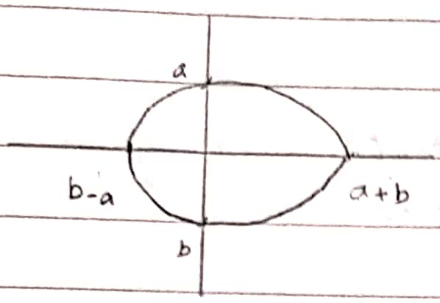
3- الشكل الثالث إذا كانت $\frac{a}{b}$ بين 1 و 2 $(1 < \frac{a}{b} < 2)$ لاهي تساويا ولاهي تساوي 2

ويسمى بـ {Dimple Limacon} النقطة أو الغمازة



Date : / / Subject:

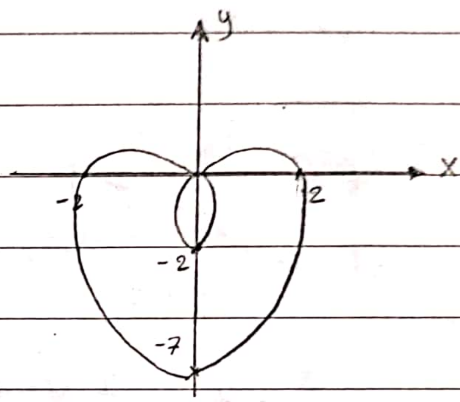
4- الشكل الرابع : عندما يكون $\frac{a}{b} \geq 2$
ويسمى {Convex limacon} المحدب



Ex: Graph the following Eq.

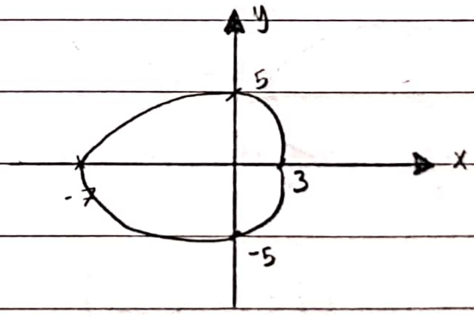
1- $r = 2 - 5\sin\theta$

$$\frac{a}{b} = \frac{2}{5}$$



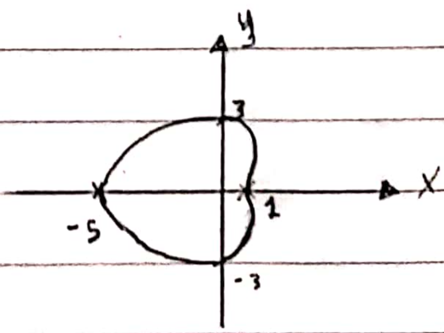
2- $r = 5 - 2\cos\theta$

$$\frac{a}{b} = \frac{5}{2}$$



3- $r = 3 - 2\cos\theta$

$$\frac{a}{b} = \frac{3}{2}$$



Date : / / Subject:

3- Rose Eq.

$$\boxed{\begin{aligned} r &= a\cos n\theta \\ r &= a\sin n\theta \end{aligned}}$$

a, n : is Constant

عدد الاوراق

No. of leaves = n odd No. of leaves $= n$; n even No. of leaves $= 2 * n$

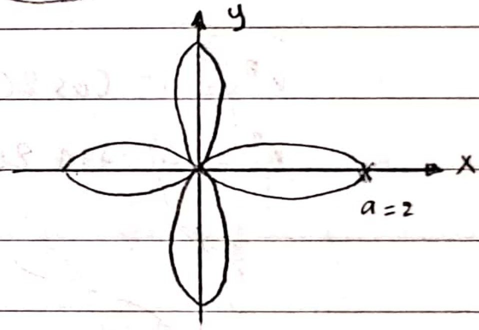
❀ Plot the following Eq.

1- $r = 2\cos 2\theta$

No. of leaves $= 4$

$\cos 2\theta = 1 \Rightarrow 2\theta = 0 \Rightarrow \theta_s = 0$ start draw

$$\boxed{Sp = \frac{360}{\text{No. of leaves}}}$$



$Sp = \frac{360}{4} = 90^\circ$

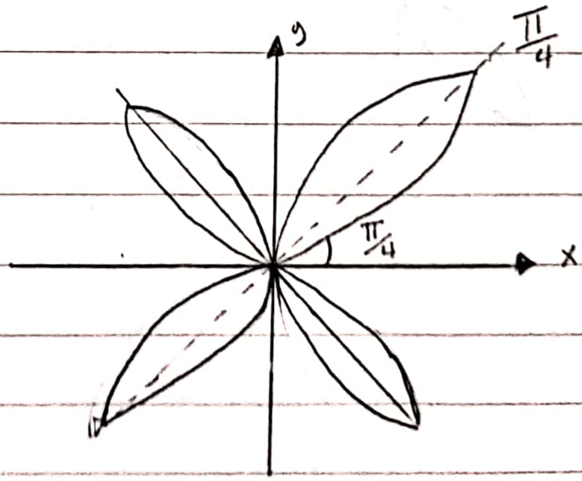
2- $r = 2\sin 2\theta$

No. of leaves $= 4$

$\sin 2\theta = 1$

$2\theta = \frac{\pi}{2}$

$$\boxed{\theta_s = \frac{\pi}{4}}$$



$Sp = \frac{360}{4} = 90^\circ$

Date : / / Subject:

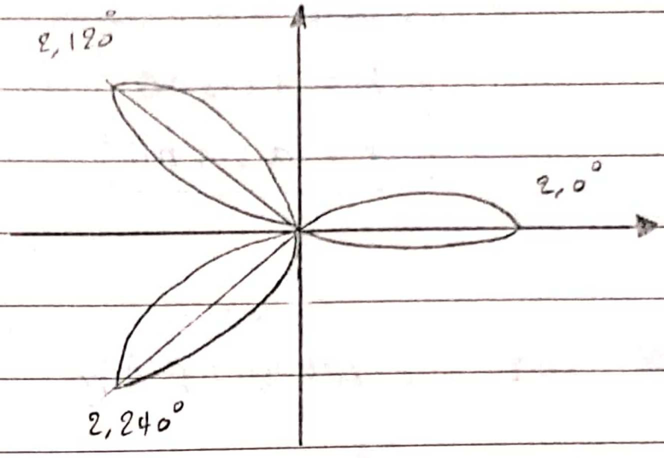
3- $r = 2\cos 3\theta$

No. of leaves = 3

$\cos 3\theta = 1$

$3\theta = 0$

$\boxed{\theta_s = 0}$

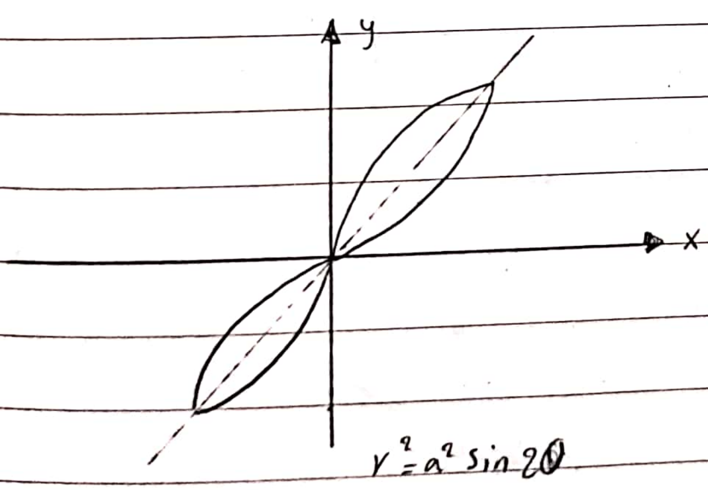


$SP = \frac{360}{3} = 120$

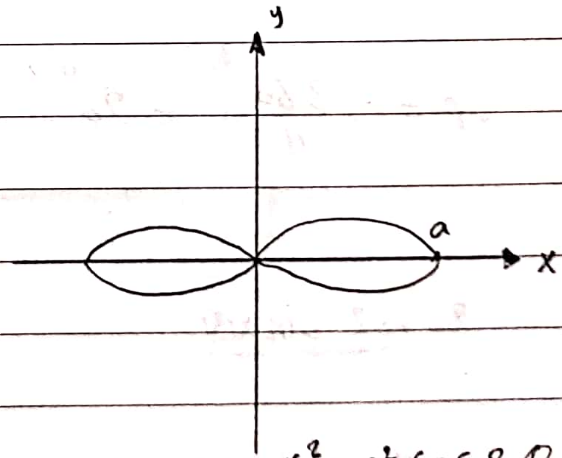
4- Lem niscale Eq. { زهرة بورقتين فقط }

$r^2 = a^2 \cos 2\theta$

$r^2 = a^2 \sin 2\theta$



$r^2 = a^2 \sin 2\theta$



$r^2 = a^2 \cos 2\theta$

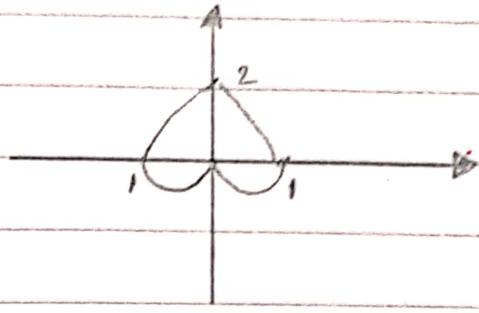
Date : / /

Subject:

Sketch the following Eq

① $r = -1 + \sin\theta$

$\frac{a}{b} = 1$



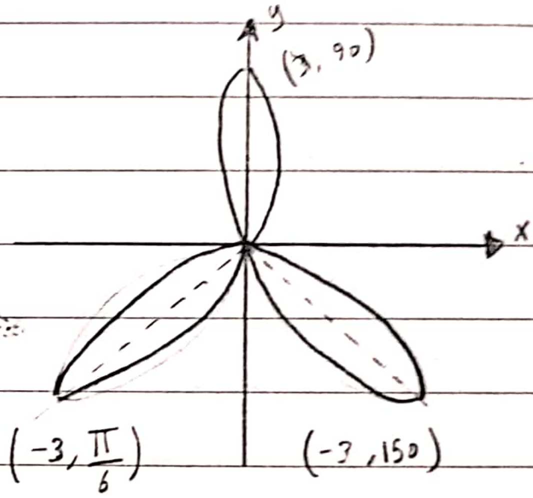
② $r = -3\sin(3\theta)$

No. of Leaves = 3

$\sin 3\theta = 1$

$3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$

$SP = \frac{360}{3} = 120^\circ$



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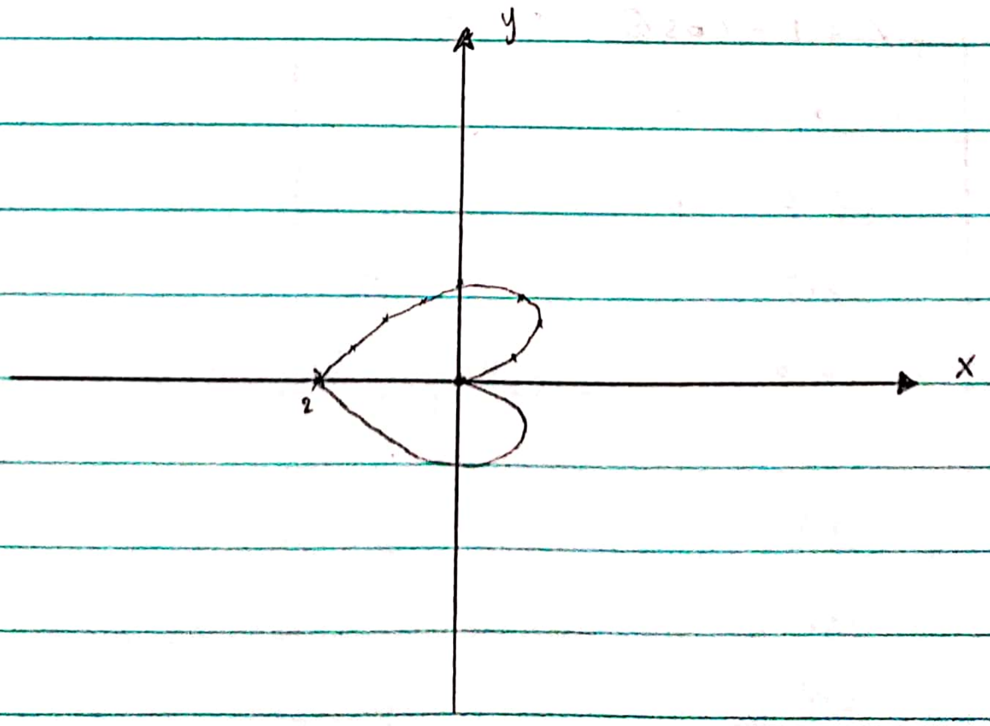
الطريقة الثانية للرسم : التعويض عن الزاوية θ

Ex : Grap the eq. $r = 4\sin 2\theta$

θ	$r = 4\sin 2\theta$
0	0
30	3.4
45	4
60	3.4
90	0
120	-3.4
135	-4
150	-3.4
180	0

θ	$r = 4\sin 2\theta$
210	3.4
225	4
240	3.4
270	0
300	-3.4
315	-4
330	-3.4
360	0



Ex: Graph the $r = 1 - \cos\theta$

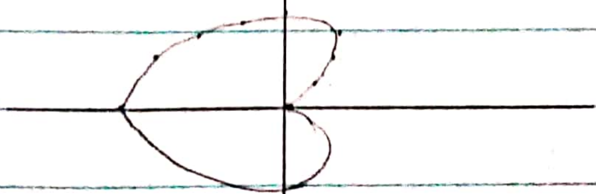
فحص التناظر

about x-axis	about y-axis	about the origin
replace θ with $-\theta$	replace r with $-r$, θ with $-\theta$	replace r with $-r$
$r_x = 1 - \cos(-\theta)$	$-r_y = 1 - \cos(-\theta)$	$-r_o = 1 - \cos\theta$
$r = 1 - \cos(\theta)$	$-r_y = 1 - \cos\theta$	$r_o = -1 + \cos\theta$
	$-r_y = -1 + \cos\theta$	$r_o \neq r$
✓ $r_x = r$	$r_y \neq r$ ✗	✗

θ	$r = 1 - \cos\theta$
0	0
30	0.2
45	0.3
60	0.5
90	1
120	1.5
135	1.7
150	1.8
180	2

ملاحظة:

$$\boxed{\begin{aligned}\cos -\theta &= \cos\theta \\ \sin -\theta &= -\sin\theta\end{aligned}}$$



فكرة السؤال :- من هذه النقاط كل نقطتين يمثلون نقطة واحدة ؟ {يكون الحل عن طريق الرسم}

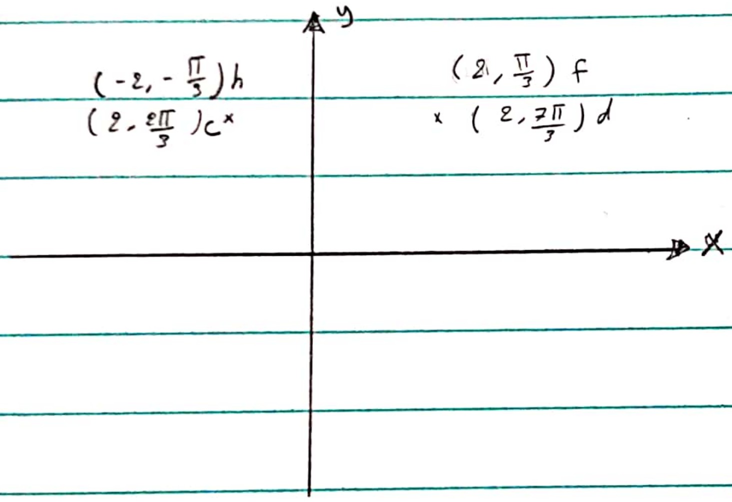
Ex: Which polar coordinate pairs label the same point?

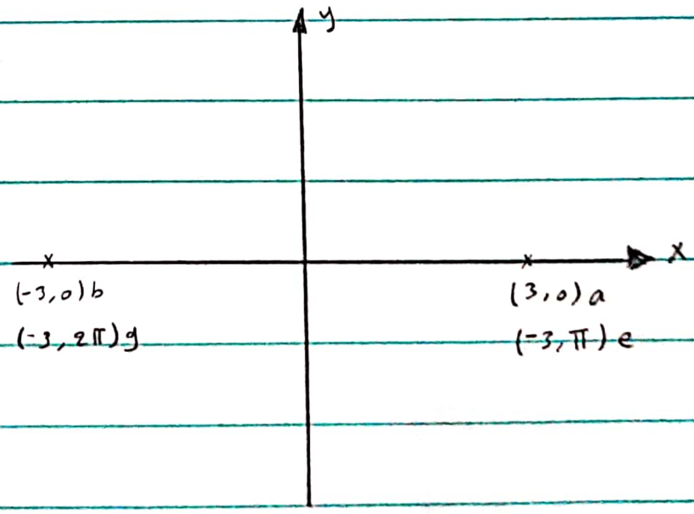
a) $(3, 0)$ b) $(-3, 0)$ c) $(2, \frac{2\pi}{3})$ d) $(2, \frac{7\pi}{3})$

e) $(-3, \pi)$ f) $(2, \frac{\pi}{3})$ g) $(-3, 2\pi)$ h) $(-2, -\frac{\pi}{3})$

ملاحظة : ما بهم السالب والموجب أهم شي القيمة

مسافة $r=2$ ومسافة $r=3$





a & e | h & c
b & g | f & d

النقطة e : نأخذ الزاوية π ثم نرجع لان 3 سالب