Ninevah University

College Of Electronics Engineering

Systems & Control Engineering Department

Subject : Control Systems Design

Class : Third

Semester : Semester II

Lecture: 1

Lecturer: Salam Ibrahim
<salam.khather@uoninevah.edu.iq>

1

ENGINEERING DESIGN

- **Engineering design** is the central task of the engineer. It is a complex process in which both creativity and analysis play major roles.
- **Design is the process of conceiving or inventing the forms, parts, and details of a system to achieve a specified purpose.**
- Design is an innovative act whereby the engineer creatively uses knowledge and materials to specify the shape, function, and material content of a system.

2

The design steps

(1) determine a need arising from the values

of various groups, covering the spectrum from public policy makers to the consumer.

(2) specify in detail what the solution to that need must be and to embody these values.

(3) develop and evaluate various alternative solutions to meet these specifications.

(4) decide which one is to be designed in detail.

3

The main approach to the most effective engineering design is parameter analysis and optimization.

- Parameter analysis is based on (1) identification of the key parameters, (2) generation of the system configuration, and (3) evaluation of how well the configuration meets the needs. These three steps form an iterative loop.
- Once the key parameters are identified and the configuration synthesized, the designer can **optimize** the parameters. Typically, the designer strives to identify a limited set of parameters to be adjusted.

4

The goal of control engineering design is to obtain the configuration, specifications, and identification of the key parameters of a proposed system to meet an actual need.

The design process consists of seven main building blocks, which we arrange into three groups:

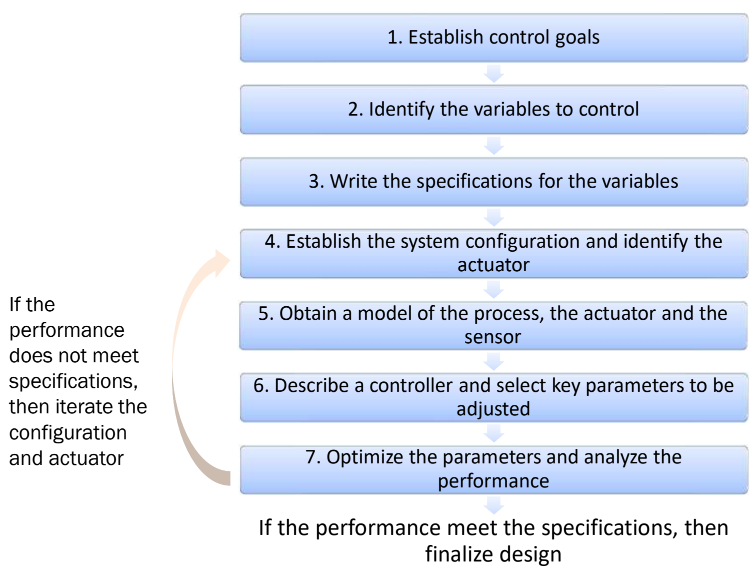
1. Establishment of goals and variables to be controlled, and definition of specifications against which to measure performance.

2. System definition and modeling.

3.Control system design and integrated system simulation and analysis.

5

Control System Design Process



6

The performance specifications will describe how the closed-loop system should perform and will include:

(1) good regulation against disturbances.

(2) desirable responses to commands.

(3) realistic actuator signals.

(4) low sensitivities.

(5) robustness.

Control System Components

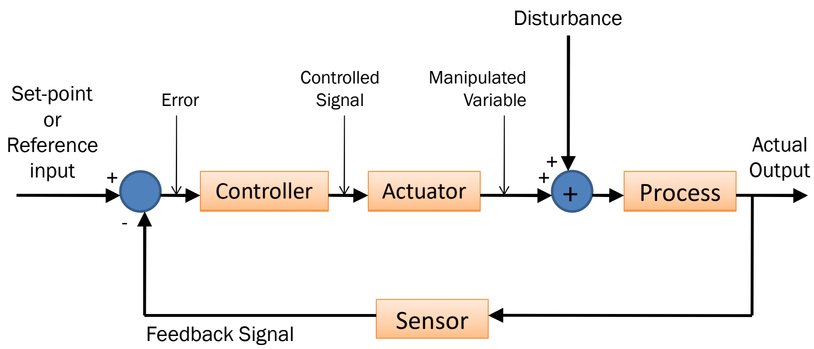
i. System, plant or process
   – To be controlled

ii. Actuators
   – Converts the control signal to a power signal

iii. Sensors
   – Provides measurement of the system output

iv. Reference input
   – Represents the desired output

General Control System



9

The design process has been dramatically affected by the advent of powerful and inexpensive computers and effective control design and analysis **software.**

- For example, the Boeing 777, which incorporates the most advanced flight avionics of any U.S. commercial aircraft, was almost entirely computer-designed.
- The Boeing 777 test pilots flew about 2400 flights in high-fidelity simulations before the first aircraft was even built.

10

- **Functionally, how do closed-loop systems differ from open-loop systems?**

Closed-loop systems compensate for disturbances by measuring the response, comparing it to the input response (the desired output), and then correcting the output response.

- **Name the three major design criteria for control systems.**

Stability, transient response, and steady-state error

11

- **Name the two parts of a system's response.**

Steady-state, transient.

- **Physically, what happens to a system that is unstable?**

It follows a growing transient response until the steady-state response is no longer visible. The system will either destroy itself, reach an equilibrium state because of saturation in driving amplifiers, or hit limit stops.

12

- **Describe a typical control system analysis task.**

Determine the transient response performance of the system.

- **Describe a typical control system design task.**

Determine system parameters to meet the transient response specifications for the system.

Where is control system design used?

- Control Systems are used **in Industrial Automation to regulate how devices operate in real time**. In a closed-loop control system the controller (RTU, PLCS, DCS) feedback (error) signal is used to adjust the control variable such that the process is constantly trying to match the operational set point.

Where is control system design used?

- Control Systems are used in **domestic applications, general industry, military and virtually every modern vehicle in the world**. Control Systems are very common in SCADA and Industrial Automation systems.
- Control Systems are used in industries such as chemical processing, pulp and paper manufacture, power generation, oil and gas processing, and telecommunications.

15

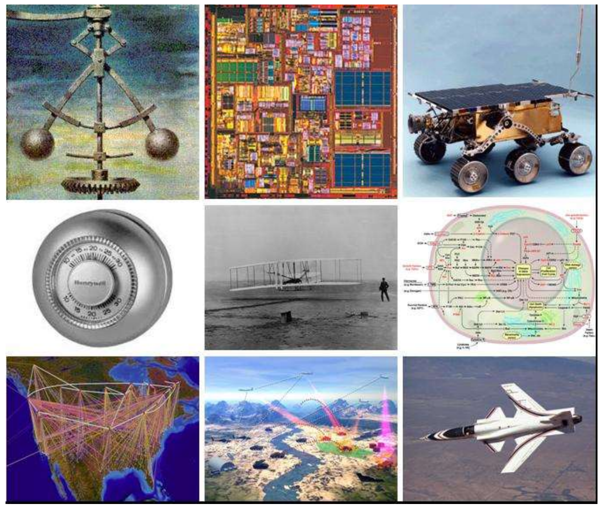
How do you create a controller control system?

*** General Tips for Designing a PID Controller**

- Obtain an open-loop response and determine what needs to be improved.
- Add a proportional control to improve the rise time.
- Add a derivative control to reduce the overshoot.
- Add an integral control to reduce the steady-state error.
- Adjust each of the gains, , and.

16

Summary



Feedback control is Every where
you just have to look for it

17

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Lecturer: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

1. PLOTTING ROOT LOCI WITH MATLAB

1.1 AIM:
To plot the root locus for a given transfer function of the system using MATLAB.

1.2 APPARATUS:
Software: MATLAB.

1.3 MATLAB PROGRAM:
We present the MATLAB approach to the generation of root-locus plots and finding relevant information from the root-locus plots.

In plotting root loci with MATLAB we deal with the characteristic equation of the negative feedback control system:

$1 + G(s)H(s) = 0$

Then rearrange this equation so that the parameter of interest appears as the multiplying Factor in the form

1 + [K(s + z1)(s + z2) …. (s + zm)]/[(s + p1))s + p2) …. (s + pn)]= 0

In the present discussions, we assume that the parameter of interest is the gain K, where K>0.

Which may be written as where num is the numerator polynomial and den is the denominator polynomial.
That is,
num = (s + z1)(s + z2) ….(s + zm)
den = (s + p1)(s + p2) …. (s + pn)
Note that both vectors num and den must be written in descending powers of s.

A MATLAB command commonly used for plotting root loci is:
rlocus(num,den)

Using this command, the root-locus plot is drawn on the screen. The gain vector K is automatically determined. (The vector K contains all the gain values for which the closed loop poles are to be computed).

If it is desired to plot the root loci with marks 'o' or 'x', it is necessary to use the following Command:
r = rlocus(num,den)
plot(r,'o') or plot(r,'x')

Plot root loci with a square aspect ratio so that a line with slope 1 is a true 45°line. Choose the region of root-locus plot to be
$-6 \leq x \leq 6$, $-6 \leq y \leq 6$
Where x and y are the real-axis coordinate and imaginary-axis coordinate, respectively.

To set the given plot region on the screen to be square, enter the command
v = [-6 6 -6 6]; axis (v); axis('square')

With this command, the region of the plot is as specified and a line with slope 1 is at a true 45°, not skewed by the irregular shape of the screen.

1.4 PROCEDURE:

- Write MATLAB program in the MATLAB specified documents.
- Save the program to run it.
- The input is to be mentioned.
- The syntax “g=tf(num,den)” gives the transfer function and is represented as g.
- The syntax “rlocus(g)” plots the rootlocus of the transfer function g.
- rlocus(g) calculates and plots the root locus of the open loop SISO model sys.
- Now we have to solve it theoretically.
- We have to compare the practical and theoretical outputs to verify each other correctly.

1.5 THEORETICAL CALCULATIONS:
enter the numerator of the transfer function
num=
enter the denominator of the transfer function
den=
Transfer function :

1.6 EXAMPLE: Consider the G(s) = [K]/[s(s + 1) (s+2)], Plot root loci. {[1] Page 273}

MATLAB Program

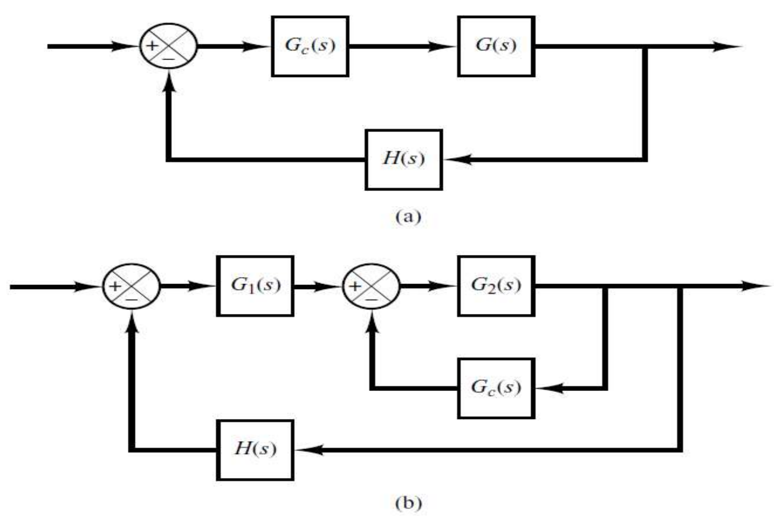
```
% --------- Root-locus plot ---------
num = [1];
den = [1 3 2 0];
g=tf(num,den)
rlocus(num,den)
v = [-4 4 -4 4];
axis(v); axis('square')
grid;
title ('Root-Locus Plot of G(s) = [K]/[s(s + 1) (s+2)]')
```

1.7 RESULTS AND DISCUSSION:

ROOT-LOCUS APPROACH TO CONTROL-SYSTEMS DESIGN

Design by Root-Locus Method. The design by the root-locus method is based on reshaping the root locus of the system by adding poles and zeros to the system's open-loop transfer function and forcing the root loci to pass through desired closed-loop poles in the s plane.

Series Compensation and Parallel (or Feedback) Compensation

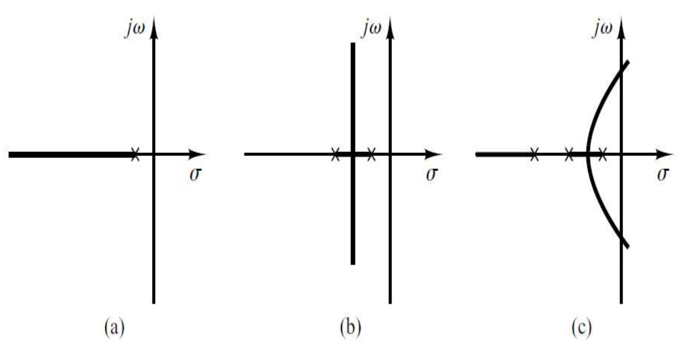


3

- The choice between series compensation and parallel compensation depends on the nature of the signals in the system, the power levels at various points, available components, the designer's experience, economic considerations, and so on.
- In general, series compensation may be simpler than parallel compensation.
- the number of components required in parallel compensation will be less than the number of components in series compensation because the energy transfer is from a higher power level to a lower level.

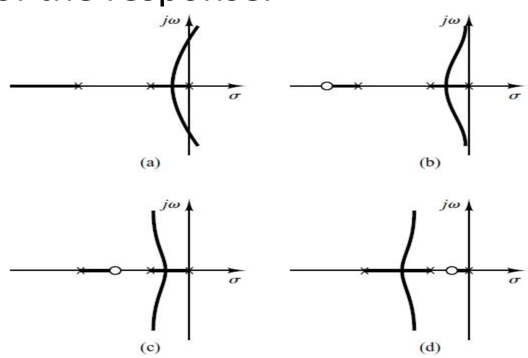
Effects of the Addition of Poles.

- The addition of a pole to the open-loop transfer function has the effect of pulling the root locus to the right, tending to lower the system's relative stability and to slow down the settling of the response.



Effects of the Addition of Zeros.

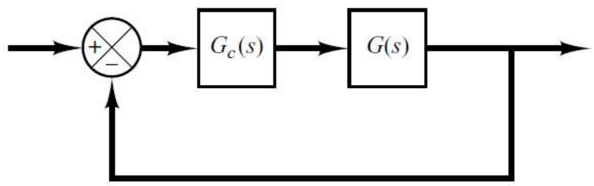
- The addition of a zero to the open-loop transfer function has the effect of pulling the root locus to the left, tending to make the system more stable and to speed up the settling of the response.



LEAD COMPENSATION

- The root-locus approach to design is very powerful when the specifications are given in terms of time-domain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time.
- Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient-response characteristics. In such a case, the reshaping of the root locus is necessary.

The procedures for designing a lead compensator for the system shown in Figure by the root-locus method may be stated as follows



1. From the performance specifications, determine the desired location for the dominant closed-loop poles.
2. By drawing the root-locus plot of the uncompensated system (original system), ascertain whether or not the gain adjustment alone can yield the desired closed loop poles. If not, calculate the angle deficiency Ø . This angle must be contributed by the lead compensator if the new root locus is to pass through the desired locations for the dominant closed-loop poles.

3. Assume the lead compensator *Gc(s) to be:*

- Gc(s) = Kc α [(T s + 1)] /[(α T s + 1)]
  = Kc (s + 1/T)/(s + 1/αT)

  (0<α<1)
- where α and T are determined from the angle deficiency (Ø). Kc is determined from the requirement of the open-loop gain.

10

4. If static error constants are not specified, determine the location of the pole and zero of the lead compensator so that the lead compensator will contribute the necessary angle (Ø). If no other requirements are imposed on the system, try to make the value of α as large as possible. A larger value of α generally results in a larger value of Kv, which is desirable. Note that

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = K_c\alpha \lim_{s\to 0} sG_c(s)$$

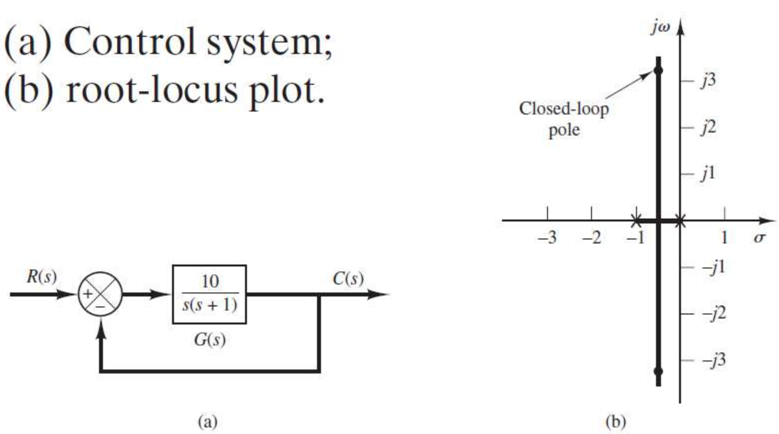
11

5. Determine the value of Kc of the lead compensator from the magnitude condition.

- Once a compensator has been designed, check to see whether all performance specifications have been met. If the compensated system does not meet the performance specifications, then repeat the design procedure by adjusting the compensator pole and zero until all such specifications are met.

- **EXAMPLE (Ogata P313):** Consider the position control system shown in Figure The feedforward transfer function is

G(s) = 10 / [s(s + 1)]



(a) Control system;
(b) root-locus plot.

The root-locus plot for this system is shown in Figure. The closed-loop transfer function for the system is :

C(s)/R(s) =10/(s^2 + s + 10)

=10/[(s + 0.5 + j3.1225)(s + 0.5 - j3.1225)]

The closed-loop poles are located at:

s = -0.5 ± j3.1225

The damping ratio of the closed-loop poles is

ζ =(1/2)/√10 = 0.1581

The undamped natural frequency of the closed-loop poles is ωn = √10 = 3.1623 rad/sec

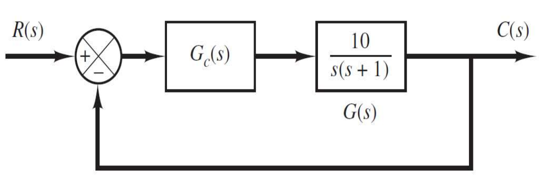
Because the damping ratio is small, this system will have a large overshoot in the step response and is not desirable.

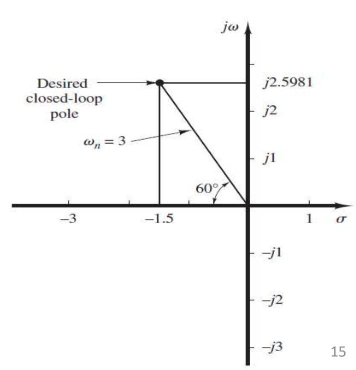
It is desired to design a lead compensator Gc(s) as shown in Figure so that the dominant closed-loop poles have the damping ratio ζ=0.5 and the undamped natural frequency ωn = 3 rad/sec.

The desired location of the dominant closed-loop poles can be determined from:

S^2 + 2ζ ωn s + ωn^2 = s^2 + 3s + 9

as follows: s = -1.5 ± j 2.5981





Assume the lead compensator *Gc(s) to be:*

$$Gc(s) = Kc\,\alpha\,[(T s + 1)] / [(\alpha T s + 1)]$$
$$= Kc\,(s + 1/T)/(s + 1/\alpha T) \quad (0<\alpha<1)$$

- The angle from the pole at the origin to the desired dominant closed-loop pole at $s = -1.5+j2.5981$ is 120°.The angle from the pole at $s=-1$ to the desired closed-loop pole is 100.894°. Hence, the angle deficiency is

Angle deficiency=$\emptyset$=180° – 120° – 100.894°= – 40.894°

- Deficit angle 40.894° must be contributed by a lead compensator. *There are infinitely many solutions.*

16

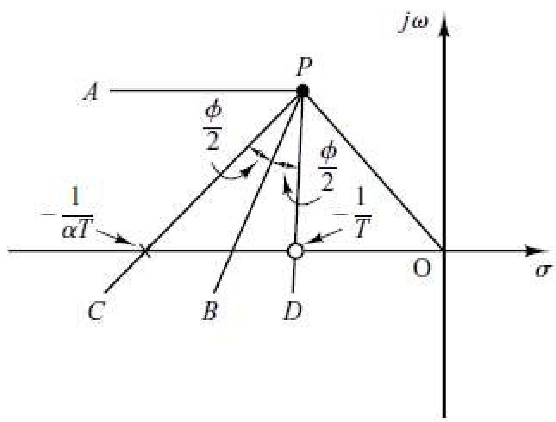
Method 1.

- First, draw a horizontal line passing through point *P, the desired* location for one of the dominant closed-loop poles. This is shown as line PA in Figure. Draw also a line connecting point P and the origin. Bisect the angle between the lines PA and PO, as shown in Figure. Draw two lines PC and PD that make angles $\pm\emptyset/2$ with the bisector PB. The intersections of PC and PD with the negative real axis give the necessary locations for the pole and zero of the lead network.

17

Determination of the pole and zero of a lead network.

$$\angle \frac{10}{s(s+1)}\bigg|_{s=-1.5+j2.5981} = -220.894°$$



18

- Thus, if we need to force the root locus to go through the desired closed-loop pole, the lead compensator must contribute Ø=40.894° at this point. By following the foregoing design procedure, we can determine the zero and pole of the lead compensator.
- Referring to last Figure, if we bisect angle APO and take 40.894°/2 each side, then the locations of the zero and pole are found as follows:

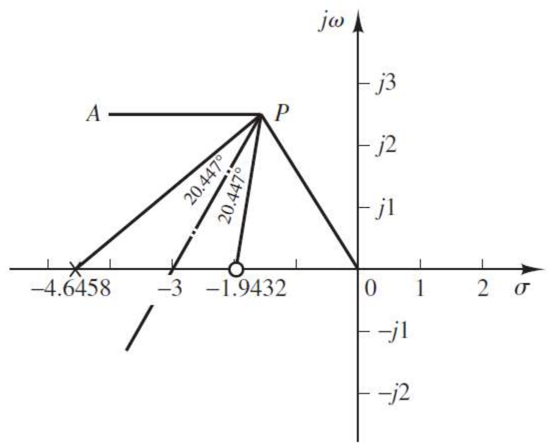
zero at s=–1.9432 & pole at s=–4.6458

(For this compensator the value of α is

α = 1.9432/4.6458 = 0.418.)

19

- Determination of the pole and zero of the lead compensator.



- Gc(s) = Kc α [(T s + 1)] /[(α T s + 1)]
  = Kc (s + 1.9432)/(s + 4.6458)

The value of K_c can be determined by use of the magnitude condition.

$$\left| K_c \frac{s + 1.9432}{s + 4.6458} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$

or

$$K_c = \left| \frac{(s + 4.6458)s(s+1)}{10(s + 1.9432)} \right|_{s=-1.5+j2.5981} = 1.2287$$

Hence, the lead compensator $G_c(s)$ just designed is given by

$$G_c(s) = 1.2287 \frac{s + 1.9432}{s + 4.6458}$$

Then, the open-loop transfer function of the designed system becomes

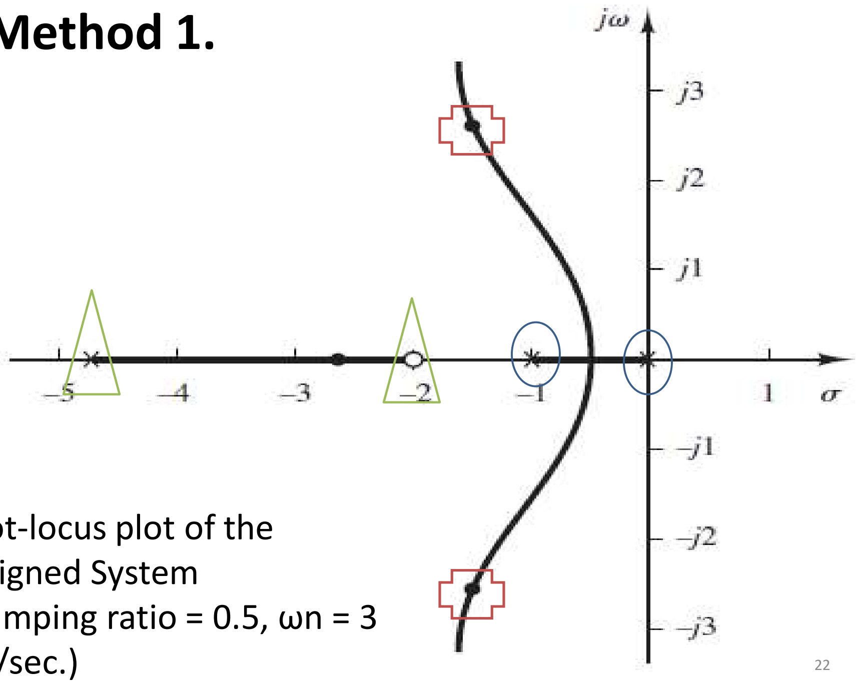
$$G_c(s)G(s) = 1.2287 \left(\frac{s + 1.9432}{s + 4.6458} \right) \frac{10}{s(s+1)}$$

and the closed-loop transfer function becomes

$$\frac{C(s)}{R(s)} = \frac{12.287(s + 1.9432)}{s(s+1)(s + 4.6458) + 12.287(s + 1.9432)}$$

$$= \frac{12.287s + 23.876}{s^3 + 5.646s^2 + 16.933s + 23.876}$$

Method 1.



Root-locus plot of the designed System
(damping ratio = 0.5, ωn = 3 rad/sec.)

22

It is worthwhile to check the static velocity error constant Kv for the system just designed.

$$K_v = \lim_{s \to 0} sG_c(s)G(s)$$

$$= \lim_{s \to 0} s\left[1.2287 \frac{s + 1.9432}{s + 4.6458} \frac{10}{s(s+1)}\right]$$

$$= 5.139$$

23

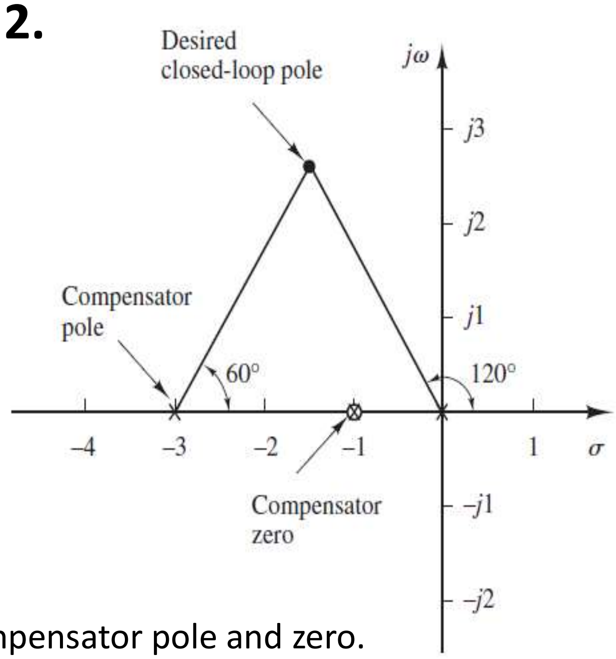
Method 2.

- If we choose the zero of the lead compensator at s = -1 so that it will cancel the plant pole at s = -1, then the compensator pole must be located at s = -3.
- Hence the lead compensator becomes

$$G_c(s) = K_c\,\alpha\,[(T s + 1)]/[(\alpha T s + 1)]$$
$$= K_c (s + 1)/(s + 3)$$

24

Method 2.



Compensator pole and zero.

25

The value of Kc can be determined by use of the magnitude condition:

$$\left| K_c \frac{s+1}{s+3} \frac{10}{s(s+1)} \right|_{s=-1.5+j2.5981} = 1$$

or

$$K_c = \left| \frac{s(s+3)}{10} \right|_{s=-1.5+j2.5981} = 0.9$$

Hence

$$G_c(s) = 0.9 \frac{s+1}{s+3}$$

The open-loop transfer function of the designed system then becomes

$$G_c(s)G(s) = 0.9 \frac{s+1}{s+3} \frac{10}{s(s+1)} = \frac{9}{s(s+3)}$$

The closed-loop transfer function of the compensated system becomes

$$\frac{C(s)}{R(s)} = \frac{9}{s^2 + 3s + 9}$$

26

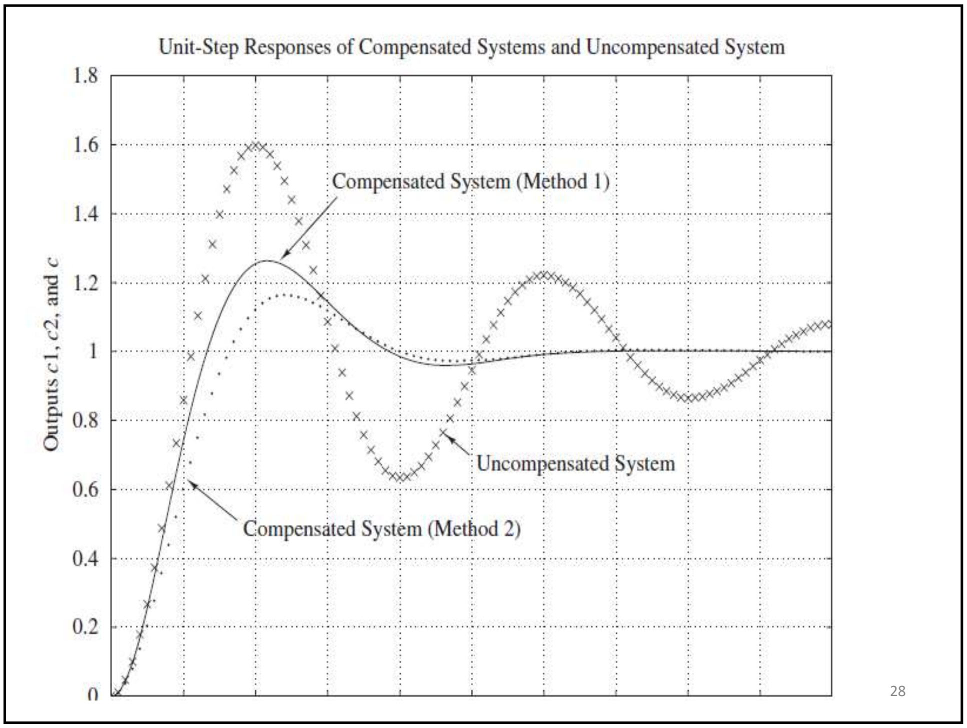
The static velocity error constant for the present case is obtained as follows:

$$K_v = \lim_{s \to 0} sG_c(s)G(s)$$

$$= \lim_{s \to 0} s \left[\frac{9}{s(s+3)} \right] = 3$$

Notice that the system designed by Method 1 gives a larger value of the static velocity error constant.
This means that the system designed by Method 1 will give smaller steady-state errors in following ramp inputs than the system designed by Method 2.

27


Unit-Step Responses of Compensated Systems and Uncompensated System
Outputs c1, c2, and c
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Compensated System (Method 1)
Uncompensated System
Compensated System (Method 2)
28

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

2. Lead Compensation Techniques Based on the Root-Locus Approach

2.1 AIM:
Lead compensation design by the root-locus method using MATLAB.

2.2 APPARATUS:
Software: MATLAB.

2.3 PROCEDURE:

- Write MATLAB program in the MATLAB specified documents.
- Then save the program to run it.
- Design a lead compensator Gc(s) for the dominant closed-loop poles have the damping ratio and the undamped natural frequencies are specified.
- Plot Root-locus of the designed System.
- Comparison of step responses of the compensated and uncompensated systems.
- Comparison of Unit-ramp response curves of designed systems.

2.4 THEORETICAL CALCULATIONS:

- Original uncompensated system

enter the numerator of the transfer function
num=

enter the denominator of the transfer function
den=
Transfer function:

- The system designed by Method 1.

enter the numerator of the transfer function
num1=
enter the denominator of the transfer function
den1=
Transfer function:

- The system designed by Method 2.

enter the numerator of the transfer function
num2=
enter the denominator of the transfer function
den2=
Transfer function:

2.5 EXAMPLE: Consider the G(s) = 10 / [s(s + 1)], Plot root loci for G(s), It is desired to design a lead compensator Gc(s) that the dominant closed-loop poles have the damping ratio ζ=0.5 and the undamped natural frequency ωn = 3 rad/sec. Plot root loci of the designed system. {[1] Page 313}

- Comparison of step responses of the compensated and uncompensated systems.
- Comparison of Unit-ramp response curves of designed systems.

MATLAB Program

```
% ***** Unit-Step Response of Compensated and Uncompensated Systems *****
num1 = [12.287 23.876];
den1 = [1 5.646 16.933 23.876];
num2 = [9];
den2 = [1 3 9];
num = [10];
den = [1 1 10];
t = 0:0.05:5;
c1 = step(num1,den1,t);
c2 = step(num2,den2,t);
c = step(num,den,t);
plot(t,c1,'-',t,c2,'.',t,c,'x')
grid
title('Unit-Step Responses of Compensated Systems and Uncompensated System')
xlabel('t Sec')
ylabel('Outputs c1, c2, and c')
text(1.51,1.48,'Compensated System (Method 1)')
text(0.9,0.48,'Compensated System (Method 2)')
text(2.51,0.67,'Uncompensated System')

% ***** Unit-Ramp Responses of Compensated Systems *****
num1 = [12.287 23.876];
den1 = [1 5.646 16.933 23.876 0];
num2 = [9];
den2 = [1 3 9 0];
t = 0:0.05:5;
c1 = step(num1,den1,t);
c2 = step(num2,den2,t);
plot(t,c1,'-',t,c2,'.',t,t,'-')
grid
title('Unit-Ramp Responses of Compensated Systems')
xlabel('t Sec')
ylabel('Unit-Ramp Input and Outputs c1 and c2')
```

```
text(2.55,3.8,'Input')
text(0.55,2.8,'Compensated System (Method 1)')
text(2.35,1.75,'Compensated System (Method 2)')
```

2.6 RESULT:

LAG COMPENSATION

- Consider the problem of finding a suitable compensation network for the case where the system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state characteristics.
- Compensation in this case essentially consists of increasing the open loop gain without appreciably changing the transient-response characteristics.
- This can be accomplished if a lag compensator is put in cascade with the given feedforward transfer function.

2

- To avoid an appreciable change in the root loci, the angle contribution of the lag network should be limited to a small amount, say less than 5°.
- To assure this, we place the pole and zero of the lag network relatively close together and near the origin of the s plane. Then the closed-loop poles of the compensated system will be shifted only slightly from their original locations. Hence, the transient-response characteristics will be changed only slightly.

3

- The main negative effect of the lag compensation is that the compensator zero that will be generated near the origin creates a closed-loop pole near the origin. This closed loop pole and compensator zero will generate a long tail of small amplitude in the step response, thus increasing the settling time.

Consider a lag compensator $G_c(s)$, where

$$G_c(s) = \hat{K}_c \beta \frac{Ts + 1}{\beta Ts + 1} = \hat{K}_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}}$$

$$|G_c(s_1)| = \left| \hat{K}_c \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\beta T}} \right| \doteqdot \hat{K}_c$$

To make the angle contribution of the lag portion of the compensator small, we require

$$-5^\circ < \angle \frac{s_1 + \frac{1}{T}}{s_1 + \frac{1}{\beta T}} < 0^\circ$$

•An increase in the gain means an increase in the static error constants. If the open loop transfer function of the uncompensated system is G(s), then the static velocity error constant *Kv of the uncompensated system is*

$$K_v = \lim_{s \to 0} sG(s)$$

Then for the compensated system with the open-loop transfer function the static velocity error constant

$$\hat{K}_v = \lim_{s \to 0} sG_c(s)G(s) = \lim_{s \to 0} G_c(s)K_v = \hat{K}_c\beta K_v$$

6

Design Procedures for Lag Compensation

1. Draw the root-locus plot for the uncompensated system whose open-loop transfer function is G(s). Based on the transient-response specifications, locate the dominant closed-loop poles on the root locus.

2. Assume the transfer function of the lag compensator to be given by

7

$$G_c(s) = \hat{K}_c\beta\frac{Ts+1}{\beta Ts+1} = \hat{K}_c\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

3. Evaluate the particular static error constant specified in the problem.

4. Determine the amount of increase in the static error constant necessary to satisfy the specifications.

8

5. Determine the pole and zero of the lag compensator that produce the necessary increase in the particular static error constant without appreciably altering the original root loci.

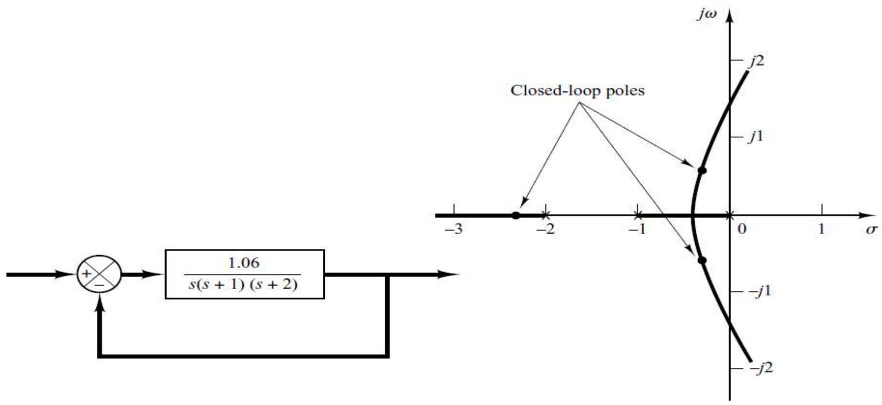
6. Draw a new root-locus plot for the compensated system. Locate the desired dominant closed-loop poles on the root locus.

7. Adjust gain Ǩc of the compensator from the magnitude condition so that the dominant closed-loop poles lie at the desired location.

9

EXAMPLE (Ogata P324): The feedforward transfer function is G(s) =[1.06]/[s(s + 1)(s + 2)]

It is desired to increase the static velocity error constant Kv to about 5 sec^–1.



(a) (b)

The closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{1.06}{s(s+1)(s+2)+1.06}$$

$$= \frac{1.06}{(s+0.3307-j0.5864)(s+0.3307+j0.5864)(s+2.3386)}$$

The dominant closed-loop poles are

$$s = -0.3307 \pm j0.5864 \quad = -\zeta\omega_n \pm j\omega_n\sqrt{(1-\zeta^2)}$$

The damping ratio of the dominant closed-loop poles is $\zeta = 0.491$. The undamped natural frequency of the dominant closed-loop poles is 0.673 rad/sec. The static velocity error constant is $0.53\ \text{sec}^{-1}$.

To meet the required specification,

let us insert a lag compensator in cascade with the given feedforward transfer function. To increase the static velocity error constant by a factor of about 10, let us choose β=10 and place the zero and pole of the lag compensator at s=–0.05 and s=–0.005, respectively. The transfer function of the lag compensator becomes :

$$G_c(s) = \hat{K}_c \frac{s + 0.05}{s + 0.005}$$

12

The angle contribution of this lag network near a dominant closed-loop pole is about 4°. Because this angle contribution is not very small, there is a small change in the new root locus near the desired dominant closed-loop poles.

The open-loop transfer function of the compensated system then becomes

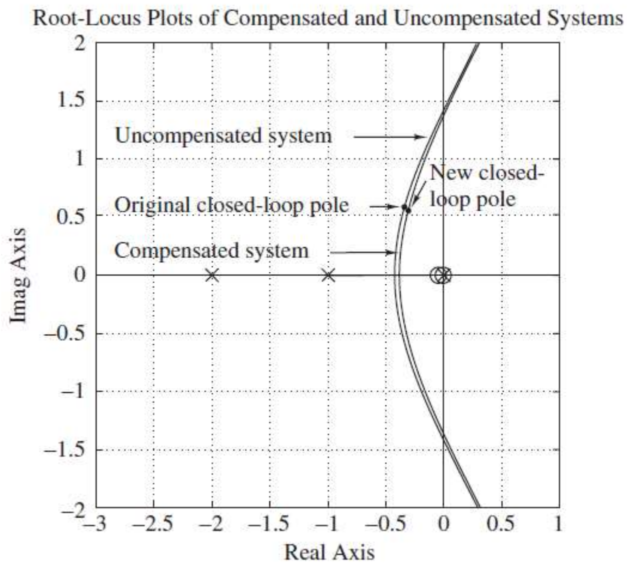
$$G_c(s)G(s) = \hat{K}_c \frac{s + 0.05}{s + 0.005} \frac{1.06}{s(s + 1)(s + 2)}$$

$$= \frac{K(s + 0.05)}{s(s + 0.005)(s + 1)(s + 2)}$$

where

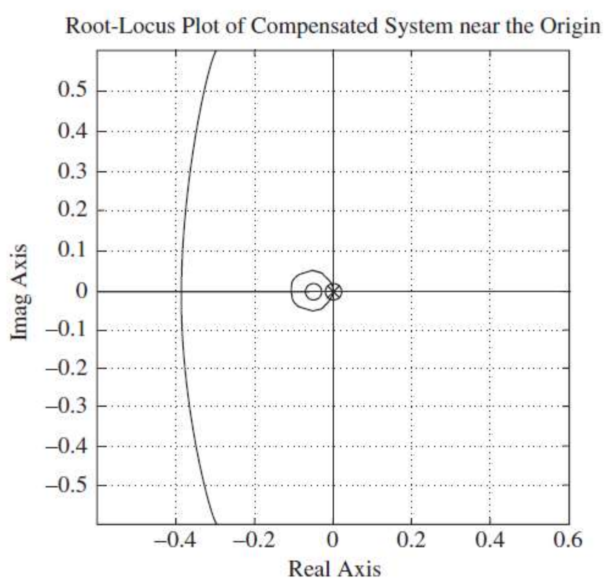
$$K = 1.06\hat{K}_c$$

13



Root-Locus Plots of Compensated and Uncompensated Systems
Uncompensated system
New closed-loop pole
Original closed-loop pole
Compensated system
Imag Axis
Real Axis

14



Root-Locus Plot of Compensated System near the Origin
Imag Axis
Real Axis

15

If the damping ratio of the new dominant closed-loop poles is kept the same, then these poles are obtained from the new root-locus plot as follows:

$$s_1 = -0.31 + j0.55, \qquad s_2 = -0.31 - j0.55$$

The open-loop gain K is determined from the magnitude condition as follows:

$$K = \left| \frac{s(s + 0.005)(s + 1)(s + 2)}{s + 0.05} \right|_{s=-0.31+j0.55}$$

Then the lag compensator gain $\hat{K}_c$ is determined as

$$\hat{K}_c = \frac{K}{1.06} = \frac{1.0235}{1.06} = 0.9656$$

Thus the transfer function of the lag compensator designed is

$$G_c(s) = 0.9656 \frac{s + 0.05}{s + 0.005} = 9.656 \frac{20s + 1}{200s + 1}$$

Then the compensated system has the following open-loop transfer function:

$$G_1(s) = \frac{1.0235(s + 0.05)}{s(s + 0.005)(s + 1)(s + 2)}$$

$$= \frac{5.12(20s + 1)}{s(200s + 1)(s + 1)(0.5s + 1)}$$

The static velocity error constant K_v is

$$K_v = \lim_{s \to 0} sG_1(s) = 5.12 \text{ sec}^{-1}$$

16

the value of the static velocity error constant of the compensated system is 9.66 times greater than that of the uncompensated system.

The two other closed-loop poles for the compensated system are found as follows:

s3 = -2.326, s4 = -0.0549

- The undamped natural frequency of the dominant closed-loop poles of the compensated system is 0.631 rad/sec. This value is about 6% less than the original value, 0.673 rad/sec. This implies that the transient response of the compensated system is slower than that of the original system.
- The maximum overshoot in the step response will increase in the compensated system.

17

- *Comments*. It is noted that under certain circumstances, however, both lead compensator and lag compensator may satisfy the given specifications (both transient response specifications and steady-state specifications.) Then either compensation may be used.

Text book

1:" Modern Control Engineering **"** By Katsuhiko Ogata.

2: “ Control Systems Engineering ” By Norman S. Nise.

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

3. Lag Compensation Techniques Based on the Root-Locus Approach

3.1 AIM:
It is desired to increase the static velocity error constant Kv to about 5 sec^{-1} without appreciably changing the location of the dominant closed-loop poles of the system using MATLAB.

3.2 APPARATUS:
Software: MATLAB.

3.3 PROCEDURE:
- Write MATLAB program in the MATLAB specified documents.
- Then save the program to run it.
- It is desired to increase the static velocity error constant Kv to about 5 sec^{-1} without appreciably changing the location of the dominant closed-loop poles
- Plot Root-locus of Compensated and Uncompensated Systems.
- The plot of the unit-ramp response curves for Compensated and Uncompensated Systems.
- The unit-step response curves of the compensated and uncompensated systems.

3.4 THEORETICAL CALCULATIONS:
Enter the numerators and denominators of the compensated and uncompensated systems.
num=
den=
Transfer function:
numc=
denc=
Transfer function:

3.5 EXAMPLE: Consider the G(s) = [1.06]/[s(s+1)(s+2)], Plot root loci.
It is desired to increase the static velocity error constant Kv to about 5 sec^{-1} without appreciably changing the location of the dominant closed-loop poles. {[1] Page 324}

```
% ***** Root-locus plots of the compensated system and uncompensated system *****
% ***** Enter the numerators and denominators of the compensated and uncompensated
systems *****
numc = [1 0.05];
denc = [1 3.005 2.015 0.01 0];
num = [1.06];
den = [1 3 2 0];
% ***** Enter rlocus command. Plot the root loci of both systems *****
rlocus(numc,denc)
hold
Current plot held
rlocus(num,den)
v = [-3 1 -2 2]; axis(v); axis('square')
grid
text(-2.8,0.2,'Compensated system')
text(-2.8,1.2,'Uncompensated system')
text(-2.8,0.58,'Original closed-loop pole')
text(-0.1,0.85,'New closed-')
text(-0.1,0.62,'loop pole')
title('Root-Locus Plots of Compensated and Uncompensated Systems')
hold
Current plot released
% ***** Plot root loci of the compensated system near the origin *****
rlocus(numc,denc)
v = [-0.6 0.6 -0.6 0.6]; axis(v); axis('square')
grid
title('Root-Locus Plot of Compensated System near the Origin')
```

```
% ***** Unit-ramp responses of compensated system and
% uncompensated system *****
% ***** Unit-ramp response will be obtained as the unit-step
% response of C(s)/[sR(s)] *****
% ***** Enter the numerators and denominators of C1(s)/[sR(s)]
% and C2(s)/[sR(s)], where C1(s) and C2(s) are Laplace
% transforms of the outputs of the compensated and un-
% compensated systems, respectively. *****
numc = [1.0235 0.0512];
```

```
denc = [1 3.005 2.015 1.0335 0.0512 0];
num = [1.06];
den = [1 3 2 1.06 0];
% ***** Specify the time range (such as t= 0:0.1:50) and enter
% step command and plot command. *****
t = 0:0.1:50;
c1 = step(numc,denc,t);
c2 = step(num,den,t);
plot(t,c1,'-',t,c2,'.',t,t,'--')
grid
text(2.2,27,'Compensated system');
text(26,21.3,'Uncompensated system');
title('Unit-Ramp Responses of Compensated and Uncompensated Systems')
xlabel('t Sec');
ylabel('Outputs c1 and c2')


% ***** Unit-step responses of compensated system and
% uncompensated system *****
% ***** Enter the numerators and denominators of the
% compensated and uncompensated systems *****
numc = [1.0235 0.0512];
denc = [1 3.005 2.015 1.0335 0.0512];
num = [1.06];
den = [1 3 2 1.06];
% ***** Specify the time range (such as t = 0:0.1:40) and enter
% step command and plot command. *****
t = 0:0.1:40;
c1 = step(numc,denc,t);
c2 = step(num,den,t);
plot(t,c1,'-',t,c2,'.')
grid
text(13,1.12,'Compensated system')
text(13.6,0.88,'Uncompensated system')
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs c1 and c2')
```

3.6 RESULT:

LAG–LEAD COMPENSATION

- Lead compensation basically speeds up the response and increases the stability of the system.
- Lag compensation improves the steady-state accuracy of the system, but reduces the speed of the response.
- If improvements in both transient response and steady-state response are desired, then both a lead compensator and a lag compensator may be used simultaneously.

2

Lag–lead Compensation Techniques Based on the Root-Locus Approach

- we use the lag–lead compensator:

$$G_c(s) = K_c \frac{\beta}{\gamma} \frac{(T_1 s + 1)(T_2 s + 1)}{\left(\frac{T_1}{\gamma} s + 1\right)(\beta T_2 s + 1)} = K_c \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right)$$

- where β>1 and Y> 1. (Consider *Kc to belong to the lead portion of the lag–lead* compensator.)

3

- In designing lag–lead compensators, we consider two cases where β≠Ɣ and β=Ɣ.
- Case 1. β≠Ɣ In this case, the design process is a combination of the design of the lead compensator and that of the lag compensator. The design procedure for the lag–lead compensator follows:

1) From the given performance specifications, determine the desired location for the dominant closed-loop poles.

4

2) Using the uncompensated open-loop transfer function G(s), determine the angle deficiency Ø if the dominant closed-loop poles are to be at the desired location. The phase-lead portion of the lag–lead compensator must contribute this angle Ø.
3) Assuming that we later choose T2 sufficiently large so that the magnitude of the lag portion equal to 1.

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right|$$

5

where s=s1 is one of the dominant closed-loop poles, choose the values of T1 and Y from the requirement that:

$$\angle \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} = \phi$$

Then determine the value of Kc from the magnitude condition:

$$\left| K_c \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} G(s_1) \right| = 1$$

6

4) If the static velocity error constant Kv is specified, determine the value of β to satisfy the requirement for Kv .The static velocity error constant Kv is given by:

$$\begin{aligned} K_v &= \lim_{s \to 0} sG_c(s)G(s) \\ &= \lim_{s \to 0} sK_c \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right) G(s) \\ &= \lim_{s \to 0} sK_c \frac{\beta}{\gamma} G(s) \end{aligned}$$

7

where Kc and Y are already determined in step 3. Hence, given the value of Kv, the value of β can be determined from this last equation.

$$= \lim_{s \to 0} sK_c \frac{\beta}{\gamma} G(s)$$

Then, using the value of β thus determined, choose the value of T2 such that:

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right| \doteq 1$$

$$-5^\circ < \angle \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} < 0^\circ$$

8

Case 1. β≠Y Summary

T1 and Y from: $\angle \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} = \phi$

Kc from : $\left| K_c \frac{s_1 + \frac{1}{T_1}}{s_1 + \frac{\gamma}{T_1}} G(s_1) \right| = 1$

9

Case 1. β≠Y Summary

β from:

$$= \lim_{s \to 0} sK_c \frac{\beta}{\gamma} G(s)$$

T2 from:

$$\left| \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} \right| \doteqdot 1$$

$$-5^\circ < \angle \frac{s_1 + \frac{1}{T_2}}{s_1 + \frac{1}{\beta T_2}} < 0^\circ$$

10

- **EXAMPLE (Ogata P335):** The feedforward transfer function is G(s) =4/[s(s + 0.5)]

  It is desired to make the damping ratio of the dominant closed-loop poles equal to **0.5** and to increase the undamped natural frequency to **5** rad/sec and the static velocity error constant to **80** sec^–1.

  Design an appropriate compensator to meet all the performance specifications.

11

This system has closed-loop poles at

s = -0.2500 ±j1.9843

The damping ratio is 0.125, the undamped natural frequency is 2 rad/sec, and the static velocity error constant is 8 sec^–1.

✓ From the performance specifications, the dominant closed-loop poles must be at

s = -2.50 ±j4.33

0.5, 5 rad/sec , 80 sec^–1.

12

Let us assume that we use a lag–lead compensator having the transfer function:

$$G_c(s) = K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right) \qquad (\gamma > 1, \beta > 1)$$

Where β≠Y .Then the compensated system will have the open-loop transfer function:

$$G_c(s)G(s) = K_c\left(\frac{s+\frac{1}{T_1}}{s+\frac{\gamma}{T_1}}\right)\left(\frac{s+\frac{1}{T_2}}{s+\frac{1}{\beta T_2}}\right)G(s)$$

13

From the performance specifications, the dominant closed-loop poles must be at:

s = -2.50 ±j4.33

$$\left/ \frac{4}{s(s+0.5)} \right|_{s=-2.50+j4.33} = -235^\circ$$

the phase-lead portion of the lag–lead compensator must contribute 55°=/180-235/ so that the root locus passes through the desired location of the dominant closed-loop poles.

To design the phase-lead portion of the compensator, we first determine the location of the zero and pole that will give 55° contribution. There are many possible choices, but we shall here choose the zero at s=–0.5 so that this zero will cancel the pole at s=–0.5 of the plant. Once the zero is chosen, the pole can be located such that the angle contribution is 55°. By simple calculation or graphical analysis, the pole must be located at s=–5.02. Thus, the phase-lead portion of the lag–lead compensator becomes: T1 = 2, Y = 5.02/0.5 = 10.04

$$K_c \frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} = K_c \frac{s + 0.5}{s + 5.02}$$

we determine the value of Kc from the magnitude condition:

$$\left| K_c \frac{s + 0.5}{s + 5.02} \frac{4}{s(s + 0.5)} \right|_{s=-2.5+j4.33} = 1$$

$$K_c = \left| \frac{(s + 5.02)s}{4} \right|_{s=-2.5+j4.33} = 6.26$$

16

The phase-lag portion of the compensator can be designed as follows: First the value of β is determined to satisfy the requirement on the static velocity error constant:

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sK_c \frac{\beta}{\gamma} G(s)$$

$$= \lim_{s\to 0} s(6.26) \frac{\beta}{10.04} \frac{4}{s(s + 0.5)} = 4.988\beta = 80$$

$$\beta = 16.04$$

17

Finally, we choose the value T2 such that the following two conditions are satisfied:

$$\left|\frac{s+\frac{1}{T_2}}{s+\frac{1}{16.04T_2}}\right|_{s=-2.5+j4.33} \doteqdot 1, \qquad -5^\circ < \angle\left.\frac{s+\frac{1}{T_2}}{s+\frac{1}{16.04T_2}}\right|_{s=-2.5+j4.33} < 0^\circ$$

We may choose several values for T2 and check if the magnitude and angle conditions are satisfied. After simple calculations we find for T2 = 5

1 > magnitude > 0.98, -2.10° < angle < 0°

T2 = 5 satisfies the two conditions.

Now the transfer function of the designed lag–lead compensator is given by:

$$G_c(s) = (6.26)\left(\frac{s+\frac{1}{2}}{s+\frac{10.04}{2}}\right)\left(\frac{s+\frac{1}{5}}{s+\frac{1}{16.04\times 5}}\right)$$

$$= 6.26\left(\frac{s+0.5}{s+5.02}\right)\left(\frac{s+0.2}{s+0.01247}\right)$$

$$= \frac{10(2s+1)(5s+1)}{(0.1992s+1)(80.19s+1)}$$

The compensated system will have the open-loop transfer function:

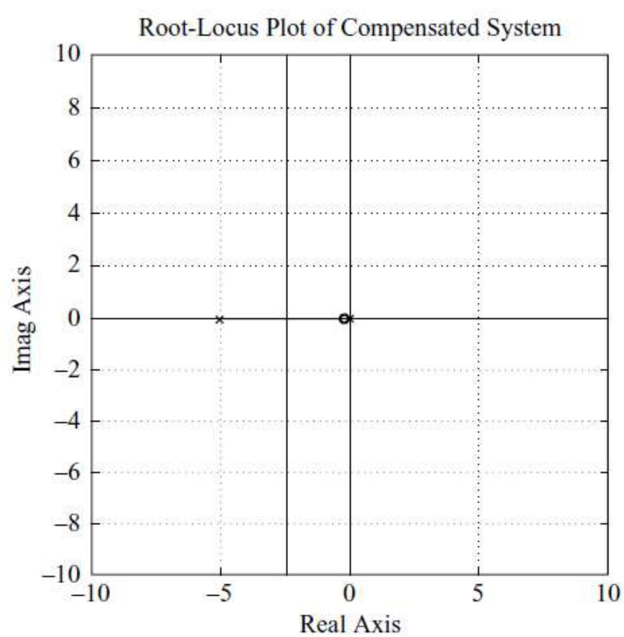
$$G_c(s)G(s) = \frac{25.04(s + 0.2)}{s(s + 5.02)(s + 0.01247)}$$

The characteristic equation for the compensated system is:

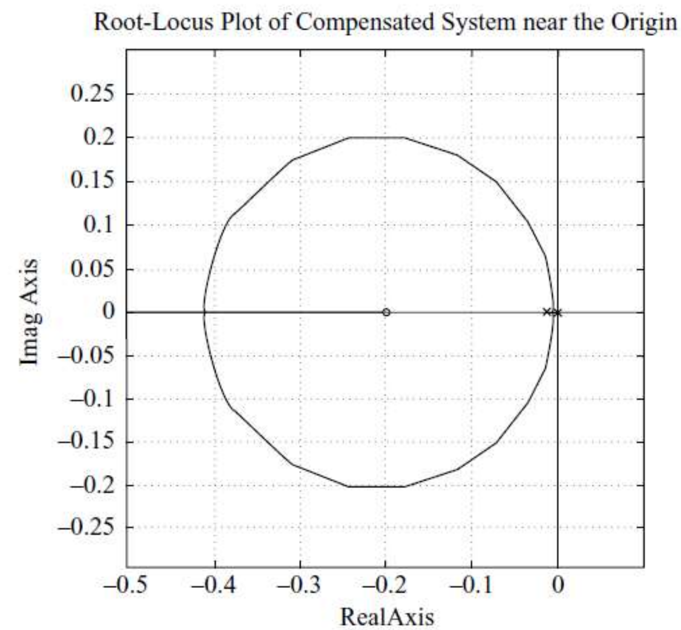
$$s^3 + 5.0325s^2 + 25.1026s + 5.008$$
$$= (s + 2.4123 + j4.2756)(s + 2.4123 - j4.2756)(s + 0.2078) = 0$$

The new damping ratio is 0.491. Therefore the compensated system meets all the required performance specifications.

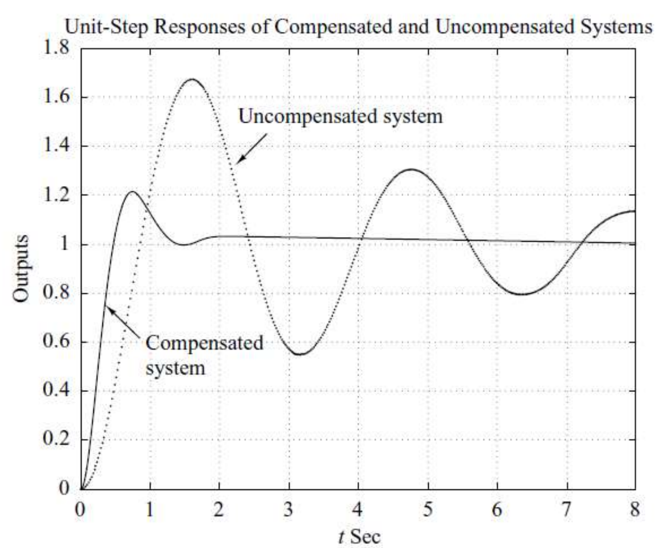
20



21


Root-Locus Plot of Compensated System near the Origin
Imag Axis
RealAxis
0.25
0.2
0.15
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.5
−0.4
−0.3
−0.2
−0.1
0

22


Unit-Step Responses of Compensated and Uncompensated Systems
Uncompensated system
Compensated
system
Outputs
t Sec
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8

23

3/24/2018College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

4. LAG–LEAD COMPENSATION Techniques Based on the Root-Locus Approach

4.1 AIM:
Lag–lead compensation combines the advantages of lag and lead compensations. Since the lag–lead, compensator possesses two poles and two zeros, such compensation increase the order of the system by 2, unless cancellation of pole(s) and zero(s) occurs in the compensated system.

4.2 APPARATUS:
Software: MATLAB.

4.3 PROCEDURE:

- Write MATLAB program in the MATLAB specified documents.
- Then save the program to run it.
- It is desired to make the damping ratio of the dominant closed-loop poles equal to 0.5 and to increase the undamped natural frequency to 5 rad/sec and the static velocity error constant to 80 sec^{-1}.
- Plot Root-locus of Compensated and Uncompensated Systems.
- The plot of the unit-ramp response curves for Compensated and Uncompensated Systems.
- The unit-step response curves of the compensated and uncompensated systems.

4.4 THEORETICAL CALCULATIONS:
Enter the numerators and denominators of the compensated and uncompensated systems.
num=
den=
Transfer function:
numc=
denc=
Transfer function:

4.5 EXAMPLE: The feedforward transfer function is G(s) = [4]/[s(s+0.5)].
It is desired to make the damping ratio of the dominant closed-loop poles equal to 0.5 and to increase the undamped natural frequency to 5 rad/sec and the static velocity error constant to 80 sec–1.{[1] Page 335}

```
% ***** Root-locus plots of the compensated system and uncompensated system *****
% ***** Enter the numerators and denominators of the compensated and uncompensated
systems *****
numc = [25.04 5];
denc = [1 5.032 0.0625 0];
num = [4];
den = [1 0.5 0];
% ***** Enter rlocus command. Plot the root loci of both systems *****
rlocus(num,den)
hold
Current plot held
rlocus(numc,denc)
v = [-10 10 -10 10]; axis(v); axis('square')
grid
title('Root-Locus Plots of Compensated and Uncompensated Systems')
hold
Current plot released
% ***** Plot root loci of the compensated system near the origin *****
rlocus(numc,denc)
v = [-0.6 0.6 -0.6 0.6]; axis(v); axis('square')
grid
title('Root-Locus Plot of Compensated System near the Origin')

% ***** Unit-ramp responses of compensated system and
% uncompensated system *****
% ***** Unit-ramp response will be obtained as the unit-step
% response of C(s)/[sR(s)] *****
% ***** Enter the numerators and denominators of C1(s)/[sR(s)]
% and C2(s)/[sR(s)], where C1(s) and C2(s) are Laplace
% transforms of the outputs of the compensated and un-
% compensated systems, respectively. *****
numc = [25.04 5];
denc = [1 5.0324 25.102 5 0];
num = [4];
den = [1 0.5 4 0];
% ***** Specify the time range (such as t= 0:0.01:10) and enter
% step command and plot command. *****
t = 0:0.01:10;
```

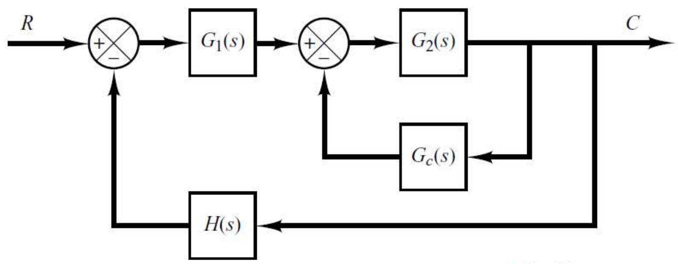
```
c1 = step(numc,denc,t);
c2 = step(num,den,t);
plot(t,c1,'-',t,c2,'.',t,t,'--')
grid
title('Unit-Ramp Responses of Compensated and Uncompensated Systems')
xlabel('t Sec');
ylabel('Outputs c1 and c2')


% ***** Unit-step responses of compensated system and
% uncompensated system *****
% ***** Enter the numerators and denominators of the
% compensated and uncompensated systems *****
numc = [25.04 5];
denc = [1 5.0324 25.102 5];
num = [4];
den = [1 0.5 4];
% ***** Specify the time range (such as t = 0:0.1:20) and enter
% step command and plot command. *****
t = 0:0.1:20;
c1 = step(numc,denc,t);
c2 = step(num,den,t);
plot(t,c1,'-',t,c2,'.')
grid
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs c1 and c2')
```

4.6 RESULT:

PARALLEL COMPENSATION

- We present a simple design problem involving parallel compensation.



$$\frac{C}{R} = \frac{G_1 G_2}{1 + G_2 G_c + G_1 G_2 H}$$

The characteristic equation is

$$1 + G_1 G_2 H + G_2 G_c = 0$$

2

- By dividing this characteristic equation by the sum of the terms that do not involve *Gc* , we obtain:

$$1 + \frac{G_c G_2}{1 + G_1 G_2 H} = 0$$

$$G_f = \frac{G_2}{1 + G_1 G_2 H}$$

$$1 + G_c G_f = 0$$

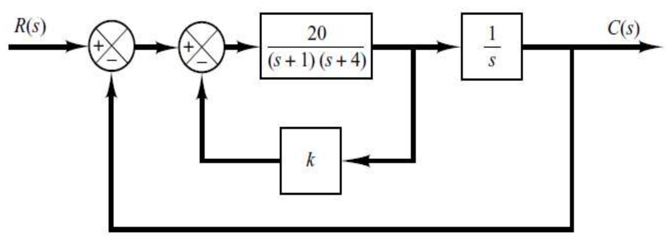
- Since Gf is a fixed transfer function, **the design of Gc becomes the same as the case of series compensation.** Hence the same design approach applies to the parallel compensated system.

3

- **EXAMPLE (Ogata P343):** Draw a root-locus diagram. Then determine the value of k such that the damping ratio of the dominant closed-loop poles is 0.4

  Here the system involves velocity feedback. The open-loop transfer function is:

$$\text{Open-loop transfer function} = \frac{20}{s(s+1)(s+4) + 20ks}$$



Notice that the adjustable variable k does not appear as a multiplying factor. The characteristic equation for the system is:

$$s^3 + 5s^2 + 4s + 20ks + 20 = 0$$

Define

$$20k = K$$

Then Equation becomes: $s^3 + 5s^2 + 4s + Ks + 20 = 0$

Dividing both sides of Equation by the sum of the terms that do not contain K, we get:

$$1 + \frac{Ks}{s^3 + 5s^2 + 4s + 20} = 0$$

$$1 + \frac{Ks}{(s + j2)(s - j2)(s + 5)} = 0$$

- We shall now sketch the root loci of the system given by last Equation. Notice that the open-loop poles are located at **s=j2, s=–j2, s=–5**, and the open-loop zero is located at **s=0**. The root locus exists on the real axis between 0 and –5.

$$\lim_{s\to\infty} \frac{Ks}{(s+j2)(s-j2)(s+5)} = \lim_{s\to\infty} \frac{K}{s^2}$$

we have

$$\text{Angles of asymptote} = \frac{\pm 180^\circ(2k+1)}{2} = \pm 90^\circ$$

The intersection of the asymptotes with the real axis can be found from

$$\lim_{s\to\infty} \frac{Ks}{s^3+5s^2+4s+20} = \lim_{s\to\infty} \frac{K}{s^2+5s+\cdots} = \lim_{s\to\infty} \frac{K}{(s+2.5)^2}$$

as

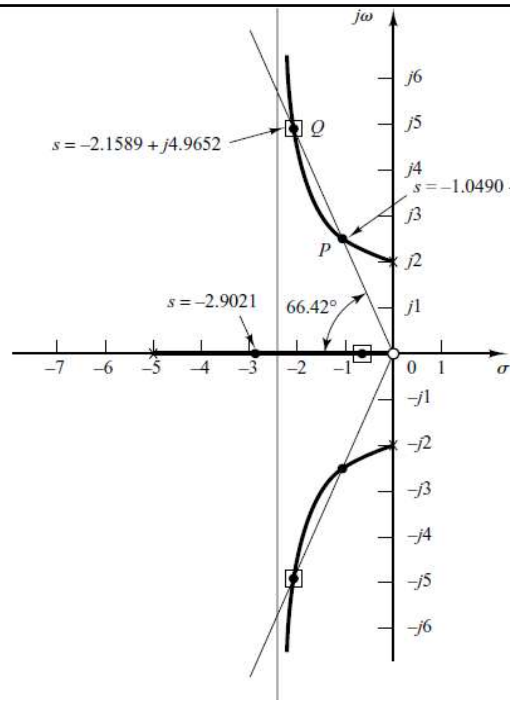
$$s = -2.5$$

- The angle of departure (angle θ) from the pole at s=j2 is obtained as follows:

θ = 180° - 90° - 21.8° + 90° = 158.2°

- Note that the closed-loop poles with **ζ=0.4** must lie on straight lines passing through the origin and making the angles **±66.42°** with the negative real axis.

7



8

- Thus, two values of K will give the damping ratio ʅ of the closed-loop poles equal to 0.4.

  At point P, the value of K is:

$$K = \left| \frac{(s + j2)(s - j2)(s + 5)}{s} \right|_{s=-1.0490+j2.4065} = 8.9801$$

$$k = \frac{K}{20} = 0.4490 \qquad \text{at point } P$$

For k=0.4490, the three closed-loop poles are located at:

s = -1.0490 + j2.4065, s = -1.0490 - j2.4065, s = -2.9021

9

- At point Q, the value of K is:

$$K = \left|\frac{(s + j2)(s - j2)(s + 5)}{s}\right|_{s=-2.1589+j4.9652} = 28.260$$

$$k = \frac{K}{20} = 1.4130 \qquad \text{at point } Q$$

For k=1.4130, the three closed-loop poles are located at:

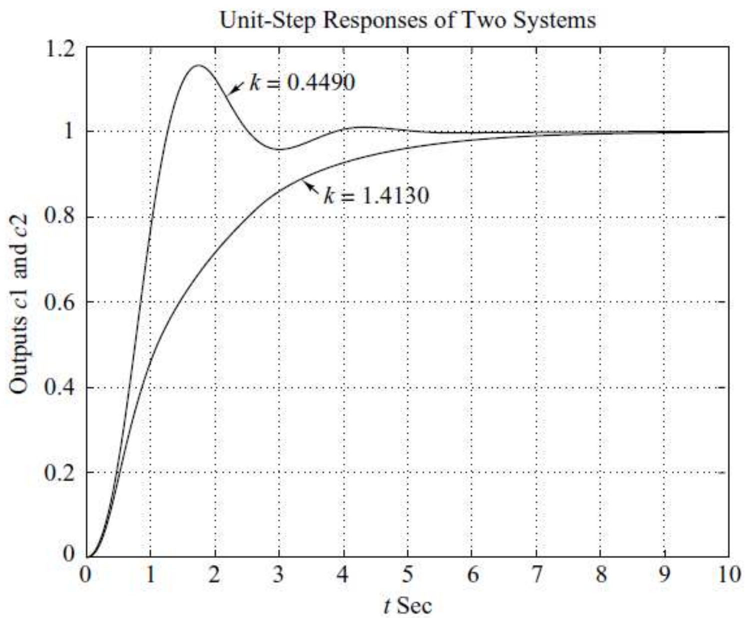
s = -2.1589 + j4.9652, s = -2.1589 - j4.9652, s = -0.6823

10

- Notice that the system with **k=0.4490** has a pair of dominant complex-conjugate closed-loop poles, while in the system with **k=1.4130** the real closed-loop pole at s=–0.6823 is dominant, and the complex-conjugate closed-loop poles are not dominant.

- The system with k=0.4490 (which exhibits a **faster response** with relatively small overshoot) has a much better response characteristic than the system with k=1.4130 (which exhibits a slow over damped response).Therefore, we should choose k=0.4490 for the present system.

11



12

- Homework: EXAMPLE -(A-6-20)- Ogata P392

13

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

5. Parallel Compensation Based on the Root-Locus Approach

5.1 AIM:
Design approach applies to the parallel compensated system.

5.2 APPARATUS:
Software: MATLAB.

5.3 PROCEDURE:
- Determine the value of k (parallel compensated) such that the damping ratio is specified.
- Plot the unit-step response curves of the systems.

5.4 THEORETICAL CALCULATIONS:
num=
den=
Transfer function:

5.5 EXAMPLE: Consider the G(s) = [20]/[(s(s+1)(s+4))+20Ks].
Plotting the unit-step response curves in one diagram, to compare the unit-step responses of both systems, for k=0.4490 and for k=1.4130

```
% ---------- Unit-step response ----------
% ***** Enter numerators and denominators of systems with
% k = 0.4490 and k = 1.4130, respectively. *****
num1 = [20];
den1 = [1 5 12.98 20];
num2 = [20];
den2 = [1 5 32.26 20];
t = 0:0.1:10;
c1 = step(num1,den1,t);
c2 = step(num2,den2,t);
plot(t,c1,t,c2)
text(2.5,1.12,'k = 0.4490')
text(3.7,0.85,'k = 1.4130')
grid
title('Unit-step Responses of Two Systems')
xlabel('t Sec')
ylabel('Outputs c1 and c2')
```

5.6 RESULT:

Frequency-Response Method

- developed in 1930s and 1940s by Nyquist, Bode, Nichols, and many others.
- By the term frequency response, we mean the steady-state response of a system to a sinusoidal input.
- advantage of the frequency-response approach is that we can use the data obtained from measurements on the physical system without deriving its mathematical model. Frequency-response tests are, in general, simple and can be made accurately by use of readily available sinusoidal signal generators and precise measurement equipment

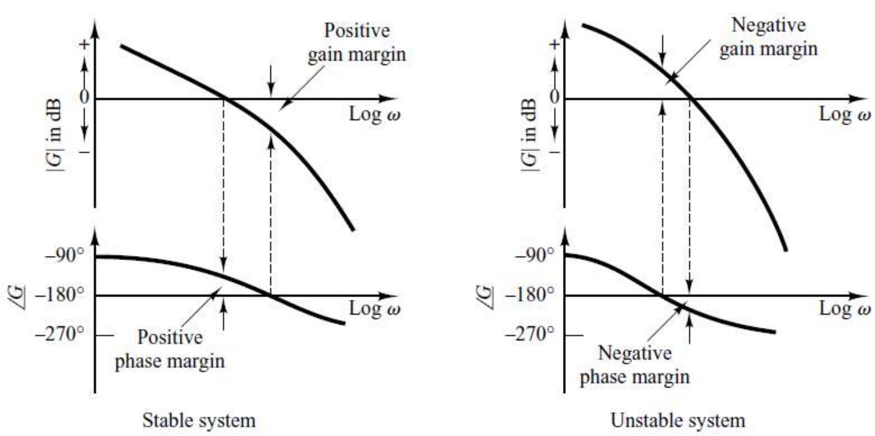
- *Phase margin: The phase margin is that amount of additional phase lag at the gain* crossover frequency required to bring the system to the verge of instability. The gain crossover frequency is the frequency at which $|G(j\omega)|$, the magnitude of the open loop transfer function, is unity. The phase margin Y is 180° plus the phase angle Ø of the open-loop transfer function at the gain crossover frequency, or Y = 180° + Ø
- The phase margin is positive for Y>0 and negative for Y<0. For a minimum phase system to be stable, the phase margin must be positive.
- In the logarithmic plots, the critical point in the complex plane corresponds to the 0 dB and –180° lines.[P464]

- Gain margin: The gain margin is the reciprocal of the magnitude |G(jω)| at the frequency at which the phase angle is –180°. Defining the phase crossover frequency ω1 to be the frequency at which the phase angle of the open-loop transfer function equals –180° gives the gain margin Kg: Kg = 1/ |G(j ω1)|
- In terms of decibels,

$$K_g \text{ dB} = 20 \log K_g = -20 \log |G(j\omega_1)|$$

- a positive gain margin (in decibels) means that the system is stable, and a negative gain margin (in decibels) means that the system is unstable.
- The gain margin expressed in decibels is positive if Kg is greater than unity and negative if Kg is smaller than unity .[P466]

4



5

CONTROL SYSTEMS DESIGN BY FREQUENCY RESPONSE APPROACH

- The frequency-response approach is very useful in designing control systems.
- a Bode diagram of the compensator can be simply added to the original Bode diagram, and thus plotting the complete Bode diagram is a simple matter.
- Also, if the open-loop gain is varied, the magnitude curve is shifted up or down without changing the slope of the curve, and the phase curve remains the same. For design purposes, therefore, it is best to work with the Bode diagram.

6

- A common approach to the design based on the Bode diagram is that we first adjust the open-loop gain so that the requirement on the steady-state accuracy is met. Then the magnitude and phase curves of the uncompensated open loop (with the open-loop gain just adjusted) are plotted. If the specifications on the phase margin and gain margin are not satisfied, then a suitable compensator that will reshape the open-loop transfer function is determined. Finally, if there are any other requirements to be met, we try to satisfy them, unless some of them are mutually contradictory.

7

Basic Characteristics of Lead, Lag, and Lag–Lead Compensation.

- Lead compensation essentially yields an appreciable improvement in transient response and a small change in steady-state accuracy. It may accentuate high-frequency noise effects.
- Lag compensation, on the other hand, yields an appreciable improvement in steady-state accuracy at the expense of increasing the transient-response time. Lag compensation will suppress the effects of high-frequency noise signals.
- Lag–lead compensation combines the characteristics of both lead compensation and lag compensation.

8

LEAD COMPENSATION

- Characteristics of Lead Compensators. Consider a lead compensator having the following transfer function:

$$K_c \alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad (0 < \alpha < 1)$$

- where α is the attenuation factor of the lead compensator. It has a zero at s=–1/T and a pole at s=–1/(αT). Since 0< α <1, we see that the zero is always located to the right of the pole in the complex plane.

9

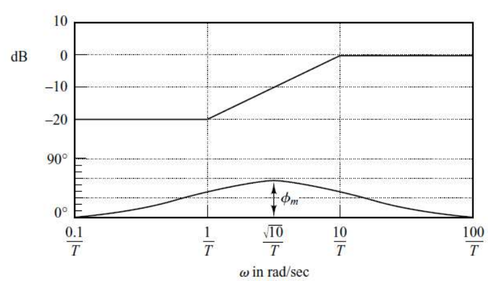
- The minimum value of α is usually taken to be about 0.05. (This means that the maximum phase lead that may be produced by a lead compensator is about 65°.)
- the maximum phase-lead angle and the value of α:

$$\sin \phi_m = \frac{\frac{1-\alpha}{2}}{\frac{1+\alpha}{2}} = \frac{1-\alpha}{1+\alpha}$$

10

- The Bode diagram of a lead compensator when Kc=1 and α =0.1. The corner frequencies for the lead compensator are ω =1/T and ω =1/(αT)=10/T. By examining Figure, we see that ωm is the geometric mean of the two corner frequencies, or

$$\log \omega_m = \frac{1}{2}\left(\log \frac{1}{T} + \log \frac{1}{\alpha T}\right)$$



- ωm = 1/ (√α .T) the lead compensator is basically a high-pass filter.

11

Lead Compensation Techniques Based on the Frequency-Response Approach.

- The primary function of the lead compensator is to reshape the frequency-response curve to provide sufficient phase-lead angle to offset the excessive phase lag associated with the components of the fixed system.
- Assume that the performance specifications are given in terms of phase margin, gain margin, static velocity error constants, and so on.

The procedure for designing a lead compensator

1. Assume the following lead compensator:

$$G_c(s) = K_c\alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad (0 < \alpha < 1)$$

Define

$$K_c\alpha = K$$

Then

$$G_c(s) = K \frac{Ts + 1}{\alpha Ts + 1}$$

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K \frac{Ts + 1}{\alpha Ts + 1} G(s) = \frac{Ts + 1}{\alpha Ts + 1} KG(s) = \frac{Ts + 1}{\alpha Ts + 1} G_1(s)$$

where

$$G_1(s) = KG(s)$$

Determine gain K to satisfy the requirement on the given static error constant.

2. Using the gain K thus determined, draw a Bode diagram of G1(jω), the gain adjusted but uncompensated system. Evaluate the phase margin.
3. Determine the necessary phase-lead angle to be added to the system. Add an additional 5° to 12° to the phase-lead angle required, because the addition of the lead compensator **shifts the gain crossover frequency to the right** and decreases the phase margin.
4. Determine the attenuation factor α by use of Equation. Determine the frequency where the magnitude of the uncompensated system G1(jω) is equal to $-20 \log(1/\sqrt{\alpha})$. Select this frequency as the new gain crossover frequency. This frequency corresponds to $\omega m = 1/ (\sqrt{\alpha} .T)$ and the maximum phase shift Øm occurs at this frequency.

5. Determine the corner frequencies of the lead compensator as follows:

$$\text{Zero of lead compensator:} \quad \omega = \frac{1}{T}$$

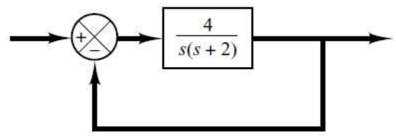
$$\text{Pole of lead compensator:} \quad \omega = \frac{1}{\alpha T}$$

6. Using the value of K determined in step 1 and that of α determined in step 4, calculate constant Kc from:

$$K_c = \frac{K}{\alpha}$$

7. Check the gain margin to be sure it is satisfactory. If not, repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained.

EXAMPLE (Ogata P496): Consider the system shown in Figure.



- It is desired to design a compensator for the system so that the static velocity error constant *Kv* is 20 sec^–1, the phase margin is at least 50°, and the gain margin is at least 10 dB.

We shall use a lead compensator of the form :

$$G_c(s) = K_c\alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}}$$

16

The compensated system will have the open-loop transfer function $G_c(s)G(s)$. Define

$$G_1(s) = KG(s) = \frac{4K}{s(s + 2)}$$

where $K = K_c\alpha$.

The first step in the design is to adjust the gain K to meet the steady-state performance specification or to provide the required static velocity error constant. Since this constant is given as 20 sec^–1, we obtain:

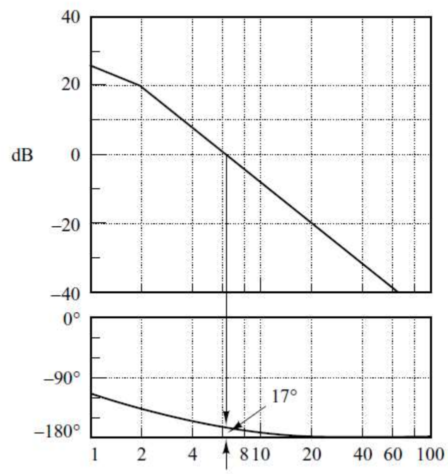
$$K_v = \lim_{s \to 0} sG_c(s)G(s) = \lim_{s \to 0} s \frac{Ts + 1}{\alpha Ts + 1} G_1(s) = \lim_{s \to 0} \frac{s4K}{s(s + 2)} = 2K = 20$$

$$K = 10$$

17

With $K = 10$, the compensated system will satisfy the steady-state requirement.
We shall next plot the Bode diagram of

$$G_1(j\omega) = \frac{40}{j\omega(j\omega + 2)} = \frac{20}{j\omega(0.5j\omega + 1)}$$



- From this plot, the phase and gain margins of the system are found to be 17° and +infinity dB, respectively.
- The specification calls for a phase margin of at least 50°.We thus find the additional phase lead necessary to satisfy the relative stability requirement is 33°.To achieve a phase margin of 50° without decreasing the value of K, the lead compensator must contribute the required phase angle.
- Noting that the addition of a lead compensator modifies the magnitude curve in the Bode diagram, we realize that the gain crossover frequency will be shifted to the right. We must offset the increased phase lag of G1(jω) due to this increase in the gain crossover frequency. Considering the shift of the gain crossover frequency, we may assume that Øm, the maximum phase lead required, is approximately **38**°. (This means that 5° has been added to compensate for the shift in the gain crossover frequency.)

the maximum phase-lead angle and the value of α:

$$\sin \phi_m = \frac{\dfrac{1-\alpha}{2}}{\dfrac{1+\alpha}{2}} = \frac{1-\alpha}{1+\alpha}$$

Øm =38° corresponds to α =0.24. Once the attenuation factor α has been determined on the basis of the required phase-lead angle, the next step is to determine the corner frequencies ω =1/T and ω =1/(αT) of the lead compensator. To do so, we first note that the maximum phase-lead angle Øm occurs at the geometric mean of the two corner frequencies or ω = 1/ (√α .T) . The amount of the modification in the magnitude curve at due to the inclusion of the term (Ts+1)/(α Ts+1) is:

20

$$\left|\frac{1 + j\omega T}{1 + j\omega\alpha T}\right|_{\omega=1/(\sqrt{\alpha}T)} = \left|\frac{1 + j\dfrac{1}{\sqrt{\alpha}}}{1 + j\alpha\dfrac{1}{\sqrt{\alpha}}}\right| = \frac{1}{\sqrt{\alpha}}$$

Note that

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.24}} = \frac{1}{0.49} = 6.2 \text{ dB}$$

and $|G_1(j\omega)| = -6.2$ dB corresponds to $\omega = 9$ rad/sec. We shall select this frequency to be the new gain crossover frequency ω_c. Noting that this frequency corresponds to $1/(\sqrt{\alpha}T)$, or $\omega_c = 1/(\sqrt{\alpha}T)$, we obtain

$$\frac{1}{T} = \sqrt{\alpha}\omega_c = 4.41$$

and

$$\frac{1}{\alpha T} = \frac{\omega_c}{\sqrt{\alpha}} = 18.4$$

21

The lead compensator thus determined is

$$G_c(s) = K_c \frac{s + 4.41}{s + 18.4} = K_c\alpha \frac{0.227s + 1}{0.054s + 1}$$

where the value of K_c is determined as

$$K_c = \frac{K}{\alpha} = \frac{10}{0.24} = 41.7$$

Thus, the transfer function of the compensator becomes

$$G_c(s) = 41.7 \frac{s + 4.41}{s + 18.4} = 10 \frac{0.227s + 1}{0.054s + 1}$$

Note that

$$\frac{G_c(s)}{K} G_1(s) = \frac{G_c(s)}{10} 10G(s) = G_c(s)G(s)$$

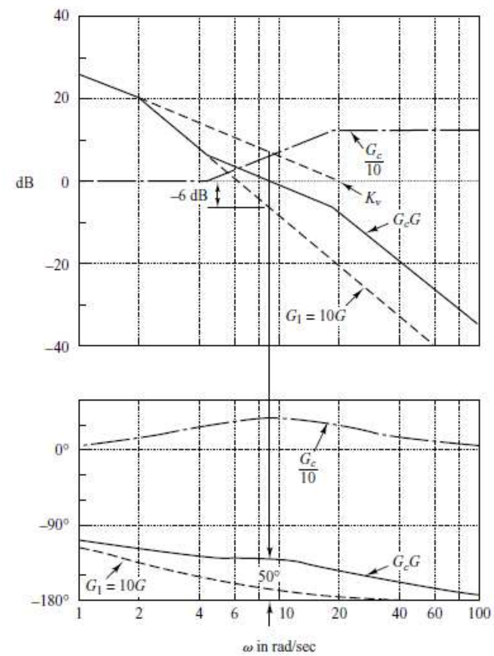
22

- The compensated system has the following open-loop transfer function:

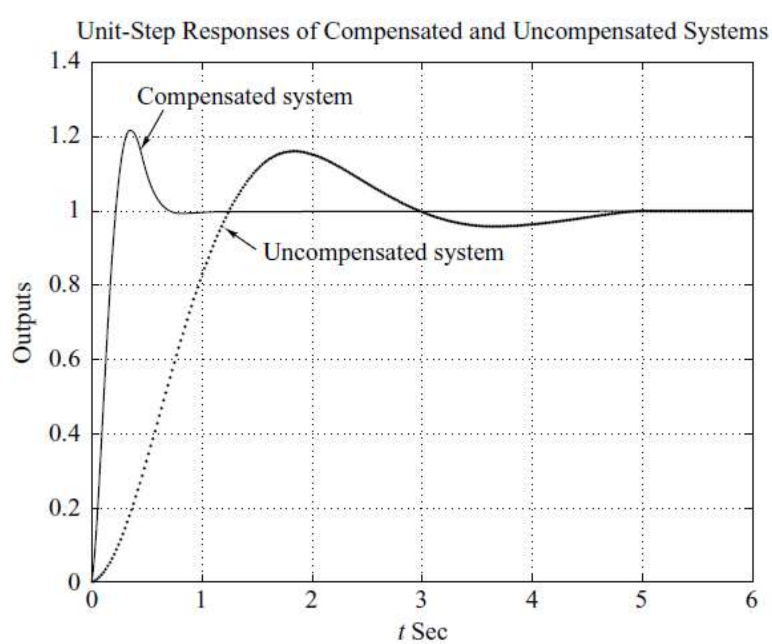
$$G_c(s)G(s) = 41.7 \frac{s + 4.41}{s + 18.4} \frac{4}{s(s + 2)}$$

- Note that the bandwidth is approximately equal to the gain crossover frequency. **The lead compensator causes the gain crossover frequency to increase from 6.3 to 9 rad/sec.** The increase in this frequency means an increase in bandwidth. This implies an increase in the speed of response.
- The phase and gain margins are seen to be approximately 50° and +infinity dB, respectively.

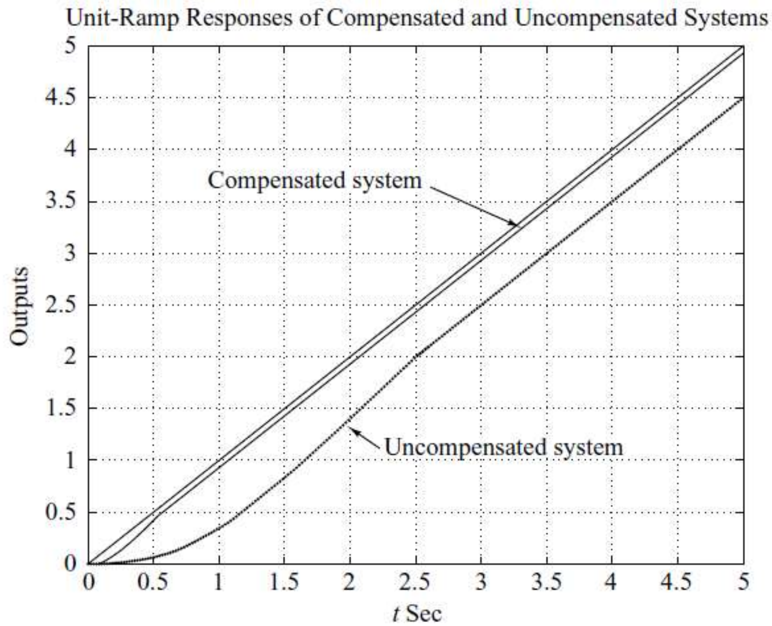
23


40
20
dB 0
−20
−40
$\frac{G_c}{10}$
K_v
G_cG
−6 dB
$G_1 = 10G$
0°
−90°
−180°
50°
1 2 4 6 10 20 40 60 100
ω in rad/sec

24


Unit-Step Responses of Compensated and Uncompensated Systems
Compensated system
Uncompensated system
Outputs
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6
t Sec

25


Unit-Ramp Responses of Compensated and Uncompensated Systems
Outputs
t Sec
Compensated system
Uncompensated system
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

26

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

6. Basic Characteristics of Lead Compensation by frequency-response approach.

6.1 AIM:
Plot bode diagram. The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems using MATLAB to examine the frequency characteristics of the lead compensator.

6.2 APPARATUS:
Software: MATLAB.

6.3 PROCEDURE:
- Plot bode diagram of Compensated and Uncompensated Systems.
- The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems.

6.4 THEORETICAL CALCULATIONS:

Enter the numerators and denominators of the compensated and uncompensated systems.
num=
den=
Transfer function:
numc=
denc=
Transfer function:
6.5 EXAMPLE: Consider the G(s) = [20]/[s(0.5s+1)], Plot bode diagram. {[1] Page 496}

```
num = [20];
den = [.5 1 0];
bode(num,den)
grid
title('Bode Diagram of G1(s) = [20]/[s(0.5s+1)], ')
```

- Plot The unit-step response curves of the system before and after compensation, G(s) = [4]/[s(s+2)], It is desired to design a compensator for the system so that the static velocity error constant Kv is 20 sec^–1, the phase margin is at least 50°, and the gain margin is at least 10 dB.

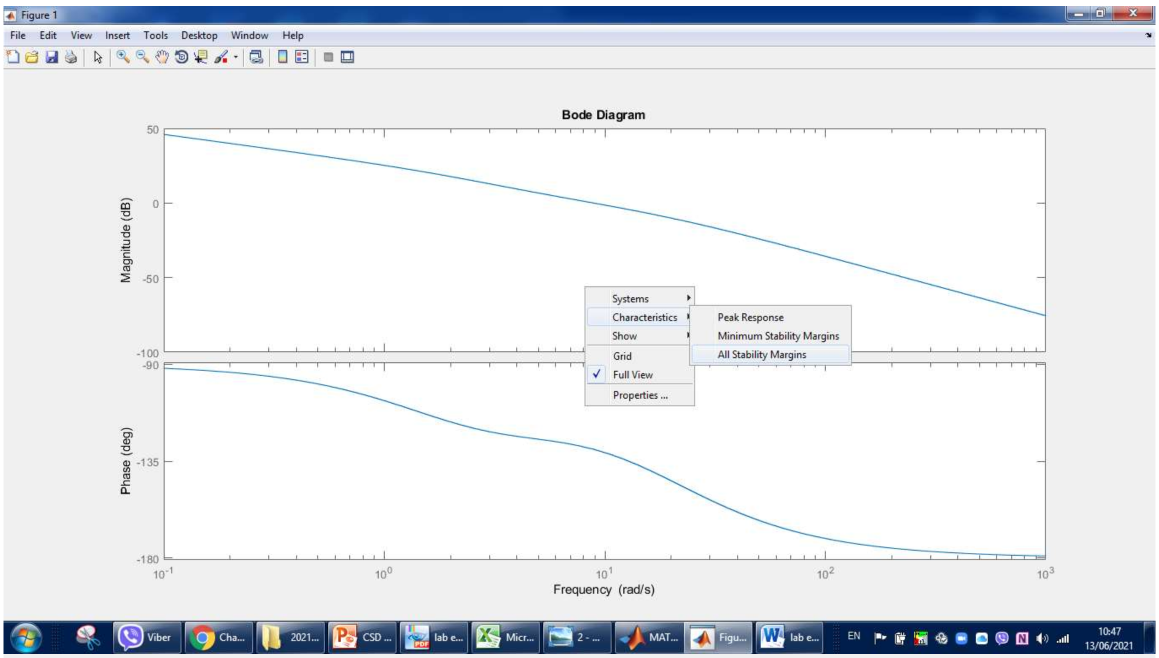
```
%*****Unit-step responses*****
num = [4];
den = [1 2 4];
numc = [166.8 735.588];
denc = [1 20.4 203.6 735.588];
t = 0:0.02:6;
[c1,x1,t] = step(num,den,t);
[c2,x2,t] = step(numc,denc,t);
plot (t,c1,'.',t,c2,'-')
grid
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')
text(0.4,1.31,'Compensated system')
text(1.55,0.88,'Uncompensated system')
```

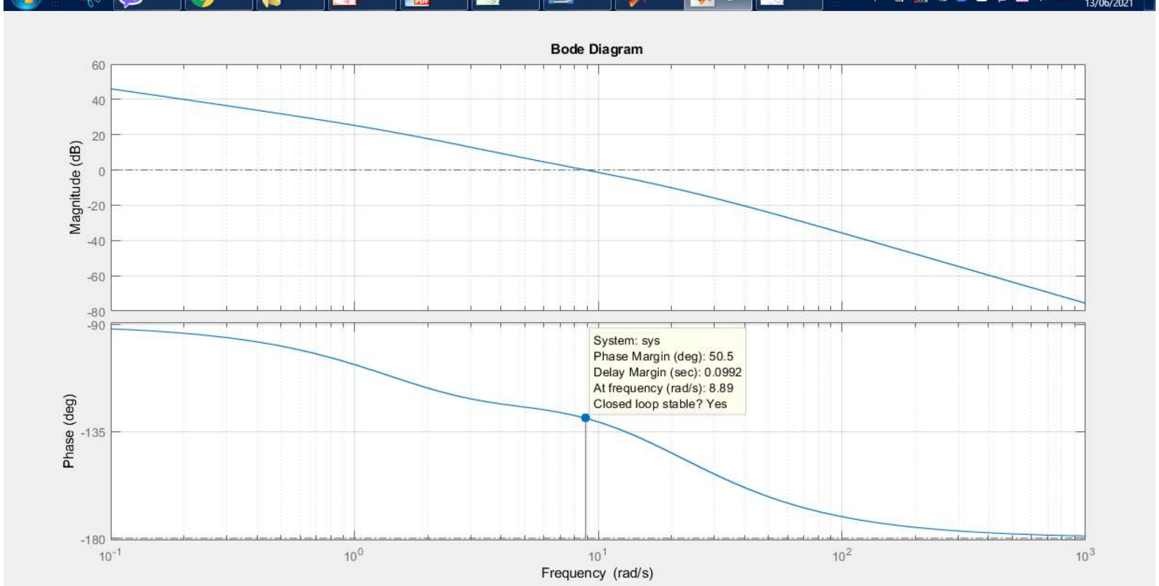
```
%*****Unit-ramp responses*****
num = [4];
den = [1 2 4 0];
numc = [166.8 735.588];
denc = [1 20.4 203.6 735.588 0];
t = 0:0.02:6;
[c1,x1,t] = step(num,den,t);
[c2,x2,t] = step(numc,denc,t);
plot (t,c1,'.',t,c2,'-')
grid
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')
text(0.4,1.31,'Compensated system')
text(1.55,0.88,'Uncompensated system')
```

6.6 RESULT:

bode diagram **of compensated system**

```
num = [166.8 735.5];
den = [1 20.4 36.8 0];
bode(num,den)
grid
```



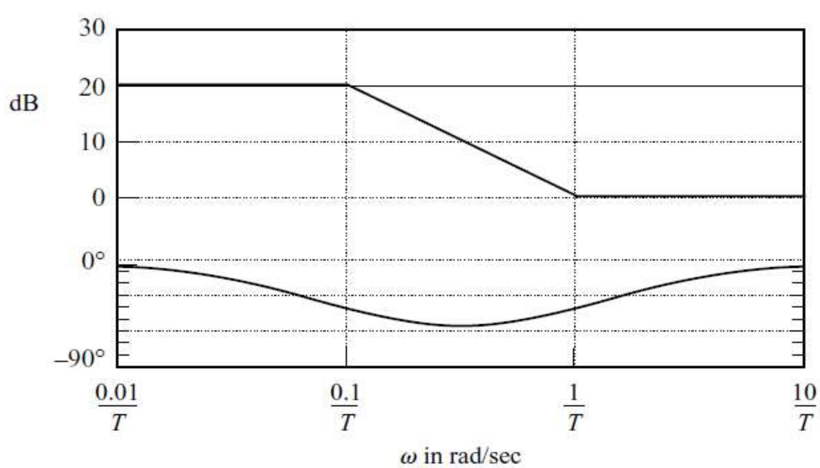


LAG COMPENSATION

- Characteristics of Lag Compensators. Consider a lag compensator having the following transfer function:

$$G_c(s) = K_c\beta \frac{Ts + 1}{\beta Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \qquad (\beta > 1)$$

- In the complex plane, a lag compensator has a zero at s=–1/T and a pole at s=–1/(β T). The pole is located to the right of the zero. Thus, the lag compensator is essentially a low-pass filter.



Bode diagram of a lag compensator β(jωT+1)/(j ω β T+1), with β =10, Kc=1

The primary function of a lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin.

The procedure for designing lag compensators

1. Assume the following lag compensator:

$$G_c(s) = K_c\beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}} \qquad (\beta > 1)$$

Define

$$K_c\beta = K$$

Then

$$G_c(s) = K\frac{Ts+1}{\beta Ts+1}$$

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = K\frac{Ts+1}{\beta Ts+1}G(s) = \frac{Ts+1}{\beta Ts+1}KG(s) = \frac{Ts+1}{\beta Ts+1}G_1(s)$$

where

$$G_1(s) = KG(s)$$

Determine gain K to satisfy the requirement on the given static velocity error constant.

2. If the gain-adjusted but uncompensated system G1(jω)=KG(jω) does not satisfy the specifications on the phase and gain margins, then find the frequency point where the phase angle of the open-loop transfer function is equal to –180° **plus** the required phase margin. The required phase margin is the specified phase margin plus 5° to 12°. Choose this frequency as the new gain crossover frequency.

3. To prevent detrimental effects of phase lag due to the lag compensator, the pole and zero of the lag compensator must be located substantially **lower** than the new gain crossover frequency. Therefore, choose the corner frequency $\omega=1/T$ (corresponding to the zero of the lag compensator) 1 octave to 1 decade below the new gain crossover frequency. (If the time constants of the lag compensator do not become too large, the corner frequency $\omega=1/T$ may be chosen 1 decade below the new gain crossover frequency.)

- Notice that we choose the compensator pole and zero sufficiently small. Thus the phase lag occurs at the low-frequency region so that it will not affect the phase margin.

6

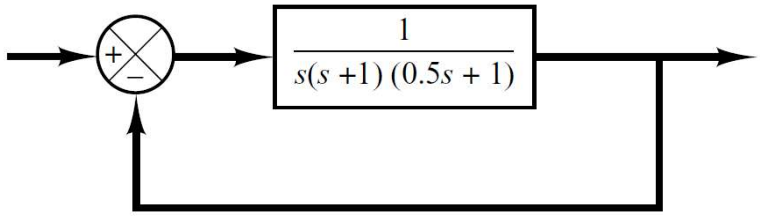
4. Determine the **attenuation** necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency. Noting that this attenuation is determine the value of β. Then the other corner frequency (corresponding to the pole of the lag compensator) is determined from $\omega = 1/(\beta T)$.

5. Using the value of K determined in step 1 and that of β determined in step 4, calculate constant Kc from:

$$K_c = K \backslash \beta$$

7

EXAMPLE (Ogata P505): Consider the system shown in Figure.



It is desired to compensate the system so that the static velocity error constant Kv is 5 sec^–1, the phase margin is at least 40°, and the gain margin is at least 10 dB.

8

We shall use a lag compensator of the form

$$G_c(s) = K_c\beta \frac{Ts + 1}{\beta Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \qquad (\beta > 1)$$

Define

$$K_c\beta = K$$

Define also

$$G_1(s) = KG(s) = \frac{K}{s(s + 1)(0.5s + 1)}$$

The first step in the design is to adjust the gain K to meet the required static velocity error constant. Thus,

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} s\frac{Ts + 1}{\beta Ts + 1}G_1(s) = \lim_{s\to 0} sG_1(s)$$

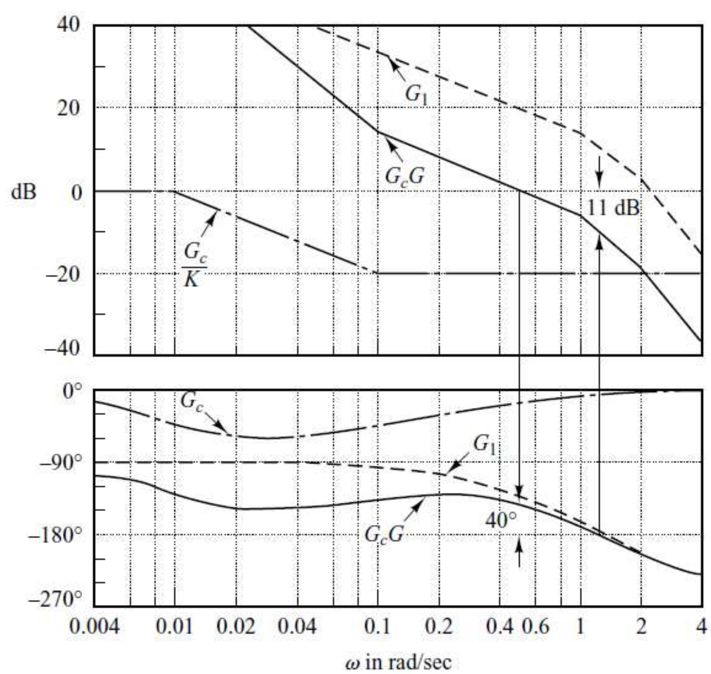
$$= \lim_{s\to 0} \frac{sK}{s(s + 1)(0.5s + 1)} = K = 5$$

With K=5, the compensated system satisfies the steady-state performance requirement. We shall next plot the Bode diagram of

$$G_1(j\omega) = \frac{5}{j\omega(j\omega + 1)(0.5j\omega + 1)}$$

The magnitude curve and phase-angle curve of G1(jω) are shown in Figure. From this plot, the phase margin is found to be –20°, which means that the gain-adjusted but uncompensated system is unstable.

10



11

Noting that the addition of a lag compensator modifies the phase curve of the Bode diagram, we must allow 5° to 12° to the specified phase margin to compensate for the modification of the phase curve. Since the frequency corresponding to a phase margin of 40° is 0.7 rad/sec, the new gain crossover frequency (of the compensated system) must be chosen near this value. To avoid overly large time constants for the lag compensator, we shall choose the corner frequency ω=1/T(which corresponds to the zero of the lag compensator) to be 0.1 rad/sec =([0.7/8]≈0.1).

12

we add about 12° to the given phase margin = 40°. The required phase margin is now 52°.The phase angle of the uncompensated open-loop transfer function is –128° at about ω=0.5 rad/sec. So we choose the new gain crossover frequency to be 0.5 rad/sec. To bring the magnitude curve down to 0 dB at this new gain crossover frequency, the lag compensator must give the necessary attenuation, which in this case is –20 dB. Hence,

$$20 \log \frac{1}{\beta} = -20$$

β= 10

13

The other corner frequency ω =1/(βT), which corresponds to the pole of the lag compensator, is then determined as:

1/(βT)= 0.01 rad/sec

Thus, the transfer function of the lag compensator is

$$G_c(s) = K_c(10)\frac{10s+1}{100s+1} = K_c\frac{s+\frac{1}{10}}{s+\frac{1}{100}}$$

Since the gain K was determined to be 5 and β was determined to be 10, we have

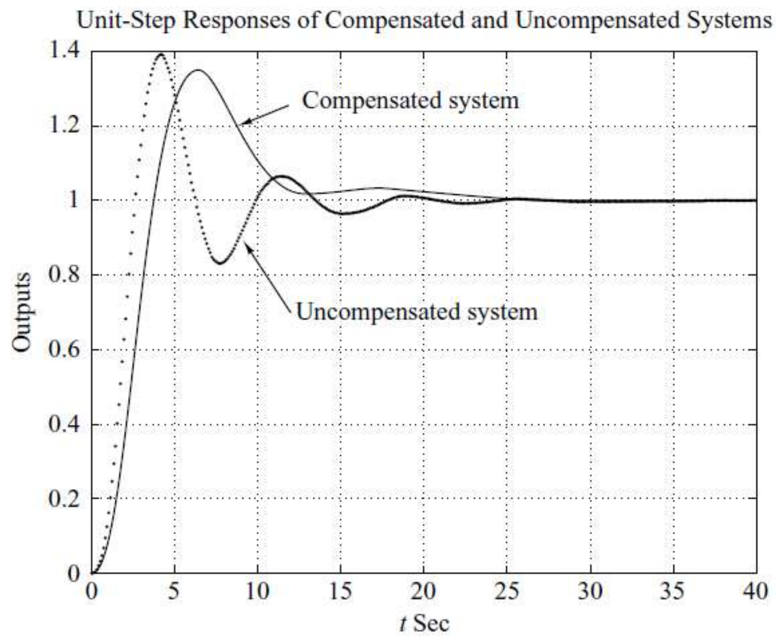
$$K_c = \frac{K}{\beta} = \frac{5}{10} = 0.5$$

The open-loop transfer function of the compensated system is

$$G_c(s)G(s) = \frac{5(10s+1)}{s(100s+1)(s+1)(0.5s+1)}$$

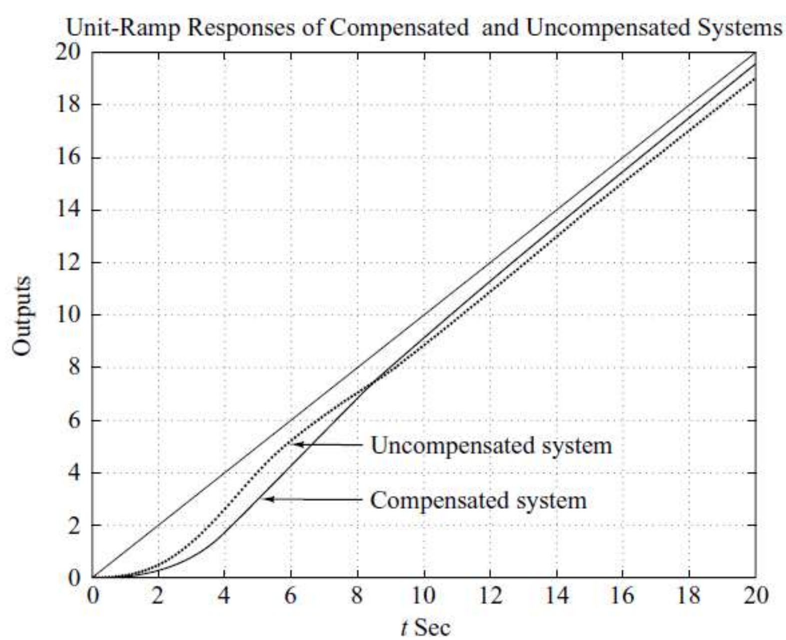
- The phase margin of the compensated system is about 40°, which is the required value. The gain margin is about 11 dB, which is quite acceptable. The static velocity error constant is 5 sec^–1, as required. The compensated system, therefore, satisfies the requirements on both the steady state and the relative stability.
- Note that the new gain crossover frequency is decreased from approximately 1 to 0.5 rad/sec. This means that the bandwidth of the system is reduced.

15



Unit-Step Responses of Compensated and Uncompensated Systems
Compensated system
Uncompensated system
Outputs
t Sec

16



Unit-Ramp Responses of Compensated and Uncompensated Systems
Uncompensated system
Compensated system
Outputs
t Sec

17

A Few Comments on Lag Compensation

1. Lag compensators are essentially low-pass filters. Therefore, lag compensation permits a high gain at low frequencies (which improves the steady-state performance) and reduces gain in the higher critical range of frequencies so as to improve the phase margin. Note that in lag compensation we utilize the attenuation characteristic of the lag compensator at high frequencies rather than the phase lag characteristic. (The phase-lag characteristic is of no use for compensation purposes.)

2. The closed-loop pole located near the origin gives a very slowly decaying transient response, although its magnitude will become very small because the zero of the lag compensator will almost cancel the effect of this pole. However, the transient response (decay) due to this pole is so slow that the settling time will be adversely affected.
3. The attenuation due to the lag compensator will shift the gain crossover frequency to a lower frequency point where the phase margin is acceptable. Thus, the lag compensator will reduce the bandwidth of the system and will result in slower transient response.

4. Since the lag compensator tends to integrate the input signal, it acts approximately as a proportional-plus-integral controller. Because of this, a lag-compensated system tends to become less stable. To avoid this undesirable feature, the time constant T should be made sufficiently larger than the largest time constant of the system.
5. Conditional stability may occur when a system having saturation or limiting is compensated by use of a lag compensator. To avoid this, the system must be designed.

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM

Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

7. Basic Characteristics of Lag Compensation by frequency-response approach.

7.1 AIM:

Plot bode diagram. The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems using MATLAB to examine the frequency characteristics of the lag compensator.

7.2 APPARATUS:

Software: MATLAB.

7.3 PROCEDURE:

- Plot bode diagram of Compensated and Uncompensated Systems.
- The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems.

7.4 THEORETICAL CALCULATIONS:

Enter the numerators and denominators of the compensated and uncompensated systems.

num=

den=

Transfer function:
numc=
denc=
Transfer function:

7.5 EXAMPLE: Consider the G(s) = [1]/ [s(s+1) (0.5s+1)], Plot bode diagram. {[1] Page 505}

- Plot The unit-step response curves of the system before and after compensation, G(s) = [1]/[s(s+1)(0.5s+1)], it is desired to design a compensator for the system so that the static velocity error constant Kv is 5 sec^–1, the phase margin is at least 40°, and the gain margin is at least 10 dB.

```
num = [5];
den = [.5 1.5 1 0];
bode(num,den)
grid

numc = [50 5];
denc = [50 150.5 101.5 1 0];
bode(numc,denc)
grid
title('Bode Diagram of G(s) = [50s+5]/[(50s^4+150.5s^3+101.5s^2+s)], ')

%*****Unit-step response*****
num = [1];
den = [0.5 1.5 1 1];
numc = [50 5];
denc = [50 150.5 101.5 51 5];
t = 0:0.1:40;
[c1,x1,t] = step(num,den,t);
[c2,x2,t] = step(numc,denc,t);
plot(t,c1,'.',t,c2,'-')
grid
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')
text(12.7,1.27,'Compensated system')
text(12.2,0.7,'Uncompensated system')

%*****Unit-ramp response*****
num1 = [1];
den1 = [0.5 1.5 1 1 0];
num1c = [50 5];
den1c = [50 150.5 101.5 51 5 0];
t = 0:0.1:20;
[y1,z1,t] = step(num1,den1,t);
[y2,z2,t] = step(num1c,den1c,t);
plot(t,y1,'.',t,y2,'-',t,t,'--');
grid
title('Unit-Ramp Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')
text(8.3,3,'Compensated system')
text(8.3,5,'Uncompensated system')
```

7.6 RESULT:

LAG–LEAD COMPENSATION

- Characteristic of Lag–Lead Compensator. Consider the lag–lead compensator given by

$$G_c(s) = K_c\left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}}\right)\left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}}\right)$$

where $\gamma > 1$ and $\beta > 1$. The term

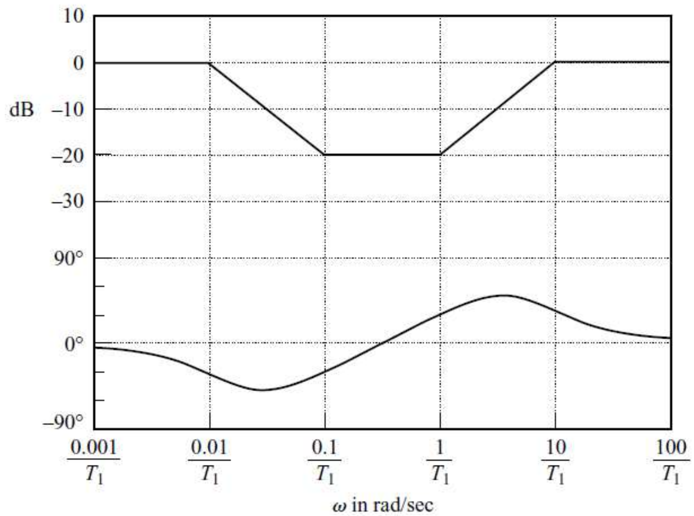
$$\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} = \frac{1}{\gamma}\left(\frac{T_1 s + 1}{\frac{T_1}{\gamma} s + 1}\right) \qquad (\gamma > 1)$$

produces the effect of the lead network, and the term

$$\frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} = \beta\left(\frac{T_2 s + 1}{\beta T_2 s + 1}\right) \qquad (\beta > 1)$$

produces the effect of the lag network.

- Figure shows the Bode diagram of a lag–lead compensator when Kc=1, Y= β =10, and T2=10T1 Notice that the magnitude curve has the value 0 dB at the low- and high-frequency regions.



In designing a lag–lead compensator, we frequently choose β= Y.

- Let us assume that the lag–lead compensator is of the following form:

$$G_c(s) = K_c \frac{(T_1 s + 1)(T_2 s + 1)}{\left(\frac{T_1}{\beta} s + 1\right)(\beta T_2 s + 1)} = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}$$

- where β>1.
- We shall illustrate the details of the procedures for designing a lag–lead compensator by an example.

- EXAMPLE [P513 Ogata]: Consider the unity-feedback system whose open-loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

It is desired that the static velocity error constant be 10 sec^–1, the phase margin be 50°, and the gain margin be 10 dB or more.

- Assume that we use the lag–lead compensator given by Equation .

$$G_c(s) = K_c \frac{(T_1 s + 1)(T_2 s + 1)}{\left(\frac{T_1}{\beta} s + 1\right)(\beta T_2 s + 1)} = K_c \frac{\left(s + \frac{1}{T_1}\right)\left(s + \frac{1}{T_2}\right)}{\left(s + \frac{\beta}{T_1}\right)\left(s + \frac{1}{\beta T_2}\right)}$$

- [Note that the phase lead portion increases both the phase margin and the system bandwidth (which implies increasing the speed of response).The phase-lag portion maintains the low-frequency gain.]

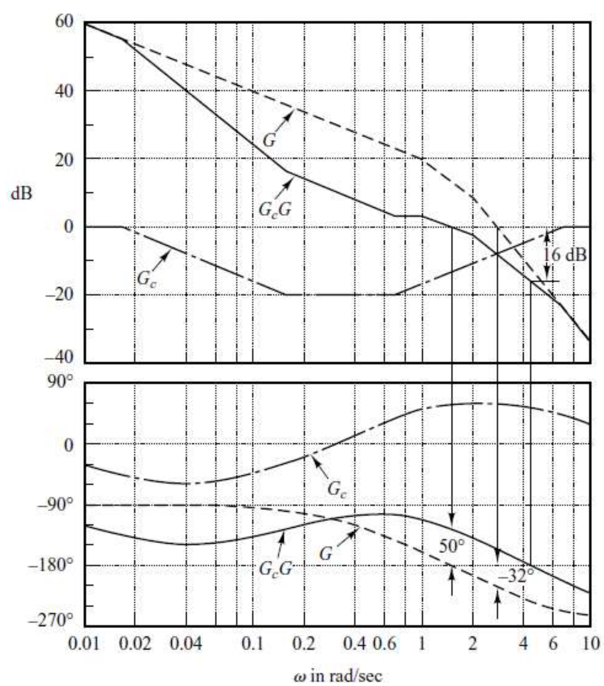
Since the gain K of the plant is adjustable, let us assume that Kc=1

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} sG_c(s)\frac{K}{s(s+1)(s+2)} = \frac{K}{2} = 10$$

$$K = 20$$

We shall next draw the Bode diagram of the uncompensated system with K=20

8



9

- The phase margin of the gain-adjusted but uncompensated system is found to be –32°, which indicates that the gain-adjusted but uncompensated system is unstable.
- The next step in the design of a lag–lead compensator is to choose a new gain crossover frequency. From the phase-angle curve for G(jω), we notice that angle of G(jω) = -180° at ω=1.5 rad/sec. It is convenient to choose the new gain crossover frequency to be 1.5 rad/sec so that the phase lead angle required at ω =1.5 rad/sec is about ***50***°, which is quite possible by use of a ***single*** lag–lead network.

- Once we choose the gain crossover frequency to be 1.5 rad/sec, we can determine the corner frequency of the phase-lag portion of the lag–lead compensator. Let us choose the corner frequency ω = 1/T2 (which corresponds to the zero of the phase-lag portion of the compensator) to be 1 decade below the new gain crossover frequency, or at ω =0.15 rad/sec.

- Recall that for the lead compensator the maximum phase-lead angle Øm is given by Equation, where a is 1/β in the present case.

$$\sin\phi_m = \frac{1-\frac{1}{\beta}}{1+\frac{1}{\beta}} = \frac{\beta-1}{\beta+1}$$

- Notice that β =10 corresponds to Øm=54.9°. Since we need a 50° phase margin, we may choose β =10. (Note that we will be using several degrees less than the maximum angle, 54.9°.) Thus, β =10

12

- Then the corner frequency ω = 1/ β T2 (which corresponds to the pole of the phase-lag portion of the compensator) becomes ω =0.015 rad/sec. The transfer function of the phase-lag portion of the lag–lead compensator then becomes :

$$\frac{s+0.15}{s+0.015} = 10\left(\frac{6.67s+1}{66.7s+1}\right)$$

13

- The phase-lead portion can be determined as follows: Since the new gain crossover frequency is ω =1.5 rad/sec, G(j1.5) is found to be 13 dB. Hence, if the lag–lead compensator contributes

  –13 dB at ω =1.5 rad/sec, then the new gain crossover frequency is as desired.
- From this requirement, it is possible to draw a straight line of slope 20 dB/decade, passing through the point (1.5 rad/sec, –13 dB). The intersections of this line and the 0-dB line and –20 dB line determine the corner frequencies. Thus, the corner frequencies for the lead portion are ω =0.7 rad/sec and ω =7 rad/sec.

14

- Thus, the transfer function of the lead portion of the lag–lead compensator becomes:

$$\frac{s+0.7}{s+7} = \frac{1}{10}\left(\frac{1.43s+1}{0.143s+1}\right)$$

- Combining the transfer functions of the lag and lead portions of the compensator, we obtain the transfer function of the lag–lead compensator. Since we chose *Kc=1, we have :*

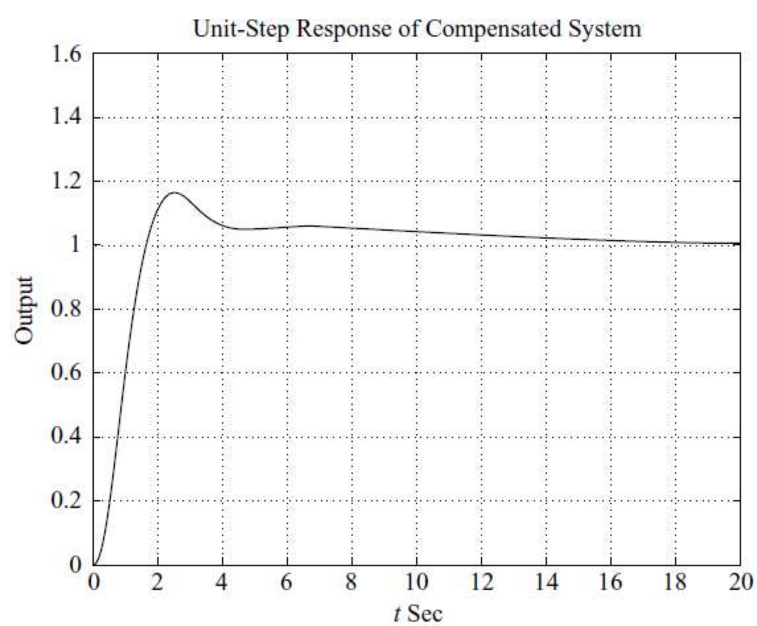
15

$$G_c(s) = \left(\frac{s + 0.7}{s + 7}\right)\left(\frac{s + 0.15}{s + 0.015}\right) = \left(\frac{1.43s + 1}{0.143s + 1}\right)\left(\frac{6.67s + 1}{66.7s + 1}\right)$$

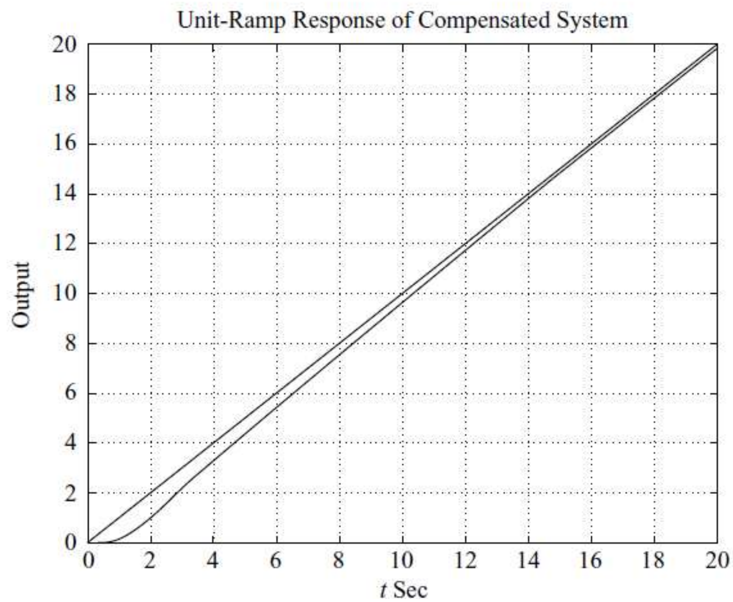
$$G_c(s)G(s) = \frac{(s + 0.7)(s + 0.15)20}{(s + 7)(s + 0.015)s(s + 1)(s + 2)}$$
$$= \frac{10(1.43s + 1)(6.67s + 1)}{s(0.143s + 1)(66.7s + 1)(s + 1)(0.5s + 1)}$$

The phase margin of the compensated system is 50°, the gain margin is 16 dB, and the static velocity error constant is 10 sec^–1. All the requirements are therefore met, and the design has been completed.

16



17


Unit-Ramp Response of Compensated System
Output
t Sec
0
2
4
6
8
10
12
14
16
18
20

18

College of Electronics Engineering
Department of Systems and Control Engineering

CONTROL SYSTEMS DESIGN LAB

III YEAR II SEM
Instructor: Mr. Salam Asmer

Summary: This course deals with practical aspects of the design and compensation of control systems.

Matlab: You will use Matlab extensively. Prior familiarity with Matlab is assumed. You are not required to purchase these packages.

Lab Reports: The Department requires formal lab reports which must satisfy the following format rules:
1. Title page: This must include a title, group member names, course and section name and date.
2. Introduction: Explain the background and objective of the lab indicating requirements and desired results.
3. Discussion: Discuss the underlying applicable theory and concepts that support the results.
4. Results: Present results in tabular, graphical or numeric form. Present results from required lab exercises.
5. Discussion of Results: Discuss results data in context of comparison to expectation, accuracy, difficulties, etc.
6. Summary and Conclusions: Discuss findings, explain errors and unexpected results; summarize and indicate conclusions.

8. Basic Characteristics of Lag- Lead Compensation by frequency-response approach.

8.1 AIM:
Plot Bode diagram. The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems using MATLAB to examine the frequency characteristics of the lag- Lead compensator.

8.2 APPARATUS:
Software: MATLAB.

8.3 PROCEDURE:

- We will follow the details of the procedures for designing a lag–lead compensator by the reference.
- Plot Bode diagram of Compensated and Uncompensated Systems.
- The plot of the unit-step response curves and unit ramp of the compensated and uncompensated systems.

8.4 THEORETICAL CALCULATIONS:

Enter the numerators and denominators of the compensated and uncompensated systems.
num=
den=
Transfer function:
numc=
denc=
Transfer function:

8.5 EXAMPLE: Consider the G(s) = [K]/ [s(s+1) (s+2)], Plot bode diagram. {[1] Page 513}

- Plot The unit-step response curves of the system before and after compensation, G(s) = [K]/ [s(s+1) (s+2)], it is desired to design a compensator for the system so that the static velocity error constant Kv is 10 sec^–1, the phase margin is at least 50°, and the gain margin is at least 10 dB.

```
num = [20];
den = [1 3 2 0];
bode(num,den)
grid

numc = [20 17 2.1];
denc = [1 10.015 23.245 14.345 0.205 0];
bode(numc,denc)
grid

num = [1];
den = [1 3 2 1];
numc = [20 17 2.1];
denc = [1 10.015 23.145 34.345 17.205 2.1];
t = 0:0.1:20;
[c1,x1,t] = step(num,den,t);
[c2,x2,t] = step(numc,denc,t);
plot(t,c1,'.',t,c2,'-')
grid
title('Unit-Step Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')

%*****Unit-ramp response*****
num1 = [1];
den1 = [1 3 2 1 0];
num1c = [20 17 2.1];
den1c = [1 10.015 23.145 34.345 17.205 2.1 0];
t = 0:0.1:20;
[y1,z1,t] = step(num1,den1,t);
[y2,z2,t] = step(num1c,den1c,t);
plot(t,y1,'.',t,y2,'-',t,t,'--');
grid
title('Unit-Ramp Responses of Compensated and Uncompensated Systems')
xlabel('t Sec')
ylabel('Outputs')
```

8.6 RESULT:

PID Controllers

•It is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

•The usefulness of PID controls lies in their general applicability to most control systems. In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful.

2

How do the PID parameters affect system dynamics?

$$U(s) = G_{PID}(s)E(s) = \left(K_P + K_I \frac{1}{s} + K_D s \right) E(s)$$

The effects of increasing each of the controller parameters K_P, K_I and K_D can be summarized as

Response	Rise Time	Overshoot	Settling Time	S-S Error
K_P	Decrease	Increase	NT	Decrease
K_I	Decrease	Increase	Increase	Eliminate
K_D	NT	Decrease	Decrease	NT

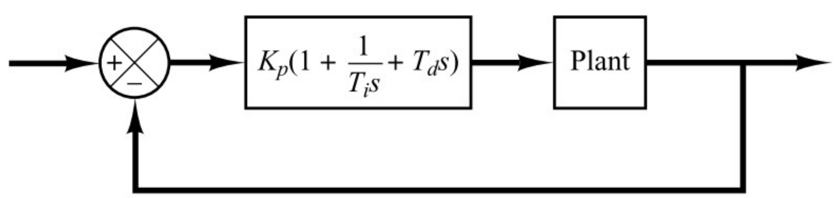
NT: No definite trend. Minor change.

3

ZIEGLER–NICHOLS RULES FOR TUNING PID CONTROLLERS

- The process of selecting the controller parameters to meet given performance specifications is known as **controller tuning**. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values of the controller) based on experimental step responses or based on the value of Kp that results in marginal stability when only proportional control action is used. Ziegler–Nichols rules, which are briefly presented in the following, are **useful when mathematical models of plants are not known**. (These rules can, of course, be applied to the design of systems with known mathematical)

PID control of a plant



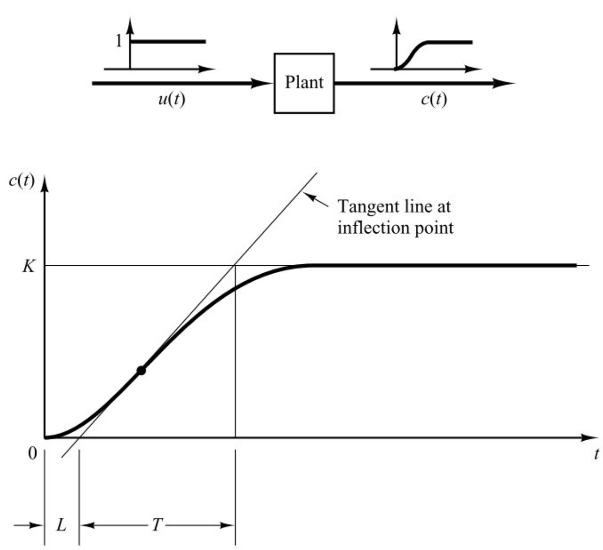
Ziegler–Nichols Rules for Tuning PID Controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature and from the manufacturers of such controllers.)

There are two methods called Ziegler–Nichols tuning rules: the first method and the second method. We shall give a brief presentation of these two methods.

First Method. In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 8–2. If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped, as shown in Figure 8–3. This method applies if the response to a step input exhibits an S-shaped curve. Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

6

Unit-step response of a plant



7

Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

8

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$
$$= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right)$$
$$= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

9

Second Method. In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 8–4), increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally

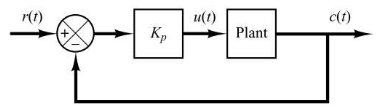


Figure 8–4 Closed-loop system with a proportional controller.

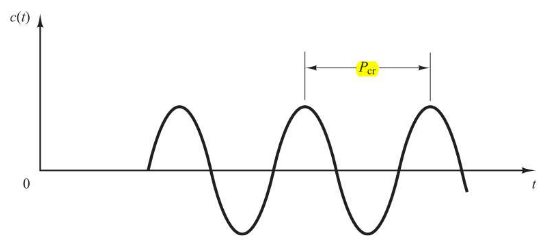


Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

determined (see Figure 8–5). Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 8–2.

Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$
\begin{aligned}
G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\
&= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\
&= 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s}
\end{aligned}
$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s+1)(s+5) + K_p}$$

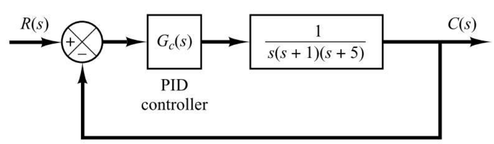


Figure 8–6 PID-controlled system.

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} $(= 30)$, the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

14

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 8–2, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 18$$

$$T_i = 0.5P_{cr} = 1.405$$

$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right)$$

$$= \frac{6.3223(s + 1.4235)^2}{s}$$

15

The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 8–7.

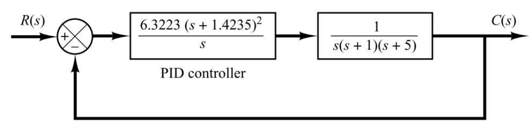


Figure 8–7 Block diagram of the system with PID controller designed by use of the Ziegler–Nichols tuning rule (second method).

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. See MATLAB Program 8–1. The resulting unit-step response curve is shown in Figure 8–8. The maximum overshoot in the unit-step response is approximately 62%. The amount of maximum overshoot is excessive. It can be reduced by fine tuning the controller parameters. Such fine tuning can be made on the computer. We find that by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$—that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\frac{(s + 0.65)^2}{s} \qquad (8–1)$$

Kcr=30

Kp=18

a=1.4

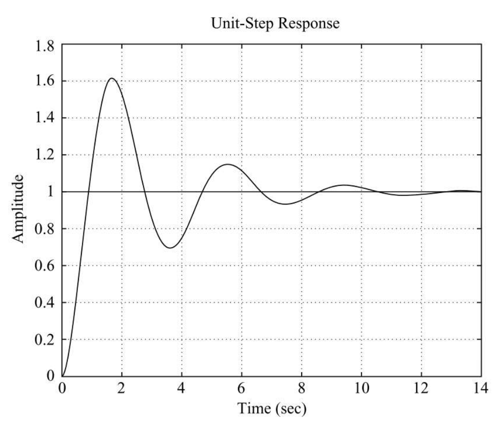


Figure 8–8 Unit-step response curve of PID-controlled system designed by use of the Ziegler–Nichols tuning rule (second method).

17

Kp=18
a=0.65

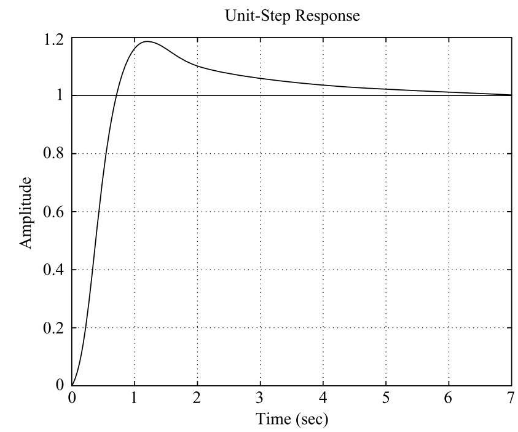


Figure 8–9
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$.

the maximum overshoot in the unit-step response can be reduced to approximately 18% (see Figure 8–9). If the proportional gain K_p is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, using the PID controller

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\,\frac{(s + 0.65)^2}{s} \tag{8–2}$$

Kp=39.42
a=0.65

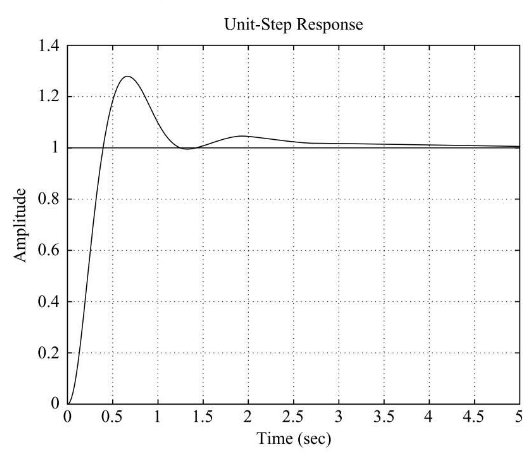


Figure 8–10
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$.

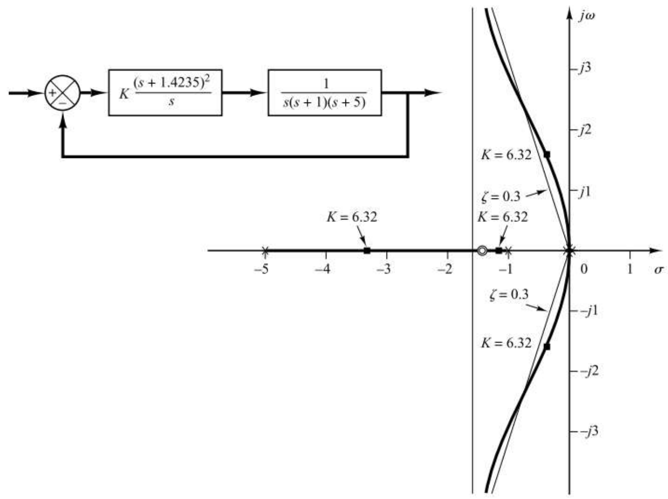


Figure 8–11
Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

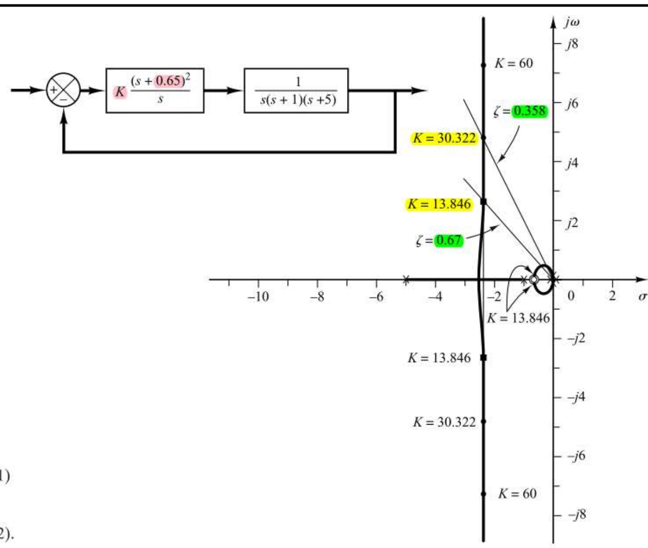


Figure 8–12
Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (8–1) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (8–2).

the values of K and a are

$$K = 29, \qquad a = 0.25$$

with the maximum overshoot equal to 9.52% and settling time equal to 1.78 sec. Another possible solution obtained there is that

$$K = 27, \qquad a = 0.2$$

with the 5.5% maximum overshoot and 2.89 sec of settling time. See Problem **A–8–12** for details.

Summary

Two things to take away from this review of Ziegler-Nichols tuning:

1. Relationships between K_P, K_I and K_D and important response characteristics, of which these three are most useful:
   - Use K_P to decrease the rise time.
   - Use K_D to reduce the overshoot and settling time.
   - Use K_I to eliminate the steady-state error.
2. The Ziegler-Nichols tuning rule (reaction curve method) for good initial estimate of parameters.

22