Introduction to Process Control

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Course Overview

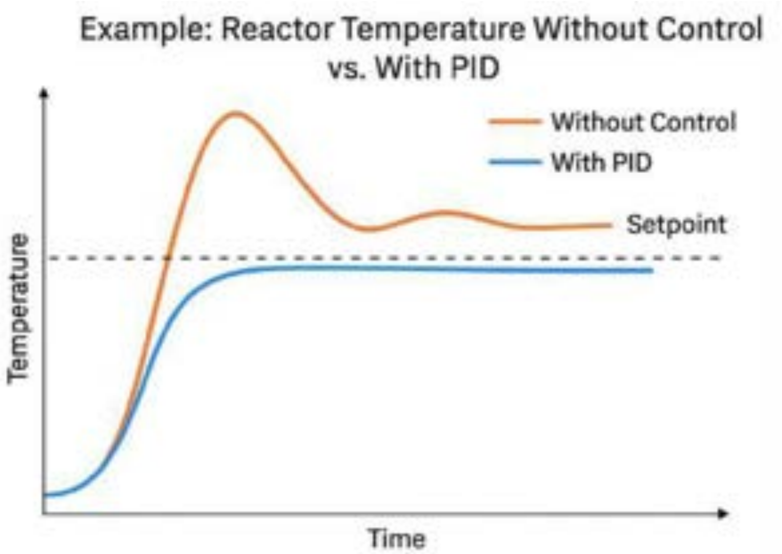
- Process dynamics
- Controllers (PID, advanced)
- Instrumentation
- Industry applications

Relevance:

- Essential knowledge for engineers working in industry.

Motivation

- Without control: instability, unsafe operations, waste.
- With control: stable, efficient, profitable processes.
- Example: Reactor temperature without control vs with PID.



Definition of Process Control

- Automatic regulation of process variables (temperature, pressure, flow, level, etc.) to achieve desired outputs.

- Key concept: Feedback.

Process Variables

1. **Controlled Variables (CVs):** The output variables of the process that we want to keep at a desired value (setpoint).

Examples: Temperature – Reactor temperature in a chemical plant. Flow rate – Steam flow to a turbine. Pressure – Boiler drum pressure.

Role: The main performance indicators of the process.

2. **Manipulated Variables (MVs):** The input variables that the controller can directly adjust to influence the controlled variables.

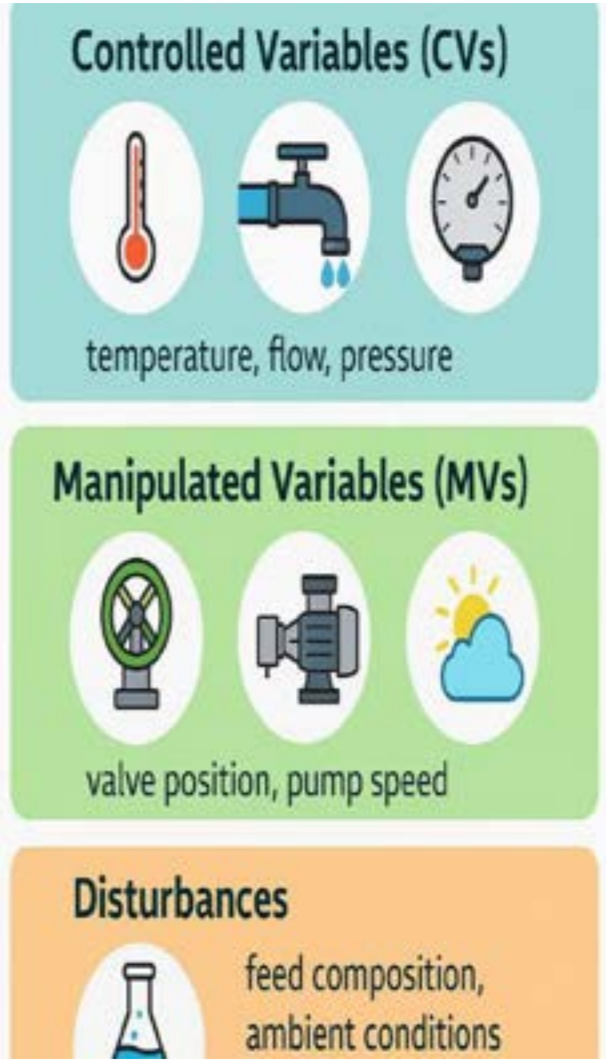
Examples: Valve position (%) – Adjusting cooling water flow through a valve to control reactor temperature. Pump speed (RPM) – Varying a pump's motor speed to regulate pipeline flow.

Role: The "knobs" the controller turns to keep CVs on target.

3. **Disturbances (DVs):** External or internal factors that affect the process but cannot be directly controlled.

Examples: Feed composition – Change in raw material concentration in a blending tank. Ambient conditions – Variations in outside temperature or humidity affecting heat exchangers.

Role: The unwanted influences that create errors and require the control system to compensate.



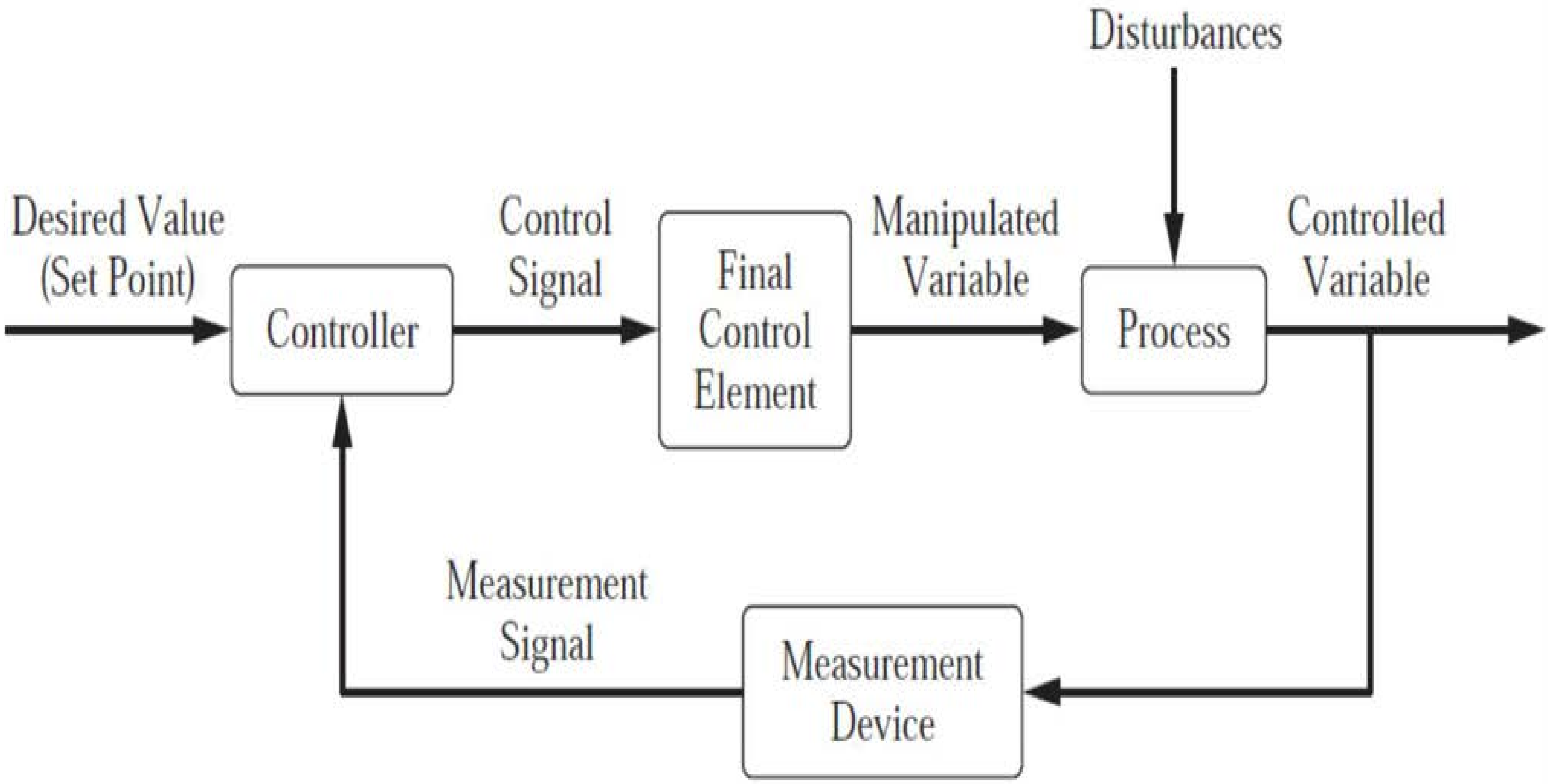
Examples of Processes

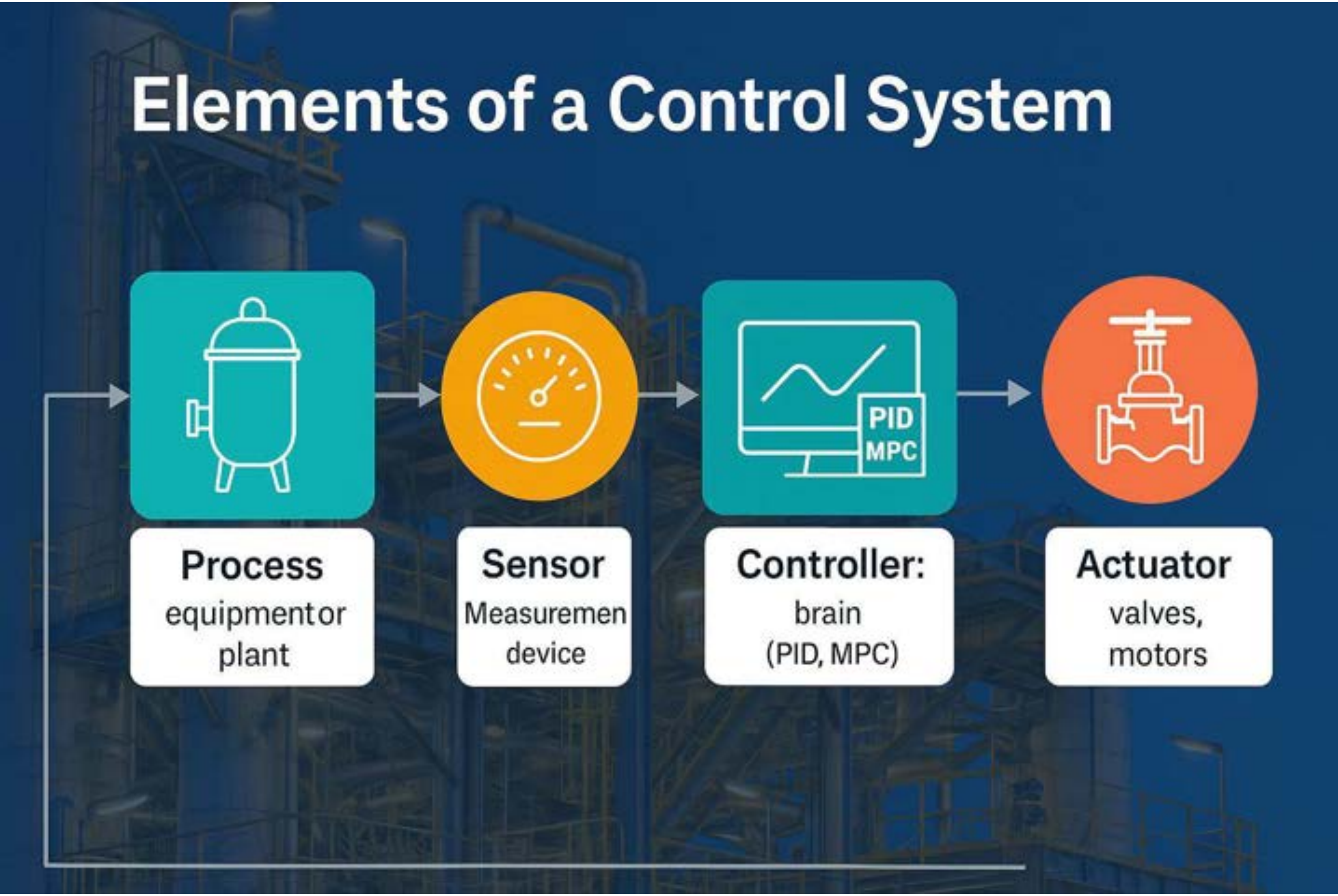
- Chemical reactor temperature control
- Boiler drum water level
- Food pasteurization temperature loop

Industry Relevance

- Oil & Gas → pipeline pressure control
- Chemical plants → distillation column
- Food industry → pasteurization
- Power plants → steam turbine
- Water treatment → pH control

General Control Loop




Elements of a Control System
PID
MPC
Process
equipmentor
plant
Sensor
Measuremen
device
Controller:
brain
(PID, MPC)
Actuator
valves,
motors

Open Loop vs Closed Loop

- Open loop: no feedback (toaster timer)
- Closed loop: feedback present (thermostat).

Disturbances in Control Systems

External factors affecting process:

- Ambient temperature
- Raw material variation
- Sensor noise

Importance: disturbance rejection.

Practical Relevance

Control Objectives

- Maintain stability
- Minimize error
- Reject disturbances
- Improve efficiency and safety

Two Components of Process Control

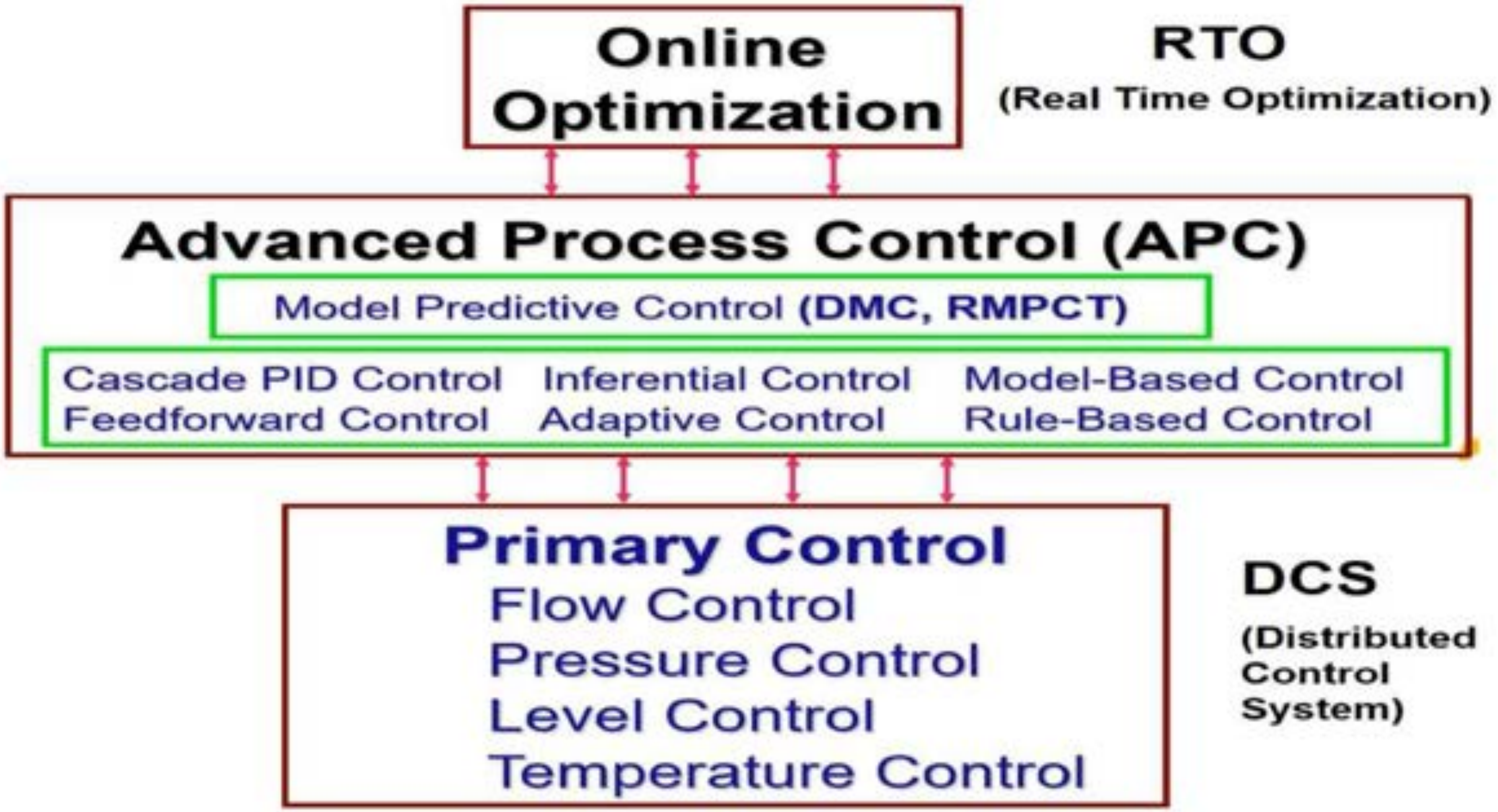
- System Identification (Characterize process dynamics)
  - Transfer Functions
    - ARMAX method (Matlab, Labview, Controlsoft, Control Station)
    - NC-GRG method (Pitops)
  - Step Response Coefficients (DMC, RMPCT)
- Controller Design and Implementation
  - DCS/PLC-resident APC (advanced process control)

    PID, Cascade PID, Feedforward, MBC (model-based controller) – all on DCS/PLC
  - DMC, RMPCT, Connoisseur – mostly on computer connected to DCS/PLC

What is Advanced Process Control (APC) ?

- Goes beyond single loop PID control
- APC includes :
  - Cascade Control
  - Feedforward Control/Decouplers
  - Constraint Override Control
  - Model-Based Control
  - Inferential Control
- APC can be implemented on DCS/PLC/PAC or an external Computer connected to DCS/PLC.

Process Control Hierarchy



Two Types of Process Control Strategies

- Continuous Control Processes
- Batch Control Processes

Continuous Control Processes

- Oil refineries, large chemical plants- olefins
- Plants are rarely shutdown
- Need PID control, Advanced Process Control like Cascade PID, Feedforward, Constraint Override, Model-based control, Multivariable control

Batch Control Processes

- Pharmaceutical plants, fermentation, alcohol
- Discrete changes in process/operating conditions
- Recipe-based logic
- Rule-based controllers in addition to PIDs.

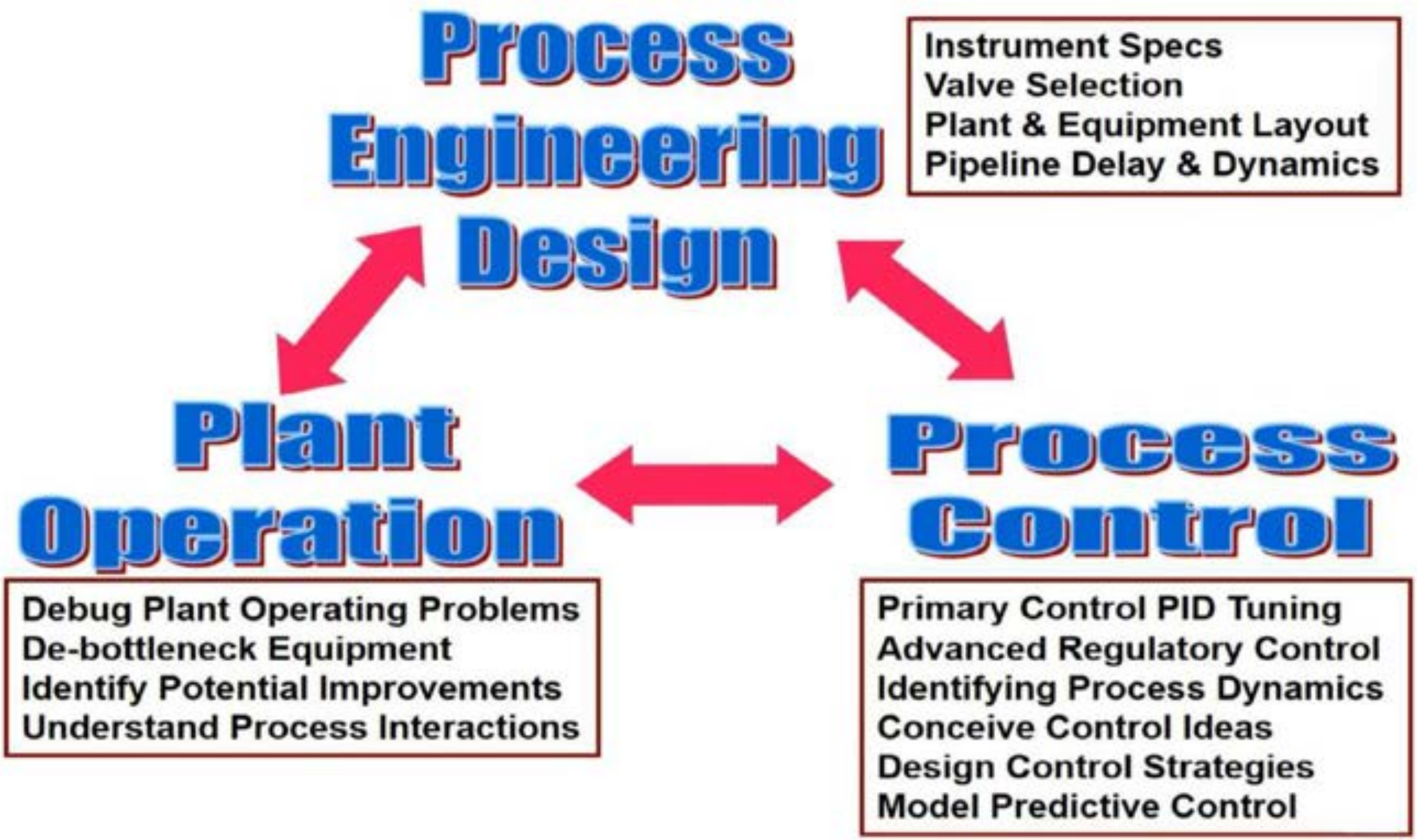
Incentive for Good Process Control

- Maximize Plant Capacity
- Minimize Utility Consumption
- Increase Plant Automation
- Operate closer to constraint limits
- Achieve smoother, more stable operation

Improved Operation | Safety Reliability | More Profits

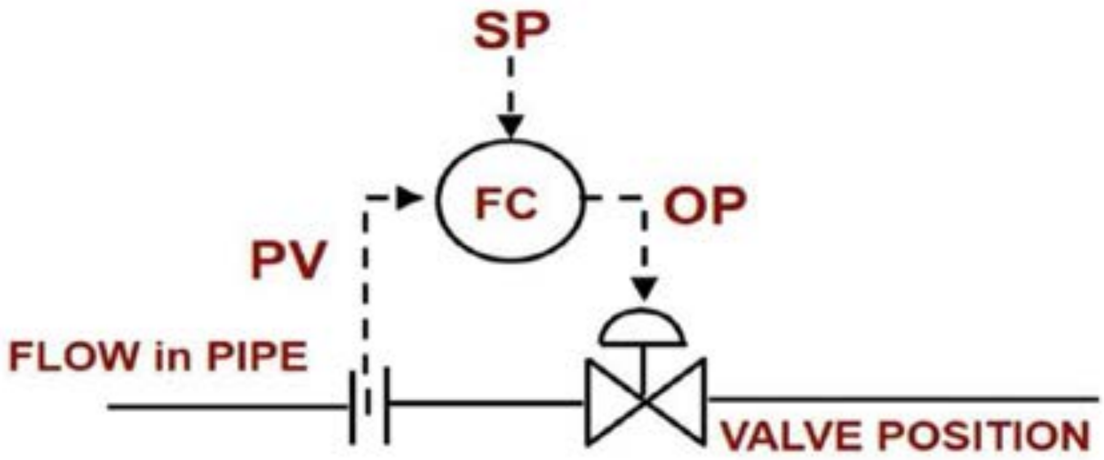
MONEY

Overlapping Elements of Process Control



Consider flow of liquid in a pipe. Flow is measured by a flowmeter and flow is controlled by manipulating a control valve.

PID I/O (input/output) Signals



FC = Flow Controller (**PID** Controller)

SP (Setpoint) is the desired value of the **CV** (set by operator).

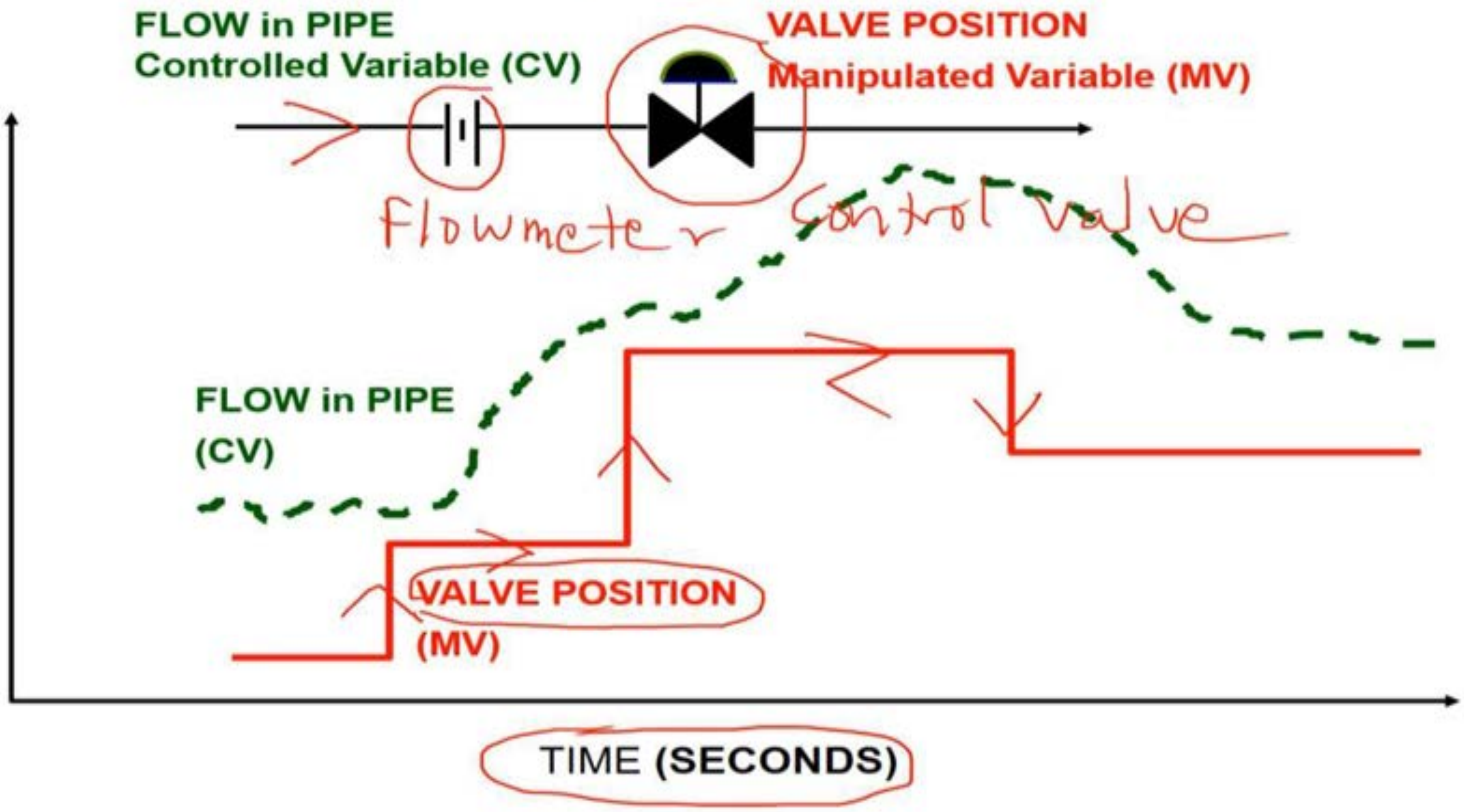
PV (Process Value) is the current measured value of the CV.

OP (Output) is the output (calculation) from the PID algorithm.

PID output will always be the same as the valve position **MV**.

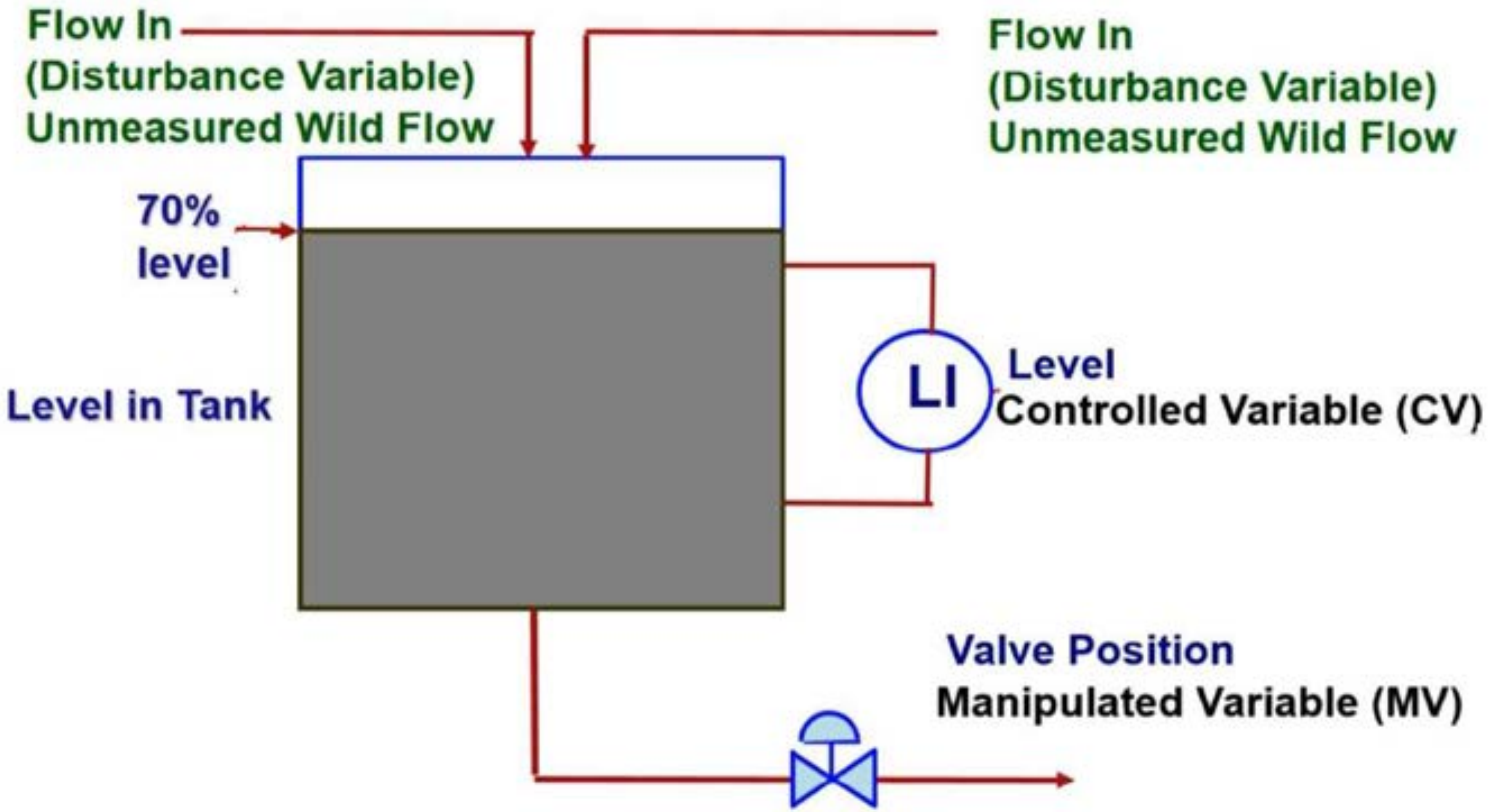
Remember, CV is Controlled Variable (Flow) and
MV is Manipulated Variable (Valve Position).

Valve Position versus Flow Dynamics

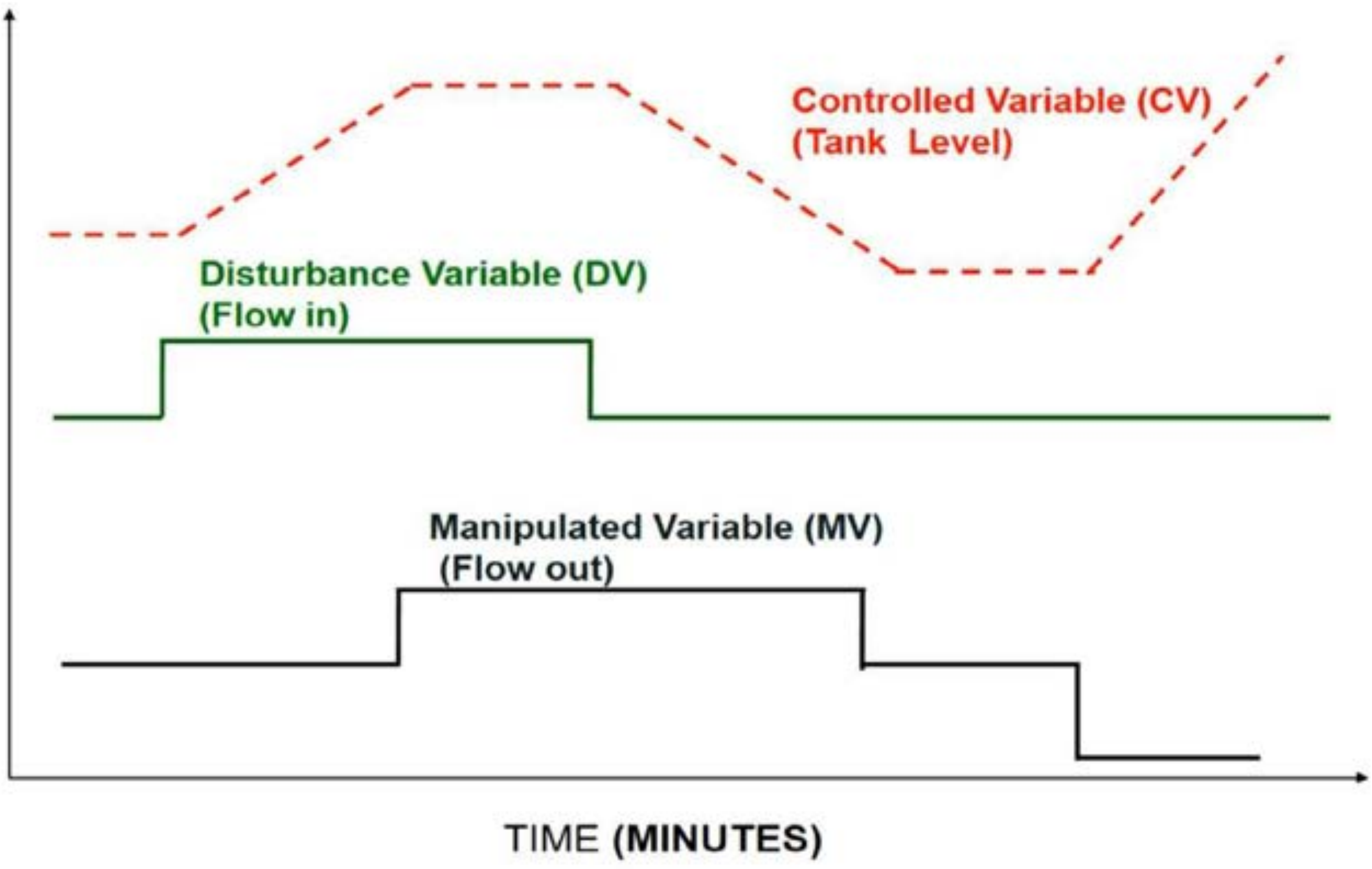


Consider level control inside a tank. Level is measured by a level indicator and the level is controlled by manipulating a control valve at the outlet of the tank.

Valve Position versus Level Dynamics



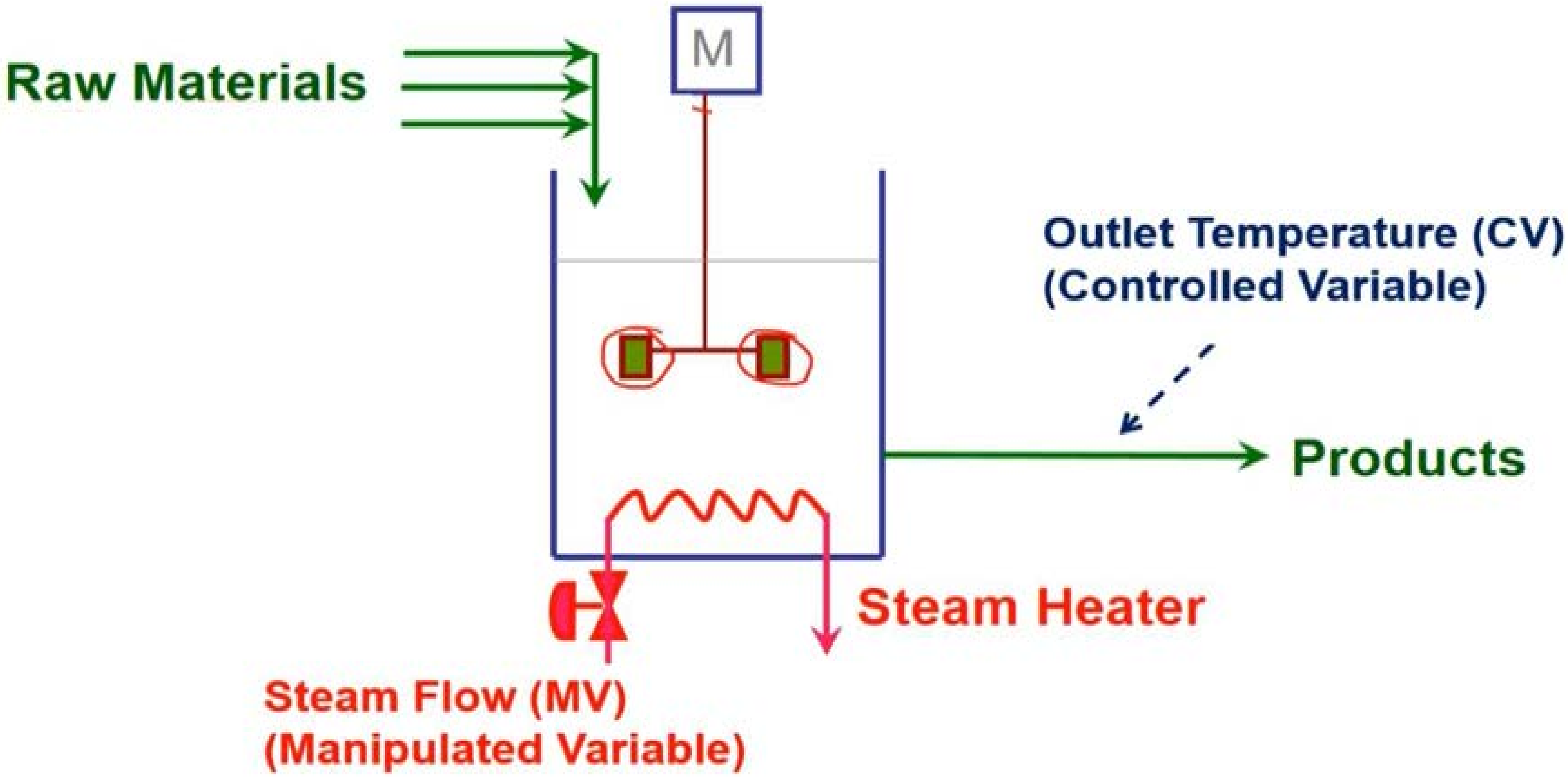
Flow versus Level Dynamics



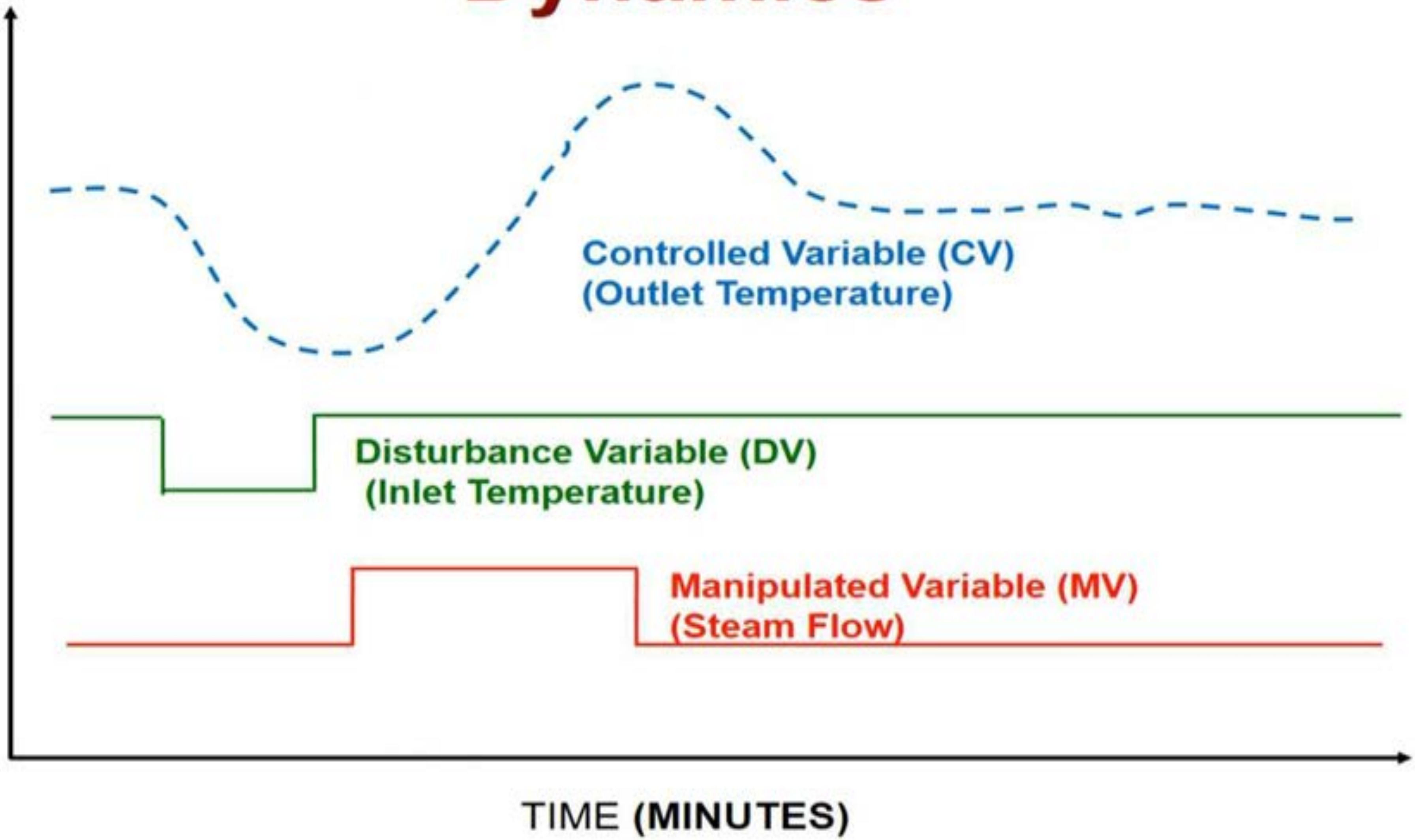
Consider temperature control in a reactor heated by an immersed steam coil.....

Steam Flow vs. Temperature Dynamics

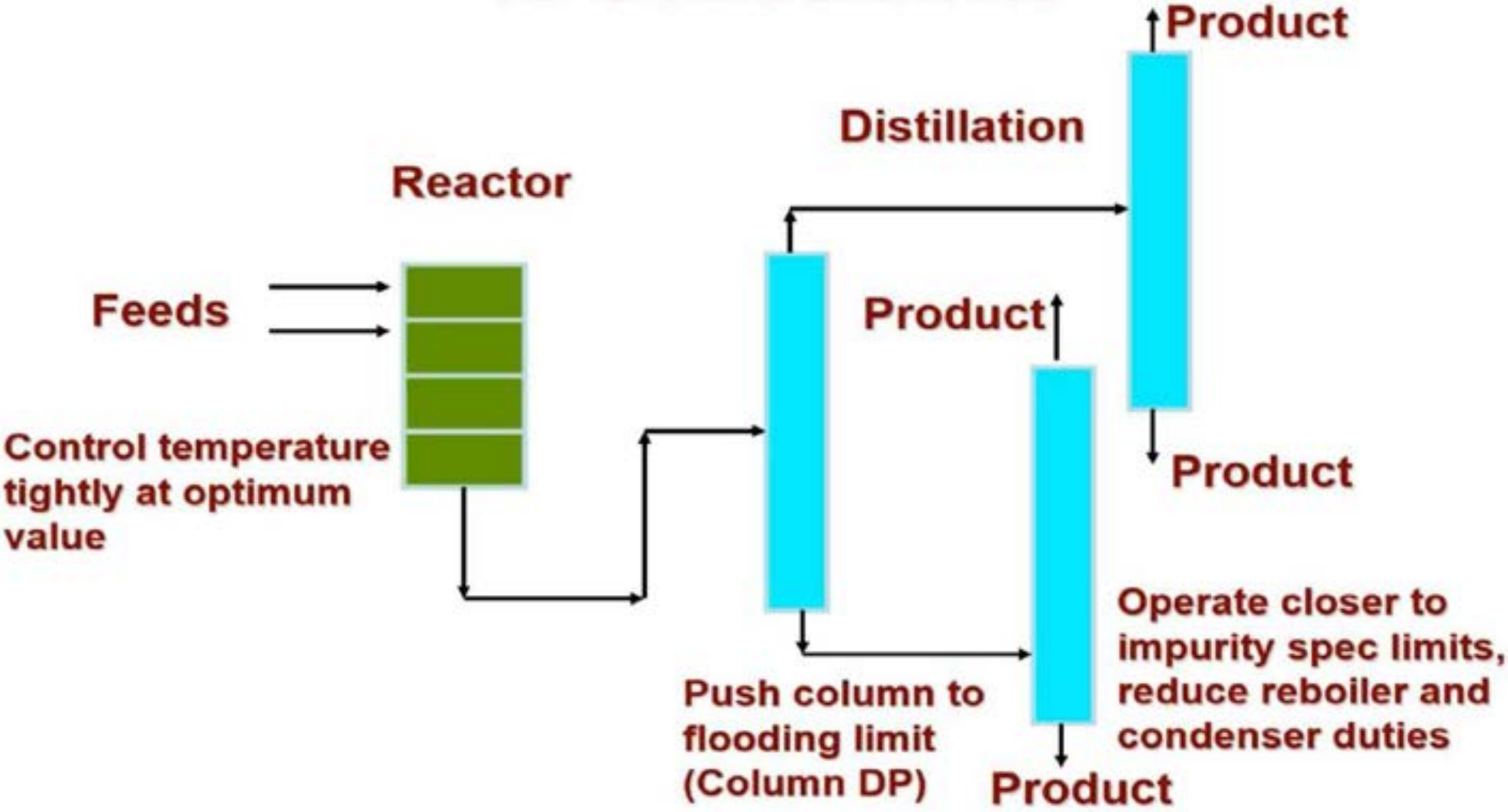
CSTR



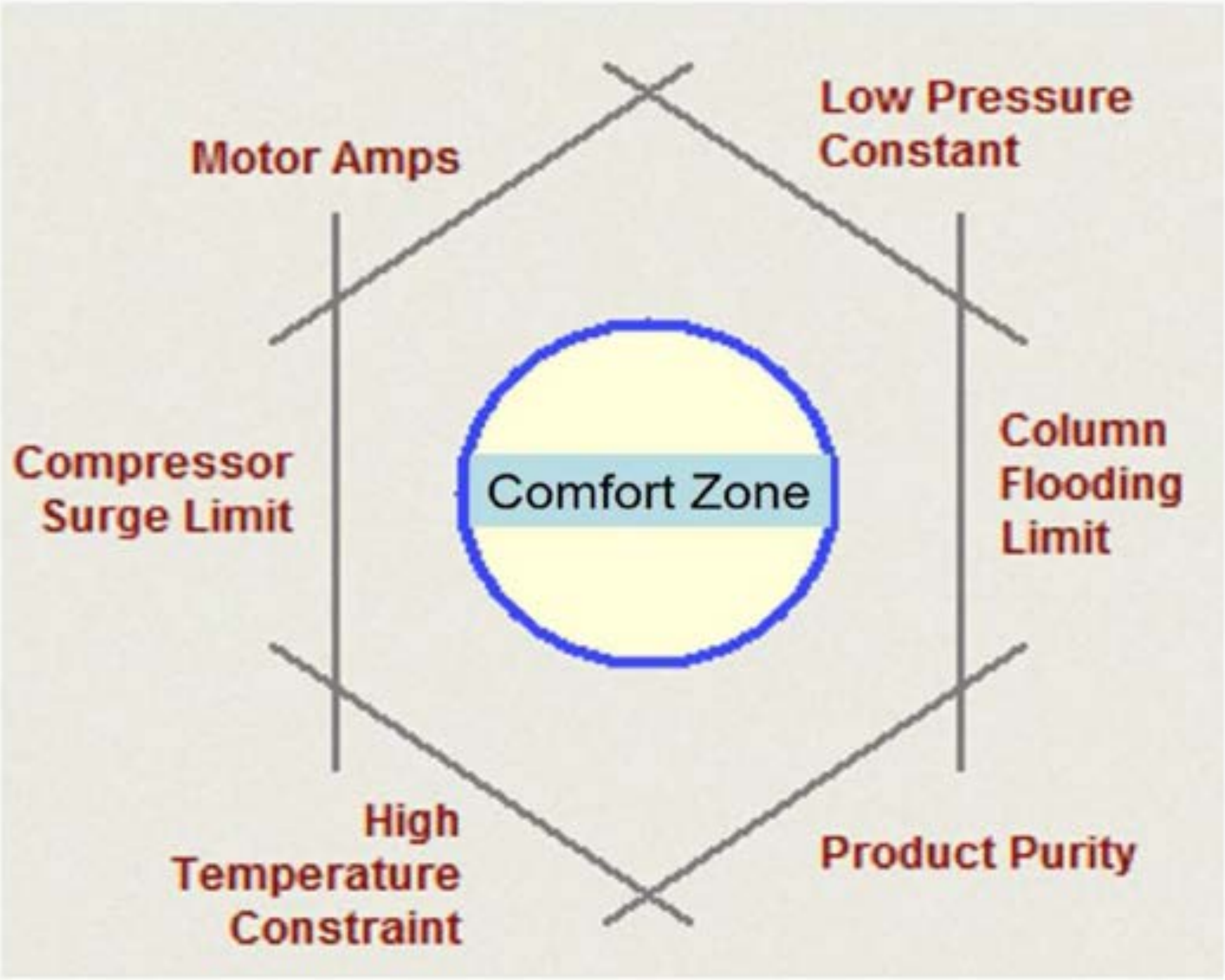
Steam Flow vs. Temperature Dynamics



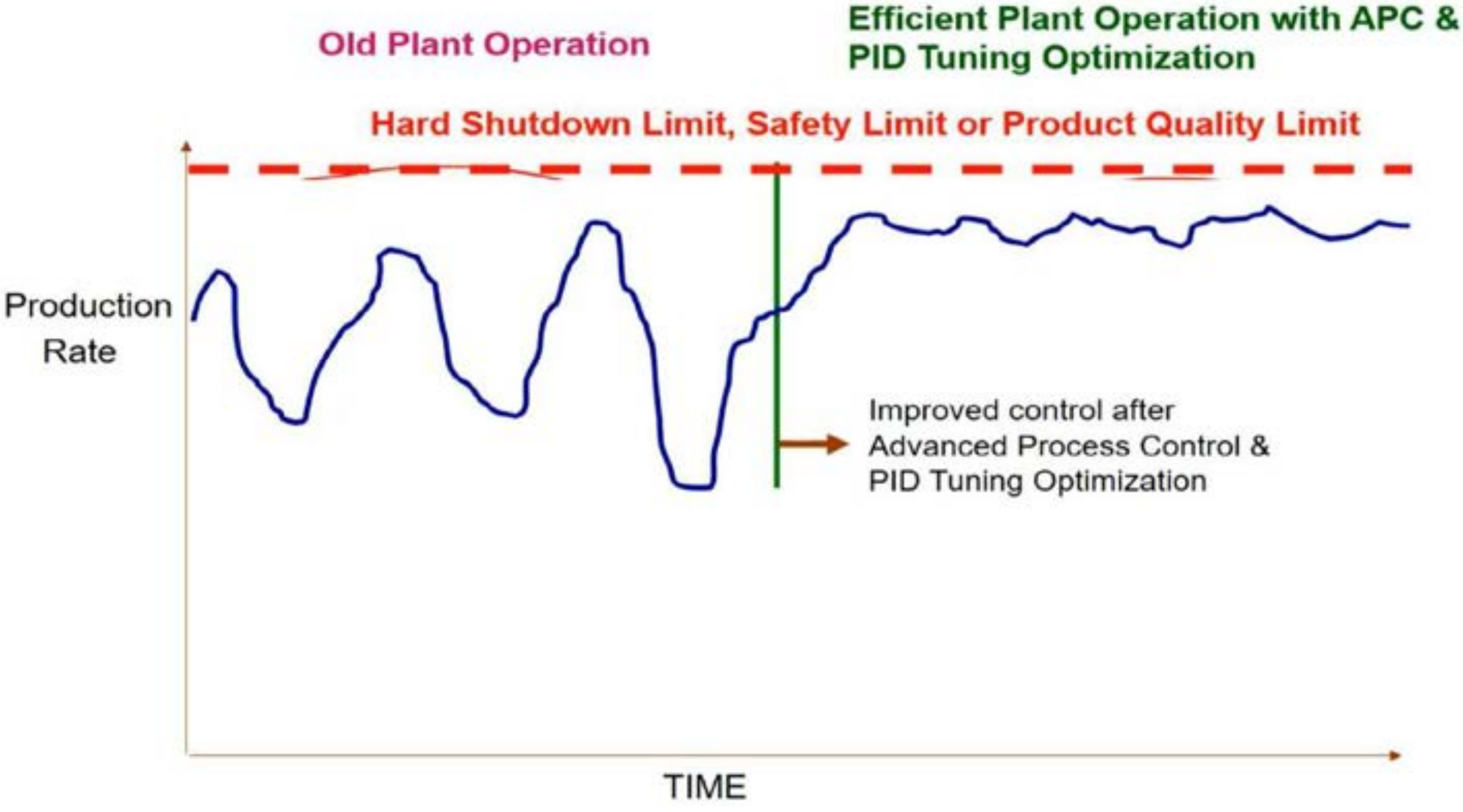
APC allows operation closer to constraints



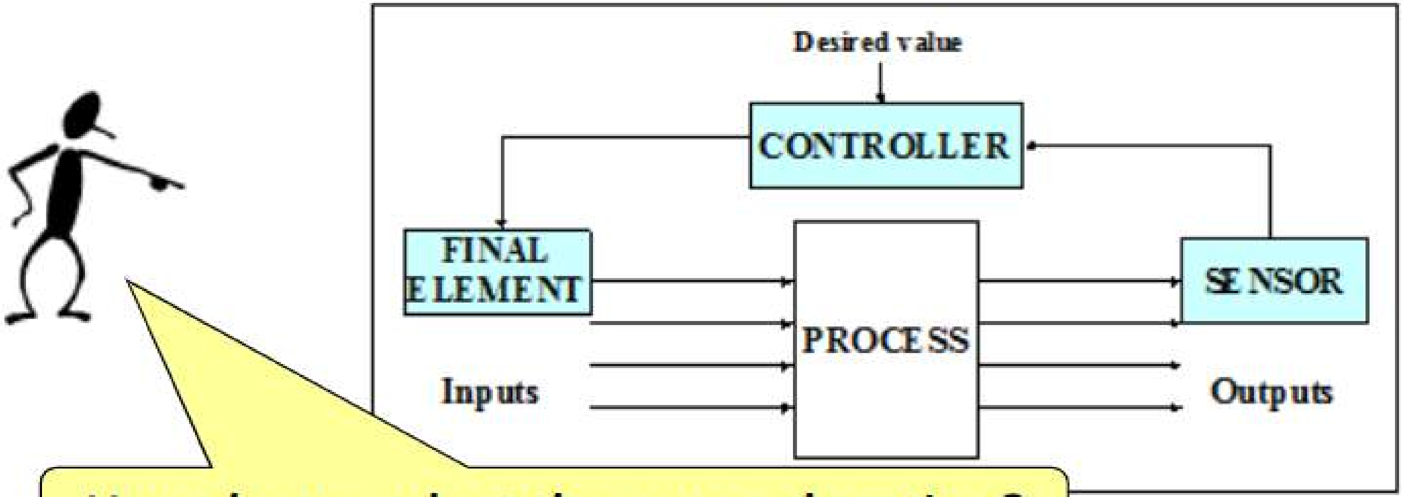
The Comfort Zone



Process Control Improvements



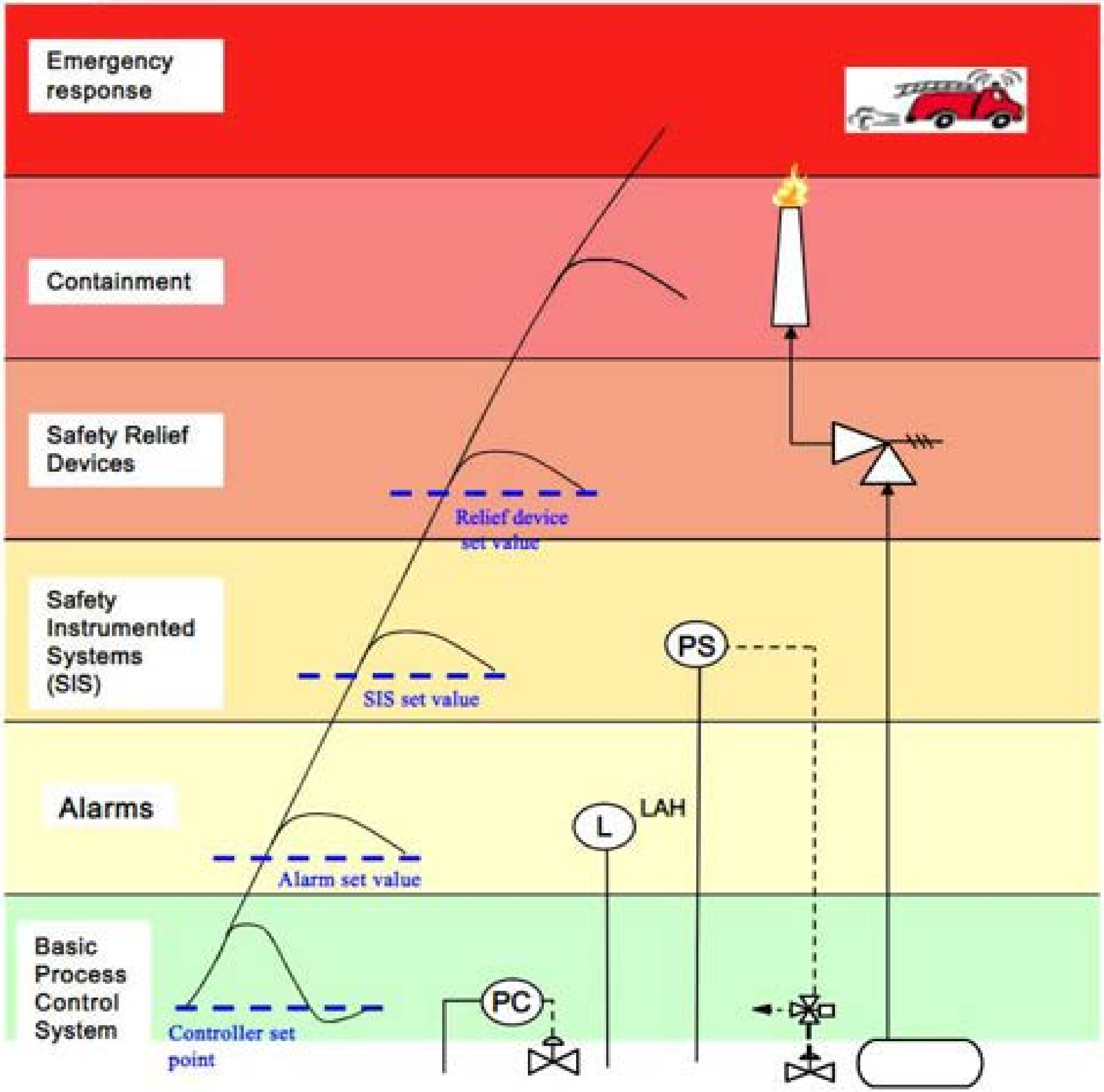
WHAT DOES A FEEDBACK SYSTEM DO?



How do we select the sensor location?

Control Objectives

1. Safety — Prevent explosion of closed vessel: Pressure
2. Environmental Protect.
3. Equipment protect.
4. Smooth operation — Constant production rate: Feed flow
5. Product quality — Water purity: Dissolved oxygen in reactor
6. Profit
7. Monitoring and diagnosis



The Control for Safety Hierarchy

Safety is implemented in independent layers

- Improve reliability
- Match equipment to functions
- Easy to understand

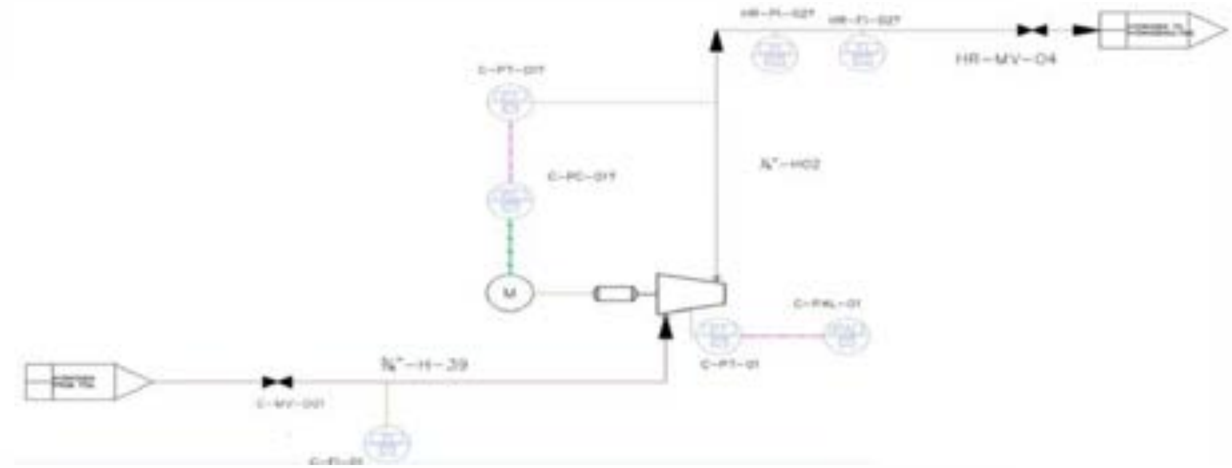
Wrap-Up & Questions

Summary:

- Definition of process control
- Block diagram & components
- Industrial relevance

Discussion & Q/A

Piping & Instrument Diagram (P&ID)



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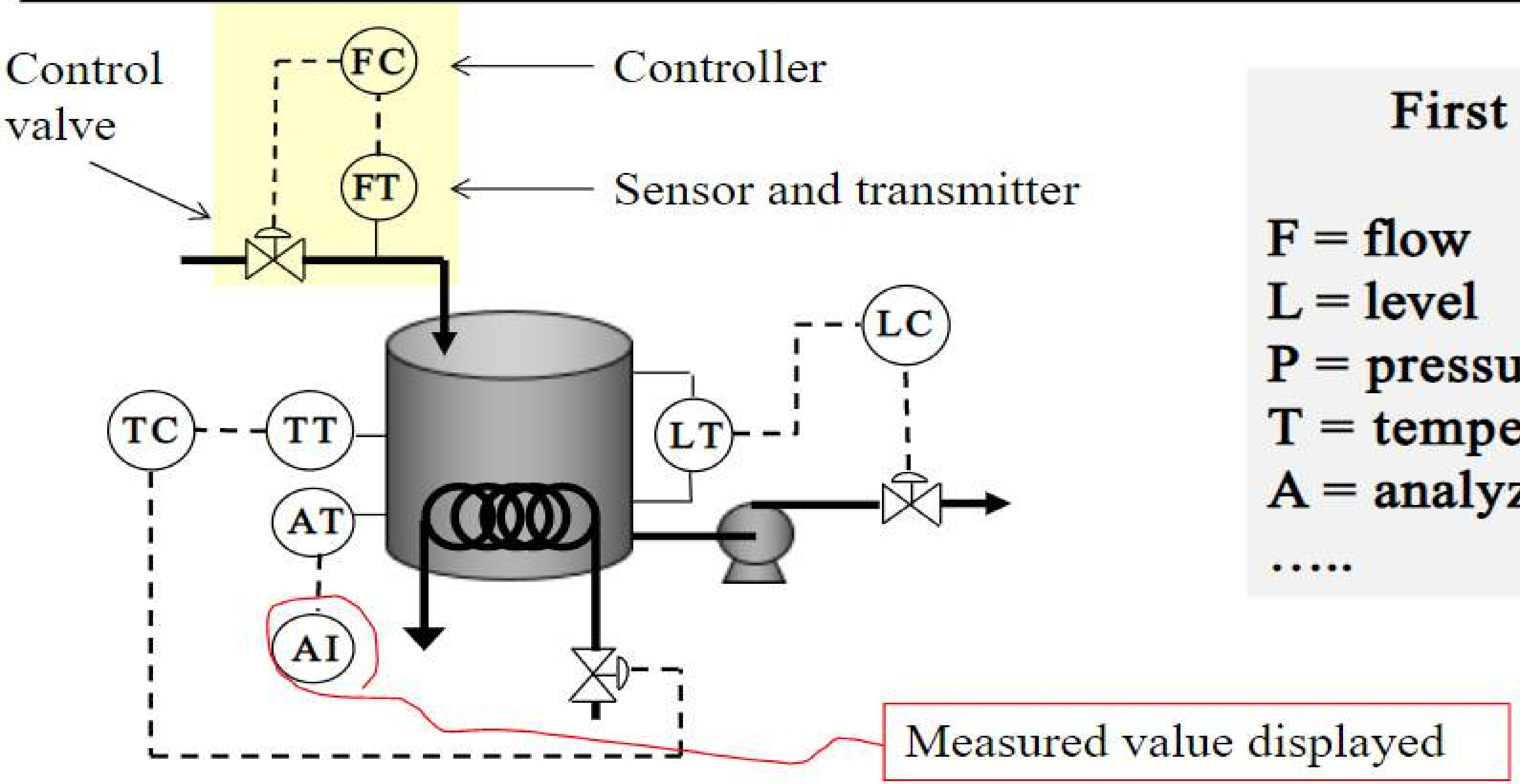
14th-Oct-2025

HOW IS CONTROL DESIGN DOCUMENTED?

Piping and Instrumentation Drawing (P&ID)

- The system is too complex to describe in text.
- We must use standard symbols.



First letter

F = flow
L = level
P = pressure
T = temperature
A = analyzer
.....

HOW IS CONTROL DESIGN DOCUMENTED?

Piping and Instrumentation Drawing (P&ID)

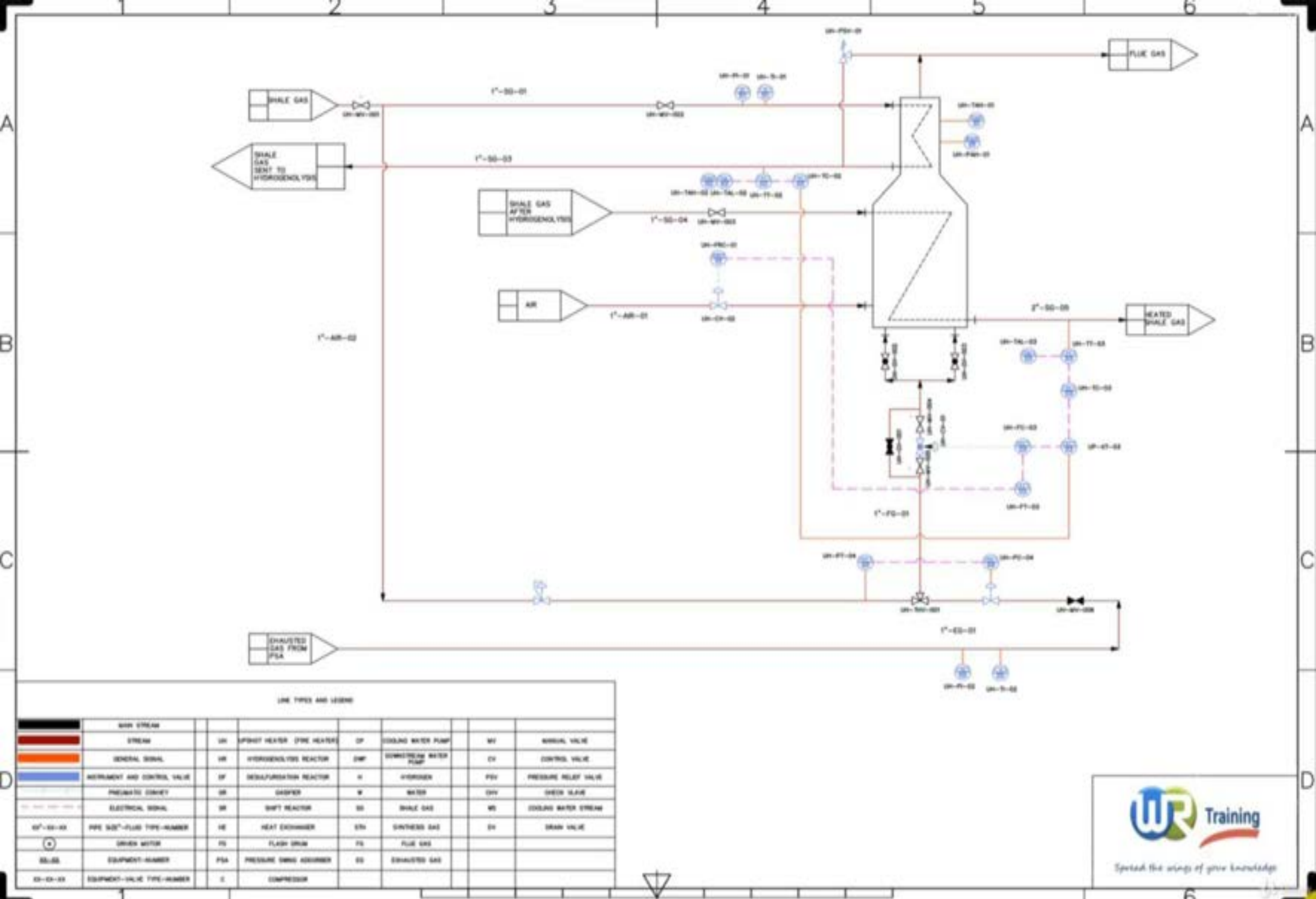
- Graphical description of the process and process equipment using standard symbols (**ANSI/ISA-S5.1 *Instrumentation Symbols and Identification***)
- The P&ID is used by field technicians, engineers and operators to better understand the process and how the instrumentation is interconnected.

HOW IS CONTROL DESIGN DOCUMENTED?

Piping and Instrumentation Diagrams (P&IDs) use specific instrumentation symbols to show the connectivity of equipment, piping, sensors, and valves within a control system, and they are most commonly used in engineering. These instrumentation symbols can represent actuators, sensors, and controllers.

Not all P&ID elements are standardized, but the instrumentation symbols follow a standard set by ANSI/ISA 5.1-2022.

Table 1: Summary of Standards for P&IDs				
Standard No.	Title	Pages	Sector	Topics Covered
ANSI/ISA 5.1	Instrumentation Symbols and Identification	128	All	Identification Letters Table Graphic Symbols Symbol Dimensions
ISA 5.3	Graphic Symbols for Distributed Control / Shared Display Instrumentation, Logic and Computer Systems	79	Computer SCADA	Symbols Identification Alarms Computer Symbols and Logic
IEC 60617	Graphical Symbols for Diagrams	1900	All	Database of Symbols
ISO 10628	Diagrams for the Chemical and Petrochemical Industry	22	Chemical Petro	Letter Symbols
ISO 14617 Parts 1 to 15	Graphical Symbols for Diagrams	300	All	Functional Links Control Loops Processing Functions Logic Functions
ISO 15519 Parts 1 & 2	Specification for Diagrams for Process Industry	25	Process	Block Diagrams, PFDs, P&ID Layout Connecting Lines Inscription, Scale, Limits
PIP PIC001	Piping and Instrumentation Diagram Documentation Criteria	79	All	Industry Standards Equipment, Piping, Instrumentation, Controls
EN 62424	Representation of Process Control Engineering	175	All	P&ID Software and Controls Interfaces

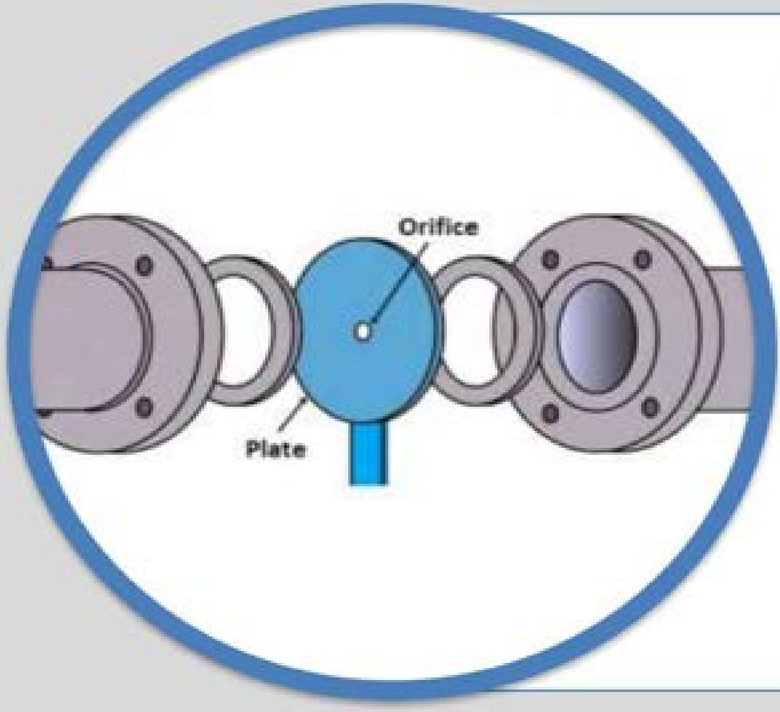

FLUE GAS
SHALE GAS
SHALE GAS SENT TO HYDROGENOLYSIS
SHALE GAS AFTER HYDROGENOLYSIS
AIR
HEATED SHALE GAS
EXHAUSTED GAS FROM PSA
Training

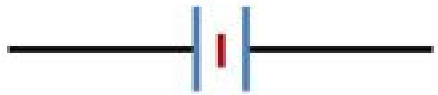
HOW IS CONTROL DESIGN DOCUMENTED?

P&ID contains the following	P&ID does not contain the following
• All piping and equipment connections	• The distance between objects. The drawing is not to scale.
• An approximate location for connections (e.g., top or bottom of tank, tray location, etc.)	• The vertical or horizontal (or 3D) position of objects
• Equipment identification (numbers)	• The sizes of objects (e.g., vessel), not even the relative size
• The size of piping	• The exact design for piping connections, including those to vessels
• All sensors (whether locally or remotely displayed and recorded) and whether used for an alarm, with priority	• Sensor details such as physical principle (e.g., thermocouple) and measurement range
• All valves (whether automated for remote operation or not), including failure position if remotely operated valves	• Details of the control calculations when involving complex logic and/or calculations
• Control strategies, as much detail as possible graphically. These can be regulatory and safety related	• Any detail about the human interface display or the type of historical data
• Whether signals and control calculations use analog or digital equipment	• Operating policy (which appears in a separate operations manual)

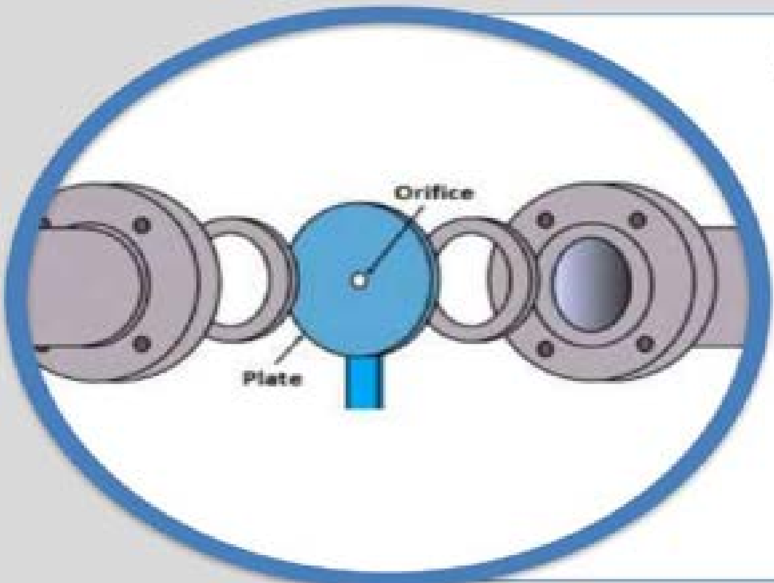
P&ID SENSING DEVICES SYMBOLS

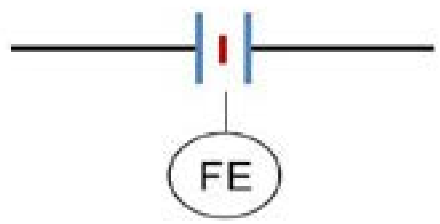
- **ORIFICE**



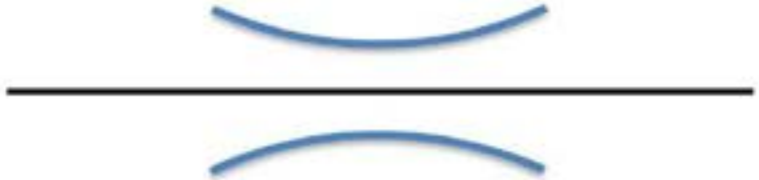


- **ORIFICE WITH FLOW ELEMENT**

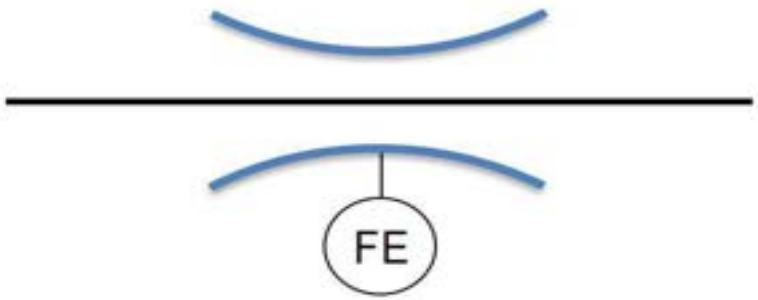




- **VENTURI**

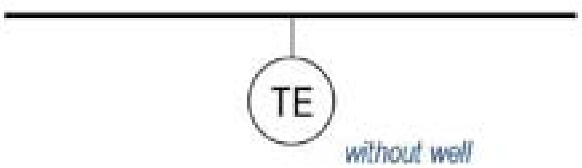


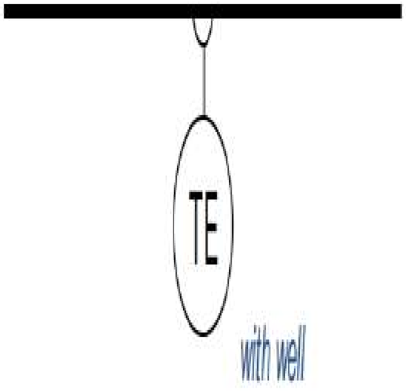
- **VENTURI WITH FLOW ELEMENT**



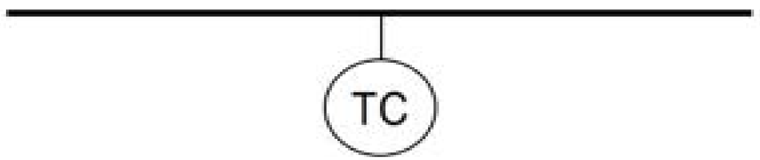
ROTOMETER

TEMPERATURE ELEMENT

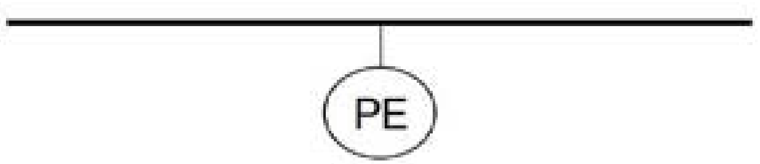




- THERMOCOUPLE

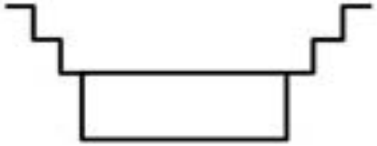


- PRESSURE ELEMENT

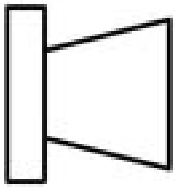


- **DIFFERENTIAL PRESSURE CELL**

- **CONDUCTIVITY CELL**



RADIATION DETECTOR



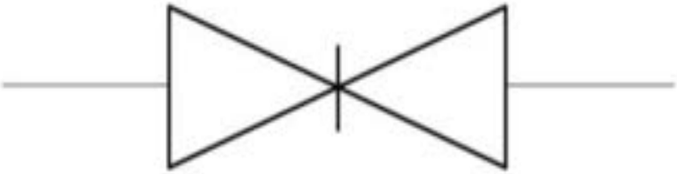
P&ID VALVE SYMBOLS

P&ID VALVE SYMBOLS

VALVES ARE USED TO CONTROL :

- THE DIRECTION
- FLOW RATE
- PRESSURE

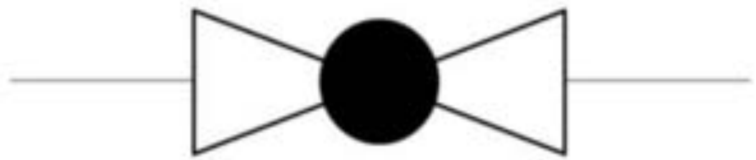
GATE VALVE



The valve is open

The valve is closed

GLOBE VALVE



The valve is closed

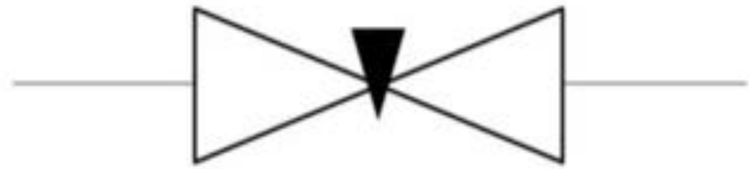
The valve is open

BALL VALVE

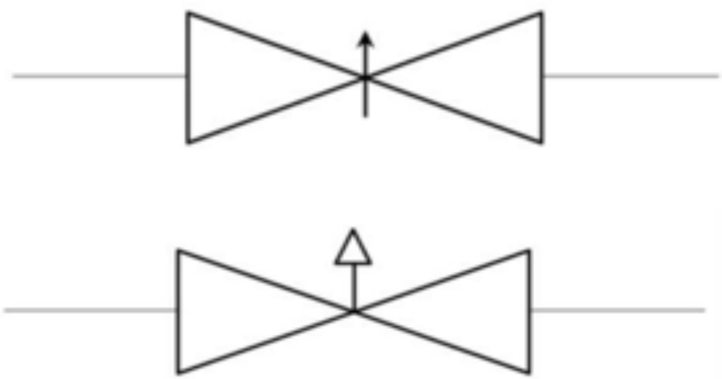
The valve is open

The valve is closed

NEEDLE VALVE

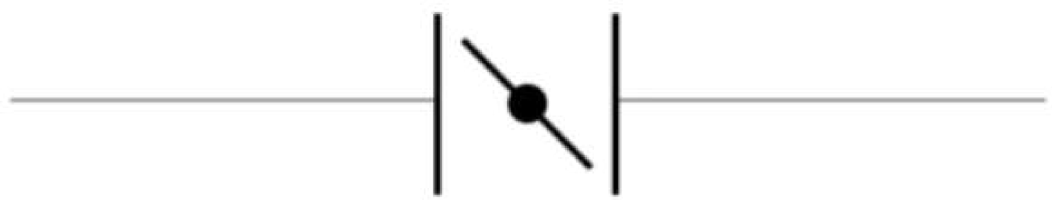


The valve is open

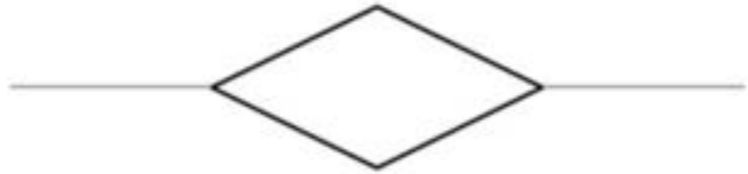


The valve is closed

BUTTERFLY VALVE



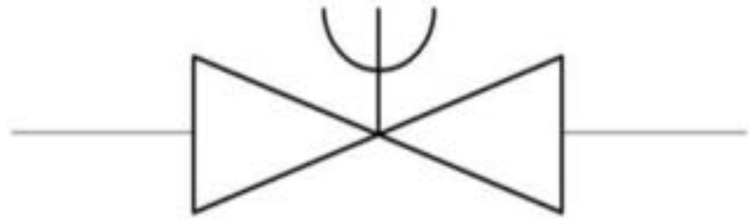
PLUG VALVE

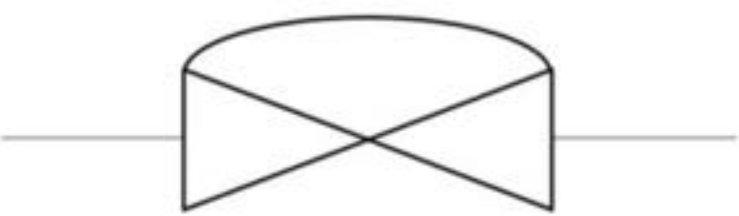


The valve is open

The valve is closed

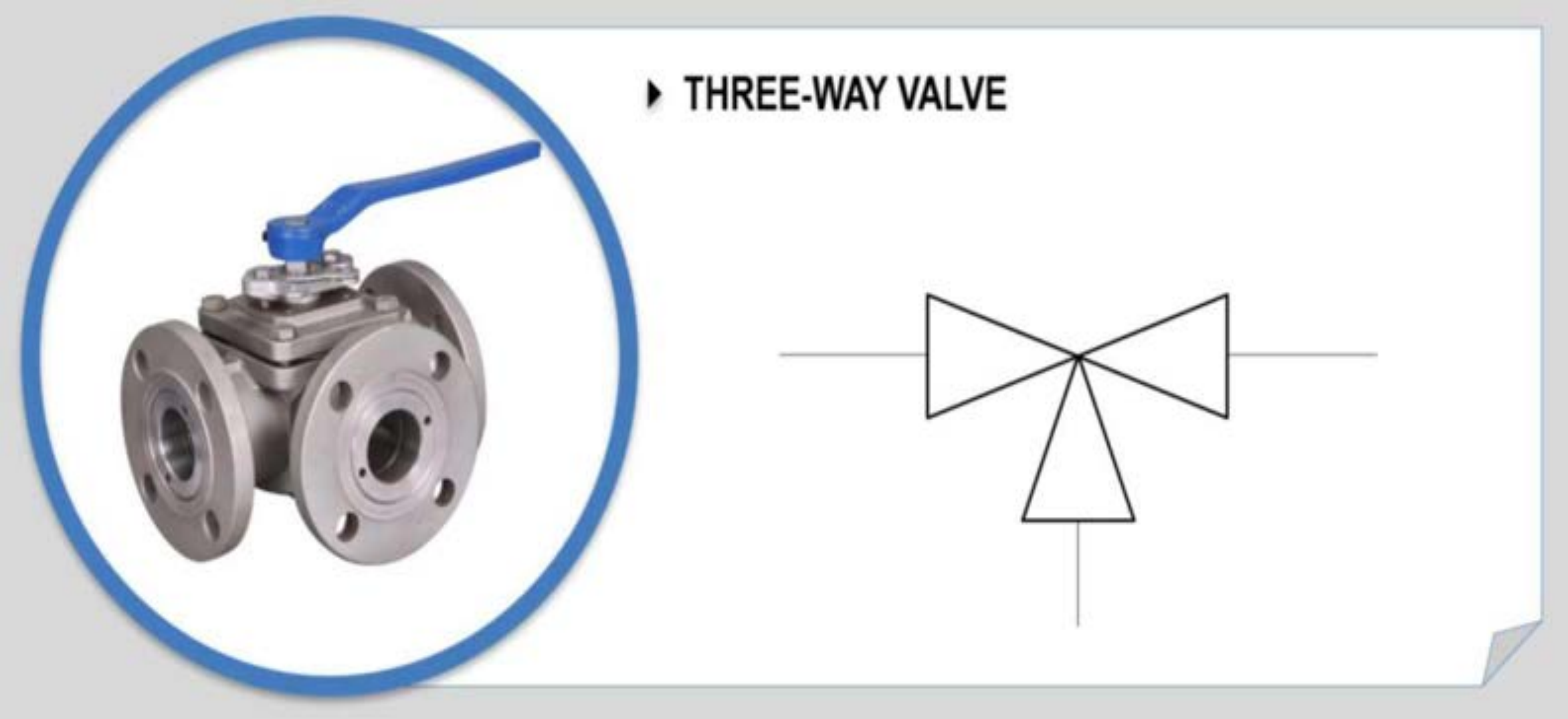
DIAPHRAGM VALVE

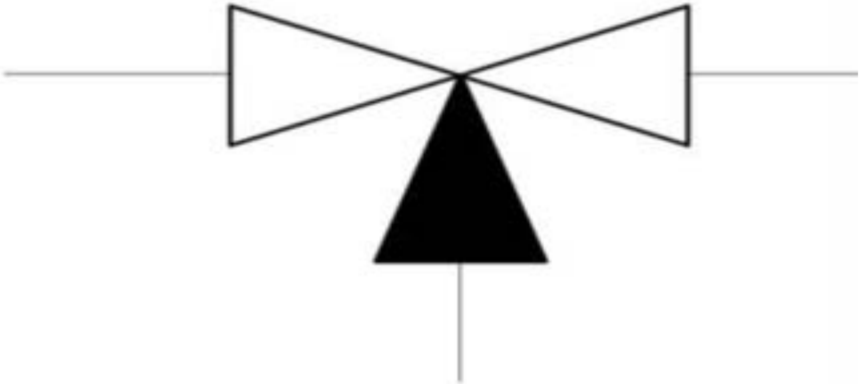


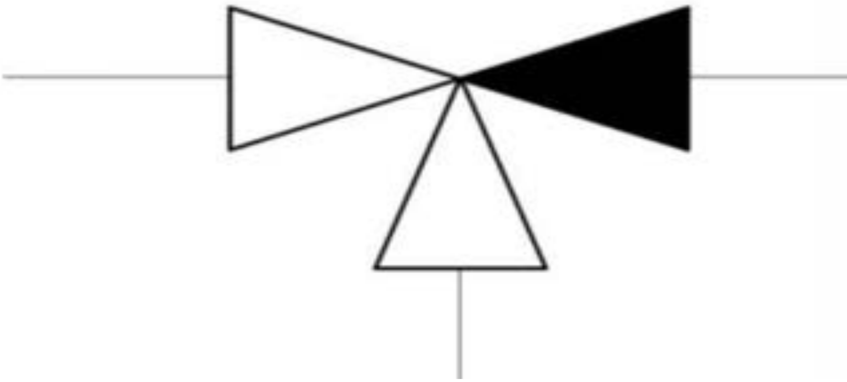


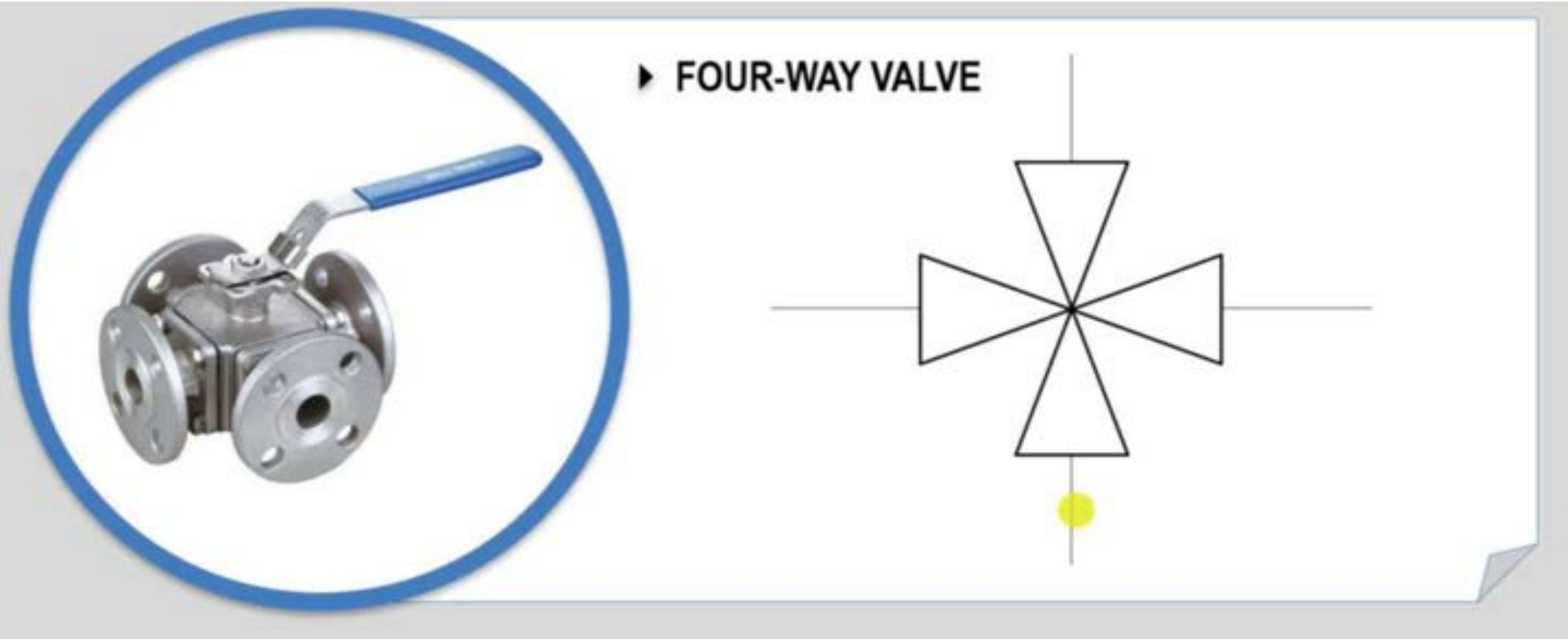
The valve is open

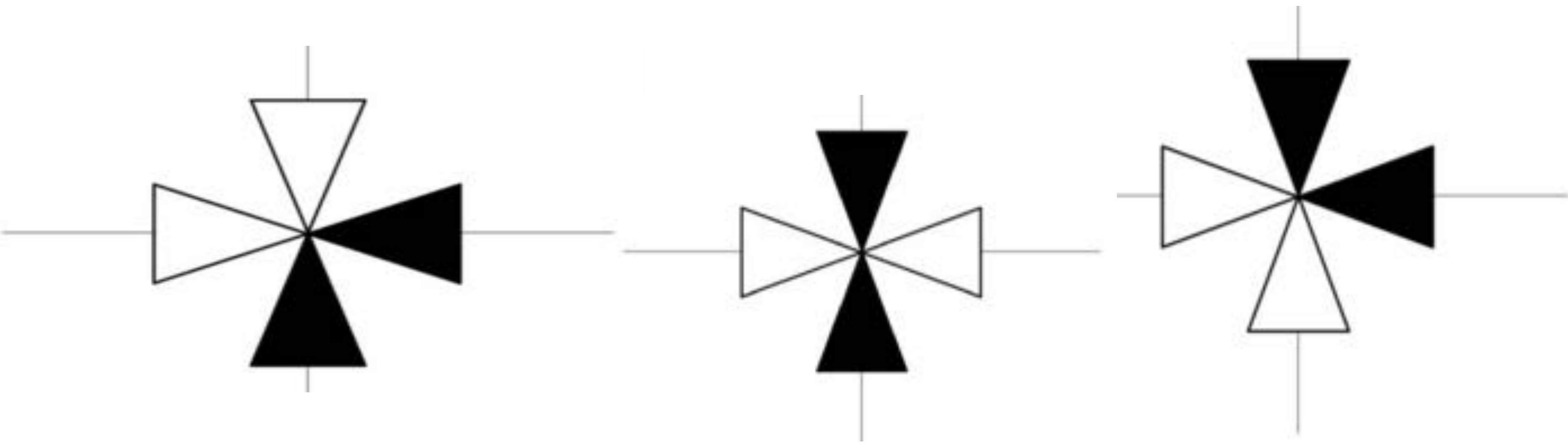
The valve is closed


THREE-WAY VALVE

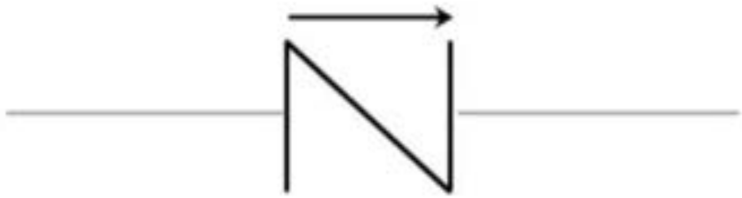


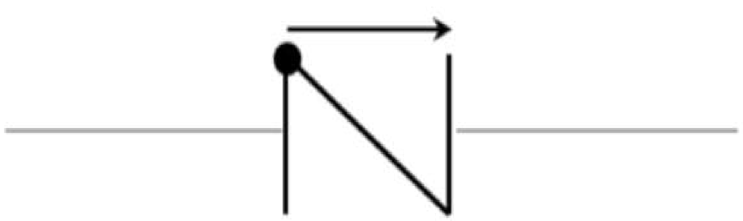


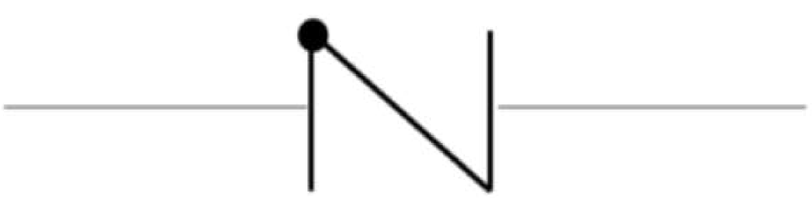

FOUR-WAY VALVE



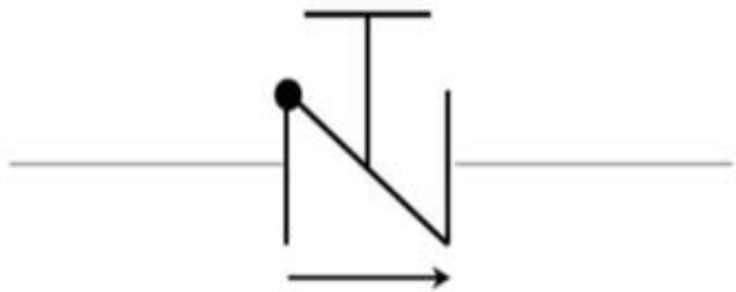
CHECK VALVE

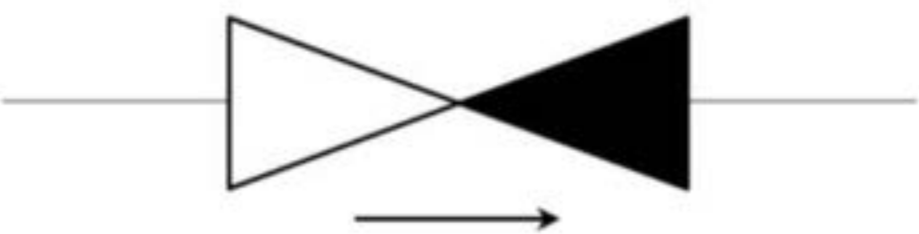


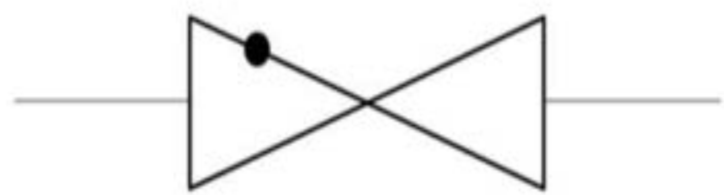




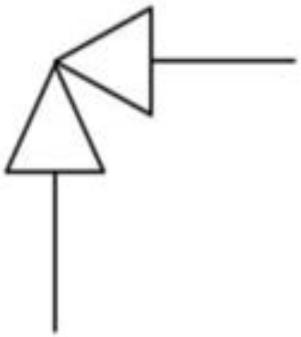
STOP CHECK VALVE



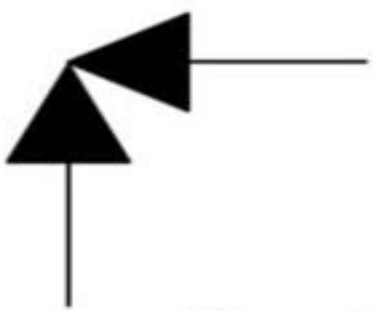




ANGLE VALVE

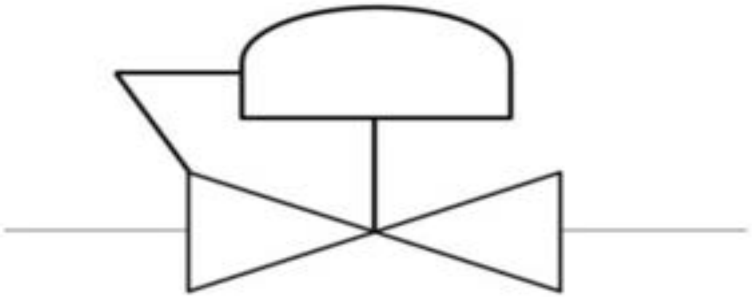


The valve is open

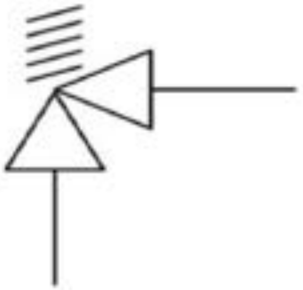


The valve is closed

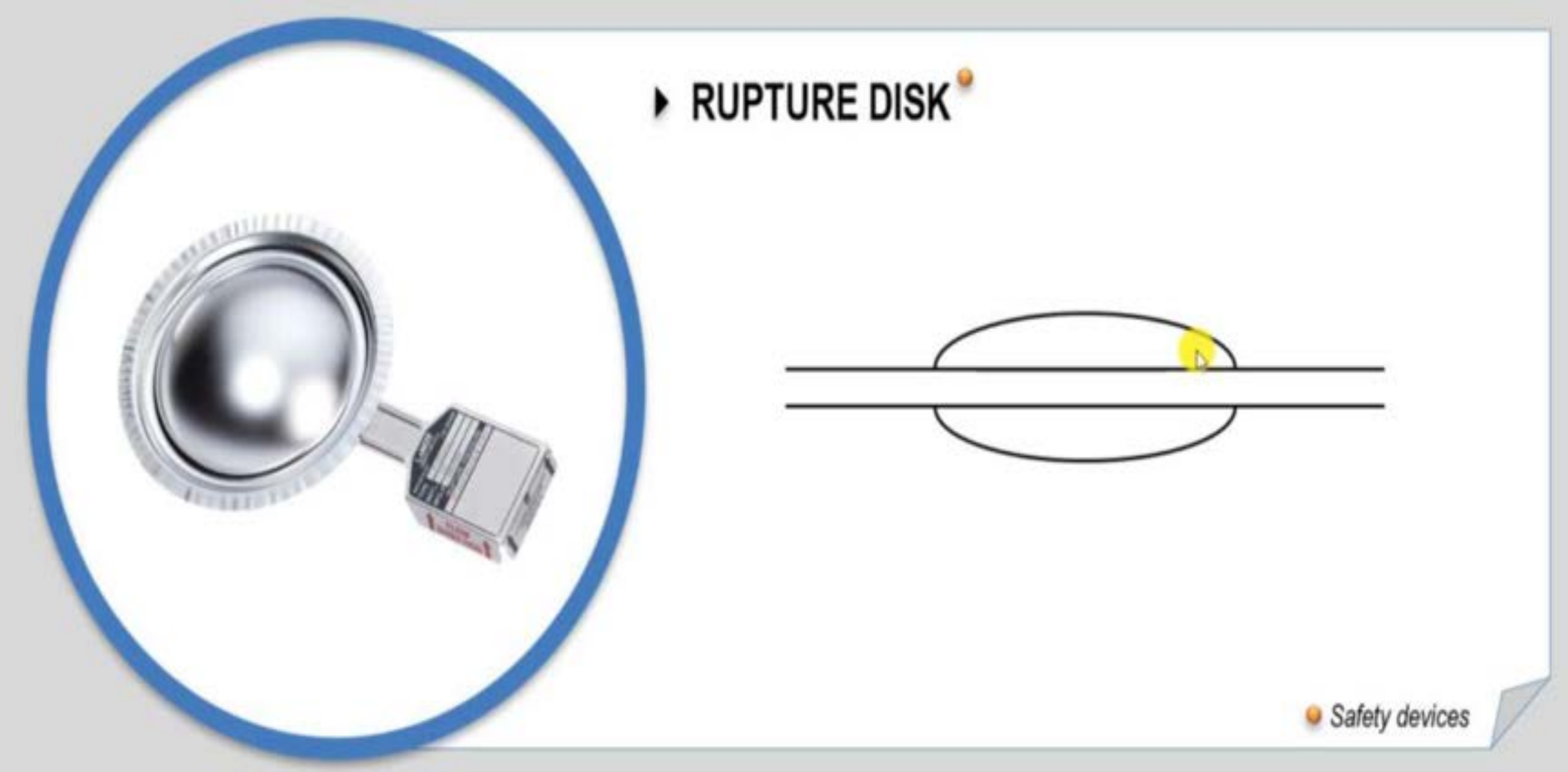
- **PRESSURE REGULATOR**

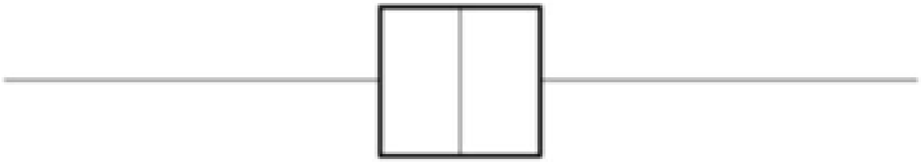


- **RELIEF VALVE**



Safety devices

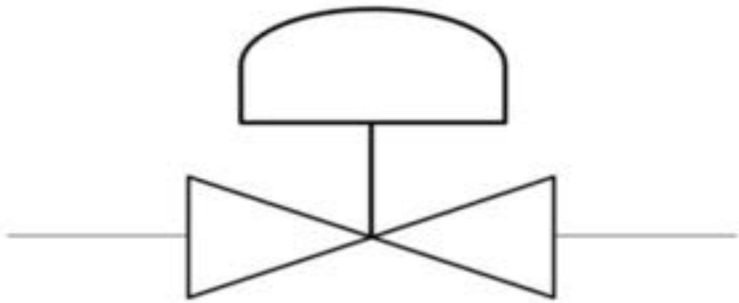

RUPTURE DISK
Safety devices



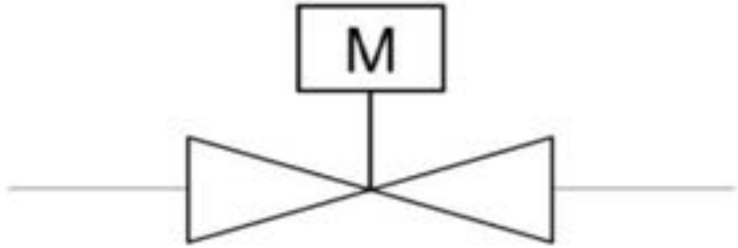
VALVES WITH ACTUATORS :

- ALLOW REMOTE OPERATION
- INCREASE MECHANICAL ADVANTAGES
- OR BOTH

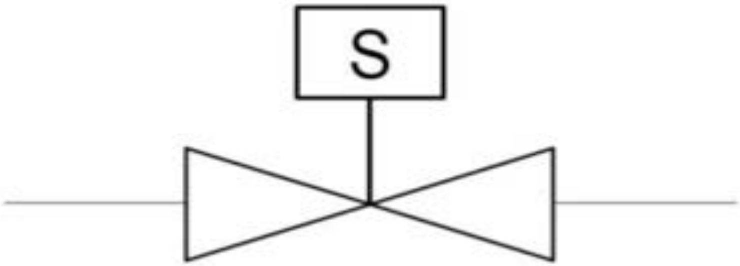
- DIAPHRAGM ACTUATOR



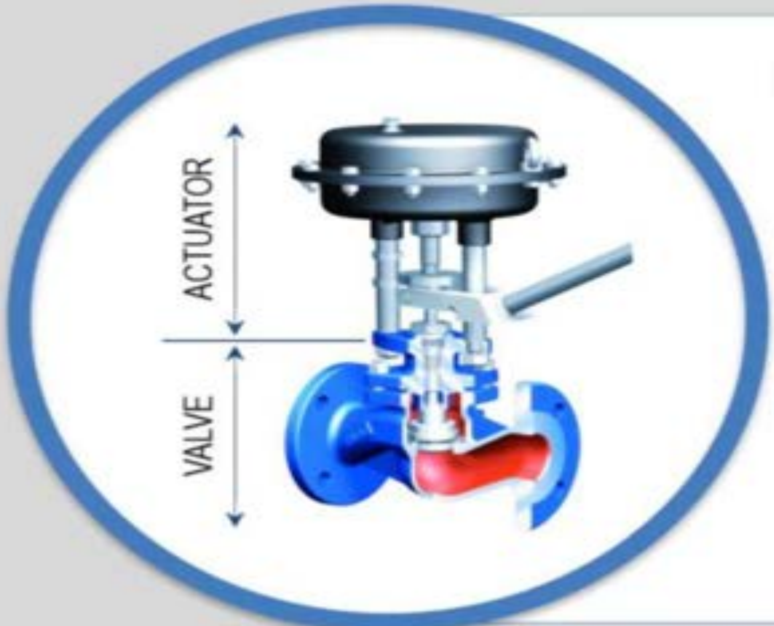
- ELECTRIC MOTOR ACTUATOR

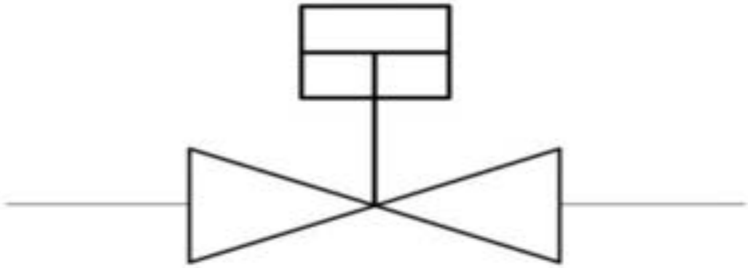


SOLENOID ACTUATOR

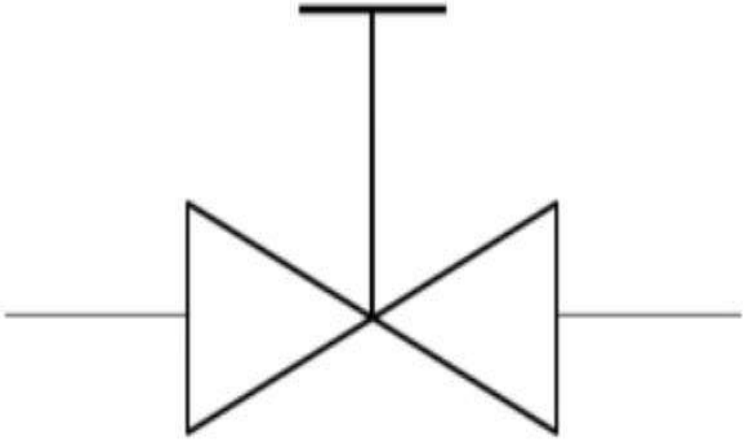


PISTON ACTUATOR

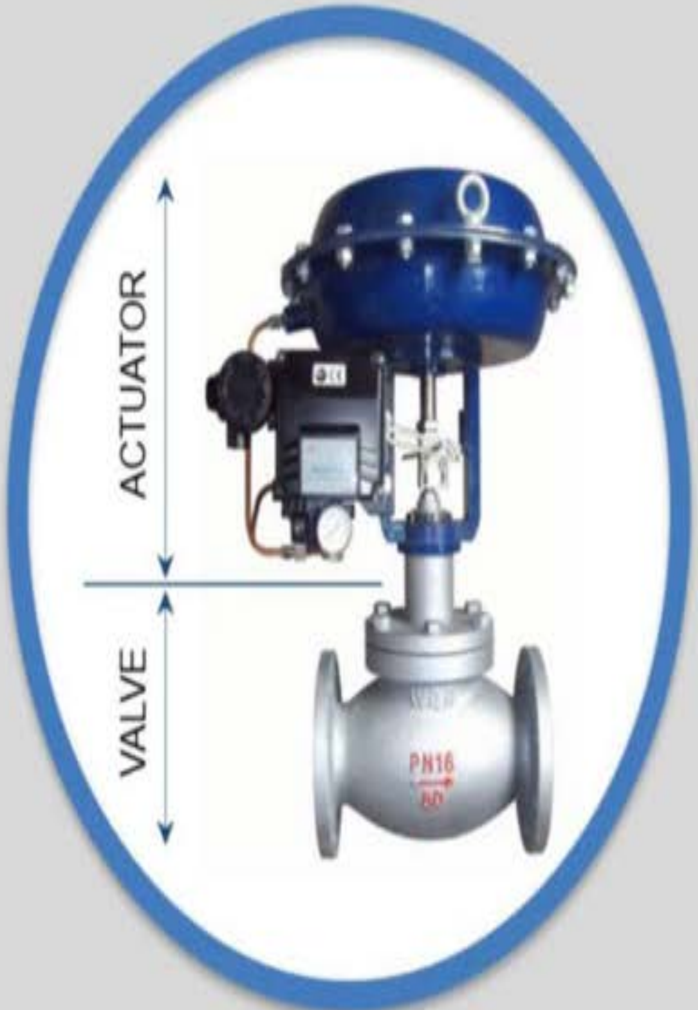




- MANUAL



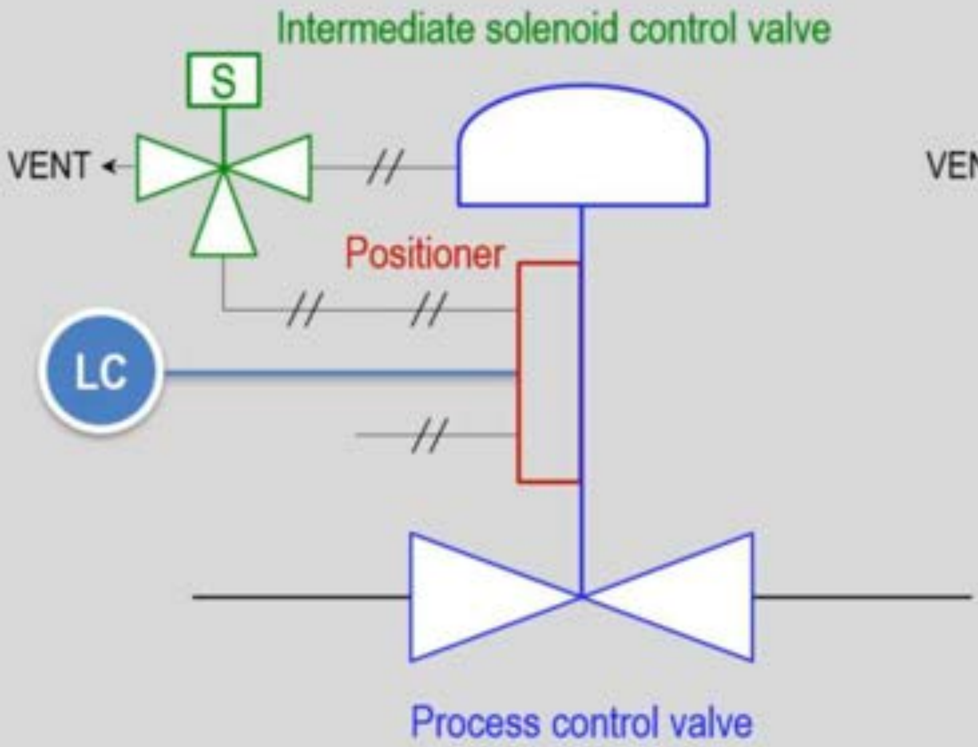
P&ID CONTROL VALVE DESIGNATIONS

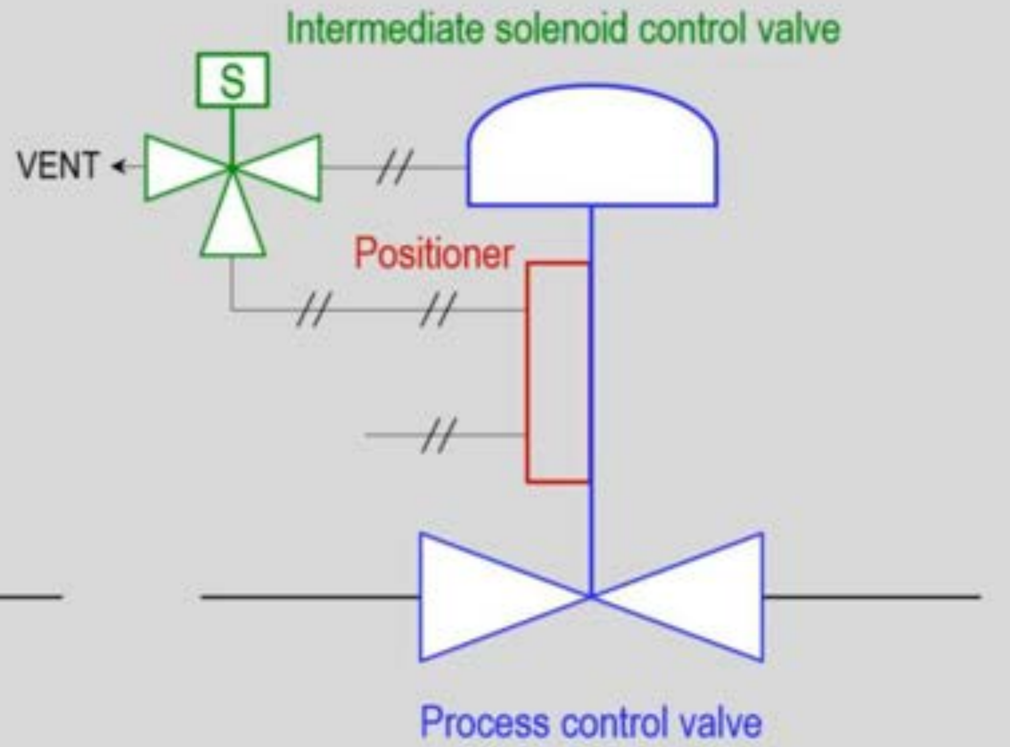


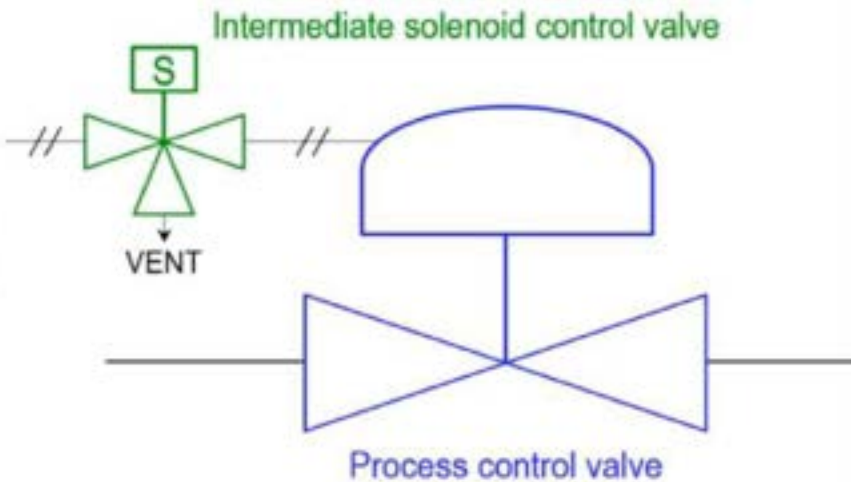
CONTROL VALVE

CONTROL VALVE CONFIGURATION :

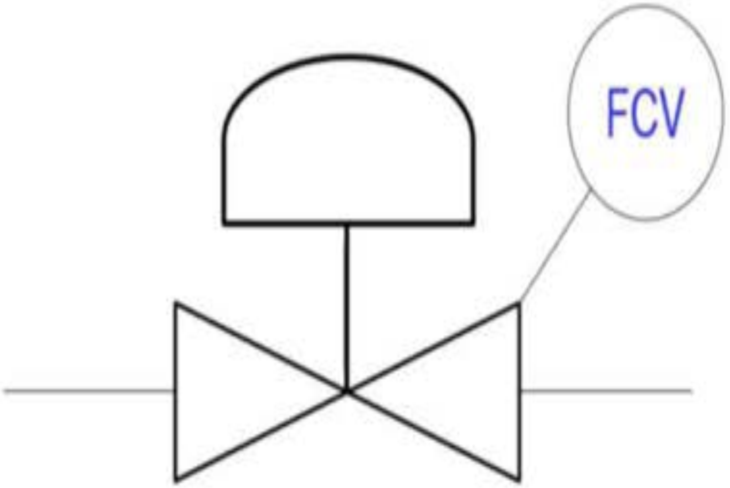
- Actuator manual control from a remote operating station
- Actuator **automatic** control from an instrument
- Or both


Intermediate solenoid control valve
S
VENT
Positioner
LC
Process control valve


Intermediate solenoid control valve
S
VENT
Positioner
Process control valve


Intermediate solenoid control valve
S
VENT
Process control valve

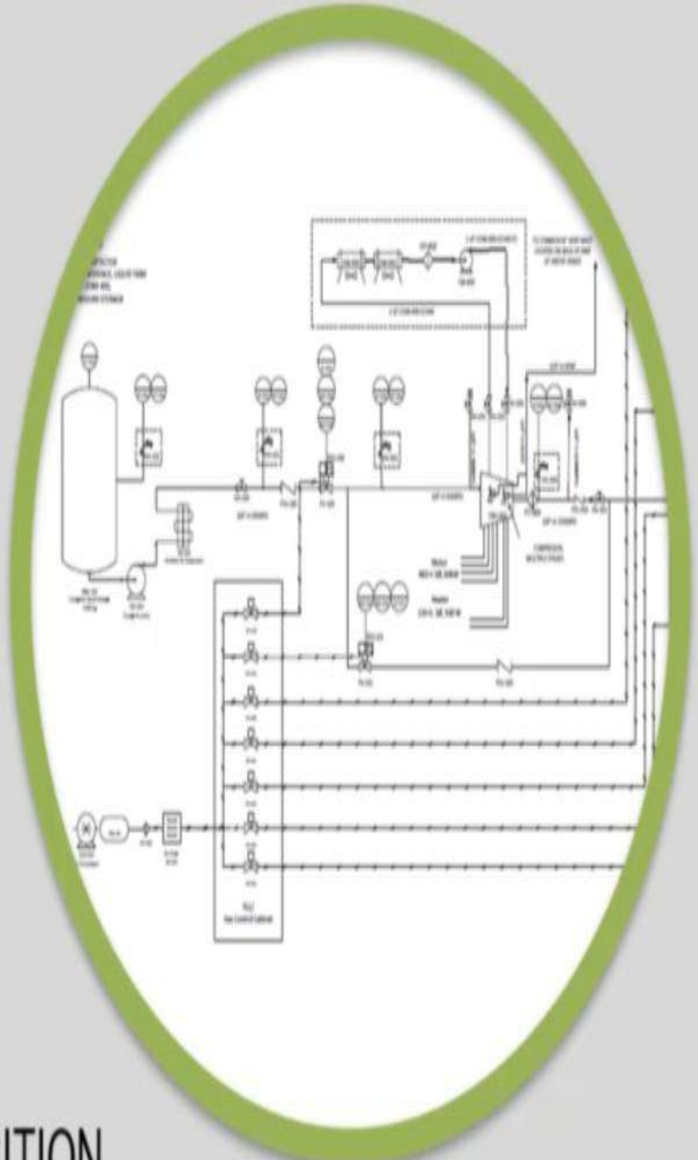
FLOW CONTROL VALVE

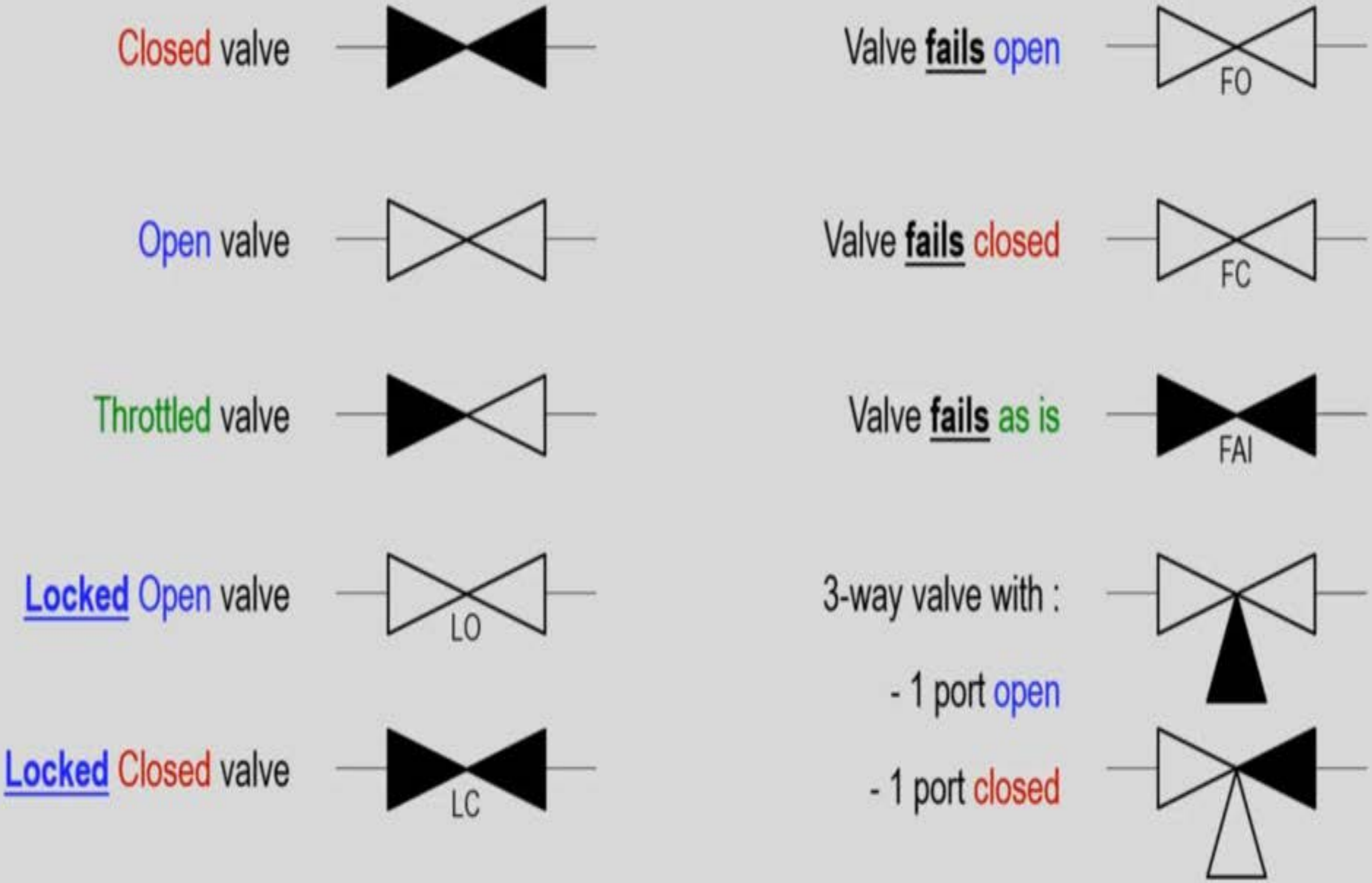


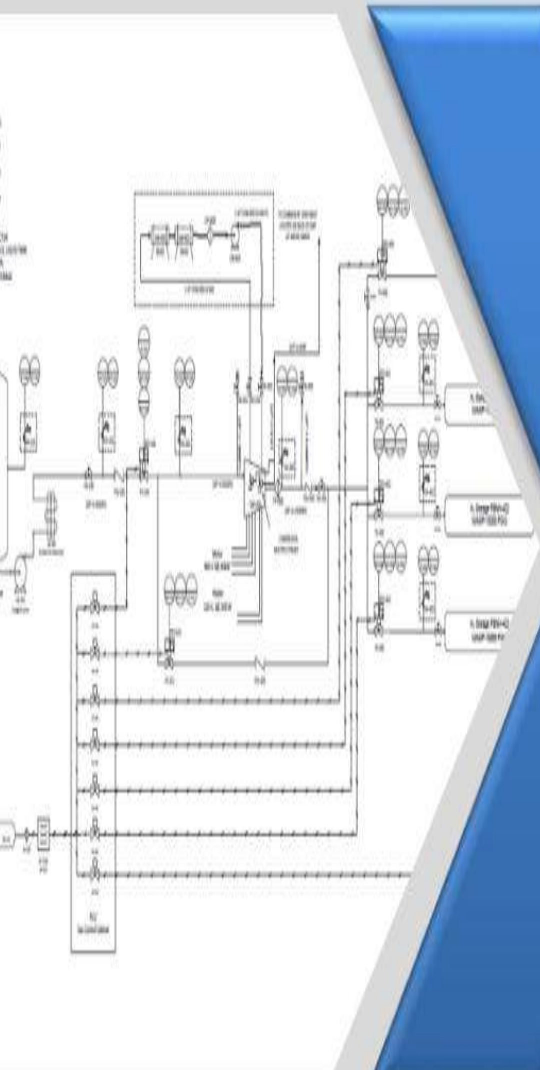
CONTROL VALVE

YOU MUST BE ABLE TO :

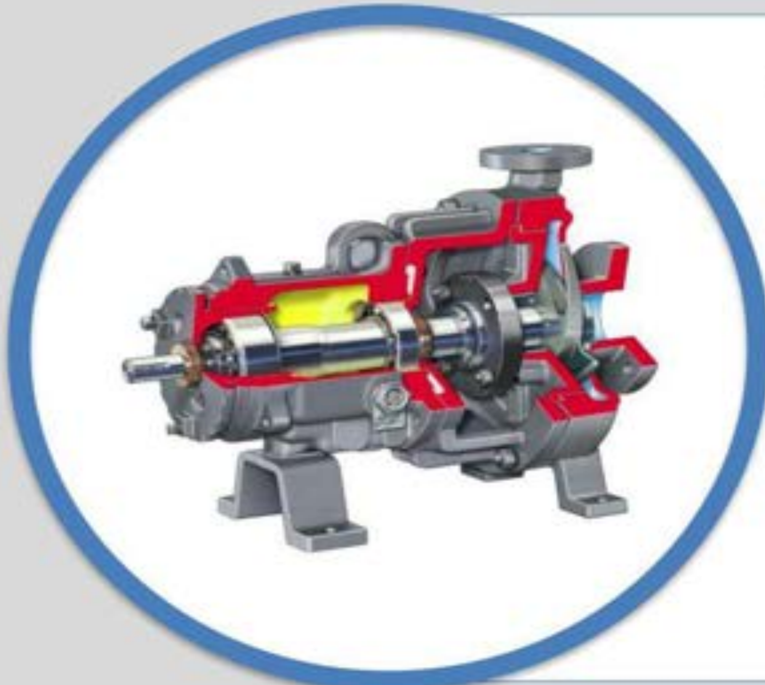
- DETERMINE THE VALVE **POSITION**
- KNOW IF THIS POSITION IS **NORMAL**
- KNOW HOW THE VALVE WILL **FAIL**
- KNOW IF THE VALVE IS NORMALLY **LOCKED** IN THAT POSITION



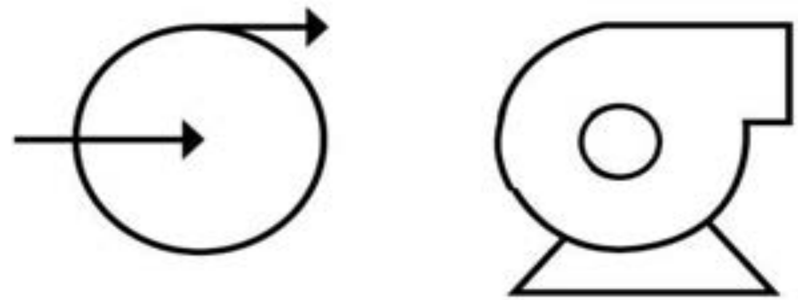

Closed valve
Open valve
Throttled valve
Locked Open valve
LO
Locked Closed valve
LC
Valve fails open
FO
Valve fails closed
FC
Valve fails as is
FAI
3-way valve with :
- 1 port open
- 1 port closed



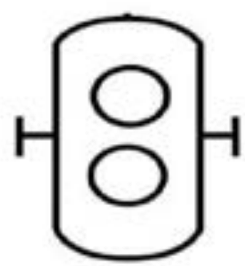
P&ID EQUIPMENT SYMBOLS



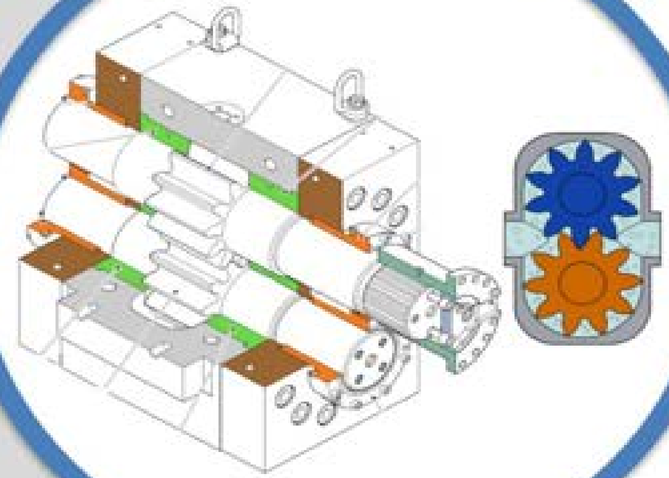
- CENTRIFUGAL PUMPS

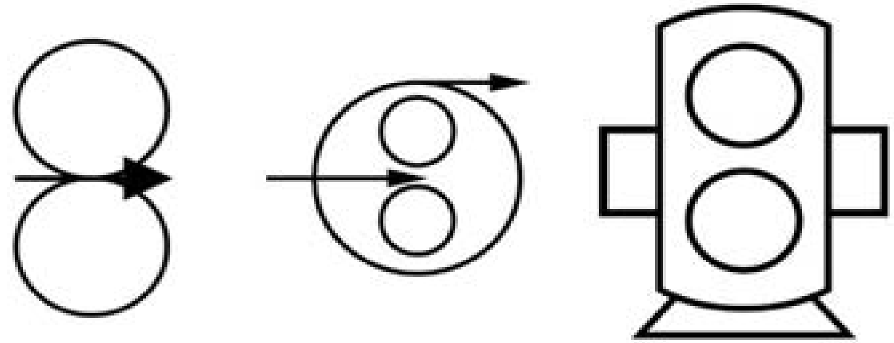


- POSITIVE DISPLACEMENT PUMPS



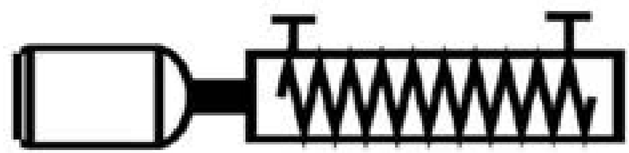
POSITIVE DISPLACEMENT PUMPS



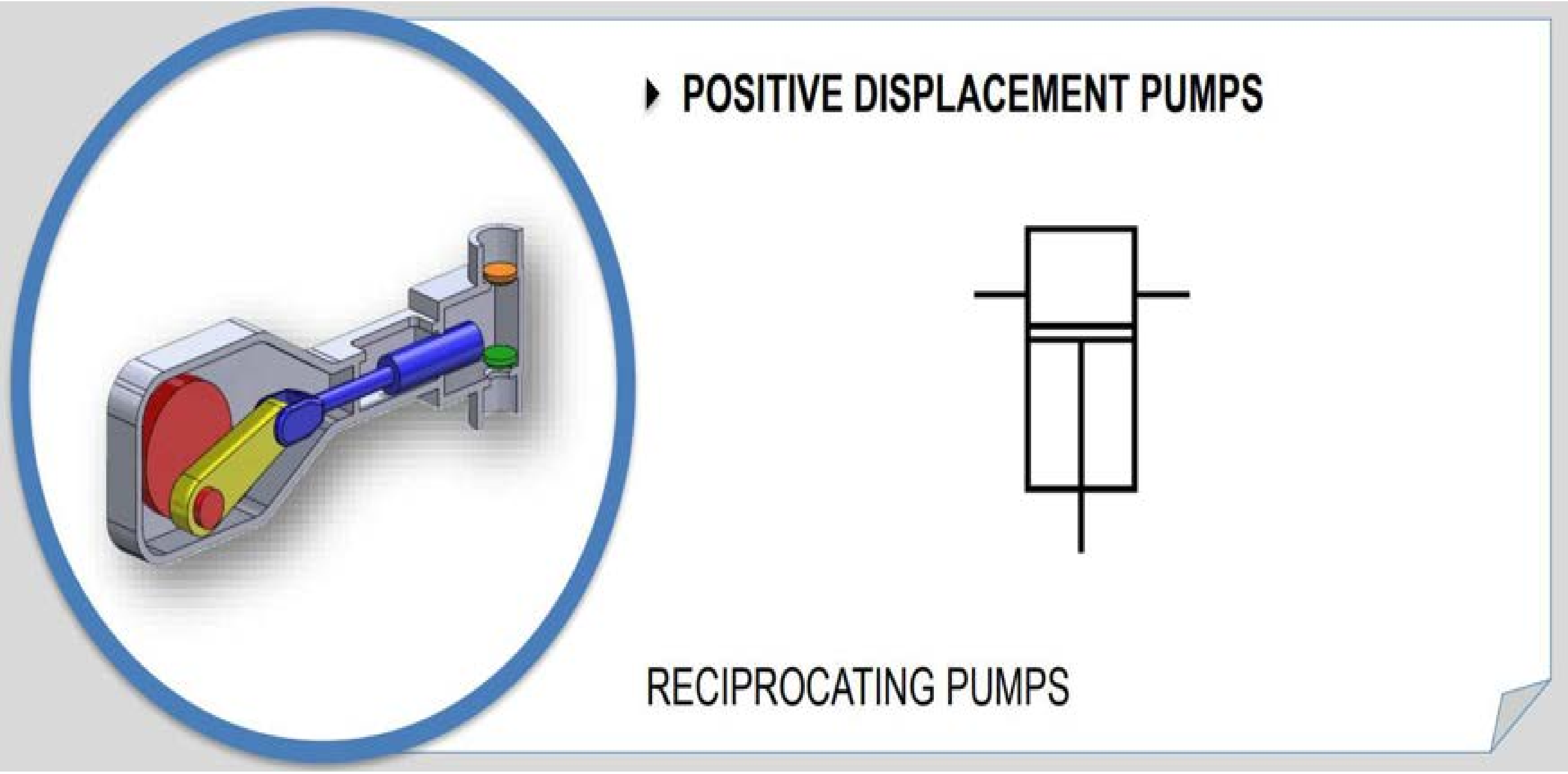


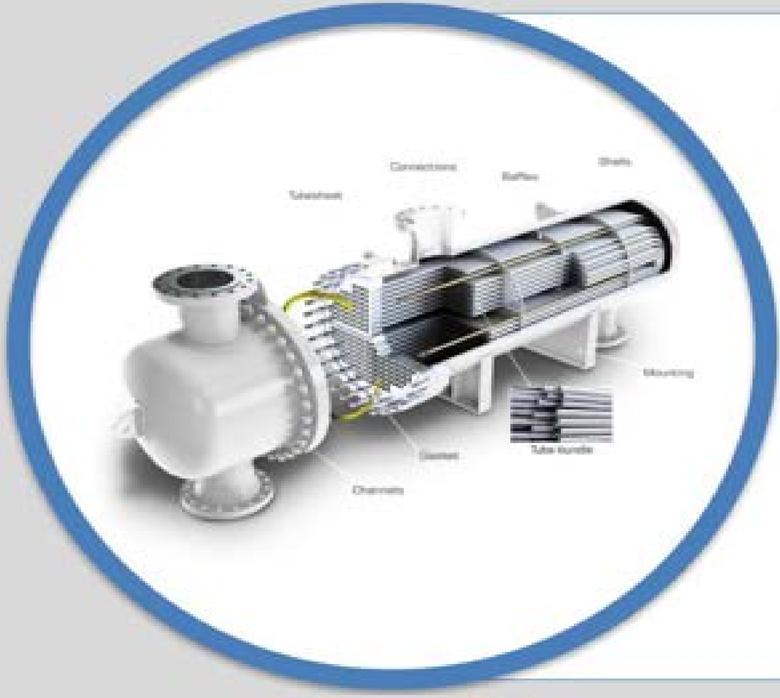
ROTARY GEAR PUMPS

POSITIVE DISPLACEMENT PUMPS

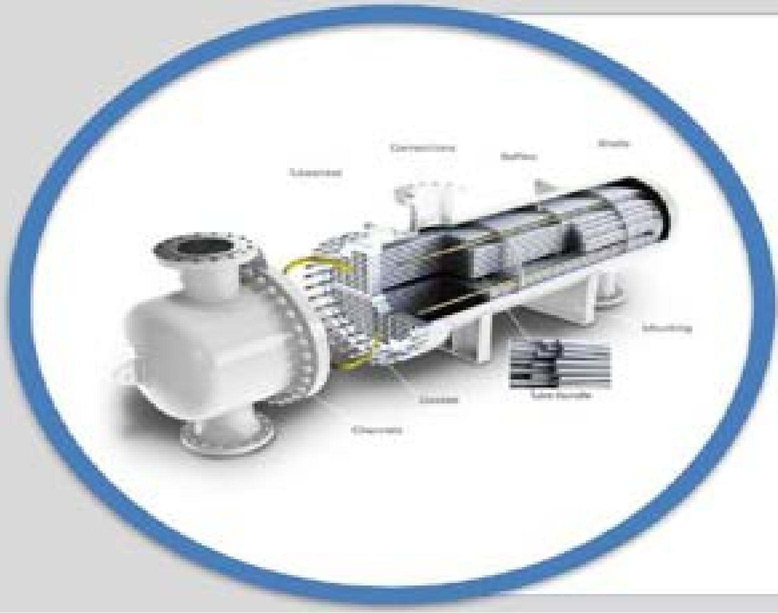


SCREW PUMPS

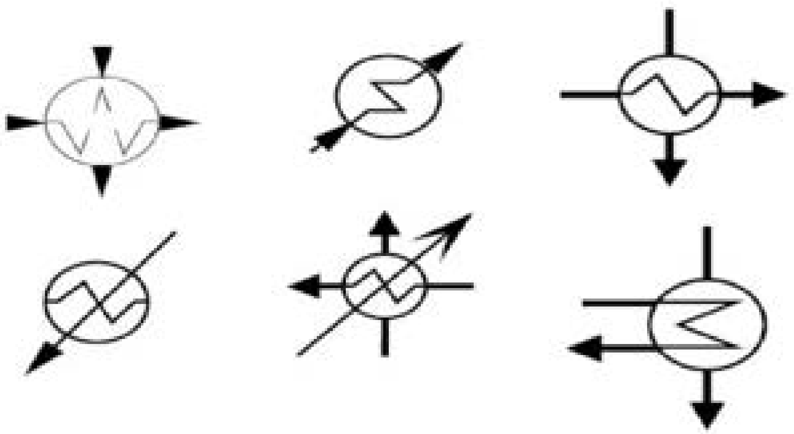

POSITIVE DISPLACEMENT PUMPS
RECIPROCATING PUMPS



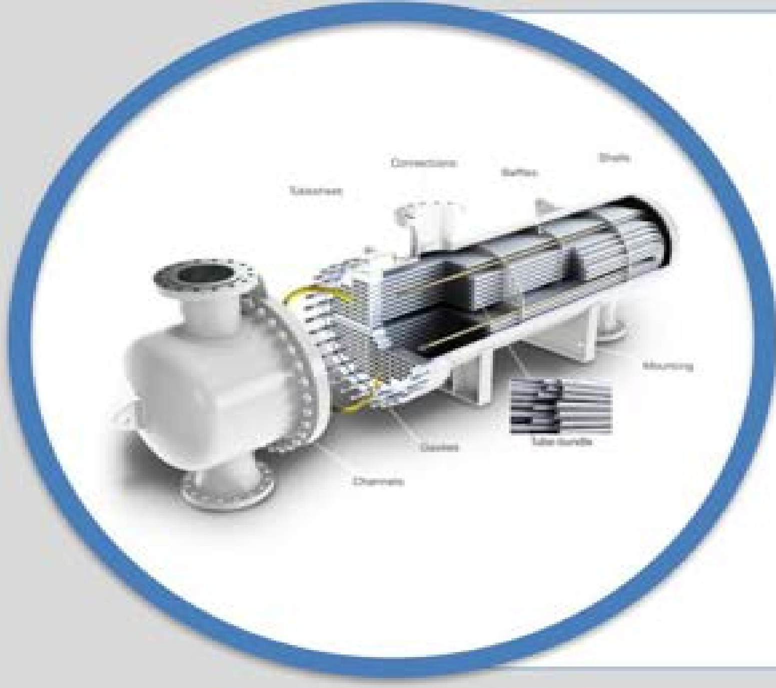
HEAT EXCHANGERS

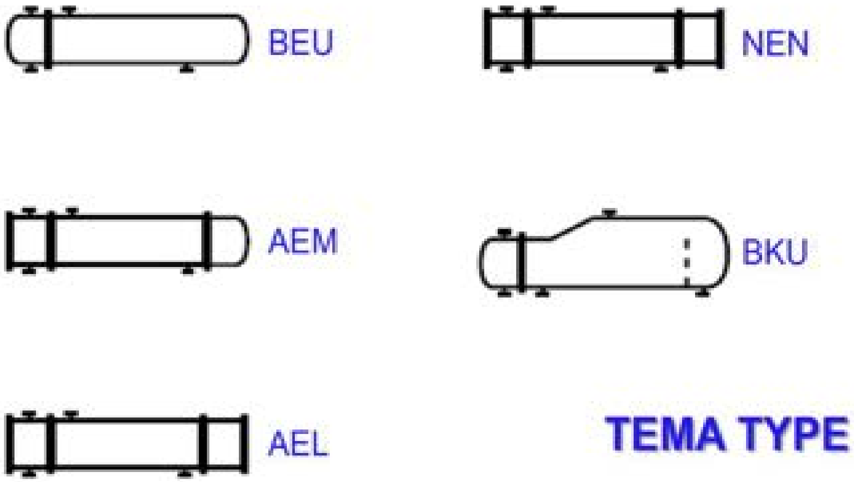


HEAT EXCHANGERS



HEAT EXCHANGERS





TEMA TYPE

HEAT EXCHANGERS

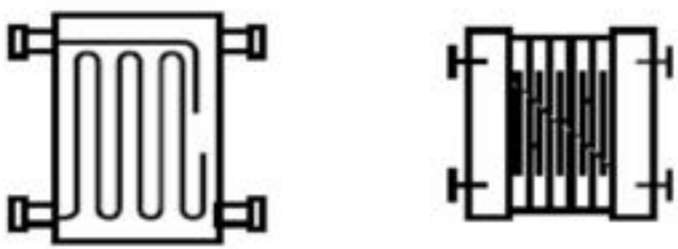
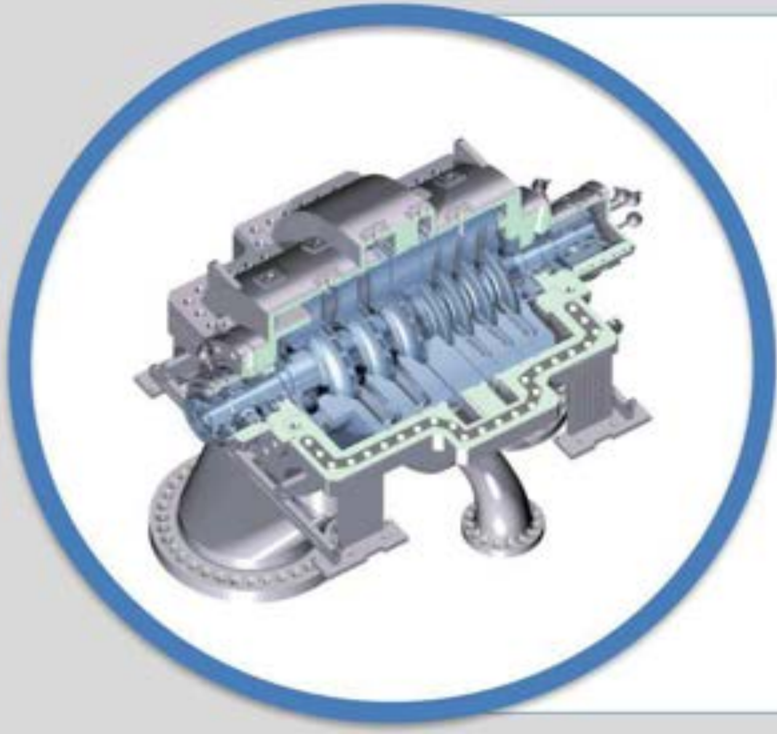
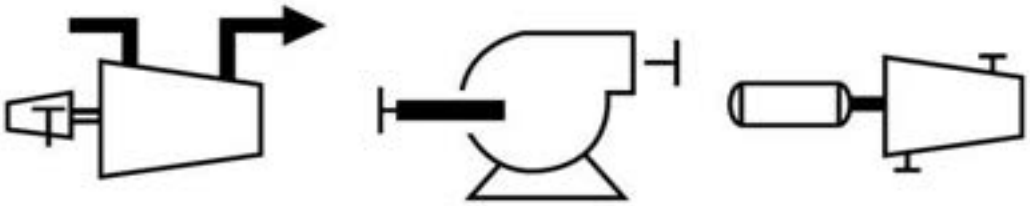


Plate Exchanger

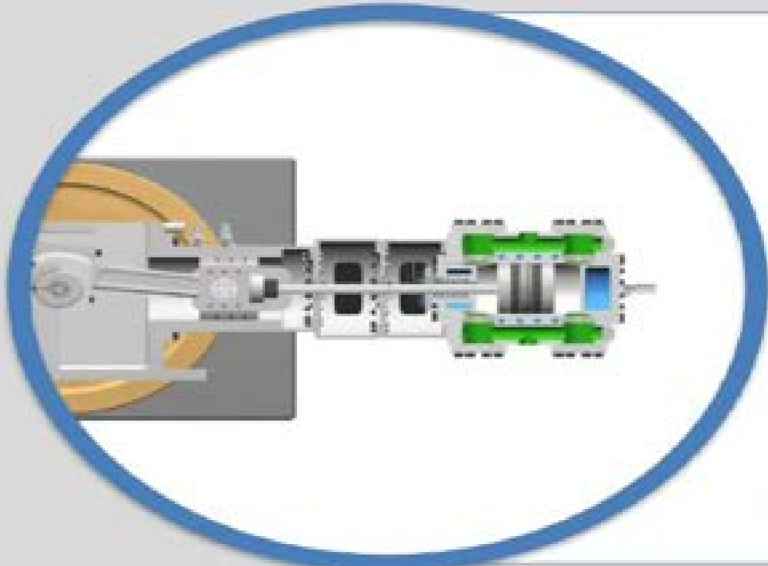
Plate & Frame Heat Exchanger

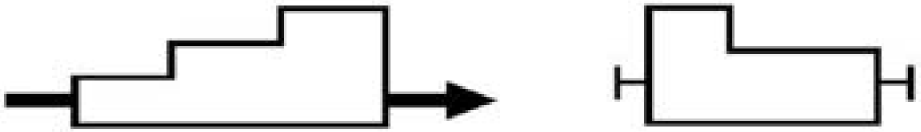
CENTRIFUGAL COMPRESSORS





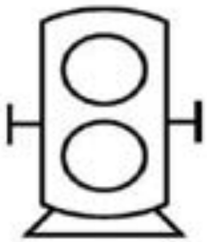
POSITIVE DISPLACEMENT COMPRESSORS





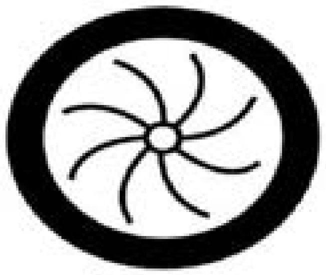
RECIPROCATING COMPRESSORS

POSITIVE DISPLACEMENT COMPRESSORS



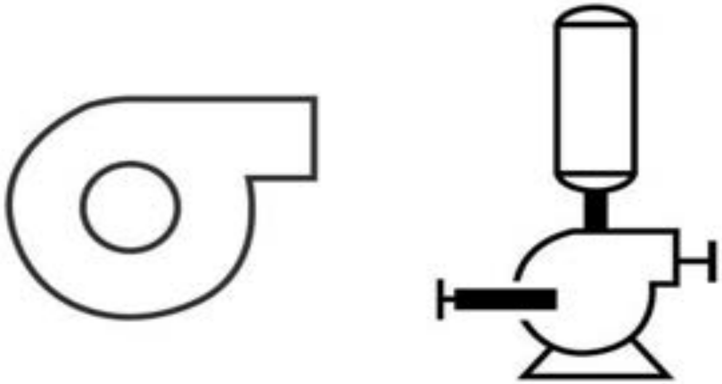
ROTARY COMPRESSORS *(Vane, Lobe, Screw)*

POSITIVE DISPLACEMENT COMPRESSORS

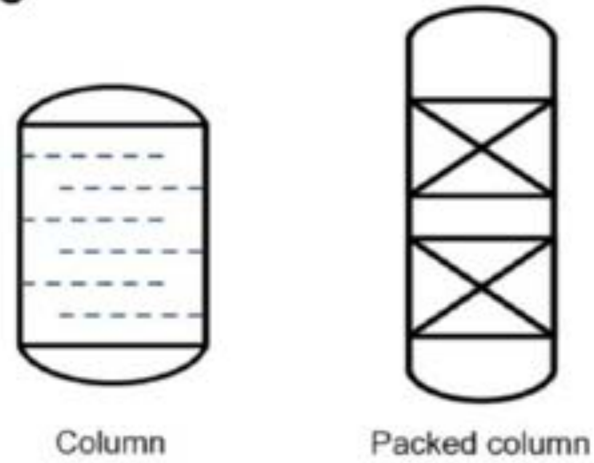


LIQUID RING COMPRESSORS

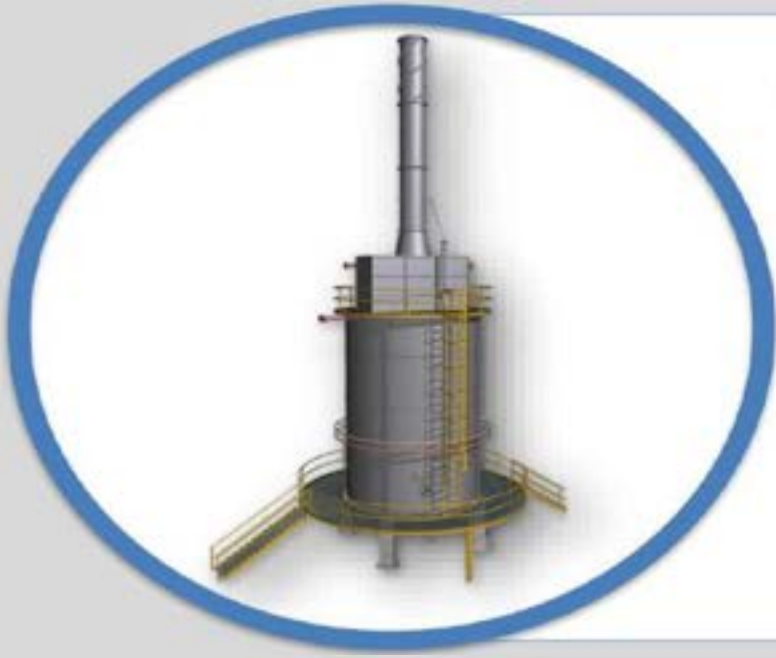
- FANS / BLOWERS

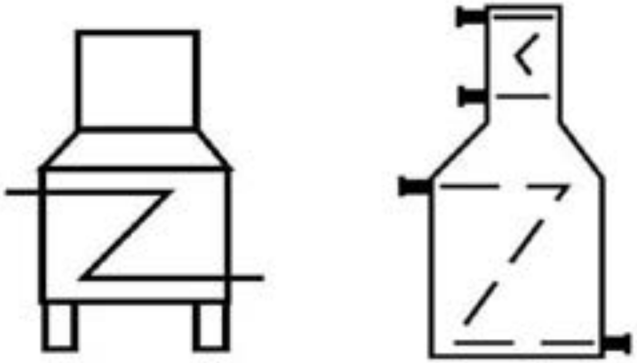


- COLUMNS

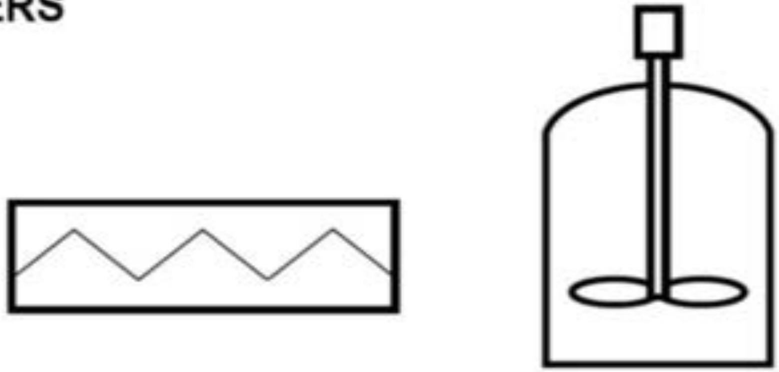


FURNACES





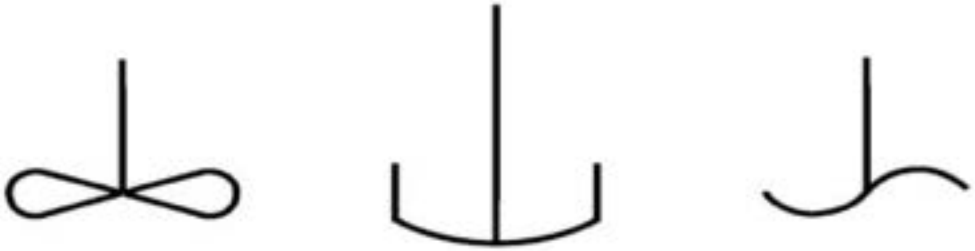
MIXERS



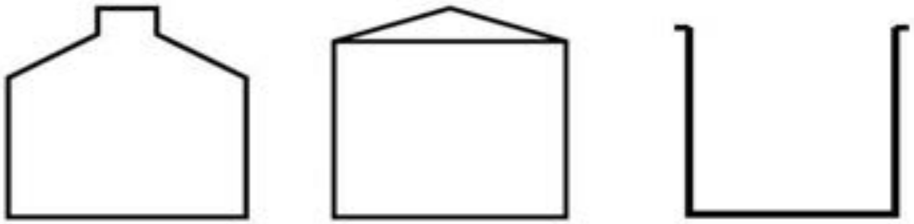
Horizontal

Vertical

- AGITATORS



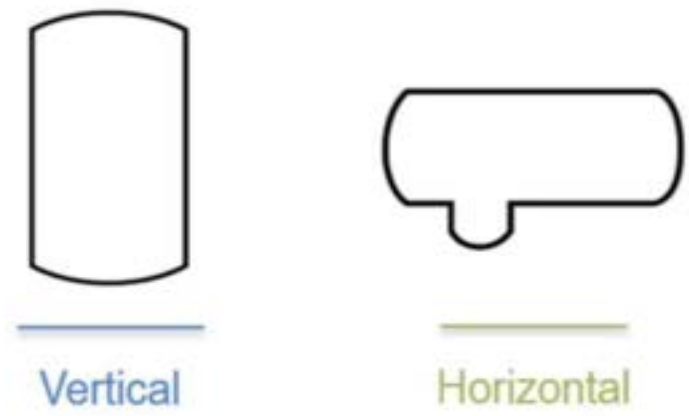
- TANKS



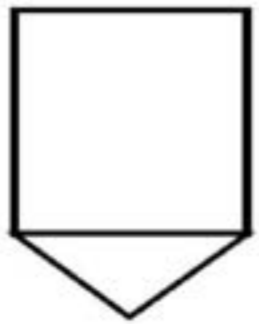
Closed tanks

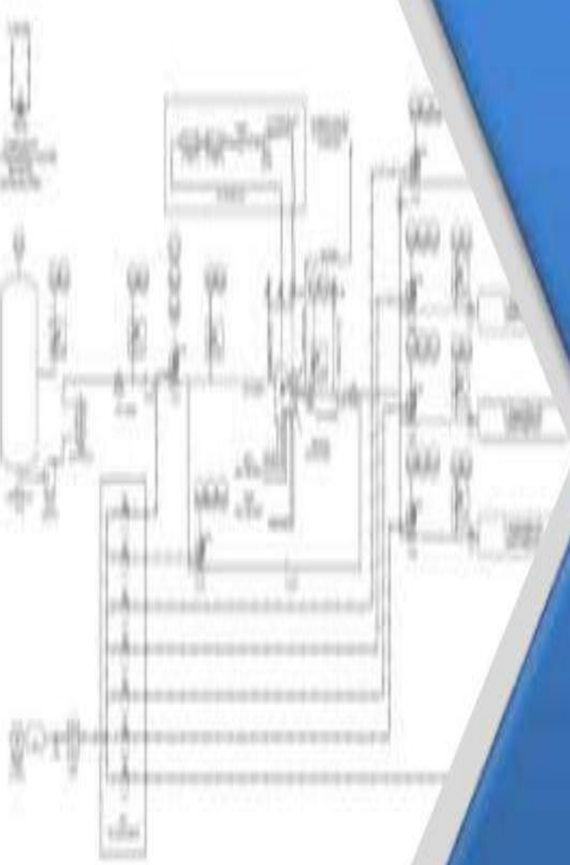
Open tanks

VESSELS



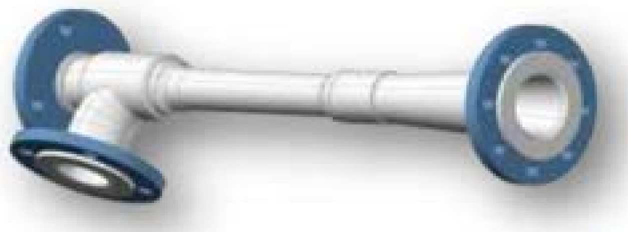
SILOS / BINS

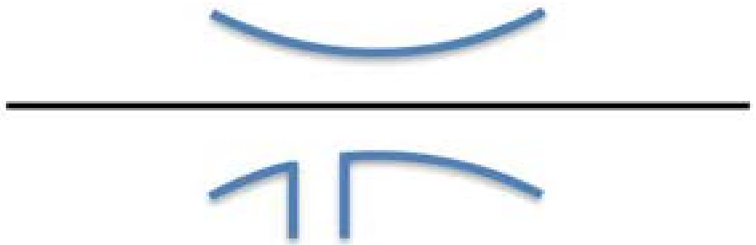




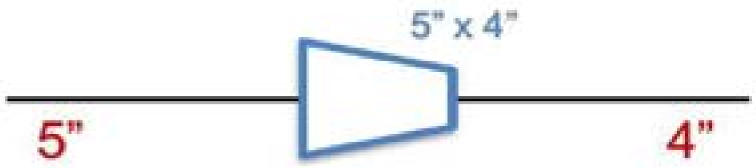
P&ID PIPE FITTING SYMBOLS

- **EJECTOR**





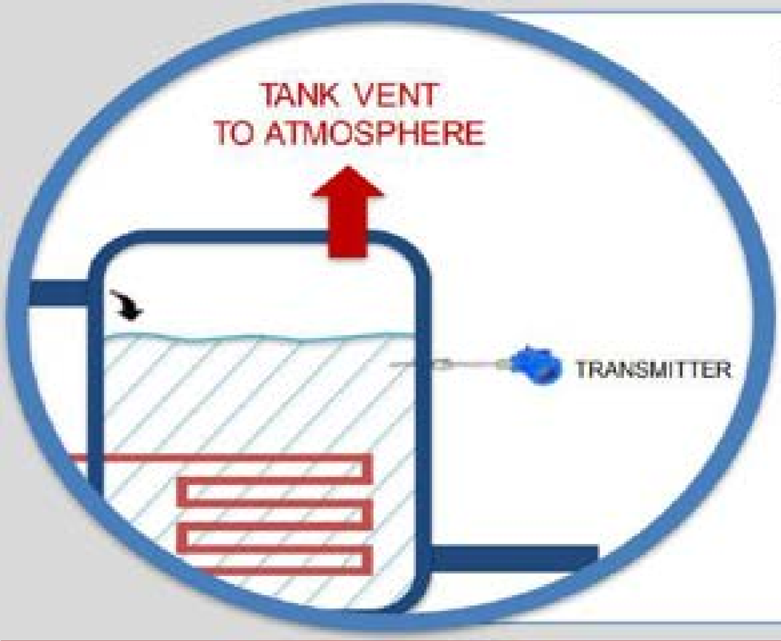
- **PIPE REDUCER**



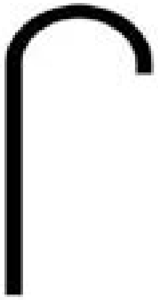
- HOSE CONNECTION

- SCREW CAP

- **WELD CAP**



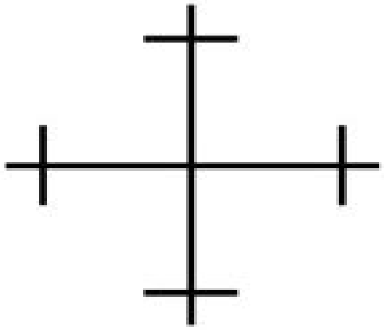
- **ATMOSPHERIC VENT**



- **FLEXIBLE CONNECTION**

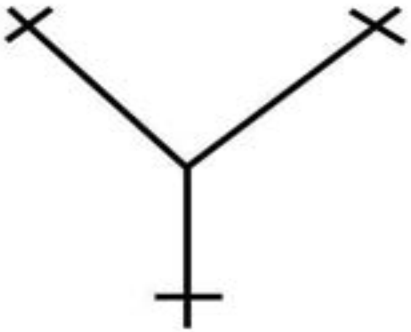
- **REMOVABLE SPOOL PIECE**

- CROSS



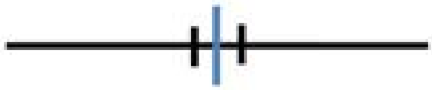
- LATERAL “Y”

- TRUE "Y"

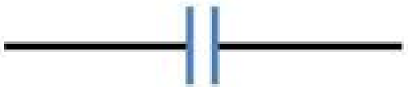


- BLANCK FLANGE

- UNION



- FLANGE



ISOLATING, VENTING & DRAINING SYMBOLS FOR EASE OF MAINTENANCE

1 EQUIPMENT DE-ENERGIZING

2 EQUIPMENT ISOLATION (LOTO)

3 EQUIPMENT VENTING & DRAINING

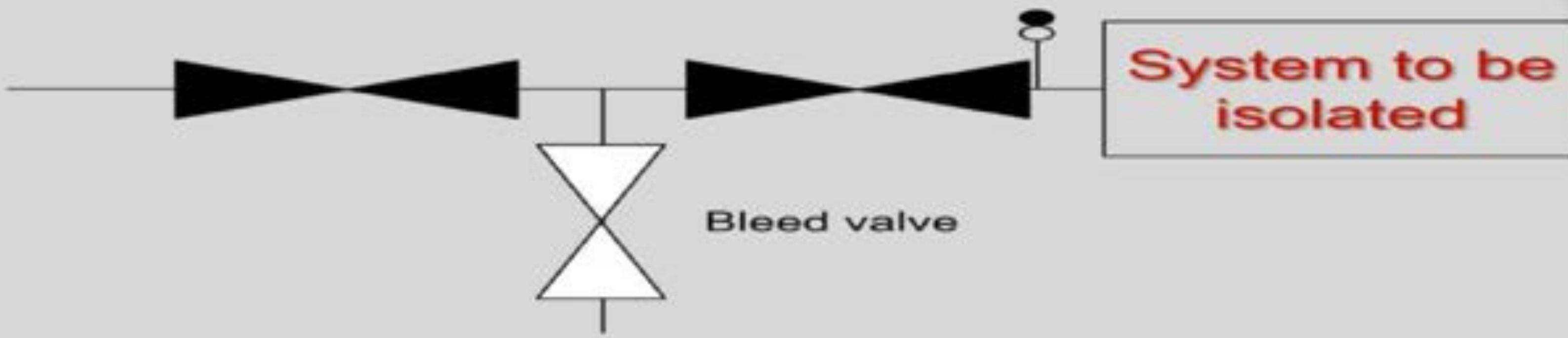
4 EQUIPMENT CLEANING

LOCK-OUT / TAG-OUT

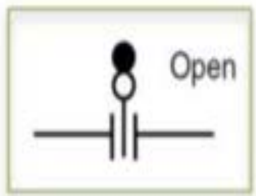
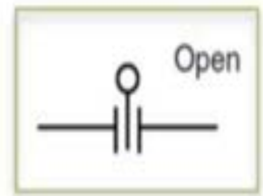
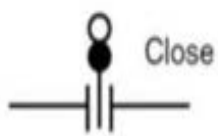
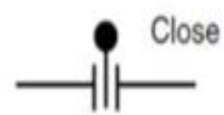
1. BLOCK VALVE AND BLIND
2. DOUBLE BLOCK AND BLEED (DB&B)
3. BLOCK VALVE AND REMOVABLE SPOOL

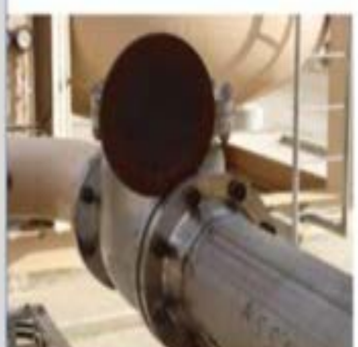
1 BLOCK VALVE AND BLIND

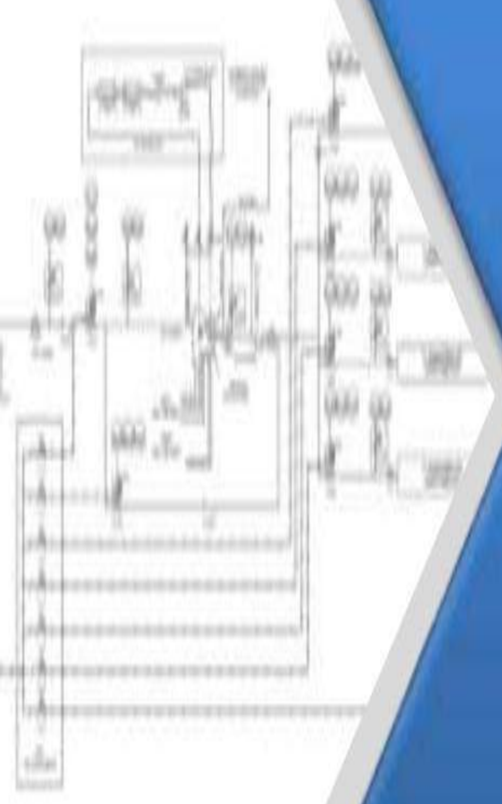
2 DOUBLE BLOCK AND BLEED (DB&B)



3 BLOCK VALVE AND REMOVABLE SPOOL

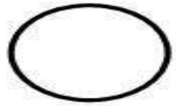
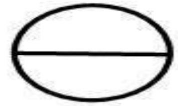
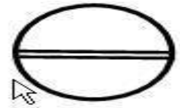
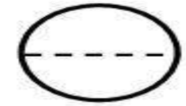
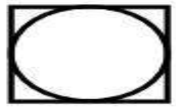
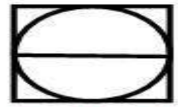
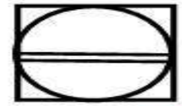
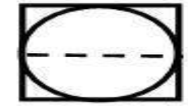
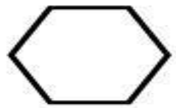
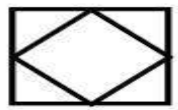
	Spectacle blind	Spade blind
Real shape		
P&ID symbol		
		





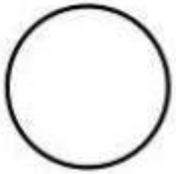
P&ID LOCATION SYMBOLS

HOW IS CONTROL DESIGN DOCUMENTED?

Instrumentation and Control Symbols				
Where → / What ↓	In the Field Locally Mounted	On a Main Panel or Screen	On a Subpanel Or Remote Location	Inaccessible, Hidden or Back/Inside Panel
Instruments & Devices	 ★★★★★	 ★★★★	 ★★★	 ★★
Graphics on a Computer Screen	 ★	 ★★★★★	 ★★★	 ★★
Computer Functions (Seldom Used)	 ★	**I Removed These!** The symbols that would normally go here do not provide any real clarity on P&IDs in my opinion. The limited use of the two at left can be adorned, as needed, to refer to specific computers and/or PLCs functions. The functional description will help clarify computer functions and control logic.		
PLC/DCS Functions (Seldom Used)	 ★			

Star Rating	"Bob's Review" Notes
★★★★★	Most commonly used symbols, always present!
★★★★	Used in most cases, maybe not all.
★★★	Used from time to time, but will be absent sometimes.
★★	Not usually needed unless there is a good reason.
★	You can avoid using this symbol most of the time.

HOW IS CONTROL DESIGN DOCUMENTED?

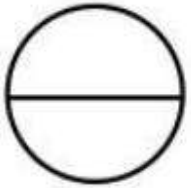


Instruments that are field mounted.

-Instruments that are mounted on process plant (i.e sensor that mounted on pipeline or process equipments.

Field mounted on pipeline

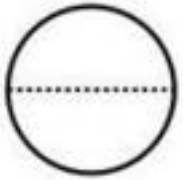
HOW IS CONTROL DESIGN DOCUMENTED?



Instruments that are board mounted

-Instruments that are mounted on control board.

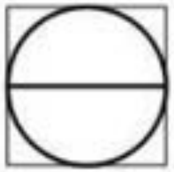
HOW IS CONTROL DESIGN DOCUMENTED?



Instruments that are board mounted (invisible).

-Instruments that are mounted behind a control panel board.

HOW IS CONTROL DESIGN DOCUMENTED?



Instruments that are functioned in Distributed Control System (DCS)

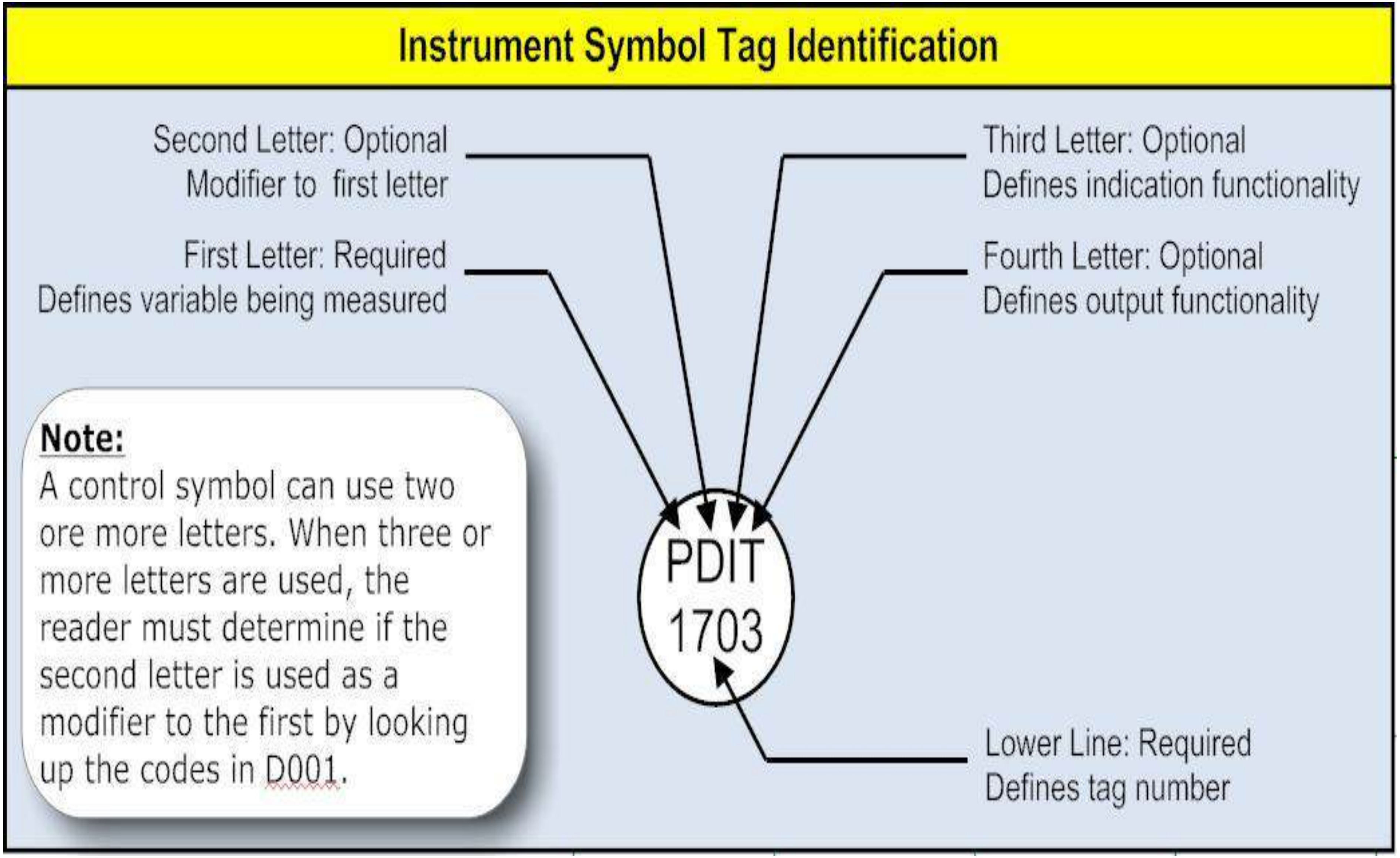
- A **distributed control system** (DCS) refers to a control system usually of a manufacturing system, process or any kind of dynamic system, in which the controller elements are not central in location (like the brain) but are distributed throughout the system with each component sub-system controlled by one or more controllers. The entire system of controllers is connected by networks for communication and monitoring.

HOW IS CONTROL DESIGN DOCUMENTED?

What are Tag Numbers? A tag number with a circle around it indicates stand alone, physical instruments.
When a first letter is used in instrumentation symbols, e.g., Pressure Indicator Controller (PIC), it defines the measured or initiating variables.
Examples include Analysis (A), Flow (F), Temperature (T), and Pressure (P).
The second letter tells the type of device being used, such as Indicator (I), Record (R), and Transmit (T).
The third, fourth, or fifth letter tells the function of the component.

INSTRUMENTATION

HOW IS CONTROL DESIGN DOCUMENTED?



INSTRUMENT IDENTIFIERS

Sensed Parameter	Type of Indicator	Type of Component	Type of Signal
F : Flow	**R** : Recorder	**T** : Transmitter	**I** : Current
T : Temperature	**I** : Indicator	**M** : Modifier	**V** : Voltage
P : Pressure	**C** : Controller	**E** : Element	**P** : Pneumatic
I : Current			
L : Level			
V : Voltage			
Z : Position			

FIC : Flow **I**ndicating **C**ontroller

TR : Temperature **R**ecorder

PT : Pressure **T**ransmitter

HOW IS CONTROL DESIGN DOCUMENTED?

ISA S5.1 IDENTIFICATION LETTERS

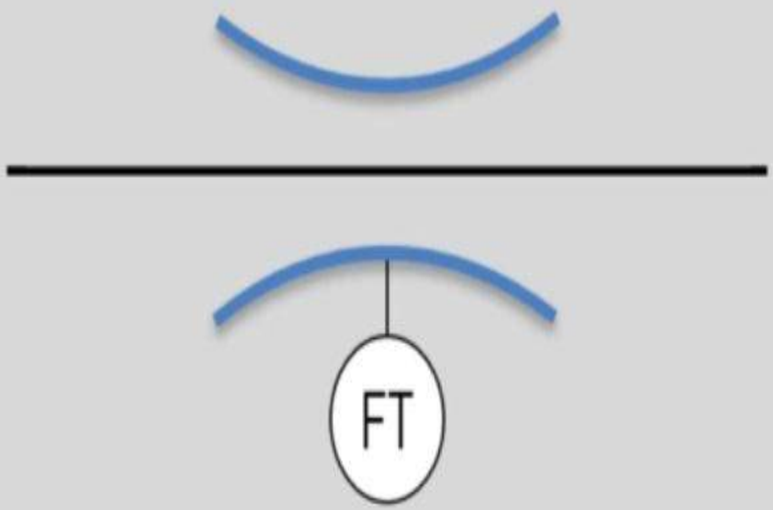
	First-letter		Succeeding- Letters		
	Measured or Initiating variable	Modifier	Readout function	Output function	Modifier
A	Analysis				
C				Control	
D		Differential			
F	Flow Rate	Ratio			
H	Hand				High
I	Current		Indicate		
L	Level				Low
P	Pressure, vacuum				
Q	Quantity	Totalizer			
S		Safety		Switch	
T	Temperature			Transmit	
V	Vibration			Valve, Damper	
z	Position			Actuator	

MODIFIERS AND TRANSMITTERS

TRANSMITTERS

- A transmitter is a device that :
  1. converts a reading from a sensor or transducer into a standard signal
  2. transmits that signal to a monitor or controller

- Transmitter types include :
  - Pressure transmitters
  - Flow transmitters
  - Temperature transmitters
  - Level transmitters



MODIFIERS : TRANSDUCERS AND CONVERTERS

- A transducer is a device that translates a mechanical signal into an electrical signal
- A converter is a device that converts one type of signal into another type of signal

Current-to-Pressure converter

MODIFIERS : TRANSDUCERS AND CONVERTERS

E/I

FM — Locally mounted Voltage to Current Flow Modifier — E/I — E/I

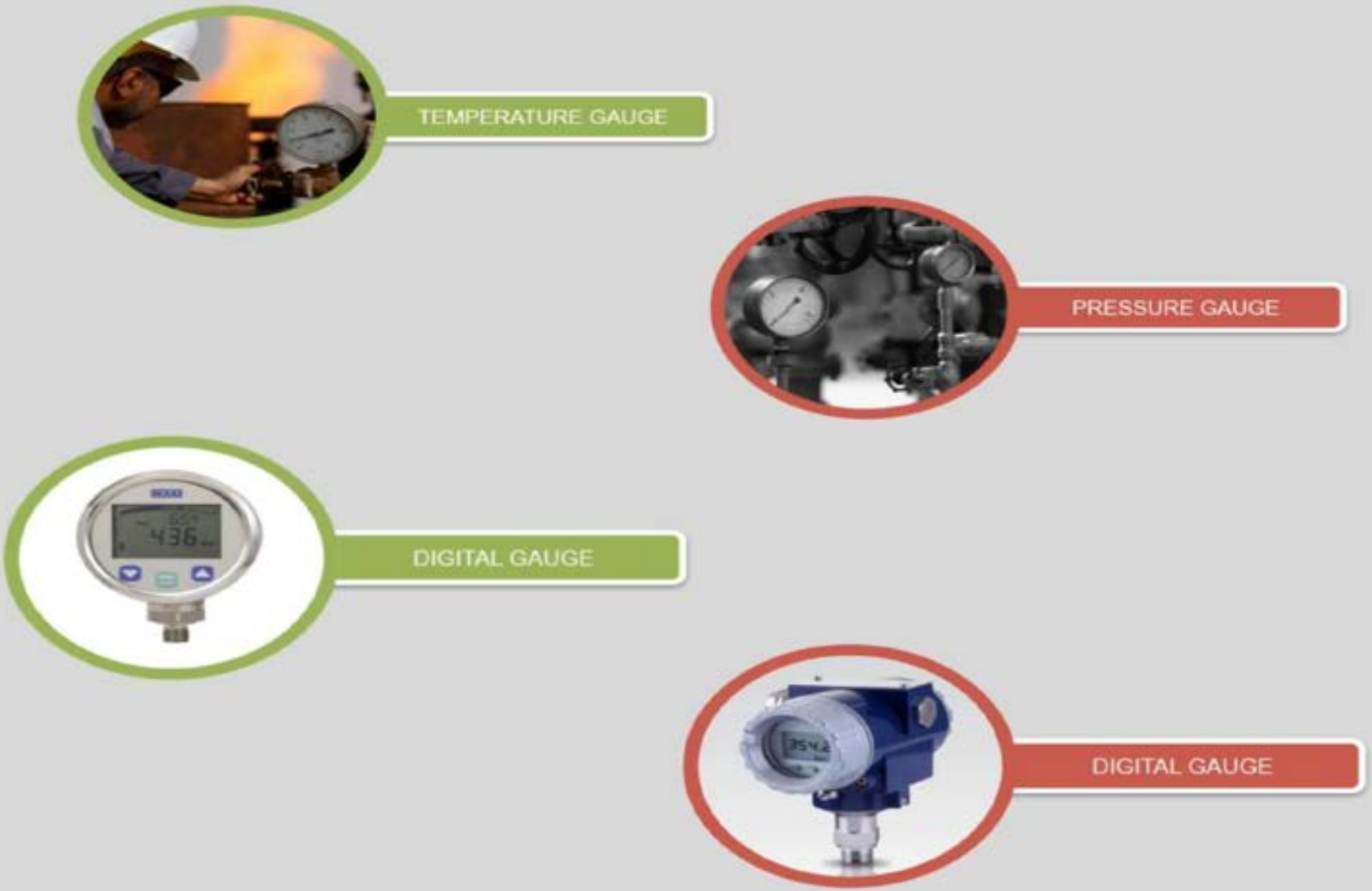
P/I

PM — Locally mounted Pneumatic to Current Pressure Modifier — P/I — P/I

√ — Locally mounted Square root extractor — √

INDICATORS AND RECORDERS

INDICATORS

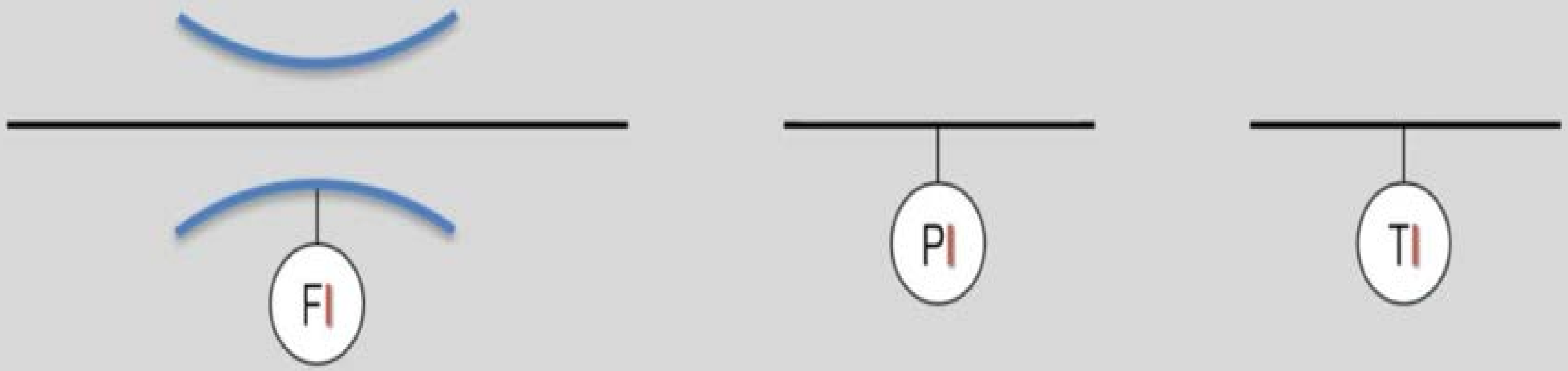

TEMPERATURE GAUGE
PRESSURE GAUGE
DIGITAL GAUGE
DIGITAL GAUGE

RECORDERS

- A recorder is a device that records the output of a measurement device

INDICATORS & RECORDERS

- Convert the signal generated by an instrument loop into a readable form
- The indicator or recorder may be : Locally or Board mounted



CONTROLLERS

CONTROLLERS

- A controller is a device that :
  1. receives data from a measurement instrument
  2. compares that data to a programmed setpoint
  3. if necessary, signals a control element to take corrective action

PLC

- Controllers always have an ability to :
  1. receive input
  2. perform a mathematical function with the input
  3. produce an output signal

DCS

CONTROLLERS

- They are denoted by placing a **"C"** in the balloon after the controlling parameter

- **FC** Flow Controller
- **TC** Temperature Controller
- **PC** Pressure Controller
- **LC** Level Controller

CONTROLLERS

- There are controllers that serve to **process** a signal and **create a new signal**

P	Proportional
PID	Proportional-Integral-Differential
PI	Proportional-Integral
I/P	Current to Pneumatic

SIGNAL CONDITIONERS

HOW IS CONTROL DESIGN DOCUMENTED?

TAG DESCRIPTORS

TI	Temp Indicator	FI	Flow Indicator	I / P	Transducer
TT	Temp Transmitter	FT	Flow Transmitter	PIC 105	Pressure Indicating Controller
TR	Temp Recorder	FR	Flow Recorder	PRC 40	Pressure Recording Controller
TC	Temp Controller	FC	Flow Controller	LA 25	Level Alarm
LI	Level Indicator	PI	Pressure Indicator	FE	Flow Element
LT 65	Level Transmitter	PT 55	Pressure Transmitter	TE	Temperature Element
LR 65	Level Recorder	PR 55	Pressure Recorder	LG	Level Gauge
LC 65	Level Controller	PC 55	Pressure Controller	AT	Analyzer Transmitter

HOW IS CONTROL DESIGN DOCUMENTED?

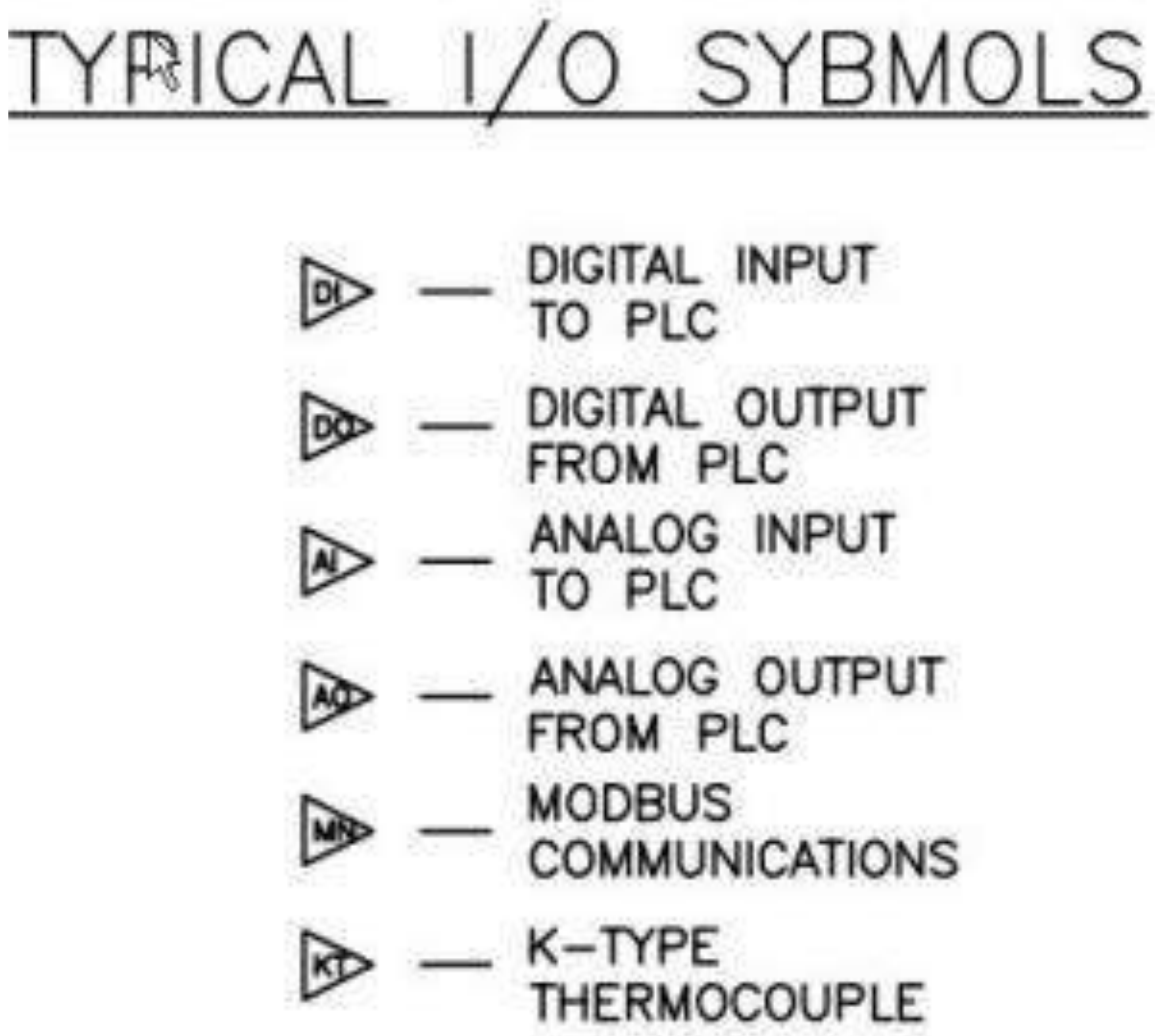
Common Primary Device Symbols

Analyzer	Level	Temperature	Pressure	What does it mean?
AI 1234	LI 1234	TI 1234	PI 1234	Indicates only
AT 1234	LT 1234	TT 1234	PT 1234	Transmits only
AIT 1234	LIT 1234	TIT 1234	PIT 1234	Indicates and transmits
ADIT 1234	LDIT 1234	TDIT 1234	PDIT 1234	Indicates and transmits (Includes first letter modifier D for Differential)

Other Common Examples

XV 1234	Actuated Valve	ZSC 1234	Limit Switch (close)
SV 1234	Solenoid Valve	ZSO 1234	Limit Switch (open)
SC 1234	Speed Controller	VS 1234	Vibration Switch
HS 1234	Hand Switch	PS 1234	Pressure Switch

HOW IS CONTROL DESIGN DOCUMENTED?

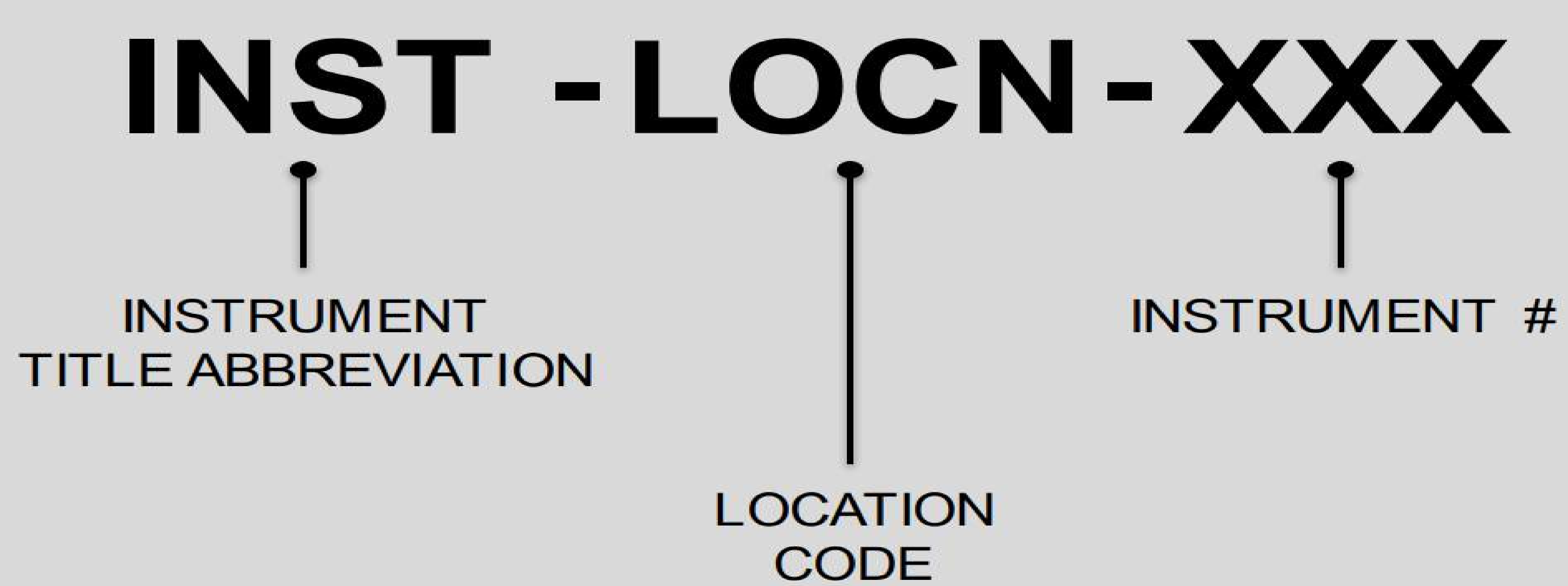


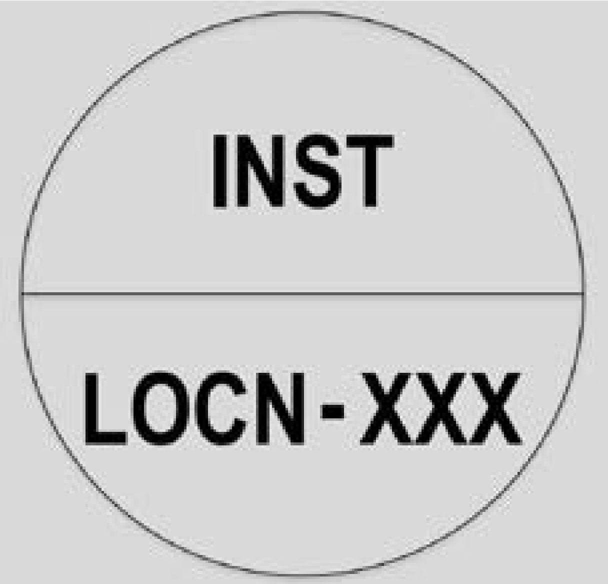
HOW IS CONTROL DESIGN DOCUMENTED?

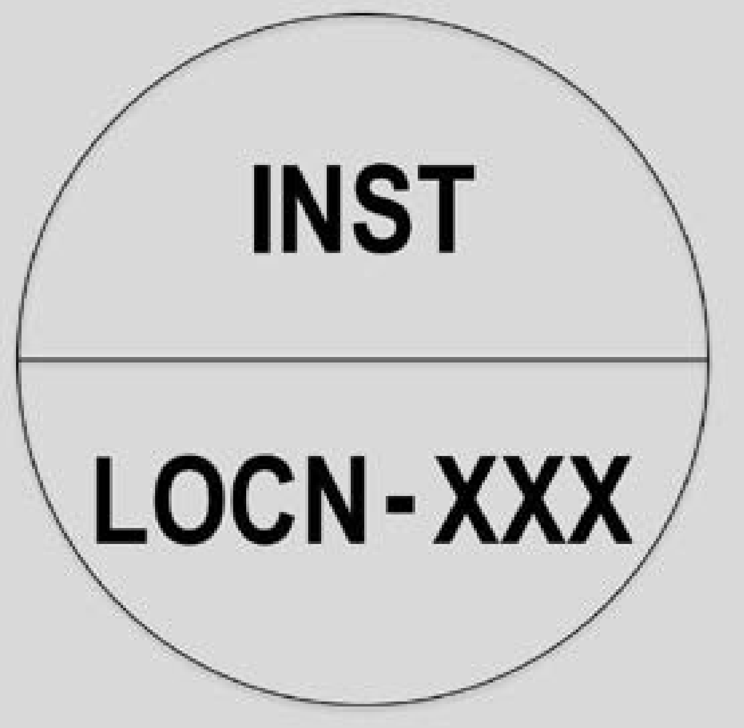
COMPUTING FUNCTION IDENTIFICATION

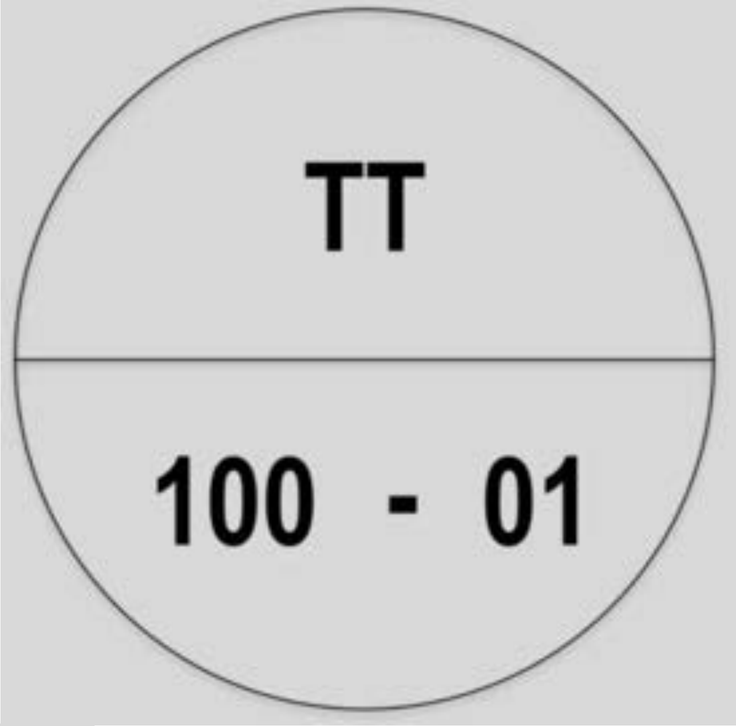
Σ — ADD	± — BIAS	∀ — VELOCITY LIMITER
Σ/n — AVERAGE	÷ — DIVIDE	-K — NEGATIVE GAIN
Δ — DIFFERENCE	> — HIGH SELECTOR	K — PROPORTIONAL GAIN
1:1 — BOOSTER	< — LOW SELECTOR	2:1 — PROPORTIONAL
≯ — HIGH LIMIT	X — MULTIPLY	f(t) — TIME FUNCTION
≮ — LOW LIMIT	∫ — INTEGRATE	ⁿ√ — ROOT EXTRACTION
d/dt — RATE OF CHANGE	x^n — EXPONENTIAL	*/* — CONVERT

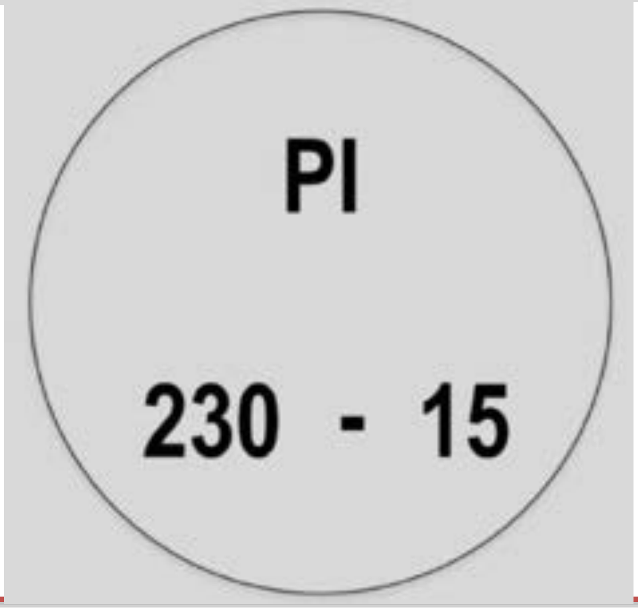
P&ID INSTRUMENT DESIGNATION CODES


INST - LOCN - XXX
INSTRUMENT
TITLE ABBREVIATION
LOCATION
CODE
INSTRUMENT #


INST
LOCN - XXX


INST
LOCN - XXX

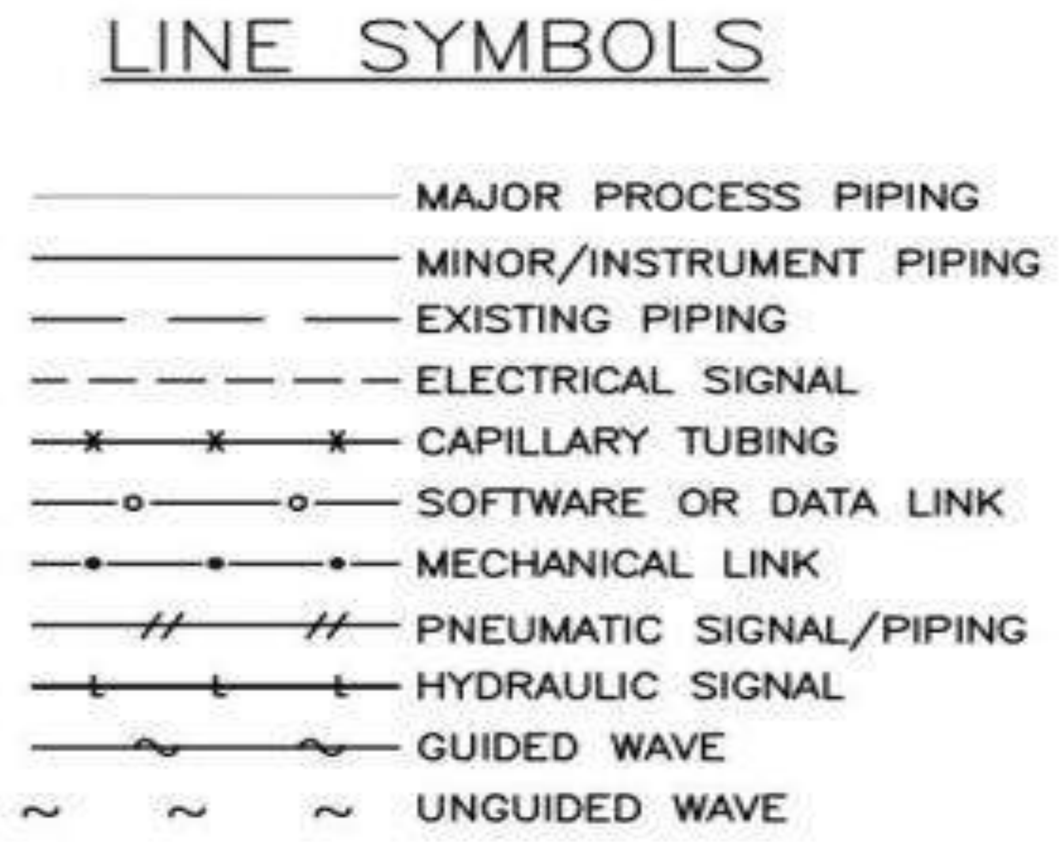

TT
100 - 01

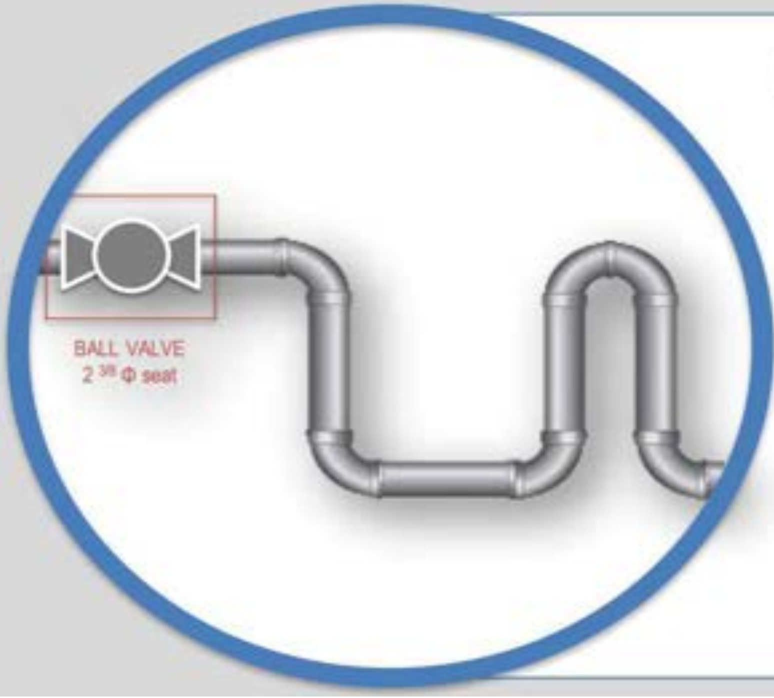

PI
230 - 15

HOW IS CONTROL DESIGN DOCUMENTED?

PIPING AND CONNECTION SYMBOLS

- Used to identify how the instruments in the process connect to each other and hat type of signal is being used.



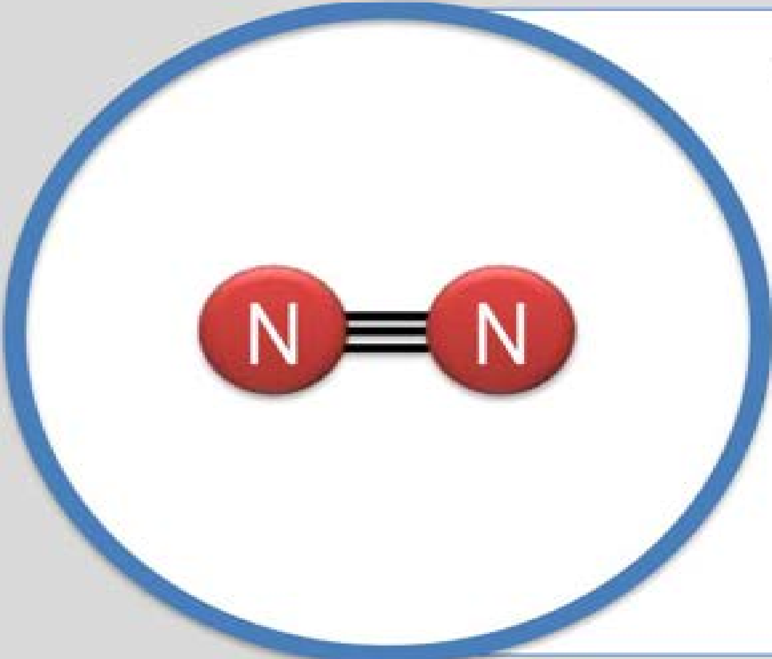


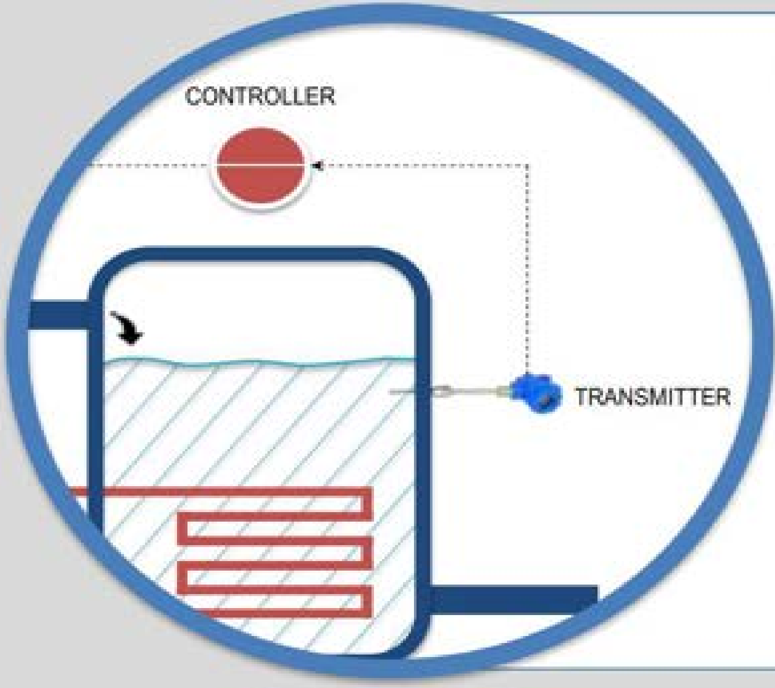
- **PROCESS FLOW LINE**

- **PNEUMATIC LINE**

- HYDRAULIC LINE

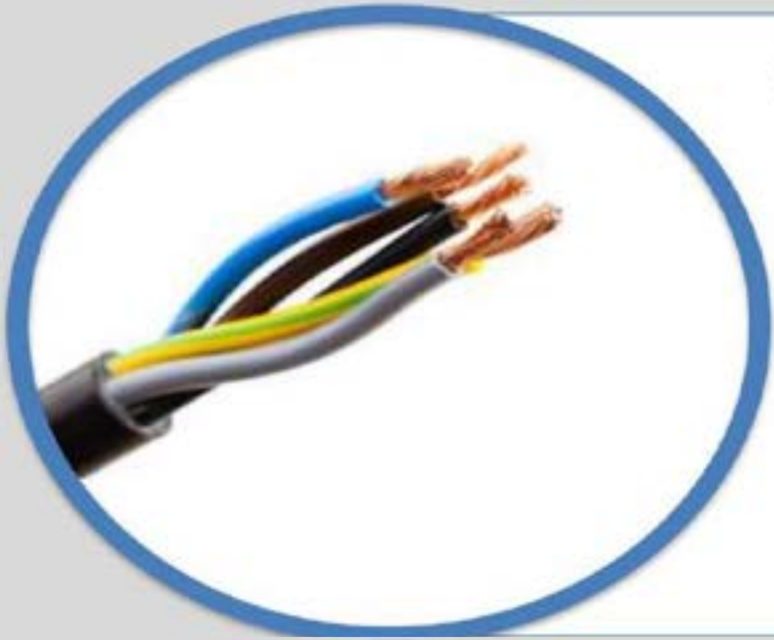
- INERT GAS LINE



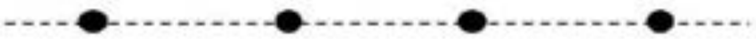


INSTURMENT SIGNAL

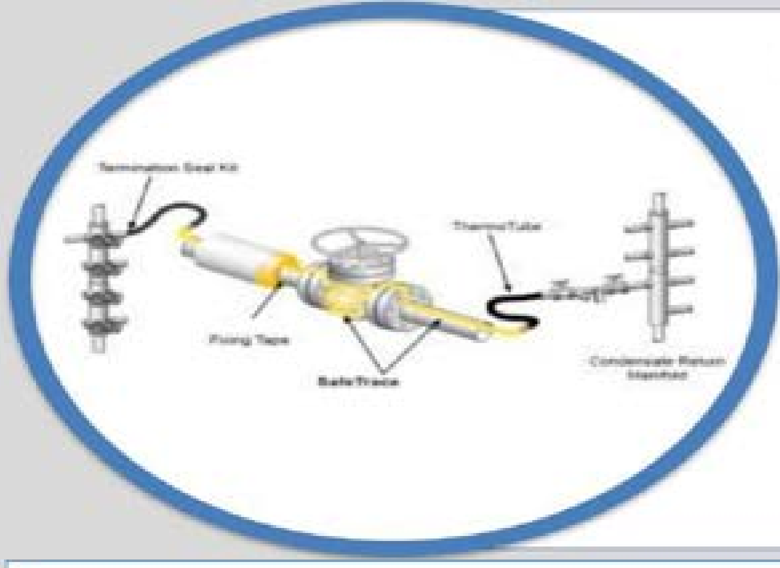
INSTURMENT CAPILLARY



- ELECTRICAL WIRES

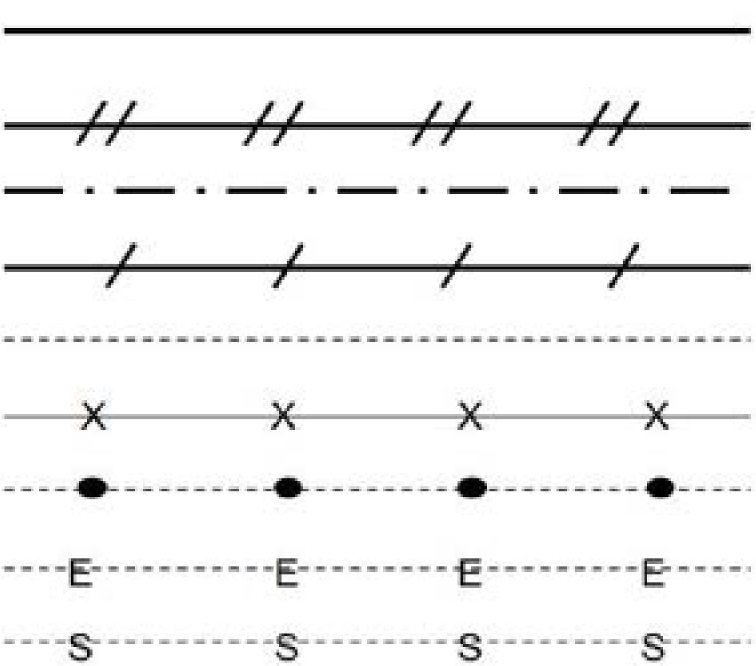


- ELECTRICAL HEAT TRACING



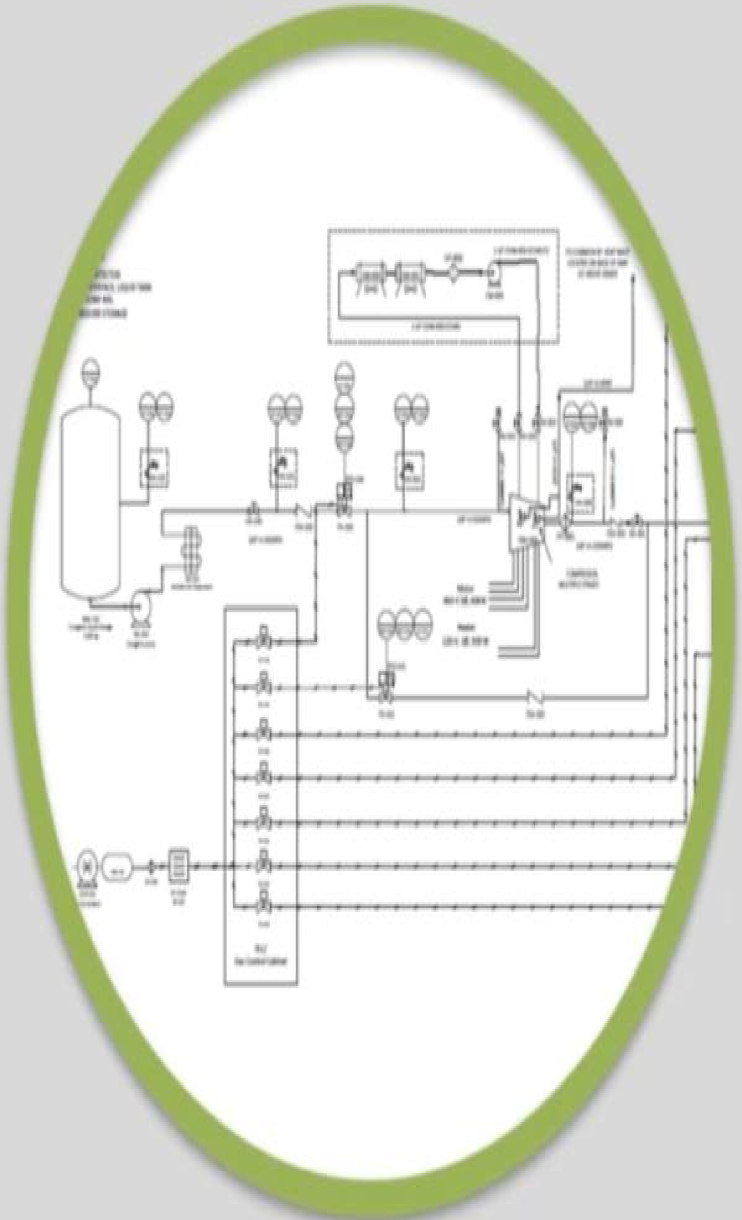
- STEAM HEAT TRACING

- PROCESS FLOW LINE
- PNEUMATIC LINE
- HYDRAULIC LINE
- INERT GAS LINE
- INSTURMENT SIGNAL
- INSTURMENT CAPILLARY
- ELECTRICAL WIRES
- STEAM HEAT TRACING
- ELECTRICAL HEAT TRACING

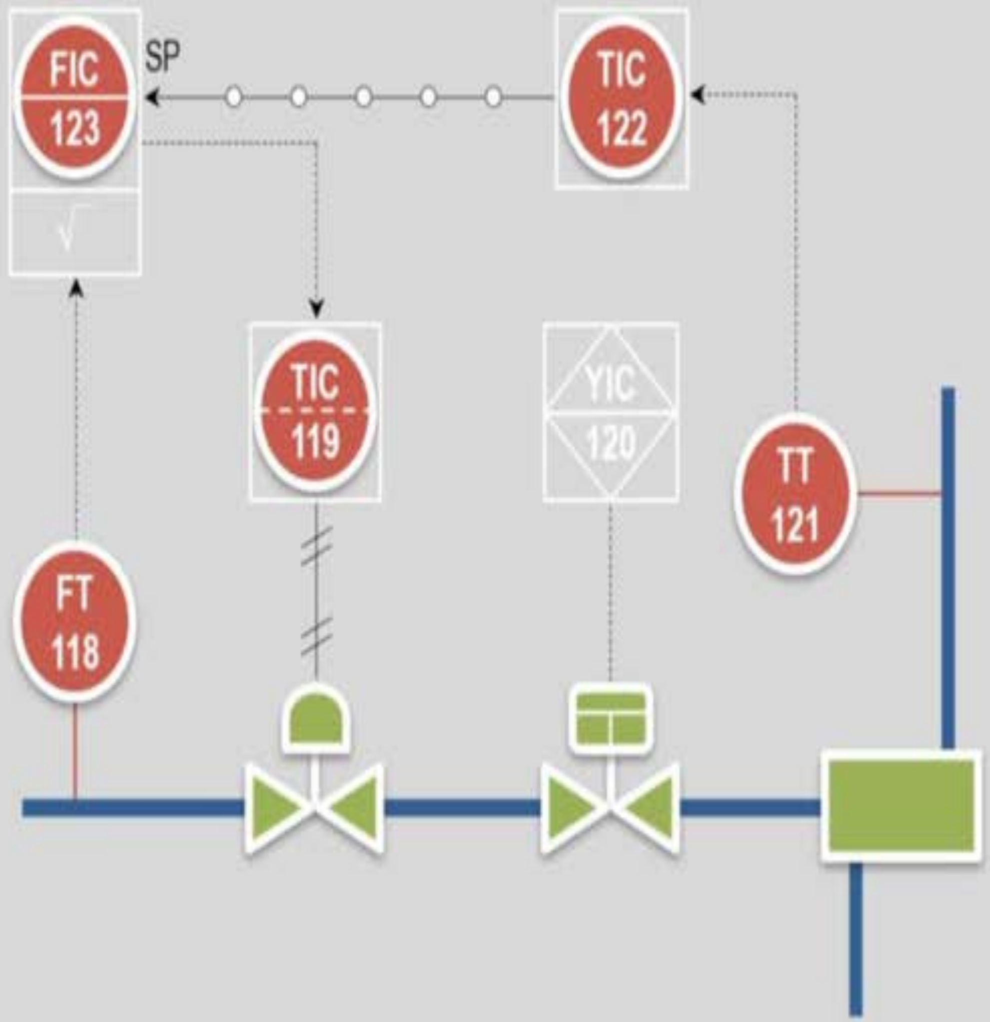


AUXILIARY PIPING MAY CARRY :

- COMPRESSED AIR
- INERT GAS
- HYDRAULIC FLUID

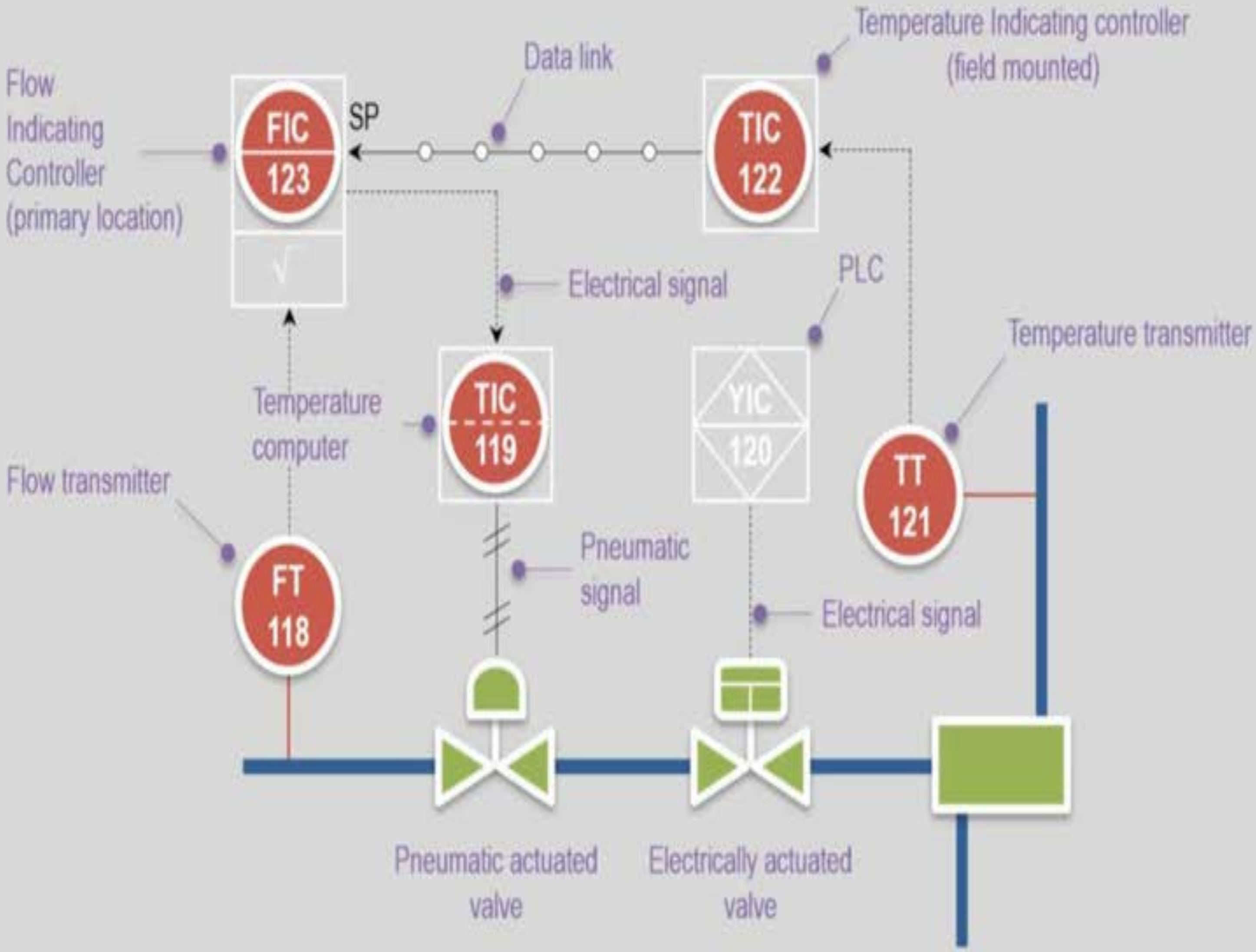


PRACTICE SESSION

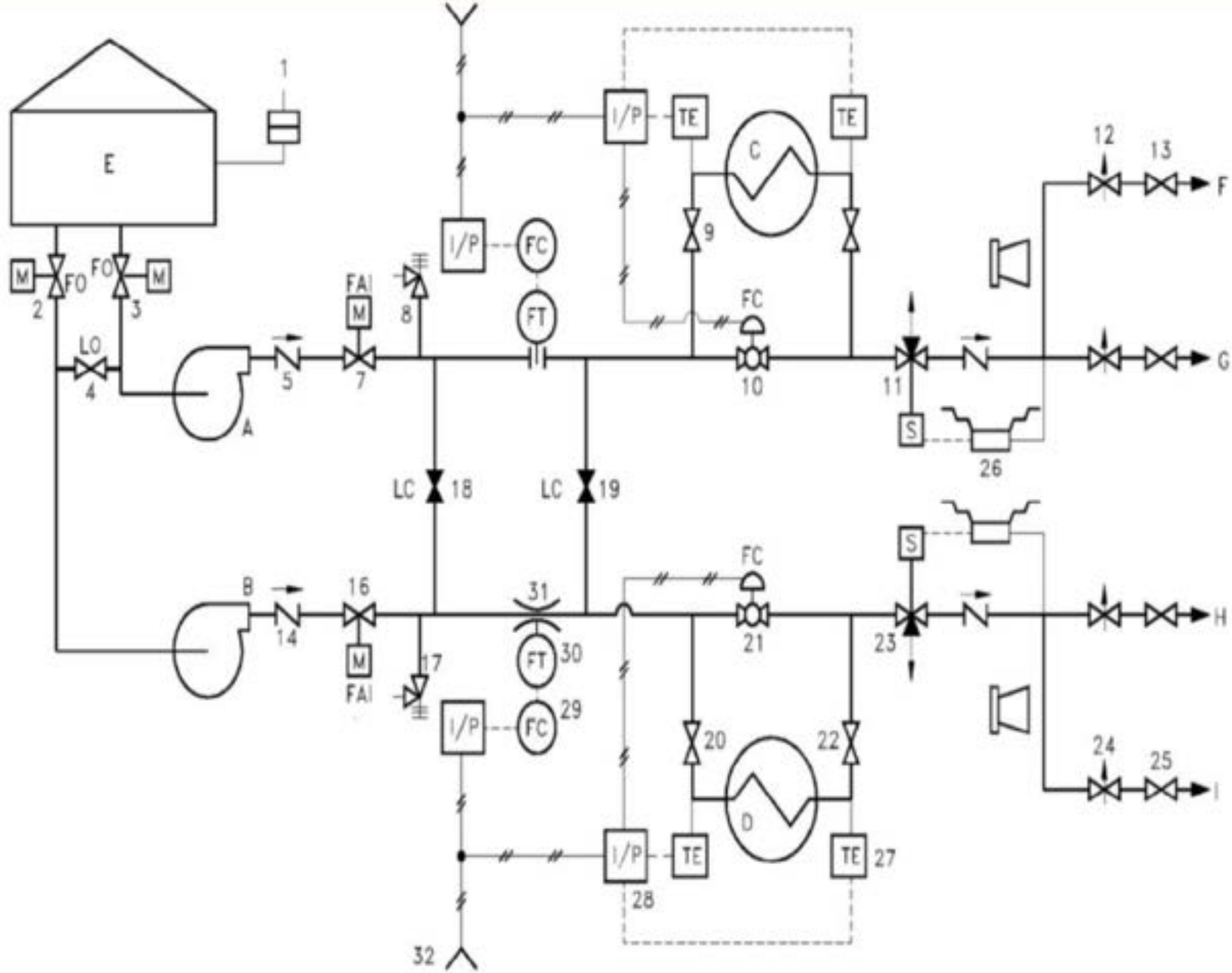


Piping and Instrumentation Drawings (P&ID)

PRACTICE SESSION



Piping and Instrumentation Drawings (P&ID)



E
1
I/P
TE
C
TE
12
13
F
I/P
FC
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FO
FO
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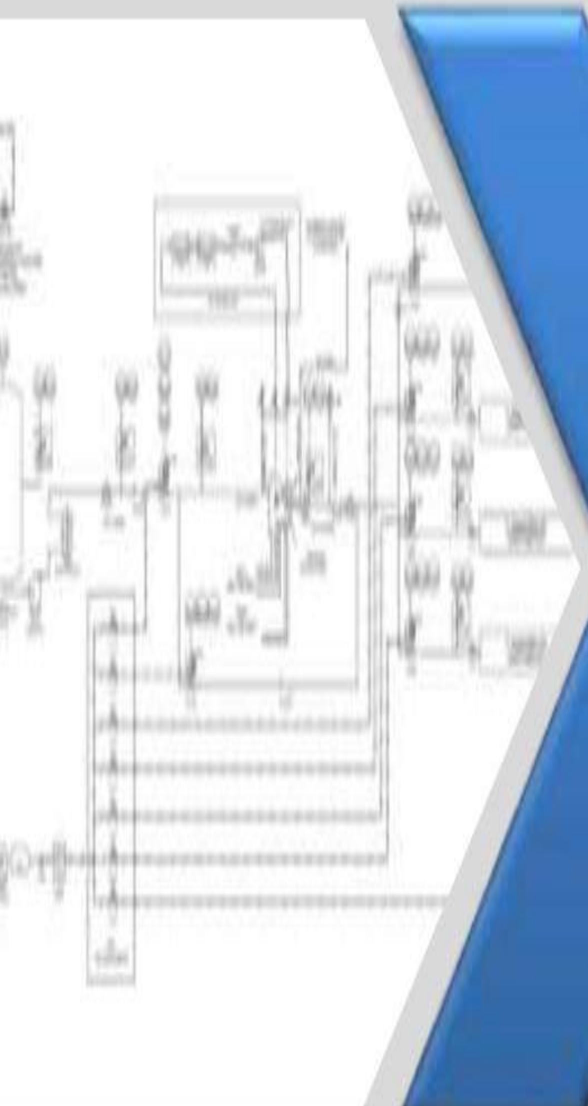
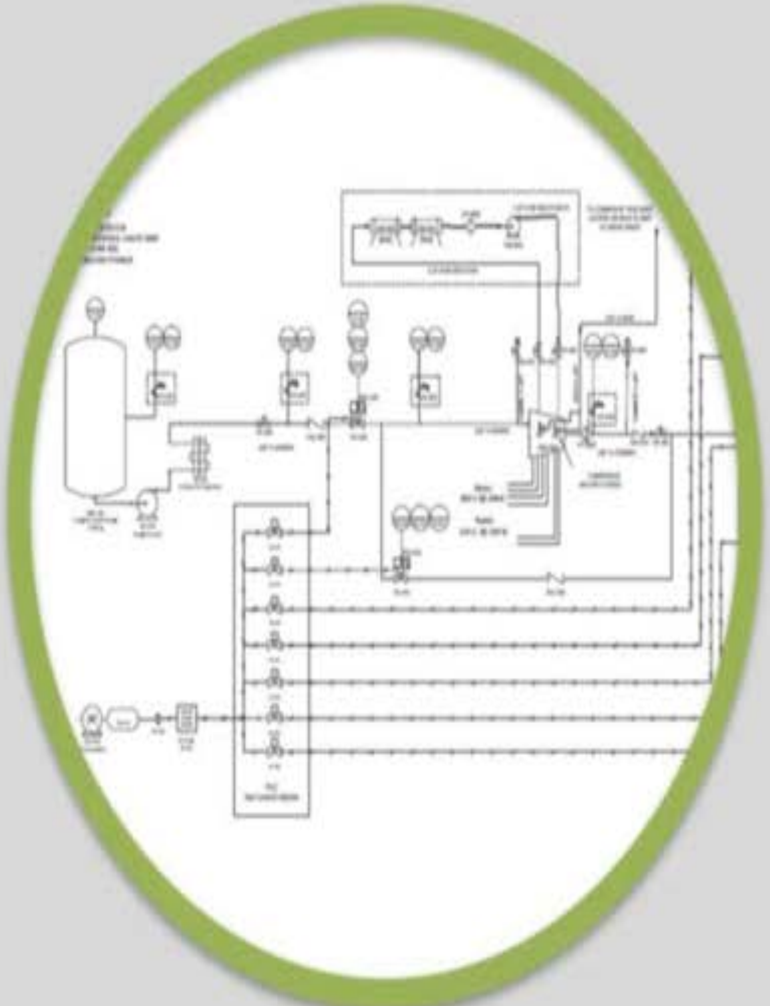
28
32



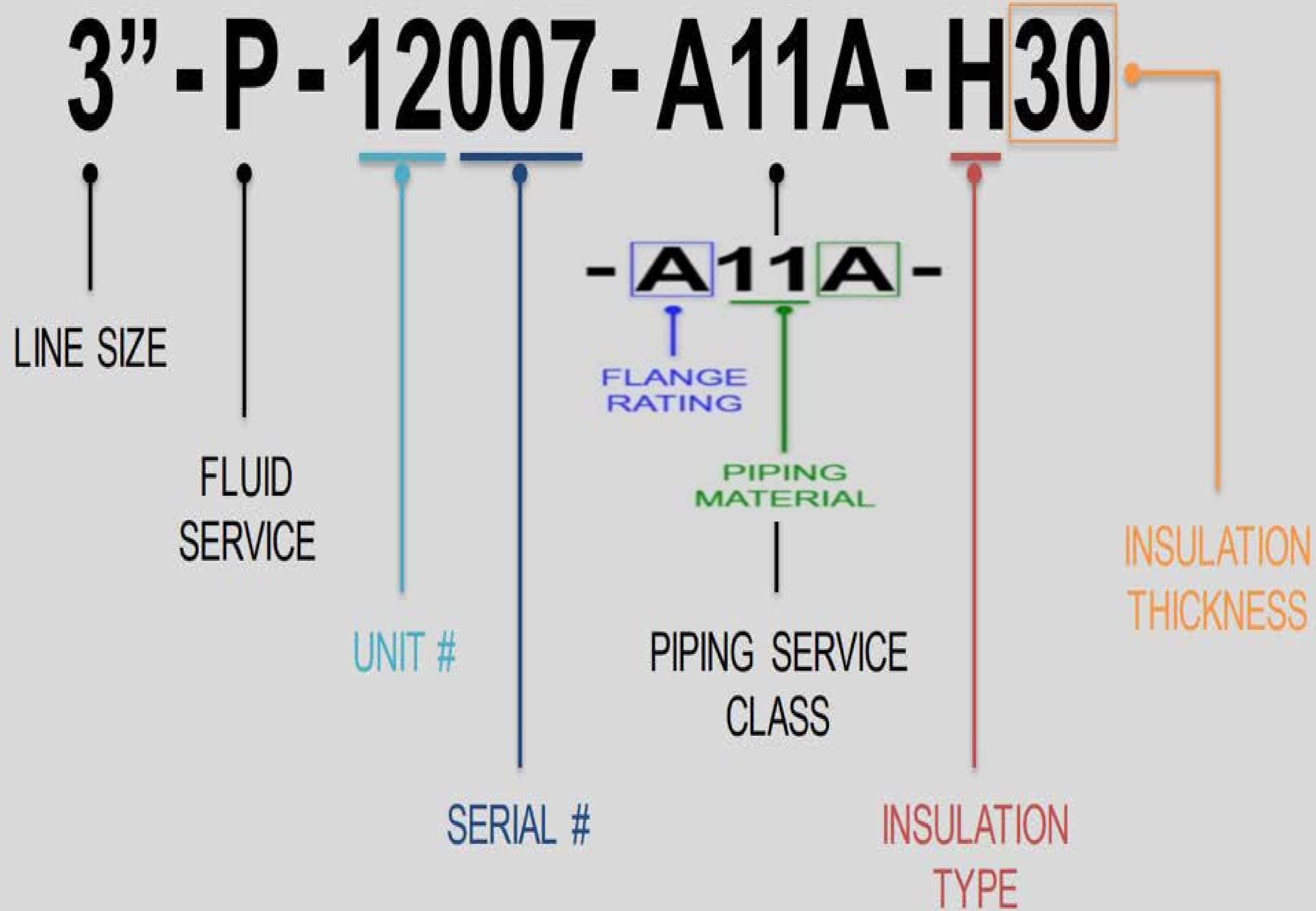
P&ID INSTRUMENT DESIGNATION CODES



- USAGE
- LINE NUMBER
- SIZE
- PIPING CLASS
- INSULATION CLASS

ASME
SETTING THE STANDARD

energy API
AMERICAN PETROLEUM INSTITUTE



3" - P - 12007 - A11A - H30
LINE SIZE
FLUID SERVICE
UNIT #
SERIAL #
- A11A -
FLANGE RATING
PIPING MATERIAL
PIPING SERVICE CLASS
INSULATION TYPE
INSULATION THICKNESS

3" - P - 12007 - A11A - H30

FLUID SERVICE

CODE	FLUID
AEA	AERATION AIR
ASW	AIR SATURATED WATER
BWW	BACKWASH WATER
CEN	CENTRATE
CMA	COMPRESSED AIR
CWS	COOLING WATER SUPPLY
CWR	COOLING WATER RETURN

CODE	FLUID
DGG	DIGESTER GAS
DGS	DIGESTED SLUDGE
DRN	DRAIN
EFF	EFFLUENT
FIL	FILTRATE
FLA	FOUL AIR
GAS	NATURAL GAS

CODE	FLUID
ALM	ALUMINUM SULFATE
CO2	CARBON DIOXIDE
FOL	FUEL OIL
HCL	HYDROCHLORIC ACID
H2	HYDROGEN
N2	NITROGEN
STM	STEAM

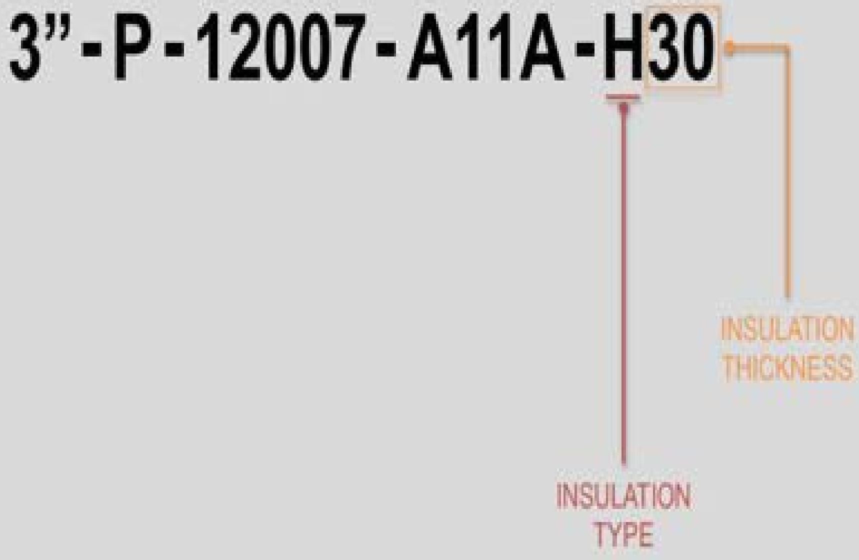
CODE	FLUID
HTW	HOT WATER
INA	INSTRUMENT AIR
MLQ	MIXED LIQUOR
MLR	MIXED LIQUOR RETURN
PER	PERMEATE
PRW	PROCESS WATER
PTW	POTABLE WATER

CODE	FLUID
RAW	RAW WATER
REW	RECYCLED WATER
SEW	RAW SEWAGE
WWW	WASTE WASHWATER

3" - P - 12007 - A11A - H30

PIPING MATERIAL

CODE	PIPE MATERIAL
A	CAST IRON
B	CARBON STEEL
C	LOW ALLOY
D	INTERMEDIATE ALLOY
E	STAINLESS STEEL
J	NON-METALLIC
U	COPPER PIPING



CODE	TYPE OF INSULATION
N	NO INSULATION
ET	ELECTRICALLY TRACED
ST	STEAM TRACED
C	COLD INSULATION
PP	PERSONNEL PROTECTION
F	FIRE PROOFING
W	HOT INSULATION

P&ID EQUIPMENT DESIGNATION CODES

optional

EQPT - LOCN - XXX

EQUIPMENT TITLE ABBREVIATION

LOCATION CODE

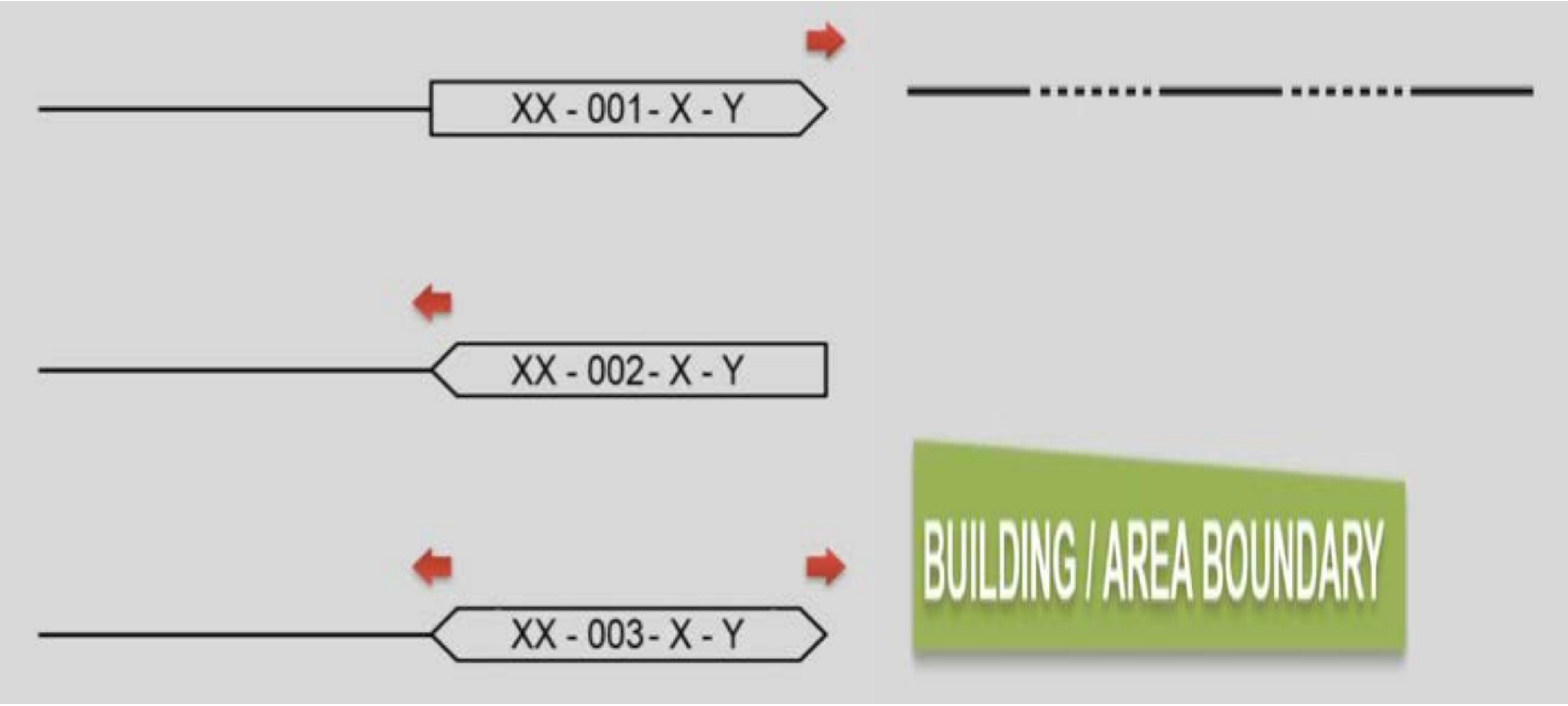
EQUIPMENT #

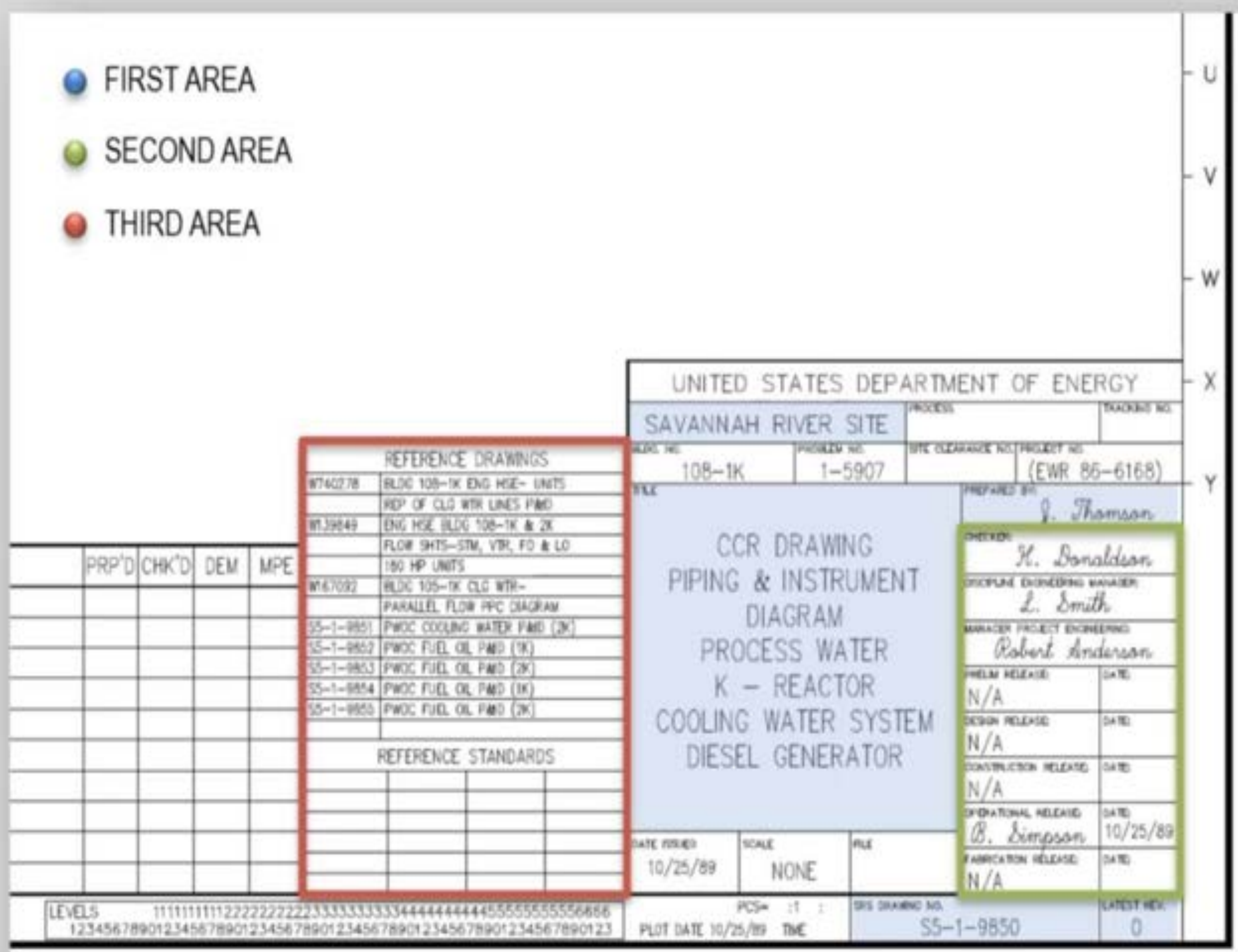
Hand Ball Valve → HV - 010

Compressor → CO - 096 - 300

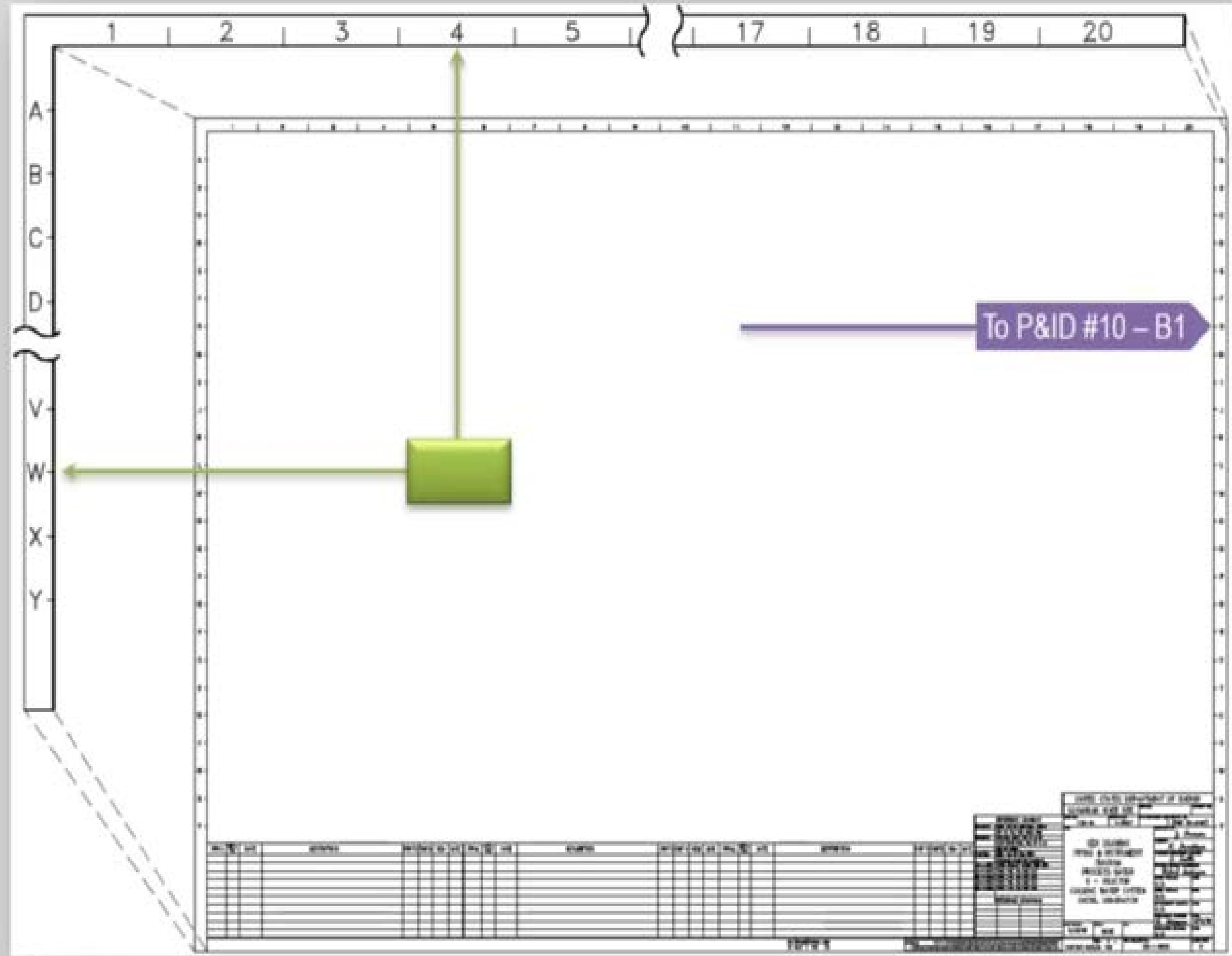
Process Water Pump 2 Chemical Dosing Area → PRWP - CHEM - 2

MISCELLANEOUS P&ID DESIGNATION CODES

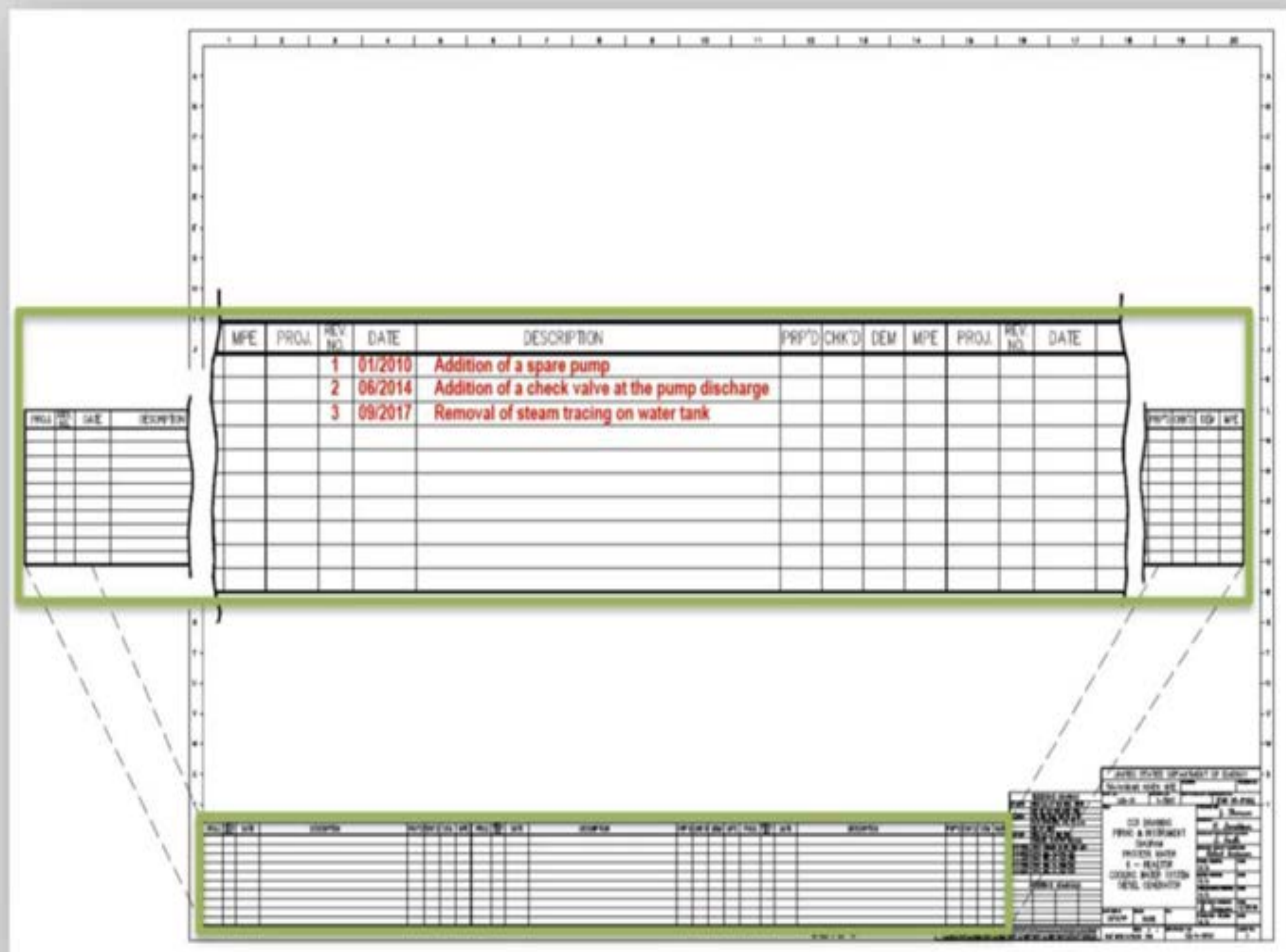




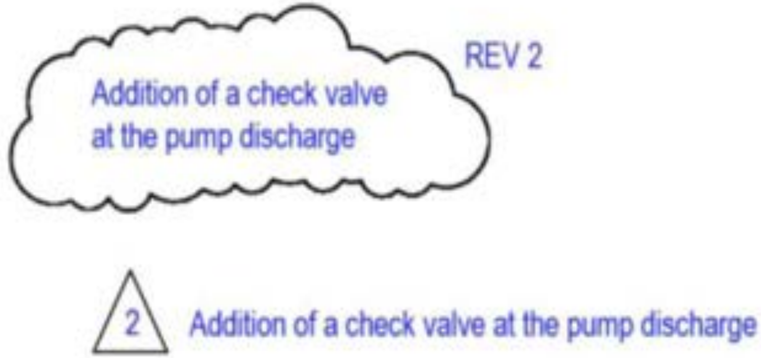
EXAMPLE OF A TITLE BLOCK



EXAMPLE OF A GRID SYSTEM



EXAMPLE OF A REVISION BLOCK



- Cloud methods indicates changes from the most recent revision **only**
- Triangle method indicates **all** revisions to the P&ID

HOW CHANGES ARE INDICATED

	Measured Variable	Readout of function	Output Function	Modifier
A	Air	Alarm		
B	Burner	Storage	Users Choice	Users Choice
C	Compressor			Closed
D	Dispenser			
E	Electrolyzer	Sensor (primary Element)		
F	Flow	Filter		
G	Chiller	Glass, viewing		
H	Hand			High
I	Current	Indicator		
J	Power			
K	Time		Control Station	
L	Level	Light		Low
M	Conductivity			Middle
N	Users Choice	Users Choice	Hydrogen	Users Choice
O	Water	Orifice, restriction	Air	Open
P	Pressure			
Q	Quantity			
R	Radiation	Record, Reduce		
S	Speed, Frequency		Switch	
T	Temperature		Transmit	
U	Multivariate	Multifunction	Multifunction	Multifunction
V	Vibration		Valve	
W	Weight	Well	Water	
X	Unclassified	Unclassified	Unclassified	Unclassified
Y	State		Relay	
Z	Position		Actuator	

Line symbols

X"-XXX-XXXX-XXX-XXX

1" – H – 001 – 15000PSI – F1

- Insulation (if applicable)
- Pipe Spec
- Line Number (if assigned)
- Service
- Line size, nominal inches

Line Service Identification codes

CCA-CLEAN COMPRESSED AIR

CHW-CHILLED WATER

H-HYDROGEN

N-NITROGEN

Insulation codes

F1- Soda-lime silicate glass (FOAMGLAS) 1" wall thickness

F2- Buna N Foam 3/8" thick

Pipe Specification Details

3000PSI	Min. pressure rating of 3,000 PSI 316/316L austenitic stainless steel tubing UNS S31600 (316), UNS S31603 (316L)
6500PSI	Min. pressure rating of 6,500 PSI 316/316L austenitic stainless steel tubing UNS S31600 (316), UNS S31603 (316L)
15000PSI	Min. pressure rating of 15,000 PSI 316/316L austenitic stainless steel tubing UNS S31600 (316), UNS S31603 (316L)
POLY	Polyamide (Nylon) pneumatic tubing, light and heat stabilized.
0.032CU	0.032" wall copper alloy 122 seamless tubing
SCH40	Schedule 40 iron pipe

VALVES

- BALL VALVE
- NEEDLE VALVE
- CHECK VALVE
- SOLENOID VALVE
- AIR ACTUATED VALVE, SPRING RETURN CLOSE
- PRESSURE REDUCING VALVE
- TWO WAY PRESSURE RELIEF
- THREE WAY PRESSURE RELIEF

EQUIPMENT

- FILTER
- PUMP
- DRYER
- HYDROGEN COMPRESSOR
- STORAGE TANK
- IR FLAME DETECTOR

PROCESS AND SIGNAL LINES

- HYDROGEN PROCESS LINE
- DEIONIZED WATER PROCESS LINE
- PNEUMATIC SIGNAL LINE
- NITROGEN SIGNAL LINE
- ELECTRICAL SIGNAL LINE

REVISIONS | FILE INFORMATION | ENGINEERING REVIEW

DESIGNER / ENGINEER / CHECKED BY / A/E APPR. BY / NREL APPR BY / BLD AREA ENG

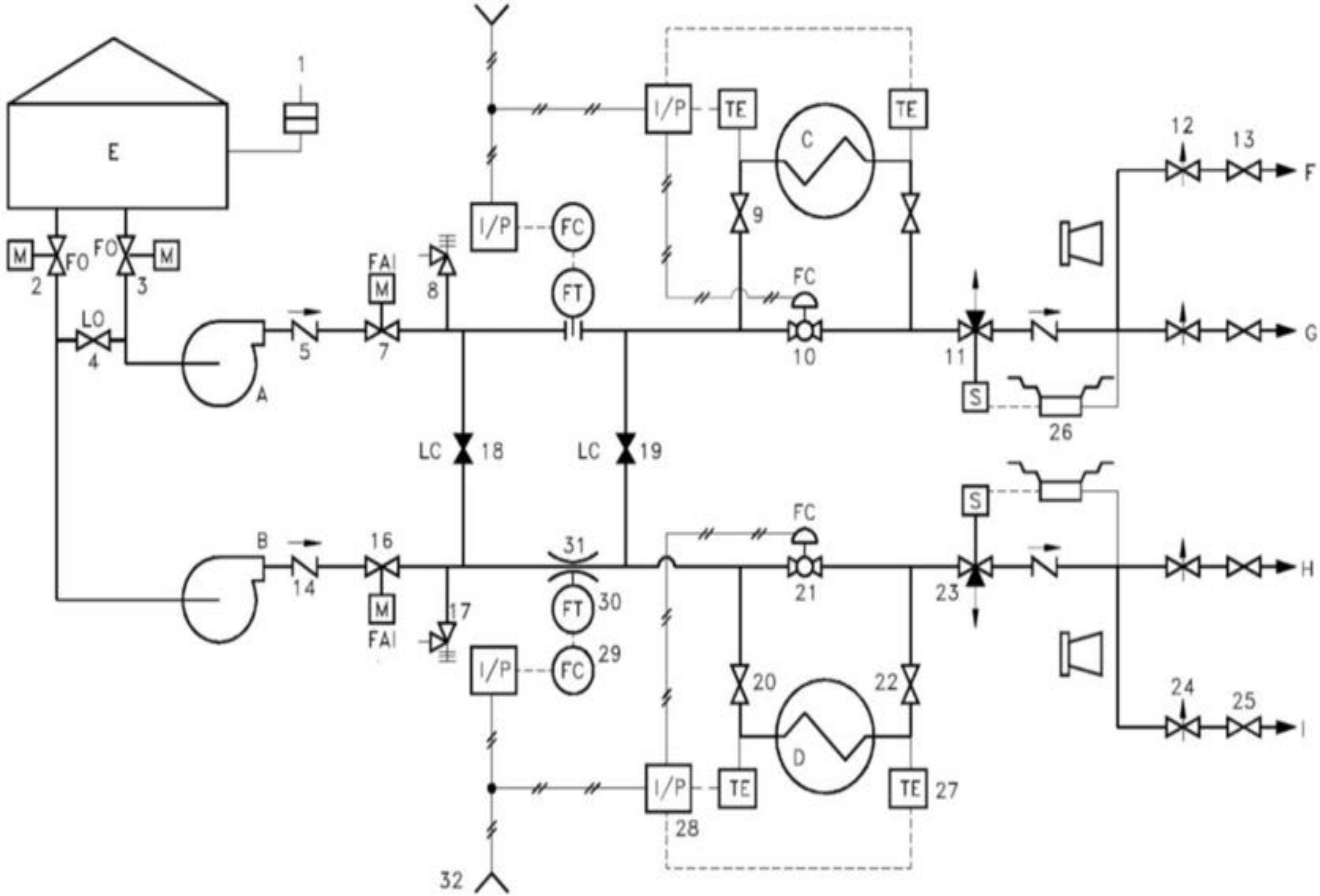
WR Training

ENERGY SYSTEMS INTEGRATION LABORATORY P&ID

LEGEND AND SPECIFICATIONS

LEG

1 OF 6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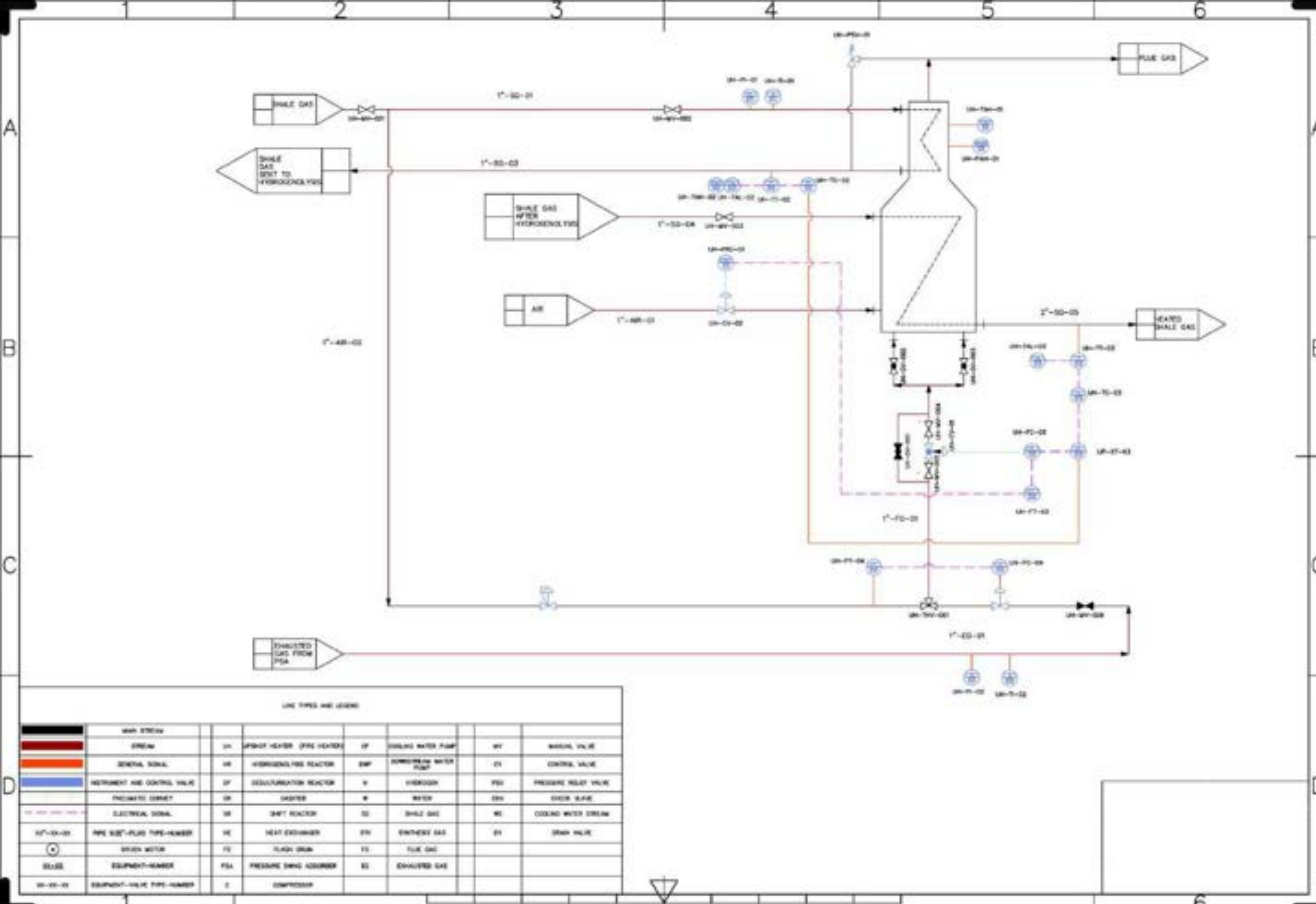
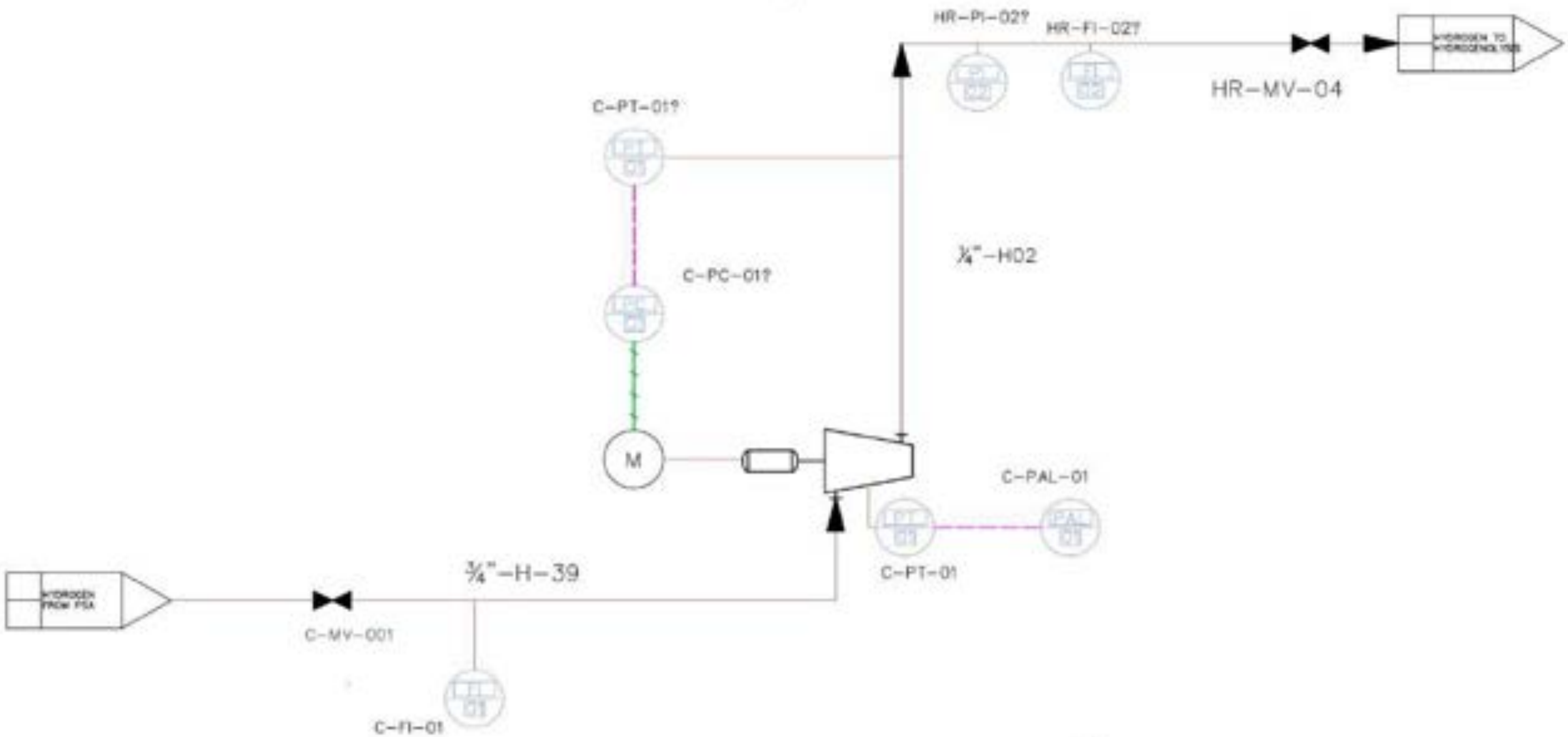


SHALE GAS
FLUE GAS
SHALE GAS SENT TO HYDROGENOLYSIS
SHALE GAS AFTER HYDROGENOLYSIS
AIR
HEATED SHALE GAS
EXHAUSTED GAS FROM PSA



LINE TYPES AND LEGEND

	MAIN STREAM						
	STREAM	UH	UPSHOT HEATER (FIRE HEATER)	CP	COOLING WATER PUMP	MV	MANUAL VALVE
	GENERAL SIGNAL	HR	HYDROGENOLYSIS REACTOR	DWP	DOWNSTREAM WATER PUMP	CV	CONTROL VALVE
	INSTRUMENT AND CONTROL VALVE	DF	DESULFURISATION REACTOR	H	HYDROGEN	PSV	PRESSURE RELIEF VALVE
	PNEUMATIC CONVEY	GR	GASIFIER	W	WATER	CHV	CHECK VLAVE
	ELECTRICAL SIGNAL	SR	SHIFT REACTOR	SG	SHALE GAS	WS	COOLING WATER STREAM
XX"-XX-XX	PIPE SIZE"-FLUID TYPE-NUMBER	HE	HEAT EXCHANGER	STH	SYNTHESIS GAS	DV	DRAIN VALVE
	DRIVEN MOTOR	FD	FLASH DRUM	FG	FLUE GAS		
XX-XX	EQUIPMENT-NUMBER	PSA	PRESSURE SWING ADSORBER	EG	EXHAUSTED GAS		
XX-XX-XX	EQUIPMENT-VALVE TYPE-NUMBER	C	COMPRESSOR				

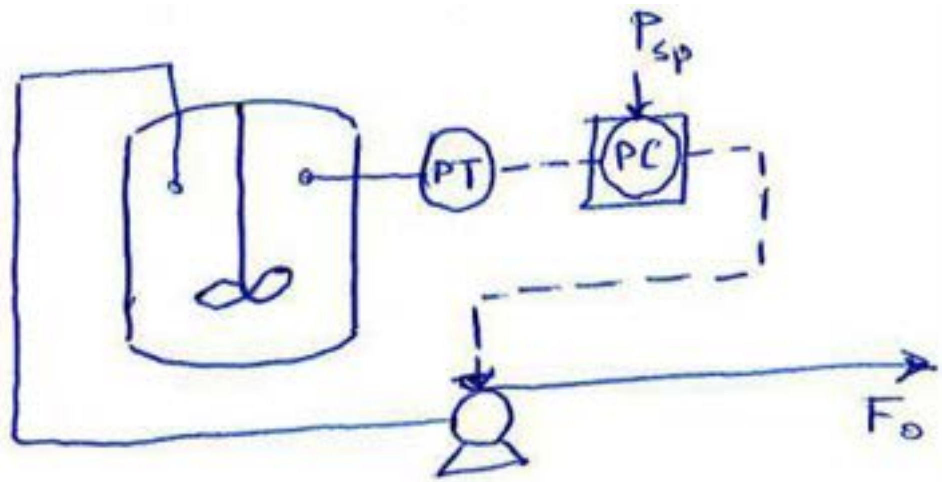
HOW IS CONTROL DESIGN DOCUMENTED?

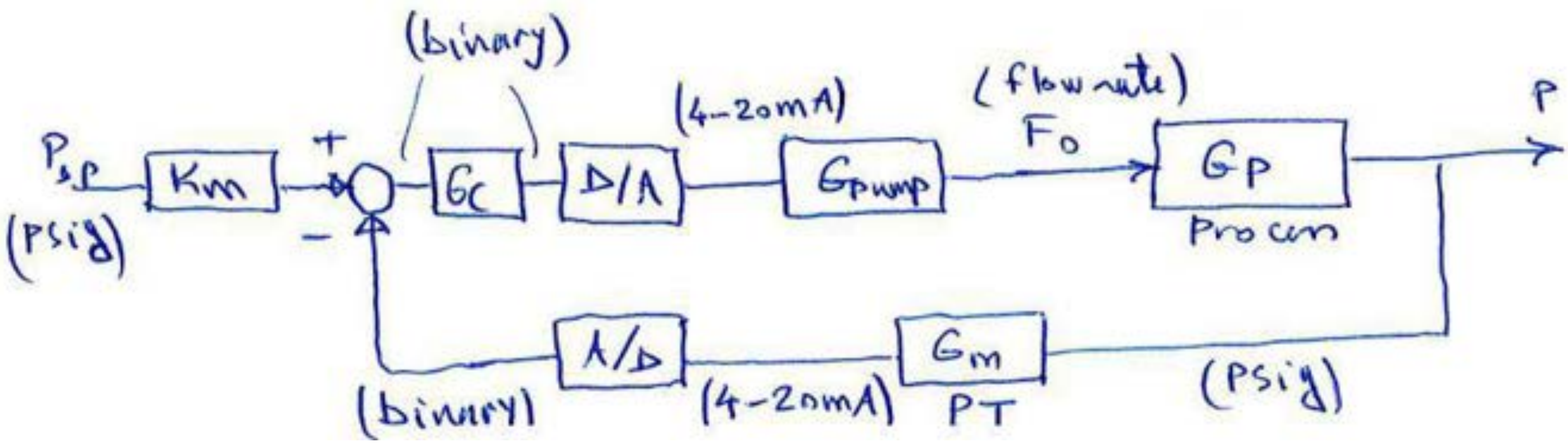
Example:

The pressure in a chemical reactor is measured by an electronic sensor/transmitter with an output ranging between 4 - 20 mA. The manipulated variable is the speed of a vacuum pump accepting a 4 - 20 mA input. The controller is a digital controller. Sketch the piping and instrumentation diagram (P&ID) and the 'block diagram' for the closed loop control system. Make sure to include all the necessary components in the control system including the AD and DA converters.

HOW IS CONTROL DESIGN DOCUMENTED?

Solution:



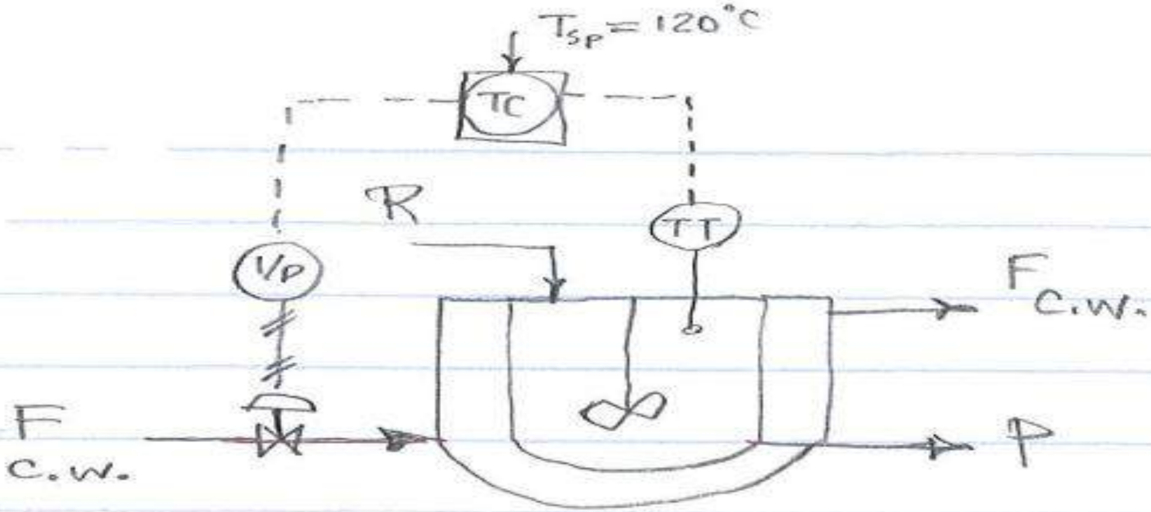


Example:

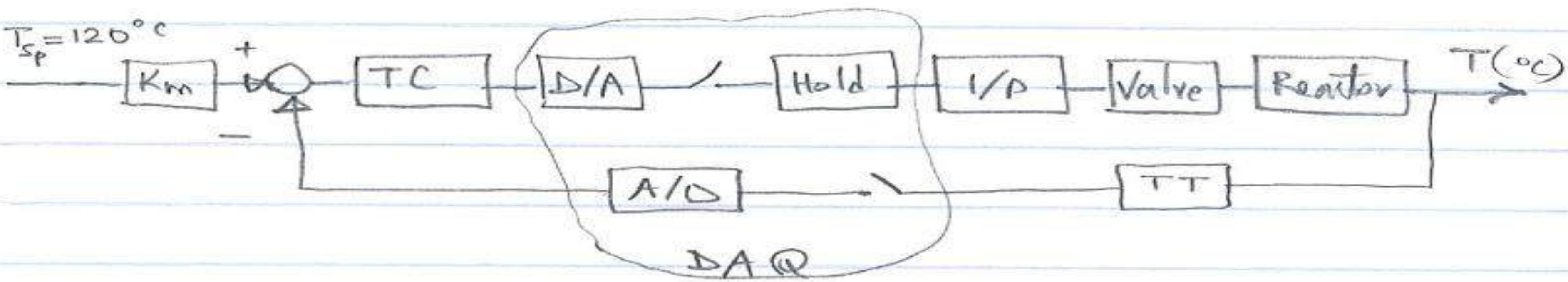
In order to control the temperature of a jacketed chemical reactor in which an exothermic reaction takes place, a DAQ system with a proper software is employed. Describe every step which is needed to read the thermocouple output (0-10 mV) and issue a signal to manipulate the cooling water flow rate supplied to the reactor jacket. The desired set-point temperature is 120C. Draw the Block diagram and the P&ID of this control system. A pneumatic control valve is used to throttle the flow of the cooling water. Make sure to include every component needed and the units of the signals in you diagrams.

HOW IS CONTROL DESIGN DOCUMENTED?

Solution:



(P & ID)



Process Dynamics

Transfer Functions Advanced Concepts

Asst. Prof. Omar Y. Ismael

PhD Stud. Control Engineering

MSc Mechatronics Engineering

Dip. Pharmacy Technician

16th-Oct-2025

Process Dynamics — Industrial Modeling Foundations

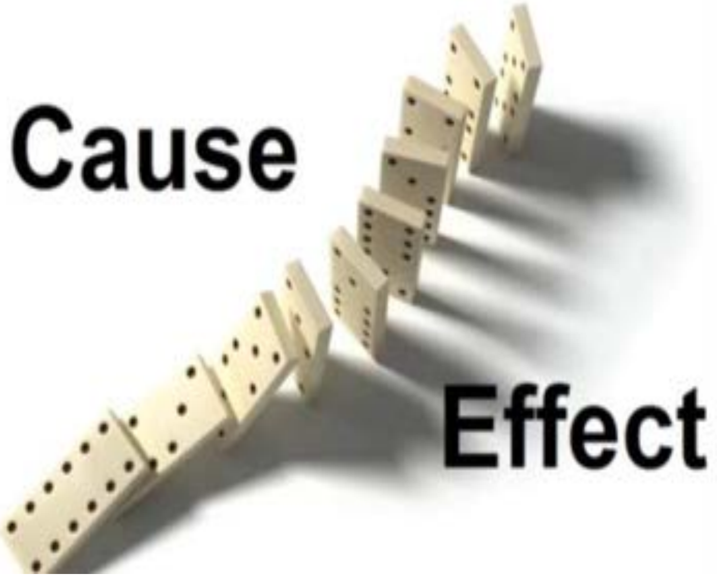
- Process dynamics describe how industrial systems respond over time to changes in inputs, disturbances, and setpoints.
- Accurate dynamic models are essential for controller design, tuning, safety, and plant optimization.
- Real processes often exhibit multiple time constants, integrating behavior, and dead time.
- Control engineers use simplified linear models (FOPDT, SOPDT, Integrating + Delay) for analysis and tuning.

Industrial examples

- Temperature control in heat exchangers → FOPDT
- Tank level → Integrating + delay
- Steam drum pressure → SOPDT + delay
- Pipelines → Pure transport delay

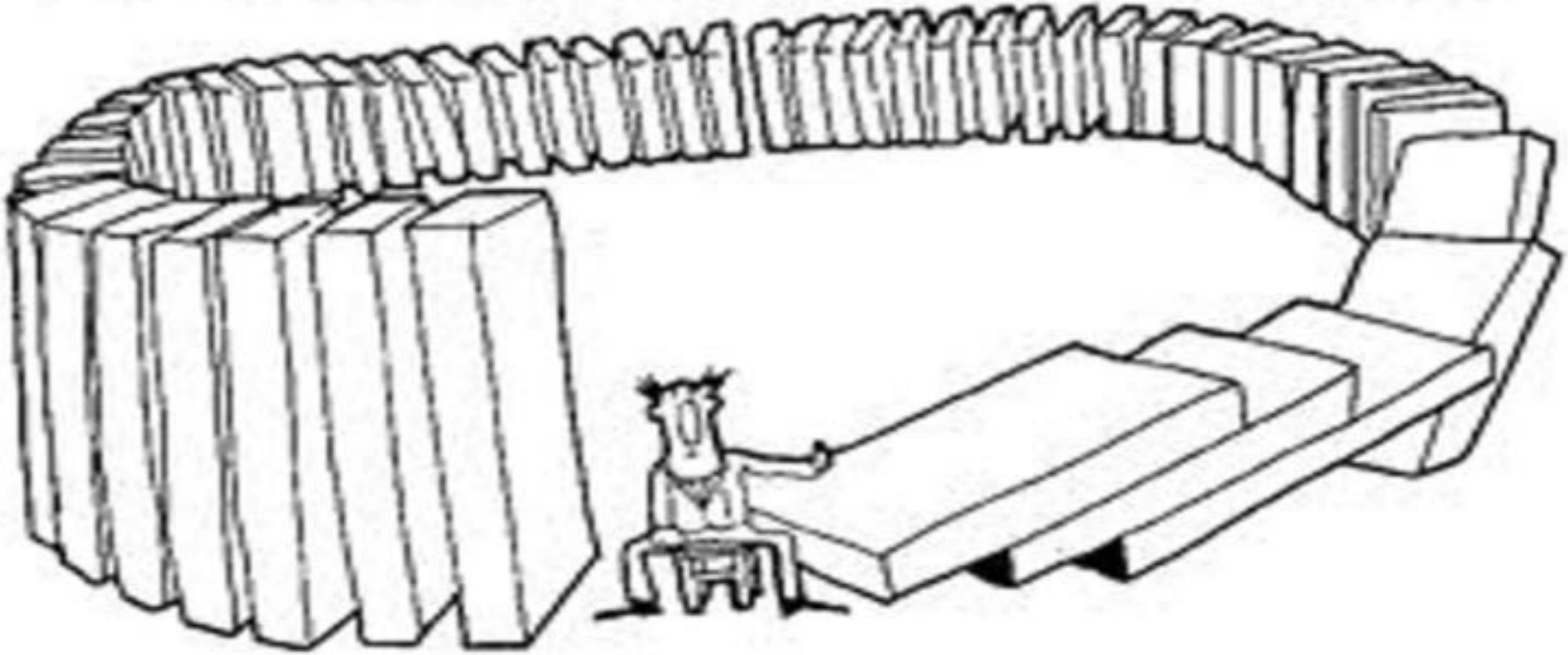
Cause and Effect

- All process control applications have Cause-and-Effect variables:
  - Open Valve (**Cause**) ➔ More Flow (**Effect**)
  - More Heat (**Cause**) ➔ Higher Temperature (**Effect**)
  - More Flow (**Cause**) ➔ Higher Level in Tank (**Effect**)
- **MV** is the **Cause**
- **CV** is the **Effect**



Cause and Effect – Time is Involved

In complex systems, cause and effect are often distant in time and space

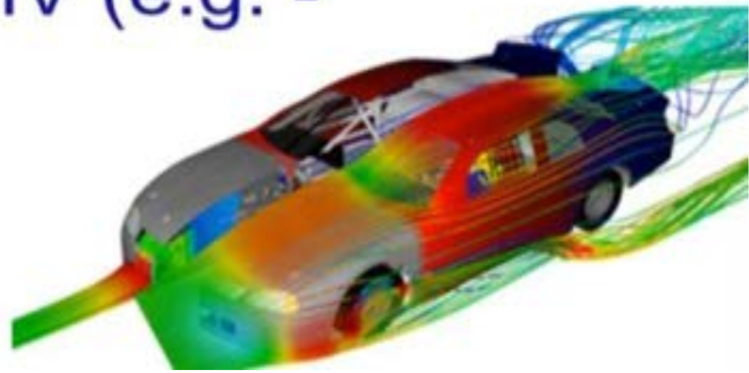


Characterizing dynamic relations between MV and CV.....

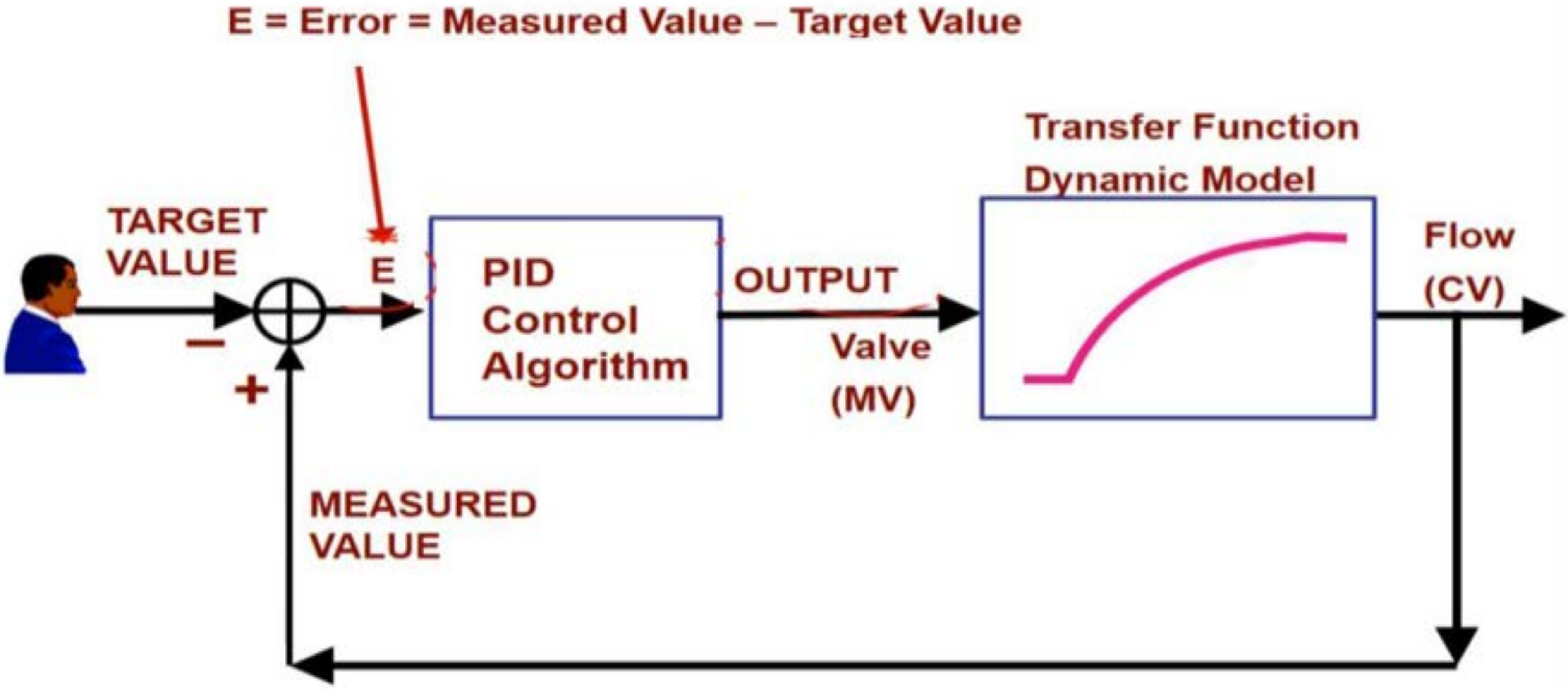
Is done commonly using Transfer functions......

Transfer Function Dynamics

- Transfer functions define the relationship between a MV and a CV.
- The MV-CV relationship is called a "*Dynamic Relationship*" because "*time element*" is involved.
- Transfer functions are commonly defined based on the CV (e.g.- flow) response due to a unit change in the MV (e.g. - control valve)



Transfer Function in Process Control Schematic



- PID Control Algorithm is an equation that calculates an OUTPUT based on the Measured Value and Target Value.
- OUTPUT moves Control Valve

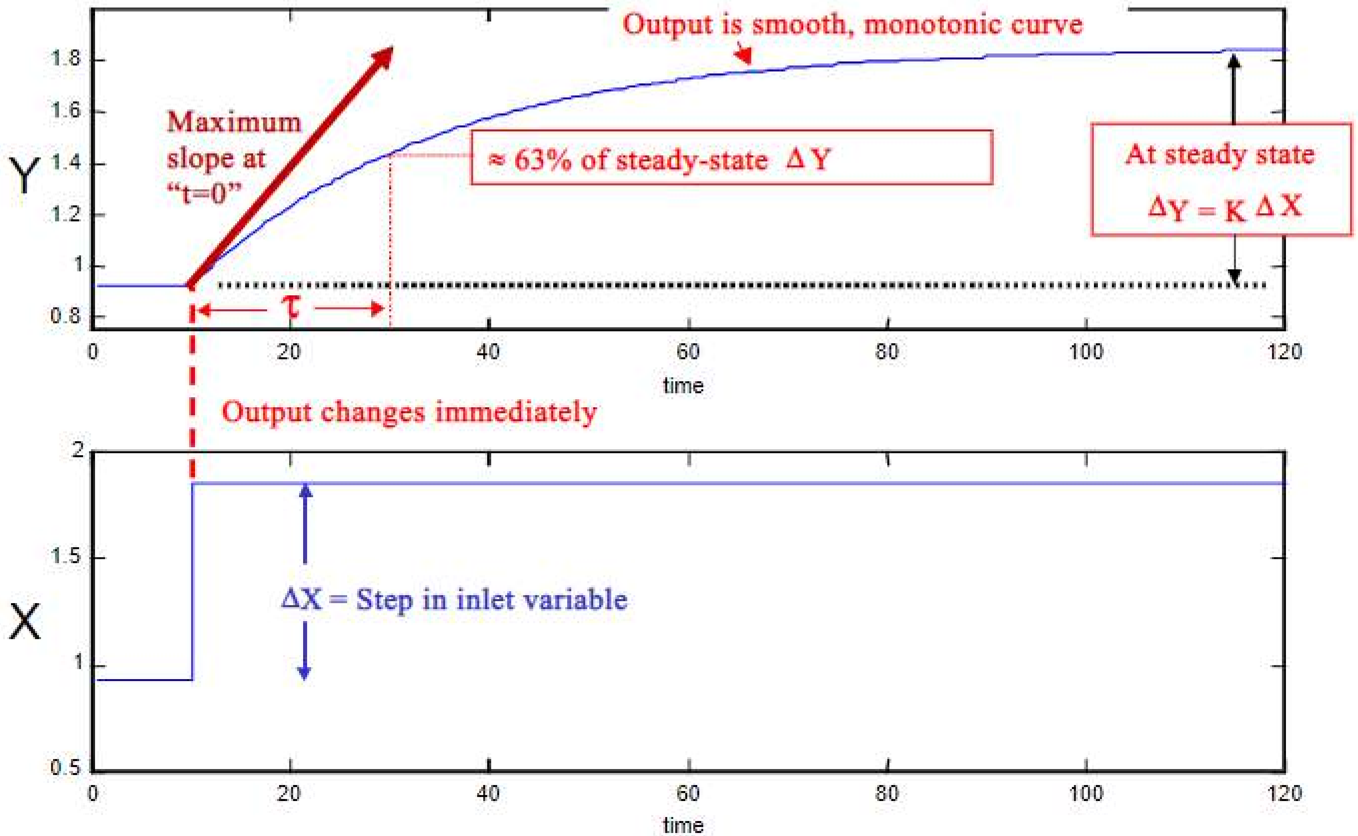
The basic equation is:

1

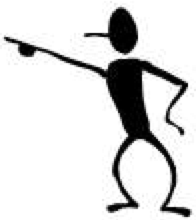
$$\tau \frac{dY(t)}{dt} + Y(t) = K\,X(t)$$

K = s-s gain

τ = time constant



Would this be easy/difficult to control?

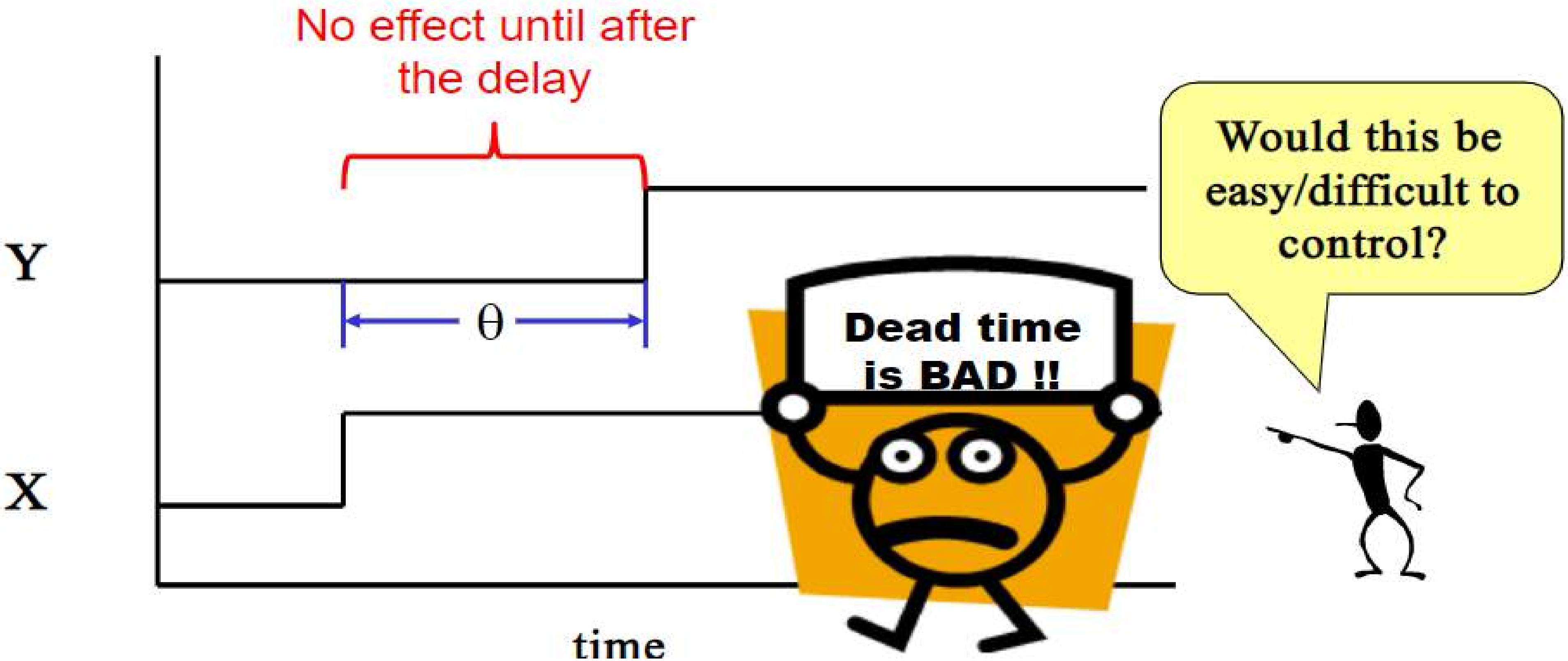


SIMPLE PROCESS SYSTEMS: DEAD TIME

Mathematical Model

$$Y(t) = X(t-\theta)$$

$$Y(s) = e^{-\theta s} X(s)$$



(d) Dead-Time Only

$$G(s) = e^{-\theta s}$$

- No dynamic change until after θ seconds.
- Occurs in **transportation pipelines**, large **thermal lags**, or **remote sensing**.
- Requires dead-time compensators (Smith Predictor, MPC).

First-Order and FOPDT Models

First-Order Model

$$\tau\frac{dy(t)}{dt}+y(t)=K_pu(t),\quad G(s)=\frac{K_p}{\tau s+1}$$

- Represents systems with a single dominant time constant (e.g., flow, some pressure loops).
- τ governs the speed of response.

First-Order Plus Dead-Time (FOPDT)

$$\tau\frac{dy(t)}{dt}+y(t)=K_pu(t-\theta),\quad G(s)=\frac{K_pe^{-\theta s}}{\tau s+1}$$

- θ represents **dead time** due to transport, measurement, or actuation delays.
- Most common industrial model for tuning and design.

Padé Approximation (1,1)

$$e^{-\theta s} \approx \frac{1 - \frac{\theta}{2}s}{1 + \frac{\theta}{2}s}$$

- Rational approximation used for simulation & state-space models.

Industrial hints:

- Flow & temperature loops behave like FOPDT.
- Large θ/τ → lower phase margin → conservative tuning or Smith Predictor/MPC.
- IMC and Cohen–Coon rely on FOPDT parameters.

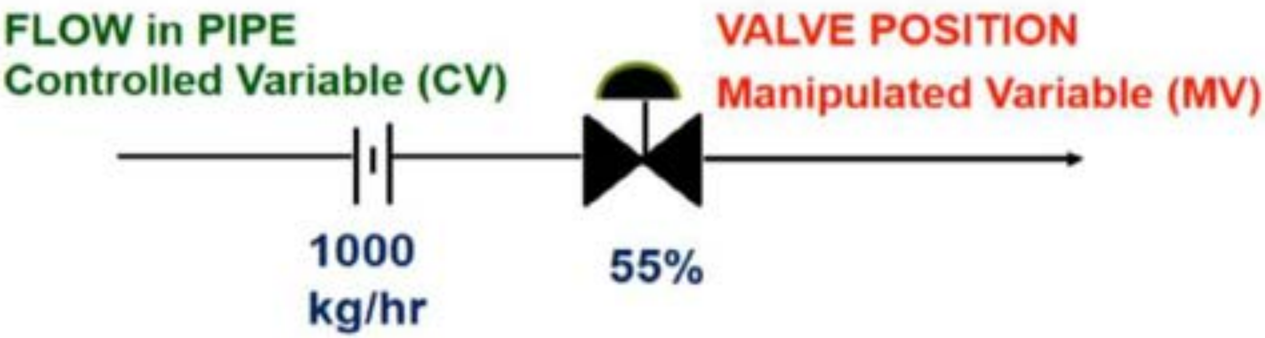
Worked Industrial Examples

A. Heat-Exchanger Outlet Temperature (FOPDT)

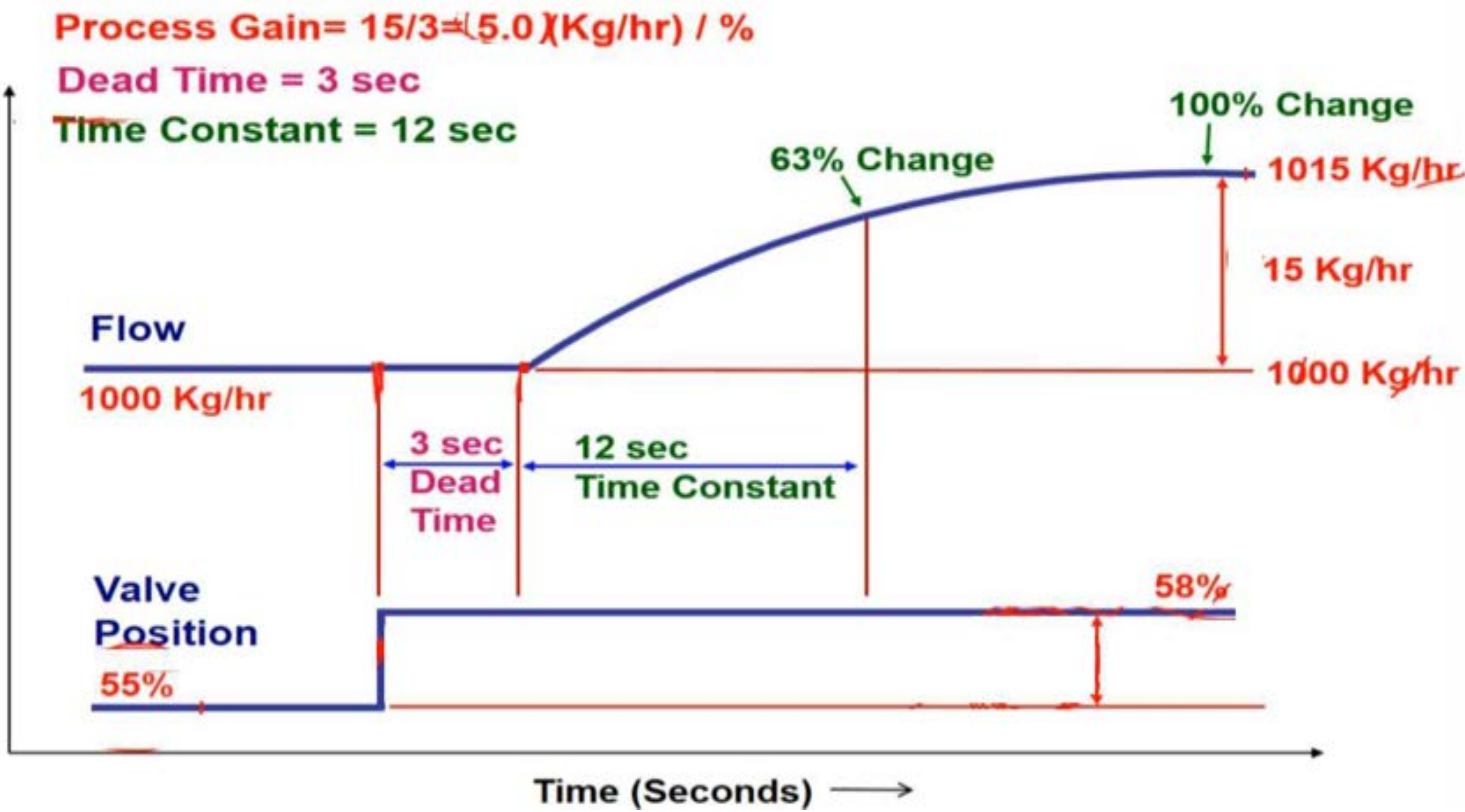
$$\tau\,\dot{T}_{out}(t) + T_{out}(t) = K_p\,F_{cw}(t-\theta), \quad G(s) = \frac{K_p e^{-\theta s}}{\tau s + 1}.$$

- MV F_{cw}: cooling-water flow (control valve)
- CV T_{out}: outlet temperature
- **Use:** IMC-PID (with θ/τ) or MPC if θ large.

Flow and Control Valve Transfer Function



Characterize VALVE versus FLOW Dynamics using Transfer Function



Transfer Function Parameters Defines Process Dynamics

- After valve is opened, no change in flow for some time. This is called **Dead Time**. After dead time has elapsed, flow starts changing. Dead time is also called *Delay*, *Time Delay*, *Transportation Delay*, often denoted by **Greek symbol *Theta* Θ**. Unit is seconds or minutes.
- Time required for the flow (CV) to reach 63% of total final change is called **Time Constant**, often denoted by **Greek symbol Tau (τ)**. Unit is seconds or minutes.
- Total final change in flow (CV) divided by change in valve (MV) is called **Process Gain**.
- This is a **First Order Transfer Function** with Dead Time.

Time to Steady State

- Often abbreviated as TTSS.
- TTSS is the total time needed for the CV to settle to a new steady state after a change in the MV.
- Generally TTSS = (2.5 x Time Constant) + Dead Time, or TTSS = $(2.5 * \tau) + \theta$
- **Note**:
  - College professors like to express TTSS as = 5 x Time Constant
  - But practically in chemical plants:
    - after 2.5 time constants have elapsed
    - The CV is within 90% of the final value (which is within the noise and disturbance/drift band)
    - So practically, the 2.5 multiplier is more realistic.

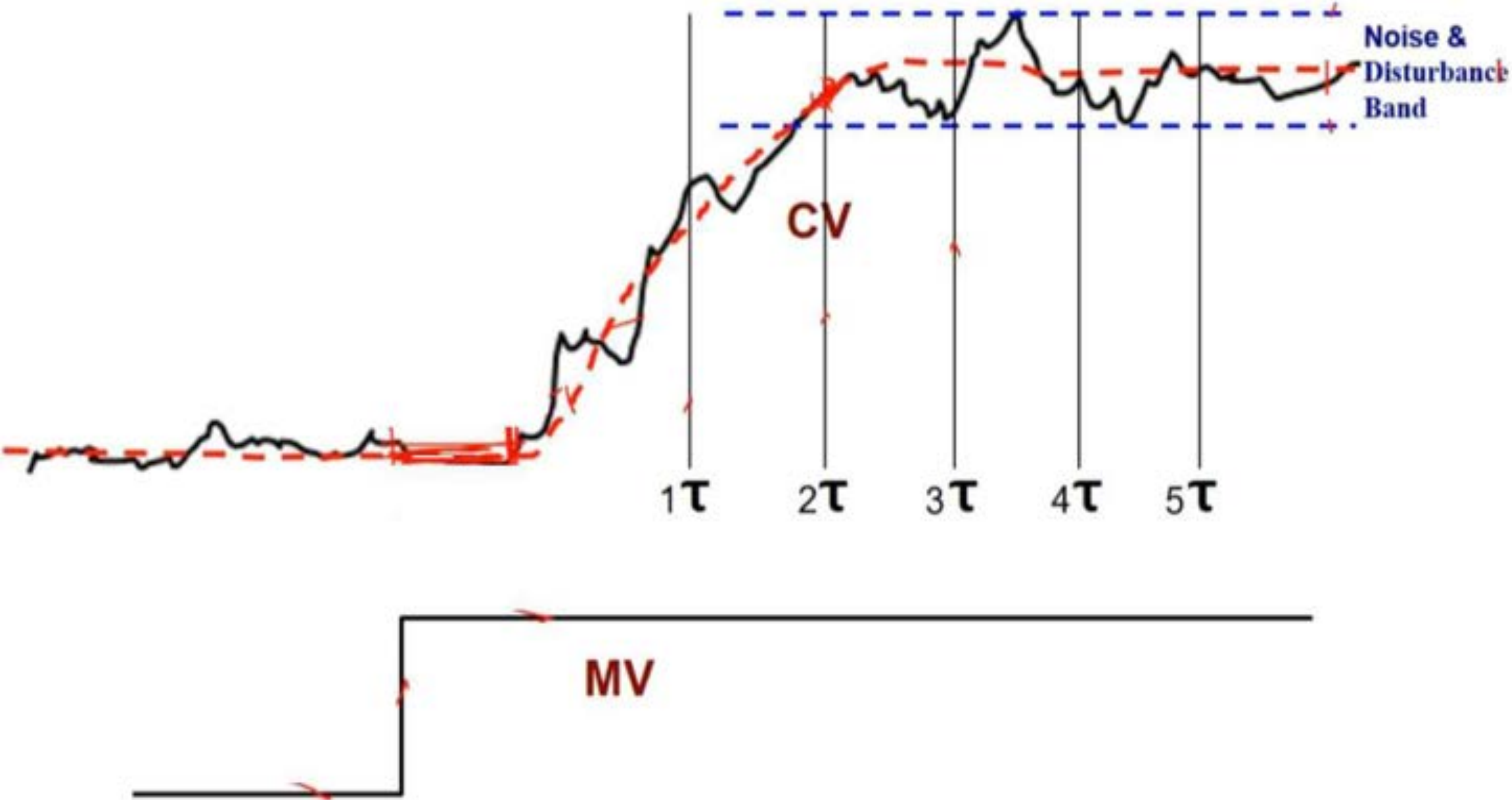
Typical Values of Transfer Function Parameters

	Flow	Pressure	Temperature	Level	Online Analysis	Motor Power	Compressor Surge
Dead Time	3 sec	20 sec	1 min	0.5 min	15 min	2 sec	5 millisec
Time Constant	20 sec	90 sec	20 min	0 min	45 min	10 sec	15 millisec
TTSS	1 min	4 min	1 h	--	2 h	30 sec	1 sec

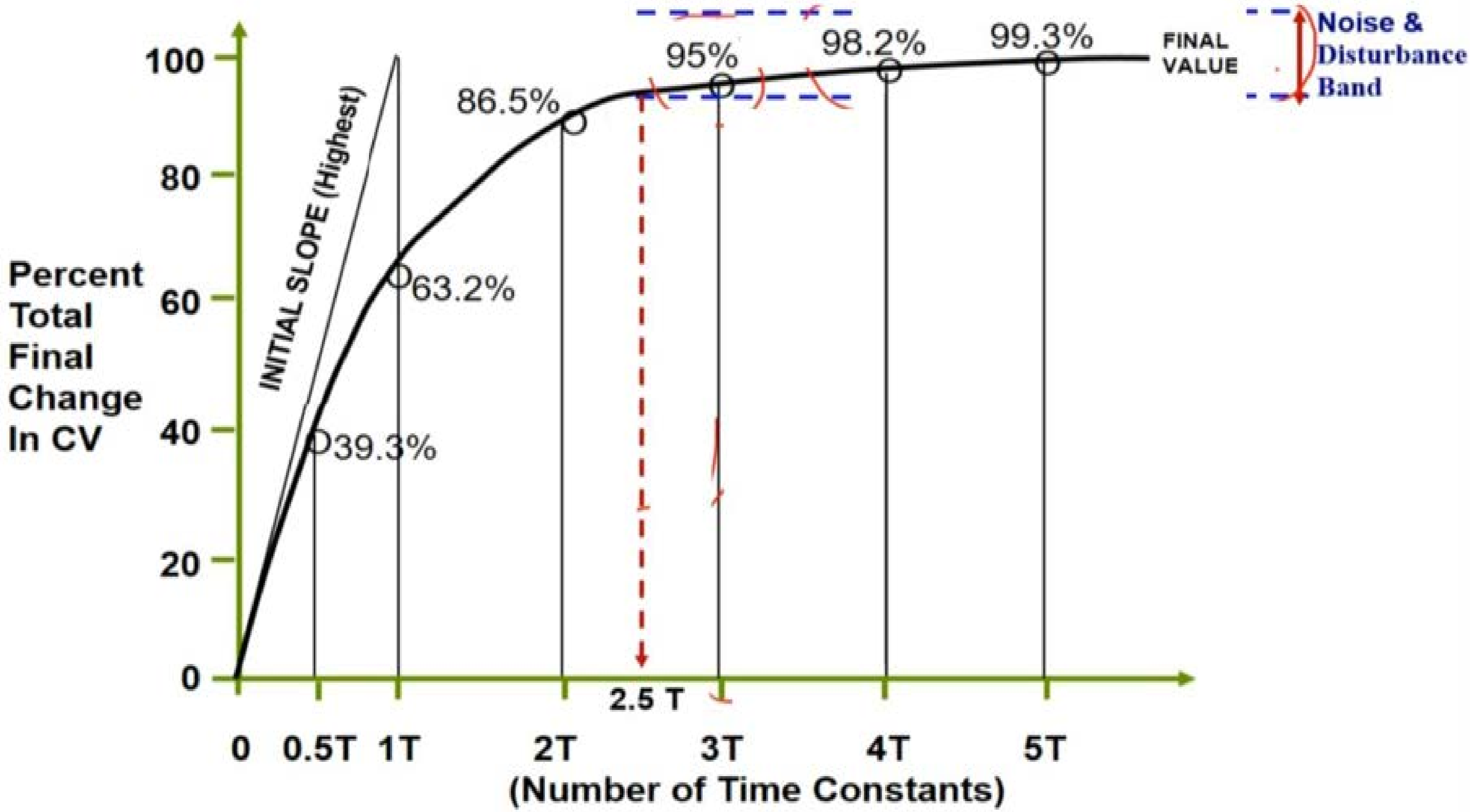
Note: **Process Gain** is **not shown** above because it is dependent on instrument range and engineering units and **cannot be stated generically**.


WELCOME TO THE
REAL WORLD

Industrial Data Showing Transfer Function Dynamics



First Order Time Constants



Transfer Function Format

$X(s)$ MV (Control Valve) → $G(s) = \frac{K}{\tau s + 1}$ → $Y(s)$ CV (Flow)

- Time Domain

$$\tau \frac{dy}{dt} + y = Kx$$

- Laplace Domain

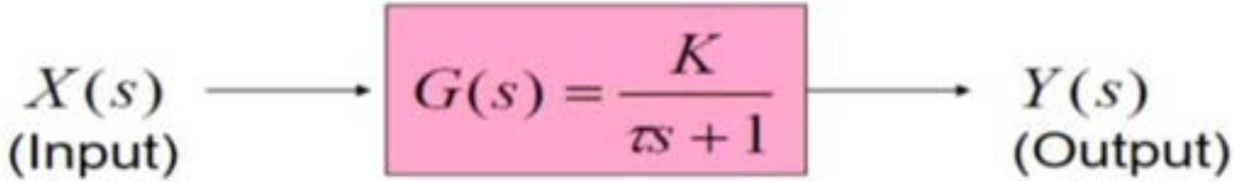
$$\frac{Y(s)}{X(s)} = G(s) = \frac{K}{\tau s + 1}$$

Process Gain

Time Constant

Laplace Operator

Transfer Function Equation
Why Time Constant is 63%



$$\frac{dY}{dt} = K * X$$

$$Y(t) = K * (1 - e^{-t/\tau})$$

If $t = \tau$, then $(1 - e^{-1}) = 0.63$

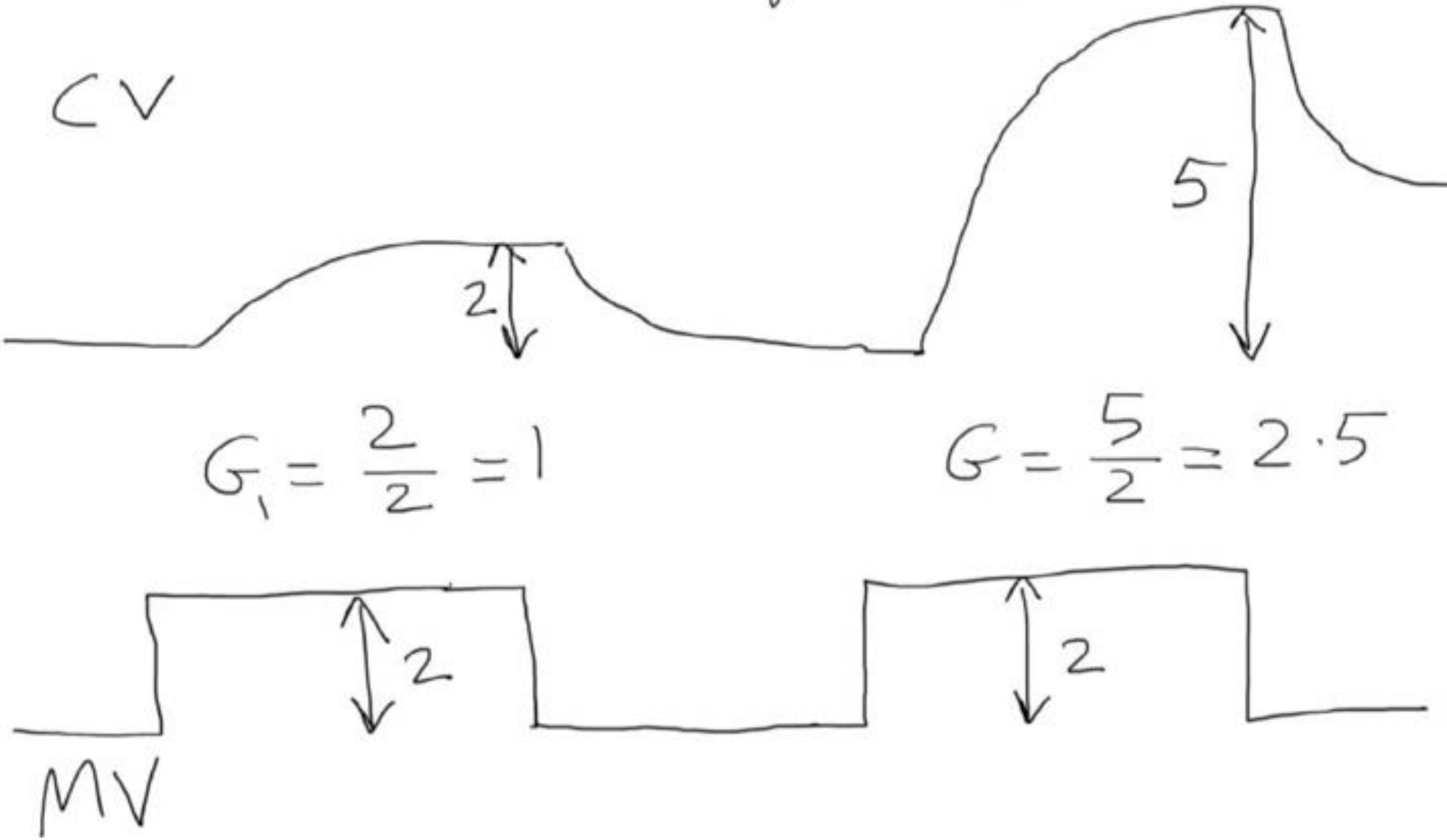
Nonlinear Transfer Functions

- If the transfer function parameters change as a function of process or operating conditions, then the transfer function is said to be Nonlinear.
- Chemical plants always have nonlinear transfer functions.
- Transfer function parameters can change:
  - Change in throughput
  - Change in operating conditions
  - Change in control valve position (<10%, >90%)
- Change in transfer function parameters is called “Nonlinearity” and will cause:
  - Cycling (aggressive or fast control)
  - Sluggish Control (slow control)

Choosing the Most Appropriate Transfer Function Parameters

- If you have CV data on multiple step tests on the MV, chose the transfer function parameters with the:
  - Highest transfer function gain, leading to less aggressive proportional gain,
  - Highest dead time leading to less aggressive proportional gain,
  - Highest integral time leading to less impatient integral action.
- What to do if transfer function parameters change a lot? Say, more than 20%??
- Then use adaptive control, where controller tuning is automatically changed as a function of change in transfer function parameters… Explained later.

Choose the higher gain



$$G_1 = \frac{2}{2} = 1 \qquad G = \frac{5}{2} = 2.5$$

Zero-Order Transfer Functions

- Zero-order transfer functions are also called:
  - Ramp Transfer Functions
  - Integrating Transfer Functions
- On a Unit Step Change in MV, the CV **NEVER** settles to a new steady state.
- Zero-order transfer functions do not have a time constant.
- Number of time constants = **zero**.
- Common example is liquid level in a tank, see example on next slide…

Integrating + Dead-Time Processes

Integrating Model

$$\frac{dy(t)}{dt} = K_i u(t), \qquad G(s) = \frac{K_i}{s}$$

- Output ramps in response to a step.
- Represents accumulation (e.g., tank level, inventories).

With Dead Time

$$G(s) = \frac{K_i e^{-\theta s}}{s}$$

- Real industrial integrating processes always have delays (sensors, transport, communication).
- For simulation, approximate delay with Padé → 2-state model (lag + integrator).

Industrial hints

- Level control uses PI with anti-windup; often cascaded with flow loop.
- Integrating + delay systems are challenging → use ramp SP, careful integral tuning.
- Long delays: Smith Predictor or MPC preferred.

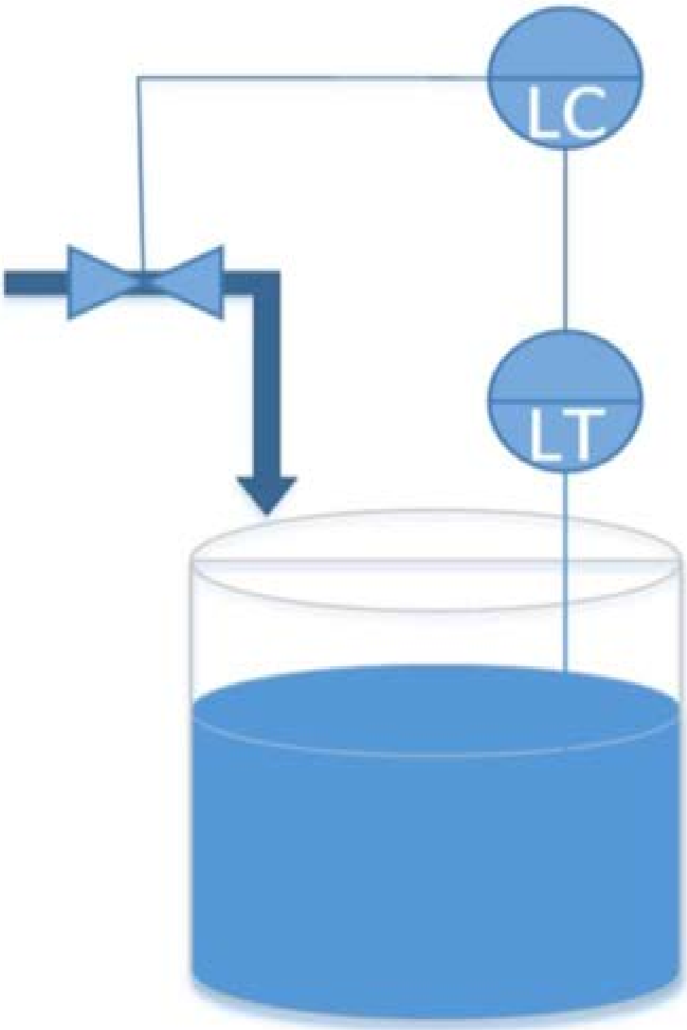
B. Tank Level (Integrating + Delay)

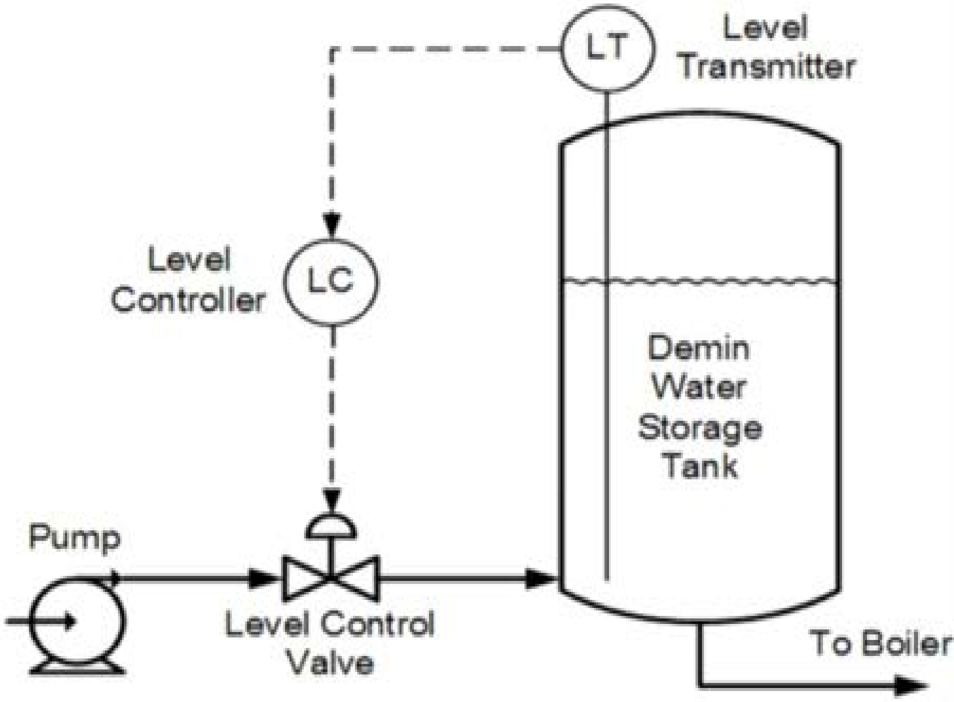
$$\dot{h}(t) = \frac{1}{A}\left(q_{in}(t-\theta) - q_{out}(t)\right) \approx \frac{K_i e^{-\theta s}}{s}, \quad K_i = \frac{1}{A}.$$

- MV: inlet valve position (affects q_{in})
- CV: level h
- **Use**: PI with anti-windup; cascade inner flow loop if available.

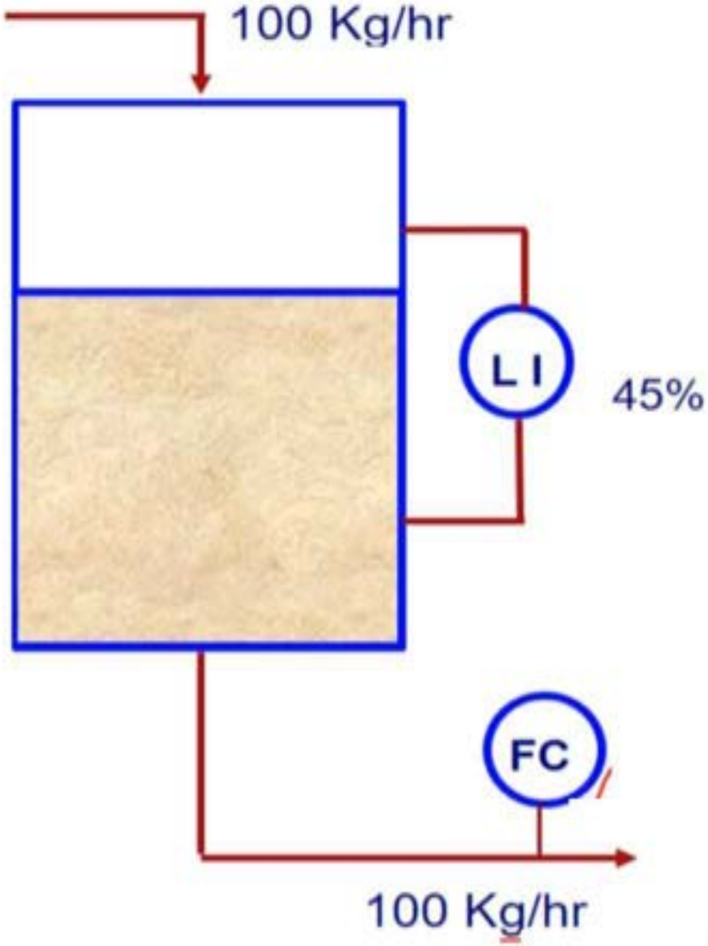
Tank Level Control Dynamics

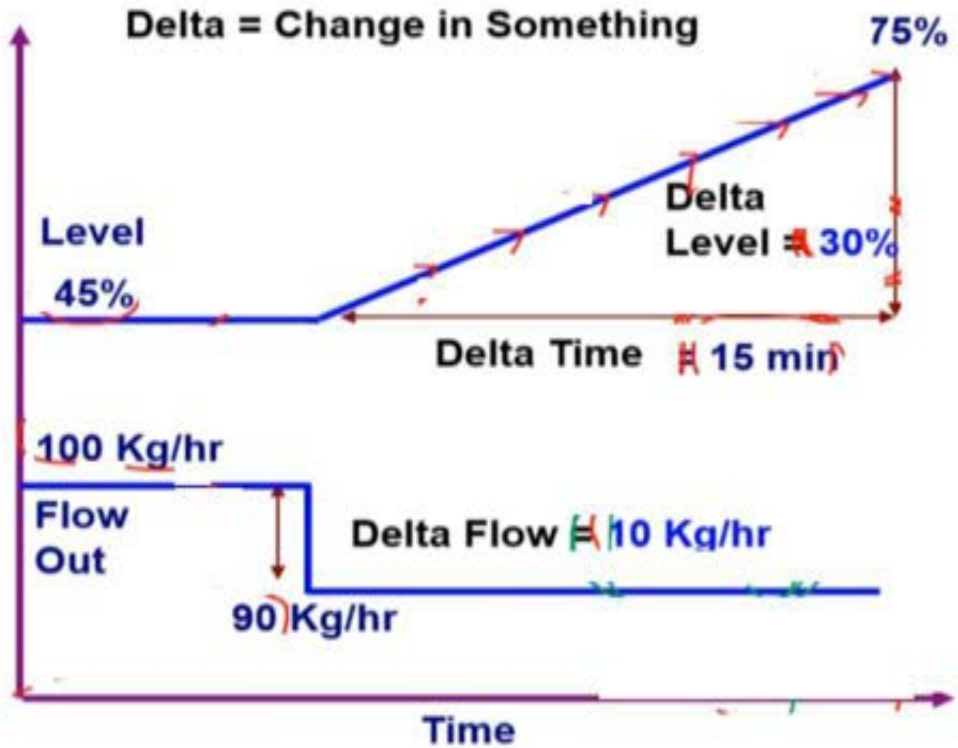
Common Zero-Order Transfer Function Example





Zero Order Transfer Function





$$\text{Ramp Rate} = \frac{\text{Delta Level / Delta Time}}{\text{Delta Flow}} = \frac{30 / 15}{10}$$

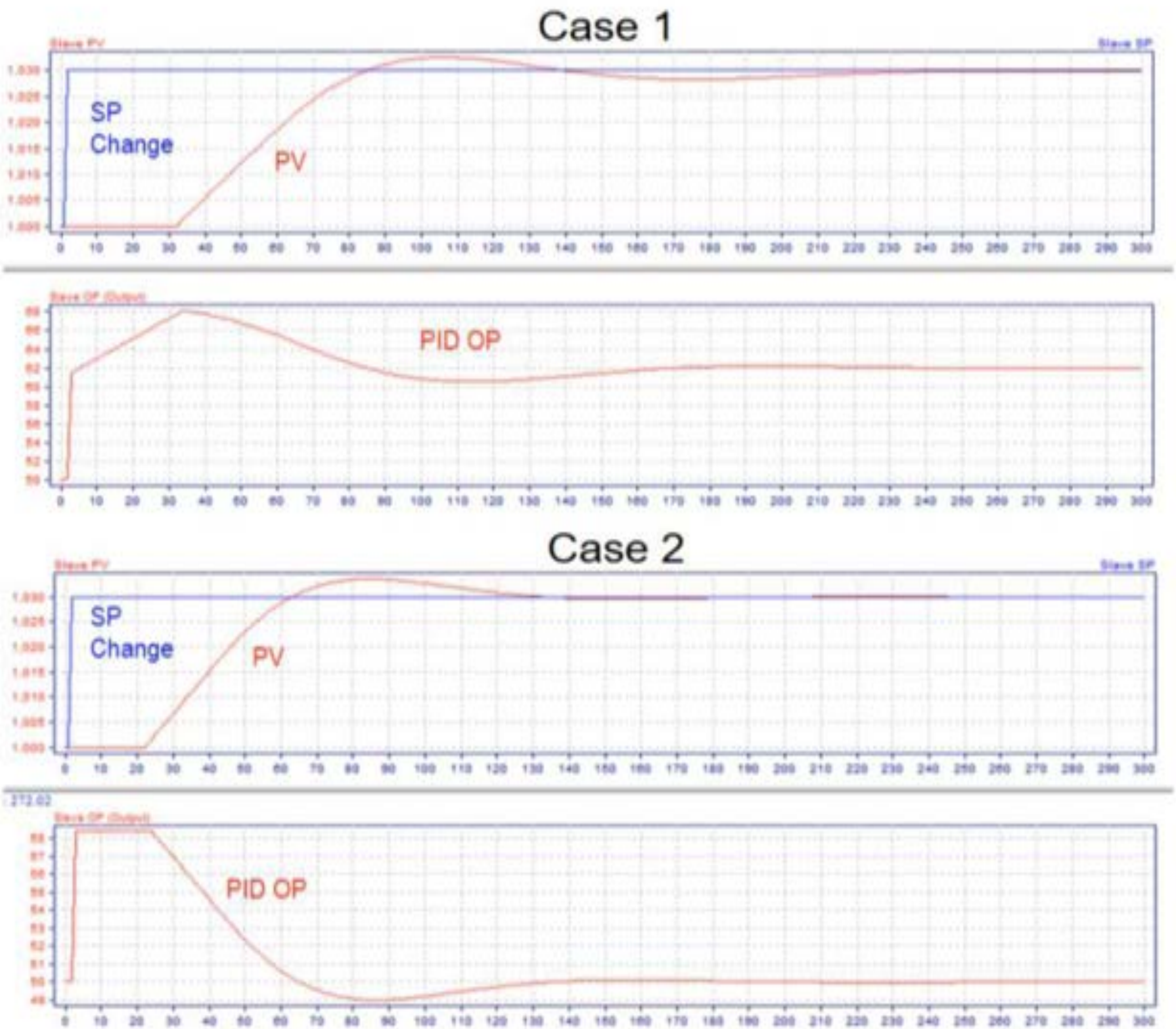
Process Gain = Ramp Rate = 0.2 (%/min/(Kg/hr))

Note: Gain and Dead Time define a zero order transfer function (Zero order transfer function does not have a time constant)

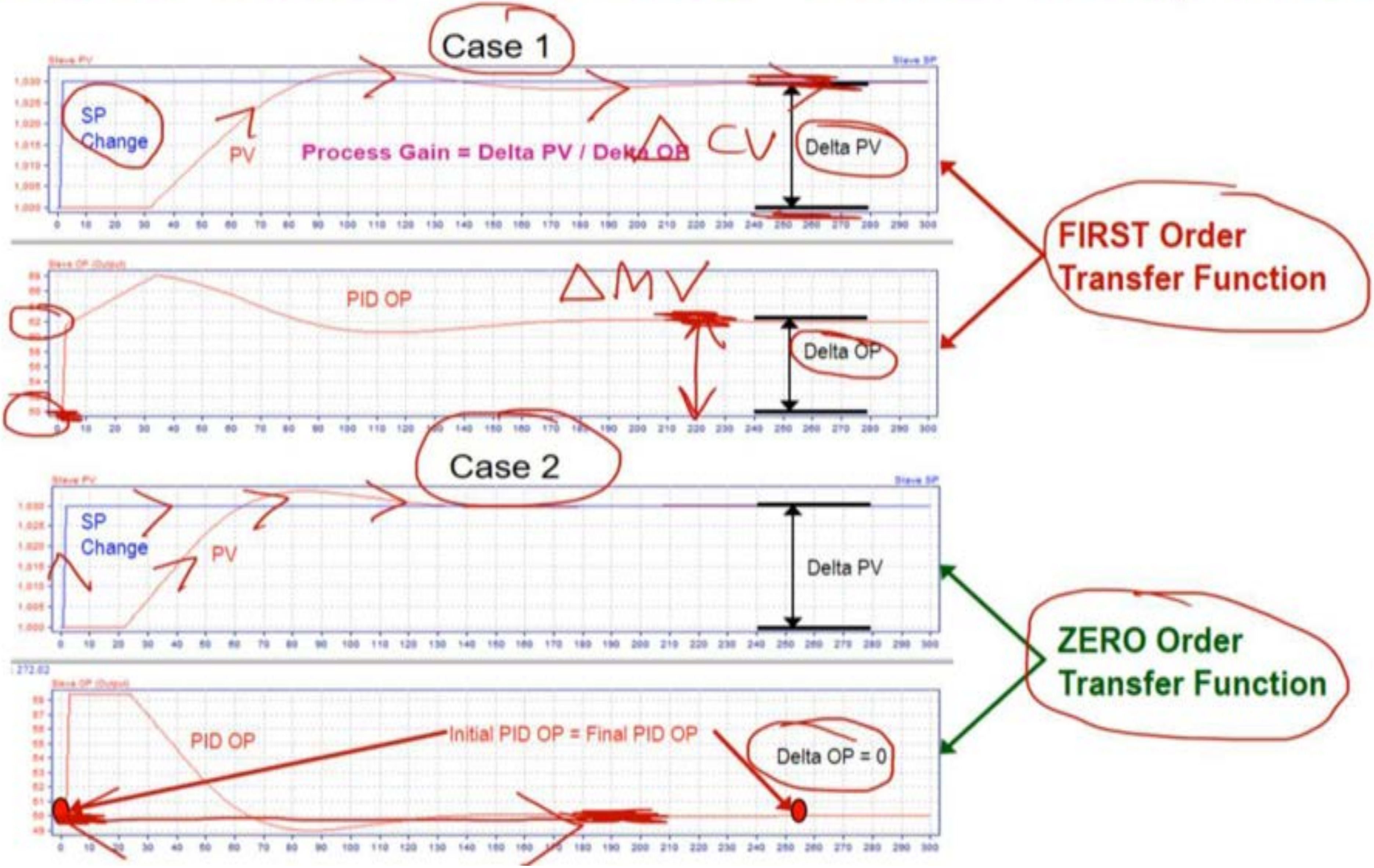
Uniqueness of Zero-Order Transfer Functions

- No time constants.
- Process gain of Zero-Order Transfer Function is the Ramp Rate.
- Process Gain of Zero-Order transfer function has unit of time (e.g. - %level/%valve position/time)
- Process Gain of First and Higher Order Transfer Functions do NOT have time unit.

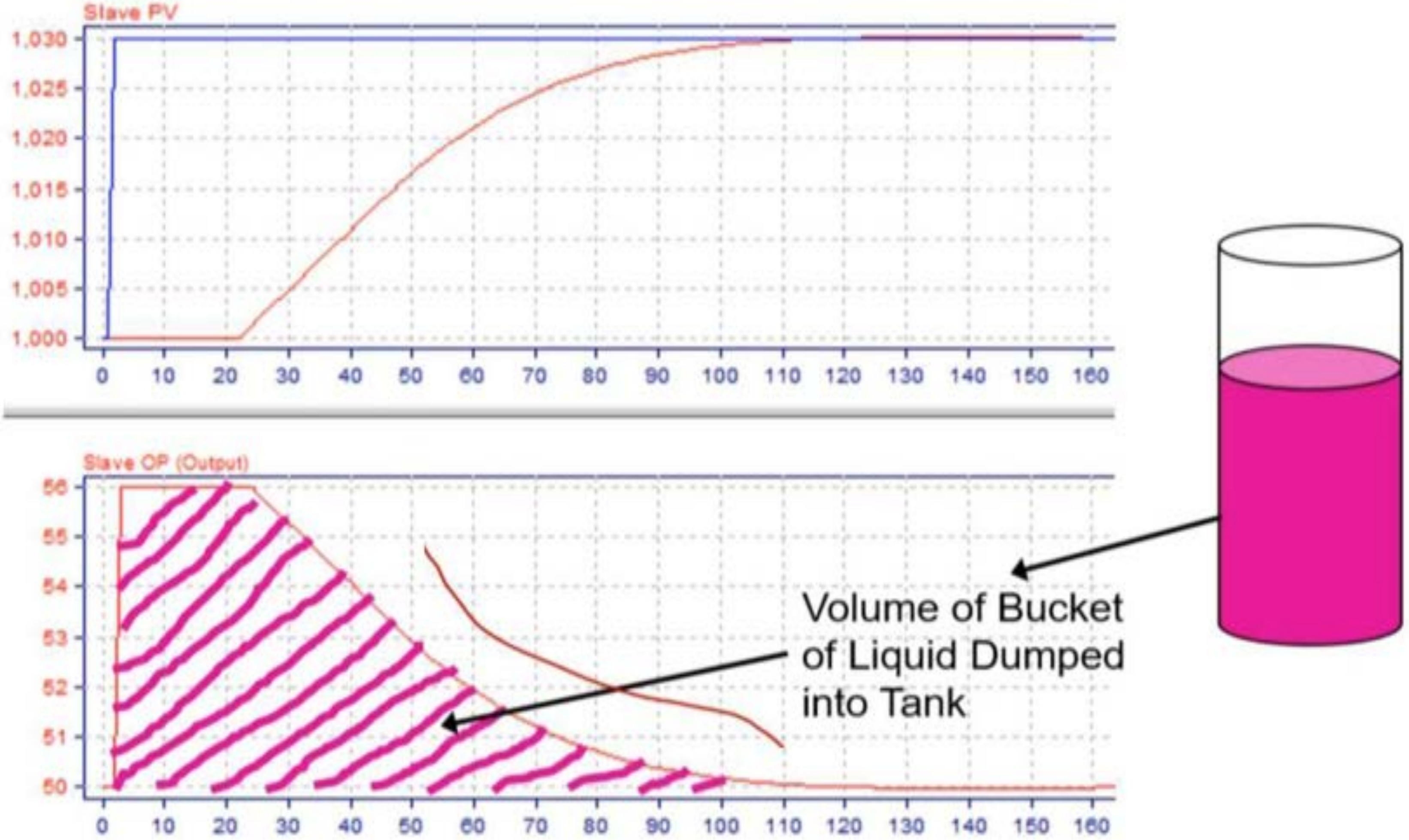
First-Order and Zero-Order Response



First-Order and Zero-Order Response



Zero-Order Transfer Function: The Concept of dumping buckets of liquid



Springs and Second Order Transfer Function

$$\frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

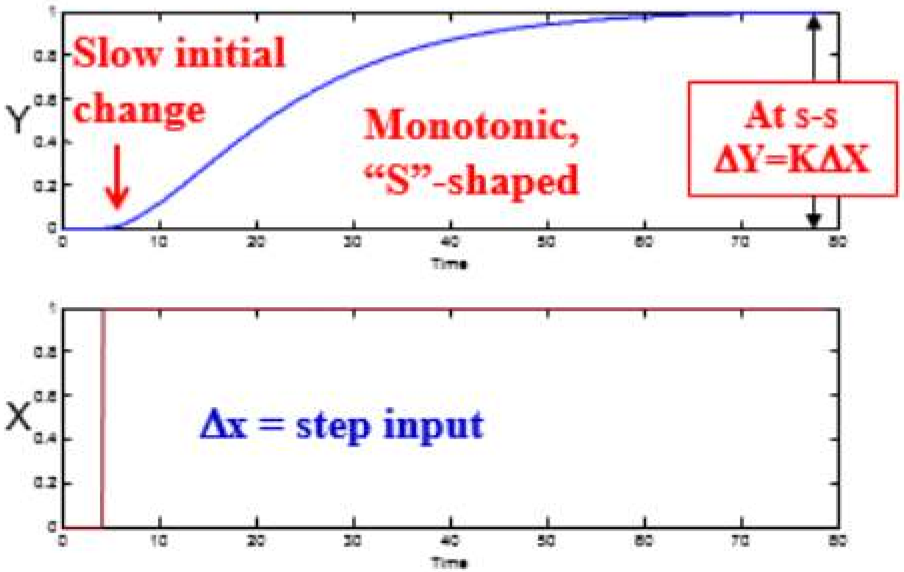
SIMPLE PROCESS SYSTEMS: 2nd ORDER

$$\tau^2 \frac{d^2Y(t)}{dt^2} + 2\xi\tau \frac{dY(t)}{dt} + Y(t) = K_p\ X(t)$$

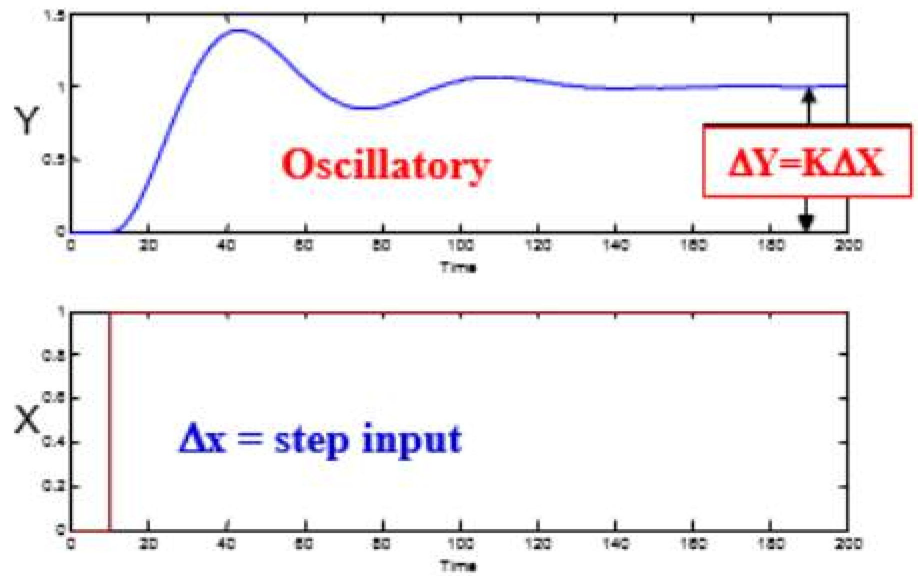
Would this be easy/difficult to control?

K_P = s-s gain , τ = time constant , ξ = damping coefficient

Overdamped (ξ>1)



Underdamped (ξ<1)



Second-Order and Underdamped Models

Model Equations

$$\tau_1\tau_2\frac{d^2y}{dt^2} + (\tau_1 + \tau_2)\frac{dy}{dt} + y = K_p u(t)$$

$$G(s) = \frac{K_p}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- Arises from two interacting lags, typical in multistage processes.
- Under low damping, shows oscillatory response.
- Adding transport delay yields SOPDT with dead time.

Second Order System

Second order system is also called quadratic lag system. The dynamics of the system in time-domain is given by a second order differential equation, as below:

$$\tau^2 \frac{d^2 Y}{dt^2} + 2\zeta\tau \frac{dY}{dt} + y = K_p X(t)$$

The corresponding transfer function is given by

$$G(s) = \frac{Y(s)}{X(s)} = \frac{K_p}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Note: this has to be 1 !

K_p = gain

τ = natural period of oscillation

ζ = damping factor (zeta)

Note: τ, $\zeta > 0$. They can't take −ve values.

Damping Factor

The characteristic equation of second order system is given by

$$\tau^2 s^2 + 2\zeta\tau s + 1 = 0$$

If	$\zeta < 1$	underdamped system, roots are complex
	$\zeta = 1$	critically damped system, real and equal roots
	$\zeta > 1$	overdamped system, roots are real
	$\zeta = 0$	undamped system, complex roots with zero real part

✅ Second-Order Plus Dead-Time (SOPDT) Model

$$G(s) = \frac{K_p\, e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

where:

- K_p = process gain
- τ_1, τ_2 = two dominant time constants
- θ = dead time (transport delay, sensor lag, actuator delay)

✅ Corresponding Differential Equation

If we write the **non-delayed part** as usual:

$$\tau_1 \tau_2 \frac{d^2 y(t)}{dt^2} + (\tau_1 + \tau_2)\frac{dy(t)}{dt} + y(t) = K_p\, u_d(t)$$

where

$$u_d(t) = u(t - \theta)$$

represents the **delayed input signal**.

So the **dead time is not inside the differential operator**, but acts as a **time shift** on the input.

✅ Why it matters

- In real industrial systems (e.g., **steam drum pressure**, **multi-tank processes**, **compressor–receiver systems**), both **second-order dynamics** and **dead time** are present.
- Dead time θ represents **transport, sensor, or actuation delays**, often significant compared to time constants.
- For **simulation and state-space models**, engineers often **approximate the delay** using Padé expansion:

$$e^{-\theta s} \approx \frac{1 - \frac{\theta}{2}s}{1 + \frac{\theta}{2}s}$$

multiplying it with the SOP denominator → gives a **third-order rational transfer function** (2 poles + 1 from Padé).

Industrial relevance

- Steam drum pressure, interacting tank systems, compressor–receiver dynamics.
- Under-damped behavior is common and requires careful tuning.
- Derivative action, cascade loops, or model-based tuning improve performance.

Control implications

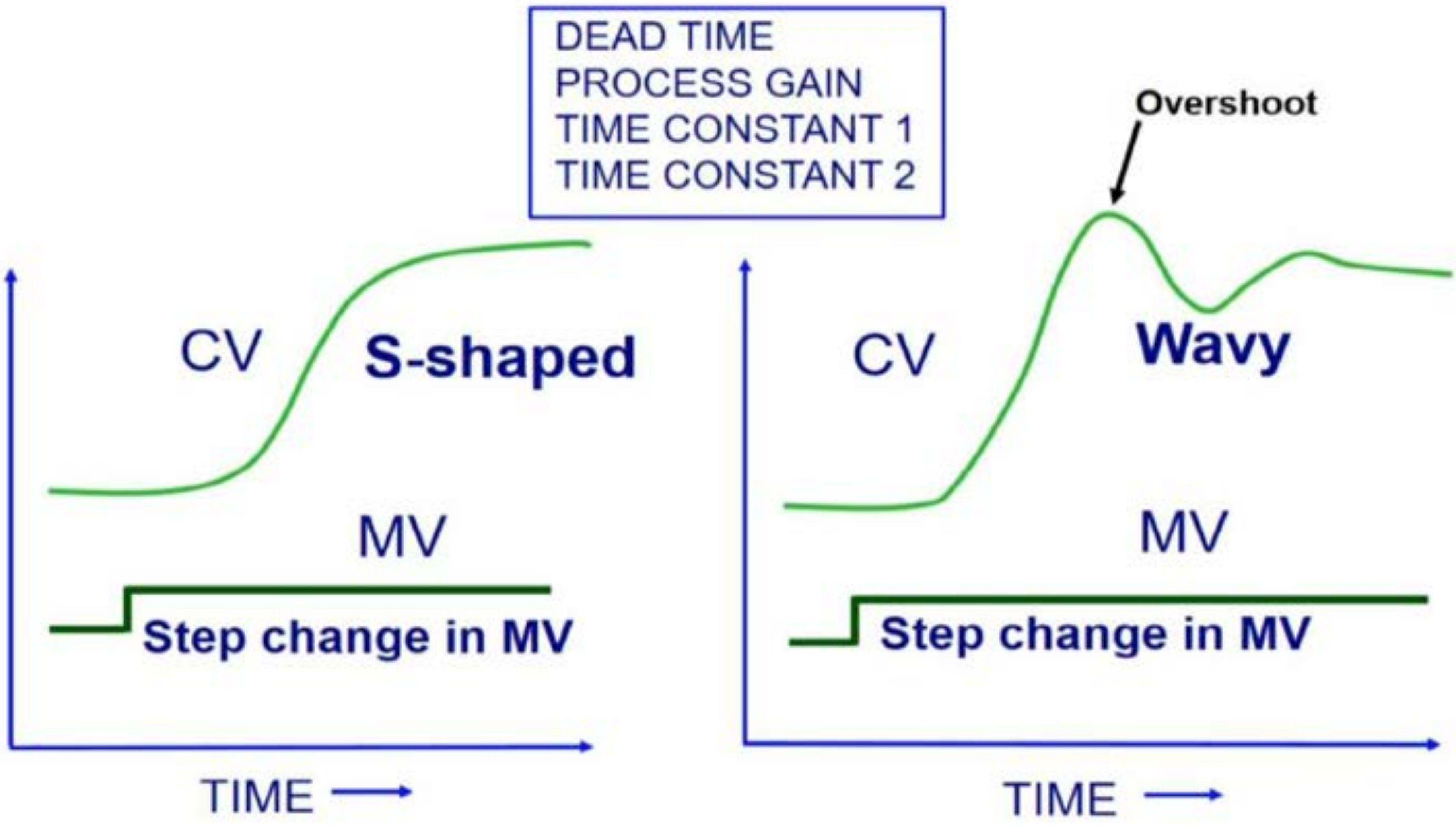
- SOPDT models give more accurate tuning than naive PID.
- Fitting $\tau 1$, $\tau 2$ improves stability margins and disturbance rejection.

C. Steam-Drum Pressure (SOPDT)

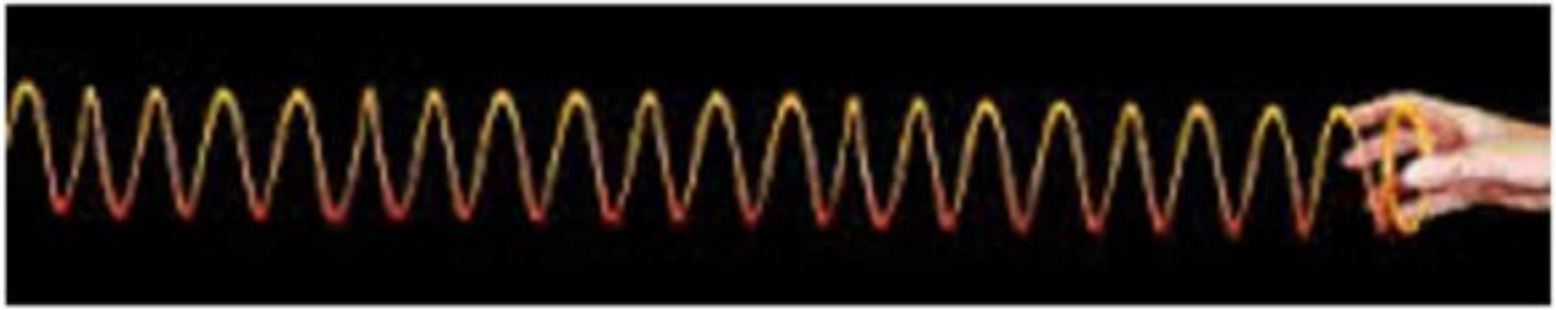
$$\tau_1 \tau_2 \ddot{p} + (\tau_1 + \tau_2)\dot{p} + p = K_p u(t - \theta), \quad G(s) = \frac{K_p e^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}.$$

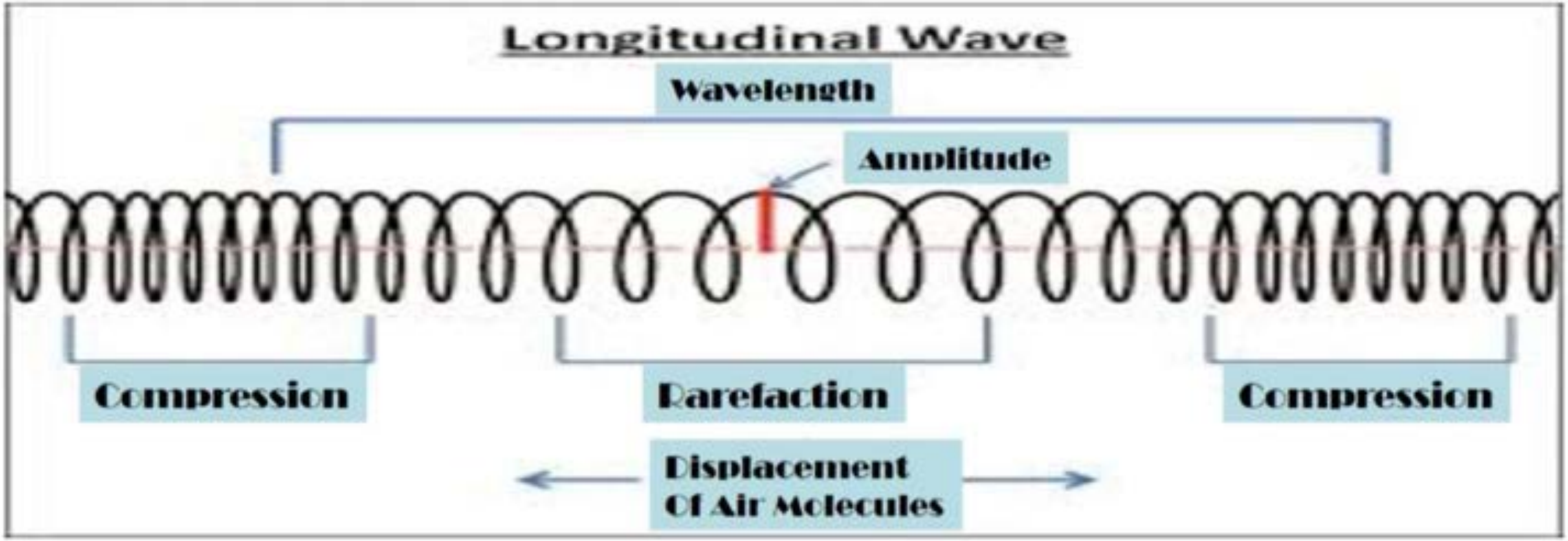
- MV: fuel/air or steam outflow trim
- CV: drum pressure
- **Use**: cascade, feedforward of load, derivative action if underdamped.

Second Order Transfer Function

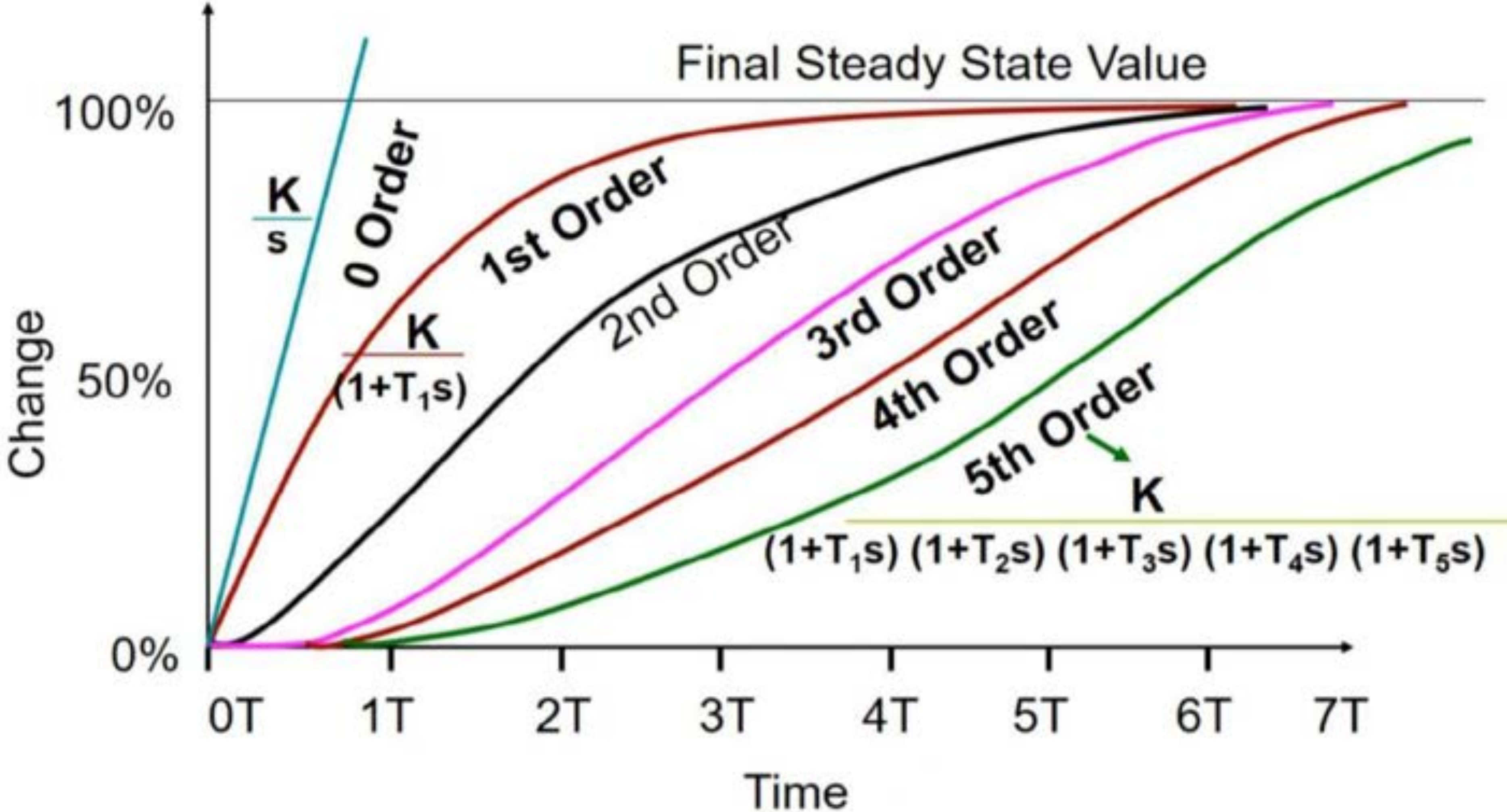


Second Order- Spring Example (Generation of Oscillations)

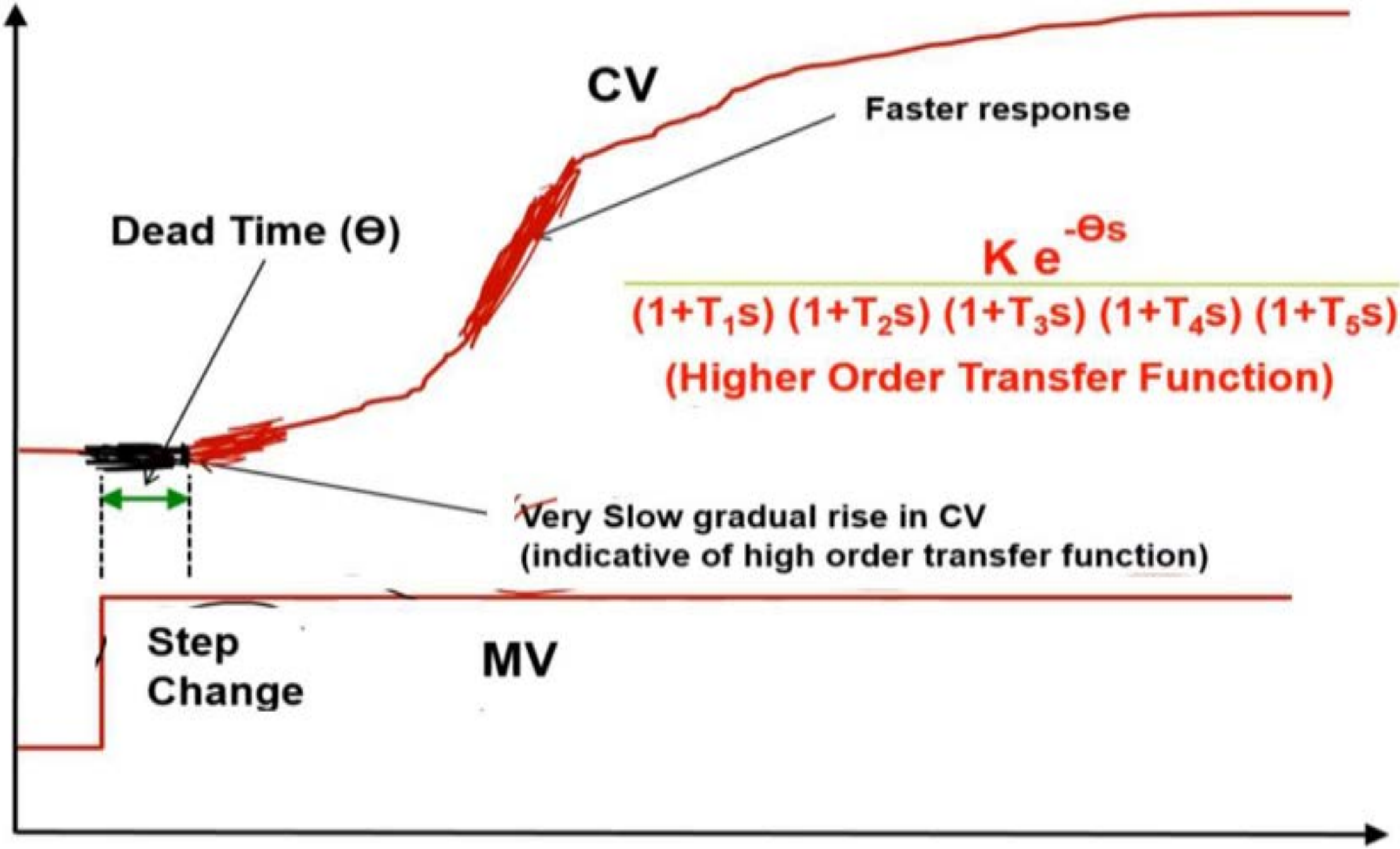




Higher Order Transfer Functions



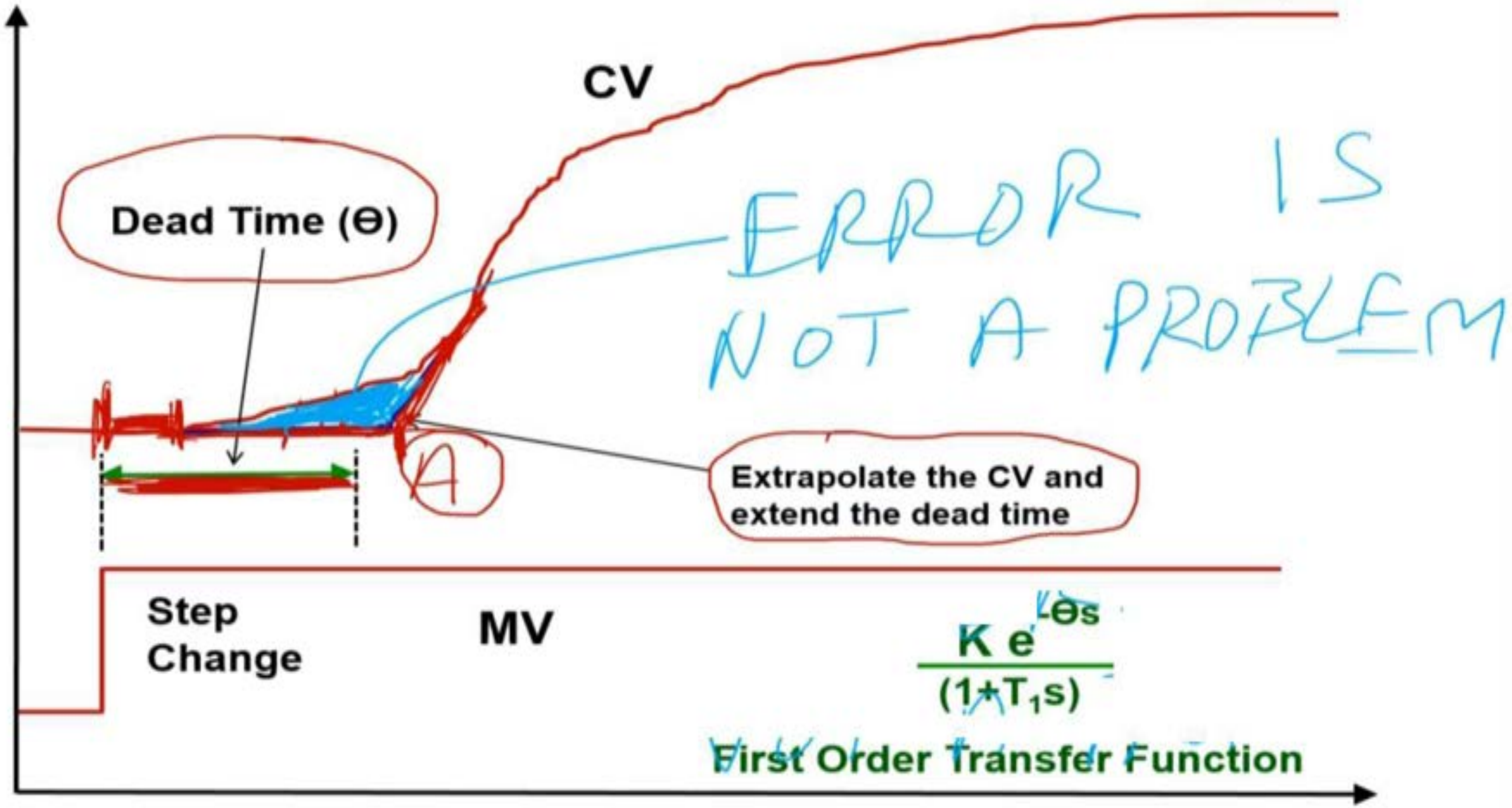
Approximating Higher Order Transfer Functions with Lower Order



$$\frac{K\,e^{-\Theta s}}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)(1+T_5 s)}$$

(Higher Order Transfer Function)

Convert Higher Order Transfer Functions to First Order



State-Space Representations

For monic transfer function:

$$G(s) = \frac{\beta_{n-1}s^{n-1} + \cdots + \beta_0}{s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_0}$$

Controllable Canonical Form

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{n-1} & \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}, \quad C = [\beta_0 \; \beta_1 \; \cdots \; \beta_{n-1}], \quad D = 0$$

Why SS form

- Useful for simulation, observer design, MPC.
- Padé delays fit naturally.
- Standard form in MATLAB / Simulink / DCS.

Industrial Modeling & Tuning Strategies

Process	Typical Model	Behavior	Control Strategy
Flow	First-Order	Fast exponential	Simple PID / IMC
Temperature	FOPDT	Exponential + lag	IMC, conservative PID
Pressure	SOPDT + delay	Oscillatory	Derivative, cascade
Level	Integrating + delay	Ramp	PI, anti-windup, cascade
Pipeline	Delay	Pure shift	Smith Predictor / MPC

Tuning guidelines

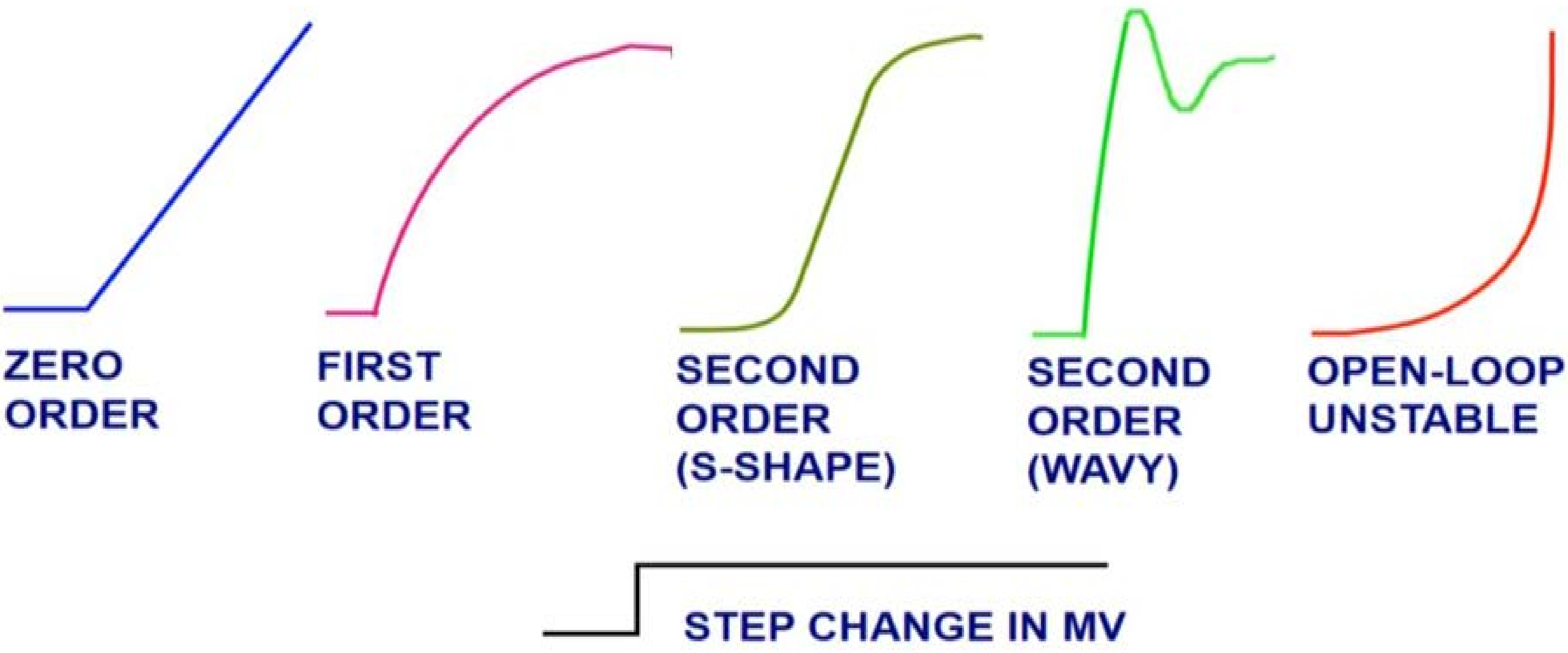
- Identify K_p, τ, θ from step tests.
- IMC tuning is more robust than classical Z–N for large delays.
- SOPDT fits handle oscillatory loops more accurately.

Transfer Functions in Chemical Plants

Types of Industrial Transfer Functions

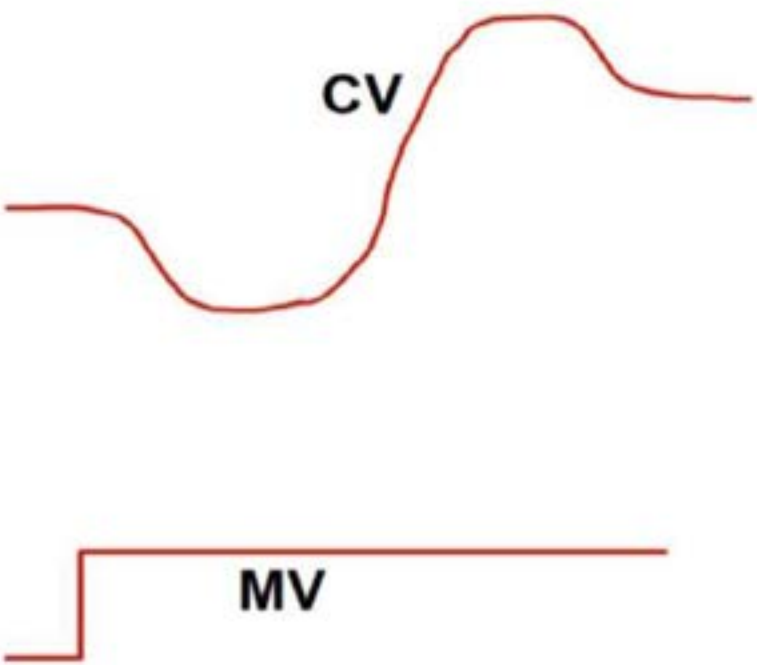
- Zero Order (comprises 20% of all cases)
- First Order (comprises 70% of all cases)
- Second Order (comprises 5% of all cases)
  - "S" shaped
  - Wavy
- Open-Loop Unstable (exothermic reactors)
- Complex (try to fit with first order approximation)

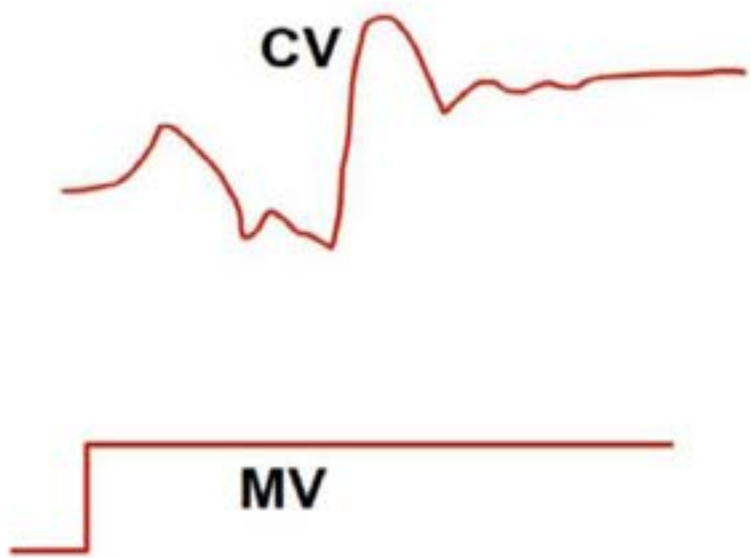
Common Transfer Functions in Plant



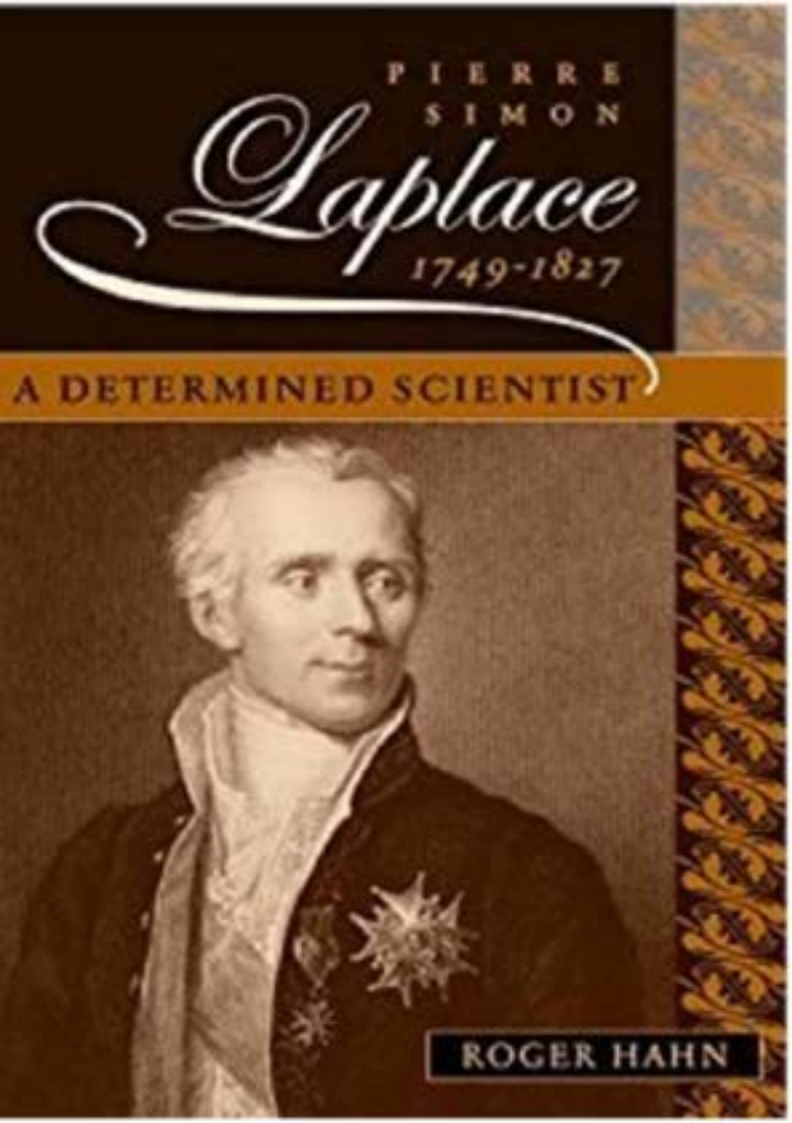
Complex – Quirky Transfer Function

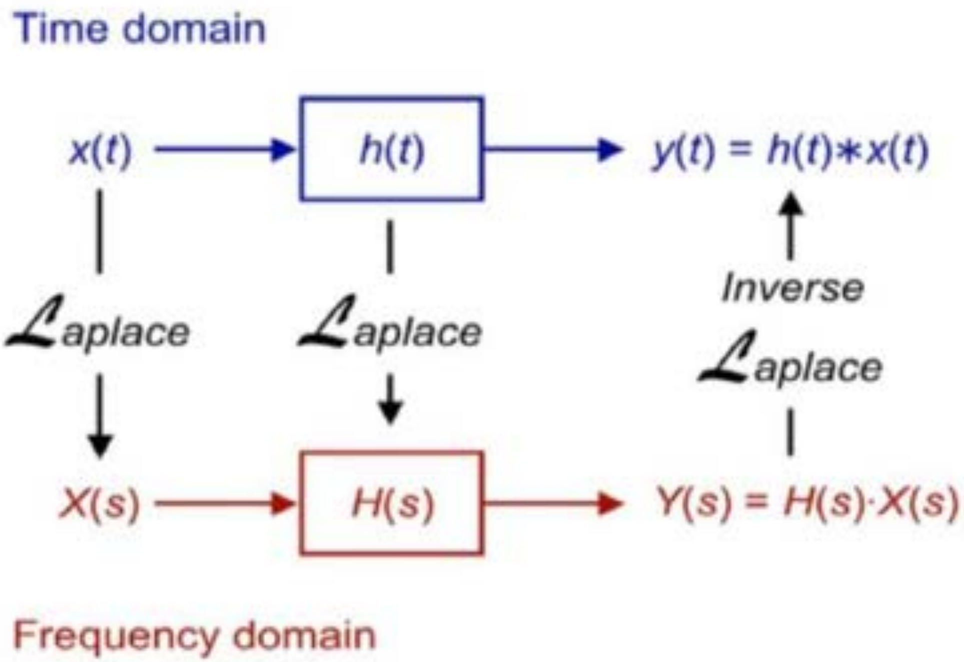
Rare and Ugly Transfer Functions:





Laplace Domain





Transfer Functions in Laplace Domain

A	= Process Gain
T	= Time Constant
$-\theta s$	= Dead Time
s	= Laplace Operator
G	= Transfer Function

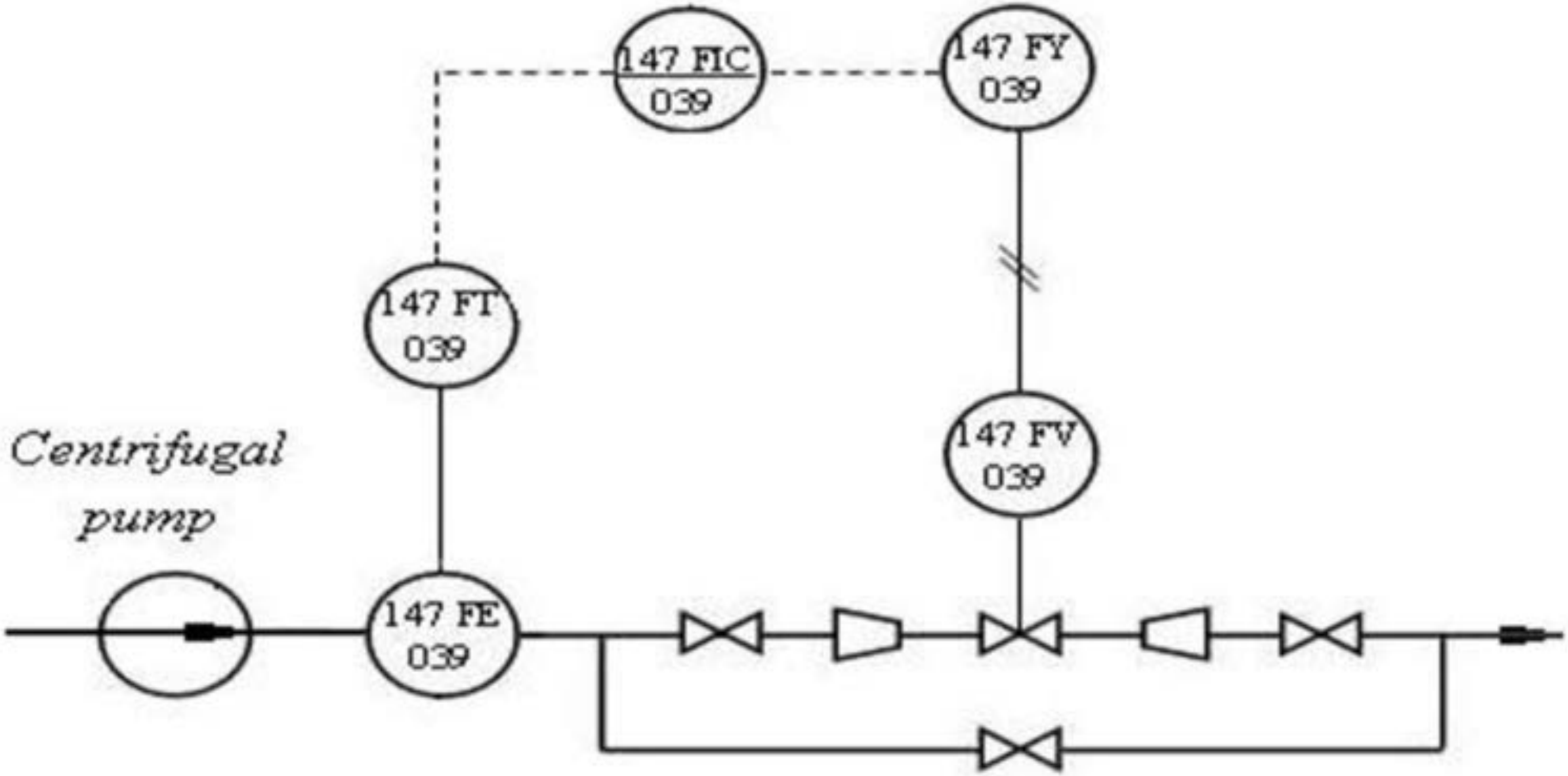
Zero Order
$$G(s) = \frac{A}{s} e^{-\theta s}$$

First Order
$$G(s) = \frac{A}{T_1 s + 1} e^{-\theta s}$$

Second Order
$$G(s) = \frac{A}{(T_1 s + 1)\ (T_2 s + 1)} e^{-\theta s}$$

Third Order
$$G(s) = \frac{A}{(T_1\, s + 1)\,(T_2\, s + 1)\,(T_3\, s + 1)} e^{-\theta s}$$

VALVE TO FLOW TRANSFER FUNCTION

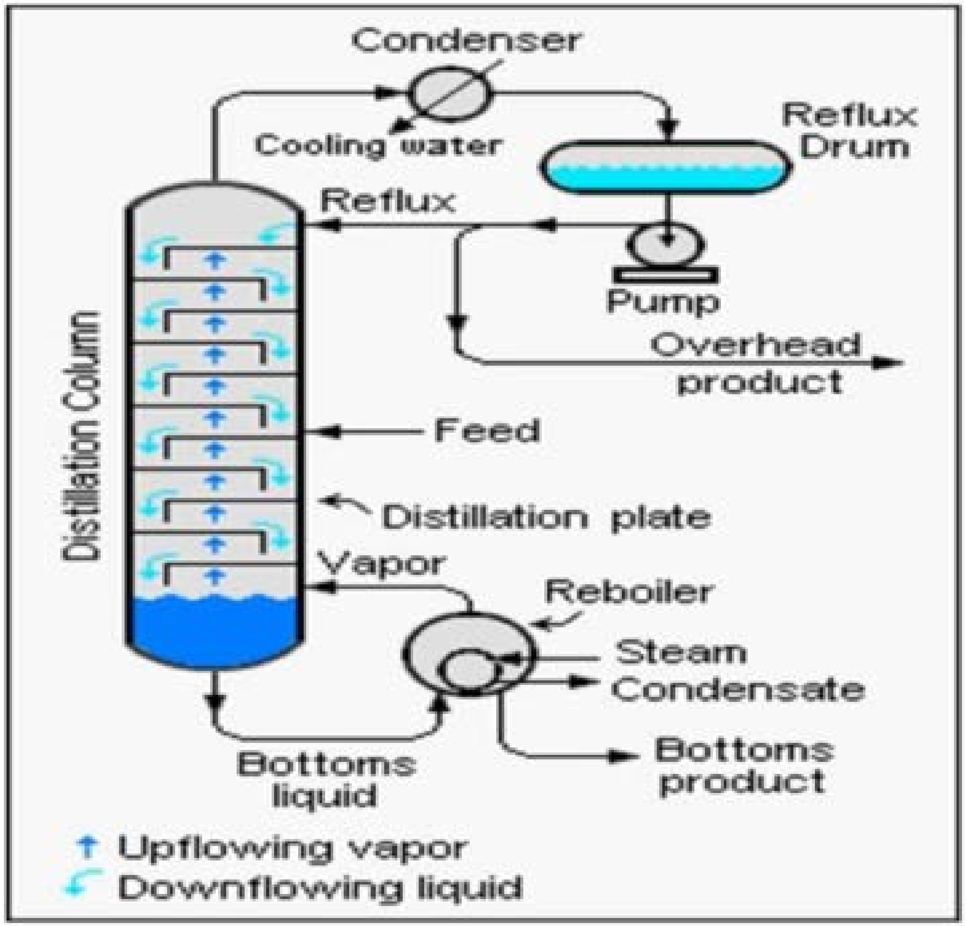


VALVE POSITION **(input signal) (MV)** →

D=5 seconds T=40 seconds G=2.5 (Kg/hr)/%

→ FLOW IN PIPE **(output signal) (CV)**

REFLUX TO PRODUCT PURITY TRANSFER FUNCTION

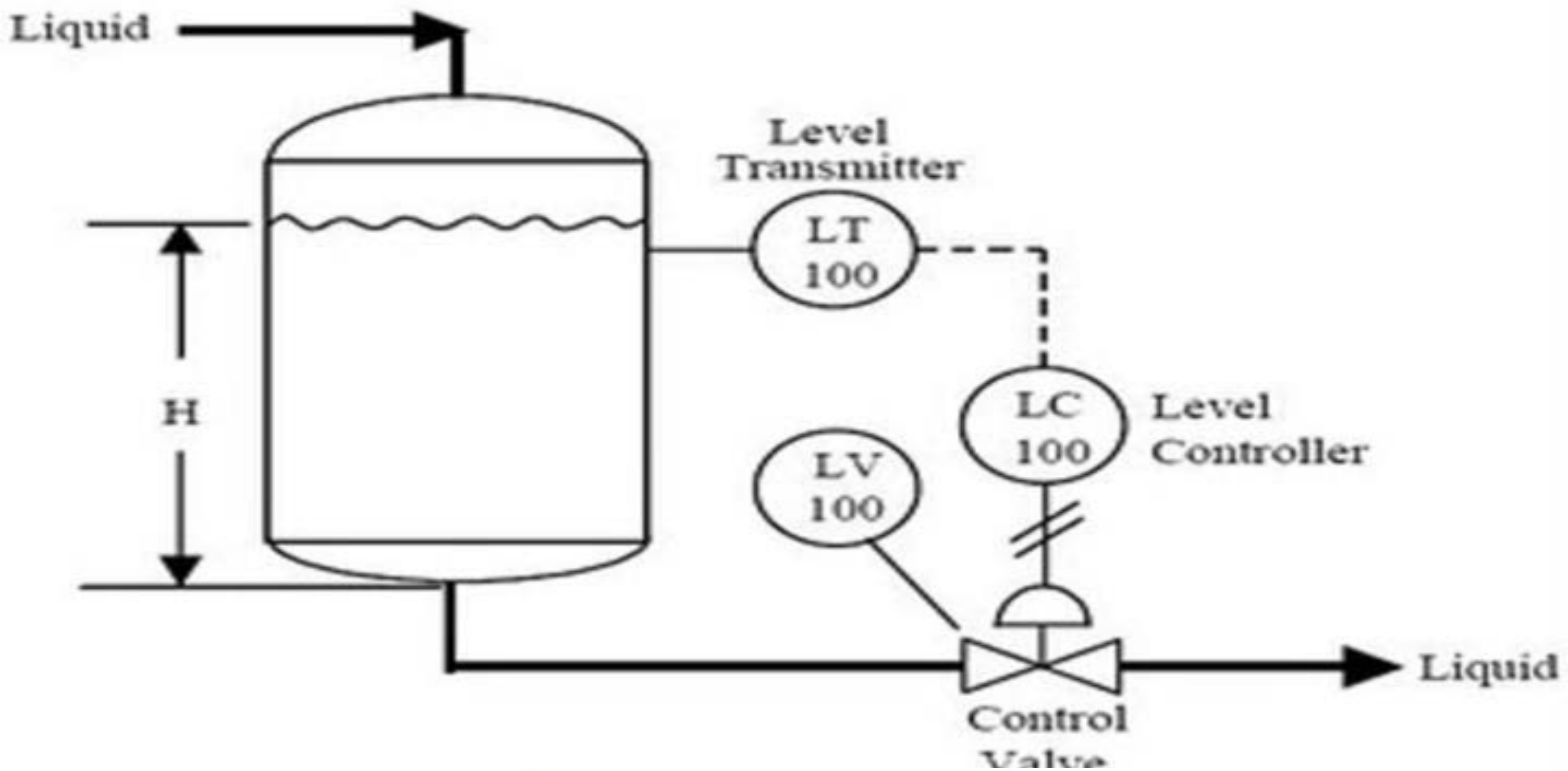


REFLUX FLOW
(input signal)
(MV)

D=8 minutes
T=50 minutes
G= 2 ppm/(Kg/hr)

OVERHEAD PURITY
(output signal)
(CV)

LEVEL TO VALVE TRANSFER FUNCTION



DRUM VALVE
(input signal)
(MV)

→

D=3 minutes
T=0 minutes
G=0.1(%/m)/%

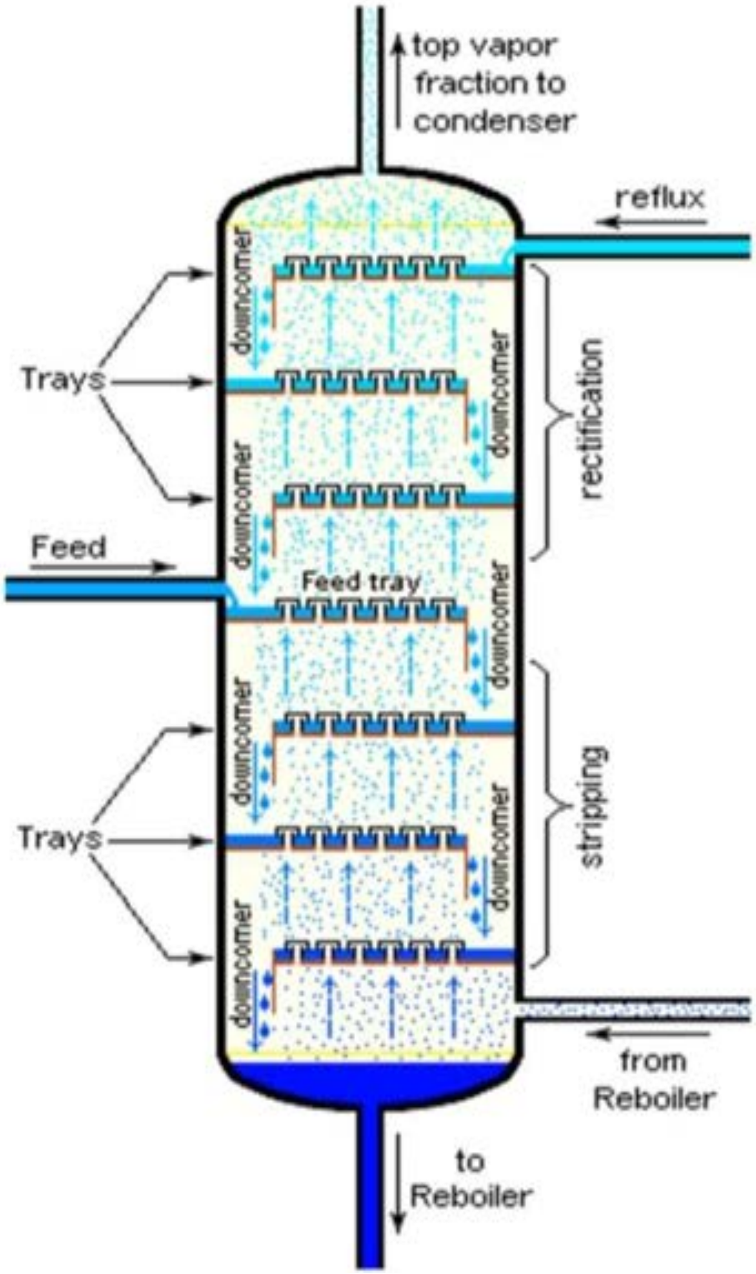
→

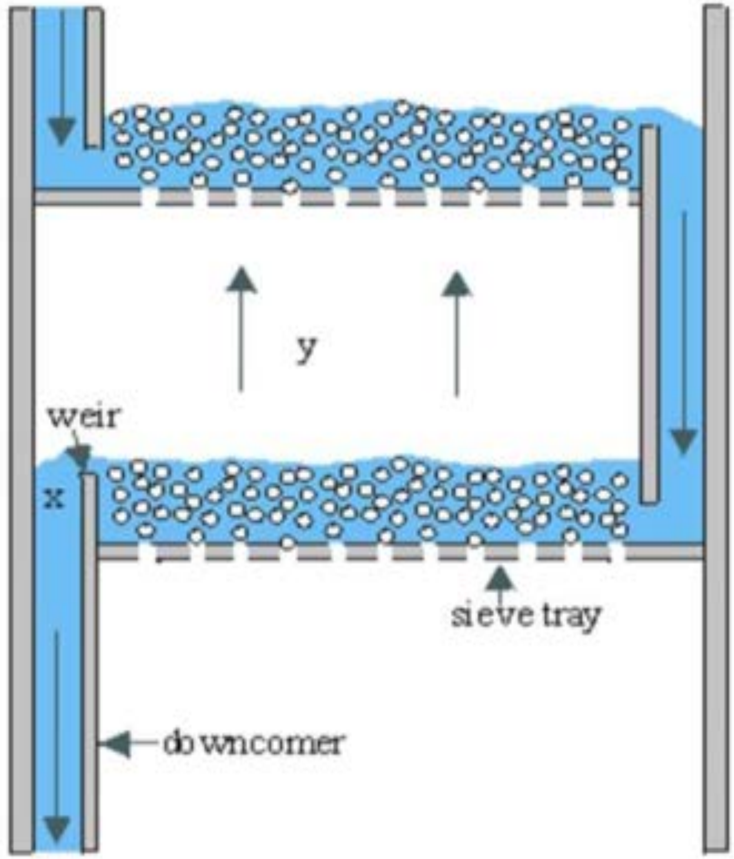
DRUM LEVEL
(output signal)
(CV)

Transfer Functions in Series

- In real life, many transfer functions are connected naturally in series.
- Distillation column – each tray is like a transfer function, so can have 100-200 transfer functions in series.
- Transfer functions in series can be approximated by a single first order or second order transfer function with dead time.
- The time constant in each transfer function in series manifests as overall dead time.

Transfer Functions in Series



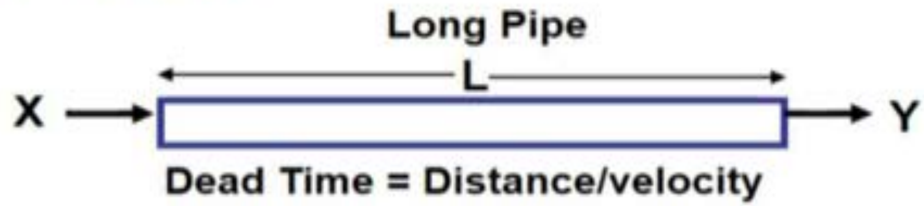


Long and Long Pipelines

Tall Distillation Columns

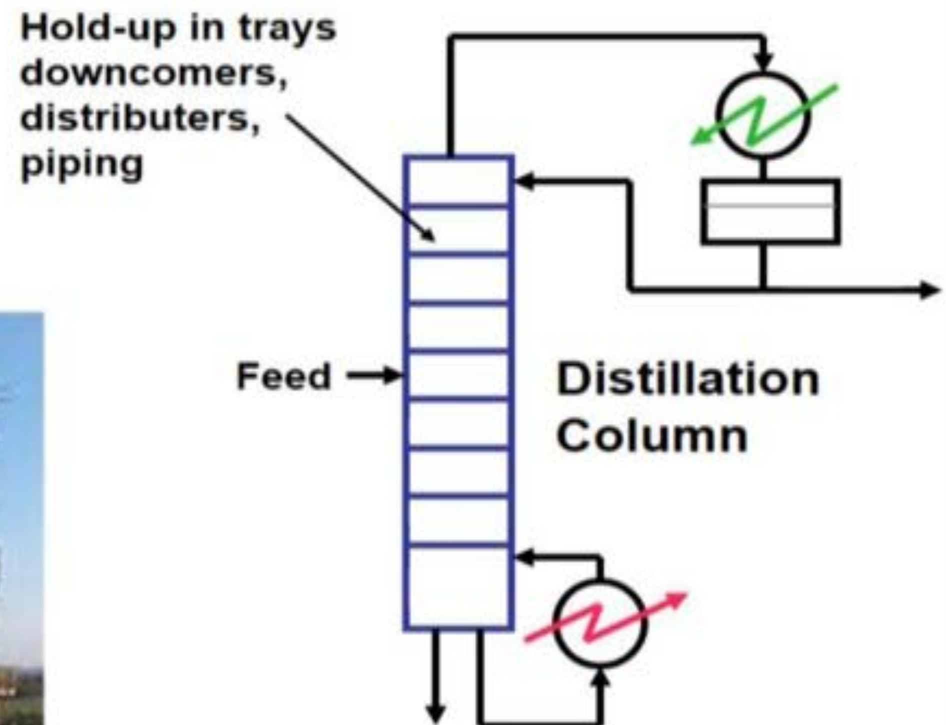
Dead Time

Dead Time is caused by transportation delay.

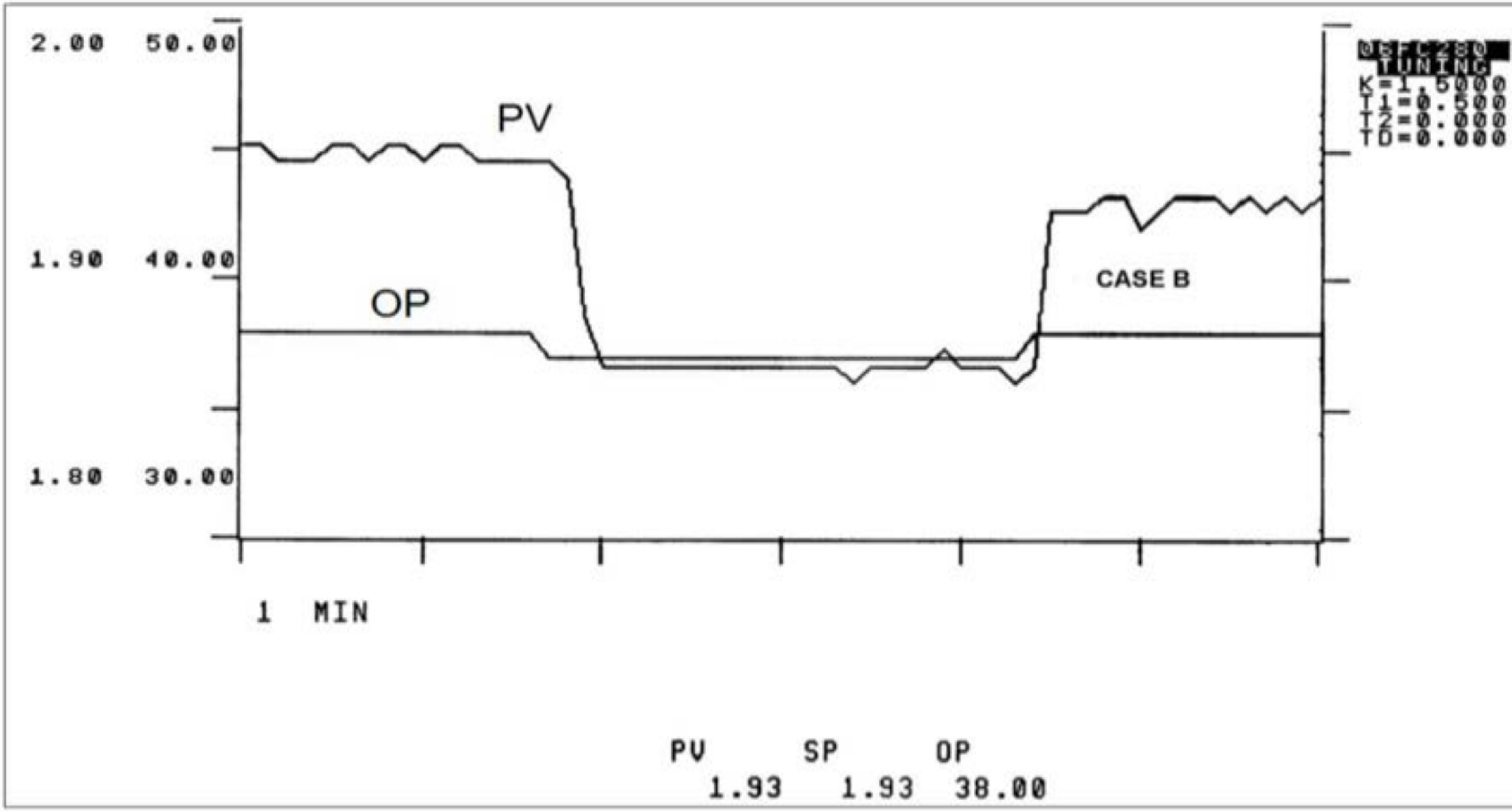


Dead Time is also called:

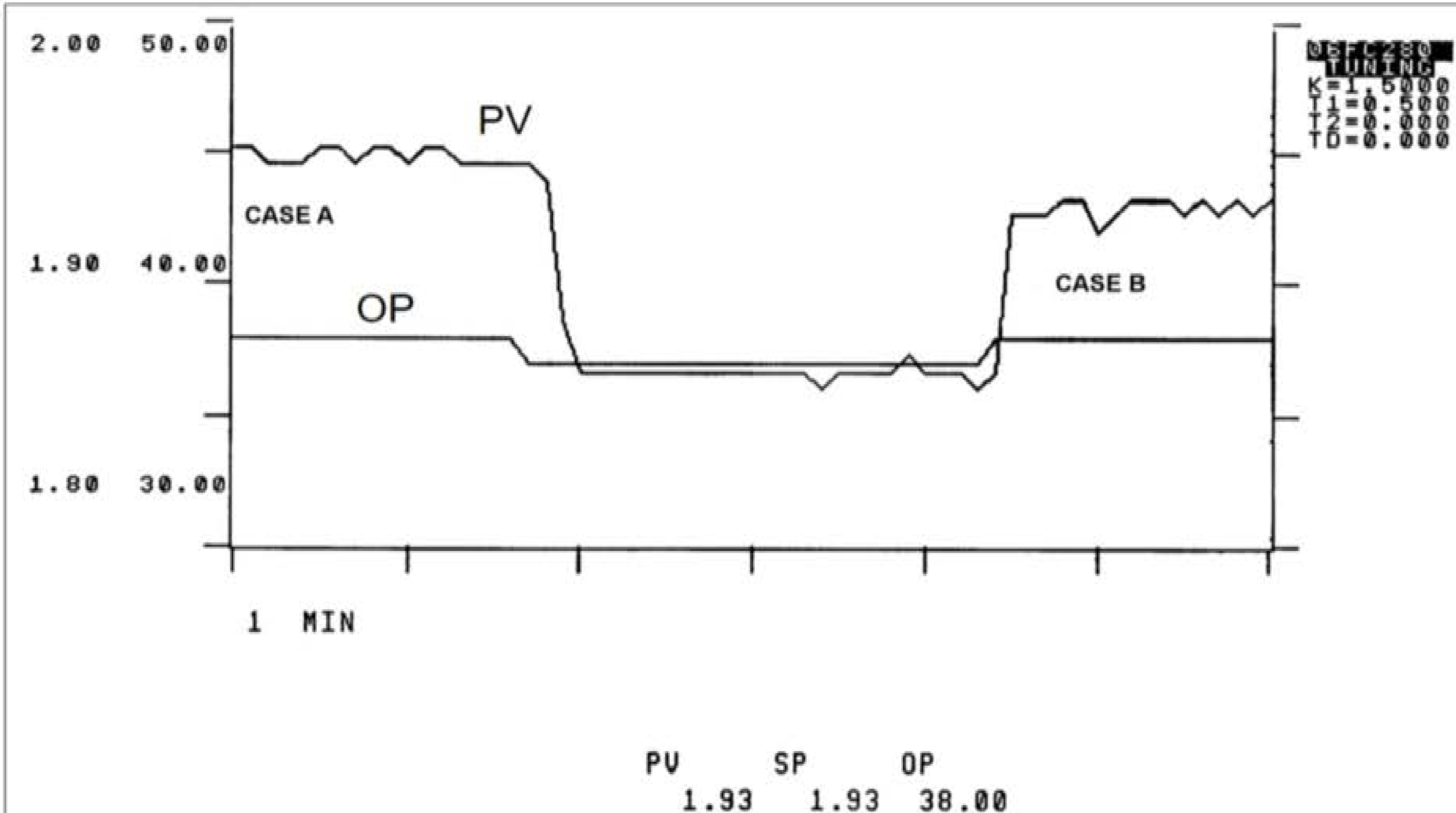
- Delay Time
- Pure Delay
- Time Delay
- Transport Delay
- Distance-Velocity Lag



Open-loop Test to Estimate Transfer Function Parameters



Process Gain Determination



Key Takeaways

- Most industrial processes are modeled as **FOPDT**, **SOPDT + dead time**, or **integrating + dead time**.
- Dead time critically affects stability and must be modeled or compensated.
- Integrating + delay systems need special strategies.
- Step testing & identification are foundational for good tuning.
- Accurate models → better control performance, safety, and efficiency.

Process Dynamics
Transfer Functions
Advanced Concepts

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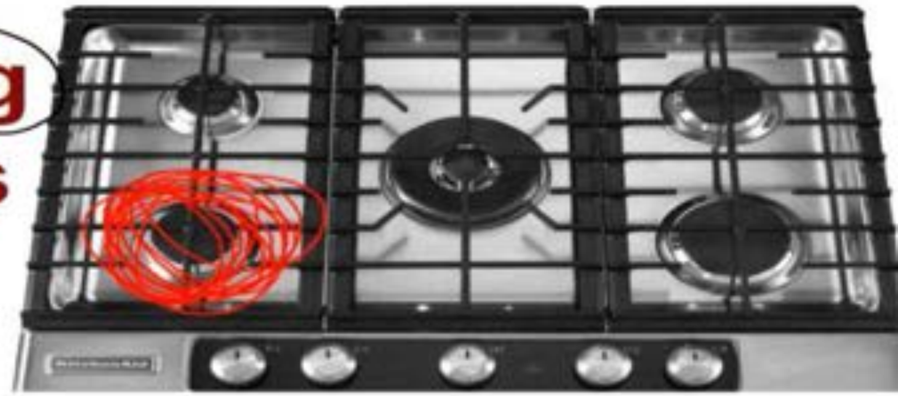
16th-Oct-2025

System Identification

- System identification involves finding the dynamic relationship between a pair of variables.
- Unlike the electric light switch example, there is dead time and time constant (inertia or lag) in the system.
- The dynamic relationship between a pair of variables can be can be slow, nonlinear and complex.
- System identification involves analyzing data from the plant, process, hospital etc. and then determining the transfer function parameters.

Boiling Water in Big & Small Containers



When we put a small container on kitchen stove and turn the heat on, it takes maybe ***10 seconds*** for water to start warming up and then may be ***2 minutes*** for water to start boiling.

When we put a large container on kitchen stove and turn the heat on, it takes maybe ***30 seconds*** for water to start warming up and then may be ***7 minutes*** for water to start boiling.

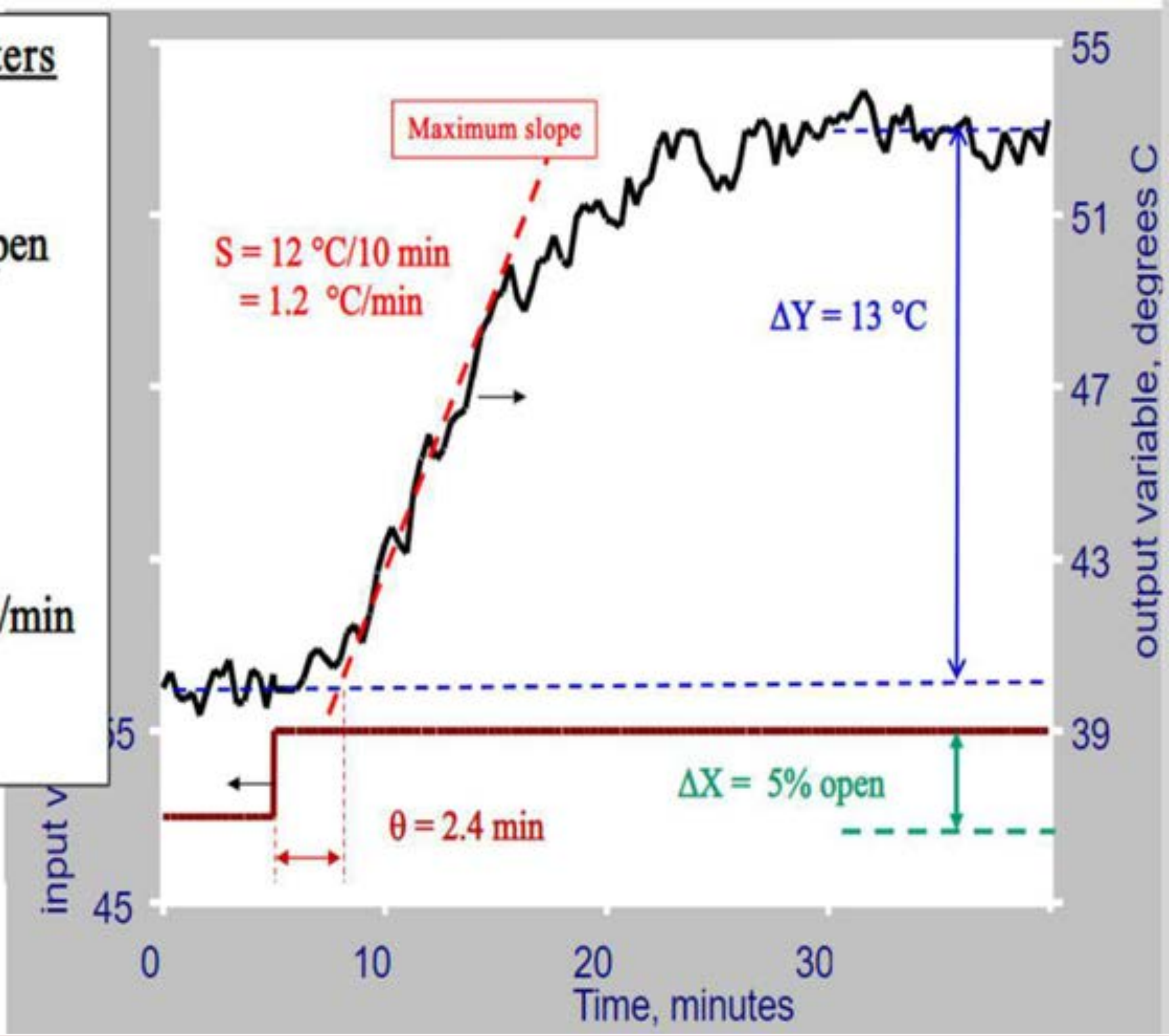
3

Model Parameters

$K_p = \Delta Y/\Delta X$
$= 13\ °C/5\ \%\ \text{open}$
$= 2.6\ °C/\%$

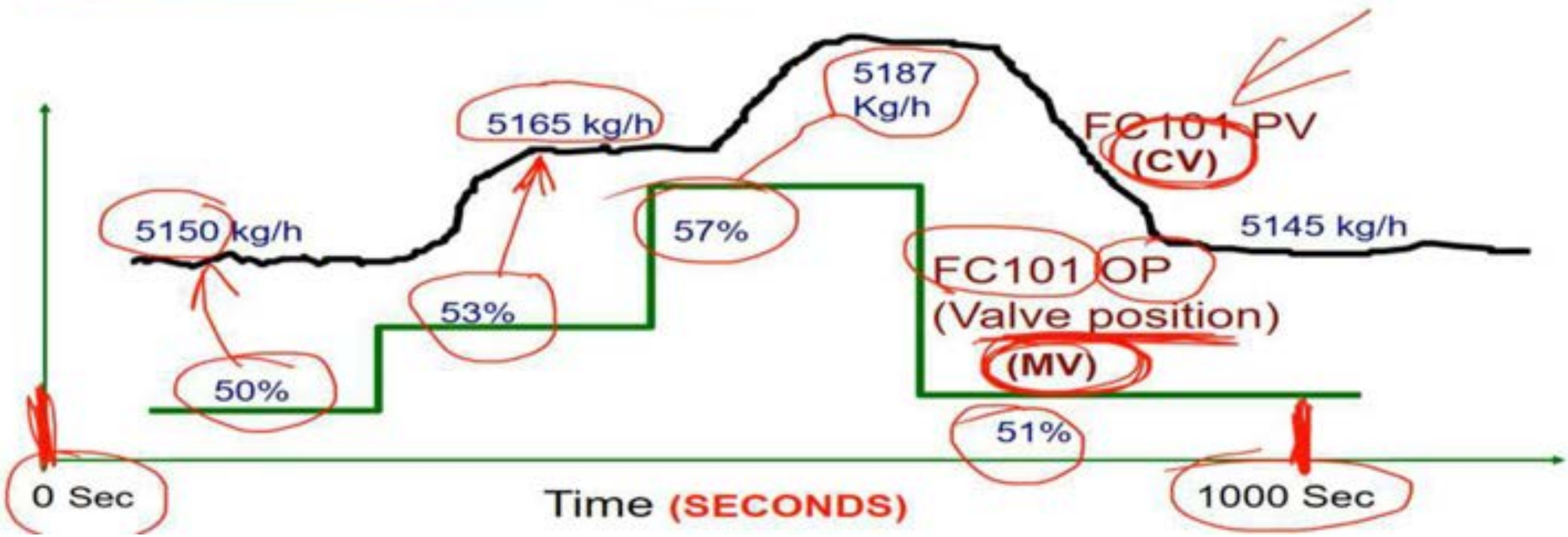
$\theta = 2.4\ \text{min}$

$\tau = \Delta Y/S$
$= 13\ °C/1.2\ °C/\text{min}$
$= 10.8\ \text{min}$

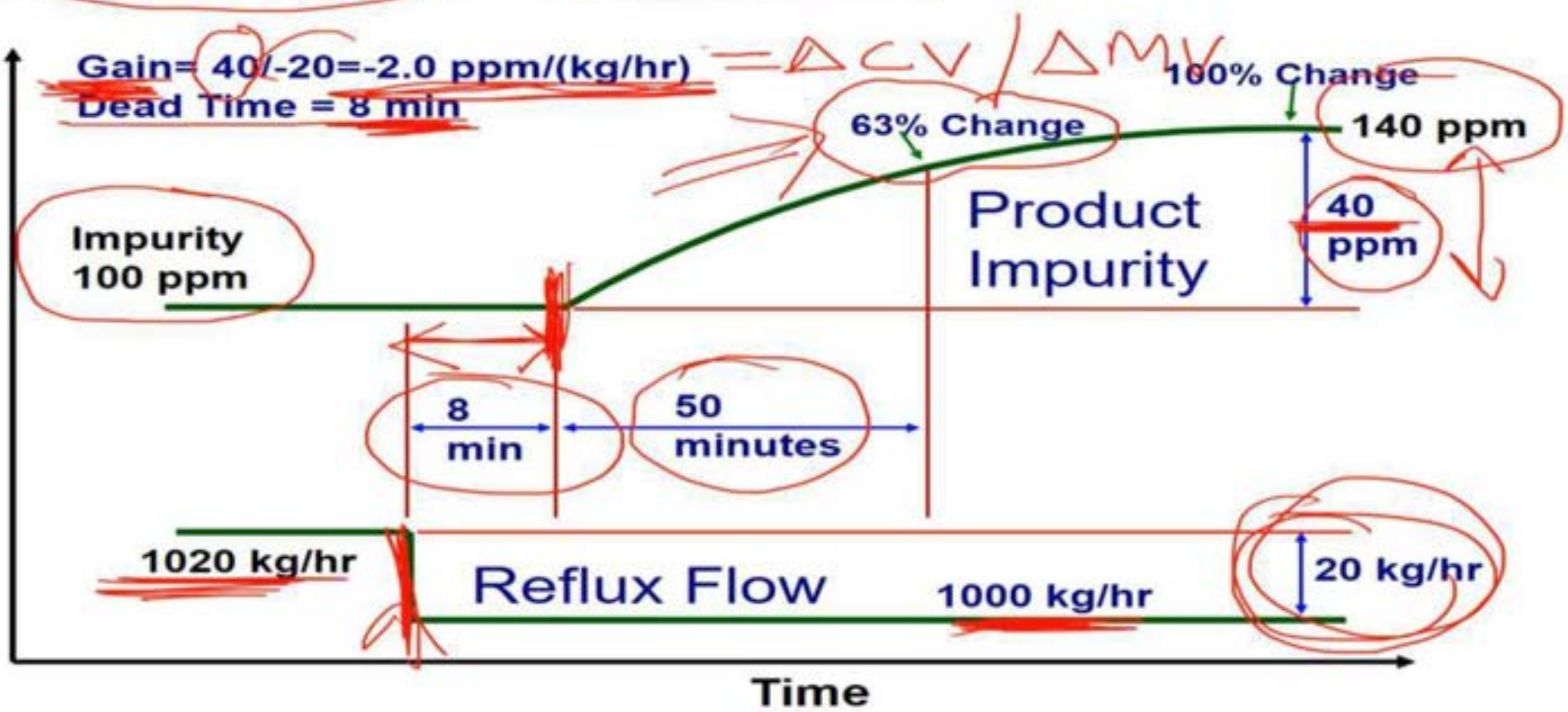


How to Determine Transfer Function Parameters For Fast Loops

Conduct open loop test. Open loop test consists of bumping the valve position (PID output) with the PID in Manual mode.

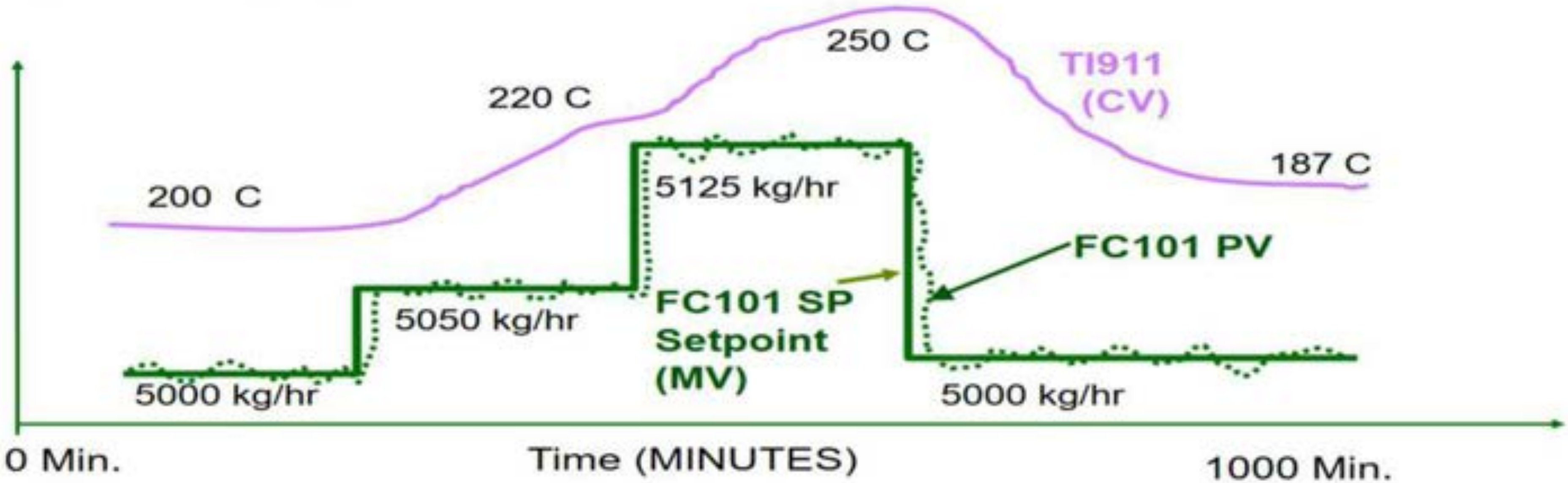


Reflux Flow vs. Impurity Transfer Function



How to Determine Transfer Function Parameters For Slow Loops

Follow similar procedure as before: e.g., bump reboiler steam flow SP and watch the temperature response:



PID Control Algorithm

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4th-Nov-2025

Learning Objectives

•Derive and interpret PID control equations in continuous and discrete form.

•Recognize how industrial PID loops are implemented in **DCS, PLC, and SCADA** systems.

•Identify field-level limitations: sensor noise, actuator dead-band, valve hysteresis.

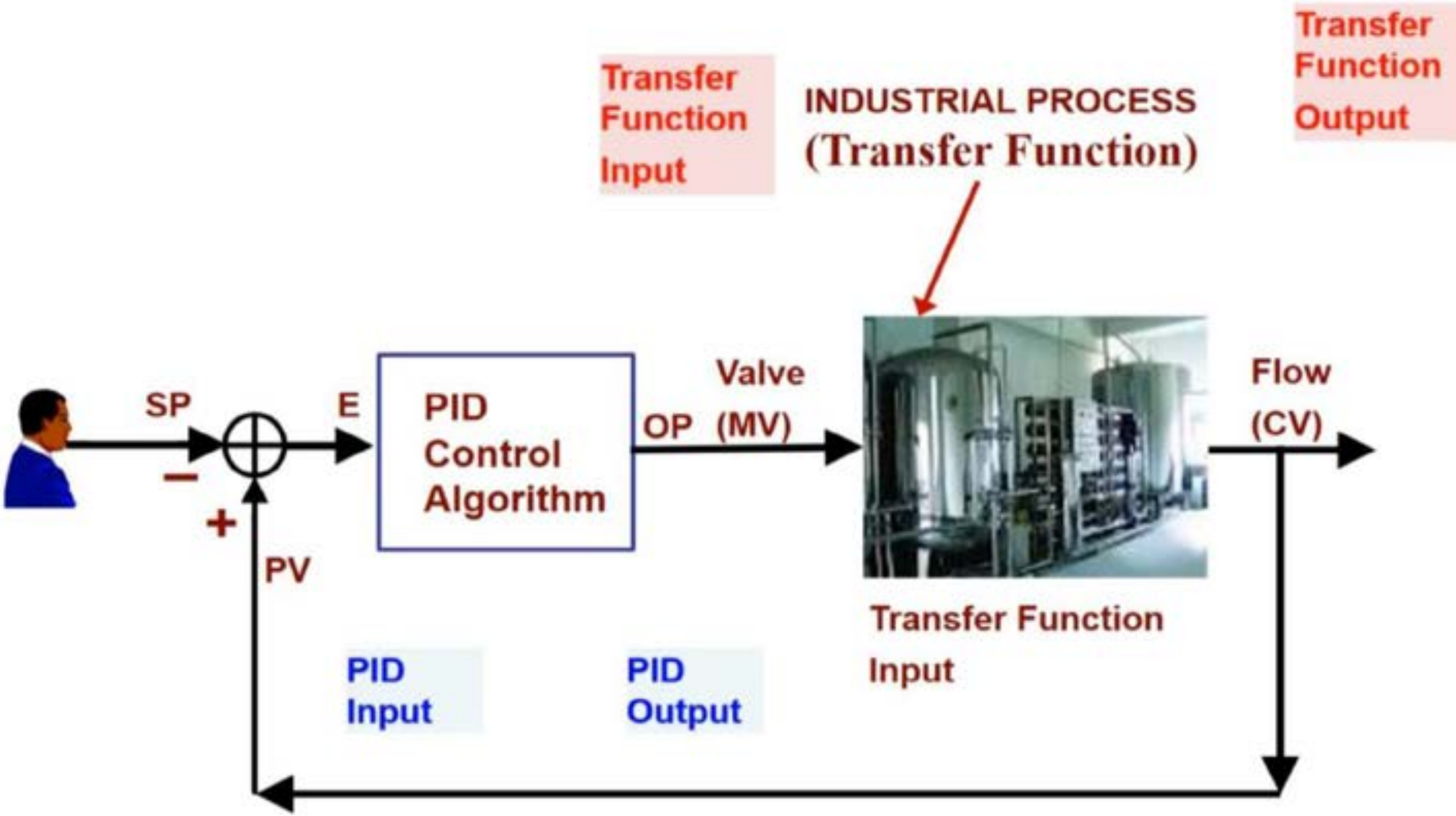
•Tune and test controllers safely following **ISA Loop Commissioning Practice**.

•Apply simulation and real-time testing using **MATLAB/Simulink**, **DeltaV Simulate**, or **Siemens S7-PID** blocks.

Who Invented PID Controller?

- Several engineer/mathematicians around the 1930's invented PID controller.
- Famous names: Bode, Ziegler, Nichols, Black, Nyquist and James.

Inputs and Outputs



Introduction to PID Control

- PID control dominates process industries (≈ 95 % of loops).
- Typical process variables: **Flow**, **Pressure**, **Level**, **Temperature**, **Composition**.
- Forms the "regulatory layer" under supervisory systems (MPC, RTO).
- Implemented as a **feedback loop**:

$$u(t) = K_p e(t) + K_i \int e(t)\,dt + K_d \frac{de(t)}{dt}$$

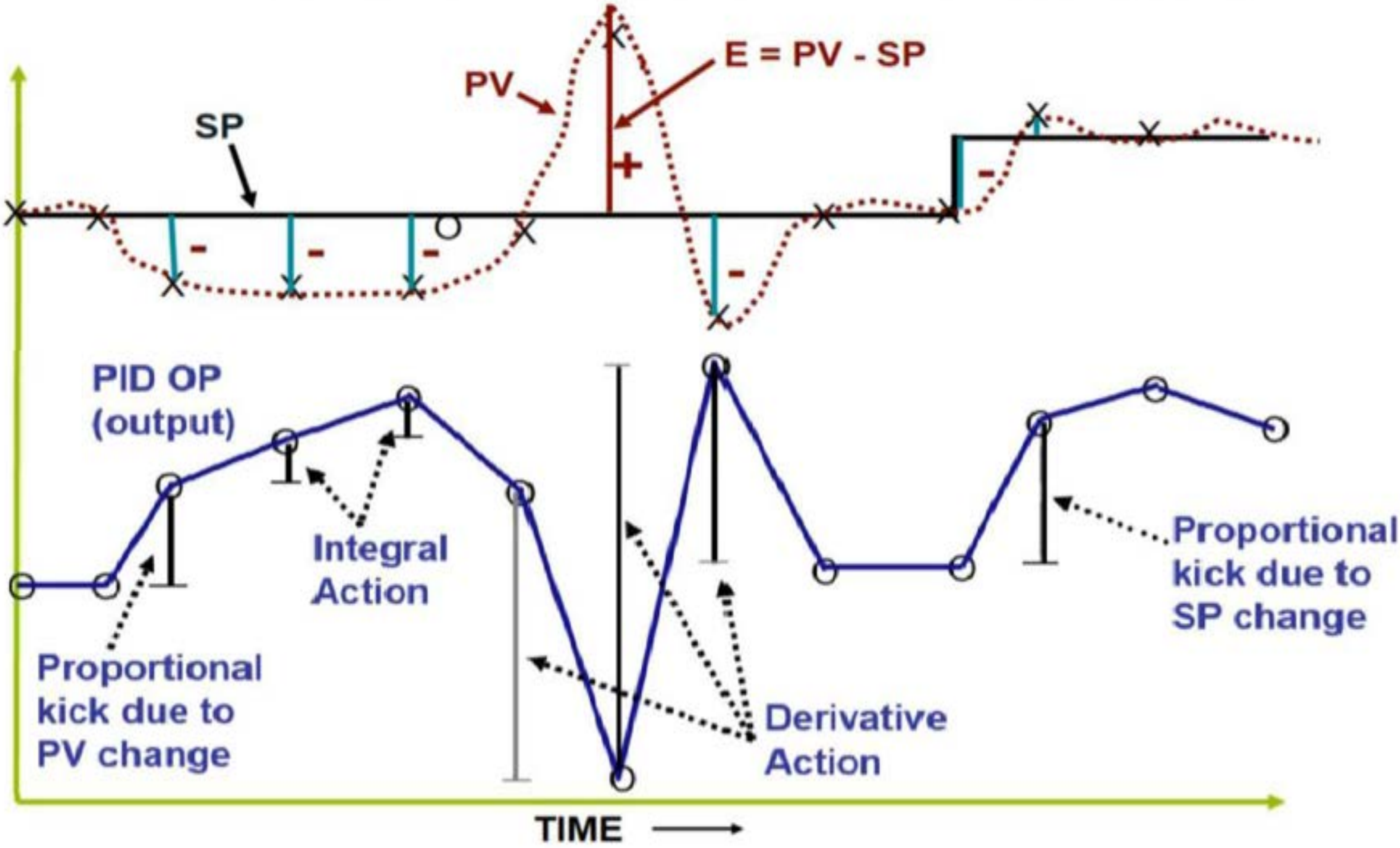
- Industrial note: modern DCS use **floating-point digital PID** with sample times of 0.1–1 s depending on loop speed.

Roles of P, I and D in Process Operations

Mode	Industrial Function	Typical Range	Field Remarks
P	Provides proportional corrective action	Gain 0.1–10 (%/%)	Too high → valve oscillation
I	Removes steady-state offset	Reset 0.1–60 min	Long reset on integrating systems (level)
D	Anticipates rate of change	0–60 s	Rarely used in noisy signals

Example: Boiler drum level — fast flow disturbances → PI only (D term suppressed).

PID Control Action Illustration



Algorithm Forms in Industrial Controllers

Ideal Form (ISA):

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{1 + \tau_D s/N}\right)$$

- N: derivative filter (8 – 20 typical).
- PID blocks in Emerson DeltaV, Yokogawa, Siemens all follow ISA format.
- **Series / Interactive** form used when loops are cascaded (inner loop faster).

Engineering Units:

- DCS vendors allow tuning in either *gain (Kc)* or *proportional band (PB %)* where $K_c = 100/PB$.

Digital PID Implementation

Discrete velocity form minimizes numerical windup:

$$u(k) = u(k-1) + K_p[(e(k) - e(k-1)) + \frac{T_s}{\tau_I}e(k) + \frac{\tau_D}{T_s}(e(k) - 2e(k-1) + e(k-2))]$$

- Sample time T_s chosen < 1/10 process time constant.
- In PLCs (e.g., Siemens S7-1500), derivative term uses internal **first-order filter** to mitigate quantization noise.
- Always check **scan time <** T_s to avoid aliasing.

PID Scan Time (dT term)

- PID Scan Time is the time period after which the PID algorithm runs and calculates a new OP that manipulates the control valve. (Final Control Element)
- PID Scan Time is how often the PID equation calculates a new OP value.
- Typical PID scan rates:
  - 10-50 milliseconds – Compressor surge control
  - 0.25 to 1 second- Motor control, FC, LC, PC…
  - 1 – 60 seconds – TC (temperature control)
  - 1 - 5 minutes – Online analyzer control (AC)

Industrial Practical Issues Overview

- **Instrumentation limits:** sensor drift, range mismatch, and A/D resolution.
- **Valve nonlinearity:** deadband ≈ 1 %, stiction ≈ 0.5 %; causes limit cycles.
- **Process dead time:** from long pipelines, thermowells, or analyzers.
- **Communication delay:** in Ethernet-based DCS can add 100–200 ms latency.
- **Controller mode coordination:** manual, auto, cascade, ratio, remote setpoint.

Sensor and Signal Conditioning

- Field transmitters (4–20 mA, HART) → analog filters ($\tau \approx 0.2$ s).
- **Anti-alias filter (hardware)** before A/D converter.
- **Digital filter (software)** tuned by control engineer per loop dynamics:

$$G_f(s) = \frac{1}{\tau_f s + 1}, \quad \tau_f < 0.05(\theta + \tau)$$

- Avoid filtering thermocouple signals twice (sensor + DCS).
- Use **shielded cables** and **proper grounding** to minimize EMI noise.

Process Noise and Frequency Spectrum

- Typical chemical process dynamics: $10^{-3} – 10^{-1}$ Hz.
- Electrical noise: 50–60 Hz + harmonics.
- Therefore, filter cut-off 0.5 – 2 Hz often adequate.
- **Spectral analysis** with FFT in MATLAB helps confirm noise bandwidth.

Controller Sense (Direct or Reverse Acting)

Action	PV ↑ → MV ↑ ?	Application Example	Verification
Direct-acting (+1)	Yes	Cooling water valve	Observe: SP ↑ → valve opens
Reverse-acting (−1)	No	Steam valve, reflux valve	SP ↑ → valve closes

Commissioning tip: always bump setpoint small (±1 %) and watch PV trend before enabling cascade link.

Integral (Reset) Windup and Prevention

- Occurs during actuator saturation, trip, or manual mode.
- Industrial solutions:
    - **Back-calculation:** internal tracking signal u_t ensures integrator stops at limits.
    - **External tracking (Track In / Track Out)** signals between cascade loops.
    - **Hold integrator** when output not in control.
- Typical parameter: Anti-windup gain $K_{aw} = 1/\tau_I$.
- In DCS, "Integral Tracking Enable" flag performs this automatically.

Bumpless Transfer

- Required when switching **Manual → Auto** or **Remote → Cascade.**
- Controller output initialized to current valve opening to prevent bump.
- Vendors:
  - DeltaV: "External Reset Feedback."
  - Siemens: "Tracking input."
- Always test in simulation before plant commissioning.

Derivative Filtering

- Differentiation amplifies high-frequency noise — use filtered D term:

$$G_D(s) = \frac{\tau_D s}{1 + \alpha \tau_D s}, \ \alpha = 0.1\text{–}0.2$$

- In Experion PKS, parameter "Derivative Filter N" = $1/\alpha$.
- Best applied to **temperature** or **pressure** loops with smooth sensors.

Output Limits and Fail-Safe Design

Fail Mode	Used For	Rationale
Fail-closed (air-to-open)	Fuel, steam, chemical feed	Safety shutdown
Fail-open (air-to-close)	Cooling water, lubrication	Prevent overheating
Fail-in-place	Compressor recycle, ratio valves	Maintain pressure balance

Control engineer must confirm pneumatic action and signal direction (3–15 psig or 4–20 mA) during FAT/SAT.

PID Tuning Methods and Industrial Rules

Method	Procedure	Industry Usage	Pros / Cons
Ziegler–Nichols	Ultimate gain K_u test	Quick loop tuning	Aggressive → overshoot
Cohen–Coon	Step-response fit (FOPDT)	Chemical plants	Good 1st approximation
IMC	Model based, choose λ	Refineries, pharma	Robust, safe margins
Lambda Tuning	Similar to IMC	Paper mills, distillation	Easy setpoint tracking
Auto-tuning	Relay oscillation	DCS built-in	Fast commissioning

Field note: Always re-verify with **disturbance test**, not only setpoint change.

Typical PID Values

	FC (Flow Control)	PC (Pressure Control)	LC (Level Control)	TC (Temperature Control)	AC (Online Analysis Control)
PID Type	PI	PI	PI or PID	PID	PID
Proportional Gain (P)	0.1 to 0.5	1 to 25	0.2 to 10	0.5 to 10	0.01 to 50
Integral (I) Minutes	0.3 to 1.0	0.5 to 10	5 to 1000	5 to 100	10 to 200
Derivative (D) Minutes	0 (never used)	0.0 to 2.0 (rarely used)	0.5 to 4 (often zero)	0.5 to 5.0	0.5 to 5.0

MATLAB / Simulink Example

matlab Copy code

```matlab
K = 2; tau = 10; theta = 2;
G = tf(K,[tau 1],'InputDelay',theta);
pidTuner(G,'PID')
```

- Use **IMC tuning** and export coefficients directly to **PLC function block**.
- Validate loop by injecting step disturbance in simulation before applying to plant.
- Observe rise time, overshoot, settling time, and IAE criterion.

Typical Industrial Applications

Process	Common PID Mode	Dynamic Behavior	Notes
Flow	P or PI	Fast, non-integrating	Primary loop for cascades
Level	PI	Integrating	Long reset to avoid oscillation
Temperature	PID	Long lag, large delay	Often cascade with flow
Pressure	PI	Compressible	Use derivative only if stable
Composition / pH	PID + Feedforward	Highly nonlinear	Apply gain scheduling

Advanced and Modern Extensions

- **Cascade Control:** inner flow loop within outer temperature loop.
- **Ratio Control:** maintains fixed proportion between two streams.
- **Feedforward:** compensates known measurable disturbance (e.g., load change).
- **Gain Scheduling:** adapts PID gains with operating conditions (e.g., reactor temp).
- **Adaptive / Self-tuning PID:** on-line identification (Relay / MRAC methods).
- **Smith Predictor PID:** compensates large dead-time ($\theta > \tau$).
- **Fuzzy-PID / RBF-NN PID:** intelligent control improving nonlinear performance.

Summary

- PID remains the **workhorse of process control**—simple, robust, and well-understood.
- Industrial success depends on **proper tuning, reliable instruments, and safety logic.**
- Control engineers must integrate theoretical understanding with:
  - Field device calibration
  - Communication diagnostics
  - Alarm management and trip logic
- Continuous improvement via simulation and historical data analysis (Plant Data Historian).

Tuning of PID Controllers in Modern Process Industries

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26th-Nov-2025

Learning Outcomes

At the end of this lecture, students should be able to:

- Recognize the industrial PID algorithm used in DCS/PLC function blocks (ISA form with filtered derivative).
- Relate academic PID form (K_p, K_i, K_d) to industrial parameters $(K_c, T_I, T_D, N, PB, R)$.
- Obtain simple dynamic models (FOPDT/SOPDT) from step-test data.
- Apply IMC/SIMC-style rules to compute initial tuning (K_c, T_I, T_D) for FOPDT and SOPDT processes.
- Interpret and fine-tune tuning parameters in actual industrial units (PB, repeats/min, filter factor).

Where PID Lives in Industry

Typical implementation platforms

- **PLC (Programmable Logic Controller)**
  - General-purpose automation; ladder/FBD logic.
- **DCS (Distributed Control System)**
  - Large process plants (refineries, chemical, power).
  - Many standard PID function blocks with auto-tune.
- **PAC / Stand-alone controllers**
  - Panel-mounted controllers, embedded controllers, etc.

Common feature

- All expose PID in industrial form: gain K_c, integral time T_I, derivative time T_D, filter factor N, and often PB and reset R.

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Review of Feedback Loop with PID

Single-loop feedback structure

- Setpoint $r(t) \rightarrow$ error $e(t) = r(t) - y(t)$.
- PID computes manipulated variable $u(t)$ from $e(t)$.
- Process (plant) produces controlled variable $y(t)$.

Roles of PID terms

- **P**: immediate correction proportional to error.
- **I**: removes steady-state offset.
- **D**: predicts error trend and adds damping / anticipatory action.

Industrial tuning objective

Choose K_c, T_I, T_D so that we get:

- Fast response with limited overshoot.
- Robustness to noise and model error.
- Acceptable actuator effort.

Industrial PID Algorithm (ISA Form with Filtered D)

Continuous "ideal" / ISA PID with derivative filter

$$G_c(s) = K_c \left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D}{N} s} \right)$$

- K_c: proportional gain
- T_I: integral time (s)
- T_D: derivative time (s)
- N: derivative filter factor (dimensionless, typically 8–20)

Notes

- Derivative is always filtered to avoid amplifying measurement noise.
- Many DCS/PLC PID blocks implement exactly this structure internally.

Other PID Forms and Mapping

1. Parallel (academic) form

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s$$

Mapping from ISA form

$$K_p = K_c, \qquad K_i = \frac{K_c}{T_I}, \qquad K_d = K_c T_D$$

2. Series / interacting form (legacy)

$$G_c(s) = K_c \left(1 + \frac{1}{T_I s}\right)(1 + T_D s)$$

3. DCS parameter units

- Proportional band:

$$PB = \frac{100}{K_c}\ (\%)$$

- Reset:

$$R = \frac{60}{T_I}\ (\text{repeats/min})$$

- Derivative: T_D (s), filter factor N (dimensionless

Why We Need Systematic PID Tuning

Trial-and-error tuning

- Time-consuming, operator-dependent.
- Risky: large excursions or sustained oscillations.

Classical Ziegler–Nichols

- Simple to apply, but often:
  - Large overshoot.
  - Poor robustness to disturbances and model uncertainty.

Modern practice (IMC/SIMC-inspired)

- Identify a simple low-order model (FOPDT/SOPDT).
- Use IMC/SIMC rules to compute robust starting values.
- Fine-tune on the real loop if needed.

Process Modelling for Tuning: FOPDT

First-Order Plus Dead Time (FOPDT) model

$$G_p(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

- K: steady-state gain $(\Delta PV / \Delta MV)$
- τ: time constant (dominant response speed)
- θ: effective dead time (transport + computation + lags)

Step-test procedure

1. Let process settle at a steady state.
2. Apply a small MV step (5–10% of range).
3. Record PV until new steady state.
4. Estimate K, τ, θ from response (graphically or by regression).

IMC/SIMC Philosophy (High-Level Idea)

Internal Model Control (IMC) viewpoint

- Use a simple low-order model of the process.
- Specify desired closed-loop time constant λ (sometimes τ_c):
  - Small λ: fast but less robust.
  - Large λ: slower but more robust to model error.
- Derive PI/PID parameters analytically from model and λ.

IMC/SIMC in practice

- IMC (Morari & Zafiriou, Rivera et al.).
- SIMC rules (Skogestad).
- Basis of many modern auto-tuning tools used in DCS/PLCs.

IMC/SIMC PI Rule for FOPDT (Self-Regulating)

Given FOPDT model

$$G_p(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

Use a PI controller in ISA form

$$G_c(s) = K_c \left(1 + \frac{1}{T_I s}\right)$$

Widely used IMC/SIMC rule

$$K_c = \frac{\tau}{K(\lambda + \theta)}, \qquad T_I = \min\left(\tau, \ 4(\lambda + \theta)\right), \qquad T_D = 0$$

- λ: desired closed-loop time constant.

Typical robust choice for self-regulating FOPDT processes

$$\lambda \approx \max(\tau, \ 3\theta)$$

For many FOPDT loops, PI is sufficient (set $T_D = 0$, ignore N).

Worked Example: FOPDT → Industrial PI

Process model (from step test)

$$G_p(s) = \frac{2e^{-2s}}{10s+1} \quad \Rightarrow \quad K = 2,\ \tau = 10\ \mathrm{s},\ \theta = 2\ \mathrm{s}$$

Choose robust $\lambda = 10$ s.

Compute IMC/SIMC PI parameters

$$K_c = \frac{\tau}{K(\lambda+\theta)} = \frac{10}{2(10+2)} \approx 0.417$$

$$T_I = \min(\tau,\ 4(\lambda+\theta)) = 10\ \mathrm{s}$$

Industrial PI (ISA)

$$G_c(s) = 0.417\left(1 + \frac{1}{10s}\right)$$

DCS interpretation

- Proportional band:

$$PB = \frac{100}{0.417} \approx 240\%$$

- Reset:

$$R = \frac{60}{T_I} = 6\ \mathrm{repeats/min}$$

From Robust PI to "Gentle" PID

Practical industrial strategy

1. First design a robust PI using IMC/SIMC.
2. If the response is too slow or under-damped, upgrade to PID with a small derivative term.

Example continuation (same process)

- Robust PI: $K_c \approx 0.417,\ T_I = 10$ s.
- Choose "gentle" PID:
  - $K_c = 0.35$ (slightly smaller for robustness)
  - $T_I = 10$ s
  - $T_D = 2$ s
  - $N = 10$

Industrial PID (ISA)

$$G_c(s) = 0.35\left(1 + \frac{1}{10s} + \frac{2s}{1 + 0.2s}\right)$$

D term increases speed and damping, but tuning is still much less aggressive than Ziegler–Nichols.

General Recipe: "Gentle PID from Robust PI"

Assume we have robust PI from IMC/SIMC with parameters (K_c^{PI}, T_I^{PI}) and known τ.

Recommended recipe for FOPDT-like loops

1. Set

$$T_I^{PID} = T_I^{PI}$$

2. Choose derivative time

$$T_D^{PID} \approx (0.1\text{–}0.25)\,\tau \quad (\text{or } (0.1\text{–}0.25)\,T_I^{PI})$$

3. Reduce proportional gain slightly

$$K_c^{PID} = (0.7\text{–}0.9)\,K_c^{PI}$$

4. Select filter factor

$$N \in [8, 20]\ (\text{e.g. } N = 10)$$

Speaker notes: Encourage students to treat this as a "rule-of-thumb box" they can bring to plant work.

SOPDT Processes

Some process loops are better approximated by a **Second-Order Plus Dead Time (SOPDT)** model:

$$G_p(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}, \quad \tau_1 \geq \tau_2 > 0$$

Examples

- Mixing plus heat-capacity effects.
- Cascaded tanks, multi-lag systems.

Comments

- If τ_2 is very small vs θ, it can be absorbed into delay → FOPDT may be sufficient.
- If both τ_1, τ_2 are significant, a PID controller usually outperforms PI.

IMC-Based PID Rule for SOPDT (ISA Form)

For SOPDT model as above, use ISA PID with filtered derivative:

$$G_c(s) = K_c \left(1 + \frac{1}{T_I s} + \frac{T_D s}{1 + \frac{T_D}{N} s}\right)$$

with:

$$K_c = \frac{\tau_1 + \tau_2}{K(\lambda + \theta)}, \qquad T_I = \tau_1 + \tau_2, \qquad T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}$$

- λ: desired closed-loop time constant (again sets speed vs robustness).
- Choose λ of the same order as the dominant time constant and delay, e.g.

$$\lambda \approx \max(\tau_1, \theta)$$

- $N \approx 8\text{–}20$ in most DCS implementations.

SOPDT Numerical Example

Process

$$G_p(s) = \frac{e^{-s}}{(4s+1)(2s+1)} \Rightarrow K = 1,\ \tau_1 = 4,\ \tau_2 = 2,\ \theta = 1\ \text{s}$$

Choose $\lambda = 4$ s.

Compute

$$\tau_1 + \tau_2 = 6, \qquad \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = \frac{8}{6} \approx 1.33\ \text{s}$$

$$K_c = \frac{\tau_1 + \tau_2}{K(\lambda + \theta)} = \frac{6}{4+1} = 1.2$$

$$T_I = 6\ \text{s}, \qquad T_D \approx 1.33\ \text{s}, \qquad N = 10$$

Industrial PID (ISA)

$$G_c(s) = 1.2\left(1 + \frac{1}{6s} + \frac{1.33\,s}{1 + 0.133s}\right)$$

This gives a well-damped closed-loop response for th., SOPDT process.

Practical Tuning Workflow in Industry

1. **Preparation**
   - Verify sensors, actuators, ranges, and limits.
   - Ensure manual control is available and safe.
2. **Step-test**
   - Perform a small MV step.
   - Record PV and estimate FOPDT/SOPDT model.
3. **Initial tuning**
   - Use IMC/SIMC rules to compute robust PI (or PID for SOPDT).
4. **Implementation**
   - Enter K_c, T_I, T_D, N into DCS/PLC.
   - Verify engineering units (PB, repeats/min, etc.).
5. **Fine-tuning on-line**
   - Adjust K_c for speed/overshoot trade-off.
   - Adjust T_I for steady-state error.
   - Adjust T_D and N only if necessary (noise vs damping).
6. **Validation**
   - Test with typical set-point changes and disturbances.
   - Always respect actuator constraints and safety limits.

Comments on Auto-Tuners

Modern controllers

- Many PLC/DCS systems have auto-tune or adaptive features.

Typical internal logic

- Perform an excitation experiment (step or relay test).
- Fit a simple FOPDT/SOPDT model.
- Apply a rule similar to IMC/SIMC to compute tuning.

Good practice

- Understand the underlying principles (this lecture).
- Use auto-tune as a starting point, then validate and adjust if needed.

Limitations and Cautions

Model limitations

- FOPDT/SOPDT are approximations:
  - Nonlinearities, changing operating points, saturation, etc.

Multivariable systems

- Strong interactions → single-loop tuning may be insufficient.
- May need multivariable control (e.g., decoupling, MPC).

Special processes

- Integrating processes (e.g., level control) and runaway reactions
  - Need modified tuning rules (not covered here).

Always check

- Safety constraints and actuator limits.
- Process specifications and operating procedures.

Summary of Key Industrial Formulas

FOPDT model

$$G_p(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

IMC/SIMC PI (ISA):

$$K_c = \frac{\tau}{K(\lambda + \theta)}, \qquad T_I = \min\left(\tau,\, 4(\lambda + \theta)\right), \qquad T_D = 0$$

SOPDT model

$$G_p(s) = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

IMC-style PID (ISA):

$$K_c = \frac{\tau_1 + \tau_2}{K(\lambda + \theta)}, \qquad T_I = \tau_1 + \tau_2, \qquad T_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2}, \qquad N \approx 8\text{–}20$$

Unit conversions

$$PB = \frac{100}{K_c}\ (\%), \qquad R = \frac{60}{T_I}\ (\text{repeats/min}) \qquad K_p = K_c, \qquad K_i = \frac{K_c}{T_I}, \qquad K_d = K_c T_D$$

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Single-loop Enhancements: Cascade Control Part 1

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GOALS

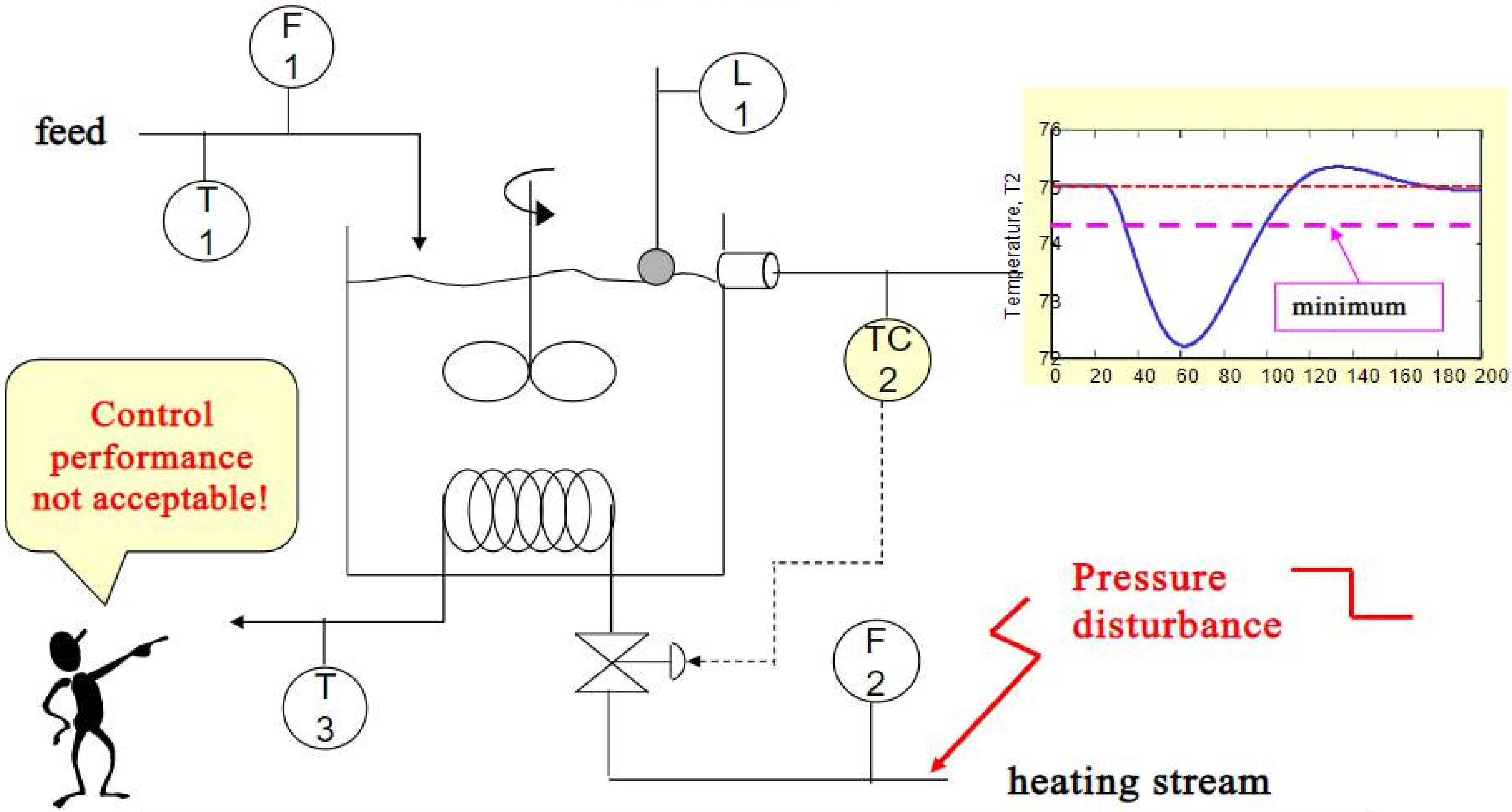
When I complete this chapter, I want to be able to do the following.

- Understand the concept of cascade control and its difference from single-loop feedback
- Determine the cascade control structure and design criteria

Review: Single-loop feedback and its limitations

- Single-loop feedback structure:
  - Process variable (PV) measured and compared with setpoint (SP).
  - Controller adjusts manipulated variable (MV) through a valve/actuator.
- Limitations for many processes:
  - Disturbances may enter **before** the measurement point.
  - Actuator and utility dynamics (e.g., steam pressure, flow) can be slow or nonlinear.
  - Sensor lags and dead time worsen response.
- Result:
  - Large deviations from SP during disturbances.
  - Slow correction; sometimes oscillatory or sluggish behavior.

MOTIVATION – Feedback does not always yield acceptable performance



We need a new control structure!

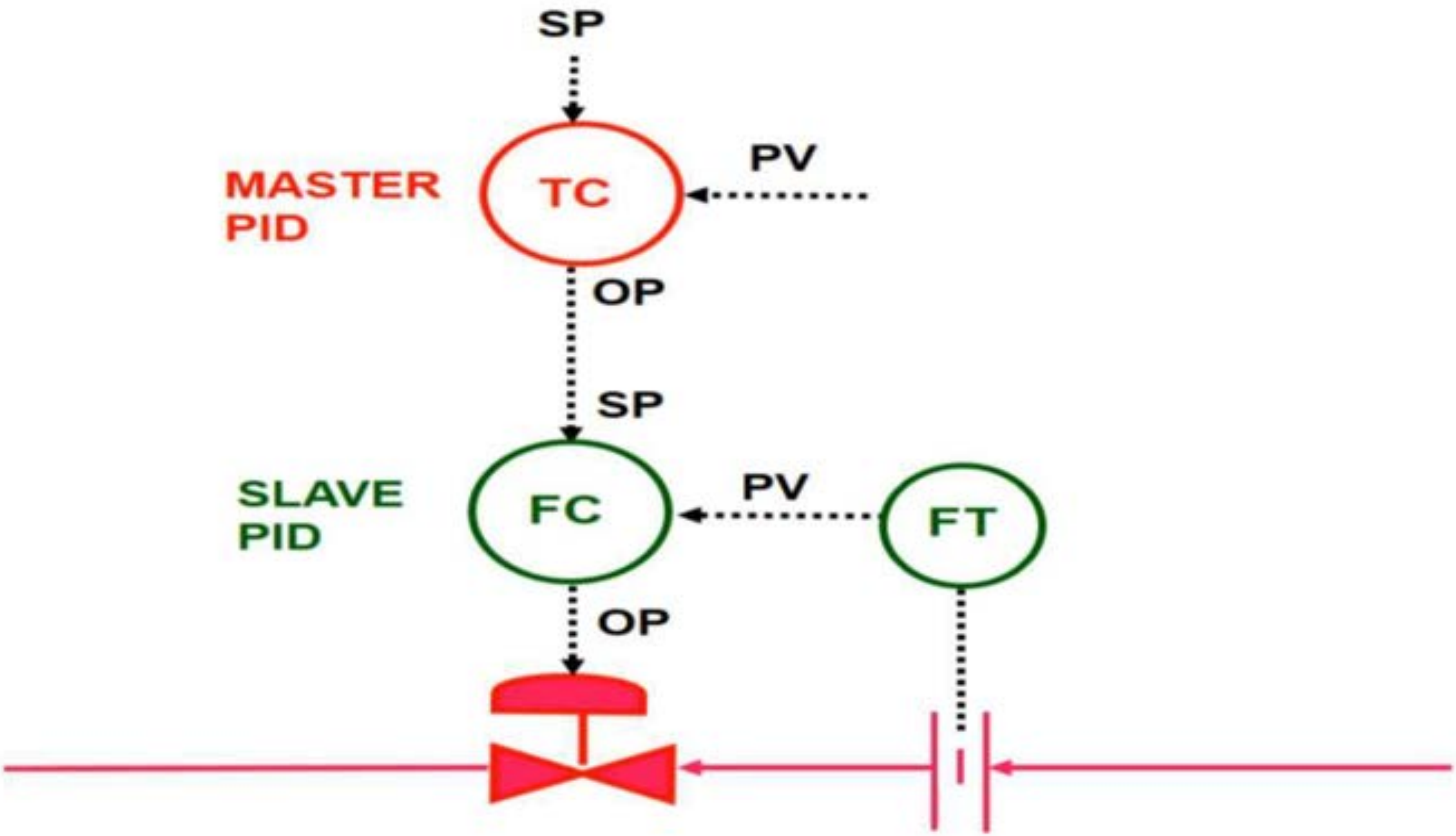
Motivating example: Continuous stirred-tank heater

- Process:
  - Cold feed enters a stirred tank.
  - Heating via steam coil/jacket.
  - Objective: maintain outlet temperature T at T_{sp}.
- Single-loop temperature control:
  - Temperature transmitter (TT) → temperature controller (TC) → control valve on steam line.
- Typical disturbance:
  - Change in steam supply pressure.
  - Change in feed flow or feed temperature.
- Observation:
  - Temperature loop reacts only **after** the tank outlet temperature has already changed.

Idea of cascade control

- **Cascade control** = two nested feedback loops:
  - **Primary (outer) loop**:
    - Regulates the main quality variable (e.g., outlet temperature).
    - Controller output is the **setpoint** of the secondary loop.
  - **Secondary (inner) loop**:
    - Regulates an intermediate variable directly related to the actuator (e.g., steam flow).
    - Its controller output drives the final control element (valve, motor drive).
- Benefit:
  - Inner loop can detect and correct certain disturbances **before** they propagate to the primary variable.

Cascade PID



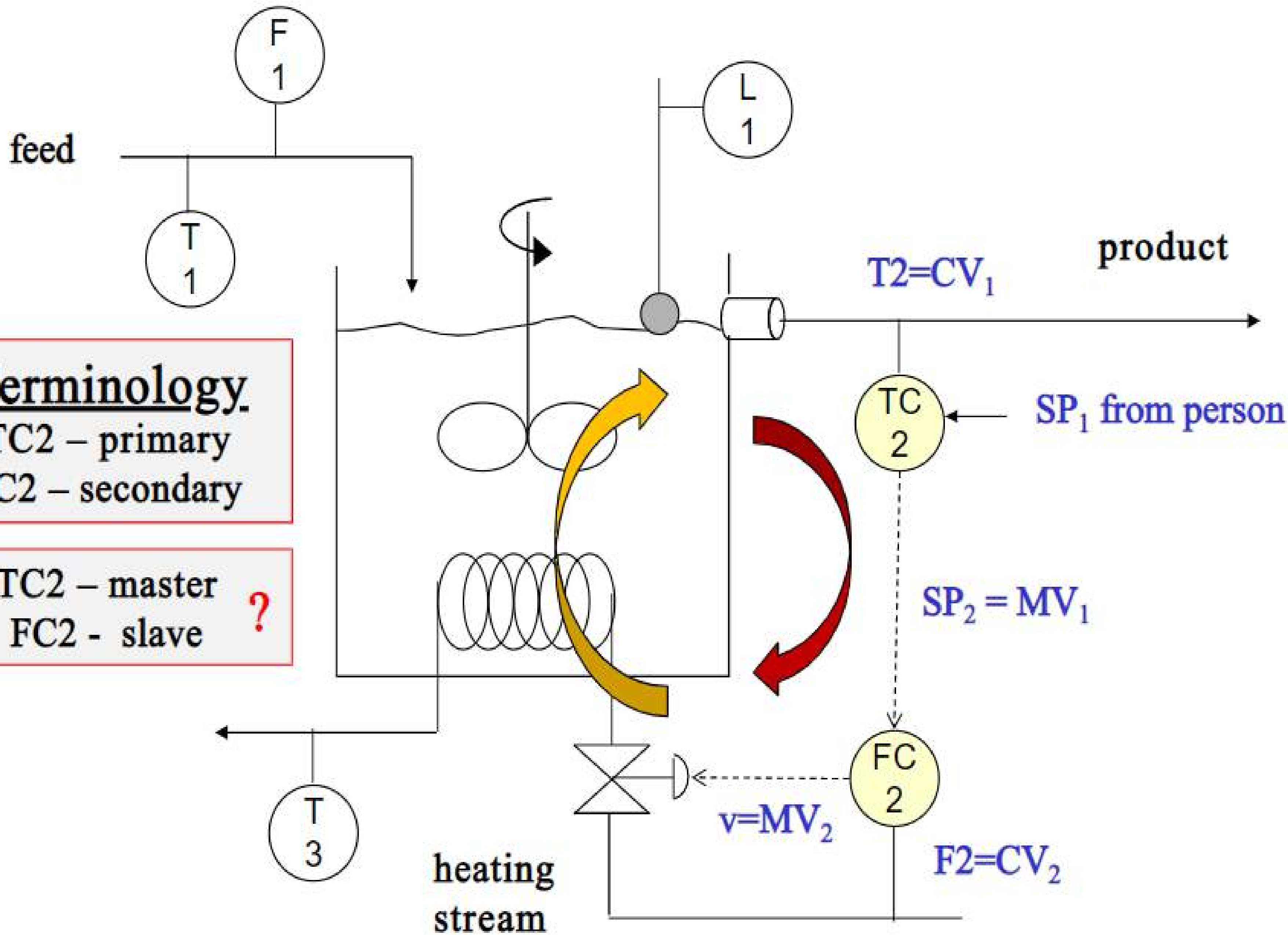
Terminology and signal structure

- Primary loop (outer):
  - Process variable: y_1 (e.g., temperature).
  - Setpoint: $y_{1,sp}$.
  - Controller: $G_{c1}(s)$ (primary controller).
- Secondary loop (inner):
  - Process variable: y_2 (e.g., steam flow or jacket temperature).
  - Setpoint: $y_{2,sp}$ (output of primary controller).
  - Controller: $G_{c2}(s)$ (secondary controller).
- Final control element:
  - Valve or actuator with dynamics $G_v(s)$.
- Disturbances:
  - d_1: act on the primary process.
  - d_2: act on the inner (secondary) process or actuator.

Cascade P&ID – CST heater example

(Use your heater P&ID diagram here.)

- Primary loop:
    - Temperature transmitter TT on tank outlet → temperature controller TC.
    - TC output = setpoint for secondary flow controller FC.
- Secondary loop:
    - Flow transmitter FT on steam line → flow controller FC.
    - FC output → I/P converter → steam valve.
- Disturbance handling:
    - Steam supply pressure change affects steam flow.
    - Inner flow loop corrects flow quickly, **before** temperature build-up.



Terminology

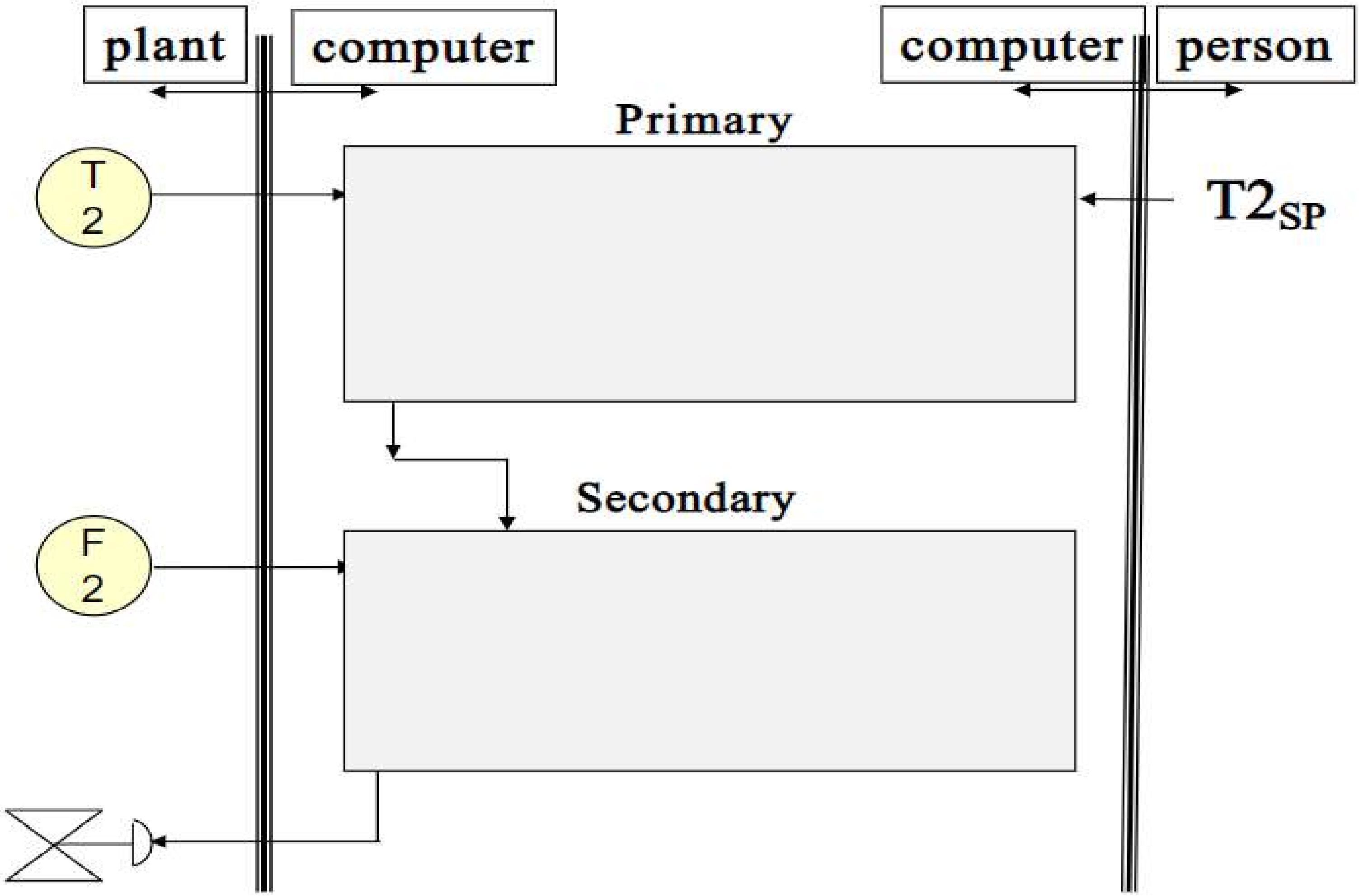
TC2 – primary

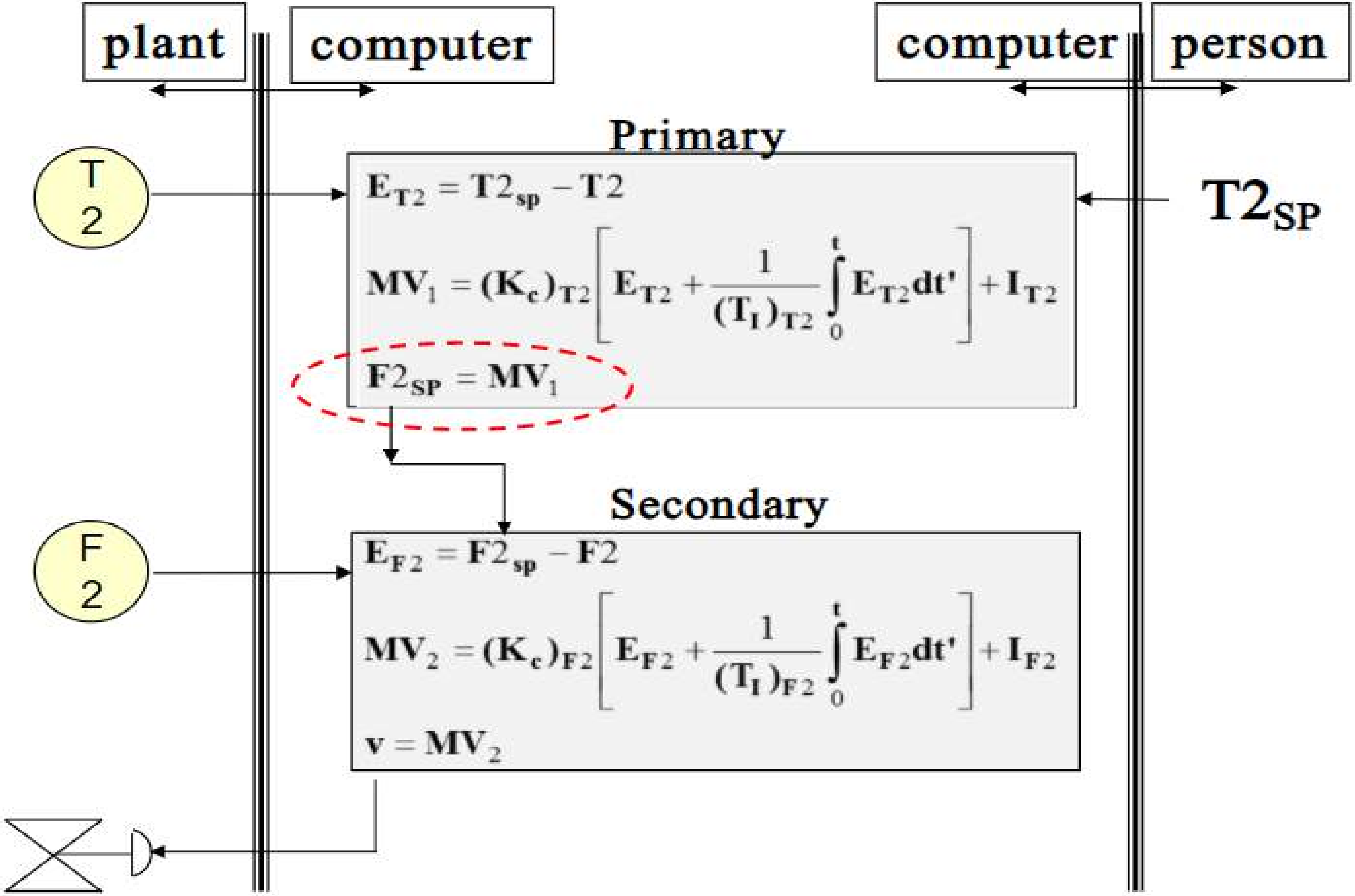
FC2 – secondary

? TC2 – master ?

FC2 - slave

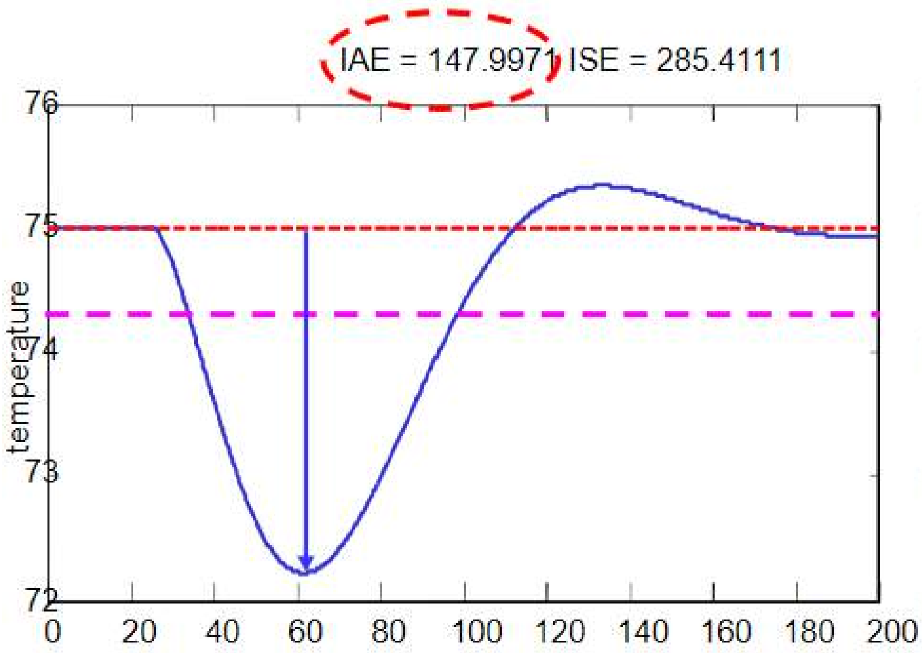
Write the equations solved for each of the two controllers in the cascade structure.



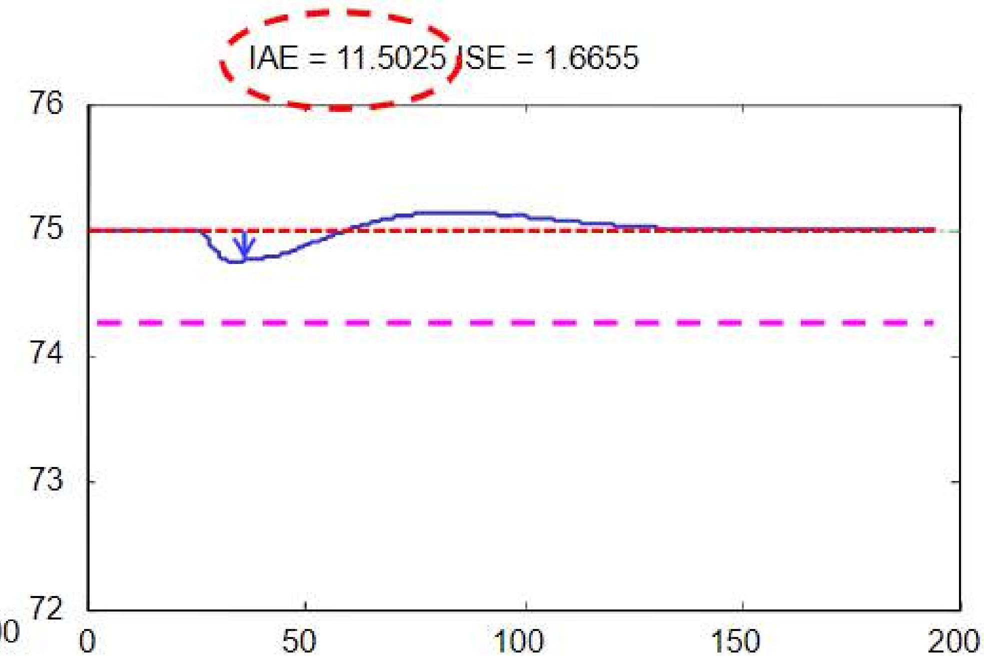

plant
computer
computer
person
Primary
T 2
$E_{T2} = T2_{sp} - T2$
$MV_1 = (K_c)_{T2}\left[E_{T2} + \frac{1}{(T_I)_{T2}}\int_0^t E_{T2}dt'\right] + I_{T2}$
$F2_{SP} = MV_1$
$T2_{SP}$
Secondary
F 2
$E_{F2} = F2_{sp} - F2$
$MV_2 = (K_c)_{F2}\left[E_{F2} + \frac{1}{(T_I)_{F2}}\int_0^t E_{F2}dt'\right] + I_{F2}$
$v = MV_2$

Control Performance Comparison for CST Heater

Single-Loop



Cascade

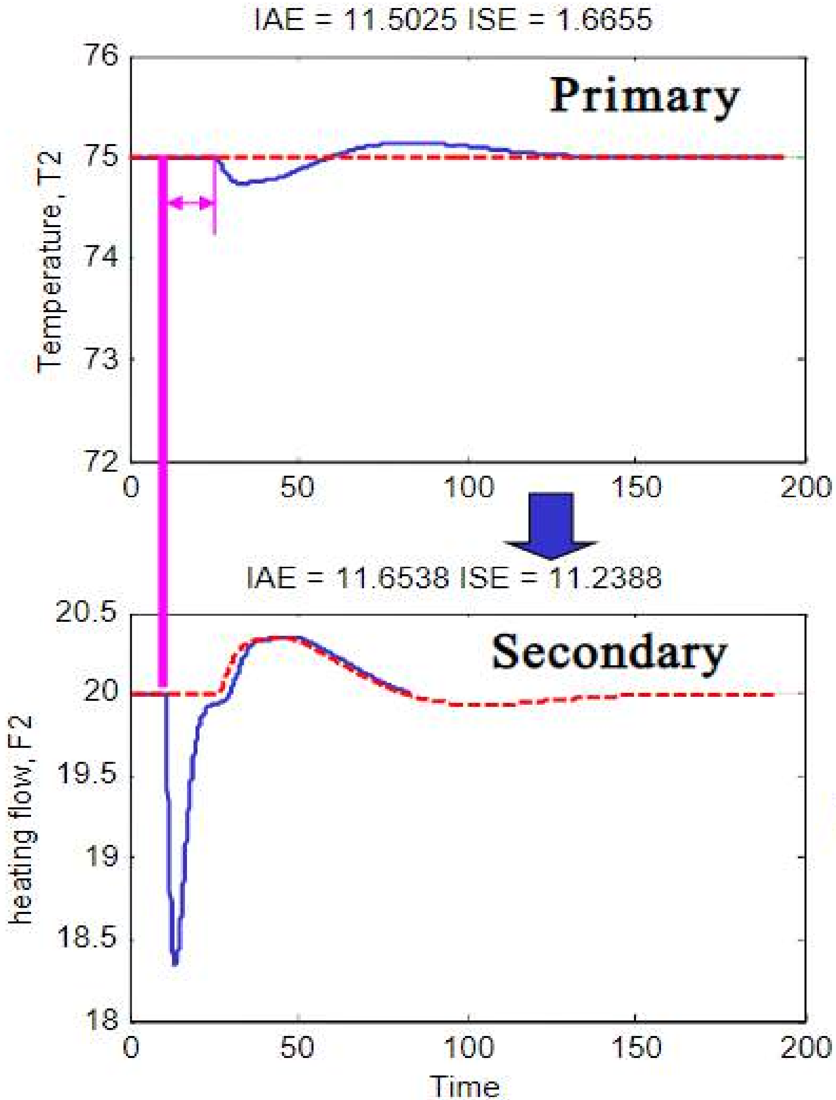


Much better performance!
WHY?

The control performance is expressed as standard deviation from the set point

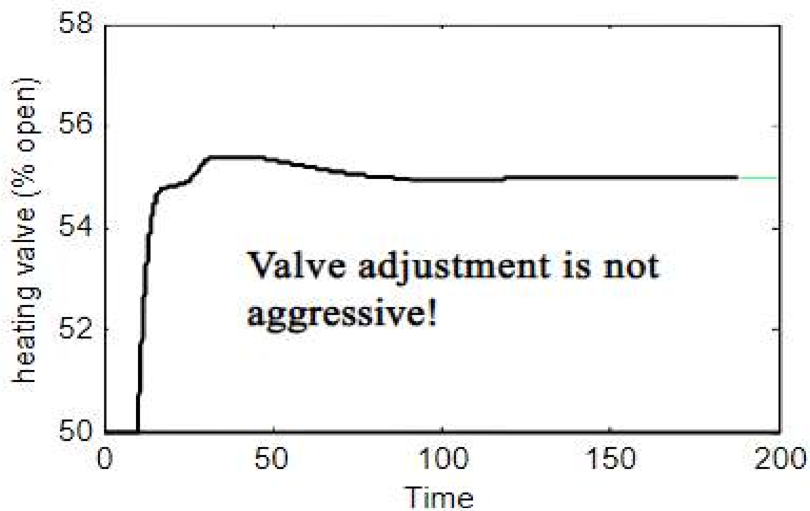
$$\sigma_{sp} = \sqrt{\sum_{i=0}^{n} \frac{(SP_i - CV_i)^2}{n}}$$

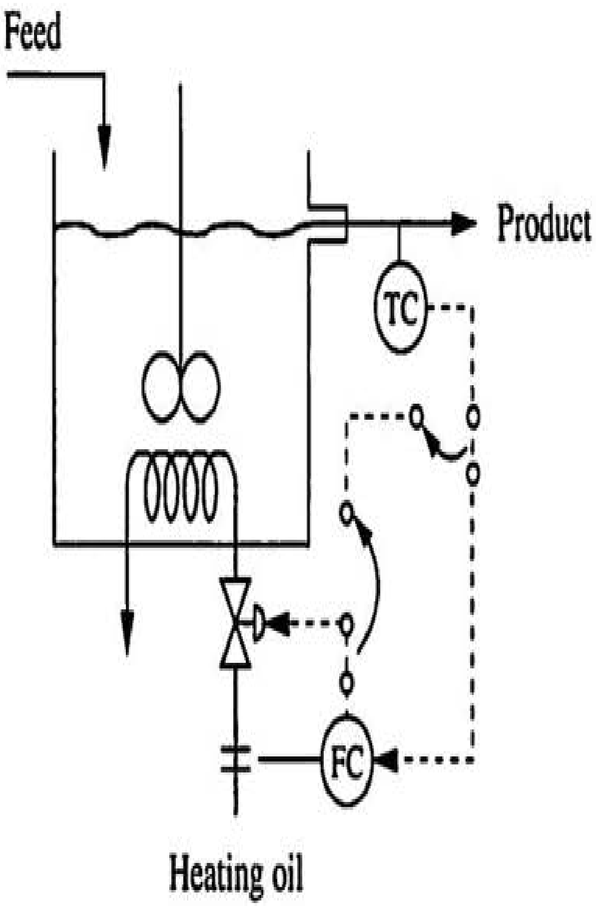
Cascade Control Performance for CST Heater (Detail)



Summary

- Flow measurement responds quickly
- Flow controlled quickly
- Temperature controller gets offset = 0
- Valve adjustment is moderate





FIGURE

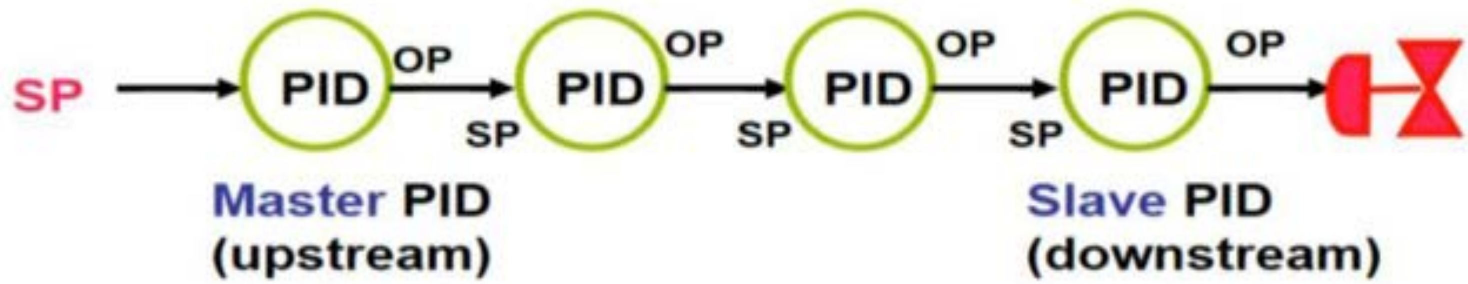
Control design with a switch to bypass the cascade and convert to single-loop control.

The cascade control system uses more control equipment—two sensors and controllers—than the equivalent single-loop system. Since the cascade requires all of this equipment to function properly, its reliability can be expected to be lower than the equivalent single-loop system, although the slightly lower reliability is not usually a deterrent to the use of cascade. If feedback control must be maintained when the secondary sensor or controller is not functioning, the flexibility to bypass the secondary and have the primary output directly to the valve can be included in the design. This option is shown in Figure , where the positions of both switches are coordinated.

Since the cascade involves more equipment, it costs slightly more than the single-loop system. The increased costs include a field sensor and transmission to the control house (if the variable were not already available for monitoring purposes), a controller (whose cost may be essentially zero if a digital system with spare capacity is used), and costs for installation and documentation. These costs are not usually significant compared to the benefits achieved through a properly designed cascade control strategy.

Cascade Control

- Two PIDs are cascaded if the output of one manipulates the setpoint of the other
- Each PID in a cascade chain has its own measured input, however only upstream PID can have an independent setpoint
- Only the ultimate secondary can have an output to the process



Conceptual block diagram of cascade control

Draw a block diagram with:

- Primary loop:
  - $y_{1,sp} \longrightarrow (+)$ summing junction → error $e_1 = y_{1,sp} - y_1$.
  - $e_1 \longrightarrow G_{c1}(s) \longrightarrow y_{2,sp}$.
- Secondary loop:
  - $y_{2,sp} \longrightarrow (+)$ summing junction → error $e_2 = y_{2,sp} - y_2$.
  - $e_2 \longrightarrow G_{c2}(s) \longrightarrow u$ (manipulated input to actuator).
  - $u \longrightarrow G_v(s) \longrightarrow G_{p2}(s) \longrightarrow y_2$.
- Primary process:
  - $u \longrightarrow G_{p1}(s) \longrightarrow y_1$.
- Disturbances:
  - d_2 entering at $G_{p2}(s)$ or $G_v(s)$.
  - d_1 entering at $G_{p1}(s)$.

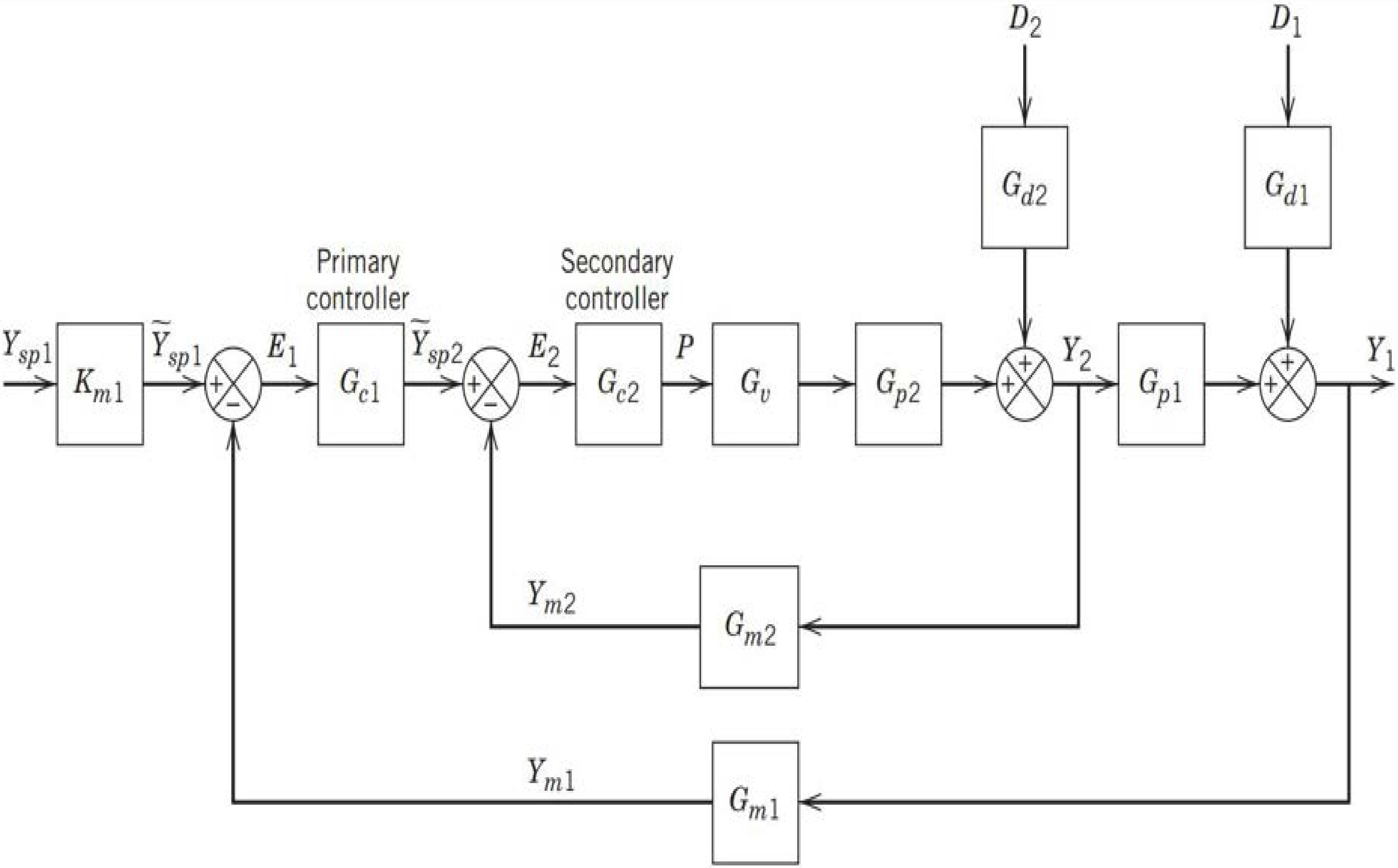


Figure 16.4 Block diagram of the cascade control system.

Simplified process models for analysis

- Use first-order (FOPDT) models for clarity:
- Inner (secondary) process (e.g. steam flow):

$$G_{p2}(s) = \frac{K_2}{\tau_2 s + 1} e^{-\theta_2 s}$$

- Primary (outer) process (e.g. tank temperature):

$$G_{p1}(s) = \frac{K_1}{\tau_1 s + 1} e^{-\theta_1 s}$$

- Actuator dynamics (valve + I/P):

$$G_v(s) = \frac{K_v}{\tau_v s + 1}$$

- Measurement elements (often ≈ 1):

$$G_{m1}(s),\ G_{m2}(s)$$

- PI controllers:

$$G_{c1}(s) = K_{c1}\left(1 + \frac{1}{\tau_{I1} s}\right), \qquad G_{c2}(s) = K_{c2}\left(1 + \frac{1}{\tau_{I2} s}\right)$$

Inner-loop closed-loop transfer function

- Close the secondary loop with its own feedback (Seborg Fig. 16.4):
- Inner-loop closed-loop transfer from $\tilde{Y}_{sp2}(s)$ to $Y_2(s)$:

$$\frac{Y_2(s)}{\tilde{Y}_{sp2}(s)} = \frac{G_{c2}(s)\,G_v(s)\,G_{p2}(s)}{1 + G_{c2}(s)\,G_v(s)\,G_{p2}(s)\,G_{m2}(s)}$$

- Define the inner closed-loop dynamics:

$$G_{2,\mathrm{cl}}(s) := \frac{G_{c2}(s)\,G_v(s)\,G_{p2}(s)}{1 + G_{c2}(s)\,G_v(s)\,G_{p2}(s)\,G_{m2}(s)}$$

- This behaves like an **effective actuator** as seen by the primary loop:

$$u_{\mathrm{eff}}(s) = Y_2(s) = G_{2,\mathrm{cl}}(s)\,\tilde{Y}_{sp2}(s)$$

- For ideal measurement $G_{m2}(s) \approx 1$:

$$G_{2,\mathrm{cl}}(s) = \frac{G_{c2}(s)\,G_v(s)\,G_{p2}(s)}{1 + G_{c2}(s)\,G_v(s)\,G_{p2}(s)}$$

Outer loop response with inner loop closed

- Equivalent process seen by the primary loop:

$$G_{\text{eq}}(s) = G_{p1}(s)\,G_{2,\text{cl}}(s)$$

- Primary closed-loop transfer function from $Y_{sp1}(s)$ to $Y_1(s)$:

$$\frac{Y_1(s)}{Y_{sp1}(s)} = \frac{G_{c1}(s)\,G_{\text{eq}}(s)\,G_{m1}(s)}{1 + G_{c1}(s)\,G_{\text{eq}}(s)\,G_{m1}(s)}$$

- Full cascade characteristic equation:

$$1 + G_{c2}G_vG_{p2}G_{m2} + G_{c1}G_{c2}G_vG_{p2}G_{p1}G_{m1} = 0$$

- With ideal sensors $G_{m1} \approx G_{m2} \approx 1$:

$$\frac{Y_1}{Y_{sp1}} = \frac{G_{c1}G_{\text{eq}}}{1 + G_{c1}G_{\text{eq}}}$$

Conditions for effective cascade control

For cascade control to significantly improve performance:

1. **Inner loop must be faster than outer loop**
   - Inner closed-loop time constant:

$$\tau_{\text{inner,cl}} \ll \tau_{\text{outer,cl}}$$

   - Rule of thumb: inner loop bandwidth about 3–5 times larger than outer loop bandwidth.
2. **Secondary variable must detect disturbances earlier**
   - The secondary PV y_2 should respond quickly to disturbances that ultimately affect y_1.
3. **Secondary measurement must be reliable and not excessively noisy**
   - High noise would force very low controller gains, reducing benefits.
4. **Strong, monotonic relationship between secondary PV and actuator**
   - Changes in valve position should produce predictable changes in y_2.

These are the main theoretical design conditions that justify cascade in practice.

ADVANTAGES OF CASCADE CONTROL

- Large improvement in performance when the secondary is much faster than primary
- Simple technology with PID algorithms
- Use of feedback at all levels. Primary has zero offset for "step-like" disturbances.
- Plant operating personnel find cascades easy to understand and operate. Open a cascade at one level, and all controllers above are inactive.
- PID tuning methods and correlations are applicable

Advantages of Cascade Control

Slave

- The secondary PID corrects the secondary loop disturbances before they affect the primary PID.
- Improves control quality and response speed of primary PID.

Master PID

Cascade PID

Single-loop Enhancements: Cascade Control Part 2

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Learning outcomes

After this lecture, you should be able to:

- Apply **design criteria** to decide when cascade control is beneficial.
- Use a **systematic procedure** to design and tune a cascade system.
- Interpret **industrial P&ID implementations** of cascade loops (heaters, boilers, reactors, motion systems).
- Recognise **practical implementation issues** in DCS/PLC (modes, bumpless transfer, failure handling).
- Identify typical **limitations and pitfalls** of cascade control.

When is cascade control worth using?

Cascade is *useful* when:

- Single-loop performance is **unsatisfactory** for important disturbances.
- There exists a **measurable intermediate variable** y_2 that:
  - Responds to disturbances **earlier** than y_1.
  - Is strongly and monotonically affected by actuator position.
- The inner loop can be tuned to be **significantly faster** than the primary loop (bandwidth ratio ≈ 3–5 or more).
- Sensor cost/complexity for y_2 is acceptable.

If these conditions are not met, cascade may give little benefit or even degrade performance.

Design criteria for choosing the secondary variable

For each candidate secondary variable y_2, check:

1. **Measurement feasibility**
   - Is a reliable transmitter available (range, accuracy, response time)?
2. **Dynamic advantage**
   - Does y_2 respond faster than y_1 to key disturbances?
   - Is the inner process G_{p2} at least 3–5× faster than G_{p1}?
3. **Disturbance sensitivity**
   - Do major disturbances act **through** G_{p2} and show up clearly in y_2?
4. **Actuator proximity**
   - Is y_2 directly affected by the actuator (valve, pump, motor)?
5. **Noise level**
   - Is the measurement noise low enough to allow reasonably high controller gain?

Students can use this as a **checklist table** in [illegible]ign problems (one column per candidate y_2).

Limitations and pitfalls of cascade control

- **No benefit or harm** when:
  - Secondary measurement is too noisy → low gain needed → weak inner loop.
  - Inner process not significantly faster than primary process.
  - Disturbances do not pass through the chosen secondary variable.
- **Potential problems:**
  - Poorly tuned secondary loop creates oscillations that propagate to primary loop.
  - Incorrect mode settings (e.g. secondary not in Cascade) → primary loop ineffective.
  - Mis-scaled or badly calibrated transmitters.

Industrial example – CST heater cascade

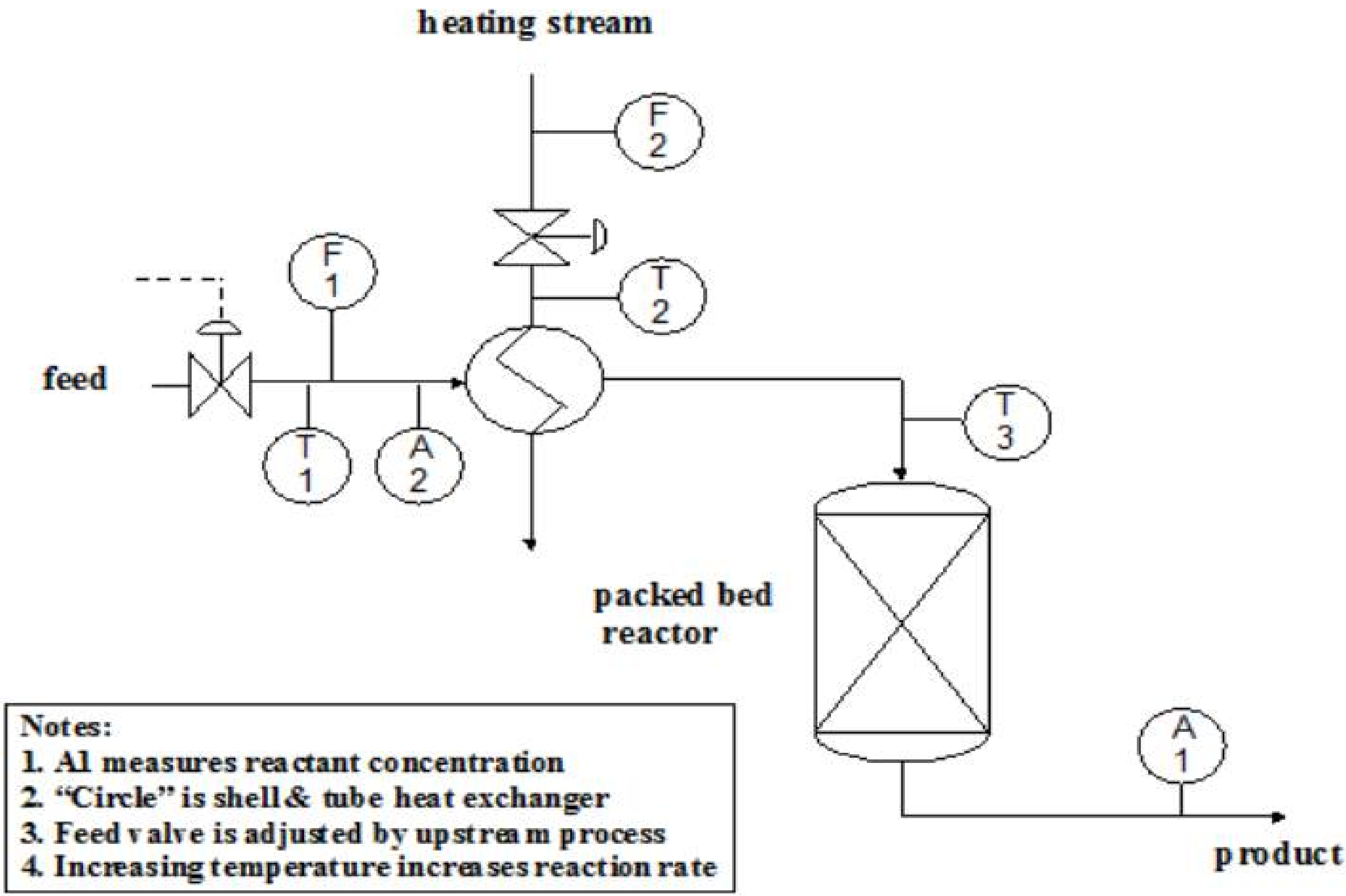
Revisit your CST heater P&ID:

- Primary loop:
  - Temperature controller (TC) → setpoint of flow controller (FC).
  - PV: outlet temperature T.
- Secondary loop:
  - Flow controller (FC) → steam valve.
  - PV: steam flow F_{steam}.
- Disturbances:
  - Steam supply pressure, feed-temperature variations.

Use design checklist:

- **Measurement**: steam flow transmitter available, reasonably fast.
- **Dynamic benefit**: flow responds quickly; temperature slower (mixing).
- **Disturbance path**: steam-pressure changes strongly affect flow.
- **Conclusion**: flow is an excellent secondary variable here.

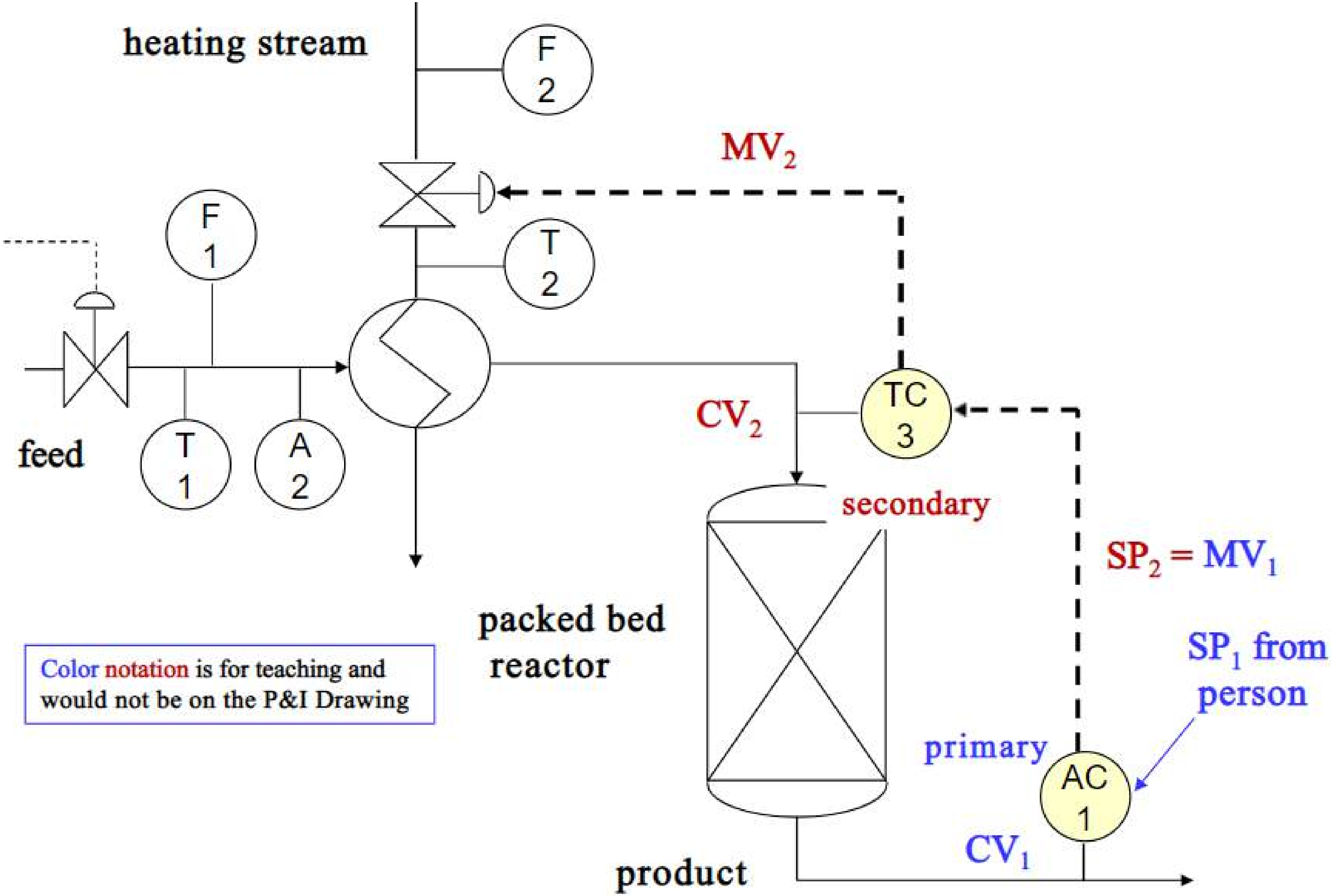
Cascade control design for packed bed chemical reactor – process introduction.



Evaluation of potential secondary variables

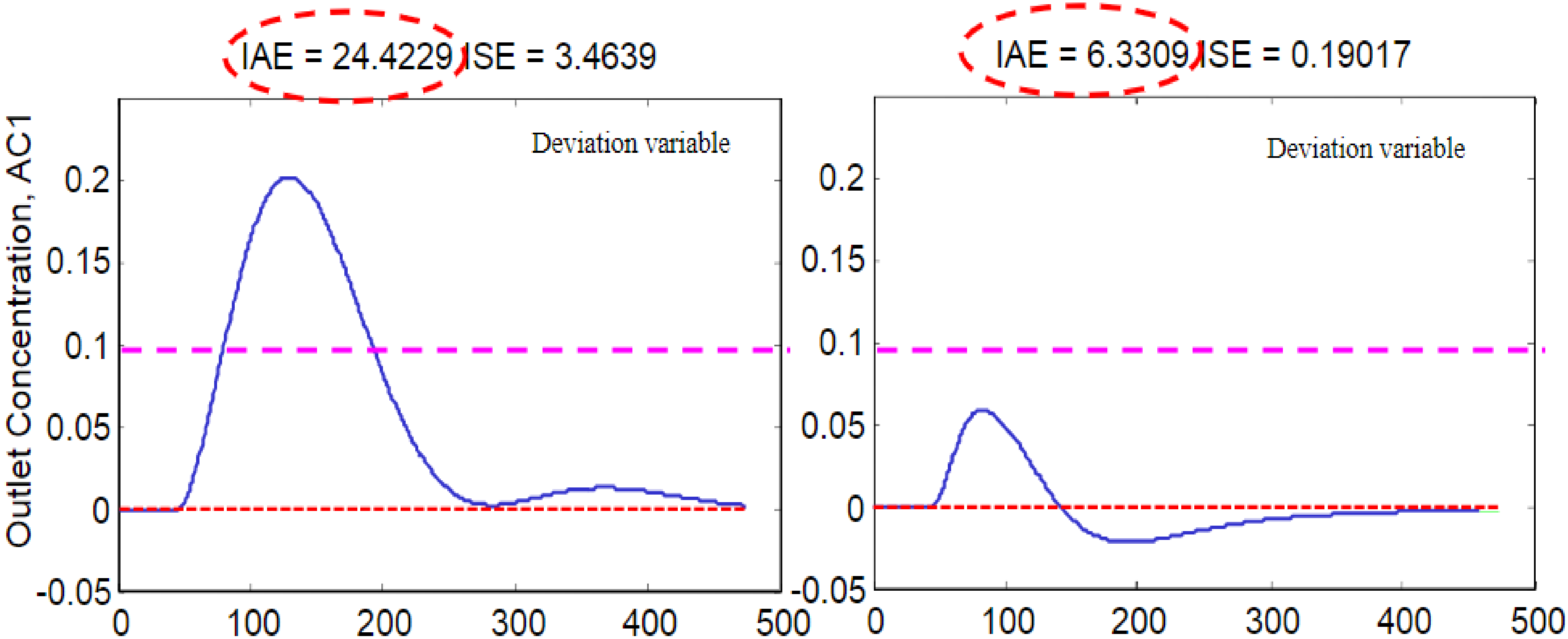
Criterion	A2	F1	F2	T1	T2	T3
1. Single-loop control is not satisfactory	Y	Y	Y	Y	Y	**Y**
2. Variable is measured	Y	Y	Y	Y	Y	**Y**
3. Indicates a key disturbance	N	N	N	N	Y	**Y**
4. Influenced by MV	N	N	Y	N	N	**Y**
5. Secondary dynamics faster	N/A	N/A	Y	N/A	N/A	**Y**

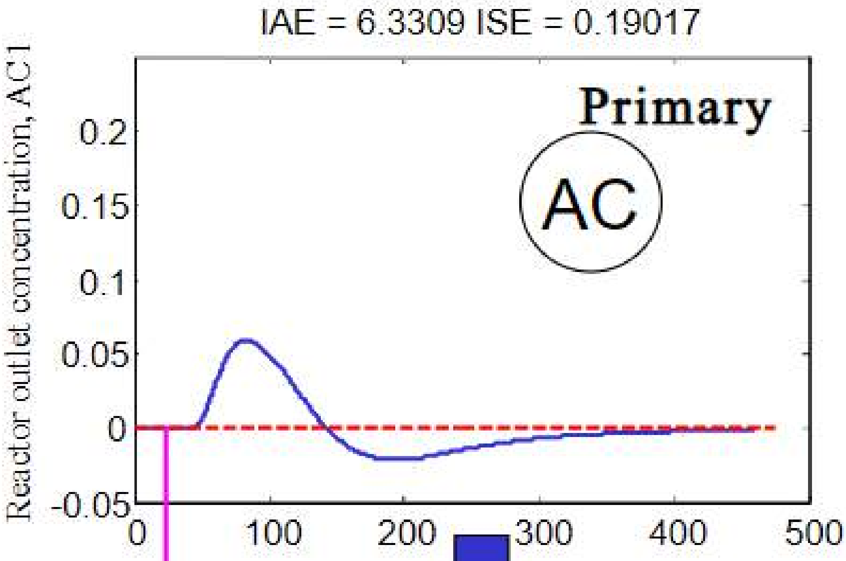
[redacted]

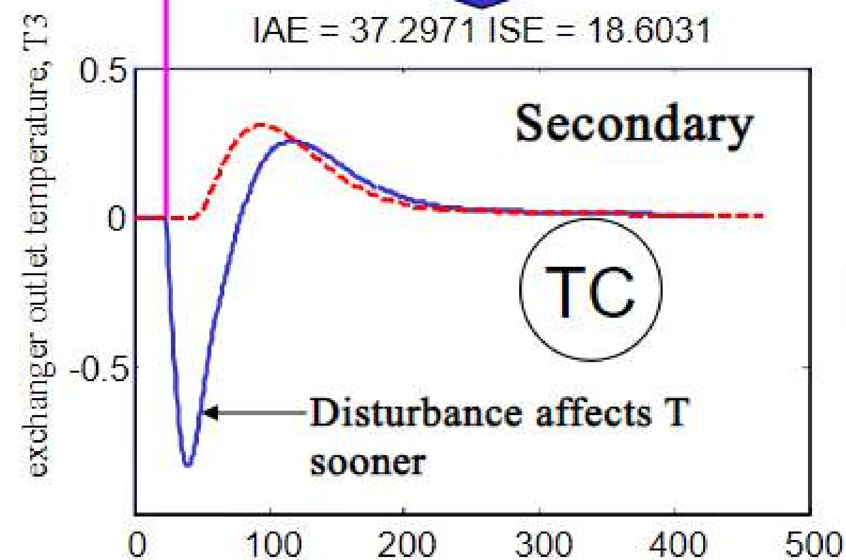

heating stream
F 2
MV2
F 1
T 2
TC 3
CV2
feed
T 1
A 2
secondary
SP2 = MV1
packed bed reactor
SP1 from person
primary
AC 1
CV1
product
Color notation is for teaching and would not be on the P&I Drawing

Control Performance Comparison for Packed Bed Reactor

Single-Loop / Cascade



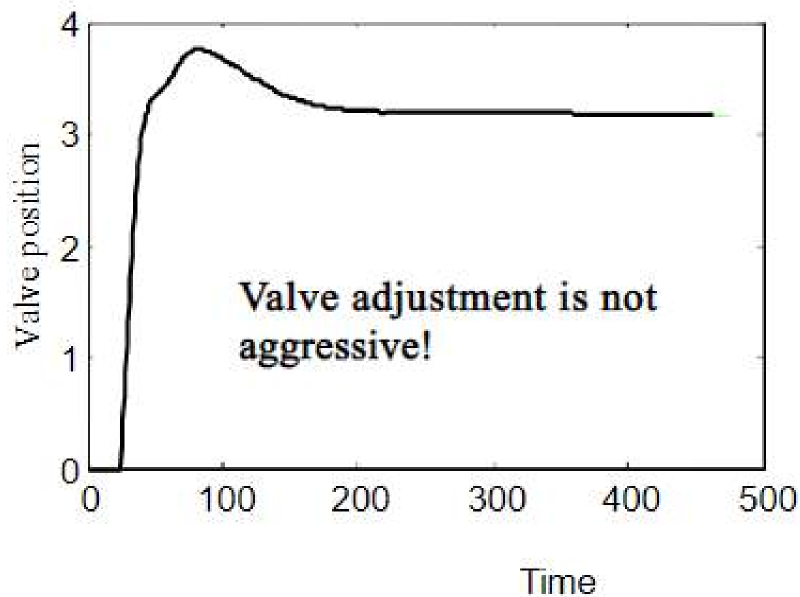




Return to question

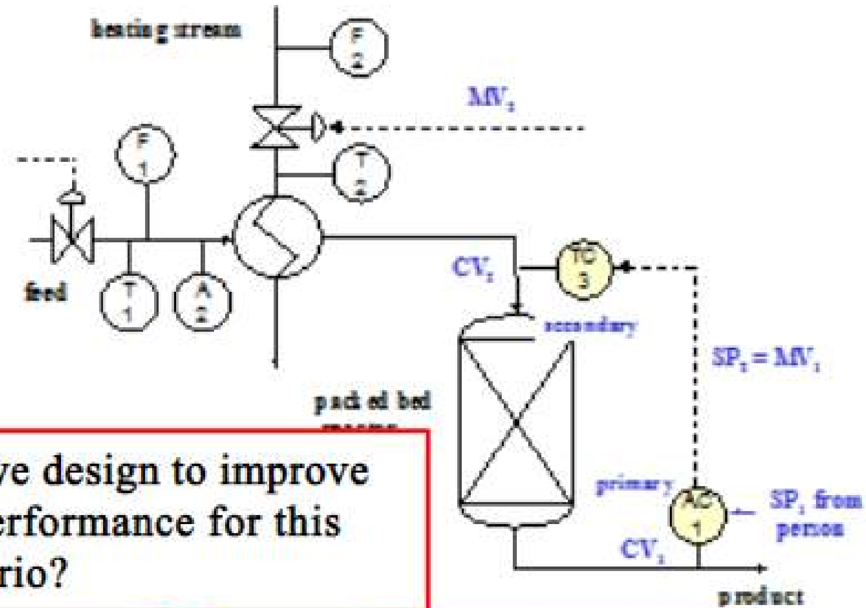
Summary

- T3 measurement responds quickly
- T3 controlled quickly
- Analyzer controller gets offset = 0
- Valve adjustment is moderate



Scenario Conclusions

- Cascade improves for several disturbances
- It does not degrade other scenarios



Can we design to improve the performance for this scenario?

- A disturbance in T1 — Cascade better
- A disturbance in heating medium inlet pressure — Cascade better
- A disturbance in feed pressure — Cascade better, but not "perfect"
- A disturbance to feed composition, A2 — Both the same
- A change to the AC-1 set point — Both the same

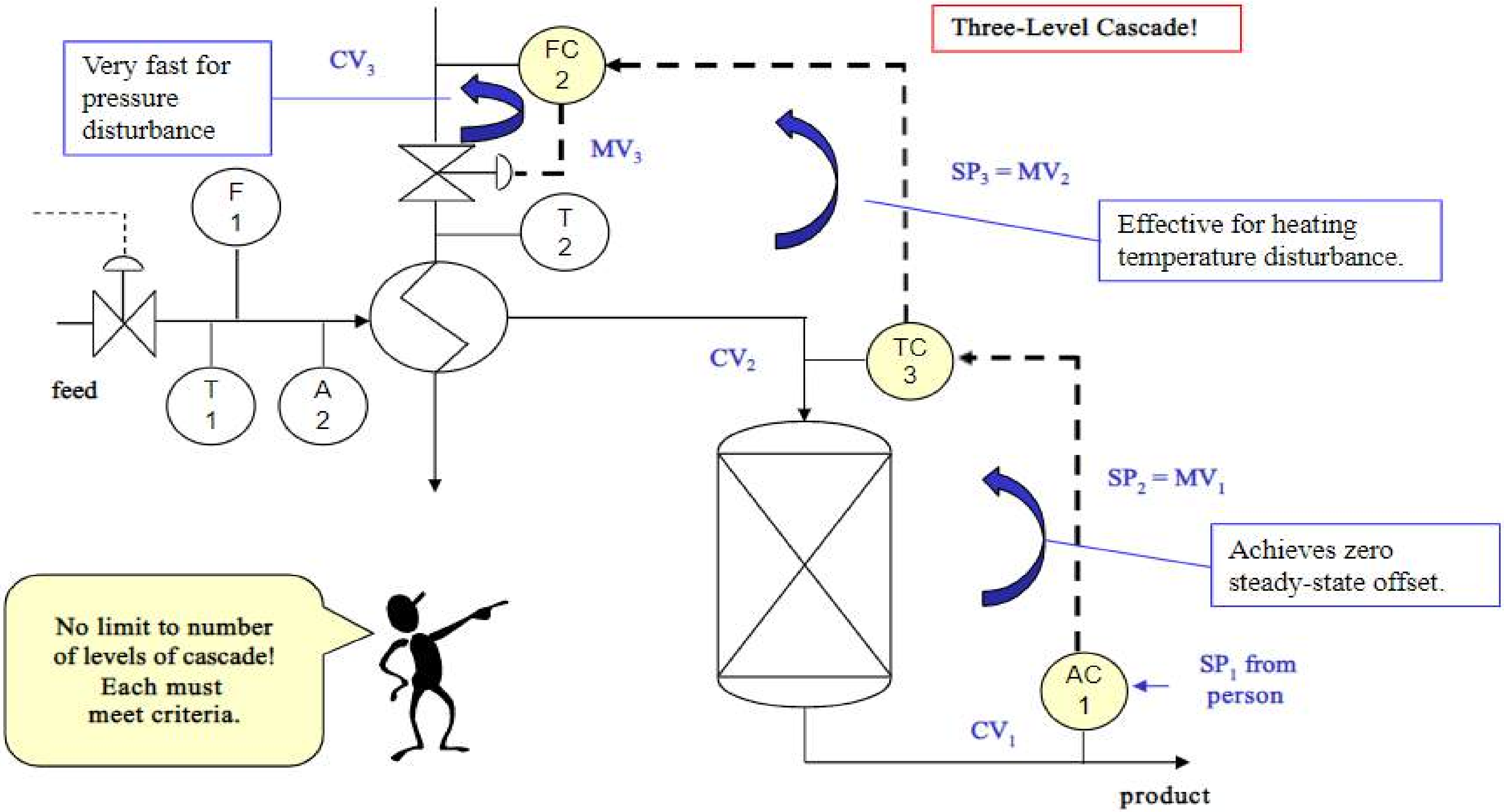
Feed temperature, T1. A change in the feed temperature affects the outlet concentration through its influence on the reactor inlet temperature, T3. Therefore, the cascade controller is effective in attenuating the feed temperature disturbance.

Heating oil pressure (not measured). A change in the oil pressure influences the oil flow and, therefore, the heat transferred. As a result the reactor inlet temperature, T3, is affected. Again, the cascade controller is effective in attenuating the oil pressure disturbance.

Feed flow rate, F1. A change in the feed flow rate influences the reactor outlet concentration in two ways: it changes the inlet temperature T3, and it changes the residence time in the reactor. The cascade controller is effective in attenuating the effect of the disturbance on T3 but is not effective in compensating for the residence time change. The residence time effect must be compensated by the primary controller, AC-1.

Feed composition, A2. A change in the feed composition clearly changes the reactor outlet concentration. The cascade has no effect on the feed composition disturbance, because the composition does not influence T3. Therefore, this disturbance must be totally compensated by the primary feedback controller, AC-1.

Improved solution for heating medium pressure disturbance



Systematic design procedure (overview)

1. **Define primary loop** – choose the main controlled variable y_1.
2. **List candidate secondary variables** y_2.
3. **Apply design criteria** to select the best y_2.
4. **Obtain a dynamic model** for the inner process (step test → FOPDT/SOPDT).
5. **Tune the secondary controller** G_{c2} with outer loop open.
6. **Close inner loop** and identify equivalent process $G_{\text{eq}} = G_{p1}G_{2,\text{cl}}$.
7. **Tune the primary controller** G_{c1} for desired overall performance.
8. **Check interaction, robustness, and constraints** (saturation, noise, etc.).

This 8-step procedure is consistent with standard treatments in Seborg and Marlin.

Inner-loop identification (step test idea)

- Put outer loop in **manual**; secondary controller in **automatic** but with local setpoint.
- Perform a **small step** in controller output (or setpoint) of the secondary loop.
- Record $y_2(t)$ and fit a **FOPDT model**:

$$G_{p2}(s) \approx \frac{K_2}{\tau_2 s + 1} e^{-\theta_2 s}$$

- Check:
  - Is the process sufficiently fast?
  - Is the dead time θ_2 small compared with τ_2?
- These parameters will be used to tune G_{c2} (PI).

Tuning the secondary controller G_{c2}

Goal: **fast, well-damped** inner loop without excessive actuator movement.

- Choose a target closed-loop time constant $\tau_{2,\mathrm{cl}}$.
  - Rule of thumb: $\tau_{2,\mathrm{cl}} \approx \frac{1}{3}\tau_{1,\mathrm{cl}}$ (outer loop).
- Use standard PI tuning rule for FOPDT (e.g. IMC-based or modified Ziegler–Nichols with conservative settings).
- Example IMC-style tuning for inner loop (no dead time, simple form):

$$K_{c2} \approx \frac{\tau_2}{K_2(\lambda_2)}, \qquad \tau_{I2} \approx \tau_2$$

  where λ_2 is desired closed-loop time constant ($\approx \tau_{2,\mathrm{cl}}$).
- Check step response of inner loop; adjust if oscillatory.

(You can choose your preferred tuning correlation to match the rest of your course.)

Time-scale separation requirement

- With inner loop closed:

$$G_{\mathrm{eq}}(s) = G_{p1}(s) G_{2,\mathrm{cl}}(s)$$

- To ensure stability and good behaviour, want:

$$\tau_{2,\mathrm{cl}} \ll \tau_{1,\mathrm{cl}}$$

often $\tau_{2,\mathrm{cl}} \approx \frac{1}{3}\tau_{1,\mathrm{cl}}$ or faster.

- Intuition:
  - Inner loop corrects disturbances almost "instantaneously" from primary perspective.
  - Primary loop then only sees the *slow* tank/plant dynamics.

Tuning the primary controller G_{c1}

With inner loop closed and tuned:

- Treat combined dynamics as a single process:
  - Perform a **setpoint step** on Y_{sp1} with G_{c1} in manual to identify $G_{\text{eq}}(s)$.
- Fit FOPDT to $y_1(t)$ versus Y_{sp1} or versus u_{eff}.
- Use the same tuning philosophy (IMC/other) to compute PI parameters for G_{c1}.
- Check that:
  - Outer loop is slower than inner loop.
  - No excessive overshoot or oscillation.
  - Valve motion remains acceptable.

Small numerical example – CST heater

(Set up as worked example or homework.)

Assume:

- Inner process (steam flow):

$$G_{p2}(s) = \frac{1}{2s+1}$$

- Primary process (tank temperature):

$$G_{p1}(s) = \frac{1}{20s+1}$$

- Valve $G_v(s) \approx 1$, sensors ideal.

Tasks for students:

1. Choose a secondary PI G_{c2} to achieve $\tau_{2,\mathrm{cl}} \approx 2\,\mathrm{s}$.
2. With inner loop closed, compute or identify $G_{\mathrm{eq}}(s)$.
3. Design primary PI G_{c1} for $\tau_{1,\mathrm{cl}} \approx 10\,\mathrm{s}$.
4. Compare **single-loop vs cascade** responses to:
   - Step in Y_{sp1} (servo).
   - Disturbance in steam pressure (modelled as bias change at inner process).

Industrial example – packed-bed reactor

- Primary variable y_1: reactor outlet composition or temperature.
- Candidate secondary variables:
  - Jacket inlet temperature.
  - Jacket outlet temperature.
  - Jacket flow rate.
- Discussion (qualitative):
  - Many disturbances (steam supply, coolant temperature) first appear in jacket variables.
  - Jacket temperature and flow are close to actuator and respond faster than reactor composition.
- Students can fill a **comparison table**:
  - Which candidate best satisfies measurement, dynamic, and disturbance criteria?

Industrial example – boiler drum level / feedwater cascade

- Primary loop:
  - Drum level controller (LC) → setpoint of feedwater flow controller (FC).
- Secondary loop:
  - Flow controller (FC) → feedwater valve.
- Why cascade?
  - Drum level is slow, integrating; direct single loop can be sluggish and sensitive.
  - Feedwater flow reacts quickly to steam demand changes.
  - Cascade improves disturbance rejection and allows safer operation in power plants.

Industrial example – motion/servo cascade

- Structure (common in drives and robotics):
  - Innermost: **current/torque loop** – very fast.
  - Middle: **velocity loop**.
  - Outer: **position loop**.
- Interpretation:
  - Position loop (outer) → velocity setpoint.
  - Velocity loop (middle) → current/torque setpoint.
- Same principle:
  - Fast inner loop rejects torque disturbances (load changes) before they corrupt position tracking.

Implementation in DCS/PLC – block-level view

- Two PID function blocks in series:
  - **Secondary PID:**
    - Input: secondary PV (y_2).
    - Output: MV (valve, pump speed, etc.).
  - **Primary PID:**
    - Input: primary PV (y_1).
    - Output: **setpoint** of secondary PID ($y_{2,sp}$).
- Modes typically available:
  - **Primary**: Auto / Manual (and sometimes Cascade from a higher-level loop).
  - **Secondary**: Auto / Cascade / Manual.

Implementation details – bumpless transfer

- When switching modes, we need **bumpless transfer:**
  - Example: Manual → Auto for secondary PID:
    - Initialize integral term so that controller output equals current MV.
  - Example: Auto → Cascade:
    - Track secondary setpoint to current PV before handing control to primary loop.
- Without bumpless transfer:
  - Large sudden changes in valve position or setpoint.
  - Risk of process upsets and trips.

Failure handling and safe operation

- If secondary transmitter fails (or goes into fault):
  - Break cascade:
    - Secondary loop may be forced to Manual or local Auto with fixed SP.
    - Primary controller may be placed in Manual or switched to a fallback single-loop mode (if available).
- Interlocks and overrides:
  - High/low pressure or temperature overrides may take over the valve.
  - Safety instrumented functions (SIS) remain separate from normal cascade logic.

Students should understand that **engineering design** includes these failure modes, not just the ideal mathematics.

Summary

- Cascade control enhances single-loop feedback by using a **fast inner loop** around a carefully chosen intermediate variable.
- Design is driven by:
  - **Dynamic considerations** (time-scale separation).
  - **Disturbance paths** and measurement availability.
- Systematic procedure:
  - Select secondary variable → identify inner model → tune inner → identify equivalent outer model → tune outer.
- Industrially, cascade is ubiquitous in:
  - Temperature control (heaters, reactors).
  - Level–flow systems (boilers, tanks).
  - Motion and drive control.
- Correct DCS/PLC implementation (modes, bumpless transfer, interlocks) is essential for safe and robust operation.

Example – Process description and models

- Process:
  - Continuous stirred-tank heater with steam coil.
  - Inner loop: steam flow control.
  - Outer loop: outlet temperature control.
- Simplified linear models (step tests around nominal operating point):
  - Inner (secondary) process – steam flow dynamics

$$G_{p2}(s) = \frac{K_2}{\tau_2 s + 1} = \frac{1}{3s + 1}$$

  units: (kg/s of steam) / (% valve opening)

  - Primary (outer) process – tank temperature dynamics

$$G_{p1}(s) = \frac{K_1}{\tau_1 s + 1} = \frac{0.5}{30s + 1}$$

  units: (°C) / (kg/s of steam)

- Assume:
  - Valve/I-P: $G_v(s) \approx 1$ (fast)
  - Measurements: $G_{m1}(s) = G_{m2}(s) = 1$ (ideal)
  - Both controllers are PI:

$$G_{c1}(s) = K_{c1}\left(1 + \frac{1}{\tau_{I1} s}\right), \quad G_{c2}(s) = K_{c2}\left(1 + \frac{1}{\tau_{I2} s}\right)$$

(You can say that numbers are adapted from typical examples in Seborg and Marlin.)

Design objectives (example)

- Inner (steam flow) loop:
  - Fast, well-damped response.
  - Target closed-loop time constant:

$$\tau_{2,\mathrm{cl}} \approx 2\ \mathrm{s}$$

- Outer (temperature) loop:
  - Smooth response, moderate speed.
  - Target closed-loop time constant:

$$\tau_{1,\mathrm{cl}} \approx 10\ \mathrm{s}$$

- Rule of thumb respected:

$$\tau_{2,\mathrm{cl}} \approx \frac{1}{5}\,\tau_{1,\mathrm{cl}}$$

$\Rightarrow$ inner loop $\approx 5\times$ faster than outer loop.

Step 1 – Tune the inner (flow) PI controller

- Model for inner process:

$$G_{p2}(s) = \frac{1}{3s+1}$$

- Use a simple IMC-style PI tuning (no dead time) with $\lambda_2 = \tau_{2,\text{cl}} = 2$ s:

$$K_{c2} \approx \frac{\tau_2}{K_2\,\lambda_2} = \frac{3}{1\cdot 2} = 1.5$$

$$\tau_{I2} \approx \tau_2 = 3\ \text{s}$$

- So choose:

$$G_{c2}(s) = 1.5\left(1 + \frac{1}{3s}\right)$$

- Closed-loop inner transfer (ideal sensor, $G_v = 1$):

$$G_{2,\text{cl}}(s) = \frac{G_{c2}G_{p2}}{1 + G_{c2}G_{p2}}$$

Substituting:

$$G_{c2}G_{p2} = 1.5\left(1 + \frac{1}{3s}\right)\frac{1}{3s+1}$$

(You don't need to simplify algebra fully in class; you can just show simulated response is ~2 s, first-order-like.)

- Qualitative result:
  - Flow responds quickly (rise time ≈ 2–3 s).
  - Minimal overshoot.

Step 2 – Equivalent process seen by the primary loop

- With inner loop closed:

$$u_{\text{eff}}(s) = Y_2(s) = G_{2,\text{cl}}(s)\, Y_{sp2}(s)$$

- Equivalent process for outer loop:

$$G_{\text{eq}}(s) = G_{p1}(s)\, G_{2,\text{cl}}(s)$$

- Approximation (for lecture level):

  If inner loop is tuned to be significantly faster than primary dynamics, then over the slower temperature time scale we can approximate:

$$G_{2,\text{cl}}(s) \approx 1$$

$\Rightarrow$

$$G_{\text{eq}}(s) \approx G_{p1}(s) = \frac{0.5}{30s+1}$$

- Interpretation:
  - From the outer controller's point of view, the "actuator" is now an almost ideal flow source with gain 1 and negligible dynamics.
  - Remaining dynamics are dominated by G_{p1} (tank).

(If you want more rigour, you can show a Bode plot where $G_{2,\text{cl}} \approx 1$ for ω in the outer loop bandwidth

Step 3 – Tune the primary (temperature) PI controller

- Equivalent process for tuning:

$$G_{\mathrm{eq}}(s) \approx \frac{0.5}{30s+1}$$

- Choose target closed-loop time constant:

$$\lambda_1 = \tau_{1,\mathrm{cl}} = 10 \text{ s}$$

- IMC-style PI tuning (FOPDT without dead time):

$$K_{c1} \approx \frac{\tau_1}{K_1\,\lambda_1} = \frac{30}{0.5 \cdot 10} = \frac{30}{5} = 6$$

$$\tau_{I1} \approx \tau_1 = 30 \text{ s}$$

- So choose:

$$G_{c1}(s) = 6\left(1 + \frac{1}{30s}\right)$$

- Closed-loop TF from temperature SP to temperature:

$$\frac{Y_1(s)}{Y_{sp1}(s)} = \frac{G_{c1}G_{\mathrm{eq}}}{1+G_{c1}G_{\mathrm{eq}}} \approx \frac{6\cdot\frac{0.5}{30s+1}}{1+6\cdot\frac{0.5}{30s+1}} = \frac{3}{30s+1+3} = \frac{3}{30s+4}$$

$\Rightarrow$ approximate closed-loop time constant: ↓

$$\tau_{1,\mathrm{cl}} \approx \frac{30}{4} \approx 7.5 \text{ s}$$

(close to the design target 10 s; tuning can be slightly relaxed if you want exactly 10 s.)

Step 4 – Compare single-loop vs cascade (qualitative)

- If we used a **single temperature loop** directly on the valve, process would be:

$$G_{\text{single}}(s) = G_v G_{p2} G_{p1} = \frac{1}{3s+1} \cdot \frac{0.5}{30s+1}$$

  - Much slower combined dynamics.
  - Harder to tune aggressively without oscillations.
- With cascade:
  - Inner flow loop removes most of G_{p2} dynamics.
  - Outer loop sees an "almost ideal" actuator and can be tuned faster with similar robustness.
- For disturbances (e.g. steam pressure changes):
  - Inner loop: attenuates disturbance via $1 + G_{c2}G_{p2}$.
  - Outer loop: corrects the remaining effect on temperature.

You can show actual MATLAB step responses in class to visualise the differences.

2. Example – Cascade control implemented in a single PLC/DCS

PLC/DCS cascade architecture in one controller

- In a real PLC or DCS, both PIDs usually sit in the **same controller**:
  - **Secondary (inner) PID block:**
    - Input: secondary PV (e.g. flow).
    - Output: MV (e.g. valve position).
    - Has its own local SP input: `SP2`.
  - **Primary (outer) PID block:**
    - Input: primary PV (e.g. temperature).
    - Output: **setpoint** for secondary PID: `SP2`.
- Operating modes:
  - Primary PID: normally in **AUTO** (closed loop).
  - Secondary PID: normally in **CASCADE** mode:
    - SP = output of primary PID.
  - Operator can switch secondary to:
    - **AUTO** (local SP) or **MANUAL** (direct MV).
- Key concept:
  - The PLC just executes two PID algorithms sequentially every scan, with the data link:

$$SP_2 \downarrow = \mathrm{OUT}_1(k)$$

Bumpless transfer and anti-windup in PLC

- **Bumpless transfer** example (secondary AUTO → CASCADE):
  - When switching from AUTO to CASCADE:
    - Initialize integral term of secondary PID so that its output equals current MV.
    - Optionally, ramp `SP2` from old value to new cascade SP to avoid a sudden jump.
- **Anti-windup:**
  - If MV saturates at 0% or 100%:
    - Freeze integral term or use back-calculation to prevent windup.
  - Both PIDs usually provide parameters like:
    - `TRK_IN_D` (track input in manual).
    - `BKCAL` (back-calculation signal for anti-windup).

You don't need to go deep into vendor-specific tags, just explain the idea.

Example – configuration in one PLC scan

- PLC scan cycle (every, say, 100 ms):
  1. Read all A/D inputs (PV1, PV2).
  2. Execute **primary PID**:
     - Compute OUT1 (temperature controller output).
  3. Depending on mode, assign SP2 for secondary loop:
     - `SP2 = OUT1` in CASCADE mode.
  4. Execute **secondary PID**:
     - Compute MV (valve command).
  5. Write MV to D/A output (I/P, valve).
  6. Handle alarms, interlocks, and mode changes.
- All of this is done **inside one PLC**, but logically it is the same structure analysed in Lecture 1:
  - Primary loop: $Y_{sp1} \rightarrow G_{c1} \rightarrow SP_2 \rightarrow Y_1$
  - Secondary loop: $SP_2 \rightarrow G_{c2} \rightarrow MV \rightarrow Y_2$

Second numeric example – setup

- We use a very simple model to see the **effect on the characteristic polynomial**:
- Primary (outer) process:

$$G_{p1}(s) = \frac{K_1}{\tau_1 s + 1}$$

- Secondary (inner) process:

$$G_{p2}(s) = \frac{1}{\tau_2 s + 1}$$

- Actuator and measurements assumed ideal:

$$G_v(s) = 1, \quad G_{m1}(s) = G_{m2}(s) = 1$$

- Controllers (for algebra simplicity we use **P only**):
  - Single-loop: one gain K_c.
  - Cascade: outer gain K_{c1}, inner gain K_{c2}.
- Goal: derive the **closed-loop characteristic polynomial** in both cases and compare coefficients.

(Structure consistent with the analysis of cascade in Seborg, Ch. 16, but specialized to P-only for clarity.)

Single-loop control – characteristic polynomial

- Single-loop structure:
  - One controller with gain K_c.
  - Open-loop transfer:

$$L_{\text{single}}(s) = K_c\, G_{p1}(s)\, G_{p2}(s) = K_c \frac{K_1}{\tau_1 s + 1} \frac{1}{\tau_2 s + 1}$$

- Closed-loop characteristic equation:

$$1 + L_{\text{single}}(s) = 0$$

- Put over a common denominator:

$$1 + \frac{K_c K_1}{(\tau_1 s + 1)(\tau_2 s + 1)} = \frac{(\tau_1 s + 1)(\tau_2 s + 1) + K_c K_1}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

- So the **characteristic polynomial** (numerator) is:

$$(\tau_1 s + 1)(\tau_2 s + 1) + K_c K_1 = \tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + (1 + K_1 K_c)$$

- Single-loop:

$$a_2 s^2 + a_1 s + a_0 = \tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + (1 + K_1 K_c)$$

Cascade control – characteristic polynomial

- Cascade structure (P–P for simplicity):
  - Inner loop controller: gain K_{c2}.

$$G_{2,\text{cl}}(s) = \frac{K_{c2}\,G_{p2}}{1 + K_{c2}\,G_{p2}}$$

  - Equivalent process seen by outer loop:

$$G_{\text{eq}}(s) = G_{p1}(s)\,G_{2,\text{cl}}(s)$$

  - Outer loop gain: K_{c1}.

    Open-loop:

$$L_{\text{casc}}(s) = K_{c1}G_{\text{eq}}(s)$$

- Direct algebra (keeping $G_v = G_{m1} = G_{m2} = 1$) gives:

$$1 + L_{\text{casc}}(s) = \frac{K_1K_{c1}K_{c2} + (\tau_1 s + 1)(K_{c2} + \tau_2 s + 1)}{(\tau_1 s + 1)(K_{c2} + \tau_2 s + 1)}$$

- The **characteristic polynomial** (numerator) is:

$$K_1 K_{c1} K_{c2} + (\tau_1 s + 1)(K_{c2} + \tau_2 s + 1)$$
$$= \tau_1 \tau_2 s^2 + \left[\tau_1 (1 + K_{c2}) + \tau_2\right] s + \left[1 + K_{c2} + K_1 K_{c1} K_{c2}\right]$$

- So for cascade:

$$a_2^{(\text{casc})} = \tau_1 \tau_2$$

$$a_1^{(\text{casc})} = \tau_1 (1 + K_{c2}) + \tau_2$$

$$a_0^{(\text{casc})} = 1 + K_{c2} + K_1 K_{c1} K_{c2}$$

- Compare with single-loop:
  - The s^2 **coefficient** is the same ($\tau_1 \tau_2$), because the physical inertias are the same.
  - The s **coefficient** is increased by the factor $(1 + K_{c2})$ on τ_1.
  - The **constant term** gets extra "gain" contributions from K_{c2} and $K_{c1} K_{c2}$.

This is exactly the mathematical expression of **extra damping and stiffness** introduced by a well-tuned inner loop.

Plugging numbers – single-loop case

Let's fix a specific process:

- $\tau_1 = 10$ s (slow tank), $\tau_2 = 2$ s (faster inner dynamics).
- $K_1 = 1$.
- Single-loop controller gain: $K_c = 1$.

Then the single-loop characteristic polynomial is:

$$a_2 s^2 + a_1 s + a_0 = 10 \cdot 2\, s^2 + (10 + 2)s + (1 + 1 \cdot 1) = 20s^2 + 12s + 2$$

Normalised form:

$$s^2 + 0.6s + 0.1$$

- Poles (numerically):

$$s_{1,2} \approx -0.3 \pm 0.1j$$

- Equivalent 2nd-order parameters:

$$\omega_n^2 = 0.1 \quad \Rightarrow \quad \omega_n \approx 0.316$$

$$2\zeta\omega_n = 0.6 \quad \Rightarrow \quad \zeta \approx 0.95$$

- Interpretation:
  - Natural frequency ≈ 0.32 rad/s.
  - Damping ratio ≈ 0.95 (mildly underdamped, very little overshoot).
  - Both poles on the real part ≈ −0.3 ⇒ relatively slow response.

Plugging numbers – cascade case

Use the same process parameters:

- $\tau_1 = 10$ s, $\tau_2 = 2$ s, $K_1 = 1$.

Choose cascade controller gains:

- Inner gain: $K_{c2} = 3$ (relatively tight inner loop).
- Outer gain: $K_{c1} = 1$ (moderate primary gain).

Characteristic polynomial for cascade:

$$a_2^{(\text{casc})} = \tau_1 \tau_2 = 20$$
$$a_1^{(\text{casc})} = \tau_1(1 + K_{c2}) + \tau_2 = 10(1+3) + 2 = 42$$
$$a_0^{(\text{casc})} = 1 + K_{c2} + K_1 K_{c1} K_{c2} = 1 + 3 + 1 \cdot 1 \cdot 3 = 7$$

So the characteristic polynomial is:

$$20s^2 + 42s + 7 \quad \Rightarrow \quad s^2 + 2.1s + 0.35$$

- Poles:

$$s_{1,2} \approx -1.92, \ -0.18$$

- One pole is **fast** (≈ −1.9), one is **slow** (≈ −0.18), both real ⇒ **overdamped**.

Compute 2nd-order parameters:

$$\omega_n^2 = 0.35 \ \Rightarrow \ \omega_n \approx 0.592$$

$$2\zeta\omega_n = 2.1 \ \Rightarrow \ \zeta \approx \frac{2.1}{2 \cdot 0.592} \approx 1.77 > 1$$

- Interpretation:
  - Natural frequency increased (≈ 0.59 vs 0.32).
  - Damping ratio > 1 ⇒ **overdamped**, no oscillations.
  - One mode is much faster, reflecting the fast inner loop dynamics.

Comparative interpretation

- **Same physical process**, same time constants τ_1, τ_2, same plant gains.
- Single-loop:

$$s^2 + 0.6s + 0.1$$

  - Slower real part (~ −0.3), lightly underdamped.
- Cascade:

$$s^2 + 2.1s + 0.35$$

  - Larger s-coefficient → effectively more damping.
  - Larger constant term → higher "stiffness" / restoring effect.
  - One fast mode and one slow mode, both stable and non-oscillatory.

This example makes visible in the characteristic polynomial what we argue qualitatively in the lecture:

> A well-tuned inner loop "moves" one pole farther left and increases the effective damping of the overall system, enabling faster, well-behaved outer-loop tuning on the same physical plant.

Cascade PID Tuning Guidelines

1. Inside Out Procedure:
   - Start tuning the ultimate secondary PID
   - Tune primary PID (master) last with all secondary (slave) PIDs active (closed-loop)

2. Secondary PID (slave) tuning guidelines:
   - Tune for fast setpoint response
   - Proportional action based on Error, not PV
   - Use PI algorithm, or PID in some cases
   - Avoid oscillatory response

Cascade PID Tuning Guidelines (more)

3. Primary PID (master) tuning guidelines:
   Try scan rate >= 4 times secondary's
   e.g., scan rate of FC = 0.5 sec
   TC = 5.0 sec
   AC = 180.0 sec

4. Allow secondary PID to meet its control objective before primary PID changes secondary PID's setpoint once again.

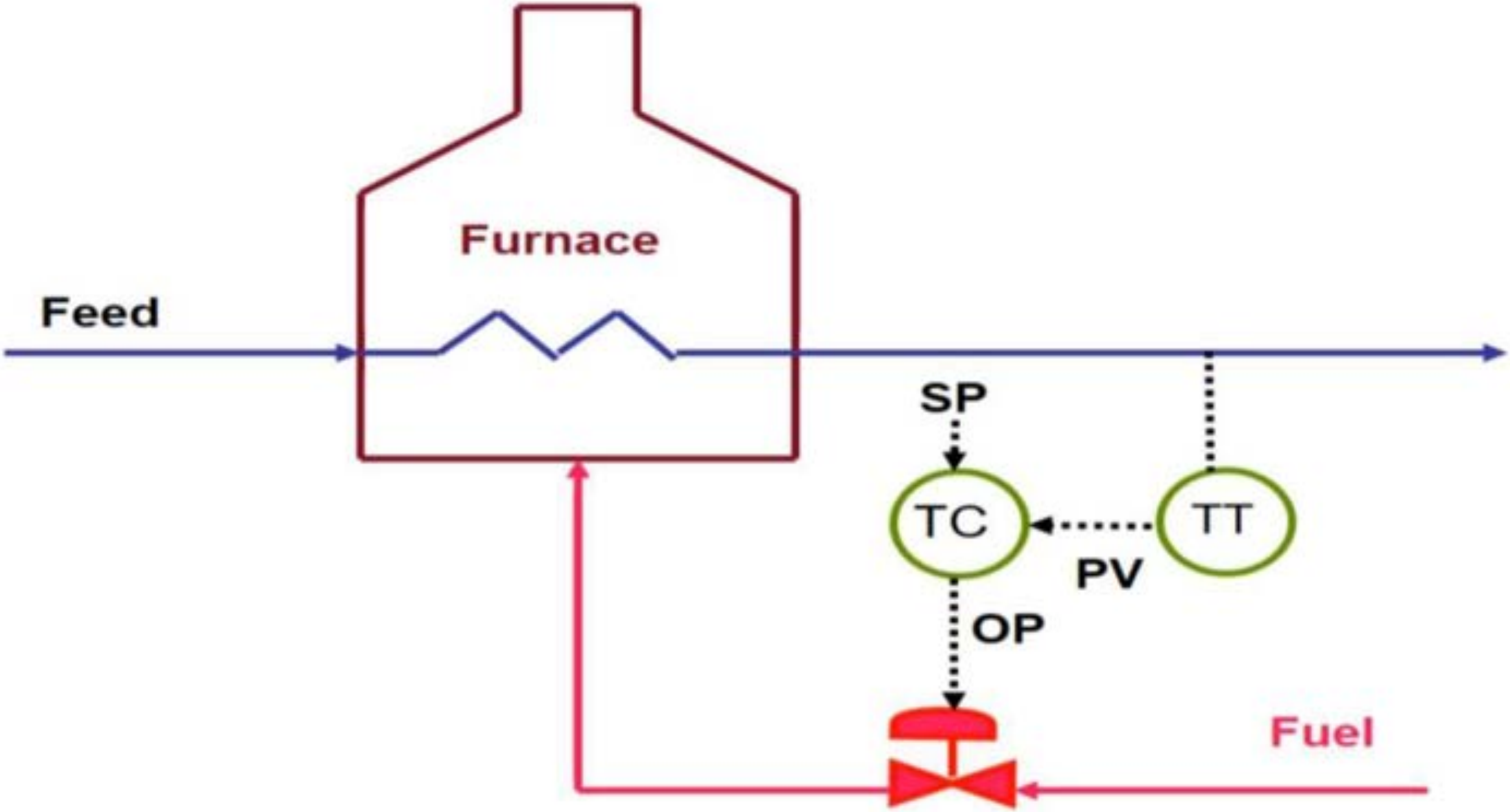
5. Retune primary PID after changes in secondary PID.

Cascade PID Tuning Guidelines (more)

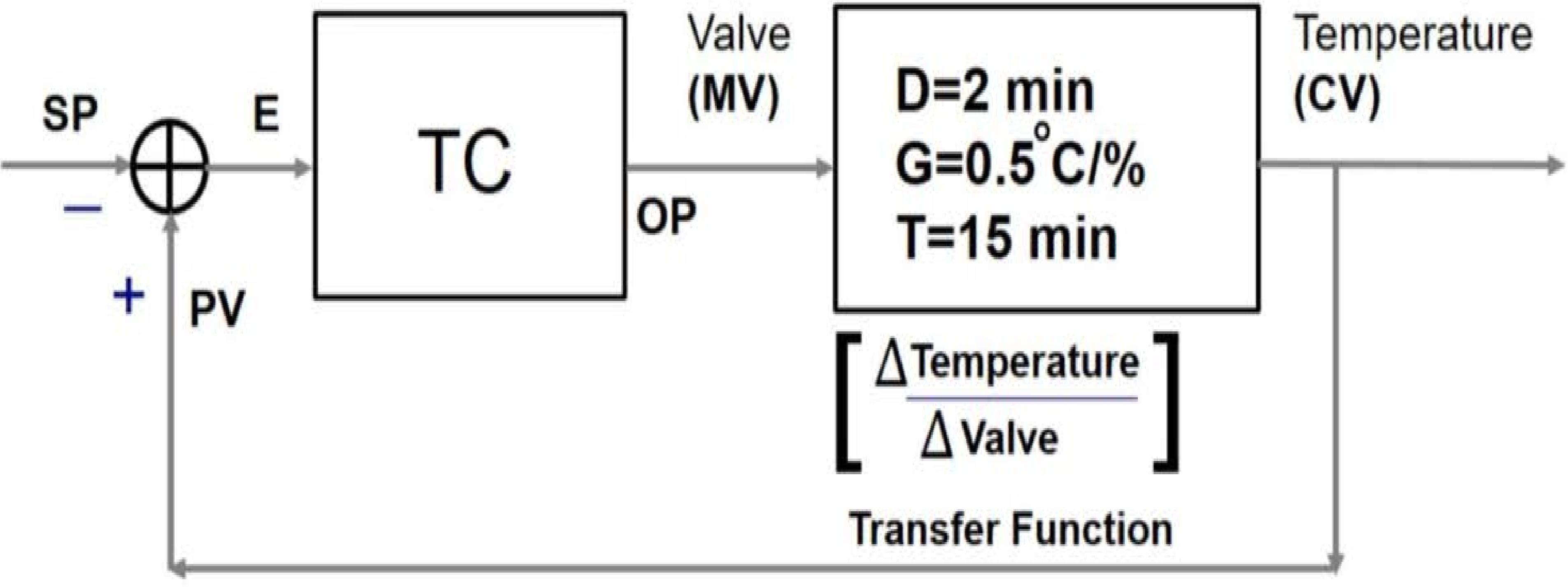
6. If secondary is not tracking its setpoint:
   - Tune secondary PID tighter
   - Reduce secondary PID's sample time
     (look-out for instability and oscillations)
   - Increase primary PID's sample time
     (look-out for instability and oscillations)
   - De-tune primary PID

Furnace Control Example

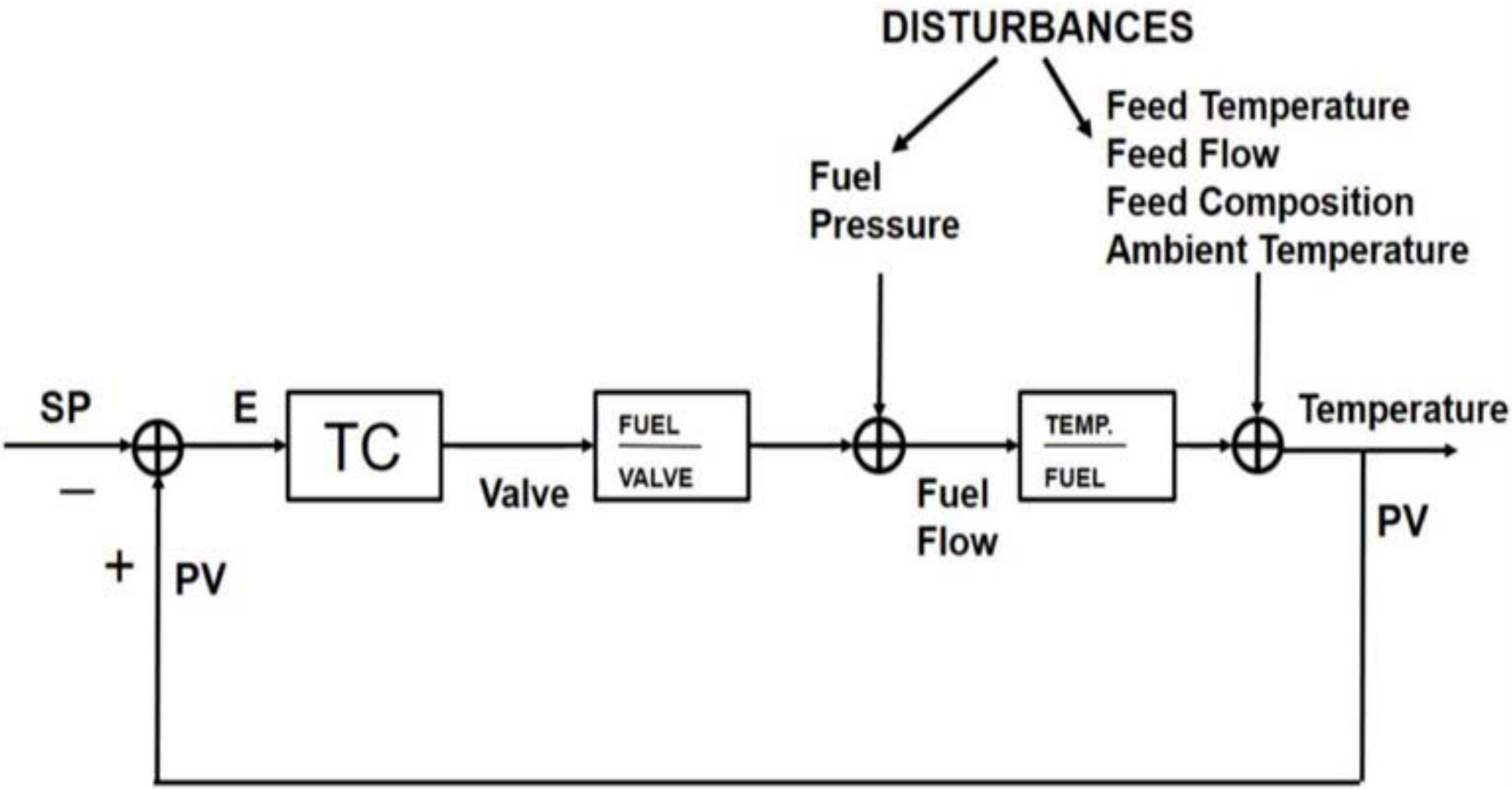
Furnace Temperature Control With Single PID



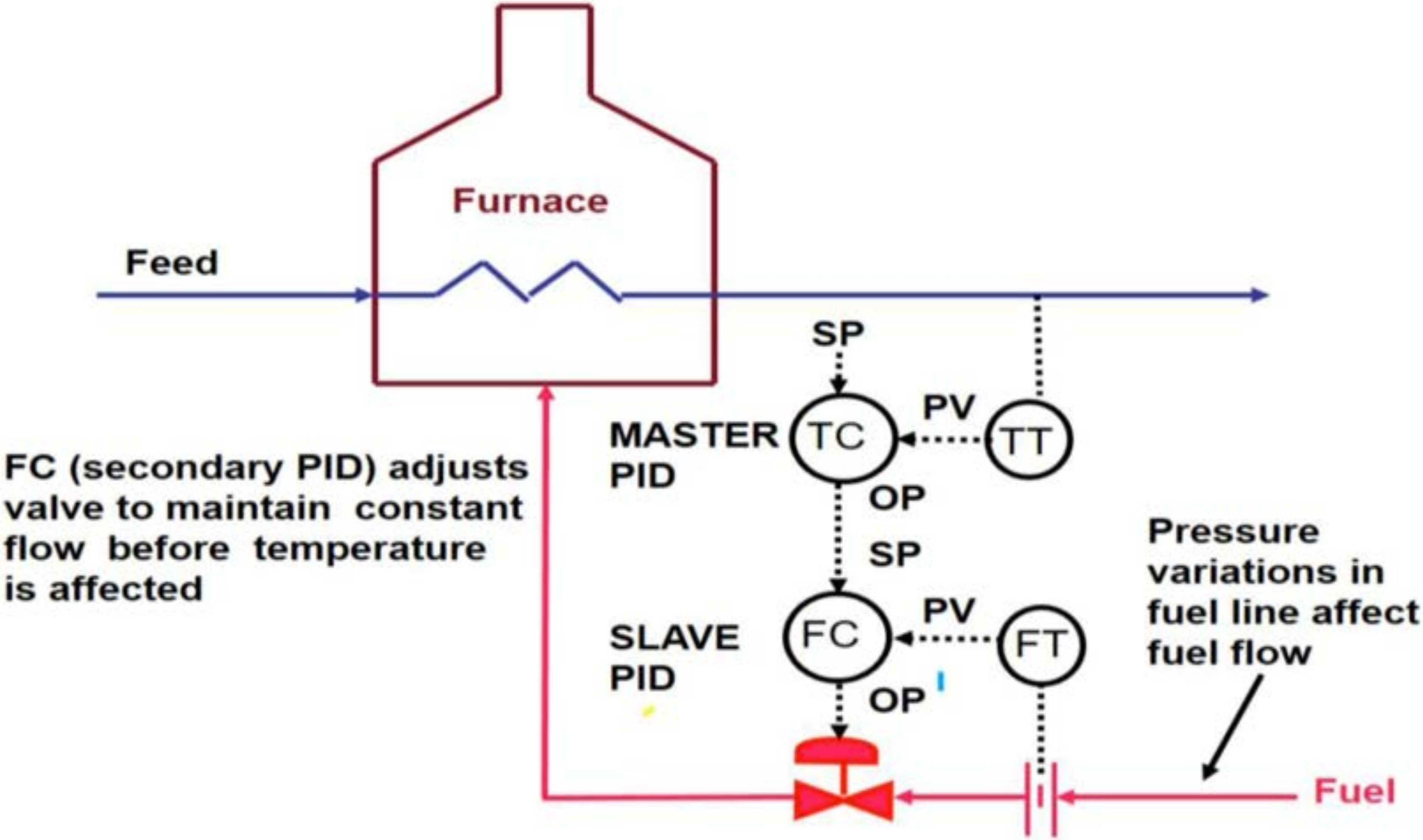
Control Schematic (simplified)
Single PID (TC)



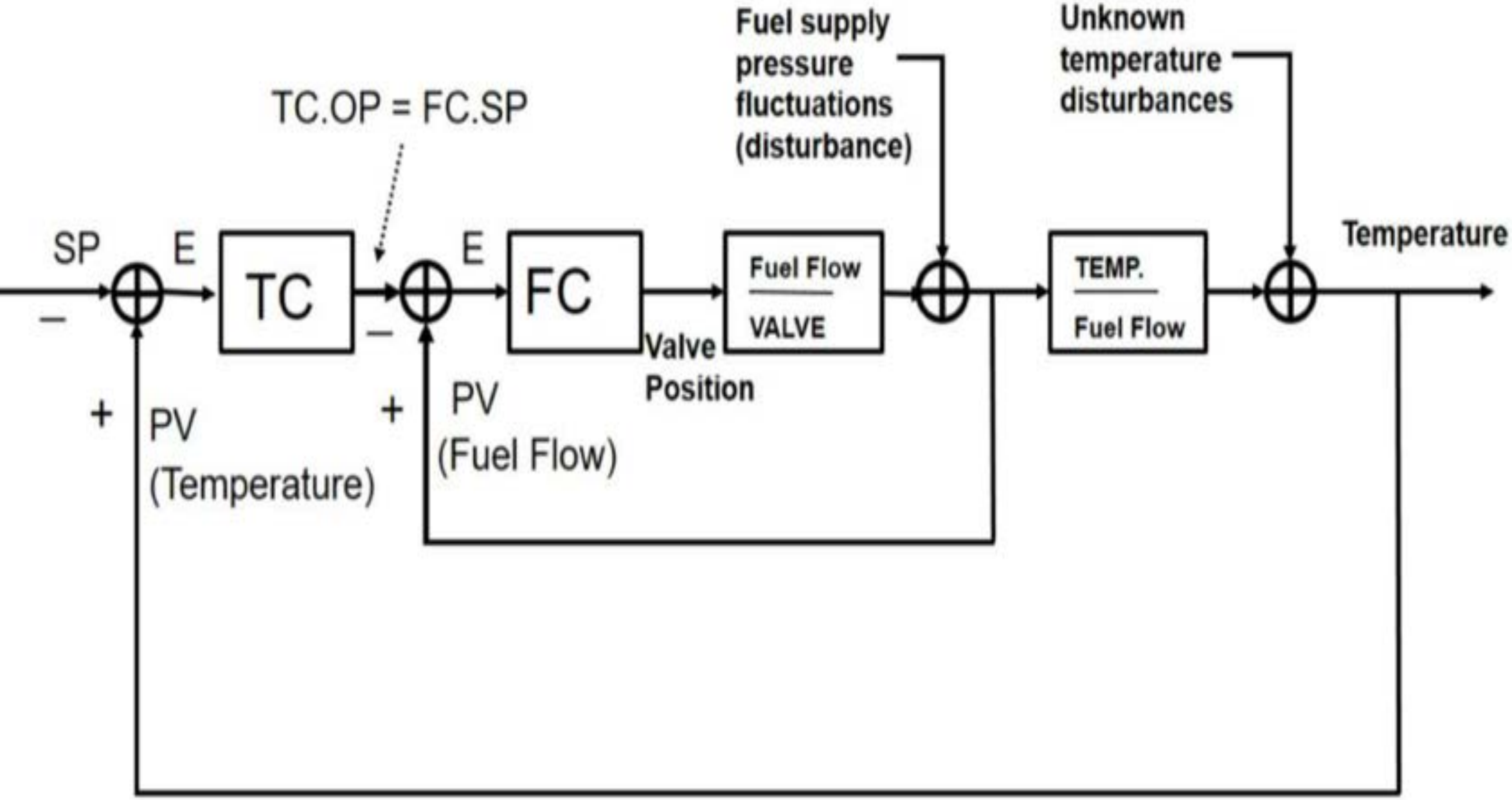
Control Schematic-Fuel Disturbance Single PID (TC)



Furnace Cascade Control

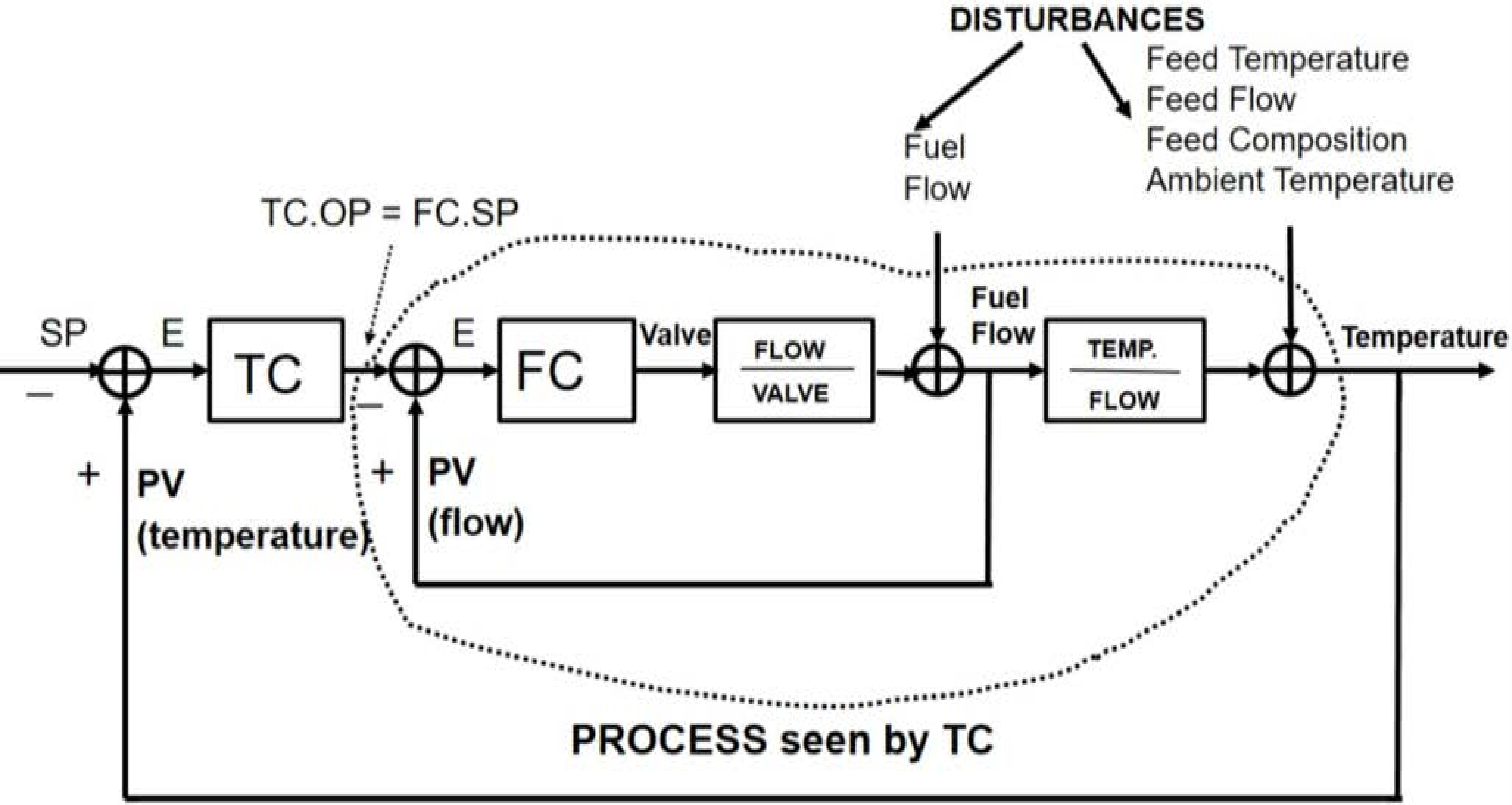


Cascade Control Schematic



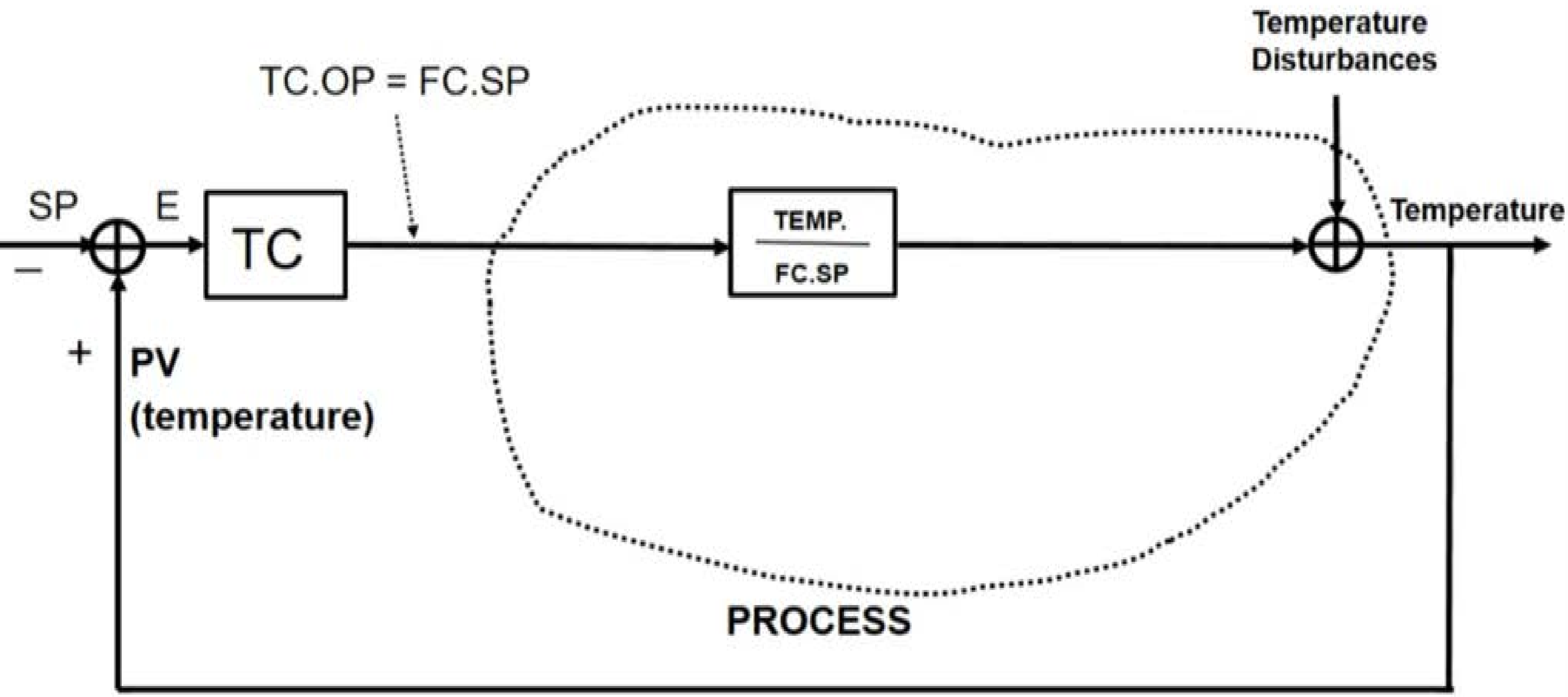
Cascade Control

Merge Slave Loop Dynamics

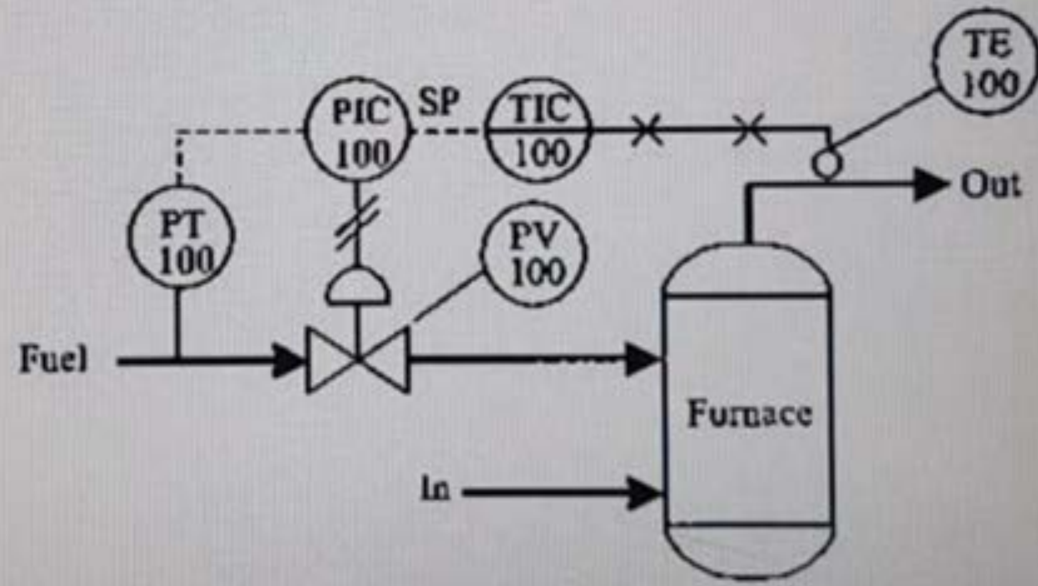


Cascade Control

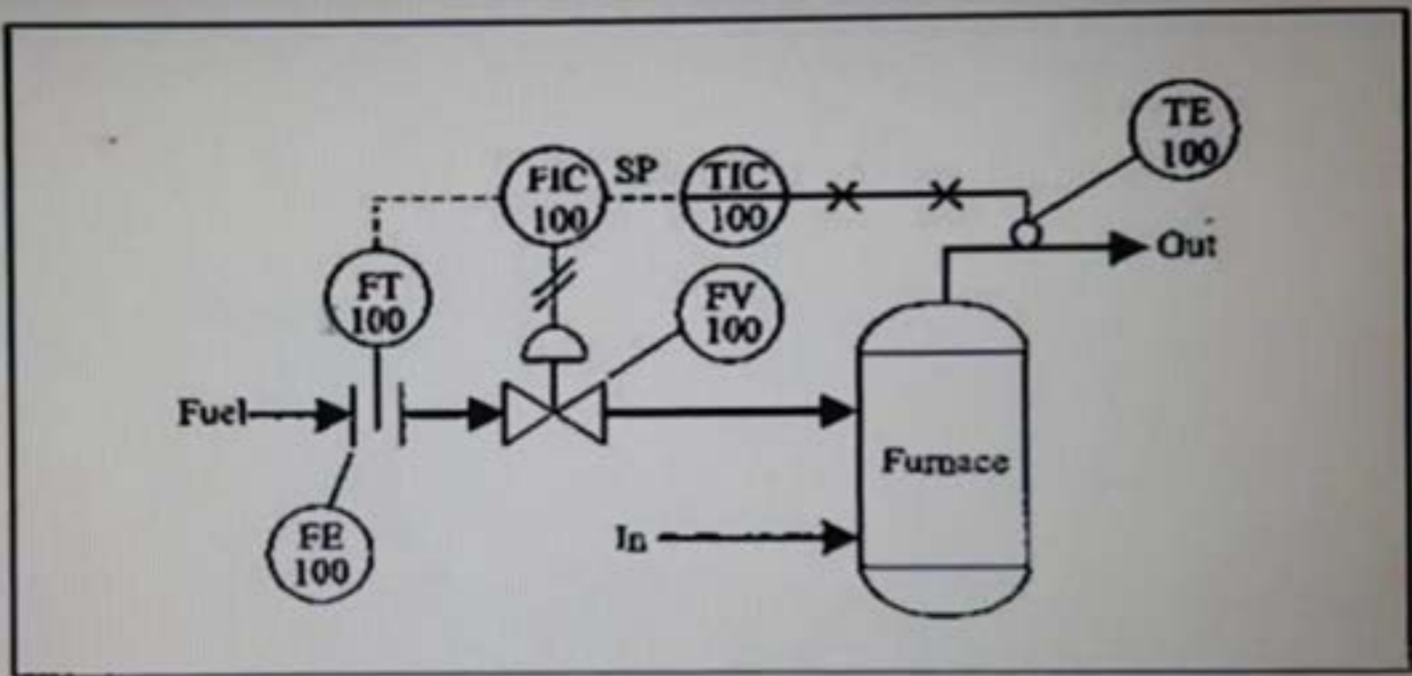
Merge Slave Loop Dynamics



Figures (3-16) Pressure and temperature cascade control

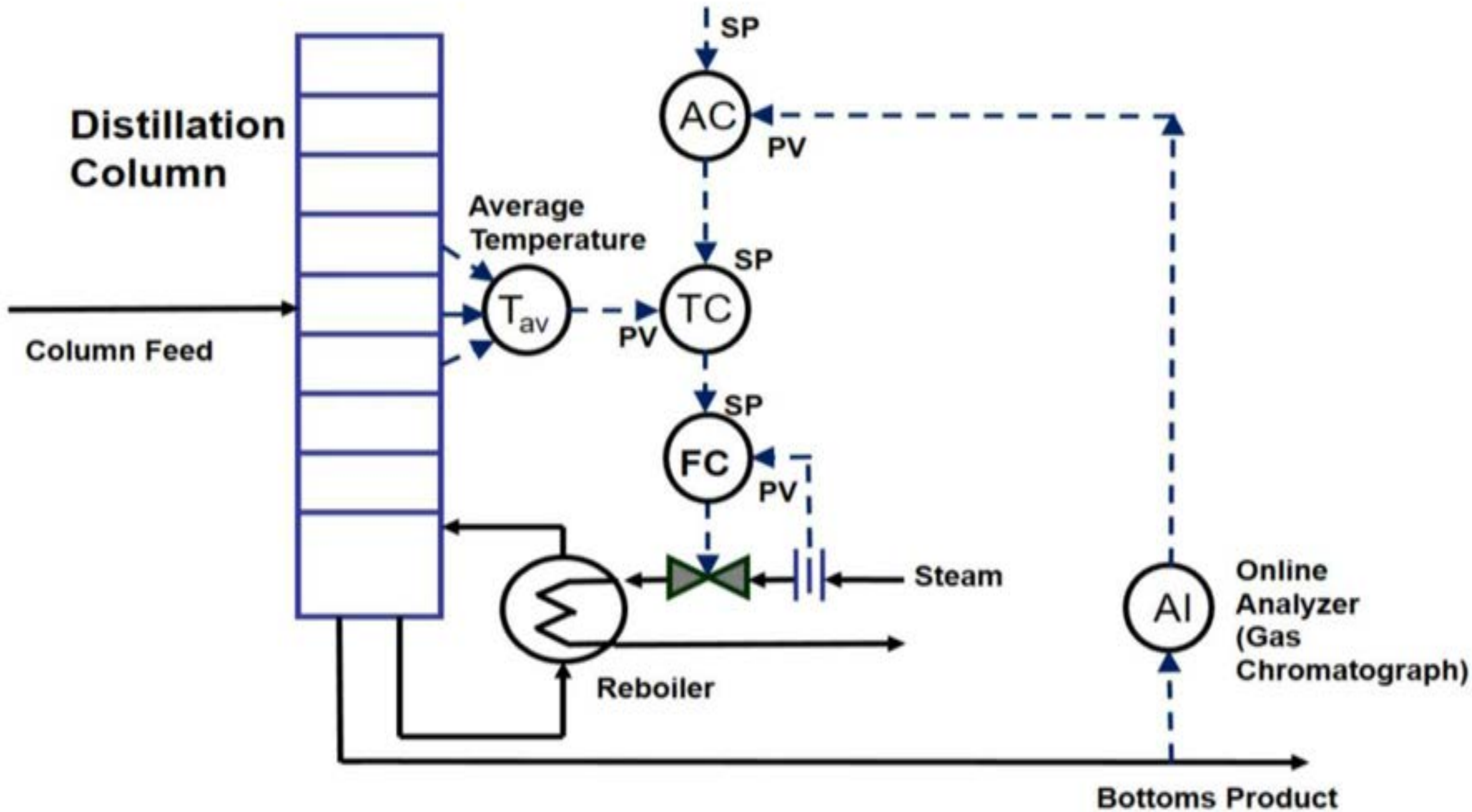


Figures (3-17) Flow and temperature cascade control



Distillation Column Triple Cascade PID Control

Distillation Cascade Control Example (Triple Cascade)



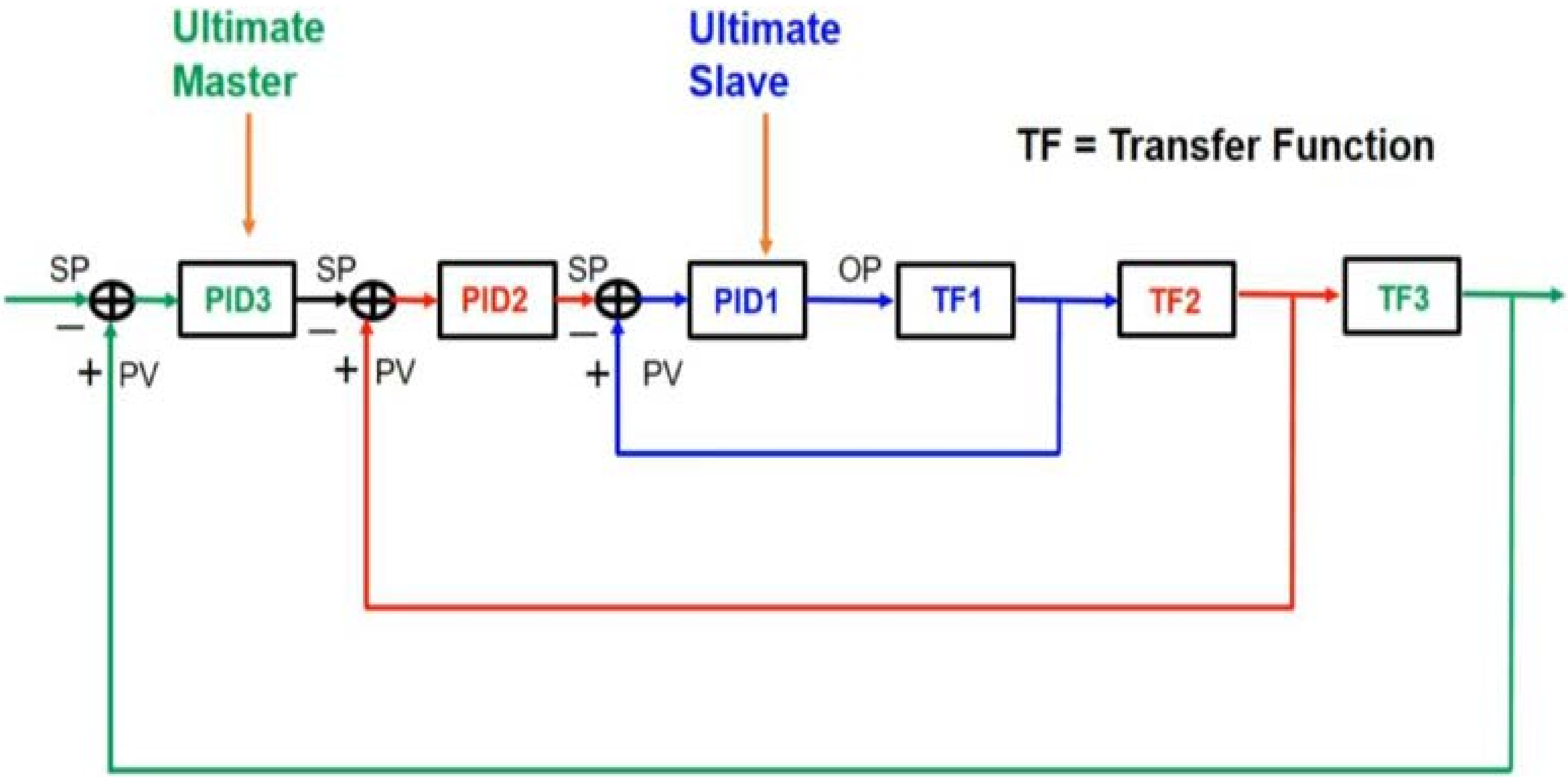
Multiple Cascade Example

AC ⟶ TC ⟶ FC			
Master	Slave		Primary Loop
	Master	Slave	Secondary Loop

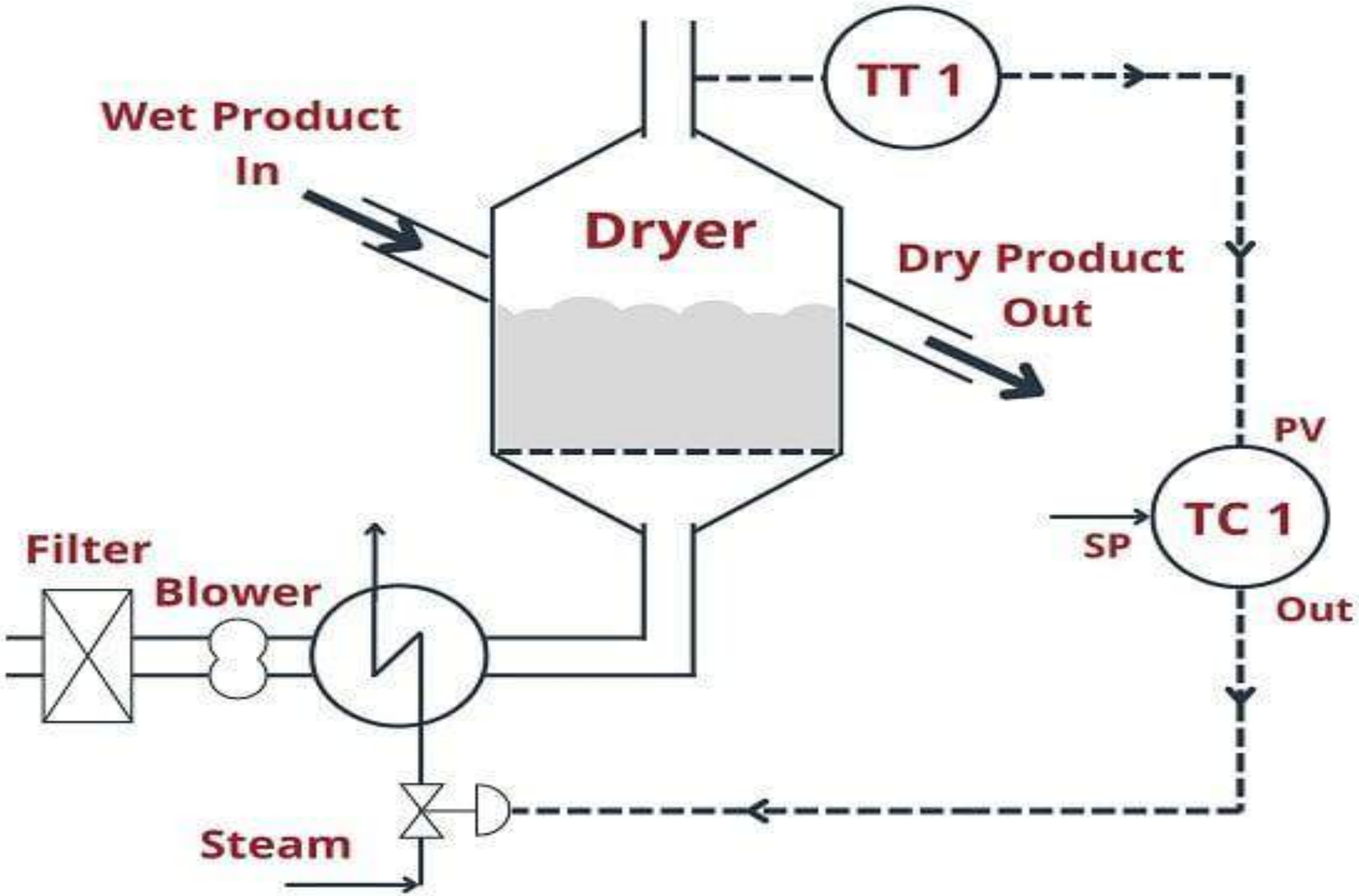
Analyzer PID cascaded to Temperature PID cascaded to Flow PID

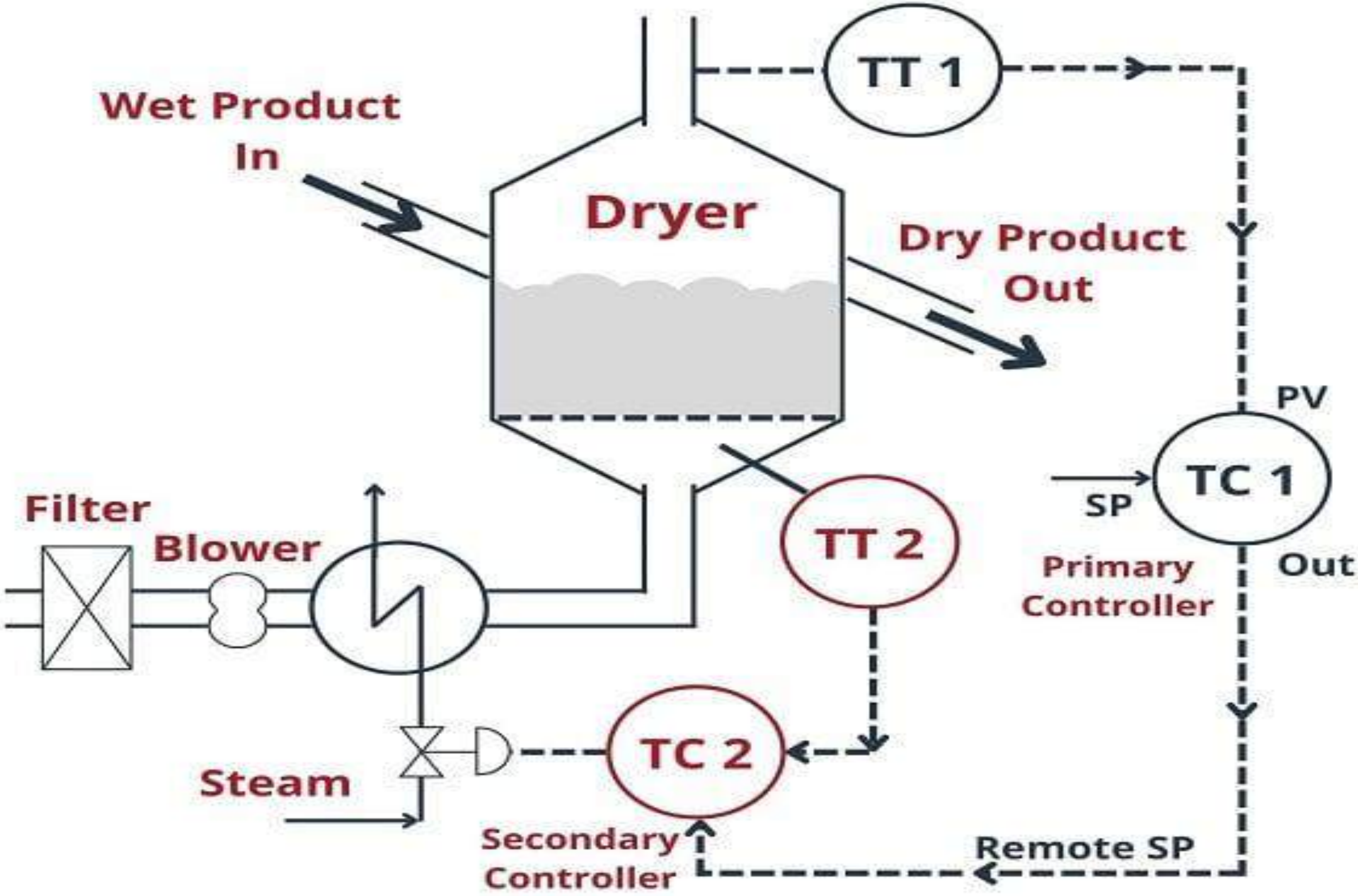
Control objective is to maintain distillate composition at an operator specified setpoint

Multiple Cascade Schematic

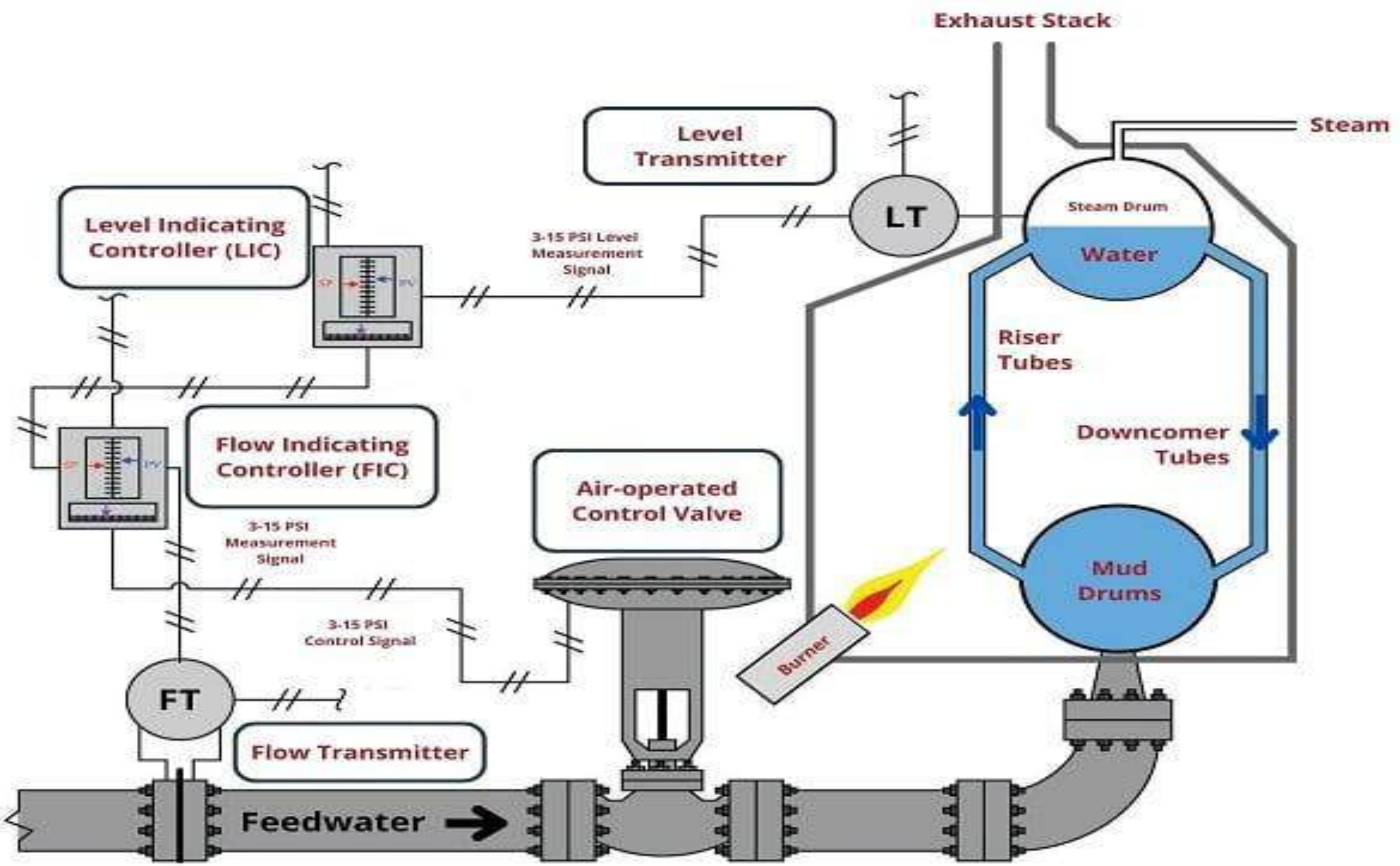


Material Dryer Process

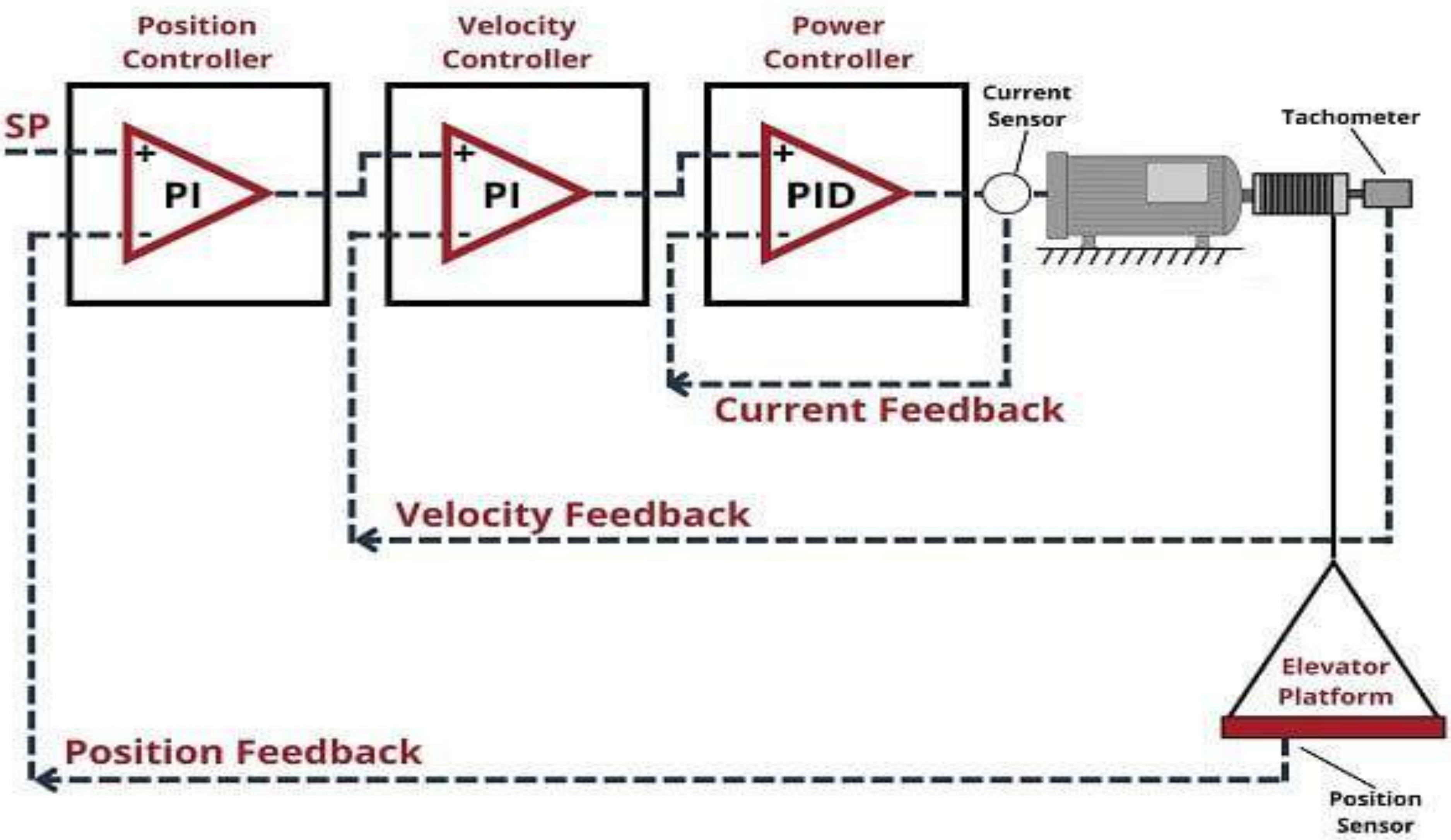



Wet Product In
Dryer
TT 1
Dry Product Out
PV
TC 1
SP
Primary Controller
Out
Filter
Blower
TT 2
Steam
TC 2
Secondary Controller
Remote SP

Cascade Control in Feedwater Systems

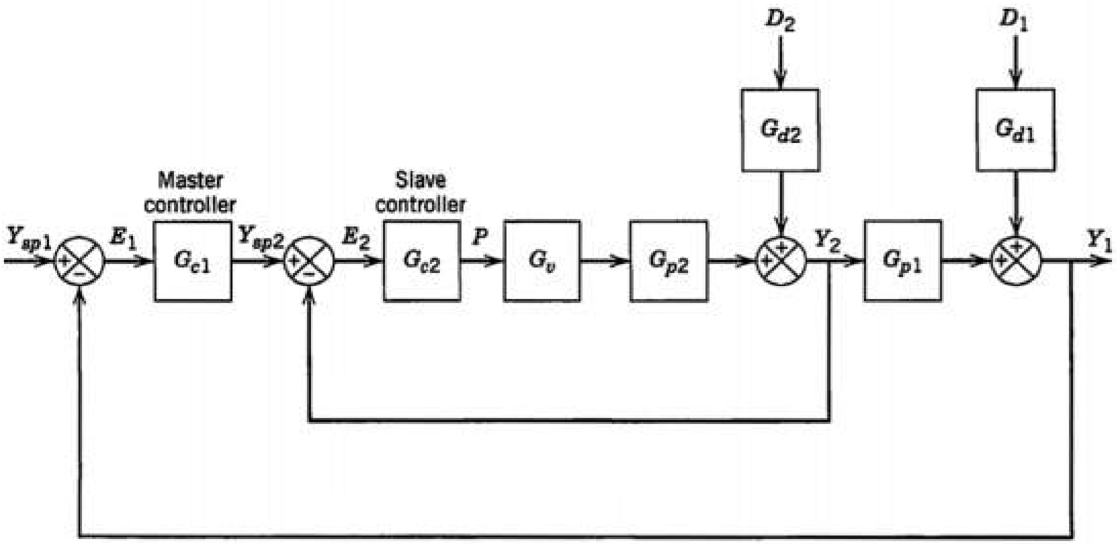


Cascade Control in Motion Systems



Question

A block diagram for a cascade loop is shown below.



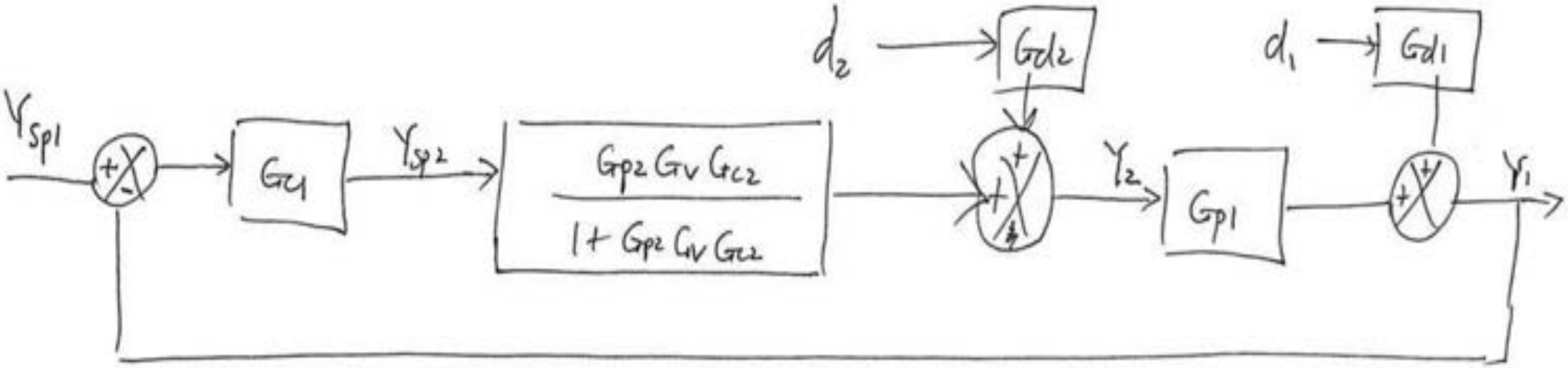
1. Derive a single transfer function from input Y_{sp2} to output Y_2, assuming $D_2 = 0$.
2. Redraw the block diagram, replacing it now with your single block transfer function at the suitable place, but still incorporate the disturbance effect from $D2$.
3. What is the characteristic equation for the inner loop?
4. If the inner loop has proportional-only controller for G_{c2}, and $G_v = 3$, and $G_{p2}(s) = \dfrac{6}{2s+1}$, derive a constraint (inequality) for the value of K_c so that the inner loop still has stable behaviour.
5. Explain whether this answer matches what we have learned earlier in the course?

Solution

1. The single transfer function is:

$$\frac{Y_2}{Y_{sp2}} = \frac{G_{p2}G_vG_{c2}}{1 + G_{p2}G_vG_{c2}}$$

2. Using this single transfer function now, we can reduce the entire inner loop to a single block. This reduction helps with analyzing the stability and properties of the outer loop, because now we don't have the complexity of an inner loop any more.



3. The characteristic equation is: $1 + G_{p2}G_vG_{c2}$

4. Stable behaviour requires the characteristic equation has roots in the left half plane:

$$1 + G_{p2}G_vG_{c2} = 0$$
$$1 + \frac{6}{2s+1} \cdot 3 \cdot K_c = 0$$
$$2s + 1 + 18K_c = 0$$
$$s = -9K_c - 0.5$$

For this to be a negative root, we require:

$$-9K_c - 0.5 < 0$$
$$9K_c + 0.5 > 0$$
$$K_c > -\frac{1}{18}$$

5. We learned that the sign of controller gain, K_c should match the sign of the process gain, K_p. This bound sets a minimum on the controller gain, in practice we would almost certainly tune it to be larger, and increase it above zero, in which case it matches our expectation. The number $-\frac{1}{18} + \epsilon$ however does lead to a stable controller and you should check this in Simulink with the three gains of:

- $K_c = -\frac{1}{18} - \epsilon$
- $K_c = -\frac{1}{18}$
- $K_c = -\frac{1}{18} + \epsilon$

to verify your answer.

The oil temperature is being controlled in a cascade control system as shown in Figure 1.

A) Highlight the benefits of using the cascade control instead of the conventional feedback control.

B) What is the Cascade control design criteria?

C) What is the primary variable in the cascade loop?

D) What is the secondary variable in the cascade loop?

E) What is the manipulated variable in the inner loop?

F) Explain whether a disturbance in the fuel gas feed pressure will be removed rapidly with the cascade loop?

G) Explain whether a disturbance in the oil temperature will be removed rapidly with the cascade loop?

H) Draw a complete block diagram of the system indicating all the inner and outer level controllers, process blocks and disturbance blocks.

I) If the inner loop has proportional-only controller for G_{c2}, and $G_V = 6$, and $G_{p2}(s) = \frac{7}{4s+1}$, derive a constraint (inequality) for the value of Kc so that the inner loop still has stable behavior. Assume that there is no disturbance effect in the inner loop.

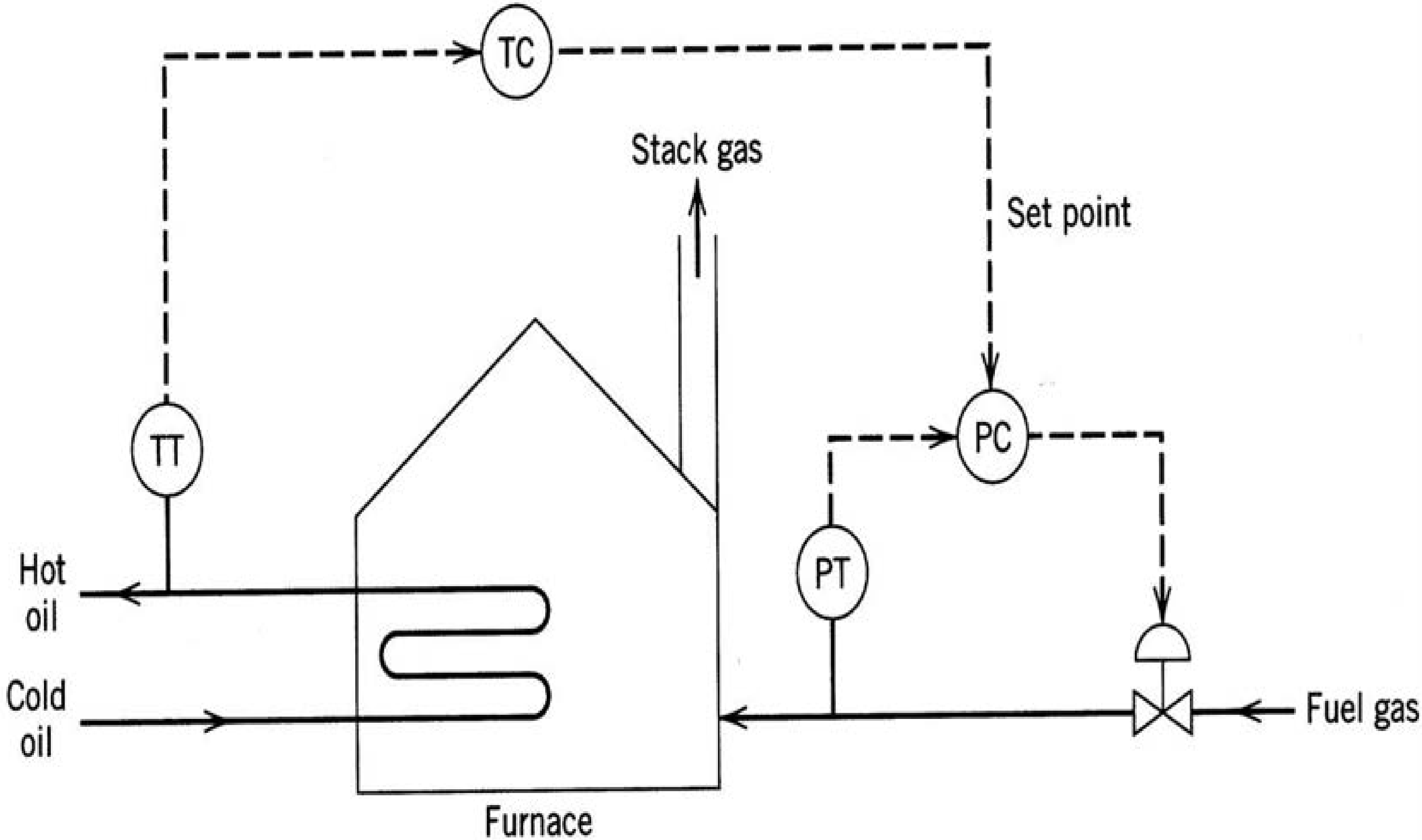

TC
Stack gas
Set point
TT
PC
PT
Hot oil
Cold oil
Fuel gas
Furnace

Figure 1

PID Modes
Manual, Auto, Cascade

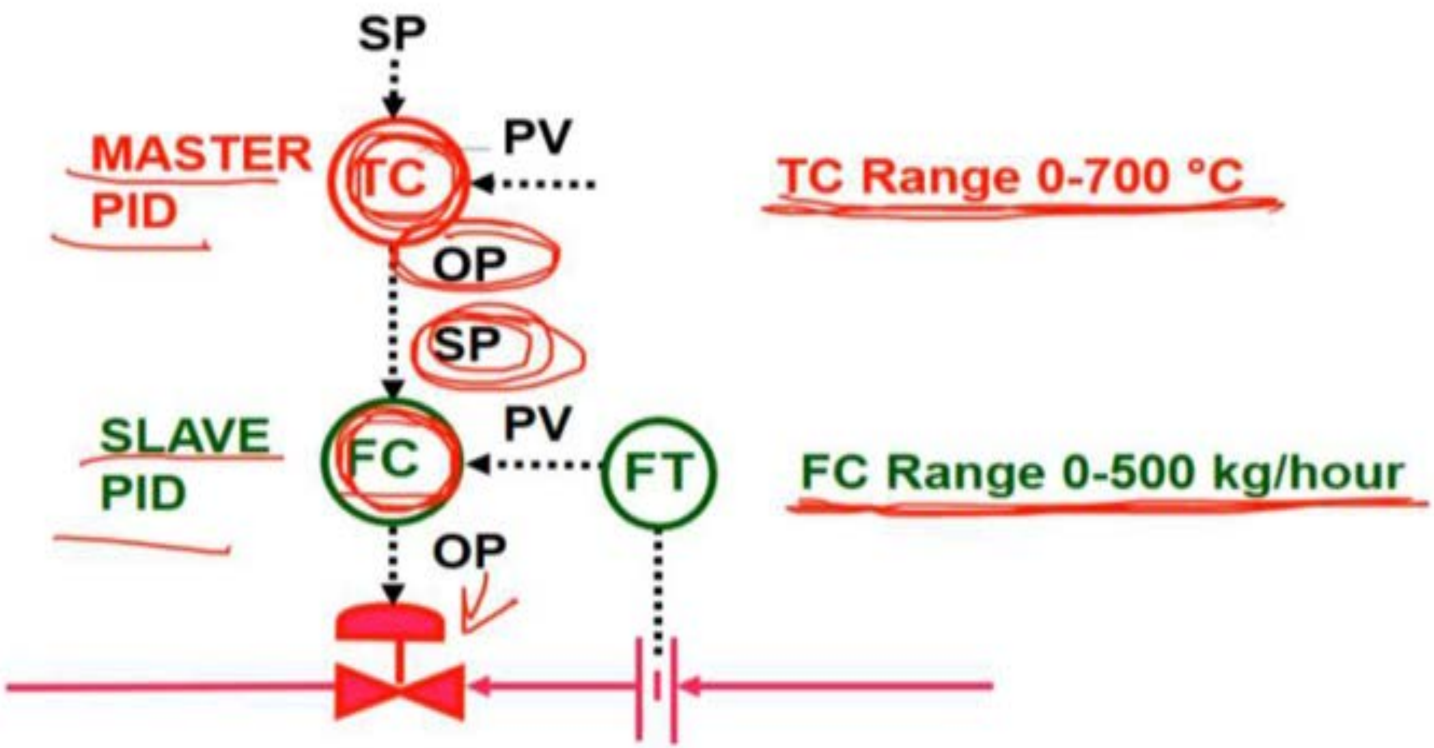
A PID has three modes:

- **Manual Mode**: PID algorithm is inactive, PID output (OP) can be changed by operator
- **Auto Mode**: PID algorithm is active, SP can be changed by operator, PID calculates new output (OP) at each sample time (ΔT)
- **Cascade Mode**: Applies to slave (secondary) PIDs only. If slave PID is in cascade mode, *only then* its setpoint can be changed by master (primary) PID.

Cascade MV and CV

- In case of single PID:
  - CV is the flow, temperature, etc.
  - MV is the Control Valve (0-100% range)
- In case of cascade PID (master):
  - MV is the Slave PID's Setpoint
  - MV range is **not 0-100%** but is the **Slave's CV range**

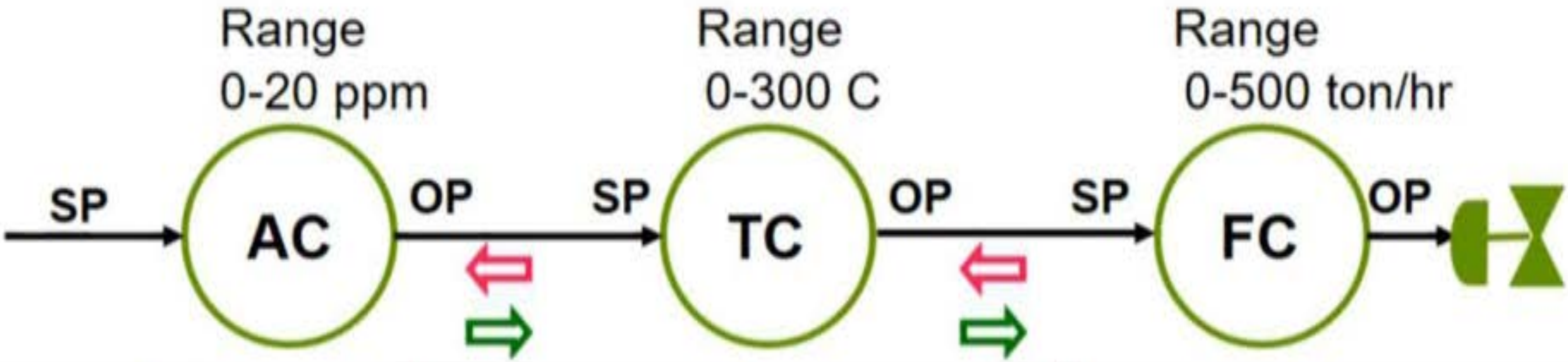
Cascade MV and CV



For Slave PID (FC):
CV is FC.PV (0-500 kg/hr)
MV is Control Valve (0-100%)

For Master PID (TC):
CV is TC.PV (0-700 °C)
MV is FC.SP (0-500 kg/hr)

Cascade Chain Activation Sequence



	AC				TC				FC		
PV Track	No				Yes				Yes		
Mode	PV	SP	OP%	Mode	PV	SP	OP%	Mode	PV	SP	OP%
Man/INIT	3	5	70	Man/INIT	210	210	40	Auto	203	200	65
Man/INIT	3	5	71	Man	213	213	40	Casc	202	200	62
Man/INIT	3	5	71	Auto	211	213	42	Casc	203	210	68
Man/INIT	3	5	73	Auto	216	219	46	Casc	222	230	73
Man	3	5	73	Casc	214	219	49	Casc	237	245	77
Man	3	5	74	Casc	214	222	49	Casc	237	245	77
Auto	3	5	78	Casc	215	234	54	Casc	264	270	93

Initialization and Bumpless Transfer

INIT Flag

- The INIT flag is shown in some DCSs to flag that the Master PID is being initialized by the slave.
- When a Master has the "INIT" flag, the Master is not in control and the cascade chain is "broken".
- The "INIT" flag on a Master is because the Slave is not in Cascade mode.

Bumpless Transfer and Output Tracking

- The **initialization** process ensures that the primary PID's OP always matches the secondary PID's SP as long as the secondary is not in cascade mode
- This is also called **output tracking**
- When the secondary is put in cascade and the primary in auto, there is smooth transition in the setpoint of the secondary.
- This feature ensures **bumpless transfer**

Initialization

- Initialization (sometimes called output tracking) ensures a master controller does not cause a sudden change to a downstream controller's SP.
- Done by forcing the master's output to equal the slave's SP.
- Required in all cascade control strategies.
- When there is fan-out (multiple slaves of one master) the master does not initialize until the last slave is taken out of cascade and initializes to the SP of the first slave put into cascade before it performs its control algorithm.
- When there are multiple masters into one slave (such as a selector) all masters whose output are not used must initialize.
- Initialization also requires that every controller calculate an initialization value (called InitVal in following discussions) to be passed up to any master(s); the calculation is always performed to ensure the initialization value will be such that the OP will not change on first calculation when automatic control is resumed.

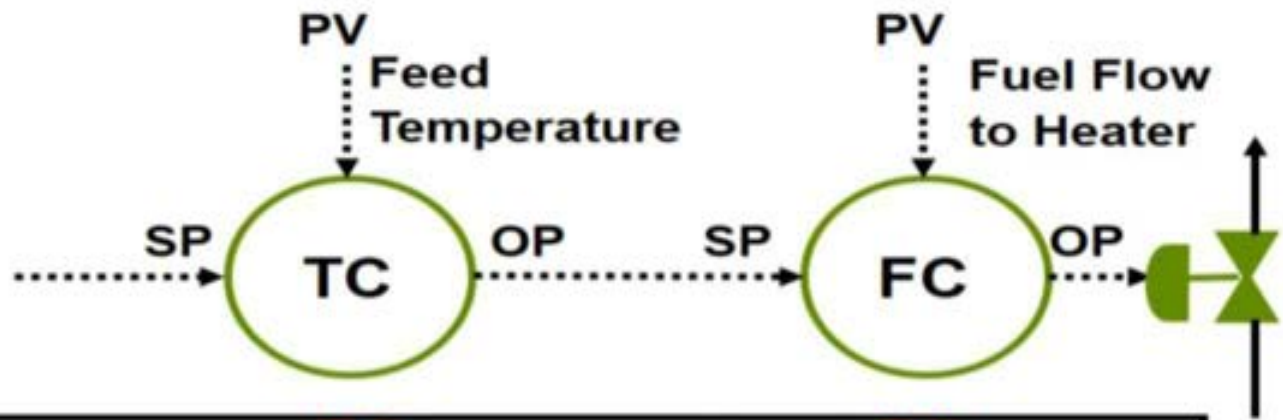
PV Tracking

(Don't confuse PV Tracking with SP/OP Tracking)

- Every PID has a PV tracking option
- If PV Track = Yes, then SP = PV when PID is in Manual mode
- If PV Track = No, then SP remains fixed at the last value entered by operator

TIME	SP	PV	OP	Mode	Comments
9:00	25.0	24.5	55	Auto	Normal operation
9:10				Manual	Plant problem, put in Manual
9:11			80	Manual	Operator bumps OP
9:11	47.3	47.3	80	Manual	PV jumps to 47.3, **SP tracks PV**
9:45	49.4	49.4	80	Auto	Problem fixed, back to Auto
9:48	42.0			Auto	Operator slowly reduces SP
9:51	37.0			Auto	in **several steps**

Anti-reset Windup



	TC			FC		
TIME	SP	PV	OP	SP	PV	OP
9:00	175	170	60	30.0	28.0	95
9:01	175	167	62	31.0	26.0	99
9:02	175	163	65	32.5	25.5	100
9:03	175	155	65	32.5	20.2	100

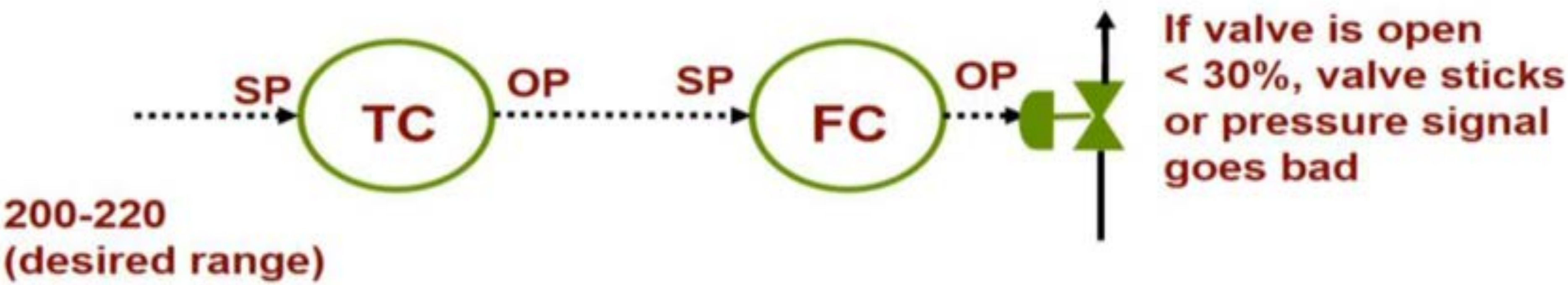
Pressure loss in fuel supply line

Output Frozen !!

Wind-up Indicator

Setpoint and Output Clamps

- Setpoint and output of a PID can be clamped with high and low clamp limits
- Default limits are 0-100% for output and 0-engineering unit range for setpoint.
- Tighter limits may be set for safety or for satisfying some process constraints.
  - TC: SP limits:200-220 deg C
  - FC: OP limits: 30-100%



Different Types of PID Equations

A0 P (dE + E dt / I + D d(dE) / dt)	A4 (P dE) + E dt / I + D d(dE) / dt
B0 P (dE + E dt / I + D d(dPV) / dt)	B4 (P dE) + E dt / I + D d(dPV) / dt
C0 P (dPV + E dt / I + D d(dPV) / dt)	C4 (P dPV) + E dt / I + D d(dPV) / dt
A1 100/PB (dE + E dt / I + D d(dE) / dt)	A5 (P dE) + E dt I + D d(dE) / dt
B1 100/PB (dE + E dt / I + D d(dPV) / dt)	B5 (P dE) + E dt I + D d(dPV) / dt
C1 100/PB (dPV + E dt / I + D d(dPV) / dt)	C5 (P dPV) + E dt I + D d(dPV) / dt
A2 P (dE + E dt I + D d(dE) / dt)	A6 (100/PB dE) + E dt / I + D d(dE) / dt
B2 P (dE + E dt I + D d(dPV) / dt)	B6 (100/PB dE) + E dt / I + D d(dPV) / dt
C2 P (dPV + E dt I + D d(dPV) / dt)	C6 (100/PB dPV) + E dt / I + D d(dPV) / dt
A3 100/PB (dE + E dt I + D d(dE) / dt)	A7 (100/PB dE) + E dt I + D d(dE) / dt
B3 100/PB (dE + E dt I + D d(dPV) / dt)	B7 (100/PB dE) + E dt I + D d(dPV) / dt
C3 100/PB (dPV + E dt I + D d(dPV) / dt)	C7 (100/PB dPV) + E dt I + D d(dPV) / dt

Which Equation to Use? A, B or C?

- Eq. B is most common and is used in 60% of industrial PIDs.
- Eq. C is used in 30% of PIDs.
- Eq. A is not recommended because Derivative component acts on Error instead of PV. On a SP change, huge Derivative contribution could bang the control valve fully open or closed.
- If your DCS/PLC gives you option of A, B or C, chose between B or C.

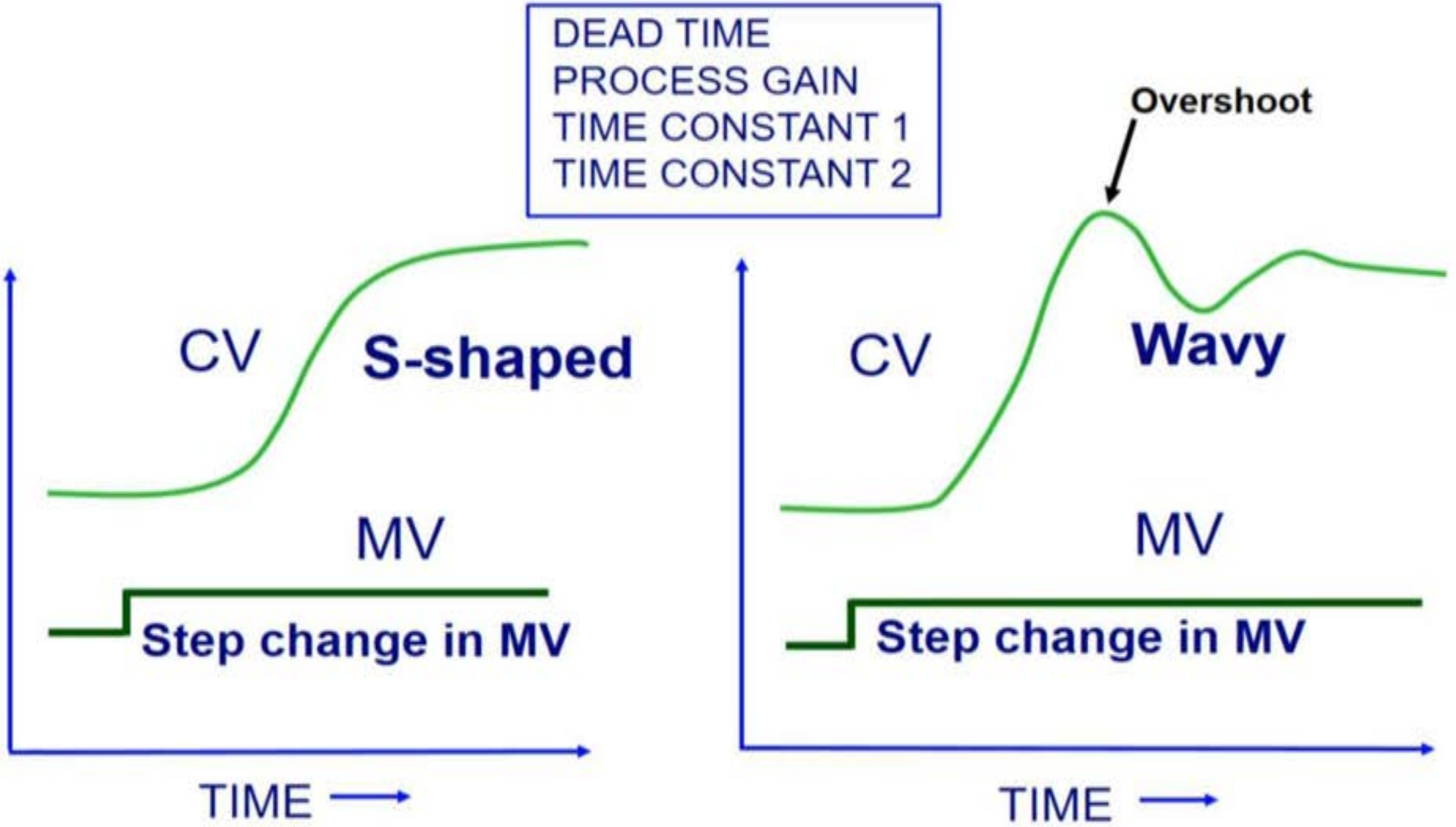
PID Eq. B Characteristics

- Proportional action acts on delta Error.
- Generates good proportional kick on a SP change.
- Control action can be tight and crisp, but....
- Downstream units could be upset by proportional kick!
- Good on slave PIDs – since proportional action can be strong, providing good slave PID control.

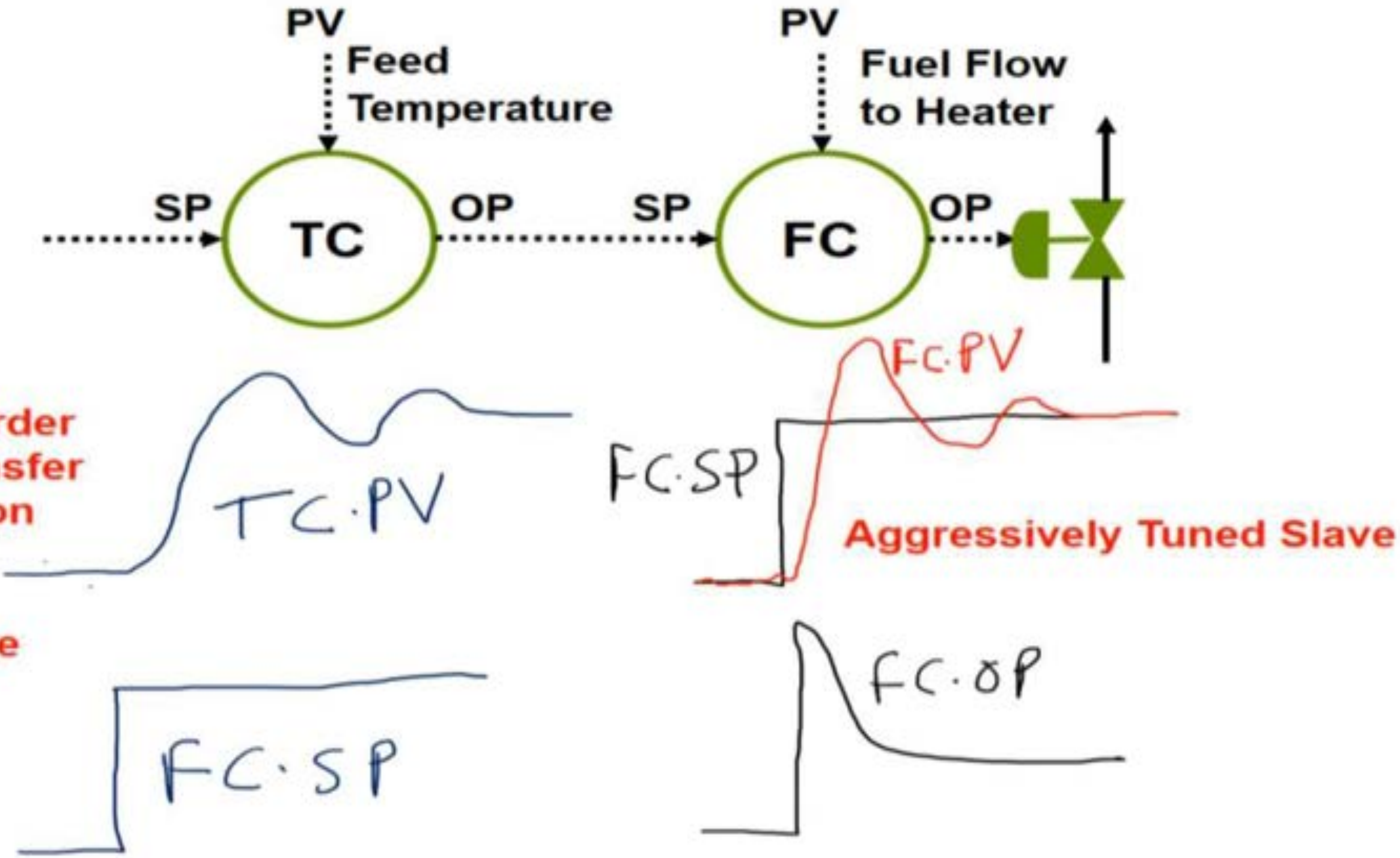
PID Eq. C Characteristics

- Proportional action acts on delta PV, so…
- There is no significant proportional kick on a SP change.
- Control action is smoother and can be less disruptive to downstream units.
- Good on LCs (level controllers) for smoother flow to downstream units.
- Bad for slave PIDs as the control action is smooth but not tight enough to help master PIDs to produce tight control.

Second Order Transfer Function



Aggressively Tuned Slave can produce wavy second order transfer function characteristics in Master



Second Order Wavy Transfer Function on Master Due to Aggressive Slave

Single-loop Enhancements: Feedforward Control , and Ratio Control

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MSc Mechatronics Engineering
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Goals

When I complete this chapter, I want to be able to do the following.

- Understand the concept of feedforward compensation and its difference from feedback
- Derive the feedforward control algorithm
- Design appropriate applications of feedforward in conjunction with feedback

Motivation: Why Feedforward? (Academic View)

- Classical feedback (FB) reacts to **error**:
  - Error = set-point – measured variable
  - Correction occurs **after** the disturbance affects the process
- Many process disturbances are **measurable**:
  - Inlet flow/temperature, feed composition, load changes, etc.
- Key idea of feedforward control:
  - **Measure disturbances** and **compensate proactively**, before they affect the controlled variable
- Used extensively in chemical, petrochemical, power, and HVAC control systems.
- Standard discussion in: Seborg et al., "Process Dynamics and Control", 3rd/4th ed., Wiley; Skogestad & Postlethwaite, "Multivariable Feedback Control", 2nd ed., Wiley.

Motivation: Industrial Perspective

- In real plants:
  - Disturbance sensors (flow, pressure, temperature, composition analyzers) are common
  - PLC/DCS systems support dedicated **feedforward function blocks**
- Typical applications:
  - **Steam heater**: feedforward from inlet temperature or flow
  - **Blending/mixing tanks**: feedforward from disturbance stream flow/composition
  - **Boilers**: feedforward from load demand (steam flow) to fuel/air flow
- Benefits for operations:
  - Faster disturbance rejection
  - Reduced product variability (quality, safety, energy efficiency)
- See: Marlin, "Process Control: Designing Processes and Control Systems for Dynamic Performance", 2nd ed., McGraw-Hill; Luyben, "Process Modeling, Simulation and Control for Chemical Engineers," McGraw-Hill.

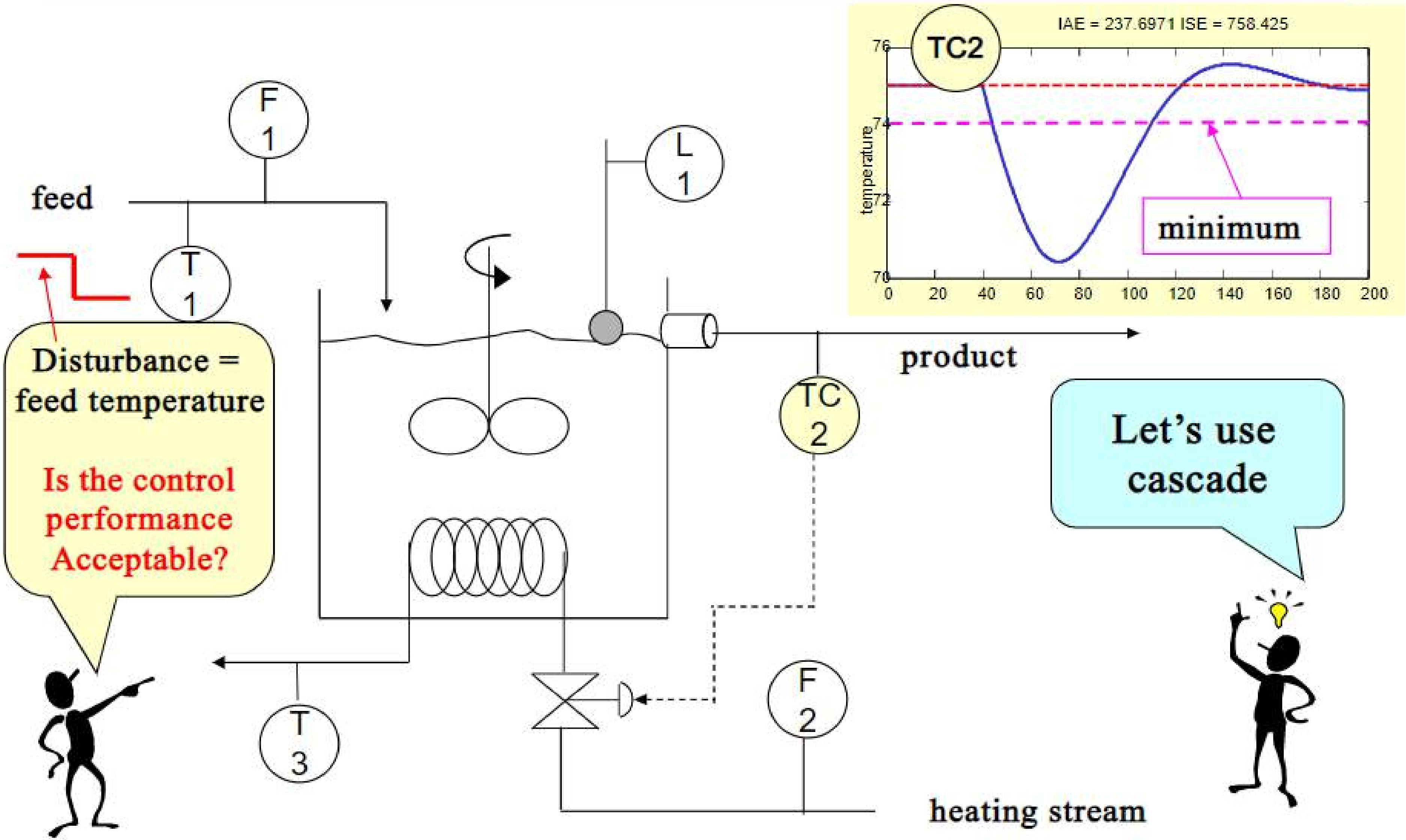
Review: Basic Feedback Control Structure

- Single-loop feedback:
  - Manipulated variable (MV): e.g. valve position, heater power
  - Controlled variable (CV): e.g. temperature, level, composition
  - Controller compares **CV** with **set-point** and sends correction to MV
- Key points:
  - Feedback compensates **all disturbances + model mismatch** (if within bandwidth)
  - Performance limited by: process dead time, time constant, actuator constraints, stability margins
- Feedback alone may be **too slow** for large or fast disturbances.

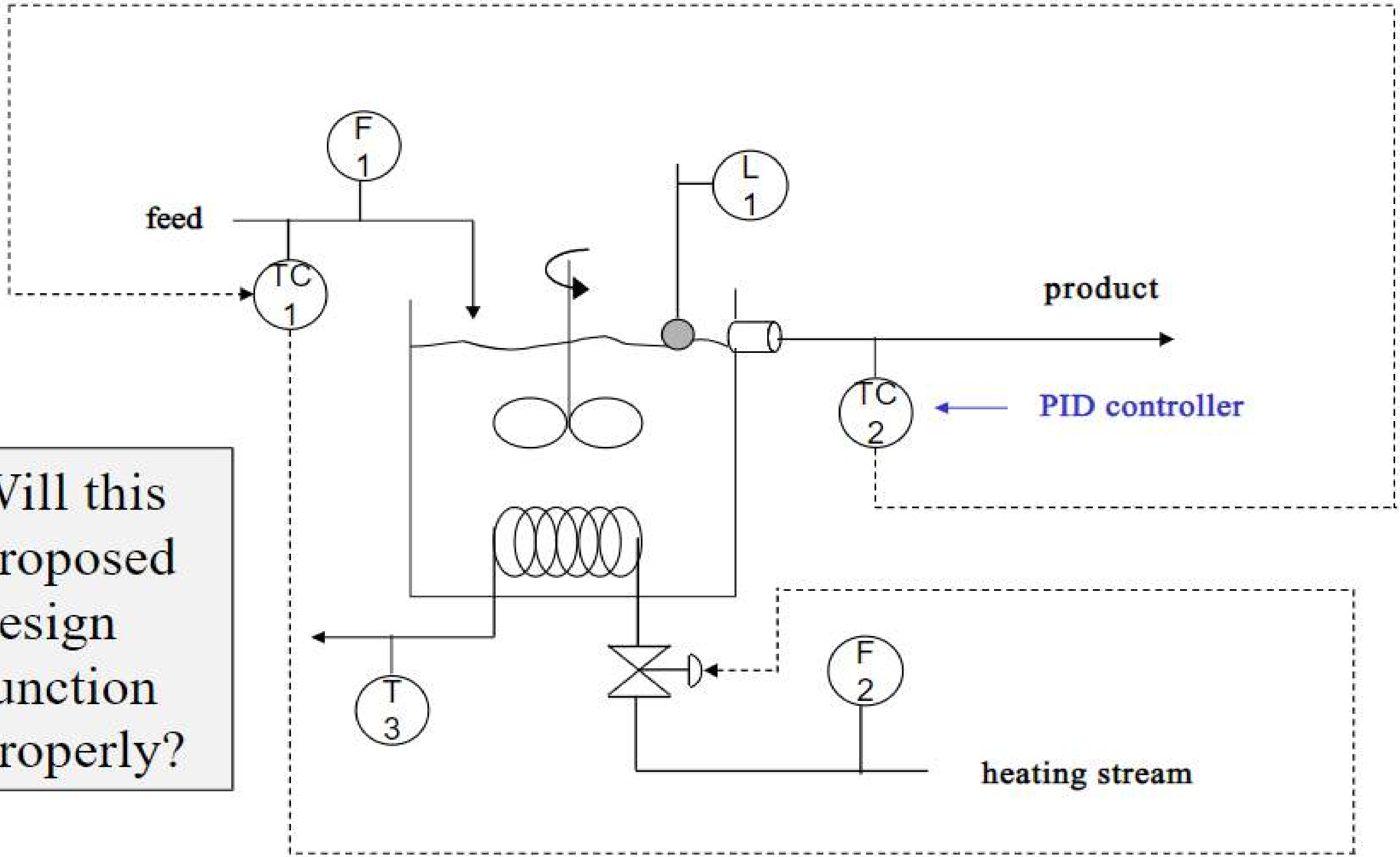
Conceptual Structure of Feedforward Control

- We distinguish:
  - **Measured disturbance:** $d(t)$ (e.g. inlet temperature)
  - **Process**: from MV → CV, and from disturbance → CV
  - **Feedforward controller**: computes a compensating action based on $d(t)$
- Block structure (for a single measured disturbance):
  - Disturbance $d(t)$ → **Feedforward block** → MV
  - Set-point $r(t)$, measurement $y(t)$ → **Feedback controller** → MV
- MV is **sum of two contributions**: feedback + feedforward.

Single-loop feedback control can give poor performance



Here is a cascade design using TC-1; will it work?



Will this proposed design function properly?

Use the cascade design criteria!

CASCADE DESIGN CRITERIA FOR T1

Cascade is desired when

OK 1. Single-loop performance unacceptable

OK 2. A measured variable is available

A secondary variable must

OK 3. Indicates the occurrence of an important disturbance

4. Have a causal relationship from valve to secondary

5. Have a faster response than the primary

NO! Cascade not possible.

We need another enhancement!

FEEDFORWARD DESIGN CRITERIA

Feedforward is desired when

1. Single-loop performance unacceptable
2. A measured variable is available

A measured disturbance variable must

3. Indicates the occurrence of an important disturbance
4. NOT have a causal relationship from valve to measured disturbance sensor
5. Not have a much faster effect on the CV than the MV (when combined with feedback)

Conditions for Effective Feedforward Control

Feedforward is particularly useful when:

- A dominant disturbance is **measurable** and sensor is reliable
- Disturbance affects the process **significantly** and **predictably**
- Process can be reasonably approximated by low-order transfer functions
- Measurement and computation delays are **small** compared to process dynamics

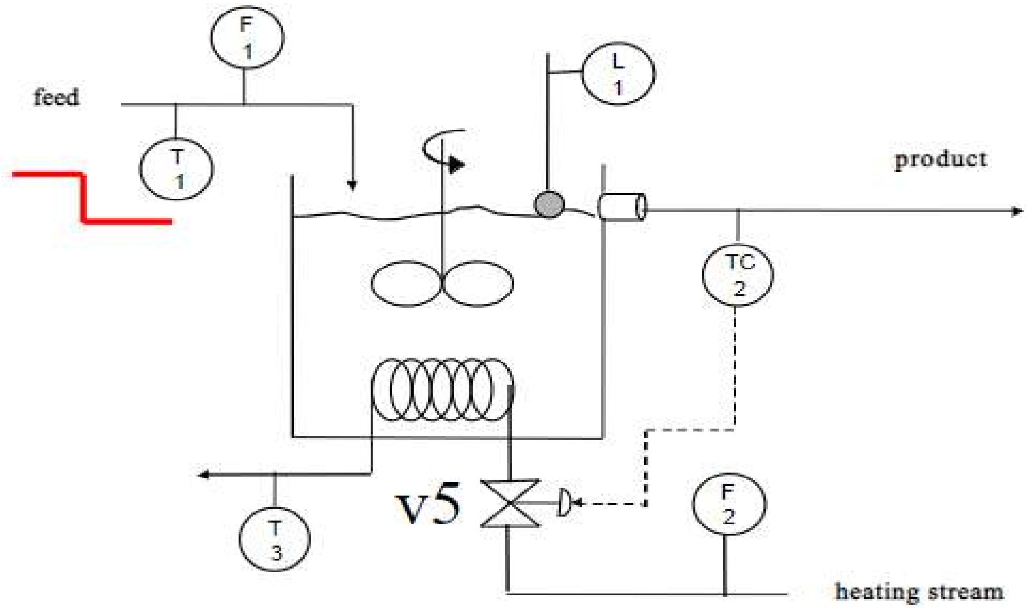
Limitations:

- If disturbance cannot be measured or correlation is poor → minimal benefit
- Model mismatch → partial or even over-compensation
- Dead time in disturbance measurement or actuation can degrade performance.

What type of compensation is needed in this case?

Let's think about the process behavior.

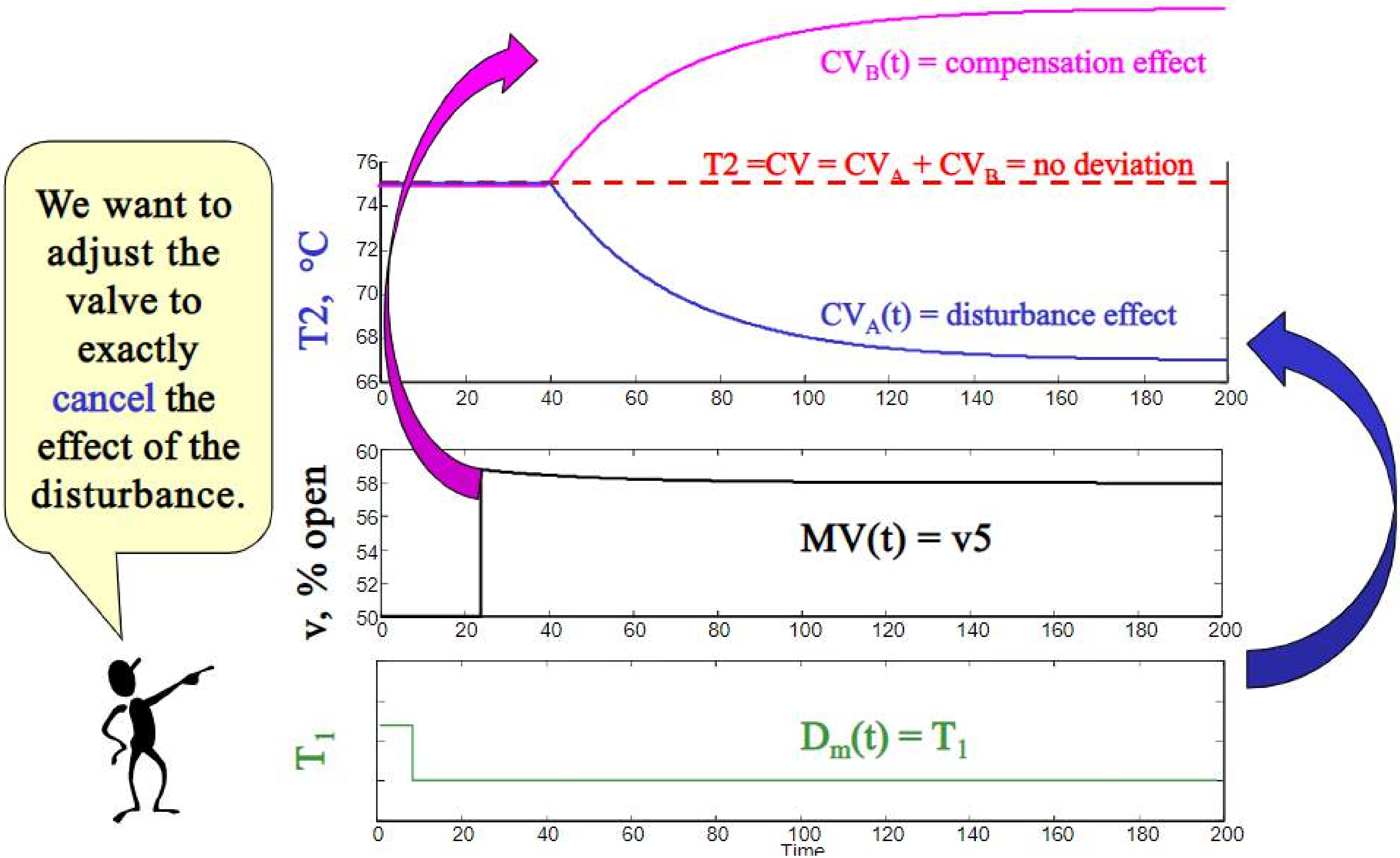
- Causal relationship from T_1 to T_2
- How can we manipulate valve to compensate?



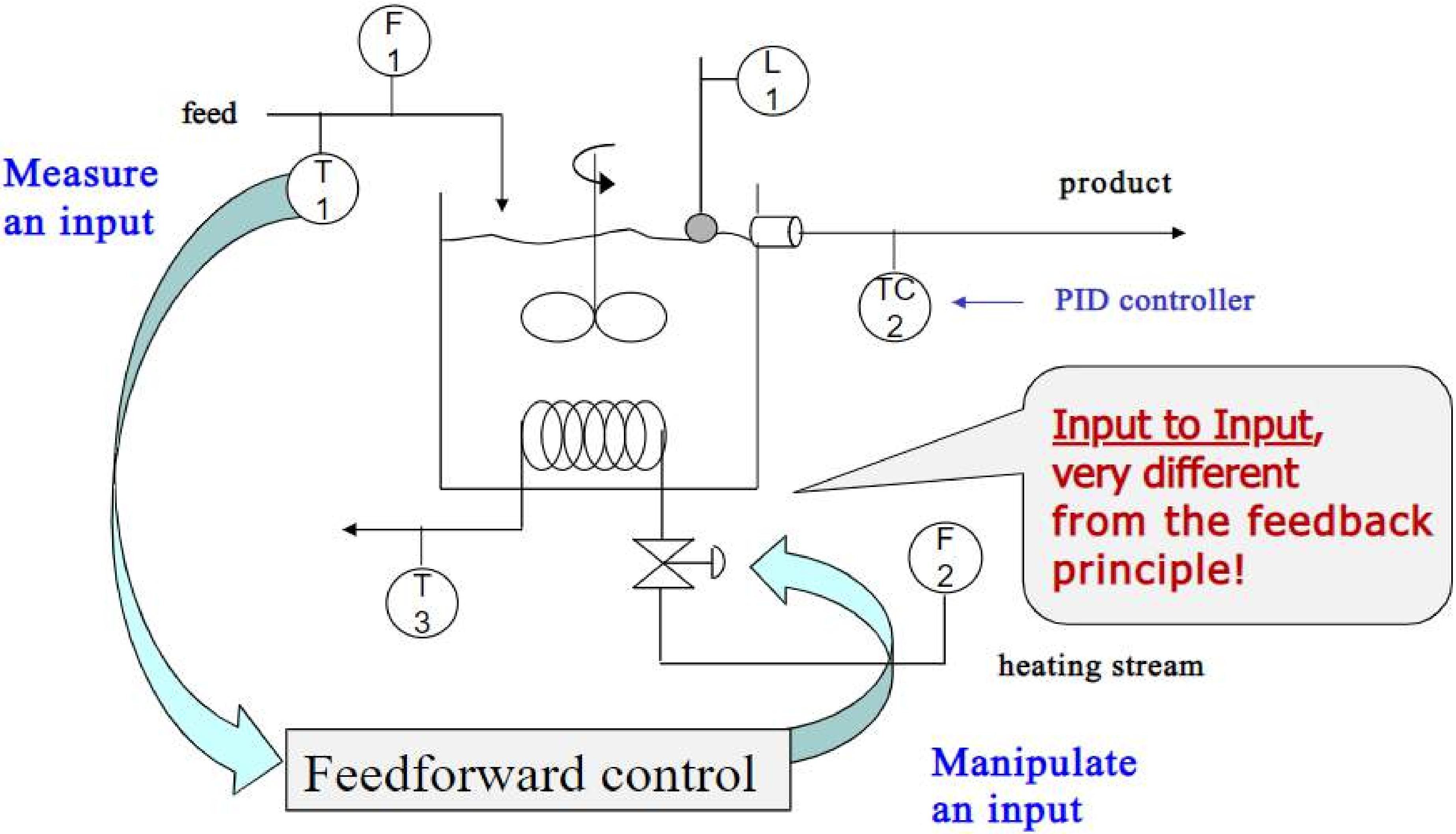
v5 (valve) → flow → heat transfer → TC2

T_1 (Feed temperature) → TC2

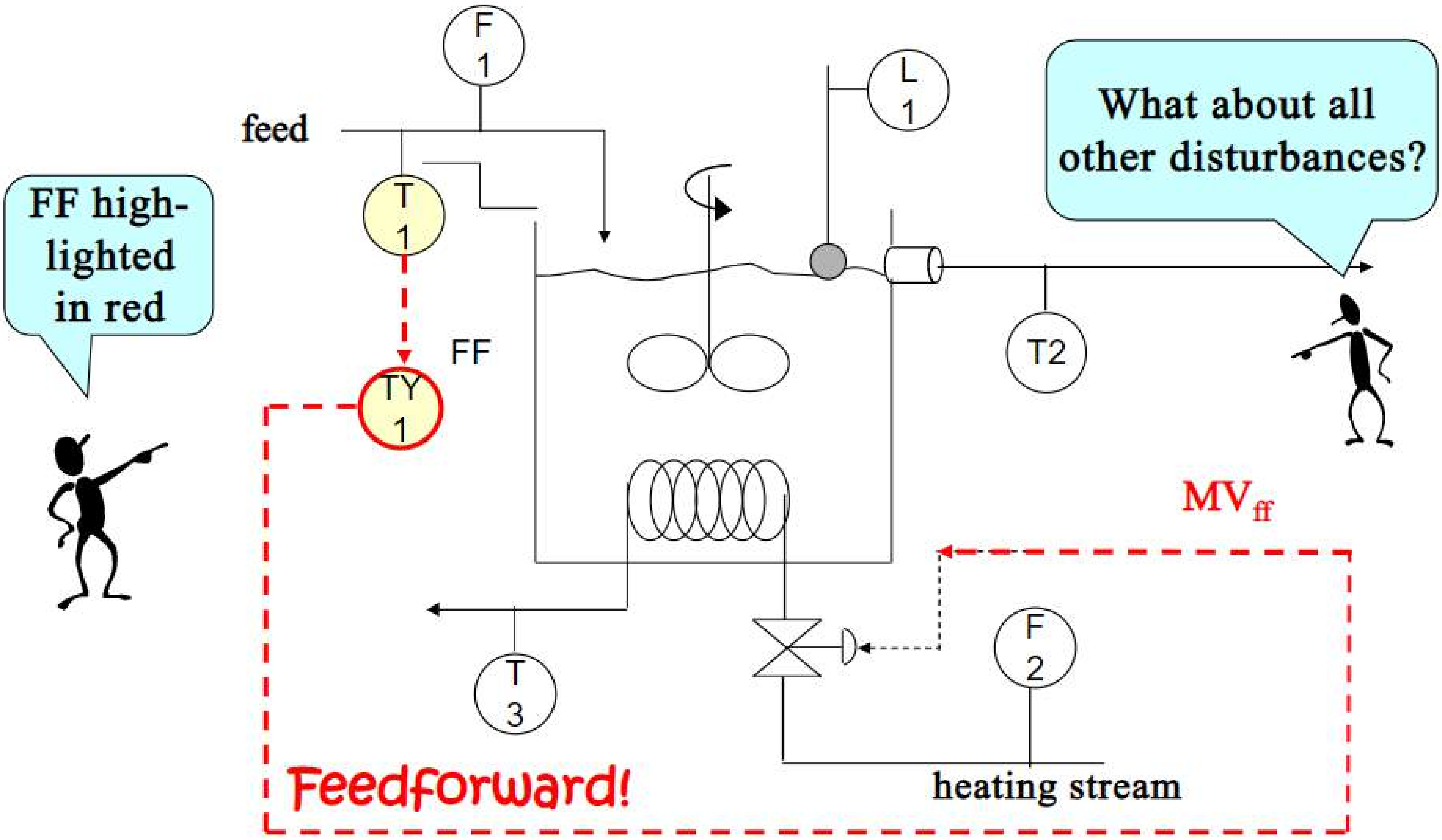
Feedforward is designed to "intercept" the disturbance



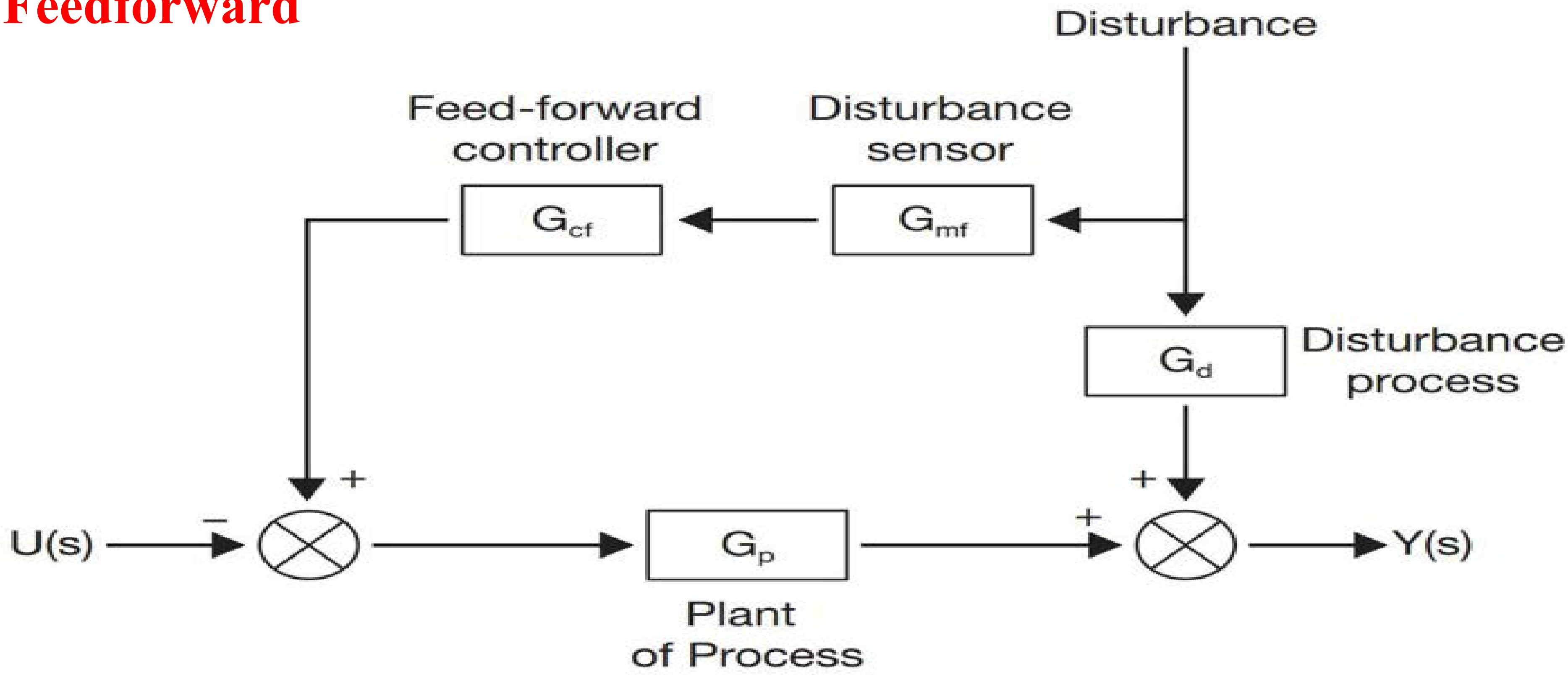
Concept of Feedforward for CST Heater



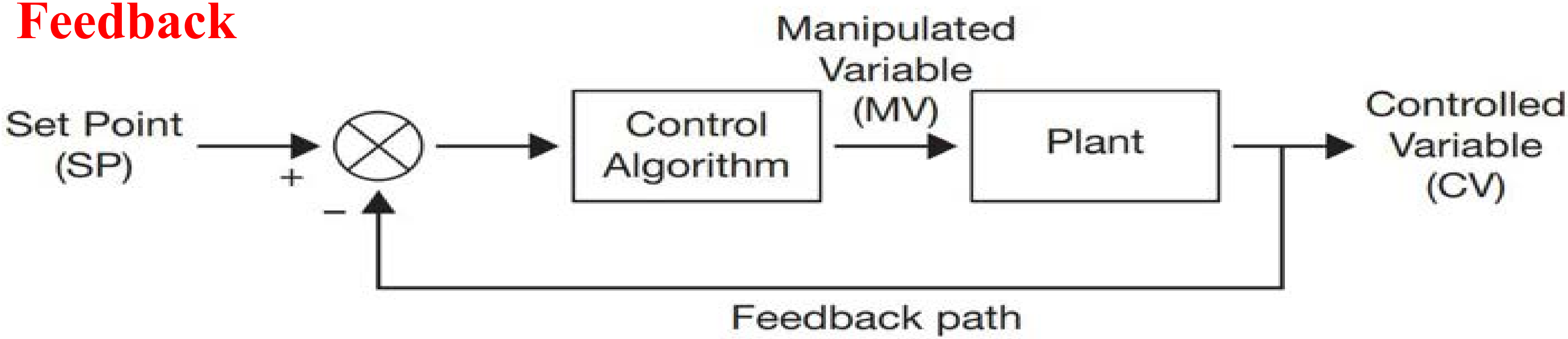
P&I Drawing with Feedforward Control



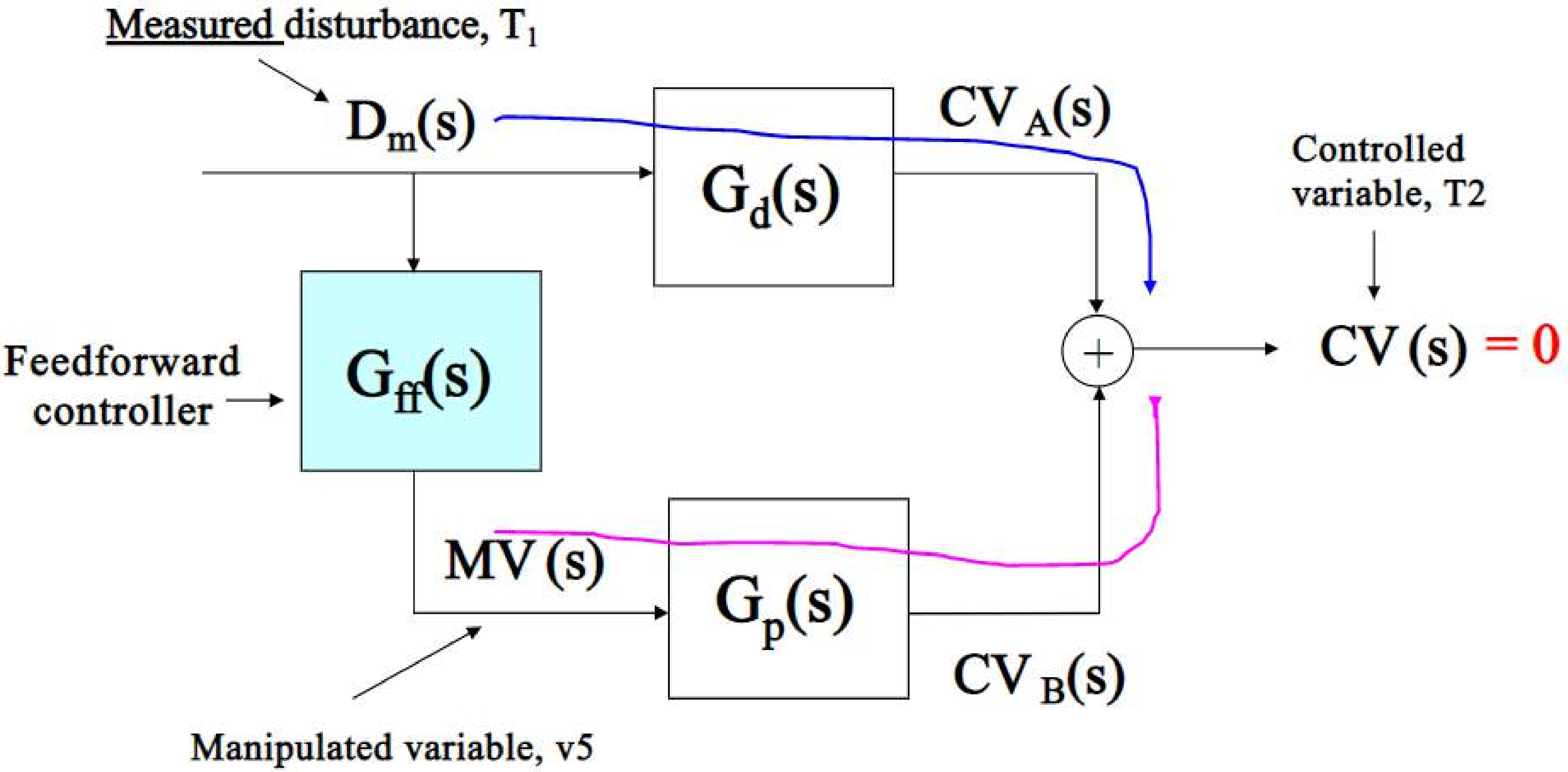
Feedforward



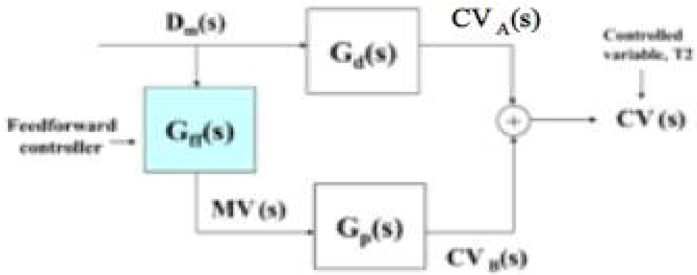
Feedback



We use block diagram algebrato determine the form of the calculation [$G_{ff}(s)$] to achieve the desired performance.



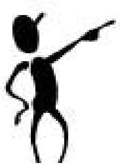
Determine $G_{ff}(s)$ so that $CV'(s) = 0$



$$CV(s) = CV_A(s) + CV_B(s)$$

$$CV(s) = \left[G_d(s)D_m(s) + G_{ff}(s)G_p(s)D_m(s)\right] = 0$$

- Not a PID algorithm!
- Depends on process models



$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = -\frac{G_d(s)}{G_p(s)}$$

This is general!

Class Exercise 1: Approximate first-order with dead time models - Solution

$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = -\frac{G_d(s)}{G_p(s)}$$

General expression for the feedforward controller

$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = -\frac{K_d e^{-\theta_d s} / (\tau_d s + 1)}{K_p e^{-\theta_p s} / (\tau_p s + 1)}$$

Special case for first order with dead time models

$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = \left[-\frac{K_d}{K_p}\right]\left[\frac{\tau_p s + 1}{\tau_d s + 1}\right] e^{-(\theta_d - \theta_p)s}$$

Rearrange to isolate gain, time-constant and dead-time elements

$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = K_{ff} \quad \frac{T_{ld} s + 1}{T_{lg} s + 1} \quad e^{-\theta_{ff} s}$$

Summarize as elements calculated by the feedforward

Gain Lead-lag Dead time

Parameters in the Feedforward Controller

Measured disturbance → Feedforward controller calculation → MV adjustment from feedforward controller

$$D_m(s) \longrightarrow G_{ff}(s) = K_{ff}\,\frac{T_{ld}s+1}{T_{lg}s+1}\,e^{-\theta_{ff}s} \longrightarrow MV_{ff}(s)$$

Lead-lag	$= (T_{ld}s+1)/T_{lg}s+1)$
FF controller gain	$= K_{ff} = -K_d/K_p$
Controller dead time	$= \theta_{ff} = \theta_d - \theta_p \geq 0$
Lead time	$= T_{ld} = \tau_p$
Lag time	$= T_{lg} = \tau_d$

Digital implementation of Feedforward Controller

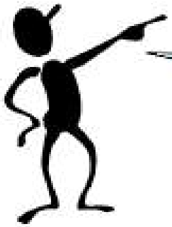
$$G_{ff}(s) = \frac{MV(s)}{D_m(s)} = K_{ff} \frac{T_{ld}s+1}{T_{lg}s+1} e^{-\theta_{ff}s}$$

$$(MV_{ff})_N = \frac{T_{lg}/\Delta t}{T_{lg}/\Delta t+1}(MV_{ff})_{N-1} + K_{ff}\left(\frac{T_{ld}/\Delta t+1}{T_{lg}/\Delta t+1}\right)(D_m)_{N-\Gamma}$$

$$-K_{ff}\left(\frac{T_{ld}/\Delta t}{T_{lg}/\Delta t+1}\right)(D_m)_{N-\Gamma-1}$$

$$\Gamma = \frac{\theta_{ff}}{\Delta t}$$

$$(MV_{ff})_N = a(MV_{ff})_{N-1} + b(D_m)_{N-\Gamma} + c(D_m)_{N-\Gamma-1}$$

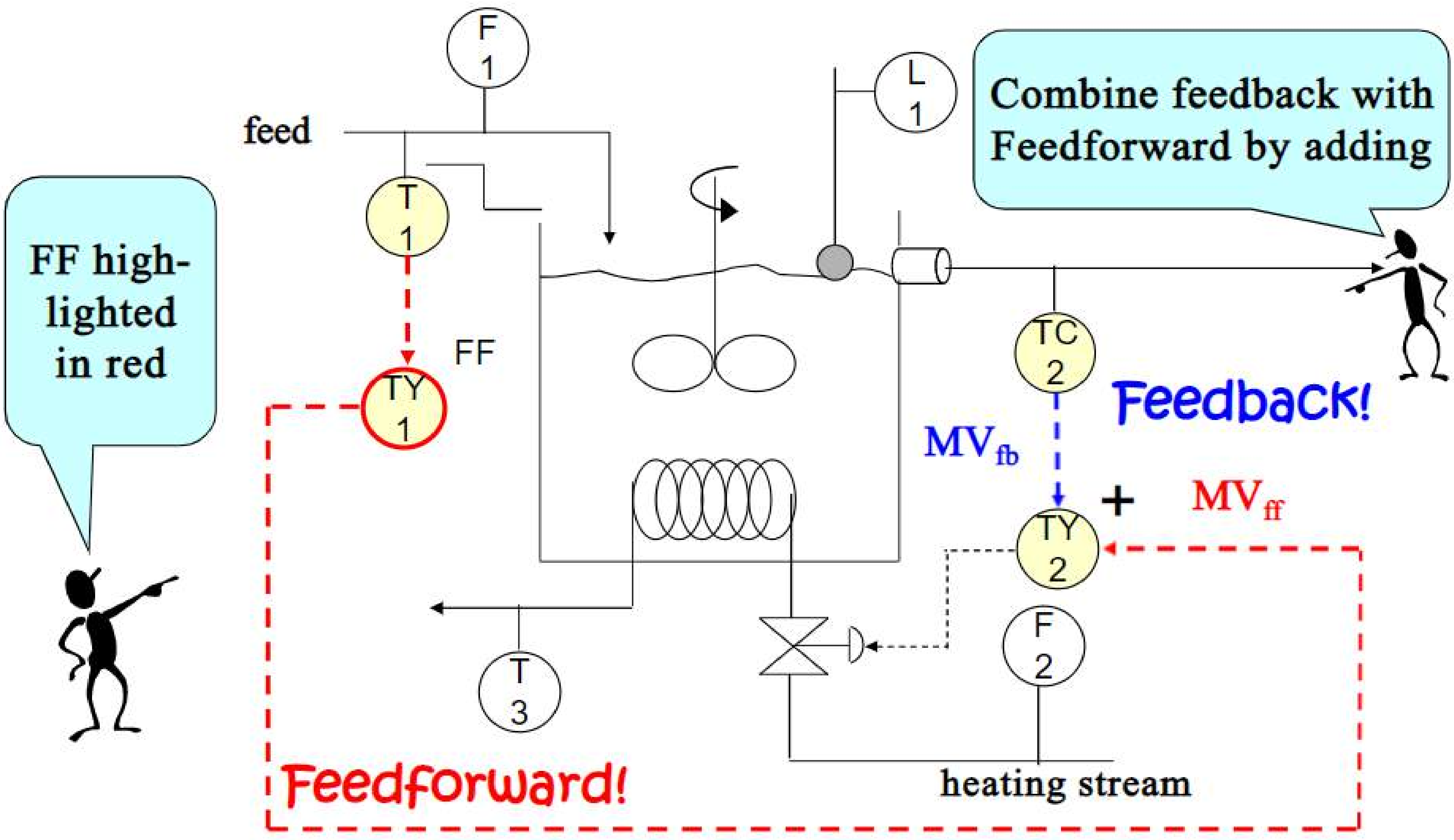


Digital implementation is straightforward.
It is derived in textbook.

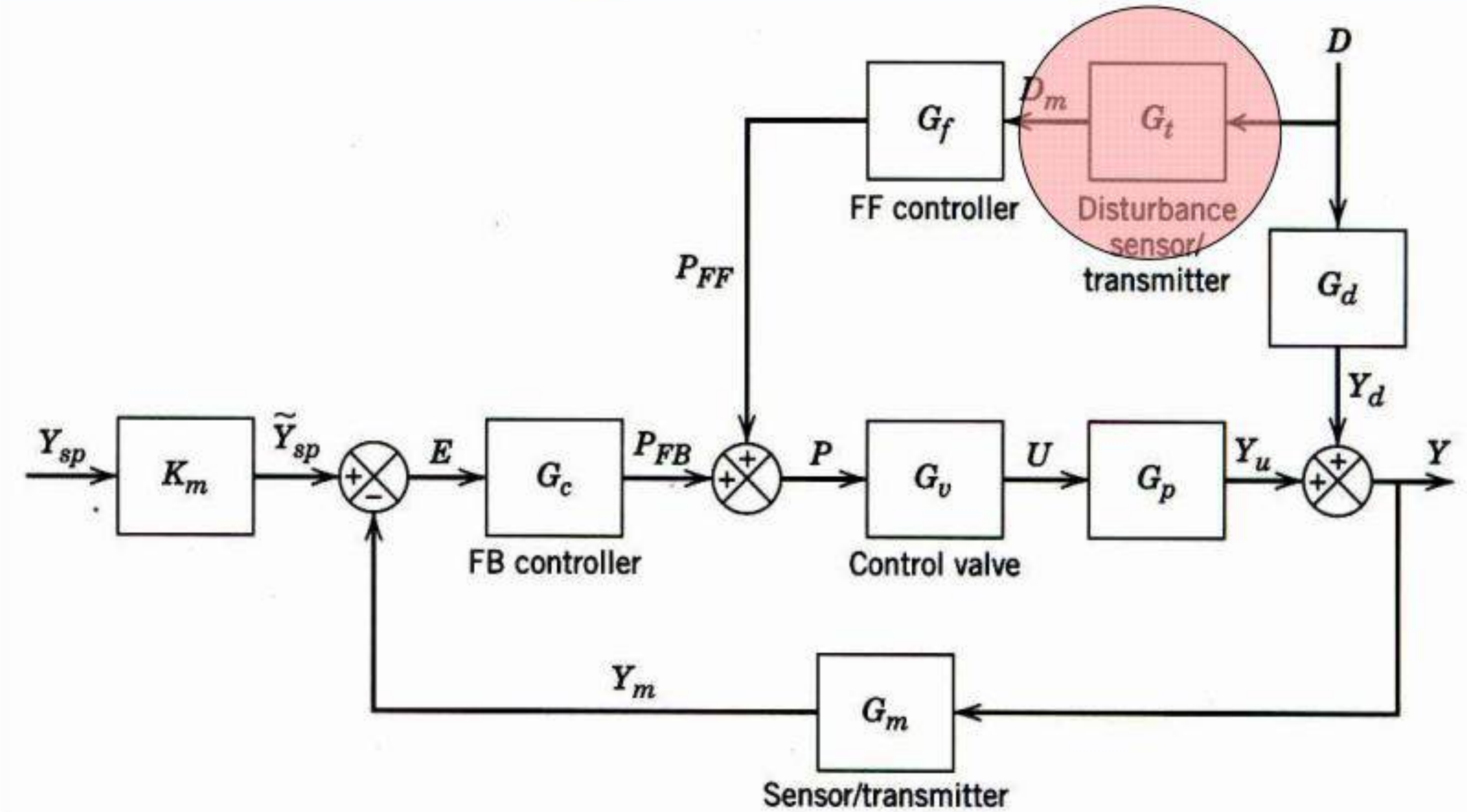
Practical Feedforward + Feedback Combination

- In practice, we **never** use feedforward alone.
- Final structure:
  - **Feedback loop** for:
    - Handling unmeasured disturbances
    - Correcting modelling errors
    - Ensuring steady-state tracking
  - **Feedforward block** for:
    - Fast compensation of **measured disturbances**
- Implementation strategy:
  - Design/tune the feedback loop (usually a PI or PID).
  - Add feedforward, designed from process models or static gains.
  - Adjust FF parameters empirically if models are approximate.

P&I Drawing with Feedforward & Feedback Control



Seborg's version



The closed-loop transfer function for disturbance changes in Eq. 15-20 can be derived using the block diagram algebra that was introduced in Chapter 11:

$$\frac{Y(s)}{D(s)} = \frac{G_d + G_t G_f G_v G_p}{1 + G_c G_v G_p G_m} \tag{15-20}$$

Ideally, we would like the control system to produce *perfect control*, where the controlled variable remains exactly at the set point despite arbitrary changes in the disturbance variable, D. Thus, if the set point is constant ($Y_{sp}(s) = 0$), we want $Y(s) = 0$, even though $D(s) \neq 0$. This condition can be satisfied by setting the numerator of (15-20) equal to zero and solving for G_f:

$$G_f = -\frac{G_d}{G_t G_v G_p} \tag{15-21}$$

Stability Considerations

To analyze the stability of the closed-loop system in Fig. 15.11, we consider the closed-loop transfer function in Eq. 15-20. Setting the denominator equal to zero gives the characteristic equation,

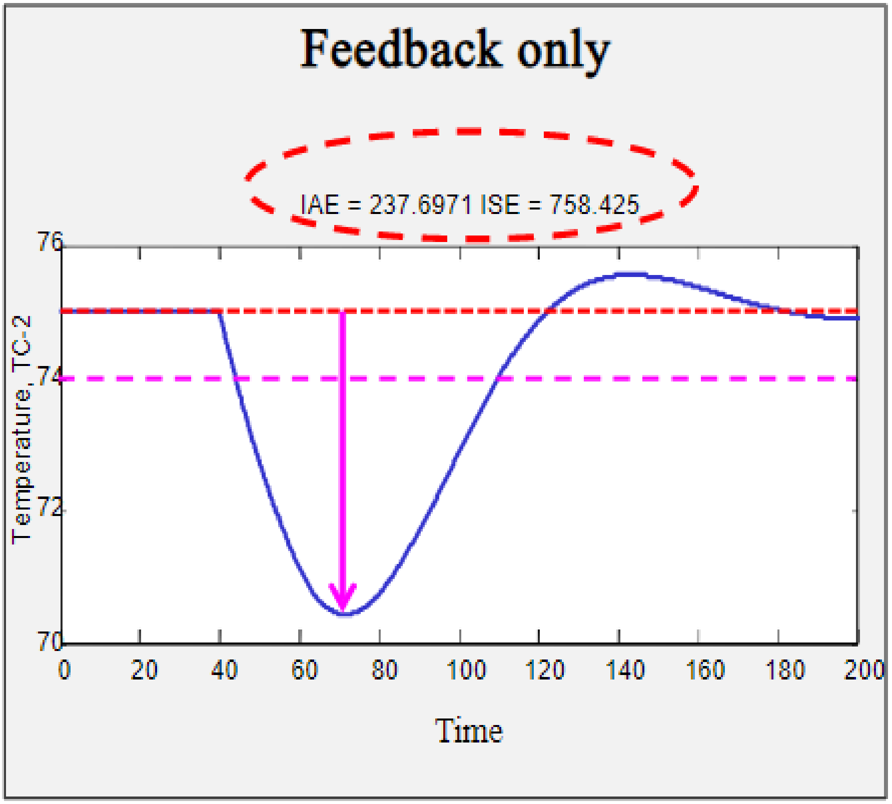
$$1 + G_c G_v G_p G_m = 0 \tag{15-28}$$

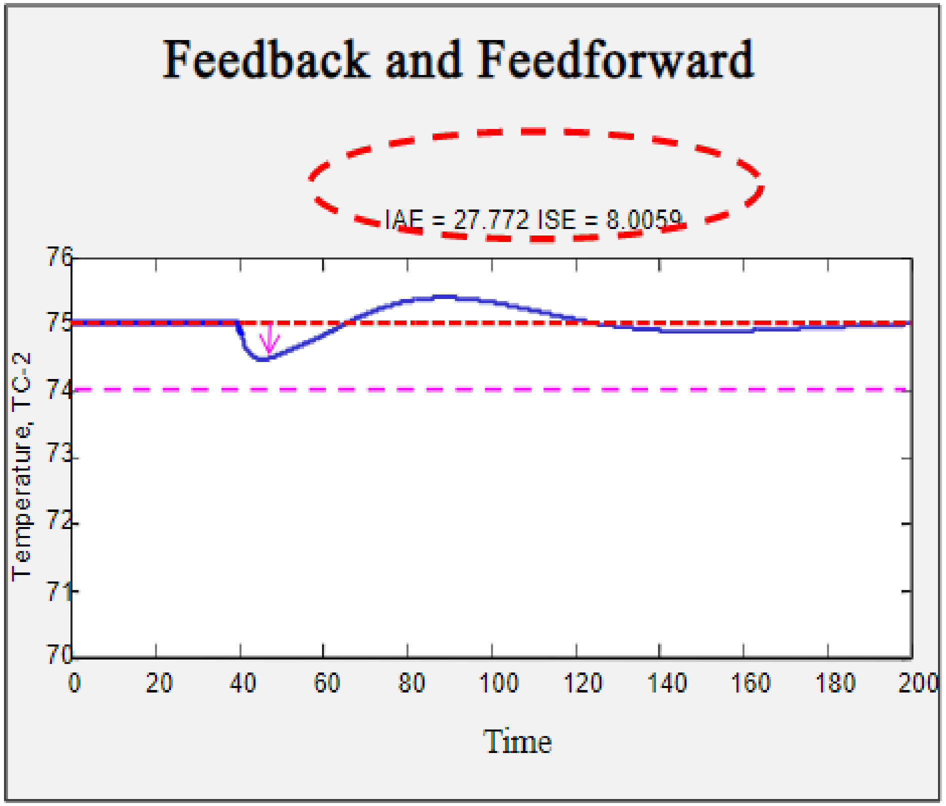
In Chapter 11 it was shown that the roots of the characteristic equation completely determine the stability of the closed-loop system. Because G_f does not appear in the characteristic equation, the feedforward controller has no effect on the stability of the feedback control system. This is a desirable situation that allows the feedback and feedforward controllers to be tuned individually.

Feedforward and Feedback are complementary

	Feedforward	Feedback
Advantages	• Compensates for disturbance before CV is affected • Does not affect the stability of the control system (if $G_{ff}(s)$ stable)	• Provides zero steady-state offset • Effective for all disturbances
Disadvantages	• Cannot eliminate steady-state offset • Effective for only measured disturbance(s) • Requires a sensor and model for each disturbance	• Does not take control action until the CV deviates from its set point • Affects the stability of the control system

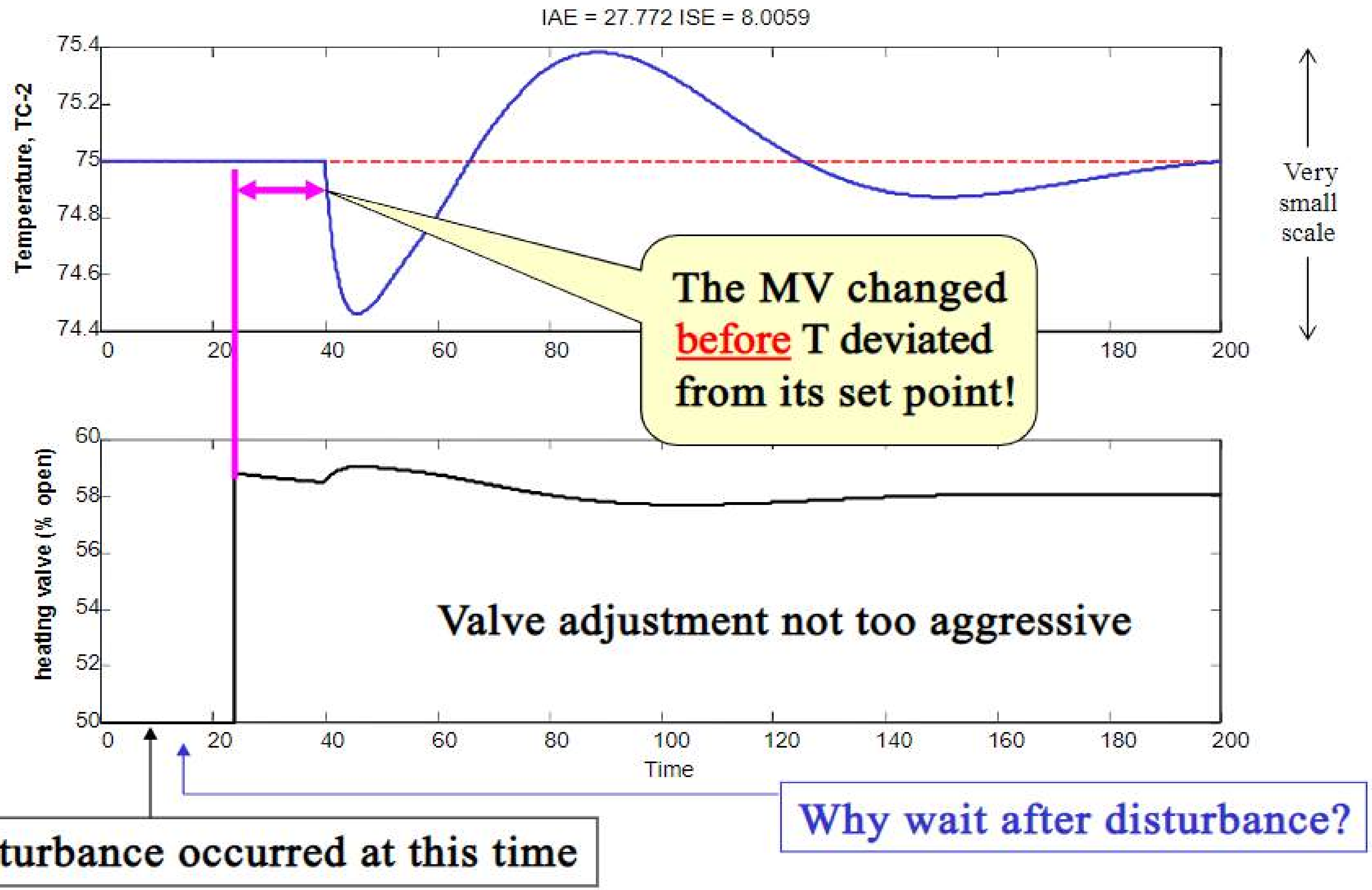
Control Performance Comparison for CST Heater



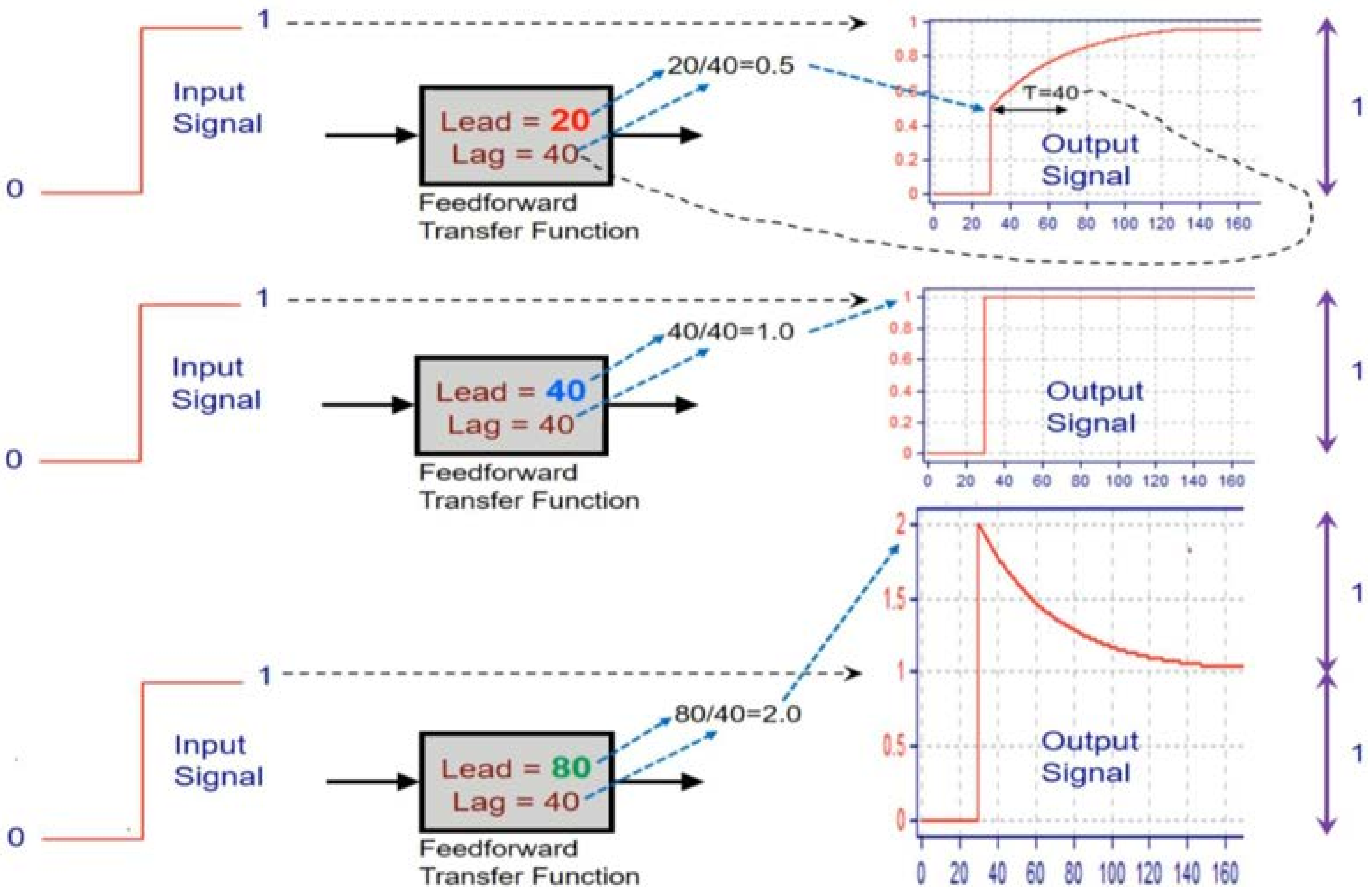


Much better performance, smaller IAE and maximum deviation! WHY?

Why does feedforward improve the control performance?



Feedforward Lead and Lag Effects



Why Perfect Feedforward Control is Never Achieved

Feedforward control is used in conjunction with PID feedback control. Cannot adopt feedforward-only control philosophy. This is because:

- Process and load transfer functions cannot be 100% accurate
- Some variables may not be accurately measured
- Process dead time may be larger than the load dead time
- There can be unmeasured disturbances and process nonlinearities

Important factors for Feedforward Control

- Very noisy PV - need to filter PV
- Excessive changes in SP - need to reduce Lead paramater
- Large changes in OP of disturbance loop in manual mode - need to deactive FeedForward completely
- Process nonlinearities - need Gain Scheduling
- Multiple flow meters - need discrete switching logic
- Instrument failure: bad PV, frozen PV, NaN values etc. - need DCS logic to prevent undesirable control action
- Field Tech. Calibration 0 to Full Scale – need DCS logic

Feedback and Feedforward Comparison

- Feedback PID
  - Reactive
  - Controlled variable must deviate from setpoint before any control action can be taken
  - Can become unstable if tuning is too aggressive and/or impatient
- Feedforward
  - Proactive
  - Corrective action is taken as soon as a disturbance is detected
  - Cannot become unstable

Simplified Feedforward Control

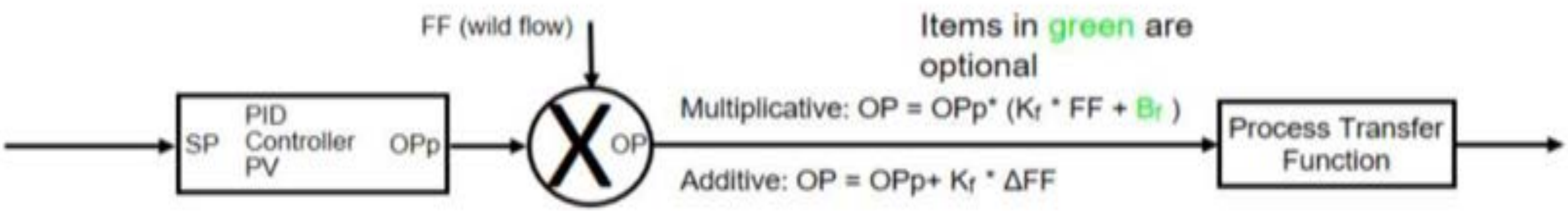
- Often the parameters of a disturbance transfer function cannot be identified or the dynamics are such that compensation will not work.
- In these situations standard (or simple) feedforward can be used.
- Though not as effective as feedforward compensation presented earlier, it can still can improve process operations significantly.
- Some DCS systems have feedforward algorithms built into or as an option of their PID algorithm.
- If there is no feedforward algorithm, simple multipliers, dividers, and/or summers will work.

PID

PIDFF

Simplified Feedforward Methods

- There are two basic methods of simplified feedforward control – multiplicative and additive.
- These are shown below.

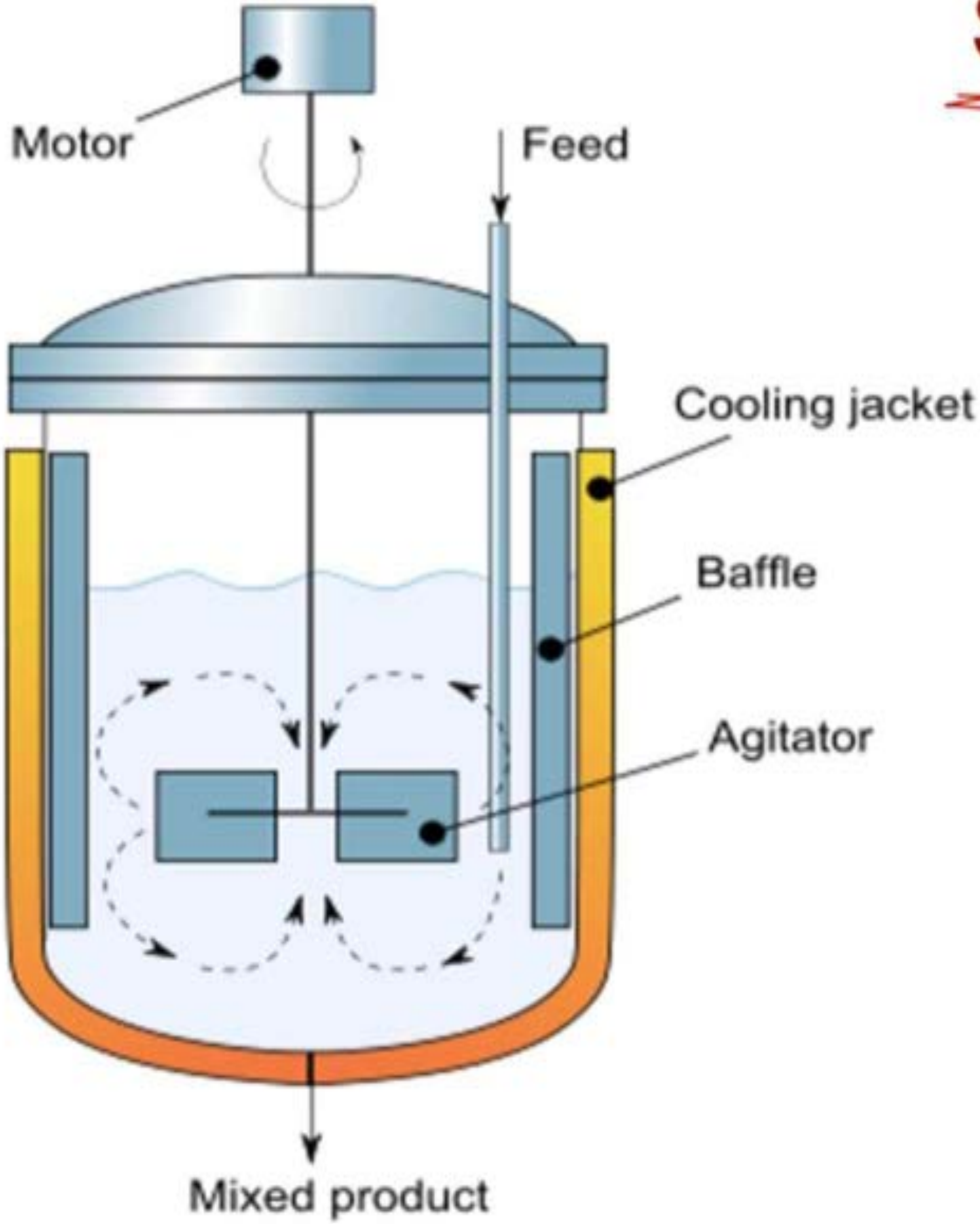


When not in cascade initialization is performed as

$$\text{Multiplicative: } OPp = InitVal = \frac{OP}{(K_f * FF + B_f)}$$

$$\text{Additive: } OPp = InitVal = OP$$

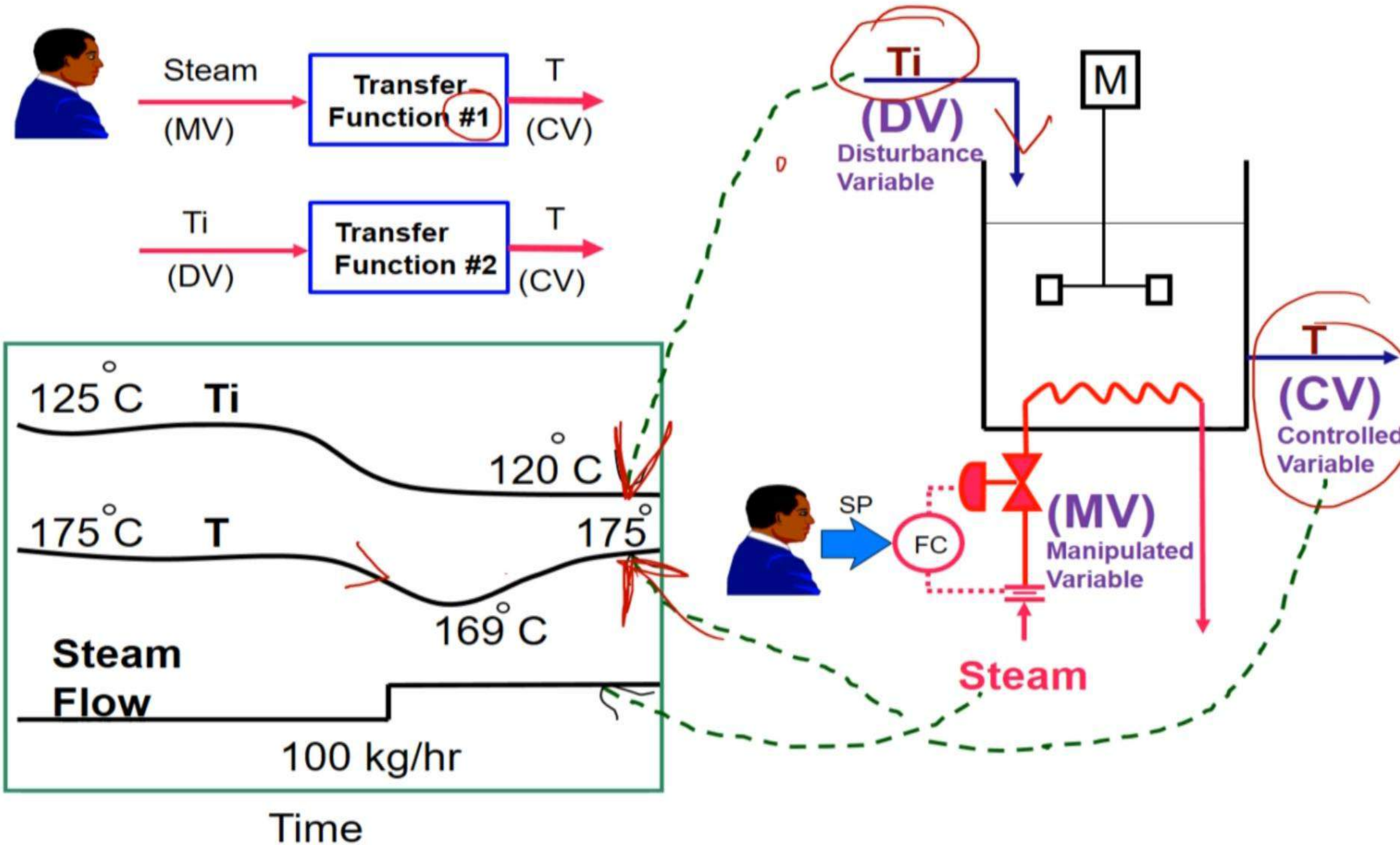
because first execution in auto sets $FF_{n-1} = FF_n$



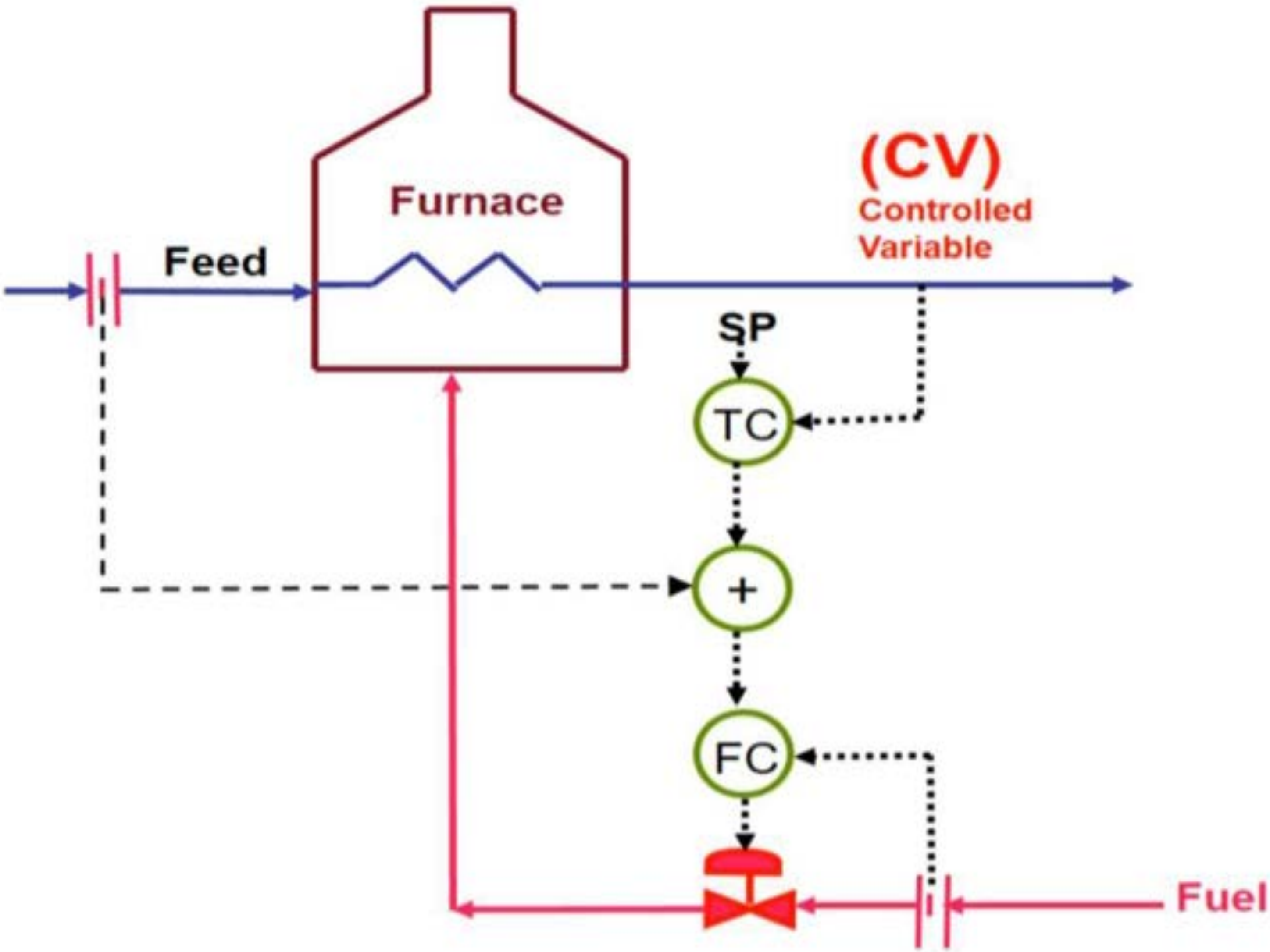
Stirred Reactor Feedforward Application

on a CSTR

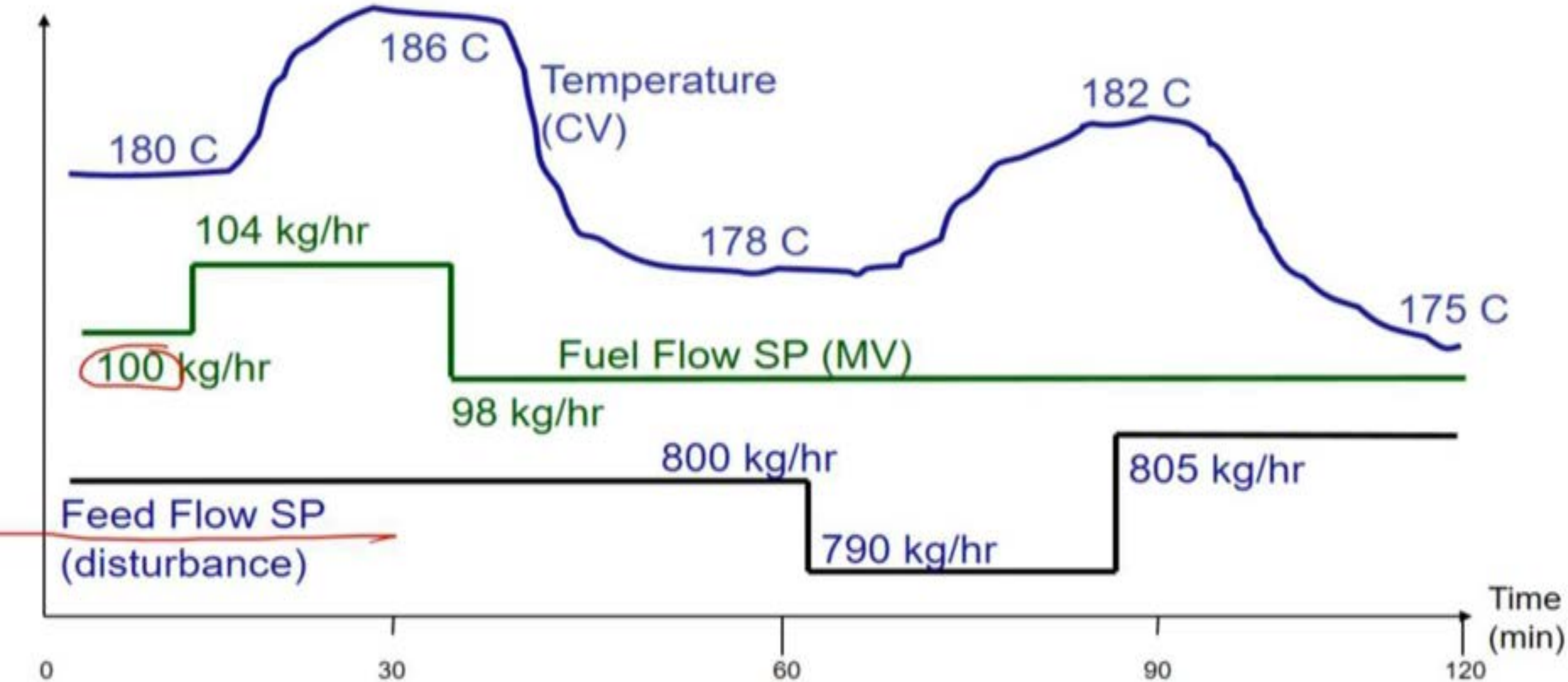
Reactor Temperature Disturbance



Furnace Feedforward Control Example



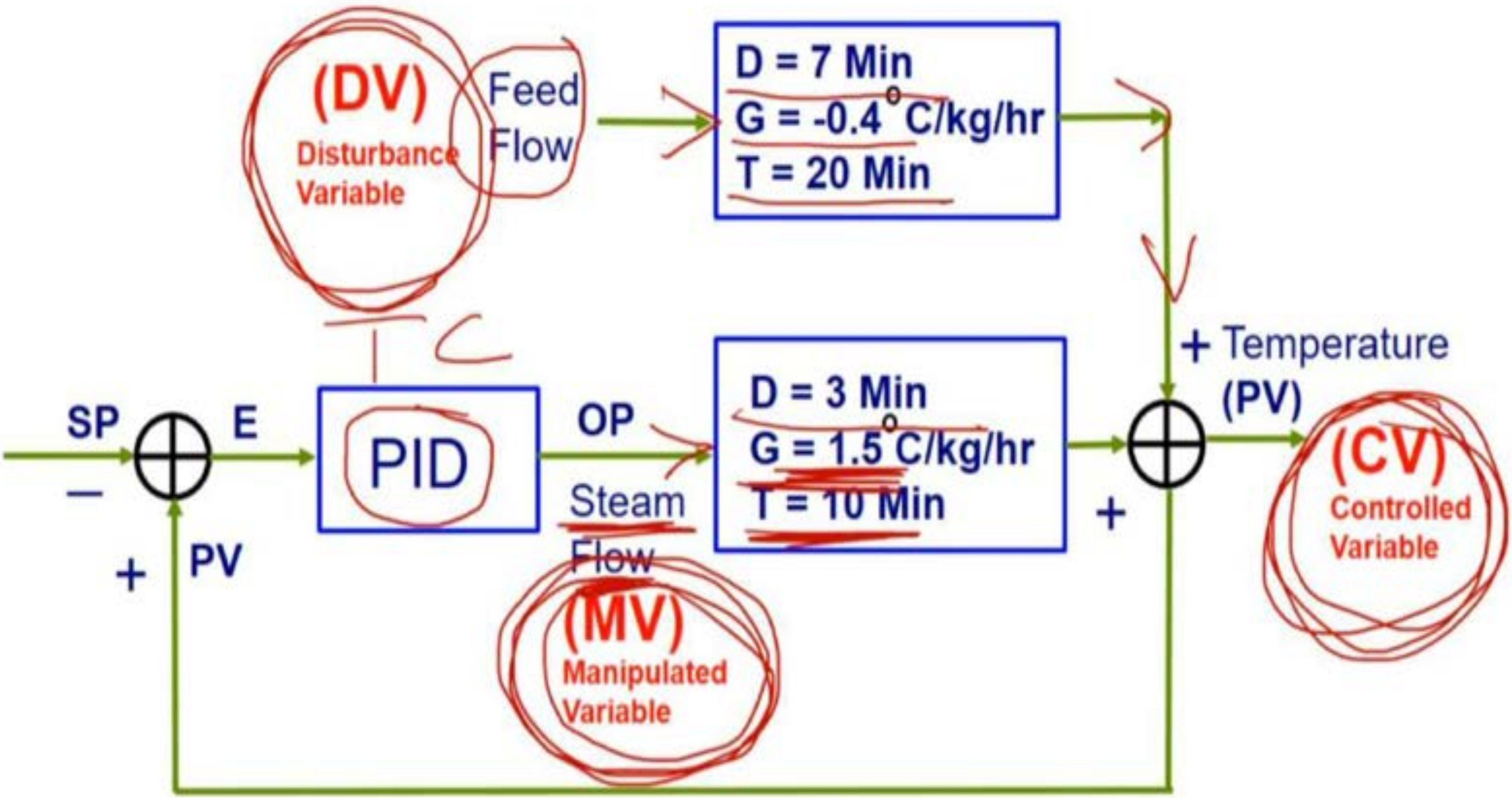
Transfer Function Calculations



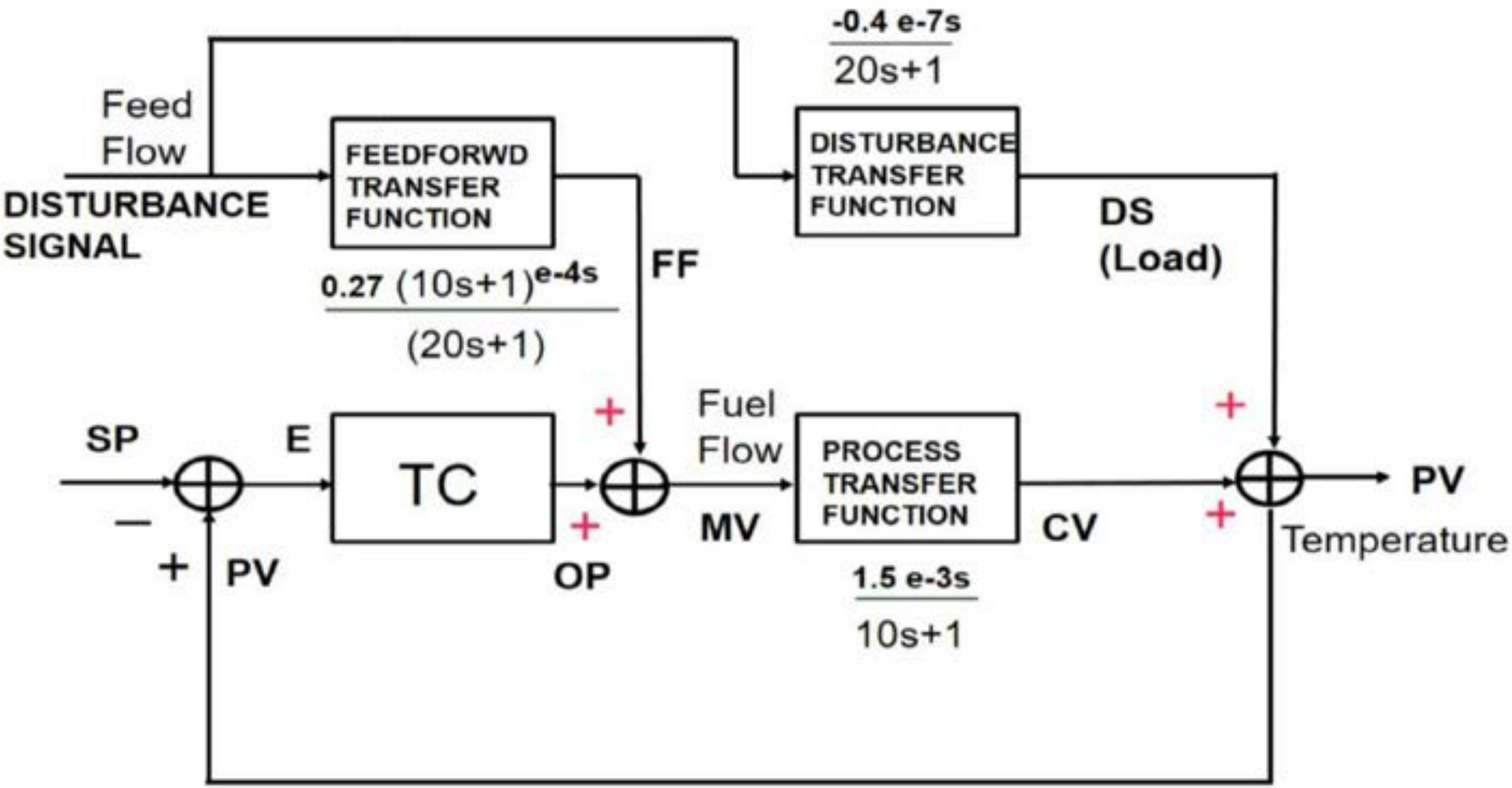
Transfer Function	Delay	Gain	Time Constant
Fuel -Temperature	3 min	1.5 C/(kg/hr)	10 min
Feed -Temperature	7 min	-0.4 C/(kg/hr)	20 min

Disturbance Signal Schematic

Process Control Schematic



Create Feedforward Transfer Function



Feedforward Parameters Delay, Gain, Lead, Lag

Dead Time = $DT_d - DT_p$
If $(DT_d - DT_{p)}) < 0$ Then Set Dead Time = 0

Gain = $- Gain_d \ / \ Gain_p$

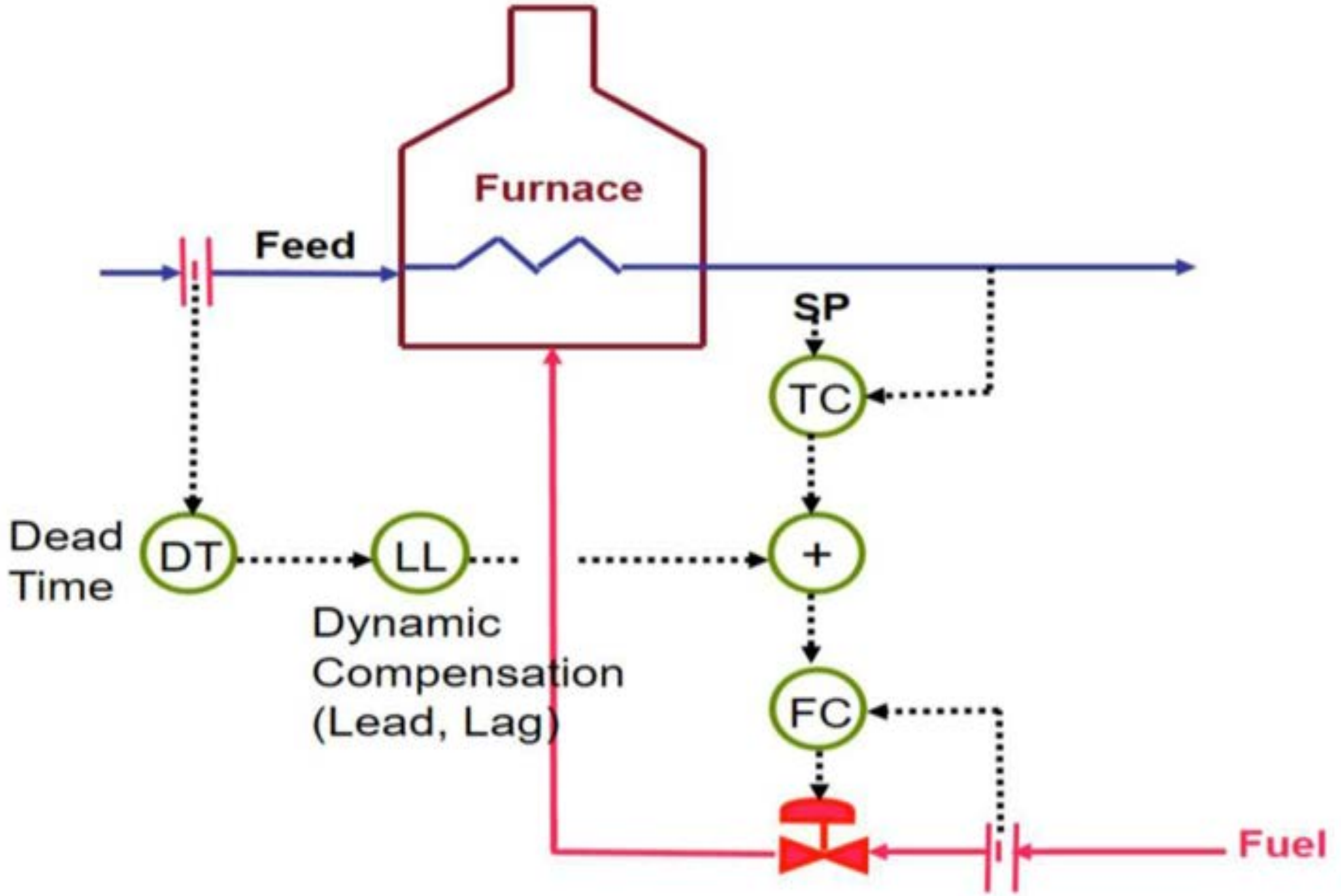
Lead = Time Constant $_p$

Lag = Time Constant $_d$

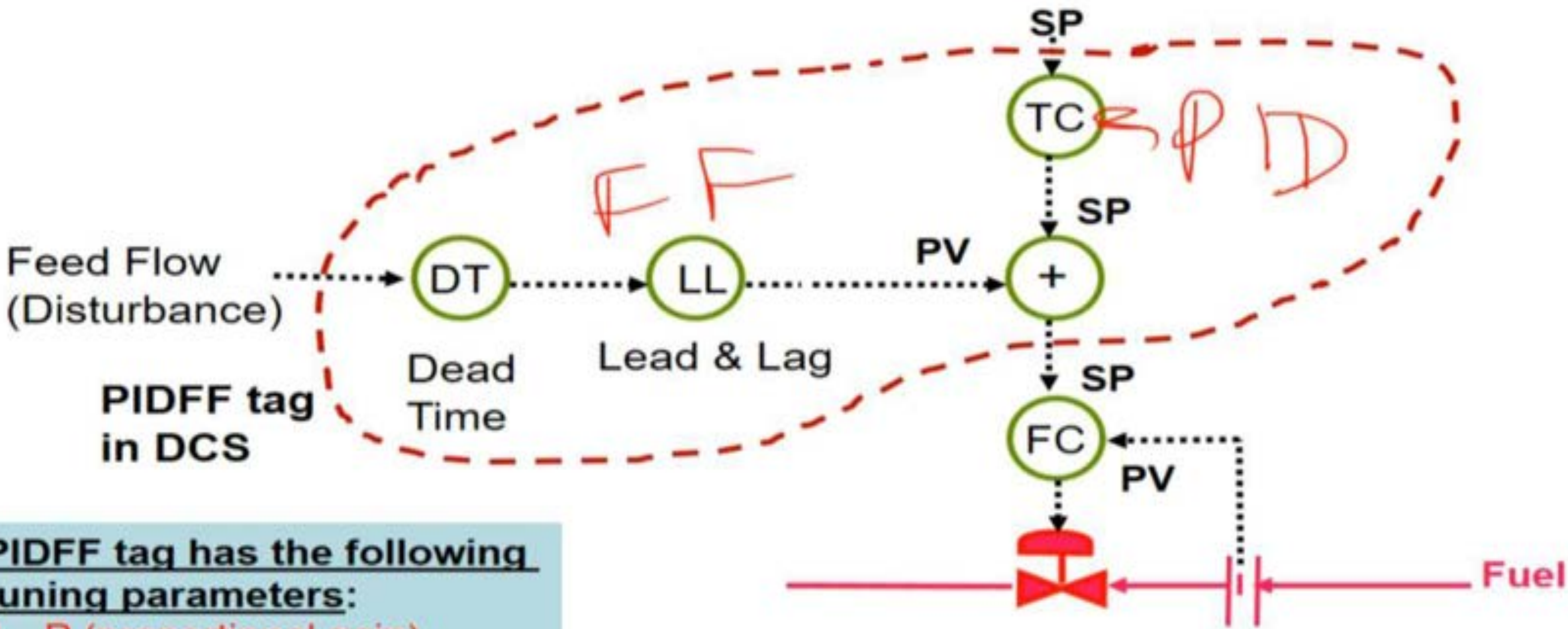
where, p=process transfer function
d=disturbance transfer function

$$\frac{-(G_d/G_p)\,(T_p s+1)e^{-(DTd\,-\,DTp)s}}{(T_d s+1)}$$

Feedforward Compensation Example



Feedforward Implementation in DCS

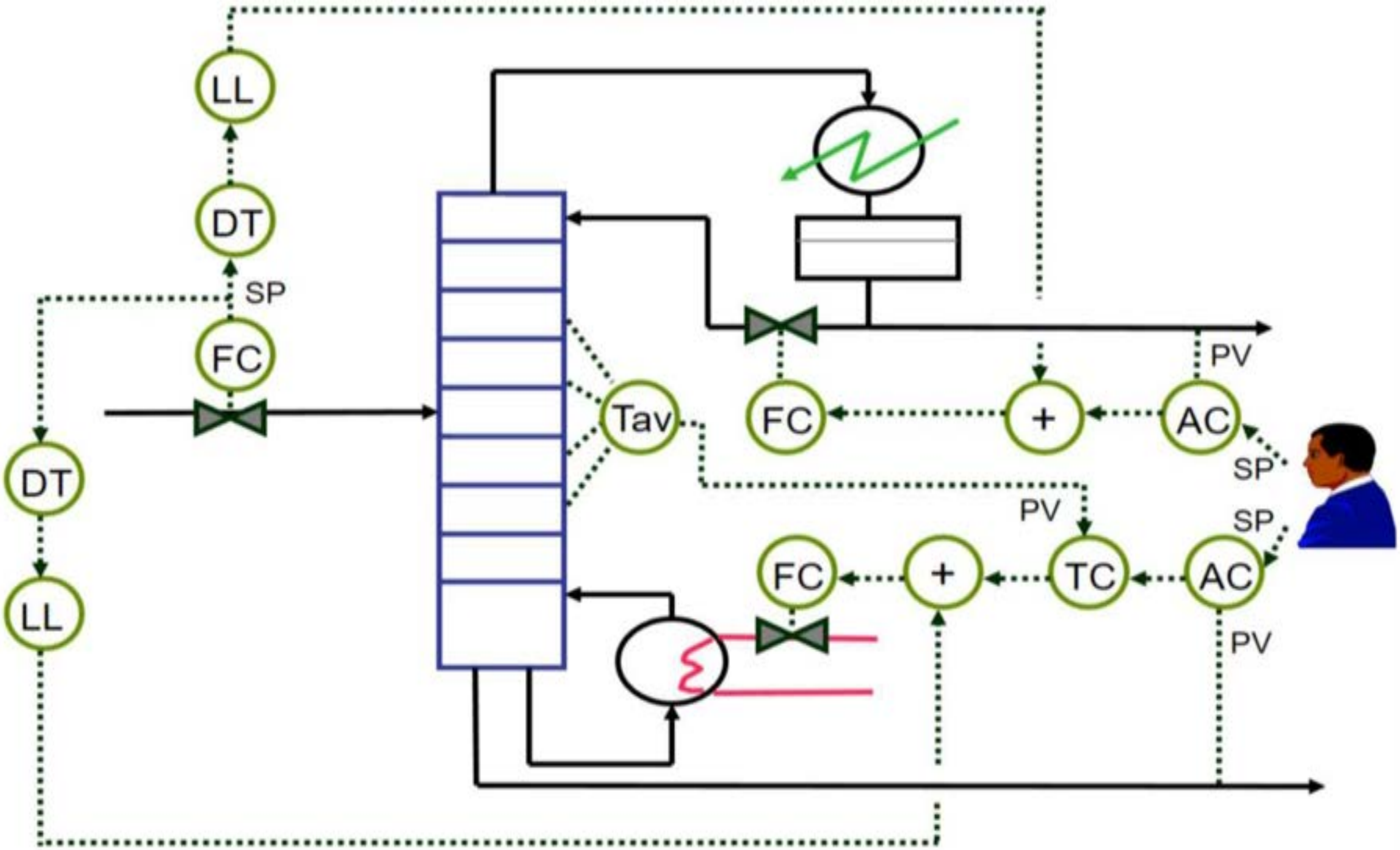


PIDFF tag has the following tuning parameters:

1. P (proportional gain)
2. I (integral)
3. D (derivative)
4. F (PV filter)
5. DT (dead time)
6. LD (lead)
7. LG (lag)
8. Kf (Feedforward Gain)

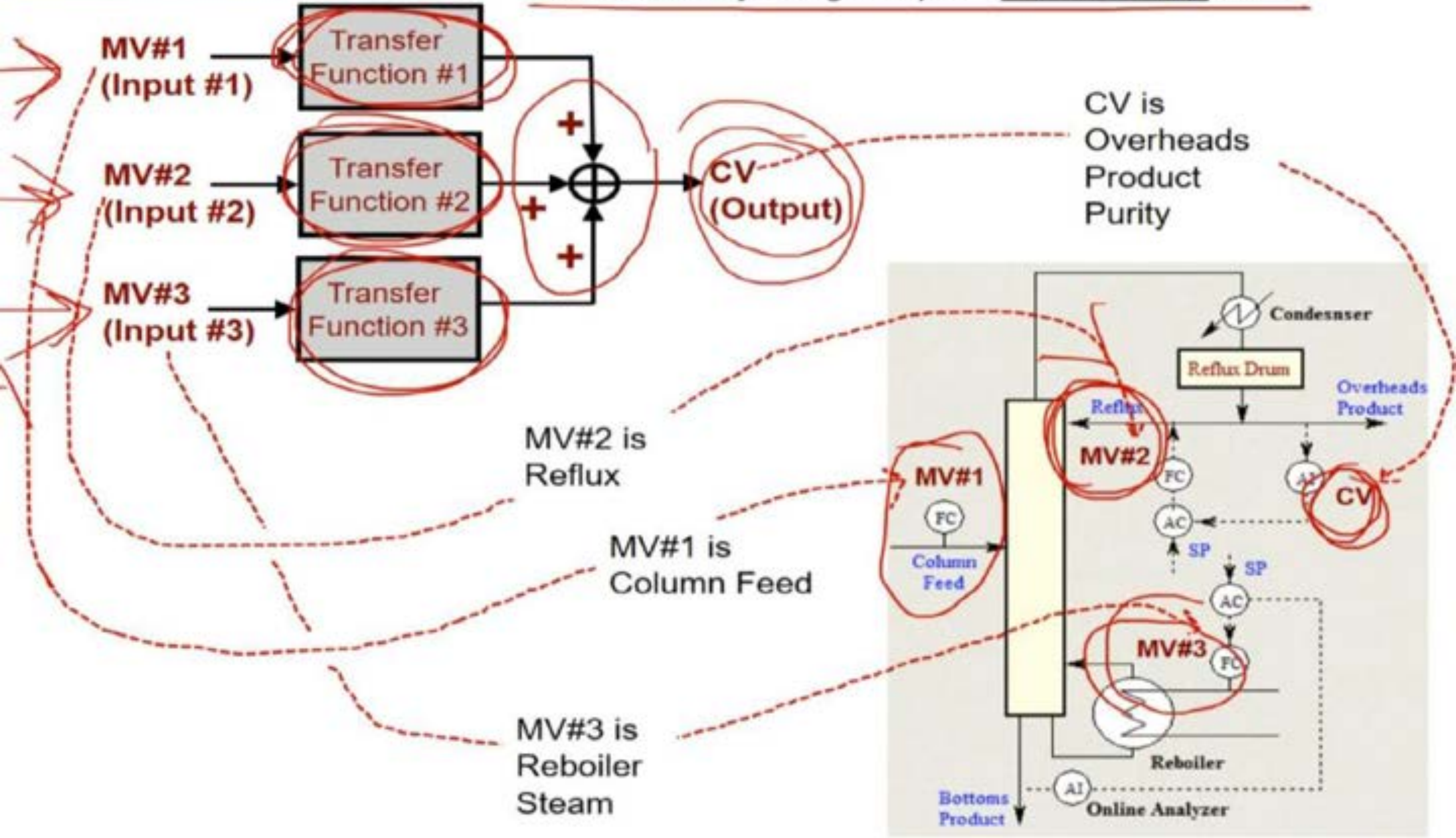
Distillation Column Feedforward Control

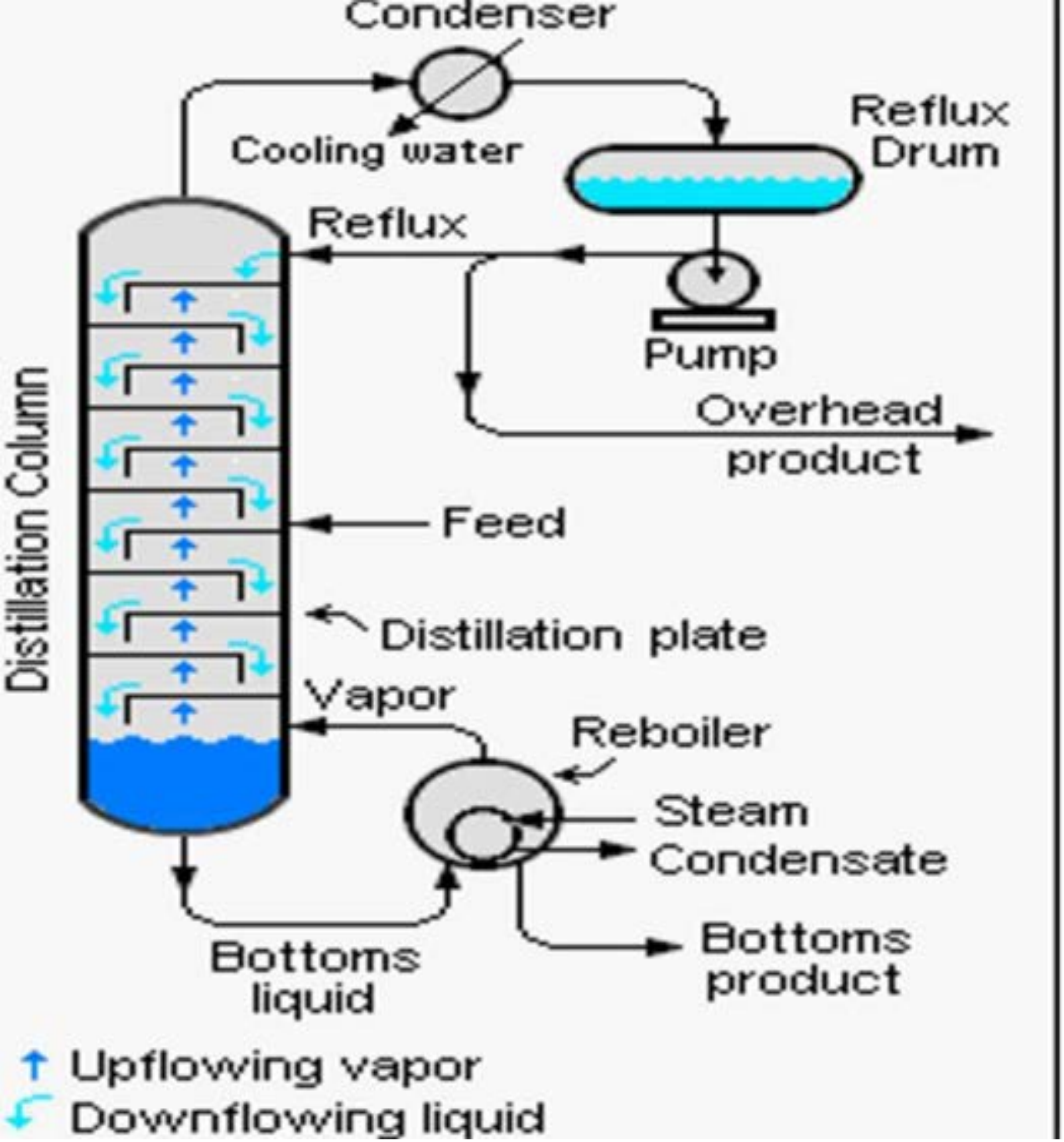
Advanced Control on Distillation Column



Multivariable Identification

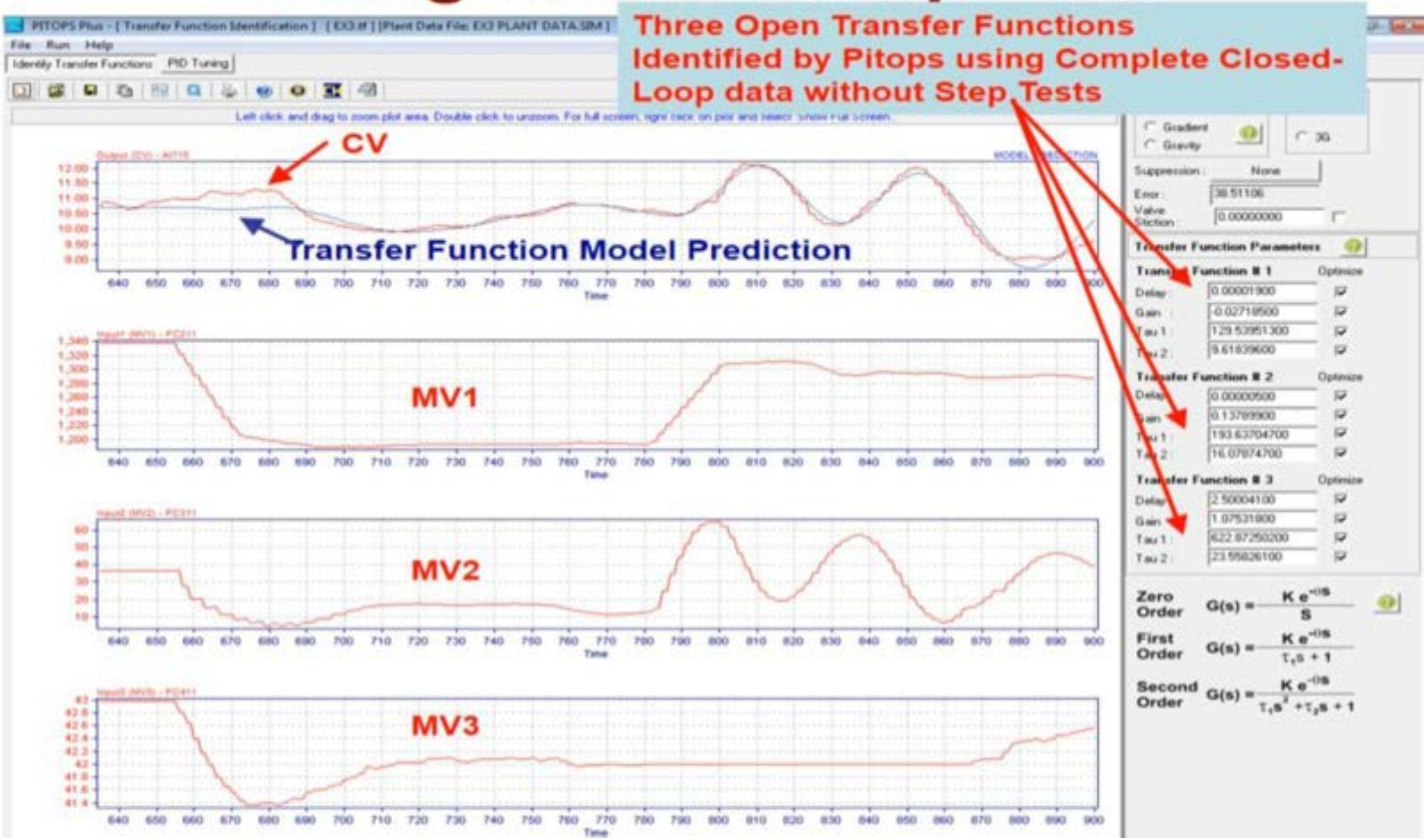
Pitops can process multivariable inputs and outputs (MIMO systems). See a three input multivariable identification below. Pitops identifies **Three** transfer functions simultaneously using complete **Closed-Loop** data.





Three Input Simultaneous Transfer Function Identification

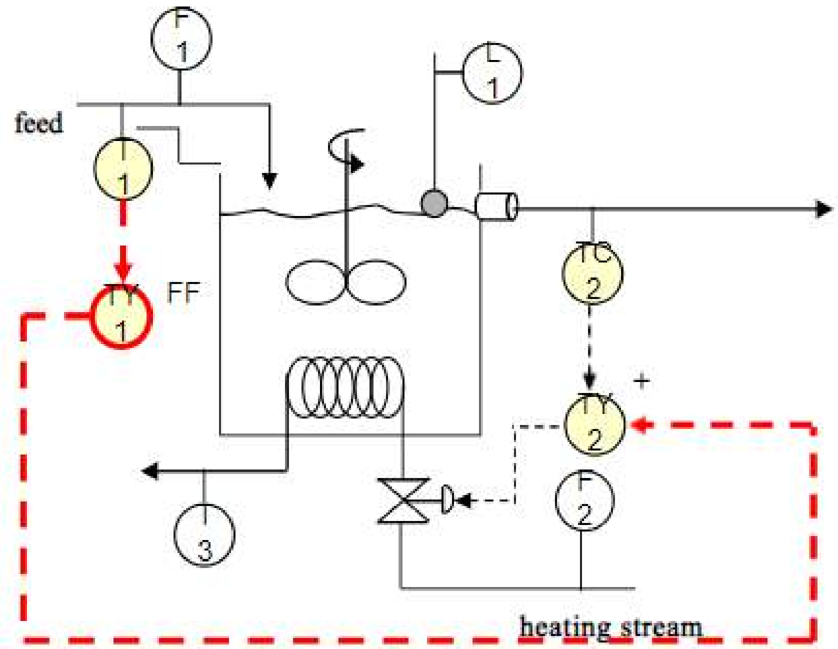
Simultaneous Identification Using Closed-Loop Data



Class Exercise 2:

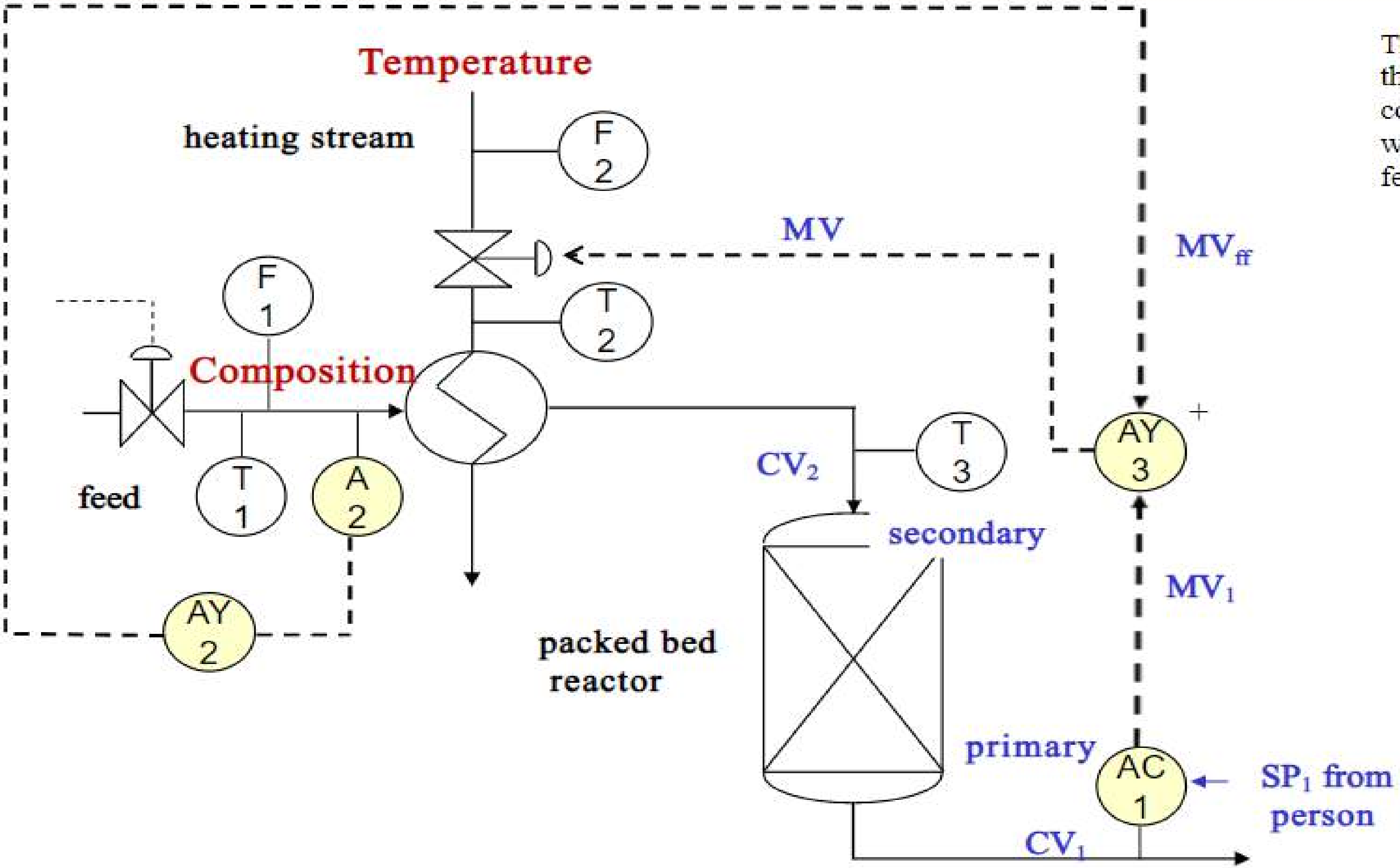
What have we gained and lost using feedforward and feedback?

For each sceanrio, is FF with FB better, same, worse than single-loop feedback (TC2 → v)?



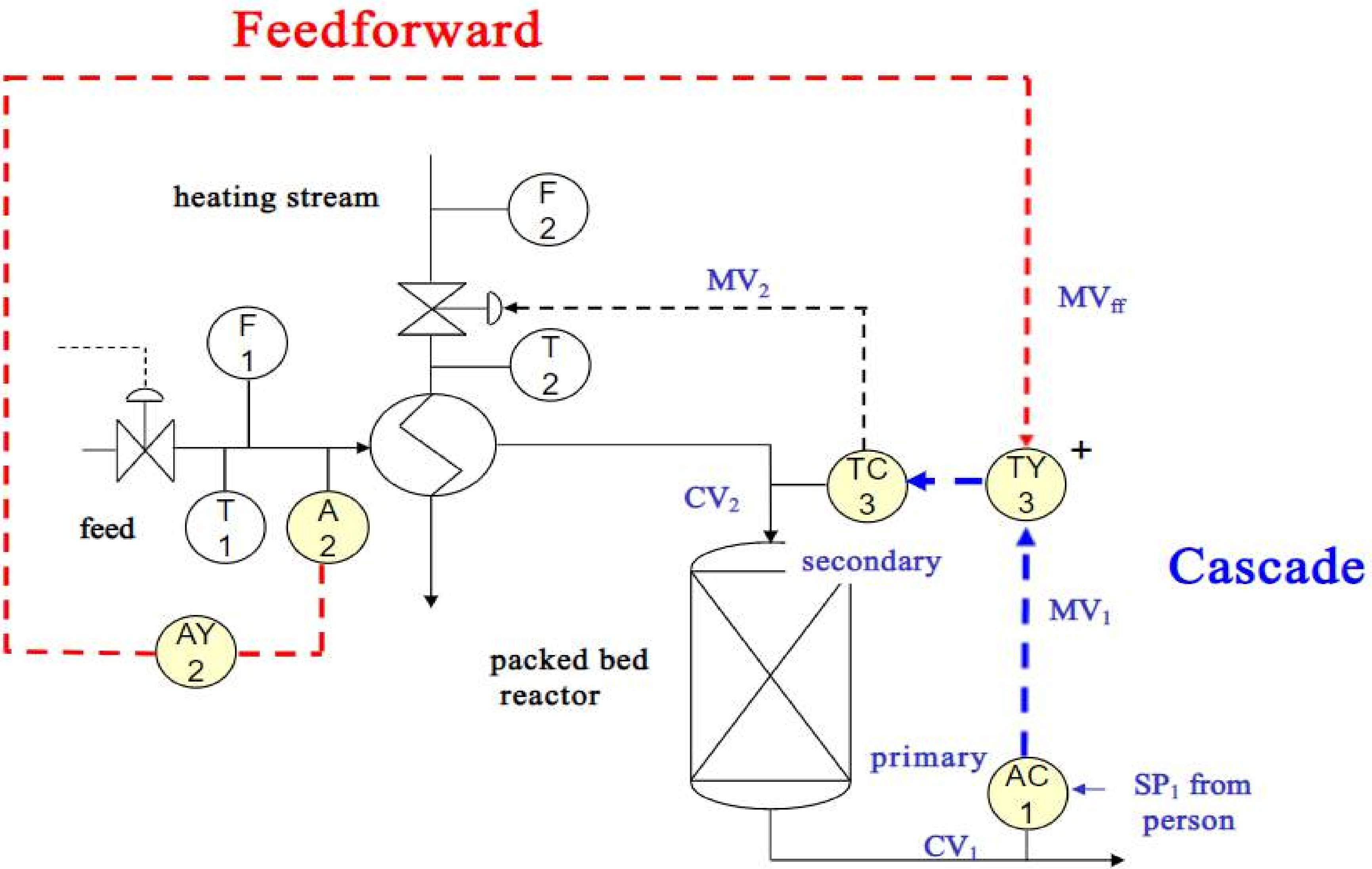
A disturbance in feed inlet temperature	FF/FB better
A disturbance in heating medium inlet pressure	Both the same
A disturbance in feed flow rate	Both the same
A change to the TC set point	Both the same

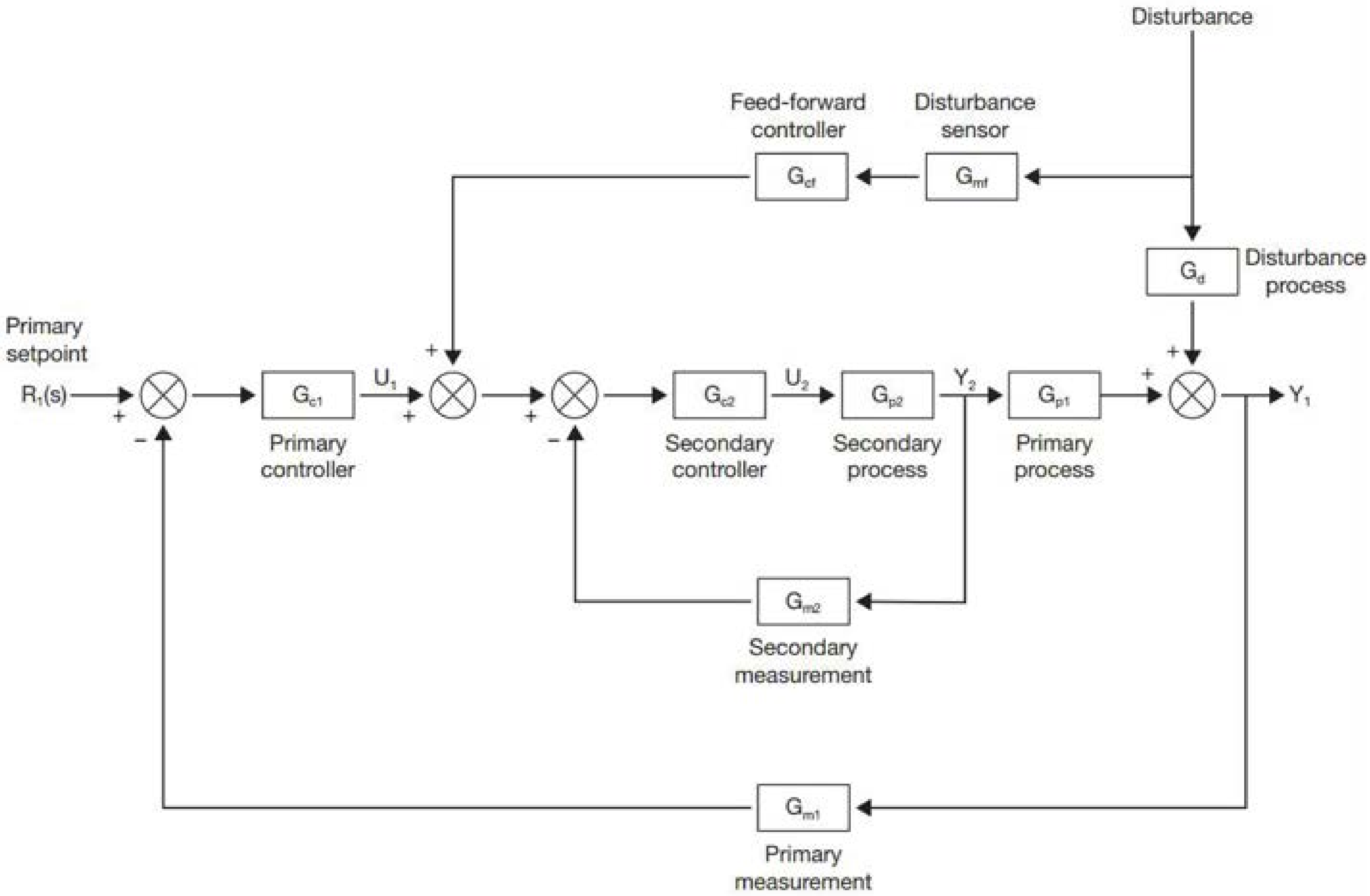
Class Exercise Combine cascade and feedforward to gain the advantages of both feed composition and heating medium temperature disturbances.



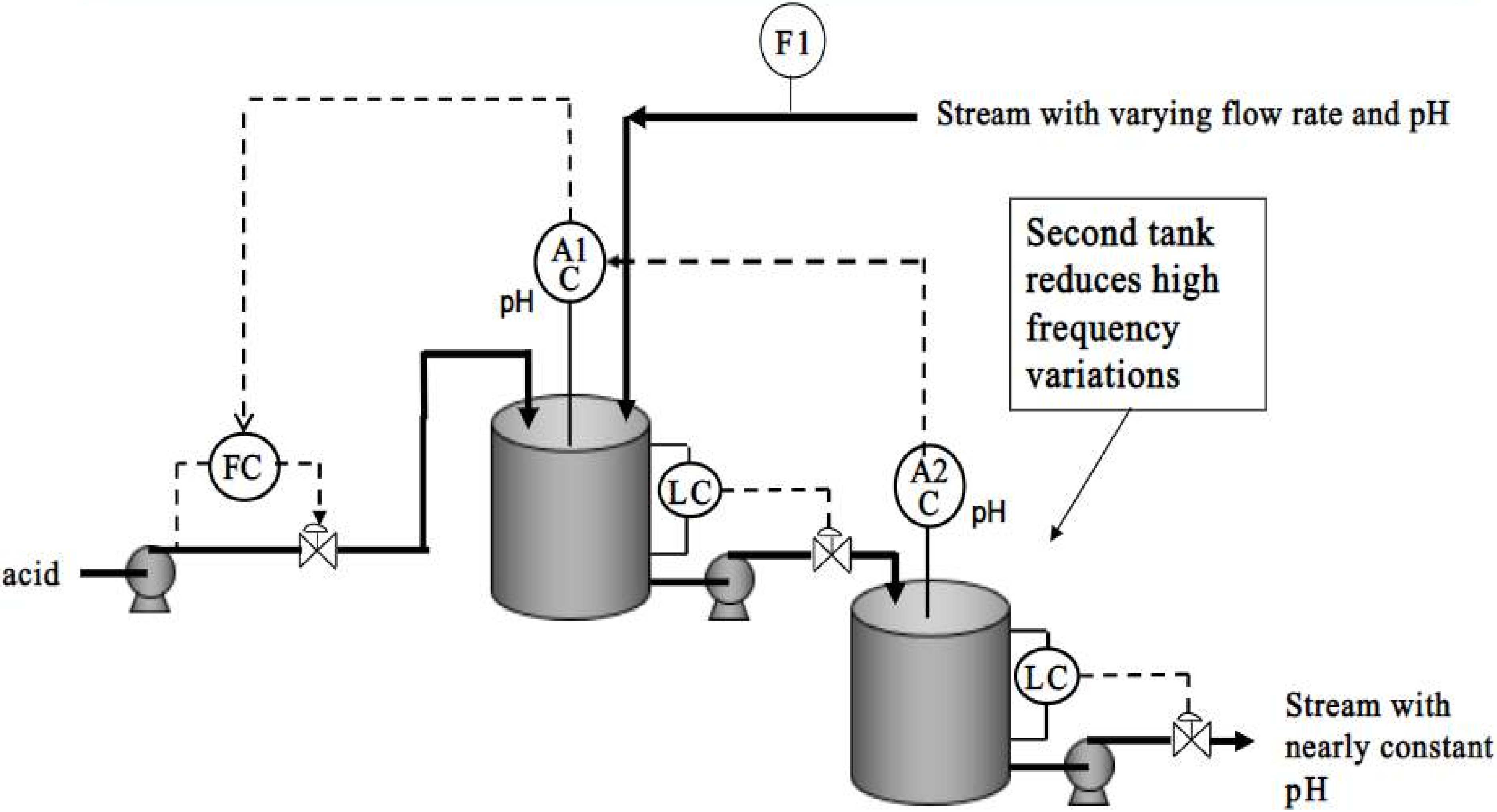
This figure shows the feedforward control design with single-loop feedback

Class Exercise Solution

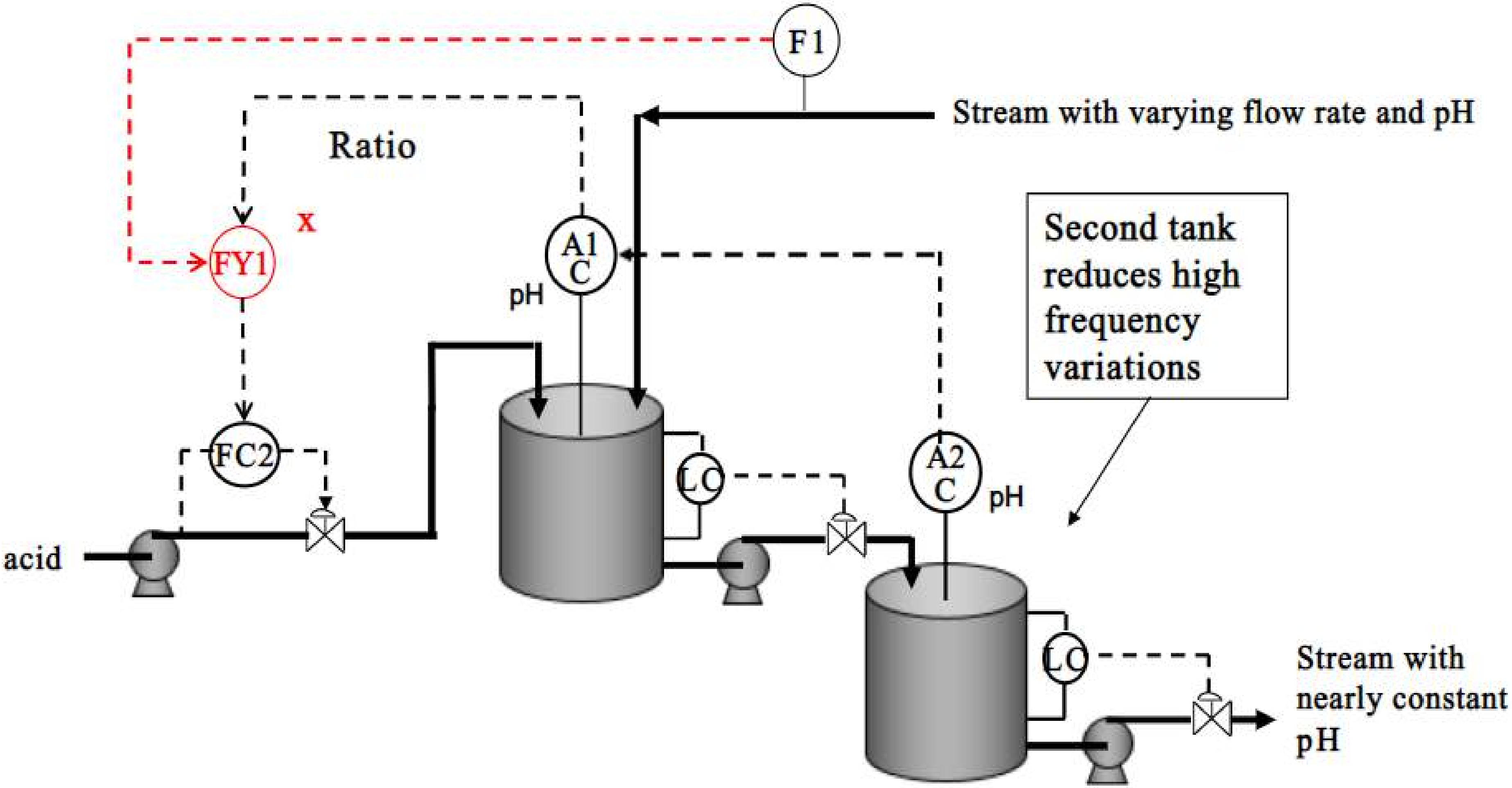



Disturbance
Feed-forward controller
Disturbance sensor
G_{cf}
G_{mf}
G_d
Disturbance process
Primary setpoint
$R_1(s)$
G_{c1}
U_1
Primary controller
G_{c2}
U_2
G_{p2}
Y_2
G_{p1}
Y_1
Secondary controller
Secondary process
Primary process
G_{m2}
Secondary measurement
G_{m1}
Primary measurement

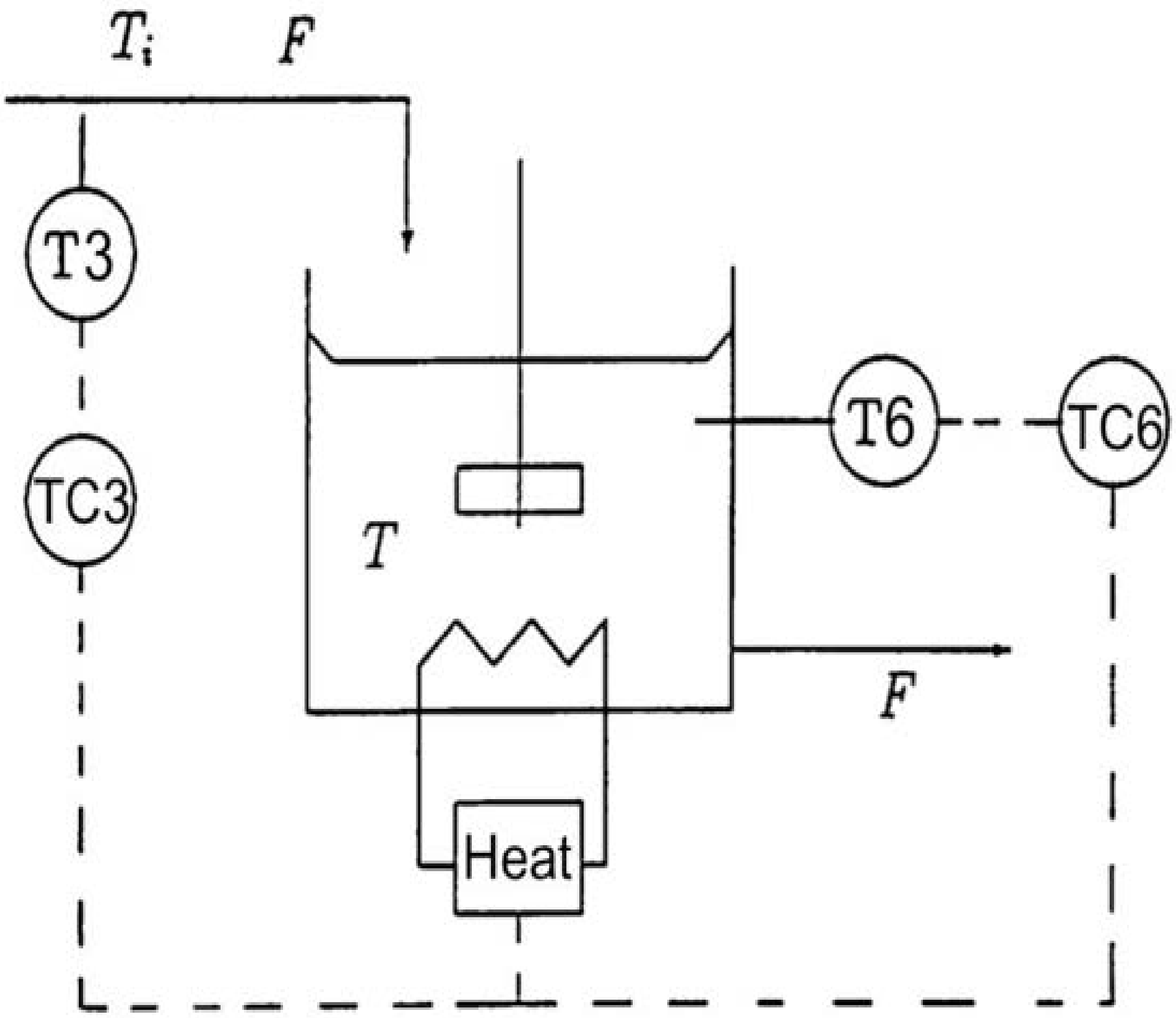
Class exercise Design a feedforward ratio controller for changes in inlet flow rate.



Class exercise Solution



The question considers a tank that is electrically heated. The goal is to maintain the temperature T6 as steady as possible.



We know that

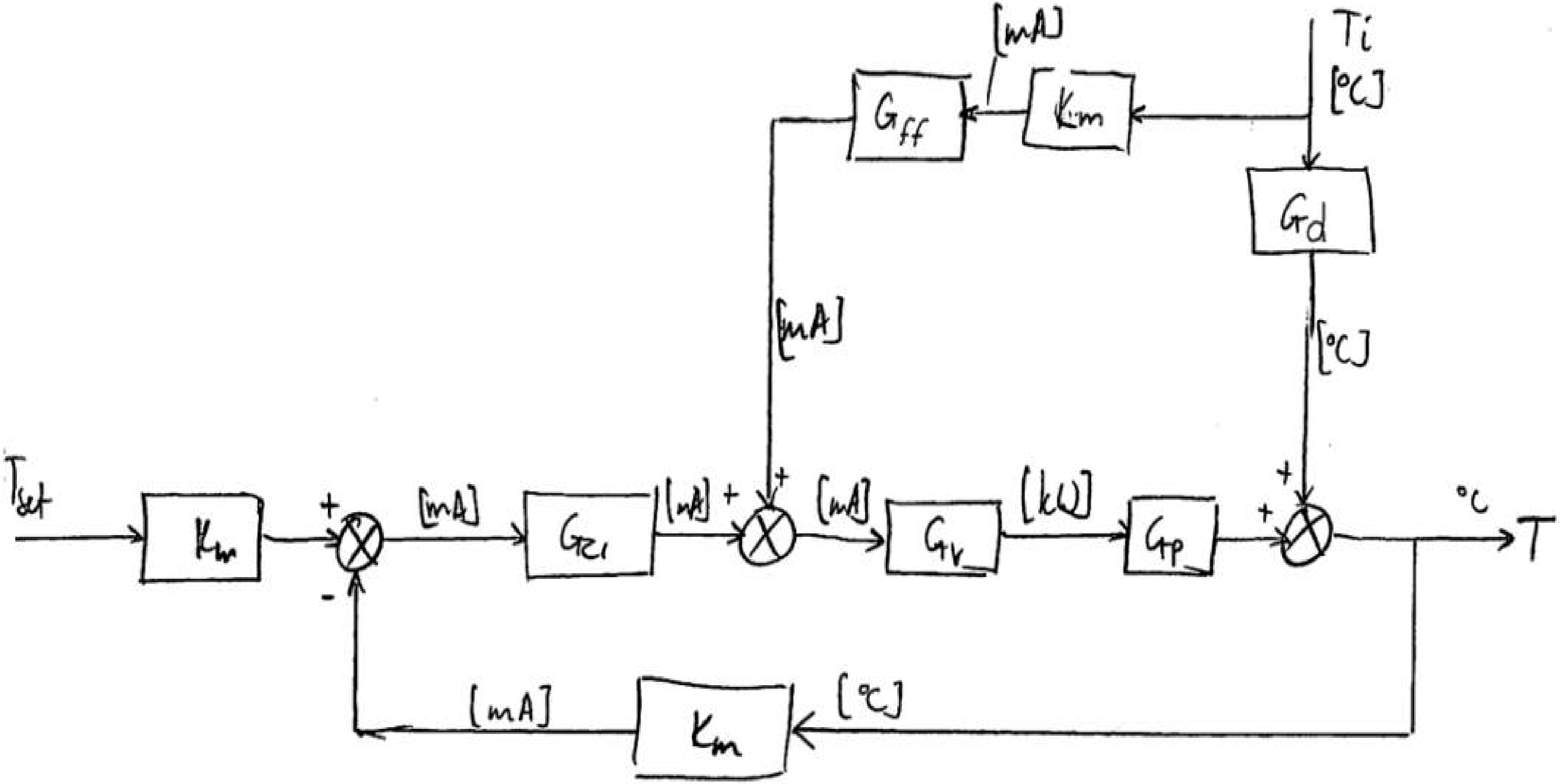
- F is constant
- A 4 to 20 mA change in input to the heater causes a change in power output from 0 to 1600 kW.
- The span of the transmitter is 64°C, with corresponding outputs of 4 to 20 mA.
- If the energy input is suddenly increased by 320 kW, it results in an eventual rise of 8°C in the tank, stabilizing after 500 seconds.
- A step change in the inlet temperature, T_i also gives rise to a complete response in about 500 seconds.

1. Which controller is the feedback controller, and which is the feedforward controller?
2. Draw a block diagram for the system, adding all elements, variables and their units to the diagram.
3. Design the feedforward controller.
4. No calculations are required: is the feedback control loop stable?
5. What is the purpose of having feedback control, in addition to feedforward control?

Solution

1. The feedback controller is `TC6` and the feedforward controller is `TC3`.

2. A diagram should include these elements:



where

- The gain $K_m = \dfrac{20-4}{64} = 0.25\,\dfrac{\text{mA}}{°\text{C}}$
- The gain $G_v = \dfrac{1600-0}{20-4} = 100\dfrac{\text{kW}}{\text{mA}}$

The other transfer functions require a derivation:

$$\rho V C_p \frac{dT}{dt} = \rho F C_p (T_i - T) + Q$$

At steady state
$$0 = \rho F C_p (T_{i,S} - T_s) + Q_s$$

Subtract
$$\rho V C_p \frac{dT'}{dt} = \rho F C_p (T_i' - T') + Q'$$

Laplace transform
$$V s T'(s) = F T_i'(s) - F T'(s) + \frac{Q'(s)}{\rho C_p}$$

$$T'(s)\,[V s + F] = F T_i(s) + \frac{Q'(s)}{\rho C_p}$$

$$T'(s) = \frac{F}{Vs + F} T_i(s) + \frac{\frac{1}{\rho C_p}}{Vs + F} Q'(s)$$

Simplify
$$T'(s) = \frac{1}{\frac{V}{F}s + 1} T_i(s) + \frac{\frac{1}{\rho C_p F}}{\frac{V}{F}s + 1} Q'(s)$$

- The process $G_p(s) = \frac{T(s)}{Q(s)} = \frac{K_p}{\tau s + 1}$ can be taken from the above derivation

(a) $K_p = \frac{1}{\rho C_p F} = \frac{8}{320} = 0.025 \frac{^\circ\text{C}}{\text{kW}}$

(b) $\tau = \frac{V}{F} = \frac{500}{5} = 100$ seconds, using the 5τ rule for first order systems

(c) There is no time delay, because electrical heat added will almost certainly show an immediate rise in temperature

(d) $G_d(s) = \frac{T(s)}{T_i(s)} = \frac{1}{\frac{V}{F}s + 1} = \frac{1}{100s + 1}$, using the same reasoning as for the process transfer function

(e) $G_{ff}(s)$ is derived below

3. The feedforward controller, $G_{ff}(s)$ is derived so that $T(s)$ shows no change when T_i changes. This requires:

$$T(s) = G_d(s)T_i(s) + K_m G_{ff}(s)G_v(s)G_p(s)T_i(s) = 0$$

$$T_i(s)\left[G_d(s) + K_m G_{ff}(s)G_v(s)G_p(s)\right] = 0$$

$$G_d(s) + K_m G_{ff}(s)G_v(s)G_p(s) = 0$$

$$G_{ff}(s) = -\frac{G_d(s)}{K_m G_v(s)G_p(s)}$$

$$G_{ff}(s) = -\frac{\dfrac{1}{100s+1}}{0.25 \cdot 100 \cdot \dfrac{0.025}{100s+1}}$$

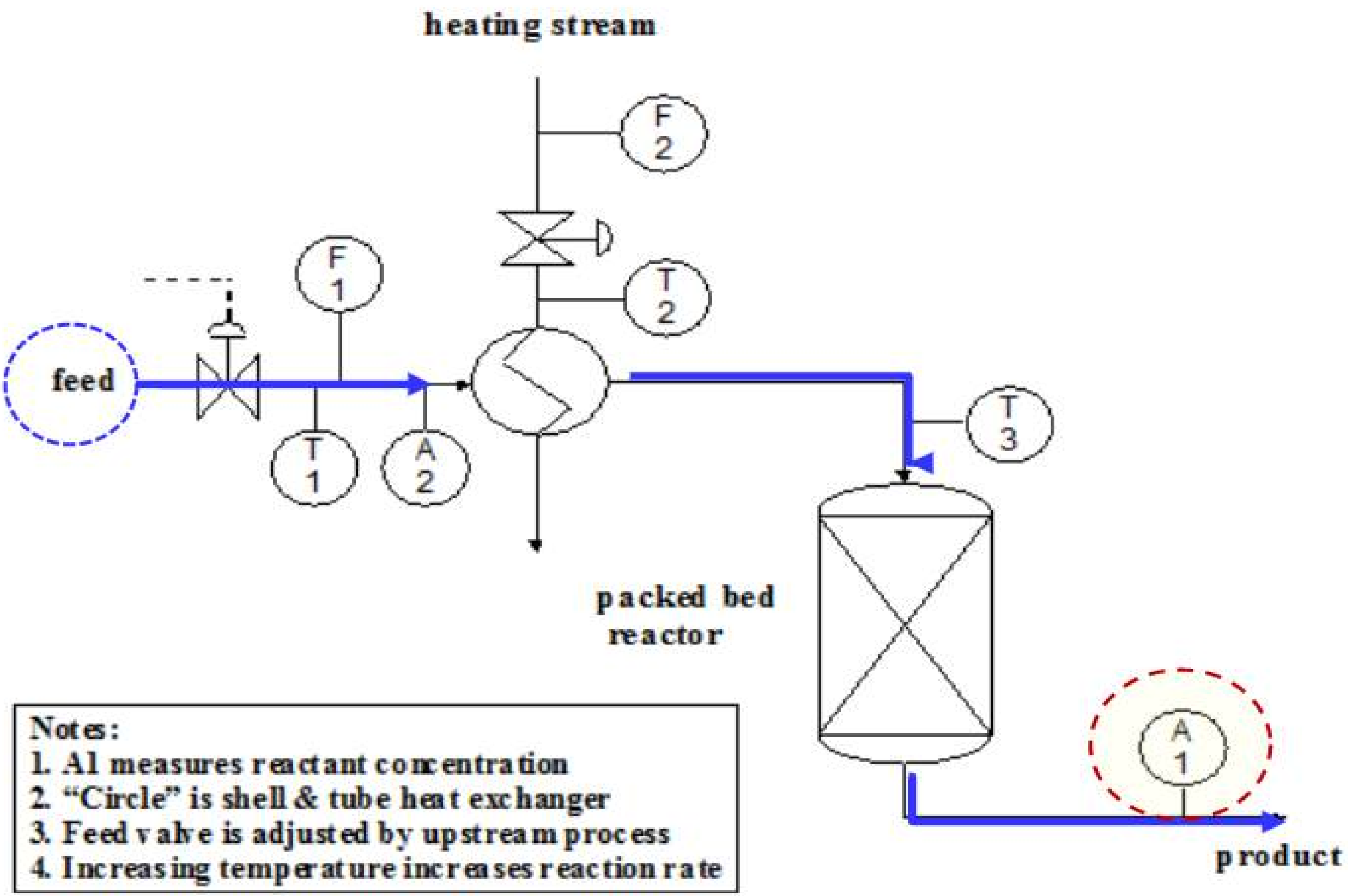
$$G_{ff}(s) = -1.6$$

which is a simple proportional-only controller.

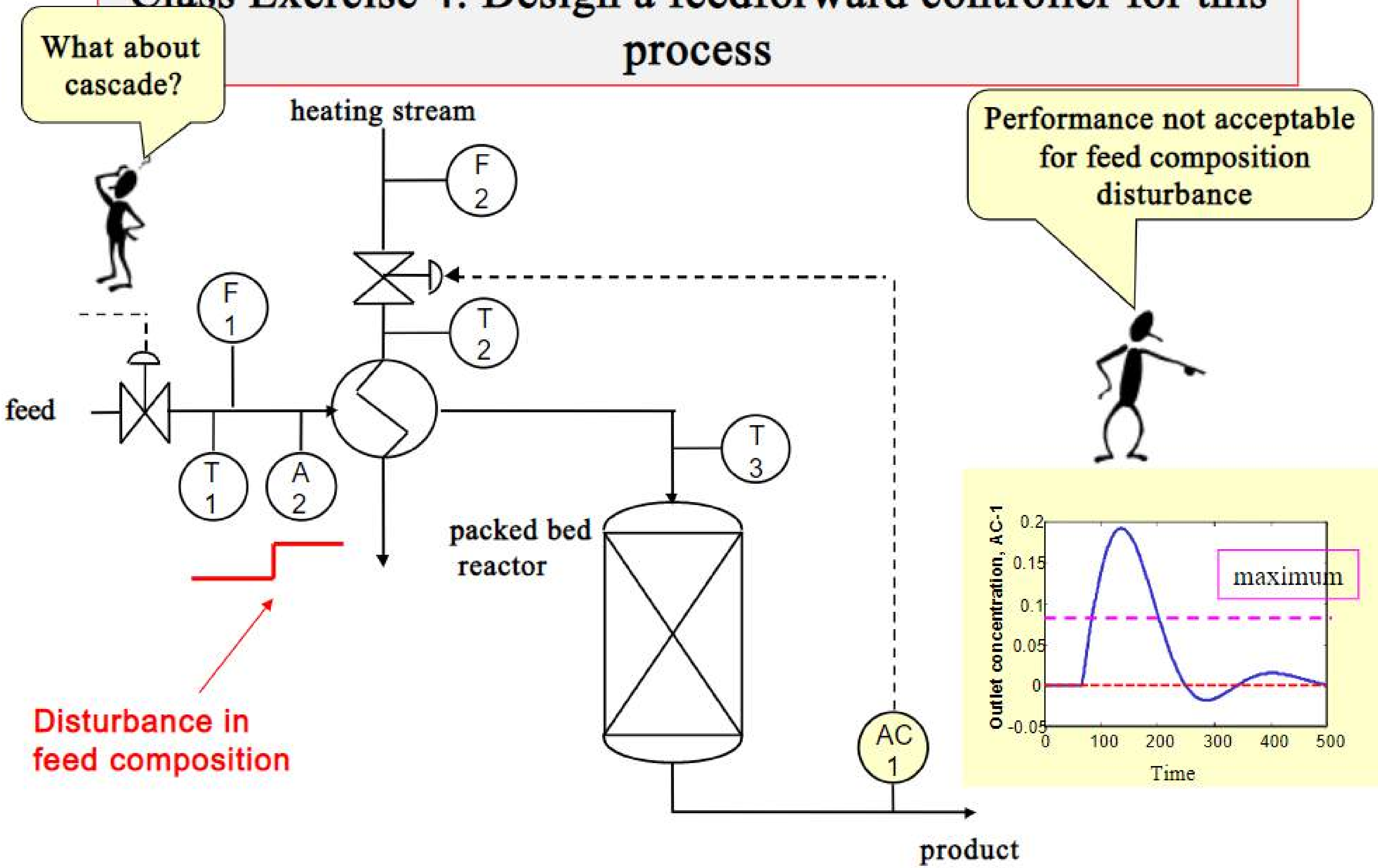
4. Yes the loop is stable; a feedforward controller does not affect the stability of the feedback loop (we proved this in the tutorial).

5. The feedback control loop maintains the desired set point of `TC6` when other disturbances (measured or unmeasured) affect the temperature, such as the inlet flow rate.

We will design a feedforward controller for this process



Class Exercise 4: Design a feedforward controller for this process



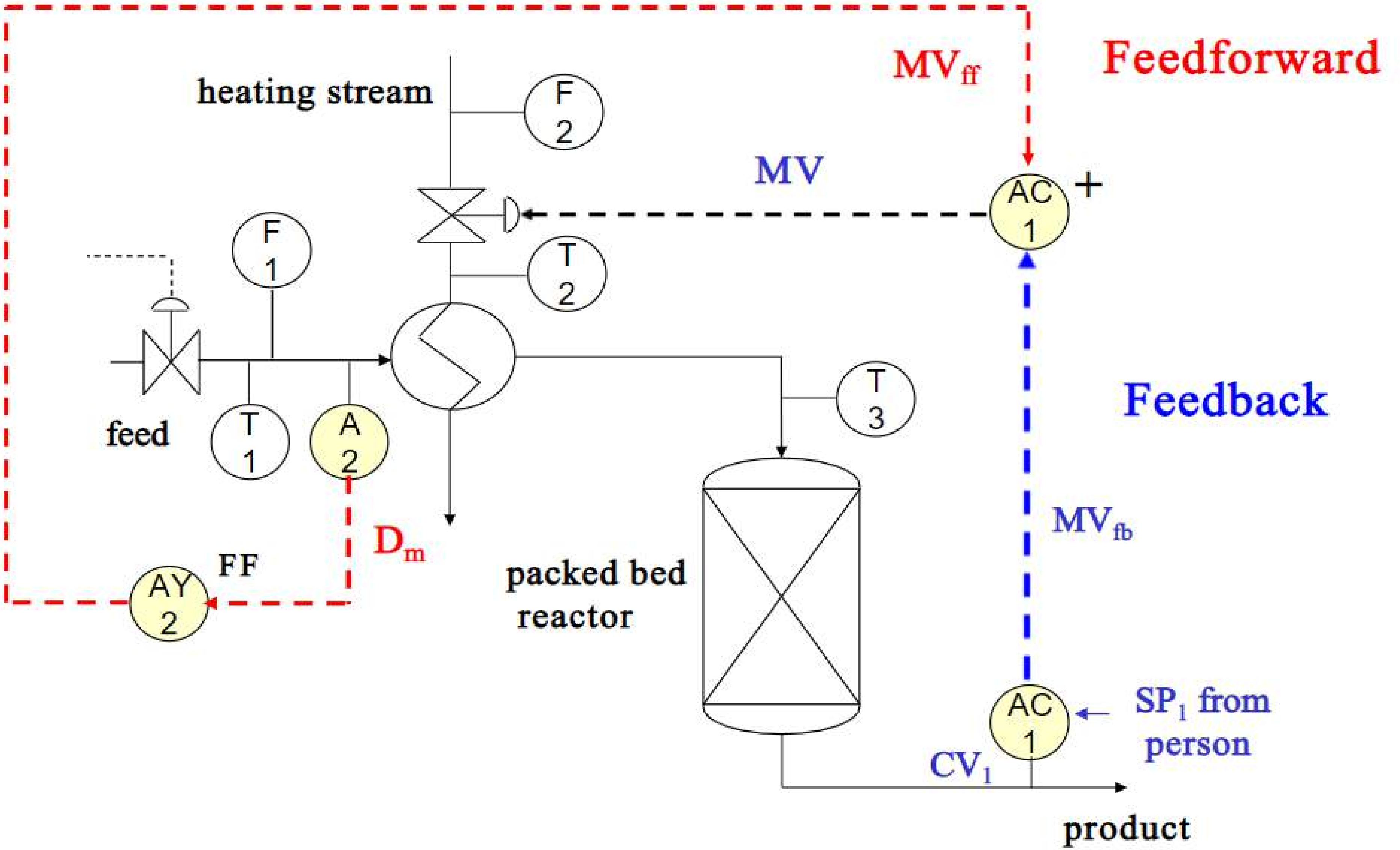
Class exercise 4: Solution – Design Criteria

Feedforward design criteria	A2	F1	F2	T1	T2	T3
1. Single-loop not acceptable	Y	Y	Y	Y	Y	Y
2. Disturbance variable is measured	Y	Y	Y	Y	Y	Y
3. Indicates a key disturbance	Y	N	N	N	N	N
4. No Causal relationship, valve → D_m	Y	Y	N	Y	Y	N
5. Disturbance dynamics not much faster than compensation	Y	N/A	N/A	N/A	N/A	N/A

A2 satisfies all of the rules; therefore, it can be used as a feedforward variable.

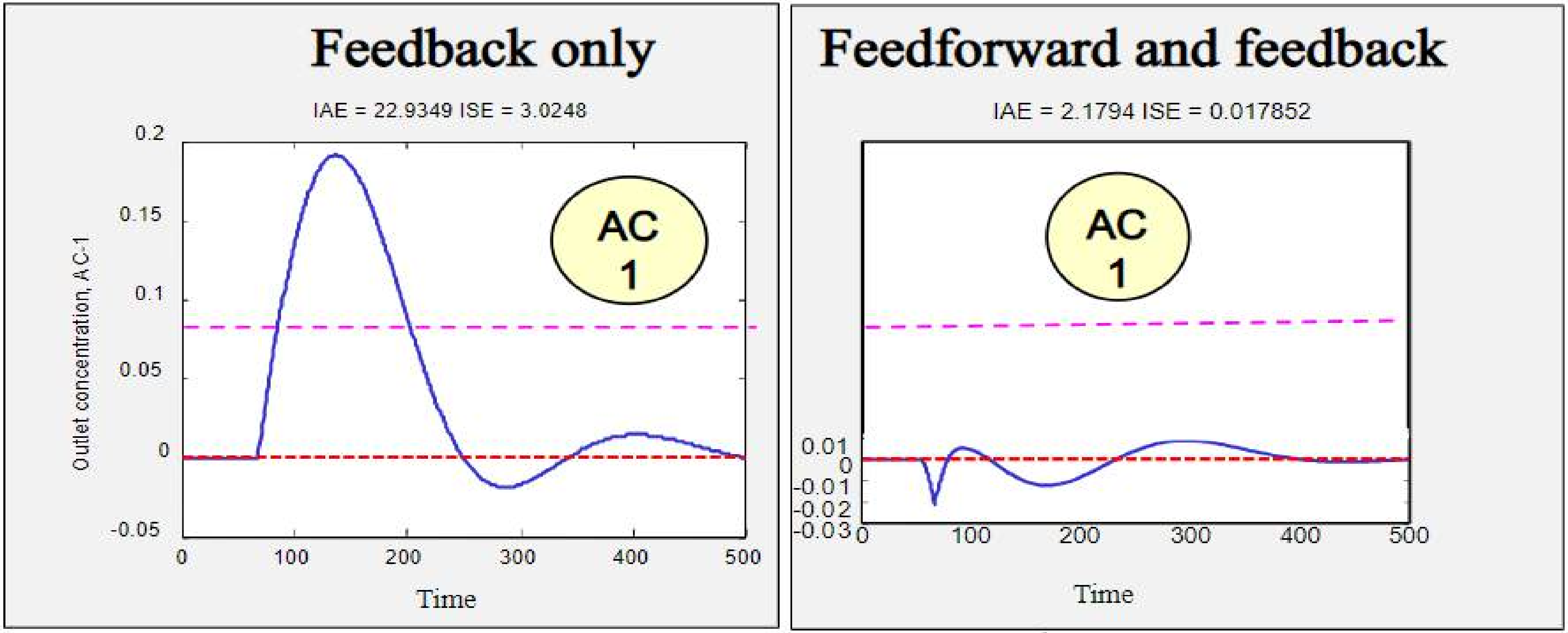
Cannot be used for feedforward!

Class exercise 4: Solution – P&I Drawing



Class exercise 4: Solution – Dynamic response

Control Performance Comparison for Packed Bed Reactor



Little model error, most experimental feedforward not this good!

Q

The question considers a tank in figure 3 that is heated by a heat stream. The goal is to maintain the temperature (T) as steady as possible by using a feedforward-feedback control scheme. The feedforward controller is overcoming the temperature disturbance in the feed line while the feedback controller is correcting the error in the product temperature (T). Both control signals are combined to control the heating stream flow rate by adjusting the valve position. Assume the following:

- F is constant
- A 4 to 20 mA change in input to the valve causes a change in the heater power output from 0 to 1500 kW.
- The span of the transmitter is 75°C, with corresponding outputs of 4 to 20 mA.
- If the energy input is suddenly increased by 250 kW, it results in an eventual rise of 6°C in the tank, stabilizing after 600 seconds.
- A step-change in the inlet temperature, Ti also gives rise to a complete response in about 600 seconds.
- There is no time delay; immediate rise in temperature by the heater.
- The system equation is:

$$T'(s) = \frac{1}{\frac{V}{F}s+1}T_i(s) + \frac{\frac{1}{\rho C_p F}}{\frac{V}{F}s+1}Q'(s)$$

A) Sketch the P&ID for the system after adding the feedforward and feedback controllers.
B) Draw a block diagram for the system in (A)
C) Design the feedforward controller.
D) No calculations are required: is the feedback control loop stable?
E) What is the purpose of having the feedback control, in addition to the feedforward control?

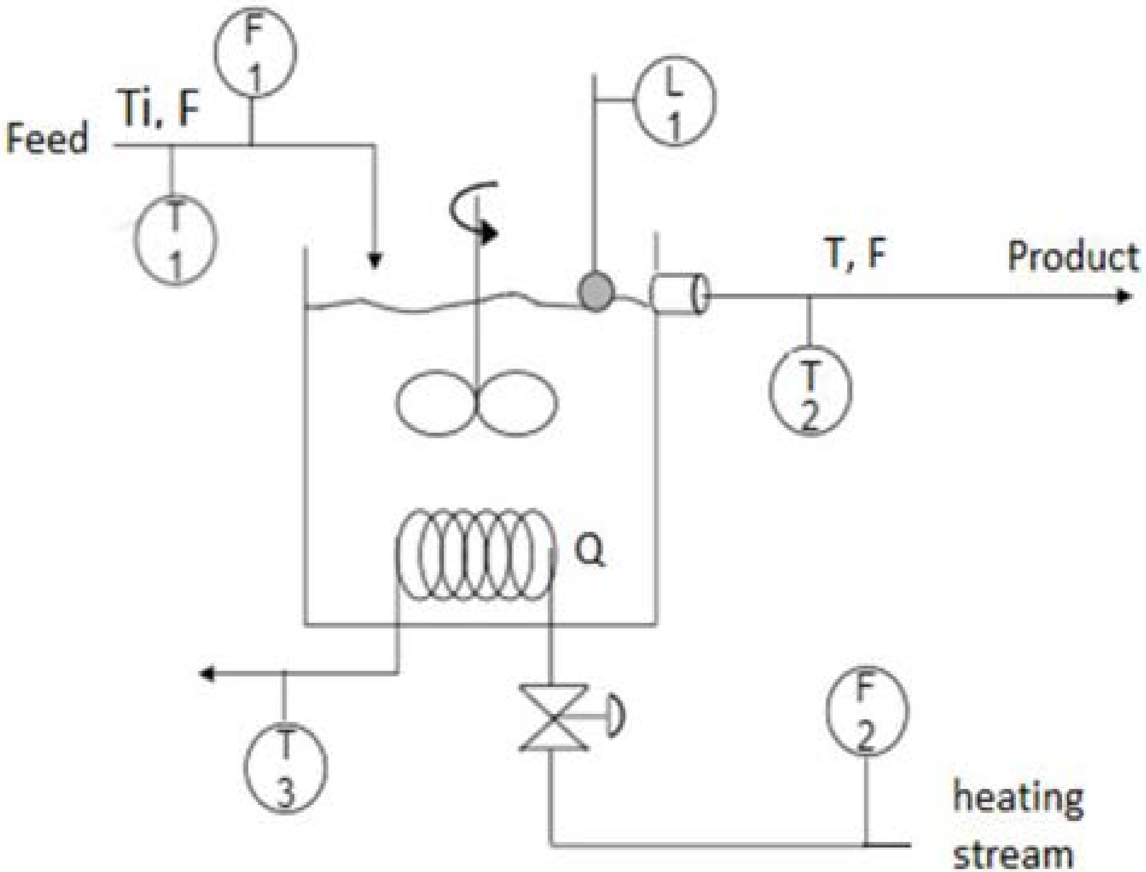


Figure 3

Example 1 – Hot-Water Heater with Measured Inlet Temperature

Process description (industrial style):

- Continuous stirred-tank heater H-101:
  - Cold water inlet at temperature $T_{in}(t)$
  - Tank is well mixed; outlet temperature $T(t)$ is controlled
  - Heat supplied by steam coil; MV is steam valve position (proportional to heat rate $Q(t)$)
- Dominant disturbance: fluctuations in inlet temperature $T_{in}(t)$, which is measured.
- CV: outlet temperature $T(t)$, SP = 80 °C
- Control goal: reduce temperature deviations caused by $T_{in}(t)$ changes.

Example 1 – Simple Dynamic Model

Energy balance (linearized, first order):

- Tank capacity: $C = M c_p$ (kJ/°C)
- Dynamics (around an operating point):

$$C \frac{dT}{dt} = k_Q \, \Delta Q(t) + k_{in} \, \Delta T_{in}(t) - C \frac{(T(t) - T_0)}{\tau}$$

For control design, we write **transfer functions** from variations:

- From MV (heat input) to CV:

$$G_p(s) = \frac{\Delta T(s)}{\Delta Q(s)} = \frac{K_p}{\tau_p s + 1}$$

- From disturbance (inlet temperature) to CV:

$$G_d(s) = \frac{\Delta T(s)}{\Delta T_{in}(s)} = \frac{K_d}{\tau_d s + 1}$$

Where K_p, K_d are process gains, τ_p, τ_d time constants.

(Similar derivations appear in Marlin, Seborg, Ogunnaike & Ray.)

Example 1 – Numerical Values (Design Data)

Assume experimentally identified transfer functions:

- From steam valve to outlet temperature:

$$G_p(s) = \frac{2.0}{5s+1} \quad [^\circ\mathrm{C}/\%\mathrm{valve}]$$

- From inlet temperature to outlet temperature:

$$G_d(s) = \frac{1.0}{3s+1} \quad [^\circ\mathrm{C}/^\circ\mathrm{C}]$$

Interpretation:

- A 1 °C step in T_{in} eventually changes outlet temperature by 1 °C.
- A 1% change in steam valve eventually changes outlet temperature by 2 °C.
- Disturbance dynamics are slightly faster than MV dynamics.

Example 1 – Ideal Feedforward Controller

Using ideal formula:

$$G_{ff}(s) = -\frac{G_d(s)}{G_p(s)} = -\frac{\frac{1}{3s+1}}{\frac{2}{5s+1}} = -\frac{1}{2}\frac{5s+1}{3s+1}$$

So:

$$G_{ff}(s) = -0.5\frac{5s+1}{3s+1}$$

- This is a **lead-lag** element:
  - Zero at $s = -1/5$
  - Pole at $s = -1/3$
- Interpretation:
  - Multiply inlet temperature deviation by ~0.5
  - Apply dynamic shaping to match the difference between disturbance and process dynamics.

Industrial simplification:

- Often approximate with a simple gain and possibly a lag:

$$G_{ff}(s) \approx -0.5 \quad \text{or} \quad -0.5 \cdot \frac{\alpha s+1}{\alpha s+1} \text{ (unity dyn.)}$$

- Then fine-tune in the plant.

Example 1 – How to Implement in a DCS/PLC

- Measured disturbance: T_{in} from RTD/thermocouple → transmitter → AI module
- Feedforward block:
  1. Compute disturbance deviation:

     $\Delta T_{in} = T_{in} - T_{in,0}$ (use engineering units)
  2. Apply a dynamic function block for $G_{ff}(s)$ (lead-lag or pure gain)
  3. Add the result to the output of the PID temperature controller:

$$MV = MV_{FB} + MV_{FF}$$

- Practical considerations:
  - MV limits (saturation): anti-windup for PID is mandatory
  - Scaling and engineering units must be consistent
  - Often the feedforward term is enabled/disabled by an operator switch.

Comparison: Control Without and With Feedforward

For the heater example, consider a step disturbance $\Delta T_{in} = +5^\circ C$:

- **Feedback only:**
  - Outlet temperature first **rises** close to +5 °C (open-loop effect)
  - PID gradually compensates; outlet temperature returns to SP with some overshoot/settling time dictated by controller tuning
- **Feedback + feedforward (ideal or well-tuned):**
  - Before the disturbance fully affects T, MV changes proactively
  - Outlet temperature deviation is much smaller (ideally almost zero)
  - Overshoot and integrated error significantly reduced
- These behaviors can be illustrated via MATLAB/Simulink simulations of the derived transfer functions.

Advantages and Disadvantages of Feedforward

Advantages:

- Potentially **very fast disturbance rejection**
- Reduces integrated absolute error (IAE) and variance of CV
- Complementary to feedback; improves quality without sacrificing robustness (if designed correctly)

Disadvantages:

- Requires **reliable measurement** of the disturbance
- Requires at least approximate **models** of G_p and G_d
- Sensitive to **model mismatch** and sensor faults
- Additional configuration/maintenance effort in PLC/DCS
- Does not eliminate the need for feedback.

Design Procedure – Academic Summary

1. **Identify dominant disturbances** that are measurable.
2. **Obtain or approximate models**:
   - $G_p(s)$: MV → CV
   - $G_d(s)$: disturbance → CV
   - Use step tests, PRBS, or empirical identification.
3. **Calculate ideal feedforward**:

$$G_{ff}(s) = -\frac{G_d(s)}{G_p(s)}$$

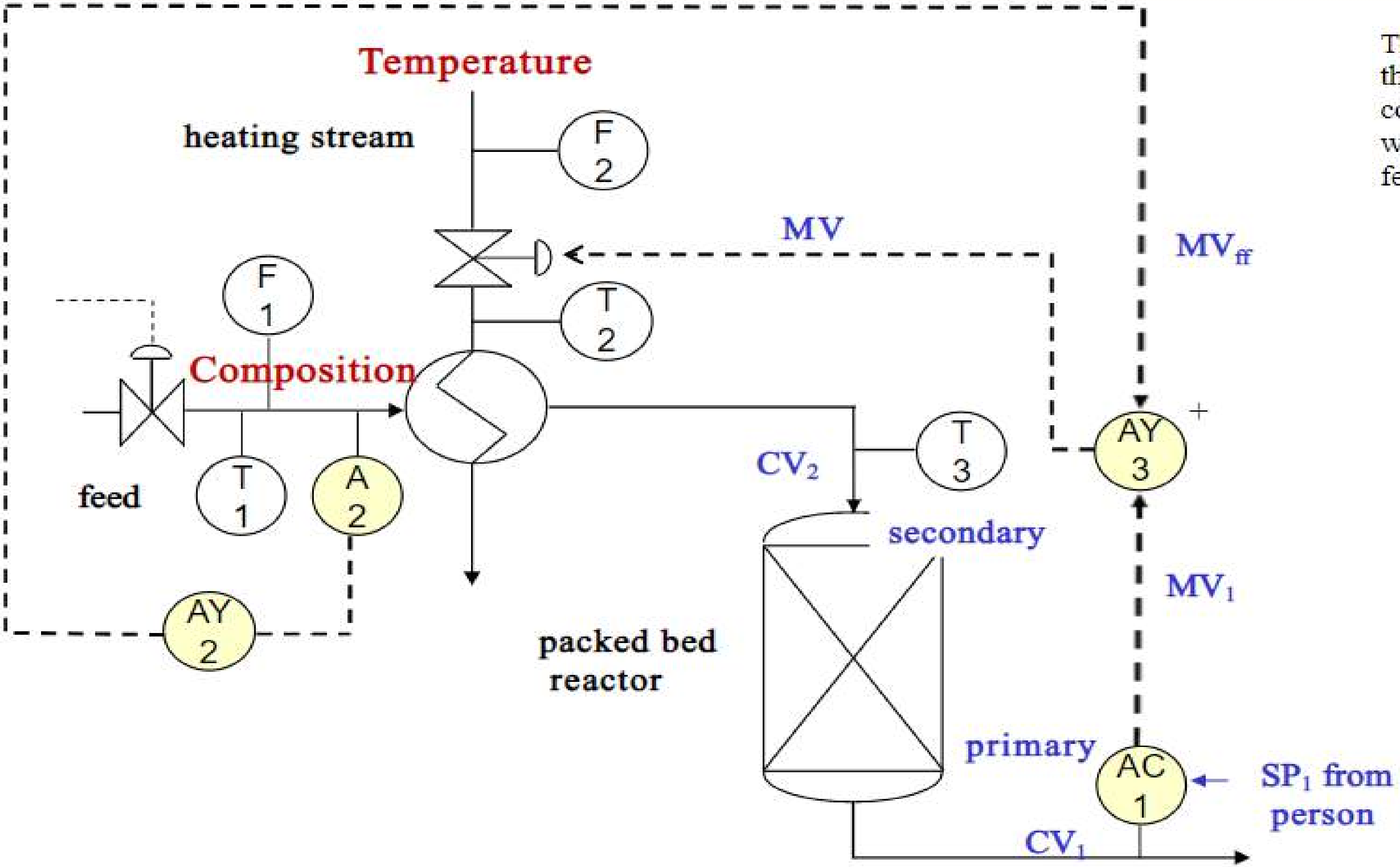
4. **Simplify** $G_{ff}(s)$ to realizable form:
   - Ensure properness (no more zeros than poles; add small lag if needed).
   - Approximate high-order terms.
5. **Combine with feedback**: tune PID for reasonable closed-loop response.
6. **Validate and retune** using simulation and plant trials.

References: Seborg et al.; Skogestad & Postlethwaite; Åström & Hägglund, "Advanced PID Control", ISA.

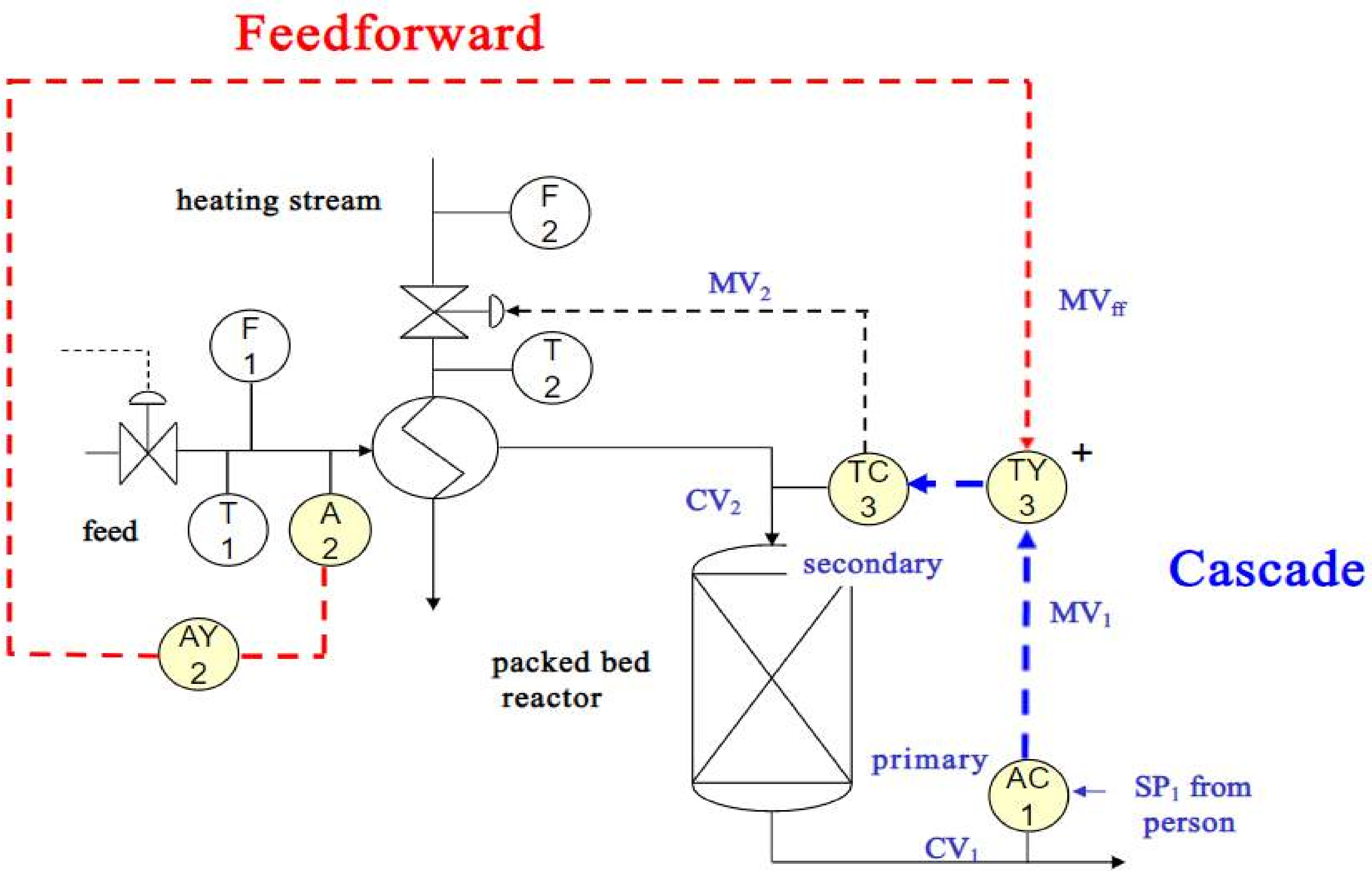
Industrial Implementation Issues

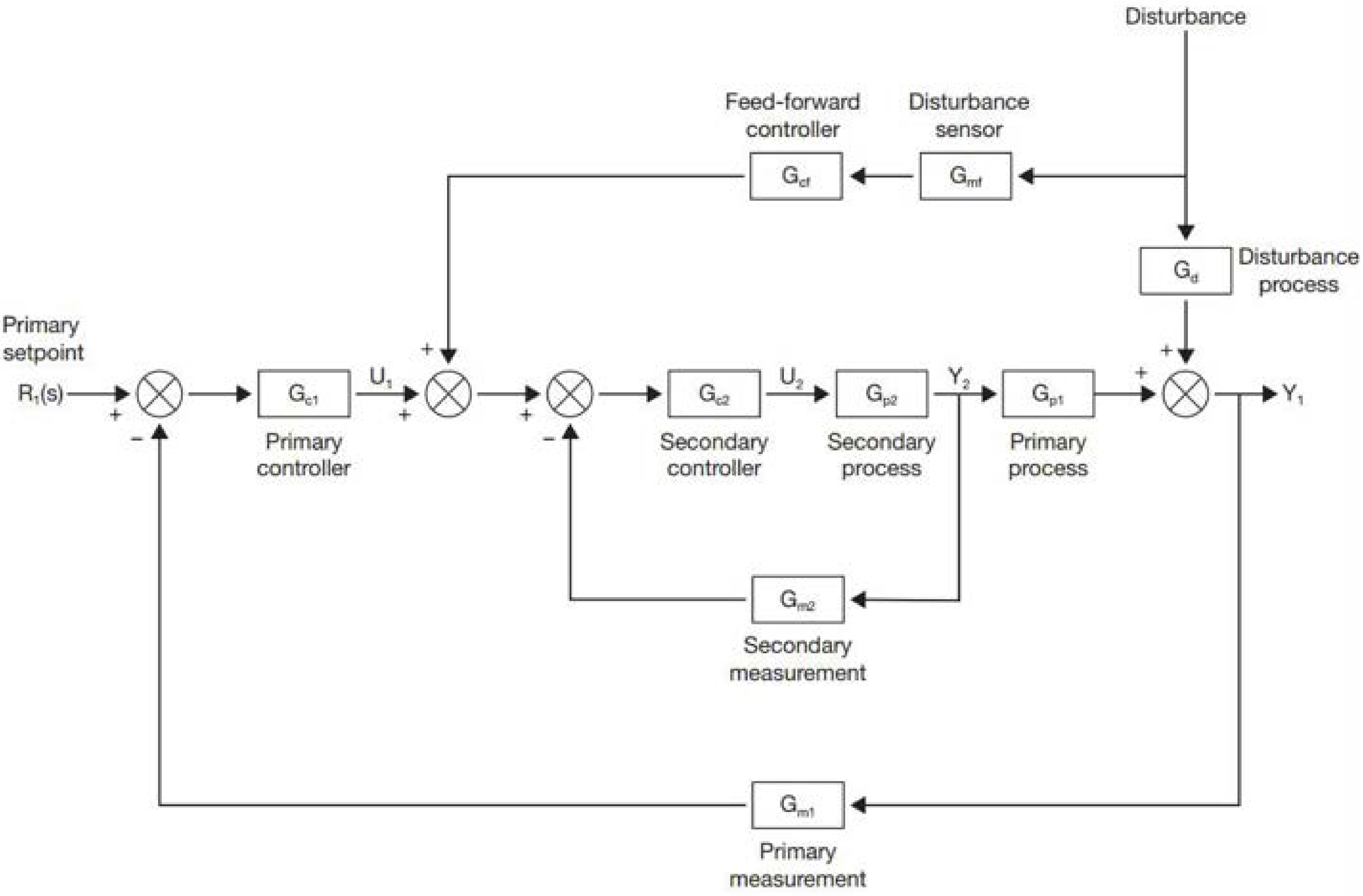
- **Dead time and delays:**
  - If disturbance measurement includes significant dead time, feedforward effectiveness is reduced; may need Smith predictor or extra lag.
- **Noise** on disturbance measurement:
  - High-frequency noise can cause MV chattering → use low-pass filtering, at the cost of slower response.
- **Actuator limits:**
  - Combine feedforward with output limiting / rate limiting.
- **Startup and mode changes:**
  - Many plants disable feedforward during startup or manual mode; re-enable after CV is close to SP.
- **Safety:**
  - For critical equipment (boilers, reactors), FF actions must be bounded and supervised by interlocks and safety instrumented systems (SIS).

Class Exercise Combine cascade and feedforward to gain the advantages of both feed composition and heating medium temperature disturbances.



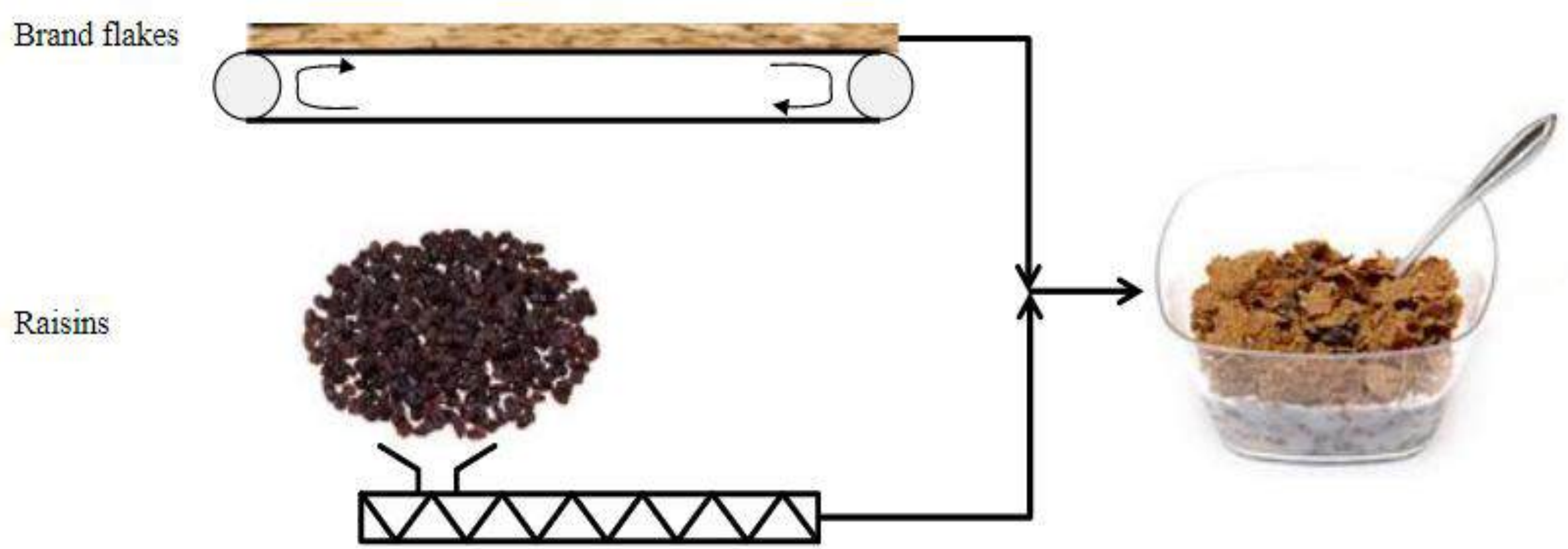
This figure shows the feedforward control design with single-loop feedback


Feedforward
heating stream
F 2
MV2
MVff
F 1
T 2
TC 3
TY 3
+
feed
T 1
A 2
CV2
secondary
Cascade
MV1
AY 2
packed bed reactor
primary
AC 1
SP1 from person
CV1

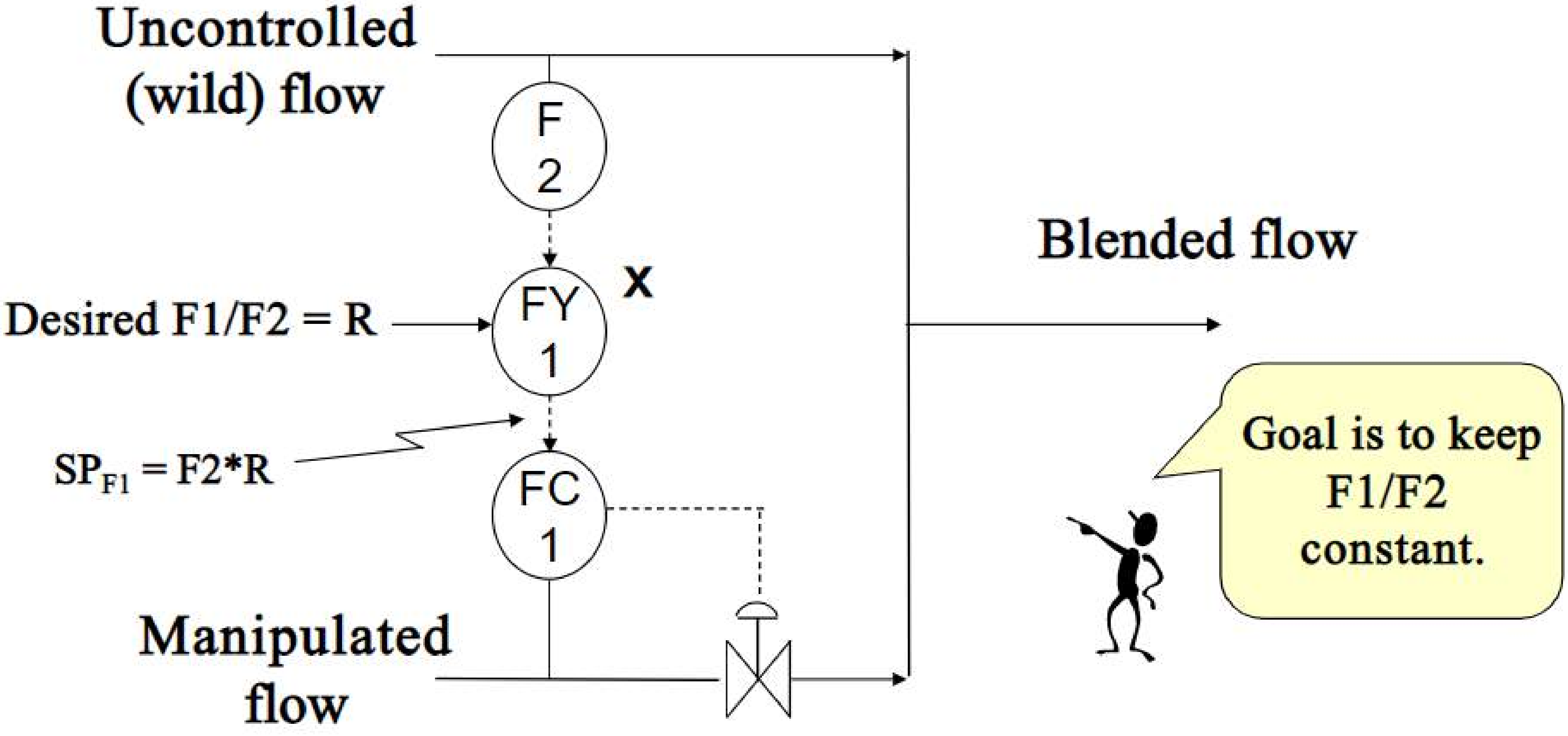

Disturbance
Feed-forward controller
Disturbance sensor
G_{cf}
G_{mf}
G_d
Disturbance process
Primary setpoint
$R_1(s)$
G_{c1}
Primary controller
U_1
G_{c2}
Secondary controller
U_2
G_{p2}
Secondary process
Y_2
G_{p1}
Primary process
Y_1
G_{m2}
Secondary measurement
G_{m1}
Primary measurement

Ratio control is a simple and frequently used feedforward application. In ratio control, the dynamics are negligible.

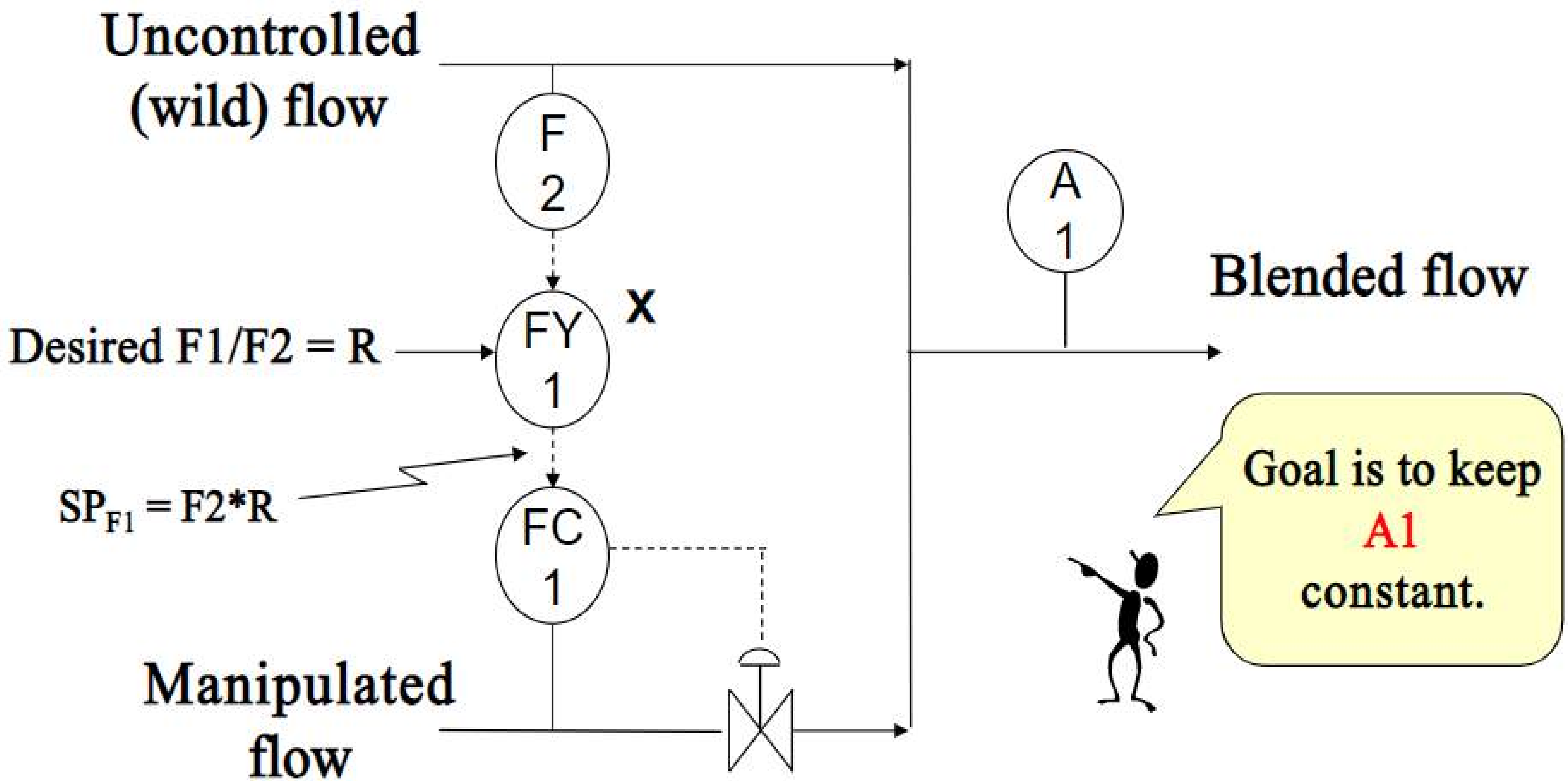
We manufacture brand flakes and mix with raisins to prepare raisin brand cereal. How do we achieve the desired blend product?



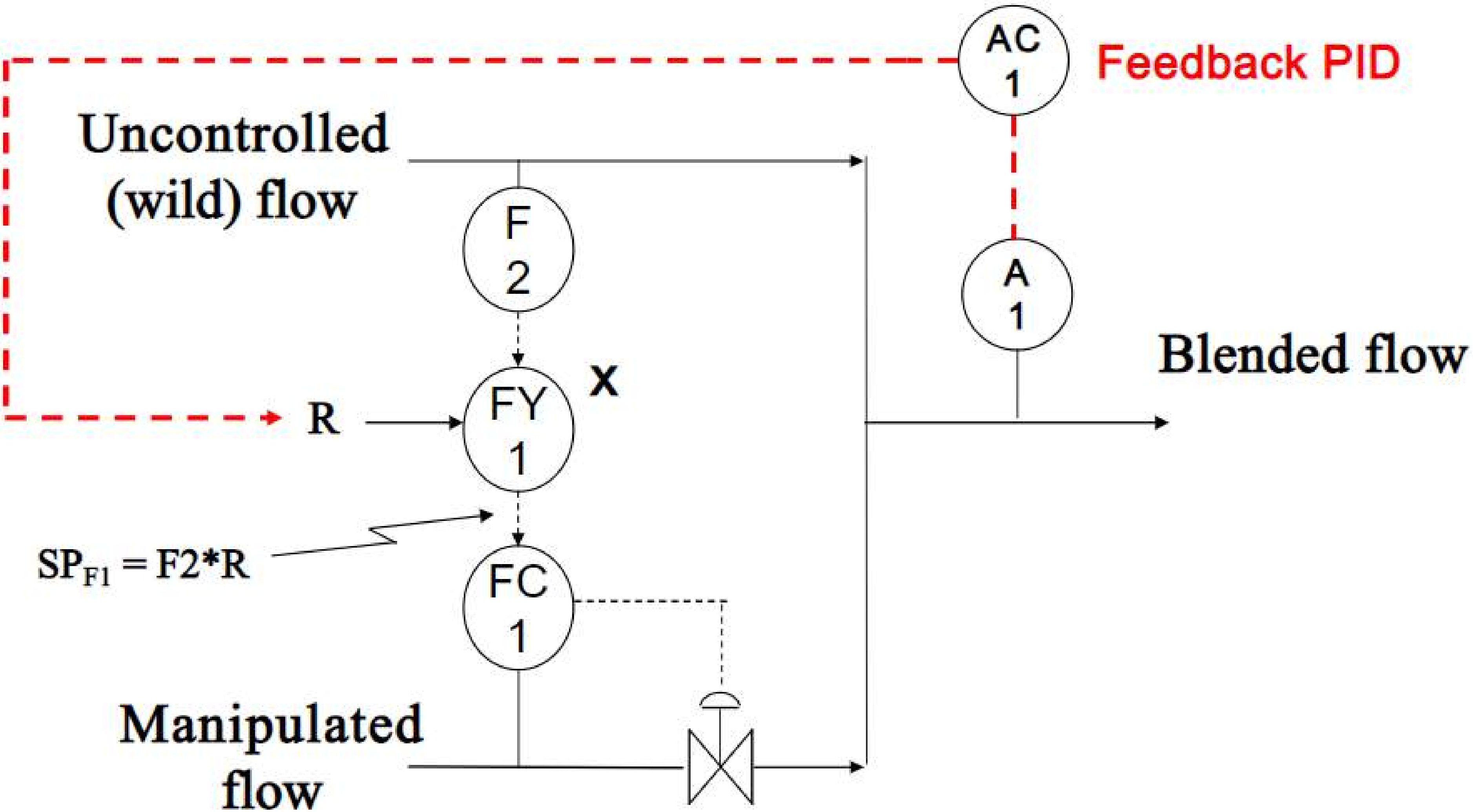
Ratio control is a simple and frequently used feedforward application. In ratio control, the dynamics are negligible, so that controller consists of only a gain element.



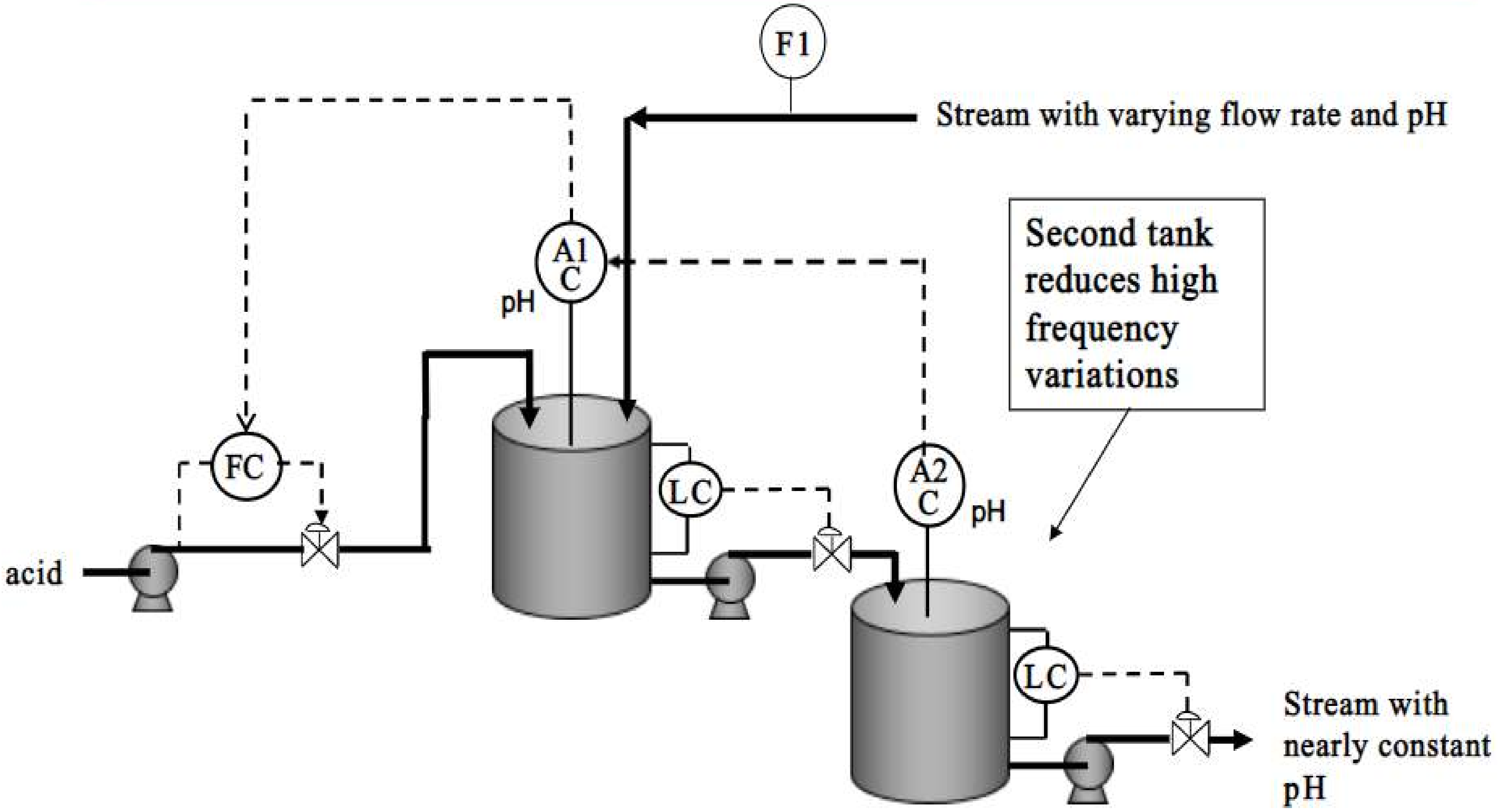
Class exercise Use analyzer for feedback control while retaining the good aspects of ratio control.



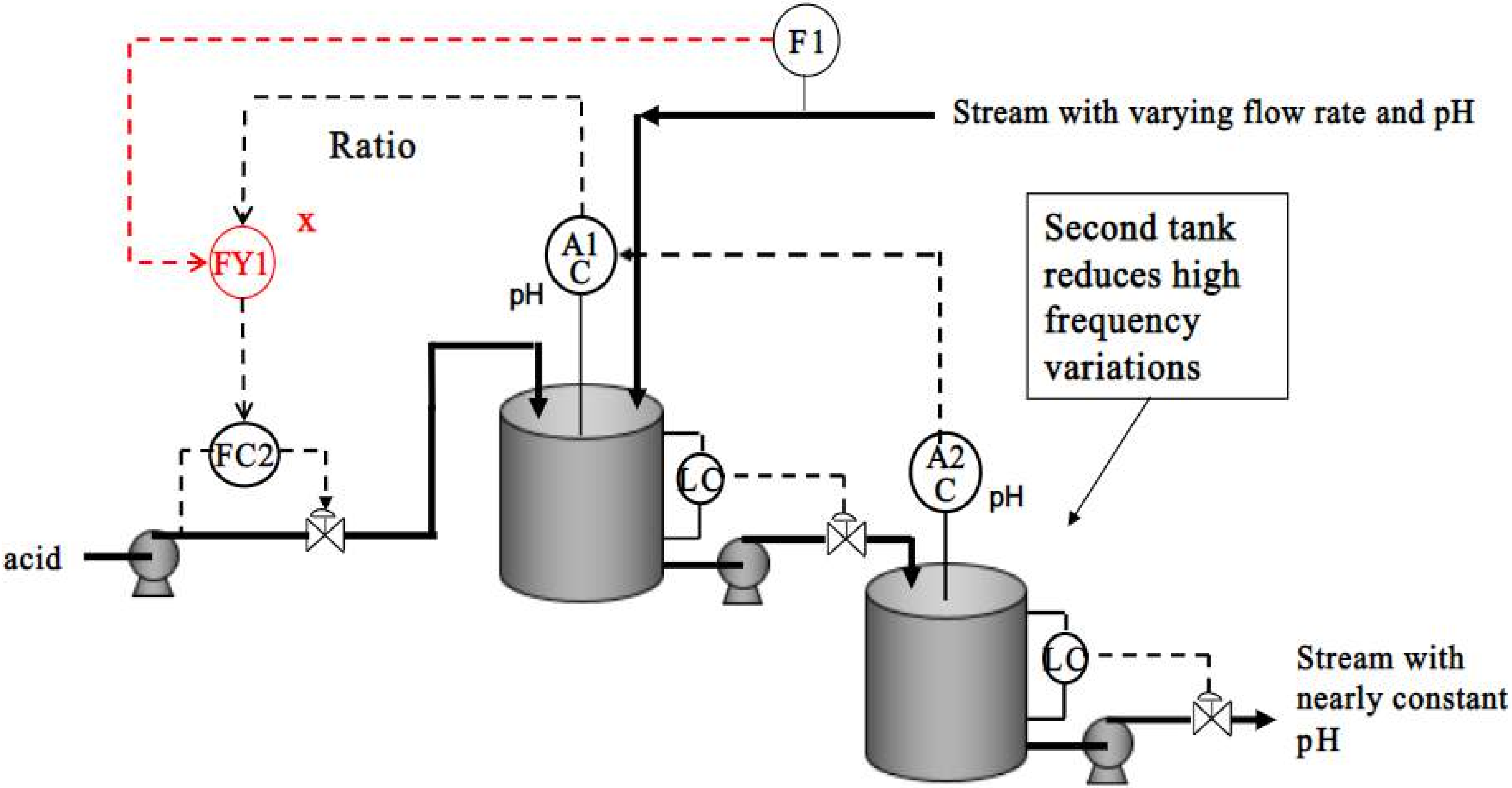
Class exercise Solution



Class exercise Design a feedforward ratio controller for changes in inlet flow rate.



Class exercise Solution



SECTION 3 — Ratio Control

Ratio control ensures that **two (or more) flows maintain a constant proportion**, regardless of upstream fluctuations.

3.1 Basic structure

Let:

- $F_2(t)$ = wild flow (master)
- $F_1(t)$ = manipulated flow (slave)
- R = desired ratio

Then:

$$\boxed{F_{1,sp}(t) = R\,F_2(t)}$$

3.2 Why ratio control?

Many industrial processes require:

- Stoichiometric mixing (reactors)
- Specified fuel–air ratio (burners)
- Constant blending ratio in food, beverage, pharmaceuticals
- Two-component dilution (polymer, adhesive, resin applications)

3.3 Dynamics

- The slave flow loop must be **faster** than the disturbances in the wild flow.
- If flow sensors have mismatched dynamics → ratio error occurs.

3.4 Industrial architecture

- Flow transmitter for wild flow
- Multiplier block computing $R \cdot F_2$
- PI/PID controller regulating the slave valve
- Optional trim controller for final composition control

3.5 Limitations

- Requires accurate and noise-free flow measurements
- Ratio alone does not guarantee final quality if feed compositions vary
- Scaling mismatches create steady-state ratio erro

SECTION 4 — Contrasting the Two Enhancement Techniques

Feature	Feedforward Control	Ratio Control
Purpose	Disturbance rejection	Maintain flow proportionality
Requires	Measured disturbance variable	Measured wild flow
Mathematical design	$G_{ff}(s) = -G_d/G_p$	$F_{1,sp} = RF_2$
Dynamic behavior	Uses dynamic models	Mostly static proportionality
Feedback required?	Yes (always)	No (for ratio only), but recommended
Industrial use	heaters, boilers, reactors	mixing, combustion, blending

Key insight

They solve **different** but complementary problems.

Feedforward = anticipatory disturbance action.

Ratio control = structural regulation of flows.

SECTION 5 — Applications That Use Both Feedforward and Ratio (Pedagogical Integration Only)

⚠️ **This is NOT combining the controllers into a single algorithm.**

Instead, this section explains **where a process uses_ratio control_ and also uses_feedforward control_ in different parts of the same plant**, so students see their roles clearly.

5.1 Example A — CSTR with Temperature Feedforward and Reactant Ratio Control

- Ratio block maintains stoichiometric flow between reactants A and B:

$$F_{A,sp} = RF_B$$

- Feedforward compensates for feed temperature disturbances:

$$Q_{ff}(t) = G_{ff}(s)\,\Delta T_{in}(t)$$

Teaching point:

Flow matching and disturbance rejection are separate tasks.

5.2 Example B — Blending Tank with Flow-Ratio + Composition Feedforward

- Ratio maintains nominal blending:

$$F_1 = RF_2$$

- Feedforward compensates for variations in x_2 (stream concentration).

Students learn:

- Ratio control maintains **relative proportions**.
- Feedforward maintains **final quality** when feed properties change.

5.3 Example C — Boiler System

- Ratio: air–fuel ratio
- Feedforward: furnace load → fuel setpoint

Again, two different enhancements appear in the same plant but for different objectives.

6.2 Ratio Example

Given:

$$R = 0.75, \quad F_2(t) = 80 + 20\sin(t)$$

$$F_{1,sp}(t) = 60 + 15\sin(t)$$

Students simulate slave flow tracking.

7. Practical Control Architecture (Industrial DCS/PLC)

Combined Loop Structure

1. **Ratio block:**

$$F_{1,sp}^{(ratio)} = RF_2$$

2. **Feedforward block:**

$$F_{1,sp}^{(ff)} = G_{ff}(s)\,d(t)$$

3. **Feedback trim:**

   $u_{\text{PID}}(t)$ (based on composition/temperature)

4. **Slave controller (FIC-1):**

   Executes final flow control

Final MV:

$$F_{1,sp} = F_{1,sp}^{(ratio)} + F_{1,sp}^{(ff)} + u_{\text{PID}}$$

8. Advantages and Limitations of Combined Control

✔ Advantages

- Excellent disturbance rejection
- Maintains consistent product quality even with fluctuating feed properties
- Reduces load on feedback loop
- Improves stability and reduces variability (IAE, ISE)

✗ Limitations

- Requires:
  - Reliable disturbance measurement
  - Accurate process/model knowledge
- Measurement dead time reduces FF effectiveness
- Ratio block amplifies noise in wild flow measurement
- Slave loop must be well-tuned and faster than both FF and ratio dynamics

Experiment 1 (Part A)

Objective: Study and use of Scientech 2476 Pressure Control Workbench hardware and software .

Equipment Needed:

- Scientech 2476 Pressure Measuring Workbench
- Ethernet Cable
- Laptop
- Air Compressor
- Software

Scientech 2476 Pressure Control Workbench

Technical Specifications

Pressure Transmitter	:	**1 No.**
Range	:	0-150 Psi
Accuracy	:	± 0.5 %
Output	:	4 to 20mA, 2 Wire System
Pressure Gauge	:	**2Nos.**
Range	:	0-110 psi
Dial	:	100 mm
Pressure Gauge	:	**1No.**
Range	:	0-150 psi
Dial	:	63 mm
Control Valve	:	**1No.**
Input Signal	:	4-20 mA
Supply Pressure	:	0 to 45kg/cm2
Type	:	Equal Percentage
Solenoid Valve	:	**1Nos.**
Supply	:	230 AC
Current Display	:	**2Nos.**
Display	:	4 Digit, 7 segment digital displays
Keys	:	3 for Digital Setting
Input Type	:	Current
Resolution	:	1 or 0.1 degree
Supply Voltage	:	230V AC
AC Ammeter	:	**1 Nos.**
Range	:	0-10 A
Voltmeter	:	**1 No.**
Range	:	0 to 300V DC
Push to ON Switch	:	4 Nos.
Indicator	:	4 Nos.
Toggle Switch	:	2 Nos.
4 – 20mA Source	:	1 No.
SS Tank	:	2 No.

Data Acquisition System (DAQ)

Analog Input	:	8
Analog Output	:	2
Digital Input	:	8
Digital Output	:	8
ADC Resolution	:	24 Bit
Two Unity Gain Amplifiers	:	0 to 5V
USB 2.0	:	Yes
Ethernet	:	Yes
Data Logging (PC Based)	:	Yes
Power Supply	:	USB Based
UART Interface	:	Yes

Human Machine Interface (HMI)

Supply	:	+24V DC
CPU	:	32-bits 400MHz RISC
Storage	:	128M FLASH + 64M DDRAM
Display size	:	7 inch
Resolution	:	800×480 TFT LCD 65,536 colors
Interface	:	RS485
Touch Screen	:	High precision four-wire resistive

Caster Wheel 2 (With Lock), 2 (Without Lock)

Size	:	4
MCB	**:**	**1 No.**
Supply	:	230V AC
Current	:	16Ampere

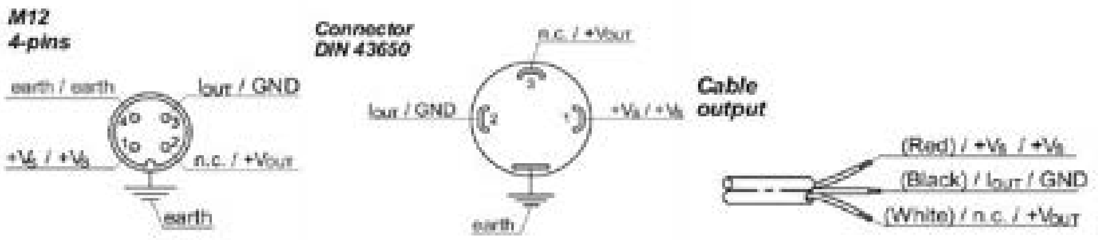
Hardware Details

Pressure Transmitter

Pressure Transmitter is a transmitter Convert pressure into current .Pressure transmitter pressure input is 0 to 150psi and current output is 4 to 20mA.

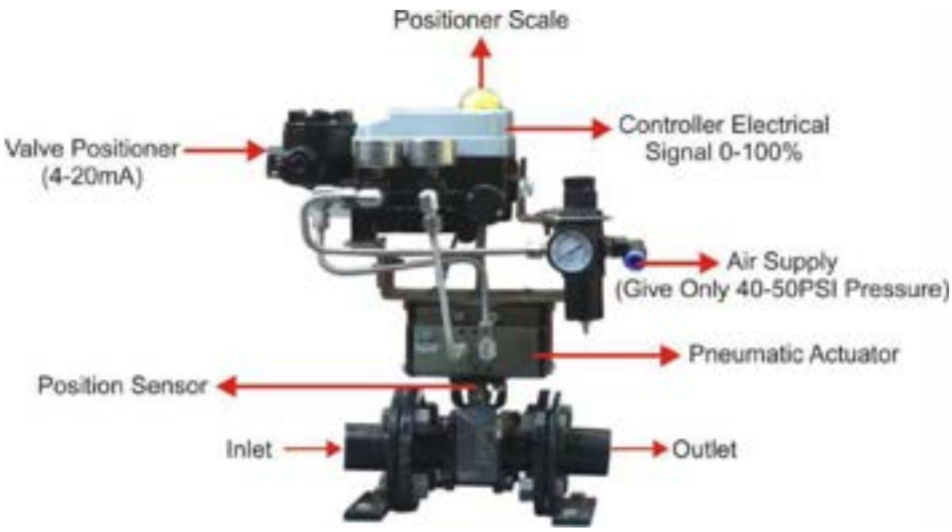
Electrical Connections:

Signal at 4……..20mA



Control Valve

A control valve is a valve used to control fluid flow by varying the size of the flow passage as directed by a signal from a controller. This enables the direct control of flow rate and the consequential control of process quantities such as pressure, temperature, and liquid level.



How the Control Valve Work?

- When you give 8.1mA Current using slider then Control valve will 25% Open.

- When you give 12mA Current using slider then Control valve will 50% Open.

- When you give 16mA Current using slider then Control valve will 75% Open.

- When you give 20mA Current using slider then Control valve will 100% Open.

Solenoid Valve:

A solenoid valve has two main parts: the solenoid and the valve. The solenoid converts electrical energy into mechanical energy which, in turn, opens or closes the valve mechanically. Solenoid valves may use metal seals or rubber seals, and may also have electrical interfaces to allow for easy control. A spring may be used to hold the valve opened or closed while the valve is not activated.

Data Acquisition System

A typical data acquisition system consists of individual sensors with the necessary signal conditioning, data conversion, data processing, multiplexing, data handling and associated transmission, storage and display systems.

In order to optimize the characteristics of the system in terms of performance, handling capacity and cost, the relevant sub systems can be combined together. Analog data is generally acquired and converted into digital form for the purpose of processing, transmission, display and storage.

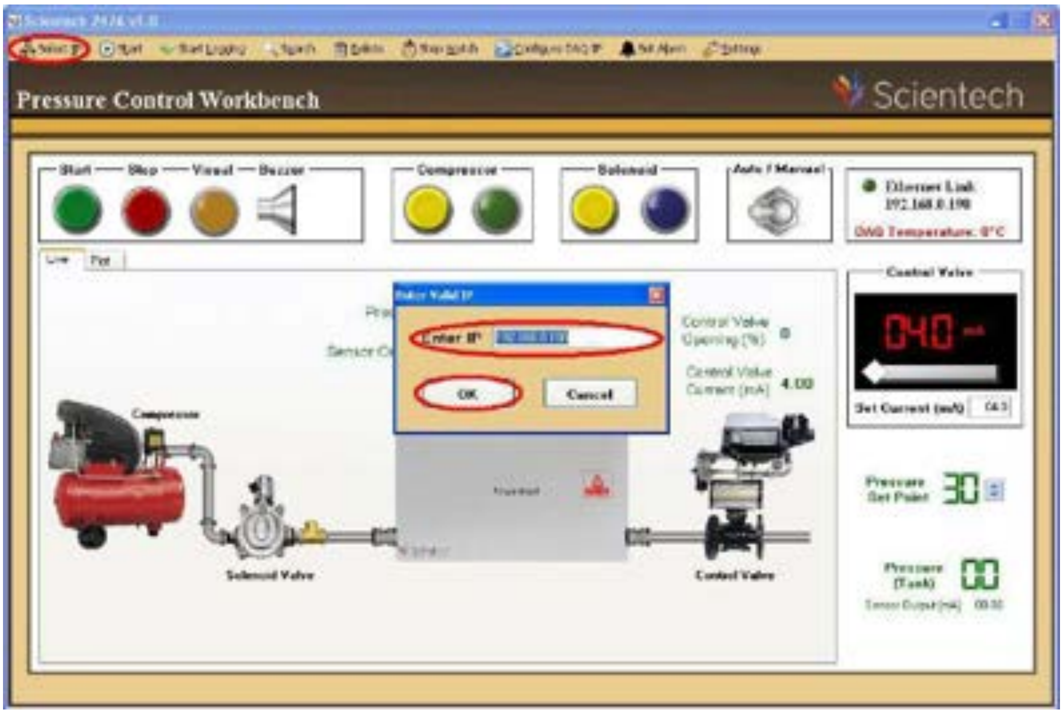
Current Display:

Air Compressor**:**

SS Tank:

Software Detail

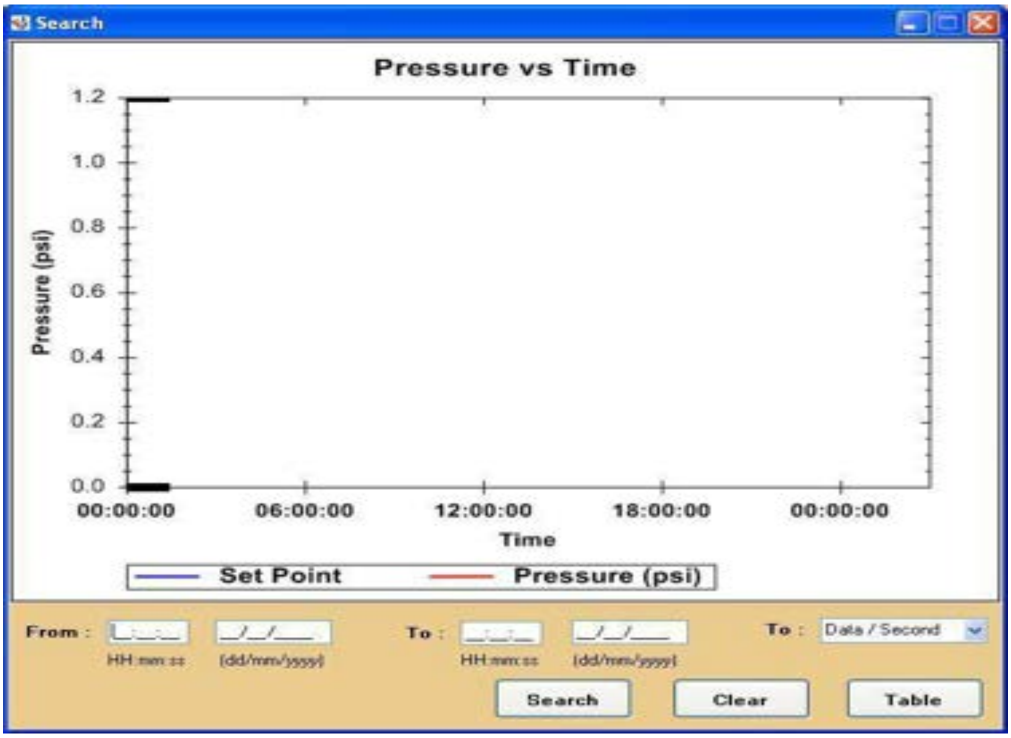
Select IP Click on Select IP then Enter IP 192.168.0.190 and click on OK button as shown below.



Start button is used for start process.

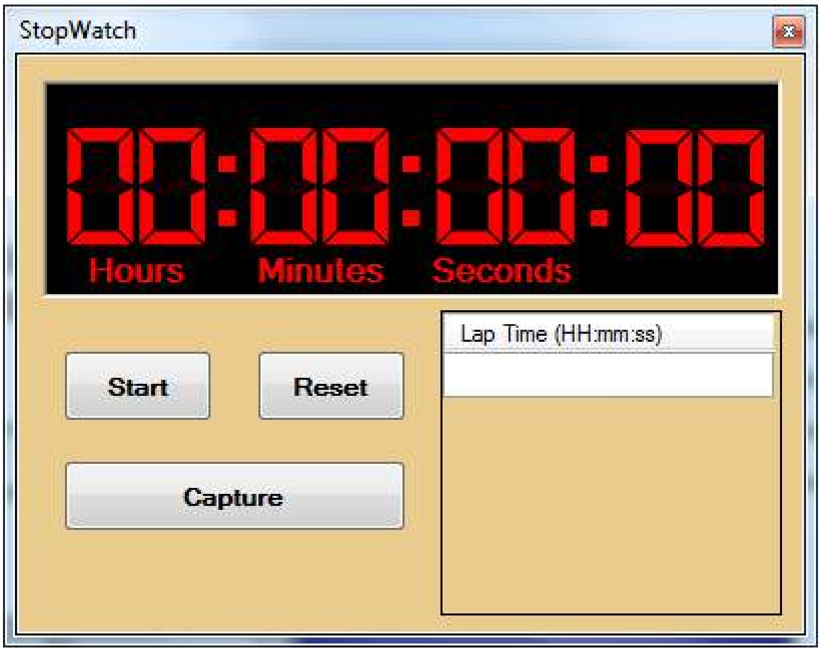
2. **Start Logging** this button is used to save data of pressure output in form of graph and buttonle.

3. **Search** this button is used to find previous/save data through date/time.

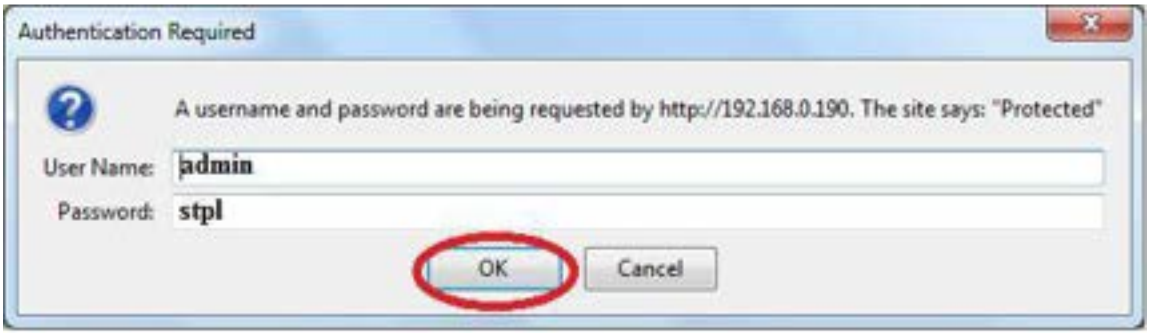


4. **Delete** this button is used to delete the save data/logging data.

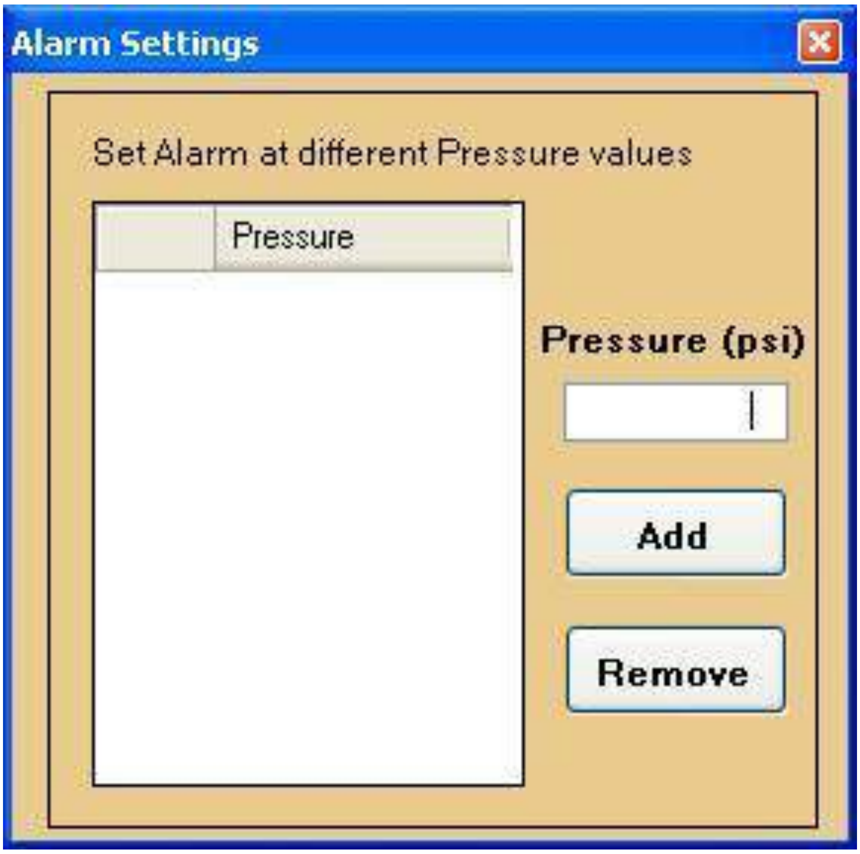
5. **Stop Watch** is used for experiment point of view.



6. **Configure DAQ IP** if you want to Configure Nvis632i according to your network, you can change the IP Address, Gateway, Subnet Mask, Primary DNS, and Secondary DNS according to your system network. If you configured Ethernet according to your system network is complete, click on to Save Configure button.

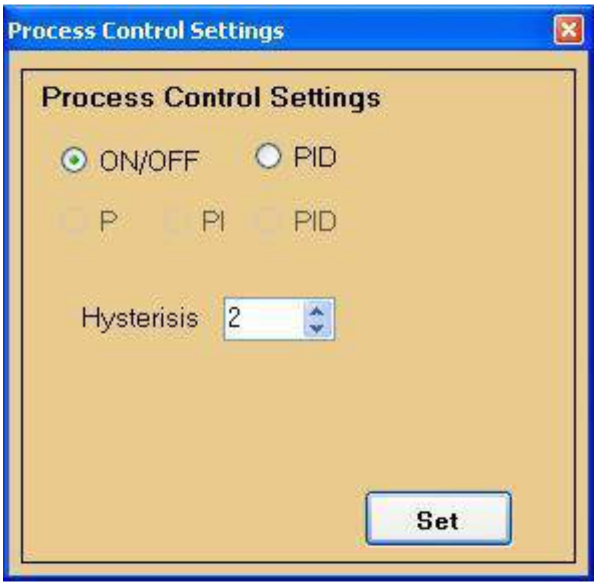


7. **Set Alarm** This button is used to set a alarm for pressure at a fixed set points.

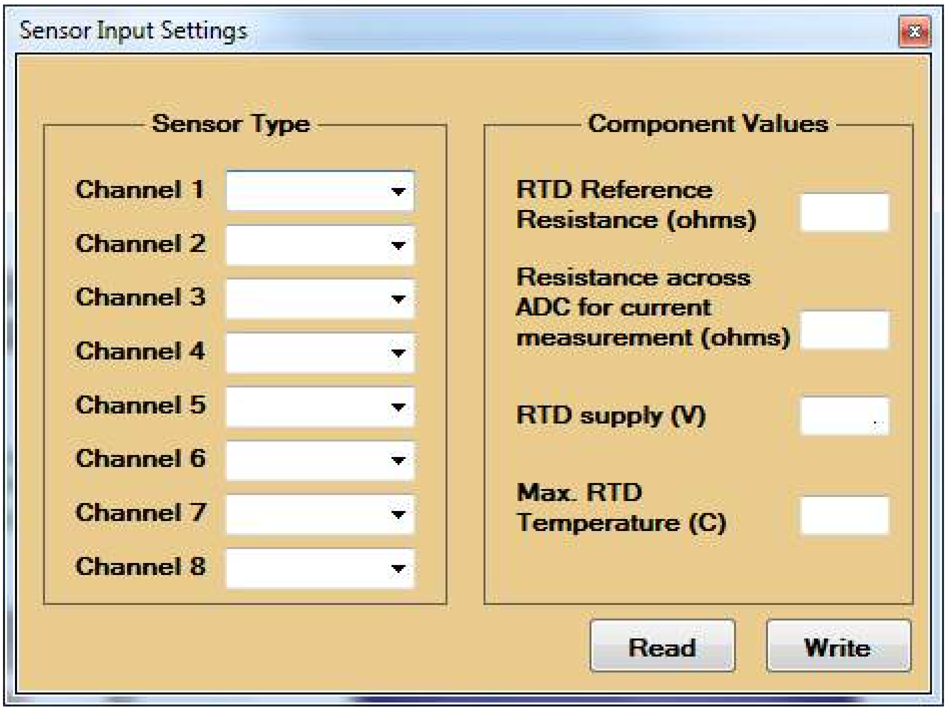


8. **Setting** this button is used to set Pressure PID values as well as sensor input settings.

Process control Setting



Sensor Input Setting



9. **Auto/Manual** from this block we can control process in auto or manual mode.

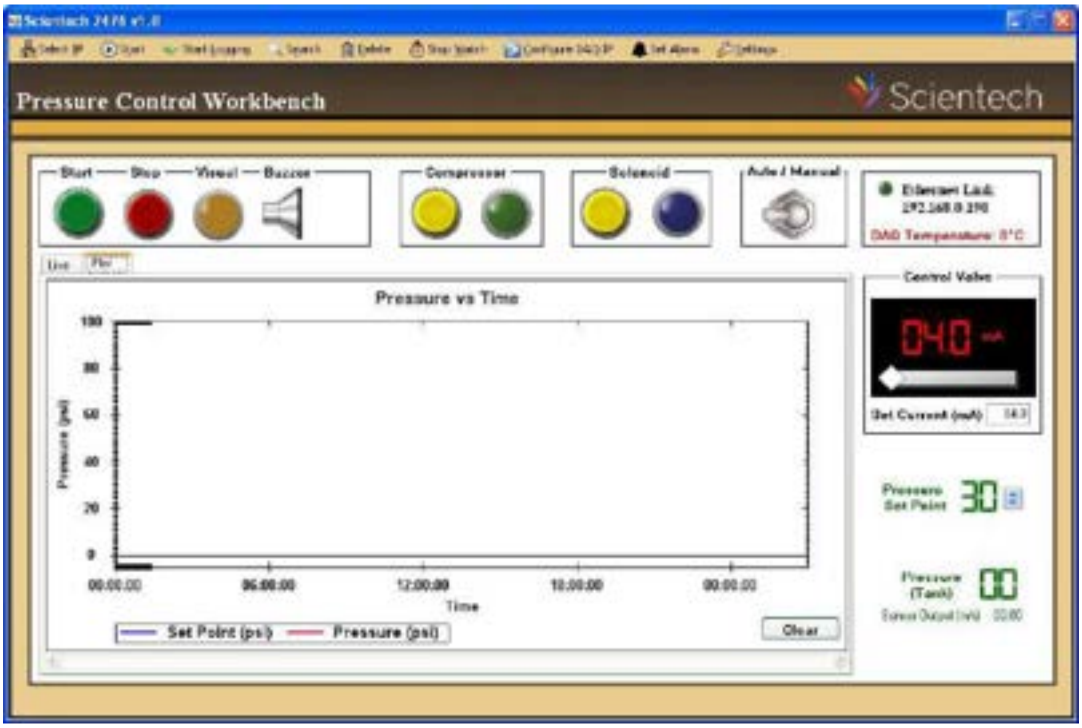
10. **Ethernet Link/ DAQ Temperature** this button indicates whether the hardware is connected to LAN or not and shows internal DAQ temperature.

11. **Control Valve** from this block we can adjust variable 4-20mA output using software.

12. **Set Value/Process Value** this block shows current reading of different kind of sensors as well as setpoint value.

13. **Live** this button is used for control and see a real time change in the Process.

15. **Plot** this button shows process value and set point value reading in the form of graph.



Experiment 1 - Part B

Objective: Study and use of Pressure Transmitter.

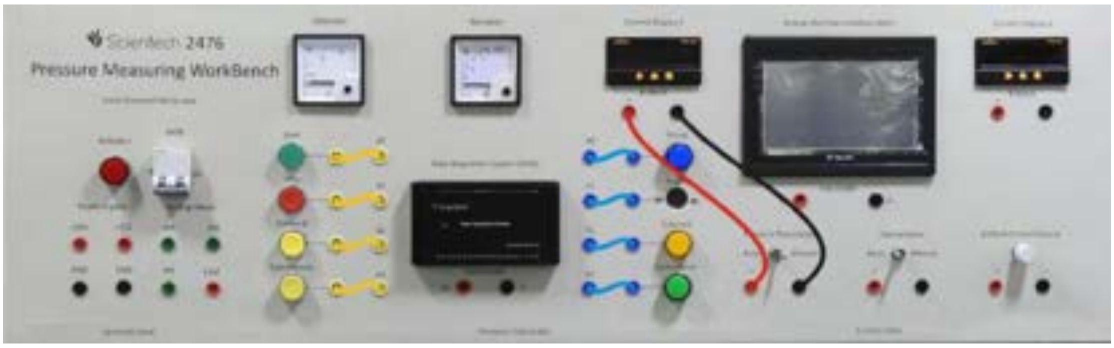
Equipment Needed:

- Scientech 2476 Pressure Measuring Workbench
- Ethernet Cable
- Patch Cords
- Air Compressor
- Patch Cord

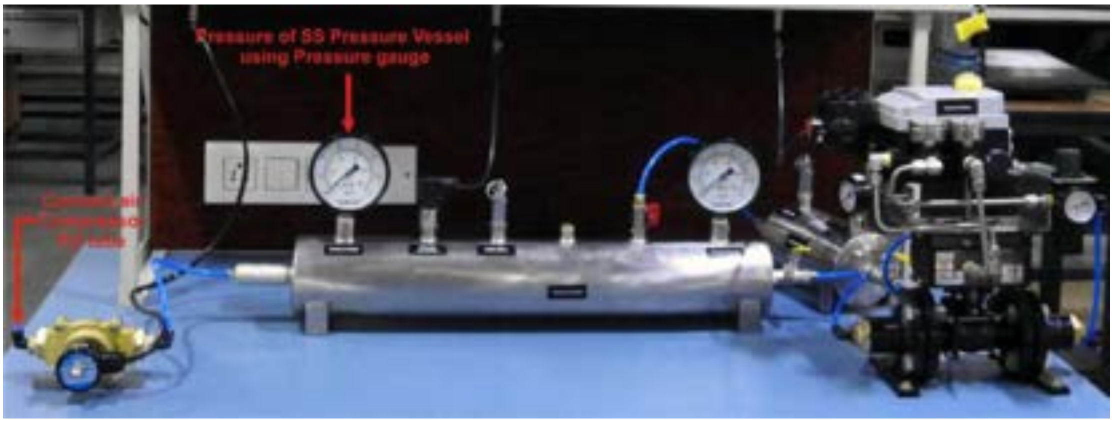
Procedure:

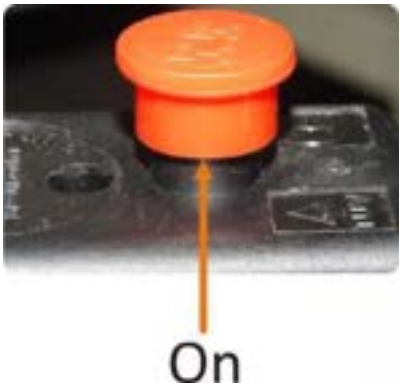
- Turn on the MCB then Power Indicator (Red Indicator) and Scientech 2476 Pressure Measuring Instrument will be on.

- Connect Ethernet Cable between Ethernet Socket of Scientech 2476 to PC Ethernet Socket.
- Make a connection diagram according to below given connection diagram. Put Pressure transmitter Toggle Switch to Manual mode

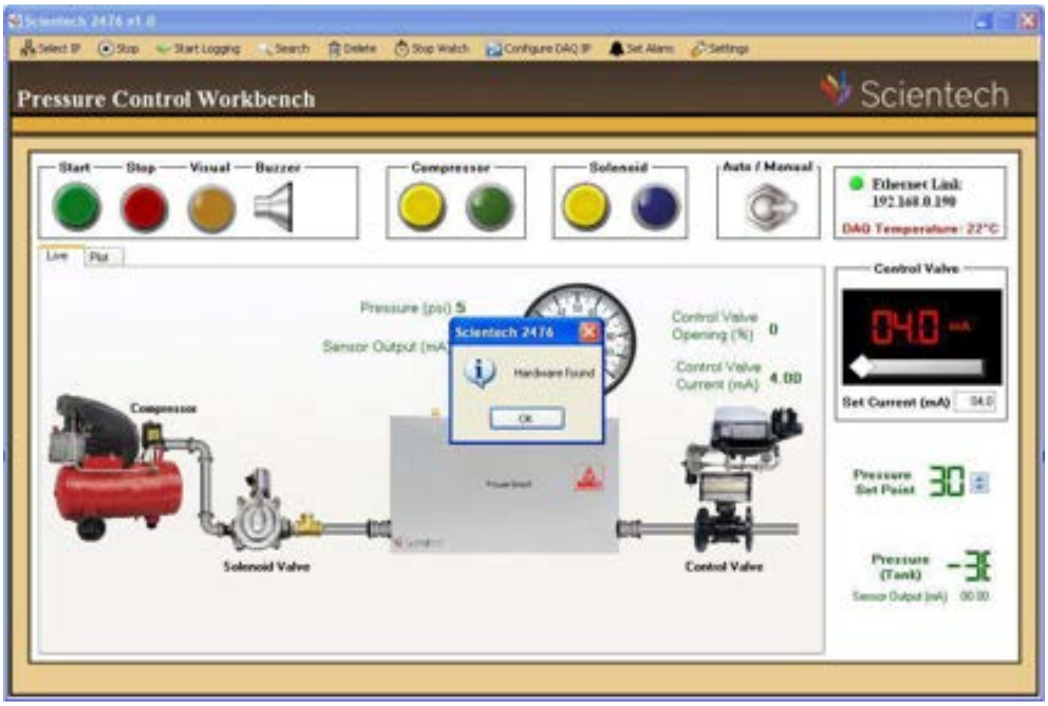


- Connect Air Compressor Input PU Tube to Input of Solenoid Valve. Pull the Red knob of Air Compressor.

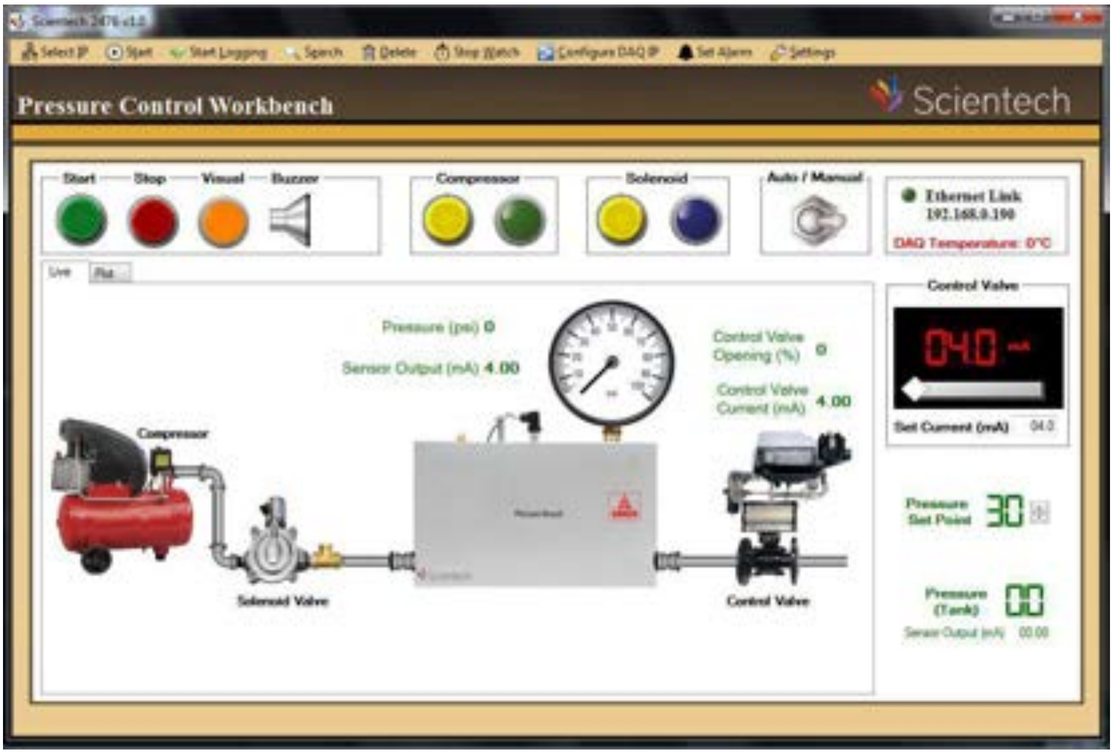




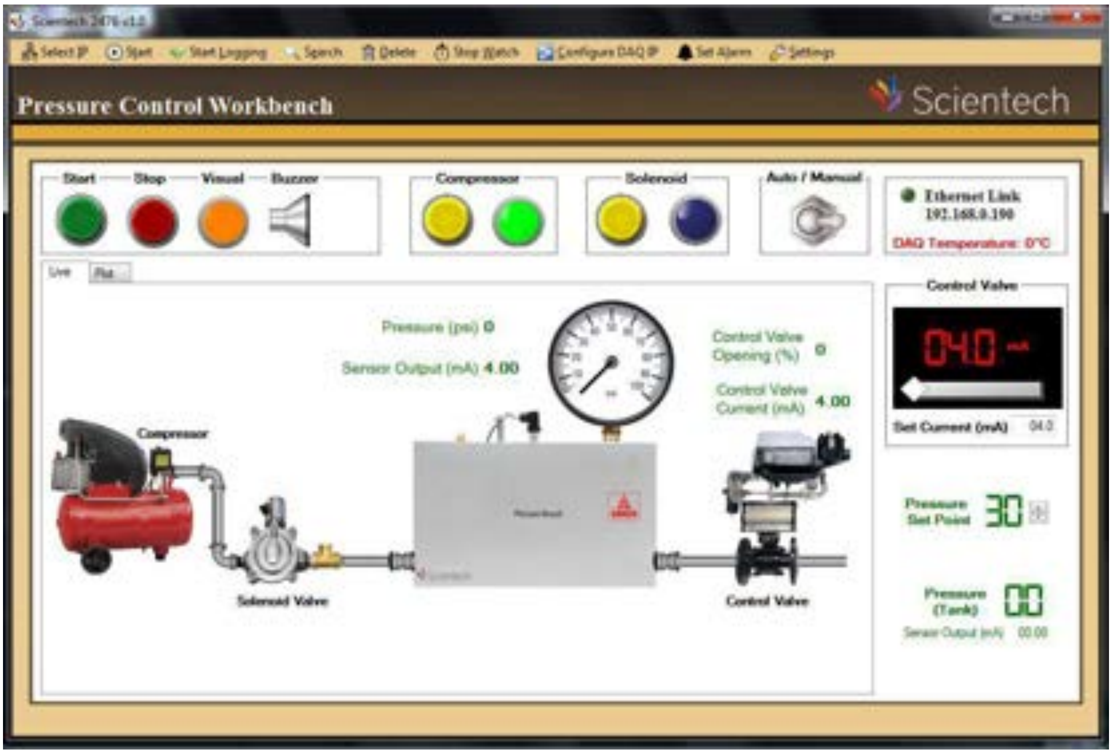
- Open the Scientech 2476 Software, Select IP and click on Start button.
- **“Hardware Found”** message will comes then click on “**OK**” Button as shown below.



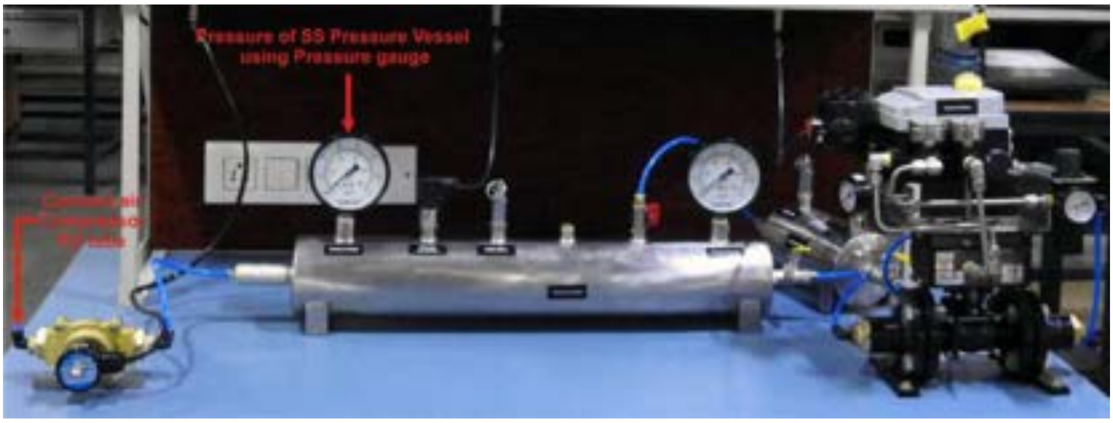
- Firstly click on Start button then Visual Indicator will ON , means your Process will ready to start.



- Click on Air Compressor Switch then your air compressor will start. In Air Compressor 100 psi air fill then Air compressor will automatically off.When you click on Air compressor button then Air Compressor will OFF.



- Click on Solenoid Valve then Solenoid Valve will open air comes start in SS Pressure Vessel. When 5 Psi air fill in SS Pressure vessel then again click on Solenoid valve button then Solenoid valve will OFF. Note down the pressure Transmitter output (in mA) using Current Display 1 in observation.



- Note down the pressure Transmitter output at every 5 Psi pressure in observation table.

Observation Buttonle

Pressure using Pressure Gauge (in Psi)	Pressure Transmitter Output (in mA)
0	
5	
10	
15	
20	
25	
30	
35	
40	
45	
50	
55	
60	
65	
70	

Experiment 1 - Part C

Objective: Study and use of Control Valve.

Equipment Needed:

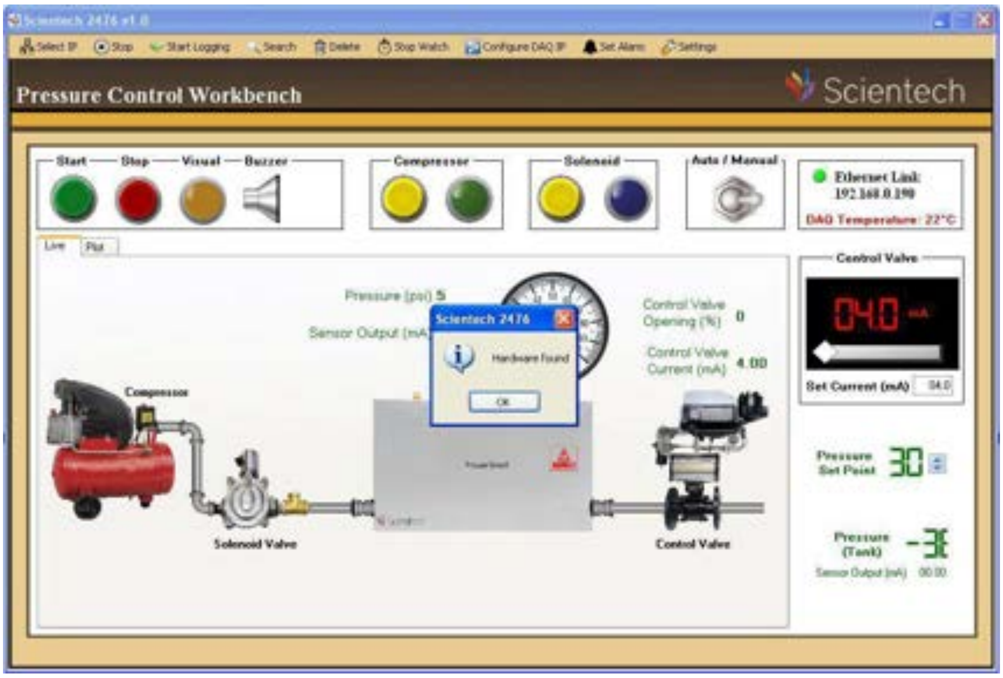
- Scientech 2476 Pressure Measuring Workbench
- Ethernet Cable
- Patch Cords
- Air Compressor
- Patch Cord

Procedure:

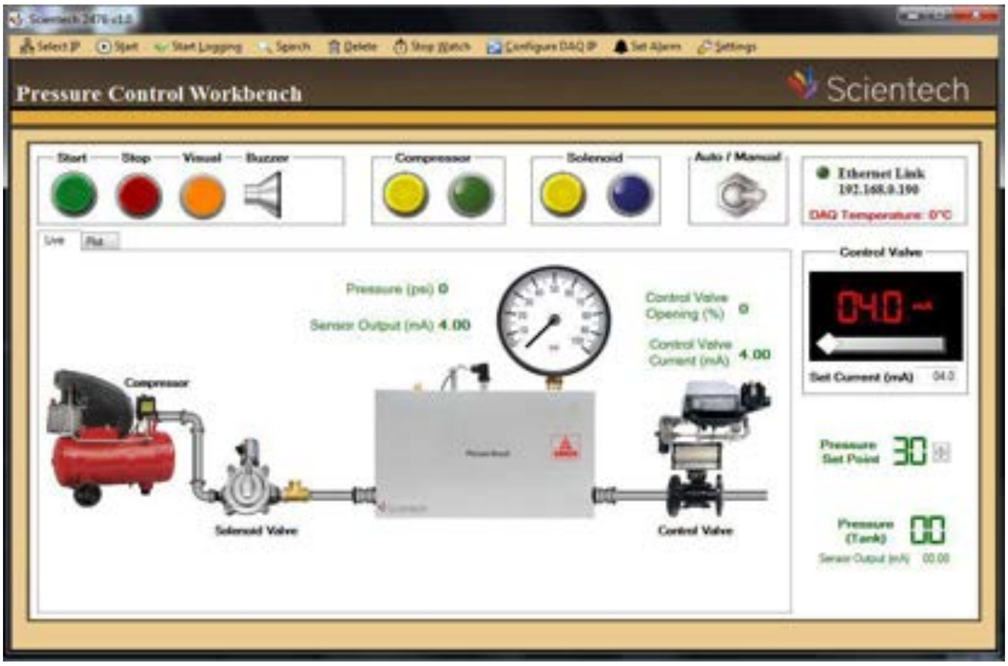
- Turn on the MCB then Power Indicator (Red Indicator) and Scientech 2476 Pressure Measuring Instrument will be on.

- Connect Ethernet Cable between Ethernet Socket of Scientech 2476 to PC Ethernet Socket.
- Make a connection diagram according to below given connection diagram.

- Open the Scientech 2476 Software, Select IP and click on Start Button.
- **“Hardware Found”** message will comes then click on **“OK”** Button as shown below.

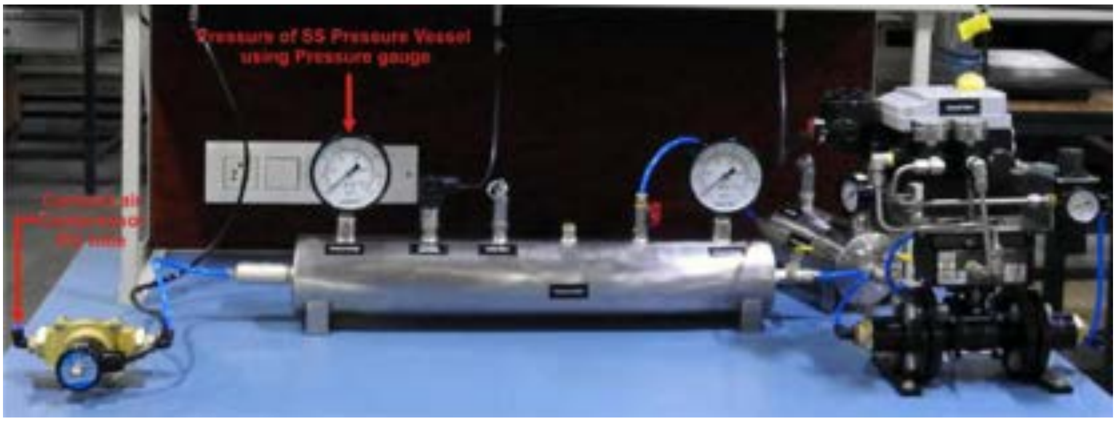


- Firstly click on Start button then Visual Indicator will ON , means your Process will ready to start.



- Click on Air Compressor Switch then your air compressor will start. In Air Compressor 100 psi air fill then Air compressor will automatically off. When you click on Air compressor button then Air Compressor will OFF.

- Click on Solenoid Valve then Solenoid Valve will open air comes start in SS Pressure Vessel. When 60 psi air fill in SS Pressure Vessel then again click on Solenoid Button then Solenoid valve will OFF.



- Make a connection according to below given connection diagram.Firstly set 4mA (approximately) using 4 to 20 mA current source Potentiometer and see the current display 2.

- Connect 4 to 20mA banana socket to Control valve banana socket using patch cord.

1- Give 4mA current to Control Valve , record the open % of control valve.

- Give 8.1mA Current using slider then record the open % of Control valve.

- Give 12mA Current using slider then record the open % of Control valve.

- Give 16mA Current using slider then record the open % of Control valve.

- Give 20mA Current using slider then record the open % of Control valve.

Observation Buttonle

Control Valve input (in mA)	Open Control Valve in %
4.3	
8	
12	
16	
20	

Note : Please do not Set Set point below 15psi and above 70 psi.

Experiment 3

Objective: Study and use of ON/OFF Controller using software.

Equipment Needed:

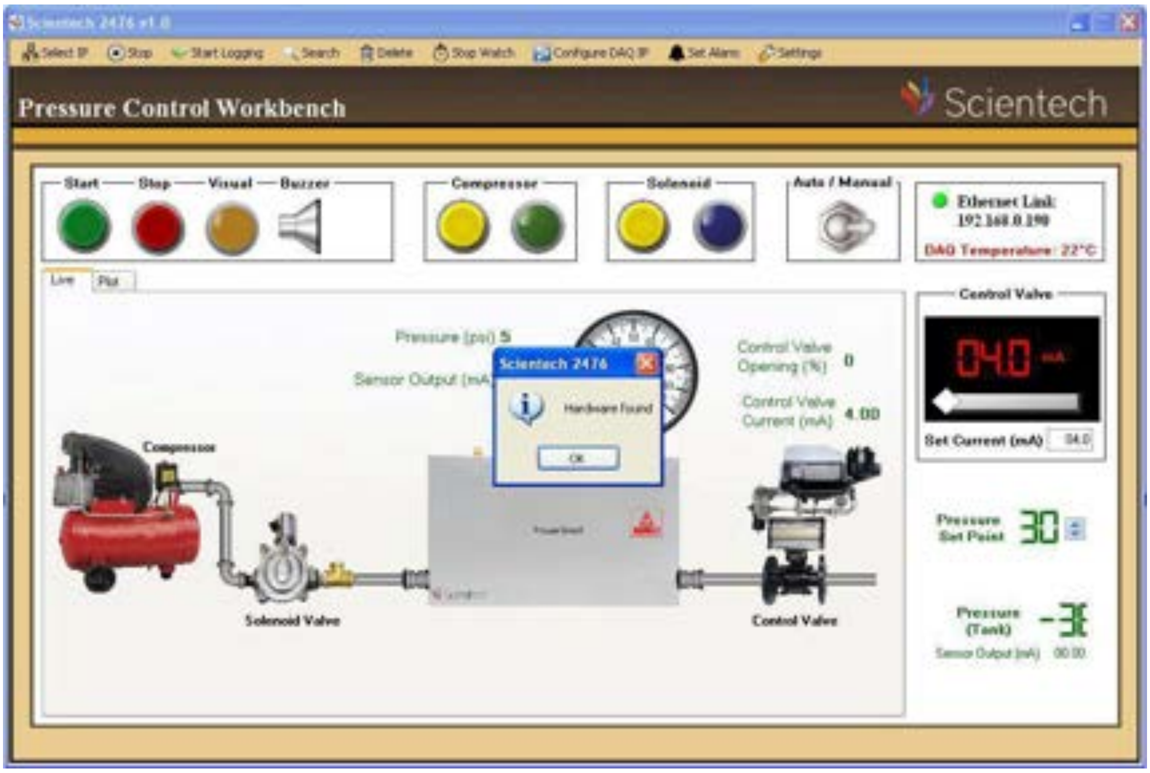
- Scientech 2476 Pressure Measuring Workbench
- Ethernet Cable
- Patch Cords
- Air Compressor
-

Procedure:

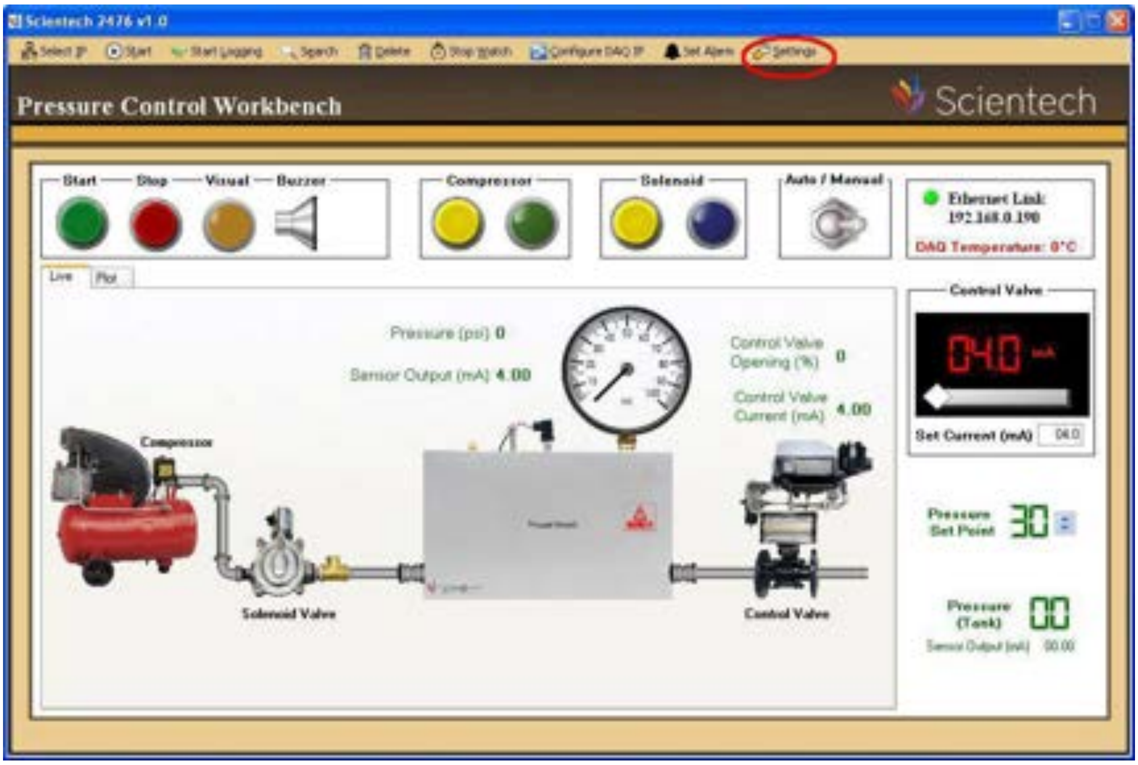
- Turn on the MCB then Power Indicator (Red Indicator) and Scientech 2476 Pressure Measuring Instrument will be on.

- Connect Ethernet Cable between Ethernet Socket of Scientech 2476 to PC Ethernet Socket.
- Turn the switches into the DAQ/Auto position as shown in the image below.

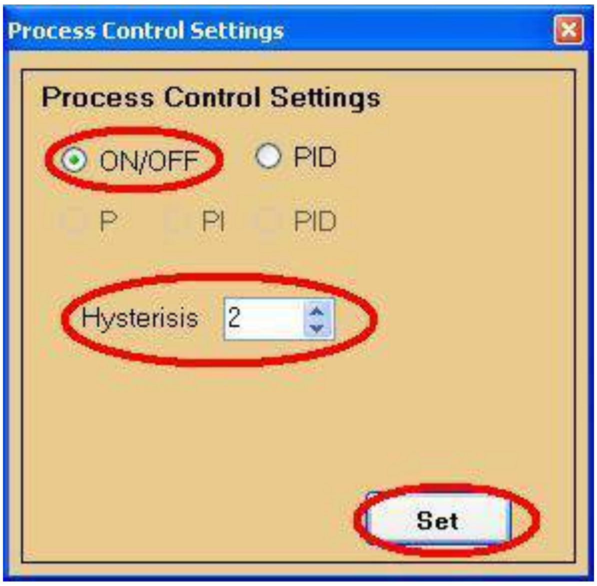
- Open the Scientech 2476 Software, Select IP and click on Start BUTTON.
- **"Hardware Found"** message will comes then click on "**OK**" Button as shown below



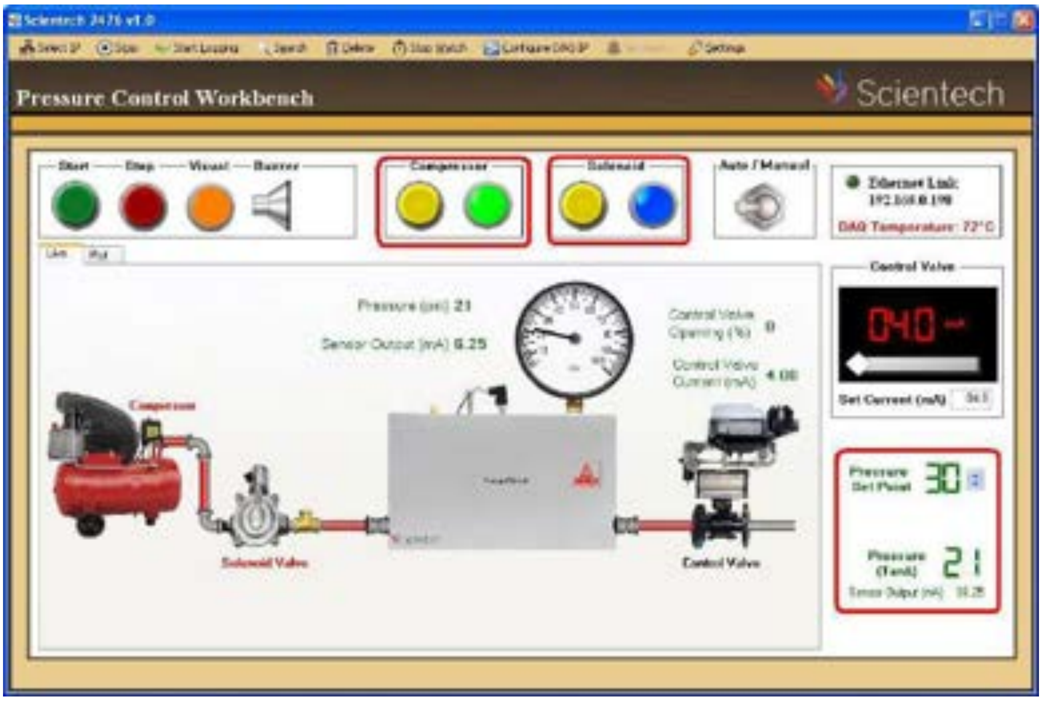
- For ON/OFF process, go to the **Setting button** then process control setting Option will open, as shown below.



- Process Control Setting window will open then click on ON/OFF>Select Hysteresis and then click OK as shown below.

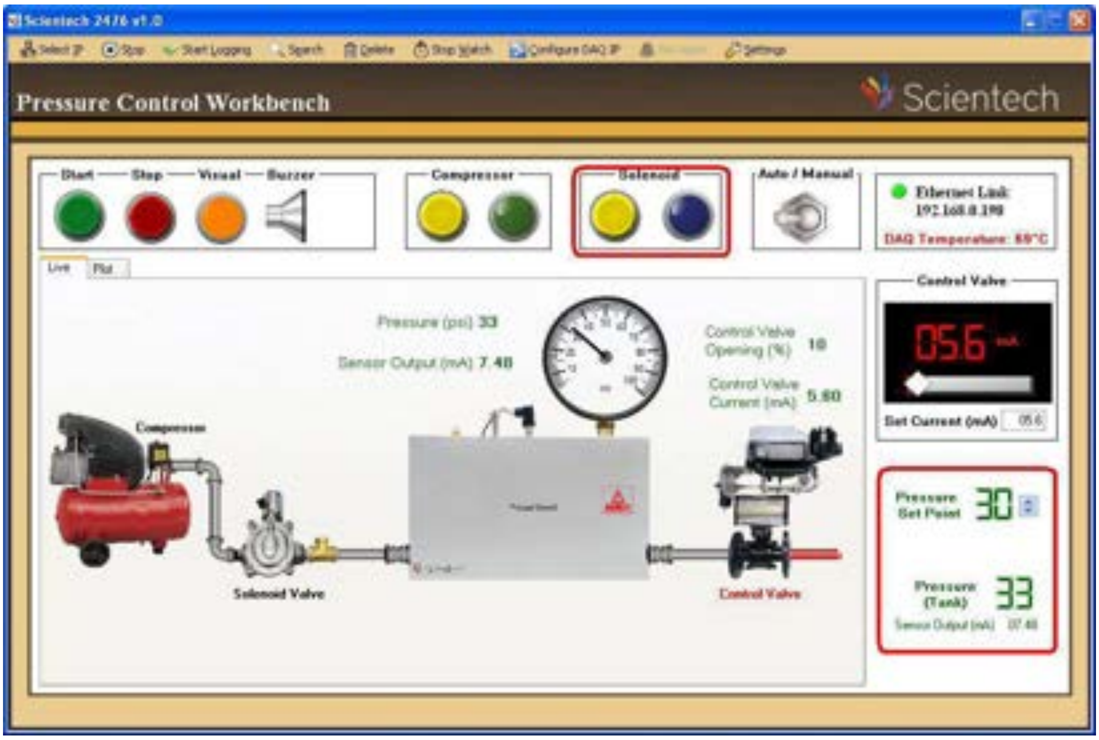


- Set the Set point of On / Off Controller as shown below. Click on Start button. When the set point is below the Current pressure then Solenoid valve will be 'ON' as shown below.

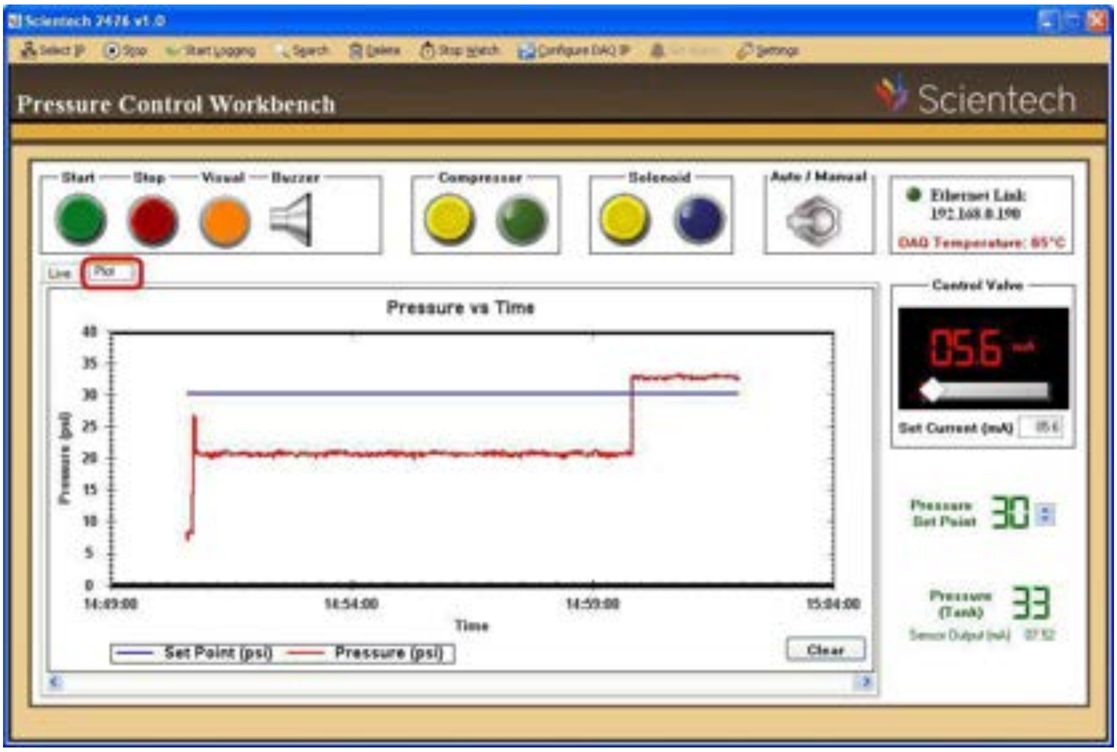


Note: When the pressure of air is below the available pressure than compressor will be on automatically**.**

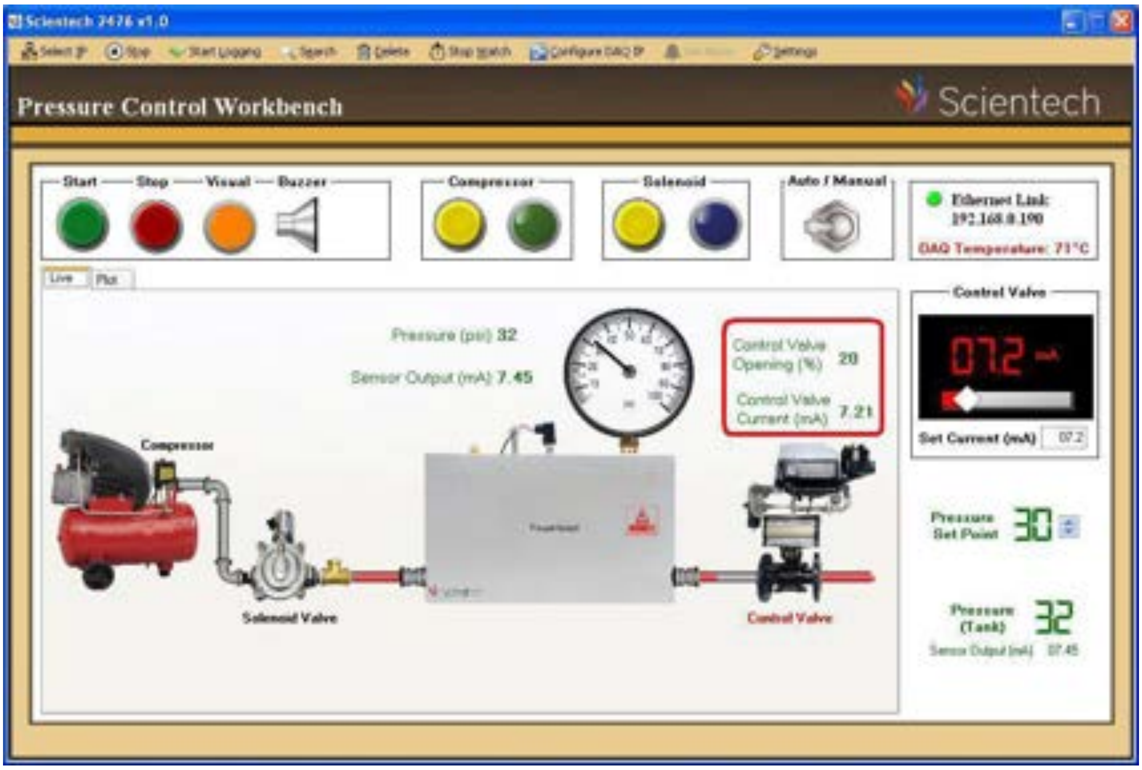
- When pressure reach to the set point then solenoid valve will be 'OFF' as shown below.



- If you want to see a graph between Pressure Vs Time, click on Plot BUTTON as shown below.



- When the process value above the set point then control valve will be open according to suitable percentage and maintain the pressure, as shown below.



Note : Please do not Set Set point below 15psi and above 70 psi.

Q1:

What are the benefits of using hysteresis in ON-OFF control?

Q2:

Why is a separate small storage tank used with the control valve?

Scientech 2476 Pressure Measuring Workbench

Experiment 4

Objective: Study and use of Proportional-Integral-Derivative using software.

Equipment Needed:

- Scientech 2476 Pressure Measuring Workbench
- Ethernet Cable
- Patch Cords
- Air Compressor
- Patch Cord

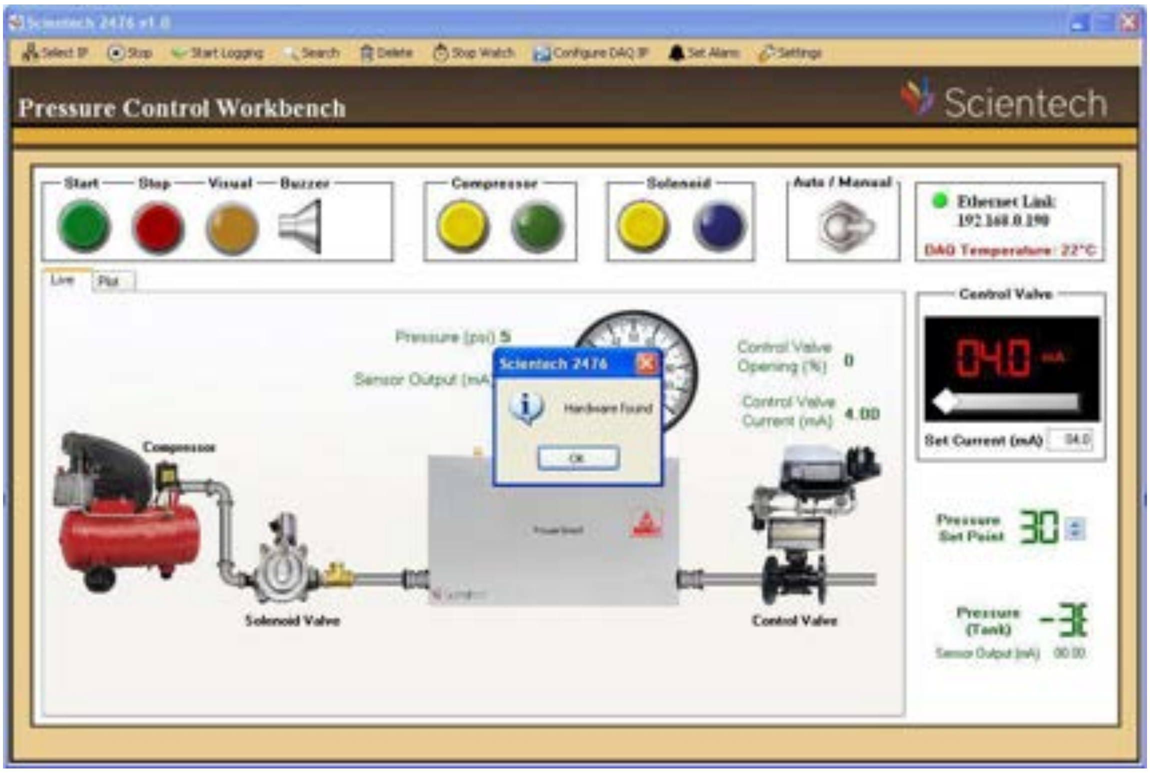
Procedure:

- Turn on the MCB then Power Indicator (Red Indicator) and Scientech 2476 Pressure Measuring Instrument will be on.

- Connect Ethernet Cable between Ethernet Socket of Scientech 2476 to PC Ethernet Socket.
- Turn the switches into the DAQ/Auto position as shown in the image below.

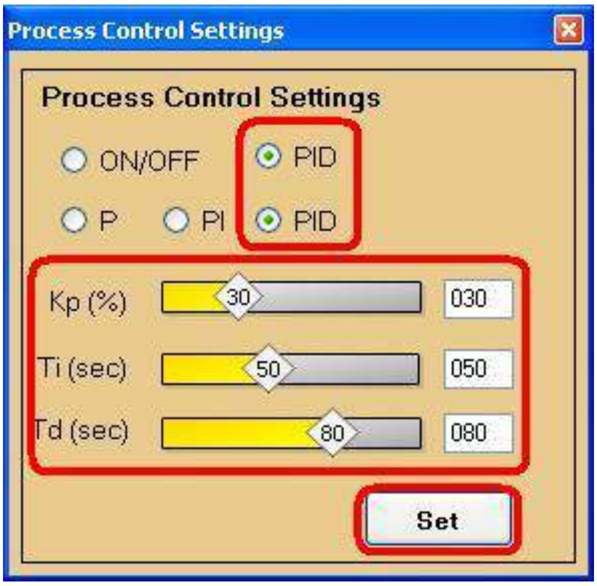
- Open the Scientech 2476 Software, Select IP and click on Start Button.
- **"Hardware Found"** message will comes then click on "**OK**" Button as shown below.

Scientech 2476 Pressure Measuring Workbench

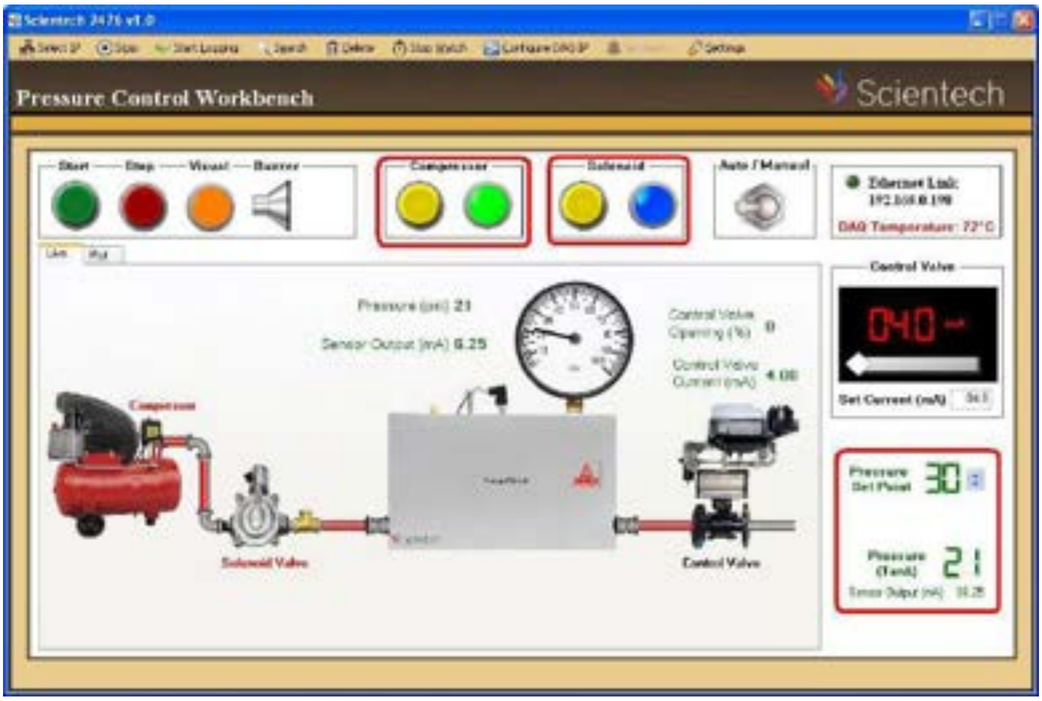


- For Proportional-Integral-De. process, go to the **Setting button** then process control setting Option will open, as shown below.

- Process Control Setting window will open then click on PID>PID set suitable gain, Integral Time and derivative time, then click OK as shown below.



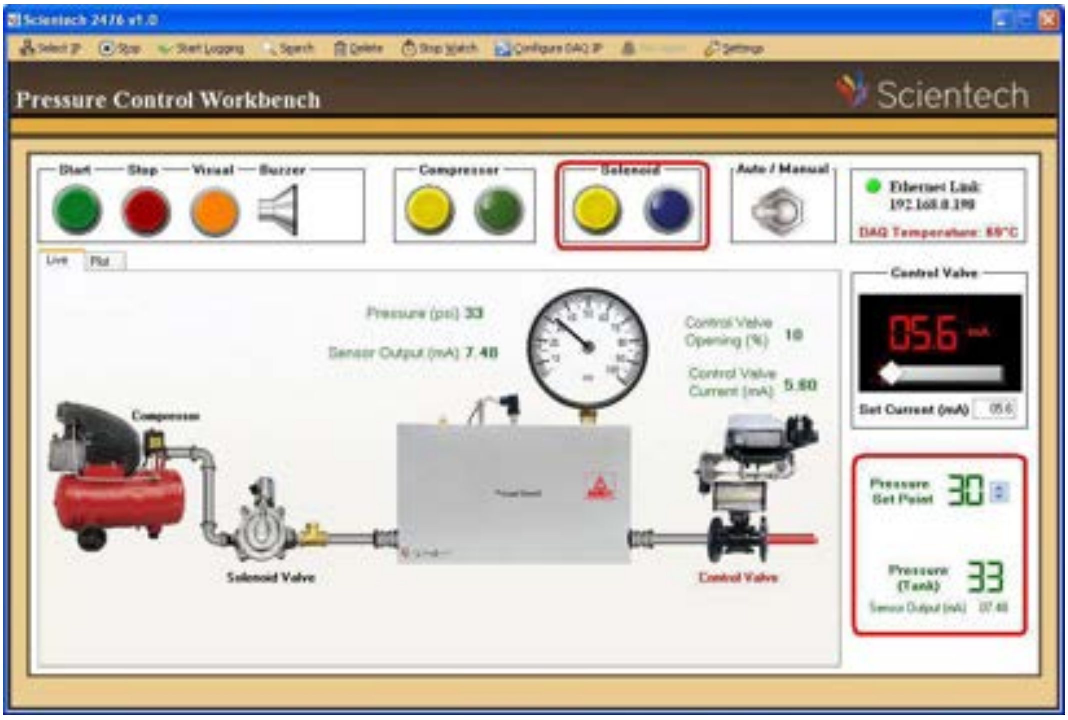
- Set the Set point of Proportional-Integral-Derivative Controller as shown below. Click on Start button. When the set point is below the Current pressure then Solenoid valve will be 'ON' as shown below.



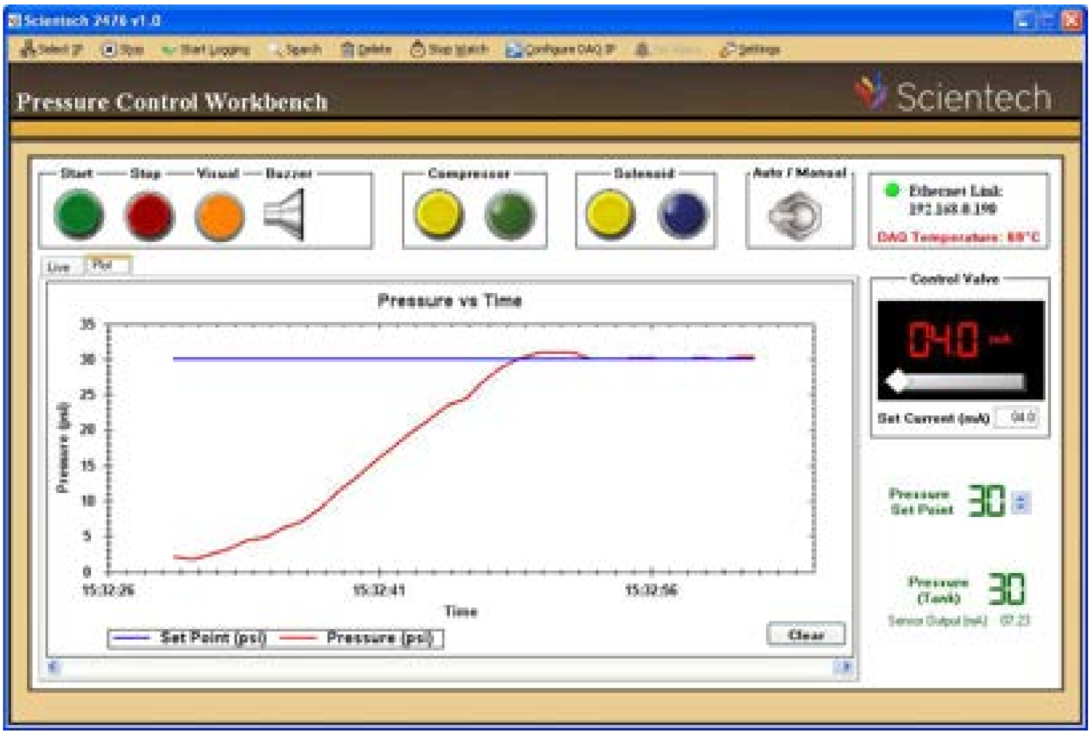
Note: When the pressure of air is below the available set pressure then compressor will be on automatically.

Scientech 2476 Pressure Measuring Workbench

- When pressure reaches to the set point then solenoid valve will be 'OFF' as shown below.

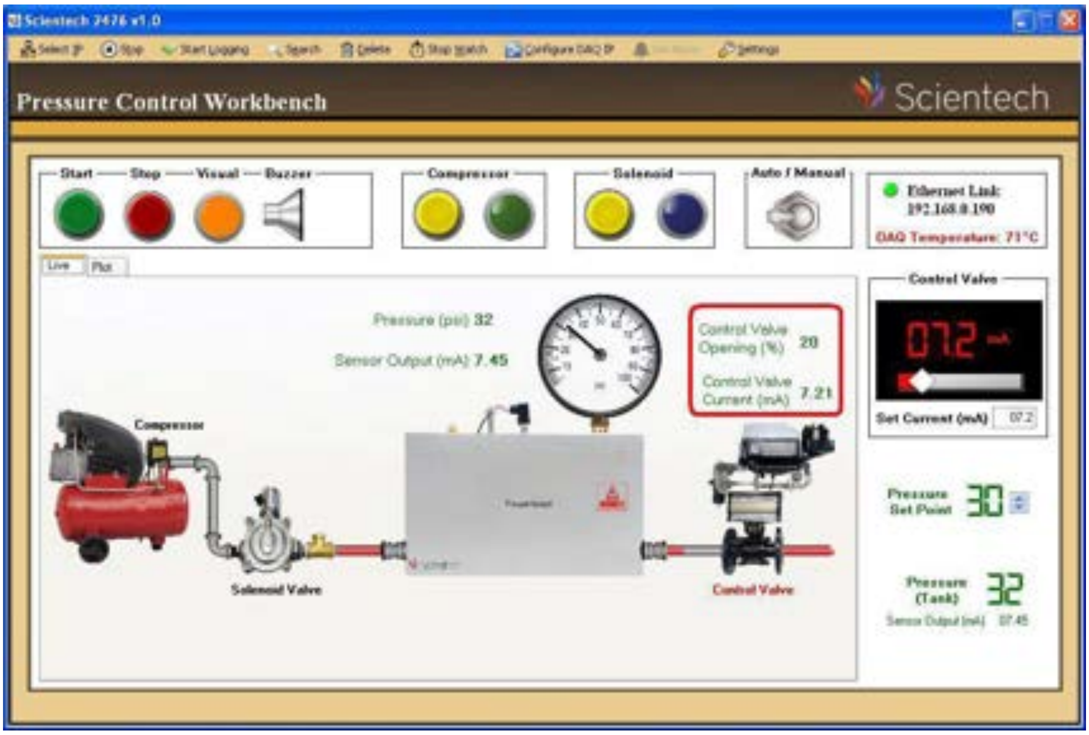


- If you want to see a graph between Pressure Vs Time, click on Plot BUTTON as shown below.



Scientech 2476 Pressure Measuring Workbench

- When the process value above the set point then control valve will be open according to suitable percentage and maintain the pressure, as shown below.



Q1:

Discuss the pressure response in figure 1 after using the PID controller in terms of the Rise Time, Steady State Error, Settling Time, and Overshoot.

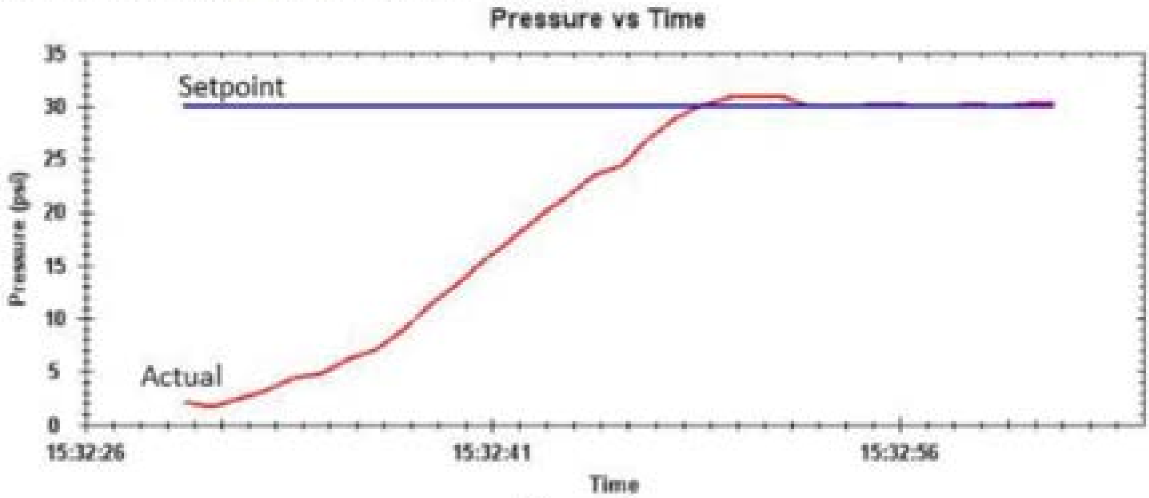


Figure 1

Q_2:

Read each statement carefully and fill in the blank(s) with the correct answer.

1- Pressure Transmitter is converting pressure into _____________.

2- When an 8.1 mA is giving to the control valve by using slider then it will open by ______%.

3- Click on ________________ button to see the graph between Pressure Vs Time.

4- When ________________ psi air fill inside the air compressor then it will automatically off.

5- When the set point is below the current pressure in the PID process control then the Solenoid valve will be ____________________.

6- In the ON/OFF process control, when the process value above the set point then control valve will be open according to ___________________________ and maintain the pressure.

Process Control LAB
Nonlinear Plant Example: Coupled-Tanks Plant

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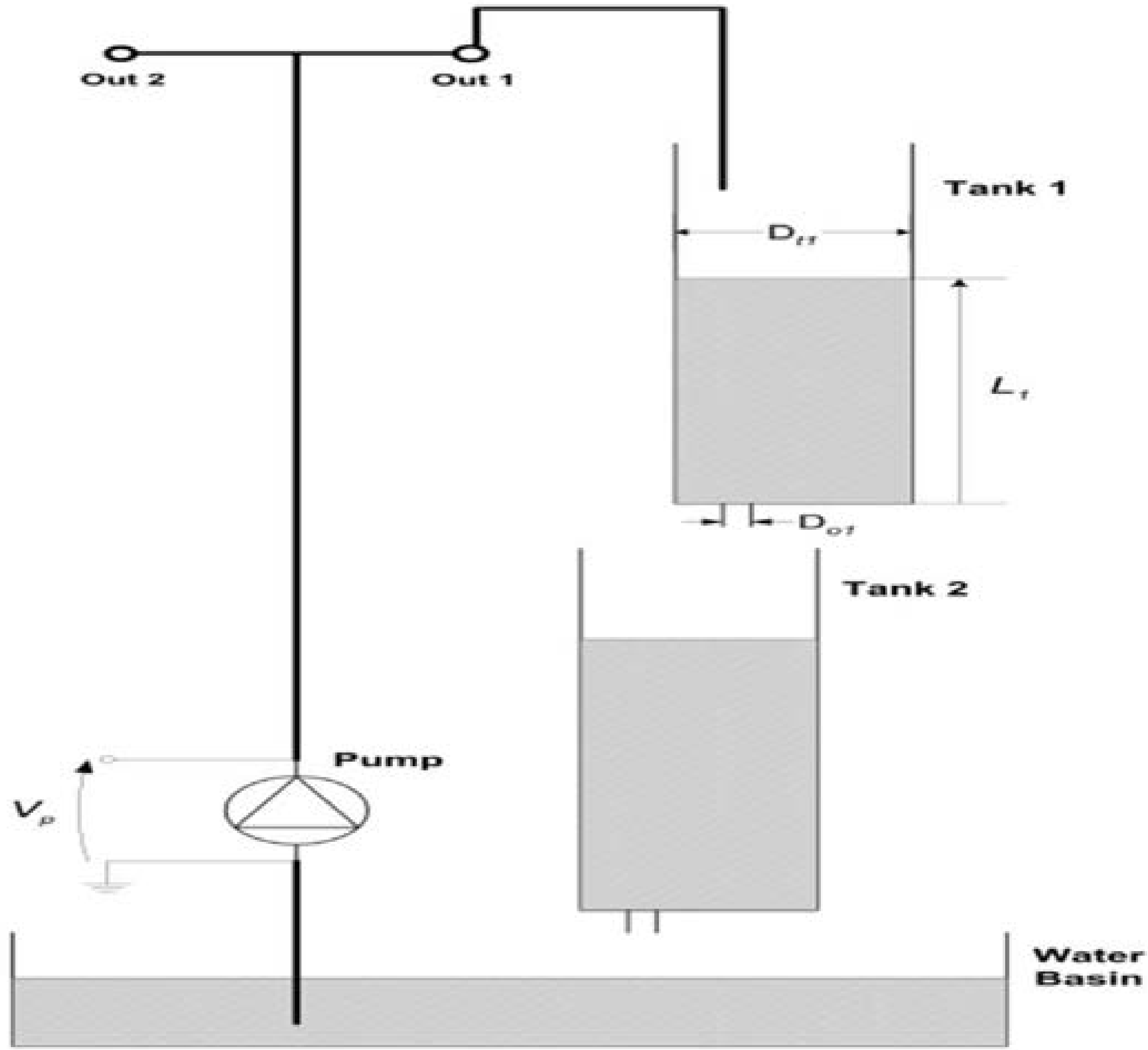
PhD Stud. Control Engineering
MSc Mechatronics Engineering
Dip. Pharmacy Technician
23rd-Oct-2025

Motivation

The problem of controlling levels of liquids in tanks and flow between tank is very common in many industries such as:

Petrochemicals, food processing and power generation.

Case Study: Quanser Coupled Tanks



INTRODUCTION

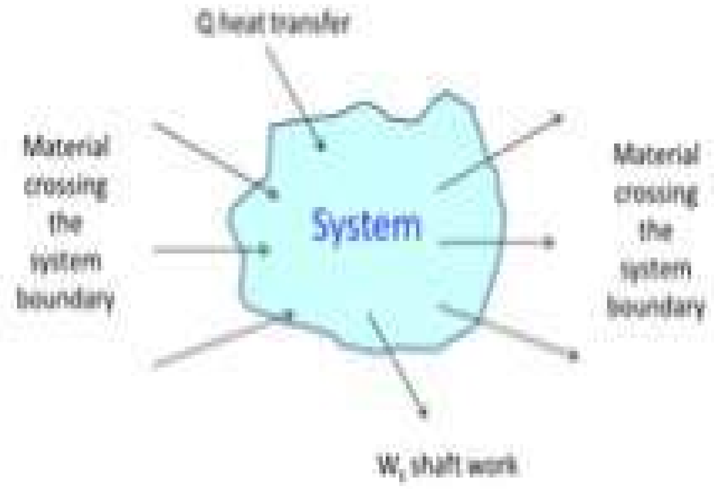
The Coupled Tanks plant is a "Two-Tank" module consisting of a pump with a water basin and two tanks. The two tanks are mounted on the front plate such that flow from the first (i.e. upper) tank can flow, through an outlet orifice located at the bottom of the tank, into the second (i.e. lower) tank. Flow from the second tank flows into the main water reservoir. The pump thrusts water vertically to two quick-connect orifices "Out1" and "Out2". The two system variables are directly measured on the Coupled-Tank rig by pressure sensors and available for feedback. They are namely the water levels in tanks 1 and 2. To name a few, industrial applications of such Coupled-Tank configurations can be found in the processing system of petro-chemical, paper making, and/or water treatment plants.

Coupled-Tank System Model Parameters and Frame Overall Dimensions

Description	Value	Unit
Overall Frame Height	0.915	m
Overall Frame Width	0.305	m
Overall Frame Depth	0.305	m

Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	$cm^3/s/V$
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

Tailoring approach to specific problem - Balance



- Which balance (which variable)?
- System Boundaries?
- Number of Balances?
- Additional equations (constitutive relationships)

1. Nonlinear Dynamic Model

For the coupled-tank system, applying mass balance and Torricelli's law gives:

Tank 1:

$$A_{t1}\frac{dL_1}{dt} = K_p V_p - A_{o1}\sqrt{2gL_1}$$

Tank 2:

$$A_{t2}\frac{dL_2}{dt} = A_{o1}\sqrt{2gL_1} - A_{o2}\sqrt{2gL_2}$$

Where:

- L_1, L_2 are liquid levels in tanks 1 and 2 (cm)
- A_{t1}, A_{t2} are tank cross-sectional areas (cm²)
- A_{o1}, A_{o2} are outlet orifice areas (cm²)
- K_p is the pump constant (cm³/s·V)
- V_p is pump voltage (V)
- g is gravitational acceleration (cm/s²)

---

2. Steady-State Operating Point

At steady-state, $\frac{dL_1}{dt} = \frac{dL_2}{dt} = 0$:

$$K_p V_{p0} = A_{o1}\sqrt{2gL_{10}}$$

$$A_{o1}\sqrt{2gL_{10}} = A_{o2}\sqrt{2gL_{20}}$$

Therefore:

$$V_{p0} = \frac{A_{o1}}{K_p}\sqrt{2gL_{10}}$$

$$L_{10} = \frac{A_{o2}^2}{A_{o1}^2}L_{20}$$

---

3. Linearized Model (about L_{10}, L_{20}, V_{p0})

Define small deviations:

$$l_1 = L_1 - L_{10}, \quad l_2 = L_2 - L_{20}, \quad v = V_p - V_{p0}$$

The linearized equations are:

$$\frac{dl_1}{dt} = -\frac{A_{o1}\sqrt{2g}}{2A_{t1}\sqrt{L_{10}}}l_1 + \frac{K_p}{A_{t1}}v$$

$$\frac{dl_2}{dt} = \frac{A_{o1}\sqrt{2g}}{2A_{t2}\sqrt{L_{10}}}l_1 - \frac{A_{o2}\sqrt{2g}}{2A_{t2}\sqrt{L_{20}}}l_2$$

4. State-Space Representation (Configuration 2)

Let the state vector $x = [l_1, l_2]^T$, input $u = v$, and output $y = Cx$.

$$\dot{x} = Ax + Bu, \quad y = Cx + Du$$

where:

$$A = \begin{bmatrix} -\frac{A_{o1}\sqrt{2g}}{2A_{t1}\sqrt{L_{10}}} & 0 \\ \frac{A_{o1}\sqrt{2g}}{2A_{t2}\sqrt{L_{10}}} & -\frac{A_{o2}\sqrt{2g}}{2A_{t2}\sqrt{L_{20}}} \end{bmatrix}$$

$$B = [\begin{matrix} \frac{K_p}{A_{t1}} \\ 0 \end{matrix}]$$

$$C = \begin{cases} [1 \ \ 0] & \text{if output is } L_1 \\ [0 \ \ 1] & \text{if output is } L_2 \end{cases}, \quad D = [0]$$

---

5. Transfer Functions

Configuration 1 (Pump → Tank 1 level):

$$G_1(s) = \frac{L_1(s)}{V_p(s)} = \frac{K_{dc1}}{\tau_1 s + 1}$$

$$\tau_1 = \frac{2A_{t1}\sqrt{L_{10}}}{A_{o1}\sqrt{2g}}, \quad K_{dc1} = \frac{2K_p\sqrt{L_{10}}}{A_{o1}\sqrt{2g}}$$

Configuration 2 (Tank 1 → Tank 2 level):

$$G_2(s) = \frac{L_2(s)}{L_1(s)} = \frac{K_{dc2}}{\tau_2 s + 1}$$

$$\tau_2 = \frac{2A_{t2}\sqrt{L_{20}}}{A_{o2}\sqrt{2g}}, \quad K_{dc2} = \frac{A_{o1}\sqrt{L_{20}}}{A_{o2}\sqrt{L_{10}}}$$

Overall system from pump to tank 2:

$$\frac{L_2(s)}{V_p(s)} = \frac{K_{dc1}}{\tau_1 s + 1} \times \frac{K_{dc2}}{\tau_2 s + 1}$$

---

6. Feedforward Control Terms

Tank 1 voltage feedforward for desired level L_{1r}:

$$V_{p,ff} = \frac{A_{o1}}{K_p}\sqrt{2gL_{1r}} = K_{ff1}\sqrt{L_{1r}}$$

$$K_{ff1} = \frac{A_{o1}\sqrt{2g}}{K_p}$$

Tank 2 feedforward reference for Tank 1:

$$L_{1,ff} = \frac{A_{o2}^2}{A_{o1}^2} L_{2r} = K_{ff2} L_{2r}$$

Feedback Controller - P, I and D

Continuous PID

$$MV(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^t E(t')dt' - T_d\,\frac{d\,CV(t)}{dt}\right] + I$$

Digital PID

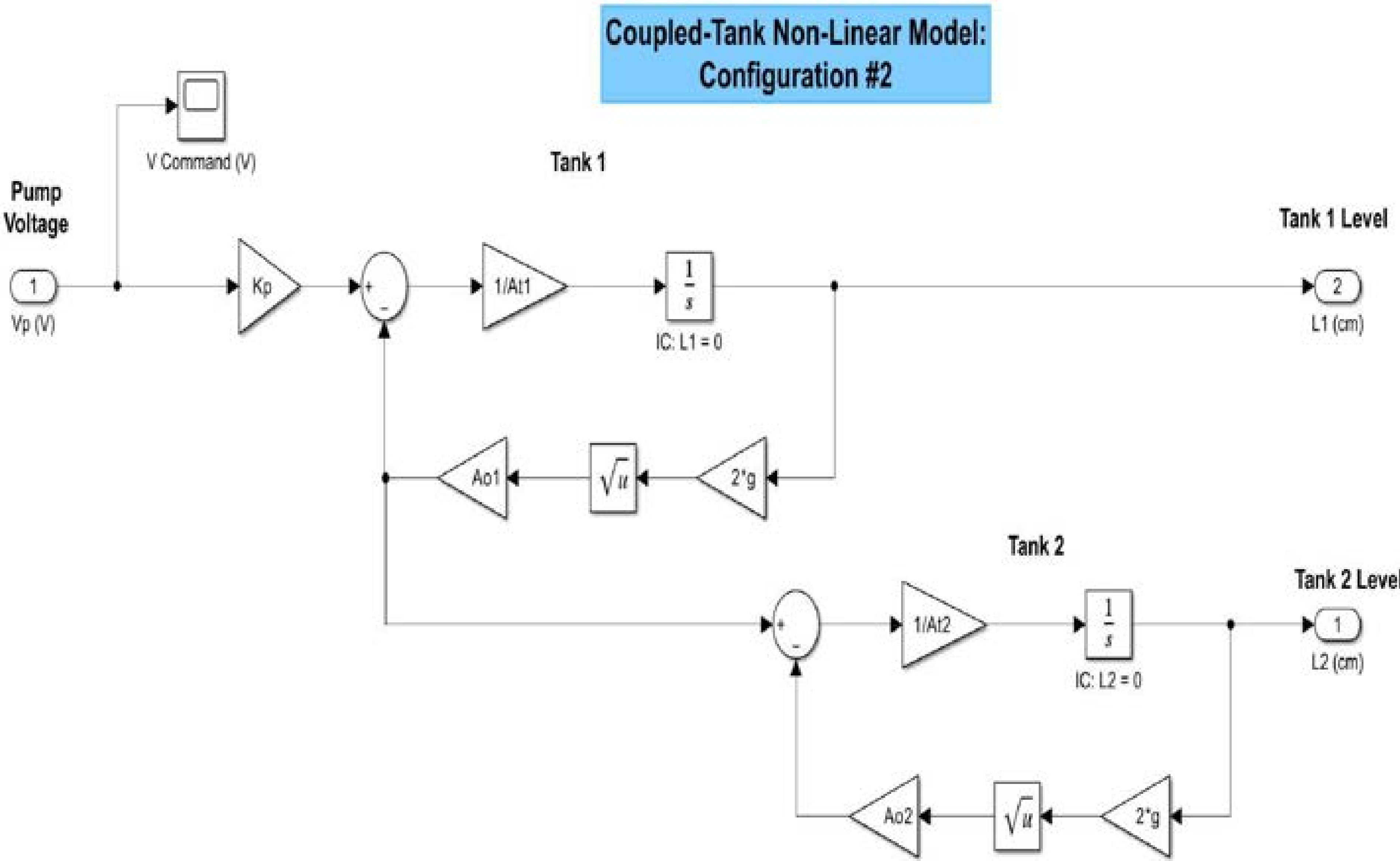
Positional form

$$MV_N = K_c\left[E_N + \frac{\Delta t}{T_I}\sum_{i=1}^{N} E_i - \frac{T_d}{\Delta t}(CV_N - CV_{N-1})\right] + I$$

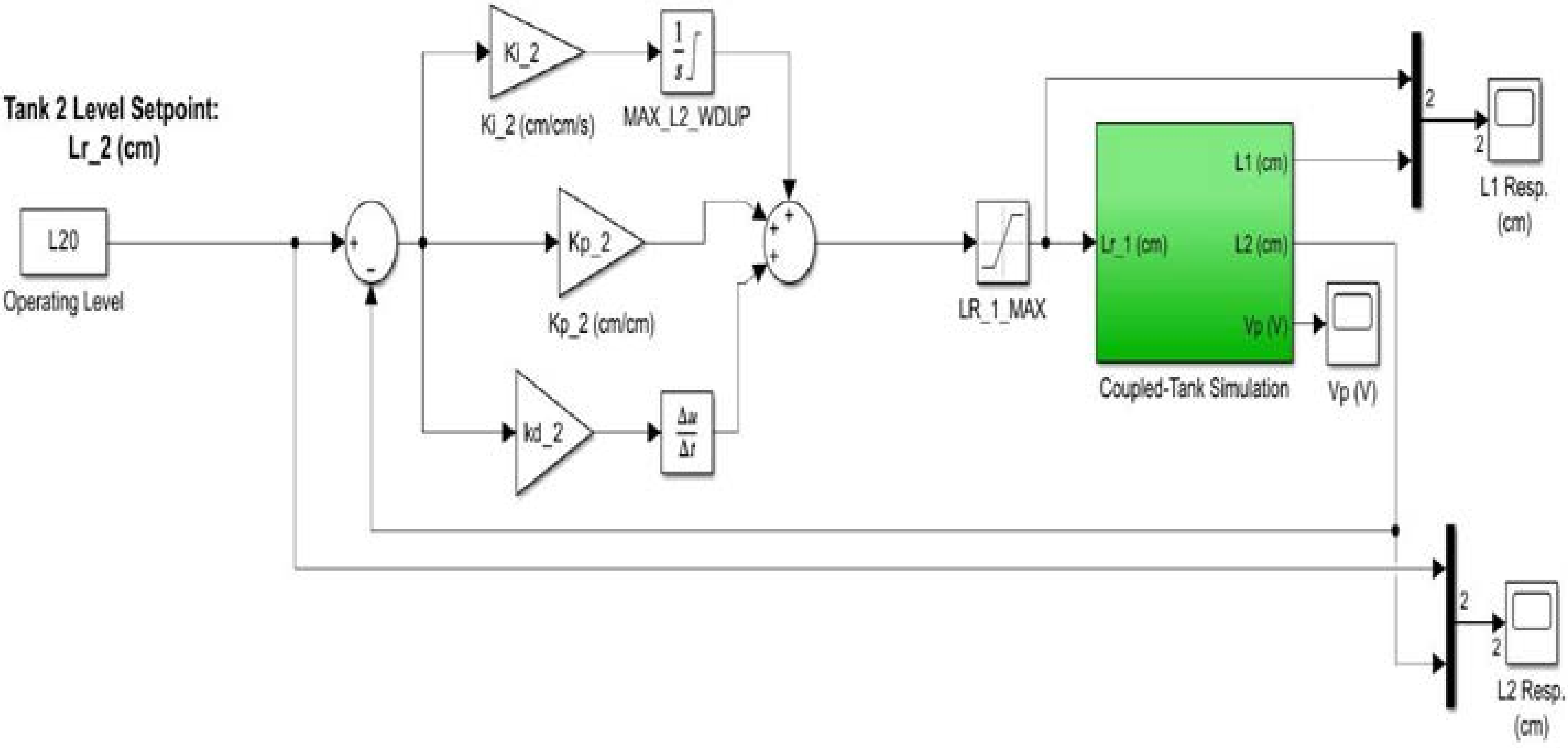
velocity form

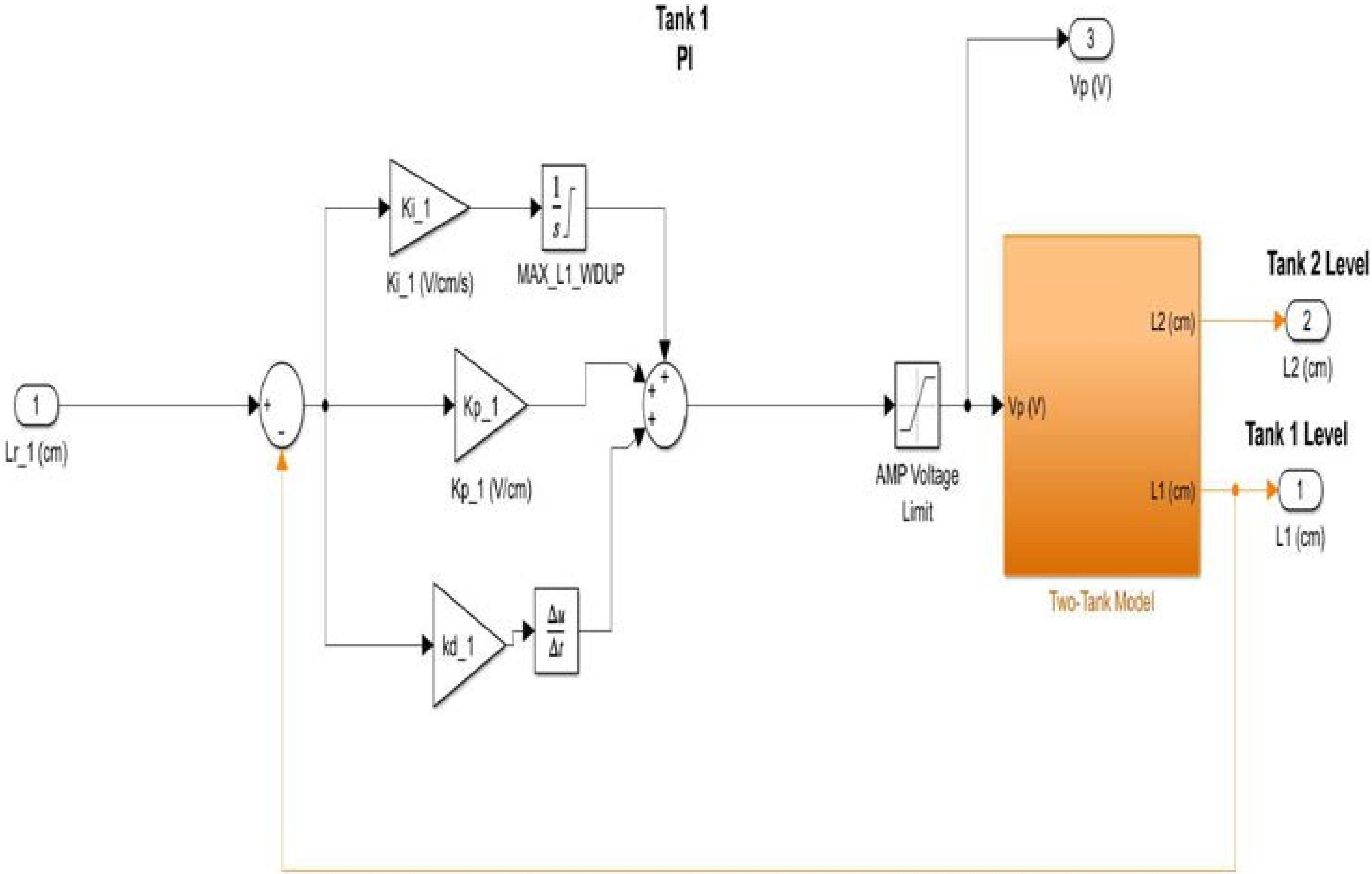
$$\Delta MV_N = K_c\left[E_N - E_{N-1} + \frac{(\Delta t)}{T_I}E_N - \frac{T_d}{\Delta t}(CV_N - 2CV_{N-1} + CV_{N-2})\right]$$

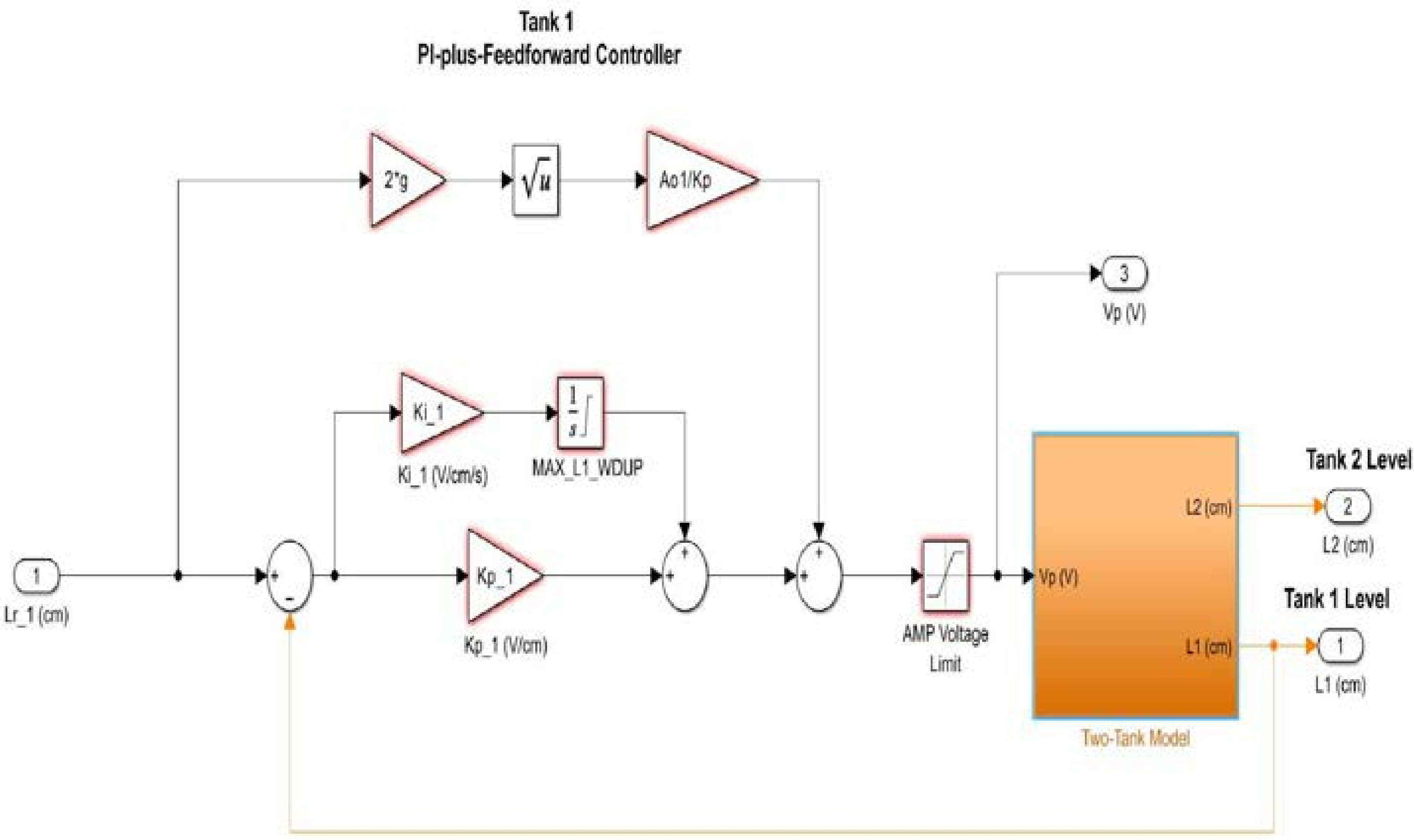
$$MV_N = MV_{N-1} + \Delta MV_N$$


Coupled-Tank Non-Linear Model:
Configuration #2
V Command (V)
Tank 1
Pump
Voltage
Tank 1 Level
1
Vp (V)
Kp
1/At1
1
s
IC: L1 = 0
2
L1 (cm)
Ao1
2*g
Tank 2
Tank 2 Level
1/At2
IC: L2 = 0
1
L2 (cm)
Ao2
2*g

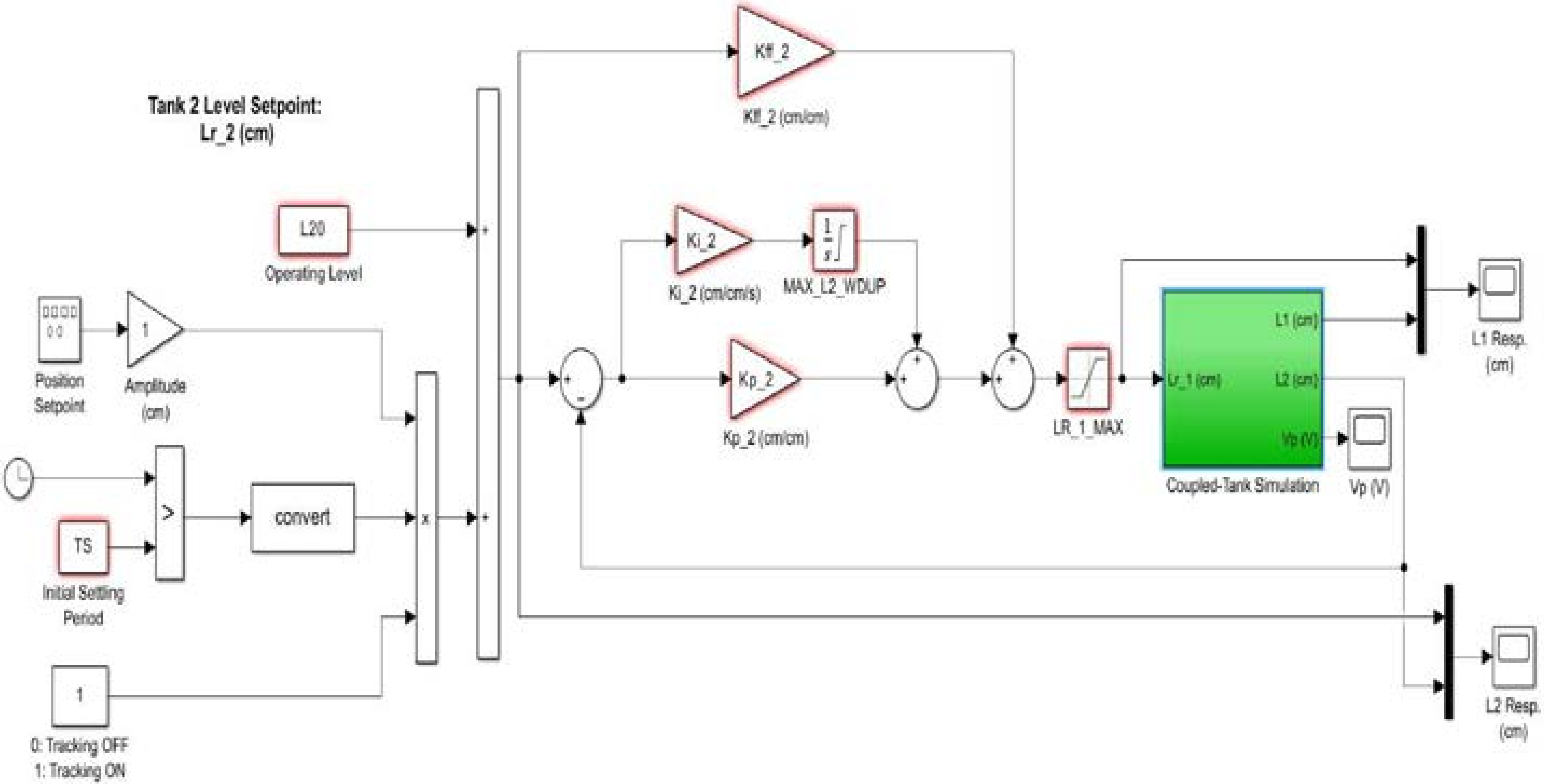
Coupled-Tank Experiment
- Configuration #2 -
Tank 2 Level Loop:
PI - Controllers

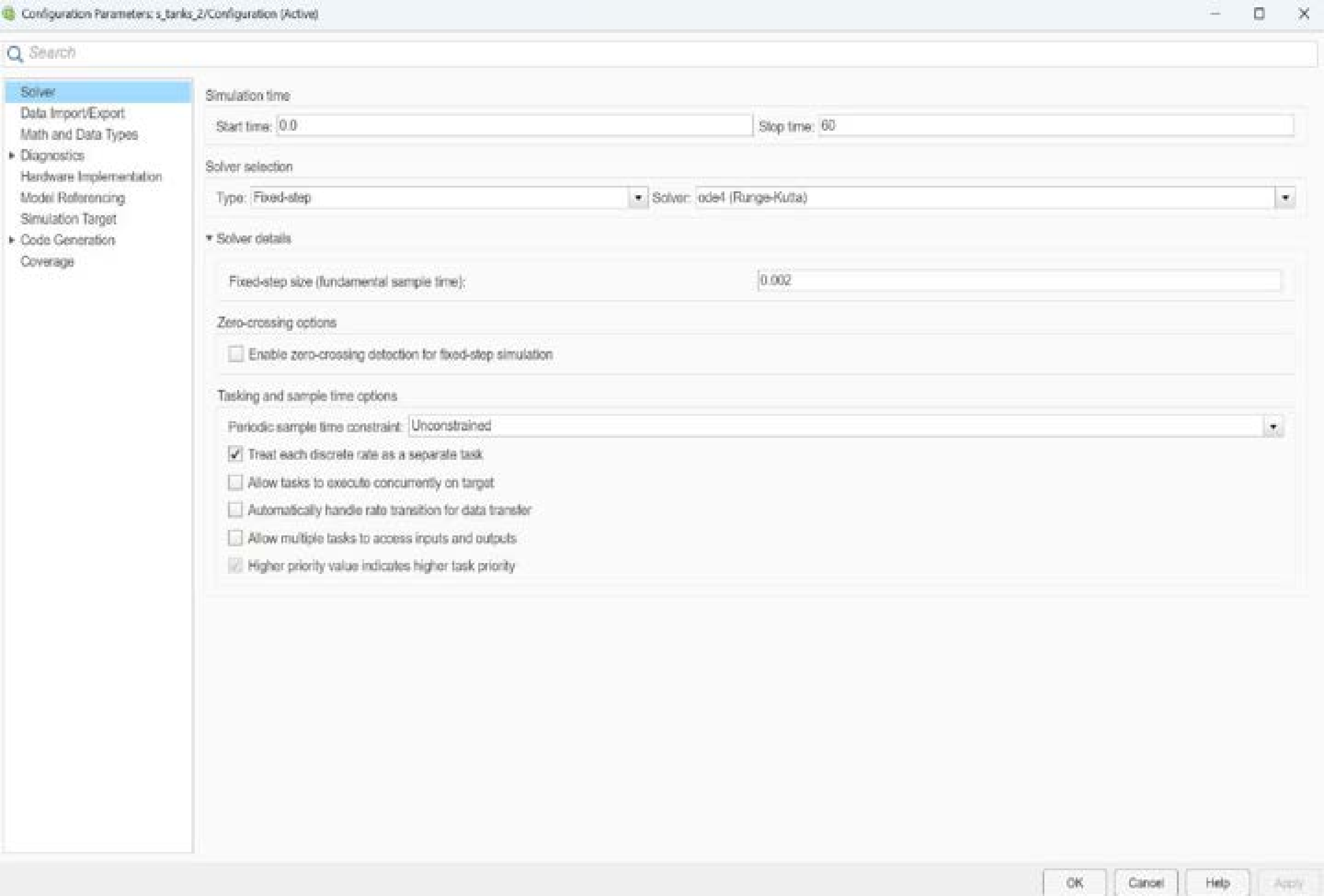



Tank 1
PI
3
Vp (V)
Ki_1
Ki_1 (V/cm/s)
MAX_L1_WDUP
Tank 2 Level
L2 (cm)
2
L2 (cm)
1
Lr_1 (cm)
Kp_1
Kp_1 (V/cm)
AMP Voltage
Limit
Vp (V)
Tank 1 Level
L1 (cm)
1
L1 (cm)
Two-Tank Model
kd_1


Tank 1
PI-plus-Feedforward Controller
2*g
Ao1/Kp
3
Vp (V)
Ki_1
Ki_1 (V/cm/s)
MAX_L1_WDUP
Tank 2 Level
L2 (cm)
2
L2 (cm)
1
Lr_1 (cm)
Kp_1
Kp_1 (V/cm)
AMP Voltage
Limit
Vp (V)
Tank 1 Level
L1 (cm)
1
L1 (cm)
Two-Tank Model

Coupled-Tank Experiment
- Configuration #2 -
Tank 2 Level Loop:
PI-plus-Feedforward Controllers




Configuration Parameters: s_tanks_2/Configuration (Active)
Search
Solver
Data Import/Export
Math and Data Types
Diagnostics
Hardware Implementation
Model Referencing
Simulation Target
Code Generation
Coverage
Simulation time
Start time: 0.0
Stop time: 60
Solver selection
Type: Fixed-step
Solver: ode4 (Runge-Kutta)
Solver details
Fixed-step size (fundamental sample time): 0.002
Zero-crossing options
Enable zero-crossing detection for fixed-step simulation
Tasking and sample time options
Periodic sample time constraint: Unconstrained
Treat each discrete rate as a separate task
Allow tasks to execute concurrently on target
Automatically handle rate transition for data transfer
Allow multiple tasks to access inputs and outputs
Higher priority value indicates higher task priority
OK
Cancel
Help
Apply

4.4 Results

B-6 Fill out Table 4.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Units
Pre Lab Questions			
Tank 1 Control Gains			
Proportional Control Gain	$k_{p,1}$	7.21	V/cm
Integral Control Gain	$k_{i,1}$	9.11	V/(cm-s)
Feed Forward Control Gain	$K_{ff,1}$	2.39	V/$\sqrt{cm}$
Tank 2 Control Gains			
Proportional Control Gain	$k_{p,2}$	5.09	cm/cm
Integral Control Gain	$k_{i,2}$	1.74	1/s
Feed Forward Control Gain	$K_{ff,2}$	1	cm/cm
Tank 2 Control Simulation			
Steady-state error	e_{ss}	0	cm
Settling time	t_s	1	s
Percent overshoot	PO	12.5	%
Tank 2 Control Implementation			
Steady-state error	e_{ss}	0.035	cm
Settling time	t_s	13.1	s
Percent overshoot	PO	18.3	%

Table 4.1: Tank 2 Level Control Results Results

```
clear all

close all

clc

%PID Controllers Parameter

Kp_1 = 1;

Ki_1 = 1;

kd_1=1;

Kp_2 = 1;

Ki_2 = 1;

kd_2=1;

% SETUP_TANKS_PARAMETERS

% Coupled Tanks' system nomenclature:

% Kpu Pump Flow Constant (cm^3/s/V)

% At1 Tank 1 Inside Cross-Section Area (cm^2)

% Ao1 Tank 1 Outlet Area (cm^2)

% At2 Tank 2 Inside Cross-Section Area (cm^2)

% Ao2 Tank 2 Outlet Area (cm^2)

% g Gravity Constant (cm/s^2)

% Dout1 "Out 1" Orifice Diameter (cm)

% Dout2 "Out 2" Orifice Diameter (cm)

Kp = 3.3;

Dout1 = 0.556; % = 1/4 in

Dout2 = 0.476; % = 3/16 in

Dt1 = 4.445; % = 1.75 inch

Dt2 = 4.445; % = 1.75 inch

At1 = pi * Dt1^2 / 4;

At2 = pi * Dt2^2 / 4;

Ao1 = pi * Dout1^2 / 4;

Ao2 = pi * Dout2^2 / 4;
```

```
% Gravitational Constant on Earth (cm/s^2)

g = 981;

K_AMP = 3;

% Digital-to-Analog Maximum Voltage (V); for MultiQ cards set to 10

VMAX_DAC = 10;

% Quiescent operating level (cm)

L20 = 15;

% Tank #1 level loop (V)

MAX_L1_WDUP = 3;

% Tank #2 level loop (cm)

MAX_L2_WDUP = 3;

% Safety Limit on tank #1 setpoint, Lr_1, (cm)

LR_1_MAX = 25;

% Control loop initial Settling Time in reaching L20 (s)

TS = 35;

% Turn on/off the safety watchdog on tank #1 level: set to 1, or 0

L1_WD_EN = 1; % enable: watchdog on

%L1_WD_EN = 0; % disable: watchdog off

% Set tank #1 level safety limits for watchdog (cm)

L1_MAX = 30;

% Turn on/off the safety watchdog on tank #2 level: set to 1, or 0

L2_WD_EN = 1; % enable: watchdog on

%L2_WD_EN = 0; % disable: watchdog off

% Set tank #2 level safety limits for watchdog (cm)

L2_MAX = 25;

% Amplifier Maximum Output Voltage (V) and Current (A)

VMAX_AMP = 22;

% Amplifier Maximum Output Voltage (V) and Current (A)

IMAX_AMP = 4;
```

1 PRESENTATION

1.1 Coupled-Tank: System Description

The typical Coupled-Tank plant is depicted in the Figure 1.1. The Coupled-Tank specialty module is a bench-top "Two-Tank" plant consisting of a pump with a water basin and two tanks of uniform cross sections. Such an apparatus forms an autonomous closed and re-circulating system. The two tanks, mounted on the front plate, are configured such that flow from the first (upper) tank can flow into the second (lower) tank. Flow from the second tank flows into the main water reservoir. In each one of the two tanks, liquid is withdrawn from the bottom through an outflow orifice (i.e. outlet). The outlet pressure is atmospheric. Both outlet inserts are configurable and can be set by changing inserts that screw into the tapped holes at the bottom of each tank. In order to introduce a disturbance flow, the first tank is also equipped with a drain tap so that, when opened, flow can be released directly into the water basin. The pump thrusts water vertically to two quick-connect orifices "Out1" and "Out2", which are **normally closed**. For configurability purpose, these two orifices, or inlets, have different diameters. Rubber tubing with appropriate couplings is supplied to enable the pump to feed water into one or both tanks. The selection of outputs from the pump controls the flow ratio between the two outlets "Out1" and "Out2". The water level in each tank is measured using a pressure-sensitive sensor located at the bottom of the tank. As detailed later in this manual, both offset and gain potentiometers of each pressure sensor are readily available for proper calibration.

Additionally, a vertical scale (in centimeters) is also placed beside each tank for visual feedback regarding each tank's water level.This single system can be configured into three main types of experiments, as listed in Table 1.1 below. Each of which can be configured with diverse parameter values (e.g. outlet diameters).

This single system can be configured into three main types of experiments, as listed in Table 1.1 below. Each of which can be configured with diverse parameter values (e.g. outlet diameters).

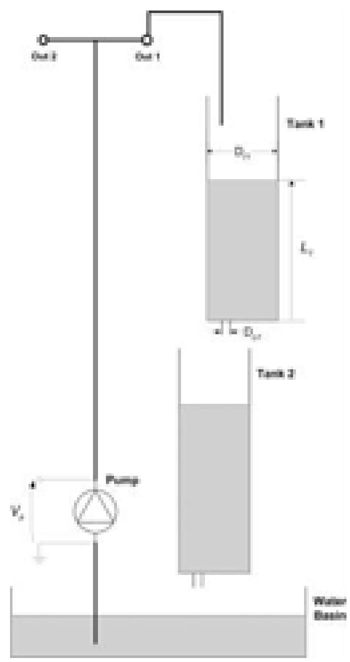
Figure 1.1: Coupled Tanks Plant

1.2 Coupled-Tank: Control Challenges

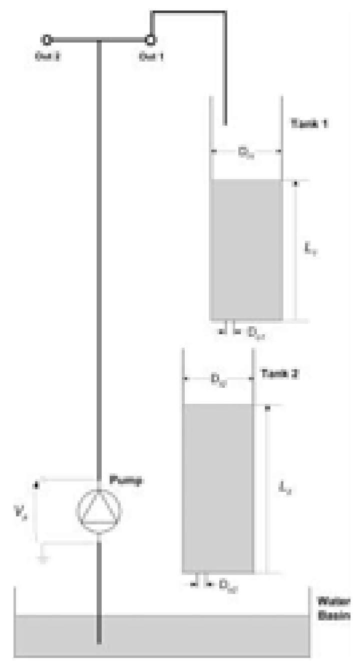
As illustrated in Figure 1.1, the purpose of the coupled-tank experiment is to design a control system that regulates the water level in a multiple coupled-tank system. The controller can then track the liquid level to a desired trajectory.

The system is supplied with different feedback-plus-feedforward controllers tuned through pole placement but, of course, you may design any other controller you wish. The complete mathematical modelling and system parameters are provided to streamline the implementation of the control theory of your choice. A single Coupled-Tank system can be configured into three types of experiments, as illustrated and described in Table 1.1, below. Each of the resulting control challenges can then be configured with various system parameters.

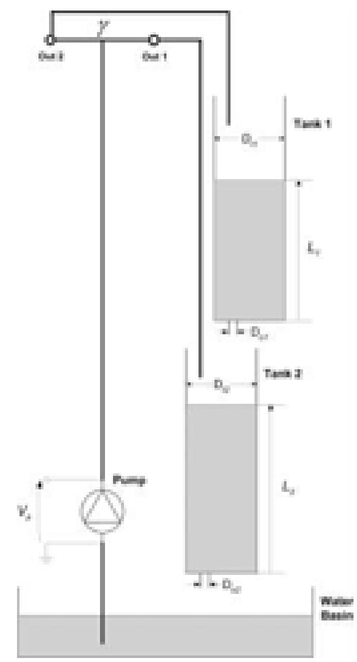
Configuration #1	**Configuration #2**	**Configuration #3**
Single Input Single Output (SISO) system.	State-coupled SISO system.	State-coupled and input-coupled SISO system.
The pump feeds into Tank 1. Tank 2 is not used at all.	The pump feeds into Tank 1, which in turn feeds into Tank 2.	The pump feeds into Tank 1 and into Tank 2 using a split flow. Tank 1 also feeds into Tank 2
A controller is designed to regulate or track the level in Tank 1.	A controller is designed to regulate or track the level in Tank 2.	A controller is designed to regulate or track the level in Tank 2.
Different inlet and outlet diameters in Tank 1 can be set up and tried.	Different inlet and outlet diameters in Tank 1 and Tank 2 can be set up and tried.	Different inlet and outlet diameters in Tank 1 and Tank 2 can be set up and tried.



(a) Configuration #1



(b) Configuration #2



(c) Configuration #3

Table 1.1: Coupled-Tank Water Level Control Configuration

Additionally, two Two-Tank plants can also be used simultaneously and coupled to obtain more complex Multi-Input-Multi-Output (MIMO) experiment. For example, Figure 1.2 below illustrates the quadruple-tank process described in the following publication: K. H. Johansson. The Quadruple-Tank Process: A Multivariable Laboratory Process with an Adjustable Zero. IEEE Transactions on Control Systems Technology, 8(3):456-465, 2000 ([**?**]) Appendix A describes how to setup two Coupled-Tank systems to be used in a quadruple-tank experiment. It can be shown that the four-interconnected-tank system has an adjustable zero, which can be moved along the real axis in the left- or right-hand-side of the s-plane. Therefore by changing the system parameters, the multivariable zero dynamics can be configured to be either minimum phase or non-minimum phase.

⚠ **Caution: This equipment is designed to be used for educational and research purposes and is not intended for use by the general public. The user is responsible to ensure that the equipment will be used by technically qualified personnel only.**

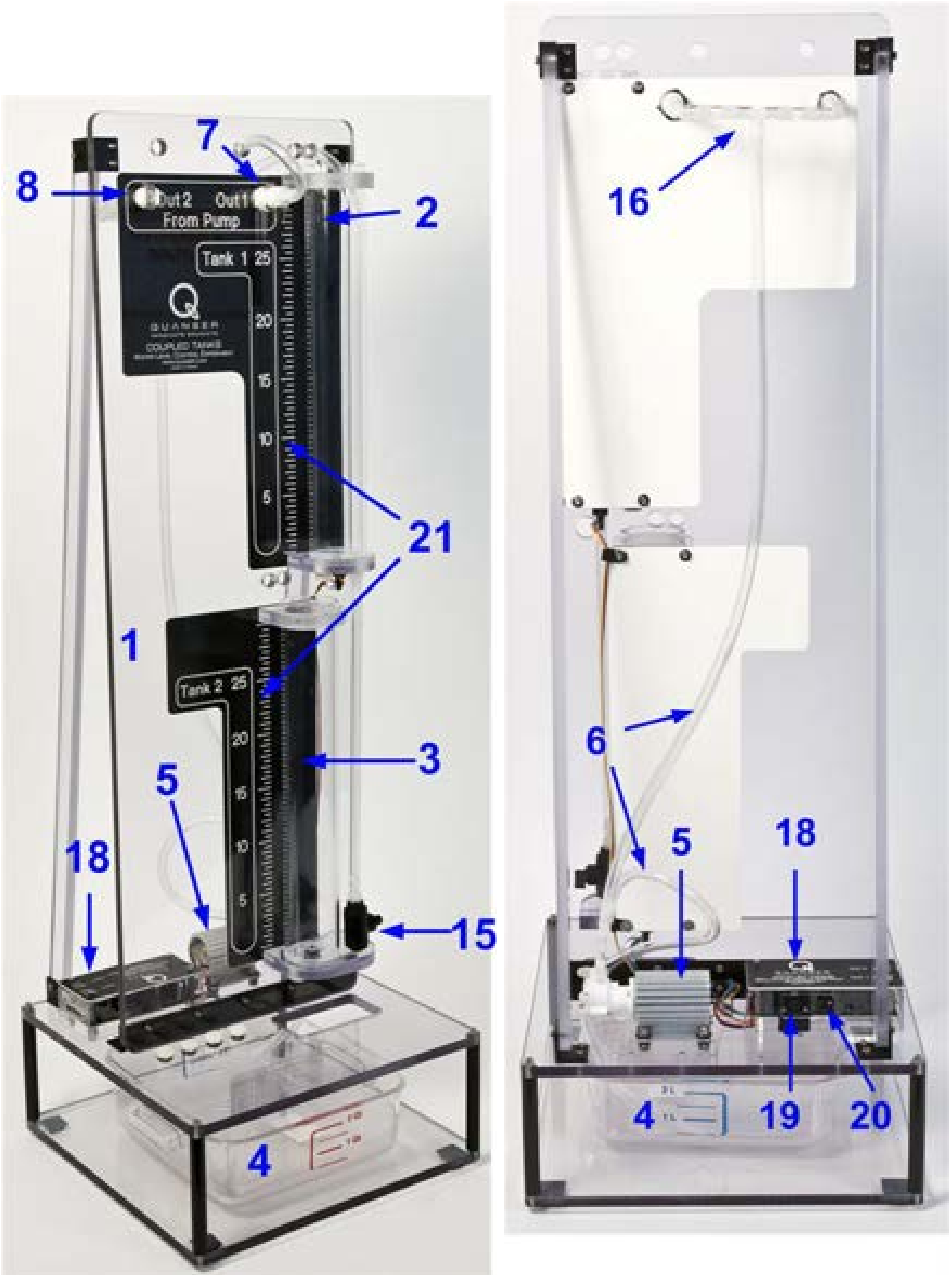
2 COUPLED-TANKS SYSTEM

2.1 Component Nomenclature

As a quick nomenclature, Table 2.1, below, provides a list of all the principal elements composing the Coupled-Tank Specialty system. Every element is located and identified, through a unique identification (ID) number, on the Coupled-Tank plant represented in Figure 2.1, Figure 2.2 and Figure 2.3, below.

ID	Component	ID	Component
1	Coupled-Tank Overall Frame	12	Medium Outlet Insert (9/16" Hexagonal Nut)
2	Tank 1	13	Large Outlet Insert (9/16" Hexagonal Nut)
3	Tank 2	14	Plain Outlet Insert (i.e. Plug)(9/16" Hexagonal Nut)
4	Main Water Basin (a.k.a. Reservoir)	15	Disturbance Tap
5	Pump	16	Flow Splitter
6	Flexible Tubing (in rubber)	17	Pressure Sensor
7	Quick-Connect Inlet Orifice "Out1"	18	Calibration And Signal Conditioning Circuit Board
8	Quick-Connect Inlet Orifice "Out2"	19	Pump Motor 4-Pin DIN Connector
9	Quick-Connect "Out1" Coupling And Hose	20	Pressure Sensors Cable 6-Pin-Mini-DIN Connector
10	Quick-Connect "Out2" Coupling And Hose	21	Tank Level Scale (in cm)
11	Small Outlet Insert (9/16" Hexagonal Nut)		

Table 2.1: Coupled-Tank Component Nomenclature



(a) Front

(b) Back

Figure 2.1: Front Back and Base of the Coupled Tank

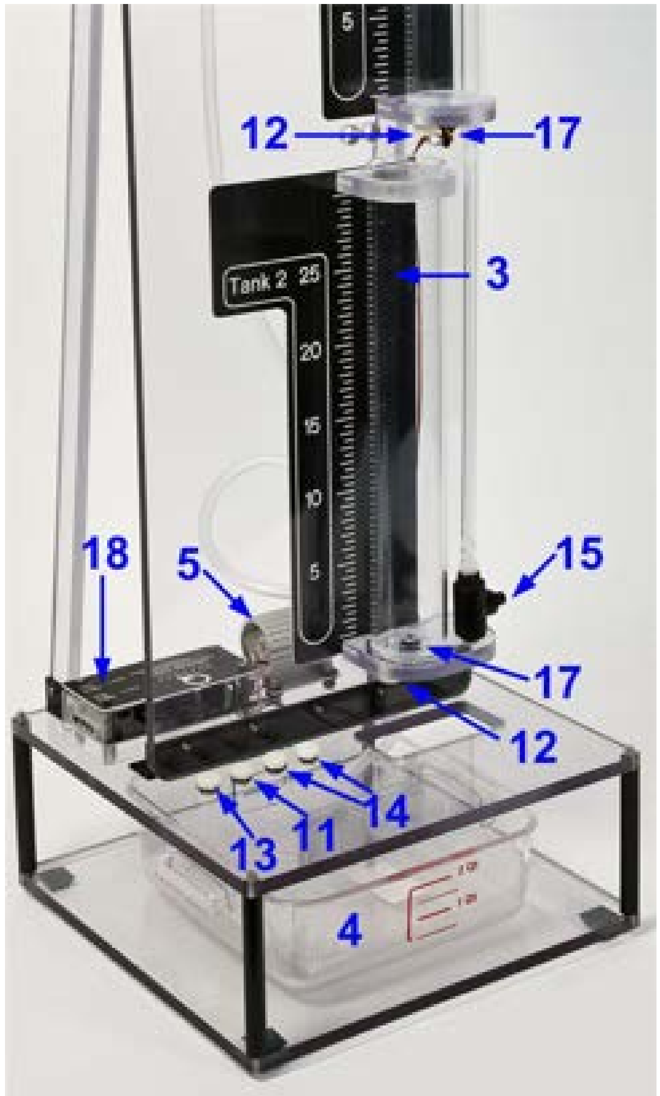
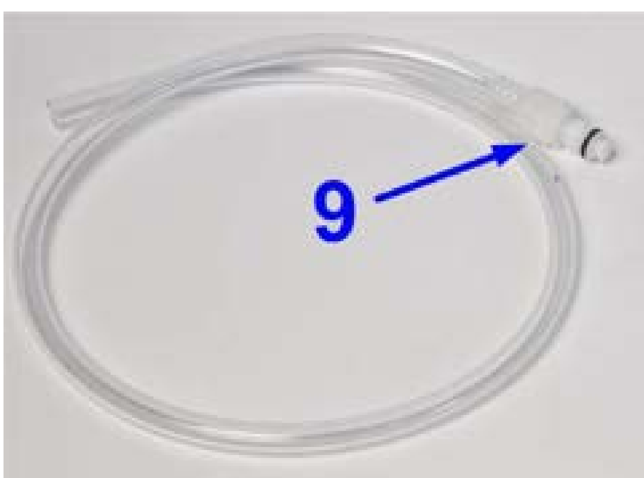
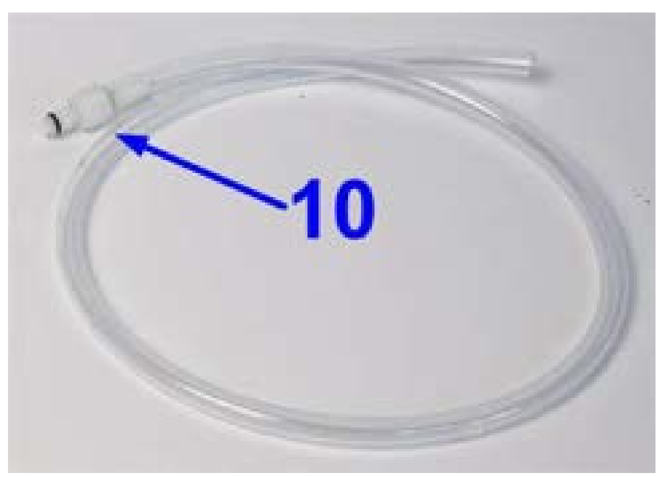


Figure 2.2: Base of the Coupled Tank



(a) Quick-Connect "Out1"



(b) Quick-Connect "Out2"

Figure 2.3: Quick Connect Couplings

2.2 Component Description

2.2.1 Overall Frame (Component #1)

The Coupled-Tank overall frame is made of Plexiglas. Its external dimensions are shown in Table 2.2, below.

Description	Value	Unit
Overall Frame Height	0.915	m
Overall Frame Width	0.305	m
Overall Frame Depth	0.305	m

Table 2.2: Coupled-Tank Frame Overall Dimensions

2.2.2 Tanks (Component #2 and #3)

The system's two water tanks are made out of Plexiglas tubes of uniform cross section.

2.2.3 Pump (Component #5)

The Coupled-Tank pump is a gear pump composed of a DC motor rated for 12 V continuous and 22 V peak with heat radiating fins. The materials that come into contact with the fluid being pumped are: two molded Delrin gears in a Delrin pump body, stainless steel shafting, a Teflon diaphragm and a Buna N seal. It is also equipped with 3/16" ID hose fittings.

⚠ **Caution:** Input +/- 24 V, 5 A peak, 3 A continuous.

2.2.4 Pressure Sensor (Component #17)

Each tank's actual liquid level is measured through a pressure sensor. Such a level sensor is located at the bottom of each tank and provides linear level readings over the complete liquid vertical level. In other words, the sensor output voltage increases proportionally to the applied pressure. Its output measurement is processed through a signal conditioning board (component #18) and made available as 0 to 5V DC signal. Its measurement sensitivity is given in Table 3.1, below. Moreover, as detailed in a following section, calibration of each pressure sensor's offset and gain potentiometers is required to keep level measurements consistent with the type of liquid used in the coupled-tank experiment.

⚠ **Caution:** Make sure the circuit board (component #18) does not get wet.

⚠ **Caution:** Depending on the duration of your experiment, the pump might get hot.

3 COUPLED-TANK MODEL PARAMETERS

Table 3.1, below, lists and characterizes the main parameters (e.g. mechanical and electrical specifications, convertion factors, constants) associated with the two-tank specialty plant. Some of these parameters can be used for mathematical modelling of the Coupled-Tank system as well as to obtain the water level's Equation Of Motion (EOM).

Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	cm^3/s/V
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

Table 3.1: Coupled-Tank System Model Paremeters

4 WIRING PROCEDURE FOR THE COUPLED-TANK SYSTEM

This section describes the standard wiring procedure for the Coupled-Tank specialty plant. The following hardware, accompanying the Coupled Tanks, is assumed:

1. **Power Amplifier**: Quanser VoltPAQ, or equivalent
2. **Data Acquisition Board**: Quanser Q2-USB, Quanser Q1-cRIO, Q8-USB, QPID, or equivalent.

4.1 Cable Nomenclature

Table 4.1, below, provides a description of the standard cables used in the wiring of the Coupled-Tank system.

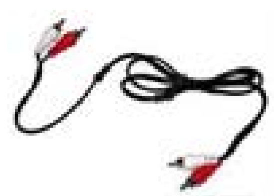
Cable	Type	Description
 (a) RCA Cable	2xRCA to 2xRCA	This cable connects an analog output of the Data Acquisition (DAQ) Device to the power module for proper power amplification.
 (b) "To Load" Cable	4-pin-DIN to 6-pin-DIN	This cable connects the output of the power module, after amplification, to the desired actuator (e.g. gear pump).
 (c) "From Analog Sensors" Cable	6-pin-mini-DIN to 6-pin-mini-DIN	This cable carries analog signals from one or two plant sensors (e.g. pressure sensors) to the amplifier, where the signals can be either monitored and/or used by an analog controller. The cable also carries a ±12VDC line from the amplifier in order to power a sensor and/or signal conditioning circuitry.
 (d) Analog-To-Digital" Cable	5-pin-DIN to 4xRCA	This cable carries the analog signals, previously taken from the plant sensors (e.g. pressure sensors), unchanged, from the amplifier to the Digital-To-Analog input channels on the Data Acquisition (DAQ) Device.

Table 4.1: Cable Nomenclature

4.2 Typical Connections

Table 4.2, below, sums up the electrical connections necessary to run the Coupled-Tank system.

Caution: If you are using the Quanser VoltPAQ, make sure the gain on the amplifier is set to 3!

Cable #	From	To	Signal
1	Analog Output **AO #0**	Amplifier **"Command"** connector	Control signal to the amplifier.
2	Amplifier **To Load** connector	Coupled-Tank's "Pump Connector" connector	Power leads to the gear pump.
3	Amplifier **"To ADC"**	Data Acquisition (DAQ) Device: 1. S1 (yellow) to Analog Input **AI #0** 2. S2 (white) to Analog Input **AI #1**	Tank 1 and tank 2 level feedback signals to the Data Acquisition (DAQ) Device, through the amplifier.
4	Coupled-Tank's "Pressure Sensors Connector" #1	Amplifier **"S1 & S2"**	Liquid level feedback signal to the amplifier.
5	Power Supply Outlet #1	Amplifier Power Socket	Amplifier Power Supply.

Table 4.2: Coupled-Tank Wiring Summary

Figure 4.1 shows the Data Acquisition Device, the back of the Coupled-Tank plant, and the amplifier, all connected with the necessary cabling to interface to and use the Coupled-Tank plant.

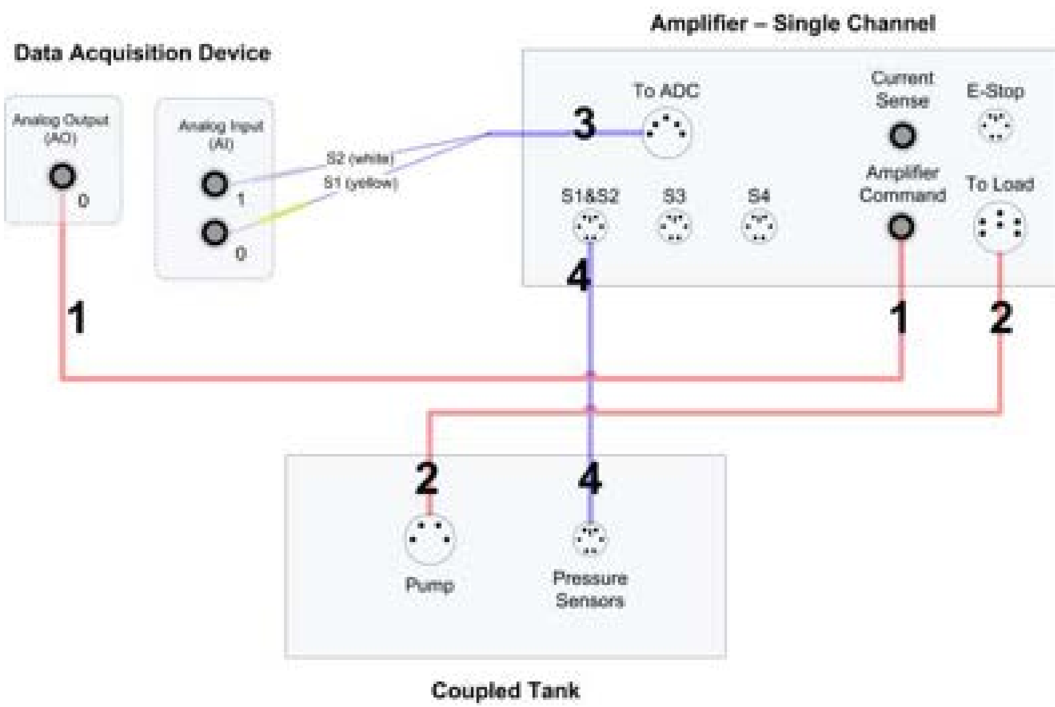


Figure 4.1: Coupled Tank Wiring Diagram

4.3 Wiring the Coupled Tanks

1. **Connect the "RCA Cable" #1**:
   Using the "RCA cable" cable described in Table 4.1, connect one end of this cable to the **Analog Output 0**(i.e. AO # 0) of your Data Acquisition (DAQ) Device and its other corresponding side to the socket labelled **"Command"** on the amplifier. These two connections are illustrated by cable #1 in Figure 4.1.

2. **Connect the "To Load" Cable #2**:
   The "To Load" cable is the 4-pin-DIN-to-6-pin-DIN cable described in Table 4.1. First, connect the cable 4-pin-DIN connector to the Coupled-Tank's **Pump** connector, which is shown as component #19 in Figure 2.1b and Figure 2.1a. Then connect the cable 6-pin-DIN connector to the amplifier socket labelled "To Load". The connection to the amplifier is illustrated by cable # 2 in Figure 4.1

3. **Connect the "Analog-To-Digital" Cable #3**:
   The "To Analog-To-Digital" cable is the 5-pin-DIN-to-4xRCA cable described in Table 4.1. First, connect the cable 5-pin-DIN connector to the amplifier socket labelled **"To ADC"**, as illustrated by cable #3 in Figure 4.1. The other end of the cable is split into four RCA connectors (yellow, white, red and black). This four RCA connectors correspond to the analog sensor signals passing through the amplifier, namely S1-yellow, S2-white, S3-red and S4-black. In order for the analog signals to be used in software, you should then connect the RCA connectors to the analog input channels of your Data Acquisition (DAQ) Device. Specifically, connect **S1 (yellow) to Analog Input 0 and S2 (white) to Analog Input 1**, S3 (red) and S4 (black) are not used in this experiment, but you can connect them to Analog Inputs 2 and 3 of your acquisition card terminal board, if it has that capability. See cable #3 in Figure 4.1.

4. **Connect the "From Analog Sensors" Cable #4**:
   The "From Analog Sensors" cable is the 6-pin-mini-DIN-to-6-pin-mini-DIN cable described in Table 4.1. First connect one end of the cable to the **Pressure Sensors Connector**, located at the back of the Coupled-Tank and which is shown as component #20 in Figure 2.1b. Then connect the cable's other end to the amplifier socket labelled **"S1 & S2"**, which is contained inside the amplifier "From Analog Sensors" front panel. These connections are illustrated by cable #4 in Figure 4.1.

In other words, the liquid level in tank 1 is sensed using A/D #0 through the amplifier analog channel S1, and the liquid level in tank 2 is sensed using A/D #1 through the amplifier analog channel S2.

Caution: If the equipment is used in a manner not specified by the manufacturer, the protection provided by the equipment may be impaired.

5 CONFIGURING THE COUPLED-TANK SYSTEM

5.1 Main Water Basin

1. Fill the Coupled-Tank water basin up to 3/4 of its height.
2. Insert the basin inside the bottom of the Coupled-Tank frame, as illustrated in Figure 2.2
3. Ensure that the pump inflow flexible tube is located inside the water basin.
4. Setup the configuration 1, 2, or 3. Table 5.1 details the inlet and outlet sizes for the three standard configurations.

Note: It is recommended to use distilled water (i.e. without mineral salts) to fill up the main basin. This is to avoid stains on the system's Plexiglas tubes and structure as the water dries out.

5.2 Flexible Tubing and Outlet Typical Setup

As previously mentioned, a single Coupled-Tank system can be configured into three different types of experiments, corresponding to the system's configurations #1, #2, and #3 as illustrated and described in Table 1.1, above. Each configuration results in a distinct control challenge and can also be modified by using different values for the system parameters. However, the default water level controllers supplied with the Coupled-Tank plant have been designed for the standard three system's configurations described hereafter.

A system configuration is defined in terms of each tank inflow and outflow characteristics, as well as the desired control variable. Table 5.1, below, details the Coupled-Tank setup for the three standard configurations.

	Configuration #1	Configuration #2	Configuration #3
Tank 1 Inflow	From "Out 1".	From "Out 1"	From "Out 1"
Tank 1 Outlet Insert Size	**Medium**	**Medium**.	**Small**
Tank 2 Inflow	From Tank 1 Outflow.	From Tank 1 Outflow.	From Tank 1 Outflow and From "Out 1"
Tank 2 Outlet Insert Size	**Medium**.	**Medium**.	**Large** .
Control Variable	Tank 1 Level	Tank 2 Level	Tank 2 Level

Table 5.1: Coupled-Tank Default Setup For Configurations #1, #2, and #3

In other words, Table 5.1, above, indicates the appropriate exit orifice and the appropriate feed from the pump to use for the three configurations corresponding to different experiments.

For all three configurations make sure:

1. The disturbance tap, directly connecting tank 1 to the main water basin, is closed.
2. The drain tap is identified by component #15 in Figure 2.1a and Figure 2.2, above. For the tap to be closed, its flap should be horizontal.
3. **Configuration 1 and 2** Figure 5.1a, illustrates the Coupled-Tank system setup in configurations #1 and #2. Note that the quick-connect "Out 1" coupling and flexible hose, depicted by component #9, is used to transport the water from the inlet orifice "Out 1" to Tank 1.

(a) Coupled-Tank Configurations #1 and #2

(b) Coupled-Tank Configurations #3

Figure 5.1: Configurations 1, 2 and 3

4. **Configuration 3** Figure 5.1b, illustrates the Coupled-Tank system setup in configuration #3. This time note that the quick-connect "Out 1" coupling and flexible hose, depicted by component #9, is used to transport the water from the inlet orifice "Out 1" to Tank 2. Similarly, the quick-connect "Out 2" coupling and flexible hose, depicted by component #10, is used to transport the water from the inlet orifice "Out 2" to Tank 1.

Note: When putting the output tube from the pump into the water tank, ensure that the water discharge to the water column occurs at atmospheric pressure. In other words, the hose tip should stand above the tank water level (e.g. above the 25-cm mark).

6 WATER LEVEL SENSOR CALIBRATION

The pressure-proportional water level voltage should be zero when the tank is empty, while it should be between 4.0 Volts and 4.2 Volts when the tank water level is at 25 centimeters (as seen on the tank scale).

Note: The pressure-sensitive water level sensor is calibrated at the factory but may need re-adjustment when you receive it, or under different water characteristics (depending on the kind of liquid used).

6.1 Calibration Circuit Board Nomenclature

To calibrate both pressure sensors signals, the bottom part of the Coupled-Tank apparatus houses a signal conditioning circuit board, depicted by component #18 in Figure 2.1, above. As a quick nomenclature, Table 6.1, below, provides a list of the different signal conditioning potentiometers to be tuned during sensors calibration. Additionally, every potentiometer is located and identified, through a unique identification (ID) number, on the circuit board close-up represented in Figure 6.1, below.

ID	Component
23	Tank 1 Sensor Offset Potentiometer
24	Tank 1 Sensor Gain Potentiometer
25	Tank 2 Sensor Offset Potentiometer
26	Tank 2 Sensor Gain Potentiometer

Table 6.1: Coupled-Tank Component Nomenclature

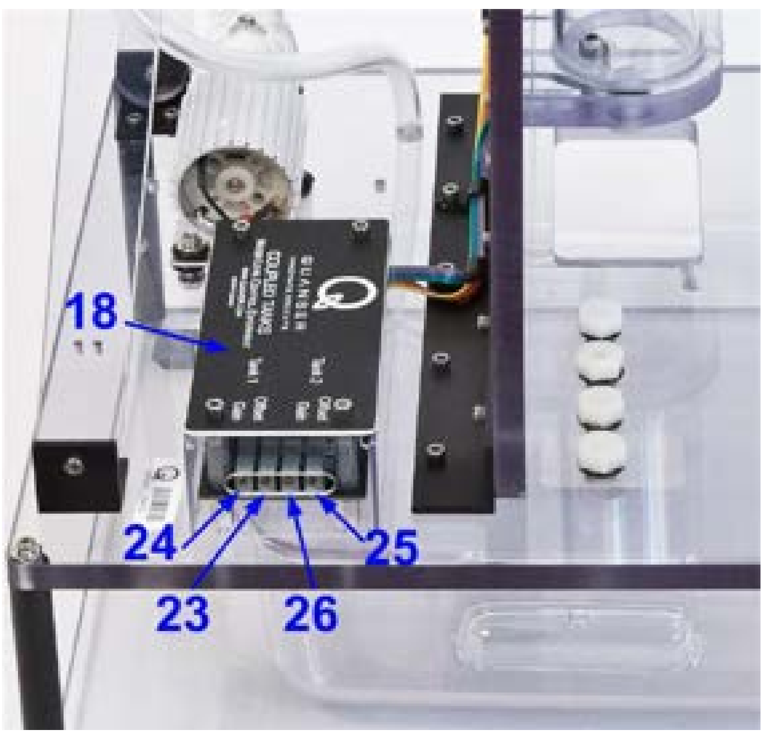


Figure 6.1: Coupled-Tank Calibration Potentiometer

6.2 Calibration Procedure

The calibration procedure detailed in the following subsections is to calibrate the circuit board's four potentiometers, which are namely "Offset 1", "Gain 1", "Offset 2", and "Gain 2", as depicted in Figure 6.1, above, as components #23, #24, #25, and #26, respectively.

In order to successfully run the calibration procedure:

1. Ensure that the Coupled-Tank system is wired as previously described.
2. Setup the Coupled-Tank apparatus to configuration #2 with the appropriate exit orifices and the appropriate feed from the pump, as previously summarized in Table 5.1 and illustrated in Figure 5.1a, above.
3. Power up the amplifier, you are now ready to proceed.

Note: Make sure the flexible tube from "Out 1" is inserted into tank 1. Do not connect a tube to "Out 2".

6.2.1 Zero "Offset" Potentiometer Calibration: With No Water

To calibrate the offset to zero to 0V for both tank pressure sensor readings do the following:

1. Ensure the Pump is **OFF** and tank is empty.
2. Run the supplied calibration software files keeping the Pump **OFF**.
3. Measure tank #1 voltage on Analog Input Channel #0. For tank #2, use Analog Input Channel #1.
4. Adjust the pots using a potentiometer adjustment tool (i.e. a small flat-end screwdriver), manually adjust tank 1 **Offset** potentiometer screw (i.e. components #23 and #25), if necessary, in order to obtain 0.0 Volts for both readings. Turning the **Offset** potentiometer screw clockwise increases the voltage and vice-versa.
5. This Voltage can be monitored in the display found in the calibration software.

When both zero-Volt offsets are achieved, you can move on the next section to calibrate the gain potentiometers.

6.2.2 "Gain" Potentiometer Calibration: At a 25-cm Water Level

The calibration of each one of the two gain potentiometers is performed with the corresponding tank containing water up to the 25-centimeter scale mark.

1. Fill up the water tank to the 25 cm mark. Plug the tank 1 outlet with your finger and apply a pump voltage using the supplied calibration software files.
2. When the 25-centimeter level mark is attained, stop the pump feed.
3. Measure tank #1 voltage on Analog Input Channel #0. For tank #2, use Analog Input Channel #1.
4. Using a potentiometer adjustment tool (i.e. a small flat-end screwdriver), manually adjust tank 1 **gain** potentiometer screw (i.e. component #24 in Figure 6.1) to obtain anywhere **between 4.0 and 4.2 Volts**. Turning the **gain** potentiometer screw clockwise increases the voltage and vice-versa.
5. This Voltage can be monitored in the display found in the calibration software.

Repeat procedure to tank 2.

1 INTRODUCTION

The Coupled Tanks plant is a "Two-Tank" module consisting of a pump with a water basin and two tanks. The two tanks are mounted on the front plate such that flow from the first (i.e. upper) tank can flow, through an outlet orifice located at the bottom of the tank, into the second (i.e. lower) tank. Flow from the second tank flows into the main water reservoir. The pump thrusts water vertically to two quick-connect orifices "Out1" and "Out2". The two system variables are directly measured on the Coupled-Tank rig by pressure sensors and available for feedback. They are namely the water levels in tanks 1 and 2. A more detailed description is provided in [5]. To name a few, industrial applications of such Coupled-Tank configurations can be found in the processing system of petro-chemical, paper making, and/or water treatment plants.

During the course of this experiment, you will become familiar with the design and pole placement tuning of Proportional-plus-Integral-plus-Feedforward-based water level controllers. In the present laboratory, the Coupled-Tank system is used in two different configurations, namely configuration #1 and configuration #2, as described in [5]. In configuration #1, the objective is to control the water level in the top tank, i.e., tank #1, using the outflow from the pump. In configuration #2, the challenge is to control the water level in the bottom tank, i.e. tanks #2, from the water flow coming out of the top tank. Configuration #2 is an example of state coupled system.

Topics Covered

- How to mathematically model the Coupled-Tank plant from first principles in order to obtain the two open-loop transfer functions characterizing the system, in the Laplace domain.
- How to linearize the obtained non-linear equation of motion about the quiescent point of operation.
- How to design, through pole placement, a Proportional-plus-Integral-plus-Feedforward-based controller for the Coupled-Tank system in order for it to meet the required design specifications for each configuration.
- How to implement each configuration controller(s) and evaluate its/their actual performance.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

1. See the system requirements in Section 5 for the required hardware and software.
2. Transfer function fundamentals, e.g., obtaining a transfer function from a differential equation.
3. Familiar with designing PID controllers.
4. Basics of Simulink®.
5. Basics of QUARC®.

2 MODELING

2.1 Background

2.1.1 Configuration # 1 System Schematics

A schematic of the Coupled-Tank plant is represented in Figure 2.1, below. The Coupled-Tank system's nomenclature is provided in Appendix A. As illustrated in Figure 2.1, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank), as represented in Figure 3.2.

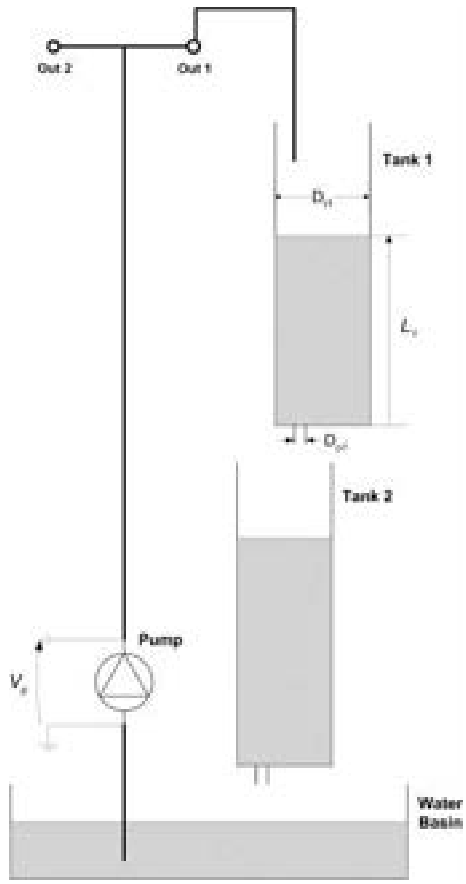


Figure 2.1: Schematic of Coupled Tank in Configuration #1.

2.1.2 Configuration # 1 Nonlinear Equation of Motion (EOM)

In order to derive the mathematical model of your Coupled-Tank system in configuration #1, it is reminded that the pump feeds into Tank 1 and that tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump V_P and its output is the water level in tank 1, L_1, (i.e. top tank).

The purpose of the present modelling session is to provide you with the system's open-loop transfer function, $G1(s)$, which in turn will be used to design an appropriate level controller. The obtained Equation of Motion, EOM, should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_1}{\partial t} = f(L_1, V_p)$$

where f denotes a function.

In deriving the Tank 1 EOM the mass balance principle can be applied to the water level in tank 1, i.e.,

$$A_{t1}\frac{\partial L_1}{\partial t} = F_{i1} - F_{o1} \tag{2.1}$$

where A_{t1} is the area of Tank 1. F_{i1} and F_{o1} are the inflow rate and outflow rate, respectively. The volumetric inflow rate to tank 1 is assumed to be directly proportional to the applied pump voltage, such that:

$$F_{i1} = K_p V_p$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, v_{o1}, can be expressed by the following relationship:

$$v_{o1} = \sqrt{2gL_1}$$

2.1.3 Configuration # 1 EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 1 system, the open-loop Laplace transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear EOM of tank 1 should be linearized around a quiescent point of operation. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{10}) is characterized by the Tank 1 level being at a constant position L_{10} due to a constant water flow generated by constant pump voltage V_{p0}.

In the case of the water level in tank 1, the operating range corresponds to small departure heights, L_{11}, and small departure voltages, V_{p1}, from the desired equilibrium point (V_{p0}, L_{10}). Therefore, L_1 and V_p can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11}, \qquad V_p = V_{p0} + V_{p1} \tag{2.2}$$

The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (V_{p0}, L_{10}). Therefore, one should express the resulting linear EOM under the following format:

$$\frac{\partial}{\partial t}L_{11} = f(L_{11}, V_{p1}) \tag{2.3}$$

where f denotes a function.

Example: Linearizing a Two-Variable Function
Here is an example of how to linearize a two-variable nonlinear function called $f(z)$. Variable z is defined

$$z^{\top} = [z_1\ z_2]$$

and $f(z)$ is to be linearized about the operating point

$$z_0{}^{\top} = [a\ b]$$

The linearized function is

$$f_z = f(z_0) + \left(\frac{\partial f(z)}{\partial z_1}\right)\Bigg|_{z=z_0}(z_1 - a) + \left(\frac{\partial f(z)}{\partial z_2}\right)\Bigg|_{z=z_0}(z_2 - b)$$

For a function, f, of two variables, L_1 and V_p, a first-order approximation for small variations at a point $(L_1, V_p) = (L_{10}, V_{p0})$ is given by the following Taylor's series approximation:

$$\frac{\partial^2}{\partial L_1 \partial V_p} f(L_1, V_p) \cong f(L_{10}, V_{p0}) + \left(\frac{\partial}{\partial L_1} f(L_{10}, V_{p0})\right) + (L_1 - L_{10})\left(\frac{\partial}{\partial V_p} f(L_{10}, V_{p0})\right)(V_p - V_{p0}) \tag{2.4}$$

Transfer Function
From the linear equation of motion, the system's open-loop transfer function in the Laplace domain can be defined by the following relationship:

$$G_1(s) = \frac{L_{11}(s)}{V_{p1}(s)} \tag{2.5}$$

The desired open-loop transfer function for the Coupled-Tank's tank 1 system is the following:

$$G_1(s) = \frac{K_{dc1}}{\tau_1 s + 1} \tag{2.6}$$

where K_{dc_1} is the open-loop transfer function DC gain, and τ_1 is the time constant.

As a remark, it is obvious that linearized models, such as the Coupled-Tank tank 1's voltage-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution, that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation 2.5 is assumed valid over the pump voltage and tank 1 water level entire operating range, V_{p_peak} and L_{1_max}, respectively.

2.1.4 Configuration #2 System Schematics

A schematic of the Coupled-Tank plant is represented in Figure 2.2, below. The Coupled-Tank system's nomenclature is provided in Appendix A. As illustrated in Figure 2.2, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank), as represented in Figure 2.2.

2.1.5 Configuration #2, Nonlinear Equation of Motion (EOM)

This section explains the mathematical model of your Coupled-Tank system in configuration #2, as described in Reference [1]. It is reminded that in configuration #2, the pump feeds into tank 1, which in turn feeds into tank 2. As far as tank 1 is concerned, the same equations as the ones explained in Section 2.1.2 and Section 2.1.3 will apply. However, the water level Equation Of Motion (EOM) in tank 2 still needs to be derived. The input to the tank 2 process is the water level, L_1, in tank 1 (generating the outflow feeding tank 2) and its output variable is the water level, L_2, in tank 2 (i.e. bottom tank). The purpose of the present modelling session is to guide you with the system's open-loop transfer function, $G_2(s)$, which in turn will be used to design an appropriate level controller. The obtained EOM should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_2}{\partial t} = f(L_2, L_1)$$

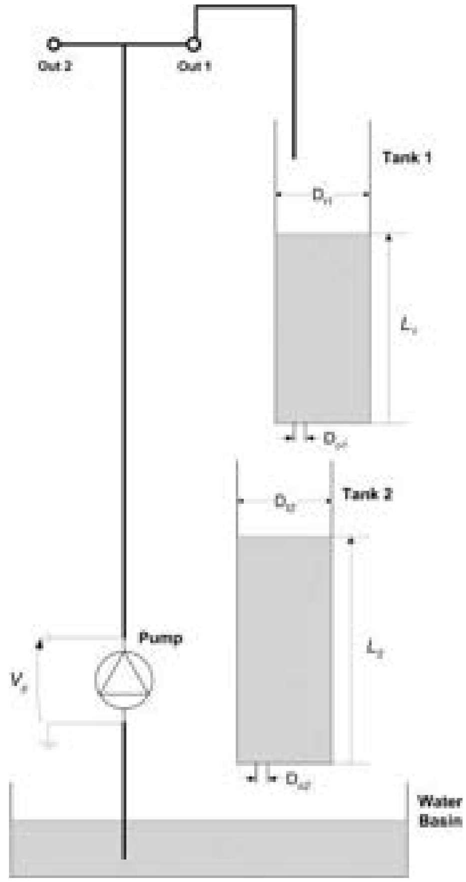


Figure 2.2: Schematic of Coupled Tank in configuration #1.

where f denotes a function.

In deriving the tank #2 EOM the mass balance principle can be applied to the water level in tank 2 as follows

$$A_{t2}\frac{\partial L_2}{\partial t} = F_{i2} - F_{o2}$$

where A_{t2} is the area of tank 2. F_{i2} and F_{o2} are the inflow rate and outflow rate, respectively.

The volumetric inflow rate to tank 2 is equal to the volumetric outflow rate from tank 1, that is to say:

$$F_{i2} = F_{o1}$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 2, v_{o2}, can be expressed by the following relationship:

$$v_{o2} = \sqrt{2gL_2}$$

2.1.6 Configuration #2 EOM Linearization and Transfer Function

In order to design and implement a linear level controller for the tank 2 system, the Laplace open-loop transfer function should be derived. However by definition, such a transfer function can only represent the system's dynamics from a linear differential equation. Therefore, the nonlinear EOM of tank 2 should be linearized around a quiescent point of operation.

In the case of the water level in tank 2, the operating range corresponds to small departure heights, L_{11} and L_{21}, from the desired equilibrium point (L_{10}, L_{20}). Therefore, L_2 and L_1 can be expressed as the sum of two quantities, as shown below:

$$L_2 = L_{20} + L_{21}, \qquad L_1 = L_{10} + L_{11} \tag{2.7}$$

The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (L_{20}, L_{10}). Therefore, you should express the resulting linear EOM under the following format:

$$\frac{\partial}{\partial t} L_{21} = f(L_{11}, L_{21}) \tag{2.8}$$

where f denotes a function.

For a function, f, of two variables, L_1 and L_2, a first-order approximation for small variations at a point $(L_1, L_2) = (L_{10}, L_{20})$ is given by the following Taylor's series approximation:

$$\frac{\partial^2}{\partial L_1 \partial L_2} f(L_1, L_2) \cong f(L_{10}, L_{20}) + \left(\frac{\partial}{\partial L_1} f(L_{10}, L_{20})\right) + (L_1 - L_{10}) \left(\frac{\partial}{\partial L_2} f(L_{10}, L_{20})\right) (L_2 - L_{20}) \tag{2.9}$$

Transfer Function
From the linear equation of motion, the system's open-loop transfer function in the Laplace domain can be defined by the following relationship:

$$G_2(s) = \frac{L_{21}(s)}{L_{11}(s)} \tag{2.10}$$

the desired open-loop transfer function for the Coupled-Tank's tank 2 system, such that:

$$G_2(s) = \frac{K_{dc2}}{\tau_2 s + 1} \tag{2.11}$$

where K_{dc_2} is the open-loop transfer function DC gain, and τ_2 is the time constant.

As a remark, it is obvious that linearized models, such as the Coupled-Tank's tank 2 level-to-level transfer function, are only approximate models. Therefore, they should be treated as such and used with appropriate caution, that is to say within the valid operating range and/or conditions. However for the scope of this lab, Equation 2.10 is assumed valid over tank 1 and tank 2 water level entire range of motion, L_{1_max} and L_{2_max}, respectively.

2.2 Pre-Lab Questions

Answer the following questions:

1. A-1, A-2, A-3 Using the notations and conventions described in Figure 2 derive the Equation Of Motion (EOM) characterizing the dynamics of tank 1. Is the tank 1 system's EOM linear?

   Hint: The outflow rate from tank 1, F_{o1}, can be expressed by:

   $$F_{o1} = A_{o1} v_{o1} \tag{2.12}$$

   Answer 2.1

Outcome	Solution
A-1	As a remark, the cross-section area of tank 1 outlet hole can be calculated by: $$A_{o1} = \frac{1}{4}\pi D_{o1}^2 \quad \text{(Ans.2.1)}$$ Using Equation Ans.2.1, the outflow rate from tank 1 given in Equation 2.12 becomes: $$F_{o1} = A_{o1}\sqrt{2gL_1} \quad \text{(Ans.2.2)}$$
A-2	Moreover, using the mass balance principle for tank 1, we obtained a first-order differential equation for L_1 in Equation 2.1. Substituting in Equation 2.1 F_{i1} and F_{o1} with their expressions given in Equation Ans.2.1 and Equation Ans.2.2, respectively, and rearranging results in the following equation of motion for the tank 1 system: $$\frac{\partial L_1}{\partial t} = \frac{K_p V_p - A_{o1}\sqrt{2}\sqrt{g L_1}}{A_{t1}} \quad \text{(Ans.2.3)}$$
A-3	The EOM of tank 1 given in Equation Ans.2.3 is nonlinear.

   □□□

2. A-1, A-2 The nominal pump voltage V_{p0} for the pump-tank 1 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{10}) is characterized by the water in tank 1 being at a constant position level L_{10} due to the constant inflow rate generated by V_{p0}. Express the static equilibrium voltage V_{p0} as a function of the system's desired equilibrium level L_{10} and the pump flow constant K_p. Using the system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 3.1.1, evaluate V_{p0} parametrically.

Answer 2.2

Outcome	Solution
A-1	At equilibrium, all time derivative terms equate zero and Equation Ans.2.3 becomes:

$$K_pV_{p0} - A_{o1}\sqrt{2}\sqrt{gL_{10}} = 0 \quad \text{(Ans.2.4)}$$

A-2 Solving Equation Ans.2.4 for V_{p0} gives the pump voltage at equilibrium. V_{p0} results to be a function of L_{10} and K_p, as expressed below:

$$V_{p0} = \frac{A_{o1}\sqrt{2}\sqrt{gL_{10}}}{K_pV} \quad \text{(Ans.2.5)}$$

Using the system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 3.1.1, the evaluation of Equation Ans.2.5 results to be:

$$V_{p0} = 9.26V$$

□□□

3. A-1 Linearize tank 1 water level's EOM found in Question #1 about the quiescent operating point (V_{p0}, L_{10}).

Answer 2.3

Outcome	Solution
A-1	Applying the Taylor's series approximation about (V_{p0}, L_{10}), Equation Ans.2.3 can be linearized as represented below:

$$\frac{\partial L_1}{\partial t} = \frac{K_pV_{p0} - A_{o1}\sqrt{2}\sqrt{gL_{10}}}{A_{t1}} - \frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} + \frac{K_pV_{p1}}{A_{t1}} \quad \text{(Ans.2.6)}$$

Substituting V_{p0} in Equation Ans.2.6 with its expression given in Equation Ans.2.5 results in the following linearized EOM for the tank 1 water level system:

$$\frac{\partial L_{11}}{\partial t} = -\frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} + \frac{K_pV_{p1}}{A_{t1}} \quad \text{(Ans.2.7)}$$

□□□

4. A-1, A-2 Determine from the previously obtained linear equation of motion, the system's open-loop transfer function in the Laplace domain as defined in Equation 2.5 and Equation 2.6. Express the open-loop transfer function DC gain, K_{dc_1}, and time constant, τ_1, as functions of L_{10} and the system parameters. What is the order and type of the system? Is it stable? Evaluate K_{dc_1} and τ_1 according to system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 3.1.1.

Answer 2.4

Outcome	Solution
A-1	Applying the Laplace transform to Equation Ans.2.7 and rearranging yields:

$$K_{dc\_1} = \frac{K_p\sqrt{2}\sqrt{gL_{10}}}{A_{o1}g}, \qquad \tau_1 = \frac{A_{t1}\sqrt{2}\sqrt{gL_{10}}}{A_{o1}g} \tag{Ans.2.8}$$

A-2 Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, $G_1(s)$ is of type zero. Evaluating Equation Ans.2.8, accordingly to the system's parameters and the desired design requirements, gives:

$$K_{dc\_1} = 3.2\frac{V}{cm}, \qquad \tau_1 = 15.2s \tag{Ans.2.9}$$

□□□

5. A-1, A-2, A-3 Using the notations and conventions described in Figure 2.2, derive the Equation Of Motion (EOM) characterizing the dynamics of tank 2. Is the tank 2 system's EOM linear?
Hint: The outflow rate from tank 2, F_{o2}, can be expressed by:

$$F_{o2} = A_{o2}v_{o2} \tag{2.13}$$

Answer 2.5

Outcome	Solution
A-1	As a remark, the cross-section area of tank 2 outlet hole can be calculated by:

$$A_{o2} = \frac{1}{4}\pi D_{o2}^2 \tag{Ans.2.10}$$

Using Equation Ans.2.10, the outflow rate from tank 2 given in Equation 2.13 becomes:

$$F_{o2} = A_{o2}\sqrt{2}\sqrt{gL_2} \tag{Ans.2.11}$$

A-2 Moreover, using the mass balance principle for tank 2, we obtain the following first-order differential equation in L_2:

$$A_{t2}\frac{\partial L_2}{\partial t} = F_{i2} - F_{o2} \tag{Ans.2.12}$$

Substituting in Equation Ans.2.12 F_{i2} and F_{o2} with their expressions given in Equation Ans.2.10 and Equation Ans.2.11, respectively, and rearranging results in the following equation of motion for the tank 2 system:

$$\frac{\partial L_2}{\partial t} = \frac{K_pV_p - A_{o2}\sqrt{2}\sqrt{gL_2}}{A_{t2}} \tag{Ans.2.13}$$

A-3 The EOM of tank 2 given in Equation Ans.2.13 is nonlinear.

□□□

6. A-1, A-2 The nominal water level L_{10} for the tank1-tank2 pair can be determined at the system's static equilibrium. By definition, static equilibrium at a nominal operating point (L_{10}, L_{20}) is characterized by the water in tank 2 being at a constant position level L_{20} due to the constant inflow rate generated from the top tank by L_{10}. Express the static equilibrium level L_{10} as a function of the system's desired equilibrium level L_{20} and the system's parameters. Using the system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 4.1.1, evaluate L_{10}.

Answer 2.6

Outcome **Solution**

A-1 At equilibrium, all time derivative terms equate zero and Equation Ans.2.3 becomes:

$$A_{o1}\sqrt{2}\sqrt{gL_{10}} - A_{o2}\sqrt{2}\sqrt{gL_{20}} = 0 \quad \text{(Ans.2.14)}$$

A-2 Solving Equation Ans.2.14 for L_{10} gives the tank 1 water level at equilibrium. L_{10} results to be a function of L_{20}, as expressed below:

$$L_{10} = \frac{A_{o2}^2 L_{20}}{A_{o1}^2} \quad \text{(Ans.2.15)}$$

Using the system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 4.1.1, the evaluation of Equation Ans.2.15 results to be:

$$L_{10} = 15 \text{ cm}$$

□□□

7. A-1 Linearize tank 2 water level's EOM found in Question #5 about the quiescent operating point (L_{10}, L_{20}).

Answer 2.7

Outcome **Solution**

A-1 Applying the Taylor's series approximation about (L_{20}, L_{10}) Equation Ans.2.13 can be linearized as represented below:

$$\frac{\partial L_2}{\partial t} = \frac{A_{o1}\sqrt{2}\sqrt{gL_{10}} - A_{o2}\sqrt{2}\sqrt{gL_{20}}}{A_{t2}} - \frac{1}{2}\frac{A_{o2}\sqrt{2}gL_{21}}{\sqrt{gL_{20}}A_{t2}} + \frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} \quad \text{(Ans.2.16)}$$

Simplifying Equation Ans.2.16 with its expression given in Equation Ans.2.15 results to the following linearized EOM for the tank 2 water level system:

$$\frac{\partial L_{21}}{\partial t} = -\frac{1}{2}\frac{A_{o2}\sqrt{2}gL_{21}}{\sqrt{gL_{20}}A_{t2}} + \frac{1}{2}\frac{A_{o1}\sqrt{2}gL_{11}}{\sqrt{gL_{10}}A_{t1}} \quad \text{(Ans.2.17)}$$

□□□

8. A-1, A-2 Determine from the previously obtained linear equation of motion, the system's open-loop transfer function in the Laplace domain, as defined in Equation 2.10 and Equation 2.11. Express the open-loop transfer function DC gain, K_{dc_2}, and time constant, τ_2, as functions of L_{10}, L_{20}, and the system parameters. What is the order and type of the system? Is it stable? Evaluate K_{dc_2} and τ_2 according to system's specifications given in the Coupled Tanks User Manual ([5]) and the desired design requirements in Section 4.1.1.

Answer 2.8

Outcome	Solution
A-1	Applying the Laplace transform to Equation Ans.2.17 and rearranging yields:

$$K_{dc\_2} = \frac{A_{o1}\sqrt{L_{20}}}{A_{o2}\sqrt{L_{10}}}, \qquad \tau_2 = \frac{A_{t2}\sqrt{2}\sqrt{gL_{20}}}{A_{o2}g} \qquad \text{(Ans.2.18)}$$

A-2	Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, $G_2(s)$ is of type zero. Evaluating Equation Ans.2.18, according to the system's parameters and the desired design requirements, gives:

$$K_{dc\_2} = 1.0\frac{\text{cm}}{\text{cm}}, \qquad \tau_2 = 15.2 \text{ s} \qquad \text{(Ans.2.19)}$$

□□□

3 TANK 1 LEVEL CONTROL

3.1 Background

3.1.1 Specifications

In configuration #1, a control is designed to regulate the water level (or height) of tank #1 using the pump voltage. The control is based on a Proportional-Integral-Feedforward scheme (PI-FF). Given a $\pm$1 cm square wave level setpoint (about the operating point), the level in tank 1 should satisfy the following design performance requirements:

1. Operating level in tank 1 at 15 cm: $L_{10} = 15$ cm.
2. Percent overshoot less than 10%: $PO_1 \leq 11$ %.
3. 2% settling time less than 5 seconds: $t_{s\_1} \leq 5.0$ s.
4. No steady-state error: $e_{ss} = 0$ cm.

3.1.2 Tank 1 Level Controller Design: Pole Placement

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 3.1, below, the voltage feedforward action is characterized by:

$$V_{p\_ff} = K_{ff\_1}\sqrt{L_{r\_1}} \tag{3.1}$$

and

$$V_p = V_{p1} + V_{p\_ff} \tag{3.2}$$

As it can be seen in Figure 3.1, the feedforward action is necessary since the PI control system is designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point (V_{p0}, L_{10}). In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 1 bottom outlet orifice, the PI controller compensates for dynamic disturbances.

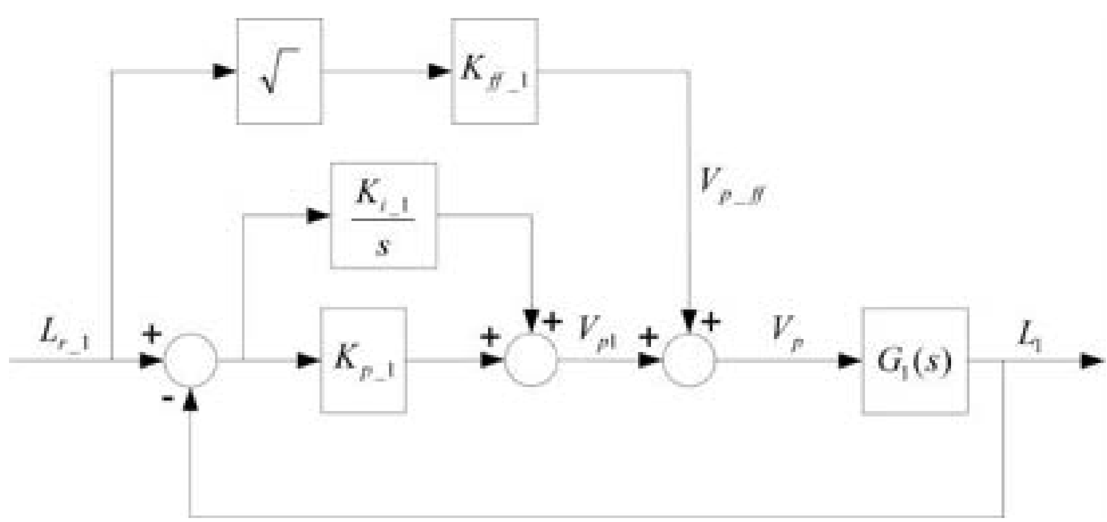


Figure 3.1: Tank 1 Water Level PI-plus-Feedforward Control Loop.

The open-loop transfer function $G_1(s)$ takes into account the dynamics of the tank 1 water level loop, as characterized by Equation 2.5. However, due to the presence of the feedforward loop, $G_1(s)$ can also be written as follows:

$$G_1(s) = \frac{L_1(s)}{V_{p1}(s)} \tag{3.3}$$

3.1.3 Second-Order Response

The block diagram shown in Figure 3.2 is a general unity feedback system with compensator, i.e., controller $C(s)$ and a transfer function representing the plant, $P(s)$. The measured output, $Y(s)$, is supposed to track the reference signal $R(s)$ and the tracking has to match to certain desired specifications.

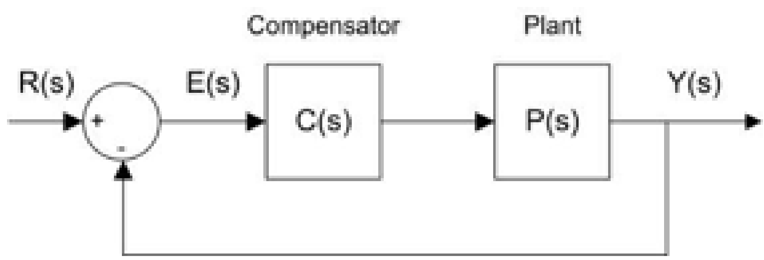


Figure 3.2: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s)\,P(s)\,(R(s) - Y(s))$$

By solving for $Y(s)$, we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

The input-output relation in the time-domain for a proportional-integral (PI) controller is

$$u = K_p(r - y) + \frac{K_i(r - y)}{s} \tag{3.4}$$

where K_p is the proportional gain and K_i is the integral gain.

In fact, when a first order system is placed in series with PI compensator in the feedback loop as in Figure 3.2, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \tag{3.5}$$

where ω_n is the natural frequency and ζ is the damping ratio. This is called the *standard second-order* transfer function. Its response properties depend on the values of ω_n and ζ.

Peak Time and Overshoot

Consider a second-order system as shown in Equation 3.5 subjected to a step input given by

$$R(s) = \frac{R_0}{s} \tag{3.6}$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 3.3, where the red trace is the response (output), $y(t)$, and the blue trace is the step input $r(t)$.

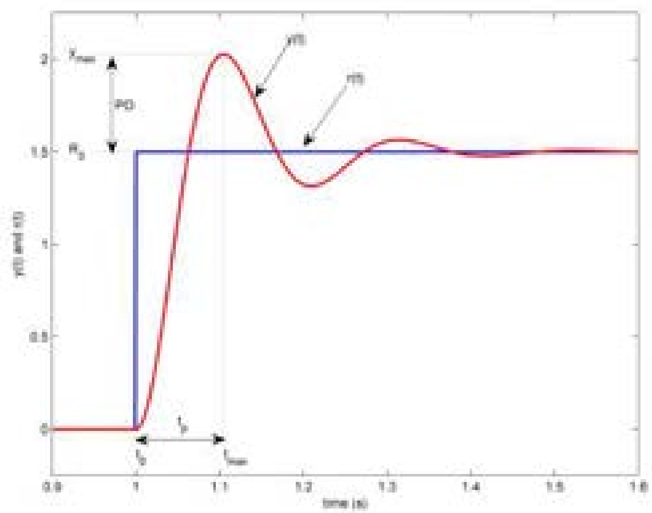


Figure 3.3: Standard second-order step response.

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 3.3, the percent overshoot is found using

$$PO = \frac{100\ (y_{max} - R_0)}{R_0} \tag{3.7}$$

From the initial step time, t_0, the time it takes for the response to reach its maximum value is

$$t_p = t_{max} - t_0 \tag{3.8}$$

This is called the *peak time* of the system.

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

$$PO = 100\, e^{\left(-\frac{\pi\,\zeta}{\sqrt{1-\zeta^2}}\right)} \tag{3.9}$$

The peak time depends on both the damping ratio and natural frequency of the system and it can be derived as

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{3.10}$$

Tank 1 level response 2% Settling Time can be expressed as follows:

$$t_s = \frac{4}{\zeta\omega} \tag{3.11}$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

3.2 Pre-Lab Questions

1. A-1, A-2 Analyze tank 1 water level closed-loop system at the static equilibrium point (V_{p0}, L_{10}) and determine and evaluate the voltage feedforward gain, K_{ff_1}, as defined by Equation 3.1.

Answer 3.1

Outcome **Solution**

A-1 By definition, at the static equilibrium point (V_{p0}, L_{10}):

$$L_1 = L_{r\_1} = L_{10}, \qquad V_p = V_{p\_ff} = V_{p0} \tag{Ans.3.1}$$

Using Equation Ans.2.5, the voltage feedforward gain results to be:

$$K_{ff\_1} = \frac{A_{o1}\sqrt{2g}}{K_p} \tag{Ans.3.2}$$

A-2 Evaluating Equation Ans.3.2 with the system's parameters given in the Coupled Tanks User Manual ([5]) leads to:

$$K_{ff\_1} = 2.39\frac{V}{\sqrt{cm}} \tag{Ans.3.3}$$

□□□

2. A-1, A-2 Using tank 1 voltage-to-level transfer function $G_1(s)$ determined in Section 2.2 and the control scheme block diagram illustrated in Figure 3.1, derive the normalized characteristic equation of the water level closed-loop system.

Hint#1: The feedforward gain K_{ff_1} does not influence the system characteristic equation. Therefore, the feedforward action can be neglected for the purpose of determining the denominator of the closed-loop transfer function. Block diagram reduction can be carried out.

Hint#2: The system's normalized characteristic equation should be a function of the PI level controller gains, K_{p_1}, and K_{i_1}, and system's parameters, K_{dc_1} and τ_1.

Answer 3.2

Outcome **Solution**

A-1 Neglecting the feedforward action and carrying out block diagram reduction using Equation 2.6 and Equation 3.4 one has

$$Y(s) = \frac{K_{dc\_1}}{\tau_1 s + 1}\left(K_{p1}(R(S) - Y(s)) + \frac{K_{i\_1}}{s}(R(S) - Y(s))\right) \tag{Ans.3.4}$$

which results in the following closed-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{K_{dc\_1}(K_{p\_1}s + K_{i\_1})}{\tau_1 s^2 + (1 + K_{dc\_1}K_{p\_1})s + K_{dc\_1}K_{i\_1}} \tag{Ans.3.5}$$

A-2 Re-arranging Equation Ans.3.5 results in tank 1 normalized characteristic polynomial:

$$s^2 + \frac{(1 + K_{dc\_1}K_{p\_1})s}{\tau_1} + \frac{K_{dc\_1}K_{i\_1}}{\tau_1} = 0 \tag{Ans.3.6}$$

□□□

3. A-2 By identifying the controller gains K_{p_1} and K_{i_1}, fit the obtained characteristic equation to the second-order standard form expressed below:

$$s^2 + 2\zeta_1\omega_{n1}s + \omega_{n1}^2 = 0 \tag{3.12}$$

Determine K_{p_1} and K_{i_1} as functions of the parameters ω_{n1}, ζ_1, K_{dc_1}, and τ_1 using Equation 3.5.

Answer 3.3

Outcome	Solution
A-2	The system's desired characteristic equation is expressed by Equation Ans.3.6. Solving for the two unknowns K_{p_1} and K_{i_1} the set of two equations resulting from identifying the coefficients of Equation Ans.3.6 with those of Equation 3.12, the PI controller gains can be expressed as follows:

$$K_{p\_1} = \frac{2\zeta_1\omega_{n1}\tau_1 - 1}{K_{dc\_1}}, \qquad K_{i\_1} = \frac{\omega_{n1}^2\tau_1}{K_{dc\_1}} \tag{Ans.3.7}$$

□□□

4. A-1, A-2 Determine the numerical values for K_{p_1} and K_{i_1} in order for the tank 1 system to meet the closed-loop desired specifications, as previously stated.

Answer 3.4

Outcome	Solution
A-1	The minimum damping ratio to meet the maximum overshoot requirement, PO_1, can be obtained by solving Equation 3.9. The following relationship results:

$$\zeta_1 = \frac{ln(\frac{1}{100}PO_1)}{\sqrt{ln(\frac{1}{100}PO_1)^2 + \pi^2}}, \qquad K_{i\_1} = \frac{\omega_{n1}^2\tau_1}{K_{dc\_1}} \tag{Ans.3.8}$$

The system natural frequency, ω_{n1}, can be calculated from Equation 3.11, as follows:

$$\omega_{n1} = \frac{4}{\zeta_1 t_{s\_1}} \tag{Ans.3.9}$$

A-2 Evaluating Equation Ans.3.8 and Equation Ans.3.9 according to the desired design requirements, then carrying out the numerical application of Equation Ans.3.7 leads to the following PI controller gains:

$$K_{p\_1} = 7.2\frac{V}{cm}, \qquad K_{i\_1} = 9.1\frac{V}{cm - s} \tag{Ans.3.10}$$

□□□

3.3 Lab Experiments

3.3.1 Objectives

- Tune through pole placement the PI-plus-feedforward controller for the actual water level in tank 1 of the Coupled-Tank system.
- Implement the PI-plus-feedforward control loop for the actual Coupled-Tank's tank 1 level.
- Run the obtained PI-plus-feedforward level controller and compare the actual response against the controller design specifications.
- Run the system's simulation simultaneously, at every sampling period, in order to compare the actual and simulated level responses.

3.3.2 Tank 1 Level Control Simulation

Experimental Setup

The *s_tanks_1* Simulink® diagram shown in Figure 3.4 is used to perform tank 1 level control simulation exercises in this laboratory.

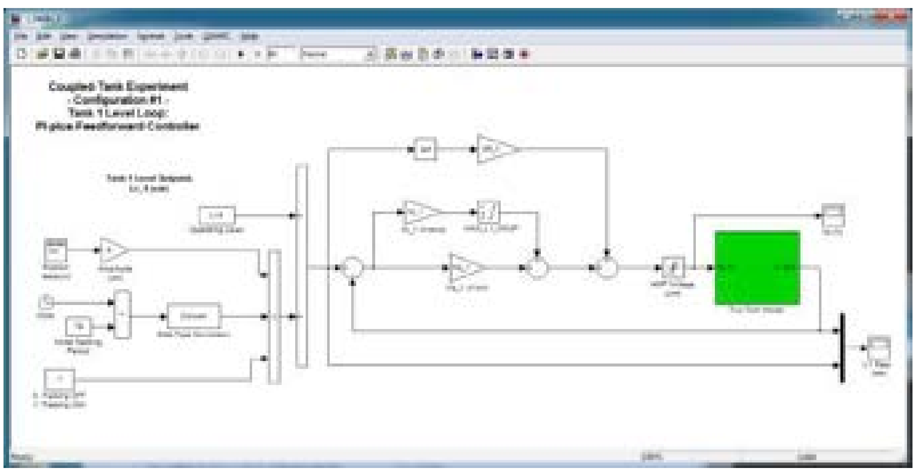


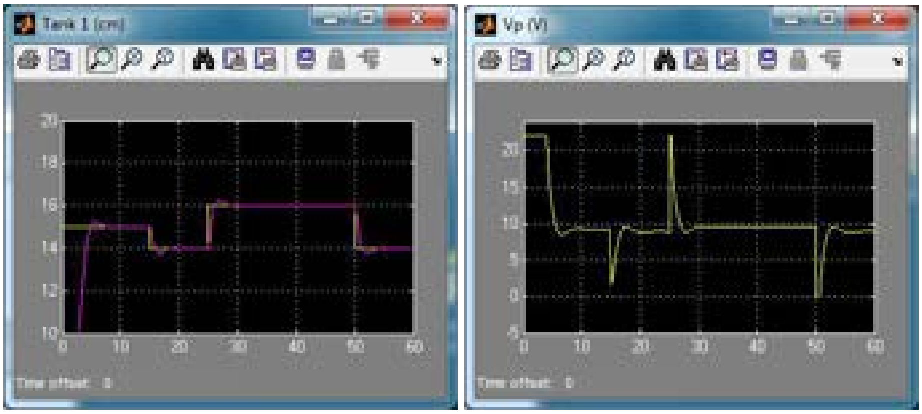
Figure 3.4: Simulink model used to simulate PI-FF control on Coupled Tanks system in configuration #1.

IMPORTANT: Before you can conduct these simulations, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the proportional, integral, and feedforward gain control gains found in Section 3.2 in Matlab as *Kp_1*, *Ki_1*, and *Kff_1*.
2. To generate a step reference, go to the Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.02 Hz
3. Set the *Amplitude (cm)* gain block to 1 to generate a square wave goes between ± 1 cm.

4. Open the pump voltage *Vp (V)* and tank 1 level response *Tank 1(cm)* scopes.
5. By default, there should be anti-windup on the Integrator block (i.e., just use the default Integrator block).
6. Start the simulation. By default, the simulation runs for 60 seconds. The scopes should be displaying responses similar to Figure 3.5. Note that in the *Tank 1 (cm)* scope, the yellow trace is the desired level while the purple trace is the simulated level.



(a) Tank 1 Level (b) Pump Voltage

Figure 3.5: Simulated closed-loop configuration #1 control response

7. B-5, K-2 Generate a Matlab® figure showing the *Simulated Tank 1* response and the pump voltage.

   Data Saving: After each simulation run, each scope automatically saves their response to a variable in the Matlab® workspace. The *Vp (V)* scope saves its response to the variable called data_Vp and the *Tank 1 (cm)* scope saves its data to the *data_L1* variable.

   - The *data_L1* variable has the following structure: *data_L1(:,1)* is the time vector, *data_L1(:,2)* is the set-point, and *data_L1(:,3)* is the simulated level.
   - For the *data_Vp* variable, *data_Vp(:,1)* is the time and *data_Vp(:,2)* is the simulated pump voltage.

   Answer 3.5

Outcome	Solution
B-5	The simulation was ran correctly if a response similar to Figure Ans.3.1 was obtained.
K-2	The closed-loop response is shown in Figure Ans.3.1. You can generate this using the plot_tanks_1_rsp.m script.

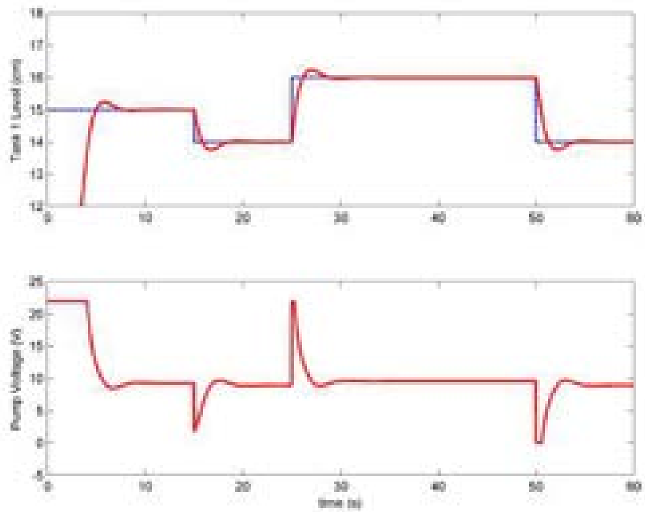


Figure Ans.3.1: Simulated tank 1 level control response.

□□□

8. K-1, B-9 Assess the actual performance of the level response and compare it to the design requirements. Measure your response actual percent overshoot and settling time. Are the design specifications satisfied? Explain. If your level response does not meet the desired design specifications, review your PI-plus-Feedforward gain calculations and/or alter the closed-loop pole locations until they do. Does the response satisfy the specifications given in Section 3.1.1?

   Hint: Use the graph cursors in the *Measure* tab to take measurements.

Answer 3.6

Outcome	**Solution**
K-1	The settling time, percentage overshoot, and steady-state error measured in the simulated response shown in Figure Ans.3.1 is

$$
\begin{aligned}
t_{s1} &= 1.9 \text{ s} \\
PO_1 &= 100 \times \frac{16.23 - 16}{2} = 11.5\% \\
e_{ss1} &= 0 \text{ cm}
\end{aligned}
$$

B-9	The settling time and steady-state error satisfy the specifications given in Section 3.1.1, but the overshoot does not. The pump does saturate at 24 V, which causes the response to overshoot slightly more than anticipated.

□□□

3.3.3 Tank 1 Level Control Implementation

The q_tanks_1 Simulink diagram shown in Figure 3.6 is used to perform the tank 1 level control exercises in this laboratory. The *Coupled Tanks* subsystem contains QUARC® blocks that interface with the pump and pressure sensors of the Coupled Tanks system.

Note that a first-order low-pass filter with a cut-off frequency of 2.5 Hz is added to the output signal of the tank 1 level pressure sensor. This filter is necessary to attenuate the high-frequency noise content of the level measurement.

Such a measurement noise is mostly created by the sensor's environment consisting of turbulent flow and circulating air bubbles. Although introducing a short delay in the signals, low-pass filtering allows for higher controller gains in the closed-loop system, and therefore for higher performance. Moreover, as a safety watchdog, the controller will stop if the water level in either tank 1 or tank 2 goes beyond 27 cm.

Experimental Setup

The *q_tanks_1* Simulink® diagram shown in Figure 3.6 will be used to run the PI+FF level control on the actual Coupled Tanks system.

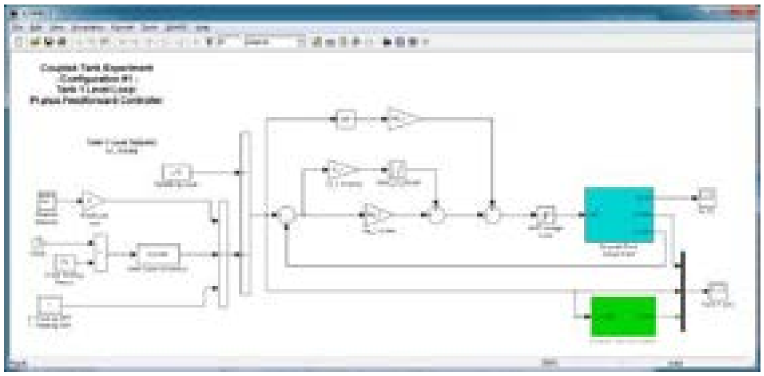


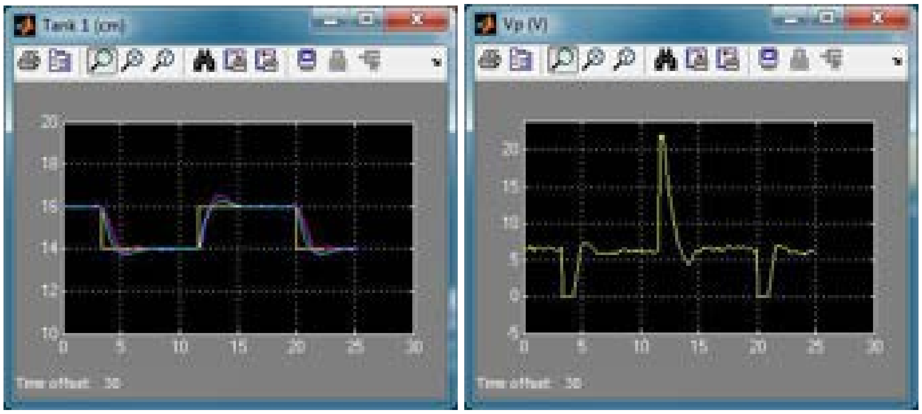
Figure 3.6: Simulink model used to run tank 1 level control on Coupled Tanks system.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the proportional, integral, and feed forward control gains found in Section 3.2 in Matlab® as *Kp_1*, *Ki_1*, and *Kff_1*.
2. To generate a step reference, go to the Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.06 Hz
3. Set the *Amplitude (cm)* gain block to 1 to generate a square wave goes between ± 1 cm.
4. Open the pump voltage *Vp (V)* and tank 1 level response *Tank 1(cm)* scopes.
5. By default, there should be anti-windup on the Integrator block (i.e., just use the default Integrator block).
6. In the Simulink diagram, go to QUARC | Build.
7. Click on QUARC | Start to run the controller. The pump should start running and filling up tank 1 to its operating level, L_{10}. After a settling delay, the water level in tank 1 should begin tracking the ± 1 cm square wave setpoint (about operating level L_{10}).
8. B-5, K-2 Generate a Matlab® figure showing the *Implemented Tank 1 Control* response and the input pump voltage.

   Data Saving: As in s_tanks_1.mdl, after each run each scope automatically saves their response to a variable in the Matlab® workspace.



(a) Tank 1 Level

(b) Pump Voltage

Figure 3.7: Measured closed-loop tank 1 control response

Answer 3.7

Outcome	**Solution**
B-5	If the procedure was followed properly, the control should have been ran on the Coupled Tanks system and the response similar to Figure Ans.3.2 should have been obtained.
K-2	The closed-loop response is shown in Figure Ans.3.2. You can generate this using the plot_tanks_1_rsp.m script.

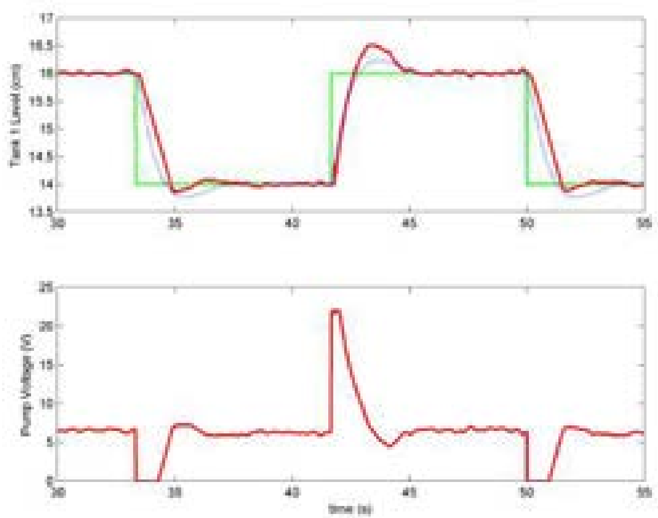


Figure Ans.3.2: Closed-loop tank 1 level control response.

□ □ □

9. K-1, B-9 Measure the steady-state error, the percent overshoot and the peak time of the response. Does the response satisfy the specifications given in Section 3.1.1? **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.

Answer 3.8

Outcome **Solution**

K-1 The settling time, percent overshoot, and steady-state error measured in the response shown in Figure Ans.3.2 are:

$$t_{ss,1} = 2.54 \text{ s}$$
$$PO_1 = 24.2\%$$
$$e_{ss,1} = 0.02 \text{ cm}.$$

B-9 The percent overshoot exceeds the maximum limit given in Section 3.1.1. This is probably due to the pump voltage (i.e., control signal) getting saturated or unmodeled effects. The settling time and steady-state error satisfy the specifications listed (i.e., the steady-state error is very low and below sensor noise).

□□□

3.4 Results

B-6 Fill out Table 3.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Units
Pre Lab Questions			
Tank 1 Control Gains			
Feed Forward Control Gain	$K_{ff,1}$	2.39	V/$\sqrt{cm}$
Proportional Control Gain	$k_{p,1}$	-7.2	V/cm
Integral Control Gain	$k_{i,1}$	-9.1	V/(cm-s)
Tank 1 Control Simulation			
Steady-state error	$e_{ss,1}$	0	cm
Settling time	$t_{s,1}$	1.9	s
Percent overshoot	PO_1	11.5	%
Tank 1 Control Implementation			
Steady-state error	$e_{ss,1}$	0.02	cm
Settling time	$t_{s,1}$	2.54	s
Percent overshoot	PO_1	24.2	%

Table 3.1: Tank 1 Level Control Results

4 TANK 2 LEVEL CONTROL

4.1 Background

4.1.1 Specifications

In configuration #2, the pump feeds tank 1 and tank 1 feeds tank 2. The designed closed-loop system is to control the water level in tank 2 (i.e. the bottom tank) from the water flow coming out of tank 1, located above it. Similarly to configuration #1, the control scheme is based on a Proportional-plus-Integral-plus-Feedforward law.

In response to a desired $\pm$ 1 cm square wave level setpoint from tank 2 equilibrium level position, the water height behaviour should satisfy the following design performance requirements:

1. Tank 2 operating level at 15 cm: $L_{20} = 15$ cm.
2. Percent overshoot should be less than or equal to 10%: $PO_2 \leq 10.0$ %.
3. 2% settling time less than 20 seconds: $t_{s,2} \leq 20.0$ s.
4. No steady-state error: $e_{ss,2} = 0$ cm.

4.1.2 Tank 2 Level Controller Design: Pole Placement

For zero steady-state error, tank 1 water level is controlled by means of a Proportional-plus-Integral (PI) closed-loop scheme with the addition of a feedforward action, as illustrated in Figure 4.1, below.

In the block diagram depicted in Figure 4.1, the water level in tank 1 is controlled by means of the closed-loop system previously designed in Section 3.1. This is represented by the tank 1 closed-loop transfer function defined below:

$$T_1(s) = \frac{L_1(s)}{L_{r\_1}(s)} \tag{4.1}$$

Such a subsystem represents an inner (or nested) level loop. In order to achieve a good overall stability with such a configuration, the inner level loop (i.e. tank 1 closed-loop system) must be much faster than the outer level loop. This constraint is met by the previously stated controller design specifications, where $t_{s\_1} \leq t_{s\_2}$.

However for the sake of simplicity in the present analysis, the water level dynamics in tank 1 are neglected. Therefore, it is assumed hereafter that:

$$L_1(t) = L_{r\_1}(t) \qquad i.e. \qquad T_1(s) = 1 \tag{4.2}$$

Furthermore as depicted in Figure 4.1, the level feedforward action is characterized by:

$$L_{ff\_1} = K_{ff\_2} L_{r\_2} \tag{4.3}$$

and

$$L_1 = L_{11} + L_{ff\_1} \tag{4.4}$$

The level feedforward action, as seen in Figure 4.1, is necessary since the PI control system is only designed to compensate for small variations (a.k.a. disturbances) from the linearized operating point L_{10}, L_{20}. In other words, while the feedforward action compensates for the water withdrawal (due to gravity) through tank 2's bottom outlet orifice, the PI controller compensates for dynamic disturbances.

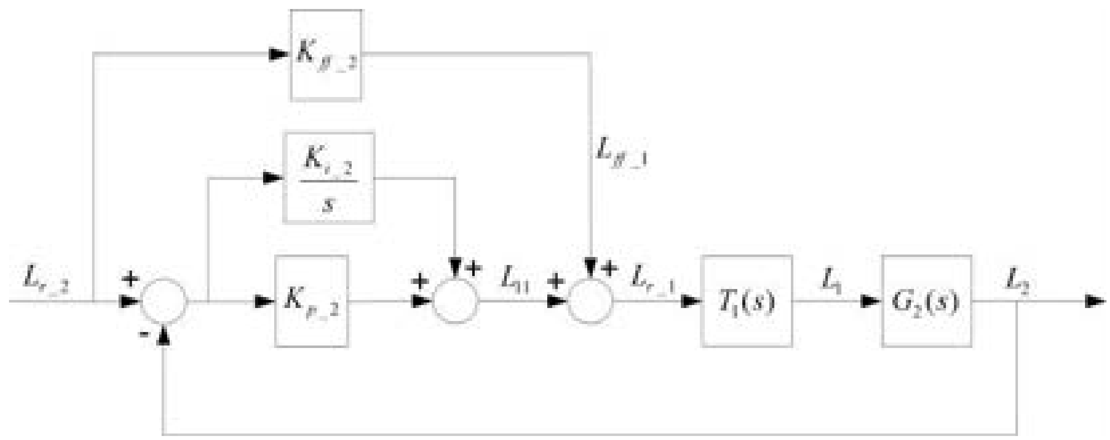


Figure 4.1: Tank 2 Water Level PI-plus-Feedforward Control Loop.

The open-loop transfer function $G_2(s)$ takes into account the dynamics of the tank 2 water level loop, as characterized by Equation 2.10. However, due to the presence of the feedforward loop and the simplifying assumption expressed by Equation 4.2, $G_2(s)$ can also be written as follows:

$$G_2(s) = \frac{L_2(s)}{L_1(s)} \tag{4.5}$$

4.2 Pre-Lab Questions

1. A-1, A-2 Analyze tank 2 water level closed-loop system at the static equilibrium point (L_{10}, L_{20}) and determine and evaluate the voltage feedforward gain, K_{ff_2}, as defined by Equation 4.3.

Answer 4.1

Outcome **Solution**

A-1 By definition, at the static equilibrium point (V_{p0}, L_{10}):

$$L_2 = L_{2\_1} = L_{20}, \qquad L_1 = L_{r\_1} = L_{ff\_1} = L_{10} \qquad \text{(Ans.4.1)}$$

Using Equation Ans.2.15, the voltage feedforward gain results to be:

$$K_{ff\_2} = \frac{A_{o2}^2}{A_{o1}^2} \qquad \text{(Ans.4.2)}$$

A-2 Evaluating Equation Ans.4.2 with the system's parameters given in Reference [5] leads to:

$$K_{ff\_2} = 1.0 \qquad \text{(Ans.4.3)}$$

□□□

2. A-1 Using tank 2 voltage-to-level transfer function $G_2(s)$ determined in Section 2 and the control scheme block diagram illustrated in Figure 4.1, derive the normalized characteristic equation of the water level closed-loop system.

Hint#1: Block diagram reduction can be carried out.

Hint#2: The system's normalized characteristic equation should be a function of the PI level controller gains, K_{p_2}, and K_{i_2}, and system's parameters, K_{dc_2} and τ_2.

Answer 4.2

Outcome **Solution**

A-1 Neglecting the feedforward action and carrying out block diagram reduction using Equation 2.11 and Equation 3.4 one has

$$Y(s) = \frac{K_{dc\_2}}{\tau_2 s + 1}(K_{p\_2}(R(S) - Y(s)) + \frac{K_{i\_2}}{s}(R(S) - Y(s))) \qquad \text{(Ans.4.4)}$$

which results in the following closed-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{K_{dc\_2}(K_{p\_2}s + K_{i\_2})}{\tau_1 s^2 + (1 + K_{dc\_2}K_{p\_2})s + K_{dc\_2}K_{i\_2}} \qquad \text{(Ans.4.5)}$$

A-2 Re-arranging Equation Ans.4.5 results in tank 2 normalized characteristic polynomial:

$$s^2 + \frac{(1 + K_{dc\_2}K_{p\_2})s}{\tau_2} + \frac{K_{dc\_2}K_{i\_2}}{\tau_2} = 0 \qquad \text{(Ans.4.6)}$$

□□□

3. A-2 By identifying the controller gains K_{p_2} and K_{i_2}, fit the obtained characteristic equation to the standard second-order equation: $s^2 + 2\zeta_2\omega_{n2}s + \omega_{n2}^2 = 0$. Determine K_{p_2} and K_{i_2} as functions of the parameters ω_{n2}, ζ_2, K_{dc_2}, and τ_2.

Answer 4.3

Outcome **Solution**

A-2 The system's desired characteristic equation is expressed by Equation 2.11. Solving for the two unknowns K_{p_2} and K_{i_2} the set of two equations resulting from identifying the coefficients of Equation 2.11 with those of Equation Ans.4.6, the PI controller gains can be expressed as follows:

$$K_{p\_2} = \frac{2\zeta_2 \omega_{n2} \tau_2 - 1}{K_{dc\_2}}, \qquad K_{i\_2} = \frac{\omega_{n2}^2 \tau_2}{K_{dc\_2}} \qquad \text{(Ans.4.7)}$$

□□□

4. A-1, A-2 Determine the numerical values for K_{p_2} and K_{i_2} in order for the tank 2 system to meet the closed-loop desired specifications, as previously stated.

Answer 4.4

Outcome **Solution**

A-1 The minimum damping ratio to meet the maximum overshoot requirement, PO_2, can be obtained by solving Equation 3.9. The following relationship results:

$$\zeta_2 = \frac{ln(\frac{1}{100} PO_2)}{\sqrt{ln(\frac{1}{100} PO_2)^2 + \pi^2}} \qquad \text{(Ans.4.8)}$$

The system natural frequency, ω_{n2}, can be calculated from Equation 3.11 (i.e. Hint #2), as follows:

$$\omega_{n2} = \frac{4}{\zeta_2 t_{s\_2}} \qquad \text{(Ans.4.9)}$$

A-2 Evaluating Equation Ans.4.8 and Equation Ans.4.9 accordingly to the desired design requirements, then carrying out the numerical application of Equation Ans.4.7 leads to the following PI controller gains:

$$\begin{aligned} K_{p\_2} &= 5.1 \text{ cm/cm} \\ K_{i\_2} &= 1.7 \text{ s}^{-1} \end{aligned} \qquad \text{(Ans.4.10)}$$

□□□

4.3 Lab Experiments

4.3.1 Objectives

- Tune through pole placement the PI-plus-Feedforward controller for the actual water level of the Coupled-Tank system's tank 2.
- Implement the PI-plus-Feedforward control loop for the actual tank 2 water level.
- Run the obtained Feedforward-plus-PI level controller and compare the actual response against the controller design specifications.
- Run the system's simulation simultaneously, at every sampling period, in order to compare the actual and simulated level responses.
- Investigate the effect of the nested PI-plus-Feedforward level control loop implemented for tank 2.

4.3.2 Tank 2 Level Control Simulation

In this section you will simulate the tank 2 level control of the Coupled Tanks system. The two-tank dynamics are modeled using the Simulink blocks and controlled using the PI+FF controller described in Section 4.1. Our goals are to confirm that the desired response specifications are satisfied and to verify that the amplifier is not saturated.

Experimental Setup

The *s_tanks_2* Simulink® diagram shown in Figure 4.2 will be used to simulate the tank 2 level control response with the PI+FF controller used earlier in Section 4.1.

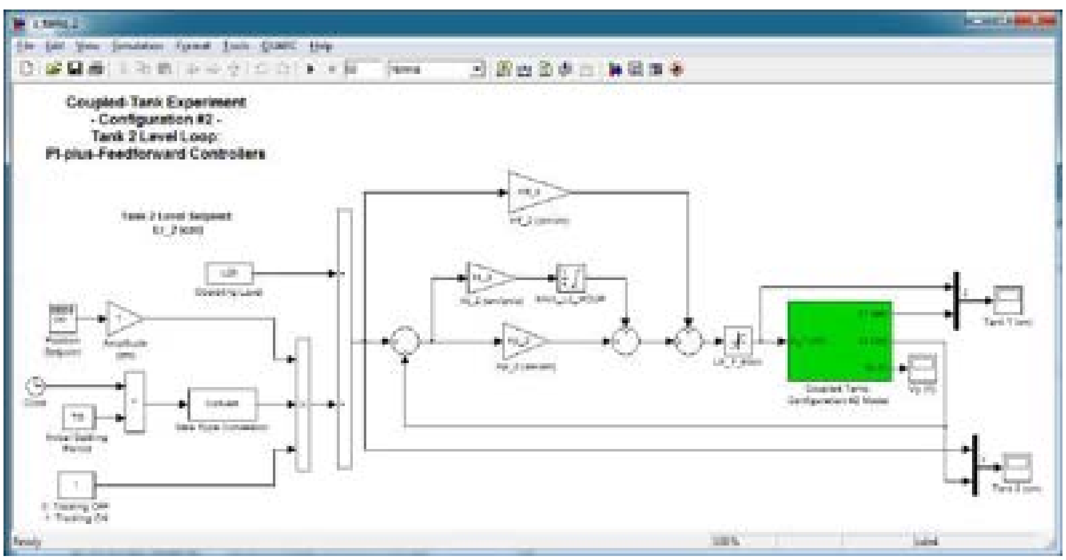
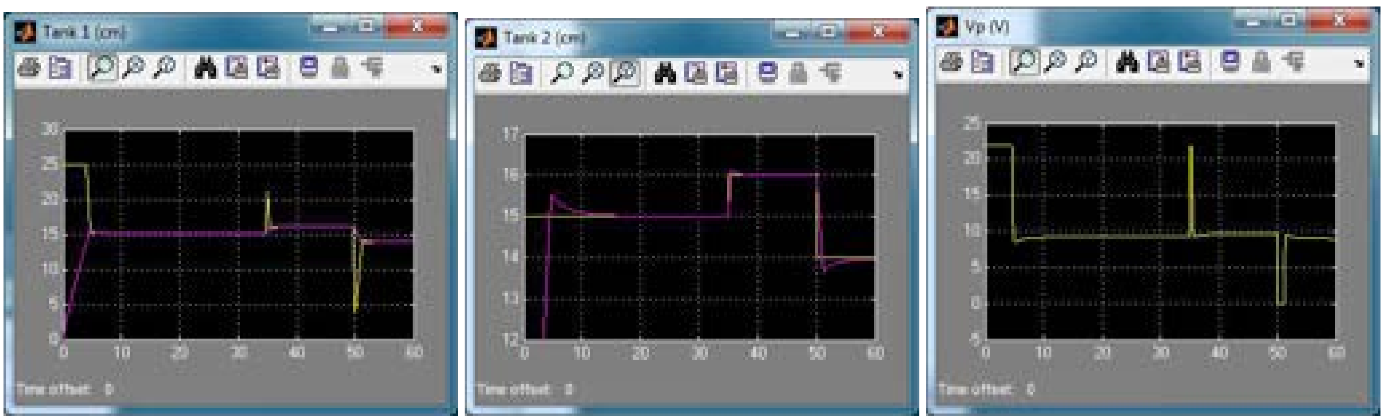


Figure 4.2: Simulink model used to simulate tank 2 level control response.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

1. Enter the proportional, integral, and feed-forward gains in Matlab found in the Tank 1 Control pre-lab questions in Section 3.2 as *Kp_1*, *Ki_1*, and *Kff_1*.
2. Enter the proportional, integral, and feed-forward control gains found in Section 4.2 in Matlab® as *Kp_2*, *Ki_2*, and *Kff_2*.

3. To generate a step reference, go to the *Position Setpoint* Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.02 Hz
4. Set the *Amplitude (cm)* gain block to 1 to generate a step that goes between 14 and 15 mm (i.e., ± 1 cm square wave with $L_{10} = 15$ cm operation point).
5. Open the *Tank 1 (cm)*, *Tank 2 (cm)*, and *Vp (V)* scopes.
6. Start the simulation. By default, the simulation runs for 120 seconds. The scopes should be displaying responses similar to Figure 4.3. Note that in the *Tank 1 (cm)* and *Tanks 2 (cm)* scopes, the yellow trace is the setpoint (or command) while the purple trace is the simulation.



(a) Tank 1 Level (b) Tank 2 Level (c) Pump Voltage

Figure 4.3: Simulated closed-loop tank 2 level control response.

7. B-5, K-2 Generate a Matlab® figure showing the *Simulated Configuration #2* response. Include both tank 1 and 2 level responses as well as the pump voltage.

   Data Saving: Similarly as with s_tanks_1, after each simulation run each scope automatically saves their response to a variable in the Matlab® workspace. The *Tank 2 (cm)* scope saves its response to the data_L2 variable. The *Tank 1 (cm)* scope saves its response to the variable called data_L1 and the *Pump Voltage (V)* scope saves its data to the *data_Vp* variable.

   Answer 4.5

Outcome	**Solution**
B-5	The simulation was ran correctly if a response similar to Figure Ans.4.1 was obtained.
K-2	The closed-loop position response is shown in Figure Ans.4.1. You can generate this using the plot_tanks_2_rsp.m script.

   □□□

8. K-1, B-9 Measure the steady-state error, the percent overshoot and the settling time of the simulated response. Does the response satisfy the specifications given in Section 2.1.4? **Hint:** Use the Matlab® *ginput* command to take measurements off the figure.

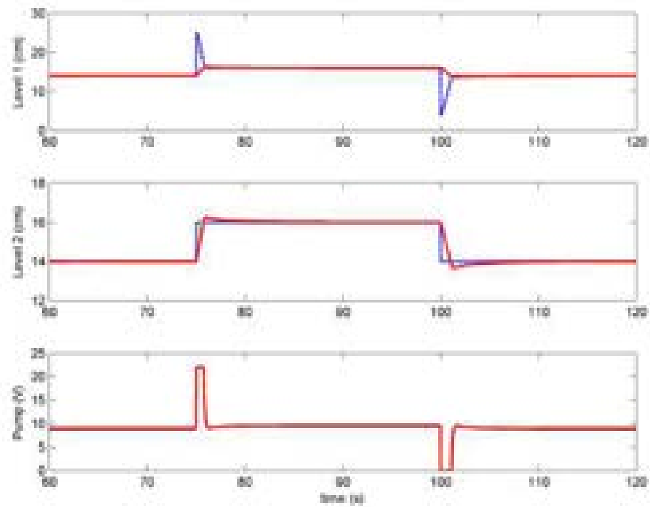


Figure Ans.4.1: Simulated configuration #2 level control response.

Answer 4.6

Outcome	Solution
K-1	From the response shown in Ans.4.1, it is clear that the steady-state error is zero, thus $$e_{ss} = 0.$$ Taking measurements from the step that begins at 35.0 seconds, the simulation settles to 2% of its final value (i.e., 16.3 mm) in 1 sec. Actually the response overshoots up to 16.25 cm at 36.0 seconds. Using the equations given in Section 2.1.1, the settling time is $$t_s = 36 - 35 = 1.0 \text{ s}.$$ Using Equation 3.7 with the measurement, we find that the percent overshoot of the simulated tank 2 level response is $$PO = 100 \times \frac{16.25 - 16}{2} = 12.5 \text{ \%}$$
B-9	Thus the settling time is acceptable. However, the overshoot goes above the desired percent overshoot listed in Section 2.1.4. Therefore the response with the PIV+FF controller does not quite match the specifications.

□□□

4.3.3 Tank 2 Level Control Implementation

The q_tanks_2 Simulink diagram shown in Figure 4.4 is used to run the Tank 2 Level control presented in Section 4.1 on the Coupled Tanks system (i.e., when set up in Configuration #2). The *Tank 1 Inner Loop* subsystem contains the PI+FF control used previously in Section 3.3.3 as well as the *Coupled Tanks* subsystem, which contains QUARC® blocks that interface with the pump and pressure sensors of the Coupled Tanks system.

Experimental Setup

The *q_tanks_2* Simulink® diagram shown in Figure 4.4 will be used to run the feed-forward and PI level control on the actual Coupled Tanks system.

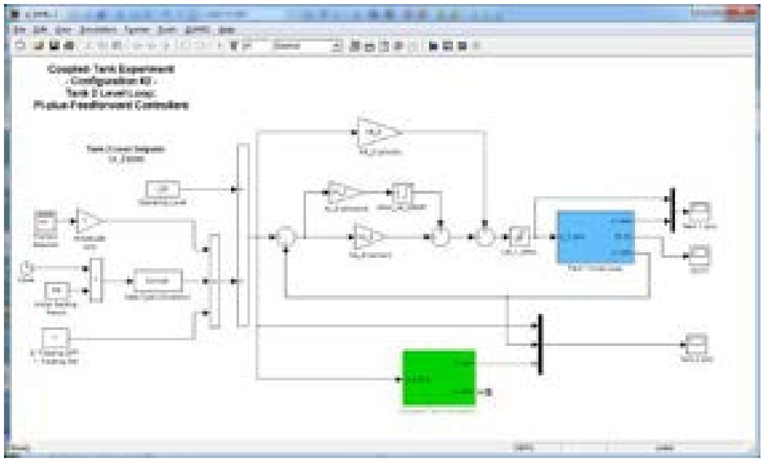


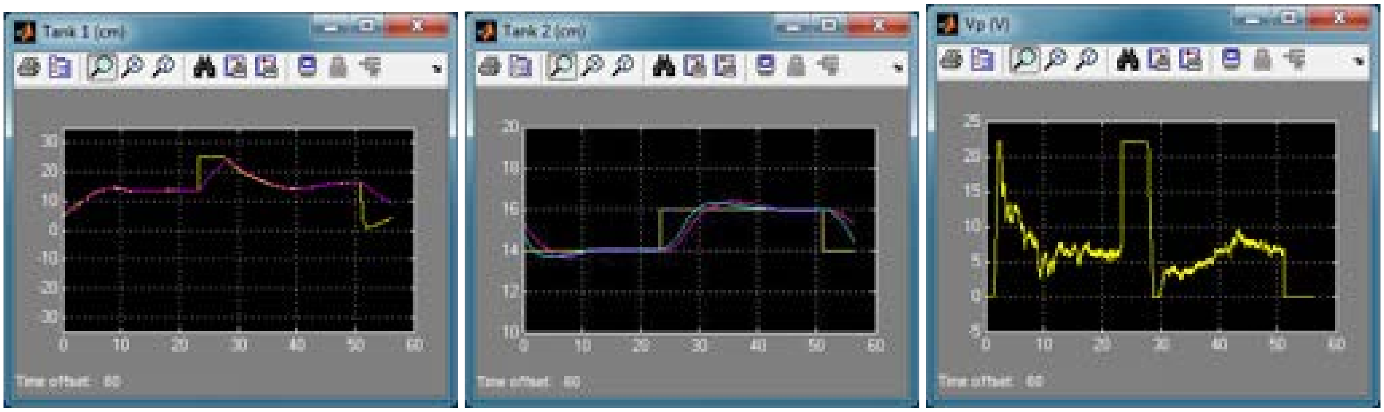
Figure 4.4: Simulink model used to run tank 2 level control on Coupled Tanks system.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 5 to configure the lab files first.

Follow this procedure:

1. Enter the proportional, integral, and feed-forward gains in Matlab used in the Tank 1 Control simulation in Section 3.3.2 as *Kp_1*, *Ki_1*, and *Kff_1*.
2. Enter the proportional, integral, and feed-forward control gains found in Section 4.2 in Matlab® as *Kp_2*, *Ki_2*, and *Kff_2*.
3. To generate a step reference, go to the *Position Setpoint* Signal Generator block and set it to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.02 Hz
4. Set the *Amplitude (cm)* gain block to 1 to generate a step that goes between 14 and 15 mm (i.e., ± 1 cm square wave with $L_{10} = 15$ cm operation point).
5. Open the *Tank 1 (cm)*, *Tank 2 (cm)*, and *Vp (V)* scopes.
6. In the Simulink diagram, go to QUARC | Build.
7. Click on QUARC | Start to run the controller. The level in tank 2 will first stabilize to the operating point tank 2 operating point. After the settling period, the $\pm$1 cm step will start. The scopes should be displaying responses similar to Figure 4.5 (after the settling period).
8. B-5, K-2 Generate a Matlab® figure showing the *Implemented Tank 2 Level Control* response, i.e., the tank 1 and 2 levels as well as the pump voltage.

   Data Saving: As with s_tanks_2, after each run the scopes automatically save their response to a variable in the Matlab® workspace. The *Tank 1 (cm)* and *Tank 2 (cm)* scopes save their response to the data_L1 and data_L2 variables. The *Pump Voltage (V)* scope saves its response to the variable called data_Vp.



(a) Tank 1 Level (b) Tank 2 Level (c) Pump Voltage

Figure 4.5: Typical response when controlling tank 2 level.

Answer 4.7

Outcome	Solution
B-5	The experimental procedure was followed correctly if a response similar to Figure Ans.4.2 was obtained.
K-2	The closed-loop position response is shown in Figure Ans.4.2. You can generate this using the plot_tanks_2_rsp.m script.

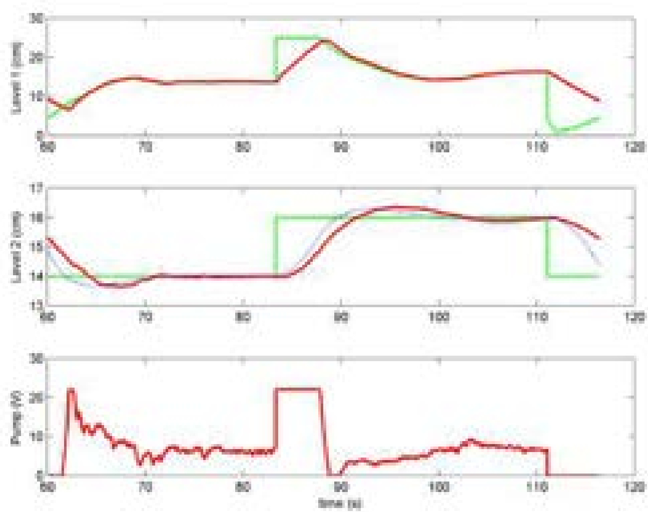


Figure Ans.4.2: Measured closed-loop ball position control response.

□□□

9. K-1, B-9 Measure the steady-state error, the percent overshoot and the peak time of the response obtained on the actual system. Does the Tank 2 response satisfy the specifications given in Section 2.1.4?

Answer 4.8

Outcome	Solution
K-1	As shown in Figure Ans.4.2, the measured response eventually settles at the setpoint, thus $$e_{ss} = 0.035 \text{ cm}$$ Looking at the step that begins at 83.3 seconds, the time it takes for the measured response to settle to 2% of its final value, i.e., in this case 16.3 cm since there is no overshoot, is approximately at 96.4 second mark. Thus $$t_s = 96.4 - 83.3 = 13.1 \text{ s}$$ Finally, the measured response peaks at 16.37 cm at the 95.3 second mark. Thus the percent overshoot of tank 2 is $$PO = 100 \times \frac{16.37 - 16}{2} = 18.3 \text{ \%}$$
B-9	The steady-state error is very small and considered negligible. The settling time satisfies the requirement given in Section 2.1.4. However, the overshoot exceeds the limit given in Section 2.1.4. This is again, probably due to the control signal being saturated.

□□□

4.4 Results

B-6 Fill out Table 4.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Units
Pre Lab Questions			
Tank 1 Control Gains			
Proportional Control Gain	$k_{p,1}$	7.21	V/cm
Integral Control Gain	$k_{i,1}$	9.11	V/(cm-s)
Feed Forward Control Gain	$K_{ff,1}$	2.39	V/$\sqrt{cm}$
Tank 2 Control Gains			
Proportional Control Gain	$k_{p,2}$	5.09	cm/cm
Integral Control Gain	$k_{i,2}$	1.74	1/s
Feed Forward Control Gain	$K_{ff,2}$	1	cm/cm
Tank 2 Control Simulation			
Steady-state error	e_{ss}	0	cm
Settling time	t_s	1	s
Percent overshoot	PO	12.5	%
Tank 2 Control Implementation			
Steady-state error	e_{ss}	0.035	cm
Settling time	t_s	13.1	s
Percent overshoot	PO	18.3	%

Table 4.1: Tank 2 Level Control Results Results

5 SYSTEM REQUIREMENTS

Required Software

- Microsoft Visual Studio (MS VS)
- Matlab® with Simulink®, Real-Time Workshop, and the Control System Toolbox
- QUARC®

See the QUARC® software compatibility chart in [3] to see what versions of MS VS and Matlab are compatible with your version of QUARC and for what OS.

Required Hardware

- Data acquisition (DAQ) device that is compatible with QUARC®. This includes Quanser DAQ boards such as Q2-USB, Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices. For a full listing of compliant DAQ cards, see Reference [1].
- Quanser Coupled Tanks.
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- QUARC®is installed on your PC, as described in [2].
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *QUARC Analog Loopback Demo*).
- Coupled Tanks and amplifier are connected to your DAQ board as described Reference [5].