Ninevah University
College of Electronics Engineering
Department of Systems and Control Engineering
Second Stage / 1st semester 2025-2026

LECTURE 1 : INTRODUCTION TO SIGNALS AND SYSTEMS

By
Abdulhameed Nabeel Hameed

Place and Date: Mosul / College of Electronics Engineering, 28/9/2025

COURSE OUTLINES

- Introduction to signals and Systems
- Signals classifications
- Systems properties
- Time-domain analysis
- Fourier Series

WHY STUDY SIGNALS AND SYSTEMS?

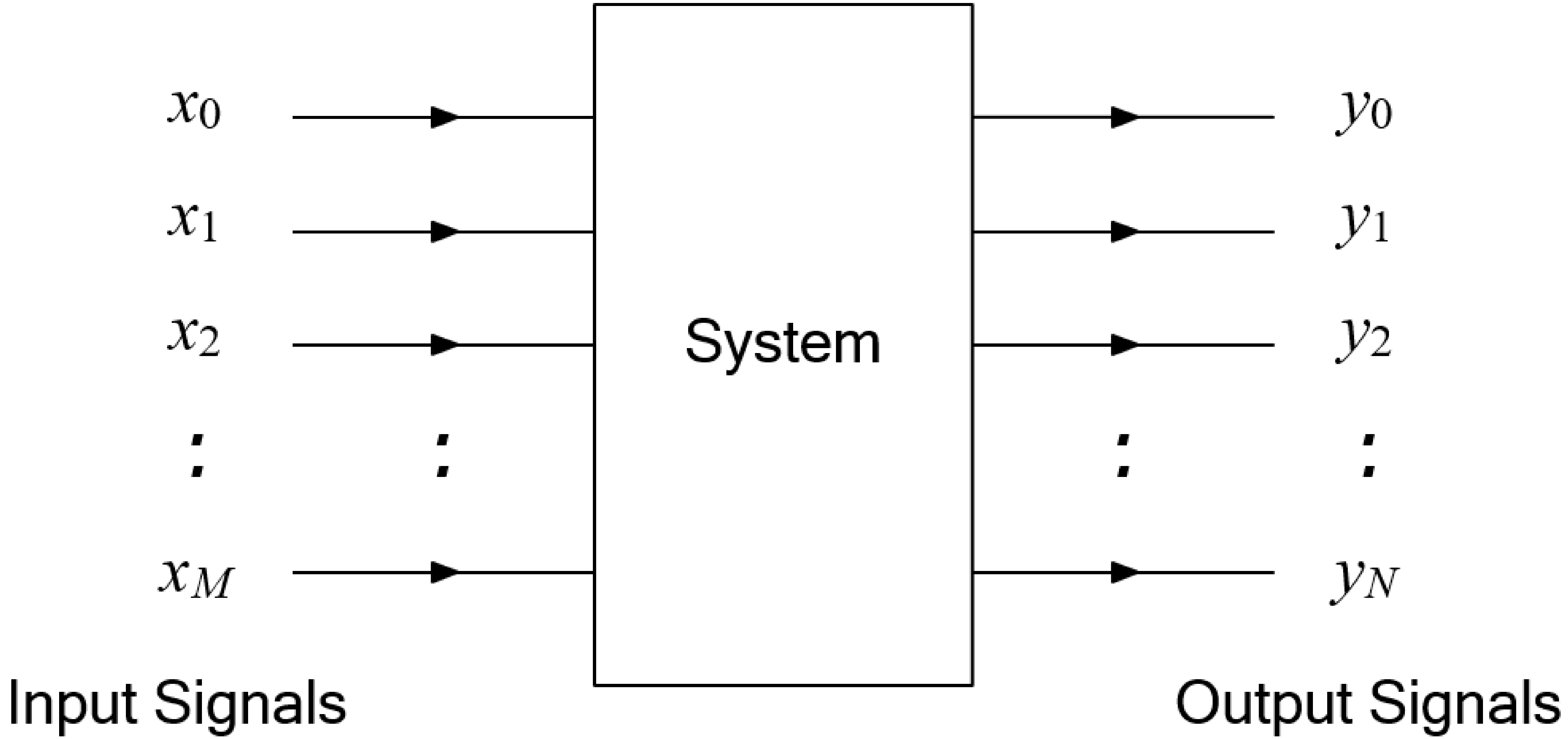
- **Fundamental to Digital World –** Every digital device (phones, computers, sensors) relies on signal representation and processing.
- **Improves Technical Intuition –** Helps engineers visualize and interpret how systems respond in time and frequency domains.
- **Optimization and Efficiency –** Enables designing systems that use less power, bandwidth, and resources.
- **Noise and Interference Handling –** Essential to understand how to filter, reduce, and manage unwanted signals.
- **Supports Integrative Fields** – Useful in AI, robotics, engineering, and seismology.
- **Simulation and Modeling Skills** – Builds capability in tools like MATLAB, Python, CST, and Simulink for real-world system analysis.

SIGNALS

- A signal is a function of one or more variables that conveys information about some (usually physical) phenomenon.
- For a function f , in the expression f (t1,t2, . . . , tn), each of the {tn} is called an independent variable, while the function value itself is referred to as a dependent variable.
- Some examples of signals include:
  - □ a voltage or current in an electronic circuit
  - □ the position, velocity, or acceleration of an object
  - □ a force or torque in a mechanical system
  - □ a flow rate of a liquid or gas in a chemical process
  - □ a digital image, digital video, or digital audio

SYSTEMS

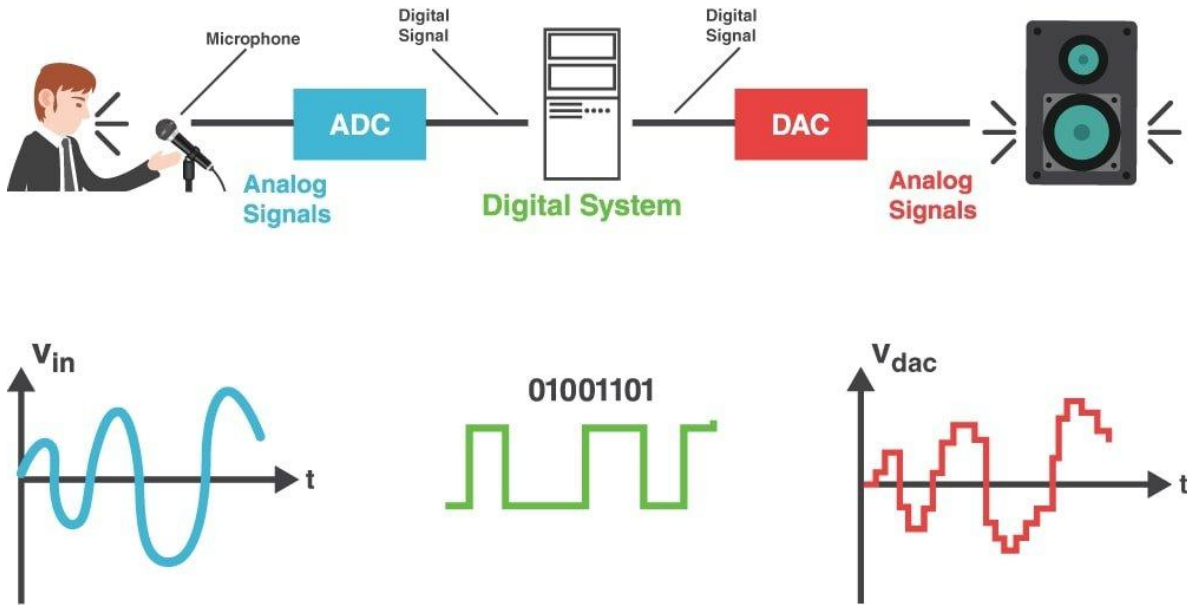
- A system is an entity that processes one or more input signals in order to produce one or more output signals.



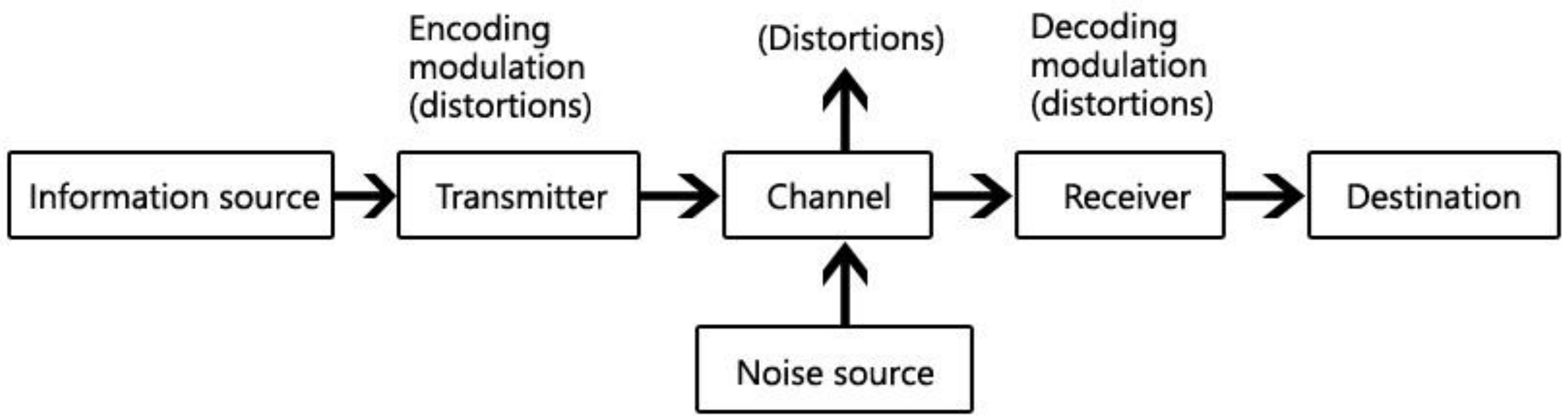
CLASSIFICATION OF SYSTEMS

- Number of inputs:
  - A system with one input is said to be single input (SI).
  - A system with more than one input is said to be multiple input (MI).
- Number of outputs:
  - A system with one output is said to be single output (SO).
  - A system with more than one output is said to be multiple output (MO).
- Types of signals processed:
  - A system can be classified in terms of the types of signals that it processes.
  - Consequently, terms such as the following (which describe signals) can also be used to describe systems:
    - one-dimensional and multi-dimensional.
    - continuous-time (CT) and discrete-time (DT), and analog and digital.
  - For example, a continuous-time (CT) system processes CT signals and a discrete-time (DT) system processes DT signals.

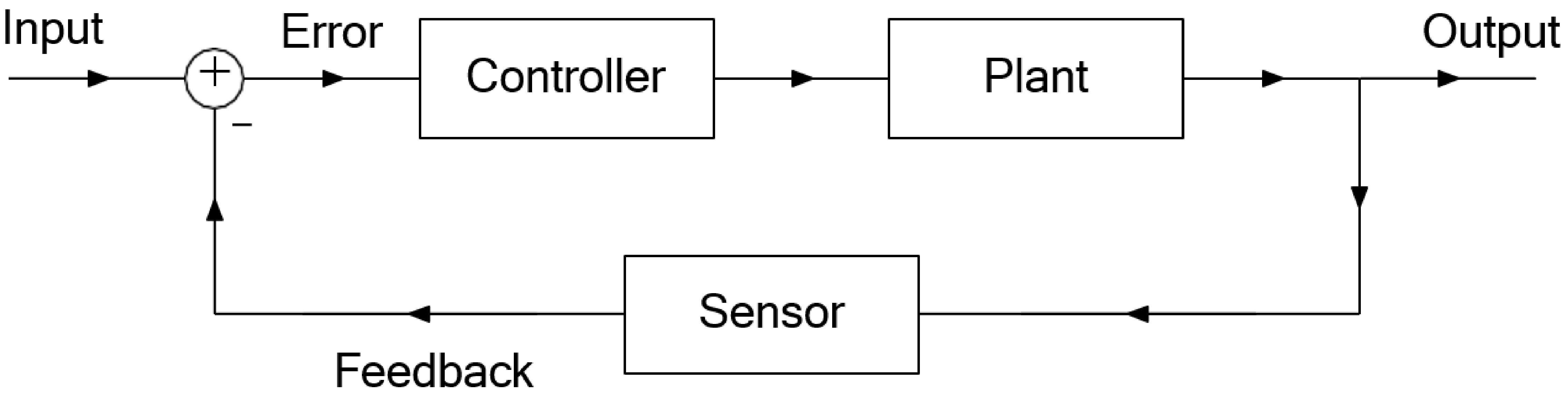
EXAMPLES FOR SOME OF SYSTEMS



Signal Processing Systems



General Structure of a Communication System



General Structure of a Feedback Control System

CLASSIFICATION OF SIGNALS

- Continuous-time and discrete-time signals.
- Analog and digital signals.
- Periodic and aperiodic signals.
- Energy and power signals.
- Deterministic and probabilistic signals.
- Even and odd signals.
- Real and imaginary signal.

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LECTURE 2 : CLASSIFICATION OF SIGNALS

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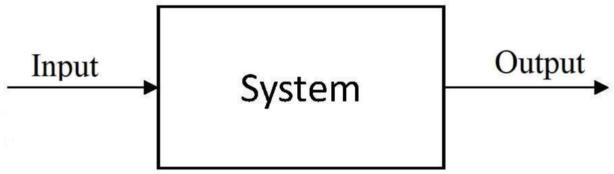
Basic Definitions

- Signal

  A signal is defined as function of one ore more variables that conveys information.

- System

  A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.

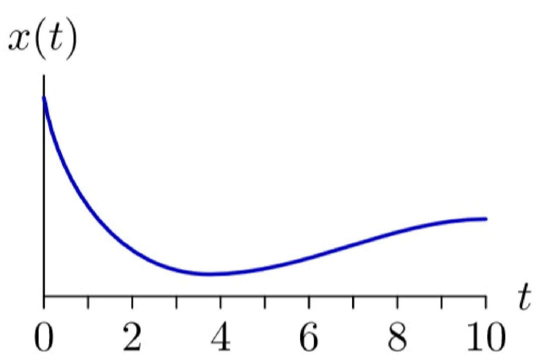


Signals Classification

Generally signals are classified as follows :

- Continuous-time signals:

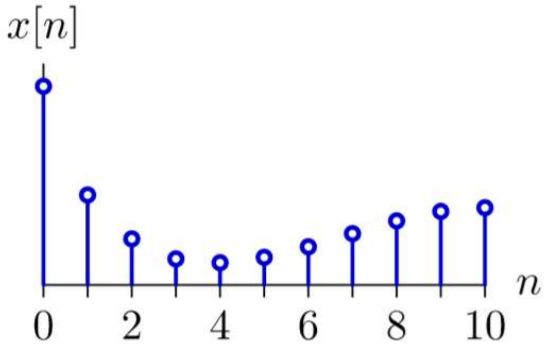
  The signals that are defined at every possible value of time as shown in the figure below. These signals are also called analog signals.



Signals Classification

- Discrete-time signals:

The signals that can be defined and represented at certain time instants of the sequence. Discrete signals are usually presented as $x(n)$ or $x[n]$.



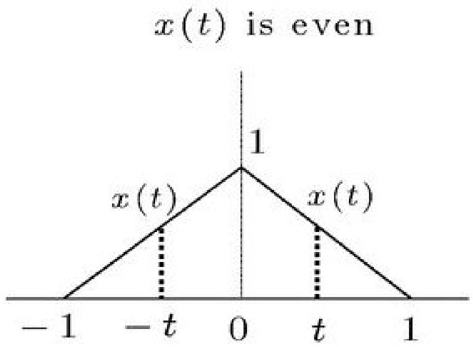
Where $n = 0, \pm 1, , \pm 2, \ldots, \pm\infty$, and n is integer

Signals Classification

- Even Signals:

A signal is said to be an even signal if it satisfies the following condition

$$x(-t) = x(t) \text{ for all } t.$$

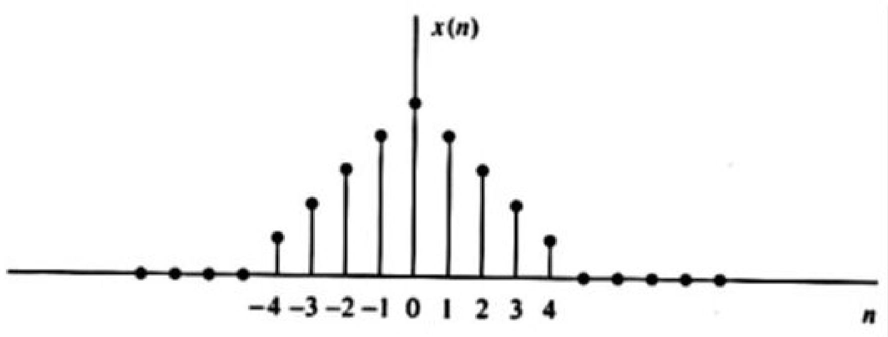


Signals Classification

- <u>Discrete Even Signals:</u>

  A discrete signal is said to be an even signal if it satisfies the following condition

$$x(-n) = x(n) \text{ for all integers } n.$$

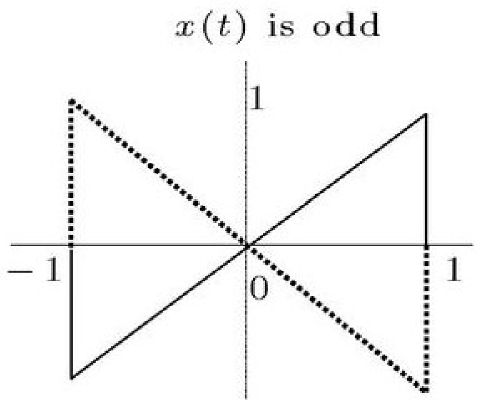


Signals Classification

- <u>Odd Signals:</u>

  A signal is said to be an odd signal if it satisfies the following condition

$$x(-t) = -x(t) \text{ for all } t.$$

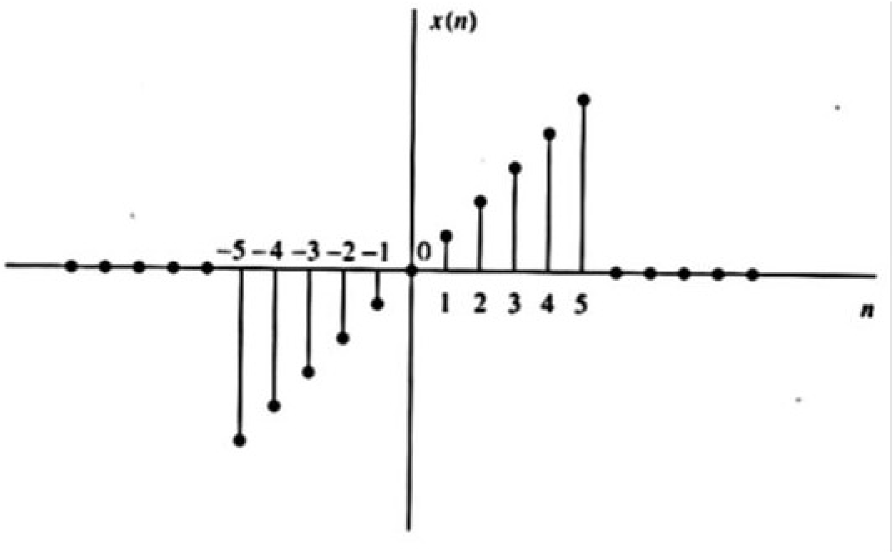


Signals Classification

- Discrete Odd Signals:

  A discrete signal is said to be an odd signal if it satisfies the following condition

$$x(-n) = -x(n) \text{ for all integers } n.$$



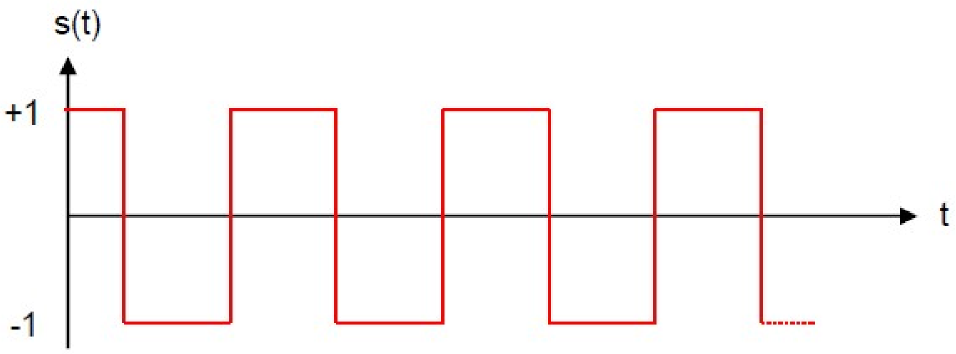
Signals Classification

- Periodic Signals:

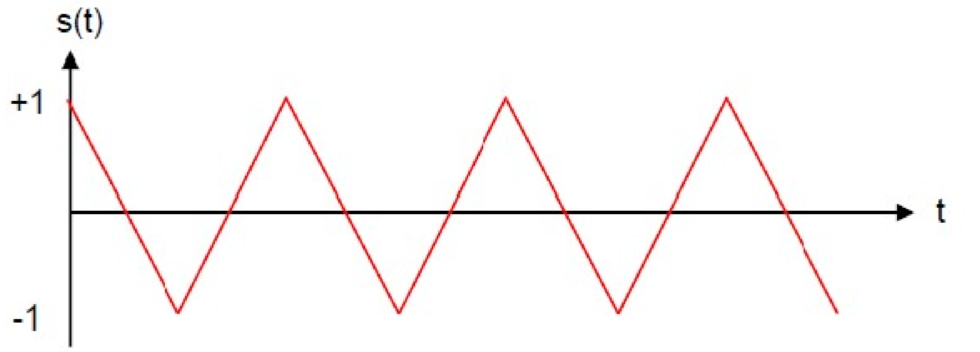
  A periodic signal is a function that satisfies the following condition

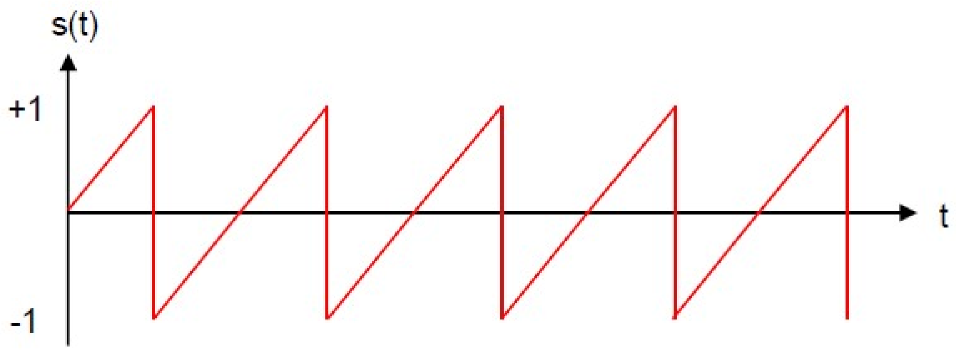
$$s(t) = s(t + nT) \text{ for all } t,$$

where n is an integer, $n = 1, 2, , \ldots, \infty$, and T is the period of the signal.



Signals Classification





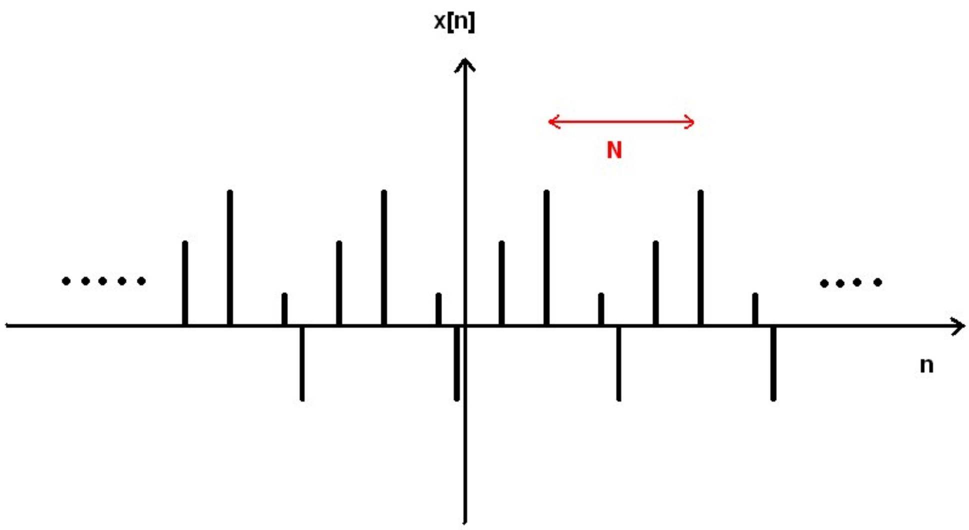
Signals Classification

- Discrete Periodic Signals:

  A discrete periodic signal is a function that satisfies the following condition

  $$x(n) = x[n] = x[n + N] \text{ for all integers } n,$$

  where N is a positive integer, which represents the fundamental period of the discrete-time signal $x[n]$.

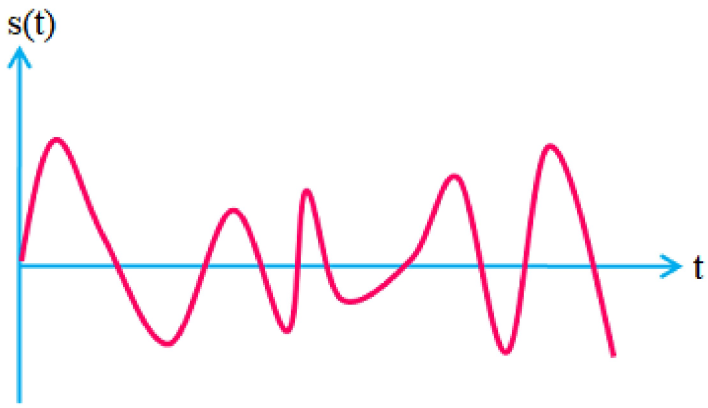


Signals Classification

- <u>Aperiodic Signals:</u>

  A signal that does not repeats itself after a specific interval of time. Hence,

$$s(t) \neq s(t + nT) \text{ for all } t,$$

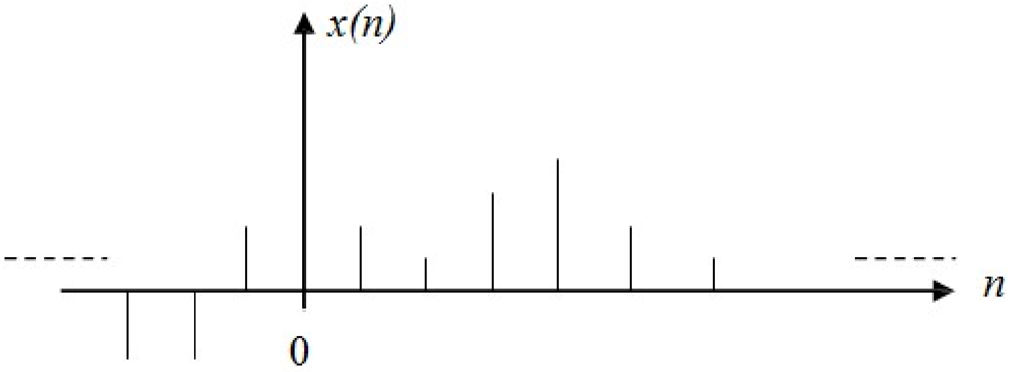


Signals Classification

- <u>Discrete Aperiodic Signals:</u>

  A discrete periodic signal is a function that satisfies the following condition

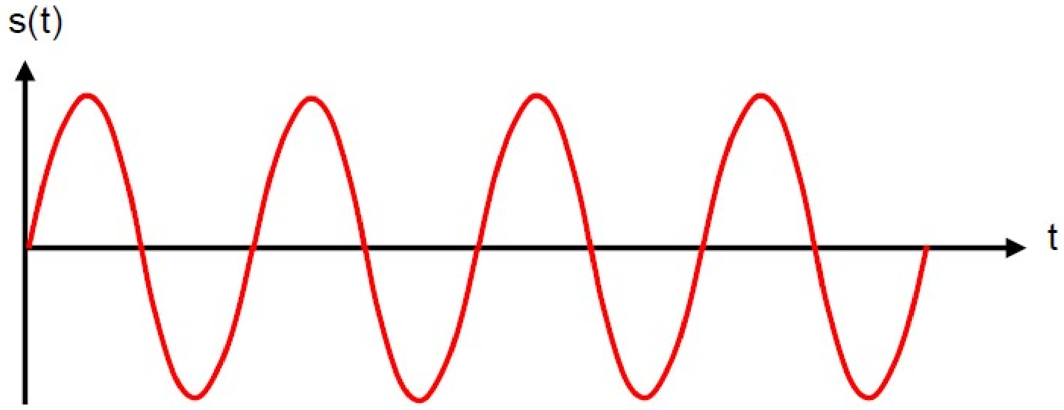
$$x(n) \neq x(n + N) \text{ for all integers } n,$$



Signals Classification

- Deterministic Signals:

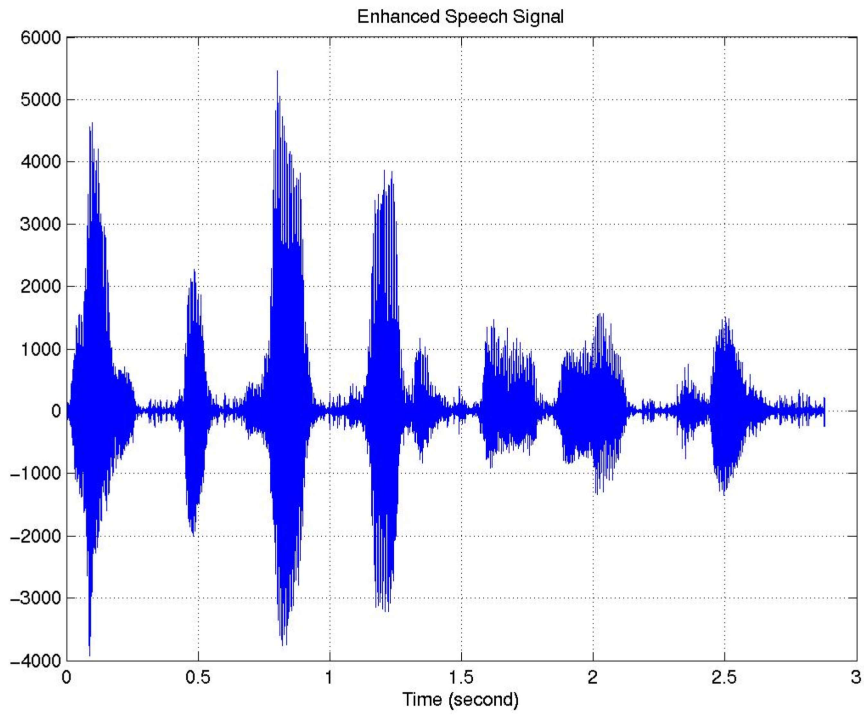
  A signal is said to be deterministic if its future values can be predicted. Therefore, deterministic signals are used by transmitters to carry information. An example of deterministic signal is $s(t) = A.cos(2\pi ft + \theta)$.



Signals Classification

- Random Signals:

  A signal is said to be random if it has some degree of uncertainty in its values before it actually occurs.



Signals Classification

- Energy Signals:

  A signal is said to be an energy signal, if and only if its total energy satisfies the condition

$$0 < E < \infty.$$

- Power Signals:

  A signal is referred to as a power signal, if and only if the average power of the signal satisfies the condition

$$0 < P < \infty.$$

Signals Classification

- Energy

  The energy of a continuous-time signal, $x(t)$, is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt.$$

- Power

  The average power of a continuous-time signal, $x(t)$, is defined as

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt = \frac{1}{T}\int_{t}^{t+T} |x(t)|^2 dt.$$

Signals Classification

- Energy

  The energy of a discrete-time signal, $x(n)$, is given as

$$E = \sum_{n=-\infty}^{\infty} x^2(n).$$

- Average Power

  The average power of a discrete-time signal, $x(n)$, is defined as

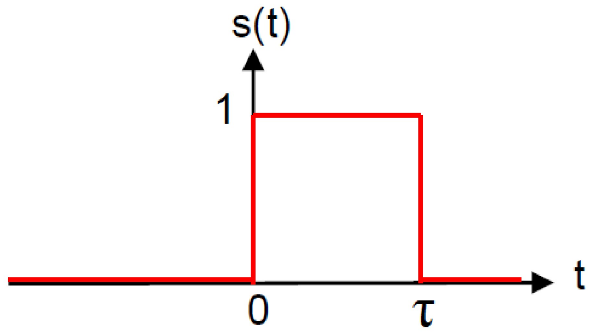
$$P = \frac{1}{N} \sum_{0}^{N-1} x^2(n).$$

Signals Classification

- The energy and power classification of the signals are mutually exclusive. In particular, an energy signal has zero average power, whereas a power signal has infinite energy.
- Periodic signals are usually viewed as power signals.
- Signals that are both deterministic and Aperiodic (nonperiodic) are energy signals

Signals Classification

- Example: Determine if the following signal is an energy or power signal.



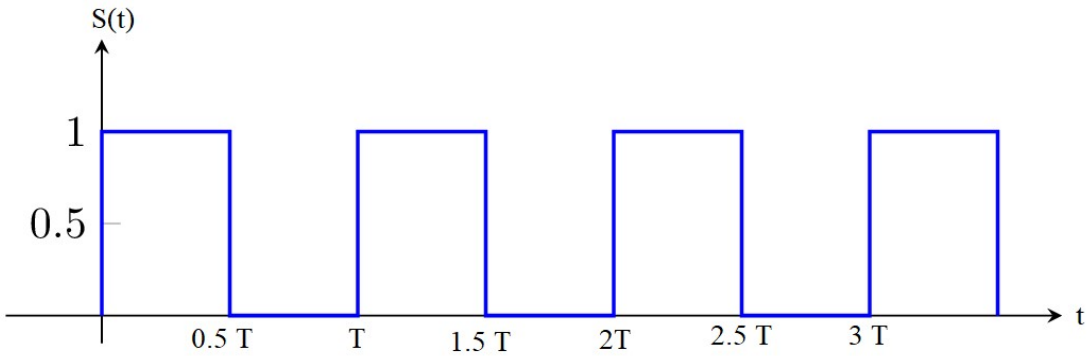
Solution:

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_0^{\tau} 1 \; dt = t\Big|_0^{\tau} = \tau \text{ joules.}$$

Hence P=0 watt

Signals Classification

- Example: Determine if the following signal is an energy or power signal.



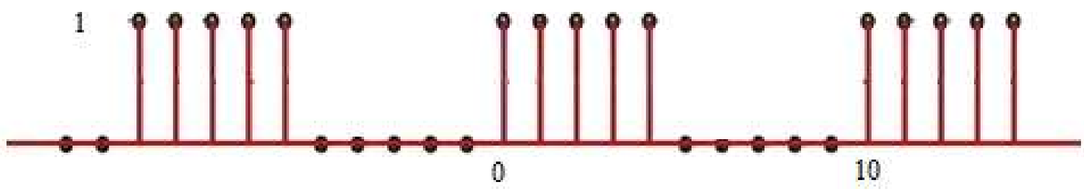
Solution:

$$P = \frac{1}{T}\int_t^{t+T} |s(t)|^2 \; dt = \frac{1}{T}\int_0^{0.5T} 1 \; dt = \frac{t}{T}\Big|_0^{0.5T} = 0.5 \text{ watt.}$$

Hence, E=∞

Signals Classification

- Example: Determine if the following signal is an energy or power signal.



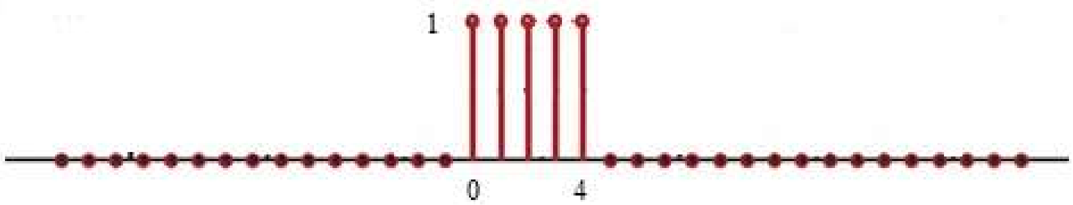
Solution:

$$P = \frac{1}{N}\sum_{0}^{N-1} x^2(n) = \frac{1}{10}\sum_{0}^{9} x^2(n)$$
$$= \frac{1}{10}\left(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 0^2 + 0^2 + 0^2 + 0^2 + 0^2\right) = 0.5 \text{ watt.}$$

Hence, E=∞

Signals Classification

- Example: Determine if the following signal is an energy or power signal.



Solution:

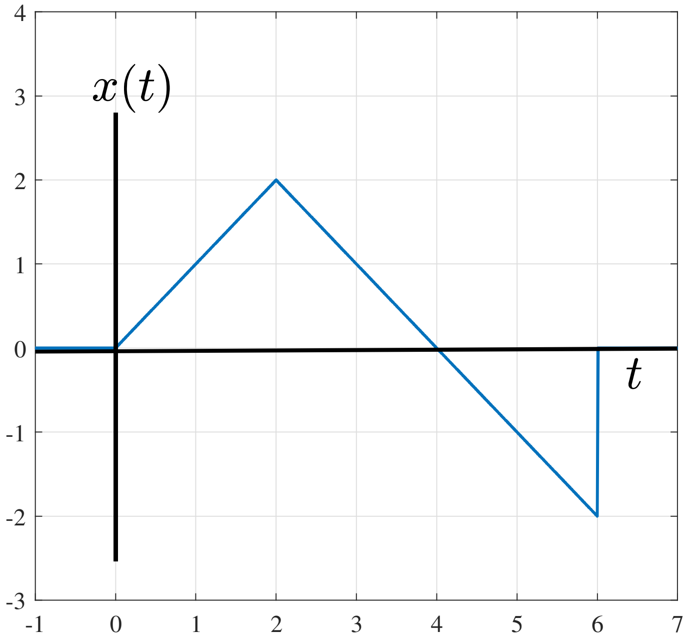
$$E = \sum_{n=-\infty}^{\infty} x^2(n) = E = \sum_{n=0}^{4} x^2(n) = \left(1^2 + 1^2 + 1^2 + 1^2 + 1^2\right)$$
$$= 5 \text{ joules.}$$

Hence, P=0 watt

Signals Classification

- Homework: Determine if the following signal is an energy or power signal.

$$x(t) = \begin{cases} t, & 0 \leq t \leq 2 \\ 4 - t, & 2 \leq t \leq 6 \\ 0, & \text{otherwise} \end{cases}$$



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LECTURE 3 SIGNAL OPERATIONS

By
Abdulhameed Nabeel Hameed

Basic Operation on Signals

An issue of major importance is the use of systems to process or manipulate signals. This issue involves a combination of some basic operations.

However, two classes of these operations can be identified that are:

① **Operation of dependent variables**

A. Amplitude scaling (Amplitude shifting, Amplification): The scaled signal $ax_{(t)}$ is $x_{(t)}$ multiplied by the factor a where a is a constant real number, such as, the physical device that performs amplitude scaling is an electronic amplifier.

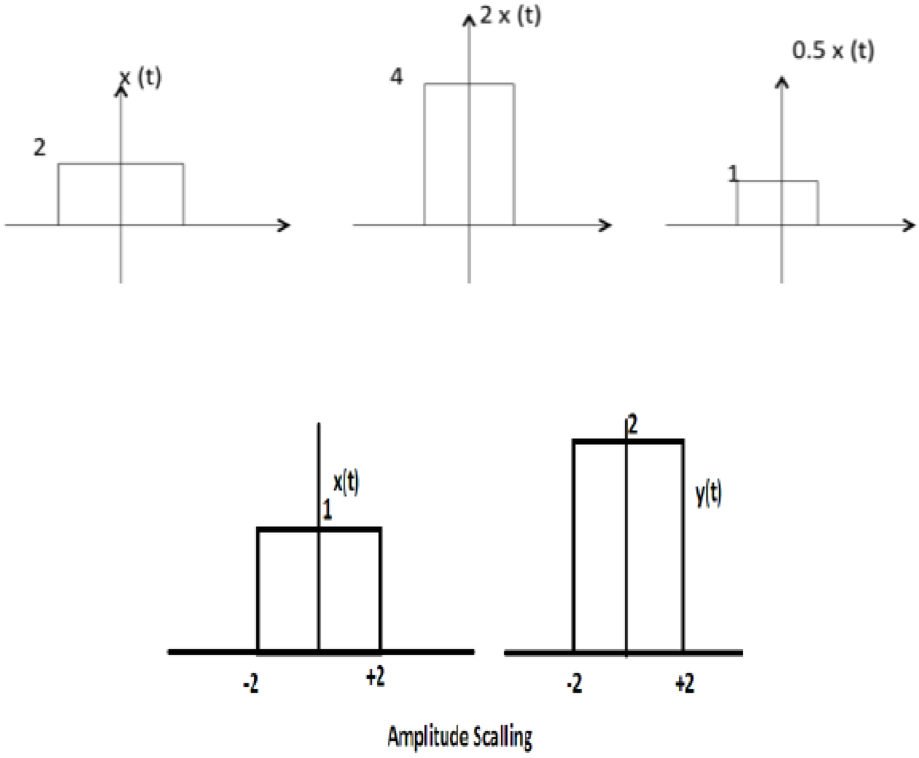


Figure 1: The amplitude scaling operation

In this case, only the values of y axis is changed since the amplitude is associated with this axis while, the values of x axis is constant.

B. Addition: If $x_1(t)$ and $x_2(t)$ denote a pair of CTSs. The signal z(t) obtained by the addition of $x_1(t)$ and $x_2(t)$ is defined by:

$z(t) = x_1(t) + x_2(t)$

In the case of DTS, it written as:

$z[n] = x_1[n] + x_2[n]$

It can be noted that the addition of two signals is nothing but addition of their corresponding amplitudes. This can be best explained by using the following example:

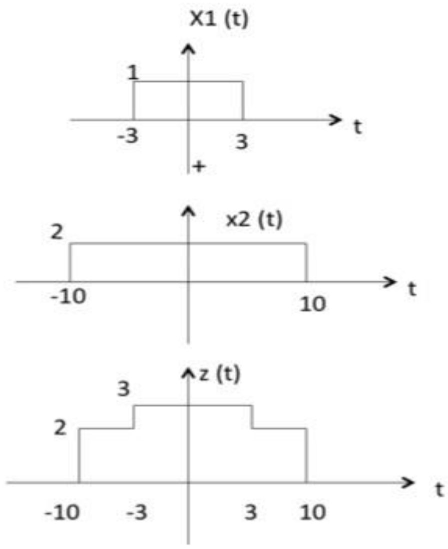


Figure 2: The addition operation

C. Subtraction: If $x_1(t)$ and $x_2(t)$ refer to a pair of CTSs. Then, the signal z(t) obtained by the subtracting of $x_1(t)$ from $x_2(t)$ is defined by:

$z(t) = x_1(t) - x_2(t)$

In the case of DTS, it written as:

$z[n] = x_1[n] - x_2[n]$

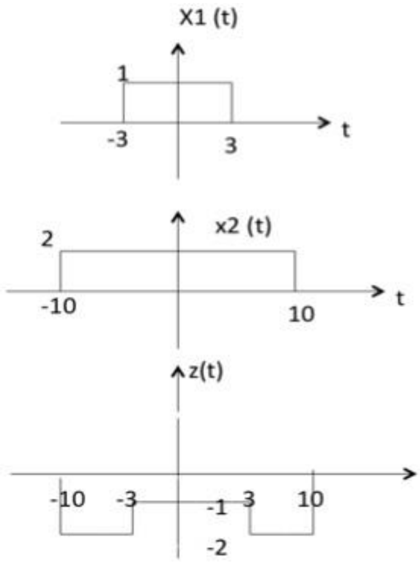


Figure 3: The subtraction operation

D. Multiplication: let $x_1(t)$ and $x_2(t)$ denote a pair of CTSs. The signal z(t) resulting from the multiplication of $x_1(t)$ and $x_2(t)$ is defined by the following equation:

$z(t) = x_1(t) * x_2(t)$

That is, for each prescribed time (t) the value of z(t) is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$.

For discrete-time signals we write:

$z[n] = x_1[n]\ x_2[n]$

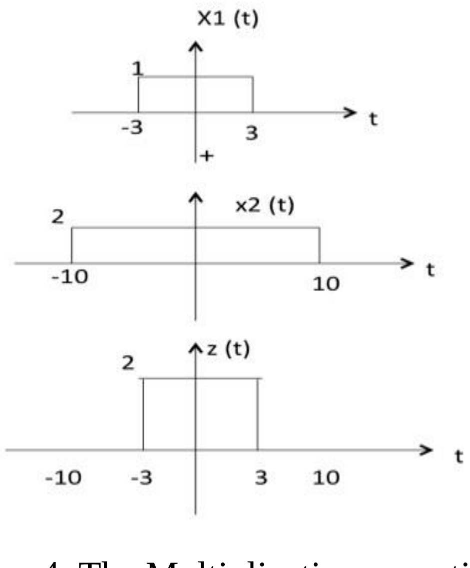


Figure 4: The Multiplication operation

② Operation of independent variables

A. Time shifting: Suppose that we have a signal x(t) and we define a new signal by adding/subtracting a finite time value to/from it. We now have a new signal, z(t). Graphically, this kind of signal operation results in a positive or negative "shift" of the signal along its time axis. However, note that while doing so, none of its characteristics are altered. This means that the time-shifting operation results in the change of just the positioning of the signal without affecting its amplitude.

If CTS is x(t), then $z(t)=x(t-T)$ is the signal x(t) shifted to the right by T units.

If CTS is x(t), then $z(t)=x(t+T)$ is the signal x(t) shifted to the left by T units.

When DTS is x[n], then $z[n]=x[n-N]$ is the signal x[n] shifted to the right by N samples.

When DTS is x[n], then $z[n]=x[n+N]$ is the signal x[n] shifted to the left by N samples.

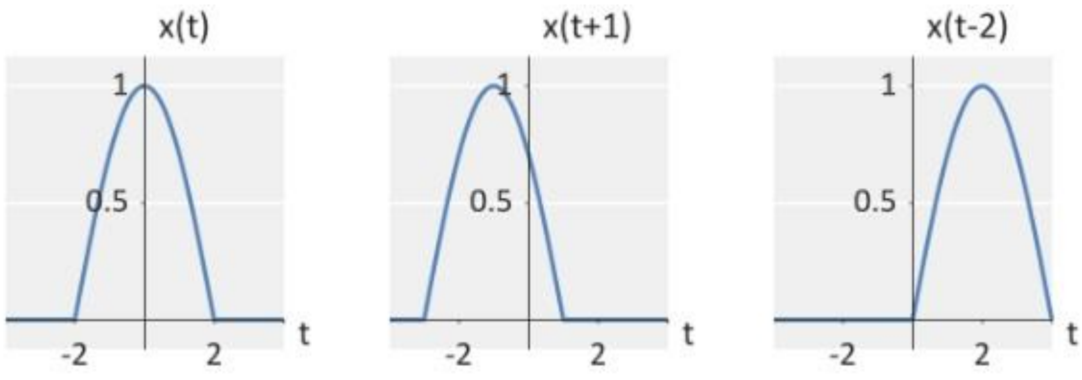


Figure 5: The time shifting operation

B. Time scaling: is a compression or expansion of a signal in time let x(t) denote a CTS, the signal z(t) obtained by scaling the independent variable, time (t), by a factor "a" is defined by two cases which are:

- If $a > 1$, the signal z(t) is a compressed version of x(t). In this case:

  $z(t) = x(at)$

- If a < 1, the signal z(t) is an expanded (stretched) version of x(t). Thus, the resulted signal z(t) is computed as:

  z(t) = x(t/a)

  Note: the factor "a" must not be equal to 0.

This mean that the signal x(t) is scaled in time by multiplying the time variable by a positive constant (a), to produce z(t).

All these cases are described in the below figure.

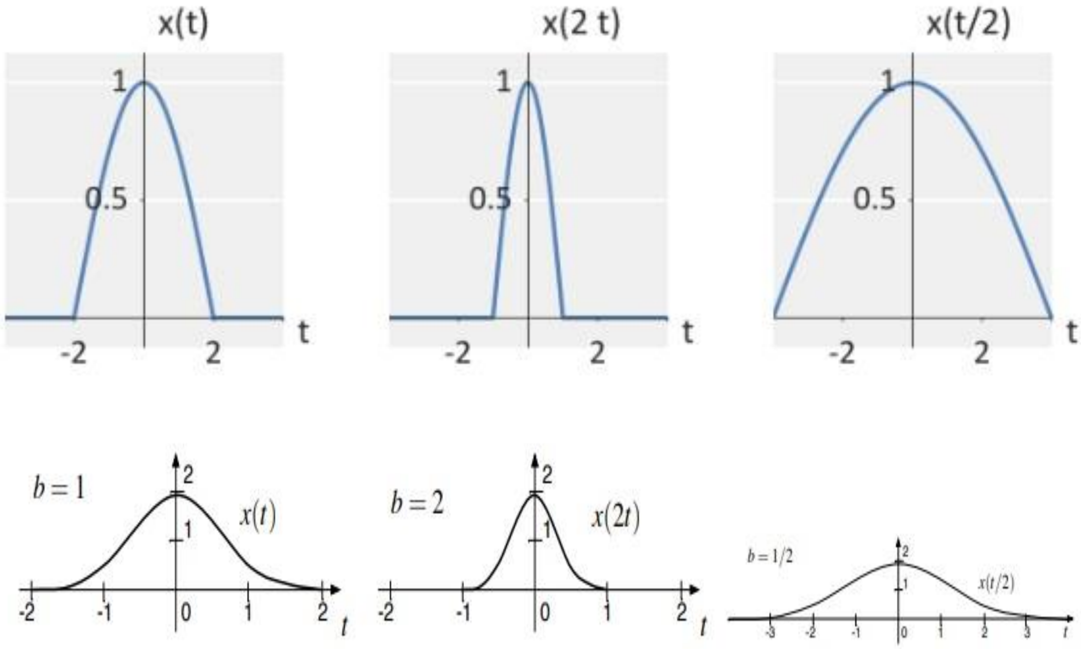


Figure 6: The time scaling operation

C. Time reversal (time inversion, reflection): The signal z(t) represents a reflected version of x(t) about the amplitude axis. Let x(t) denotes a CTS signal and z(t) denotes the signal obtained by replacing time (t) with (–t), as shown in the next two cases:

- The CTS is represented as:

y(t)=x(−t)

- While the DTS is as follows:

y[n]=x[−n]

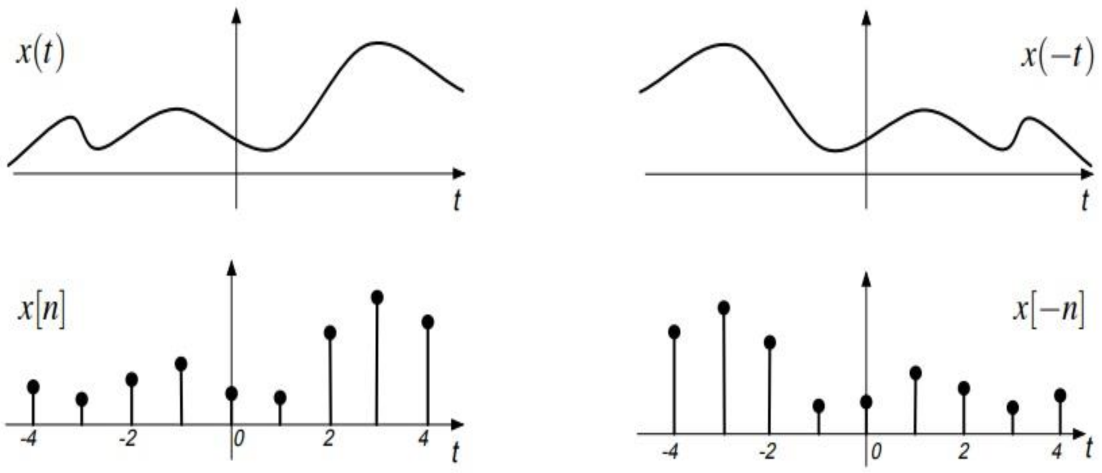
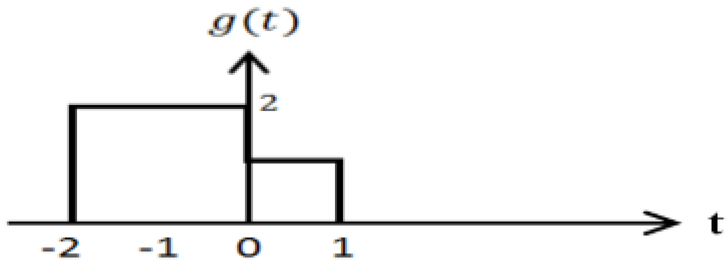


Figure 7: The time reversal operation

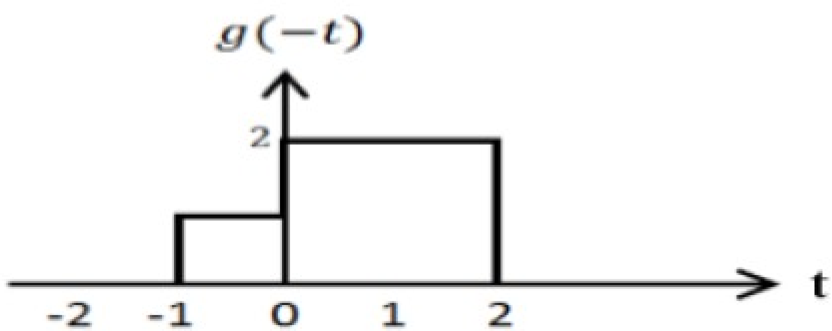
Example: For the signal shown in the figure below, sketch $g\left(-1-\frac{t}{2}\right)$.



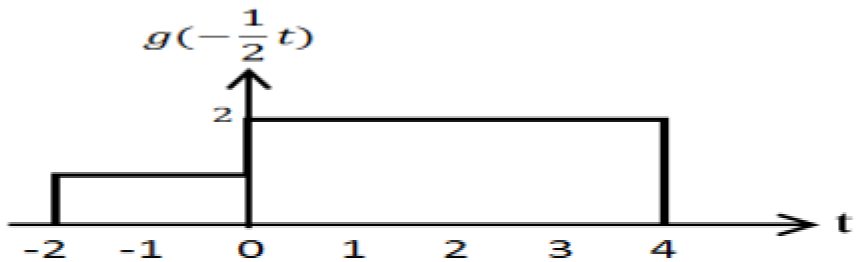
Solution:

$$g\left(-1-\frac{t}{2}\right)=g\left(-\frac{t}{2}-1\right)=g\left(-\frac{1}{2}(t+2)\right)$$

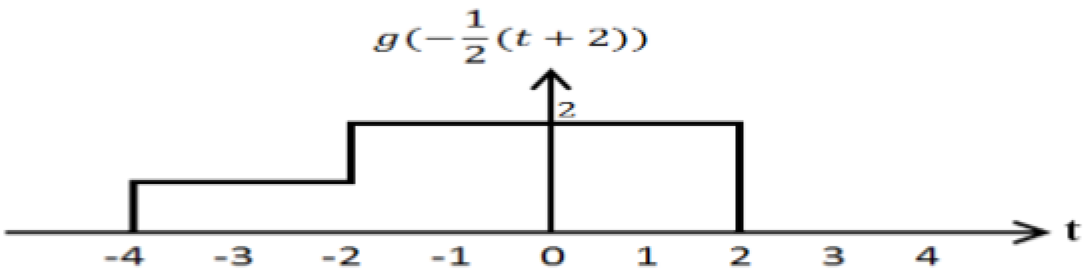
Time-inverse



Time-scaling by $\frac{1}{2}$



Shifting to the left by 2



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LECTURE 4 : BASIC TYPES OF COMMON SIGNALS

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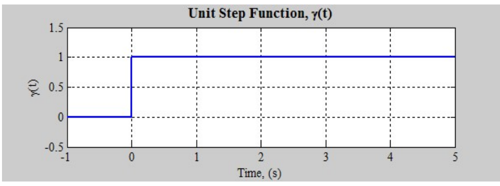
BASIC TYPES OF COMMON SIGNALS

- Unit step signals.
- Unit impulse signals.
- Ramp function.
- Complex exponentials.
- Sinusoidal Signals.
- Sinc function signals.
- Signum function signals.
- Rectangular signals
- Triangular signals

Unit step signals

Unit step function is denoted by u(t). It is defined as

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



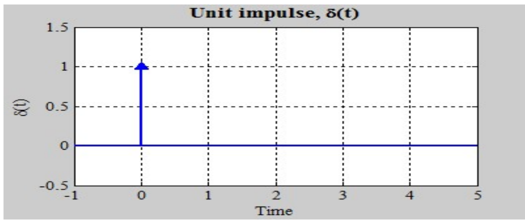
Unit impulse signals

Impulse function is denoted by δ(t). and it is defined as

$$\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \qquad \int_{-\infty}^{\infty} \delta(t)dt = u(t)$$

For any continuous-time function f(t) we have $f(t)\delta(t) = f(0)\delta(t)$

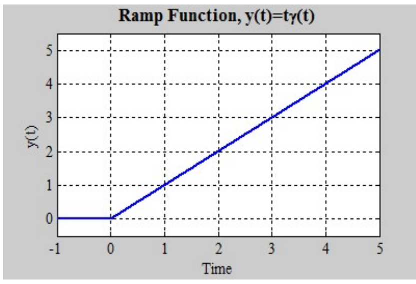
and



Ramp function signals

Ramp signal is denoted by r(t), and it is defined as

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

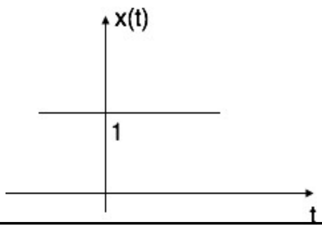


Real and Complex exponentials signals

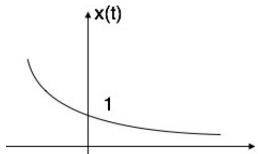
Exponential signal is in the form of x(t) = $e^{\alpha t}$

The shape of exponential can be defined

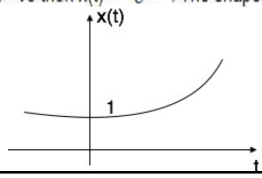
by Case i: if $\alpha = 0 \rightarrow x(t) = e^{0} = 1$



Case ii: if $\alpha < 0$ i.e. -ve then x(t) = $e^{-\alpha t}$. The shape is called decaying exponential.



Case iii: if $\alpha > 0$ i.e. +ve then x(t) = $e^{\alpha t}$. The shape is called raising exponential.



The complex exponential signal is defined as $z(t) = Ae^{j(\omega_0 t+\phi)}$

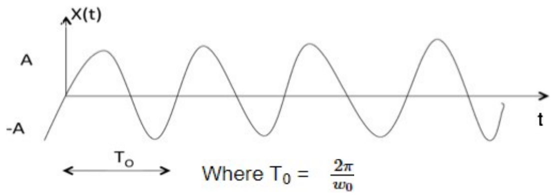
Using Euler's formula $e^{j\theta} = \cos\theta + j\sin\theta$

$$= A\cos(\omega_0 t + \phi) + jA\sin(\omega_0 t + \phi)$$

Sinusoidal Signals

Sinusoidal signal is in the form of

$x(t) = A\cos(w_0 \pm \phi)$ or $A\sin(w_0 \pm \phi)$

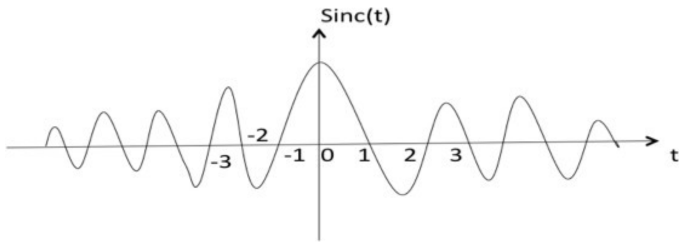


Sinc function

It is denoted as sinc(t) and it is defined as sinc

$$(t) = \frac{\sin\pi t}{\pi t}$$

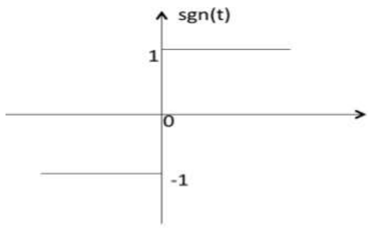
$= 0$ for $t = \pm1, \pm2, \pm3\ldots$



Signum function signals

Signum function is denoted as sgn(t). It is defined as

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



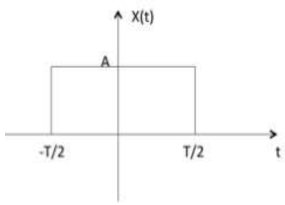
sgn(t) = 2u(t) – 1

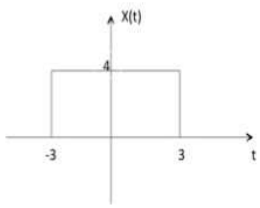
Rectangular Signal

Let it be denoted as x(t) and it is defined as

$x(t) = A\, rect\left[\frac{r}{T}\right]$

ex: $4\, rect\left[\frac{r}{6}\right]$



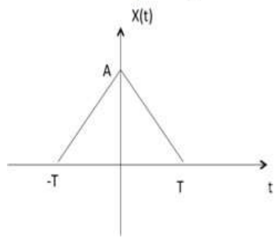


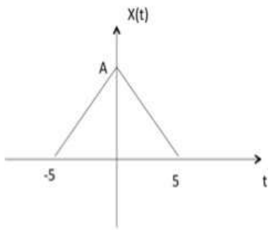
Triangular Signal

Let it be denoted as x(t)

$x(t) = A\left[1 - \frac{|t|}{T}\right]$

ex: $x(t) = A\left[1 - \frac{|t|}{5}\right]$



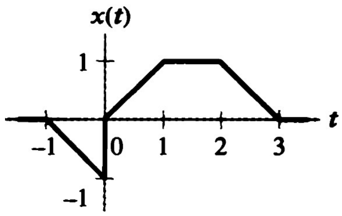


Example 1: Let x(t) and y(t) be given in Figs. (a) and (b), respectively. Carefully :sketch the following signals

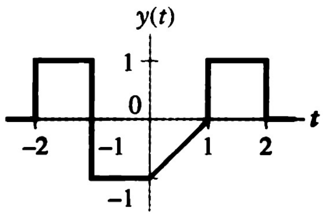
(a) $x(t)y(t-1)$

(b) $x(t-1)y(-t)$

(c) $x(t+1)y(t-2)$



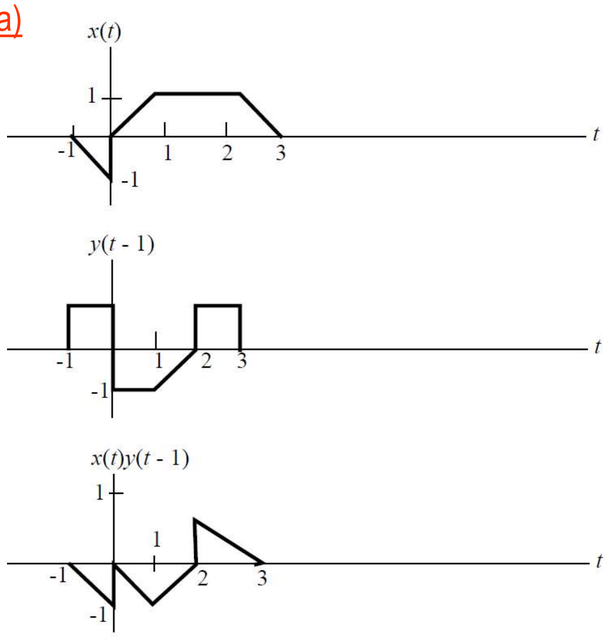
(a)



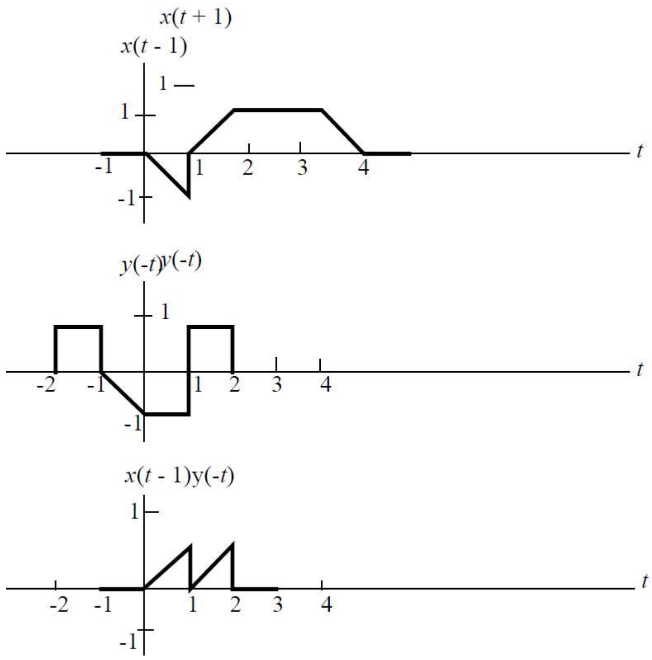
(b)

Solution

(a)

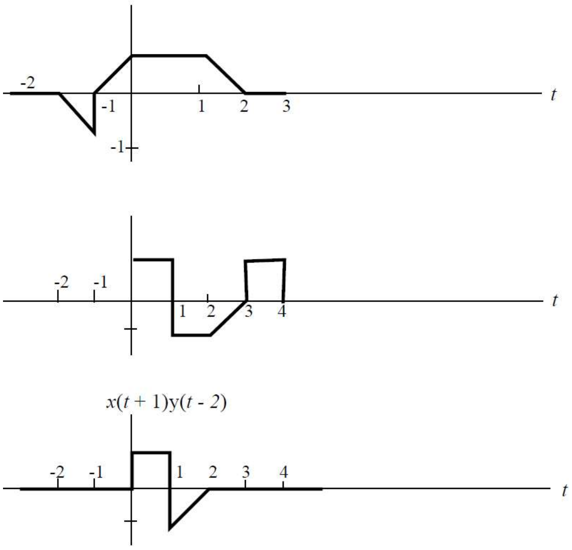


(b)



Solution

(c)



:Example 2: Sketch the waveforms of the following signals

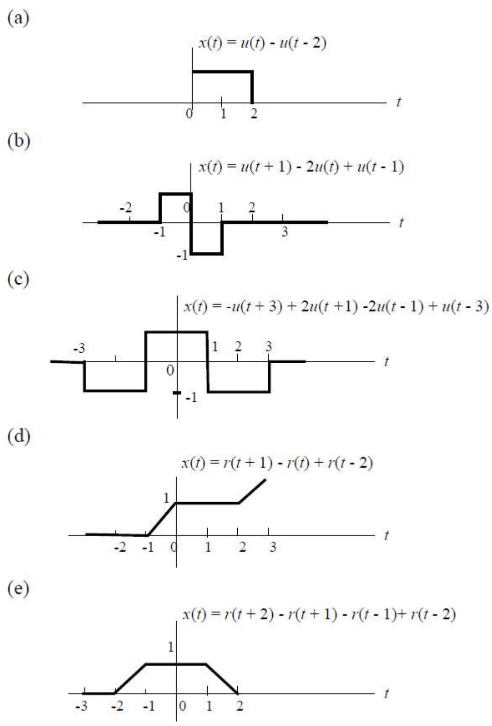
(a) $x(t) = u(t) - u(t-2)$

(b) $x(t) = u(t+1) - 2u(t) + u(t-1)$

(c) $x(t) = -u(t+3) + 2u(t+1) - 2u(t-1) + u(t-3)$

(d) $y(t) = r(t+1) - r(t) + r(t-2)$

(e) $y(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$



Homework: Let x[n] and y[n] be given in Figs. (a) and (b), respectively. Carefully sketch :the following signals

(a) $x[2n]$

(b) $x[3n - 1]$

(c) $y[1 - n]$

(d) $y[2 - 2n]$

(e) $x[n - 2] + y[n + 2]$

(f) $x[2n] + y[n - 4]$

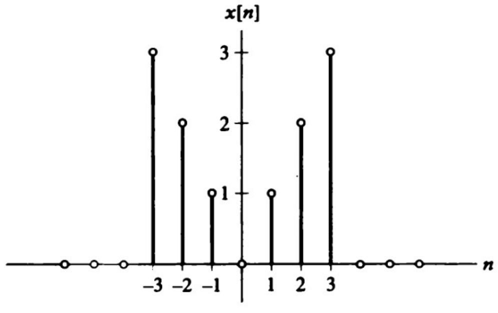
(g) $x[n + 2]y[n - 2]$

(h) $x[3 - n]y[n]$

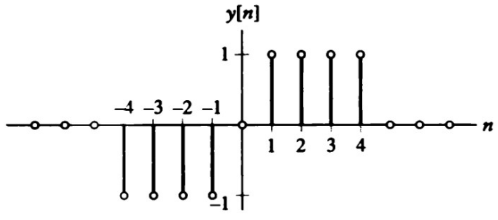
(i) $x[-n]y[-n]$

(j) $x[n]y[-2 - n]$

(k) $x[n + 2]y[6 - n]$



(a)



(b)

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LECTURE 5 : INTRODUCTION TO SYSTEMS

By
Abdulhameed Nabeel Hameed

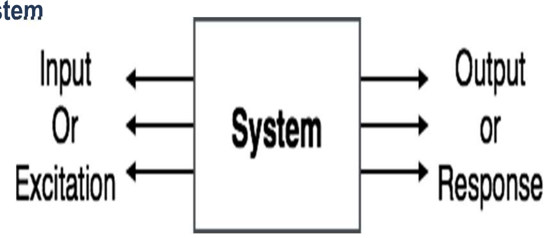
SYSTEM DEFINITION

What is System?

System is a device or combination of devices, which can operate on signals and produces corresponding response. Input to a system is called as excitation and output from it is called as response.

For one or more inputs, the system can have one or more outputs.

Example: Communication System



CLASSIFICATION OF SYSTEMS

Systems are classified into the following categories:

- linear and Non-linear Systems
- Time Variant and Time Invariant Systems
- linear Time variant and linear Time invariant systems
- Static and Dynamic Systems
- Causal and Non-causal Systems
- Invertible and Non-Invertible Systems
- Stable and Unstable Systems

• Linear and Non-linear Systems

A system is said to be linear when it satisfies superposition and homogenate principles. Consider two systems with inputs as x1(t), x2(t), and outputs as y1(t), y2(t) respectively. Then, according to the superposition and homogenate principles,

Condition for linear system. $$a_1 x_1(t) + a_2 x_2(t) = a_1 y_1(t) + a_2 y_2(t)$$

Determine whether the following systems are linear (or) non linear.

1. y(t) = x(3-t)

Solution:

$$y_1(t) = x_1(3-t)$$
$$y_2(t) = x_2(3-t)$$

Condition for linear system.

$$a_1 x_1(t) + a_2 x_2(t) = a_1 y_1(t) + a_2 y_2(t)$$

L.H.S

$$a_1 x_1(t) + a_2 x_2(t) = a_1 x_1(3-t) + a_2 x_2(3-t)$$

R.H.S

$$a_1 y_1(t) + a_2 y_2(t) = a_1 x_1(3-t) + a_2 x_2(3-t)$$

L.H.S = R.H.S

Hence the given system is linear.

2. $y(t) = x^2(t)$

Solution

$$y_1(t) = x_1^2(t)$$
$$y_2(t) = x_2^2(t)$$

Condition for linear system

3. y(t) = 5x(t) + 3

Solution

$$y_1(t) = 5x_1(t) + 3$$
$$y_2(t) = 5x_2(t) + 3$$

Condition for linear system.

$$a_1 x_1(t) + a_2 x_2(t) = a_1 y_1(t) + a_2 y_2(t)$$

L.H.S.

$$a_1 x_1(t) + a_2 x_2(t) = 5a_1 x_1(t) + 5a_2 x_2(t) + 3$$

R.H.S

$$a_1 y_1(t) + a_2 y_2(t) = a_1[5x_1(t) + 3] + a_2[5x_2(t) + 3] = 5a_1 x_1(t) + 3a_1 + 5a_2 x_2(t) + 3a_2$$

L.H.S ≠ R.H.S

Hence the given system is non linear system

4. y(t) = x(t) cos 50πt

Solution

$$y_1(t) = x_1(t)\cos 50\pi t$$
$$y_2(t) = x_2(t)\cos 50\pi t$$

Condition for linear system

$$a_1 x_1(t) + a_2 x_2(t) = a_1 y_1(t) + a_2 y_2(t)$$

L.H.S

$$a_1 x_1(t) + a_2 x_2(t) = [a_1 x_1(t) + a_2 x_2(t)]\cos 50\pi t$$

R.H.S

$$a_1 y_1(t) + a_2 y_2(t) = a_1 x_1(t)\cos 50\pi t + a_2 x_2(t)\cos 50\pi t$$

L.H.S = R.H.S

Hence the given system is linear.

- **Time Variant and Time Invariant Systems**

A system is said to be time variant if its input and output characteristics vary with time.

Otherwise, the system is considered as time invariant. The condition for time invariant system is: y (n , t) = y(n-t)

The condition for time variant system is:

y (n , t) ≠ y(n-t)

Where y (n , t) = T[x(n-t)] = input change

y (n-t) = output change

Example:

y(n) = x(-n)

y(n, t) = T[x(n-t)] = x(-n-t)

y(n-t) = x(-(n-t)) = x(-n + t)

∴ y(n, t) ≠ y(n-t). Hence, the system is time variant.

If a system is both liner and time variant, then it is called liner time variant (LTV) system.

If a system is both liner and time Invariant then that system is called liner time invariant (LTI) system.

- **Static and Dynamic Systems**

Static system is memory-less whereas dynamic system is a memory system.

Example 1: y(t) = 2 x(t)

For present value t=0, the system output is y(0) = 2x(0). Here, the output is only dependent upon present input. Hence the system is memory less or static.

Example 2: y(t) = 2 x(t) + 3 x(t-3)

For present value t=0, the system output is y(0) = 2x(0) + 3x(-3).

Here x(-3) is past value for the present input for which the system requires memory to get this output. Hence, the system is a dynamic system.

- **Causal and Non-Causal Systems**

A system is said to be causal if its output depends upon present and past inputs, and does not depend upon future input.

For non causal system, the output depends upon future inputs also.

Example 1: y(t) = 2 x(t) + 3 x(t-3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2). Here, the system output only depends upon present and past inputs. Hence, the system is causal.

Example 2: y(t) = 2 x(t) + 3 x(t-3) + 6x(t + 3)

For present value t=1, the system output is y(1) = 2x(1) + 3x(-2) + 6x(4) Here, the system output depends upon future input. Hence the system is non-causal system.

- **Invertible and Non-Invertible systems**

A system is said to invertible if the input of the system appears at the output.

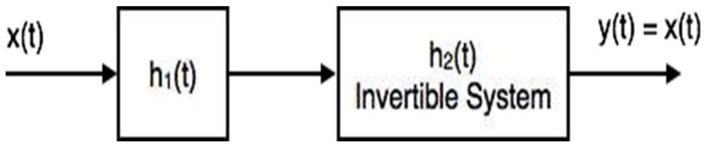
Y(S) = X(S) H1(S) H2(S)

= X(S) H1(S) · 1/(H1(S))

Since H2(S) = 1/(H1(S))

∴ Y(S) = X(S)

→ y(t) = x(t)



Hence, the system is invertible.

If y(t) ≠ x(t), then the system is said to be non-invertible.

- **Stable and Unstable Systems**

The system is said to be stable only when the output is bounded for bounded input. For a bounded input, if the output is unbounded in the system then it is said to be unstable.

Note: For a bounded signal, amplitude is finite. Example 1: y (t) = x^2(t)

Let the input is u(t) (unit step bounded input) then the output y(t) = u^2(t) = u(t) = bounded output.

Hence, the system is stable.

Example 2: y (t) = ∫x(t)dt. Let the input is u (t) (unit step bounded input) then the output y(t) = ∫u(t)dt = ramp signal (unbounded because amplitude of ramp is not finite it goes to infinite when t → infinite). Hence, the system is unstable.

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LECTURE 6 : Representation of Systems Using Block Diagrams and System Equations

By
Abdulhameed Nabeel Hameed

Representation of LTI Systems

In Signals and Systems, a **system** is any device or process that transforms an input signal $x(t)$or $x[n]$into an output signal $y(t)$or $y[n]$.

A system can be represented in two major ways:

1. Block Diagram Representation
2. Mathematical (Input–Output) Equation

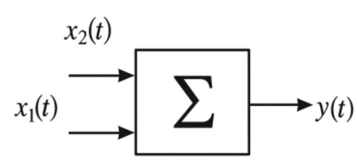
Understanding how to move **from a block diagram → system equation** is essential for analyzing or implementing systems.

Fundamental Block Diagram Elements:

1- Summation / Addition (Σ)

A summing block adds multiple inputs.

$$y(t) = x_1(t) + x_2(t) + \cdots$$



Summation may also include **subtraction**:

$$y(t) = x_1(t) - x_2(t)$$

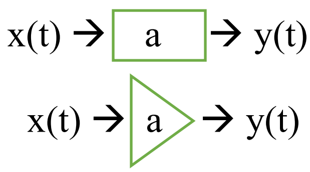
Typically shown by adding a "+" or "–" sign at each input port.

2- Multiplication by a Constant

A system that scales a signal:

$$y(t) = a\,x(t)$$

In block diagrams, this is shown as:



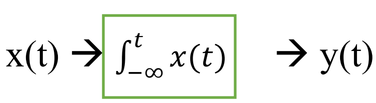
3- Delay Unit

A system that delays a signal in time:

$$y(t) = x(t - To)$$

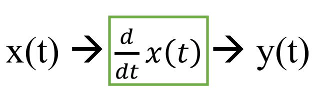
4- Integrator

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$



5- Differentiator

$$y(t) = \frac{d}{dt} x(t)$$



Example 1

Using basic building blocks introduced above, sketch the block diagram representation of the discrete-time system described by the input–output relation

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$$

where $x(n)$ is the input and $y(n)$ is the output of the system.

Solution. According to (A), the output $y(n)$ is obtained by multiplying the input $x(n)$ by 0.5, multiplying the previous input $x(n-1)$ by 0.5, adding the two products, and then adding the previous output $y(n-1)$ multiplied by $\frac{1}{4}$. Figure (4.7.a) illustrates this block diagram realization of the system. A simple rearrangement of (A),

$$y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}[x(n) + x(n-1)]$$

leads to the block diagram realization shown in Fig. 4. 7(b).

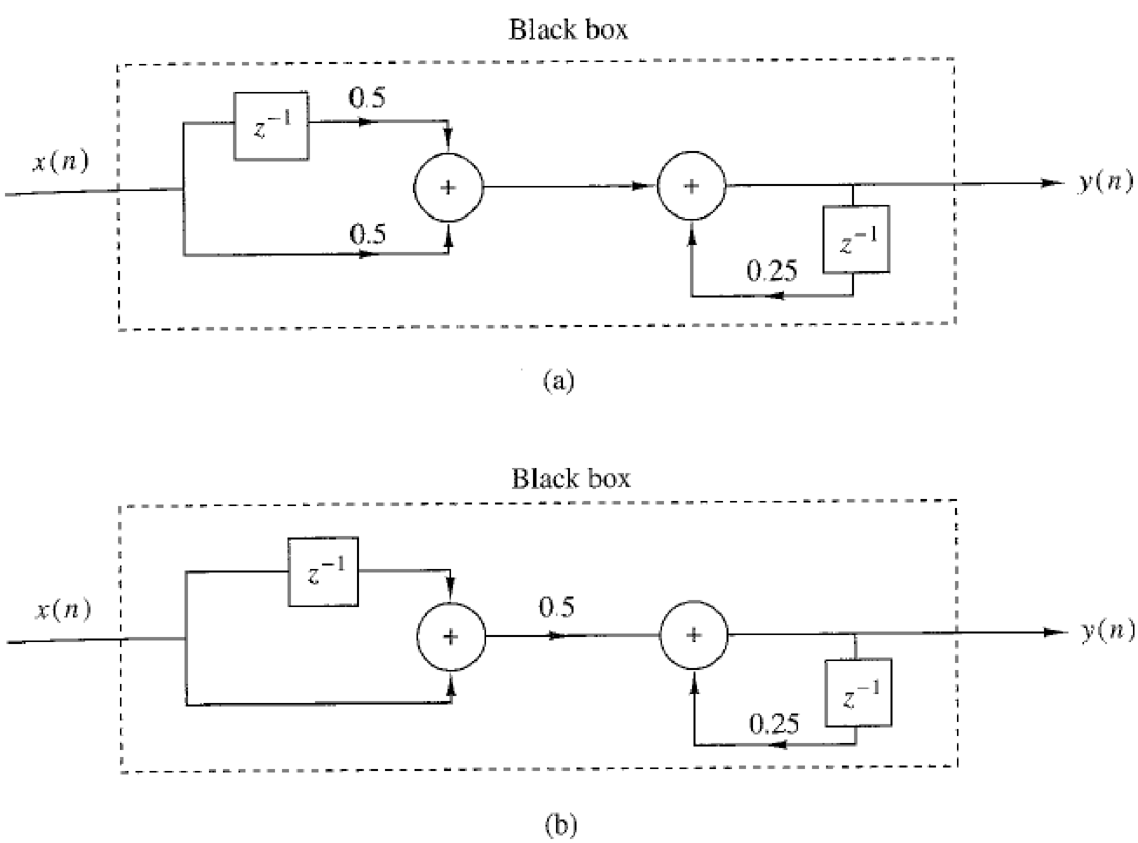


Figure 4.7 Block diagram realizations of the system $y(n) = 0.25y(n-1) + 0.5x(n) + 0.5x(n-1)$.

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LECTURE 7 : Impulse Response of LTI Systems

By
Abdulhameed Nabeel Hameed

Time Domain Analysis of LTI Systems

Impulse Response of LTI Systems

Linear Time-Invariant (LTI) systems are one of the most important classes of systems in Signals and Systems.

Any LTI system can be **fully described** by its *impulse response*, denoted by **h(t)** for continuous time or **h[n]** for discrete time.

The **impulse response** (h(t)/h[n]) of an LTI system is defined as the response of the system when the input signal (x(t)/x[n]) is a delta function (δ(t)/δ[n]).

- Continuous-time:

$$x(t) = \delta(t) \Rightarrow y(t) = h(t)$$

- Discrete-time:

$$x[n] = \delta[n] \Rightarrow y[n] = h[n]$$

If we know **h(t)**, we can find the output for **any input**.

For LTI systems:

Input → System → Output

Because of **linearity** and **time invariance**, any signal can be represented as a combination of shifted impulses.

Thus:

$$y(t) = (x * h)(t)$$

$$y[n] = (x * h)[n]$$

This means the system is completely described by its impulse response.

$x[n]\ \delta[\text{n}]=$

$x[n]\ \delta[\text{n-k}]=\ x[k]\ \delta[\text{n-k}]$

$x[n] = \quad \ldots\ldots. + x[-2]\ \delta[\text{n+2}] + x[-1]\ \delta[\text{n+1}] + x[0]\ \ \delta[\text{n}]\ \ + x[1]\ \ \delta[\text{n-1}]\ \ + x[2]\ \ \delta[\text{n-2}] + \ldots\ldots$

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

So, the LTI system output can be expressed as:

$$\boldsymbol{y[n] = x[n]\ \circledast\ h[n]}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

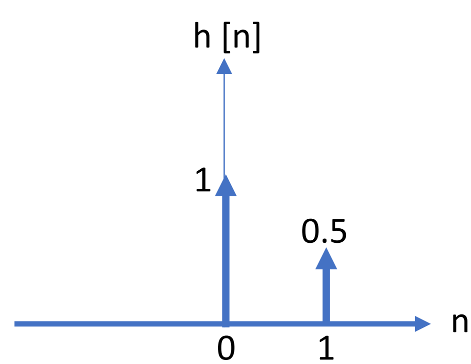
This operation is called <u>**Convolution**</u>

Example 1: Find and sketch the impulse response of the following system:

$y[n] = x[n] + \frac{1}{2}x[n-1]$

Solution:

$h[n] = \delta[n] + \frac{1}{2}\delta[n-1]$



Time Domain Analysis of LTI Systems

Example 2: Find and sketch the impulse response of the following system:

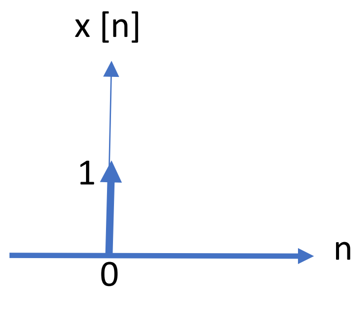
$y[n] = 2x[n] - x[n-1]$

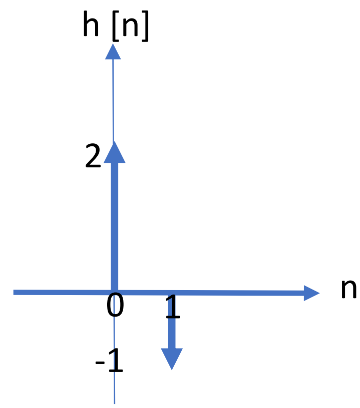
Solution:

X[n]= $\delta[n]$

y[n] → h[n]

h[n]= $2\delta[n] - \delta[n-1]$





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LECTURE 8 : Continous Time and Discrete Time Convolution

By
Abdulhameed Nabeel Hameed

Computing the Output of LTI Systems (Convolution)

Convolution is a mathematical operation used to determine the **output of a linear time-invariant (LTI) system** when the input and the system's impulse response are known.
It combines two signals to produce a third signal that shows **how one signal modifies the other**.

$$\boldsymbol{y(t) = x(t) \circledast h(t)}$$

$$\boldsymbol{y[n] = x[n] \circledast h[n]}$$

1- For continuous LTI systems:

$\boldsymbol{y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau}$ .... **Continuous-time convolution**

$-\infty$

Or

$$y(t) = x(t) \circledast h(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

There are several methods to compute continuous-time convolution, such as:

1- Analytical Integration

2- **Graphical Method (Most Common)**

3- Using Properties

The steps for the graphical method include:

(1) Sketch the waveform for input $x(\tau)$ by changing the independent variable from t to τ.

(2) Sketch the waveform for the impulse response $h(\tau)$ by changing the independent variable from t to τ.

(3) Reflect $h(\tau)$ about the vertical axis to obtain the time-inverted impulse response $h(-\tau)$.

(4) Shift the time-inverted impulse function $h(-\tau)$ by a selected value of "t" . The resulting function represents $h(t-\tau)$.

(5) Multiply function $x(\tau)$ by $h(t-\tau)$ and plot the product function $x(\tau)h(t-\tau)$.

(6) Calculate the total area under the product function $x(\tau)h(t-\tau)$ by integrating it over $\tau = [-\infty,\infty]$.

(7) Repeat steps 4−6 for different values of t to obtain $y(t)$ for all time, $-\infty \leq t \leq \infty$.

2- For discrete LTI systems:

$$\boldsymbol{y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}$$ **............... Discrete-time convolution**

There are several methods to compute discrete-time convolution, such as:

1- Summation Formula
2- **Graphical Method (Most Common)**
3- Table method

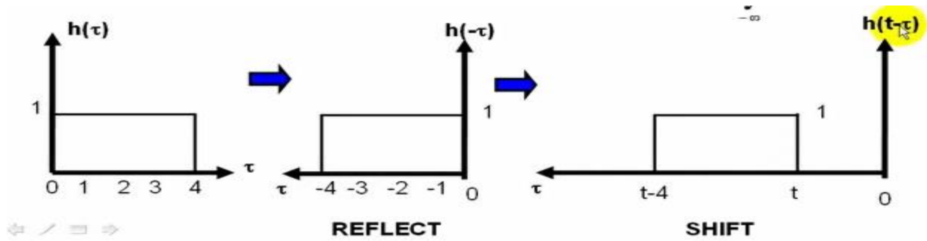
The steps for the graphical method include:

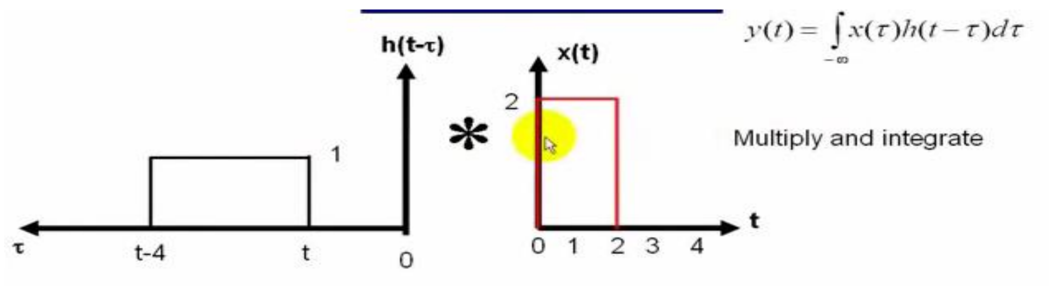
1. Flip: $h[k] \rightarrow h[-k]$
2. Shift by $n \rightarrow h[n-k]$
3. Multiply and sum overlapping values.

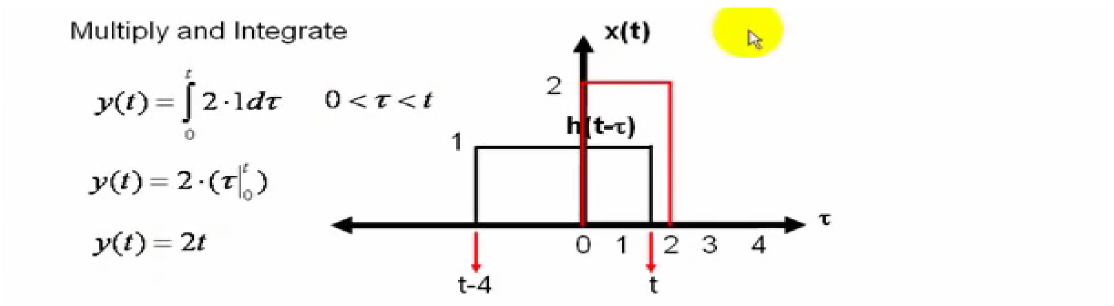
Example

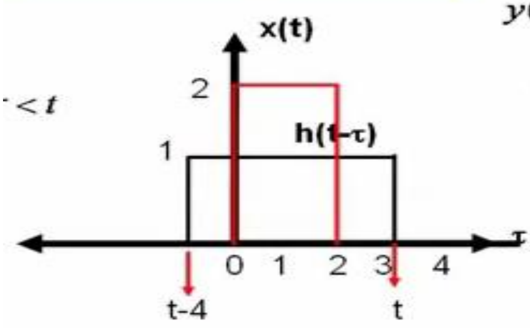
Determiner and sketch the output y (t) of a LTI system with the impulse response h (t) and the input signal x (t) are given in Figure below.

solution









$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$t-4<0 \qquad t>2$

$2<t<4$

$0<\tau<2$

$$y(t) = \int_{0}^{2} 2d\tau$$

$y(t) = 4$

Multiply and Integrate

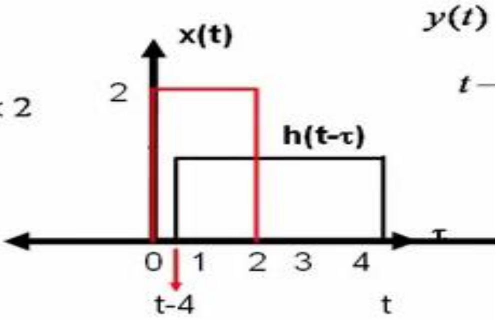
$$y(t) = \int_{t-4}^{2} 2\cdot 1 d\tau \qquad t-4<\tau<2$$

$t<6$

$y(t) = 2\cdot(\tau\big|_{t-4}^{2})$

$y(t) = 2(2-(t-4))$

$y(t) = -2t+12$



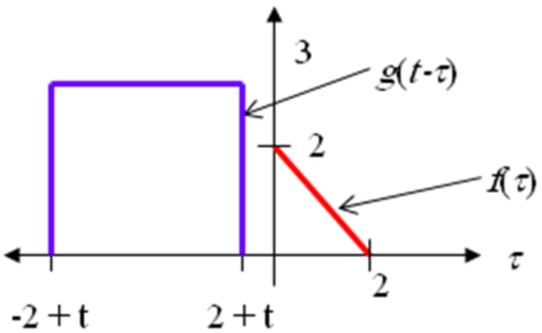
- **Convolve the following two functions:**

- **Replace t with τ in $f(t)$ and $g(t)$**
- **Choose to flip and slide $g(\tau)$ since it is simpler and symmetric**
- **Functions overlap like this:**

- **Convolution can be divided into 5 parts**

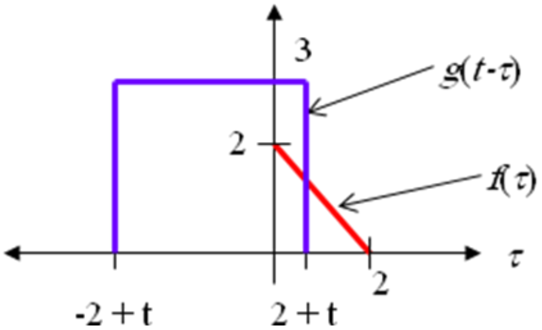
I. $t < -2$

- Two functions do not overlap
- Area under the product of the functions is zero



II. $-2 \le t < 0$

- Part of **g(t)** overlaps part of **f(t)**
- Area under the product of the functions is

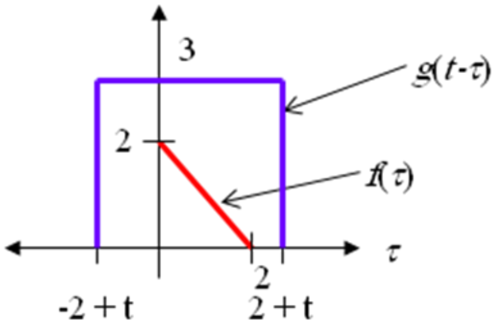


$$\int_0^{2+t} 3(-\tau+2)\,d\tau = 3\left(-\frac{\tau^2}{2}+2\tau\right)\Bigg|_0^{2+t} = -\frac{3(2+t)^2}{2}+6(2+t) = -\frac{3t^2}{2}+6$$

III. $0 \le t < 2$

- Here, **g(t)** completely overlaps **f(t)**
- Area under the product is just

$$\int_0^{2} 3(-\tau+2)\,d\tau = 3\left(-\frac{\tau^2}{2}+2\tau\right)\Bigg|_0^{2} = 6$$

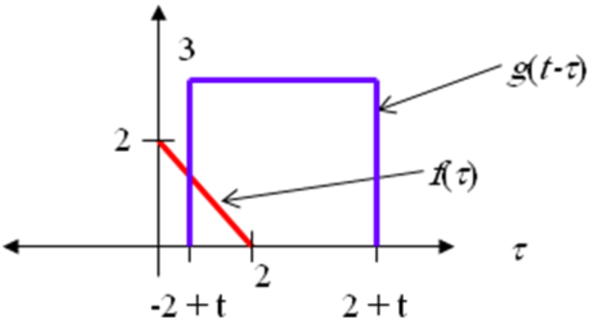


IV. $2 \le t < 4$

- Part of $g(t)$ and $f(t)$ overlap
- Calculated similarly to $-2 \le t < 0$

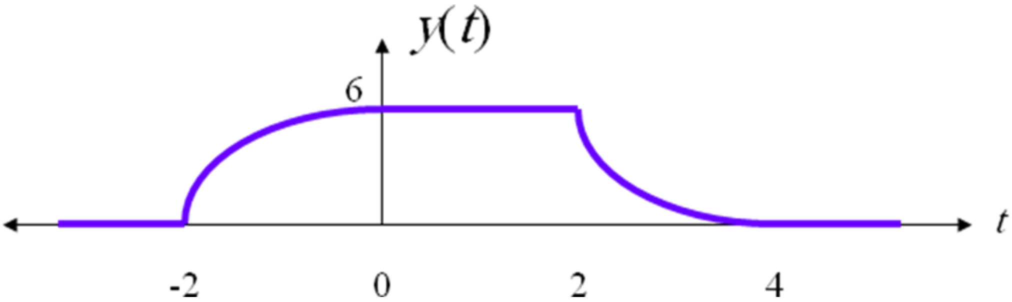
V. $t \ge 4$

- $g(t)$ and $f(t)$ do not overlap
- Area under their product is zero



- **Result of convolution (5 intervals of interest):**

$$y(t) = f(t) * g(t) = \begin{cases} 0 & \text{for } t < -2 & \text{No Overlap} \\ -\frac{3}{2}t^2 + 6 & \text{for } -2 \le t < 0 & \text{Partial Overlap} \\ 6 & \text{for } 0 \le t < 2 & \text{Complete Overlap} \\ \frac{3}{2}t^2 - 12t + 24 & \text{for } 2 \le t < 4 & \text{Partial Overlap} \\ 0 & \text{for } t \ge 4 & \text{No Overlap} \end{cases}$$



Note:

We found the function f(τ) by calculate the slop as shown in these equations.

$$m = \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$(y_1, x_1) = (2, 0)$$

$$(y_2, x_2) = (0, 2)$$

$$m = \frac{y - 2}{x - 0} = \frac{0 - 2}{2 - 0}$$

$$\frac{y - 2}{x} = \frac{-2}{2}$$

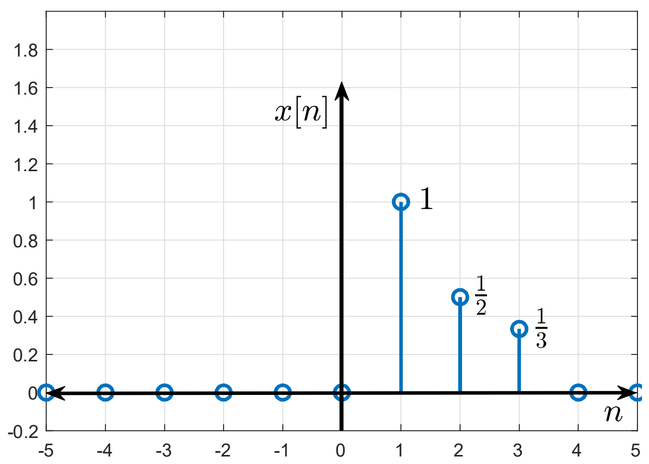
$$\frac{y - 2}{x} = \frac{-2}{2}$$

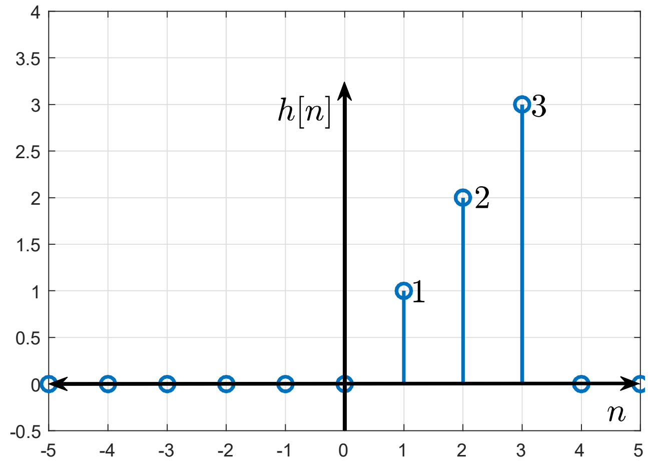
$$y - 2 = -x$$

$$y = -x + 2$$

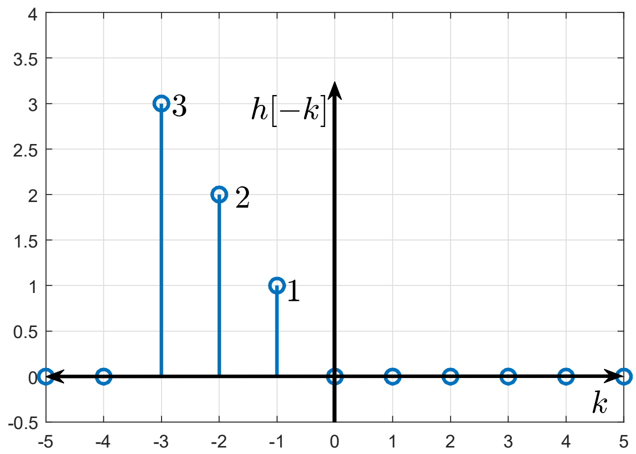
$$or \quad y = -\tau + 2$$

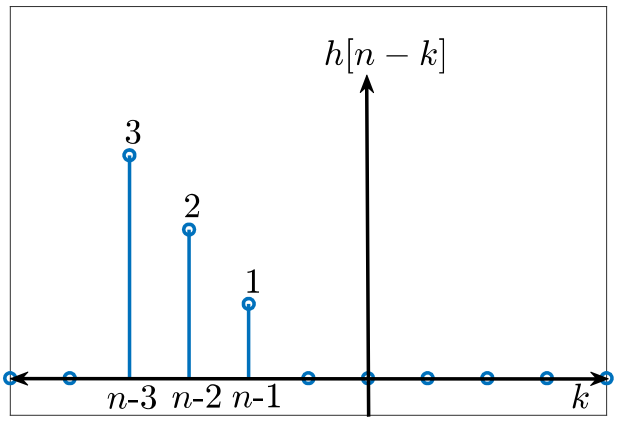
Example : Find the output of an LTI system if $x[n]$ and the impulse response of that system, $h[n]$ are given such that



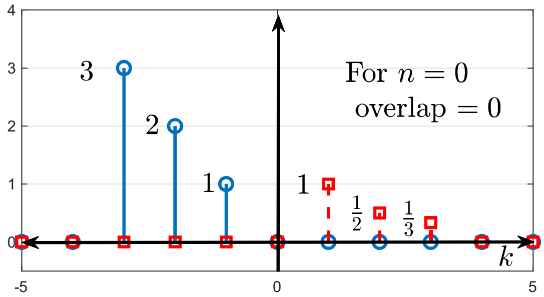


Sol: First flip $h[n]$



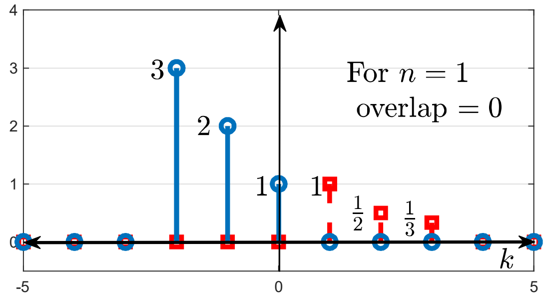


- Second, we start shifting the flipped signal.



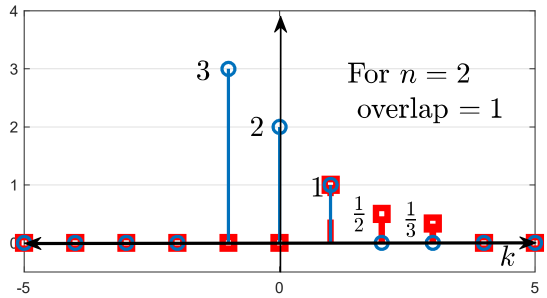
$$y[0] = 0$$

- For $n = 1$.



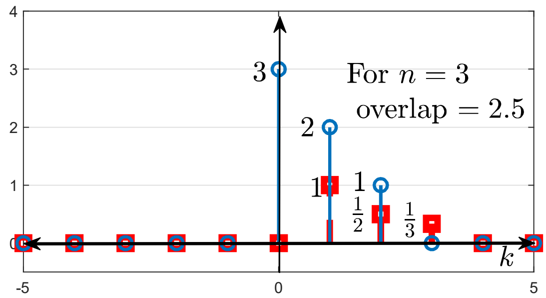
$$y[1] = 0$$

- Thus, for $n \leq 1$, $y[n] = 0$
- For $n = 2$



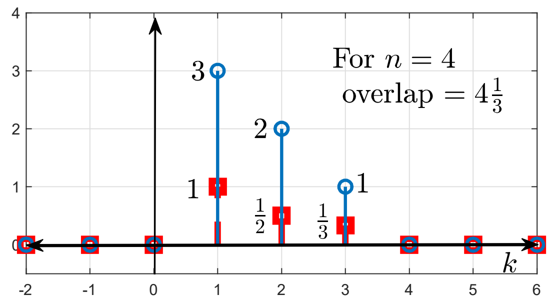
$$y[2] = (1 \times 1) = 1$$

- For $n = 3$.



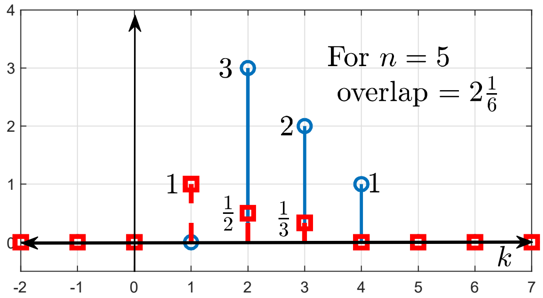
$$y[3] = (2 \times 1) + (1 \times 0.5) = 2.5$$

- For $n = 4$



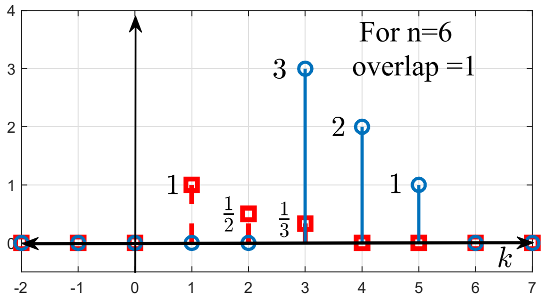
$$y[4] = (3 \times 1) + (2 \times \frac{1}{2}) + (1 \times \frac{1}{3}) = 4\frac{1}{3}$$

- For $n = 5$.



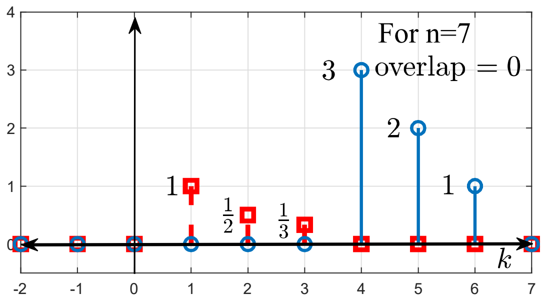
$$y[5] = (3 \times \frac{1}{2}) + (2 \times \frac{1}{3}) + (1 \times 0) = 2\frac{1}{6}$$

- For $n = 6$



$$y[6] = (3 \times \frac{1}{3}) + (2 \times 0) + (1 \times 0) = 1$$

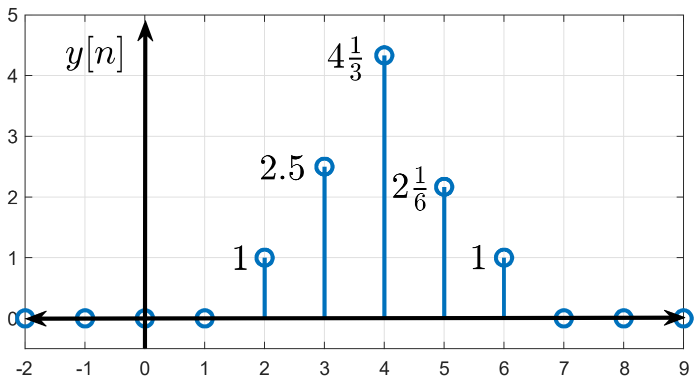
- For $n = 7$.



$$y[7] = 0$$

- Hence, for $n \geq 7$, $y[n] = 0$

- Finally, the output of the LTI system, i.e. $y[n] = x[n] * h[n]$ or $y[n] = x[n] \circledast h[n]$, is given as

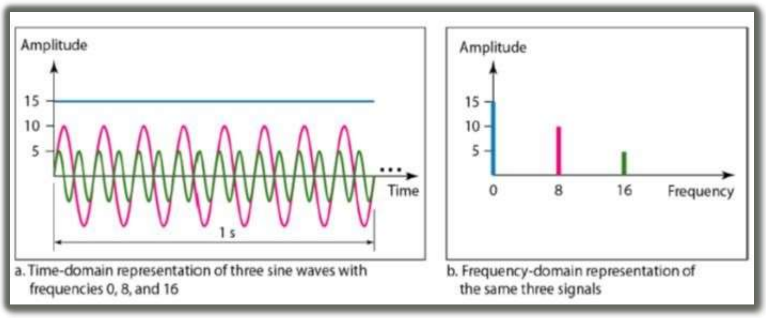


Ninevah University
College of Electronics Engineering
Second Stage / 1st semester 2025-2026

LECTURE 9 : CONTINUOUS-TIME FOURIER SERIES

By
Abdulhameed Nabeel Hameed

The frequency domain is more compact and useful when we are dealing with more than one sine wave. For example, Figure below shows three sine waves, each with different amplitude and frequency. All can be represented by three spikes in the frequency domain.



The time domain and frequency domain of three sine waves

TABLE 3.1 ***Relationship between Time Properties of a Signal and the Appropriate Fourier Representation.***

Time Property	*Periodic*	*Nonperiodic*
Continuous (t)	Fourier Series (FS)	Fourier Transform (FT)
Discrete [n]	Discrete-Time Fourier Series (DTFS)	Discrete-Time Fourier Transform (DTFT)

Continuous-Time Fourier Series

Introduction

- One very important tool in the study of signals and systems is the Fourier series.
- A very large class of functions can be represented using Fourier series, namely most practically useful periodic functions.
- The Fourier series represents a periodic function as a (possibly infinite) linear combination of complex sinusoids.
- This is often desirable since complex sinusoids are easy functions with which to work. For example, complex sinusoids are easy to integrate and differentiate.
- Also, complex sinusoids have important properties in relation to LTI systems. In particular, complex sinusoids are eigenfunctions of LTI systems.

<u>Where is Fourier series used?</u>

The Fourier series has many such applications:

- Electrical engineering,
- Vibration analysis,
- Acoustics, (علم الصوت)
- Optics, (بصريات)
- Signal processing,
- Image processing,
- Quantum mechanics,
- Econometrics,
- Shell theory (نظرية القشرة), etc.

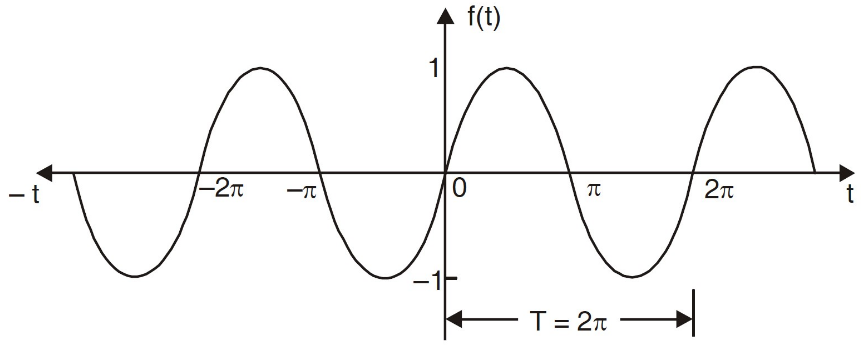
Periodic Functions

If the value of each ordinate $f(t)$ repeats itself at equal intervals in the abscissa, then $f(t)$ is said to be a periodic function.

If $f(t) = f(t + T) = f(t + 2T) = ...$ then T is called the period of the function $f(t)$.

For example :

$\sin x = \sin(x + 2\pi) = \sin(x + 4\pi) = ...$ so $\sin x$ is a periodic function with the period 2.
This is also called sinusoidal periodic function.



Methods of Fourier Series

In electronics applications, we have been familiar with some periodic signals such as the square wave, rectangular wave, triangular wave, sinusoid, sawtooth wave, and so on. These periodic signals can be analyzed in frequency domain with the help of the Fourier series expansion

According to Fourier theory, a periodic signal can be represented by a Fourier series that contains the sum of a series of sine and/or cosine functions (harmonics) plus a direct-current (DC) term. There are three forms of Fourier series:

(1) sine-cosine
(2) amplitude phase
(3) complex exponential

Types of Fourier Series

1. Trigonometric Fourier Series

Here we will express a non-sinusoidal periodic function into a fundamental and its harmonics. A series of sines and cosines of an angle and its multiples of the form.

$$\frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + ... + a_n \cos nx + ...$$

$$+ b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + ... + b_n \sin nx + ...$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx.$$

is called the *Fourier series, where* $a_1, a_2, .. a_n, ... b_1, b_2, b_3. . b_n$. .. are constants.

A periodic function $f(x)$ can be expanded in a Fourier Series. The series consists of the following:

(*i*) A constant term a_0(called d.c. component in electrical work).

(*ii*) A component at the fundamental frequency determined by the values of a_1, b_1.

(*iii*) Components of the harmonics (multiples of the fundamental frequency) determined by $a_2, a_3 ... b_2, b_3 ...$. And $a_0, a_1, a_2 ..., b_1, b_2$. .. are known as *Fourier coefficients* or Fourier constants.

USEFUL INTEGRALS

The following integrals are useful in Fourier Series.

(i) $\int_0^{2\pi} \sin nx \, dx = 0$

(ii) $\int_0^{2\pi} \cos nx \, dx = 0$

(iii) $\int_0^{2\pi} \sin^2 nx \, dx = \pi$

(iv) $\int_0^{2\pi} \cos^2 nx \, dx = \pi$

(v) $\int_0^{2\pi} \sin nx \cdot \sin mx \, dx = 0$

(vi) $\int_0^{2\pi} \cos nx \cos mx \, dx = 0$

(vii) $\int_0^{2\pi} \sin nx \cdot \cos mx \, dx = 0$

(viii) $\int_0^{2\pi} \sin nx \cdot \cos nx \, dx = 0$

(ix) $\int uv \, dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \ldots$

where $v_1 = \int v \, dx, v_2 = \int v_1 \, dx$ and so on $u' = \frac{du}{dx}$, $u'' = \frac{d^2u}{dx^2}$ and so on and

(x) $\sin n\pi = 0$, $\cos n\pi = (-1)^n$ where $n \in I$

Fourier Series Formulas

For a periodic signal $x(t)$with period T:

Formula

$$x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)]$$

Where:

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\,\omega_0 t)\, dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\,\omega_0 t)\, dt$$

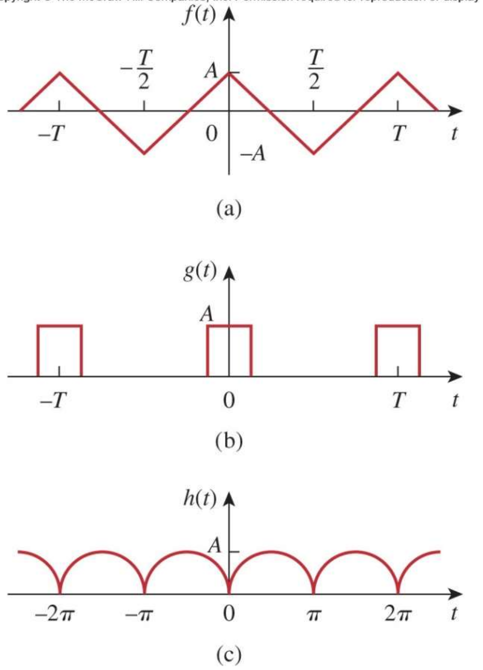
Even Symmetry

- In the case of even symmetry, the function is symmetrical about the vertical axis:

$$f(t) = f(-t)$$

- A main property of an even function is that:

$$\int_{-T/2}^{T/2} f_e(t)\,dt = 2\int_{0}^{T/2} f_e(t)\,dt$$



Even Symmetry

- The Fourier coefficients for an even function become:

$$a_0 = \frac{1}{T}\int_0^{T/2} f(t)\,dt \qquad a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\,dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_0 t\,dt \qquad a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos n\omega_0 t\,dt$$

$$b_n = 0 \qquad b_n = 0$$

- Note that this become a Fourier cosine series.

- Since Cosine is an even function, one can see how this series is called even.

Odd Symmetry

- A function is said to be odd if its plot is antisymmetrical about the vertical axis.

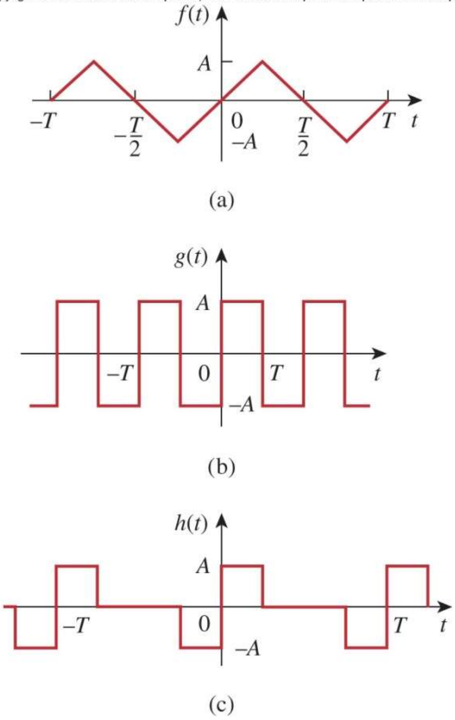
$$f(-t) = -f(t)$$

- Examples of odd functions are t, t^3, and sin t

- An add function has this major characteristic:

$$\int_{-T/2}^{T/2} f_0(t)\,dt = 0$$

Odd Symmetry



- This comes about because the integration from –T/2 to 0 is the negative of the integration from 0 to T/2

- The coefficients are:

$$a_0 = 0 \quad a_n = 0$$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin n\omega_0 t\,dt$$

$$a_0 = 0 \quad a_n = 0$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin n\omega_0 t\,dt$$

- This gives the Fourier sine series,

Example 1:

Considering the following square waveform $x(t)$, shown in Figure B.1, where T_0 represents a period, find the Fourier series expansions in terms of (a) the sine-cosine form, (b) the amplitude-phase form, and (c) the complex exponential form.

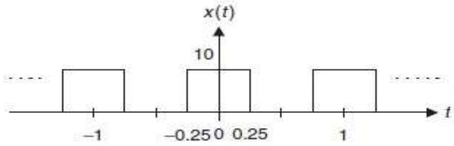


FIGURE B.1 **Square waveform in Example B.1.**

Solution:

From Figure B.1, we notice that $T_0 = 1$ second and $A = 10$. The fundamental frequency is

$$f_0 = 1/T_0 = 1\,\text{Hz or } \omega_0 = 2\pi \times f_0 = 2\pi\ \text{rad/sec}.$$

a. Using Equations (B.1) to (B.3) yields

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt = \frac{1}{1}\int_{-0.25}^{0.25} 10dt = 5$$

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos(n\omega_0 t)dt$$

$$= \frac{2}{1}\int_{-0.25}^{0.25} 10\cos(n2\pi t)dt$$

$$= \frac{2}{1}\frac{10\times\sin(n2\pi t)}{n2\pi}\bigg|_{-0.25}^{0.25} = 10\frac{\sin(0.5\pi n)}{0.5\pi n}$$

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\sin(n\omega_0 t)dt$$

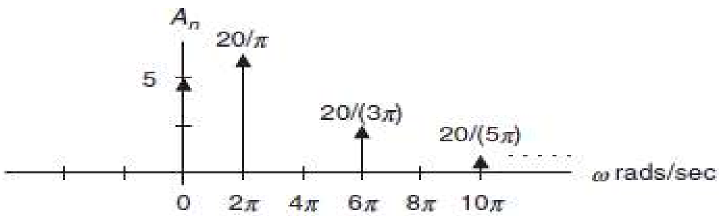
$$= \frac{2}{1}\int_{-0.25}^{0.25} 10\times\sin(n2\pi t)dt$$

$$= \frac{2}{1}\frac{-10\cos(n2\pi t)}{n2\pi}\bigg|_{-0.25}^{0.25} = 0$$

Thus, the Fourier series expansion in terms of the sine-cosine form is written as

$$x(t) = 5 + \sum_{n=1}^{\infty} 10\frac{\sin(0.5\pi n)}{0.5\pi n}\cos(n2\pi t)$$

$$= 5 + \frac{20}{\pi}\cos(2\pi t) - \frac{20}{3\pi}\cos(6\pi t) + \frac{4}{\pi}\cos(10\pi t) - \frac{20}{7\pi}\cos(14\pi t) + \ldots$$

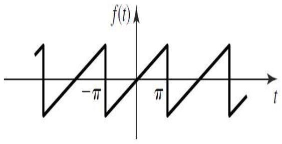


One-sided spectrum of the square waveform in Example B.2.

Example 2:

Write the Fourier series representation of the periodic function $f(t)$ if in one period

$$f(t) = t, \quad -\pi < t < \pi$$



▶ **Solution**

For this example, $T = 2\pi$. For a_n we have

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\,dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t\,dt = \frac{t^2}{2\pi}\bigg|_{-n}^{\pi} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt, \quad n = 1, 2, 3, \ldots$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} t\cos nt\,dt = \frac{1}{\pi}\left[\frac{t}{n}\sin nt + \frac{1}{n^2}\cos nt\right]_{-\pi}^{\pi} = 0$$

recognizing that $\cos n\pi = \cos(-n\pi)$ and $\sin n\pi = -\sin(-n\pi) = 0$. For b_n we have

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin nt\,dt, \quad n = 1, 2, 3, \ldots$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi} t\sin nt\,dt = \frac{1}{\pi}\left[-\frac{t}{n}\cos nt + \frac{1}{n^2}\sin nt\right]_{-\pi}^{\pi} = -\frac{2}{n}\cos n\pi$$

The Fourier series representation has only sine terms. It is given by

$$f(t) = -2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nt$$

where we have used $\cos n\pi = (-1)^n$. Writing out several terms, we have

$$f(t) = -2[-\sin t + \tfrac{1}{2}\sin 2t - \tfrac{1}{3}\sin 3t + \cdots]$$

$$= 2\sin t - \sin 2t + \tfrac{2}{3}\sin 3t - \cdots$$