CHAPTER ONE
STATICS FUNDAMENTALS

<u>Engineering mechanics definition:</u>

Engineering mechanics: It is the physical science that describes the state of motion of bodies (rest or motion) under the action of the forces exerted on them. It is divided into two branches, as shown in the figure below.

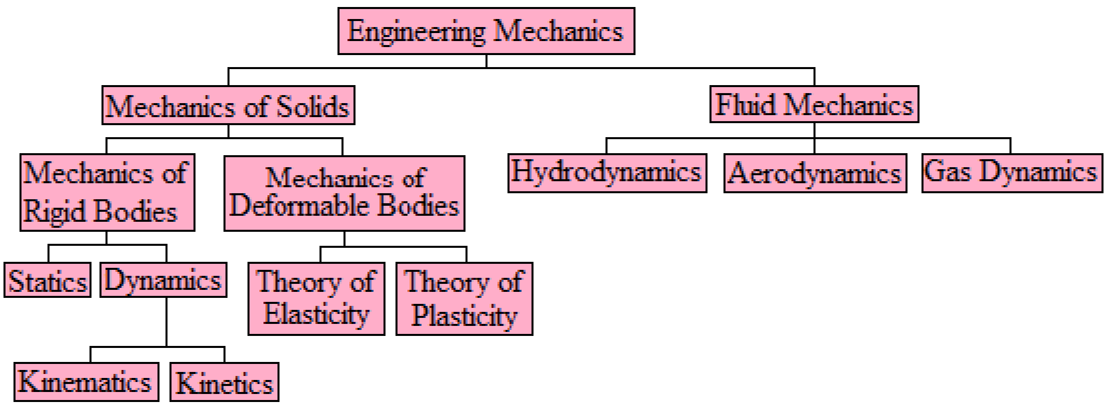


Fig. (1-1) Engineering mechanics branches

Statics:

Is the mechanical science which study the states of bodies under the effect of forces at rest (stopping or regular motion).

Dynamics:

Is the mechanical science which study the states of bodies under the effect of forces at irregular motion.

stopping	Accelarated Motion	Regular Motion	Decelarated Motion	stopping
$v = 0$	$v = \uparrow$	$v = \text{Constant}$	$v = \downarrow$	$v = 0$
$a = 0$	$a = +$	$a = 0$	$a = -$	$a = 0$

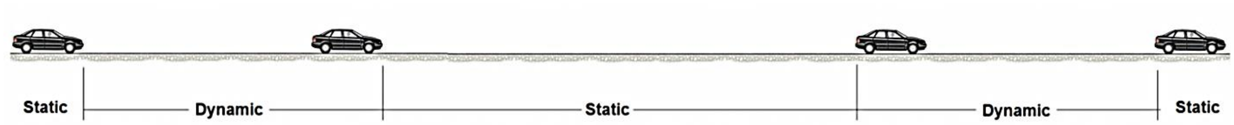


Fig. (1-2) The statics and dynamics states during a car traveling from starting motion to stopping

stopping Accelarated Flight Cruse Flight Decelarated Flight stopping

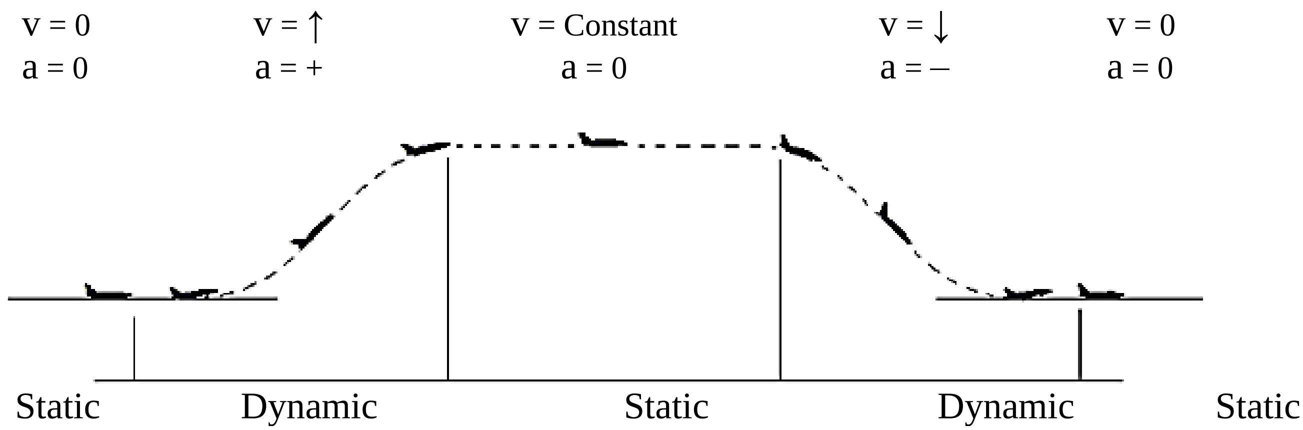


Fig. (1-3) The statics and dynamics states during aircraft travel from starting motion to stopping

Basic concepts:

Space: It is the geometric space or location occupied by the body, where this space or location is described by linear and angular measurements depending on the coordinate system followed.

Time (t): A measure that expresses the intervals of successions of events, and is a basic quantity in the analysis of dynamic problems. It is not directly depended on the analysis of static issues.

Mass (m): It is a measure of a body's resistance to a change in its state of motion (the inertia of the body). Also the mass can be defined as the amount of matter in the body.

Force (F): The external or internal action on the bodies or between bodies. Or an external action that change or tends to change the body shape or its state of motion.

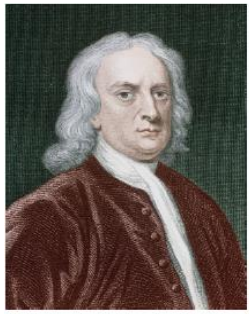
Particle: An object with negligible dimensions in the mathematical sense, or an object whose dimensions are close to zero so that it can be analyzed as a point mass (which can be represented by a point).

Rigid body: A body is expressed as a rigid body when the change in the distance between any two points of it as a result of the forces acting on it is negligible.

Concentrated Force: Is the force acting on a body and effected on a single point on it.

Distributed force: Is the force acting on a body and its effect is distributed over a specific distance or area on the body.

Newton's three fundamental laws in motion:



Sir Isaac Newton

Newton's first law

The body remains in a state of rest or continues in its state of motion at a constant speed unless unbalanced forces act on it (Resultant force = zero).

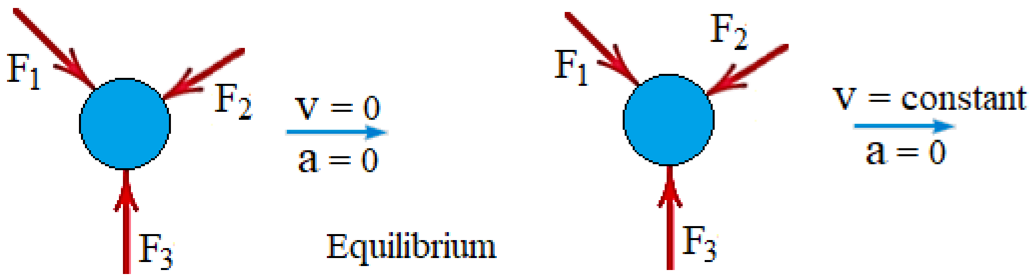


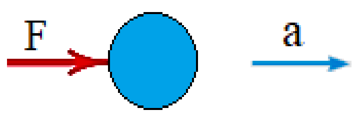
Fig. (1-4) Newton's first law aplication

Newton's second law

The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$$F = ma \quad \ldots\ldots\ldots\ldots \quad (1\text{-}1)$$

Fig. (1-5) Newton's second law aplication

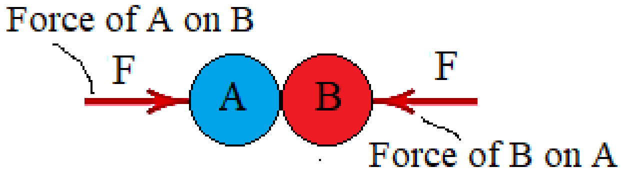


Accelerated motion direction

Newton's therd law

For every action force there is a reaction force that is equal in magnitude and opposite in direction, and is on the same line of effect with it (Collinear).

Fig. (1-6) Newton's therd law aplication



Action Reaction

Newton's law of gravitation:

There is an attractive force between any two bodies in the universe, proportional to the multiplication of their masses, and inversely proportional to the square of the distance between their centers.

The mathematical law of gravitation is expressed by the equation:

$$F = G\frac{m_1 m_2}{r^2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}2)$$

Where:
F : mutual force of attraction between two bodies.
G : The constant of gravitation = 66.73×10^{-12} $m^3/(Kg.s^2)$.
m_1, m_2 : masses of the two attractive particles.
r : distance between the centers of the two attractive particles.

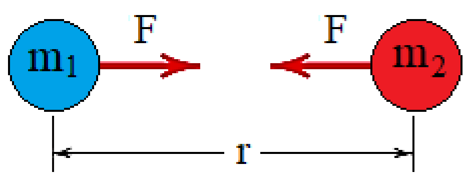


Fig. (1-7) Aplication of newton's law of gravitation

Weight (w): It is the gravitational force of the Earth for the body.

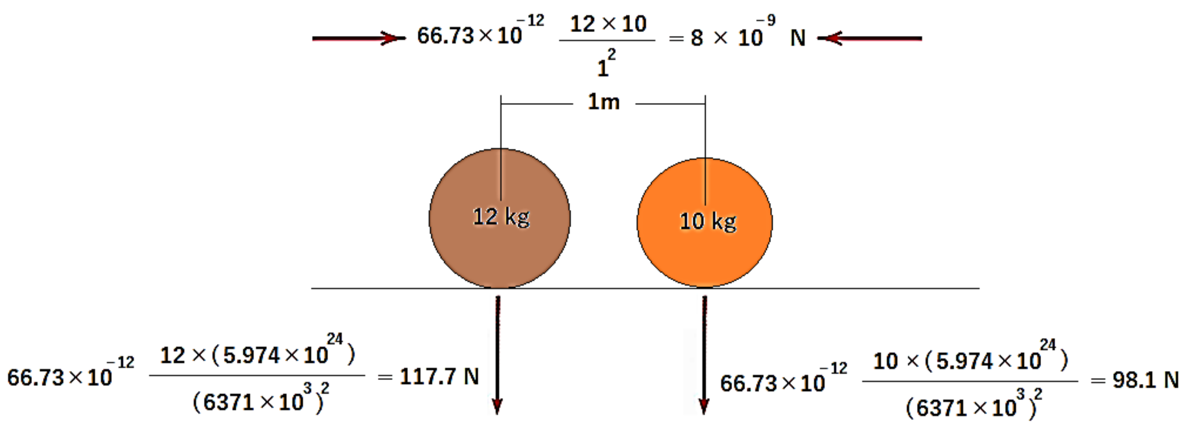


Fig. (1-8) The defference between the gravitational force between the bodies and the Earth gravity force (Weight)

- Any two bodies in nature have a mutual gravitational force.
- To find the weight (W) of any particle on the earth's surface it's mass can be considered as ($m_1 = m$).
- If we assume the earth to be a non-rotating sphere of constant density and having a mass of ($m_2 = M_e$).
- If (r) is the distance between the earth's center and the particle center which represents the radius of the earth, we have:

$$W = G\frac{M_e m}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}3)$$

The gravitational constant (G), the Earth's mass (M_e) and the Earth's radius (r) in equation (1-3) are constant values and can been replaced by a single constant value called { Earth gravitational acceleration (g) }.

$$g = \frac{GM_e}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}4)$$

So the weight in equation (1-3) is as follows:

$$W = mg \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}5)$$

For most engineering calculations, Earth gravitational acceleration (g) is determined at sea level and at a latitude of (45°), which is considered the "standard location.":

$$g = 9.81 \text{ m/s}^2$$

Table (1-1) Orbital and physical properties for the sun, the moon, and some planets

The Sun and The Planets			Mass $\times 10^{24}$ kg	Average radius km	Distance to the sun $\times 10^{6}$ km	Orbital average speed km/s	Notes
The sun			1989000	696000			
The moon			0.0735	1737		1.022 **(Around the earth)**	**Distance to the earth: 384399 km**
Planets	**Internal (rocky)**	**Mercury**	0.33	2430	57.91	47.87	
		Venus	4.869	6052	108.21	35.02	
		Earth	5.9736	6371	149.6	29.78	
		Mars	0.642	3387	227.99	24.077	
	External (gaseous)	**Jupiter**	1898.6	69910	778.55	13.07	
		Saturn	578.46	57320	1433.4	9.69	
		Uranus	86.81	25270	2876.7	6.81	
		Neptune	102.43	24550	4498.3	5.432	

Equilibrium between the earth and the moon:

Newton's law of gravitation:

$$F = G\,\frac{Mm}{r^2}$$

$$F = 66.73 \times 10^{-12}\ \frac{5.9736 \times\ 10^{24} \times 7.3477 \times\ 10^{22}}{(\,384399\ \times\ 10^{3}\,)^{2}}$$

$$= 66.73\,\frac{5.9736 \times 7.3477 \times\ 10^{28}}{384399^{\,2}}\ =\ 2 \times 10^{20}\ \text{N}$$

Centrifugal force law:

$F_c = mr\omega^2$ ……………..….. (1-6) $\quad v = \omega r\ ,\ \ \omega = v/r$

$F_c = mr\,(v/r)^2$ …………. (1-7)

$F_c = mv^2/r$ ……………… (1-8)

$$F_c = \frac{7.3477 \times\ 10^{22}\ \times\ 1022^{2}}{384399\ \times\ 10^{3}} = \frac{7.3477\ \ \times\ 1022^{2}\ \times\ 10^{19}}{384399}$$

$$= 2 \times 10^{20}\ \text{N}$$

Equilibrium between the sun and the earth:

Newton's law of gravitation:

$$F = G\,\frac{Mm}{r^2}$$

$$F = 66.73 \times 10^{-12}\ \frac{5.9736 \times\ 10^{24} \times 1.989 \times\ 10^{30}}{(149.6\ \times\ 10^{9}\,)^{2}}$$

$$= 66.73\,\frac{5.9736 \times 1.989 \times\ 10^{24}}{149.6^{\,2}}\ =\ 3.54 \times 10^{22}\ \text{N}$$

Centrifugal force law:

$$F_c = mv^2 / r$$

$$F_c = \frac{5.9736 \times\ 10^{24} \times\ 29780^{2}}{149.6\ \times\ 10^{9}} = \frac{5.9736\ \ \times\ 29780^{2}\ \times\ 10^{15}}{149.6}$$

$$= 3.54 \times 10^{22}\ \text{N}$$

Gravitational acceleration at the surface of the Earth:

$$g = \frac{Mm}{r^2} = \frac{66.73 \times 10^{-12} \times 5.9736 \times 10^{24}}{(6371 \times 10^3)^2} = \frac{66.73 \times 5.9736 \times 10^6}{(6371)^2}$$

$$= 9.80665 \text{ m/s}^2 = 9.81 \text{ m/s}^2$$

$$= \frac{9.81}{0.3048} = 32.2 \text{ ft/s}^2$$

Gravitational acceleration at the surface of the Moon:

$$g_m = \frac{Gm}{{r_m}^2} = \frac{66.73 \times 10^{-12} \times 7.3477 \times 10^{22}}{(1737.35 \times 10^3)^2} = \frac{66.73 \times 7.3477 \times 10^4}{(1737.35)^2}$$

$$= 1.6244 \text{ m/s}^2$$

$$= 5.33 \text{ ft/s}^2$$

$$= 0.1656\ g_e$$

Gravitational acceleration at the surface of the Sun:

$$g_s = \frac{Gm}{{r_s}^2} = \frac{66.73 \times 10^{-12} \times 1.989 \times 10^{30}}{(696 \times 10^6)^2} = \frac{66.73 \times 1.989 \times 10^6}{(696)^2}$$

$$= 274 \text{ m/s}^2$$

$$= 899.368 \text{ ft/s}^2$$

$$= 27.93\ g_e$$

Table (1-2) Gravitational acceleration at the surfaces of the planets and the gravity force between the sun and the planet

THE PLANETS	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
The central gravitational acceleration (m/s^2)	**3.73**	**8.87**	**9.81**	**3.73**	**25.92**	**11.75**	**9.07**	**11.34**
The gravity force between the sun and the planet ($\times 10^{22}$ N)	**1.31**	**5.52**	**3.54**	**0.163**	**41.6**	**3.7**	**0.14**	**0.0672**

Example (1-1):

Calculate the gravitational force generated between two bodies whose masses are (10 kg) and (15 kg) respectively, and the distance between their centers is (750 mm), then calculate the weight of each body.

Solution:

$$F = G\,\frac{m_1 m_2}{r^2}$$

Where $G = 66.73 \times 10^{-12}\ m^3 / (Kg\ .\ s^2)$

$$F = 66.73 \times 10^{-12}\left[\frac{(10)(15)}{(0.75)^2}\right] = 17.795\,(10^{-9})\ N = 17.8\ nN$$

$$W_1 = (10)(9.81) = 98.1\ N$$

$$W_2 = (15)(9.81) = 147.15\ N$$

Example (1-2):

A satellite weighing (700 Ib) on the surface of the Earth. Calculate the Earth's gravitational force for this satellite when it is positioned at a distance of (36000 km) from the surface of the Earth, where its rotation speed around the Earth is equal to the speed of the Earth's rotation around itself.

- Earth mass ($M_e = 5.97 \times 10^{24}$ kg).

- Radius of the Earth ($R_e = 6371$ km).

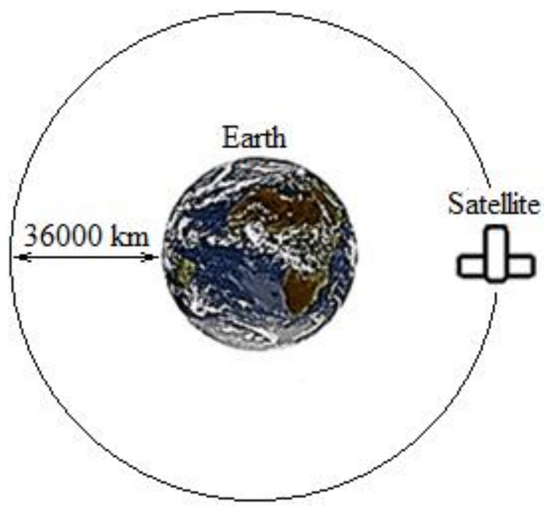


Fig. (Ex. 1-2)

Solution:

$$W = 700\ Ib = 700 \times 4.448 = 3113.6\ N$$

$$m = \frac{W}{g} = \frac{3113.6}{9.81} = 317.4\ kg$$

$$r = R_e + 36000 = 6371 + 36000 = 42371\ km = 42371 \times 10^3\ m$$

$$F = G\,\frac{M_e \times m}{r^2} = 66.73 \times 10^{-12}\,\frac{5.97 \times 10^{24} \times 317.4}{(42371 \times 10^3)^2}$$

$$= 70.43\ N$$

Example (1-3):

A spacecraft of mass (3000 kg) is launched from earth to the moon.

1. **Calculate the distance from the earth's surface at which the force of gravity between the spacecraft and both the Earth and the moon is equal.**
2. **Calculate the gravitational force between the spacecraft and both the earth and the moon at this distance.**

- **The distance between the centers of the earth and the moon = 384400 km.**
- **The mass of the Earth = 5.97×10^{24} kg.**
- **The mass of the Moon = 7.35×10^{22} kg.**
- **Radius of the Earth = 6371 km.**
- **Radius of the Moon = 1737 km.**

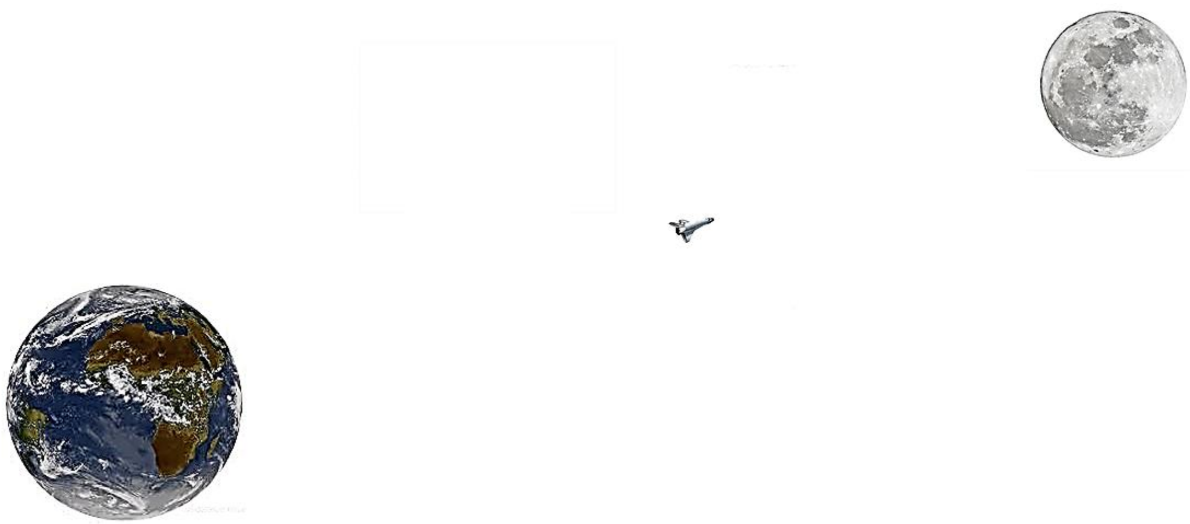


Fig. (Ex. 1-3)

Solution:

$$F_e = G\ \frac{M_e\ M_s}{r_e^{\,2}} \qquad\qquad F_m = G\ \frac{M_m\ M_s}{r_m^{\,2}}$$

$$F_e = 66.73 \times 10^{-12}\ \frac{5.9736 \times 10^{24} \times 3 \times 10^{3}}{r_e^{\,2}} = \frac{1.196 \times 10^{18}}{r_e^{\,2}}$$

$$F_m = 66.73 \times 10^{-12}\ \frac{7.3477 \times 10^{22} \times 3 \times 10^{3}}{r_m^{\,2}} = \frac{1.471 \times 10^{16}}{r_m^{\,2}}$$

$F_e = F_m$

$$\frac{1.196 \times 10^{18}}{r_e^{\,2}} = \frac{1.471 \times 10^{16}}{r_m^{\,2}}$$

$$\frac{1.196 \times 10^{18}}{1.471 \times 10^{16}} = \frac{r_e^{\,2}}{r_m^{\,2}}$$

$$\frac{{r_e}^2}{{r_m}^2} = 81.3 \qquad \Rightarrow \qquad \frac{r_e}{r_m} = 9.02$$

$r_e = 9.02\ r_m$ .... (1)

$r_e + r_m = 384400$ .... (2)

$9.02\ r_m + r_m = 384400$

$10.02\ r_m = 384400 \quad \Rightarrow \quad r_m = 38363.27$ km

$r_e = 9.02 \times 38363.27 = 346036.73$ km

$h_e = r_e - R_e = 346036.73 - 6371 = 339665.73$ km

$$F_e = \frac{1.196 \times 10^{18}}{(346036730)^2} = 10\ \text{N} \qquad\qquad F_m = \frac{1.471 \times 10^{16}}{(38363270)^2} = 10\ \text{N}$$

r_m : Distance between the spacecraft and the Moon's center.
r_e : Distance between the spacecraft and the Earth's center.
R_e : Radius of the Earth.
h_e : Distance between the spacecraft and the Earth's surface.
F_m : gravitational force between the spacecraft and the Moon.
F_e : gravitational force between the spacecraft and the Earth.

Example (1-4):

Calculate the distance between the satellite and the Earth's surface that would cause the satellite to constantly face one region of the Earth's surface.

- **Earth mass = 5.97×10^{24} kg.**
- **Radius of the Earth = 6371 km.**
- **The linear speed of the Earth's rotation around its axis = 1674.4 km/hr.**

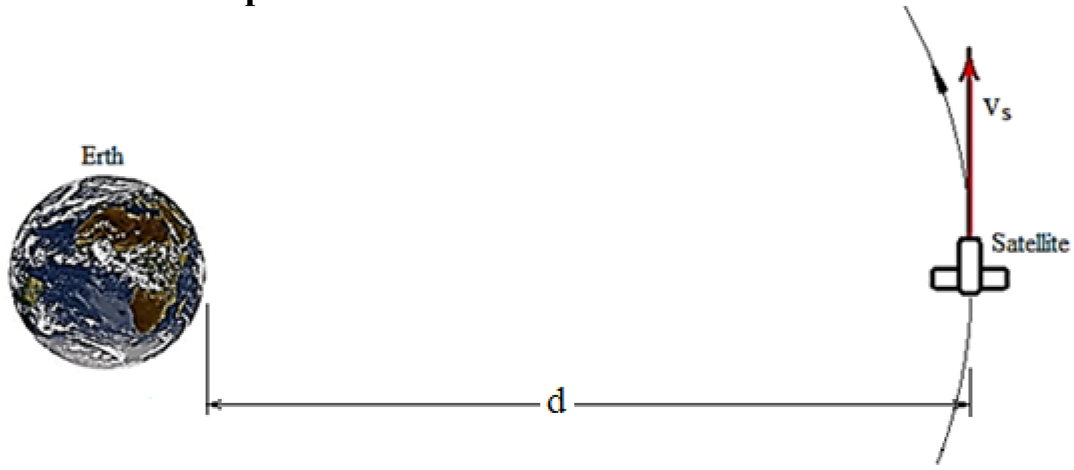
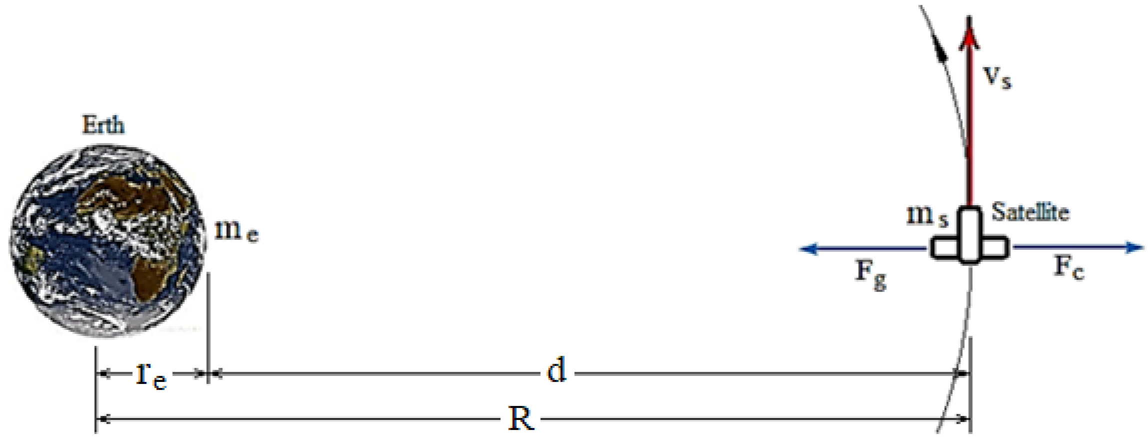


Fig. (Ex. 1-4)

Solution:



$F_g = G \dfrac{m_e\ m_s}{R^2}$ (Gravity force)

$F_c = m_s R\, \omega_s^2$ (Centrifugal force)

$G \dfrac{m_e\ m_s}{R^2} = m_s R\, \omega_s^2$

$\dfrac{G\, m_e}{R^2} = R\, \omega_s^2$

$G\, m_e = R^3\, \omega_s^2$

V_e = 1674.4 km/hr = 465 m/s

(Linear speed of the Earth's rotation around its axis)

r_e = 6371 km = 6371000 m (Radius of the Earth)

$\omega_e = V_e / r_e$ (Angular velocity of the Earth about its center)

$\omega_e = 465 / 6371000 = 7.3 \times 10^{-5}$ rad/s $= \omega_s$

$m_e = 5.97 \times 10^{24}$ kg (Earth mass)

$G = 66.73 \times 10^{-12}$ $m^3/(Kg.s^2)$ (Constant of gravitation)

$66.73 \times 10^{-12} \times 5.97 \times 10^{24} = R^3\, (7.3\times10^{-5})^2$

$R^3 = \dfrac{66.73 \times 5.97 \times 10^{12}}{(7.3\times10^{-5})^2} = 7.48 \times 10^{22}$

R = 42125970 m

$d = R - r_e$ = 42125970 – 6371000 = 35754970 m = 35755 km

Coordinates systems:

- Cartesian coordinate system:

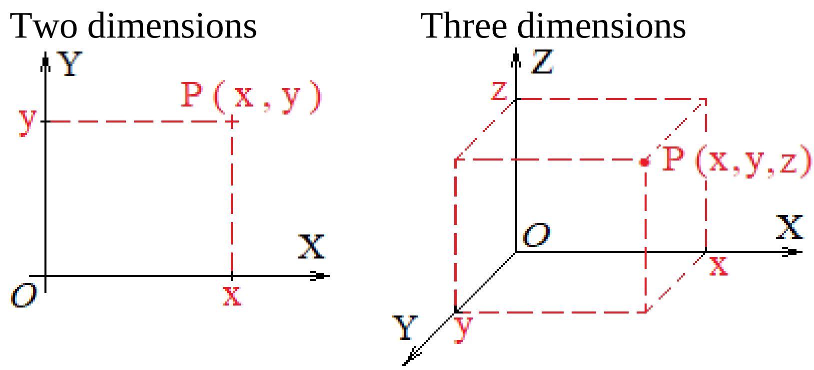


Fig. (1-9)
Cartesian coordinate system

- Polar coordinate system:

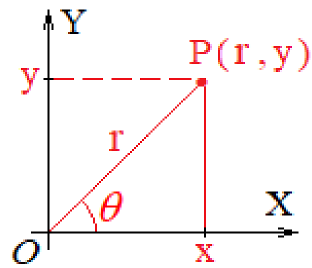


Fig. (1-10)

Polar coordinate system

- Spherical coordinate system:

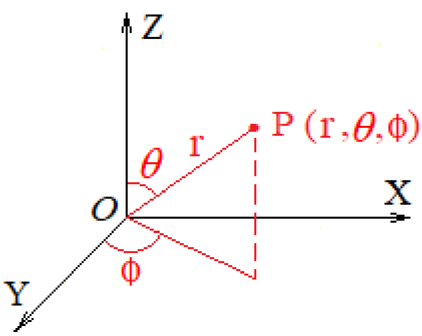


Fig. (1-11)

Spherical coordinate system

- Cylindrical coordinate system:

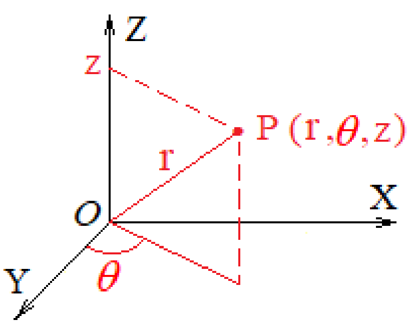


Fig. (1-12)

Cylindrical coordinate system

System of units:

The units of length, mass and time are the basic units from which other units are derived, the force unit was added due to its importance in the subject of engineering mechanics.

	Length	**Mass**	**Time**	**Force**
(SI) Units System International Units	Meter (m)	Kilogram (kg)	Second (s)	Newton (N)
(FPS) Units System British Units	Foot (ft)	Slug (slug)	Second (s)	Pound (Ib)

1 Ib = 4.448 N
1 slug = 14.59 kg
1 ft = 0.3048 m

Conversion of Units:

1 ft (foot) = 12 in. (inches).
1 yd (yard) = 3 ft = 36 in.
1 mi (mile) = 1760 yd = 5280 ft = 63360 in.
1 kip (kilo-pound) = 1000 lb (pound)
1 Ib = 0.453 kg.
1 kg = 2.205 Ib.
1 ton = 2205 Ib = 2.205 kip.
1 ton = 1000 kg
1 in. = 2.54 cm = 25.4 mm
1 ft = 0.3048 m = 30.48 cm
1 yd = 91.44 cm
1 mi = 1609.34 m = 1.609 km

Prefixes:

If the quantity to be measured is too large or too small, it can be expressed in multiples or fractions of the units to determine its value logically.

Table (1-3) shows the prefixes used in the global system. Where each represents a multiple or fraction of a particular unit. *The kilogram is the only basic unit defined by a prefix.*

For example:

4 000 000 N = 4 000 kN (kilo-newton) = 4 MN (mega-newton).

0.005 m = 5 mm (milli-meter).

Table (1-3) The multiple or submultiples of units

Multiplication Factor		Prefix	Symbol
1 000 000 000 000	$= 10^{12}$	Tera	T
1 000 000 000	$= 10^{9}$	Giga	G
1 000 000	$= 10^{6}$	Mega	M
1 000	$= 10^{3}$	Kilo	k
100	$= 10^{2}$	Hecto	h
10	$= 10^{1}$	Deka	da
0.1	$= 10^{-1}$	Deci	d
0.01	$= 10^{-2}$	Centi	c
0.001	$= 10^{-3}$	Milli	m
0.000 001	$= 10^{-6}$	Micro	μ
0.000 000 001	$= 10^{-9}$	Nano	n
0.000 000 000 001	$= 10^{-12}$	Pico	p

Example (1-5):

Express the following units in the correct international units (SI) form using an appropriate prefix:

(a) MN/μs.
(b) Gg/mN.
(c) GN/(kg.ms).

Solution:

(a) $\mathrm{MN/\mu s} = \dfrac{(10^6)\mathrm{N}}{(10^{-6})\mathrm{s}} = \dfrac{(10^{12})\mathrm{N}}{\mathrm{s}} = \mathrm{TN/s}$

(b) $\mathrm{Gg/mN} = \dfrac{(10^9)\mathrm{g}}{(10^{-3})\mathrm{N}} = \dfrac{(10^{12})\mathrm{g}}{\mathrm{N}} = \mathrm{Tg/N}$

(c) $\mathrm{GN/(kg.ms)} = \dfrac{(10^9)\mathrm{N}}{\mathrm{kg}(10^{-3})\mathrm{s}} = \dfrac{(10^{12})\mathrm{N}}{\mathrm{kg.s}} = \mathrm{TN/(kg.s)}$

Example (1-6):

What is the density of wood expressed in the international units (SI - units), if its value according to the british system units (FBS - units) is (4.7 slug/ft³)?

Solution:

$$(4.7 \text{ slug/ft}^3) = 4.7\,\frac{(14.59)}{(0.3048)^3} = 2421.6 \text{ kg/m}^3 = 2.42 \text{ Mg/m}^3$$

Example (1-7):

Find the speed of the car shown in Figure (Ex. 1-7) in (kilometers per hour) and (meters per second) units, as the car is traveling at a speed of (60 mi/h).

Fig. (Ex. 1-7)

Solution:

60 mi/h = (60)(5280)(0.3048)/(1000) = 96.56 km/h

96.56 km/h = (96.56)(1000)/(3600) = 26.8 m/s

= 96.56 /3.6 = 26.8 m/s

Example (1-8):

A car of (1400 kg) mass.

(a)- Determine the weight of the car in newtons.
(b)- Convert the mass of the car to slugs.
(c) Determine its weight in pounds.

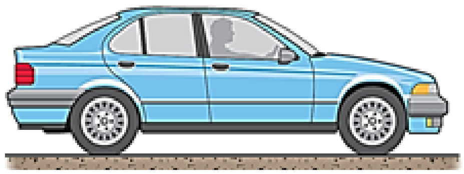


Fig. (Ex. 1-8)

Solution:

(a)- W = mg = 1400 × 9.81= 13730 N

(b)- $m = \frac{1400}{14.59} = 95.96$ slugs

(c)- W = mg = 95.96 × 32.2 = 3090 Ib

Example (1-9):

The spacecraft shown in Fig. (Ex. 1-9) has a mass of (15×10^3) slugs on earth surface. Specify:

(a) its mass in (SI – units).

(b) its weight in (SI – units).

If the spacecraft is on the moon surface, where the acceleration due to gravity is ($g_m = 5.3\ ft/s^2$), determine:

(c) its weight in (SI – units).

(d) its mass in (SI – units).

Fig. (Ex. 1-9)

Solution:

(a) 15×10^3 slugs = $(15 \times 10^3) (14.59)$

$= 218.85 \times 10^3$ kg = 218.85 Mg

(b) $W_e = m.g = (218.85 \times 10^3) (9.81)$

$= 2.147 \times 10^6$ N = 2.147 MN

(c) $W_m = m.g_m = (15 \times 10^3) (5.3) = 79.5 \times 10^3$ Ib

$= (79.5 \times 10^3) (4.448) = 353.6 \times 10^3$ N = 0.35 MN

Or $W_m = W_e (g_m / g) = (2.147\ MN) \left(\dfrac{5.3\ ft/s^2}{32.2\ ft/s^2} \right) = 0.35\ MN$

(d) Since the mass is independent of its location, then:

$m_m = m_e = 218.85 \times 10^3$ kg = 218.85 Mg

Example (1-10):

A man weighs (180 lb) on Earth surface.

(a)- Specify his mass in slugs.
(b)- Specify his mass in kilograms.
(c)- Specify his weight in newtons.

If the man is on the surface of the Moon, where the acceleration due to gravity is (g_m = 5.3 ft/s^2).

(d)- Specify his weight in pounds.
(e)- Specify his mass in kilograms.

Solution:

(a) $m = \frac{180}{32.2} = 5.59$ slug

(b) $m = 5.59 \times 14.59 = 81.56$ kg

(c) $W = 180 \times 4.4482 = 800$ N or $81.56 \times 9.81 = 800$ N

(d) $W = 5.59 \times 5.3 = 29.63$ Ib

(e) $m = 5.59 \times 14.59 = 81.56$ kg

Problems:

1-1) Express each of the following units in the correct (SI – form) using an appropriate prefix:

a) µm/ms, (b) mkm, (c) Gs /mg (d) µN.Gm.

Ans. mm/s, m, Ts/g, kN.m

1-2) The density of brass is (8.33 Mg/m^3). Determine its specific weight (wt. / vol.) in English units. Use an appropriate prefix.

Ans. 81.717 kN/m^3

1-3) Convert the following quantities from the international system (SI) to the English system (FPS), using an appropriate prefix:

(a)- (27.5 kN/m^3) to (lb/ft^3). (b)- (0.5 mm/s) to (ft/h).

(c)- (1.13 kN.m) to (Ib.ft).

Ans. 175 Ib/ft^3, 5.9 ft/h, 833.5 Ib.ft

1-4) Find the mass of an object that has a weight of:

(a)- 35 mN. (b)- 200 kN. (c)- 50 MN.

Ans. 3.57 g, 20.39 Mg, 5.1 Gg

1-5) Determine the weight in (SI) units of a body that has a mass of:

(a) - 15 kg (b) - 0.75 g (c) - 7.5 Mg

Ans. 147.15 N , 7.36 mN , 73.6 kN

1-6) Two balls with a mass of (250 kg) and radius of (350 mm) for each ball, each ball are touching each other. Determine the gravitational force acting between the two balls.

Ans. 8.5 µN

1-7) The density of the water is (1 Mg/m^3). What is the density of it expressed in English units?

Ans. 1.94 $slug/ft^3$

1-8) A spacecraft of mass (3000 kg) is launched from Earth to the Moon.

1- Calculate the weight and mass of the spacecraft on the surface of the Earth and on the surface of the Moon.
2- Calculate the gravitational force between the spacecraft and both the Earth and the Moon when the spacecraft is at a distance of (100000 km) from the surface of the Earth.

- The distance between the centers of the Earth and the Moon = 384400 km.
- The mass of the Earth = 5.97×10^{24} kg.
- The mass of the Moon = 7.35×10^{22} kg.
- The radius of the Earth = 6371 km.
- The radius of the Moon = 1737 km.

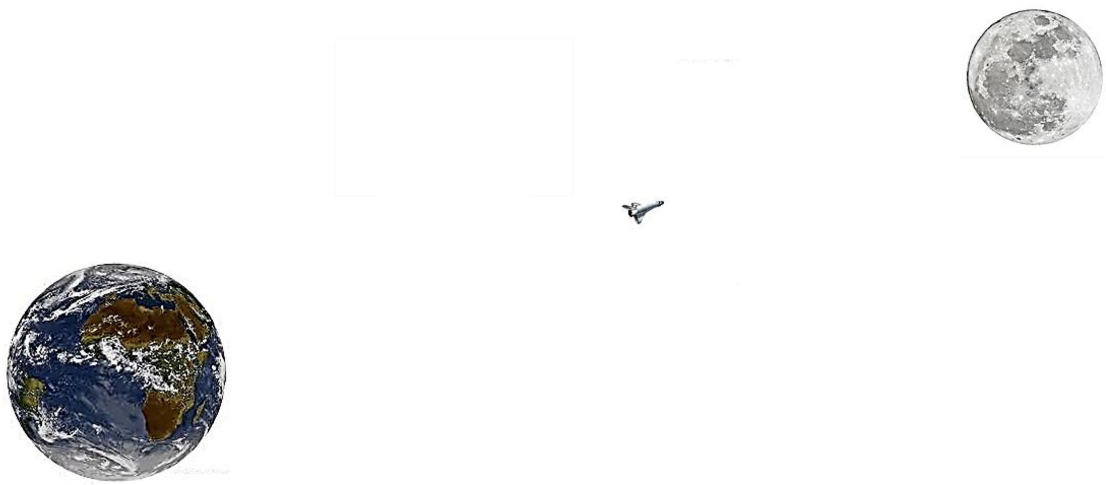


Fig. (Pr. 1-8)

Ans:

For the Earth		For the Moon	
$W_e = 29430\ N$	$m = 3000\ kg$	$W_m = 4873\ N$	$m = 3000\ kg$
$F_{se} = 105.6\ N$		$F_{sm} = 0.19\ N$	

CHAPTER TWO
FORCE ANALYSIS

Scalars and Vectors:

Scalar (A)

A scalar is any positive or negative physical quantity that can be completely expressed by its *magnitude*.

for example: length, mass, time, density, volume.

Vector ($\vec{A}$)

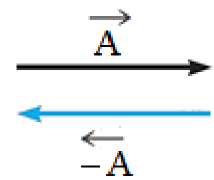
A vector is any physical quantity that requires both a *magnitude and a direction* for its complete description.

for example: force, position, moment, weight, velocity, displacement, acceleration.

- The *magnitude* of the vector is represented by the length of the arrow.
- The *direction* of the vector line of action is represented by the angle (θ) between the vector and a fixed axis.

Vector direction:

The negative sign means the opposite direction.



Types of vectors:

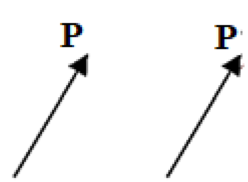
There are three types of a vector:

1- **Free vector:**
It is a vector which may be freely moved creating couples in space.

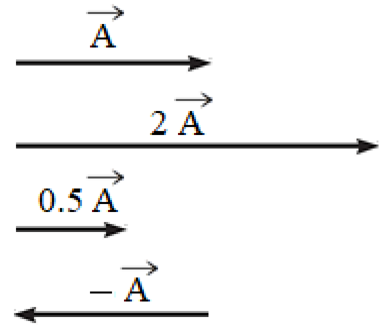
2- **Sliding vector**:
It is a vector that can represent the force acting on a rigid body and can be moved along the line of action of the force without any effect on the body.

3- **Bound vector or Fixed vector:**
It is a vector that its moving requires changing the conditions of the problem.

For two vectors to be equal they must have the same value and direction, they do not need to have the same point of application.:



Multiplication and Division of a vector by a scalar:



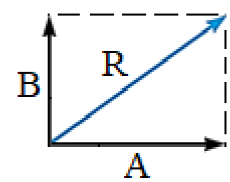
- Multiplying a vector by a positive absolute number leads to an increase or decrease in its value by that number.
- Multiplying a vector by a negative absolute number, leads to a change in its direction in the opposite direction, with an increase or decrease in its value according to the magnitude of this number.

Vector Addition:

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are collinear, i.e., both have the same line of action, the algebraic addition is used, as shown:

$$(\vec{R} = \vec{A} + \vec{B}) \quad \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}1)$$

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are perpendicular, then they can be summed by using Pythagoras' theory.

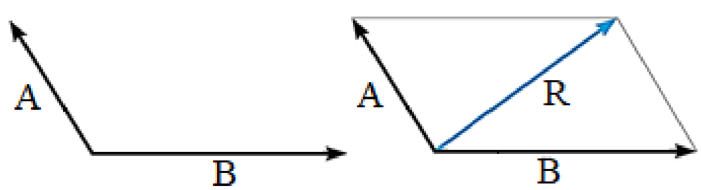


$$R = \sqrt{A^2 + B^2} \quad \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are not collinear, there are two methods for addition:

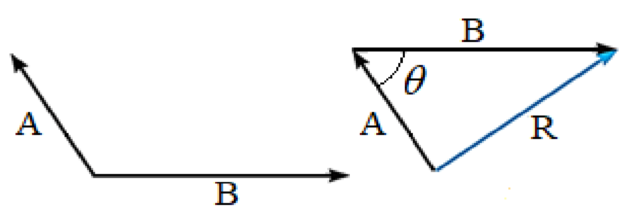
1- Parallelogram law.

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta} \quad \ldots\ldots\ldots\ldots \quad (2\text{-}3)$$



2- Triangle rule.

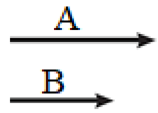
$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

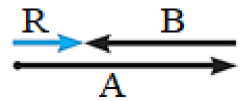


Vector Subtraction:

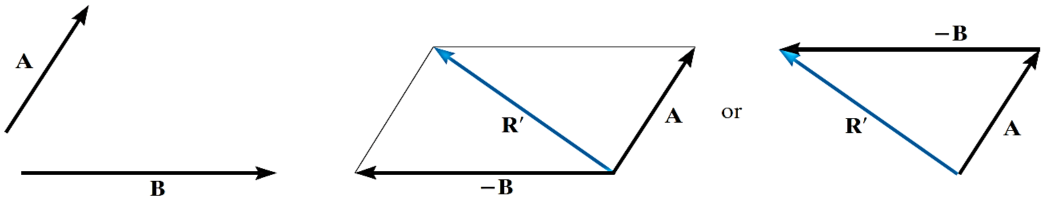
If the two vectors ($\vec{A}$) and ($\vec{B}$) are in the same direction, the difference between them can be found as follows:

$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ …….. (2-4)



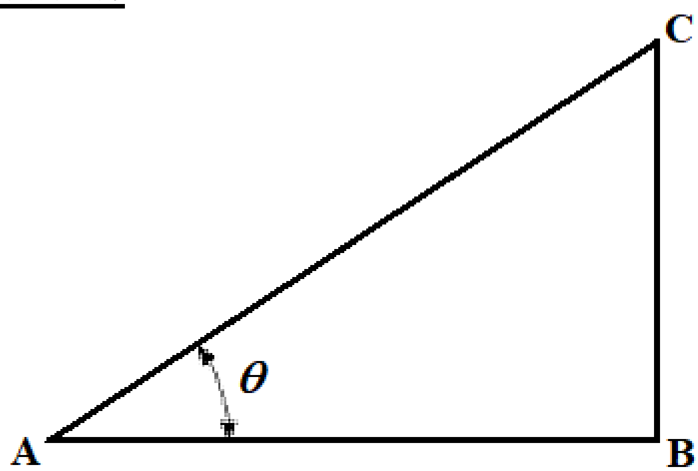


That is, subtraction is a special case of addition, and vector addition laws can be used to subtract vectors by reversing the direction of the subtracted vector.



Parallelogram law Triangle rule

Trigonometric relations:

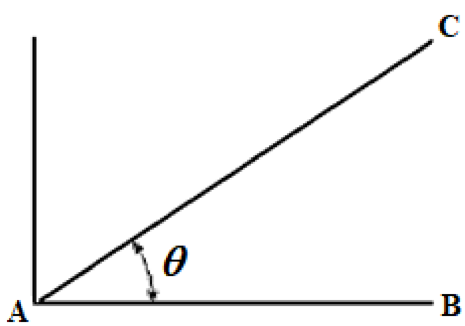


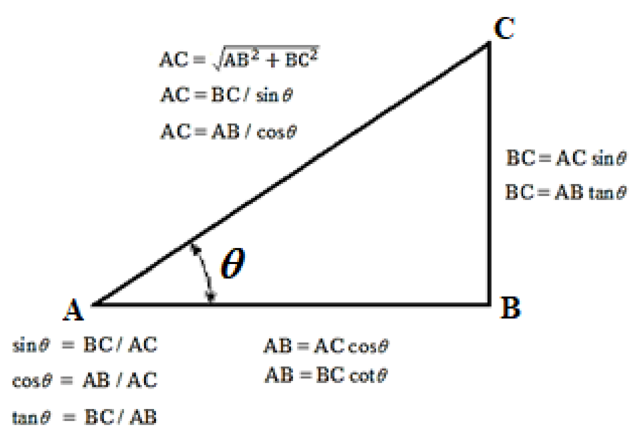
$AB = AC \cos\theta \qquad BC = AC \sin\theta$

$AB = BC \cot\theta \qquad BC = AB \tan\theta$

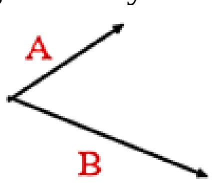
$$\left.\begin{aligned} AC &= \sqrt{AB^2 + BC^2} \\ AC &= BC / \sin\theta \\ AC &= AB / \cos\theta \end{aligned}\right\} \quad \ldots\ldots\ldots \text{(2-5)}$$

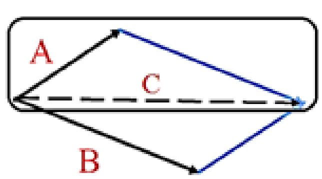
$$\left.\begin{aligned} \sin\theta &= BC / AC \\ \cos\theta &= AB / AC \\ \tan\theta &= BC / AB \end{aligned}\right\} \quad \ldots\ldots\ldots \text{(2-6)}$$

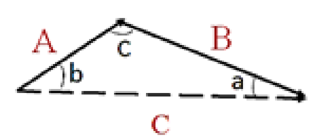




Trigonometry can be used to find the resultant force:







From Cosine Law: $\quad C = \sqrt{A^2 + B^2 - 2AB\cos(c)}$ ……………. (2-7)

From Sines Law: $\quad \dfrac{A}{\sin a} = \dfrac{B}{\sin b} = \dfrac{C}{\sin c}$ ………………..…. (2-8)

Types of force systems:

A force system is a group of forces (two or more forces) that affect a body or group of bodies in a specific situation, and it can be classified into:

1- The system of forces located on one line of action (Collinear system).
2- The system of parallel forces (Parallel system).
3- The system of forces located on one plane (Coplanar system).
4- The system of converging forces (Concurrent system).
5- The system of parallel forces located in one plane (Parallel, Coplanar system), in which the lines of action of the forces are parallel and located in one plane.
6- The system of converging forces located in one plane (Concurrent, Coplanar system), in which the lines of action of the forces intersect at a common point and are located in one plane.
7- The system of convergent and non-parallel forces and located in one plane (Concurrent, Nonparallel, Coplanar system), in which the lines of action of the forces are intersecting and non-parallel and located in one plane.
8- The system of parallel forces that do not locate in one plane (Parallel, Noncoplanar system), in which the lines of action of the forces are parallel and do not locate in one plane.
9- The system of converging forces that do not locate in one plane (Concurrent, Noncoplanar system), in which the lines of action of the forces intersect at a common point and do not locate in one plane.
10- The system of non-parallel, non-intersecting forces that do not locate in one plane (Nonparallel, Nonconcurrent, Noncoplanar system), where the lines of action of the forces are non-parallel, non-intersecting and do not locate in one plane.

Principle of force transmissibility on its line of action:

If a force (F) acting on a specific body is transmitted along its line of action without changing its direction, the effect of the force on the body does not change.

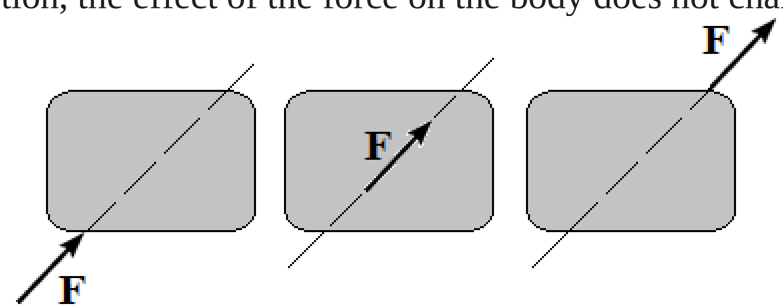


Fig. (2-1) Principle of Force transmissibility on its line of action

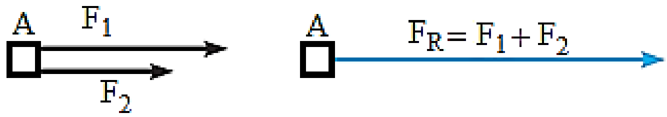
Resultant of forces:

If two forces (F_1) and (F_2) acting on a particle (A) may be replaced by a single force (F_R), which has the same effect on the particle, this force is called the *resultant* of the forces (F_1) and (F_2).

If the forces located on one line of action (Collinear):

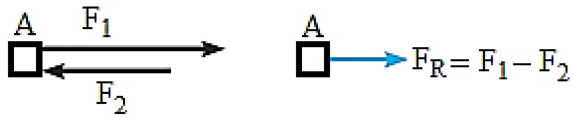
If the forces are in the same direction, then their resultant is according to equation (2-1):

$$F_R = F_1 + F_2$$



If the forces are in opposite directions, then their resultant is according to Eq. (2-4):

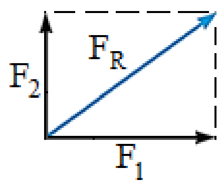
$$F_R = F_1 - F_2$$



If the forces are converging, perpendicular forces located in one plane (Concurrent, perpendicular, Coplanar):

The resultant can be determined by using Pythagoras' theory, and from Eq. (2-2):

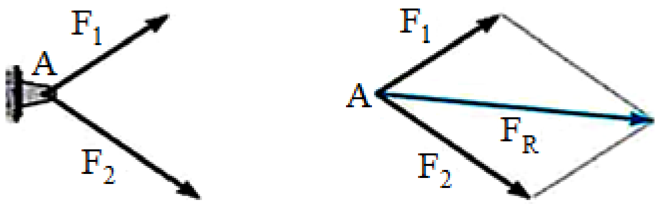
$$R = \sqrt{A^2 + B^2}$$



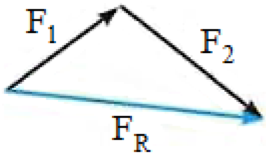
If the forces are converging forces located in one plane (Concurrent, Coplanar):

The resultant may be obtained by:

1- A parallelogram, using (F_1) and (F_2) as two sides of the parallelogram. The diagonal that pass through point (A) represents the resultant. This is known as the *parallelogram law*.



2- The triangle of forces, where the force (F_1) is drawn, and from the end of it the force (F_2) is drawn, so the resultant is the force that starts from the beginning of (F_1) and ends at the end of (F_2). This is known as the *triangle law*.



$$F_R = F_1 + F_2$$

3- Using the triangle of forces, the value of the resultant can be found by using the law of cosines, and finding its direction from the law of sines. Equations (2-7) and (2-8).

Example (2-1):

Two forces (180 N) and (120 N) are applied to the ring shown in Fig. (Ex. 2-1). Determine the magnitude and direction of the resultant force.

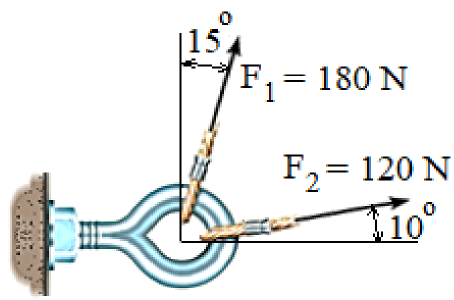
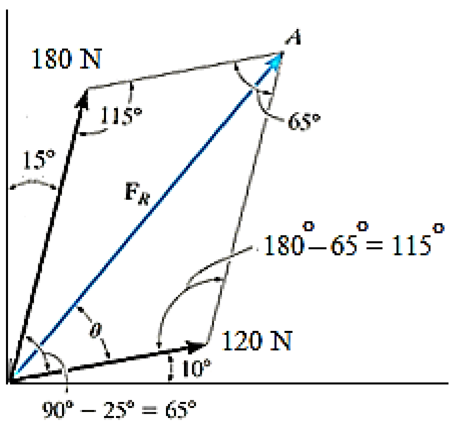


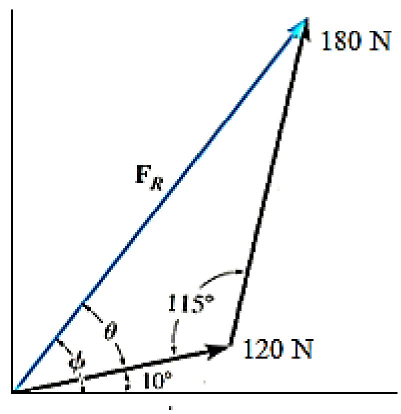
Fig. (Ex. 2-1)

Solution:

Parallelogram Law.

A parallelogram is drawn from drawing a line starting from the head of force (180 N) and parallel to the force (120 N), and another line starting from the head of force (120 N) and parallel to the force (180 N). The resultant of the two forces (F_R) will be from the starting point of the two forces to the point of intersection of these two lines at point (A).





Trigonometry laws.

From the parallelograms. A triangle of forces is created.

Use the law of cosines.

$$F_R = \sqrt{F_1{}^2 + F_2{}^2 - 2\,F_1\,F_2 \cos\alpha}$$

$$F_R = \sqrt{(180)^2 + (120)^2 - 2\,(180)\,(120)\cos 115}$$

$$= \sqrt{32400 + 14400 - 43200\,(\,-0.4226\,)} \;=\; 255\text{ N}$$

Applying the law of sines to determine (θ),

$$\frac{180}{\sin\theta} = \frac{255}{\sin 115^{o}} \quad\Rightarrow\quad \sin\theta = \frac{180\ \sin 115^{o}}{255} = 0.64 \quad\Rightarrow\quad \theta = 39.8^{o}$$

Thus the direction (ϕ) of the resultant force (F_R) measured from the horizontal is:

$$\theta = 39.8^{o} + 10^{o} = 49.8^{o}$$

Example (2-2):

In the fixed structure shown in Fig. (Ex. 2-2).

P = 400 N
T = 150 N

Replace the two forces (P) and (T) by a single force (R) which has the same effect on the fixed structure.

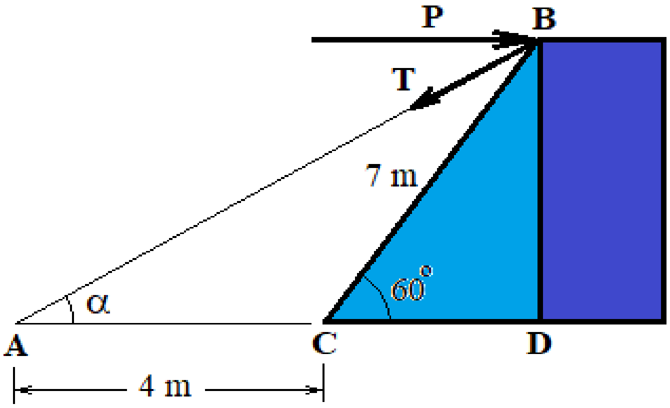
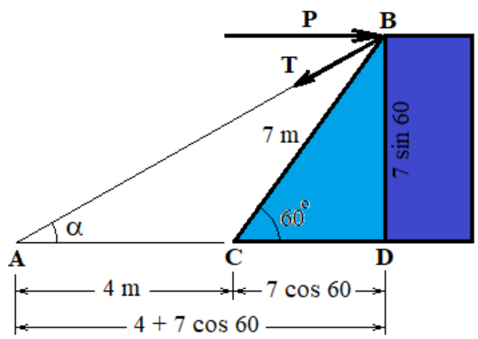


Fig. (Ex. 2-2)

Solution:

$$\tan \alpha = \frac{BD}{AD} = \frac{7 \sin 60°}{4 + 7 \cos 60°} = 0.81$$

$\alpha = 38.9°$

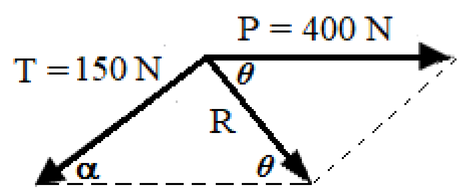


Law of cosines:

$$R = \sqrt{T^2 + P^2 - 2TP \cos 38.9°}$$

$$= \sqrt{150^2 + 400^2 - 2(150)(400) \cos 38.9°}$$

$= 298.5$ N

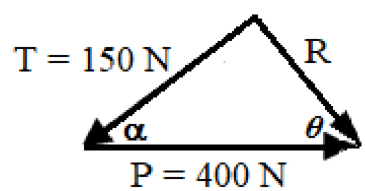


Law of sines:

$$\frac{150}{\sin \theta} = \frac{298.5}{\sin 38.9°}$$

$$\sin \theta = \frac{150 \sin 38.9°}{298.5} = 0.316$$

$\theta = 18.4°$



R = 298.5 N 18.4°

Example (2-3):

A structure consisting of two members, which is affected downward by a force (F) of (1500 N) at point (A). Find the two components of the force acting along the two members of the structure (AB) and (AC).

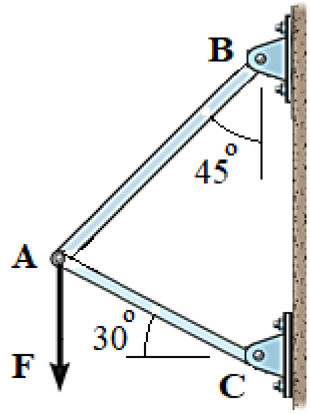


Fig. (Ex. 2-3)

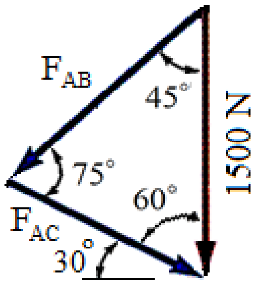
Solution:

$$\frac{F_{AB}}{\sin 60} = \frac{1500}{\sin 75}$$

$F_{AB} = 1344.86$ N

$$\frac{F_{AC}}{\sin 45} = \frac{1500}{\sin 75}$$

$F_{AC} = 1098.08$ N



Example (2-4):

The (I) beam is hoisted by two cables, as shown in Figure (Ex. 2-4). Find the value of the tensile forces (F_A) and (F_B) in each cable so that their resultant magnitude is (3000 Ib) directed towards the positive vertical axis (y-axis).

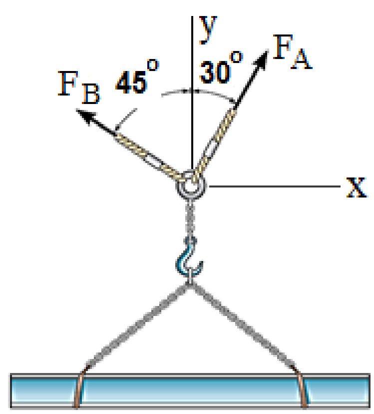


Fig. (Ex. 2-4)

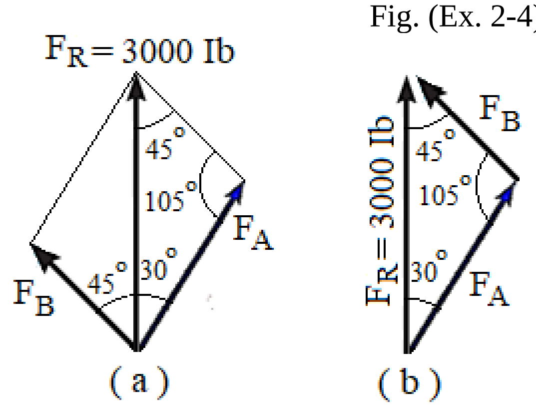
Solution:

$$\frac{F_A}{\sin 45°} = \frac{3000}{\sin 105°}$$

$$F_A = \frac{3000 \sin 45°}{\sin 105°} = 2196 \text{ Ib}$$

$$\frac{F_B}{\sin 30°} = \frac{3000}{\sin 105°}$$

$$F_B = \frac{3000 \sin 30°}{\sin 105°} = 1553 \text{ Ib}$$



Example (2-5):

A barge is pulled by two tugboats as shown in Fig. (Ex. 2-5). If the resultant of the forces exerted by the tugboats is (2500 N) directed along the axis of the barge. Determine:

a) The tension in each of the ropes for ($\alpha = 40^o$),

b) The value of (α) for which the tension in rope (2) is a minimum.

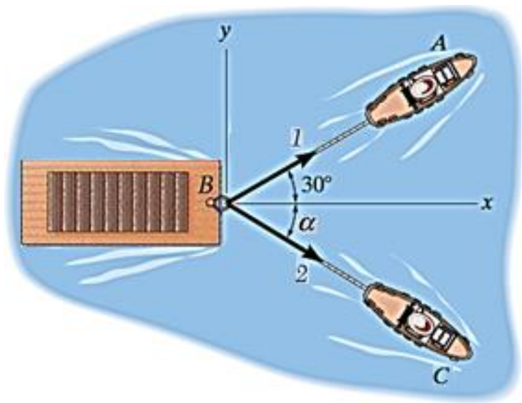


Fig. (Ex. 2-5)

Solution:

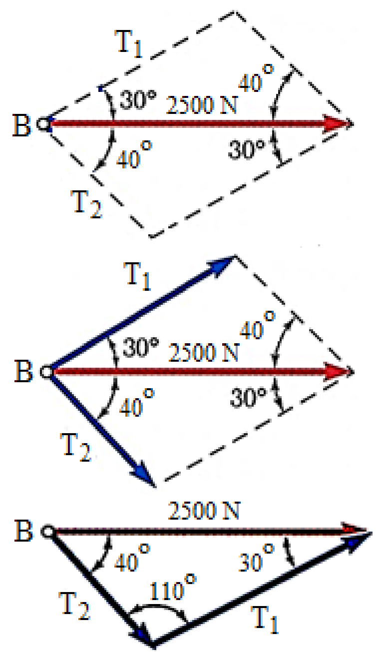
a)

$$\frac{2500}{\sin 110} = \frac{T_1}{\sin 40}$$

$$T_1 = 1710 \text{ N}$$

$$\frac{2500}{\sin 110} = \frac{T_2}{\sin 30}$$

$$T_2 = 1330 \text{ N}$$

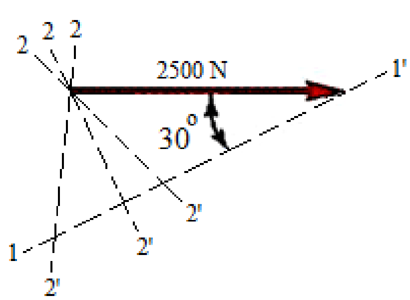


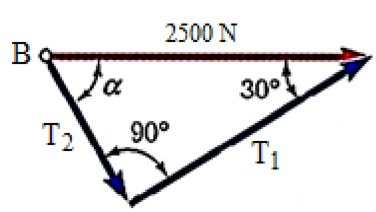
b)

$$T_1 = 2500 \cos 30 = 2165 \text{ N}$$

$$T_2 = 2500 \sin 30 = 1250 \text{ N}$$

$$\alpha = 90 - 30 = 60^o$$





Example (2-6):

Analyze the force (F = 15 lb) acting on the tooth of the gear shown in Fig. (Ex. 2-6). into two components towards the axes (a - a) and (b - b).

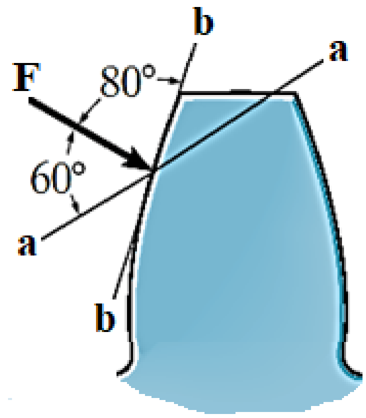
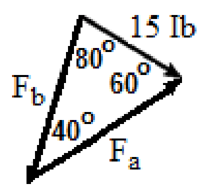


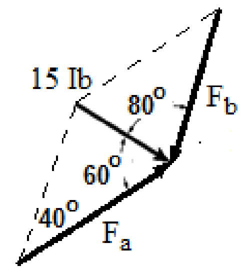
Fig. (Ex. 2-6)

Solution:

$$\frac{15}{\sin 40°} = \frac{F_a}{\sin 80°} \quad \Rightarrow \quad F_a = 22.98 \text{ lb}$$

$$\frac{15}{\sin 40°} = \frac{F_b}{\sin 60°} \quad \Rightarrow \quad F_b = 20.21 \text{ lb}$$





Example (2-7):

Two forces of (8 kN) and (6 kN) are applied to the structure shown in Fig. (Ex. 2-7). Find the magnitude and direction of the resultant force, measured clockwise from the horizontal axis.

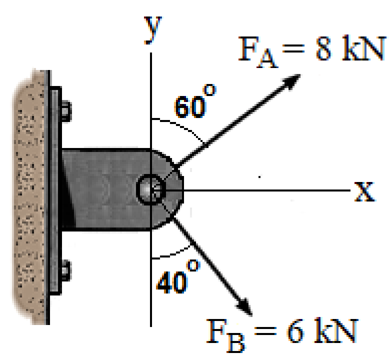


Fig. (Ex. 2-7)

Solution:

$$F_R = \sqrt{F_1{}^2 + F_2{}^2 - 2(F_1)(F_2)\cos\phi}$$

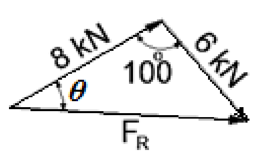
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°} = 10.8 \text{ kN}$$

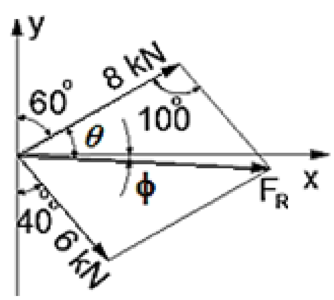
$$\frac{6}{\sin\theta} = \frac{10.8}{\sin 100°}$$

$\sin\theta = 0.547$

$\theta = 33.17°$

$\phi = 33.17° - 30° = 3.17°$





Example (2-8):

Express the magnitude of the resultant (F_R) and its direction (θ) in terms of the values of the components (F_1) and (F_2) and the angle (ϕ).

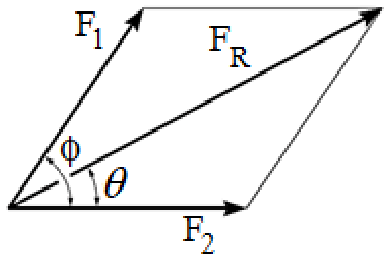


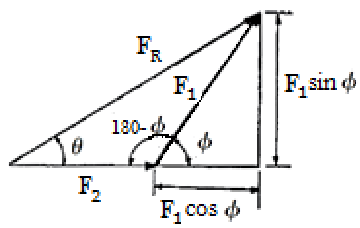
Fig. (Ex. 2-8)

Solution:

$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} - 2F_1F_2\cos(180 - \phi)}$$

Since { $\cos(180 - \phi) = -\cos\phi$ }:

$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos\phi}$$



From the figure:

$$\tan\theta = \frac{F_1 \sin\phi}{F_2 + F_1\cos\phi}$$

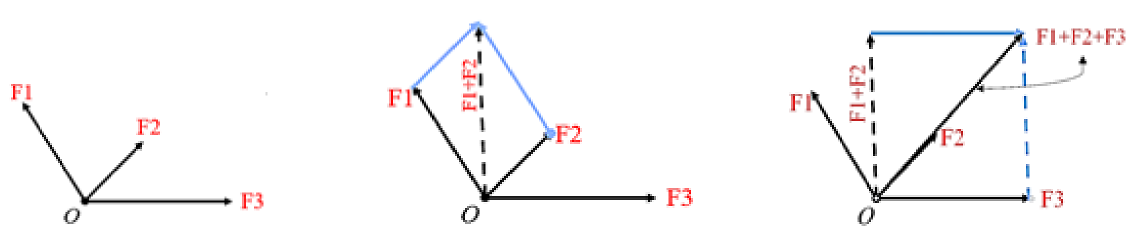
$$\theta = \tan^{-1}\left(\frac{F_1 \sin\phi}{F_2 + F_1\cos\phi}\right)$$

Resultant of Several Forces (More than two Forces):

Th resultant of more than two forces can be found by applying the law of parallelograms in two stages:

1- Finding the resultant of the first and second forces.
2- Taking the resultant of these two forces as a force, and then finding its Resultant with the third force, so we get the resultant of the three forces.

For example, if three forces (F_1) , (F_2) , (F_3) act at a point (O), the resultant will be as follows:



(1) (2) (3)

Example (2-9):

Three chains attached to a bracket ring, as shown in the figure (Ex. 2-9), a tensile force is applied to each chain so that the resultant of the three forces is (2500 N). If two chains of them are affected by known forces, find the angle of the third chain measured clockwise from the positive (x-axis) so that the magnitude of force (F) in this chain is a minimum, then find the magnitude of (F).

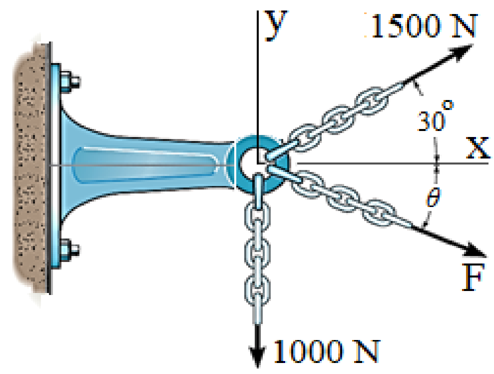


Fig. (Ex. 2-9)

Hint: First find the magnitude and direction of the resultant of the two known forces, force (F) acts in this direction.

Solution:

Cosine law:

$$F_{R1} = \sqrt{1500^2 + 1000^2 - 2(1500)(1000)\cos 60°} = 1322.9 \text{ N}$$

Sine law:

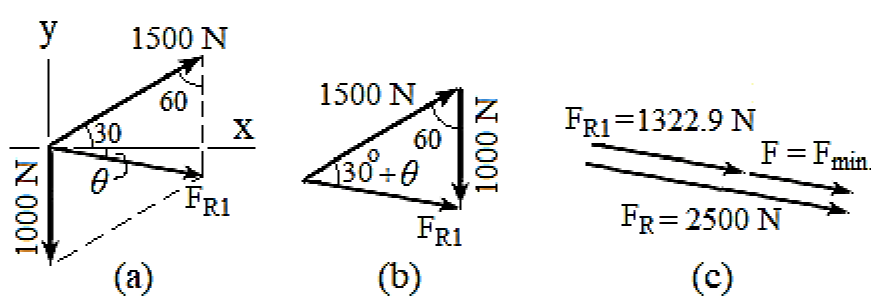
$$\frac{1000}{\sin(30+\theta)} = \frac{1322.9}{\sin 60}$$

$$\sin(30+\theta) = \frac{1000 \sin 60}{1322.9} = 0.65$$

$$30 + \theta = 40.89° \quad \Rightarrow \quad \theta = 10.89°$$

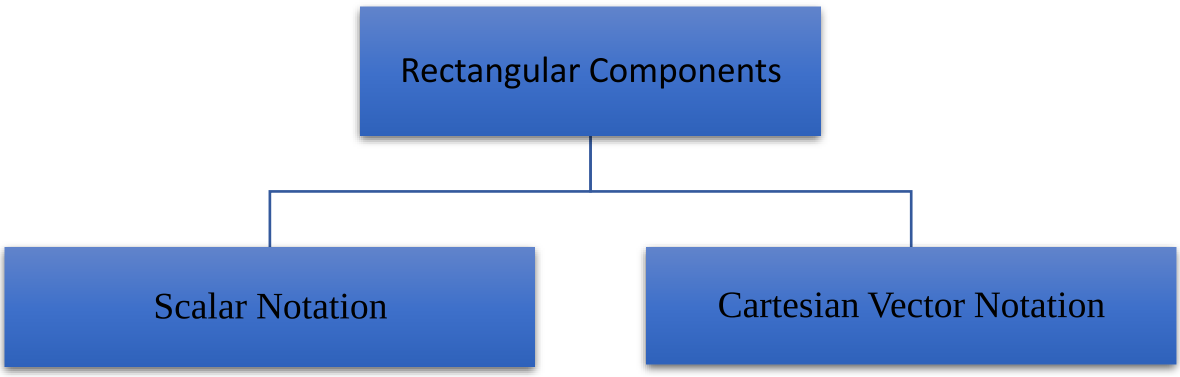
To obtain the required resultant at minimum value of the force (F), the direction of the force (F) must be the same as the direction of the resultant of the two given forces (F_{R1}).

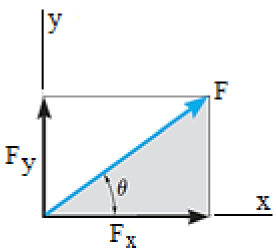
$$F_R = F_{R1} + F$$
$$2500 = 1322.9 + F_{min}$$
$$F_{min} = 1177.1 \text{ N}$$



The resultant of two-dimensional forces by analysis method:

It is possible to analyze any force in a specific plane, such as (x – y) plane, into two perpendicular components along the (x) and (y) axes. These two components are called " Cartesian (rectangular) components ".





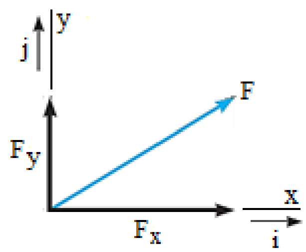
$F_x = F \cos\theta$

$F_y = F \sin\theta$

$F = \sqrt{(Fx)^2 + (Fy)^2}$

$\theta = \tan^{-1}\dfrac{Fy}{Fx}$

F_x and F_y are Force componentes



$F = F_x i + F_y j$

i: unit vector in x-direction

j: unit vector in y-direction

Fig. (2-2) Vector Notation and Scalar Notation for the force

$$F_x = F \cos\theta \quad \text{..............................} \quad (2\text{-}9)$$

$$F_y = F \sin\theta \quad \text{..............................} \quad (2\text{-}10)$$

Directional value:

$$F = F_x i + F_y j \quad \text{..............................} \quad (2\text{-}11)$$

Absolute value:

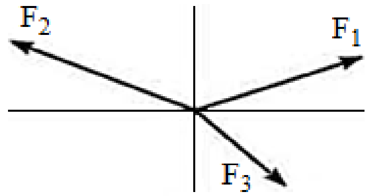
$$F = \sqrt{(Fx)^2 + (Fy)^2} \quad \text{......................} \quad (2\text{-}12)$$

Direction:

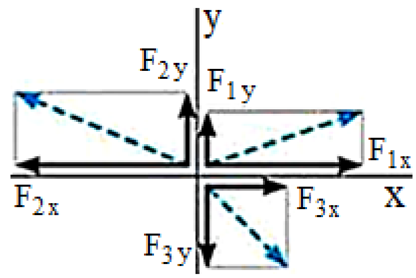
$$\theta = \tan^{-1}\frac{Fy}{Fx} \quad \text{............................} \quad (2\text{-}13)$$

Finding the resultant:

1- Each force is resolved into its two components with the axes (x) and (y).
2- The components applied on the x-axis summed to be the horizontal component of the resultant.
3- The components applied to the y-axis are summed to be the vertical component of the resultant.
4- Using the Cartesian vector, the resultant is represented as a Cartesian vector.



$F_R = (F_{Rx})\, i + (F_{Ry})\, j$
$F_{Rx} = \sum F_x$
$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$
$F_{Ry} = \sum F_y$
$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$
$F_R = (F_{1x} - F_{2x} + F_{3x})\, i + (F_{1y} + F_{2y} - F_{3y})\, j$

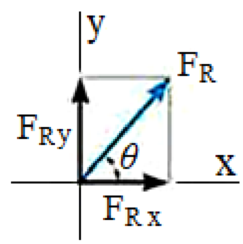


5- Finding the value of the resultant (F_R) by using the Pythagorean theory:

$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}}$$

6- The angle (θ) that represents the direction of the resultant can be found from the trigonometric laws:

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$



Example (2-10):

Calculate the horizontal and vertical components of the forces (F_1) and (F_2) on the arm shown in the Fig. (Ex. 2-10). Then express each force as a Cartesian vector. Then find the magnitude and direction of the resultant force.

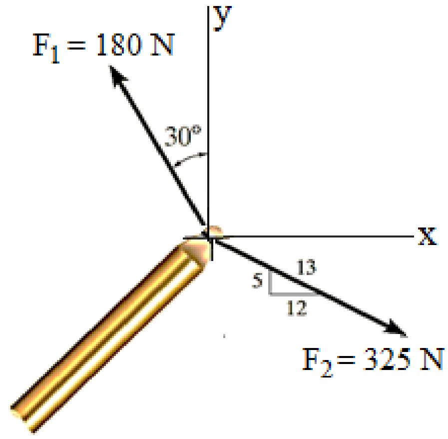


Fig. (Ex. 2-10)

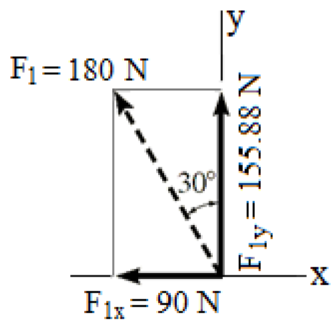
Solution:

$F_{1x} = -180 \sin 30° = -90 \text{ N} = 90 \text{ N} \leftarrow$

$F_{1y} = 180 \cos 30 = 155.88 \text{ N}$

$F_{2x} = 325\ (12/13) = 300 \text{ N}$

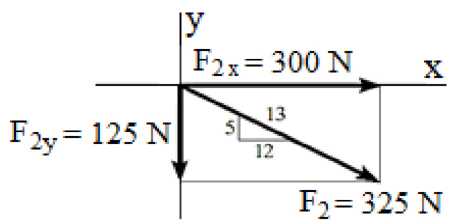
$F_{2y} = -325\ (5/13) = -125 \text{ N} = 125 \text{ N} \downarrow$



Cartesian vector notation.

$F_1 = \{ -90\, i + 155.88\, j \}$ N

$F_2 = \{ 300\, i - 125\, j \}$ N



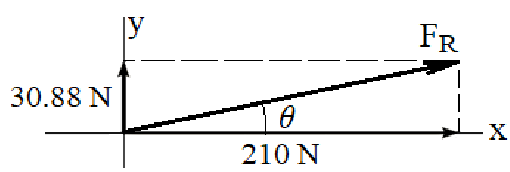
The resultant force:

$$F_R = \sqrt{F_{Rx}{}^2 + F_{Ry}{}^2}$$

$F_{Rx} = F_{1x} + F_{2x} = -90 + 300 = 210$ N

$F_{Ry} = F_{1y} + F_{2y} = 155.88 - 125 = 30.88$ N

$F_R = \sqrt{210^2 + 30.88^2} = 212.258$ N



The direction (θ):

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{30.88}{210}\right) = 8.37°$$

Example (2-11):

Determine the magnitude and direction of the resultant force of the three forces acting on the ring (A) measured counterclockwise from the positive (x-axis).

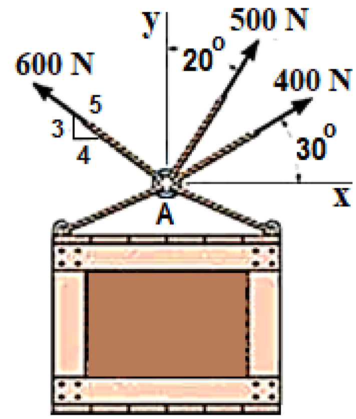


Fig. (Ex. 2-11)

Solution:

$F_R = \sum F$

$F_{Rx} = \sum F_x = -\,600\,(\frac{4}{5}) + 500 \sin 20^\circ + 400 \cos 30^\circ = 37.42$ N

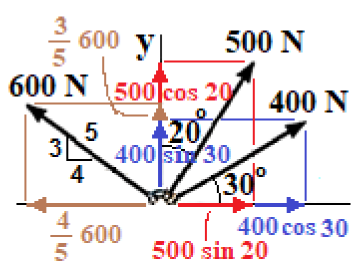
$F_{Ry} = \sum F_y = 600\,(\frac{3}{5}) + 500 \cos 20^\circ + 400 \sin 30^\circ = 1029.85$ N

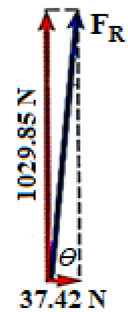
$F_R = \sqrt{(37.42)^2 + (1029.85)^2} = 1030.5$ N

$\tan\theta = \dfrac{F_{Ry}}{F_{Rx}}$

$\theta = \tan^{-1}\dfrac{1029.85}{37.42}$

$= 87.92^\circ$





Example (2-12):

Determine the components of the resultant of the forces acting on the gusset plate of a bridge truss in the direction of (x - axis) and (y- axis). Then show that the resultant is zero.

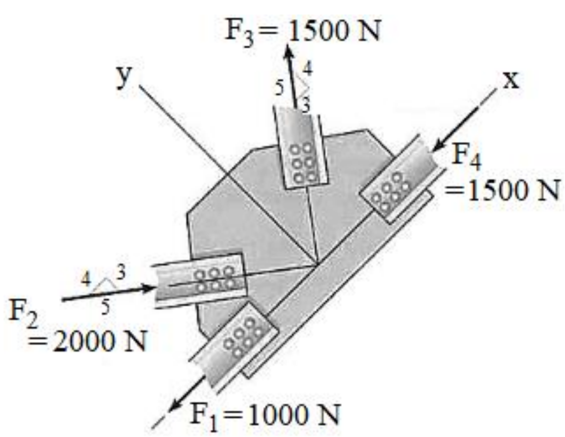


Fig. (Ex. 2-12)

Solution:

$F_R = \sum F$

$F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$= -\,1000 + 2000\,(\frac{4}{5}) + 1500\,(\frac{3}{5}) - 1500 = 0$

$F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$= 0 - 2000\,(\frac{3}{5}) + 1500\,(\frac{4}{5}) - 0 = 0$

Example (2-13):

Four forces act on fixed frame as shown in Fig. (Ex. 2-13). Determine the magnitude and direction of the resultant of these forces.

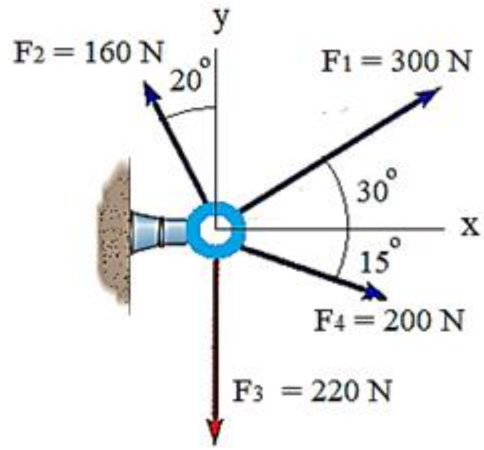


Fig. (Ex. 2-13)

Solution:

$(F_1)_x = 300 \cos 30° = 259.8$ N

$(F_1)_y = 300 \sin 30° = 150$ N

$(F_2)_x = -160 \sin 20° = -54.7$ N

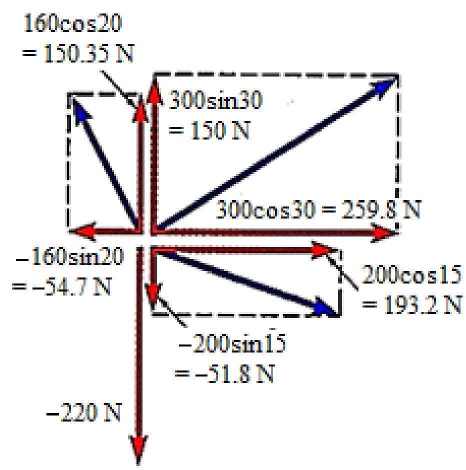
$(F_2)_y = 160 \cos 20° = 150.35$ N

$(F_3)_x = 0$

$(F_3)_y = -220$ N

$(F_4)_x = 200 \cos 15° = 193.2$ N

$(F_4)_y = -200 \sin 15° = -51.8$ N

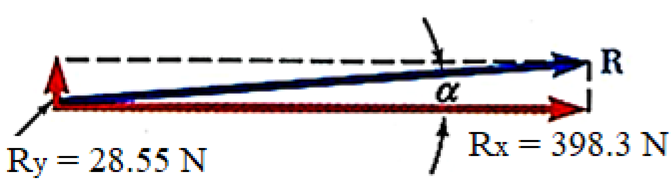


$R_x = 259.8 - 54.7 + 0 + 193.2 = 398.3$ N

$R_y = 150 + 150.35 - 220 - 51.8 = 28.55$ N

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{398.3^2 + 28.55^2} = 399.3 \text{ N}$$

$$\tan \alpha = \frac{R_y}{R_x} \quad \Rightarrow \quad \alpha = \tan^{-1} \frac{28.55}{398.3} = 4.1°$$



Example (2-14):

Express the forces (F_1), (F_2), and (F_3) acting on the body shown in Fig. (Ex. 2-14) in the form of a Cartesian vector, then find the magnitude and direction of the resultant of the three forces.

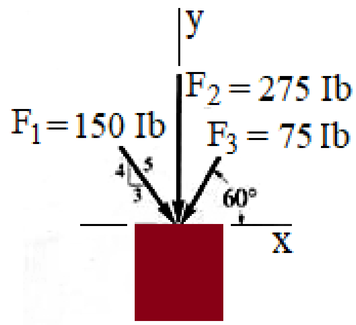


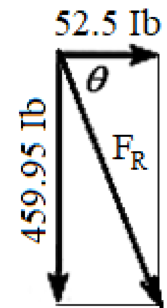
Fig. (Ex. 2-14)

Solution:

$$F_1 = 150\left(\frac{3}{5}\right) i - 150\left(\frac{4}{5}\right) j = 90 \text{ Ib } i - 120 \text{ Ib } j$$

$$F_2 = 0\, i - 275 \text{ Ib } j$$

$$F_3 = -75 \cos 60^\circ\, i - 75 \sin 60^\circ\, j = -37.5 \text{ Ib } i - 64.95 \text{ Ib } j$$



$$F_R = \sum F$$

$$F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} = 90 + 0 - 37.5 = 52.5 \text{ Ib}$$

$$F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} = -120 - 275 - 64.95$$
$$= -459.95 \text{ Ib}$$

$$F_R = \sqrt{(52.5)^2 + (-459.95)^2} = 462.94 \text{ Ib}$$

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{-459.95}{52.5} = -83.49^\circ$$

Example (2-15):

A wooden dowel rotates in a lathe and a force of (100 N) is applied to it by the cutting stylus of the lathe, as shown in Fig. (Ex. 2-15). Resolve this force into its components acting:
(a) along the (x) and (y) axes.
(b) along the (x') and (y') axes.

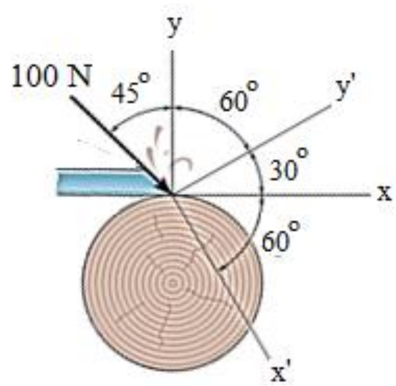
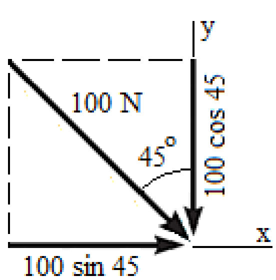


Fig. (Ex. 2-15)

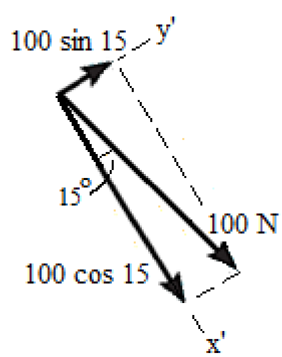
Solution:

a) $F_x = 100 \sin 45^\circ = 70.7$ N
   $F_y = -100 \cos 45^\circ = -70.7$ N

b) $F_{x'} = 100 \cos 15^\circ = 96.6$ N
   $F_{y'} = 100 \sin 15^\circ = 25.88$ N



(a)



(b)

Example (2-16):

It is required to remove the screw from the wood by applying force along its horizontal axis. The obstruction (A) prevents direct access, so that two forces are applied, one of (1000 N) and the other of (P), by cables as shown in Fig. (Ex.2-16). Determine the magnitude of the force (P) that is required to get the resultant force (T) directed along the screw axis. Also find the magnitude of (T).

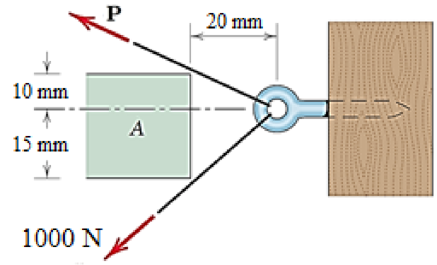


Fig. (Ex. 2-16)

Solution:

Method (I):

$$\theta_1 = \tan^{-1}\frac{10}{20} = 26.57°$$

$$\theta_2 = \tan^{-1}\frac{15}{20} = 36.87°$$

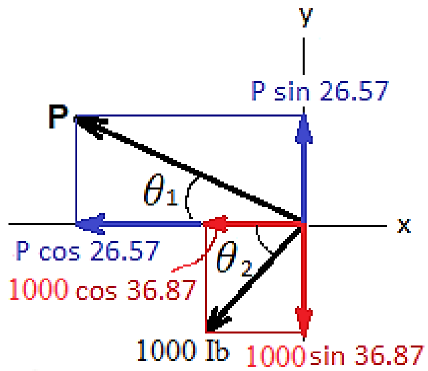
$R_y = \sum F_y = 0$

$$P \sin 26.57° - 1000 \sin 36.87° = 0$$

$$P \sin 26.57° = 1000 \sin 36.87°$$

$$P = \frac{1000 \sin 36.87°}{\sin 26.57°} = 1341.4 \text{ N}$$

$$T = R_x = \sum F_x = 1341.4 \cos 26.57° + 1000 \cos 36.87° = 2000 \text{ N}$$



Method (II):

$$\theta_1 = \tan^{-1}\frac{10}{20} = 26.57°$$

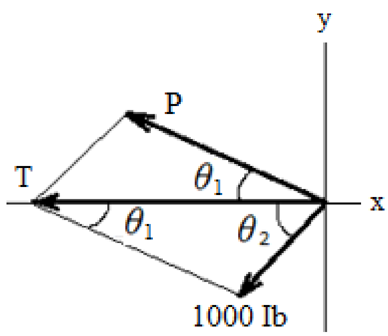
$$\theta_2 = \tan^{-1}\frac{15}{20} = 36.87°$$

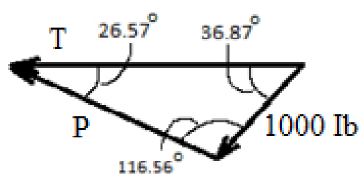
$$\frac{P}{\sin 36.87} = \frac{1000}{\sin 26.57}$$

$$P = \frac{1000 \sin 36.87°}{\sin 26.57°} = 1341.4 \text{ N}$$

$$\frac{T}{\sin 116.56} = \frac{1000}{\sin 26.57}$$

$$T = \frac{1000 \sin 116.56}{\sin 26.57} = 2000 \text{ N}$$





Example (2-17):

Three forces are applied to the bracket shown Fig. (Ex. 2-17). Find the value and direction of the force (F_3) that makes the resultant of the three forces (100 lb) directed along the positive (u-axis).

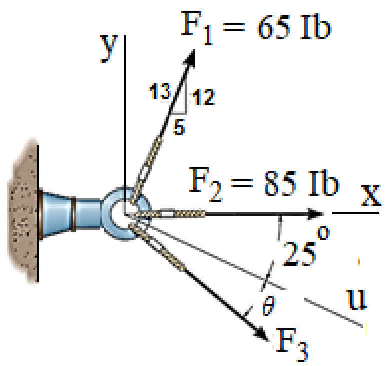


Fig. (Ex. 2-17)

Solution:

$+ \uparrow \sum F_{Rx} = \sum F_x$ $\quad$ $100 \cos 25° = 85 + 65 \left(\frac{5}{13} \right) + F_3 \cos (25° + \theta)$

$F_3 \cos (25° + \theta) = -19.37$ ………..... (1)

$+ \rightarrow \sum F_{Ry} = \sum F_y$ $\quad$ $-100 \sin 25° = 65 \left(\frac{12}{13} \right) - F_3 \sin (25° + \theta)$

$F_3 \sin (25° + \theta) = 102.26$ ………….. (2)

Solving Eqs. (1) and (2):

Method (1):

From Eq. (1) $\quad$ $F_3 = \dfrac{-19.37}{\cos (25° + \theta)}$

Sub. in Eq. (2) $\quad$ $\dfrac{-19.37}{\cos (25° + \theta)} \sin (25° + \theta) = 102.26$

$-19.37 \tan (25° + \theta) = 102.26$

$(25° + \theta) = \tan^{-1} \dfrac{102.26}{-19.37} = -79.27° = 79.27°$

$\theta = 79.27 - 25 = 54.27°$

$F_3 = \dfrac{-19.37}{\cos (25° + 54.27°)} = 104$ Ib

Method (2):

$F_3 \sin (25° + \theta) = 102.26$ ………….. (2)

$F_3 \cos (25° + \theta) = -19.37$ ………..... (1)

---------------------------------- Division

$\tan (25° + \theta) = -5.28$

$25° + \theta = \tan^{-1} -5.28 = -79.27° = 79.27°$

$\theta = 79.27 - 25 = 54.27°$

Sub. in Eq. (1) $\quad$ $F_3 \cos (-79.27°) = -19.37$

$F_3 = \dfrac{-19.37}{\cos (-79.27°)} = 104$ Ib

Example (2-18):

The forces (F_1), (F_2), and (F_3) are acted on the bracket shown in Fig. (Ex. 2-18) so that their resultant is (120 Ib) in the positive direction of the (u) axis. Find the value of the unknown force (F_1) and its direction (ϕ).

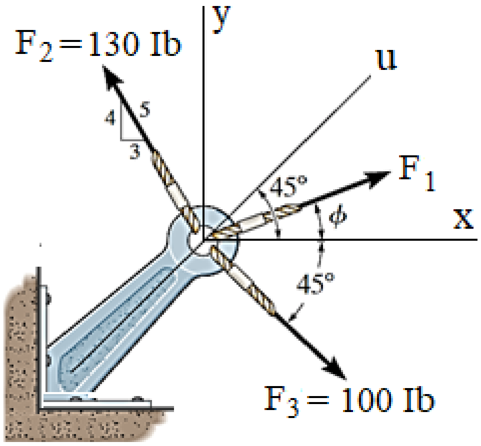


Fig. (Ex. 2-18)

Solution:

$(F_1)_x = F_1 \cos \phi$ $\qquad$ $(F_1)_y = F_1 \sin \phi$

$(F_2)_x = -130\ (\frac{3}{5}) = -78$ Ib $\qquad$ $(F_2)_y = 130\ (\frac{4}{5}) = 104$ Ib

$(F_3)_x = 100 \cos 45° = 70.7$ Ib $\qquad$ $(F_3)_y = -100 \sin 45° = -70.7$ Ib

$(F_R)_x = 120 \cos 45° = 84.84$ Ib $\qquad$ $(F_R)_y = 120 \sin 45° = 84.84$ Ib

$\overset{+}{\rightarrow} F_{Rx} = \sum F_x$ $\qquad$ $84.84 = F_1 \cos \phi \ - 78 + 70.7$

$F_1 \cos \phi = 92.14$ ……………..…….. (1)

$+\uparrow F_{Ry} = \sum F_y$ $\qquad$ $84.84 = F_1 \sin \phi + 104 - 70.7$

$F_1 \sin \phi = 51.54$ …………..……….. (2)

From Eq. (1): $\qquad F_1 = \dfrac{92.14}{\cos \phi}$

Sub. in Eq. (2): $\qquad \dfrac{92.14}{\cos \phi} \sin \phi = 51.54$

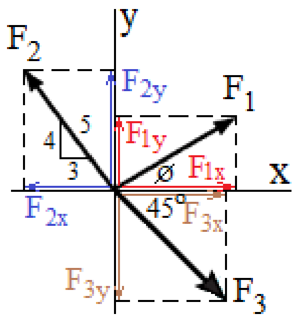
$92.14 \tan \phi = 51.54$

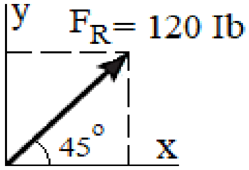
$\tan \phi = \dfrac{51.54}{92.14} = 0.56$

$\phi = \tan^{-1} 0.56 = 29.22°$

$F_1 = \dfrac{92.14}{\cos \phi}$

$= \dfrac{92.14}{\cos 29.23°} = 105.57$ Ib





Example (2-19):

If the resultant of the two forces shown in Fig. (Ex. 2-19) is directed along the positive (y-axis) and has a magnitude of (300 Ib), determine the magnitude of (F_B) and its direction (θ).

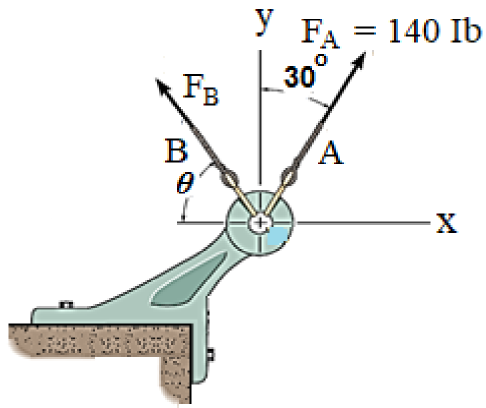


Fig. (Ex. 2-19)

Solution:

Metod (1):

$+\rightarrow F_{Rx} = \sum F_x \qquad 0 = 140 \sin 30° - F_B \cos\theta$

$F_B \cos\theta = 70$ ……………..…….. (1)

$+\uparrow F_{Ry} = \sum F_y \qquad 300 = 140 \cos 30° + F_B \sin\theta$

$F_B \sin\theta = 178.76$ ..……………….. (2)

$F_B \sin\theta = 178.76$ ..……………….. (2)

$F_B \cos\theta = 70$ ..………………..…….. (1)

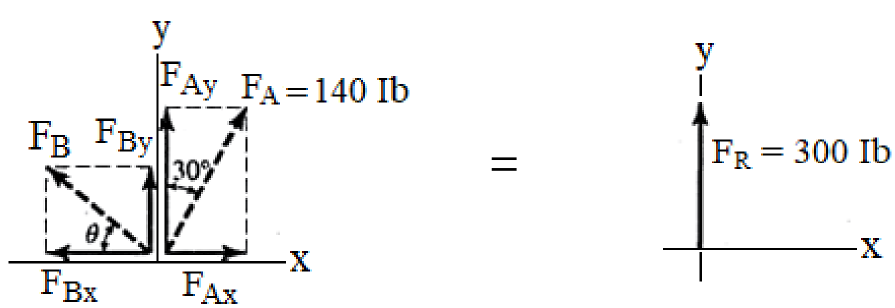
---------------------------------- Division

$\tan\theta = 2.55$

$\theta = 68.6°$

Sub. in Eq. (2): $F_B \sin(68.6°) = 178.76$

$F_B = 192$ Ib



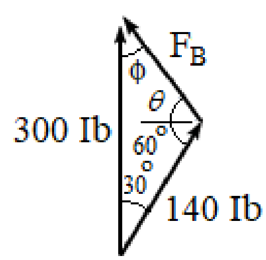
Metod (2):

$F_B = \sqrt{(300)^2 + (140)^2 - 2(300)(140)\cos 30} = 192$ Ib

$\dfrac{140}{\sin\phi} = \dfrac{192}{\sin 30°} \quad \Rightarrow \quad \phi = 21.4°$

$\theta + 60 = 180 - 30 - 21.4 = 128.6°$

$\theta = 128.6 - 60 = 68.6°$



Example (2-20):

If the forces (F_1 = 7 kN), (F_2) and (F_3 = 4 kN) acting on the bracket as shown in Fig. (Ex. 2-20). Determine the magnitude of force (F_2) so that the resultant force of the three forces is as small as possible. Then find the magnitude of the resultant force.

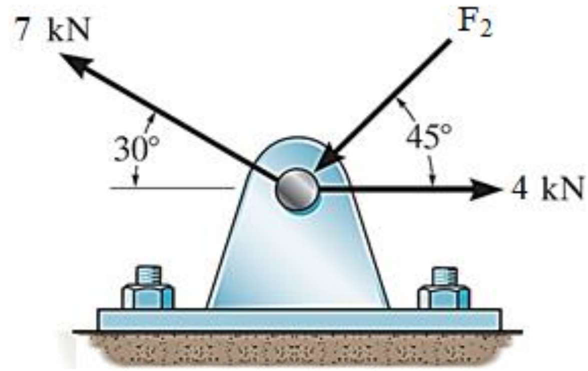


Fig. (Ex. 2-20)

Solution:

$+\rightarrow F_{Rx} = \sum F_x$

$F_{Rx} = 4 - F_2 \cos 45° - 7 \cos 30°$

$F_{Rx} = -2.06 - 0.707\ F_2$

$+\uparrow\ F_{Ry} = \sum F_y$

$F_{Ry} = -F_2 \sin 45° + 7 \sin 30°$

$F_{Ry} = 3.5 - 0.707\ F_2$

$F_R^2 = (-2.06 - 0.707\ F_2)^2 + (3.5 - 0.707\ F_2)^2$ …………… (1)

$2F_R \dfrac{dF_R}{dF} = 2(-2.06 - 0.707\ F_2)(-0.707)$

$+ 2(3.5 - 0.707\ F_2)(-0.707) = 0$

$2(1.456 + 0.5\ F_2 - 2.475 + 0.5\ F_2) = 0$

$1.456 + 0.5\ F_2 - 2.475 + 0.5\ F_2 = 0$

$F_2 = 1.02$ kN

Sub. in Eq. (1):

$F_R^2 = [-2.06 - 0.707(1.02)]^2 + [3.5 - 0.707(1.02)]^2$

$F_R^2 = (-2.78)^2 + (2.78)^2$

$F_R = 3.93$ kN

Problems:

2-1) Find the magnitude of the resultant force acting on the plate shown in the Fig. (Pr. 2-1) and its direction, measured clockwise from the positive horizontal axis (x).

Ans.: $R = 1353$ Ib , $\phi = 132.8°$

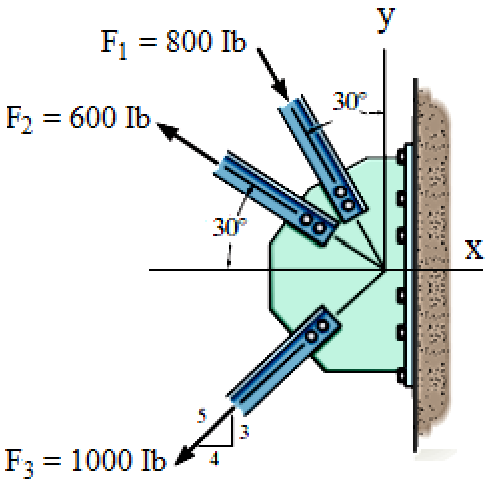


Fig. (Pr. 2-1)

2-2) Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive (x – axis).

Ans.: $R = 2313.6$ N , $\phi = 140.4°$

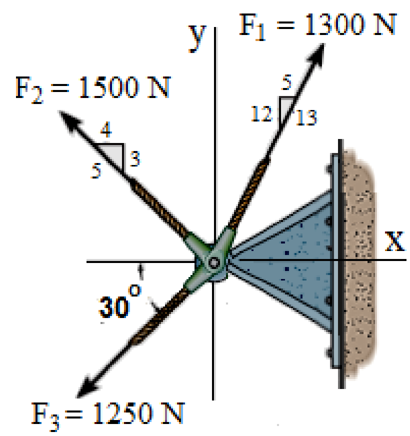
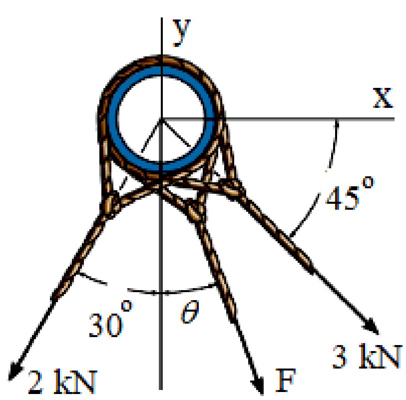


Fig. (Pr. 2-2)

2-3) A pipe pulled by three ropes with tensile forces shown in Fig. (Pr. 2-3) so that a resultant force of (5 kN) is generated. If the tensile forces in two of them are known, find the angle of the third rope (θ) so that the magnitude of the tensile force (F) in it is at its minimum. What is the magnitude of the force?

Ans.: $F = 1$ kN , $\theta = 16.12°$



2-4) Determine the magnitude of the resultant force acting on the eyebolt shown in Fig. (Pr. 2-4) and its direction measured counter-clockwise from the positive (x – axis).

Ans.: $F_R = 1733.82$ Ib , $\phi = 93°$

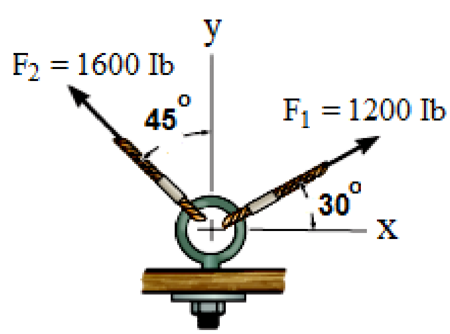


Fig. (Pr. 2-4)

Fig. (Pr. 2-3)

2-5) Determine the magnitude of the resultant force acting on the bracket shown in Fig. (Pr. 2-5) and its direction measured counter-clockwise from the negative (x – axis).

Ans.: $F_R = 1083.5\ N$, $\theta = 3°$

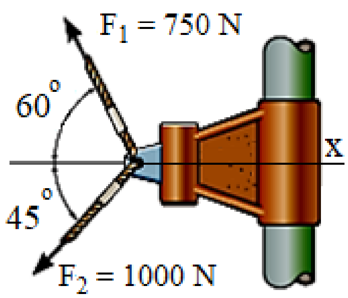


Fig. (Pr. 2-5)

2-6) Find the magnitude of the force (F) and its direction (θ) to achieve a resultant force acting the bracket shown in Fig. (Pr. 2-6), with a value of (100 N) directed along the positive (y - axis).

Ans.: $F = 192\ Ib$, $\theta = 45.2°$

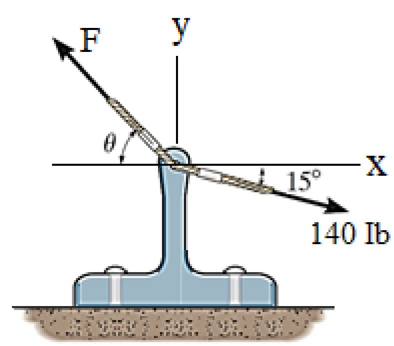


Fig. (Pr. 2-6)

2-7) A force of (4 kN) acts on the frame shown in Fig. (Pr. 2-7), so if its component acting along the member (BC) has a value of (3 kN), directed from (B) towards (C), what is the magnitude of the required angle (θ) and its component that affected along the member (AB).

Ans.: $F_{AB} = 2.05\ kN$, $\theta = 32°$

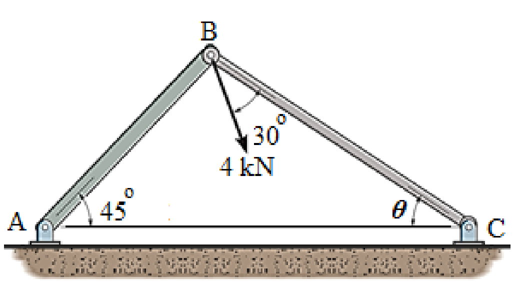


Fig. (Pr. 2-7)

2-8) Determine the angle (θ) between the two forces that act on the screw eye, so that the resultant force has a magnitude of (160 Ib).

Ans.: $\theta = 75.5°$

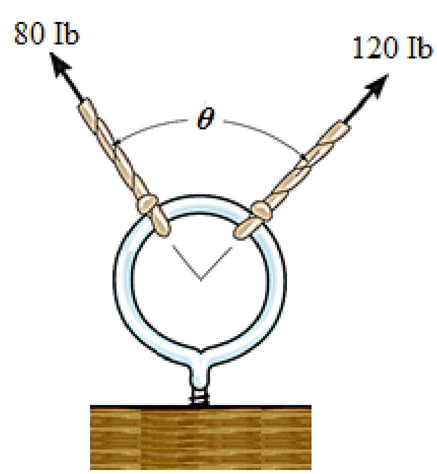


Fig. (Pr. 2-8)

2-9) Determine the angle (θ) required in the design of the struts shown in Figure (Pr. 2-9) so that the (2 kN) horizontal force has a component of (2.5 kN) directed from (A) towards (C). What is the component of the force acting along the member (AB)?

Ans.: $\theta = 62.1°$, $F_{AB} = 2.7$ kN

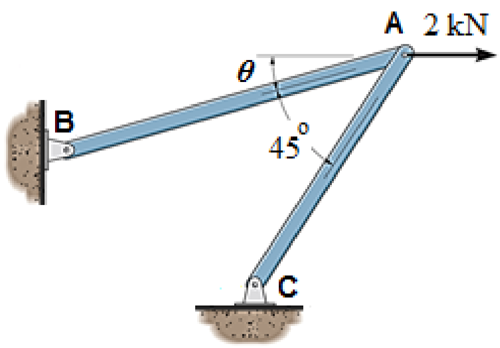


Fig. (Pr. 2-9)

2-10) Determine the angle (θ) required in the design of the struts shown in Figure (Pr. 2-10) so that the horizontal force (5 kN) has a component of (3 kN) directed from (B) towards (A).

Ans.: $\theta = 53.1°$

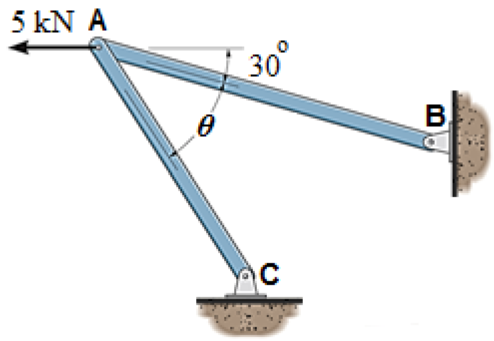


Fig. (Pr. 2-10)

2-11) Determine the magnitude of the force (F_1) and its direction (θ), if the resultant force of the three forces is (1000 N) directed (45°) counterclockwise from the positive (x – axis).

Ans.: $F_1 = 753.66$ N, $\theta = 45°$

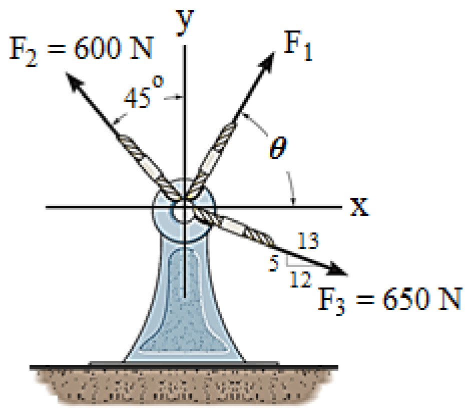


Fig. (Pr. 2-11)

2-12) Find the magnitude of the force (F_3) and its direction (θ) to achieve a resultant force affecting the bracket shown in Fig. (Pr. 2-12) with value of (200 Ib) directed along the positive (x' - axis).

Ans.: $F_3 = 178$ Ib, $\theta = 37°$

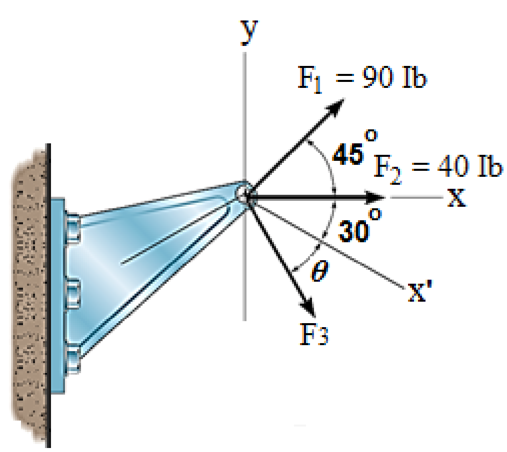


Fig. (Pr. 2-12)

2-13) Determine the magnitude of the resultant force acting on the plate shown in Fig. (Pr. 2-13) and its direction measured counter-clockwise from the posative (x- axis).

Ans.: $F_R = 433\ N$, $\theta = 183.68°$

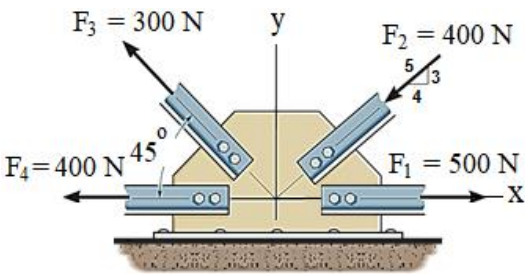


Fig. (Pr. 2-13)

2-14) The forces (F_1), (F_2), and (F_3) are applied to the brakes shown in Fig. (Pr. 2-14). What is the magnitude and direction of the resultant force, measured counter-clockwise from the positive (x- axis) of the bracket?

Ans.: $F_R = 557.48\ N$, $\theta = 67.55°$

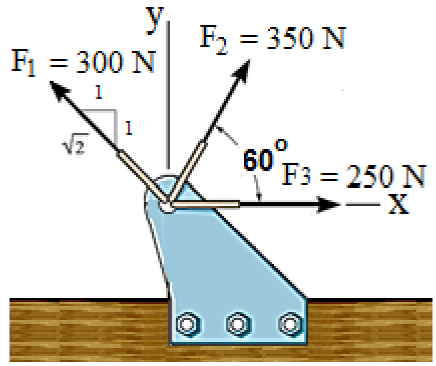


Fig. (Pr. 2-14)

2-15) Determine the magnitude of the resultant force acting on the bracket shown in Fig. (Pr. 2-15) and its direction measured clockwise from the positive (y – axis).

Ans.: $F_R = 235.79\ Ib$, $\phi = 7.06°$

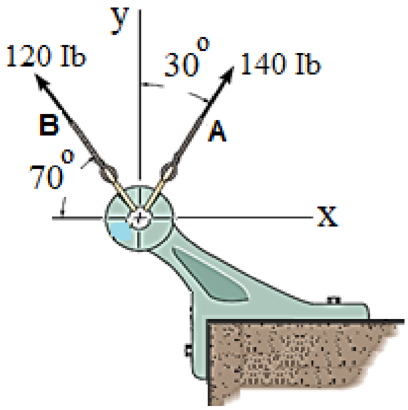


Fig. (Pr. 2-15)

2-16) If the lift force on wing cross-section (airfoil) is (600 N) and the ratio of the lift force (L) to the drag force (D) for the airfoil is (L/D = 12), compute the magnitude of the resultant force (R) and the angle (θ) which it makes with the horizontal.

Ans.: $R = 602\ N$, $\theta = 85.24°$

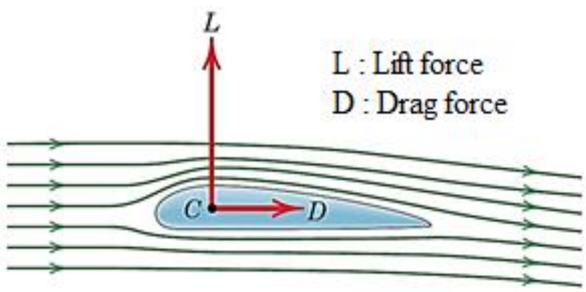


Fig. (Pr. 2-16)

2-17) Two forces applied to a bracket as shown in Fig (Pr. 2-17). Determine the resultant (F_R) of the two forces, and then find (F_R) in terms of unit vectors along the (x' – axis) and (y' – axis).

Ans.: $F_R = 520.5\ N$
$F_R = -334.57\ N\, i + 398.73\ N\, j$

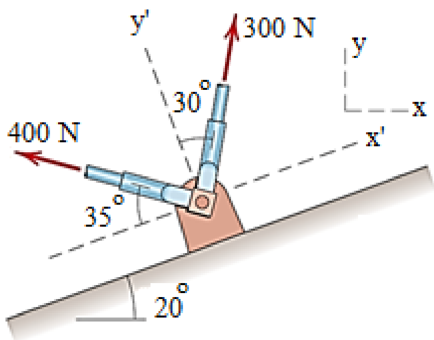


Fig. (Pr. 2-17)

2-18) In the structure shown in Fig. (Pr. 2-18), find the value of the unknown force (F) that makes the resultant (F_R) of the three forces as small as possible. Then find the value of the resultant (F_R).

Ans.: $F = 2.825\ kN$, $F_R = 2.18\ kN$

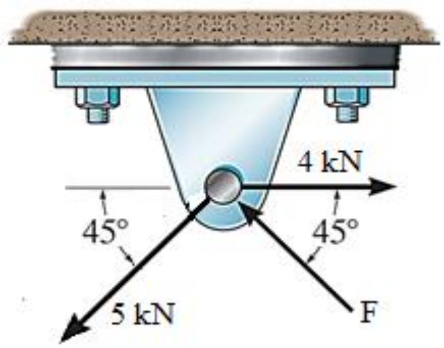


Fig. (Pr. 2-18)

2-19) The tension in the cable (AB) that prevents the bar (OA) from rotating counterclockwise about the pivot (*O*) is (150 Ib). Determine the (n) and (t) components of this force acting on point (A) of the bar.

Ans.: $F_n = 66.46\ Ib$, $F_t = 134.47\ Ib$

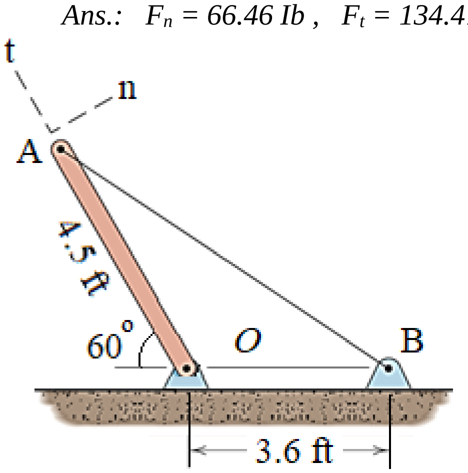


Fig. (Pr. 2-19)

2-20) Find the magnitude of (T) and its direction (θ) for which the eye bolt under a resultant of (7.5 kN) horizontally to the left.

Ans.: $T = 6.4\ k\,N$, $\theta = 38.66°$

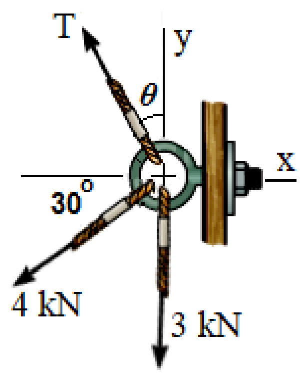


Fig. (Pr. 2-20)

CHAPTER THREE
THE MOMENTS

Definition of the moment:

When a force is applied to a specific body, the body moves in the direction of the line of action of that force. If the line of action of the force passes through the center of gravity of the body, it will move in a linear motion only by the effect of the force, but if the line of action of the force does not pass through its center of gravity, then it will move in angular motion around its center of gravity under the effect of the force in addition to the linear motion. This effect that generates angular motion is called the *moment* **(M)** of the force. Moment is also referred to as *torque*.

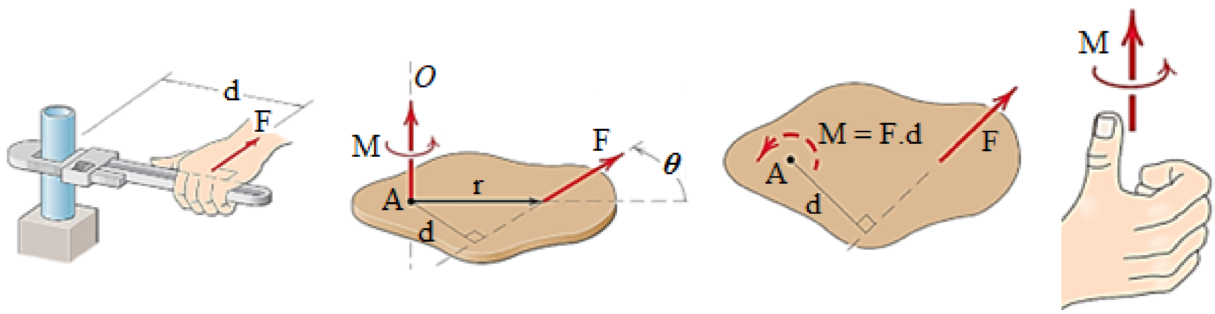


Fig. (3-1) Illustrative diagrams of moments

The diagrams in Fig. (3-1) show the effect of force (F) on a two-dimensional body in its dimensional plane. This force generates a moment that the body rotates around the axis (*O* – *O*) perpendicular to the plane of its dimensions, the magnitude of the moment is equal to the product of the magnitude of the force (F) with the perpendicular distance between this axis and the line of action of the force called the force arm (d). Therefore, the magnitude of the moment is:

$$M = F.d \quad \text{……………..} \quad (3\text{-}1)$$

Methods of solution:

- **Varignon's theorem:**

This theory states that (*the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point*).

- **Normal distance:**

In this principle, (*the moment of any force around a given axis can be found by multiplication of the force by the normal distance between the line of action of the force and the given axis*).

- **Principle of transmissibility:**

In this principle, (*the moment can be found by transferring the force to the horizontal axis or the vertical axis of the point where the moment required about it*).

Example (3-1):

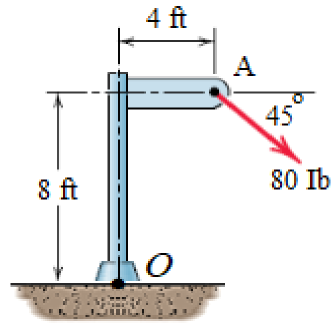


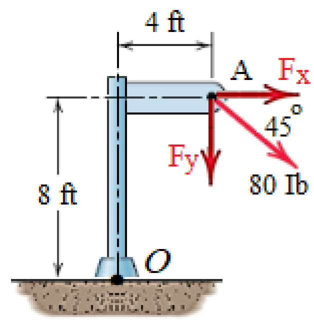
Fig. (Ex. 3-1)

Calculate the magnitude of the moment about the base point (*O*) of the (80 Ib) force in four different methods.

Solution:

(I) Replace the force by its rectangular (cartesian) components:

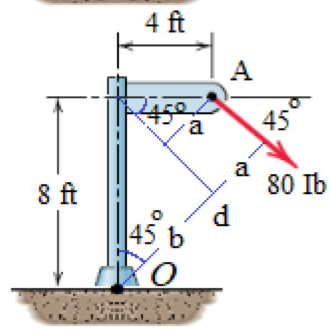
$$F_x = 80 \cos 45^o = 56.57 \text{ Ib}$$
$$F_y = 80 \sin 45^o = 56.57 \text{ Ib}$$



By using Varignon's theorem:

$$M_o = (-56.57 \times 8) - (56.57 \times 4) = -678.8 \text{ Ib.ft}$$
$$= 678.84 \text{ Ib.ft} \qquad \text{(C.W)}$$

(II) By using the normal distance method, where the moment arm of the (80 Ib) force is:

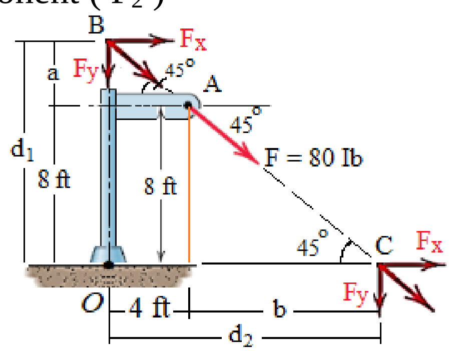


$$d = a + b = 4 \sin 45^o + 8 \cos 45^o = 8.485 \text{ ft}$$

$$M = Fd$$
$$M_o = -80 \times 8.484 = -678.8 \text{ Ib.ft} = 678.8 \text{ Ib.ft}$$

(C.W)

(III) By using the principle of transmissibility, The force (80 Ib) can be transferred along its line of action to point (B), the vertical component (F_2) is neglected because its line of action passes through point (*O*). The moment arm of (F_1) will be:



$$d_1 = 8 + 4 \tan 45^o = 12 \text{ ft}$$
$$F_1 = 80 \cos 45^o = 56.57 \text{ Ib}$$

The moment is:

$$M_o = -56.57 \times 12 = -678.8 \text{ Ib.ft}$$
$$= 678.8 \text{ Ib.ft} \qquad \text{(C.W)}$$

(IV) The force (80 Ib) can be transferred along its line of action to point (C), neglecting the horizontal component (F_1) because its line of action passes through point (*O*). The moment arm of (F_2) will be:

$$d_2 = 4 + 8 \cot 45^o = 12 \text{ ft} \quad , \quad F_2 = 80 \sin 45^o = 56.57 \text{ Ib}$$

The moment is:

$$M_o = -56.57 \times 12 = -678.8 \text{ Ib.ft} = 678.8 \text{ Ib.ft} \qquad \text{(C.W)}$$

Resultant moment:

In two-dimensional problems, all forces lie in one plane, let it be the (x – y) plane, as shown in Fig. (3-2), the resultant moments (M_R)$_o$ about the point (*O*) (z - axis) can be determined by finding the algebraic summation of the moments produced by each force in the system. The magnitude of the moment is positive if the direction of the moment is counterclockwise as it is directed about the positive (z) axis (outside the page) and vice versa, the magnitude of the moment is negative if the direction of the moment is clockwise.

$$+ (M_R)_o = \sum F\, d\,; \qquad (M_R)_O = F_1\, d_1 - F_2\, d_2 + F_3\, d_3 \;\ldots\ldots \quad (3\text{-}2)$$

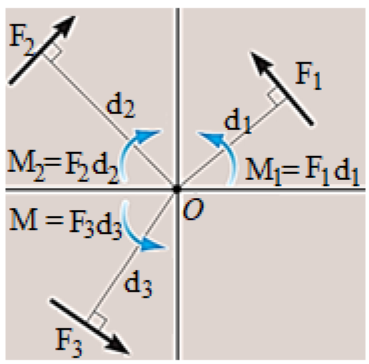


Fig. (3-2) Resultant moment

Example (3-2):

Determine the resultant moment of the four forces acting on the rod shown in Fig. (Ex. 3-2) about point (*O*).

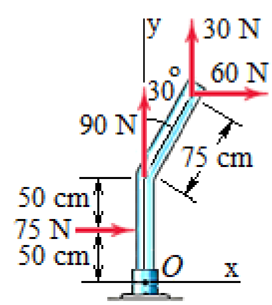


Fig. (Ex .3-2)

Solution:

$$(M_R)_o = -75 \times 0.5 \;+\; 90 \times 0$$
$$+\; 30 \times 0.75 \sin 30^o$$
$$-\,60 \times (\,1 + 0.75 \cos 30^o\,)$$
$$= -\,125.22 \text{ N.m} = 125.22 \text{ N.m}$$

Example (3-3):

Three forces acting on a cantilever beam as shown in Fig. (Ex. 3-3). Determine the moment about point (A) of each of the three forces and the resultant moment of these forces about the same point.

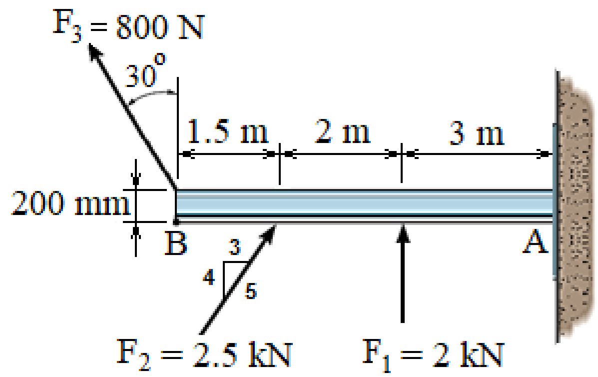


Fig. (Ex. 3-3)

Solution:

$+ \; (M_{F1})_A = -(2000)(3)$
$\quad = -6000$ N.m $= 6$ kN.m (Clockwise)

$+ \; (M_{F2})_A = -(2500)(4/5)(5)$
$\quad = -10000$ N.m $= 10$ kN.m (Clockwise)

$+ \; (M_{F3})_A = -(800 \cos 30^\circ)(6.5) + (800 \sin 30^\circ)(0.2)$
$\quad = -4423$ N.m $= 4.4$ kN.m (Clockwise)

$+ \; (M_R)_A = 6 + 10 + 4.4 = 20.4$ kN.m (Clockwise)

Example (3-4):

Determine the moment of the forces (F_A) and (F_B) about the bolt located at point (C).

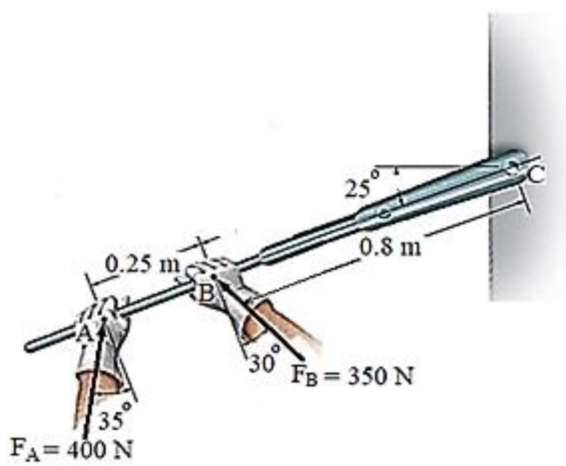


Fig. (Ex. 3-4)

Solution:

$+ \; M_C = -(400 \cos 35°)(1.05) - (350 \cos 30°)(0.8) = -586.53$ N.m
$\quad = 586.53$ N.m

Example (3-5):

The boom of the crane shown in Fig. (Ex. 3-5) can be specified at an angle (θ) confined between (0^o) and (90^o) within a specified elongation (x) between (0 ft) and (12 ft). If the suspended mass is (120 kg), find the moment resulting from this mass at point (B) in terms of (x) and (θ), then find the values of (x) and (θ) to achieve the maximum possible moment at point (B)? What is the value of this moment? Neglect the pulley size at point (A).

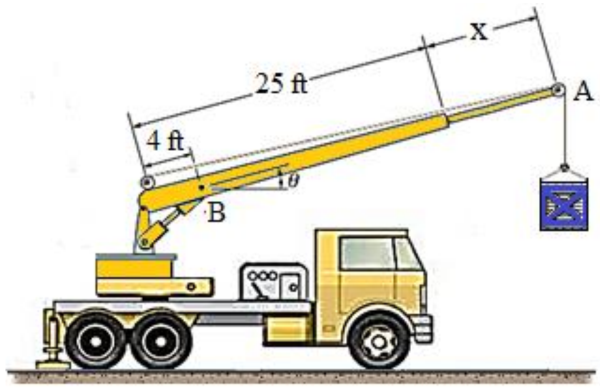


Fig. (Ex. 3-5)

Solution:

$$W = m\,g = 120 \times 9.81 = 1177.2 \text{ N} = \frac{1177.2}{4.448} = 264.66 \text{ Ib}$$

$$+ M_A = -(264.66)(21 + x)\cos\theta$$
$$= (264.66 \cos\theta)(21 + x) \text{ Ib.ft} \qquad (\text{Clockwise})$$

The maximum moment at (A) occurs when ($\theta = 0^o$) and (x = 12 ft).

$$+ (M_A)_{max} = \{(264.66 \cos 0^o)(21 + 12)\} \text{ Ib.ft} = 8733.78 \text{ Ib.ft} \qquad (\text{Clockwise})$$

Example (3-6):

The gate shown in Fig. (Ex. 3-6) consists of an arm with a mass of (75 kg) and a center of mass at (G_a) and a balance weight of (200 kg) with a center of mass at (G_W). Determine the magnitude and direction of the moment produced by the weights of the gate parts about point (A).

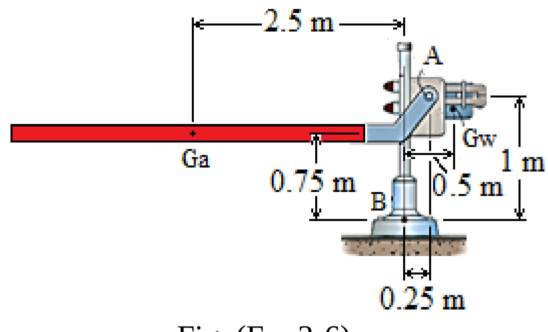


Fig. (Ex. 3-6)

Solution:

$$+(M_R)_A = \sum F\,d$$
$$(M_R)_A = (75)(9.81)(2.5 + 0.25) - (200)(9.81)(0.5 - 0.25)$$
$$= 1532.8 \text{ N.m} = 1.53 \text{ kN.m} \qquad (\text{Counterclockwise})$$

Example (3-7):

A hinged gate at point (C). pushed on both sides by two boys with two forces of different value and direction (F_A = 150 N), and (F_B = 250 N), as shown in Fig. (Ex. 3-7). Determine the moment of each force about point (C). Then indicate in which way will the gate rotate? Neglect the thickness of the gate.

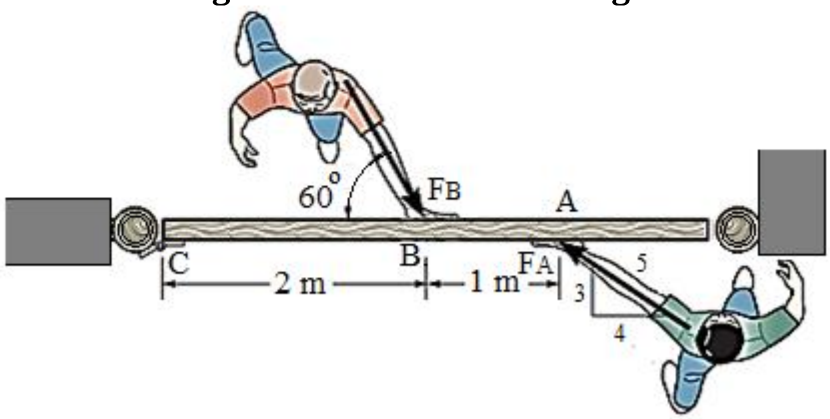


Fig. (Ex. 3-7)

Solution:

$\circlearrowleft + (M_{FA})_C = 150 \times \dfrac{3}{5} \times 3 = 270$ N.m (Counterclockwise)

$\circlearrowleft + (M_{FB})_C = -250 \sin 60^o \times 2 = -433 = 433$ N.m (Clockwise)

Since { $(M_{FB})_C > (M_{FA})_C$ }, the gate will rotate clockwise.

Example (3-8) :

A force of (100 N) is subjected to a handle of the hammer. Determine the moment of this force about the point (A).

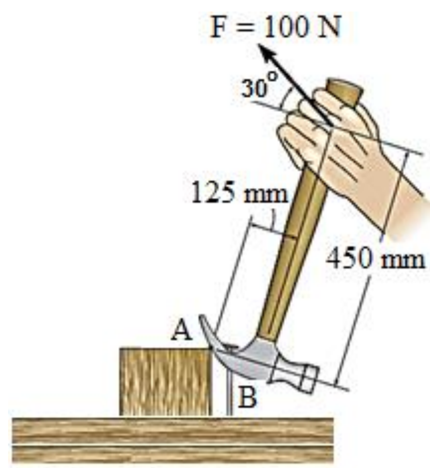
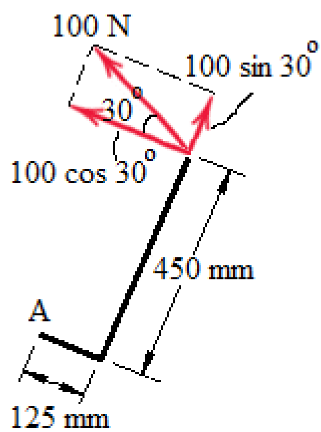


Fig. (Ex. 3-8)

Solution:



$\circlearrowleft + \sum M_A = (100 \cos 30^o)(0.45) + (100 \sin 30^o)(0.125)$

$= 45.22$ N.m (Counterclockwise)

Example (3-9):

The figure (Ex. 3-9) shows the members of the lower arm. The weight of the forearm is (25 N) and its center of gravity is at the point (G). When the palm is carrying a mass of (4 kg), calculate the biceps tension (T) so that the resultant moments about the point (*O*) equals to zero, (equilibrium state).

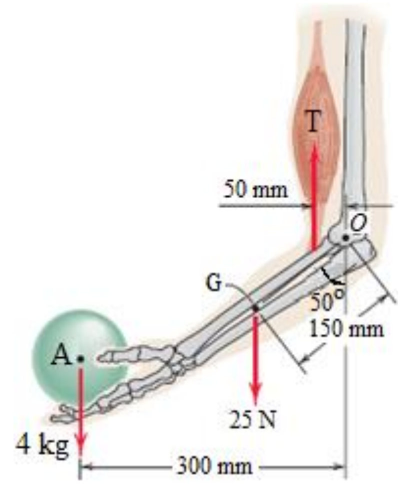


Fig. (Ex. 3-9)

Solution:

$\circlearrowleft + (\sum M_R)_o = \sum F\, d$

$0 = (4) (9.81) (300)$

$+ (25) (150 \sin 50)$

$- (T) (50)$

$50\, T = 14644.67$

$T = 292.9$ N

Example (3-10):

When a man tried to stand on his toes, the Achilles tendon moved with a force of (F_t = 145 Ib), and the reaction force of the ground on each of his feet was (N_f = 90 Ib). Find the resultant moments of the two forces (F_t) and (N_f) about the ankle joint (A).

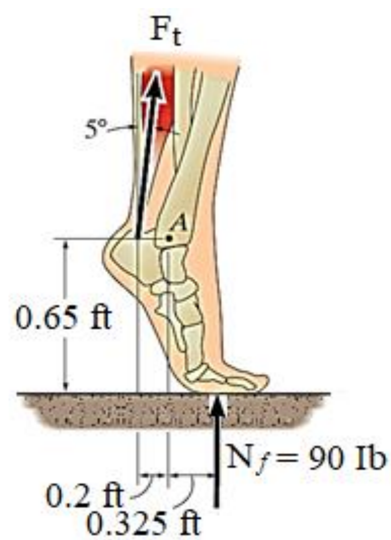


Fig. (Ex. 3-10)

Solution:

$\circlearrowleft + (\sum M_R)_A = \sum F\, d$

$(M_R)_A = (90) (0.325)$

$- (145 \cos 5^o) (0.2)$

$= 0.36$ Ib.ft (Counterclockwise)

Example (3-11):

A specialized crane in the field of electrical maintenance, the mass of its arm (AB) is (750 kg), the mass of the cage (BCD) is (100 kg), the mass of the electrician is (80 kg), and the centers of gravity are located at points (G_1), (G_2) and (G_3) respectively. Determine the moment produced by each part about the point (A), then find the resultant moments about the same point.

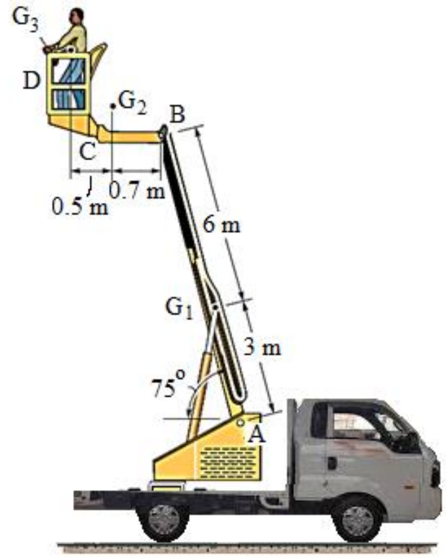


Fig. (Ex. 3-11)

Solution:

$\circlearrowleft + (M_{ar})_A = (750\times9.81)(3 \cos 75°) = 5712.8$ N.m

$= 5.7$ kN.m (Counterclockwise)

$\circlearrowleft + (M_c)_A = (100\times9.81)(9 \cos 75° + 0.7) = 2971.8$ N.m

$= 3$ kN.m (Counterclockwise)

$\circlearrowleft + (M_m)_A = (80\times9.81)(9 \cos 75° + 1.2) = 2769.8$ N.m

$= 2.8$ kN.m (Counterclockwise)

$\circlearrowleft + (M_R)_A = 5.7 + 3 + 2.8 = 11.5$ kN.m (Counterclockwise)

Example (3-12):

The tower crane shown in Fig. (Ex. 3-12) is used to hoist the (2000 kg) load upward at constant velocity. Its main arm (BD) has mass (1500 kg), and center of gravity at point (G_1), and the counterweight arm (BC) of mass (500 kg) and center of gravity at point (G_2), The (7000 kg) counterweight at point (C) have centers of mass at (G_3). Determine the resultant moment produced by the load and the weights of the tower crane arms and the counterweight about point (A) and about point (B).

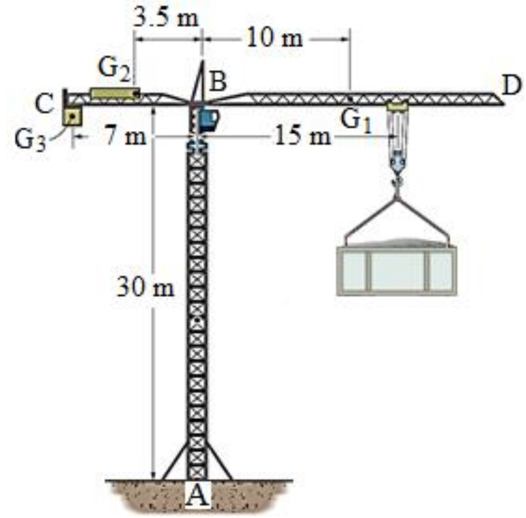


Fig. (Ex. 3-12)

Solution:

Since the moment arms of the weights and the load measured to points (A) and (B) are the same, the resultant moments produced by the load and the weights about points (A) and (B) are the same.

$$\circlearrowleft + (M_R)_A = (M_R)_B = \sum Fd$$

$$(M_R)_A = (M_R)_B = (7000)(9.81)(7) + (500)(9.81)(3.5) - (1500)(9.81)(10) - (2000)(9.81)(15)$$

$$= 480690 + 17167.5 - 147150 - 294300$$

$$= 56407.5 \text{ N.m} = 56.4 \text{ kN.m} \qquad (\text{Counterclockwise})$$

Example (3-13):

A crane with an arm length of (15 m), The towline exerts a force of (P = 3 kN) at the end of the arm. Determine the arm angle (*θ*) of the arm so that this force creates a maximum moment about point (*O*), then find the magnitude of this moment.

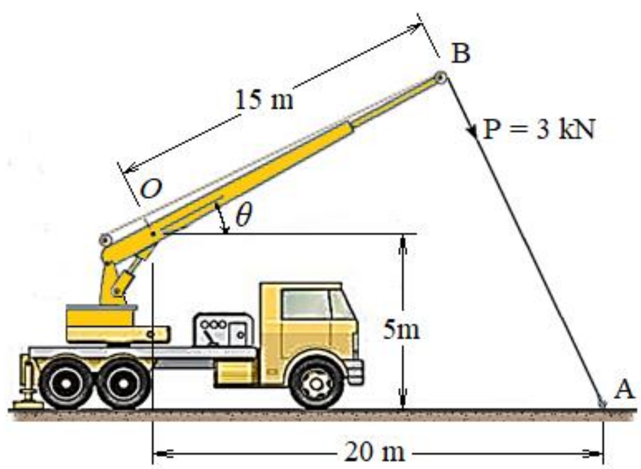
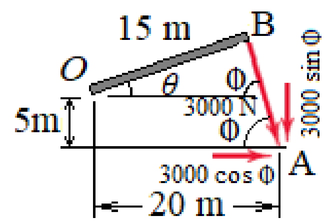


Fig. (Ex. 3-13)

Solution:



At maximum moment, OB ⊥ BA

$\circlearrowright$ + $(M_o)_{max} = -(3000)(15) = -45000$ N.m

$-(3000 \sin\phi)(20) + (3000 \cos\phi)(5) = -45000$

$-60000 \sin\phi + 15000 \cos\phi = -45000$ $\qquad \div -15000$

$4 \sin\phi - \cos\phi = 3$

$\sin^2\phi + \cos^2\phi = 1 \quad \Rightarrow \quad \sin\phi = \sqrt{1-\cos^2\phi}$

$4\sqrt{1-\cos^2\phi} - \cos\phi = 3$ $\qquad$ Let $\quad x = \cos\phi$

$4\sqrt{1-x^2} - x = 3$

$4\sqrt{1-x^2} = x + 3$

$16(1 - x^2) = x^2 + 6x + 9$

$16 - 16x^2 - x^2 - 6x - 9 = 0$

$-17x^2 - 6x + 7 = 0$ $\qquad \div -17$

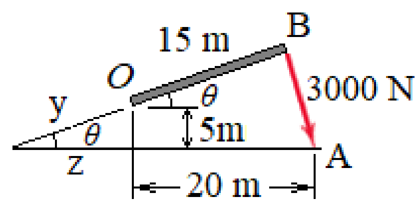
$x^2 + 0.353x - 0.412 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(0.353) \pm \sqrt{(0.353)^2 - 4(1)(-0.412)}}{2(1)} = 0.489 \quad \text{or} \quad -0.842$$

$\phi = \cos^{-1} 0.489 = 60.7^o$

$\theta = 90^o - 60.7^o = 29.3^o$

Method (2):



$(5)^2 + z^2 = y^2$

$25 + z^2 = y^2$

Similar triangles:

$\dfrac{15+y}{z} = \dfrac{20+z}{y} \quad \Rightarrow \quad 15y + y^2 = 20z + z^2$

$15(\sqrt{25+z^2}) + 25 + z^2 = 20z + z^2$

$15(\sqrt{25+z^2}) = 20z + z^2 - 25 - z^2$

$15(\sqrt{25+z^2}) = 20z - 25 \quad \Rightarrow \quad 225(25 + z^2) = 400z^2 - 1000z + 625$

$5625 + 225z^2 - 400z^2 + 1000z - 625 = 0$

$-175z^2 + 1000z + 5000 = 0$

$z^2 - 5.7z - 28.6 = 0 \quad \Rightarrow \quad (z - 8.91)(z + 3.21) = 0$

$z = 8.91$ m

$y = \sqrt{25+z^2} = \sqrt{25 + (8.91)^2} = 10.22$ m

$\theta = \cos^{-1}\left(\dfrac{8.91}{10.22}\right) = 29.3^o$

The moment of couple:

A moment of couple is formed by two parallel forces that have the same magnitude, and opposite direction, and not on the same line of action, i.e. separated by a distance (d) perpendicular to their lines of action, as shown in the figures (3-3) and (3-4). Since the two forces in the couple are equal and in opposite directions, their resultant will be equal to zero, so the only effect of it is to produce a rotation or tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn, one hand will push the steering wheel up while the other hand pulls it down, and that produce a couple moment on the center of the steering wheel that turns it in the direction of the desired rotation of the car.

$$M = \left(\frac{d}{2} \times F \right) + \left(\frac{d}{2} \times F \right)$$

$$M = d \times F \quad \ldots\ldots\ldots\ldots \quad (3\text{-}3)$$

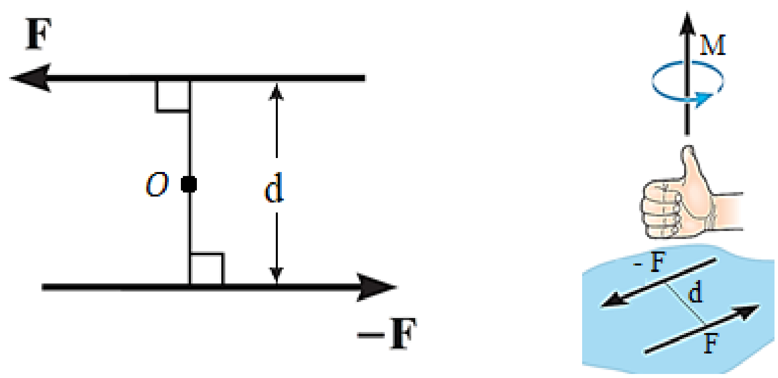


Fig. (3-3) The moment of couple

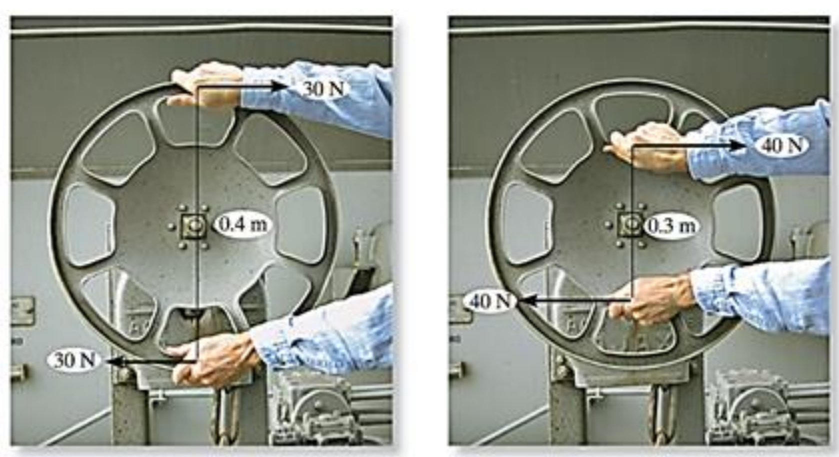


Fig. (3-4) The moment of couple

Example (3-14):

Determine the resultant couple moment of the three couples acting on the plate shown in Fig. (Ex. 3-14).

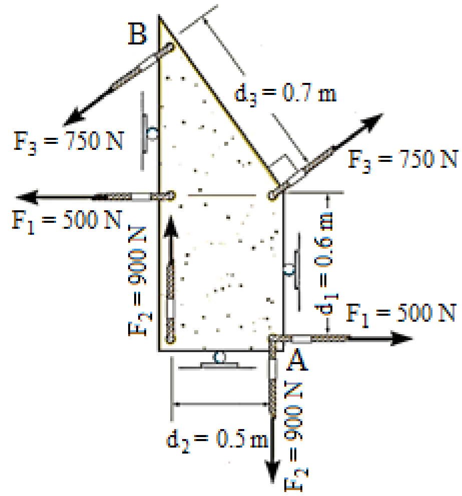


Fig. (Ex. 3-14)

Solution:

$d_1 = 0.6$ m , $d_2 = 0.5$ m , and $d_3 = 0.7$ m.

$\circlearrowleft + M_R = \sum M$ $\quad\quad$ $M_R = F_1 d_1 - F_2 d_2 + F_3 d_3$

$= (500)(0.6) - (900)(0.5) + (750)(0.7)$

$= 375$ N.m $\quad\quad$ (Counterclockwise)

Example (3-15):

A couple moment of (7 N.m) is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces (F) exerted on the handle end and (P) exerted on the blade end.

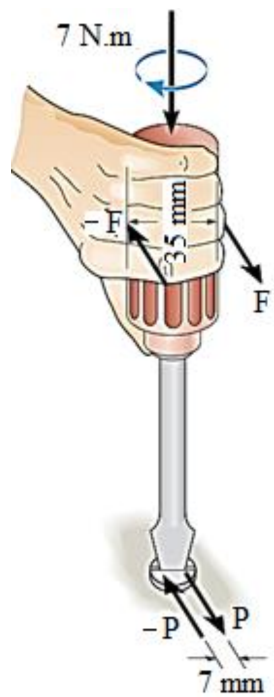


Fig. (Ex. 3-15)

Solution:

For the handle:

$M_C = F.d$ $\quad\quad$ $(F)(0.035) = 7$

$F = 200$ N

For the blade:

$M_C = P.d$ $\quad\quad$ $(P)(0.007) = 7$

$P = 1000$ N

Example (3-16):

A devices carrier wheel is subjected to the two couples. Determine the forces (F) that the bearings exert on the shaft so that the resultant couple moment on the wheel is zero.

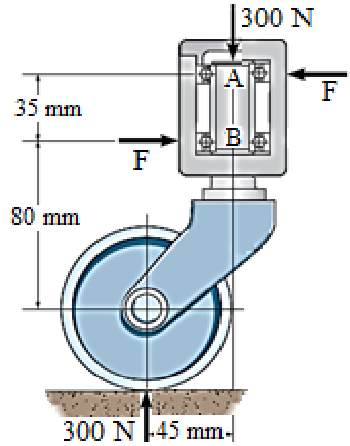


Fig. (Ex. 3-16)

Solution:

$\circlearrowleft + \sum M_C = 0 \qquad (F)(35) - (300)(45) = 0$

$35\,F = 13500$

$F = 385.7\ N$

Example (3-17):

If the resultant couple moments on the frame shown in Fig. (Ex. 3-17) is (500 N.m) clockwise, determine the magnitude of the force (F).

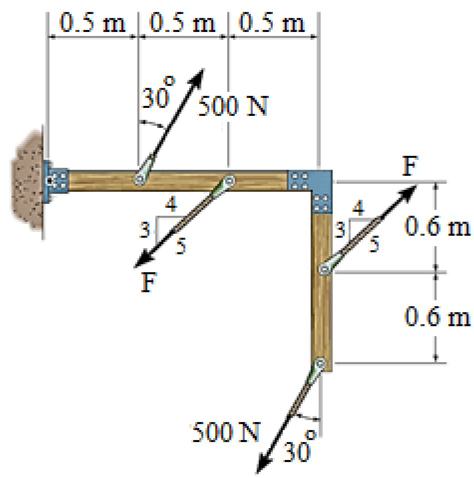
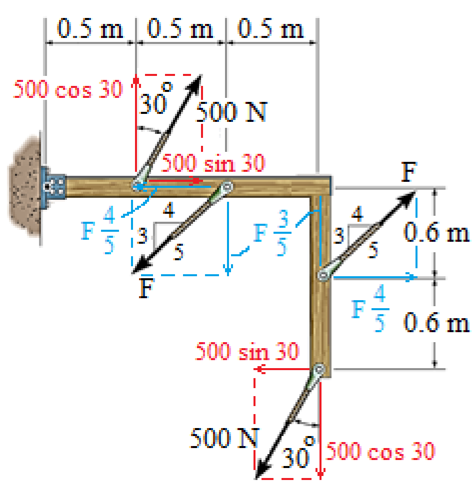


Fig. (Ex. 3-17)



Solution:

$\circlearrowleft + (M_C)_1 = (F)(4/5)(0.6) + (F)(3/5)(0.5)$

$= 0.78\ F$

$\circlearrowleft + (M_C)_2 = -(500 \cos 30^o)(1)$

$- (500 \sin 30^o)(1.2)$

$= -433 - 300$

$= -733\ N.m = 733\ N.m \circlearrowright$

$\circlearrowleft + (M_C)_R = (M_C)_1 + (M_C)_2$

$-500 = 0.78\ F - 733$

$733 - 500 = 0.78\ F$

$0.78\ F = 233$

$F = 298.7\ N$

Example (3-18):

The vertical reaction of the ground on the two main wheels of an aircraft at points (A) and (B) before the aircraft engine is running is (275 N) for each of the two wheels, and when the engine is running, the reaction is (350 N) at point (A).

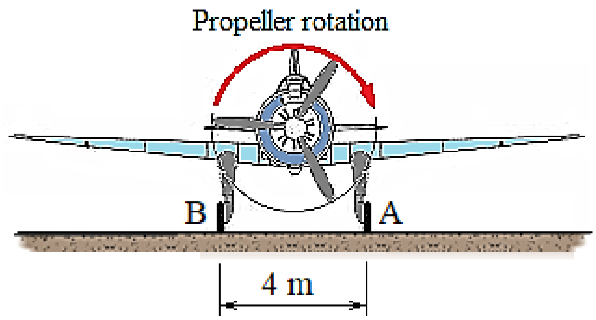


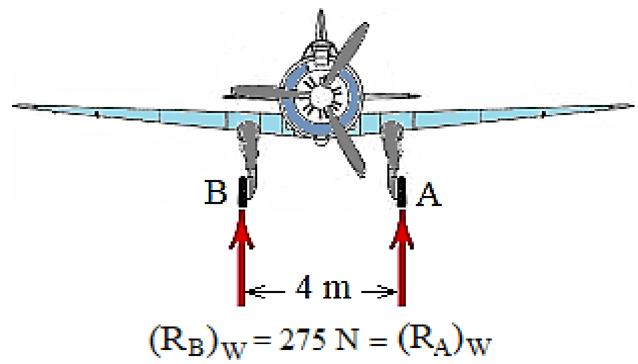
Fig. (Ex. 3-18)

The difference in reaction at point (A) is due to the torque of the dual propeller when the engine is running and its direction is clockwise, as shown in Fig. (Ex. 3-18). Find the magnitude of this torque and the magnitude of the reaction force of the ground acting at point (B) while the engine is running.

Solution:

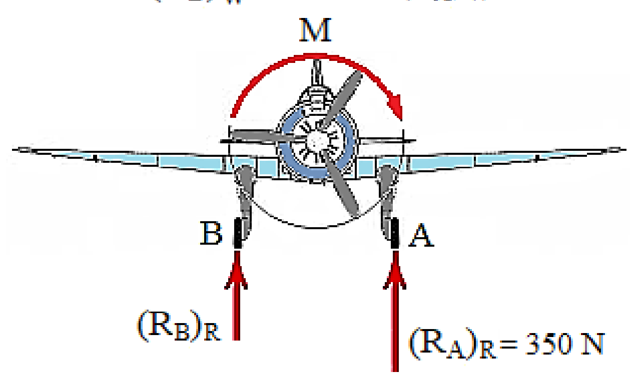
Due to weight:

$(R_A)_W = 275$ N
$(R_B)_W = 275$ N



Due to weight and propeller couple:

$(R_A)_R = 350$ N
$(R_B)_R = ?$



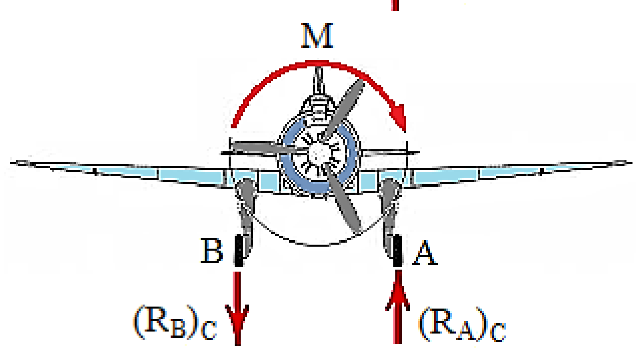
Due to propeller couple:

$(R_A)_C = 350 - 275 = 75$ N
$(R_B)_C = 75$ N

$(R_B)_R = 275 - 75 = 200$ N

$\curvearrowright + \sum M_C = 0$

$75 \times 4 = 300$ N.m



Force transformation to a line of action parallel to its line of action:

If a force moves on a certain body from one point to another point that is not on the same line of action, it is transferred in the form of a force with the same value and direction and a moment equal to the force multiplied by the distance perpendicular to the line of action of the force between the two points.

Example (3-19):

Replace the horizontal (500 N) force acting at point (A) on the lever by equivalent system consisting of a force and a couple at (*O*).

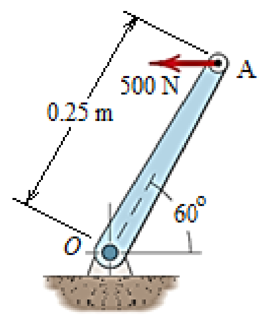


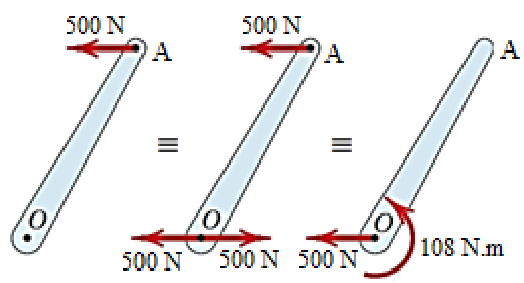
Fig. (Ex. 3-19)

Solution:

When a two forces with a value of (500 N) are applied to point (*O*) in opposite directions, so their resultant is equal to zero, so the two forces (500 N) on point (A) and opposite to it in the direction at point (*O*) generate a couple counterclockwise.

$M = F.d$

$M = 500 \times 0.25 \sin 60^{o} = 108$ N.m



Thus, the force (500 N) on point (A) will be equivalent to a force of (500 N) in the same direction and a torque of (108 N.m) counterclockwise at point (*O*).

The resultant of a system of coplanar non-concurrent forces (forces and moments):

When several non-concurrent forces located in the same plane act on a body and are at specific distances from a specific point, let it be a point (O), then it is possible to calculate the resultant of those forces on the known point and the resultant of the moments around that point, and then these forces can be converted by a single force that is a distance away calculated from the known point, as shown in Fig. (3-5).

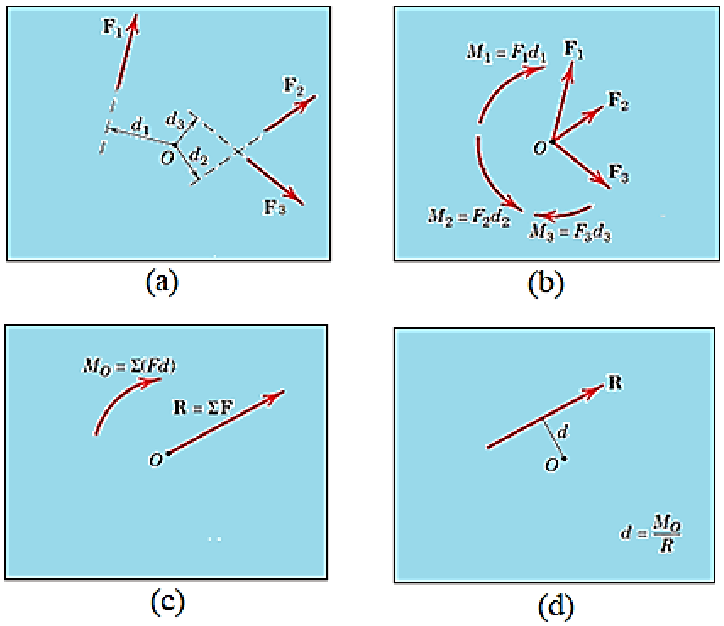


Fig. (3-5) The resultant of a system of coplanar non-concurrent forces

The resultant (value and direction) can be calculated according to the following equations:

$$R = F_1 + F_2 + F_3 + \ldots\ldots = \sum F \quad \ldots\ldots\ldots\ldots \quad (3\text{-}4)$$

$$R_x = \sum F_x \qquad R_y = \sum F_y \qquad R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \quad \ldots\ldots \quad (3\text{-}5)$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\sum F_y}{\sum F_x} \quad \ldots\ldots\ldots\ldots \quad (3\text{-}6)$$

The value of the moment and the perpendicular distance (its location) can be calculated according to the following equations:

$$R = \sum F \quad \ldots\ldots\ldots\ldots \quad (3\text{-}7)$$

$$M_o = \sum M = \sum (Fd) \quad \ldots\ldots\ldots\ldots \quad (3\text{-}8)$$

$$Rd = M_O \quad \ldots\ldots\ldots\ldots \quad (3\text{-}9)$$

Example (3-20):

Four forces and one couple act on the bracketed shown in Fig. (Ex. 3-20). Determine the resultant of the forces and the couple, and then indicate the point of action of the resultant on the horizontal axis with respect to the origin (*O*).

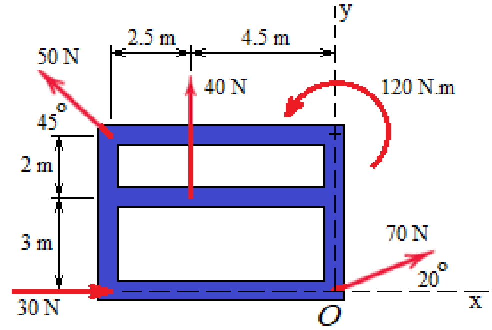
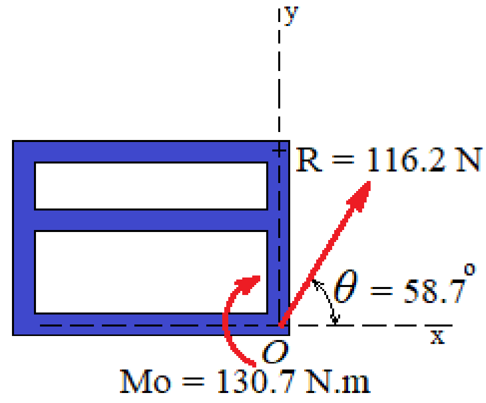


Fig. (Ex. 3-20)

Solution:



$R_x = \sum F_x$

$R_x = 30 + 70 \cos 20° - 50 \cos 45° = 60.4$ N

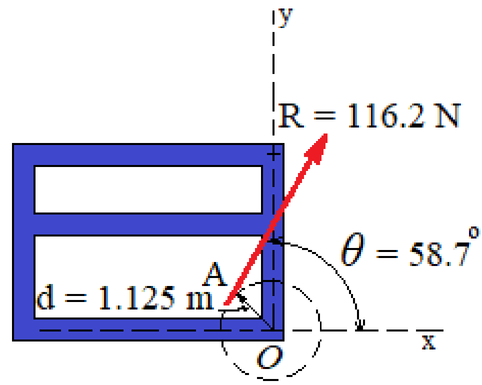
$R_y = \sum F_y$

$R_y = 40 + 70 \sin 20° + 50 \sin 45° = 99.3$ N

$R = \sqrt{Rx^2 + Ry^2}$

$R = \sqrt{(60.4)^2 + (99.3)^2} = 116.2$ N

$\theta = \tan^{-1} \dfrac{R_y}{R_x} \qquad \theta = \tan^{-1} \dfrac{99.3}{60.4} = 58.7°$

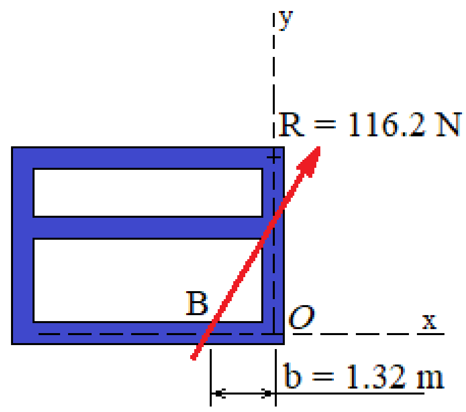


$M_o = \sum (Fd)$

$M_o = 120 - (40 \times 4.5) + (50 \cos 45° \times 5) - (50 \sin 45° \times 7)$

$= -130.7 = 130.7$ N.m (C.W.)

$R\, d = M_o, \quad 116.2\, d = 130.7, \quad d = 1.125$ m



$R_y\, b = M_o$

$R_y = 99.3$ N

$b = \dfrac{130.7}{99.3} = 1.32$ m

Example (3-21):

Replace the two forces acting on the bracket shown in Fig. (Ex. 3-21) by an equivalent resultant force and couple moment at point (*O*).

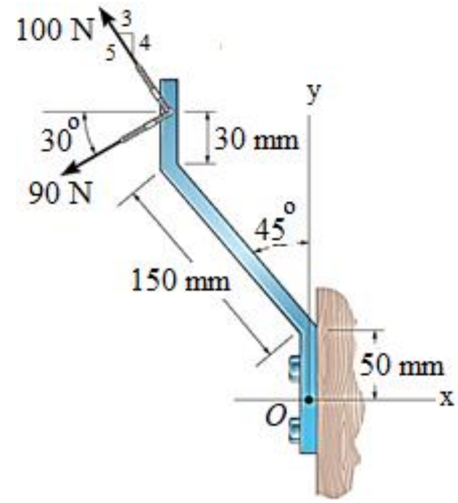


Fig. (Ex. 3-21)

Solution:

$\xrightarrow{+}$ $F_{Rx} = \sum F_x$ $\qquad$ $F_{Rx} = -(100)(3/5) - 90\cos 30^\circ = -138 = 138 \text{ N} \leftarrow$

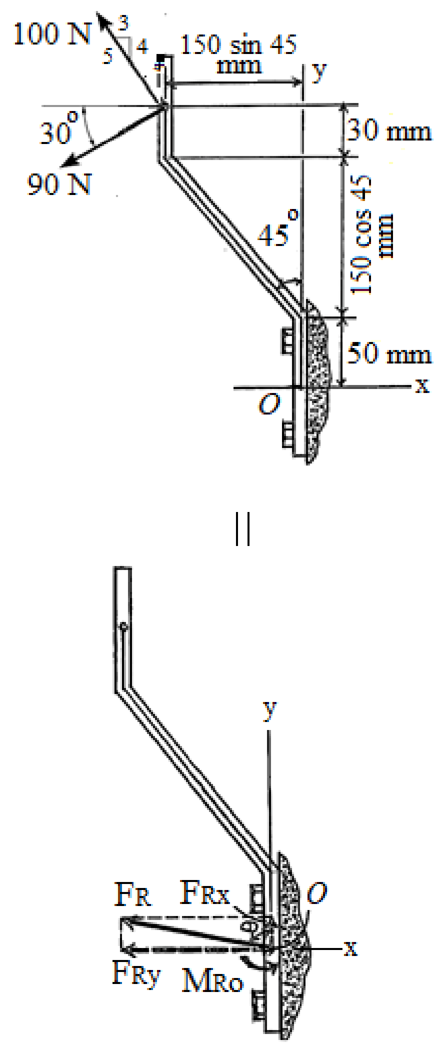
$+\uparrow$ $F_{Ry} = \sum F_y$ $\qquad$ $F_{Ry} = (100)(4/5) - 90\sin 30^\circ = 35 \text{ N}$

$$F_R = \sqrt{F_{Rx}^{\,2} + F_{Ry}^{\,2}} = \sqrt{(138)^2 + (35)^2} = 142.4 \text{ N}$$

$$\theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{35}{138} = 14^\circ$$

$\circlearrowleft + M_{Ro} = \sum M_o$

$$\begin{aligned} M_{Ro} &= (90\sin 30^\circ)(0.15\sin 45^\circ) \\ &\quad + (90\cos 30^\circ)(0.08 + 0.15\cos 45^\circ) \\ &\quad - (100)(4/5)(0.15\sin 45^\circ) \\ &\quad + (100)(3/5)(0.08 + 0.15\cos 45^\circ) \\ &= 4.773 + 14.502 - 8.485 + 11.164 \\ &= 22 \text{ N.m} \circlearrowleft \end{aligned}$$



Example (3-22):

The bulldozer shown in Fig. (Ex. 3-22) consists of four main parts, the engine part has a mass of (2 tons) and its center of gravity (G_1), the cabin part has a mass of (0.8 ton) and its center of gravity (G_2), the drivetrain part has mass of (1.2 ton) and its center of gravity (G_3), the kit part has a mass of (1 ton) and its center of gravity (G_4). Replace the forces produced by these masses with an equivalent resultant and indicate the position of this resultant measured from point (A).

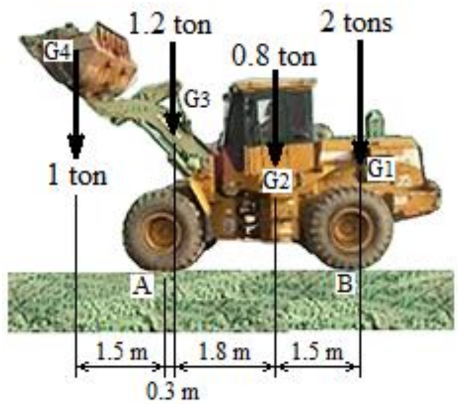


Fig. (Ex. 3-22)

Solution:

$+\uparrow F_R = \sum F_y$

$$F_R = -(1000 \times 9.81) - (1200 \times 9.81) - (800 \times 9.81) - (2000 \times 9.81)$$
$$= -49050 = 49050 \text{ N} \downarrow$$

$\curvearrowright + \quad M_{RA} = \sum M_A$

$$-(49050 \times d) = (1000 \times 9.81 \times 1.5) - (1200 \times 9.81 \times 0.3)$$
$$- (800 \times 9.81 \times 2.1) - (2000 \times 9.81 \times 3.6)$$

$d = 1.548$ m

Problems:

3-1) Calculate the moment of the (50 Ib) force on the handle of the monkey wrench about the center of the bolt.

Ans.: M = 370.8 Ib.in (CW)

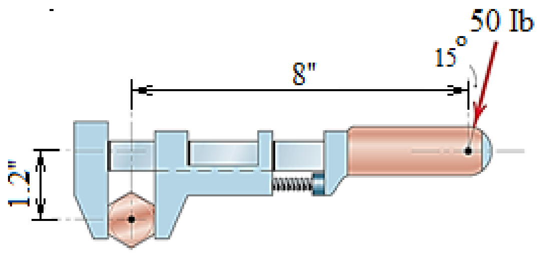


Fig. (Pr. 3-1)

3-2) A rod bent at an obtuse angle as shown in Fig. (Pr. 3-2), and a force of (25 N) is applied perpendicular to the axis of the (BC) part of it. Find the moment of this force about point (B) and about point (A).

Ans.: $M_B = 25$ *N.m (CW)*
$M_A = 46.2$ *N.m (CW)*

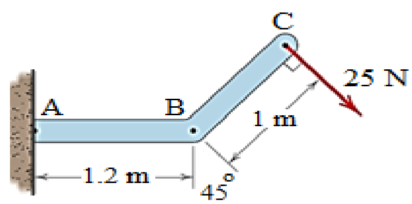


Fig. (Pr. 3-2)

3-3) The (25 Ib) force is applied to one end of the curved wrench, as shown in Fig. (Pr. 3-3). If ($\alpha = 30°$), calculate the moment of (F) about the center (*O*) of the bolt. Determine the value of (α) which would maximize the moment about (*O*), and determine the value of this maximum moment.

Ans.: $M_o = 359.8$ *Ib.in. (CW)*
$\alpha = 33.7°$
$(M_o)_{max} = 360$ *Ib.in. (CW)*

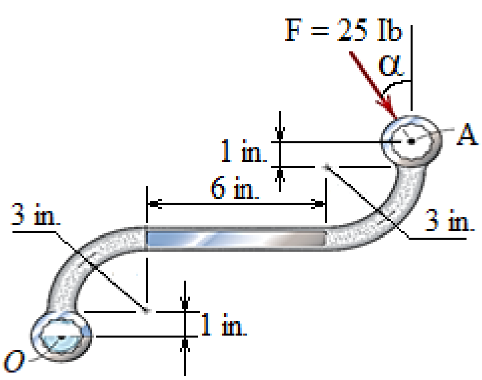


Fig. (Pr. 3-3)

3-4) For the frame shown in Fig. (Pr. 3-4). Determine the resultant moment of the three forces about point (A). Neglect the thickness of the frame.

Ans.: $M_A = 600$ *N.m (CW)*

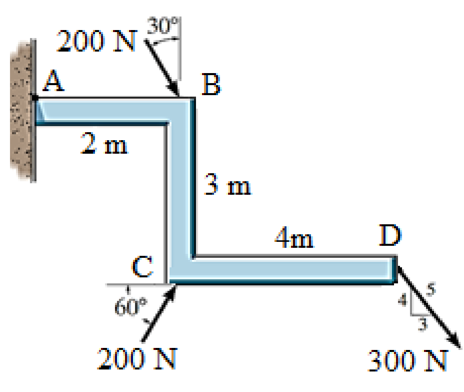


Fig. (Pr. 3-4)

3-5) The center of gravity of the gate arm, shown in Fig. (Pr. 3-5), is located at point (G_a) and the center of gravity of the counterweightv (200 kg) is located at point (G_W). If the resultant moment about point (A) is (4.6 kN.m) counter-clockwise, determine the magnitude of the gate arm mass.

Ans.: m_G = 188.5 kg

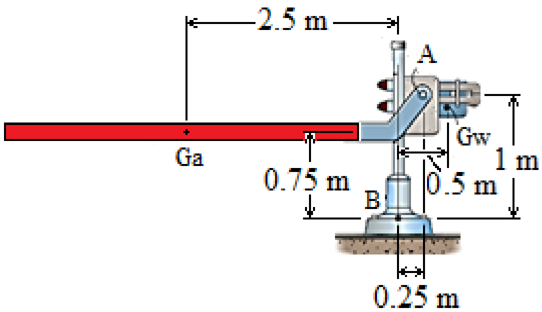


Fig. (Pr. 3-5)

3-6) The tool shown in Figure (Pr. 3-6) is used to hold the lawn mower blade while loosening the nut with a wrench. If a force of (60 N) is applied to the wrench at (B) in the direction shown in the figure, determine the moment it creates about the nut at (C). What is the magnitude of force (F) at (A) so that it creates the opposite moment about (C) ?

Ans.: M_C = 13 N.m, F = 40.2 N

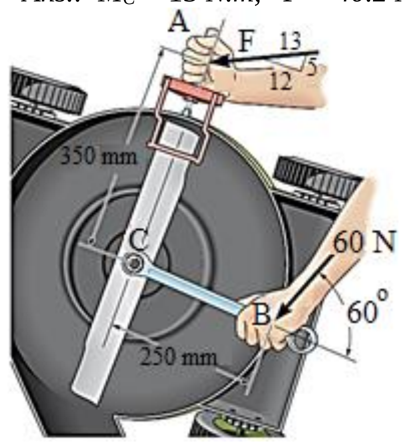


Fig. (Pr. 3-6)

3-7) Two persons push a gate from both sides of it, as shown in Fig. (Pr. 3-7). If the force applied by the person at point (B) is (F_B = 150 N), determine the magnitude of the force (F_A) required by the person at point (A) to prevent the gate from turning. Neglect the thickness of the gate.

Ans.: F_A = 144.3 N

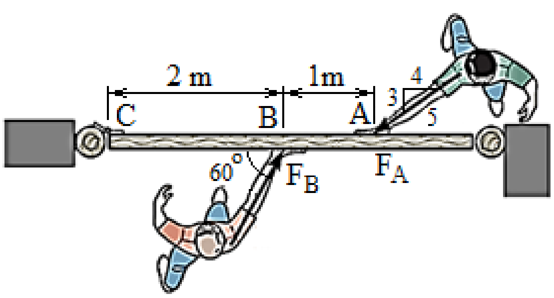


Fig. (Pr. 3-7)

3-8) To increase the torque required to unscrew the screw at point (A), the screwdriver arm is lengthened using the rod (BC) as shown in Fig. (Pr. 3-8). Determine the moment produced by the force (250 N) about the axis of the bolt at point (A).

Ans.: M_A = 102 N.m (CW)

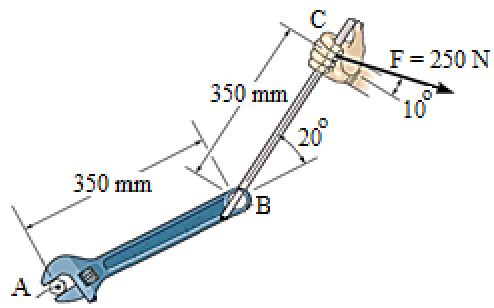


Fig. (Pr. 3-8)

3-9) In a load car (trailer), when the trailer is towed in the forward direction, a force (600 N) is applied to the trailer hitch ball, as shown in Fig. (Pr. 3-9). Determine the moment of this force force about point (*O*).

Ans.: M_O = 167.88 N.m (CCW)

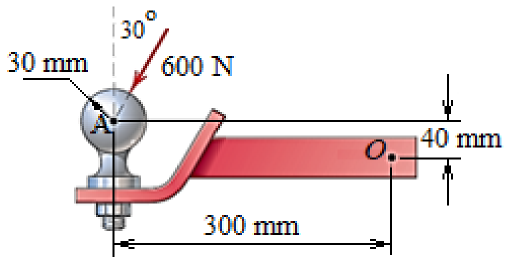


Fig. (Pr. 3-9)

3-10) In Fig. (Pr. 3-10), a force of magnitude (100 N) is exerted on an automobile parking-brake lever. Replace the force by an equivalent force – couple system at the pivot point (*O*).

Ans.: M_O = 34.58 N.m (CCW)

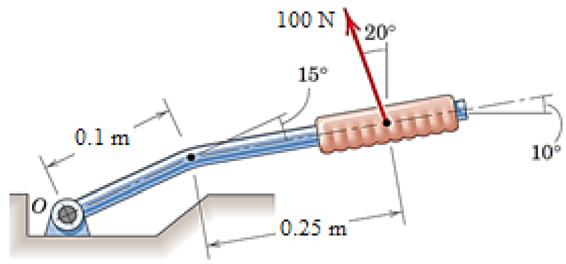


Fig. (Pr. 3-10)

3-11) During the test of the work of the aircraft, its two engines are accelerated and the direction of the two propellers is adjusted so that it results in forward thrust and rear thrust as shown in Fig. (Pr. 3-11). Calculate the friction force (F) that the ground exerts on each of the main wheels at points (A) and (B) to resist the effect of the two thrust forces. Ignore the effect of the nose wheel (C) which turns at an angle of (90º).

Ans.: F = 5 kN

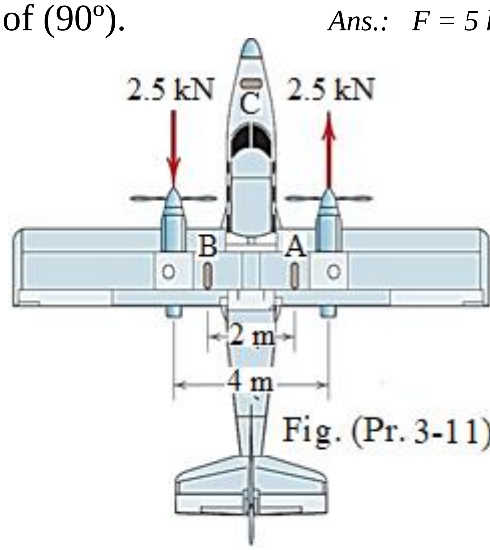


Fig. (Pr. 3-11)

3-12) The figure (Pr. 3-12) shows the top view of the entrance revolving door. Two persons approach the door at the same time and exert two forces of the same magnitudes as shown in the figure. If the resultant moment about the door pivot axis at (*O*) is (30 N.m), determine the magnitude of the force (F).

Ans.: F = 15.5 N

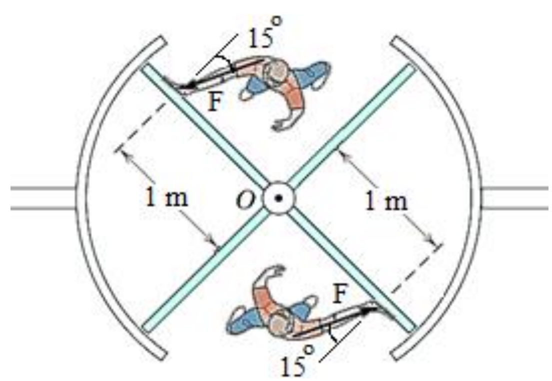


Fig. (Pr. 3-12)

3-13) Each propeller of the twin-engine ship rotates in a speed that generate thrust (300 kN). During the ship's maneuvering motion, one propeller rotates at full forward speed and the other at full rearward speed, as shown in Fig. (Pr. 3-13). Each boat applies a force of (50 kN) to the ship to counteract the action of the ship's propellers. Find the distance (x).

Ans.: X = 35 m

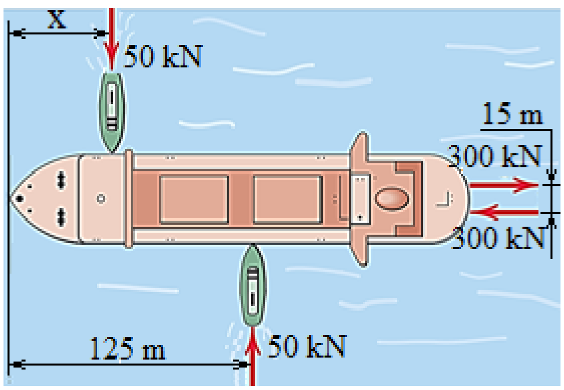


Fig. (Pr. 3-13)

3-14) The airplane shown in Fig. (Pr. 3-14) with four jet engines, each producing (100 kN) of forward thrust. At cruise flight, the engine number (3) suddenly fails. Determine and locate the resultant of the three remaining engines thrust forces.

Ans.: R = 300 kN, x = 4 m

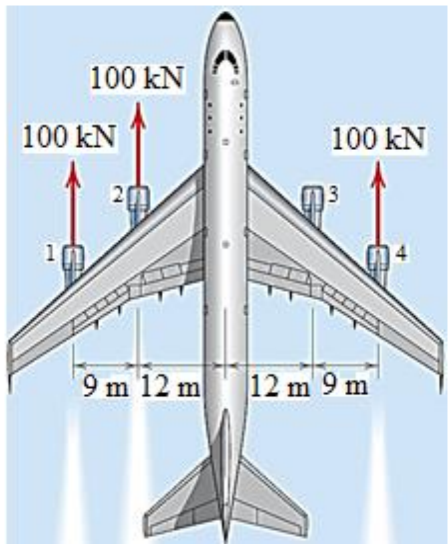


Fig. (Pr. 3-14)

3-15) In Fig. (Pr. 3-15) Two couples act on the cantilever beam. Find the resultant couple moment.

Ans.: $(M_C)_R = 1.8$ *kN. m*

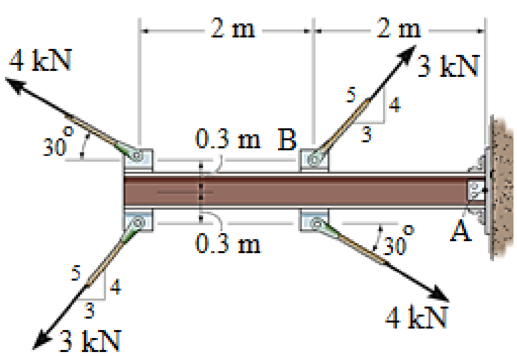


Fig. (Pr. 3-15)

3-16) Fig. (Pr. 3-16) shows a valve for opening and closing the water pipe. A man tried to open the valve by applying couple forces of (F = 100 N) on the lever of the valve. Find the couple moment of the two forces.

Ans.: Mc = 30 N. m (CW)

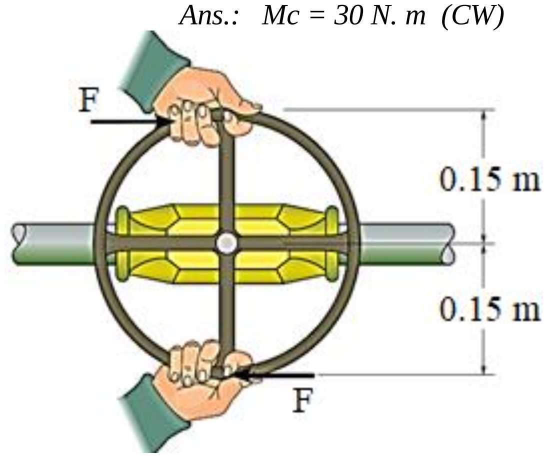


Fig. (Pr. 3-16)

3-17) If the resultant of the two forces and the couple (M) passes through point (O), determine the magnitude of the couple (M).

Ans. $M = 160.6\ N.m$

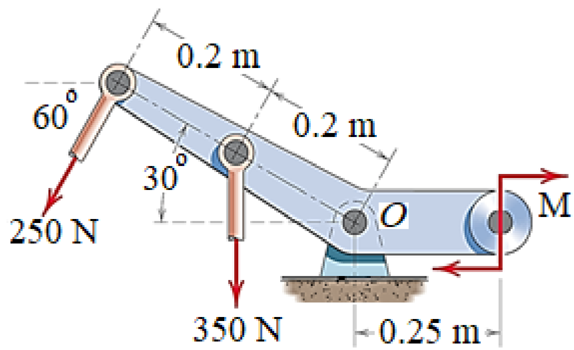


Fig. (Pr. 3-17)

3-18) The specialized truck shown in Fig. (Pr. 3-18) consists of three main parts, and its wights and center of gravity is indicated on each part. Replace the system of forces resulting from the weights of these parts with an equivalent resultant force and locate it relative to point (A).

Ans.: $F_R = 49\ kN \downarrow,\ d = 3\ m$

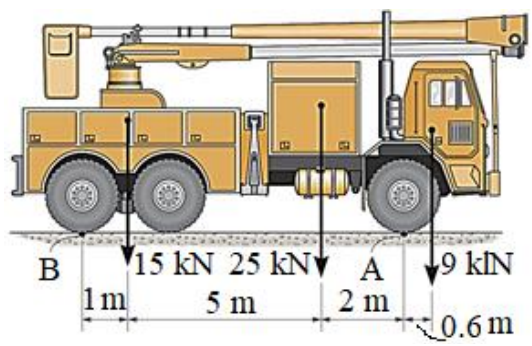


Fig. (Pr. 3-18)

3-19) Replace the forces and couple system shown in Fig. (Pr. 3-19) by an equivalent force and couple moment at point (O).

Ans.
$F_R = 2.07\ kN,\ \theta = 8.5^\circ,\ M = 10.62\ k\ N.m$ *CW*

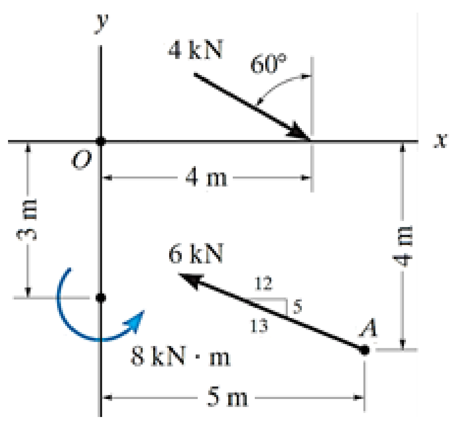


Fig. (Pr. 3-19)

3-20) Replace the force system acting on the frame shown in Fig. (Pr. 3-20) by a resultant force, and specify where its line of action intersects member (AB), measured from point (A).

Ans.: $F_R = 462\ Ib,\ \theta = 50.1^\circ,\ d = 3.07\ ft$

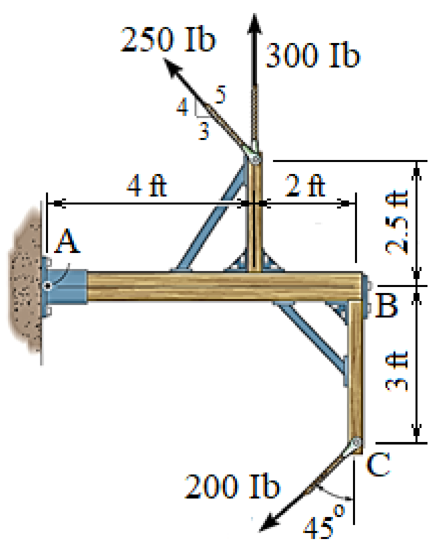


Fig. (Pr. 3-20)

CHAPTER FOUR
EQUILIBRIUM

In the previous two chapters, the study of the analysis of forces and moments on particles and rigid bodies was discussed, and how to conclude the resultant of forces in different values and directions on these rigid bodies and particles, and how to conclude the resultant of moments, and the resultant of forces and moments together.

In this chapter, the equilibrium states between these forces and moments on rigid bodies and particles will be discussed, and this topic will be divided into two parts, the first part shows the state of equilibrium in forces on particles, and the second part shows the state of equilibrium in forces and moments on rigid bodies.

A body is in equilibrium if the resultant of the forces acting on it is zero. This case is in stationary objects and objects moving in uniform motion (motion at constant velocity).

PART (1) : EQUILIBRIUM OF THE PARTICLES:

In this part of the equilibrium, the dimensions of the body are not taken into account, and the body is assumed as a point, so the body is in a state of equilibrium when the resultant of the forces acting on it is equal to zero, and the moments are not taken into consideration due to the neglect of the effect of the dimensions and the assumption of the body as a point, so the force acting on it is hypothetically concurrent. So the equilibrium equation is:

$$R = \sum F_x = \sum F_y = \sum F_z = 0 \quad \ldots\ldots\ldots\ldots \quad (4\text{-}1)$$

Conditions for the equilibrium of a particle:

If the particle is originally in a state of rest (without motion), it is said to be in equilibrium if it continues in its state of rest, and if it is originally in a state of uniform motion with a constant velocity and zero acceleration, it is said to be in equilibrium if it continues in its state of uniform motion without change. Most often, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe a particular body at rest. To maintain equilibrium, it is necessary to apply Newton's first law of motion which is the basic law of equilibrium equations in the field of statics, and Newton's first law of motion requires that the force or the resultant of forces applied to a particle be equal to zero. This can be expressed mathematically as:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

The free-body diagram:

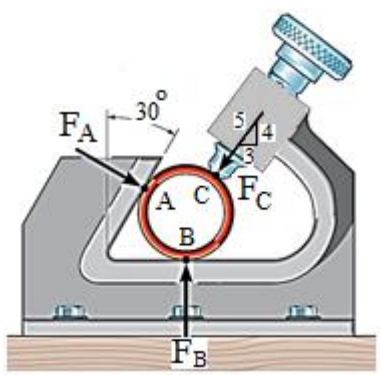
The best way to explain the known and unknown forces acting on the particle, and apply the equilibrium equations to account for the unknown forces ($\sum Fx = 0$), ($\sum F_y = 0$) is to think of the particle as isolated and "free" from its surroundings. A drawing that shows a body with all the forces acting on it is called a *free body diagram* (FBD).

Procedure for drawing a free-body diagram:

In order to calculate all the forces acting on the particle when applying the equilibrium equations, the free-body diagram must be drawn first.

The following steps are necessary to create a free-body diagram.

1- We assume that the particle is isolated from its surroundings and then draw its specific shape (free body diagram).
2- Placing known and unknown forces on the particle diagram.
3- Draw the required dimensions and angles.
4- Apply the equations of equilibrium to find the unknown forces.



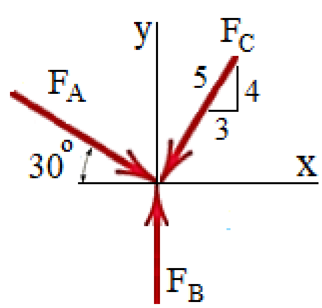


Fig. (4-1) Procedure for drawing a free-body diagram

Example (4-1):

Find the magnitude of each of the two unknown forces (F_1) and (F_2) required to achieve equilibrium in the truss members shown in Fig. (Ex. 4-1), that are hinged at the (*O*) joint.

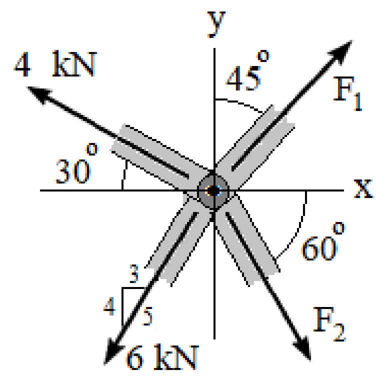


Fig. (Ex. 4-1)

Solution:

$+\rightarrow \sum F_x = 0$

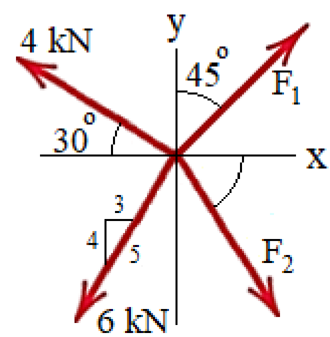
$$F_1 \sin 45° + F_2 \cos 60° - 4 \cos 30° - 6\left(\frac{3}{5}\right) = 0$$

$$0.707\, F_1 + 0.5\, F_2 = 7.064 \quad \ldots\ldots\ldots\ldots (1)$$

$+\uparrow \sum F_y = 0$

$$F_1 \cos 45° + 4 \sin 30° - F_2 \sin 60° - 6\left(\frac{4}{5}\right) = 0$$

$$0.707\, F_1 - 0.866\, F_2 = 2.8 \quad \ldots\ldots\ldots\ldots (2)$$



$$0.707\, F_1 + 0.5\, F_2 = 7.064$$
$$0.707\, F_1 - 0.866\, F_2 = 2.8$$

-------------------------------- Subtraction

$$1.366\, F_2 = 4.264$$
$$F_2 = 3.12 \text{ kN}$$

Sub. in Eq. (1): $0.707\, F_1 + 0.5\,(3.12) = 7.064$

$0.707\, F_1 = 5.504 \quad \Rightarrow \quad F_1 = 7.78 \text{ kN}$

Example (4-2):

The pipe is held in place by the vise. If the fixing bolt exerts a force of (250 N) on the pipe in the direction shown in Fig. (Ex. 4-2), determine the forces (F_A) and (F_B) that the smooth surfaces at (A) and (B) exert on the pipe.

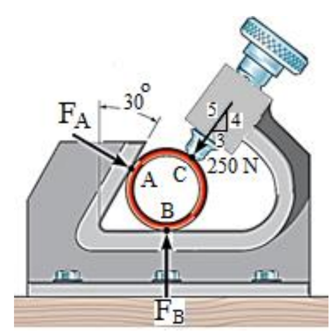


Fig. (Ex. 4-2)

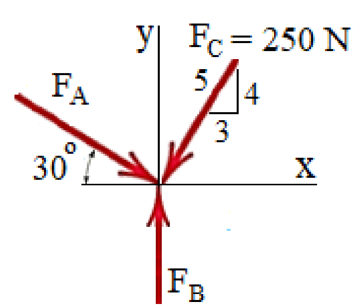
Solution:

$+\rightarrow \sum F_x = 0, \qquad F_A \cos 30° - 250\left(\frac{3}{5}\right) = 0$

$F_A = 173.2 \text{ N}$

$+\uparrow \sum F_y = 0, \qquad F_B - 173.2 \sin 30° - 250\left(\frac{4}{5}\right) = 0$

$F_B = 286.6 \text{ N}$



Example (4-3):

Determine the tension in each of the ropes (AB) and (AC) used to lift a container of mass (650 kg) as a function of angle (θ). If the maximum allowable tension in each rope is (6.5 kN), find the shortest length of ropes (AB) and (AC) that can be used for lifting. Since the container's center of gravity is at point (G).

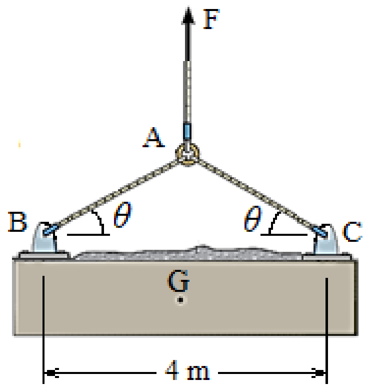
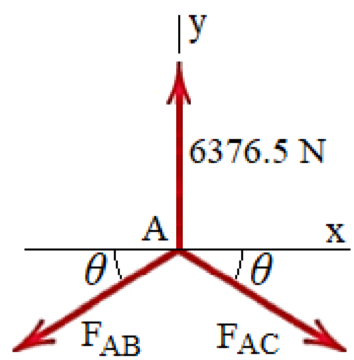


Fig. (Ex. 4-3)

Solution:

$$W = m\,g = 650 \times 9.81 = 6376.5 \text{ N}$$



$+\rightarrow \sum F_x = 0$ $\qquad F_{AC} \cos\theta - F_{AB} \cos\theta = 0$

$$F_{AC} = F_{AB} = F$$

$+\uparrow \sum F_y = 0$ $\qquad 6376.5 - 2\,F \sin\theta = 0$

$$2\,F \sin\theta = 6376.5$$

$$F = \frac{6376.5}{2\ \sin\theta} = \frac{3188.25}{\sin\theta}$$

Thus:

$$F_{AC} = F_{AB} = F = \frac{3188.25}{\sin\theta}\ \text{N}$$

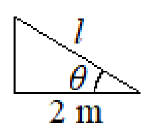
If the maximum allowable tension in the rope is (6.5 kN), then:

$$\frac{3188.25}{\sin\theta} = 6500$$

$$3188.25 = 6500 \sin\theta$$

$$\theta = \sin^{-1}\frac{3188.25}{6500} = 29.37°$$

From the geometry, ($l = \dfrac{2}{\cos\theta}$) and ($\theta = 29.37°$):



$$l = \frac{2}{\cos 29.37°} = 2.3 \text{ m}$$

Example (4-4):

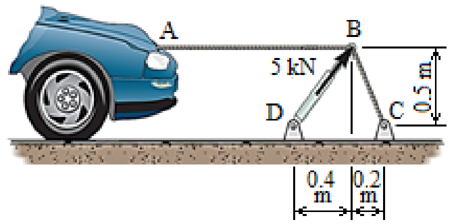
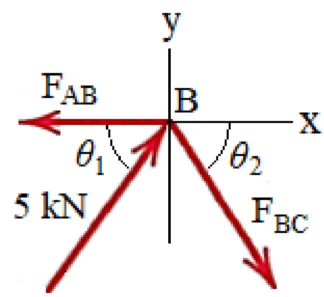


Fig. (Ex. 4-4)

The device shown in Fig. (Ex. 4-4) is used to straighten the bodies of wrecked cars. It consists of a chain that bears great forces, attached on one tip to a fixed point and connected on the other tip to the part of the car to be straightened.

A force applied by a hydraulic cylinder is applied to a point in the central part. Find the tension force of each part of the chain (AB) and (BC), if the force exerted by the hydraulic cylinder (DB) on point (B) is (5 kN).

Solution:



$$\theta_1 = \tan^{-1}\frac{0.5}{0.4} = 51.3° \quad , \quad \theta_2 = \tan^{-1}\frac{0.5}{0.2} = 68°$$

$+\uparrow \sum F_y = 0$

$5 \sin 51.3° - F_{BC} \sin 68° = 0$

$F_{BC} = 4.21 \text{ kN}$

$+\rightarrow \sum F_x = 0 \qquad 5 \cos 51.3° + 4.21 \cos 59° - F_{AB} = 0 \quad \Rightarrow \quad F_{AB} = 5.29 \text{ kN}$

Example (4-5):

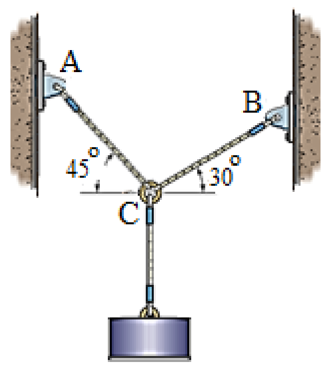


Fig. (Ex. 4-5)

In the system of wires shown in Fig. (Ex. 4-5), if the mass of the cylinder is (15 kg), Determine the required tensile force in the wires (CA) and (CB) to achieve equilibrium.

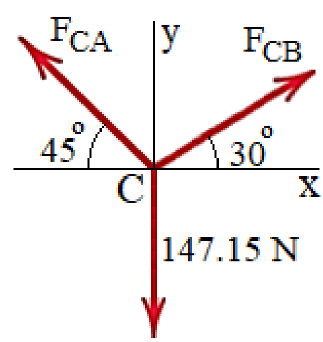
Solution:

$W = m\,g = 15 \times 9.81 = 147.15 \text{ N}$

$+\rightarrow \sum F_x = 0$

$F_{CB} \cos 30° - F_{CA} \cos 45° = 0$

$0.866\, F_{CB} - 0.707\, F_{CA} = 0 \quad \ldots\ldots\ldots \quad (1)$



$+\uparrow \sum F_y = 0$

$F_{CB} \sin 30° + F_{CA} \sin 45° - 147.15 = 0$

$0.5\, F_{CB} + 0.707\, F_{CA} - 147.15 = 0 \ldots\ldots (2)$

From Eq. (1): $\quad F_{CB} = 0.816\, F_{CA}$

Sub. in Eq. (2):

$0.5\,(0.816\, F_{CA}) + 0.707\, F_{CA} - 147.15 = 0$

$1.115\, F_{CA} = 147.15 \quad \Rightarrow \quad F_{CA} = 132 \text{ N} \qquad F_{CB} = 0.816\,(132) = 107.7 \text{ N}$

Example (4-6):

Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of (250 N) in either cable (AB) or (AC).

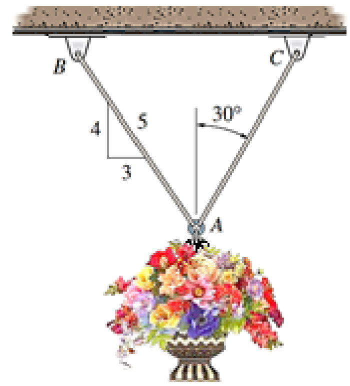


Fig. (Ex. 4-6)

Solution:

$+ \rightarrow \sum F_x = 0$

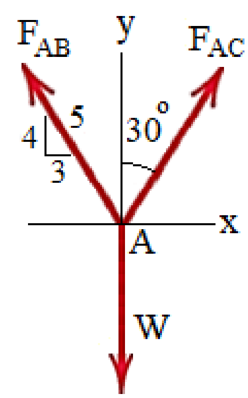
$$F_{AC} \sin 30° - F_{AB} \left(\frac{3}{5} \right) = 0$$

$$F_{AC} = 1.2\, F_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$+ \uparrow \sum F_y = 0$

$$F_{AC} \cos 30° + F_{AB} \left(\frac{4}{5} \right) - W = 0$$

$$0.866\, F_{AC} + 0.8\, F_{AB} = W \quad \ldots\ldots\ldots\ldots \quad (2)$$



Since ($F_{AC} > F_{AB}$) failure will occur first at cable (AC) with (F_{AC} = 250 N). Then solving Eqs. (1) and (2) yields:

$$250 = 1.2\, F_{AB} \quad \Rightarrow \quad F_{AB} = 208.33$$

$$(0.866)(250) + (0.8)(208.33) = W$$

$$W = 383.17 \text{ N}$$

Example (4-7):

Find the main tensile force (F) and the tensile force in each of the two cables (AB) and (AC) necessary to support the container which has a mass of (200 kg) and it's center of gravity located at point (G).

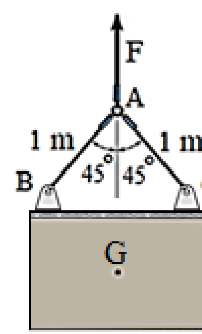


Fig. (Ex. 4-7)

Solution:

$$F = W = m\, g = 200 \times 9.81 = 1962 \text{ N}$$

$+ \rightarrow \sum F_x = 0$

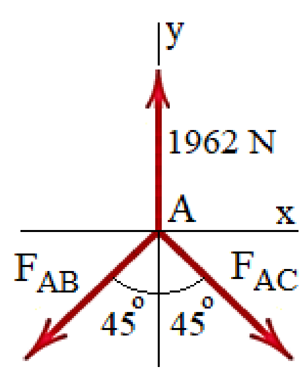
$$F_{AC} \sin 45° - F_{AB} \sin 45° = 0$$

$$F_{AC} = F_{AB}$$

$+ \uparrow \sum F_y = 0$

$$1962 - 2\, F_{AB} \cos 45° = 0$$

$$F_{AB} = F_{AC} = 1387.34 \text{ N}$$



Example (4-8):

If the mass of each of cylinders (D) and (F) is (2 kg) and the mass of cylinder (E) is (3 kg). Determine the distance (d) for equilibrium. Neglect the size of the pulleys.

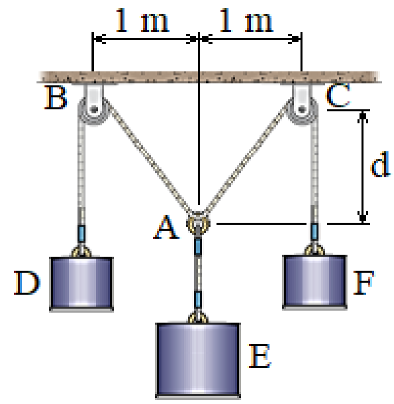


Fig. (Ex. 4-8)

Solution:

$$W_D = 2 \times 9.81 = 19.62 \text{ N}$$
$$W_E = 3 \times 9.81 = 29.43 \text{ N}$$
$$W_F = 2 \times 9.81 = 19.62 \text{ N}$$

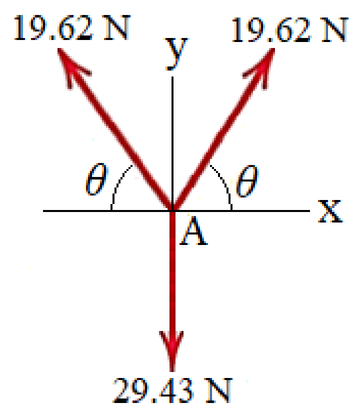
$+\uparrow \sum F_y = 0$

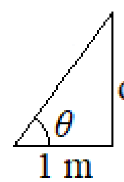
$$2\,(19.62) \sin\theta - 29.43 = 0$$

$$\theta = \sin^{-1}(\,0.75\,) = 48.6°$$

$$\tan\theta = \frac{d}{1}$$

$d = \tan 48.6°$ ٩





Example (4-9):

If you know that the maximum tension that both ropes (AB) and (AC) can withstand is (750 Ib), and that the mass of the drum is (25 slugs), find the smallest angle (*θ*) at which the drum can be lifted within the limits that the ropes withstand.

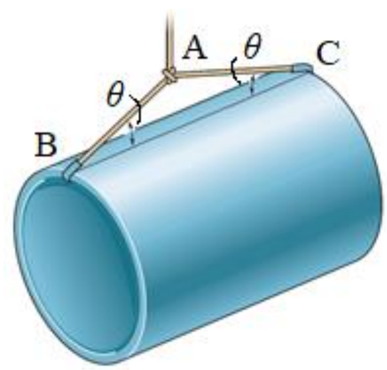


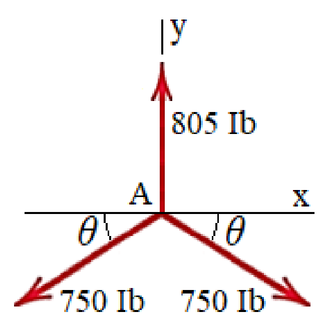
Fig. (Ex. 4-9)

Solution:

$$W = m\,g = 25 \times 32.2 = 805 \text{ Ib}$$

$+\uparrow \sum F_y = 0$ $\qquad 805 - 2(750)\sin\theta = 0$

$$\theta = 32.5°$$



Example (4-10):

Find the tension force in each cord in the system of cords shown in Fig. (Ex. 4-10) so that equilibrium is achieved with the load (25 kg).

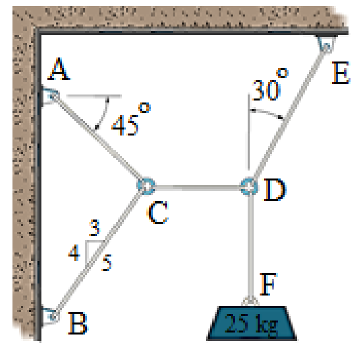
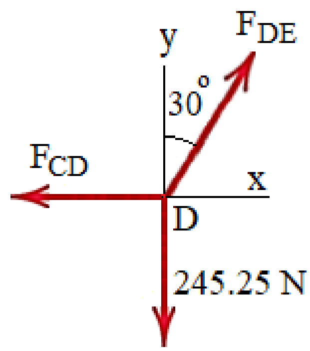


Fig. (Ex. 4-10)

Solution:

$$W = m\,g = 25 \times 9.81 = 245.25 \text{ N}$$

Equilibrium at point (D):



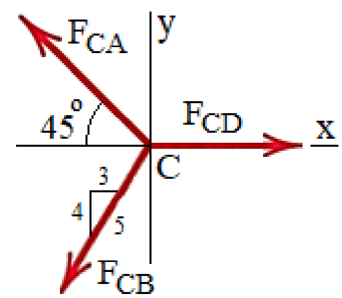
$+\uparrow \sum F_y = 0$ $\quad\quad$ $F_{DE} \cos 30° - 245.25 = 0$

$F_{DE} = 283.2$ N

$+\rightarrow \sum F_x = 0$ $\quad\quad$ $283.2 \sin 30° - F_{CD} = 0$

$F_{CD} = 141.6$ N

Equilibrium at point (C):



$+\rightarrow \sum F_x = 0$

$$141.6 - F_{CA} \cos 45° - F_{CB} \left(\frac{3}{5}\right) = 0$$

$$141.6 - 0.707\, F_{CA} - 0.6\, F_{CB} = 0$$

$$0.707\, F_{CA} + 0.6\, F_{CB} = 141.6 \quad \ldots\ldots\ldots \quad (1)$$

$+\uparrow \sum F_y = 0$ $\quad\quad$ $F_{CA} \sin 45° - F_{CB} \left(\frac{4}{5}\right) = 0$

$$0.707\, F_{CA} - 0.8\, F_{CB} = 0 \quad \ldots\ldots \quad (2)$$

From Eq. (2): $\quad F_{CA} = 1.13\, F_{CB}$

Sub. in Eq. (1): $\quad 0.707\,(1.13\, F_{CB}) + 0.6\, F_{CB} = 141.6$

$1.4\, F_{CB} = 141.6$

$F_{CB} = 101.15$ N

$F_{CA} = 1.13\,(101.15) = 114.3$ N

Or:

$$0.707\, F_{CA} + 0.6\, F_{CB} = 141.6 \quad \ldots\ldots\ldots \quad (1)$$

$$0.707\, F_{CA} - 0.8\, F_{CB} = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

---------------------------------- Subtraction

$1.4\, F_{CB} = 141.6 \quad \Rightarrow \quad F_{CB} = 101.15$ N

Sub. in Eq. (1): $\quad 0.707\, F_{CA} + 0.6\,(101.15) = 141.6$

$0.707\, F_{CA} + 60.69 = 141.6$

$0.707\, F_{CA} = 141.6 - 60.69 = 80.91$

$F_{CA} = 114.3$ N

Problems:

4-1) Find the tensile force in each wire of the wire system shown in Fig. (Pr. 4-1).

Ans.: $F_{ED} = 60.6$ Ib , $F_{EB} = 87.27$ Ib
$F_{BC} = 120.9$ Ib, $F_{BA} = 139.6$ Ib

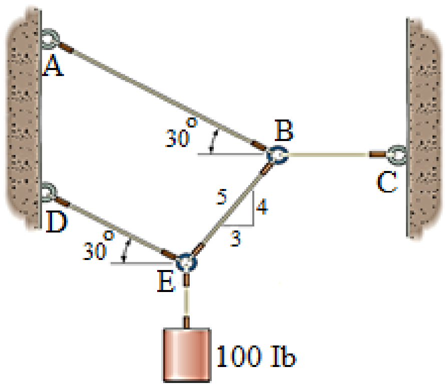


Fig. (Pr. 4-1)

4-2) Find the tensile force in each wire of the wire system shown in Fig. (Pr. 4-2).

Ans.: $F_{ED} = F_{EC} = 57.74$ Ib
$F_{DB} = F_{CA} = 70.7$ Ib, $F_{CD} = 21.13$ Ib

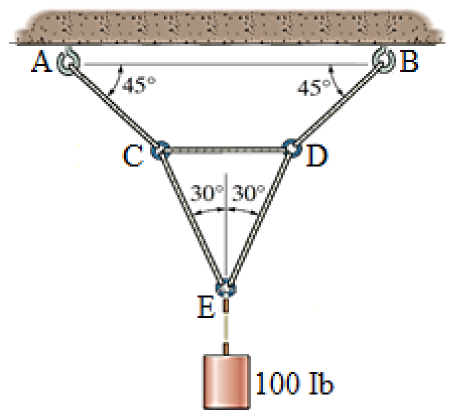


Fig. (Pr. 4-2)

4-3) The length of the wire (ABC) is (5 m). Find the distance (x) and the tension force applied in the wire (ABC) required for equilibrium with the mass of the cylinder of (100 kg). Neglect the size of the pully at (B).

Ans.: $x = 1.38$ m, $T = 686.87$ N

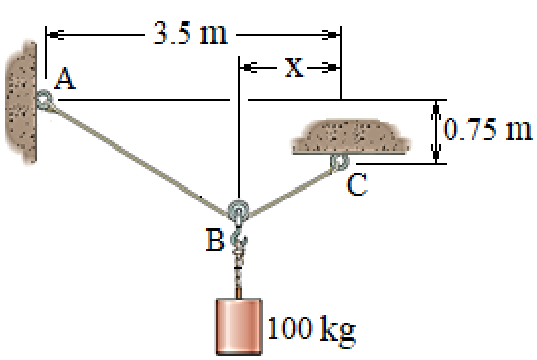


Fig. (Pr. 4-3)

4-4) The total length of the rope (ABCD) shown in Fig. (Pr. 4-4) is (6 m), determine the magnitude of the angle (θ) and the force (F) for equilibrium with the (10 kg) mass.

Ans.: $\theta = 36.87°$, $F = 61.3$ N

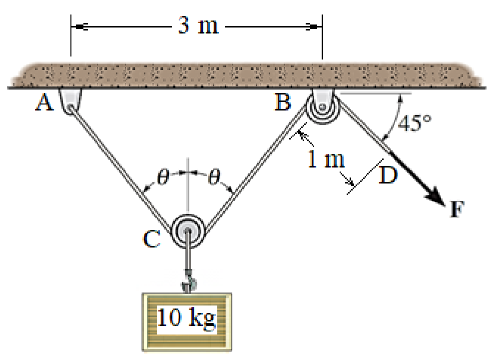


Fig. (Pr. 4-4)

4-5) A container of (100 kg) is lifted by the sling (BAC) with constant velocity. Determine the force in the sling (F), and find the value of the tension (T) of each of the robs (AB) and (AC) as a function of its orientation (θ),where ($0° \le \theta \le 90°$).

Ans.: $F = 981$ N, $T = \dfrac{981}{2 \sin \theta}$

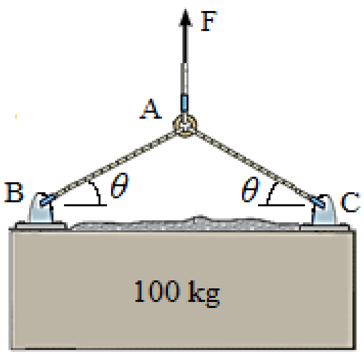


Fig. (Pr. 4-5)

4-6) The box (D) has a mass of (15 kg). If a force of (F = 80 N) is applied horizontally to the ring at (C), determine the largest dimension (d) so that the force in cable (CB) is zero.

Ans.: $d = 2.17$ m

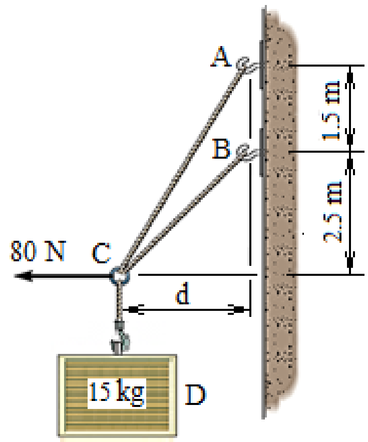


Fig. (Pr. 4-6)

4-7) Calculate the values of the tensile force (T) and the compressive force (P) in the bracket shown in Fig. (Pr. 4-7) to achieve equilibrium.

Ans.: $P = 714.3$ N, $T = 606.2$ N

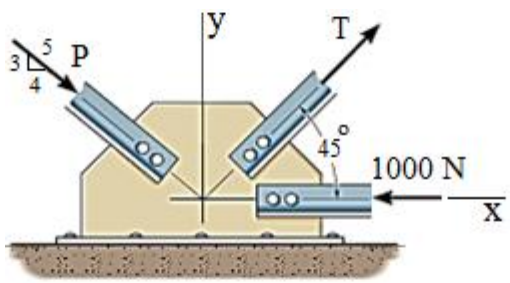


Fig. (Pr. 4-7)

4-8) An engine suspended by the system of chains shown in Fig. (Pr. 4-8). Determine the maximum weight of the engine that can be suspended without exceeding the tensile force of (500 Ib) in both chains (AB) and (AC).

Ans.: $W = 500$ Ib

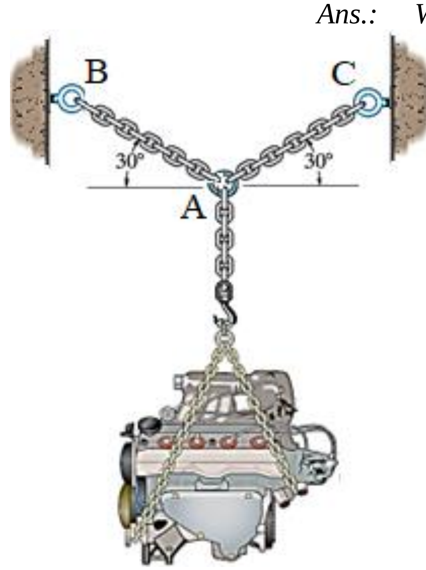


Fig. (Pr. 4-8)

4-9) In the cables arrangement shown in Fig. (Pr. 4-9), determine the tension forces in cables (AC) and (BC) caused by the weight the (25 kg) box.

Ans.: $T_{AC} = 179.6\ N$, $T_{BC} = 219.9\ N$

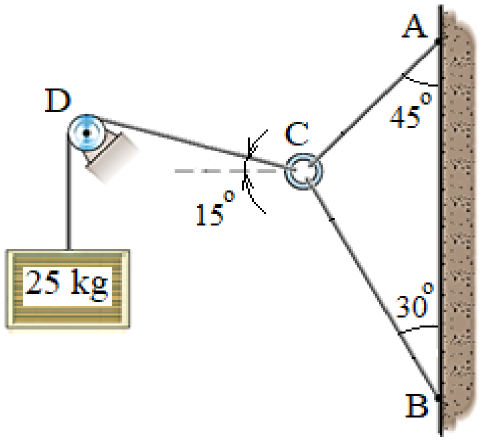


Fig. (Pr. 4-9)

PART (2): EQUILIBRIUM OF THE RIGID BODIES:

In this part of the equilibrium, the dimensions of the body are taken into account and have an effect in calculating the forces and moments applied to it, and the body is in a state of equilibrium when the sum of all forces and moments or couples applied to it is equal to zero. So the equilibrium equations will be:

$$F_R = \sum F = 0 \quad \ldots\ldots\ldots\ldots\ldots.. \quad (4\text{-}2)$$

$$M_R = \sum M = 0 \quad \ldots\ldots\ldots\ldots.. \quad (4\text{-}3)$$

System isolation and the free-body diagram (FBD):

A mechanical system can be virtually isolated from its surroundings by a so-called free-body diagram (FBD). A mechanical system may be a single body or a combination of bodies connected in a manner appropriate to the desired purpose of the system. The bodies may be solid or non-solid. The system may also be an identifiable fluid mass, either a liquid or a gas, or a mixture of liquids and solids.

In the statics branch of engineering mechanics, we mainly study the forces acting on solid bodies at rest.

To draw a free body diagram, you must follow these steps:

1- We assume that the rigid body is isolated from its surroundings and then draw its isolated shape (free body diagram).
2- Placing known and unknown forces on the free body diagram.
3- Replace the supports with reaction forces.
4- Draw the required dimensions and angles.
5- Applying the equations of equilibrium to find the unknown forces.

Modeling the action of forces:

The following figures shows the common types of supports used in mechanical systems and the corresponding forces of reactions on the free-body diagram (FBD) for analysis in a two-dimensional plane. Where each of the following examples shows the forces acting on the body as an isolated body from its surroundings. Newton's third law must be taken into account when replacing the supports on which the bodies rest with reaction forces, which states that for every action force there is a reaction force equal in magnitude and opposite in direction. The reaction forces that is applied to a specific body as a result of its contact with a support or with another body is always opposite to the direction of motion of the isolated body that would occur if the support or the contacting body were removed.

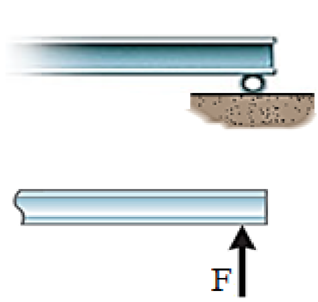


Fig. (4-2) Roller support

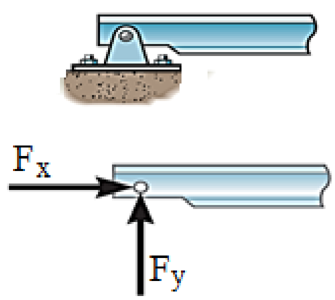


Fig. (4-3) Pin support

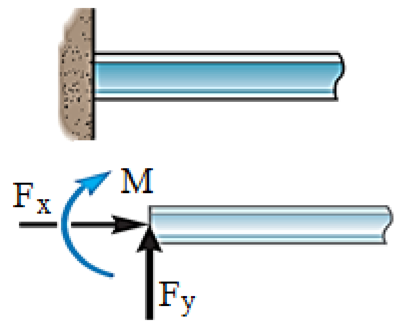
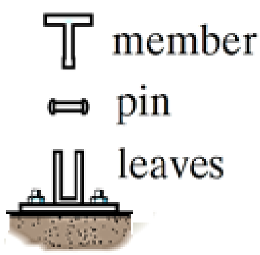


Fig. (4-4) Fixed support



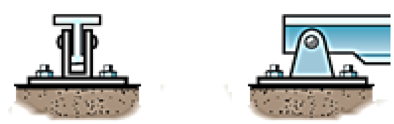


Fig. (4-5) Pin connection assembly

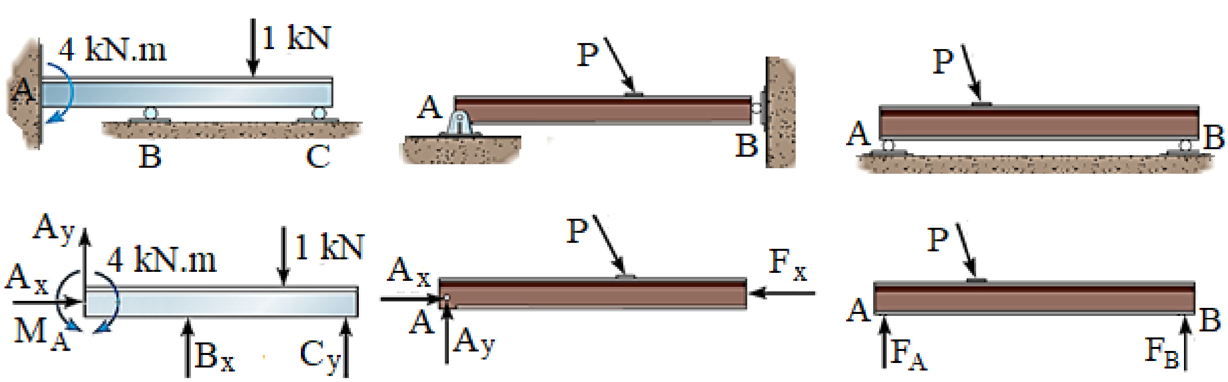


Fig. (4-6) Types of supports

Modeling the action of forces in two-dimensional analysis:

Table (4-1) Modeling the action of forces in two-dimensional analysis:

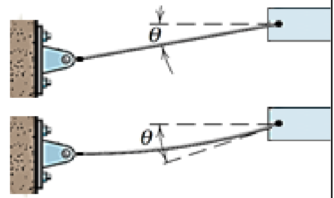
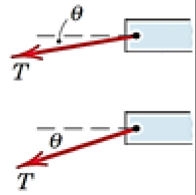
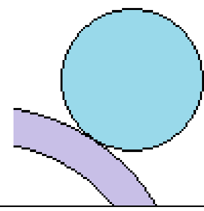
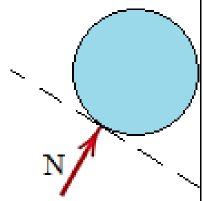
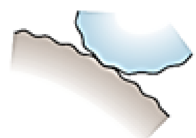
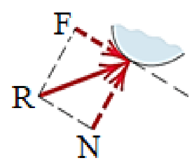
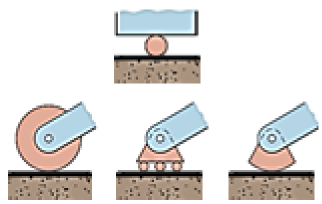
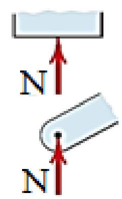
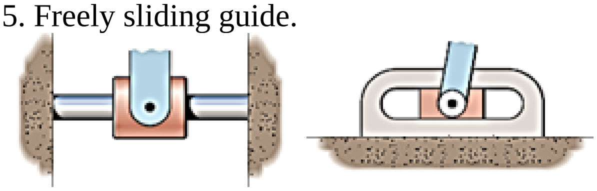
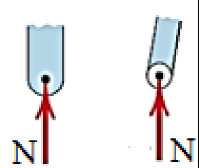
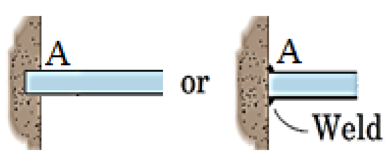
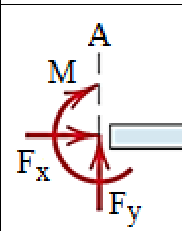
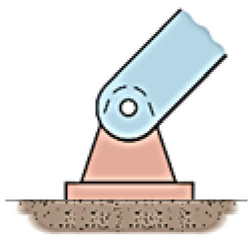
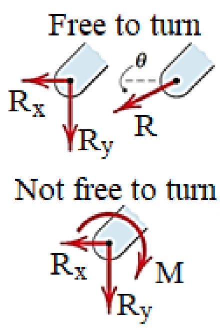
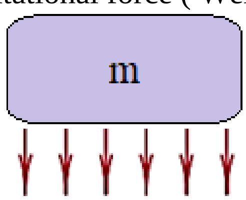
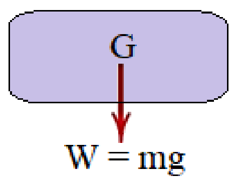
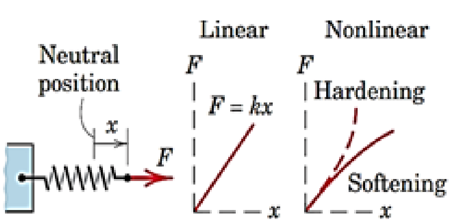
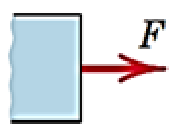
Type of contact and force origin		**Representation of forces on a body as an isolated body**	
1. Flexible cable, belt, chain, or rope. Weight of cable negligible. Weight of cable considerable.			The force that a flexible cable, belt, chain, or rope exerts on a body is represented by a tensile force starting from the body in the direction of the cable or the direction of the tangent to the cable.
2. Smooth surfaces.			Reaction force as a result of contact represented by a compressive force and is normal to the surface.
3. Rough surfaces.			Reaction force as a result of contact is inclined force (R), and it is resolved into two components, the tangential component (F) (friction force) and the normal component (N).
4. Roller supports.			Roller, rocker, or ball support represented by a compressive force normal to the supporting surface.
5. Freely sliding guide. 			Collar or slider free to move along smooth guides, can be represented by a support force normal to the guide.
6. Fixed support. 			The fixed support represented by an axial force (F_x), and a lateral force (F_y) (shear force), and a moment (M) about the fixation point to prevent the rotation.

Table (4-1) Modeling the action of forces in two-dimensional analysis:

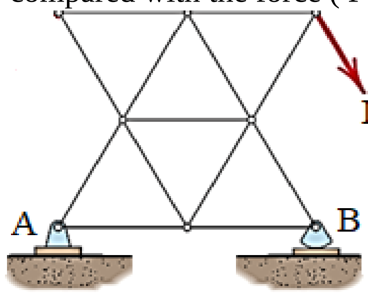
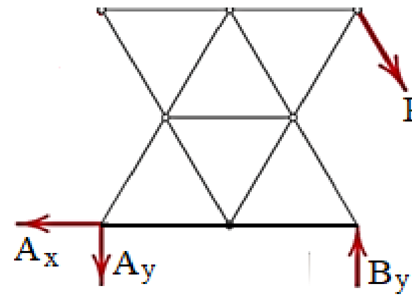
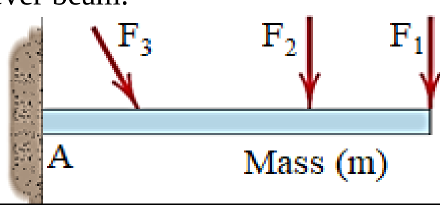
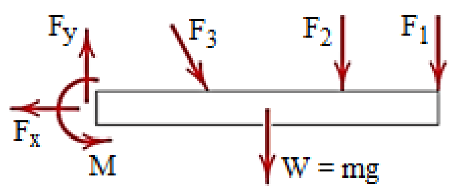
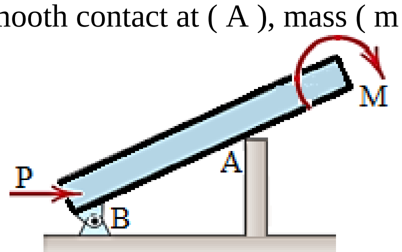
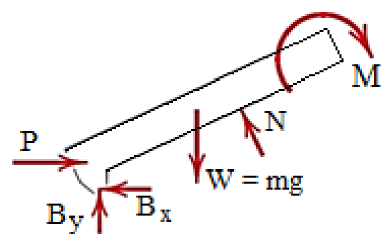
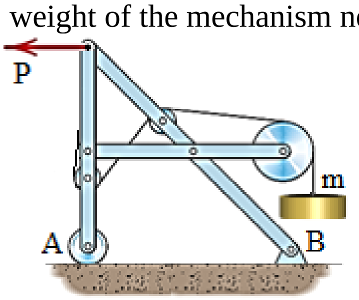
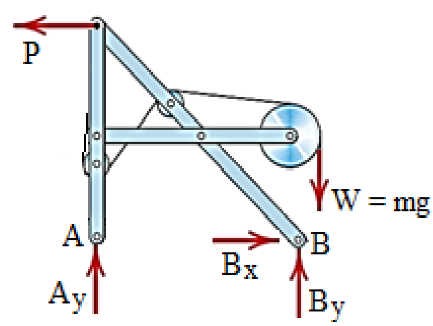
Type of contact and force origin	**Representation of forces on a body as an isolated body**	
7. Pin connection. 		In a free-rotational joint, the reaction force can be represented as either a horizontal (R_x) and a vertical (R_y) component, or a force (R) with its direction (θ). As for the hinge with restricted rotational motion, to the above reactions, a torque (M) about the fixation point is added.
8. Gravitational force (Weight). 		The resultant gravitational forces resulting from the elements of a body of mass (m) is the weight (W = mg). The weight is represented by a force directed towards the center of the earth, starting from the center of gravity of the body (G).
9. Spring force. 		The force of the spring results from multiplying the elongation by the stiffness of the spring, and it is a tensile force if the spring is elongated and a compressive force if it contracts. The spring's stiffness (k) is the force required to deform the spring by an elongation per unit distance.

Construction of free – body diagrams:

Examples of free - body diagrams:

Table (4-2) shows four examples of mechanisms and structures with their free-body diagrams. Dimensions and magnitudes omitted as general examples. In each case the entire mechanism or structure is treated as a single body, so that internal forces do not appear. The four examples shown in the table show the known and unknown forces and the reactions of the various types of supports.

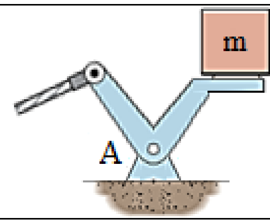
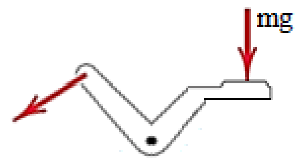
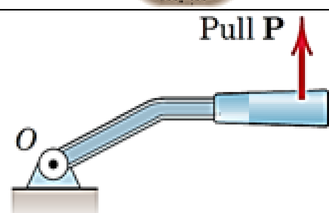
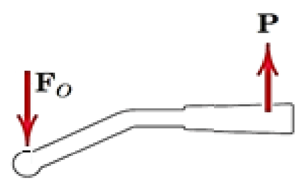
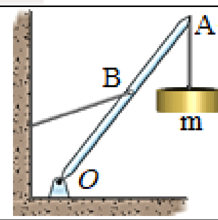
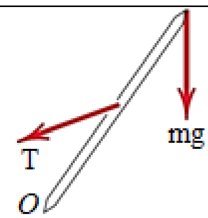
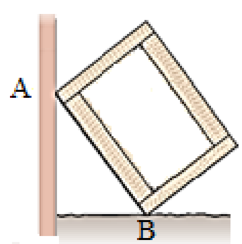
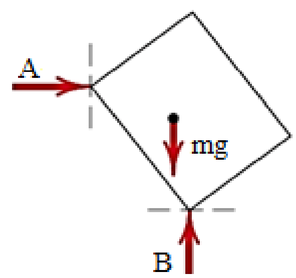
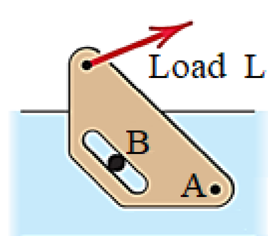
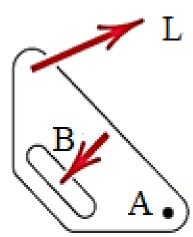
Table (4-2) Sample of free-body diagrams

Mechanical System	**Free-Body Diagram of Isolated structure**
1- Plane truss: Weight of truss assumed to be negligible compared with the force (P).  	
2- Cantilever beam.  	
3- Beam: smooth contact at (A), mass (m).  	
4- System of rigid interconnected bodies analyzed as a single unit, weight of the mechanism negligible  	

Exercises of free - body diagrams:

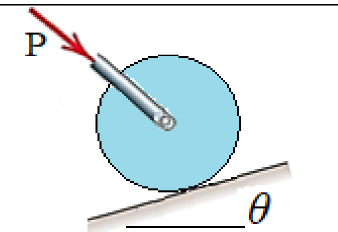
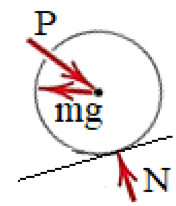
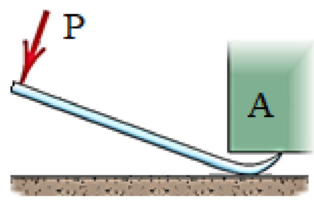
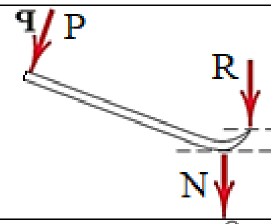
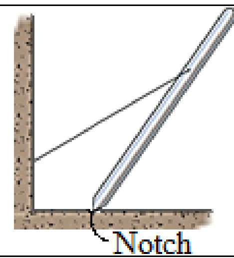
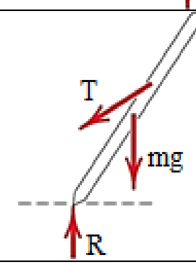
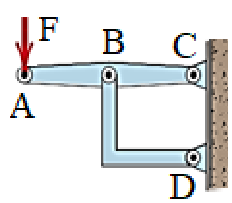
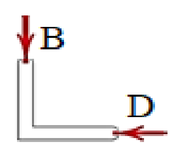
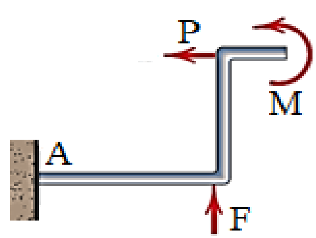
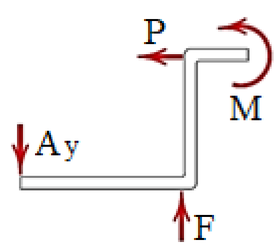
In Table (4-3), the middle column represents specific structures, the left column represents brief details of these structures, and the right column represents an incomplete free body diagrams (FBD) of the isolated body. It is required to complete the free body diagrams of the structures in the right column.
Body weights are negligible unless otherwise noted.
Dimensions and numerical values have been omitted for simplicity.

Table (4-3) Free-body diagram exercises

	Body	**Incomplete (FBD)**
a- Bell crank supporting mass (m) with pin support at (A).		
b- Control lever applying torque to the shaft at (*O*).		
c- The boom (OA), of negligible mass compared with the mass (m). The boom hinged at (*O*) and taut by cable at (B).		
d- Uniform crate of mass (m) leaning against smooth vertical wall and supported on a rough horizontal surface.		
e- Loaded bracket supported by pin connection at (A) and fixed pin in smooth slot at (B).		

In Table (4-4), the middle column represents specific structures, the left column represents brief details of these structures, and the right column represents the free body diagrams (FBD) of the isolated body, which is incorrect or incomplete. It is required to correct or complete the free body diagrams of the structures in the right column.
Body weights are negligible unless otherwise noted.
Dimensions and numerical values have been omitted for simplicity.

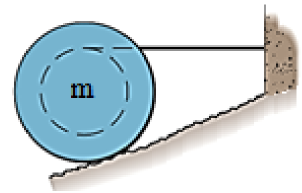
Table (4-4) Free-body diagram exercises

	Body	**Wrong or Incomplete (FBD)**
a- A cylinder of mass (m) is pushed up a slope inclined at an angle (θ).		
b-Pry-bar lifting body (A) which have a smooth horizontal surface. The bar rests on horizontal rough surface.		
c- Uniform pole of mass (m) being hoisted into position by cable. Horizontal supporting surface notched to prevent slipping of pole.		
d- The member (BD) in the form of a right angle is hinged with the horizontal member at the point (B).		
e- Bent rod welded to a wall at (A) and subjected to two forces and couple.		

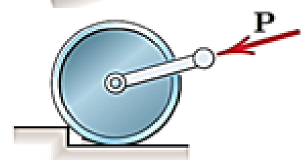
Draw a free-body diagram for each of the structures shown below showing all the known and unknown forces, knowing that the weights of the bodies are not required unless the mass is specified.

a- Uniform horizontal bar of mass (m) suspended by vertical cable at (A) and supported on a rough inclined surface at (B).

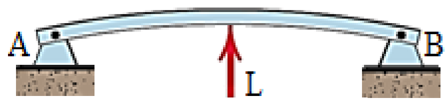
b- A notched disk of uniform mass, of mass (m), is drawn with a horizontal wire and rests on a rough surface.



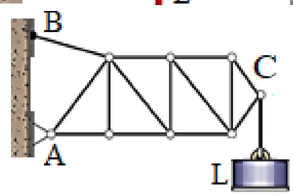
c- A disk of mass (m) is about to tip over on to the pavement due to an inclined pushing force (P).



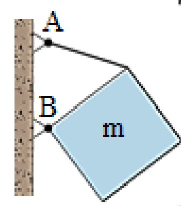
d- A horizontal bar bent due to load (L). Fixed at both ends with articulated anchors.



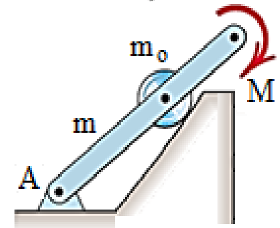
e- A truss hinged at point (A) and taut with a cable at point (B) and curing a weight (L) at point (C).



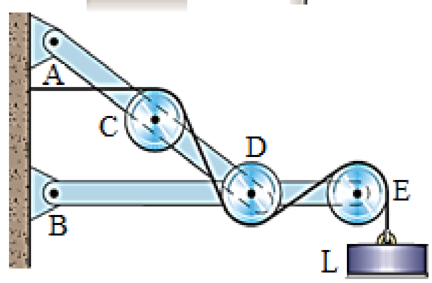
f- A plate of uniform mass, of mass (m), hinged at point (B) and taut by a wire at point (A).



g- A structure consisting of a uniform rod of mass (m) and a pulley of mass (m_o). A torque (M) is applied to it and hinged at point (A).



h- A structure consisting of rods and pulleys jointly linked with each other, and a connecting wire that carries the mass (L).



Example (4-11):

The smooth disk (A) with a mass of (50 kg) and the smooth disk (B) with a mass of (100 kg), a horizontal force of (F = 1000 N) was applied to the center of the disk (A). Find the normal reactions to the support surfaces at points (C), (D), and (E).

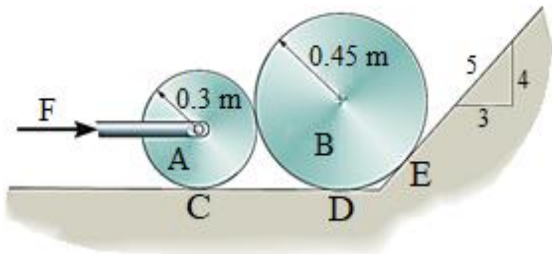
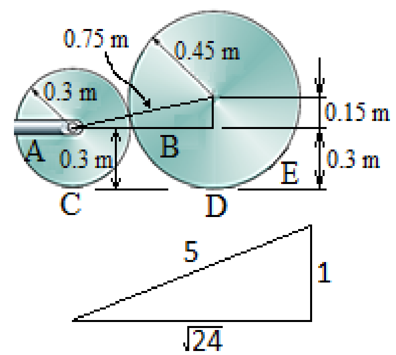


Fig. (Ex. 4-11)

Solution:

Disk (A):

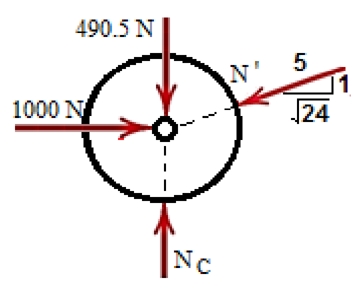


$W = 50 \times 9.81 = 490.5$ N

$+ \rightarrow \sum F_x = 0$

$$1000 - N' \left(\frac{\sqrt{24}}{5}\right) = 0$$

$N' = 1020.6$ N



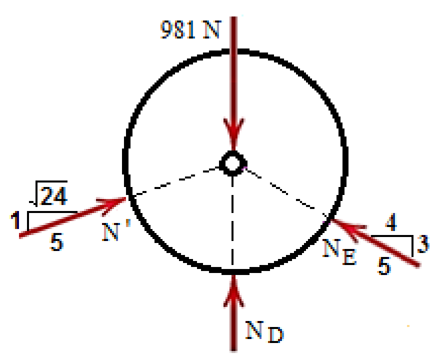
$+ \uparrow \sum F_y = 0$

$$N_C - 490.5 - 1020.6 \left(\frac{1}{5}\right) = 0$$

$N_C = 694.6$ N

Disk (B):

$W = 100 \times 9.81 = 981$ N



$+ \rightarrow \sum F_x = 0$

$$N_E \left(\frac{4}{5}\right) - 1020.6 \left(\frac{\sqrt{24}}{5}\right) = 0$$

$N_E = 1250$ N

$+ \uparrow \sum F_y = 0$

$$1250 \left(\frac{3}{5}\right) + N_D - 981 + 1020.6 \left(\frac{1}{5}\right) = 0$$

$N_D = 26.88$ N

<u>Example (4-12):</u>

For the jib crane shown in Fig. (Ex. 4-12), determine the magnitude of the tension (T) in the supporting cable and the magnitude of the reaction force on the pin at (A). The beam (AB) is a standard (0.4-m / I-beam) with a mass of (80 kg) per meter of length.

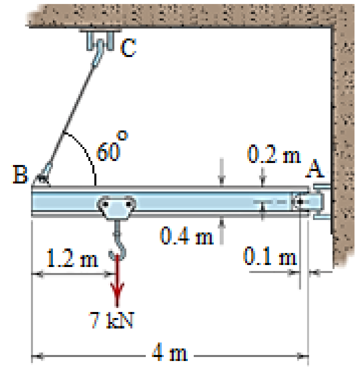
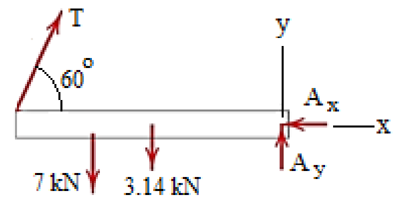


Fig. (Ex. 4-12)

Solution:

Algebraic solution:

$W = m\,g = (80 \times 4)(9.81) = 3139.2\ N = 3.14\ kN$



$\sum M_A = 0,$

$$-T\cos 60^\circ \times 0.2 - T\sin 60^\circ \times (4 - 0.1) + 7\,(4 - 1.2 - 0.1) + 3.14\,(2 - 0.1) = 0$$

$$-0.1\,T - 3.38\,T + 18.9 + 5.97 = 0$$

$$3.48\,T = 24.87$$

$$T = 7.15\ kN$$

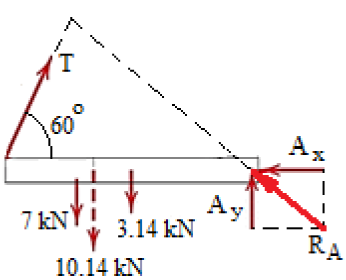
$\sum F_x = 0$ $\qquad 7.15\cos 60^\circ - A_x = 0 \qquad A_x = 3.6\ kN$

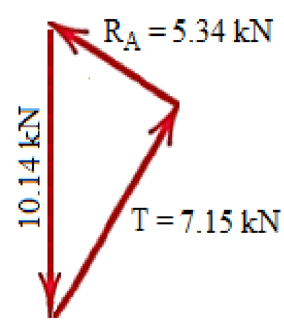
$\sum F_y = 0$ $\qquad A_y + 7.15\sin 60^\circ - 3.14 - 7 = 0$

$A_y = 3.95\ kN$

$$R_A = \sqrt{Ax^2 + Ay^2} = \sqrt{(3.6)^2 + (3.95)^2} = 5.34\ kN$$

Graphical solution:





Example (4-13):

The system shown in Fig. (Ex. 4-13) carry a cylinder of mass (10 kg). If the mass of the uniform shaft is (6 kg), and the pulley at point (D) is frictionless, determine the tension in the cable and the components of reactions at the fixed point (A).

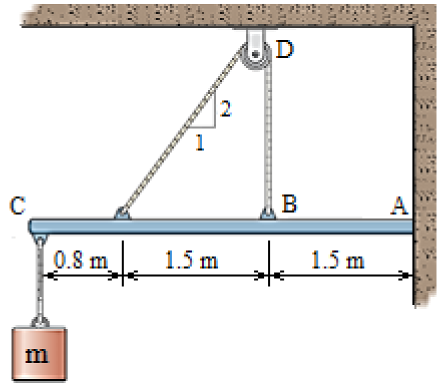
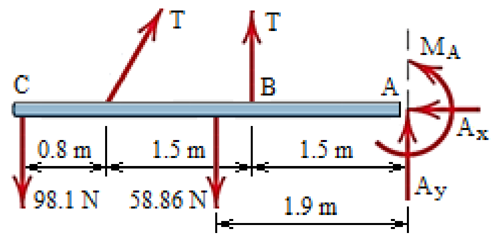


Fig. (Ex. 4-13)

Solution:

$W_{cy} = 10 \times 9.81 = 98.1 \text{ N}$
$W_{sh} = 6 \times 9.81 = 58.86 \text{ N}$



$\circlearrowleft + \sum M_A = 0$

$$-(T)(1.5) - (T)\left(\frac{2}{\sqrt{5}}\right)(3) + (98.1)(3.8) + (58.86)(1.9) = 0$$

$4.18\, T = 484.61$
$T = 115.94 \text{ N}$

$+\rightarrow \sum F_x = 0 \qquad (115.94)\left(\frac{1}{\sqrt{5}}\right) - A_x = 0$

$A_x = 51.85 \text{ N}$

$+\uparrow \sum F_y = 0 \qquad A_y + 115.94 + (115.94)\left(\frac{2}{\sqrt{5}}\right) - 98.1 - 58.86 = 0$

$A_y = -62.68 = 62.68 \text{ N} \downarrow$

$\circlearrowleft + \sum M_A = 0$

$$M_A - (115.94)(1.5) - (115.94)\left(\frac{2}{\sqrt{5}}\right)(3) + (98.1)(3.8) + (58.86)(1.9) = 0$$

$M_A - 173.91 - 311.1 + 372.78 + 111.83 = 0$
$M_A = 0$

Example (4-14):

On the lever shown in Fig. (Ex. 4-14), determine the horizontal and vertical components of reaction force at the pin at point (A) and the reaction force of the roller at point (B).

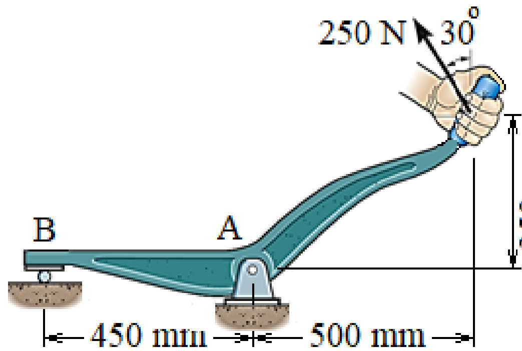
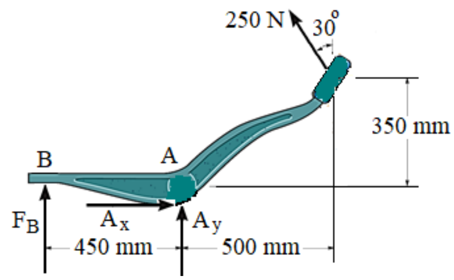


Fig. (Ex. 4-14)

Solution:



$\circlearrowleft + \sum M_A = 0$ (250 cos 30°) (0.5) + (250 sin 30°) (0.35) – (F_B) (0.45) = 0

152 = 0.45 F_B

F_B = 337.78 N

$+ \rightarrow \sum F_x = 0$ A_x – 250 sin 30° = 0

A_x = 125 N

$+ \uparrow \sum F_y = 0$ A_y + 250 cos 30° + 337.78 = 0

A_y = – 554.29 = 554.29 N ↓

Example (4-15):

On the rod shown in Fig. (Ex. 4-15), determine the reactions at the supports (A) and (B).

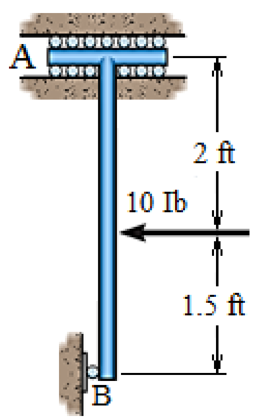
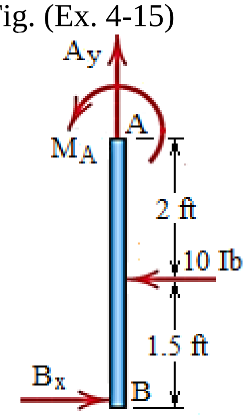


Fig. (Ex. 4-15)

Solution:



$+ \rightarrow \sum F_x = 0$ B_x – 10 = 0

B_x = 10 Ib

$+ \rightarrow \sum F_y = 0$ A_y = 0

$\circlearrowleft + \sum M_A = 0$ M_A + 10 (3.5) – 10 (2) = 0

M_A = – 15 = 15 Ib.ft (C.W)

Example (4-16):

Calculate the reactions of the Earth to the wheels of the bulldozer shown in Fig. (Ex. 4-16) at points (A) and (B). Consider the problem as a two-dimensional problem.

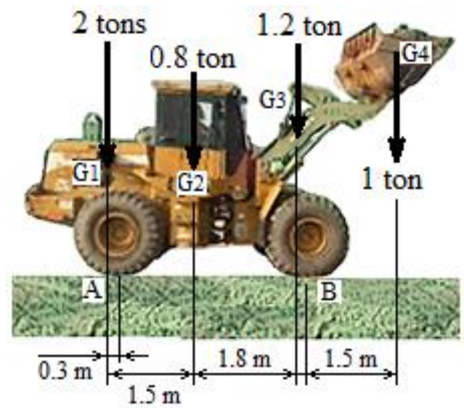
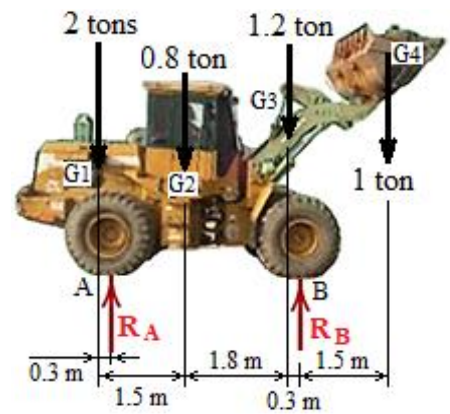


Fig. (Ex. 4-16)



Solution:

$\circlearrowleft + \sum M_B = 0$

$$(2000 \times 9.81 \times 3.6) + (800 \times 9.81 \times 2.1) + (1200 \times 9.81 \times 0.3) - (1000 \times 9.81 \times 1.5) - (R_A \times 3.3) = 0$$

$$70632 + 16480.8 + 3531.6 - 14715 = 3.3\, R_A$$

$$R_A = 23009 \text{ N} = 23 \text{ kN}$$

$+\uparrow \sum F_y = 0$

$$R_B + 23009 - (2000 \times 9.81) - (800 \times 9.81) - (1200 \times 9.81) - (1000 \times 9.81) = 0$$

$$R_B = 26041 \text{ N} = 26 \text{ kN}$$

Example (4-17):

On the cantilevered structure shown in Fig. (Ex. 4-17), determine the components of the support reactions at the fixed support (A).

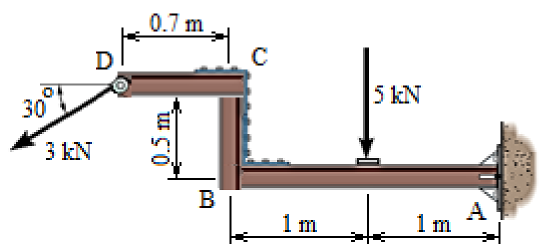
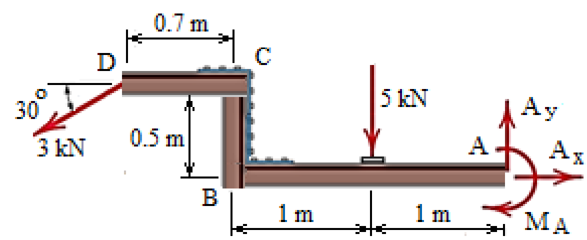


Fig. (Ex. 4-17)



Solution:

$+\rightarrow \sum F_x = 0$

$$A_x - 3 \cos 30^\circ = 0$$

$$A_x = 2.6 \text{ kN}$$

$+\uparrow \sum F_y = 0$

$$A_y - 5 - 3 \sin 30^\circ = 0$$

$$A_y = 6.5 \text{ kN}$$

$\circlearrowleft + \sum M_A = 0$

$$(5)(1) + (3 \cos 30^\circ)(0.5) + (3 \sin 30^\circ)(2.7) - M_A = 0 \quad \Rightarrow \quad M_A = 10.35 \text{ kN.m}$$

Example (4-18):

Calculate the tensile force (T) in the cable that carries a mass of (300 kg) with the pulley system shown in Figure (Ex. 4-18), and find the magnitude of the total force acting on the support of the pulley (C).

All part weights are negligible compared to the load, and all pulleys rotate freely on their axis.

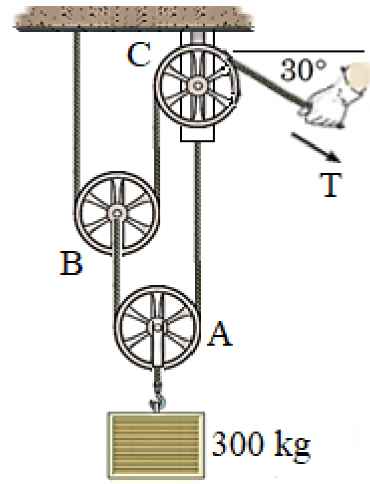


Fig. (Ex. 4-18)

Solution:

$$W = m\,g = 300 \times 9.81 = 2943 \text{ N}$$

pulley A:

$\sum M_o = 0$ $\quad T_2\, r - T_1\, r = 0$ $\quad T_1 = T_2$

$\sum F_y = 0$ $\quad T_1 + T_2 - 2943 = 0$

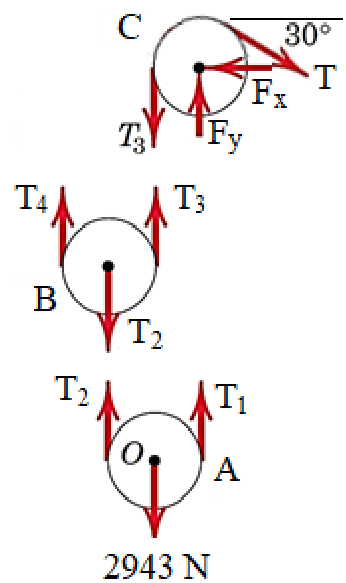
$T_1 = T_2 = 1471.5 \text{ N}$

pulley B:

$$T_3 = T_4 = T_2 / 2 = 735.75 \text{ N}$$

pulley C:

$T = T_3$ $\quad$ or $\quad$ $T = 735.75 \text{ N}$



Equilibrium of the pulley in the x-and y-directions requires:

$\sum F_x = 0$ $\quad 735.75 \cos 30^o - F_x = 0$

$F_x = 637.2 \text{ N}$

$\sum F_y = 0$ $\quad F_y - 735.75 \sin 30^o - 735.75 = 0$

$F_y = 1103.6 \text{ N}$

$$F = \sqrt{Fx^2 + Fy^2} = \sqrt{(637.2)^2 + (1103.6)^2} = 1274.4 \text{ N}$$

Example (4-19):

Calculate the magnitude of the force supported by the pin at point (A) under the action of the (1.5 kN) load applied to the bracket. Neglect friction in the slot.

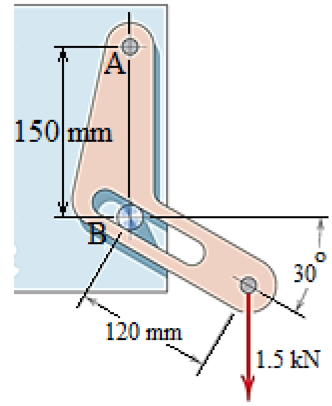


Fig. (Ex. 4-19)

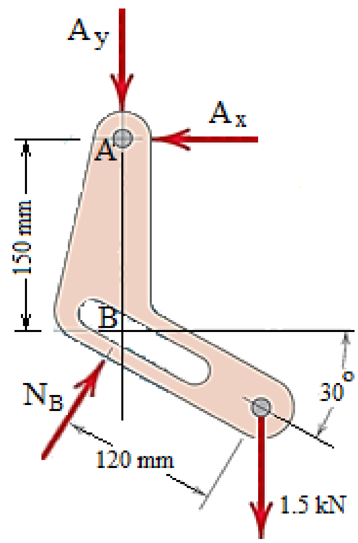
Solution:

$\circlearrowleft + \sum M_A = 0$

$(N_B \sin 30°) (0.15)$
$\quad - (1.5) (0.12 \cos 30°) = 0$

$N_B = 2.1 \text{ kN}$

$+ \rightarrow \sum F_x = 0 \qquad 2.1 \sin 30° - A_x = 0$
$\qquad A_x = 1.05 \text{ kN}$



$+ \uparrow \sum F_y = 0$

$2.1 \cos 30° - A_y - 1.5 = 0$
$A_y = 0.3 \text{ kN}$

$$R_A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(1.05)^2 + (0.3)^2} = 1.09 \text{ kN}$$

Example (4-20):

In Fig. (Ex. 4-20), a gerl is training on the rowing machine. If she exerts a pulling force of (F = 300 N) on the handle of the machine (ABC), determine the force exerted by the hydraulic cylinder (BD) on the handle, and the horizontal and vertical components of the reaction force at the joint (C).

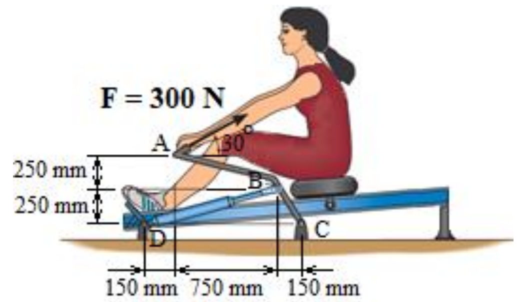
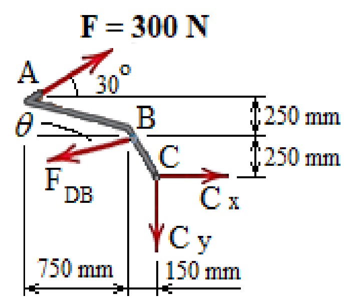


Fig. (Ex. 4-20)

Solution:

$$\theta = \tan^{-1}\frac{0.25}{0.75} = 18.4$$



$\curvearrowright + \sum M_C = 0$

$$-(300\cos 30)(0.5) - (300\sin 30)(0.9) + (F_{DB}\cos 18.4)(0.25) + (F_{DB}\sin 18.4)(0.15) = 0$$

$$-129.9 - 135 + 0.237\,F_{DB} + 0.047\,F_{DB} = 0$$

$$0.284\,F_{DB} = 232.7$$

$$F_{DB} = 819.4\text{ N}$$

$+\rightarrow \sum F_x = 0$

$$C_x + 300\cos 30 - 819.4\cos 18.4 = 0$$

$$C_x = 777.51 - 259.81 = 517.7\text{ N} \rightarrow$$

$+\uparrow \sum F_y = 0$

$$300\sin 30 - 819.4\sin 18.4 - C_y = 0$$

$$C_y = 150 - 258.64 = -108.64 = 108.64\text{ N} \uparrow$$

Example (4-21):

The total mass of the floor crane and its driver is (5 tons) with the center of gravity at point (G). If the crane is to lift a box of (250 kg), determine the reaction forces of the ground to both wheels at (A) and both wheels at (B) when the boom is in the position shown in Fig. (Ex. 4-21).

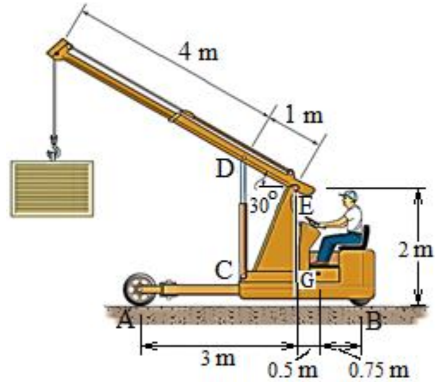
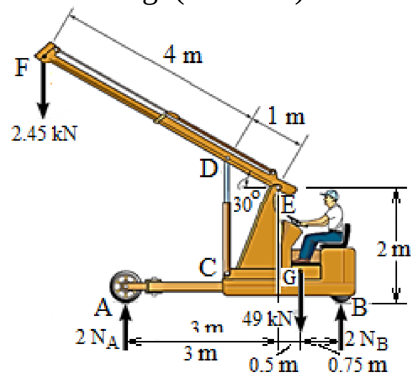


Fig. (Ex. 4-21)



Solution:

$W_{cr} = 5000 \times 9.81 = 49050$ N = 49 kN
$W_b = 250 \times 9.81 = 2452.5$ N = 2.45 kN

$\circlearrowleft + \sum M_B = 0 \qquad (2.45)(5\cos 30 + 1.25) + (49)(0.75) - (2N_A)(4.25) = 0$

$2N_A = 11.86$ kN $\Rightarrow$ $N_A = 5.93$ kN

$+\uparrow \sum F_y = 0 \qquad 11.86 - 49 - 2.45 + 2N_B = 0$

$2N_B = 39.59$ kN $\Rightarrow$ $N_A = 19.79$ kN

Example (4-22):

The mass of the mobile crane is (60 tons) with a center of gravity at (G_1), and the mass of the boom is (15 tons) with a center of gravity at (G_2). Determine the smallest angle of tilt (θ) of the boom, without causing the crane to overturn if the suspended load is (W = 200 kN). Neglect the thickness of the tracks at (A) and (B).

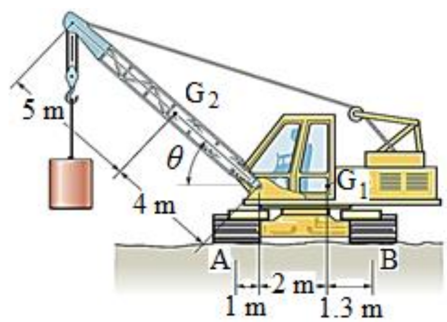
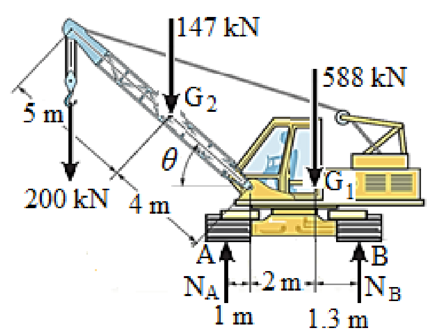


Fig. (Ex. 4-22)



Solution:

$W_c = 60000 \times 9.81 = 588600$ N = 588 kN
$W_b = 15000 \times 9.81 = 147150$ N = 147 kN

$\circlearrowleft + \sum M_A = 0$

$(200)(9\cos\theta - 1) + (147)(4\cos\theta - 1) + (0)(4.3) - (588)(3) = 0$

$1800\cos\theta - 200 + 588\cos\theta - 147 - 1764 = 0$
$2388\cos\theta - 2111 = 0 \quad \Rightarrow \quad 2388\cos\theta = 211$
$\cos\theta = 0.884 \quad \Rightarrow \quad \theta = \cos^{-1}(0.884) = 27.87°$

Example (4-23):

The crane shown in Fig. (Ex. 4-23) consists of three parts, which have masses of (m_1 = 1750 kg), (m_2 = 450 kg), and (m_3 = 750 kg) and centers of gravity at (G_1), (G_2) and (G_3) respectively. Determine:

(a) The reaction of the earth on each of the four wheels if the weight of the suspended load is (4 kN) and it is pulled at a constant speed.

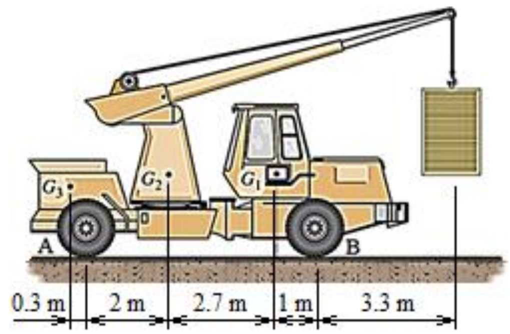


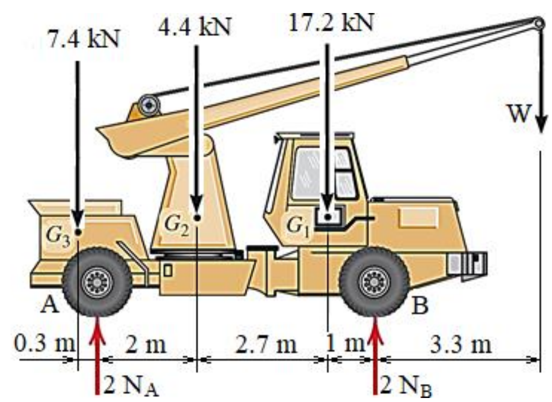
Fig. (Ex. 4-23)

(b) The maximum load the crane can lift without tipping over, When the boom held in the position shown.

Neglect the weight of the boom.

Solution:

$W_1 = 1750 \times 9.81 = 17167.5$ N = 17.2 kN
$W_2 = 450 \times 9.81 = 4414.5$ N = 4.4 kN
$W_3 = 750 \times 9.81 = 7357.5$ N = 7.4 kN



$\circlearrowright + \sum M_A = 0$

$(2N_B)(5.7) - (W)(9) - (17.2)(4.7) - (4.4)(2) + (7.4)(0.3) = 0$

$11.4\, N_B - 9W - 80.84 - 8.8 + 2.22 = 0$

$11.4\, N_B = 9W + 87.42$

$N_B = 0.79W + 7.67$ .... (1)

Using the result ($N_B = 0.79W + 7.67$):

$+\uparrow \sum F_y = 0$

$2N_A + 2N_B - W - 17.2 - 4.4 - 7.4 = 0$

$2N_A + 2\,(0.79W + 7.67) - W - 17.2 - 4.4 - 7.4 = 0$

$2N_A + 1.58W + 15.34 - W - 29 = 0$

$2N_A = - 0.58W + 13.66$

$N_A = - 0.29W + 6.83$ .... (2)

a) By substituting the load (W = 4 kN) in equations (1) and (2):

$N_A = - 0.29(4) + 6.83 = 5.67$ kN
$N_B = 0.79(4) + 7.67 = 10.83$ kN

b) At the overturning moment of the lever, ($N_A = 0$). And from equation (2):

$0 = - 0.29W + 6.83 \quad \Rightarrow \quad W = 23.55$ kN

Problems:

4-10) If the mass of the bicycle is (15 kg) with center of gravity at (G). Determine the normal reactions at (A) and (B) when the bicycle is in equilibrium.

Ans.: $R_A = 81.75\ N$, $R_B = 65.4\ N$

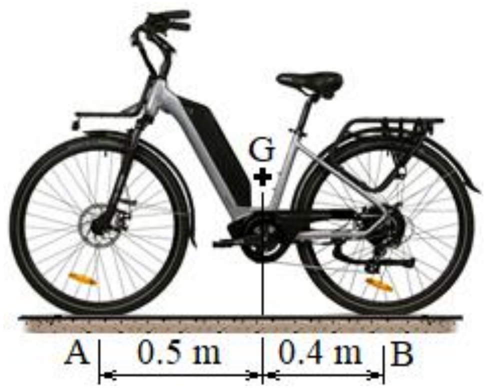


Fig. (Pr. 4-10)

4-11) The cantilever beam shown in Fig. (Pr. 4-11) is subjected to a two external forces and one couple. Compute the reactions at the support point (*O*).

Ans.:
$O_x = 1.73\ kN$, $O_y = 2.5\ kN$, $M_o = 0.5\ k\,N.m\ (CW)$

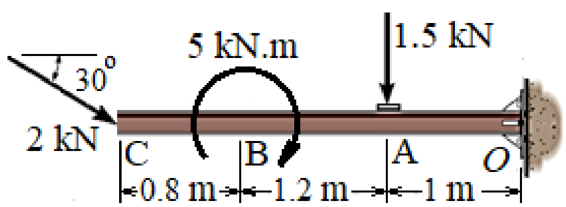


Fig. (Pr. 4-11)

4-12) The total mass of the wheel-barrow and its load is (120 kg) with center of gravity at point (G). Determine the magnitude of the minimum vertical force (F) required to lift the wheelbarrow free of the ground at point (B).

Ans.: $F = 327\ N$

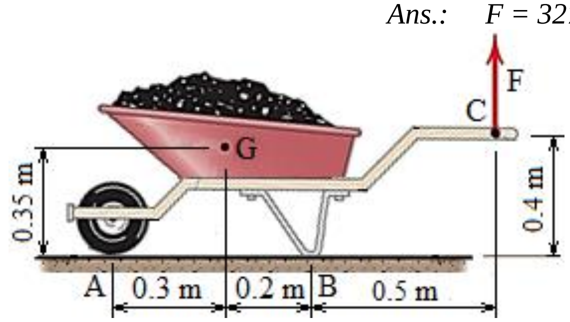


Fig. (Pr. 4-12)

4-13) The center of gravity of the (1.6 tons) pickup truck located at point (G) for the unloaded condition. If a load whose center of gravity is (0.4 m) in front of the rear wheels axle is added to the truck, determine the load weight (W_L) for which the normal reactions under the front and rear wheels are equal.

Ans.: $W_L = 4.13\ kN$

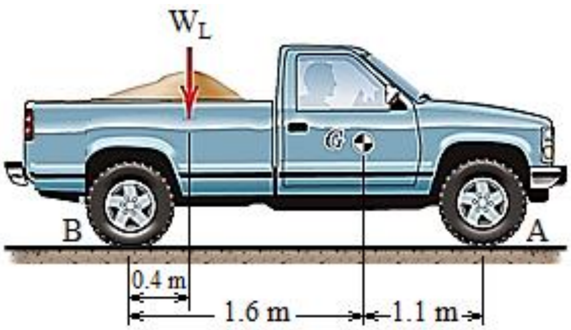


Fig. (Pr. 4-13)

4-14) The total mass of the floor crane and its driver is (5 tons) with the center of gravity at point (G). Determine the largest weight of the box that can be lifted without causing the crane to overturn when its boom is in the position shown in Fig. (Pr. 4-14).

Ans.: $W_b = 64\ kN$

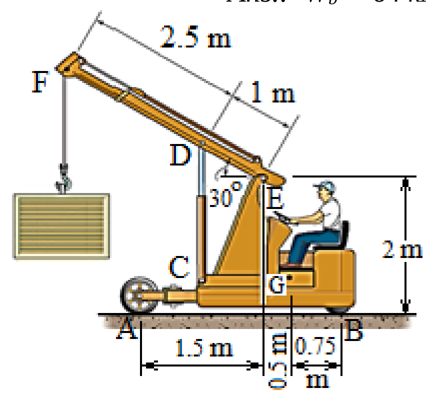


Fig. (Pr. 4-14)

4-15) Excavator weighing (400 kN) at center of gravity (G_1) and its boom weight (50 kN) at center of gravity (G_2) and its arm weight (30 kN) at center of gravity (G_3) and bucket weight (10 kN) at center of gravity (G_4). Determine the reactions at (A) and (B).

Ans.: $N_A = 238.3\ kN$, $N_B = 251.7\ kN$

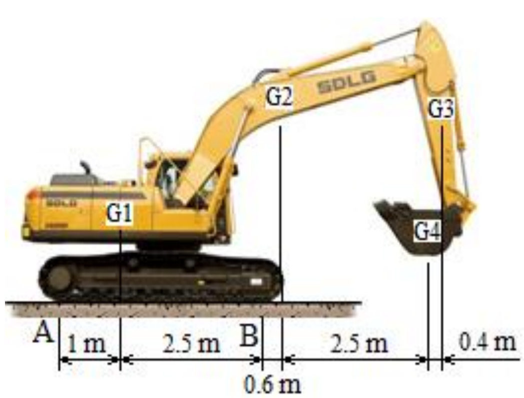


Fig. (Pr. 4-15)

4-16) The ramp shown in Fig. (Pr. 4-16) has a mass of (100 kg) and a center of gravity at (G). Determine the tension in cable (CD) needed to just start lifting the ramp, and the horizontal and vertical components of reaction force at the hinge (pin) at (A).

Ans.:
$T = 955.3\ N$, $A_x = 477.65\ N$, $A_y = 153.7\ N$

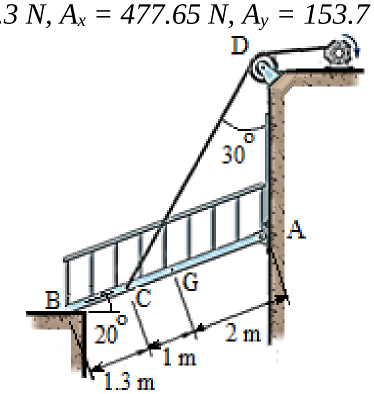


Fig. (Pr. 4-16)

4-17) The dump truck's payload container has a weight of (25 kN) and its center of gravity at point (G), it is hinged with the truck body at point (A), and the hydraulic cylinder is hinged with the truck body at point (C) and with the container hold at point (B). Find the hydraulic cylinder force (F_{CB}) required for equilibrium and the horizontal and vertical components of the reaction force at the join (A).

Ans.:
$F_{CB} = 23.03$ kN, $A_x = 19.95$ kN, $A_y = 13.49$ kN

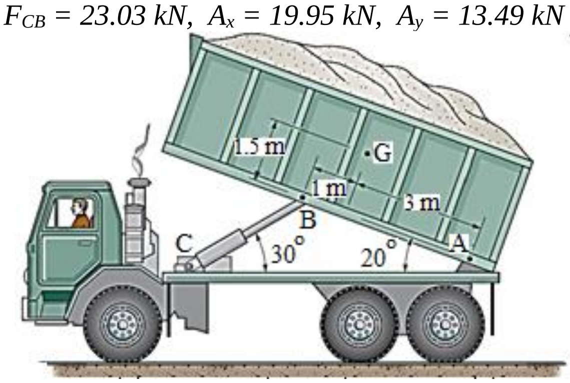


Fig. (Pr. 4-17)

4-18) A worker holds the handles of a building materials wheelbarrow upward, and pushes it forward with a force of (250 N). If the wheelbarrow and its contents have a mass of (50 kg) and center of mass at (G), determine the reactions on the tire.

Ans.: $A_y = 85.9$ N, $B_x = 250$ N, $B_y = 404.6$ N

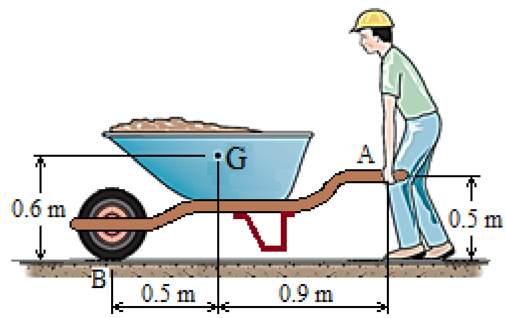


Fig. (Pr. 4-18)

4-19) The beam shown in Fig. (Pr. 4-19) carry a box of (400 kg) mass. Determine the horizontal and vertical components of reaction force at the hinge (pin) at (A) and the reaction force of the rocker (B) on the beam.

Ans.:
$N_B = 3.4$ k N, $A_x = 1.7$ k N, $A_y = 0.98$ k N

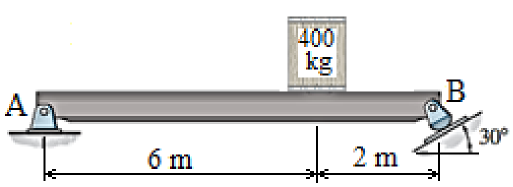


Fig. (Pr. 4-19)

4-20) The jib crane is fixed at (A) and carry a box of (250 kg) mass, as shown in Fig. (Pr. 4-20). Determine the reactions on the jib crane at point (A).

Ans.: $A_x = 1.225$ kN, $A_y = 2.122$ kN
$M_A = 8.73$ kN.m (C.W)

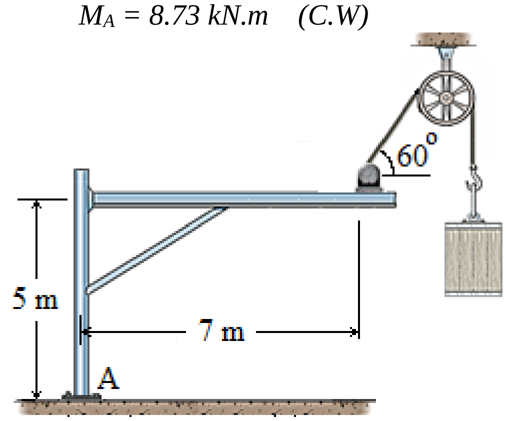


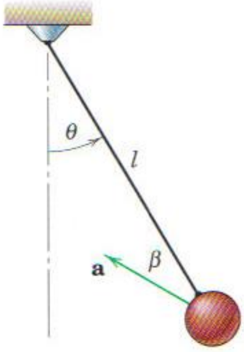
Fig. (Pr. 4-20)

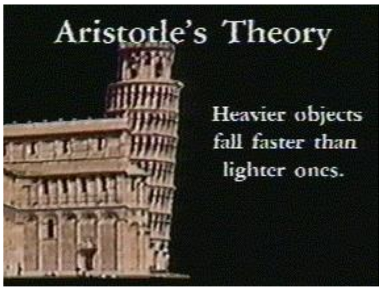
DYNAMICS FUNDAMENTALS

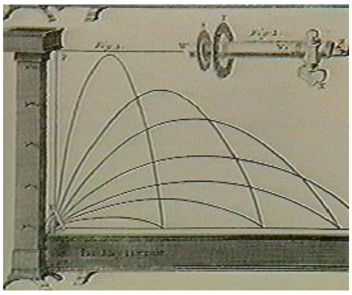
Historical Background:

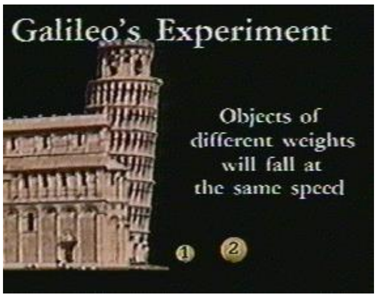
Galileo Galilei (1564-1642):

- Motion of the pendulum
- Bodies in free fall



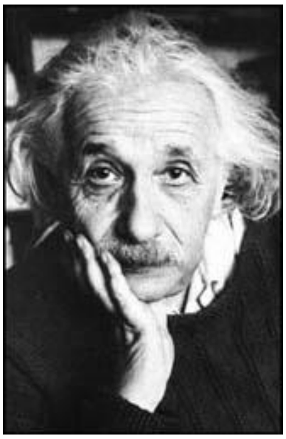


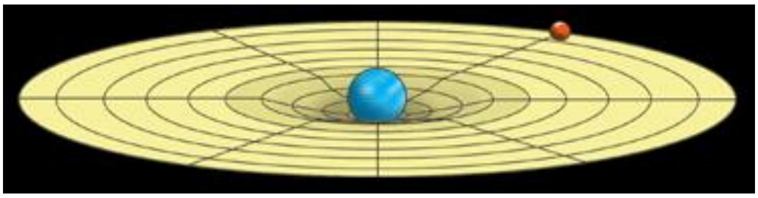




Albert Einstein (1879-1955)

Came up with a new idea describing the gravity (General Relativity).





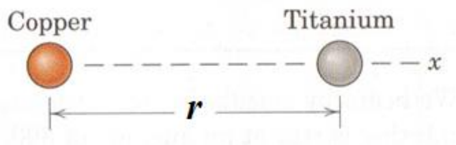
Newton (1642-1727)

Three Laws of motion and law of universal gravitation

Law I: A body remains at rest or continues to move with uniform velocity if there is no unbalanced force acting on it.

Law II: The acceleration of a body is proportional to the resultant force acting on it and is in the direction of this force ($\boldsymbol{F} = m\boldsymbol{a}$).

Law III: The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and collinear.



$$F = G\frac{m_1 m_2}{r^2}$$

F = mutual force of attraction between the two particles.
G = the constant gravitation.
m_1 & m_2 = masses of the two particles.
r = distance between centers of particles.

What is Dynamics?

Mechanics

Branch of physical sciences that deals with the state of rest or motion of bodies subjected to the action of forces

Statics

Deals with the action of forces on bodies at rest

$$\sum F = 0$$
$$\sum M = 0$$

Dynamics

Deals with the motion of bodies under the action of forces

$$\sum F \neq 0$$
$$\sum M \neq 0$$

Dynamics

Kinematics

Study the **motion** without reference to the forces that cause it

Displacement / Distance
Velocity / Speed
Acceleration / Deceleration

Including Time

Kinetics

Relates the action of **forces** on bodies to their resulting **motions**

Kinematics:

- Rectilinear kinematics: Continuous motion.
- Rectilinear kinematics: Graphic representation of the motion.
- Motion of projectile.
- Curvilinear motion.
- Relative - motion of two particles.
- Absolute dependent motion analysis of two particles.

Kinetics:

- Newton's Second Law of Motion.
- Work & Kinetic Energy.
- Potential Energy.
- Impulse-Momentum.
- Impact.

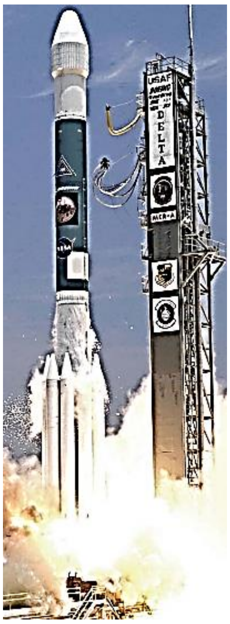
Part 1 / Kinematics of a particle:

Chapter 1 / **Rectilinear kinematics: Continuous motion:**



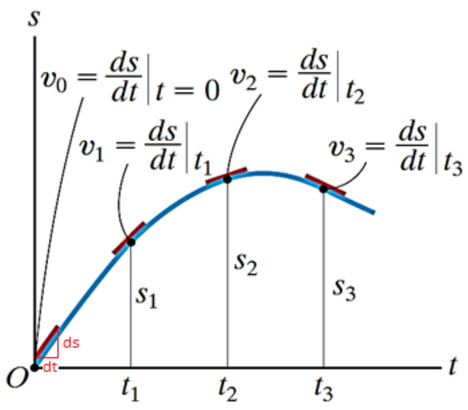




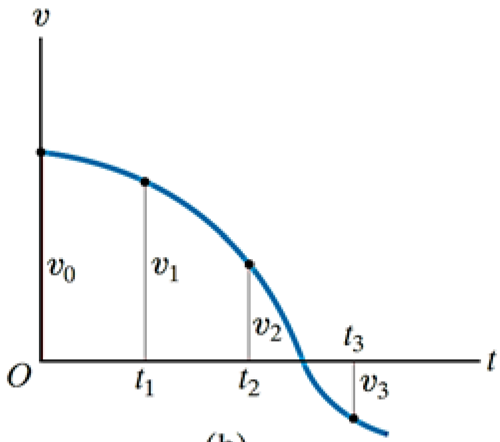


Chapter 2 /

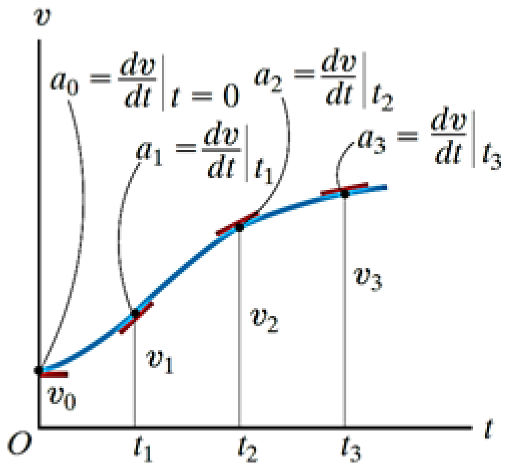
Rectilinear kinematics : Graphic representation of the motion:



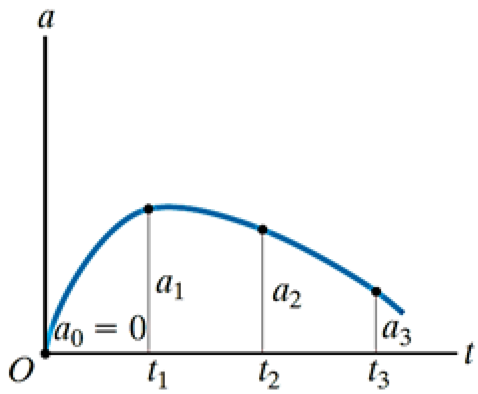
(a)



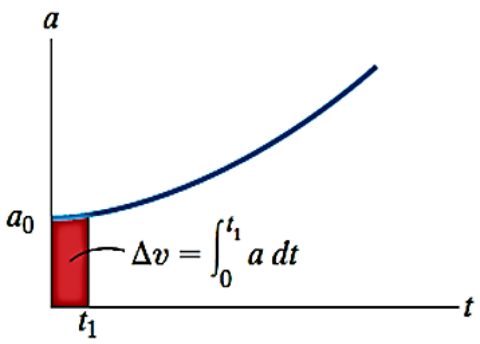
(b)



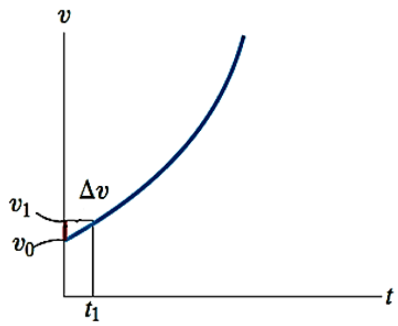
(a)



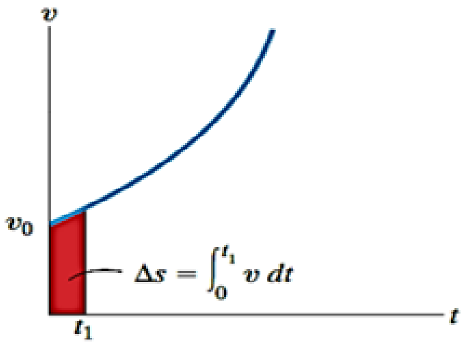
(b)



(a)

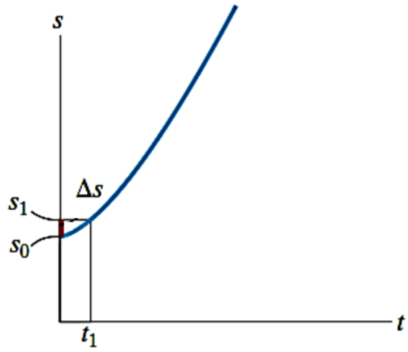


(b)



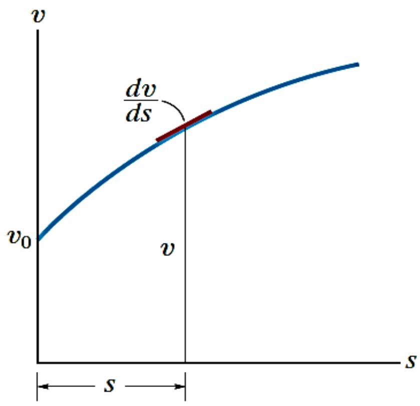
v
v_0
$\Delta s = \int_0^{t_1} v\,dt$
t_1
t

(a)



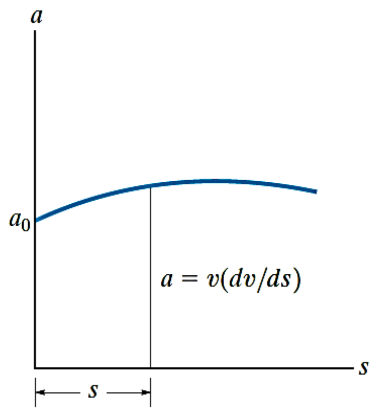
s
Δs
s_1
s_0
t_1
t

(b)



v
$\frac{dv}{ds}$
v_0
v
s
s

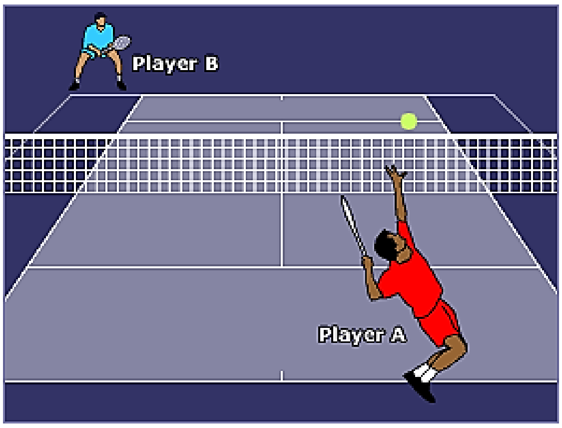
(a)



a
a_0
$a = v(dv/ds)$
s
s

(b)

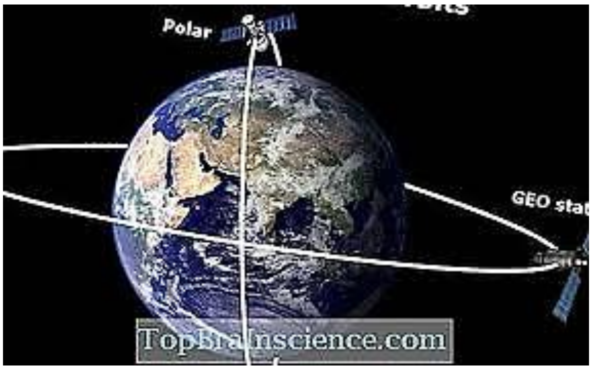
Chapter 3 / **Motion of projectiles:**

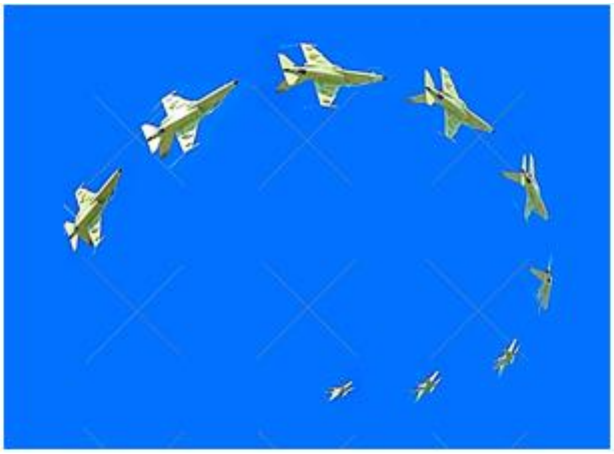


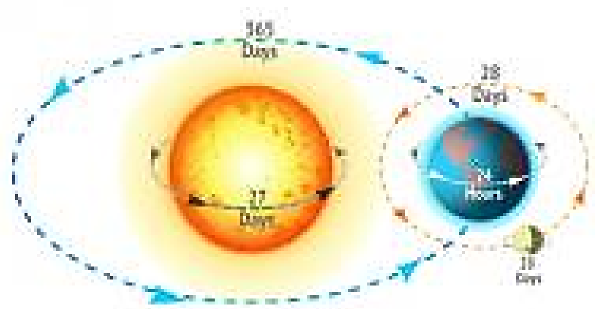

123RF


wikiHow

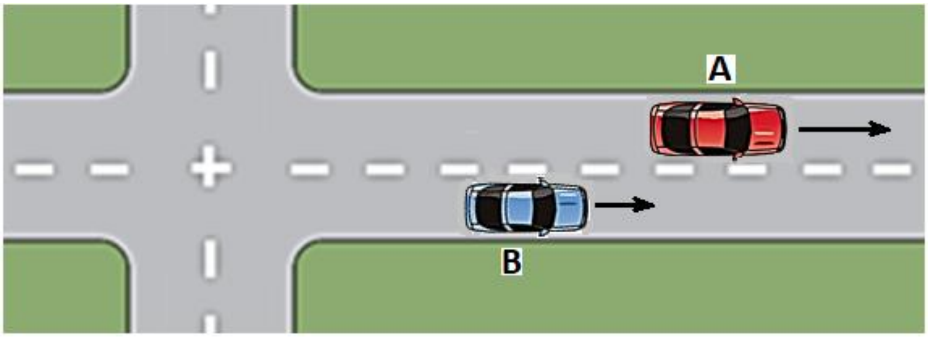
Chapter 4 / **Curvilinear motion:**

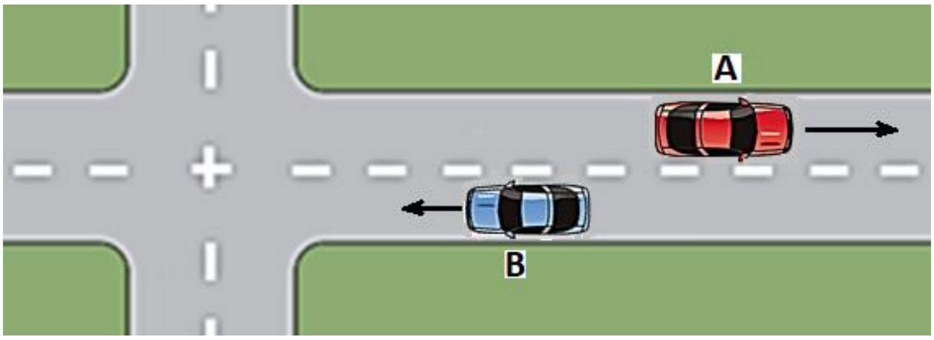


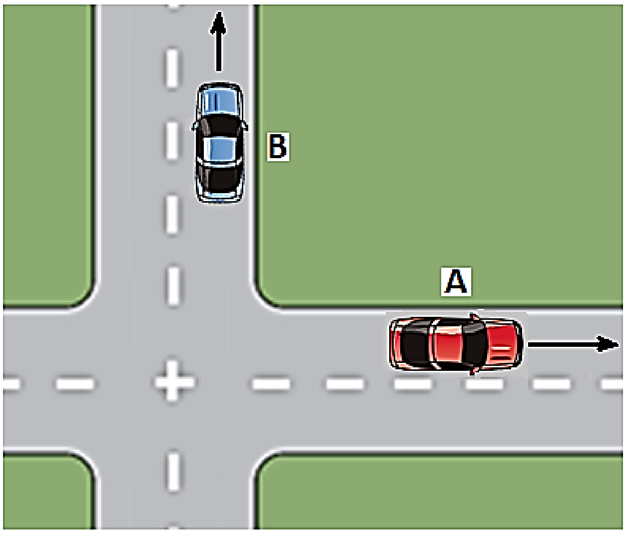




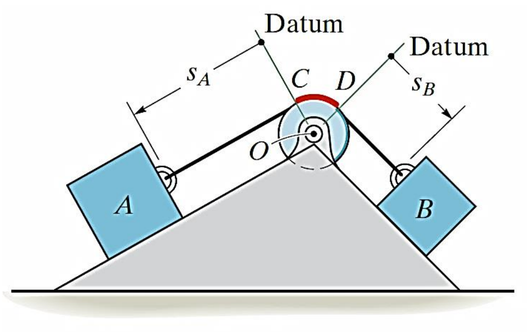
Chapter 5 / **Relative - motion of two particles:**

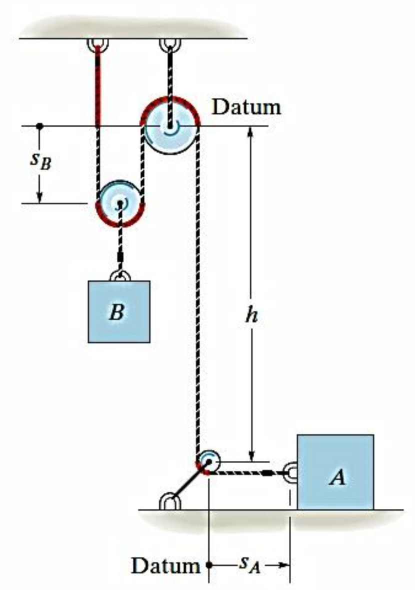


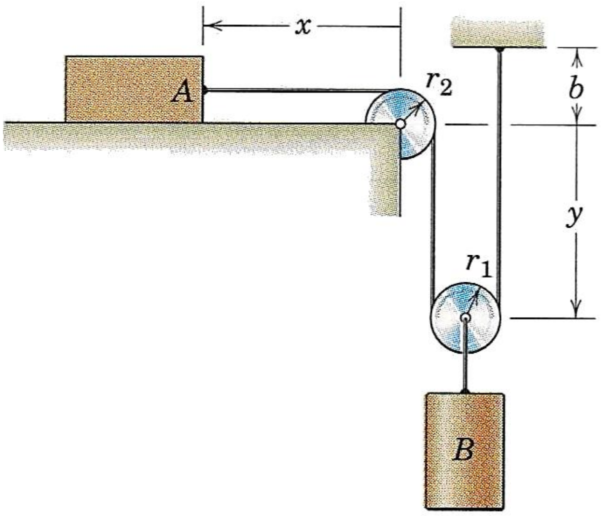


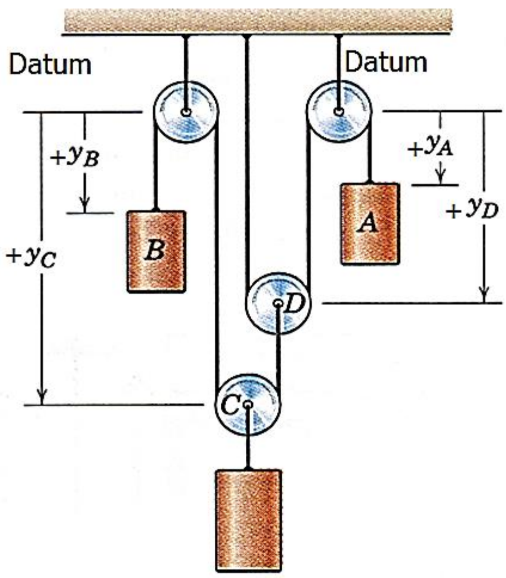


Chapter 6 / **Absolute dependent motion analysis of two particles:**









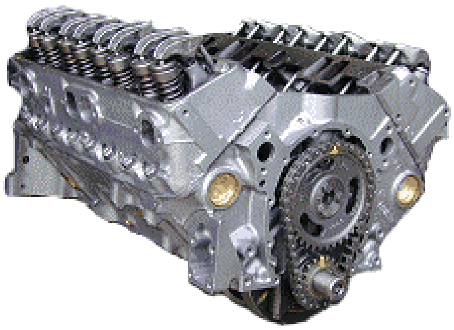
Why do we need to study dynamics ?

Dynamics principles are basic to the analysis and design of moving structures in:

- Automotive industries
- Aerospace industries
- Automatic control systems
- Turbines, pumps &machine tools.



- How do we decide how big to make the pistons?
- Where should they be placed in the engine block?
- How do we make the engine run smoothly?



Well, we could answer these questions by trial and error. But the 'errors' would be expensive exercises. Why not study the dynamics of engines and make some predictions instead!

Or suppose we want to build a robot;

- How do we decide how big to make the motors?
- How fast can we expect it to move from one place to another?
- How accurate will it be while it moves and stops?
- How many joints should it have and where?

Fundamental concepts:

- **Length (Space) :** needed to locate position of a point in space, & describe size of the physical system
    - Position in space is determined relative to some geometric reference system by means of linear and angular measurements.
    - This reference system may be:
        - Fixed in space: measurements are said to be absolute.
        - Moving in space: measurements are made relative to fixed reference system.

- **Time (t):** measure of succession of events.
- **Mass (m):** The quantity of matter in a body as well as the property which gives rise to gravitational attraction. Weight is related to the mass by: *mg.; where g* is the gravitational acceleration.
- **Force**: represents the action of one body on another characterized by its magnitude, direction of its action, and its point of application

Particles : A body of negligible dimensions (when its dimensions are irrelevant to the description of its motion or the action of the forces on it). Bodies of finite size, such as rockets, projectiles, or vehicles.

Rigid Bodies : A combination of large number of particles in which all particles remain at a fixed distance (practically) from one another before and after applying a load.

Basic vector agebra:

Scalars are quantities having only magnitude.

- Length or distance, speed, mass, time, work, energy, power, etc.

Vectors are quantities having both a magnitude and a direction.

- Position, displacement, velocity, acceleration, force, moment, impulse, momentum, etc.

Derived quantities related dynamics and their (SI) units:

Velocity = $\dfrac{\text{Displacement}}{\text{Time}}$ m/s

Speed = $\dfrac{\text{Distance}}{\text{Time}}$ m/s

Acceleration = $\dfrac{\text{Velocity}}{\text{Time}}$ m/s^2

Force = Mass × Acceleration $kg.m/s^2$ = N (Newton)

Work = Energy = Force × Distance N.m = J (Joule)

Power = $\dfrac{\text{Work}}{\text{Time}}$ J/s = W (Watt)

Impulse = Momentum = Force × Time N.s

Angular quantities needed in dynamics:

Angular distance $\theta\,° = \dfrac{\theta \times \pi}{180}$ rad.

Angular speed $\dot{\theta} = \dfrac{\text{deg.}}{\text{s}} = \dfrac{\dot{\theta} \times \pi}{180}$ rad/s.

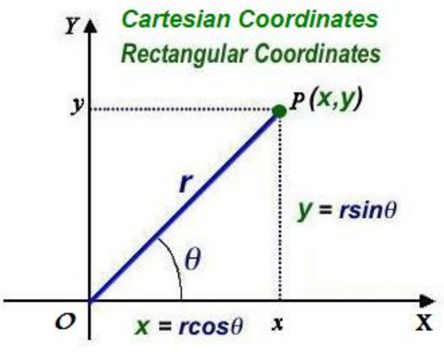
$\dot{\theta}$ rpm $= \dfrac{2\pi \times \dot{\theta}}{60}$ rad/s.

Angular speed $\ddot{\theta} = \dfrac{\text{deg.}}{\text{s}^2} = \dfrac{\ddot{\theta} \times \pi}{180}$ rad/s^2.

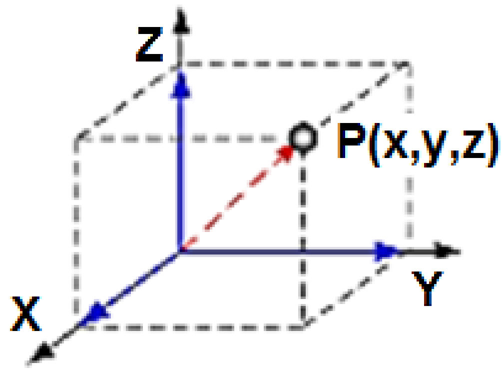
Coordinates system :

- **Cartesian coordinate system:**

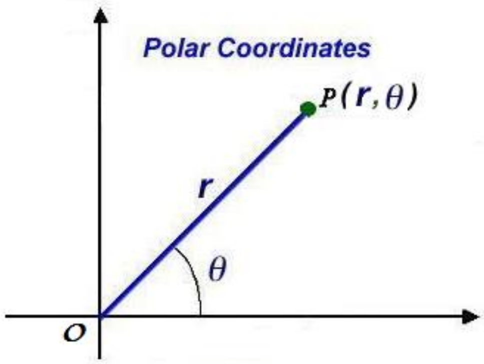
Two dimensions



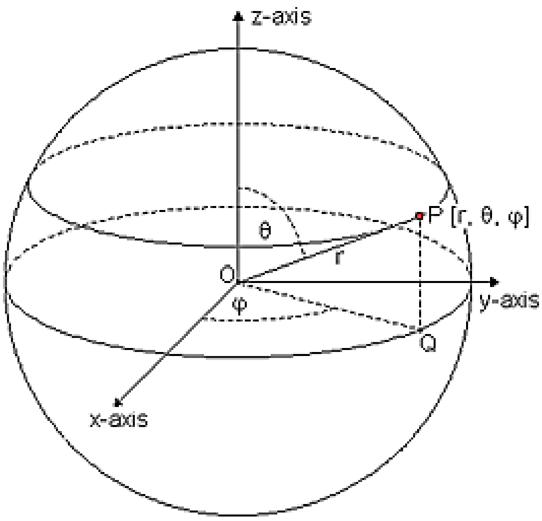
Three dimensions



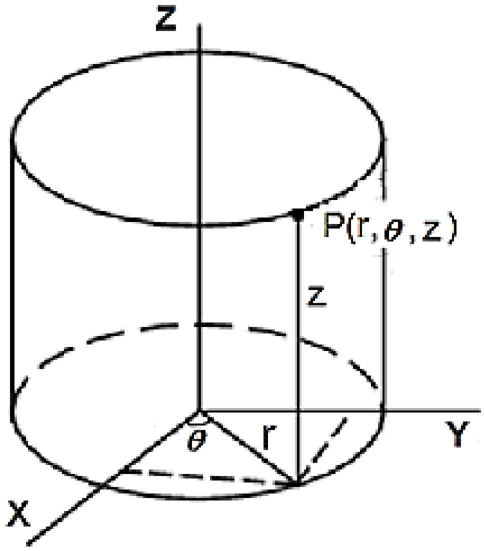
- **Polar coordinate system:**



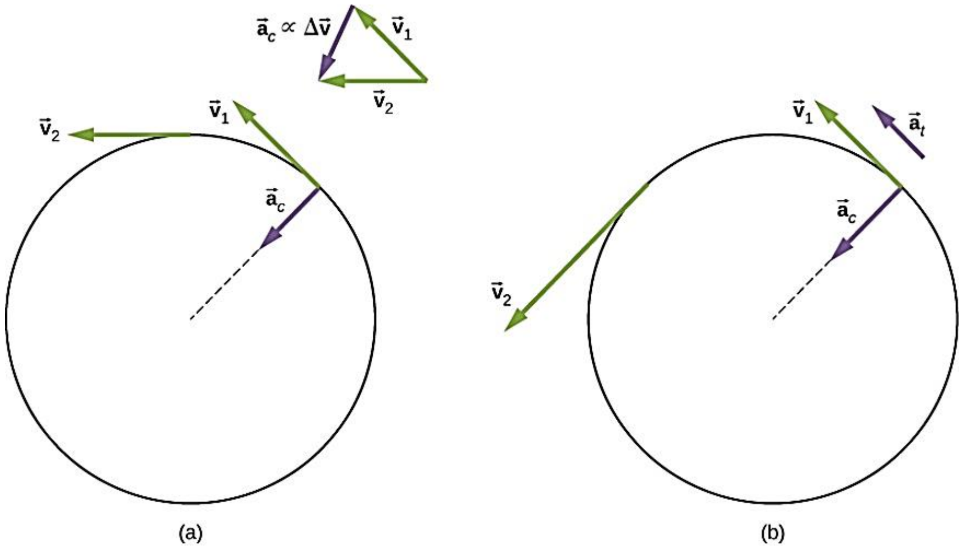
- **Spherical coordinate system:**



- **Cylindrical coordinate system:**



- **Normal - Tangential coordinate system:**

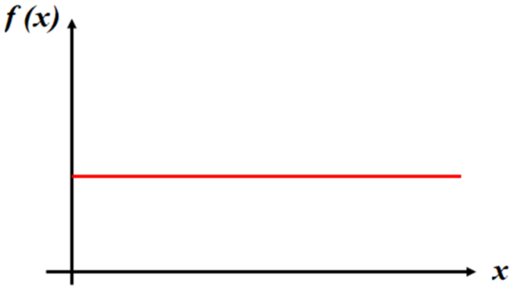


Math Revision:

Functions:

Zero order function
Constant value

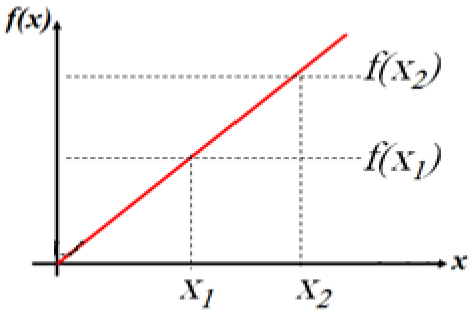
$F(x) = c$



$$Slope = m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

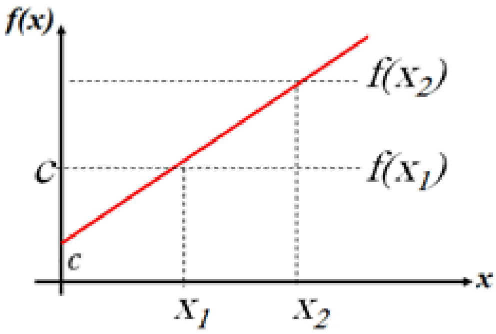
1^{st} order function
Linearly Variable

$F(x) = mx$



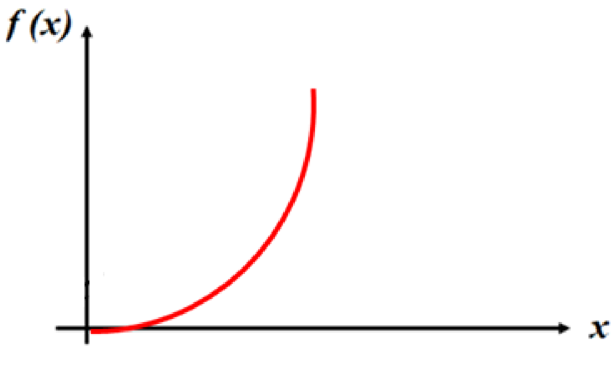
$$Slope = m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$F(x) = mx + c$



2^{nd} order function
Curvilinearly Variable

$F(x) = x^2$



Differential Calculus:

Derivatives Table

$f(x)$	$f'(x)$	$f(x)$	$f'(x)$
$af(x)$	$af'(x)$	a^u	$(\ln a)a^u \dfrac{du}{dx}$
$u(x)+v(x)$	$u'(x)+v'(x)$	$\ln u$	$\dfrac{1}{u}\dfrac{du}{dx}$
$f(u)$	$f'(u)\dfrac{du}{dx}=\dfrac{df(u)}{du}\dfrac{du}{dx}$	$\log_a u$	$(\log_a e)\dfrac{1}{u}\dfrac{du}{dx}$
a	0	$\sin u$	$\cos u\left(\dfrac{du}{dx}\right)$
$x^n \quad (n\neq 0)$	nx^{n-1}	$\cos u$	$-\sin u\dfrac{du}{dx}$
$u^n \quad (n\neq 0)$	$nu^{n-1}\dfrac{du}{dx}$	$\tan u$	$\sec^2 u\dfrac{du}{dx}$
uv	$u\dfrac{dv}{dx}+v\dfrac{du}{dx}$	$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx} \quad \left(-\dfrac{\pi}{2}\leq \sin^{-1} u\leq \dfrac{\pi}{2}\right)$
$\dfrac{u}{v}$	$\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$	$\cos^{-1} u$	$\dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx} \quad \left(0\leq \cos^{-1} u\leq \pi\right)$
e^u	$e^u\dfrac{du}{dx}$	$\tan^{-1} u$	$\dfrac{1}{1+u^2}\dfrac{du}{dx} \quad \left(-\dfrac{\pi}{2}< \tan^{-1} u< \dfrac{\pi}{2}\right)$

Integral Table

$f(x)$	$F(x)=\int f(x)dx$	$f(x)$	$F(x)=\int f(x)dx$
$af(x)$	$aF(x)$	$\cos^2 ax$	$\dfrac{1}{2}x+\dfrac{1}{4a}\sin 2ax$
$u(x)+v(x)$	$\int u(x)dx+\int v(x)dx$	$x\sin ax$	$\dfrac{1}{a^2}\sin ax-\dfrac{x}{a}\cos ax$
a	ax	$x\cos ax$	$\dfrac{1}{a^2}\cos ax+\dfrac{x}{a}\sin ax$
$x^n \quad (n\neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sin ax\cos ax$	$\dfrac{1}{2a}\sin^2 ax$
e^{ax}	$\dfrac{e^{ax}}{a}$	$\sin ax\cos bx$ for $a^2\neq b^2$	$-\dfrac{\cos(a-b)x}{2(a-b)}-\dfrac{\cos(a+b)x}{2(a+b)}$
$\dfrac{1}{x}$	$\ln x$	xe^{ax}	$\dfrac{e^{ax}}{a^2}(ax-1)$
$\sin ax$	$-\dfrac{1}{a}\cos ax$	$\ln x$	$x(\ln x-1)$
$\cos ax$	$\dfrac{1}{a}\sin ax$	$\dfrac{1}{ax^2+b}$	$\dfrac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\dfrac{a}{b}}\right)$
$\sin^2 ax$	$\dfrac{1}{2}x-\dfrac{1}{4a}\sin 2ax$		

Part 1

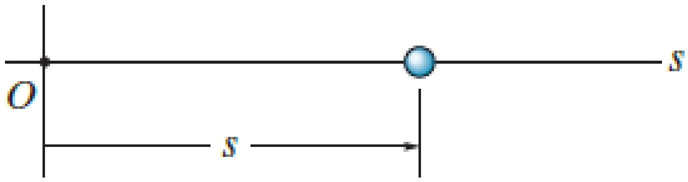
Kinematics of a particle

Chapter 1

Rectilinear kinematics: Continuous motion

Rectilinear motion: particle moving along a straight line.

1. **Position (S):** defined by positive or negative distance from a fixed origin on the line.

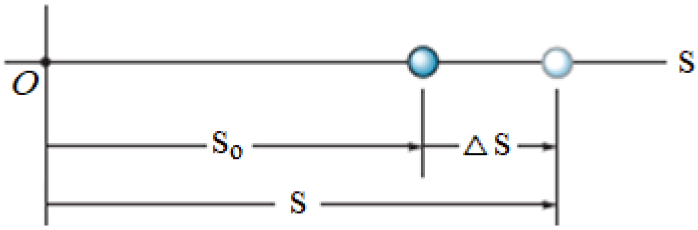


Position

The magnitude of (S) is the distance from (*O*) to the particle, usually measured in meters (m) or feet (ft), and the sense of direction is defined by the algebraic sign on (S).

2. **Displacement (ΔS):** The displacement of the particle is defined as the change in its position. For example, if the particle moves from one point to another, the displacement is:

$$\Delta s = s - s_o$$



- Δs is *positive* since the particle's final position is to the *right* of its initial position.
- Δs is *negative* since the particle's final position is to the *left* of its initial position.

3. **Distance (d):**

The distance is the total path length traveled by an object during its motion. It is a scalar quantity, so it only has magnitude and no direction. It is always non-negative, as it represents the total length traveled.

4. **Velocity (V):** If the particle moves through a displacement (ΔS) during the time interval (Δt), the **average velocity** of the particle during this time interval is:

$$\mathbf{V_{avg}} = \frac{\Delta S}{\Delta t}$$

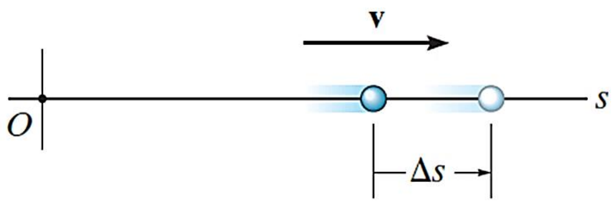
- If we take smaller and smaller values of (Δt), the magnitude of (ΔS) becomes smaller and smaller. Consequently, we get an **instantaneous velocity**:

$$+ \longrightarrow \qquad v_{instant} = \lim_{\Delta t \to \mathbf{0}} \frac{\Delta s}{\Delta t}$$

- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as **particle speed**.
- From the definition of a derivative,

$$v = \lim_{\Delta t \to \mathbf{0}} \frac{\Delta s}{\Delta t} = \frac{\mathbf{ds}}{\mathbf{dt}}$$

- The magnitude of the velocity is generally expressed in units of (m/s) or (ft/s).
- If the particle is moving to the right, the velocity is positive; whereas if it is moving to the left, the velocity is negative.

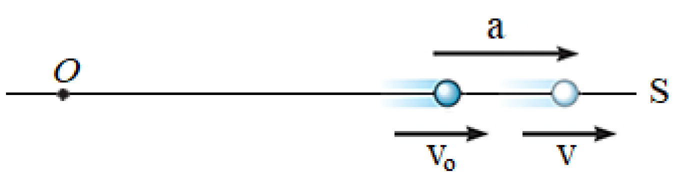


5. **Speed (V_{sp}):** Speed is the rate at which an object changes its position. It is a scalar quantity and is defined as the distance traveled divided by the time taken to travel that distance.

Mathematically, $\quad V_{sp} = \frac{d}{t} \qquad (V_{sp})_{avg} = \frac{d}{\Delta t}$

6. **Acceleration:** Provided the velocity of the particle is known at two points, the average acceleration of the particle during the time interval (Δt) is defined as

$$a_{avg} = \frac{\Delta V}{\Delta t} \qquad \text{when,} \quad \Delta v = v - v_o$$



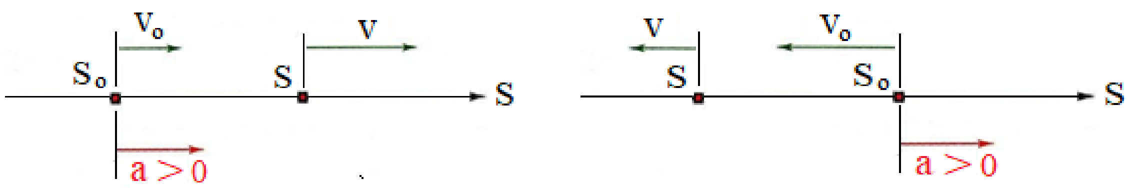
- From the definition of a derivative, $a = \lim\limits_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = \frac{d^2 s}{dt^2}$

• If the velocity is constant, then the acceleration is **zero**,

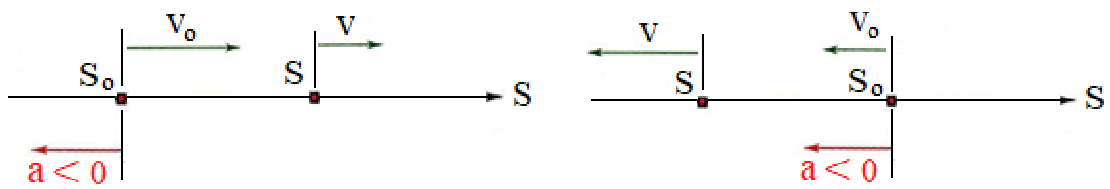
$$(\Delta v = v - v_o = 0)$$

- Instantaneous acceleration may be:

- **positive:** increasing positive velocity or decreasing negative velocity



- **negative:** decreasing positive velocity or increasing negative velocity



7- Equations of Motion:

Velocity:

$$v = \frac{dS}{dt} \quad \text{..........................} \quad (1)$$

$ds = v\,dt$

$\int_{s_o}^{s} ds = \int_0^t v\,dt$

Acceleration:

$$a = \frac{dv}{dt} \quad \text{..........................} \quad (2)$$

$dv = a\,dt$

$\int_{v_o}^{v} dv = \int_0^t a\,dt$

$d\,t = \frac{ds}{v} = \frac{dv}{a}$

$$a\,ds = v\,dv \quad \text{....................} \quad (3)$$

$\int_{s_o}^{s} a\,ds = \int_{v_o}^{v} v\,dv$

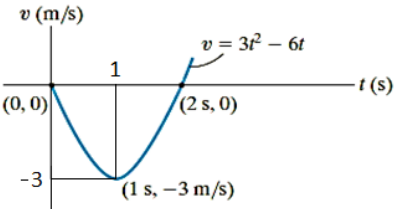
Example 1 - 1:

A particle moves along a horizontal path with a velocity of [v = (3t^2 - 6t) m/s], where (t) is the time in seconds. If it is initially located at the origin (*O*), determine the distance traveled in (3.5 s), and the particle's average velocity and average speed during the time interval.

Solution:

Coordinate System. Here positive motion is to the right, measured from the origin *O*, Fig. *a*.

Distance Traveled. Since v = f(t), the position as a function of time may be found by integrating v = ds/dt with $t = 0$, $s = 0$.



$\overset{+}{\rightarrow}$

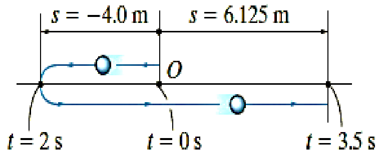
$$ds = v\,dt = (3t^2 - 6t)\,dt$$

$$\int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$$

$$s = (t^3 - 3t^2)\text{ m} \quad \ldots\ldots\ (1)$$

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. *b*, then it reveals that for 0 s *to* 2 s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that v = 0 at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can be determined from Eq. 1. This yields:

$$s\big|_{t=0} = 0\ , \quad s\big|_{t=2\text{ s}} = -4\text{ m}\ , \quad s\big|_{t=3.5\text{ s}} = 6.125\text{ m}$$



(a)

The path is shown in Fig. *a*. Hence, the distance traveled in 3.5 s is:

$$s_T = 4 + 4 + 6.125 = 14.125\text{ m} = 14.1\text{ m}$$

Velocity, The displacement from (t = 0) to (t = 3.5 s) is:

$$\Delta s = s\big|_{t=3.5\text{ s}} - s\big|_{t=0} = 6.125 - 0 = 6.125\text{ m} \rightarrow$$

And so the average velocity is:

$$V_{avg} = \frac{\Delta S}{\Delta t} = \frac{6.125}{3.5 - 0} = 1.75\text{ m/s} \rightarrow$$

The average speed is defined in terms of the distance traveled (S_T).

$$(V_{sp})_{avg} = \frac{S_T}{\Delta t} = \frac{14.125}{3.5 - 0} = 4.04\text{ m/s}$$

Constant acceleration, ($a = a_c$):

When the acceleration is constant, each of the three kinematic equations:

$$a_c = \frac{dv}{dt} \qquad\qquad v = \frac{ds}{dt} \qquad\qquad a_c\, ds = v\, dv$$

can be integrated to obtain formulas that relate (a_c , v, s, and t).

Velocity as a function of time.

Integrate ($a_c = \frac{dv}{dt}$), assuming that initially ($v = v_o$) when (t = 0).

$$dv = a_c\ dt$$

$$\int_{v_o}^{v} dv = \int_0^t a_c\ dt = a_c \int_0^t dt$$

$$v - v_o = a_c\ t$$

$$v = v_o + a_c\ t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Position as a function of time.

Integrate ($v = \frac{ds}{dt} = v_o + a_c\ t$), assuming that initially ($s = s_o$) when (t = 0).

$$ds = (v_o + a_c\ t)\, dt$$

$$\int_{s_o}^{s} ds = \int_0^t (v_o + a_c\ t)\, dt$$

$$s - s_o = v_o\, t + \frac{1}{2} a_c\, t^2$$

$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Velocity as a function of position.

Either solve for (t) in Eq. (4) and substitute into Eq. (5), or integrate ($v\, dv = a_c\, ds$) assuming that initially ($v = v_o$) at ($s = s_o$).

Metod (1):

From Eq. (4) $\qquad t = \dfrac{v - v_o}{a_c}$

Sub. in Eq. (5) $\qquad s = s_o + v_o \left(\dfrac{v - v_o}{a_c}\right) + \dfrac{1}{2} a_c \left(\dfrac{v - v_o}{a_c}\right)^2$

$$s - s_o = \frac{v\, v_o - v_o 2}{a_c} + \frac{1}{2}\, \frac{v^2 - 2v\, v_o + v_o 2}{a_c} \qquad \times\, 2a_c$$

$$2a_c\, (s - s_o) = 2vv_o - 2v_o^2 + v^2 - 2vv_o + v_o^2$$

$$2a_c\, (s - s_o) = v^2 - v_o^2$$

$$v^2 = v_o^2 + 2a_c\, (s - s_o) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

Metod (2):

$$\mathrm{v\,dv} = \mathrm{a_c\,ds}$$

$$\int_{\mathrm{v_o}}^{\mathrm{v}} \mathrm{v\,dv} = \int_{\mathrm{s_o}}^{\mathrm{s}} \mathrm{a_c ds} = \mathrm{a_c} \int_{\mathrm{s_o}}^{\mathrm{s}} \mathrm{ds}$$

$$\frac{1}{2}\,\mathrm{v}^2 - \frac{1}{2}\,\mathrm{v_o}^2 = \mathrm{a_c}\,(\,\mathrm{s} - \mathrm{s_o}\,)$$

$$\frac{1}{2}\,(\,\mathrm{v}^2 - \mathrm{v_o}^2\,) = \mathrm{a_c}\,(\,\mathrm{s} - \mathrm{s_o}\,)$$

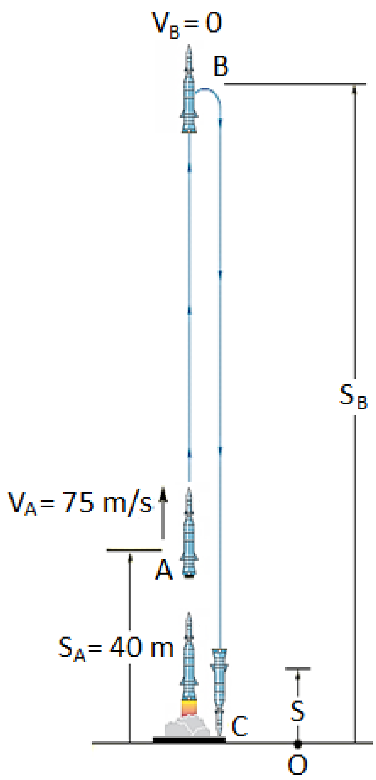
$$\mathrm{v}^2 - \mathrm{v_o}^2 = 2\mathrm{a_c}\,(\,\mathrm{s} - \mathrm{s_o}\,)$$

$$\mathrm{v}^2 = \mathrm{v_o}^2 + 2\mathrm{a_c}\,(\,\mathrm{s} - \mathrm{s_o}\,) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\,6\,)$$

Example 1 - 2:

During a test a rocket travels upward at (75 m/s), and when it is (40 m) from the ground its engine fails. Determine the maximum height (S_B) reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of (9.81 m/s^2) due to gravity. Neglect the effect of air resistance.

Solution:



Coordinate System. The origin *O* for the position coordinate *s* is taken at ground level with positive upward.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$ m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points *A* and *B* on the path by using Eq. 6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2\,a_c\,(s_B - s_A)$$

$$0 = (75)^2 + 2\,(-9.81)\,(s_B - 40)$$

$$s_B = 327 \text{ m}$$

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 6 between points *B* and *C*,

$$(+\downarrow) \qquad v_C^2 = v_B^2 + 2\,a_c\,(s_C - s_B)$$

$$= (0)^2 + 2\,(9.81)\,(0 - (-327))$$

$$v_C = 80.1 \text{ m/s} \downarrow$$

The negative root was chosen since the rocket is moving downward. Similarly, Eq. 6 may also be applied between points A and C, i.e.,

$$(+\downarrow) \qquad v_C^2 = v_A^2 + 2\,a_c\,(s_C - s_A)$$

$$= (75)^2 + 2\,(9.81)\,(0 - (-40))$$

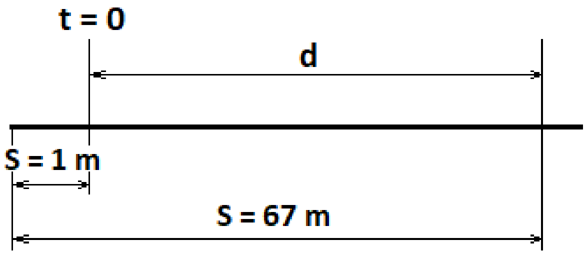
$$v_C = 80.1 \text{ m/s} \downarrow$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from *A* to *B* of 9.81 m/s^2, and then from *B* to *C* it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at *B* ($v_B = 0$) the acceleration at *B* is still 9.81 m/s^2 downward!

Example 1 - 3:

The acceleration of a particle as it moves along a straight line is given by [$a = (2t - 1)\ m/s^2$], where (t) is in seconds. If (s = 1 m) and (v = 2 m/s) when (t = 0), determine the particle's velocity and position when (t = 6 s). Also, determine the total distance the particle travels during this time period.

Solution:



$$a = 2t - 1$$
$$a = \frac{dv}{dt}$$
$$dv = a\ dt$$
$$\int_2^v dv = \int_0^t (2t - 1)\ dt$$
$$v = t^2 - t + 2$$
$$v = \frac{ds}{dt}$$
$$ds = v\ dt$$
$$\int_1^s ds = \int_0^t (t^2 - t + 2)\ dt$$
$$s = \frac{1}{3}t^3 - \frac{1}{2}t^2 + 2t + 1$$

When $t = 6$ s

$v = (6)^2 - (6) + 2$ $\qquad$ $v = 32$ m/s

$s = \frac{1}{3}(6)^3 - \frac{1}{2}(6)^2 + 2(6) + 1$ $\qquad$ $s = 67$ m

Since $v \neq 0$ for $0 \leq t \leq 6$ s, then:

$d = 67 - 1 = 66$ m

Example 1 - 4:

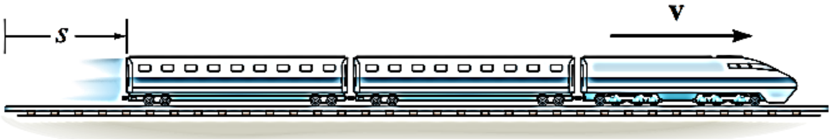
If a jeep has an initial velocity of (v_o = 12 ft/s) to the right, at (s_o = 0), determine its position when (t = 10 s), if ($a = 2\ ft/s^2$) to the lift.

Solution:

$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2$$
$$= 0 + 12\,(10) + \frac{1}{2}(-2)(10)^2 = 20 \text{ ft}$$

Example 1 - 5:

When a train is traveling along a straight track at (2 m/s), it begins to accelerate at [$a = (60\, v^{-4})\ m/s^2$], where (v) is in (m/s). Determine its velocity (v) and the position (3 s) after the acceleration.



Solution:

$$a = \frac{dv}{dt} \quad \Rightarrow \quad dt = \frac{dv}{a}$$

$$\int_0^3 dt = \int_2^v \frac{dv}{60\, v^{-4}}$$

$$= \int_2^v \frac{1}{60}\, v^4\, dv$$

$$3 - 0 = \frac{1}{300} v^5 - \frac{1}{300} (2)^5$$

$$3 = \frac{1}{300} v^5 - \frac{32}{300} = \frac{1}{300} (v^5 - 32)$$

$$v = 3.93 \text{ m/s}$$

$$a\, ds = v\, dv \quad \Rightarrow \quad ds = \frac{v\, dv}{a} = \frac{1}{60} v^5\, dv$$

$$\int_0^s ds = \frac{1}{60} \int_2^{3.93} v^5\, dv$$

$$s = \frac{1}{60} \left(\frac{v^6}{6} \right) \Big|_2^{3.93}$$

$$= \frac{1}{60} \left(\frac{(3.93)^6}{6}\right) - \frac{1}{60} \left(\frac{(2)^6}{6}\right) = 9.98 \text{ m}$$

Example 1 - 6:

Traveling with an initial speed of (70 km/h), a car accelerates at (6000 km/h^2) along a straight road. How long will it take to reach a speed of (120 km/h) ? Also, through what distance does the car travel during this time ?

Solution:

$$v = v_o + a_c\, t$$

$$120 = 70 + 6000\, t \quad \Rightarrow \quad 50 = 6000\, t$$

$$t = 8.33 \times 10^{-3} \text{ hr} \quad = 30 \text{ s}$$

$$v^2 = v_o^2 + 2\, a_c\, (s - s_1)$$

$$(120)^2 = (70)^2 + 2\, (6000)\, (s - 0)$$

$$s = 0.792 \text{ km} = 792 \text{ m}$$

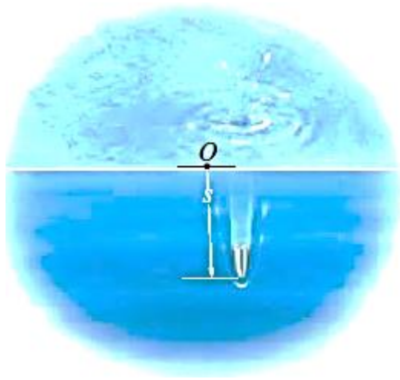
$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2$$

$$s = 0 + (70)(8.33 \times 10^{-3}) + \frac{1}{2} (6000)(8.33 \times 10^{-3})^2$$

$$s = 0.5831 + 0.2082 = 0.7913 \text{ km} = 791 \text{ m}$$

Example 1 - 7:

A small projectile is fired vertically downward into a fluid medium with an initial velocity of (60 m/s). Due to the drag resistance of the fluid the projectile experiences a deceleration of [a = (– 0.4 v^3) m/s^2], where (v) is in (m/s). Determine the projectile's velocity and position (4 s) after it is fired.



Solution:

Velocity:

Since the motion is downward, the position coordinate is positive downward, with origin location at *O*.

$$+\downarrow \quad a = \frac{dv}{dt} = -0.4\, v^3$$

$$\int_{60}^{v} \frac{dv}{-0.4\, v^3} = \int_0^t dt \quad \Rightarrow \quad \int_{60}^{v} \frac{1}{-0.4} v^{-3}\, dv = \int_0^t dt$$

$$\frac{1}{-0.4} \times \frac{v^{-2}}{-2} \Bigg|_{60}^{v} = t - 0 \quad \Rightarrow \quad \frac{1}{0.8} \times \frac{1}{v^2} \Bigg|_{60}^{v} = t$$

$$\frac{1}{0.8} \left[\frac{1}{v^2} - \frac{1}{(60)^2} \right] = t \qquad \times 0.8$$

$$\frac{1}{v^2} - \frac{1}{(60)^2} = 0.8\, t \quad \Rightarrow \quad \frac{1}{v^2} = \frac{1}{(60)^2} + 0.8\, t$$

$$v^2 = \frac{1}{\left[\frac{1}{(60)^2} + 0.8\, t \right]} \quad \Rightarrow \quad v = \left\{ \frac{1}{\sqrt{\frac{1}{60^2} + 0.8\, t}} \right\} \text{ m/s}$$

At (t = 4 s)

$$v = \frac{1}{\sqrt{\frac{1}{60^2} + 0.8\,(4)}} = \frac{1}{\sqrt{\frac{1}{60^2} + 3.2}} = 0.559 \text{ m/s} \downarrow$$

$$+\downarrow \quad v = \frac{ds}{dt} = \frac{1}{\sqrt{\frac{1}{60^2} + 0.8\, t}} = \left[\frac{1}{(60)^2} + 0.8\, t \right]^{-1/2}$$

Position:

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8\, t \right]^{-1/2} dt$$

$$s = \frac{1}{1/2} \left[\frac{1}{(60)^2} + 0.8\, t \right]^{1/2} \frac{1}{0.8} \Bigg|_0^t = \frac{2}{0.8} \left[\frac{1}{(60)^2} + 0.8\, t \right]^{1/2} \Bigg|_0^t$$

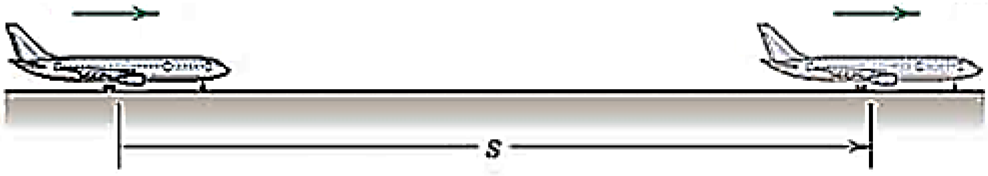
$$s = \frac{1}{0.4} \left\{ \left[\frac{1}{(60)^2} + 0.8\, t \right]^{1/2} - \frac{1}{60} \right\} = \frac{1}{0.4} \left\{ \sqrt{\frac{1}{(60)^2} + 0.8\, t} - \frac{1}{60} \right\} \text{ m}$$

At (t = 4 s): $\quad s = 4.43$ m

Example 1 - 8:

On its take-off roll, the airplane starts from rest and accelerates at (a = 1.8 m/s^2), determine the design length of runway required for the airplane to reach the take-off speed of (250 km/h).

$v_o = 0$ $\qquad\qquad\qquad\qquad$ $v = 250$ km/h



Solution I:

$$v = 250 \text{ km/h} = \frac{250}{3.6} = 69.44 \text{ m/s}$$

$$v = v_o + a_c\, t$$

$$69.44 = 0 + 1.8\, t \quad \Rightarrow \quad t = \frac{69.44}{1.8} = 38.58 \text{ s}$$

$$s = s_o + v_o\, t + \frac{1}{2}\, a_c\, t^2$$

$$s = 0 + 0 + \frac{1}{2}(1.8)(38.58)^2 = 1340 \text{ m} = 1.34 \text{ km}$$

Solution II:

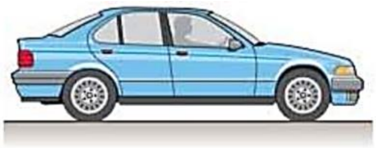
$$v^2 = v_o^2 + 2\, a_c\, (\, s - s_o\,)$$

$$(69.44)^2 = 0^2 + 2\,(1.8)\,(\, s - 0\,) \quad \Rightarrow \quad (69.44)^2 = 3.6\, s$$

$$s = 1340 \text{ m} = 1.34 \text{ km}$$

Example 1 - 9:

An automobile starting from rest, speeds up to (40 ft/s) with a constant acceleration of (4 ft/s²), runs at this speed for a time, and finally comes to rest with a deceleration of (5 ft/s²). If the total distance traveled is (1000 ft), find the total time required.



Solution:

t = 0

v = 0   a = 4 ft/s²   v = 40 ft/s   a = 0   v = 40 ft/s   a = -5 ft/s²   v = 0

```
+----------------------------+-----------------------------------+--------------------------------+
A                            B                                   C                                D
```

First stage

$$v = v_o + a_c\, t$$

$40 = 0 + 4\, t_1 \quad \Rightarrow \quad t_1 = 10$ s

$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2$$

$$s_1 = 0 + 0 + \frac{1}{2}(4)(10)^2 = 200 \text{ ft}$$

Final stage

$$v = v_o + a_c\, t$$

$0 = 40 - 5\, t_3 \quad \Rightarrow \quad t_3 = 8$ s

$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2$$

$$s_3 = 0 + (40)(8) - \frac{1}{2}(5)(8)^2 = 160 \text{ ft}$$

Mid. stage

$s_2 = 1000 - 200 - 160 = 640$ ft

$$s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2$$

$s_2 = 0 + 40\, t + \frac{1}{2}(0){t_2}^2 \quad \Rightarrow \quad s_2 = 40\, t_2$

$640 = 40\, t_2 \quad \Rightarrow \quad t_2 = 16$ s

Final result

$t_T = t_1 + t_2 + t_3 = 10 + 16 + 8 = 34$ s

t = 0   t =10 s   t =16 s   t =8 s

v = 0   a = 4 ft/s²   v = 40 ft/s   a = 0   v = 40 ft/s   a = -5 ft/s²   v = 0

```
+------------------------------+------------------------------------+-------------------------------+
A                              B                                    C                               D
```

S=200m   S=640m   S=160

Example 1 - 10:

A particle is moving along a straight line such that its position is defined by [$s = (10t^2 + 20)$ mm], where (t) is in seconds. Determine:

(a) the displacement of the particle during the time interval from (t = 1 s) to (t = 5 s).
(b) the average velocity of the particle during this time interval.
(c) the acceleration when (t = 1 s).

Solution:

$$s = 10\,t^2 + 20$$

(a) at (t = 1 s), $s = 10(1)^2 + 20 = 30$ mm

at (t = 5 s), $s = 10(5)^2 + 20 = 270$ mm

$$\Delta s = 270 - 30 = 240 \text{ mm}$$

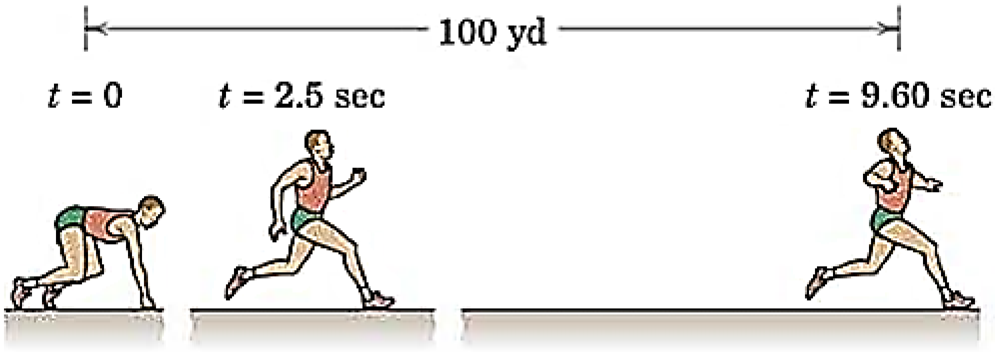
(b) $\Delta t = 5 - 1 = 4$ s

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{240}{4} = 60 \text{ mm/s}$$

(c) $a = \dfrac{d^2s}{dt^2} = 20 \text{ mm/s}^2$

Example 1 - 11:

A sprinter reaches his maximum speed (v_{max}) in (2.5 s) from rest with constant acceleration. He then maintains that speed and finishes the (100 yards) in the overall time of (9.6 s). Determine his maximum speed (v_{max}).



Solution:

$$v = v_o + a_c t$$

$$v_{max} = 0 + a_c (2.5) = 2.5 a_c$$

$$s = s_o + v_o t + \frac{1}{2} a_c t^2$$

$$s = 0 + 0 + \frac{1}{2} a_c (2.5)^2 = 3.125 a_c$$

$$s = s_o + v_o t + \frac{1}{2} a_c (2.5)^2$$

$$100 = s + v_{max} (9.6 - 2.5) + 0$$

$$v_{max} = \frac{100-s}{9.6-2.5} = \frac{100-3.125a_c}{7.1}$$

$$2.5 a_c = \frac{100-3.125a_c}{7.1}$$

$$17.75 a_c = 100 - 3.125 a_c$$

$$20.875 a_c = 100 \quad \Rightarrow \quad a_c = 4.79 \text{ y/s}^2$$

$$v_{max} = 2.5 a_c = 2.5 (4.79) = 11.976 \text{ y/s} = 11.976 \times 3 = 35.928 \text{ ft/s}$$

Example 1 - 12:

As a space probe carrier rocket is projected to a high altitude above the earth's surface, the variation of the acceleration of gravity with respect to altitude (y) must be taken into account. Neglecting air resistance, this acceleration is determined from the formula { $a = -g_o [R^2 / (R + y)^2]$ }, where (g_o) is the constant gravitational acceleration at sea level, (R) is the radius of the earth, and the positive direction is measured upward. If ($g_o = 9.81$ m/s^2) and (R = 6356 km), determine the minimum initial velocity (escape velocity) at which a rocket should be shot vertically from the earth's surface so that it does not fall back to the earth. Hint: This requires that (v = 0) as ($y \to \infty$).

Solution:

$$v\,dv = a\,dy$$

$$\int_v^0 v\,dv = \int_0^\infty -g_o \frac{R^2}{(R+y)^2}\,dy = -g_o R^2 \int_0^\infty \frac{dy}{(R+y)^2}$$

$$= -g_o R^2 \int_0^\infty (R+y)^{-2} dy$$

$$0 - \frac{v^2}{2} = -g_o R^2 \left(\frac{(R+y)^{-1}}{-1 \times 1} \right) \Big|_0^\infty$$

$$-\frac{v^2}{2} = g_o R^2 (R+y)^{-1} \Big|_0^\infty = \frac{g_o R^2}{R+y} \Big|_0^\infty$$

$$-\frac{v^2}{2} = \frac{g_o R^2}{R+\infty} - \frac{g_o R^2}{R+0} = -\frac{g_o R^2}{R} \quad \Rightarrow \quad \frac{v^2}{2} = \frac{g_o R^2}{R}$$

$$\frac{v^2}{2} = g_o R \quad \Rightarrow \quad v^2 = 2g_o R$$

$$v = \sqrt{2g_o R} = \sqrt{2(9.81)(6356)(10)^3} = 11167 \text{ m/s} = 11.2 \text{ km/s}$$

PROBLEMS:

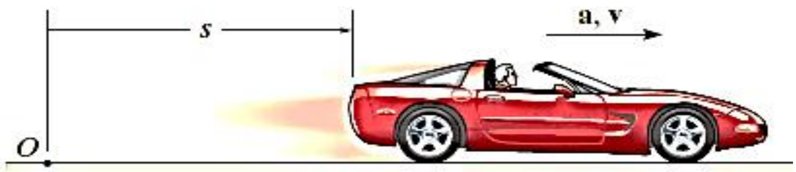
(1 - 1):

Starting from rest, a motorcycle moving in a straight line has an acceleration of [$a = (2t - 6)\ m/s^2$], where (t) in seconds. What is the motorcycle's velocity when (t = 6 s), and what is its position when (t = 11 s)?

Ans. : (v = 0) , (s = 80.7 m)

(1 - 2):

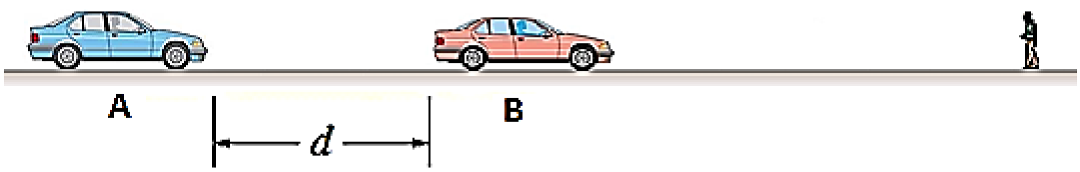
The car in the figure below moves in a straight line such that for a short time its velocity is defined by [$v = (3t^2 + 2t)$ ft/s], where (t) is in seconds. Determine its position and acceleration when (t = 3 s). When (t = 0), (s = 0).



Ans. : (s = 36 ft) , ($a = 20\ ft/s^2$)

(1 - 3):

Car (B) is traveling at distance (d) ahead of car (A). Both cars are traveling at (60 ft/s), when the driver of (B) suddenly applies the brakes causing his car to decelerate at ($12\ ft/s^2$). It takes the driver of car (A) (0.75 s) to react (this is the normal reaction time for drivers). When he applies his brakes, he decelerates at ($15\ ft/s^2$). Determine the minimum distance (d) between the cars so as to avoid a collision.



Ans. : (d = 16.9 ft)

(1 - 4):

The position of a bicycle along a straight line is given by [$S = (1.5\ t^3 - 13.5\ t^2 + 22.5\ t)$ ft], where (t) in seconds. Determine the position of the bicycle when (t = 6 s), and the total distance it travels during the (6) seconds time intervals.

Ans. : (s = – 27 ft) , ($s_T = 69$ ft)

(1 - 5):

A particle travels along a straight line with a constant acceleration. When (s = 4 ft), (v = 3 ft/s) and when (s = 10 ft), (v = 8 ft/s). Determine the velocity as a function of position.

Ans. : $v = \sqrt{9.166\, s - 27.664}$ ft/s

(1 - 6):

A bicycle travels along a straight line with a velocity [$v = (12 - 3t^2)$ m/s], where (t) is in seconds. When (t = 1 s), the bicycle is located (10 m) to the left of the origin. Determine the acceleration when (t = 4 s), the displacement from (t = 0) to (t = 10 s), and the distance the bicycle travels during this time period.

Ans. : ($a = -24$ m/s^2) , ($\Delta s = -880$ m) , ($s_T = 912$ m)

(1 - 7):

The velocity of a particle traveling in a straight line is given by [$v = (6t - 3t^2)$ m/s], where (t) is in seconds. If (s = 0) when (t = 0), determine the particle's deceleration and position when (t = 3 s). How far has the particle traveled during the (3 s) time interval, and what is its average speed?

Ans. : ($a = -12$ m/s^2) , ($s = 0$) , ($s_T = 8$ m) , ($(v_{sp})_{avg} = 2.67$ m/s)

(1 – 8):

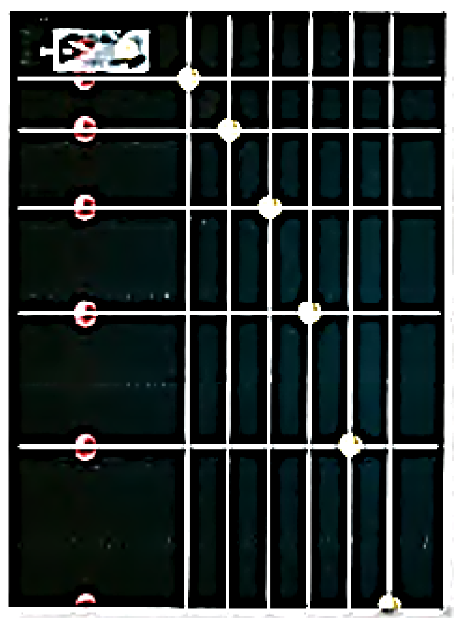
A train starts from rest at station (A) and accelerates at (0.5 m/s^2) for (60 s). Afterwards it travels with a constant velocity for (15 min). It then decelerates at (1 m/s^2) until it is brought to rest at station (B). Determine the distance between the stations.

Ans. ($s = 28.4$ km)

Part 1
Kinematics of a particle

Chapter 3
Motion of projectiles

The free-flight motion of a projectile is often studied in terms of its rectangular components. To illustrate the kinematic analysis, consider a projectile launched at point (x_o , y_o), with an initial velocity of ($\mathbf{v}_o$), having components $(\mathbf{v}_o)_x$ and $(\mathbf{v}_o)_y$. When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a *constant downward acceleration* of approximately ($a_c = g = 9.81\ \mathrm{m/s^2}$) or ($g = 32.2\ \mathrm{ft/s^2}$).

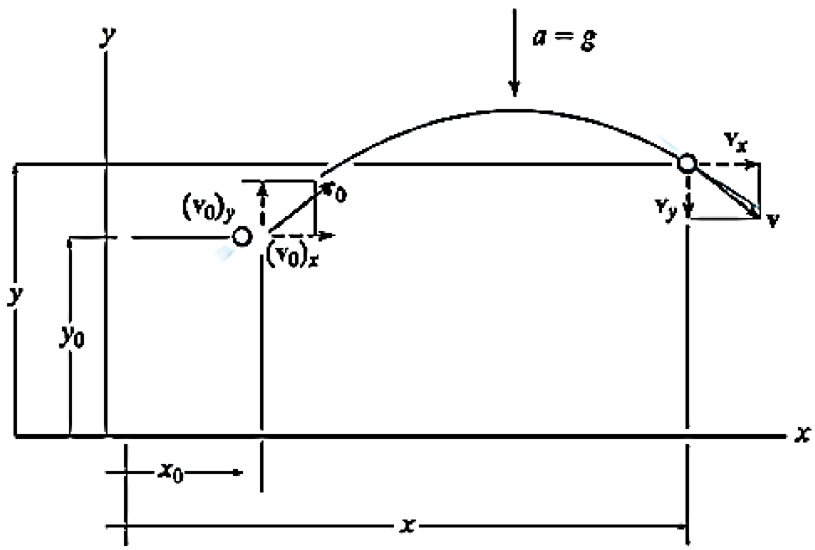


Each picture in this sequence is taken after the same time interval. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity when released.

Both balls accelerate downward at the same rate, and so they remain at the same elevation at any instant. This acceleration causes the difference in elevation between the balls to increase between successive photos.

Also, note the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction remains constant.

(R.C. Hibbeler)



Horizontal Motion.
Since $a_x = 0$, application of the constant acceleration equations, (4 – 6), yields:

$$+\rightarrow \quad v = v_o + a_c\, t \qquad v_x = (v_o)_x$$
$$+\rightarrow \quad s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2 \qquad x = x_o + (v_o)_x\, t$$
$$+\rightarrow \quad v^2 = v_o^2 + 2\, a_c\, (\, s - s_o\,) \qquad v_x = (v_o)_x$$

Vertical Motion.
Since the positive y axis is directed upward, then $a_y = -\, g$. Applying Eqs. (4–6), we get:

$$+\uparrow \quad v = v_o + a_c\, t \qquad v_y = (v_o)_y - g\, t$$
$$+\uparrow \quad s = s_o + v_o\, t + \frac{1}{2} a_c\, t^2 \qquad y = y_o + (v_o)_y\, t - \frac{1}{2} g\, t^2$$
$$+\uparrow \quad v^2 = v_o^2 + 2\, a_c\, (\, s - s_o\,) \qquad v_y^2 = (v_o)_y^2 - 2\, g\, (\, y - y_o\,)$$

Recall that the last equation can be formulated on the basis of eliminating the time t from the first two equations, and therefore *only two of the above three equations are independent of one another.*

To summarize, problems involving the motion of a projectile can have at most three unknowns since only three independent equations can be written; that is, *one* equation in the *horizontal direction* and *two* in the *vertical direction.* Once $\mathbf{v}_x$ and $\mathbf{v}_y$ are obtained, the resultant velocity $\mathbf{v}$, which is *always tangent* to the path, can be determined by the *vector sum*

Once thrown, the basketball follows a parabolic trajectory. (© R.C. Hibbeler

Gravel falling off the end of this conveyor belt follows a path that can be predicted using the equations of constant acceleration. In this way the location of the accumulated pile can be determined.
Rectangular coordinates are used for the analysis since the acceleration is only in the vertical direction. (© R.C. Hibbeler)

Procedure for Analysis

Coordinate System.

- Establish the fixed *x, y* coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns*. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2 or 32.2 ft/s^2. The particle's initial and final velocities should be represented in terms of their *x* and *y* components.
- Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

Kinematic Equations.

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.

Horizontal Motion.

□ The *velocity* in the horizontal or *x* direction is *constant*, i.e., $v_x = (v_0)_x$, and $x = x_o + (v_o)_x\, t$

Vertical Motion.

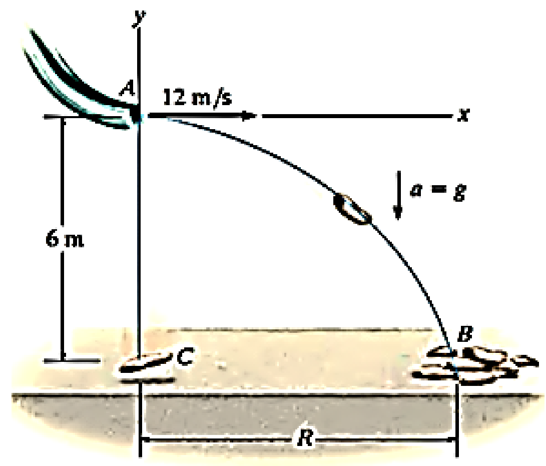
□ In the vertical or y direction only two of the following three equations can be used for solution.

$$v_y = (v_o)_y - g\,t$$
$$y = y_o + (v_o)_y\, t - 1/2\, g\, t^2$$
$$v_y^2 = (v_o)_y^2 - 2g(\, y - y_o)$$

For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.

Example 3 - 1:

A sack slides off the ramp, shown in the figure, with a horizontal velocity of (12 m/s). If the height of the ramp is (6 m) from the floor, determine the time needed for the sack to strike the floor and the range (R) where sacks begin to pile up.



SOLUTION:

Coordinate System:

The origin of coordinates is established at the beginning of the path, point *A*. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81$ m/s^2. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, *R*, and the time of flight (t_{AB}). Here we do not need to determine $(v_B)_y$.

Vertical Motion:

The vertical distance from *A* to *B* is known, and therefore we can obtain a direct solution for t_{AB} by using the equation:

$$y_B = y_A + (v_A)_y\, t_{AB} - \frac{1}{2}\, g\, (t_{AB})^2$$

$$-6 = 0 + 0 - \frac{1}{2}\,(9.81)\,(t_{AB})^2$$

$$t_{AB} = 1.11 \text{ s}$$

Horizontal Motion:

Since t_{AB} has been calculated, *R* is determined as follows:

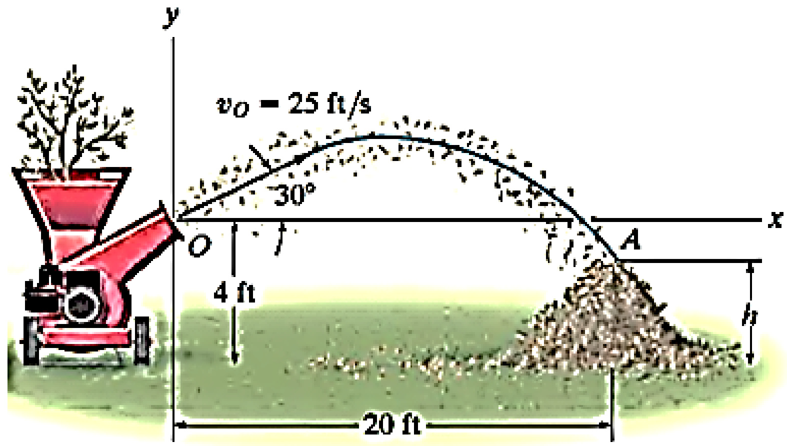
$$x_B = x_A + (v_A)_x\, t_{AB}$$

$$R = 0 + 12\,(1.11) = 13.3 \text{ m}$$

NOTE: The calculation for t_{AB} also indicates that if a sack were released *from rest* at *A*, it would take the same amount of time to strike the floor at *C*.

Example 3 - 2:

The chipping machine is designed to eject wood chips at ($v_o = 25$ ft/s) as shown in the figure below. If the tube is oriented at (30°) from the horizontal, determine how high (h) the chips strike the pile if at this instant they land on the pile (20 ft) from the tube.



SOLUTION:

Coordinate system:

When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(V_A)_x = (V_O)_x$]. With the origin of coordinates at O, the initial velocity of a chip has components of

$$(V_o)_x = (25 \cos 30°) = 21.65 \text{ ft/s} \rightarrow$$
$$(V_o)_y = (25 \sin 30°) = 12.5 \text{ ft/s} \uparrow$$

Also, $(V_A)_x = (V_o)_x = 21.65$ ft/s , and $g = -32.2$ ft/s^2.

Since we do not need to determine $(V_A)_y$, we have:

Horizontal motion:

$+\rightarrow$

$$x_A = x_o + (V_o)_x\, t_{OA}$$
$$20 = 0 + (21.65)\, t_{OA}$$
$$t_{OA} = 0.9238 \text{ s}$$

Vertical motion:

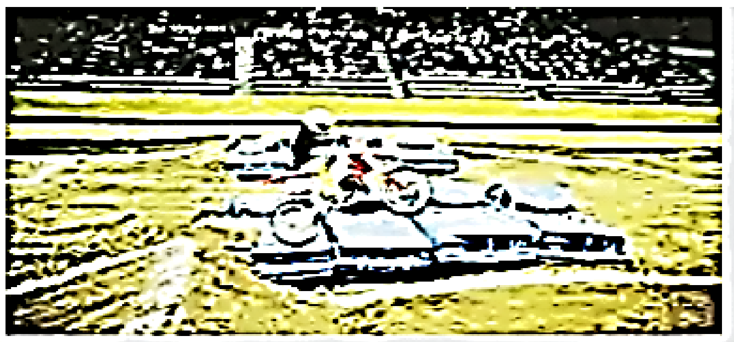
$+\uparrow$

$$y_A = y_o + (V_o)_y\, t_{OA} - \frac{1}{2}\, g\, t_{OA}^2$$
$$-(4 - h) = (h - 4) = 0 + (12.5)\,(0.9238) - \frac{1}{2}\,(32.2)\,(0.9238)^2$$
$$h = 1.81 \text{ ft}$$

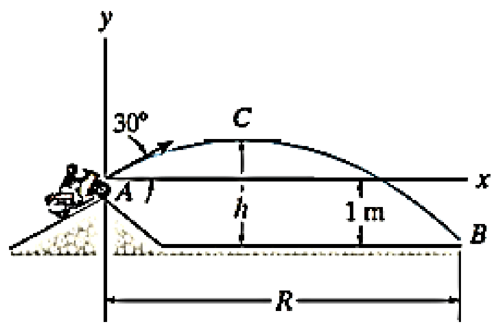
Note: We can determine $(V_A)_y$ by using $[(V_A)_y = (V_o)_y - g\, t_{OA}]$.

Example 3 - 3:

The track for this racing event was designed so that riders jump off the slope at (30°) from a height of (1 m). During a race it was observed that the rider shown in Figure, remained in mid air for (1.5 s). Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.



SOLUTION:



Coordinate System:

The origin of the coordinates is established at A. Between the end points of the path *AB* the three unknowns are the initial speed v_A, range R, and the vertical component of velocity $(v_B)_y$.

Vertical Motion:

Since the time of flight and the vertical dista

$+\uparrow$

$$y_B = y_A + (v_A)_y\, t_{AB} - \frac{1}{2}\, g\, t_{AB}^2$$

$$-1 = 0 + v_A \sin 30^\circ\,(1.5) - \frac{1}{2}\,(9.81)\,(1.5)^2$$

$$v_A = 13.38 \text{ m/s}$$

Horizontal Motion:

The range *R* can now be determined.

$+\rightarrow$

$$x_B = x_A + (v_A)_x\, t_{AB}$$

$$R = 0 + 13.38 \cos 30^\circ\,(1.5) = 17.4 \text{ m}$$

In order to find the maximum height *h* we will consider the path *AC*. Here the three unknowns are the time of flight t_{AC}, the horizontal distance from *A* to *C*, and the height *h*. At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine *h directly* without considering t_{AC} using the following equation.

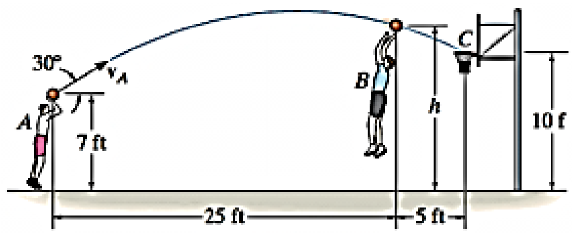
$$(v_C)_y^2 = (v_A)_y^2 - 2\, g\, [\, y_C - y_A\,]$$

$$0 = (\,13.38 \sin 30^\circ\,)^2 - 2\,(9.81)\,[\,(h-1) - 0\,]$$

$$h = 3.28 \text{ m}$$

Example 3 - 4:

The basketball passed through the hoop even though it barely cleared the hands of the player (B) who attempted to block it. Neglecting the size of the ball, determine the magnitude of its initial velocity (v_A) and the height (h) of the ball when it passes over player (B).



SOLUTION:

$(+ \rightarrow)$ $x = x_o + (v_o)_x\ t$

$30 = 0 + v_A \cos 30\ t_{AC}$

$$t_{AC} = \frac{30}{0.866\ v_A} = \frac{34.64}{v_A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

$(+ \uparrow)$ $y = y_o + (v_o)_y - 0.5\ g\ t^2$

$10 = 7 + v_A \sin 30\ t_{AC} - 0.5\ (\ 32.2\ )\ t_{AC}^2$

$16.1\ (\ t_{AC}\ )^2 - 0.5\ v_A\ t_{AC} + 3 = 0 \quad \ldots\ldots\ldots\ldots\ (2)$

Sub. Eq. (1) in (2)

$$16.1\ (\ \frac{34.64}{v_A}\ )^2 - 0.5\ v_A\ \frac{34.64}{v_A} + 3 = 0$$

$$\frac{19318.87}{v_A{}^2} - 14.32 = 0 \quad \Rightarrow \quad \frac{19318.87}{v_A{}^2} = 14.32$$

$$v_A{}^2 = \frac{19318.87}{14.32} = 1349.08 \quad \Rightarrow \quad v_A = 36.73 \text{ ft/s}$$

$$t_{AC} = \frac{34.64}{36.73} = 0.943 \text{ s}$$

$(+ \rightarrow)$ $x = x_o + (v_o)_x\ t$

$25 = 0 + 36.73 \cos 30°\ t_{AB}$

$t_{AB} = 0.786$ s

$(+ \uparrow)$ $y = y_o + (v_o)_y - 0.5\ g\ t^2$

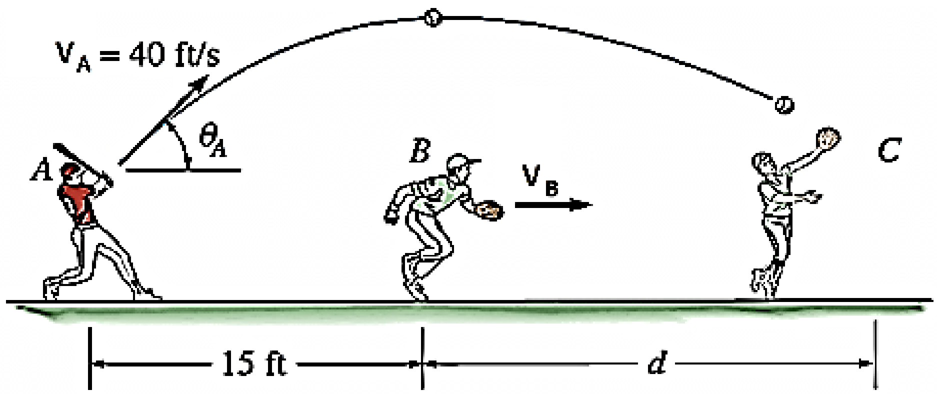
$$h = 7 + 36.73 \sin 30°\ t_{AB} - \frac{1}{2}(32.2)(\ t_{AB})^2$$

$$= 7 + 36.73 \sin 30°\ (0.786) - \frac{1}{2}(32.2)\ (0.786)^2$$

$= 11.5$ ft

<u>Example 3 - 5:</u>

The baseball player (A) hits the baseball at (v_A = 40 ft/s) and (θ_A = 60°) from the horizontal. When the ball is directly overhead of player (B) he begins to run under it. Determine the constant speed at which (B) must run and the distance (d) in order to make the catch at the same elevation at which the ball was hit.



SOLUTION:

y - motion: Here, $(v_o)_y = 40 \sin 60° = 34.64$ ft/s , [$y_o = 0$] and [$y = 0$].

$+\uparrow$

$$y = y_o + (v_o)_y\, t - \frac{1}{2}\, g\, t^2$$

$$0 = 34.64\, t - \frac{1}{2}\,(32.2)\, t^2$$

$$t = 2.152 \text{ s}$$

x - motion: Here, $(v_o)_x = 40 \cos 60° = 20$ ft/s , [$x_o = 0$] and [$X = R$].

$+\rightarrow$

$$x = x_o + (v_o)_x\, t$$

$$R = 0 + 20\,(2.152) = 43.03 \text{ ft}$$

The distance for which player (B) must travel in order to catch the baseball is:

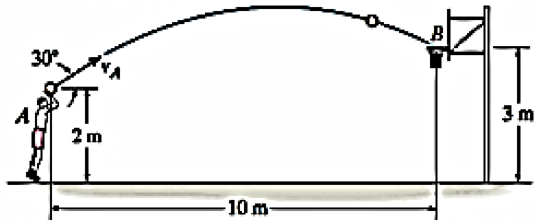
$$d = R - 15 = 43.03 - 15 = 28.03 \text{ ft}$$

Player (B) is required to run at a same speed as the horizontal component of velocity of the baseball in order to catch it.

$$v_B = 20 \text{ ft/s}$$

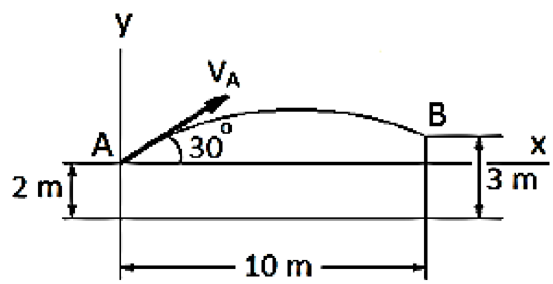
Example 3 - 6:

Neglecting the size of the ball, determine the magnitude of initial velocity (v_A) of the basketball and its velocity when it passes through the basket.



SOLUTION:

x - motion: Here, $(v_A)_x = v_A \cos 30°$, [$x_A = 0$] and [$x_B = 10$ m]. Thus,



$+\rightarrow$
$$x_B = x_A + (v_A)_x\, t$$
$$10 = 0 + (v_A \cos 30°)\, t$$
$$t = \frac{10}{v_A \cos 30}$$
$$t = \frac{10}{0.866\ v_A} = \frac{11.55}{v_A} \quad \text{...... (1)}$$

Also, $(v_B)_x = (v_A)_x = v_A \cos 30°$
$$(v_B)_x = 0.866\ v_A \quad \text{....... (2)}$$

y - motion: Here, $(v_A)_y = v_A \sin 30°$, [$y_A = 0$] and [$y_B = 3 - 2 = 1$ m], and ($g = 9.81$ m/s^2).

$+\uparrow$
$$y_B = y_A + (v_A)_y\, t - \frac{1}{2}\, g\, t^2$$
$$1 = 0 + (v_A \sin 30°)\, t - \frac{1}{2}\, (9.81)\, t^2$$
$$4.905\, t^2 - 0.5\, v_A\, t + 1 = 0 \quad \text{....... (3)}$$

Also,

$+\uparrow$
$$(v_B)_y = (v_A)_y - g\, t$$
$$(v_B)_y = v_A \sin 30° - (9.81)\, t$$
$$(v_B)_y = 0.5\, v_A - 9.81\, t \quad \text{....... (4)}$$

Substitute Eq. (1) in Eq. (3)

$$4.905\, \left(\frac{11.55}{v_A}\right)^2 - 0.5\, v_A\, \frac{11.55}{v_A} + 1 = 0$$
$$\frac{654.34}{v_A{}^2} - 5.775 + 1 = 0$$
$$\frac{654.34}{v_A{}^2} = 4.775$$

$$v_A^{\,2} = \frac{654.34}{4.775} = 137.03$$
$$v_A = 11.705 \text{ m/s}$$

Substitute in Eq. (1)

$$t = \frac{11.55}{v_A} = \frac{11.55}{11.705} = 0.987 \text{ s}$$

Substitute these results in Eq. (2) and Eq. (4)

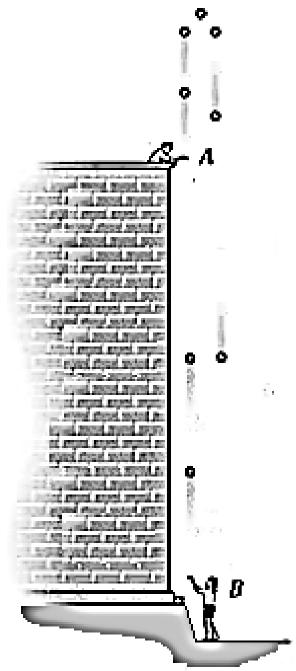
$$(v_B)_x = 0.866\,(11.705) = 10.14 \text{ m/s} \rightarrow$$
$$(v_B)_y = 0.5\,(11.705) - 9.81\,(0.987) = -3.83 = 3.83 \text{ m/s} \downarrow$$

Thus, the magnitude and direction of (v_B) is:

$$v_B = \sqrt{(v_B)_x^{\,2} + (v_B)_y^{\,2}} = \sqrt{(10.14)^2 + (3.83)^2} = 10.8 \text{ m/s}$$
$$\theta_B = \tan^{-1}\frac{(v_B)_y}{(v_B)_x} = \tan^{-1}\frac{3.83}{10.14} = 20.7^\circ$$

Example 3 - 7:

A ball (A) is thrown vertically upward from the top of a (30 m) high building with an initial velocity of (5 m/s). At the same instant another ball (B) is thrown upward from the ground with an initial velocity of (20 m/s). Determine the height from the ground and the time at which they pass.



SOLUTION:

Origin at roof:

Ball A:

$+\uparrow$

$$y = y_o + (v_o)_y\, t - \frac{1}{2} g\, t^2$$

$$h = 30 + 5\,t - \frac{1}{2}\,(9.81)\, t^2$$

$$h = 30 + 5\,t - 4.905\, t^2 \quad \ldots\ldots \quad (1)$$

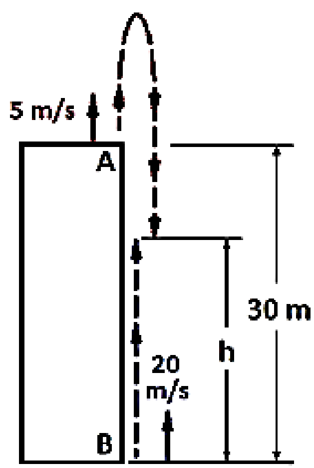
Ball B:

$+\uparrow$

$$y = y_o + (v_o)_y\, t - \frac{1}{2} g\, t^2$$

$$h = 0 + 20\,t - \frac{1}{2}\,(9.81)\, t^2$$

$$h = 0 + 20\,t - 4.905\, t^2 \quad \ldots\ldots \quad (2)$$



$$h = 0 \quad + 20\,t - 4.905\, t^2$$

$$h = 30 \quad + 5\,t \quad - 4.905\, t^2$$

-- Subtraction

$$0 = -30 + 15\,t$$

$$15\,t = 30 \qquad \Rightarrow \qquad t = 2\ \text{s}$$

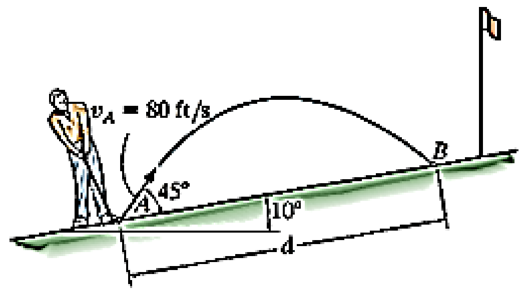
Sub. in (1)

$$h = 30 + 5\,t - 4.905\, t^2$$

$$h = 30 + 5\,(2) - 4.905\,(2)^2 = 20.38\ \text{m}$$

Example 3 - 8:

A golf ball is struck with a velocity of (80 ft/s) as shown. Determine the distance (d) to where it will land.



SOLUTION:

x - motion: $(v_o)_x = 80\cos 55° = 45.89$ ft/s

$x_o = 0 \qquad x = d\cos 10°$

$(+\rightarrow)$

$$x = x_o + (v_o)_x\, t$$

$$d\cos 10° = 0 + 45.89\, t$$

$$0.985\, d = 45.89\, t \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

y - motion: $(v_o)_y = 80\sin 55° = 65.53$ ft/s

$y_o = 0 \qquad y = d\sin 10°$

$(+\uparrow)$

$$y = y_o + (v_o)_y\, t + \frac{1}{2}(a_c)_y\, t^2$$

$$d\sin 10° = 0 + 65.53\, t + \frac{1}{2}(-32.2)\, t^2$$

$$0.174\, d = 65.53\, t - 16.1\, t^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

From Eq. (1): $d = \dfrac{45.89\, t}{0.985} = 46.6\, t$

Sub. in Eq. (2) $0.174\,(46.6\, t) = 65.53\, t - 16.1\, t^2$

$$8.1\, t = 65.53\, t - 16.1\, t^2$$

$$16.1\, t^2 - 57.43\, t = 0$$

$$t\,(16.1\, t - 57.43) = 0$$

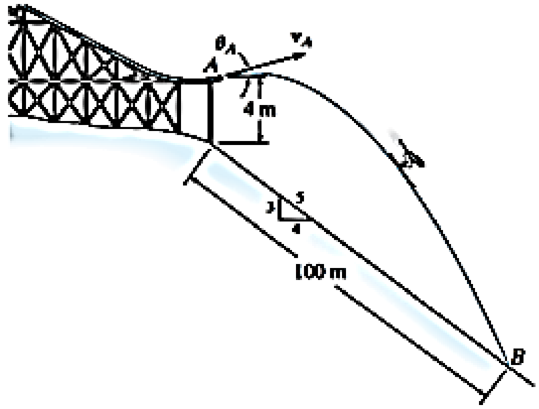
$$t = 0 \quad \text{or} \quad 16.1\, t - 57.43 = 0$$

$$16.1\, t = 57.43 \quad \Rightarrow \quad t = \frac{57.43}{16.1} = 3.57 \text{ s}$$

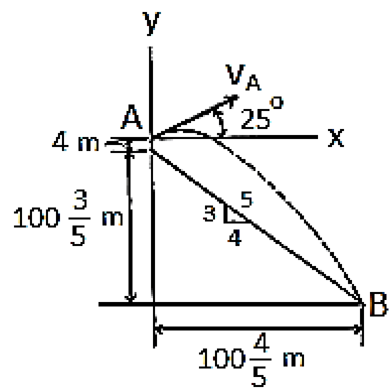
$$d = 46.6\, t = 46.6\,(3.57) = 166.2 \text{ ft}$$

Example 3 - 9:

It is observed that the skier leaves the ramp (A) at an angle ($\theta_A = 25^\circ$) with the horizontal. If he strikes the ground at (B), determine his initial speed (v_A) and the time of flight (t_{AB}).



SOLUTION:



($+\rightarrow$) $\quad x = x_o + (v_o)_x\, t$

$$100\,\frac{4}{5} = 0 + v_A \cos 25^\circ\, t_{AB}$$

$$80 = 0.9\,(v_A)\,(t_{AB}) \;\ldots\ldots\; (1)$$

($+\uparrow$) $\quad y = y_o + (v_o)_y\, t - \frac{1}{2}\, g\, t^2$

$$-(4 + 100\,\frac{3}{5}) = 0 + v_A \sin 25^\circ\, t_{AB} - \frac{1}{2}\,(9.81)\, t_{AB}^{\,2}$$

$$-64 = 0.42\,(v_A)\,(t_{AB}) - 4.905\, t_{AB}^{\,2} \;\ldots\ldots\ldots\; (2)$$

From Eq. (1): $\quad v_A = \dfrac{80}{0.9\, t_{AB}} = \dfrac{88.9}{t_{AB}}$

Sub. in Eq. (2) $\quad -64 = 0.42\,(\dfrac{88.9}{t_{AB}})\,(t_{AB}) - 4.905\, t_{AB}^{\,2}$

$$-64 = 37.3 - 4.905\, t_{AB}^{\,2}$$

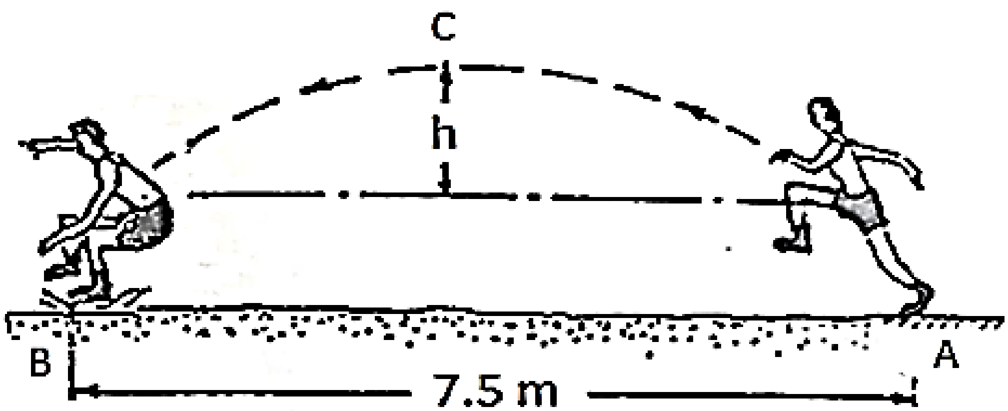
$$4.905\, t_{AB}^{\,2} = 101.3 \quad \Rightarrow \quad t_{AB}^{\,2} = \frac{101.3}{4.905} = 20.66$$

$$t_{AB} = 4.55\ \text{s}$$

$$v_A = \frac{88.9}{4.55} = 19.56\ \text{m/s}$$

Example 3 - 10:

The horizontal component of the initial velocity (v_o)$_x$ for the jumping player shown in the figure is (10 m/s). Calculate the vertical component of his initial velocity (v_o)$_y$ at point (A) for obtaining a horizontal distance of (7.5 m), and find the height (h).



SOLUTION - I:

A → B

$x = x_o + (v_o)_x\, t$

$7.5 = 0 + 10\, t$

$t = 0.75$ s

A → C

$v_y = (v_o)_y - g\, t$

$0 = (v_o)_y - (9.81)\,(0.375)$

$(v_o)_y = 3.68$ m/s

$y = y_o + v_o\, t - \frac{1}{2}\, g\, t^2$

$h = 0 + (3.68)\,(0.375) - \frac{1}{2}\,(9.81)\,(0.375)^2$

$h = 0.69$ m

SOLUTION - I:

A → B

$x = x_o + (v_o)_x\, t$

$7.5 = 0 + 10\, t$

$t = 0.75$ s

$v_y = (v_o)_y - g\, t$

$(v_B)_y = (v_A)_y - g\, t$

$(v_B)_y = -\,(v_A)_y$

$-\,(v_A)_y = (v_A)_y - g\, t$

$-\,(v_A)_y - (v_A)_y = (9.81)\,(0.75)$

$-\,2(v_A)_y = -\,7.32$

$(v_A)_y = 3.67$ m/s

A → C

$v_y^2 = (v_o)_y^{\,2} - 2\, g\, (\, y - y_o\,)$

$(v_C)_y^2 = (v_A)_y^2 - 2g\, (\, y_C - y_A\,)$

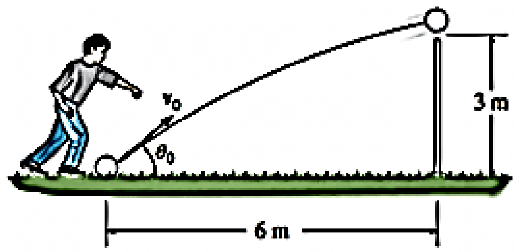
$0 = (3.67)^2 - 2 \times 9.81\, (\, h - 0\,)$

$0 = 13.46 - 19.62\, h$

$h = 0.68$ m

Example 3 - 11:

Determine the minimum initial velocity (v_o) and the corresponding angle (θ_o) at which the ball must be kicked in order for it to just cross over the (3 m) high fence.



SOLUTION:

Coordinate System:

The coordinate system will be set so that its origin coincides with the ball's initial position.

x - motion: Here, [$(v_o)_x = v_o \cos\theta$] , ($x_o = 0$) and ($x = 6$ m). Thus,

$+\rightarrow$

$$x = x_o + (v_o)_x\, t$$
$$6 = 0 + (v_o \cos\theta)\, t$$
$$t = \frac{6}{v_0 \cos\theta} \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

y - motion: Here, [$(v_o)_y = v_o \sin\theta$] , ($a_y = -g = -9.81$ m/s^2) and ($y_o = 0$). Thus,

$+\uparrow$

$$y = y_o + (v_o)_y\, t - \frac{1}{2}\, g\, t^2$$
$$3 = 0 + (v_o \sin\theta)\, t - \frac{1}{2}\,(9.81)\, t^2$$
$$3 = (v_o \sin\theta)\, t - 4.905\, t^2 \quad \ldots\ldots\ldots\ldots\ldots\ (2)$$

Sub. Eq. (1) in Eq. (2)

$$3 = v_0\ (\sin\theta)\, \frac{6}{v_0 \cos\theta} - 4.905\, \left(\frac{6}{v_0 \cos\theta}\right)^2$$
$$3 = (\sin\theta)\, \frac{6}{\cos\theta} - \frac{176.58}{v_0^{\,2} \cos^2\theta}$$
$$3 - (\sin\theta)\, \frac{6}{\cos\theta} = -\frac{176.58}{v_0^{\,2} \cos^2\theta}$$
$$\left(3 - 6\,\frac{\sin\theta}{\cos\theta}\right)\, v_0^{\,2} \cos^2\theta = -176.58$$
$$\left(3 \cos^2\theta - 6\,\frac{\sin\theta}{\cos\theta}\, \cos^2\theta\right)\, v_0^{\,2} = -176.58$$
$$3\, (\cos^2\theta - 2 \sin\theta \cos\theta)\, v_0^{\,2} = -176.58$$
$$3\, (\cos^2\theta - \sin 2\theta)\, v_0^{\,2} = -176.58 \qquad \times\ -1$$
$$3\, (\sin 2\theta - \cos^2\theta)\, v_0^{\,2} = 176.58$$

$$v_0{}^2 = \frac{176.58}{3\,(\sin 2\theta - cos^2\theta)} = \frac{58.86}{(\sin 2\theta - cos^2\theta)}$$

$$v_0 = \sqrt{\frac{58.86}{(\sin 2\theta - cos^2\theta)}} \quad \text{..........................} \quad (3)$$

From Eq. (3), we notice that (v_o) is minimum when [$f(\theta) = \sin 2\theta - \cos^2\theta$] is maximum. This requires $\left(\frac{d\,f(\theta)}{d\theta} = 0 \right)$

$$f(\theta) = \sin 2\theta - \cos^2\theta = 2\sin\theta\,\cos\theta - \cos^2\theta$$

$$\frac{d\,f(\theta)}{d\theta} = 2\,[\,\sin\theta(-\sin\theta) + \cos\theta\,(\cos\theta)\,] - [\,2\cos\theta\,(-\sin\theta\,)\,]$$

$$= 2\,(-\sin^2\theta + \cos^2\theta\,) - (-2\sin\theta\,\cos\theta)$$

$$= 2\,(\cos^2\theta - \sin^2\theta\,) + 2\sin\theta\,\cos\theta = 2\cos 2\theta + \sin 2\theta$$

$$2\cos 2\theta + \sin 2\theta = 0$$

$$\sin 2\theta = -2\cos 2\theta \qquad \div\ \cos 2\theta$$

$$\tan 2\theta = -2$$

$$2\theta = 116.57^\circ \quad \Rightarrow \quad \theta = 58.28^\circ$$

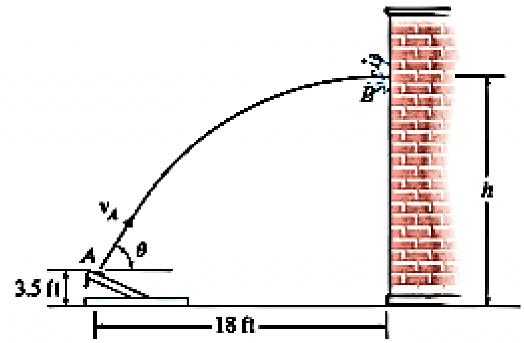
Substituting the result of (θ) into Eq. (2), we have:

$$(v_o)_{min} = \sqrt{\frac{58.86}{\sin 116.57 - cos^2 58.28}} = 9.76\ \text{m/s}$$

PROBLEMS:

(3 - 1):

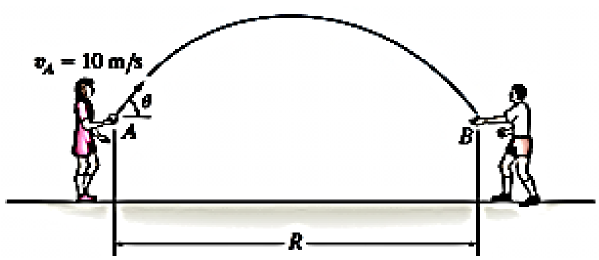
The catapult is used to launch a ball such that it strikes the wall of the building at the maximum height of its trajectory. If it takes (1.5 s) to travel from (A) to (B), determine the velocity (v_A) at which it was launched, the angle of release (θ), and the height (h).



Ans. : ($v_A = 49.8$ ft/s) , ($h = 39.7$ ft)

(3 - 2):

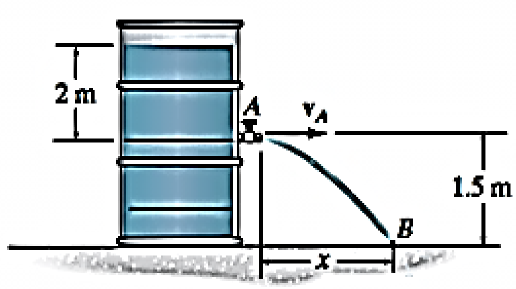
The girl at (A) can throw a ball at (v_A = 10 m/s). Calculate the maximum possible range ($R = R_{max}$) and the associated angle (θ) at which it should be thrown. Assume the ball is caught at (B) at the same elevation from which it is thrown.



Ans. : (R = 102 m) , ($\theta = 45°$)

(3 - 3):

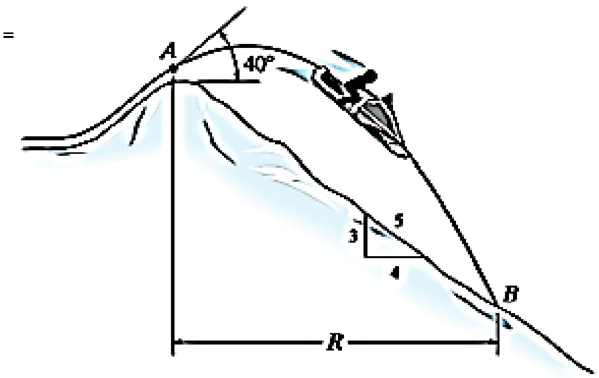
The velocity of the water jet discharging from the orifice can be obtained from ($v = \sqrt{2gh}$, where (h = 2m) is the depth of the orifice from the free water surface. Determine the time for a particle of water leaving the orifice to reach point (B) and the horizontal distance (x) where it hits the surface.



Ans. : ($t_A = 0.553$ s) , ($x = 3.46$ m)

(3 - 4):

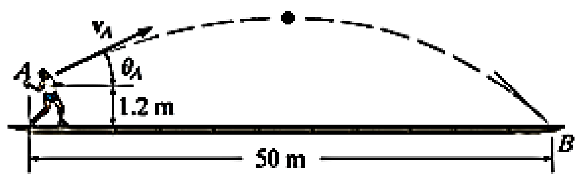
The snowmobile is traveling at (10 m/s) when it leaves the embankment at (A). Determine the time of flight from (A) to (B) and the range (R) of the trajectory.



Ans. : (t = 2.48 s) , (R = 19 m)

(3 - 5):

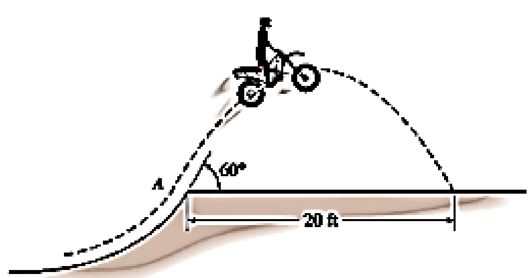
It is observed that the time for the ball to strike the ground at (B) is (2.5 s). Determine the speed (v_A) and angle (θ_A) at which the ball was thrown.



Ans. : ($\theta_A = 30.5°$) , ($v_A = 23.2$ m/s)

(3 - 6):

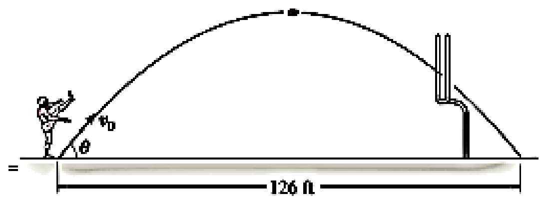
During a race the dirt bike was observed to leap up off the small hill at (A) at an angle of (60°) with the horizontal. If the point of landing is (20 ft) away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.



Ans. : ($v_A = 27.3$ ft/s)

(3 - 7):

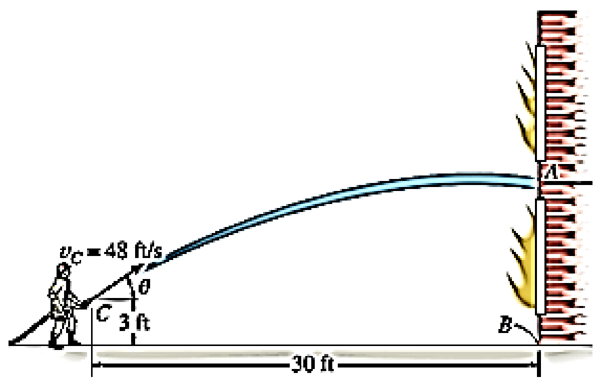
From a videotape, it was observed that a pro football player kicked a football (126 ft) during a measured time of (3.6) seconds. Determine the initial speed of the ball and the angle (θ) at which it was kicked.



Ans. : ($v_o = 67.7$ ft/s) , ($\theta = 58.9°$)

(3 - 8):

Determine the maximum height on the wall to which the firefighter can project water from the hose, if the speed of the water at the nozzle is ($v_C = 48$ ft/s).



Ans. : (h = 11.1 ft)

Part 1
Kinematics of a particle

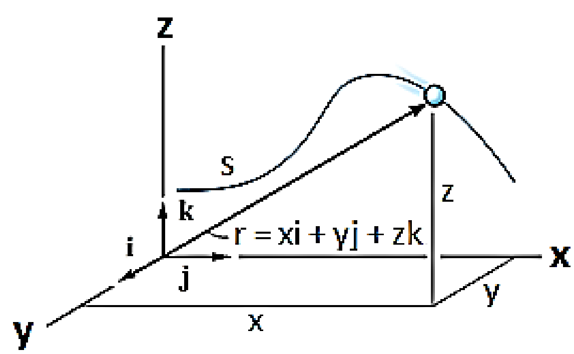
Chapter 4
Curvilinear motion

Curvilinear motion: Rectangular components

Occasionally the motion of a particle can best be described along a path that can be expressed in terms of its (x, y, z) coordinates.

Position:

If the particle is at point (x, y, z) on the curved path *s* shown in the figure, then its location is defined by the *position vector*

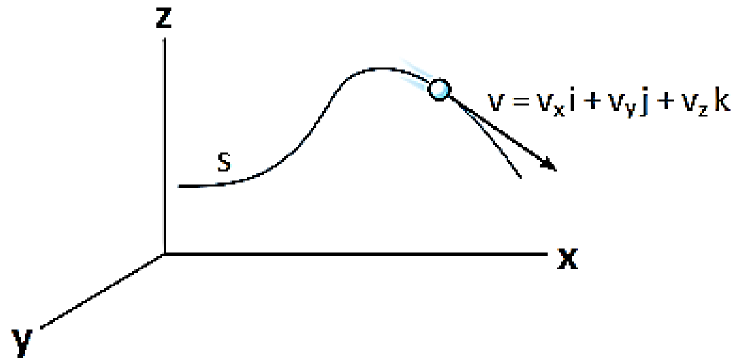


Position

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad\qquad r = \sqrt{x^2 + y^2 + z^2}$$

Velocity:

The first time derivative of **r** yields the velocity of the particle. Hence,



Velocity

$$\mathbf{v} = \frac{dr}{dt} = \frac{d}{dt}(\,xi\,) + \frac{d}{dt}(\,yj\,) + \frac{d}{dt}(\,zk\,)$$

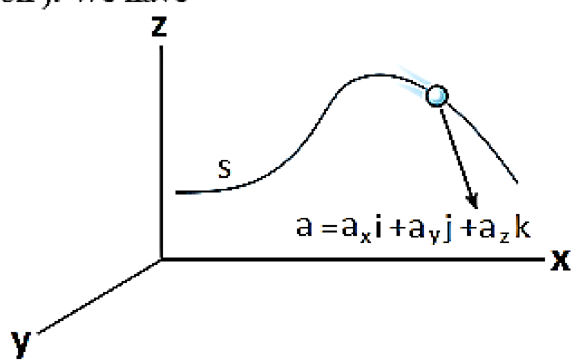
$$\mathbf{v} = \frac{dr}{dt} = v_x\mathbf{i} + v_y\,\mathbf{j} + v_z\mathbf{k}$$

$$v_x = \dot{x} \;,\quad v_y = \dot{y} \;,\quad v_z = \dot{z}$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Acceleration:

The acceleration of the particle is obtained by taking the first of the velocity (or the second derivative of the position). We have



Acceleration

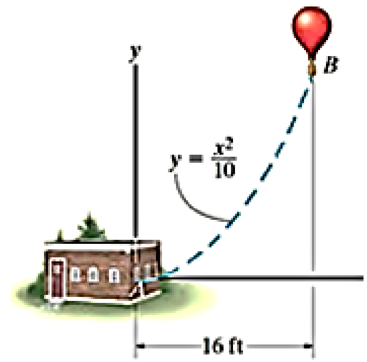
$$\mathbf{a} = \frac{dv}{dt} = a_x\mathbf{i} + a_y\,\mathbf{j} + a_z\mathbf{k}$$

$$a_x = \dot{v}_x = \ddot{x} \;,\quad a_y = \dot{v}_y = \ddot{y} \;,\quad a_z = \dot{v}_z = \ddot{z}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Example 4 - 1:

At any instant the horizontal position of the weather balloon in (Fig. a) is defined by [x = (8t) ft], where (t) is in seconds. If the equation of the path is ($y = x^2/10$), determine the magnitude and direction of the velocity and the acceleration when (t = 2 s).



a

SOLUTION:

Velocity. The velocity component in the x direction is

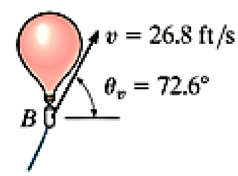
$$v_x = \dot{x} = \frac{d}{dt}(8\,t) = 8 \text{ ft/s} \rightarrow$$

When (t = 2 s), $\quad x = 8\,(2) = 16$ ft

$$v_y = \dot{y} = \frac{d}{dt}\left(\frac{x^2}{10}\right) = \frac{2\,x\,\dot{x}}{10} = \frac{2\,(16)(8)}{10} = 25.6 \text{ ft/s} \uparrow$$

When (t = 2 s), the magnitude of velocity is therefore

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(8)^2 + (25.6)^2} = 26.8 \text{ m/s}$$



(b)

The direction is tangent to the path, Fig. *b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6°$$

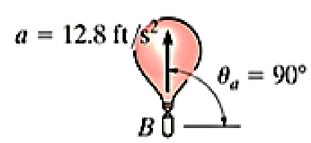
Acceleration. The relationship between the acceleration components is determined using the chain rule. We have:

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}\left(\frac{2\,x\,\dot{x}}{10}\right) = \frac{2}{10}(x\,\ddot{x} + \dot{x}\,\dot{x})$$

$$= \frac{2}{10}(x\,a_x + {v_x}^2)$$

$$= \frac{2}{10}[\,(16)(0) + (8)^2 = 12.8 \text{ ft/s}^2 \uparrow$$



(c)

Thus,

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ m/s}^2$$

The direction of **a**, as shown in Fig. *c*, is:

$$\theta_a = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{12.8}{0} = 90°$$

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (8t)2/10 = 6.4t2$ and then taking successive time derivatives.

Example 4 - 2:

For a short time, the path of the plane in the figure is described by [$y = (0.001x^2)$ m]. If the plane is rising with a constant upward velocity of (10 m/s), determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of ($y = 100$m).

SOLUTION:

When ($y = 100$ m),

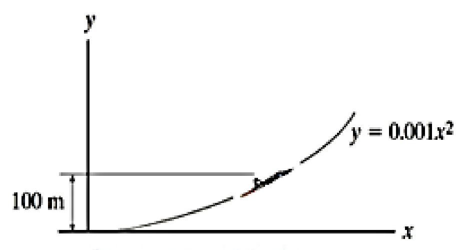
Then, $100 = 0.001x^2 \quad \Rightarrow \quad x = 316.2$ m.

Also, due to constant velocity ($v_y = 10$ m/s), so:

$$y = v_y\, t; \qquad 100\ \text{m} = (10\ \text{m/s})\, t \qquad t = 10\ \text{s}$$

Velocity. Using the chain rule to find the relationship between the velocity components, we have

$$y = 0.001\, x^2$$
$$v_y = \dot{y} = \frac{d}{dt}(0.001\, x^2) = (0.002\, x)\, \dot{x}$$
$$v_y = 0.002\, x\, v_x$$
$$10 = 0.002\,(316.2)\,(v_x)$$
$$v_x = 15.81\ \text{m/s}$$



(a)

The magnitude of the velocity is therefore

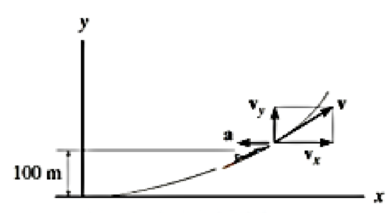
$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(15.81)^2 + (10)^2} = 18.7\ \text{m/s}$$

Acceleration. Using the chain rule, the time derivative of the velocity gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\, \dot{x})\, \dot{x} + 0.002\, x\, (\ddot{x})$$
$$= 0.002\, ({v_x}^2 + x\, a_x)$$

When, ($x = 316.2$ m), ($\dot{v}_x = 15.81$ m/s^2), $v_y = a_y = 0$

$$0 = 0.002\,[\,(15.81)^2 + (316.2\, a_x)\,]$$
$$a_x = -\,0.791\ \text{m/s}^2$$



(b)

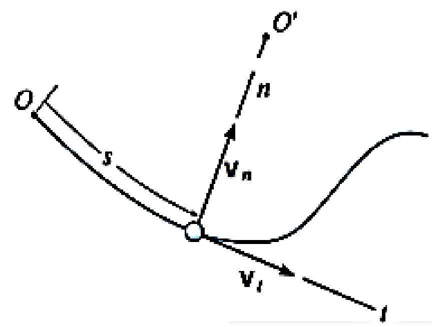
The magnitude of the plane's acceleration is therefore:

$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{(-\,0.791)^2 + (0)^2} = 0.791\ \text{m/s}^2$$

Curvilinear motion: Normal and tangential components

Coordinate system:

- Provided the path of the particle is known, we can establish a set of (n) and (t) coordinates having a fixed origin, which is coincident with the particle at the instant considered.
- The positive tangent axis acts in the direction of motion and the positive normal axis is directed toward the path's center of curvature.



Velocity:

- The particle's velocity is always tangent to the path.
- The magnitude of velocity is found from the time derivative of the path function.

$$v = \dot{s}$$

Tangential acceleration:

- The tangential component of acceleration is the result of the time rate of change in the magnitude of velocity. This component acts in the positive (s) direction if the particle's speed is increasing or in the opposite direction if the speed is decreasing.
- The relations between at (v) , (t) , and (s) are the same as for rectilinear motion, namely

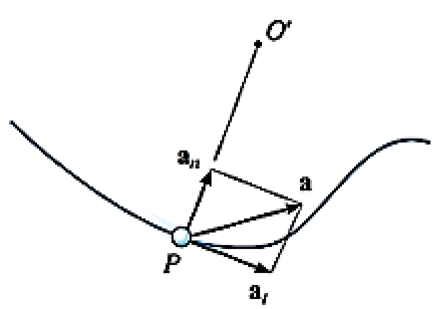
$$a_t = \dot{v} \qquad a_t\, ds = v\, dv$$

- If (a_t) is constant, [$a_t = (a_t)_c$], the above equations, when integrated, yield:

$$v = v_o + (a_t)_c\, t$$

$$s = s_o + v_o\, t + \frac{1}{2}(a_t)_c\, t^2$$

$$v^2 = v_o^2 + 2\,(a_t)_c\,(\, s - s_o\,)$$



Normal acceleration:

- The normal component of acceleration is the result of the time rate of change in the direction of the velocity. This component is always directed toward the center of curvature of the path, i.e., along the positive (n – axis).
- The magnitude of this component is determined from

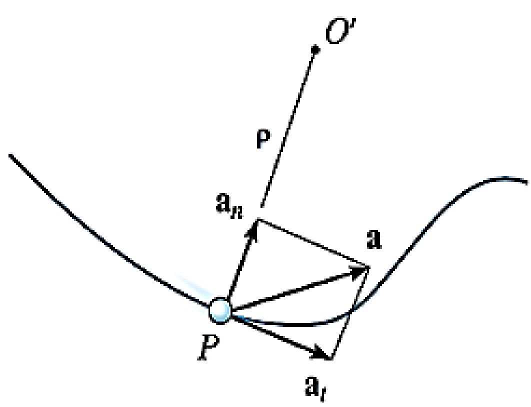
$$a_n = \frac{v^2}{\rho}$$

ρ : radius of curvature.

- If the path is expressed as (y = f(x)), the radius of curvature (r) at any point on the path is determined from the equation

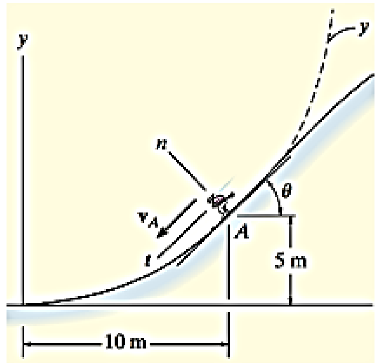
$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

The derivation of this result is given in any standard calculus text.



Example 4 - 3:

When the skier reaches point (A) along the parabolic path in the figure, he has a speed of (6 m/s) which is increasing at (2 m/s²). Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.



SOLUTION:

Velocity:

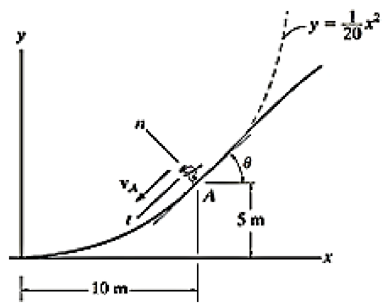
$$y = \frac{1}{20}x^2 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{10}x$$

at point (A), $\quad x = 10$ m, $\quad \frac{dy}{dx} = 1$

$$\theta = \tan^{-1} 1 = 45° \quad \text{with the (x-axis)}.$$

Therefore,

$$v = 6 \text{ m/s} \quad 45°$$

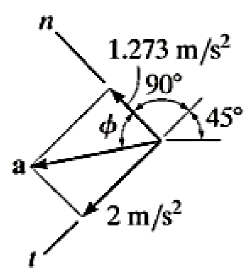


Acceleration:

$$\frac{dy}{dx} = \frac{1}{10}x \quad \Rightarrow \quad \frac{d^2y}{dx^2} = \frac{1}{10}$$

$$\rho = \frac{[1+\left(\frac{dy}{dx}\right)^2]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+\left(\frac{1}{10}x\right)^2]^{3/2}}{\left|\frac{1}{10}\right|}$$

At (x = 10 m), $\quad \rho = 28.28$ m



$$a_n = \frac{v^2}{\rho} = \frac{6^2}{28.28} = 1.273 \text{ m/s}^2$$

$$a_t = 2 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 1.273^2} = 2.37 \text{ m/s}^2$$

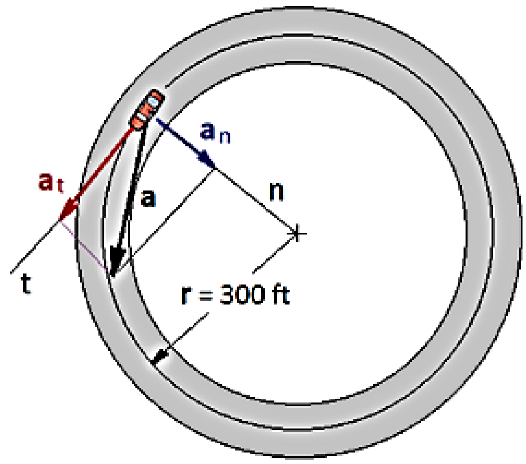
$$\phi = \tan^{-1}\frac{2}{1.273} = 57.5°$$

Thus , $\quad 45° + 90° + 57.5° - 180° = 12.5°$

So that: $\quad a = 2.37 \text{ m/s}^2 \quad 12.5°$

Example 4 - 4:

A race car travels around the horizontal circular track that has a radius of (300 ft). If the car increases its speed at a constant rate of (7 ft/s^2), starting from rest, determine the time needed for it to reach an acceleration of (8 ft/s^2). What is its speed at this instant?



SOLUTION:

Acceleration:

$$a = \sqrt{a_t^2 + a_n^2}$$

$$a_t = 7 \text{ ft/s}^2 \qquad a_n = \frac{v^2}{\rho}$$

$$v = v_o + (a_t)_c\, t = 0 + 7\,t = 7\,t$$

$$a_n = \frac{v^2}{\rho} = \frac{(7t)^2}{300} = 0.163\, t^2 \text{ ft/s}^2$$

The time needed for the acceleration to reach (8 ft/s^2) is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$

$$8 = \sqrt{(7)^2 + (0.163\, t^2)^2}$$

$$64 = 49 + 0.0266\, t^4$$

$$t = \sqrt[4]{\frac{15}{0.0266}} = 4.87 \text{ s}$$

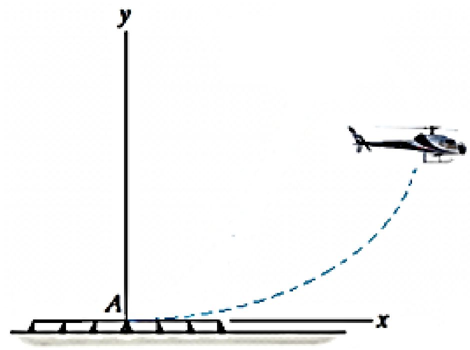
Velocity:

The speed at time ($t = 4.87$ s) is:

$$v = 7\,t = 7\,(4.87) = 34.1 \text{ ft/s.}$$

Example 4 - 5:

The flight path of the helicopter as it takes off from (A) is defined by the parametric equations [$x = (2t^2)$ m] and [$y = (0.04t^3)$ m], where (t) is the time in seconds. Determine the distance the helicopter is from point (A) and the magnitudes of its velocity and acceleration when (t = 10 s).



SOLUTION:

$x = 2t^2 \qquad y = 0.04t^3$

At (t = 10 s), $x = 200$ m $\qquad y = 40$ m

$d = \sqrt{(200)^2 + (40)^2} = 204$ m

$v_x = \dfrac{dx}{dt} = 4t$

$a_x = \dfrac{dv_x}{dt} = 4$

$v_y = \dfrac{dy}{dt} = 0.12t^2$

$a_y = \dfrac{dv_y}{dt} = 0.24t$

At (t = 10 s),

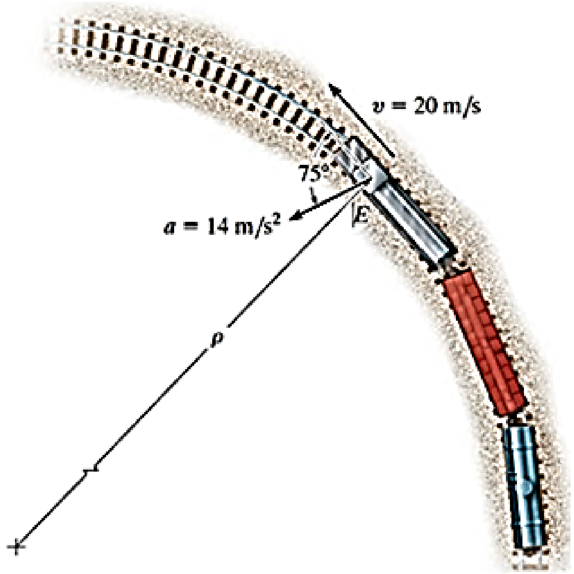
$v_x = 40$ m/s , $v_y = 12$ m/s , $a_x = 4$ m/s^2 , $a_y = 2.4$ m/s^2

$v = \sqrt{(40)^2 + (12)^2} = 41.8$ m/s

$a = \sqrt{(4)^2 + (2.4)^2} = 4.66$ m/s^2

Example 4 - 6:

At a given instant the train engine at (E) has a speed of (20 m/s) and an acceleration of (14 m/s²) acting in the direction shown. Determine the rate of increase in the train's speed and the radius of curvature (ρ) of the path.



SOLUTION:

$a_t = 14 \cos 75° = 3.62 \text{ m/s}^2$

$a_n = 14 \sin 75° = 13.52 \text{ m/s}^2$

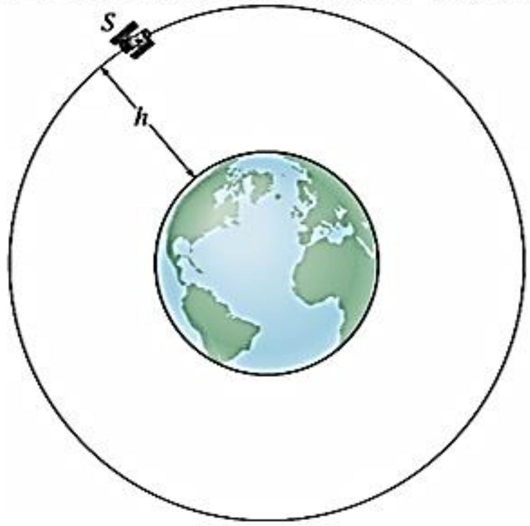
$$a_n = \frac{v^2}{\rho}$$

$$13.52 = \frac{(20)^2}{\rho} = \frac{400}{\rho}$$

$$\rho = \frac{400}{13.52} = 29.6 \text{ m}$$

Example 4 - 7:

The satellite (s) travels around the earth in a circular path with a constant speed of (20 Mm/h). If the acceleration is (2.5 m/s²), determine the altitude (h). Assume the earth's diameter to be (12713 km).



SOLUTION:

$$v = 20\ \text{Mm/h} = \frac{20\times10^{6}}{3600} = 5.56\times10^{3}\ \text{m/s}$$

Since $a_t = \frac{dv}{dt} = 0$ then, $a = a_n = 2.5\ \text{m/s}^3$

$$a_n = \frac{v^2}{\rho} \quad \Rightarrow \quad \rho = \frac{v^2}{a_n} = \frac{(5.56\times10^{3})^2}{2.5} = 12.35\times10^{6}\ \text{m}$$

The radius of the earth is:

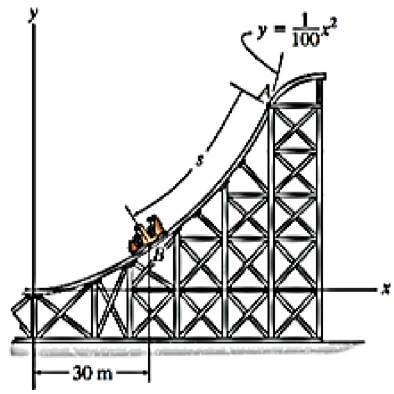
$$\frac{12713\times10^{3}}{2} = 6.36\times10^{6}\ \text{m}$$

Hince,

$$h = 12.35\times10^{6} - 6.36\times10^{6} = 5.99\times10^{6}\ \text{m} = 5.99\ \text{Mm}$$

Example 4 - 8:

When the roller coaster is at (B), it has a speed of (25 m/s), which is increasing at (a_t = 3 m/s^2). Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the (x – axis).



SOLUTION:

Radius of curvature:

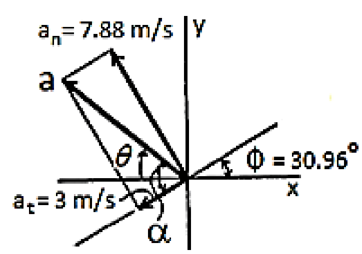
$$y = \frac{1}{100}x^2$$
$$\frac{dy}{dx} = \frac{1}{50}x$$
$$\frac{d^2y}{dx^2} = \frac{1}{50}$$
$$\rho = \frac{[1+\left(\frac{dy}{dx}\right)^2]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1+\left(\frac{1}{50}x\right)^2]^{3/2}}{\left|\frac{1}{50}\right|}$$

At (x = 30 m), ρ = 79.3 m

Acceleration:

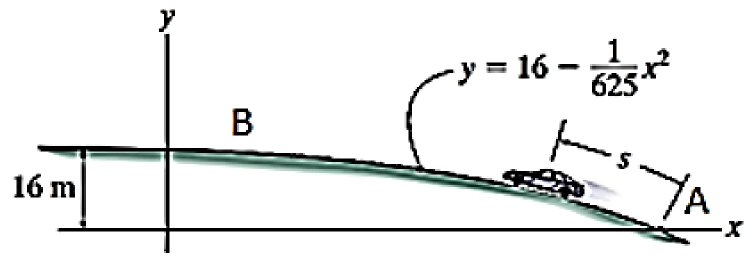
$$a_t = \dot{v} = 3 \text{ m/s}^2$$
$$a_n = \frac{{v_B}^2}{\rho} = \frac{(25)^2}{79.3} = 7.881 \text{ m/s}^2$$
$$a = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{(3)^2 + (7.881)^2}$$
$$= 8.43 \text{ m/s}^2$$

$$\phi = \tan^{-1}\frac{dy}{dx} = \tan^{-1}\frac{1}{50}x$$
$$= \tan^{-1}\frac{1}{50}(30) = 30.96°$$
$$\alpha = \tan^{-1}\frac{a_n}{a_t} = \tan^{-1}\frac{7.881}{3} = 69.16°$$
$$\theta = \alpha - \phi = 69.16° - 30.96°$$
$$= 38.2°$$



Example 4 - 9:

If the car passes point (A) with a speed of (20 m/s) and begins to increase its speed at a constant rate of (a_t = 0.5 m/s^2), determine the magnitude of the car's acceleration at (B) when (s = 100 m).



SOLUTION:

Velocity:

The speed of the car at (B) is:

$$v_B^2 = v_A^2 + 2\,a_t\,(s_B - s_A)$$
$$v_B^2 = (20)^2 + 2\,(0.5)\,(100 - 0)$$
$$v_B = 22.361 \text{ m/s}$$

Radius of Curvature:

$$y = 16 - \frac{1}{625}x^2$$
$$\frac{dy}{dx} = -3.2 \times 10^{-3}\,x$$
$$\frac{d^2y}{dx^2} = -3.2 \times 10^{-3}$$
$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\left[1+\left(-3.2 \times 10^{-3}\,x\right)^2\right]^{3/2}}{\left|-3.2 \times 10^{-3}\right|}$$

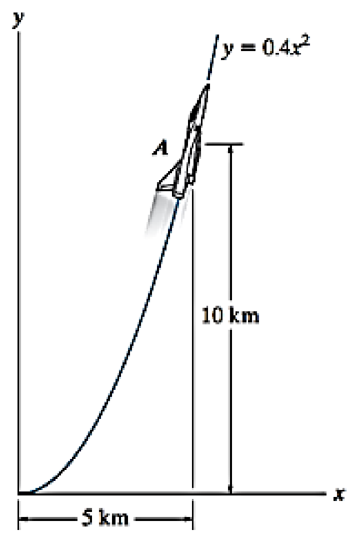
At (x = 0), $\rho = 312.5$ m

Acceleration:

$$a_t = \dot{v} = 0.5 \text{ m/s}^2$$
$$a_n = \frac{v_B^2}{\rho} = \frac{(22.361)^2}{312.5} = 1.6 \text{ m/s}^2$$

The magnitude of the car's acceleration at (B) is:

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{(0.5)^2 + (1.6)^2} = 1.68 \text{ m/s}^2$$



Example 4 - 10:

The jet plane travels along the vertical parabolic path. When it is at point (A) it has a speed of (200 m/s), which is increasing at the rate of (0.8 m/s^2). Determine the magnitude of acceleration of the plane when it is at point (A).

SOLUTION:

Radius of Curvature:

$$y = 0.4\,x^2$$

$$\frac{dy}{dx} = 0.8\,x$$

$$\frac{d^2y}{dx^2} = 0.8$$

$$\rho = \frac{\left[\,1+\left(\frac{dy}{dx}\right)^2\,\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[\,1+(4)^2\,]^{3/2}}{|0.8|} = 87.62\text{ km}$$

Acceleration:

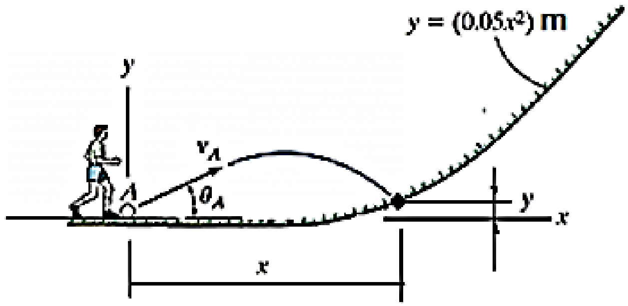
$$a_t = 0.8\text{ m/s}^2$$

$$a_n = \frac{{v_B}^2}{\rho} = \frac{(200)^2}{87.62\times 10^3} = 0.457\text{ m/s}^2$$

$$a = \sqrt{{a_t}^2 + {a_n}^2} = \sqrt{(0.8)^2 + (0.457)^2} = 0.921\text{ m/s}^2$$

Example 4 - 11:

The ball at (A) is kicked with a speed ($v_A = 25$ m/s) and at an angle ($\theta_A = 30^o$). Determine the point (x , y) where it strikes the ground. Assume the ground has the shape of a parabola as shown.



SOLUTION:

$(v_A)_x = 25 \cos 30 = 21.65$ m/s

$(v_A)_y = 25 \sin 30 = 12.5$ m/s

$+\rightarrow$ $\quad x = x_o + v_o\, t$

$x = 0 + 21.65\, t$

$x = 21.65\, t$ ………………………… (1)

$+\uparrow$ $\quad y = y_o + v_o\, t - 0.5\, g\, t^2$

$y = 0 + 12.5\, t - 0.5\,(9.81)\, t^2$

$y = 12.5\, t - 4.905\, t^2$ ………………… (2)

$y = 0.05\, x^2$ ………………………… (3)

From Eq. (1) $\quad t = \dfrac{x}{21.65}$

Sub. in Eq. (2) $\quad y = 12.5\left(\dfrac{x}{21.65}\right) - 4.905\left(\dfrac{x}{21.65}\right)^2$

$y = 0.5774\, x - 0.01\, x^2$

Sub. in Eq. (3) $\quad 0.05\, x^2 = 0.5774\, x - 0.01\, x^2$

$0.06\, x^2 = 0.5774\, x$

$0.06\, x = 0.5774$

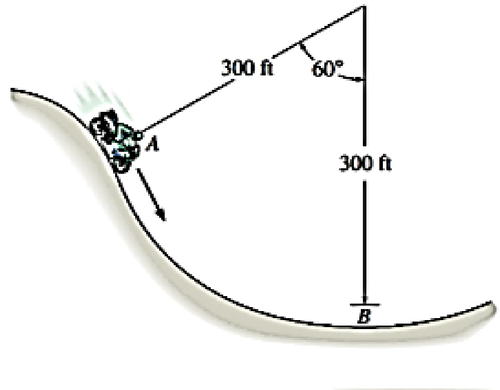
$x = 9.623$ m

$y = 0.05\,(9.623)^2 = 4.63$ m

The point is: (9.623 , 4.63)

Example 4 - 12:

When the motorcyclist is at (A), he increases his speed along the vertical circular path at the rate of [a = (0.3 t) ft/s²], where (t) is in seconds. If he starts from rest at (A), determine the magnitudes of his velocity and acceleration when he reaches (B).



SOLUTION:

$$\int_0^v dv = \int_0^t 0.3\,t\,dt$$

$$v = 0.15\,t^2$$

$$\int_0^s ds = \int_0^t 0.15\,t^2\,dt$$

$$s = 0.05\,t^3$$

$$s = \frac{1}{6}\,[\,2\pi\rho\,] = \frac{1}{6}\,[\,2\pi(300)\,] = \frac{\pi}{3}\,(300) = 100\,\pi$$

$$0.05\,t^3 = 100\,\pi \quad \Rightarrow \quad t^3 = \frac{100\,\pi}{0.05} = 2000\,\pi$$

$$t = 18.453\text{ s}$$

$$v = 0.15\,t^2 = 0.15\,(18.453)^2 = 51.1\text{ ft/s}$$

$$a_t = 0.3\,t = 0.3\,(18.453) = 5.536\text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{51.1^2}{300} = 8.7\text{ ft/s}^2$$

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{(5.536)^2 + (8.7)^2} = 10.3\text{ ft/s}^2$$

Circular motion:

Circular motion is a special and important case of plane curvilinear motion, where the radius of curvature (ρ) becomes the constant radius of the circle (r).

Components of velocity and acceleration of circular motion become:

(θ): Angular displacement.
(ω): Angular velocity.
(α): Angular acceleration.

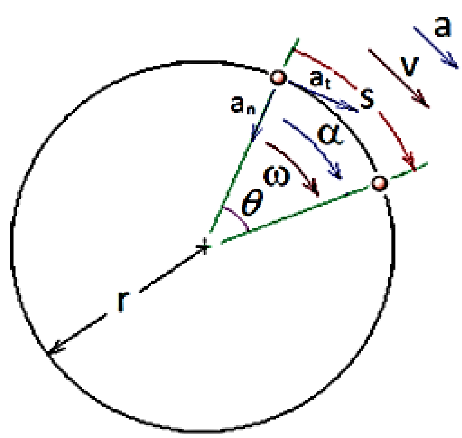
$$v = r\,\omega$$

$$a_n = \frac{v^2}{\rho} = r\,\omega^2 = v\,\omega$$

$$a_t = \frac{dv}{dt} = r\,\alpha$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$



Common examples of circular motion:

The flywheel on a stationary machine, the pulleys on axles, the satellites in circular orbits, the car traveling in a circular path of a given radius, the centrifuge.

Angular displacement (θ):

Angular displacement is the central angle formed by the arc of motion, and is measured in the radial unit, and it is measured in the radial unit (radian).

1 radian = 57.3°.

Angular velocity (ω):

Angular velocity is the time rate of change in angular displacement, and it is measured by (rad/s) or (rpm) revolutions per minuet.

Angular acceleration (α):

Angular acceleration is the time rate of change in angular velocity, and it is measured by (rad/s^2).

Equations of circular motion:

The equations describing circular motion are similar to the linear motion equations, but the symbols used are the symbols for circular motion, (θ, ω , α) instead of (s , v , a).

Circular motion	**Linear motion**
$\omega = \frac{d\theta}{dt}$	$v = \frac{ds}{dt}$
$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$	$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
$\omega\, d\omega = \alpha\, d\theta$	$v\, dv = a\, ds$
$\omega = \omega_o + \alpha\, t$	$v = v_o + a\, t$
$\theta = \theta_o + \omega_o\, t + \frac{1}{2}\alpha\, t^2$	$s = s_o + v_o\, t + \frac{1}{2} a\, t^2$
$\omega^2 = \omega_o^2 + 2\,\alpha\,(\,\theta - \theta_o\,)$	$v^2 = v_o^2 + 2\,a\,(\,s - s_o\,)$
$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$, $\Delta\theta = \theta - \theta_o$, $\Delta t = t - t_o$	$v_{avg} = \frac{\Delta S}{\Delta t}$, $\Delta S = S - S_o$

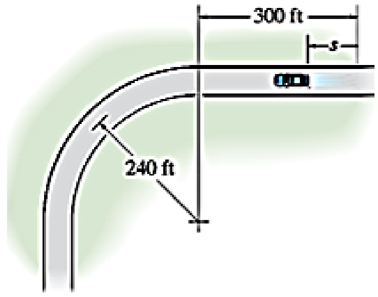
θ : Final angular displacement.
θ_o : Initial angular displacement.
ω : Final angular velocity.
ω_o : Initial angular velocity.
α : Angular acceleration.
ω_{avg} : Average angular velocity.
t : Final time

s : Final linear displacement.
s_o : Initial linear displacement.
v : Final linear velocity.
v_o : Initial linear velocity.
a : Linear acceleration.
v_{avg} : Average linear velocity.
t_o : Initial time

PROBLEMS:

(4 - 1):

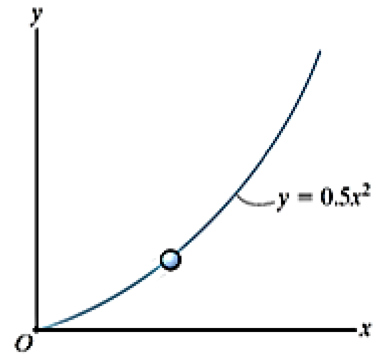
The automobile is originally at rest at (s = 0). If its speed is increased by [($\dot{v}$ = 0.05 t^2) ft/s^2], where (t) is in seconds, determine the magnitudes of its velocity and acceleration when (t = 18 s).



Ans. : (v = 97.2 ft/s) , (a = 42.6 ft/s^2)

(4 - 2):

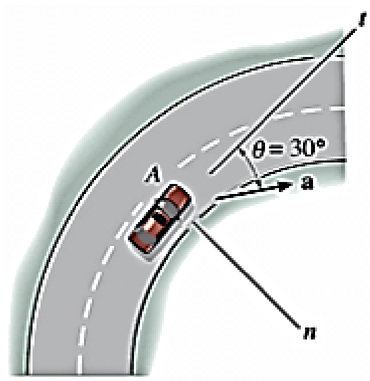
The particle travels along the path defined by the parabola (y = 0.5 x^2). If the component of velocity along the (x - axis) is [v_x = (5t) ft/s], where (t) is in seconds, determine the particle's distance from the origin (*O*) and the magnitude of its acceleration when (t = 1 s). When (t = 0), (x = 0), (y = 0).



Ans. : (d = 4 ft) , (a = 37.8 ft/s^2)

(4 - 3):

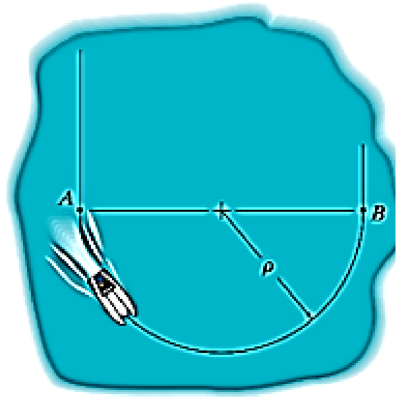
The automobile has a speed of (80 ft/s) at point (A) and an acceleration (a) having a magnitude of (10 ft/s^2), acting in the direction shown. Determine the components of acceleration at point (A) and the radius of curvature of the path.



Ans. : (a_t = 8.66 ft/s^2) , (a_n = 5 ft/s^2) , (ρ = 1280 ft)

(4 - 4):

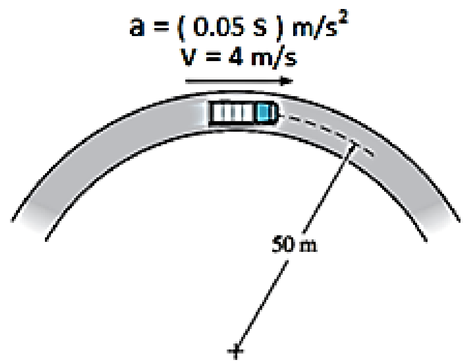
The speedboat travels at a constant speed of (15 m s) while making a turn on a circular curve from (A) to (B). If it takes (45 s) to make the turn, determine the magnitude of the boat's acceleration during the turn.



Ans. : ($a = 1.05 \text{ m/s}^2$)

(4 - 5):

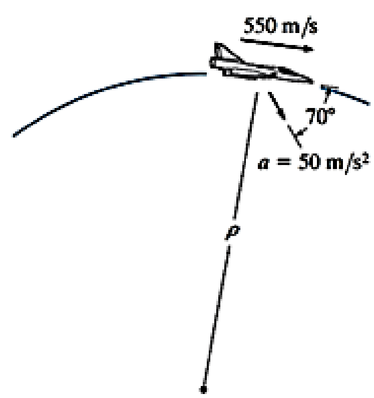
The truck travels in a circular path having a radius of (50 m) at a speed of (v = 4 m/s). For a short distance from (s = 0), its speed is increased by [$a = (0.05\ s)\ \text{m/s}^2$) where (s) is in meters. Determine its speed and the magnitude of its acceleration when it has moved (s = 10 m).



Ans. : ($v = 4.58 \text{ m/s}$) , ($a = 0.653 \text{ m/s}^2$)

(4 - 6):

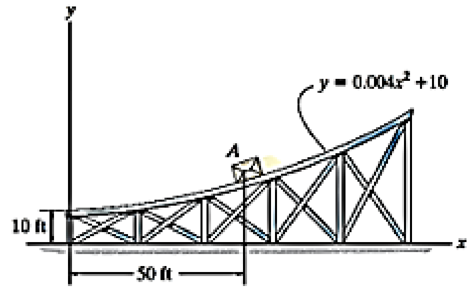
At a given instant the jet plane has a speed of (550 m/s) and an acceleration of (50 m/s^2) acting in the direction shown. Determine the radius of curvature (ρ) of the path.



Ans. : ($\rho = 6.44 \text{ km}$)

(4 - 7):

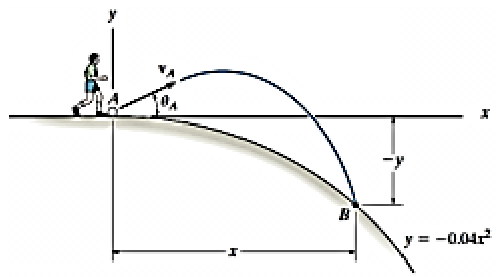
If the speed of the box at point (A) on the track is (30 ft/s) which is increasing at the rate of ($a = 5\ ft/s^2$) , determine the magnitude of the acceleration of the box at this instant.



Ans. : ($a = 7.63\ ft/s^2$)

(4 - 8):

The ball at (A) is kicked with a speed ($v_A = 80$ ft/s) and at an angle ($\theta_A = 30°$). Determine the point (x , –y) where it strikes the ground. Assume the ground has the shape of a parabola as shown.



The point is: (13.3 , − 7.09)

Part 2 - Kinetics
Chapter 1
Newton's Second Law
The Equation of Motion

- Kinetics is the study of the relations between the unbalanced forces and the changes in motion that they produce.
- Newton's 2nd law states that the particle will accelerate when it is subjected to unbalanced forces. The acceleration of the particle is always in the direction of the applied forces.
- Newton's 2nd law is also known as the equation of motion.
- To solve the equation of motion, the choice of an appropriate coordinate systems depends on the type of motion involved.
- Two types of problems are encountered when applying this equation:
  - The acceleration of the particle is either specified or can be determined directly from known kinematic conditions. Then, the corresponding forces, which are acting on the particle, will be determined by direct substitution.
  - The forces acting on the particle are specified, then the resulting motion will be determined. Note that, if the forces are constant, the acceleration is also constant and is easily found from the equation of motion. However, if the forces are functions of time, position, or velocity, the equation of motion becomes a differential equation which must be integrated to determine the velocity and displacement.
- In general, there are three general approaches to solve the equation of motion: the direct application of Newton's 2nd law, the use of the work & energy principles, and the impulse and momentum method.

Free-body Diag Kinetic Diagram

Note: The equation of motion has to be applied in such way that the measurements of acceleration are made from a Newtonian or inertial frame of reference. This coordinate does not rotate and is either fixed or translates in a given direction with a constant velocity (zero acceleration).

Rectilinear Motion -----------------------------------

$$\begin{cases} \sum F_x = m\,a_x \\ \sum F_y = 0 \end{cases}$$

Curvilinear Motion

Rectangular Coordinates

$$\begin{cases} \sum F_x = m\,a_x \\ \sum F_y = m\,a_y \\ a = a_x\,i + a_y\,j \\ |a| = \sqrt{a_x^2 + a_y^2} \\ \sum F = \sum F_x\,i + \sum F_y\,j \\ \left|\sum F\right| = \sqrt{\sum F_x^2 + \sum F_y^2} \end{cases}$$

(n - t) Coordinates

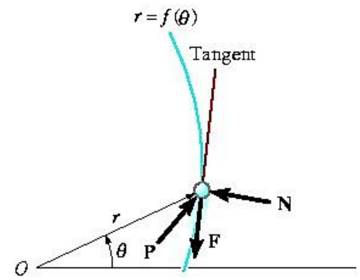
$$\begin{cases} \sum F_t = m\,a_t \\ \sum F_n = m\,a_n \\ a = a_t\,e_t + a_n\,e_n \\ |a| = \sqrt{a_t^2 + a_n^2} \\ \sum F = \sum F_t\,e_t + \sum F_n\,e_n \\ \left|\sum F\right| = \sqrt{\sum F_t^2 + \sum F_n^2} \end{cases}$$

Polar Coordinates

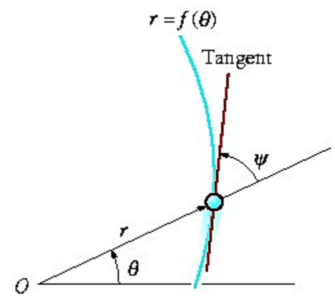
$$\begin{cases} \sum F_r = m\,a_r \\ \sum F_\theta = m\,a_\theta \\ a = a_r\,e_r + a_\theta\,e_\theta \\ |a| = \sqrt{a_r^2 + a_\theta^2} \\ \sum F = \sum F_r\,e_r + \sum F_\theta\,e_\theta \\ \left|\sum F\right| = \sqrt{\sum F_r^2 + \sum F_\theta^2} \end{cases}$$

In Polar Coordinates

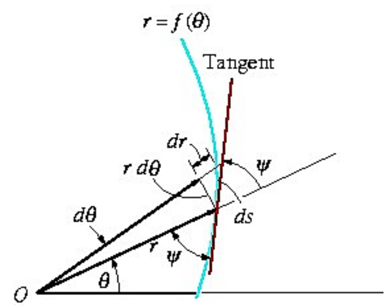
- Consider the force (**P**) that causes the particle to move along a path (r = f(θ)).



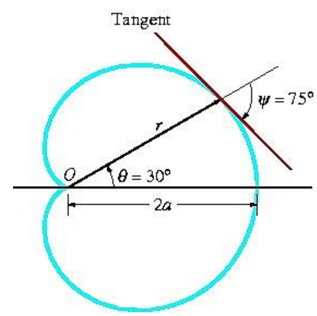
- The *normal force* (**N**) which the path exerts on the particle is always *perpendicular to the tangent of the path.*
- Frictional force (**F**) always acts along the tangent in the opposite direction of motion.
- The *directions* of (**N)** and (**F)** can be specified relative to the radial coordinate by using the angle (ψ), which is defined between the *extended* radial line and the tangent to the curve.



$$\tan \psi = \frac{r}{dr/d\theta}$$



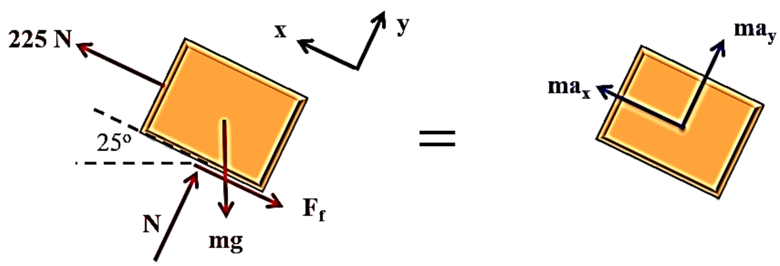
- If (ψ) is positive, it is measured from the extended radial line to the tangent in a (CCW) sense or in the positive direction (θ).
- If it is negative, it is measured in the opposite direction to positive (θ).



Free Body Diagrams and Kinetic Diagrams

The free body diagram is the same as you have done in statics; we will add the kinetic diagram in our dynamic analysis.

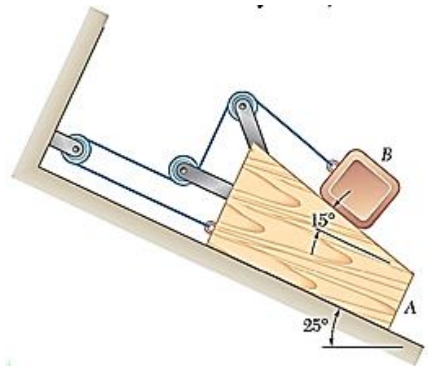
1. Isolate the body of interest (free body).
2. Draw your axis system (e.g., Cartesian, polar, path).
3. Add in applied forces (e.g., weight, [225 N] pulling force).
4. Replace supports with forces (e.g., normal force).
5. Draw appropriate dimensions (usually angles for particles).



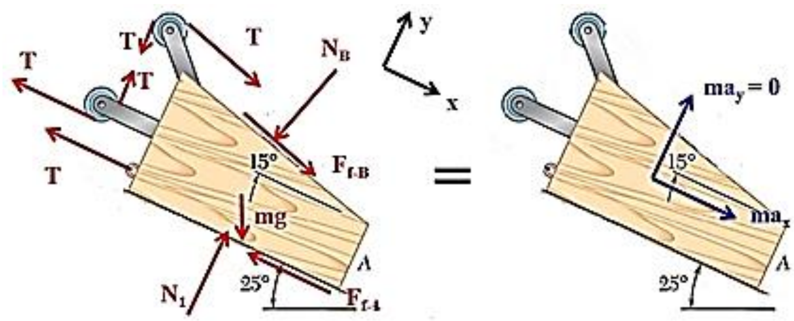
$$\sum \mathbf{F} \qquad = \qquad \mathbf{m\,a}$$

<u>**Example 2 - 1 - 1:**</u>

Draw the (FBD) and (KD) for block (A) [note that the massless, frictionless pulleys are attached to block (A) and should be included in the system].



1. Isolate body.
2. Axes.
3. Applied forces.
4. Replace supports with forces.
5. Dimensions (already drawn).
6. Kinetic diagram.



<u>**Example 2 - 1 - 2:**</u>

A body at rest of mass (10 kg) is exerted by a force of (20 N) for a period of (5 s). find:

a – Resulted acceleration.

b – Velocity after (5 s).

SOLUTION:

$$\sum F = m\,a \qquad\qquad v = v_o + a\,t$$

$$20 = 10\,a \qquad\qquad = 0 + (2)(5)$$

$$a = 2\ \text{m/s}^2 \qquad\qquad = 10\ \text{m/s}$$

<u>**Example 2 - 1 - 3:**</u>

Calculate the mass of a body on which a force of magnitude (20 N) was applied, which caused it to accelerate at (5 m/s^2).

SOLUTION:

$$\sum F = m\,a$$

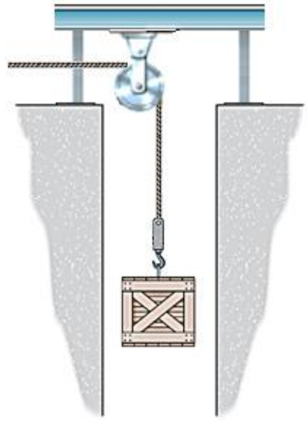
$$20 = 5\,m$$

$$m = \frac{20}{5} = 4\ \text{kg}$$

Example 2 - 1 - 4:

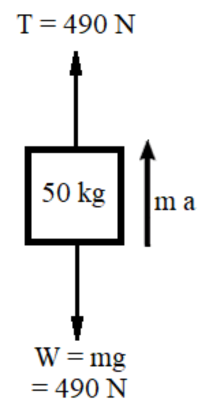
A box of mass (50 kg) suspended at the end of a rope. Find the acceleration of the box if the tension in the rope is:

a - (490 Newtons).
b - (240 Newtons).
c - (890 Newtons).



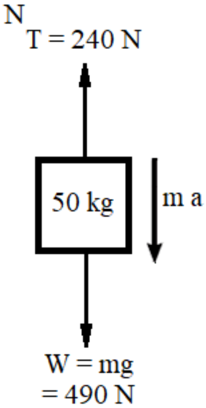
SOLUTION:

$W = mg = 50 \times 9.8 = 490\ N$
$\sum F = T - W$



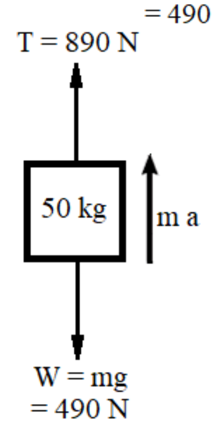
a- $W = T = 490\ N$
$\sum F = T - W = 490 - 490 = 0$

$\sum F = m\ a$
$0 = 50\ a$
$a = 0$ (Static state)



b- $W = 490\ N$, $T = 240\ N$
$\sum F = T - W = 240 - 490 = -250\ N = 250\ N \downarrow$

$\sum F = m\ a$
$250 = 50\ a$
$a = \frac{250}{50} = 5\ m/s^2 \downarrow$

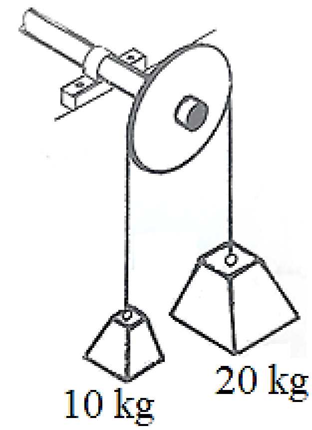


c- $W = 490\ N$, $T = 890\ N$
$\sum F = T - W = 890 - 490 = 400\ N$

$\sum F = m\ a$
$400 = 50\ a$
$a = \frac{400}{50} = 8\ m/s^2 \uparrow$

Example 2 - 1 - 5:

In the figure, the rope passes on a completely smooth pulley and is suspended to one end (10 kg) and to the other end (20 kg), calculate the approximate acceleration of the system and the tension in the rope.



SOLUTION:

$W_1 = 10 \times 9.8 = 98$ N

$W_2 = 20 \times 9.8 = 196$ N

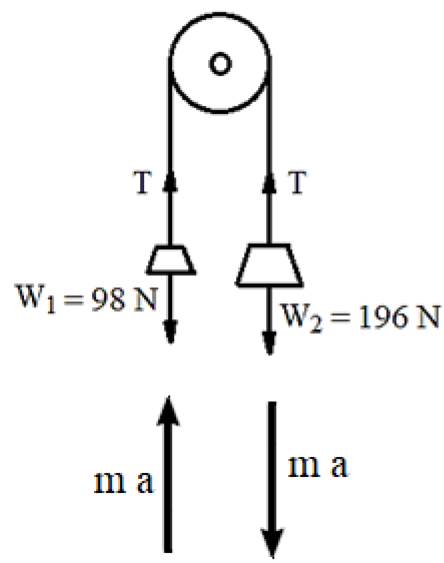
Lift side:

$\sum F = T - W_1 = m\,a$

$T - 98 = 10\,a$

$T = 10\,a + 98$ …….. (1)

Right side:



$\sum F = W_2 - T = m\,a$

$196 - T = 20\,a$

$T = 196 - 20\,a$ …….. (2)

$10\,a + 98 = 196 - 20\,a$

$30\,a = 98$

$a = 3.27\ \text{m/s}^2$

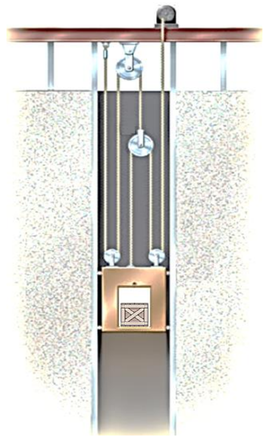
Sub. in Eq. (1):

$T = 10\,a + 98 = 10\,(3.27) + 98$

$= 130.7$ N

<u>Example 2 - 1 - 6:</u>

A box of mass (30 kg) placed on the floor of an elevator. Calculate the force that the box exerts on the elevator floor in the following cases:



- **a- When the elevator moves up at a constant velocity.**
- **b - When the elevator moves up with an acceleration of ($1.2\ m/s^2$).**
- **c - When the elevator moves down with an acceleration of ($1.2\ m/s^2$).**

SOLUTION:

$$W = m\,g = 30 \times 9.8 = 294\ N$$

$$\sum F = m\,a$$

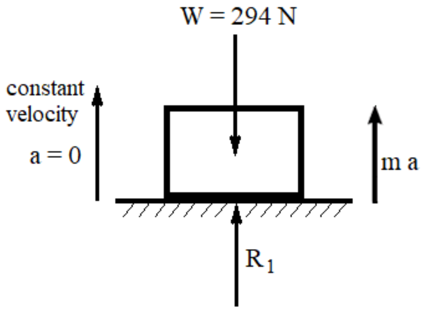
a-

$a = 0$ (constant velocity)

$R_1 - W = m\,a$

$R_1 - 294 = 0$

$R_1 = 294\ N$



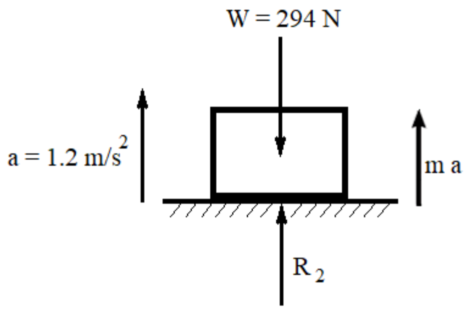
b-

$a = 1.2\ m/s^2$

$R_2 - W = m\,a$

$R_2 - 294 = 30 \times 1.2$

$R_2 = 36 + 294 = 330\ N$



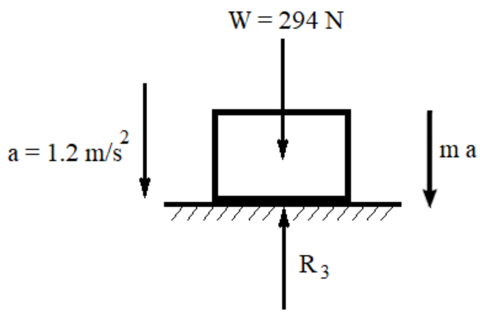
c-

$a = 1.2\ m/s^2$

$R_3 - W = -\,(\,m\,a\,)$

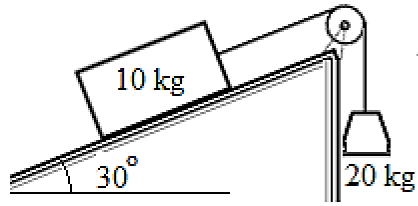
$R_3 - 294 = -\,(\,30 \times 1.2\,)$

$R_3 = -\,36 + 294 = 258\ N$



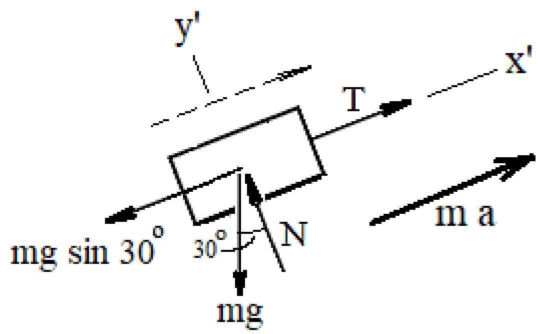
Example 2 - 1 - 7:

In the figure, the weights (10 kg) and (20 kg) are tied with a rope that passes the bug of the pulley. Ignore the friction. Find the tension force in the rope and the acceleration of the system.



SOLUTION:

For (10 kg) mass:



$\sum F_{x'} = m\,a$

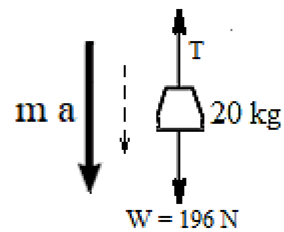
$T - mg \sin 30° = m\,a$

$T - (10)(9.8) \sin 30° = 10\,a$

$T - 49 = 10\,a$

$T = 10\,a + 49$ ………… (1)

For (20 kg) mass:



$W = mg = (20)(9.8) = 196$ N

$\sum F_y = m\,a$

$196 - T = 20\,a$

$T = 196 - 20\,a$ ………… (2)

$196 - 20\,a = 10\,a + 49$

$196 - 49 = 10\,a + 20\,a$

$147 = 30\,a$

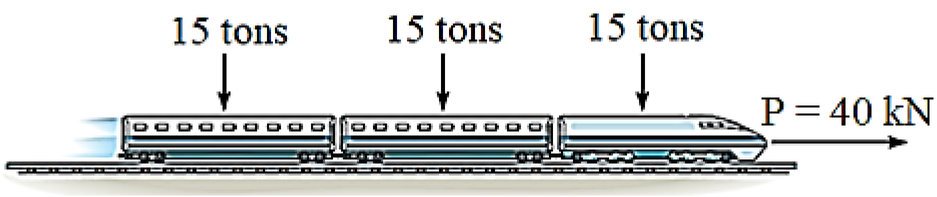
$$a = \frac{147}{30} = 4.9 \text{ m/s}^2$$

Sub. in Eq. (1):

$T = 10\,a + 49 = 10(4.9) + 49 = 98$ N

<u>Example 2 - 1 - 8:</u>

A train consisting of three cars, each car weighing (15 tons). The first cart works as a machine and exerts a pulling force of (40 kN) on the rail. The frictional resistance of each cart on the rail is (1 kN). Find the acceleration of the train and the tension in the connections between the carts.



SOLUTION:

$$\sum F_x = m\,a$$

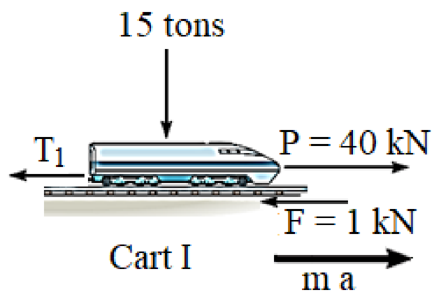
Cart I: $P - F - T1 = m\,a$

$40000 - 1000 - T_1 = 15000\,a$

$39000 - T_1 = 15000\,a$ .... (1)

Cart II: $T_1 - 1000 - T_2 = 15000\,a$ .... (2)

Cart III: $T_2 - 1000 = 15000\,a$ .... (3)



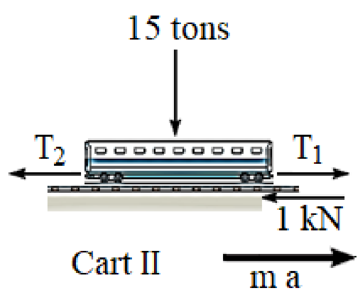
From Eq. (3) $T_2 = 15000\,a + 1000$

Sub. in Eq. (2)

$T_1 - 1000 - (15000\,a + 1000) = 15000\,a$

$T_1 - 2000 - 15000\,a = 15000\,a$

$T_1 = 30000\,a + 2000$



Sub. in Eq. (1)

$39000 - (30000\,a + 2000) = 15000\,a$

$39000 - 30000\,a - 2000 = 15000\,a$

$37000 = 45000\,a$

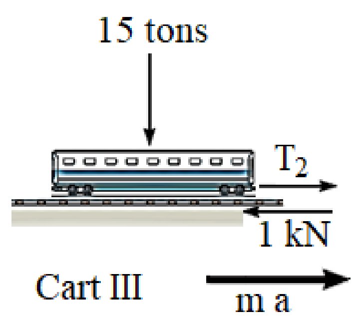
$$a = \frac{37000}{45000} = 0.822\ m/s^2$$

$T_1 = 30000 (0.822) + 2000$

$= 26666\ N = 26.67\ kN$

$T_2 = 15000 (0.822) + 1000$

$= 13333\ N = 13.33\ kN$



Example 2 - 1 - 9:

A man of mass (75 kg) is lifted by a rope hanging from a helicopter that is above him. If the mass of the hook is (10 kg), find the tension in the rope when the man is lifting:

a - at a constant velocity.

b - with a constant acceleration of (0.5 m/s²).

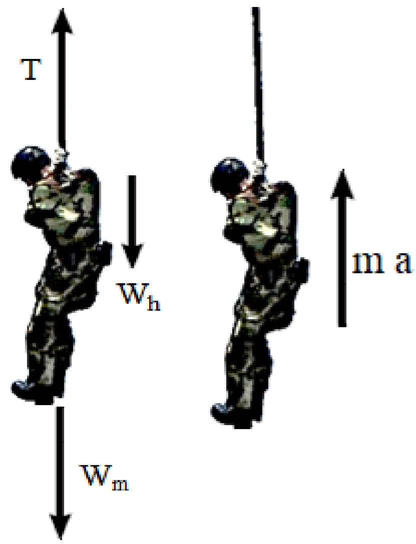
Ignore the rope mass.

SOLUTION:

$$W_m = 75 \times 9.8 = 735\ N$$
$$W_h = 10 \times 9.8 = 98\ N$$

a- at a constant velocity, $a = 0$

$$\sum F_y = m\,a$$
$$T - W_m - W_h = m\,a$$
$$T - 735 - 98 = 0$$
$$T = 833\ N$$



b- at ($a = 0.5\ m/s^2$)

$$\sum F_y = m\,a$$
$$T - W_m - W_h = m\,a$$
$$T - 735 - 98 = (75 + 10) (0.5)$$
$$T = 42.5 + 735 + 98 = 875.5\ N$$

Example 2 - 1 - 10:

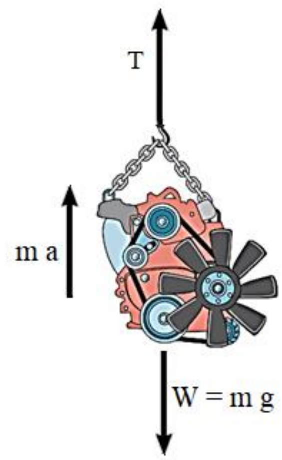
A machine of mass (4 tons) was lifted vertically to the top by a chain, a distance of (3 m) during (4 s) of rest. Assuming the acceleration is constant, find the tension in the chain.

SOLUTION:

$s = s_o + v_o + 0.5\ a\ t^2$

$3 = 0 + 0 + 0.5\ a\ (4)^2$

$3 = 8\ a \quad \Rightarrow \quad a = \frac{3}{8} = 0.375\ m/s^2$



$W = 4000 \times 9.8 = 39200\ N$

$\sum F_y = m\ a$

$T - W = m\ a$

$T - 39200 = 4000 \times 0.375$

$T = 1500 + 39200 = 40700\ N = 40.7\ kN$

Example 2 - 1 - 11:

During the test of a vertically upward rocket, the thrust of the engine was (5 kN) for a period of (15 s). If you know that the minimum vertical acceleration of the rocket on which the engine will be installed is (65 m/s^2), what is the largest mass of the rocket? neglect the engine mass.

SOLUTION:

$\sum F_y = m\,a$

$F - W = m\,a$

$F - m\,g = m\,a$

$F = m\,g + m\,a = m\,(g + a)$

$5000 = m\,(9.8 + 65)$

$5000 = 74.8\,m$

$m = \frac{5000}{74.8} = 66.8$ kg

PROBLEMS:

(2 - 1 - 1):

A person whose mass is (80 kg) is located in a building at a height of (10 m) above the ground. The person was forced to go down on a rope with a maximum tension force of (650 N). Calculate the minimum acceleration he must have on the rope so that it does not break.

a = 1.7 m/s^2 ↓

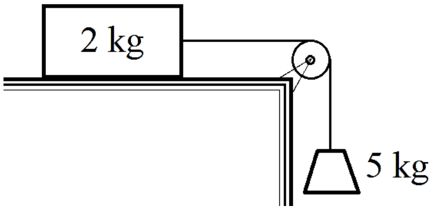
(2 - 1 - 2):

A body traveling towards the east at a speed of (30 m/s), was subjected to the effect of a force of (250 N) towards the north for a period of (2 s). If the mass of the body is (50 kg), what is its final velocity?

v = 31.62 m/s
θ = 18.435°

(2 - 1 - 3):

In the system of weights shown in the figure, the horizontal surface is smooth (frictionless), and the friction in the pulley is negligible. Find acceleration of the system and tension in the rope.



a = 7 m/s^2 T = 14 N

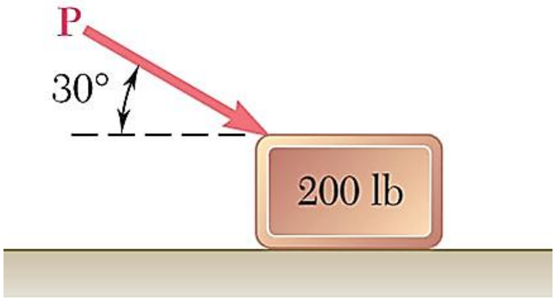
(2 - 1 - 4):

An air-to-air missile with a mass of (500 kg) was launched horizontally from a fighter aircraft. If the acceleration of the missile is (90 m/s^2) relative to the aircraft, what is the thrust of the missile engine?

F = 45 kN

(2 - 1 - 5):

A (200 lb) block rests on a horizontal plane. Find the magnitude of the force (P) required to give the block an acceleration of (10 ft/s^2) to the right. Neglect the friction between the block and plane.



P = 71.7 lb

(2 - 1 - 6):

A car of mass (1 ton) is traveling at a speed of (30 km/h) in a straight line. Calculate the magnitude of resistance that must be applied by the brakes to stop the car within (75 m).

F = 6 kN ←

(2 - 1 - 7):

A spacecraft landed on the surface of the moon with a vertical deceleration of (1 m/s^2). If the mass of the spacecraft is (13×10^3 kg) and the gravitational acceleration on the moon is (1.67 m/s^2). Find the thrust for the landing machine during this stage.

T = 8.7 kN