CHAPTER ONE
STATICS FUNDAMENTALS

Engineering mechanics definition:

Engineering mechanics: It is the physical science that describes the state of motion of bodies (rest or motion) under the action of the forces exerted on them. It is divided into two branches, as shown in the figure below.

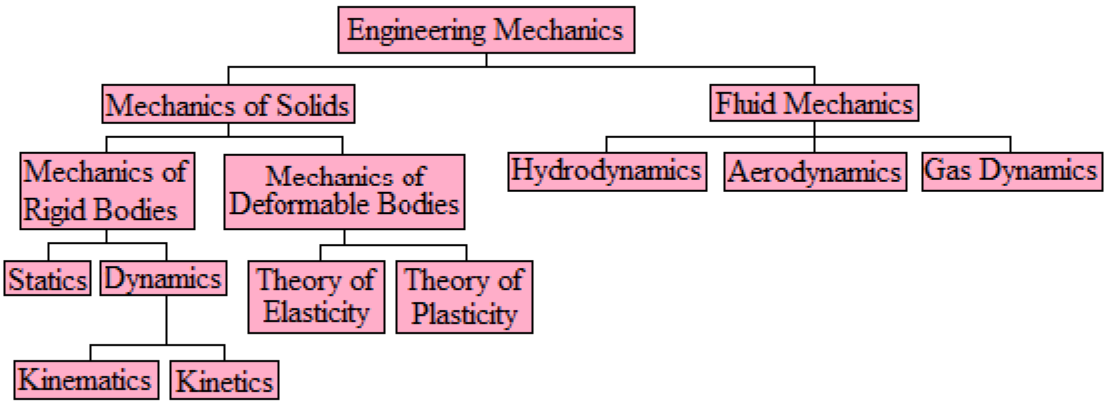


Fig. (1-1) Engineering mechanics branches

Statics:

Is the mechanical science which study the states of bodies under the effect of forces at rest (stopping or regular motion).

Dynamics:

Is the mechanical science which study the states of bodies under the effect of forces at irregular motion.

stopping	Accelarated Motion	Regular Motion	Decelarated Motion	stopping
v = 0	v = ↑	v = Constant	v = ↓	v = 0
a = 0	a = +	a = 0	a = –	a = 0

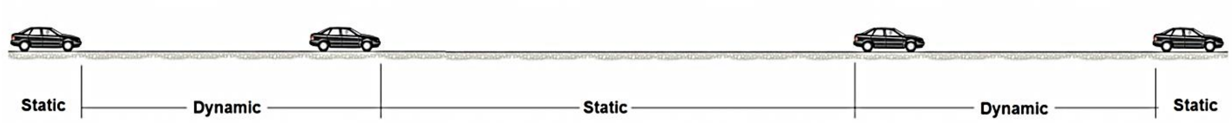


Fig. (1-2) The statics and dynamics states during a car traveling from starting motion to stopping

stopping　　Accelarated Flight　　Cruse Flight　　Decelarated Flight　　stopping

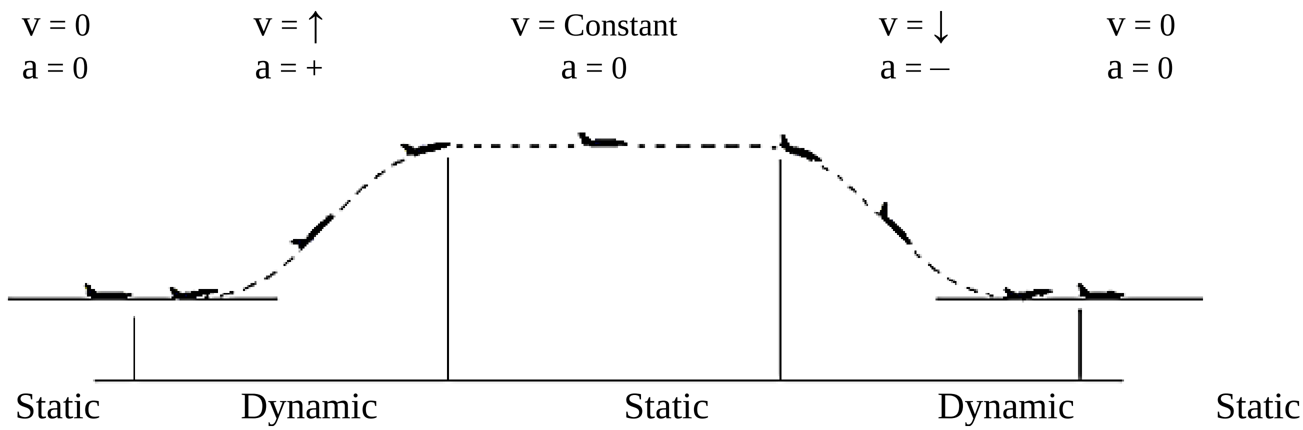


Fig. (1-3) The statics and dynamics states during aircraft travel from starting motion to stopping

Basic concepts:

Space: It is the geometric space or location occupied by the body, where this space or location is described by linear and angular measurements depending on the coordinate system followed.

Time (t): A measure that expresses the intervals of successions of events, and is a basic quantity in the analysis of dynamic problems. It is not directly depended on the analysis of static issues.

Mass (m): It is a measure of a body's resistance to a change in its state of motion (the inertia of the body). Also the mass can be defined as the amount of matter in the body.

Force (F): The external or internal action on the bodies or between bodies. Or an external action that change or tends to change the body shape or its state of motion.

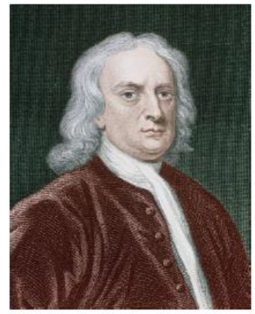
Particle: An object with negligible dimensions in the mathematical sense, or an object whose dimensions are close to zero so that it can be analyzed as a point mass (which can be represented by a point).

Rigid body: A body is expressed as a rigid body when the change in the distance between any two points of it as a result of the forces acting on it is negligible.

Concentrated Force: Is the force acting on a body and effected on a single point on it.

Distributed force: Is the force acting on a body and its effect is distributed over a specific distance or area on the body.

Newton's three fundamental laws in motion:



Sir Isaac Newton

Newton's first law

The body remains in a state of rest or continues in its state of motion at a constant speed unless unbalanced forces act on it (Resultant force = zero).

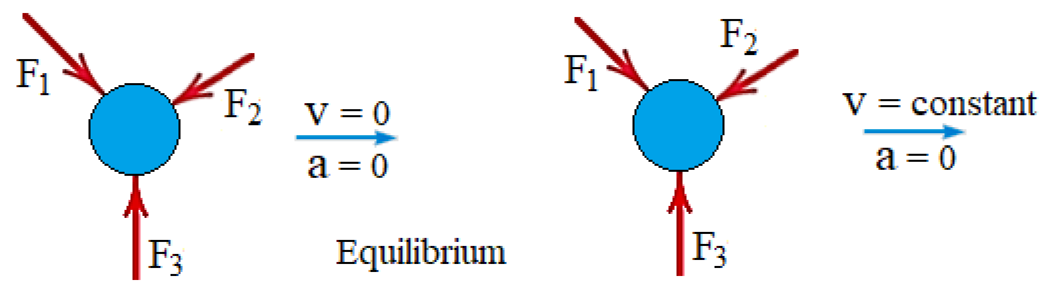


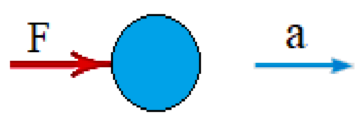
Fig. (1-4) Newton's first law aplication

Newton's second law

The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.

$$F = ma \quad \ldots\ldots\ldots\ldots \quad (1\text{-}1)$$

Fig. (1-5) Newton's second law aplication

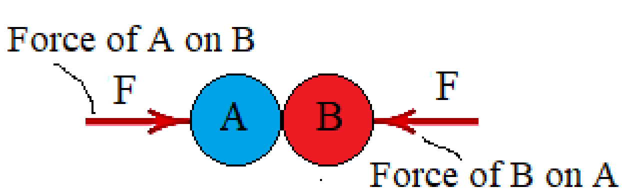


Accelerated motion direction

Newton's therd law

For every action force there is a reaction force that is equal in magnitude and opposite in direction, and is on the same line of effect with it (Collinear).

Fig. (1-6) Newton's therd law aplication



Action Reaction

Newton's law of gravitation:

There is an attractive force between any two bodies in the universe, proportional to the multiplication of their masses, and inversely proportional to the square of the distance between their centers.

The mathematical law of gravitation is expressed by the equation:

$$F = G\frac{m_1 m_2}{r^2} \quad \text{……………..} \quad (1\text{-}2)$$

Where:
F : mutual force of attraction between two bodies.
G : The constant of gravitation = 66.73×10^{-12} $m^3/(Kg.s^2)$.
m_1, m_2 : masses of the two attractive particles.
r : distance between the centers of the two attractive particles.

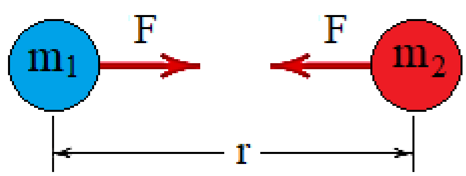


Fig. (1-7) Aplication of newton's law of gravitation

Weight (w): It is the gravitational force of the Earth for the body.

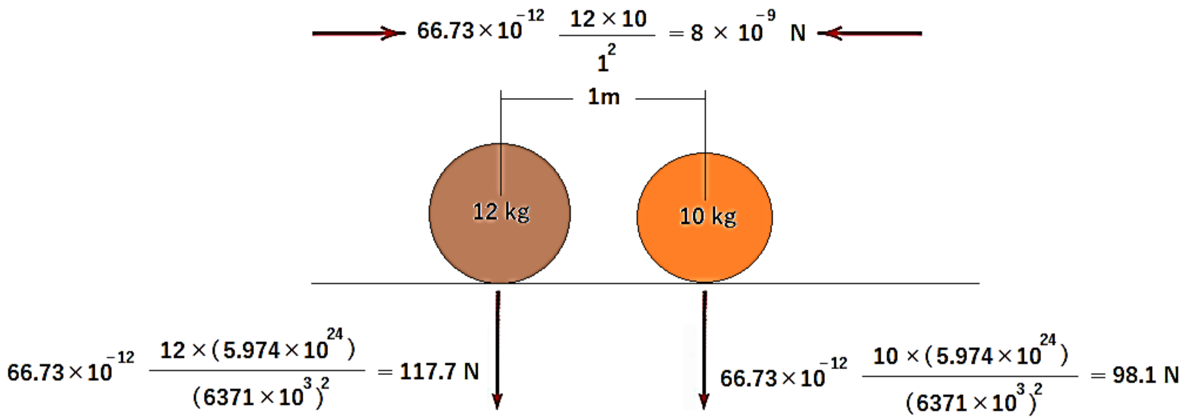


Fig. (1-8) The defference between the gravitational force between the bodies and the Earth gravity force (Weight)

- Any two bodies in nature have a mutual gravitational force.
- To find the weight (W) of any particle on the earth's surface it's mass can be considered as ($m_1 = m$).
- If we assume the earth to be a non-rotating sphere of constant density and having a mass of ($m_2 = M_e$).
- If (r) is the distance between the earth's center and the particle center which represents the radius of the earth, we have:

$$W = G\frac{M_e m}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}3)$$

The gravitational constant (G), the Earth's mass (M_e) and the Earth's radius (r) in equation (1-3) are constant values and can been replaced by a single constant value called { Earth gravitational acceleration (g) }.

$$g = \frac{GM_e}{r^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}4)$$

So the weight in equation (1-3) is as follows:

$$W = mg \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}5)$$

For most engineering calculations, Earth gravitational acceleration (g) is determined at sea level and at a latitude of (45°), which is considered the "standard location.":

$$g = 9.81 \text{ m/s}^2$$

Table (1-1) Orbital and physical properties for the sun, the moon, and some planets

The Sun and The Planets			**Mass** $\times 10^{24}$ kg	**Average radius** km	**Distance to the sun** $\times 10^{6}$ km	**Orbital average speed** km/s	**Notes**
The sun			1989000	696000			
The moon			0.0735	1737		1.022 **(Around the earth)**	**Distance to the earth: 384399 km**
Planets	**Internal (rocky)**	**Mercury**	0.33	2430	57.91	47.87	
		Venus	4.869	6052	108.21	35.02	
		Earth	5.9736	6371	149.6	29.78	
		Mars	0.642	3387	227.99	24.077	
	External (gaseous)	**Jupiter**	1898.6	69910	778.55	13.07	
		Saturn	578.46	57320	1433.4	9.69	
		Uranus	86.81	25270	2876.7	6.81	
		Neptune	102.43	24550	4498.3	5.432	

Equilibrium between the earth and the moon:

Newton's law of gravitation:

$$F = G\,\frac{Mm}{r^2}$$

$$F = 66.73 \times 10^{-12}\ \frac{5.9736 \times\ 10^{24} \times 7.3477 \times\ 10^{22}}{(\ 384399\ \times\ 10^{3}\ )^{2}}$$

$$= 66.73\,\frac{5.9736 \times 7.3477 \times\ 10^{28}}{384399^{\ 2}}\ =\ 2 \times 10^{20}\ \text{N}$$

Centrifugal force law:

$F_c = mr\omega^2$ …………...….. (1-6) $\qquad v = \omega r\ ,\ \ \omega = v/r$

$F_c = mr\ (v/r)^2$ …………. (1-7)

$F_c = mv^2/r$ …………..…. (1-8)

$$F_c = \frac{7.3477 \times\ 10^{22}\ \times\ 1022^{2}}{384399\ \times\ 10^{3}} = \frac{7.3477\ \ \times\ 1022^{2}\ \times\ 10^{19}}{384399}$$

$$= 2 \times 10^{20}\ \text{N}$$

Equilibrium between the sun and the earth:

Newton's law of gravitation:

$$F = G\,\frac{Mm}{r^2}$$

$$F = 66.73 \times 10^{-12}\ \frac{5.9736 \times\ 10^{24} \times 1.989 \times\ 10^{30}}{(149.6\ \times\ 10^{9}\ )^{2}}$$

$$= 66.73\,\frac{5.9736 \times 1.989 \times\ 10^{24}}{149.6^{\ 2}}\ =\ 3.54 \times 10^{22}\ \text{N}$$

Centrifugal force law:

$$F_c = mv^2\,/\,r$$

$$F_c = \frac{5.9736 \times\ 10^{24} \times\ 29780^{2}}{149.6\ \times\ 10^{9}} = \frac{5.9736\ \ \times\ 29780^{2}\ \times\ 10^{15}}{149.6}$$

$$= 3.54 \times 10^{22}\ \text{N}$$

Gravitational acceleration at the surface of the Earth:

$$g = \frac{Mm}{r^2} = \frac{66.73 \times 10^{-12} \times 5.9736 \times 10^{24}}{(6371 \times 10^3)^2} = \frac{66.73 \times 5.9736 \times 10^6}{(6371)^2}$$

$$= 9.80665 \text{ m/s}^2 = 9.81 \text{ m/s}^2$$

$$= \frac{9.81}{0.3048} = 32.2 \text{ ft/s}^2$$

Gravitational acceleration at the surface of the Moon:

$$g_m = \frac{Gm}{{r_m}^2} = \frac{66.73 \times 10^{-12} \times 7.3477 \times 10^{22}}{(1737.35 \times 10^3)^2} = \frac{66.73 \times 7.3477 \times 10^4}{(1737.35)^2}$$

$$= 1.6244 \text{ m/s}^2$$

$$= 5.33 \text{ ft/s}^2$$

$$= 0.1656\ g_e$$

Gravitational acceleration at the surface of the Sun:

$$g_s = \frac{Gm}{{r_s}^2} = \frac{66.73 \times 10^{-12} \times 1.989 \times 10^{30}}{(696 \times 10^6)^2} = \frac{66.73 \times 1.989 \times 10^6}{(696)^2}$$

$$= 274 \text{ m/s}^2$$

$$= 899.368 \text{ ft/s}^2$$

$$= 27.93\ g_e$$

Table (1-2) Gravitational acceleration at the surfaces of the planets and the gravity force between the sun and the planet

THE PLANETS	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
The central gravitational acceleration (m/s^2)	**3.73**	**8.87**	**9.81**	**3.73**	**25.92**	**11.75**	**9.07**	**11.34**
The gravity force between the sun and the planet ($\times 10^{22}$ N)	**1.31**	**5.52**	**3.54**	**0.163**	**41.6**	**3.7**	**0.14**	**0.0672**

Example (1-1):

Calculate the gravitational force generated between two bodies whose masses are (10 kg) and (15 kg) respectively, and the distance between their centers is (750 mm), then calculate the weight of each body.

Solution:

$$F = G \frac{m_1 m_2}{r^2}$$

Where $G = 66.73 \times 10^{-12}\ m^3 / (Kg\ .\ s^2)$

$$F = 66.73 \times 10^{-12} \left[\frac{(10)(15)}{(0.75)^2} \right] = 17.795\ (10^{-9})\ N = 17.8\ nN$$

$$W_1 = (10)(9.81) = 98.1\ N$$

$$W_2 = (15)(9.81) = 147.15\ N$$

Example (1-2):

A satellite weighing (700 Ib) on the surface of the Earth. Calculate the Earth's gravitational force for this satellite when it is positioned at a distance of (36000 km) from the surface of the Earth, where its rotation speed around the Earth is equal to the speed of the Earth's rotation around itself.

- Earth mass ($M_e = 5.97 \times 10^{24}$ kg).

- Radius of the Earth ($R_e = 6371$ km).

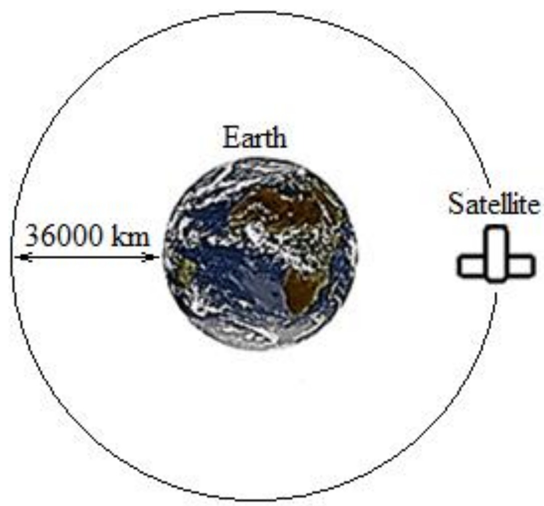


Fig. (Ex. 1-2)

Solution:

$W = 700\ Ib = 700 \times 4.448 = 3113.6\ N$

$$m = \frac{W}{g} = \frac{3113.6}{9.81} = 317.4\ kg$$

$r = R_e + 36000 = 6371 + 36000 = 42371\ km = 42371 \times 10^3\ m$

$$F = G\frac{M_e \times m}{r^2} = 66.73 \times 10^{-12} \frac{5.97 \times 10^{24} \times 317.4}{(42371 \times 10^3)^2}$$

$$= 70.43\ N$$

Example (1-3):

A spacecraft of mass (3000 kg) is launched from earth to the moon.

1. **Calculate the distance from the earth's surface at which the force of gravity between the spacecraft and both the Earth and the moon is equal.**
2. **Calculate the gravitational force between the spacecraft and both the earth and the moon at this distance.**

- **The distance between the centers of the earth and the moon = 384400 km.**
- **The mass of the Earth = 5.97 × 10^{24} kg.**
- **The mass of the Moon = 7.35 × 10^{22} kg.**
- **Radius of the Earth = 6371 km.**
- **Radius of the Moon = 1737 km.**

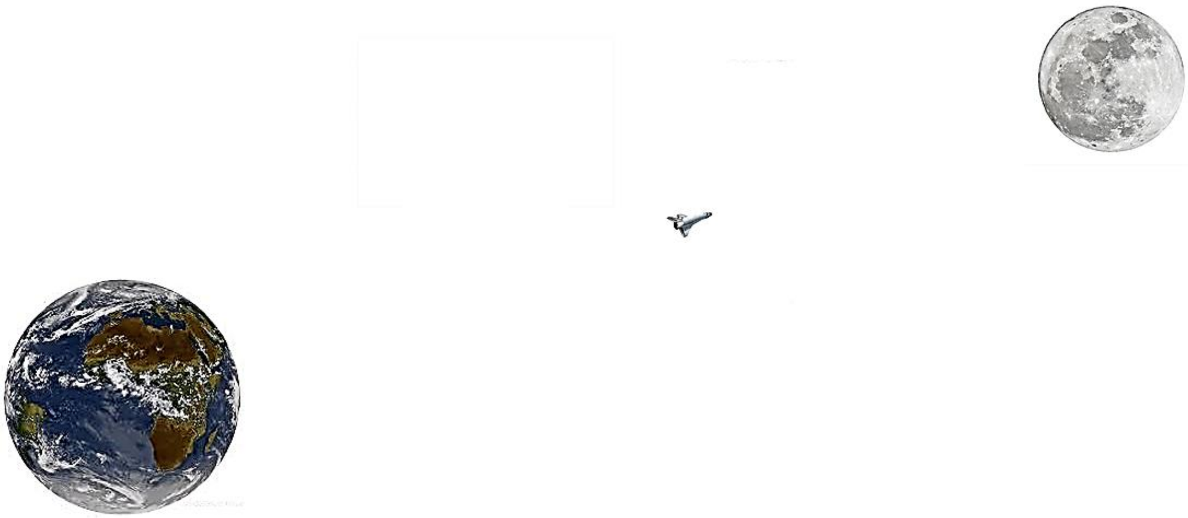


Fig. (Ex. 1-3)

Solution:

$F_e = G\ \dfrac{M_e\ M_s}{{r_e}^2}$ $\qquad$ $F_m = G\ \dfrac{M_m\ M_s}{{r_m}^2}$

$$F_e = 66.73 \times 10^{-12}\ \frac{5.9736 \times 10^{24} \times 3 \times 10^{3}}{{r_e}^2} = \frac{1.196 \times 10^{18}}{{r_e}^2}$$

$$F_m = 66.73 \times 10^{-12}\ \frac{7.3477 \times 10^{22} \times 3 \times 10^{3}}{{r_m}^2} = \frac{1.471 \times 10^{16}}{{r_m}^2}$$

$F_e = F_m$

$$\frac{1.196 \times 10^{18}}{{r_e}^2} = \frac{1.471 \times 10^{16}}{{r_m}^2}$$

$$\frac{1.196 \times 10^{18}}{1.471 \times 10^{16}} = \frac{{r_e}^2}{{r_m}^2}$$

$$\frac{r_e^2}{r_m^2} = 81.3 \qquad \Rightarrow \qquad \frac{r_e}{r_m} = 9.02$$

$r_e = 9.02\ r_m$ .... (1)

$r_e + r_m = 384400$ .... (2)

$9.02\ r_m + r_m = 384400$

$10.02\ r_m = 384400 \quad \Rightarrow \quad r_m = 38363.27$ km

$r_e = 9.02 \times 38363.27 = 346036.73$ km

$h_e = r_e - R_e = 346036.73 - 6371 = 339665.73$ km

$$F_e = \frac{1.196 \times 10^{18}}{(346036730)^2} = 10\ N \qquad F_m = \frac{1.471 \times 10^{16}}{(38363270)^2} = 10\ N$$

r_m : Distance between the spacecraft and the Moon's center.
r_e : Distance between the spacecraft and the Earth's center.
R_e : Radius of the Earth.
h_e : Distance between the spacecraft and the Earth's surface.
F_m : gravitational force between the spacecraft and the Moon.
F_e : gravitational force between the spacecraft and the Earth.

Example (1-4):

Calculate the distance between the satellite and the Earth's surface that would cause the satellite to constantly face one region of the Earth's surface.

- **Earth mass = 5.97 × 10^{24} kg.**
- **Radius of the Earth = 6371 km.**
- **The linear speed of the Earth's rotation around its axis = 1674.4 km/hr.**

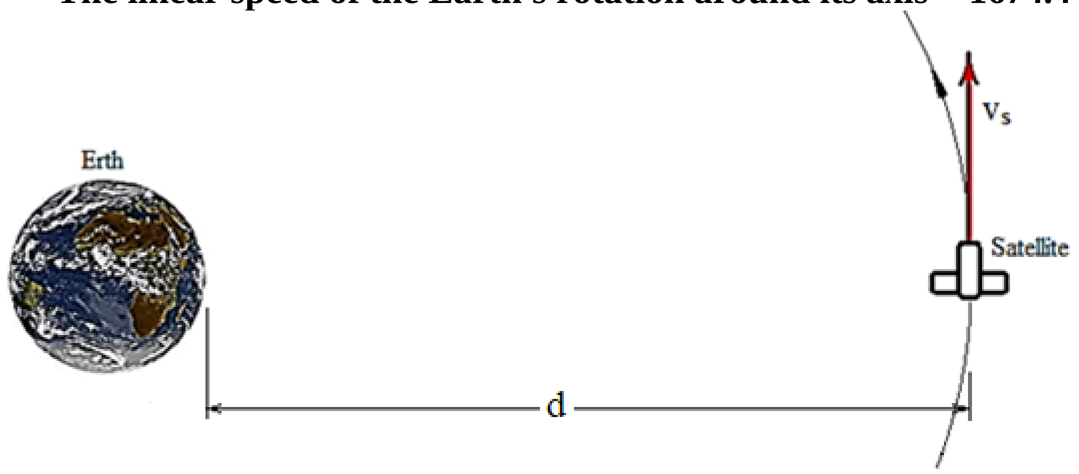
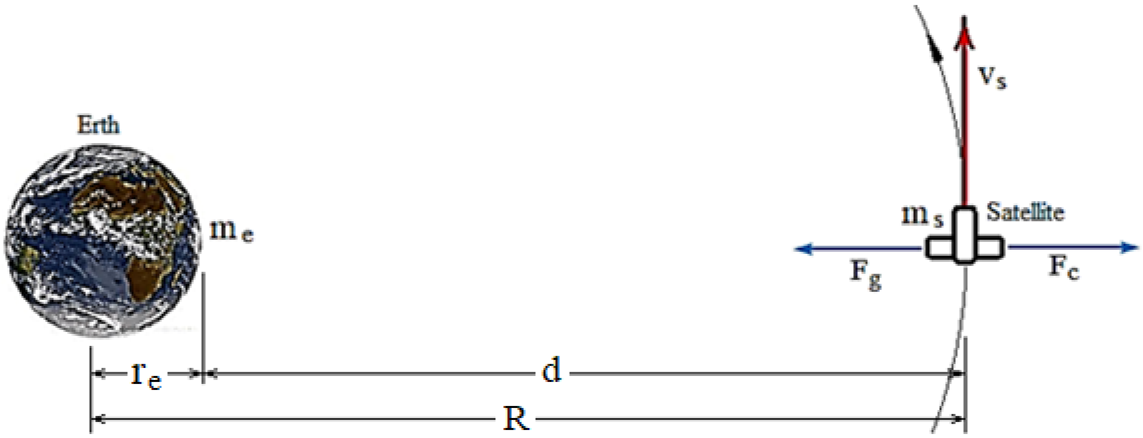


Fig. (Ex. 1-4)

Solution:



$F_g = G \dfrac{m_e \ m_s}{R^2}$ (Gravity force)

$F_c = m_s R \, \omega_s^2$ (Centrifugal force)

$G \dfrac{m_e \ m_s}{R^2} = m_s R \, \omega_s^2$

$\dfrac{G \, m_e}{R^2} = R \, \omega_s^2$

$G \, m_e = R^3 \, \omega_s^2$

V_e = 1674.4 km/hr = 465 m/s

(Linear speed of the Earth's rotation around its axis)

r_e = 6371 km = 6371000 m (Radius of the Earth)

$\omega_e = V_e / r_e$ (Angular velocity of the Earth about its center)

$\omega_e = 465 / 6371000 = 7.3 \times 10^{-5}$ rad/s $= \omega_s$

$m_e = 5.97 \times 10^{24}$ kg (Earth mass)

$G = 66.73 \times 10^{-12}$ $m^3/(Kg.s^2)$ (Constant of gravitation)

$66.73 \times 10^{-12} \times 5.97 \times 10^{24} = R^3 \, (7.3 \times 10^{-5})^2$

$R^3 = \dfrac{66.73 \times 5.97 \times 10^{12}}{(7.3 \times 10^{-5})^2} = 7.48 \times 10^{22}$

R = 42125970 m

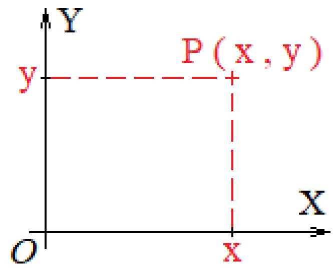
$d = R - r_e$ = 42125970 – 6371000 = 35754970 m = 35755 km

Coordinates systems:

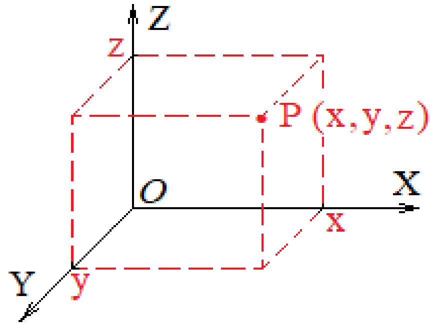
- Cartesian coordinate system:

Fig. (1-9)
Cartesian coordinate system

Two dimensions



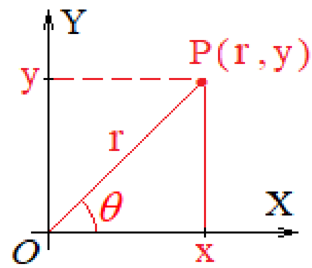
Three dimensions



- Polar coordinate system:

Fig. (1-10)

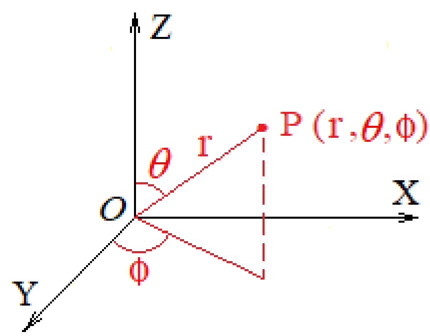
Polar coordinate system



- Spherical coordinate system:

Fig. (1-11)

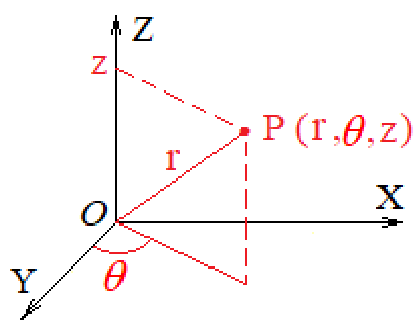
Spherical coordinate system



- Cylindrical coordinate system:

Fig. (1-12)

Cylindrical coordinate system



System of units:

The units of length, mass and time are the basic units from which other units are derived, the force unit was added due to its importance in the subject of engineering mechanics.

	Length	**Mass**	**Time**	**Force**
(SI) Units System International Units	Meter (m)	Kilogram (kg)	Second (s)	Newton (N)
(FPS) Units System British Units	Foot (ft)	Slug (slug)	Second (s)	Pound (Ib)

1 Ib = 4.448 N
1 slug = 14.59 kg
1 ft = 0.3048 m

Conversion of Units:

1 ft (foot) = 12 in. (inches).
1 yd (yard) = 3 ft = 36 in.
1 mi (mile) = 1760 yd = 5280 ft = 63360 in.
1 kip (kilo-pound) = 1000 lb (pound)
1 Ib = 0.453 kg.
1 kg = 2.205 Ib.
1 ton = 2205 Ib = 2.205 kip.
1 ton = 1000 kg
1 in. = 2.54 cm = 25.4 mm
1 ft = 0.3048 m = 30.48 cm
1 yd = 91.44 cm
1 mi = 1609.34 m = 1.609 km

Prefixes:

If the quantity to be measured is too large or too small, it can be expressed in multiples or fractions of the units to determine its value logically.

Table (1-3) shows the prefixes used in the global system. Where each represents a multiple or fraction of a particular unit. *The kilogram is the only basic unit defined by a prefix.*

For example:

4 000 000 N = 4 000 kN (kilo-newton) = 4 MN (mega-newton).

0.005 m = 5 mm (milli-meter).

Table (1-3) The multiple or submultiples of units

Multiplication Factor		Prefix	Symbol
1 000 000 000 000	$= 10^{12}$	Tera	T
1 000 000 000	$= 10^{9}$	Giga	G
1 000 000	$= 10^{6}$	Mega	M
1 000	$= 10^{3}$	Kilo	k
100	$= 10^{2}$	Hecto	h
10	$= 10^{1}$	Deka	da
0.1	$= 10^{-1}$	Deci	d
0.01	$= 10^{-2}$	Centi	c
0.001	$= 10^{-3}$	Milli	m
0.000 001	$= 10^{-6}$	Micro	µ
0.000 000 001	$= 10^{-9}$	Nano	n
0.000 000 000 001	$= 10^{-12}$	Pico	p

Example (1-5):

Express the following units in the correct international units (SI) form using an appropriate prefix:

(a) MN/µs.
(b) Gg/mN.
(c) GN/(kg.ms).

Solution:

(a) $\text{MN/µs} = \dfrac{(10^6)\text{N}}{(10^{-6})\text{s}} = \dfrac{(10^{12})\text{N}}{\text{s}} = \text{TN/s}$

(b) $\text{Gg/mN} = \dfrac{(10^9)\text{g}}{(10^{-3})\text{N}} = \dfrac{(10^{12})\text{g}}{\text{N}} = \text{Tg/N}$

(c) $\text{GN/(kg.ms)} = \dfrac{(10^9)\text{N}}{\text{kg}(10^{-3})\text{s}} = \dfrac{(10^{12})\text{N}}{\text{kg.s}} = \text{TN/(kg.s)}$

<u>Example (1-6):</u>

What is the density of wood expressed in the international units (SI - units), if its value according to the british system units (FBS - units) is (4.7 slug/ft^3)?

Solution:

$$(4.7\ \text{slug/ft}^3) = 4.7\ \frac{(14.59)}{(0.3048)^3} = 2421.6\ \text{kg/m}^3 = 2.42\ \text{Mg/m}^3$$

<u>Example (1-7):</u>

Find the speed of the car shown in Figure (Ex. 1-7) in (kilometers per hour) and (meters per second) units, as the car is traveling at a speed of (60 mi/h).

Fig. (Ex. 1-7)

Solution:

60 mi/h = (60)(5280)(0.3048)/(1000) = 96.56 km/h

96.56 km/h = (96.56)(1000)/(3600) = 26.8 m/s

= 96.56 /3.6 = 26.8 m/s

<u>Example (1-8):</u>

A car of (1400 kg) mass.

(a)- Determine the weight of the car in newtons.

(b)- Convert the mass of the car to slugs.

(c) Determine its weight in pounds.

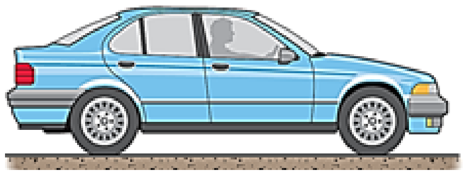


Fig. (Ex. 1-8)

Solution:

(a)- W = mg = 1400 × 9.81= 13730 N

(b)- $m = \frac{1400}{14.59} = 95.96$ slugs

(c)- W = mg = 95.96 × 32.2 = 3090 Ib

Example (1-9):

The spacecraft shown in Fig. (Ex. 1-9) has a mass of (15×10^3) slugs on earth surface. Specify:

(a) its mass in (SI – units).

(b) its weight in (SI – units).

If the spacecraft is on the moon surface, where the acceleration due to gravity is ($g_m = 5.3\ ft/s^2$), determine:

(c) its weight in (SI – units).

(d) its mass in (SI – units).

Fig. (Ex. 1-9)

Solution:

(a) 15×10^3 slugs = $(15 \times 10^3) (14.59)$
$= 218.85 \times 10^3$ kg = 218.85 Mg

(b) $W_e = m.g = (218.85 \times 10^3) (9.81)$
$= 2.147 \times 10^6$ N = 2.147 MN

(c) $W_m = m.g_m = (15 \times 10^3) (5.3) = 79.5 \times 10^3$ Ib
$= (79.5 \times 10^3) (4.448) = 353.6 \times 10^3$ N = 0.35 MN

Or $W_m = W_e (g_m / g) = (2.147\ MN) \left(\dfrac{5.3\ ft/s^2}{32.2\ ft/s^2} \right) = 0.35\ MN$

(d) Since the mass is independent of its location, then:
$m_m = m_e = 218.85 \times 10^3$ kg = 218.85 Mg

Example (1-10):

A man weighs (180 lb) on Earth surface.

(a)- Specify his mass in slugs.
(b)- Specify his mass in kilograms.
(c)- Specify his weight in newtons.

If the man is on the surface of the Moon, where the acceleration due to gravity is (g_m = 5.3 ft/s^2).

(d)- Specify his weight in pounds.
(e)- Specify his mass in kilograms.

Solution:

(a) $m = \frac{180}{32.2} = 5.59$ slug
(b) $m = 5.59 \times 14.59 = 81.56$ kg
(c) $W = 180 \times 4.4482 = 800$ N or $81.56 \times 9.81 = 800$ N
(d) $W = 5.59 \times 5.3 = 29.63$ Ib
(e) $m = 5.59 \times 14.59 = 81.56$ kg

Problems:

1-1) Express each of the following units in the correct (SI – form) using an appropriate prefix:

a) µm/ms, (b) mkm, (c) Gs /mg (d) µN.Gm.

Ans. mm/s, m, Ts/g, kN.m

1-2) The density of brass is (8.33 Mg/m^3). Determine its specific weight (wt. / vol.) in English units. Use an appropriate prefix.

Ans. 81.717 kN/m^3

1-3) Convert the following quantities from the international system (SI) to the English system (FPS), using an appropriate prefix:

(a)- (27.5 kN/m^3) to (lb/ft^3). (b)- (0.5 mm/s) to (ft/h).

(c)- (1.13 kN.m) to (Ib.ft).

Ans. 175 Ib/ft^3, 5.9 ft/h, 833.5 Ib.ft

1-4) Find the mass of an object that has a weight of:

(a)- 35 mN. (b)- 200 kN. (c)- 50 MN.

Ans. 3.57 g, 20.39 Mg, 5.1 Gg

1-5) Determine the weight in (SI) units of a body that has a mass of:

(a) - 15 kg (b) - 0.75 g (c) - 7.5 Mg

Ans. 147.15 N , 7.36 mN , 73.6 kN

1-6) Two balls with a mass of (250 kg) and radius of (350 mm) for each ball, each ball are touching each other. Determine the gravitational force acting between the two balls.

Ans. 8.5 µN

1-7) The density of the water is (1 Mg/m^3). What is the density of it expressed in English units?

Ans. 1.94 $slug/ft^3$

1-8) A spacecraft of mass (3000 kg) is launched from Earth to the Moon.

1- Calculate the weight and mass of the spacecraft on the surface of the Earth and on the surface of the Moon.
2- Calculate the gravitational force between the spacecraft and both the Earth and the Moon when the spacecraft is at a distance of (100000 km) from the surface of the Earth.

- The distance between the centers of the Earth and the Moon = 384400 km.
- The mass of the Earth = 5.97×10^{24} kg.
- The mass of the Moon = 7.35×10^{22} kg.
- The radius of the Earth = 6371 km.
- The radius of the Moon = 1737 km.

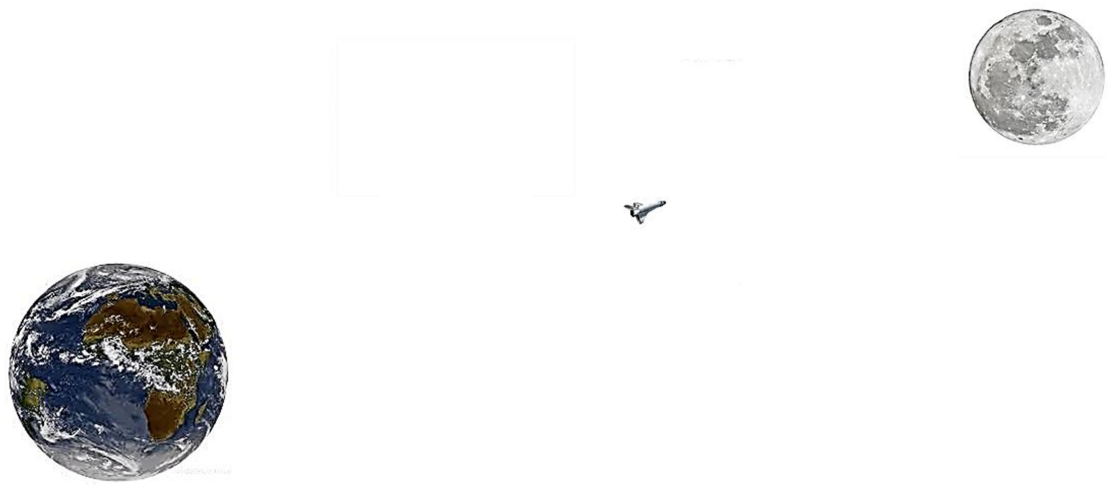


Fig. (Pr. 1-8)

Ans:

For the Earth	*For the Moon*
$W_e = 29430\ N$ $m = 3000\ kg$	$W_m = 4873\ N$ $m = 3000\ kg$
$F_{se} = 105.6\ N$	$F_{sm} = 0.19\ N$

CHAPTER TWO
FORCE ANALYSIS

Scalars and Vectors:

Scalar (A)

A scalar is any positive or negative physical quantity that can be completely expressed by its *magnitude*.

for example: length, mass, time, density, volume.

Vector ($\vec{A}$)

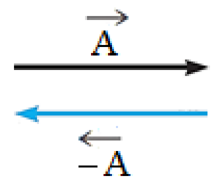
A vector is any physical quantity that requires both a *magnitude and a direction* for its complete description.

for example: force, position, moment, weight, velocity, displacement, acceleration.

- The *magnitude* of the vector is represented by the length of the arrow.
- The *direction* of the vector line of action is represented by the angle (θ) between the vector and a fixed axis.

Vector direction:

The negative sign means the opposite direction.



Types of vectors:

There are three types of a vector:

1- Free vector:

It is a vector which may be freely moved creating couples in space.

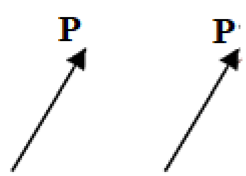
2- Sliding vector:

It is a vector that can represent the force acting on a rigid body and can be moved along the line of action of the force without any effect on the body.

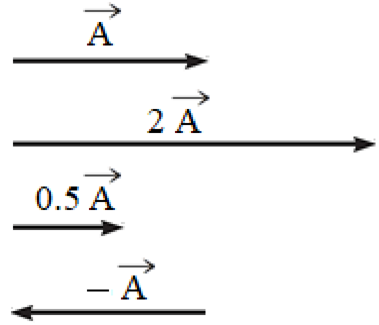
3- Bound vector or Fixed vector:

It is a vector that its moving requires changing the conditions of the problem.

For two vectors to be equal they must have the same value and direction, they do not need to have the same point of application.:



Multiplication and Division of a vector by a scalar:



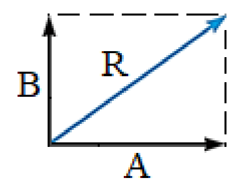
- Multiplying a vector by a positive absolute number leads to an increase or decrease in its value by that number.
- Multiplying a vector by a negative absolute number, leads to a change in its direction in the opposite direction, with an increase or decrease in its value according to the magnitude of this number.

Vector Addition:

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are collinear, i.e., both have the same line of action, the algebraic addition is used, as shown:

$$(\vec{R} = \vec{A} + \vec{B}) \ldots\ldots\ldots\ldots.. \quad (2\text{-}1)$$

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are perpendicular, then they can be summed by using Pythagoras' theory.

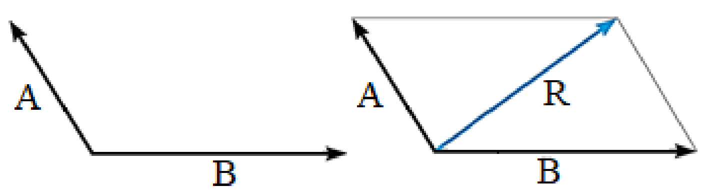


$$R = \sqrt{A^2 + B^2} \quad \ldots\ldots\ldots\ldots.. \quad (2\text{-}2)$$

- If the two vectors ($\vec{A}$) and ($\vec{B}$) are not collinear, there are two methods for addition:

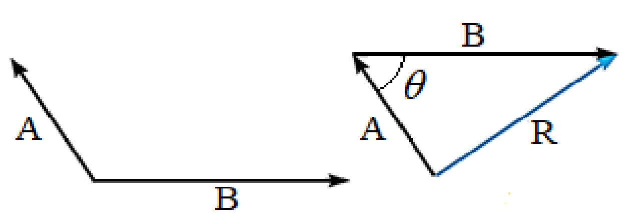
1- Parallelogram law.

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta} \quad \ldots\ldots\ldots\ldots \quad (2\text{-}3)$$



2- Triangle rule.

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

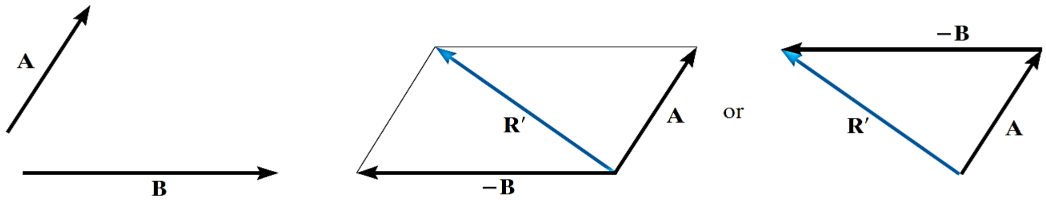


Vector Subtraction:

If the two vectors ($\vec{A}$) and ($\vec{B}$) are in the same direction, the difference between them can be found as follows:

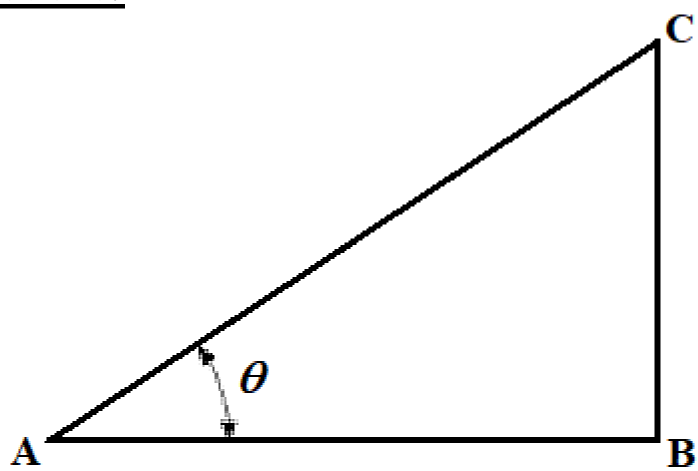
$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ …….. (2-4)

That is, subtraction is a special case of addition, and vector addition laws can be used to subtract vectors by reversing the direction of the subtracted vector.



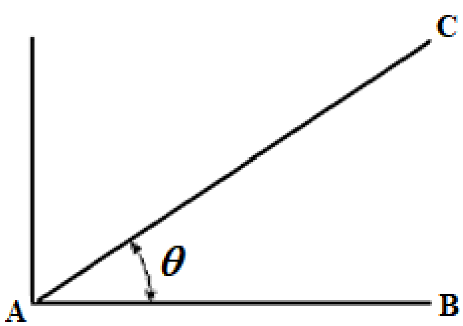
Parallelogram law   Triangle rule

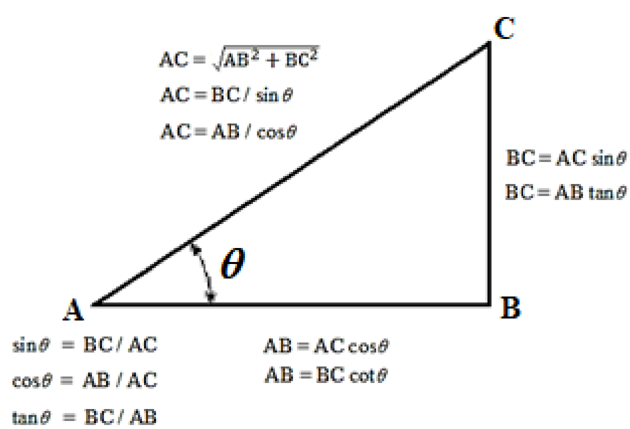
Trigonometric relations:



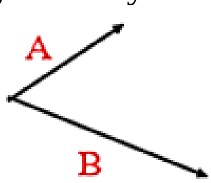
$$\left.\begin{array}{ll} AB = AC\cos\theta & BC = AC\sin\theta \\ AB = BC\cot\theta & BC = AB\tan\theta \end{array}\right. \qquad \left.\begin{array}{l} AC = \sqrt{AB^2 + BC^2} \\ AC = BC / \sin\theta \\ AC = AB / \cos\theta \end{array}\right\} \quad \ldots\ldots\ldots (2\text{-}5)$$

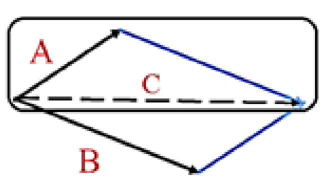
$$\left.\begin{array}{l} \sin\theta = BC / AC \\ \cos\theta = AB / AC \\ \tan\theta = BC / AB \end{array}\right\} \quad \ldots\ldots\ldots (2\text{-}6)$$

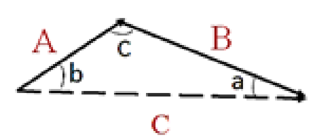




Trigonometry can be used to find the resultant force:







From Cosine Law: $$C = \sqrt{A^2 + B^2 - 2AB\cos(c)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}7)$$

From Sines Law: $$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}8)$$

Types of force systems:

A force system is a group of forces (two or more forces) that affect a body or group of bodies in a specific situation, and it can be classified into:

1- The system of forces located on one line of action (Collinear system).
2- The system of parallel forces (Parallel system).
3- The system of forces located on one plane (Coplanar system).
4- The system of converging forces (Concurrent system).
5- The system of parallel forces located in one plane (Parallel, Coplanar system), in which the lines of action of the forces are parallel and located in one plane.
6- The system of converging forces located in one plane (Concurrent, Coplanar system), in which the lines of action of the forces intersect at a common point and are located in one plane.
7- The system of convergent and non-parallel forces and located in one plane (Concurrent, Nonparallel, Coplanar system), in which the lines of action of the forces are intersecting and non-parallel and located in one plane.
8- The system of parallel forces that do not locate in one plane (Parallel, Noncoplanar system), in which the lines of action of the forces are parallel and do not locate in one plane.
9- The system of converging forces that do not locate in one plane (Concurrent, Noncoplanar system), in which the lines of action of the forces intersect at a common point and do not locate in one plane.
10- The system of non-parallel, non-intersecting forces that do not locate in one plane (Nonparallel, Nonconcurrent, Noncoplanar system), where the lines of action of the forces are non-parallel, non-intersecting and do not locate in one plane.

Principle of force transmissibility on its line of action:

If a force (F) acting on a specific body is transmitted along its line of action without changing its direction, the effect of the force on the body does not change.

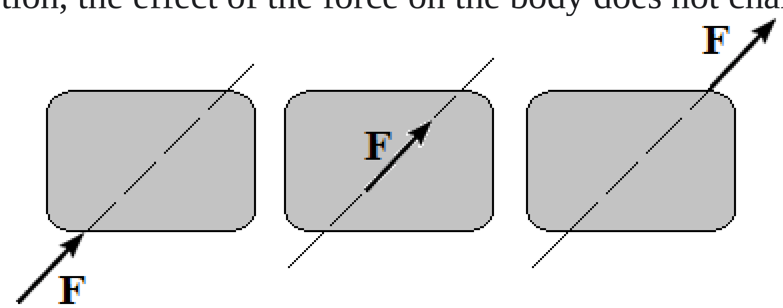


Fig. (2-1) Principle of Force transmissibility on its line of action

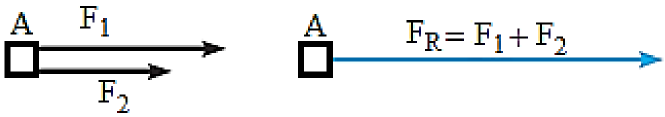
Resultant of forces:

If two forces (F_1) and (F_2) acting on a particle (A) may be replaced by a single force (F_R), which has the same effect on the particle, this force is called the *resultant* of the forces (F_1) and (F_2).

If the forces located on one line of action (Collinear):

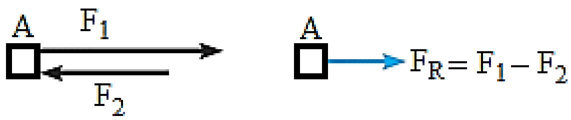
If the forces are in the same direction, then their resultant is according to equation (2-1):

$$F_R = F_1 + F_2$$



If the forces are in opposite directions, then their resultant is according to Eq. (2-4):

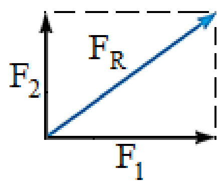
$$F_R = F_1 - F_2$$



If the forces are converging, perpendicular forces located in one plane (Concurrent, perpendicular, Coplanar):

The resultant can be determined by using Pythagoras' theory, and from Eq. (2-2):

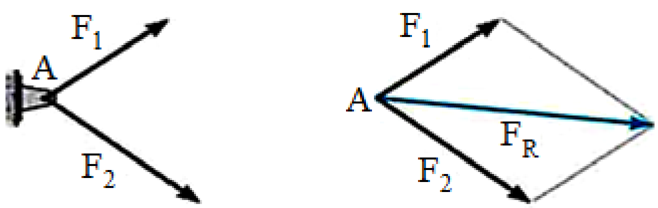
$$R = \sqrt{A^2 + B^2}$$



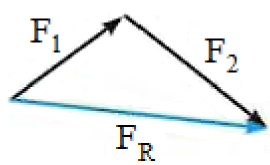
If the forces are converging forces located in one plane (Concurrent, Coplanar):

The resultant may be obtained by:

1- A parallelogram, using (F_1) and (F_2) as two sides of the parallelogram. The diagonal that pass through point (A) represents the resultant. This is known as the *parallelogram law*.



2- The triangle of forces, where the force (F_1) is drawn, and from the end of it the force (F_2) is drawn, so the resultant is the force that starts from the beginning of (F_1) and ends at the end of (F_2). This is known as the *triangle law*.



$$F_R = F_1 + F_2$$

3- Using the triangle of forces, the value of the resultant can be found by using the law of cosines, and finding its direction from the law of sines. Equations (2-7) and (2-8).

Example (2-1):

Two forces (180 N) and (120 N) are applied to the ring shown in Fig. (Ex. 2-1). Determine the magnitude and direction of the resultant force.

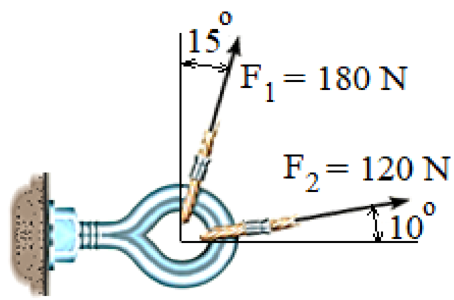
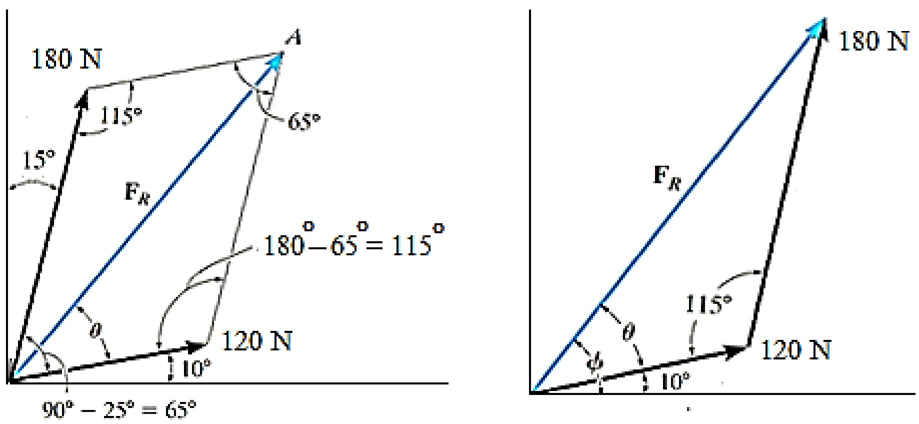


Fig. (Ex. 2-1)

Solution:

Parallelogram Law.

A parallelogram is drawn from drawing a line starting from the head of force (180 N) and parallel to the force (120 N), and another line starting from the head of force (120 N) and parallel to the force (180 N). The resultant of the two forces (F_R) will be from the starting point of the two forces to the point of intersection of these two lines at point (A).



Trigonometry laws.

From the parallelograms. A triangle of forces is created.

Use the law of cosines.

$$F_R = \sqrt{F_1{}^2 + F_2{}^2 - 2\,F_1\,F_2 \cos\alpha}$$

$$F_R = \sqrt{(180)^2 + (120)^2 - 2\,(180)\,(120)\cos 115}$$

$$= \sqrt{32400 + 14400 - 43200\,(-0.4226)} = 255\text{ N}$$

Applying the law of sines to determine (θ),

$$\frac{180}{\sin\theta} = \frac{255}{\sin 115^o} \quad\Rightarrow\quad \sin\theta = \frac{180\ \sin 115^o}{255} = 0.64 \quad\Rightarrow\quad \theta = 39.8^o$$

Thus the direction (ϕ) of the resultant force (F_R) measured from the horizontal is:

$$\theta = 39.8^o + 10^o = 49.8^o$$

<u>Example (2-2):</u>

In the fixed structure shown in Fig. (Ex. 2-2).

P = 400 N
T = 150 N

Replace the two forces (P) and (T) by a single force (R) which has the same effect on the fixed structure.

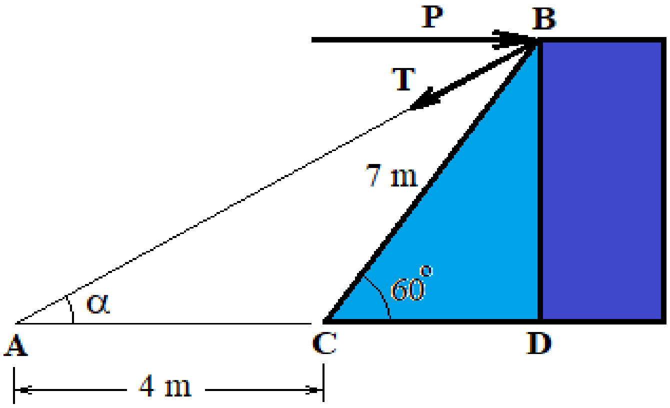
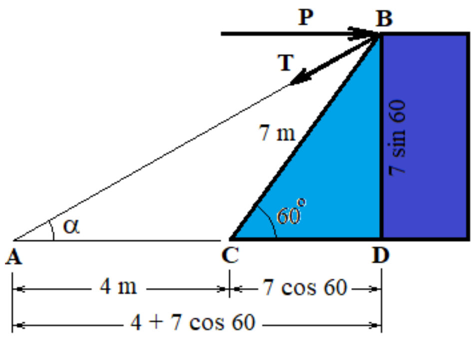


Fig. (Ex. 2-2)

Solution:

$$\tan\alpha = \frac{BD}{AD} = \frac{7\sin 60°}{4 + 7\cos 60°} = 0.81$$

$$\alpha = 38.9°$$

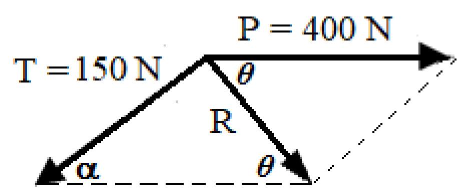


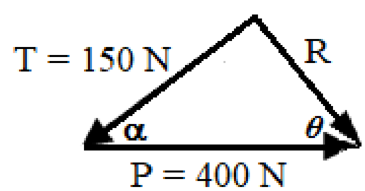
Law of cosines:

$$R = \sqrt{T^2 + P^2 - 2TP\cos 38.9°}$$

$$= \sqrt{150^2 + 400^2 - 2(150)(400)\cos 38.9°}$$

$$= 298.5 \text{ N}$$





Law of sines:

$$\frac{150}{\sin\theta} = \frac{298.5}{\sin 38.9°}$$

$$\sin\theta = \frac{150\sin 38.9°}{298.5} = 0.316$$

$$\theta = 18.4°$$

R = 298.5 N  18.4°

Example (2-3):

A structure consisting of two members, which is affected downward by a force (F) of (1500 N) at point (A). Find the two components of the force acting along the two members of the structure (AB) and (AC).

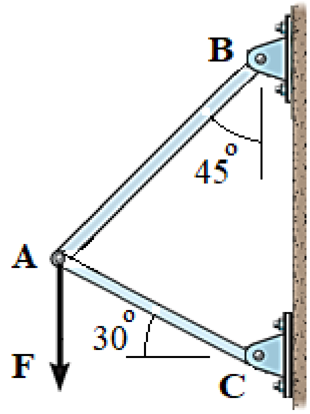


Fig. (Ex. 2-3)

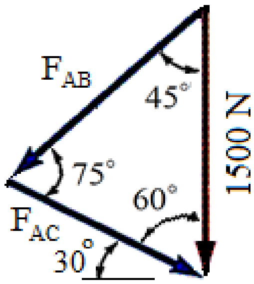
Solution:

$$\frac{F_{AB}}{\sin 60} = \frac{1500}{\sin 75}$$

$F_{AB} = 1344.86$ N

$$\frac{F_{AC}}{\sin 45} = \frac{1500}{\sin 75}$$

$F_{AC} = 1098.08$ N



Example (2-4):

The (I) beam is hoisted by two cables, as shown in Figure (Ex. 2-4). Find the value of the tensile forces (F_A) and (F_B) in each cable so that their resultant magnitude is (3000 Ib) directed towards the positive vertical axis (y-axis).

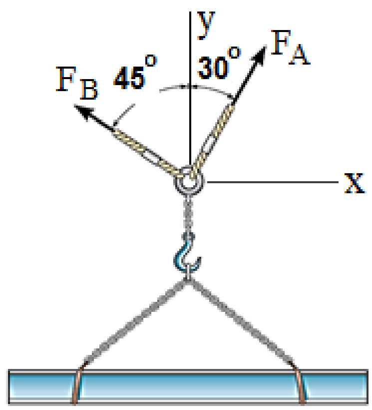


Fig. (Ex. 2-4)

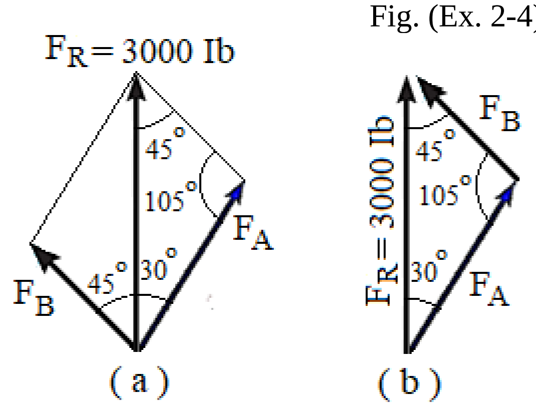
Solution:

$$\frac{F_A}{\sin 45°} = \frac{3000}{\sin 105°}$$

$$F_A = \frac{3000 \sin 45°}{\sin 105°} = 2196 \text{ Ib}$$

$$\frac{F_B}{\sin 30°} = \frac{3000}{\sin 105°}$$

$$F_B = \frac{3000 \sin 30°}{\sin 105°} = 1553 \text{ Ib}$$



Example (2-5):

A barge is pulled by two tugboats as shown in Fig. (Ex. 2-5). If the resultant of the forces exerted by the tugboats is (2500 N) directed along the axis of the barge. Determine:

a) The tension in each of the ropes for (α = 40º),

b) The value of (α) for which the tension in rope (2) is a minimum.

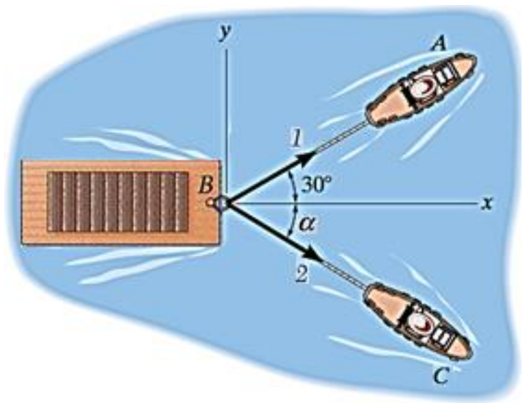


Fig. (Ex. 2-5)

Solution:

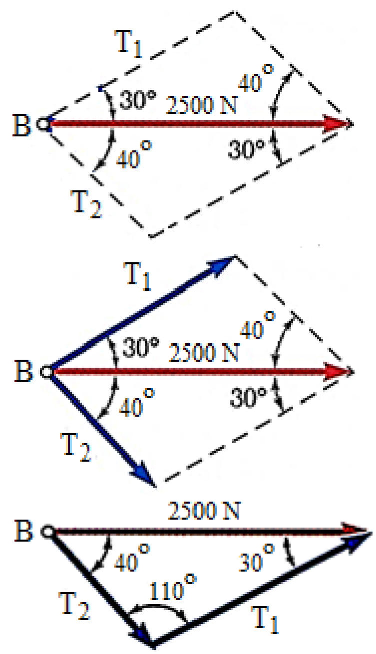
a)

$$\frac{2500}{\sin 110} = \frac{T_1}{\sin 40}$$

$T_1 = 1710$ N

$$\frac{2500}{\sin 110} = \frac{T_2}{\sin 30}$$

$T_2 = 1330$ N

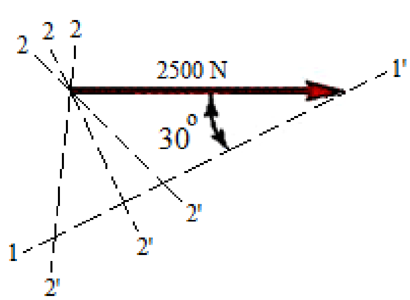


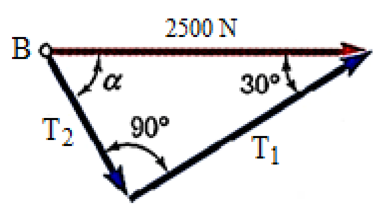
b)

$T_1 = 2500 \cos 30$
$= 2165$ N

$T_2 = 2500 \sin 30$
$= 1250$ N

$\alpha = 90 - 30 = 60º$





<u>Example (2-6):</u>

Analyze the force (F = 15 lb) acting on the tooth of the gear shown in Fig. (Ex. 2-6). into two components towards the axes (a - a) and (b - b).

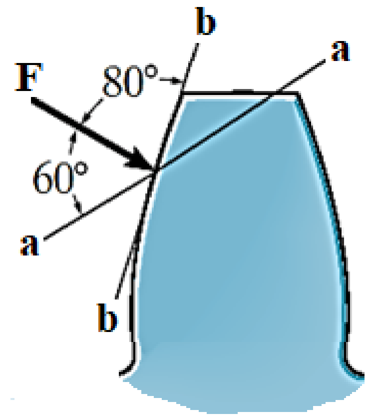
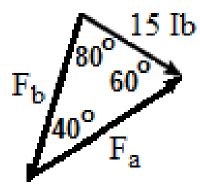


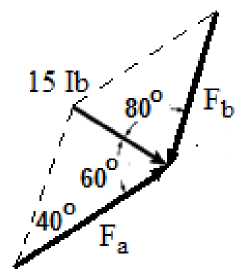
Fig. (Ex. 2-6)

Solution:

$$\frac{15}{\sin 40°} = \frac{F_a}{\sin 80°} \quad \Rightarrow \quad F_a = 22.98 \text{ lb}$$

$$\frac{15}{\sin 40°} = \frac{F_b}{\sin 60°} \quad \Rightarrow \quad F_b = 20.21 \text{ lb}$$





<u>Example (2-7):</u>

Two forces of (8 kN) and (6 kN) are applied to the structure shown in Fig. (Ex. 2-7). Find the magnitude and direction of the resultant force, measured clockwise from the horizontal axis.

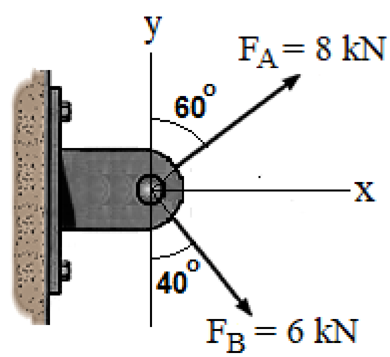


Fig. (Ex. 2-7)

Solution:

$$F_R = \sqrt{{F_1}^2 + {F_2}^2 - 2(F_1)(F_2)\cos\phi}$$

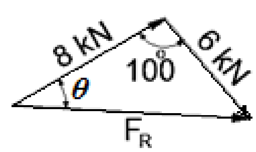
$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100°} = 10.8 \text{ kN}$$

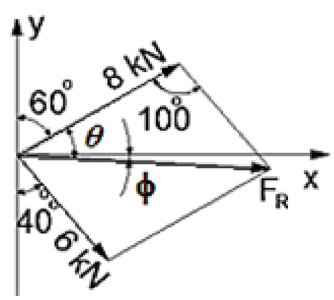
$$\frac{6}{\sin\theta} = \frac{10.8}{\sin 100°}$$

$\sin\theta = 0.547$

$\theta = 33.17°$

$\phi = 33.17° - 30° = 3.17°$





Example (2-8):

Express the magnitude of the resultant (F_R) and its direction (θ) in terms of the values of the components (F_1) and (F_2) and the angle (ϕ).

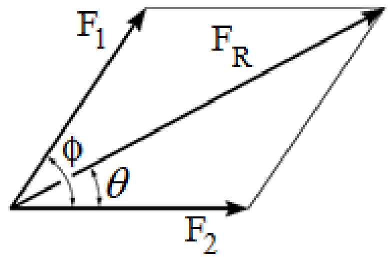


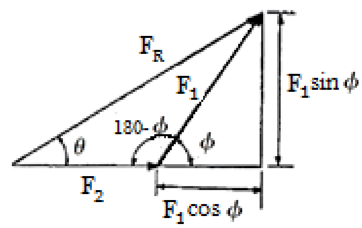
Fig. (Ex. 2-8)

Solution:

$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} - 2F_1F_2\cos(180-\phi)}$$

Since { $\cos(180-\phi) = -\cos\phi$ }:

$$F_R = \sqrt{F_1^{\,2} + F_2^{\,2} + 2F_1F_2\cos\phi}$$



From the figure:

$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$$

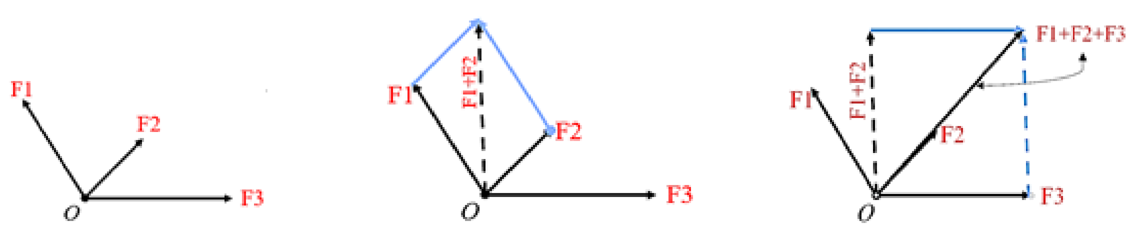
Resultant of Several Forces (More than two Forces):

Th resultant of more than two forces can be found by applying the law of parallelograms in two stages:

1- Finding the resultant of the first and second forces.

2- Taking the resultant of these two forces as a force, and then finding its Resultant with the third force, so we get the resultant of the three forces.

For example, if three forces (F_1) , (F_2) , (F_3) act at a point (O), the resultant will be as follows:



(1) (2) (3)

<u>Example (2-9):</u>

Three chains attached to a bracket ring, as shown in the figure (Ex. 2-9), a tensile force is applied to each chain so that the resultant of the three forces is (2500 N). If two chains of them are affected by known forces, find the angle of the third chain measured clockwise from the positive (x-axis) so that the magnitude of force (F) in this chain is a minimum, then find the magnitude of (F).

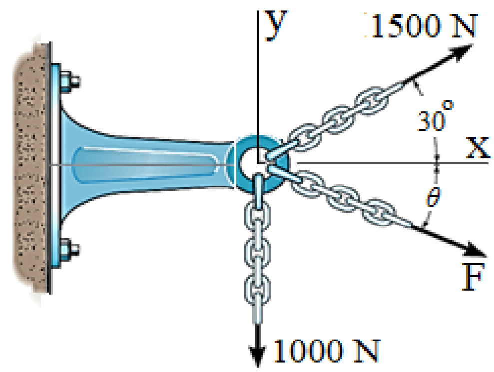


Fig. (Ex. 2-9)

Hint: First find the magnitude and direction of the resultant of the two known forces, force (F) acts in this direction.

Solution:

Cosine law:

$$F_{R1} = \sqrt{1500^2 + 1000^2 - 2(1500)(1000)\cos 60^\circ} = 1322.9 \text{ N}$$

Sine law:

$$\frac{1000}{\sin(30+\theta)} = \frac{1322.9}{\sin 60}$$

$$\sin(30+\theta) = \frac{1000 \sin 60}{1322.9} = 0.65$$

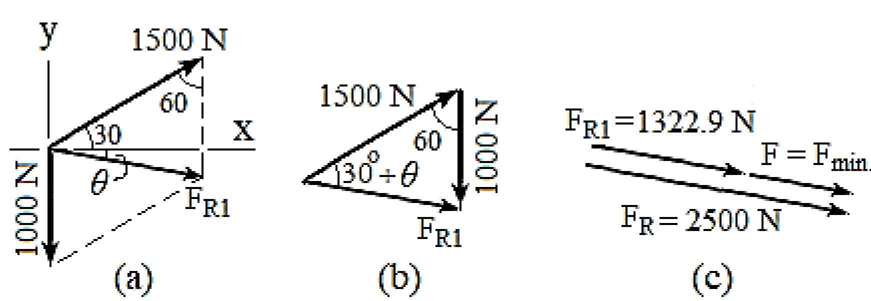
$$30 + \theta = 40.89^\circ \quad \Rightarrow \quad \theta = 10.89^\circ$$

To obtain the required resultant at minimum value of the force (F), the direction of the force (F) must be the same as the direction of the resultant of the two given forces (F_{R1}).

$F_R = F_{R1} + F$

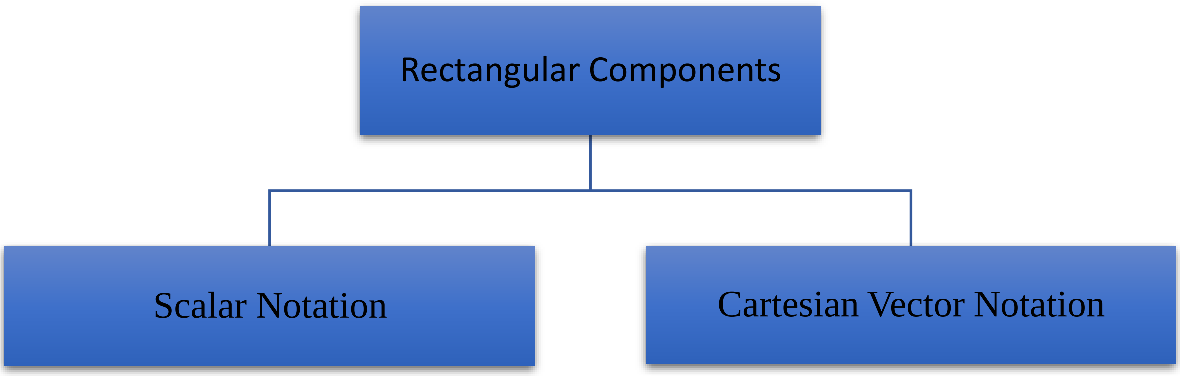
$2500 = 1322.9 + F_{min}$

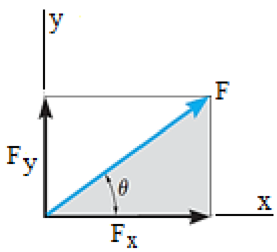
$F_{min} = 1177.1$ N



The resultant of two-dimensional forces by analysis method:

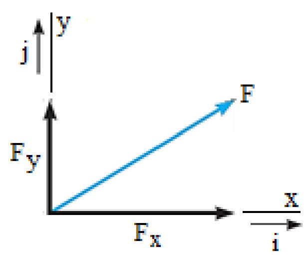
It is possible to analyze any force in a specific plane, such as (x – y) plane, into two perpendicular components along the (x) and (y) axes. These two components are called " Cartesian (rectangular) components ".





$F_x = F \cos \theta$
$F_y = F \sin \theta$
$F = \sqrt{(Fx)^2 + (Fy)^2}$
$\theta = \tan^{-1} \frac{Fy}{Fx}$

F_x and F_y are Force componentes



$F = F_x i + F_y j$

i: unit vector in x-direction

j: unit vector in y-direction

Fig. (2-2) Vector Notation and Scalar Notation for the force

$F_x = F \cos \theta$ ………………………..………… (2-9)
$F_y = F \sin \theta$ ……………………...………… (2-10)

Directional value:

$F = F_x i + F_y j$ …………………………… (2-11)

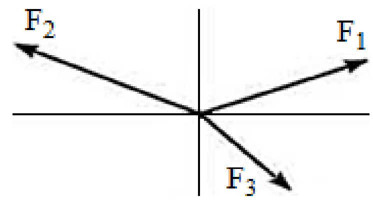
Absolute value:

$F = \sqrt{(Fx)^2 + (Fy)^2}$ …………………… (2-12)

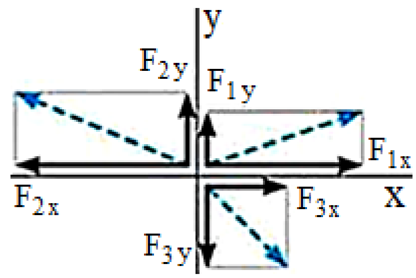
Direction:

$\theta = \tan^{-1} \frac{Fy}{Fx}$ ………………………..…… (2-13)

Finding the resultant:



1- Each force is resolved into its two components with the axes (x) and (y).
2- The components applied on the x-axis summed to be the horizontal component of the resultant.
3- The components applied to the y-axis are summed to be the vertical component of the resultant.
4- Using the Cartesian vector, the resultant is represented as a Cartesian vector.



$F_R = (F_{Rx})\, i + (F_{Ry})\, j$

$F_{Rx} = \sum F_x$

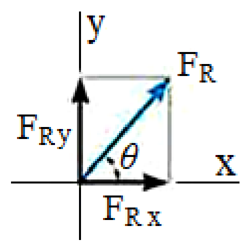
$F_{Rx} = F_{1x} - F_{2x} + F_{3x}$

$F_{Ry} = \sum F_y$

$F_{Ry} = F_{1y} + F_{2y} - F_{3y}$

$F_R = (F_{1x} - F_{2x} + F_{3x})\, i + (F_{1y} + F_{2y} - F_{3y})\, j$

5- Finding the value of the resultant (F_R) by using the Pythagorean theory:



$$F_R = \sqrt{F_{Rx}^{\ 2} + F_{Ry}^{\ 2}}$$

6- The angle (θ) that represents the direction of the resultant can be found from the trigonometric laws:

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right)$$

Example (2-10):

Calculate the horizontal and vertical components of the forces (F_1) and (F_2) on the arm shown in the Fig. (Ex. 2-10). Then express each force as a Cartesian vector. Then find the magnitude and direction of the resultant force.

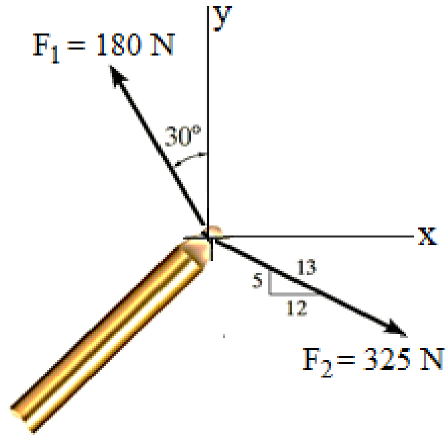


Fig. (Ex. 2-10)

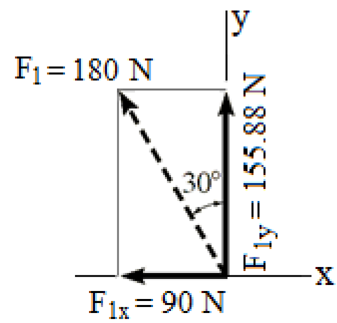
Solution:

$F_{1x} = -180 \sin 30° = -90$ N $= 90$ N $\leftarrow$

$F_{1y} = 180 \cos 30 = 155.88$ N

$F_{2x} = 325\,(12/13) = 300$ N

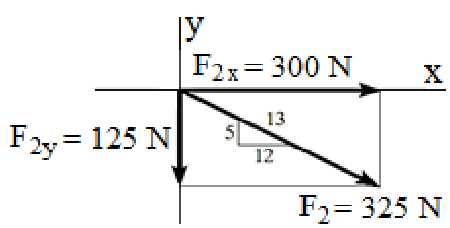
$F_{2y} = -325\,(5/13) = -125$ N $= 125$ N $\downarrow$



Cartesian vector notation.

$F_1 = \{ -90\, i + 155.88\, j \}$ N

$F_2 = \{ 300\, i - 125\, j \}$ N



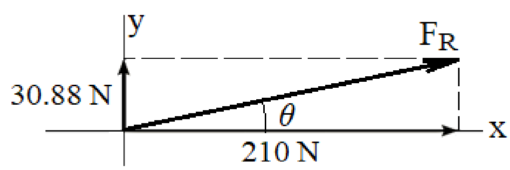
The resultant force:

$$F_R = \sqrt{{F_{Rx}}^2 + {F_{Ry}}^2}$$

$F_{Rx} = F_{1x} + F_{2x} = -90 + 300 = 210$ N

$F_{Ry} = F_{1y} + F_{2y} = 155.88 - 125 = 30.88$ N

$F_R = \sqrt{210^2 + 30.88^2} = 212.258$ N



The direction (θ):

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{30.88}{210}\right) = 8.37^{o}$$

Example (2-11):

Determine the magnitude and direction of the resultant force of the three forces acting on the ring (A) measured counterclockwise from the positive (x-axis).

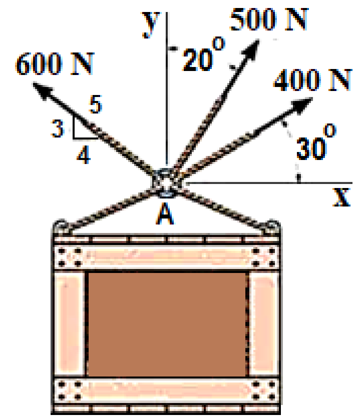


Fig. (Ex. 2-11)

Solution:

$F_R = \sum F$

$F_{Rx} = \sum F_x = -600\left(\frac{4}{5}\right) + 500 \sin 20^\circ + 400 \cos 30^\circ = 37.42\text{ N}$

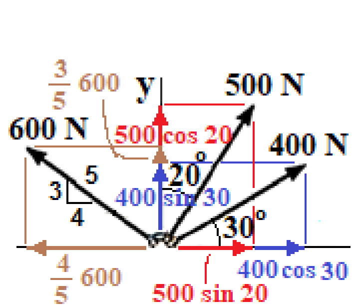
$F_{Ry} = \sum F_y = 600\left(\frac{3}{5}\right) + 500 \cos 20^\circ + 400 \sin 30^\circ = 1029.85\text{ N}$

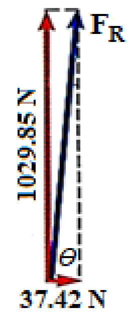
$F_R = \sqrt{(37.42)^2 + (1029.85)^2} = 1030.5\text{ N}$

$\tan\theta = \dfrac{F_{Ry}}{F_{Rx}}$

$\theta = \tan^{-1}\dfrac{1029.85}{37.42}$

$= 87.92^\circ$





Example (2-12):

Determine the components of the resultant of the forces acting on the gusset plate of a bridge truss in the direction of (x - axis) and (y- axis). Then show that the resultant is zero.

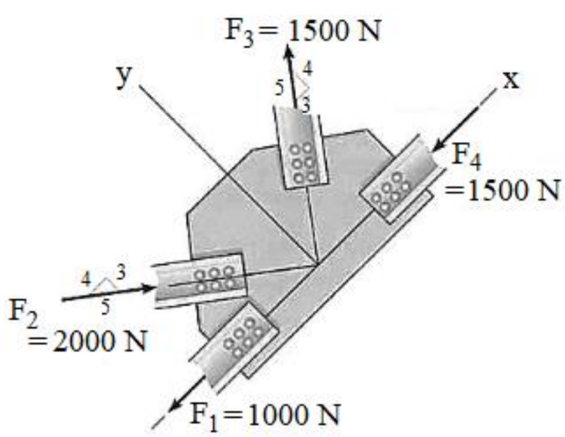


Fig. (Ex. 2-12)

Solution:

$F_R = \sum F$

$F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} + F_{4x}$

$= -1000 + 2000\left(\frac{4}{5}\right) + 1500\left(\frac{3}{5}\right) - 1500 = 0$

$F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} + F_{4y}$

$= 0 - 2000\left(\frac{3}{5}\right) + 1500\left(\frac{4}{5}\right) - 0 = 0$

Example (2-13):

Four forces act on fixed frame as shown in Fig. (Ex. 2-13). Determine the magnitude and direction of the resultant of these forces.

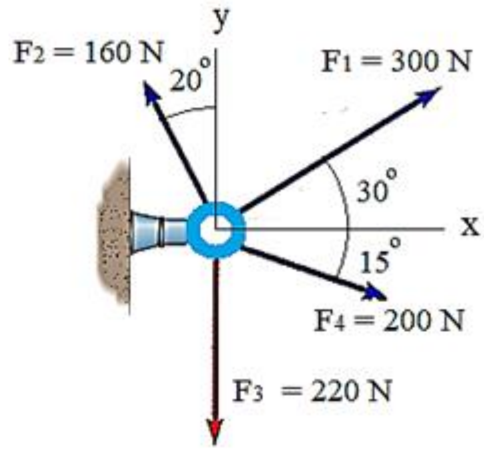
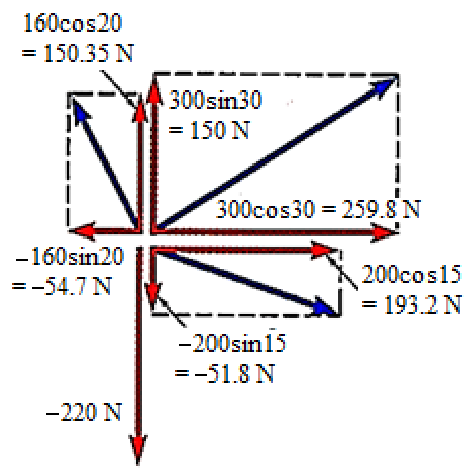


Fig. (Ex. 2-13)

Solution:



$(F_1)_x = 300 \cos 30° = 259.8$ N

$(F_1)_y = 300 \sin 30° = 150$ N

$(F_2)_x = -160 \sin 20° = -54.7$ N

$(F_2)_y = 160 \cos 20° = 150.35$ N

$(F_3)_x = 0$

$(F_3)_y = -220$ N

$(F_4)_x = 200 \cos 15° = 193.2$ N

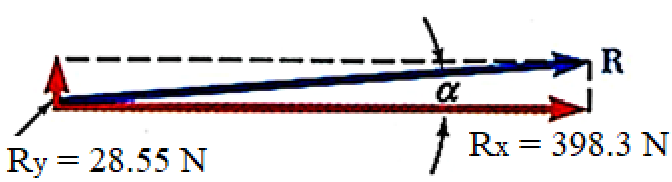
$(F_4)_y = -200 \sin 15° = -51.8$ N

$R_x = 259.8 - 54.7 + 0 + 193.2 = 398.3$ N

$R_y = 150 + 150.35 - 220 - 51.8 = 28.55$ N

$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{398.3^2 + 28.55^2} = 399.3$ N

$\tan\alpha = \dfrac{R_y}{R_x} \quad \Rightarrow \quad \alpha = \tan^{-1}\dfrac{28.55}{398.3} = 4.1°$



Example (2-14):

Express the forces (F_1), (F_2), and (F_3) acting on the body shown in Fig. (Ex. 2-14) in the form of a Cartesian vector, then find the magnitude and direction of the resultant of the three forces.

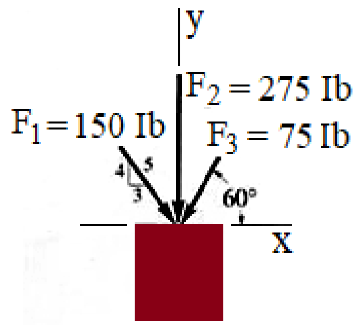


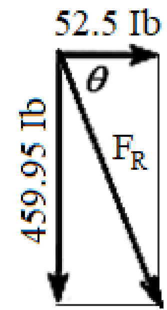
Fig. (Ex. 2-14)

Solution:

$$F_1 = 150\left(\frac{3}{5}\right) i - 150\left(\frac{4}{5}\right) j = 90 \text{ Ib } i - 120 \text{ Ib } j$$

$$F_2 = 0\, i - 275 \text{ Ib } j$$

$$F_3 = -75 \cos 60^\circ\, i - 75 \sin 60^\circ\, j = -37.5 \text{ Ib } i - 64.95 \text{ Ib } j$$



$$F_R = \sum F$$

$$F_{Rx} = \sum F_x = F_{1x} + F_{2x} + F_{3x} = 90 + 0 - 37.5 = 52.5 \text{ Ib}$$

$$F_{Ry} = \sum F_y = F_{1y} + F_{2y} + F_{3y} = -120 - 275 - 64.95$$

$$= -459.95 \text{ Ib}$$

$$F_R = \sqrt{(52.5)^2 + (-459.95)^2} = 462.94 \text{ Ib}$$

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}} \Rightarrow \theta = \tan^{-1}\frac{-459.95}{52.5} = -83.49^\circ$$

Example (2-15):

A wooden dowel rotates in a lathe and a force of (100 N) is applied to it by the cutting stylus of the lathe, as shown in Fig. (Ex. 2-15). Resolve this force into its components acting:
(a) along the (x) and (y) axes.
(b) along the (x') and (y') axes.

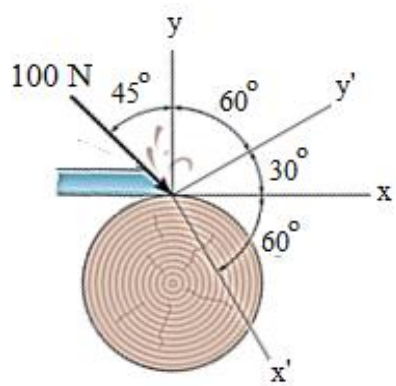
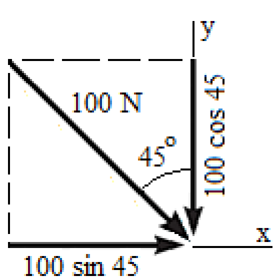


Fig. (Ex. 2-15)

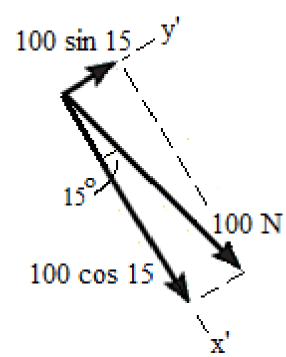
Solution:

a) $F_x = 100 \sin 45^\circ = 70.7$ N
$F_y = -100 \cos 45^\circ = -70.7$ N

b) $F_{x'} = 100 \cos 15^\circ = 96.6$ N
$F_{y'} = 100 \sin 15^\circ = 25.88$ N



(a)



(b)

Example (2-16):

It is required to remove the screw from the wood by applying force along its horizontal axis. The obstruction (A) prevents direct access, so that two forces are applied, one of (1000 N) and the other of (P), by cables as shown in Fig. (Ex.2-16). Determine the magnitude of the force (P) that is required to get the resultant force (T) directed along the screw axis. Also find the magnitude of (T).

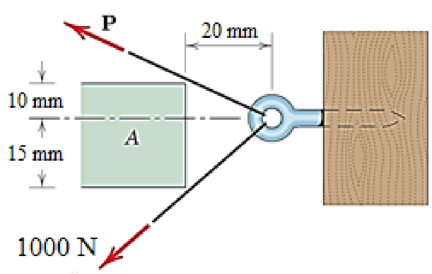


Fig. (Ex. 2-16)

Solution:

Method (I):

$$\theta_1 = \tan^{-1}\frac{10}{20} = 26.57°$$

$$\theta_2 = \tan^{-1}\frac{15}{20} = 36.87°$$

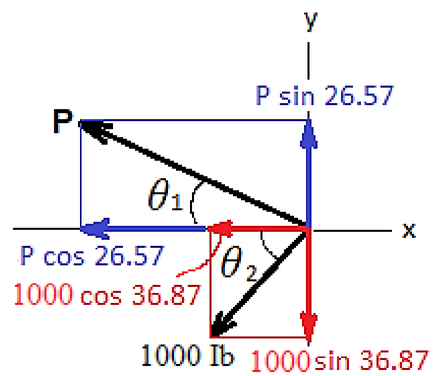
$R_y = \sum F_y = 0$

$$P \sin 26.57° - 1000 \sin 36.87° = 0$$

$$P \sin 26.57° = 1000 \sin 36.87°$$

$$P = \frac{1000 \sin 36.87°}{\sin 26.57°} = 1341.4 \text{ N}$$

$$T = R_x = \sum F_x = 1341.4 \cos 26.57° + 1000 \cos 36.87° = 2000 \text{ N}$$



Method (II):

$$\theta_1 = \tan^{-1}\frac{10}{20} = 26.57°$$

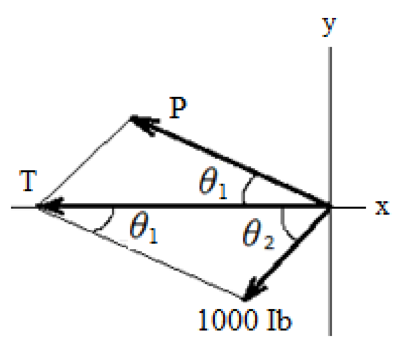
$$\theta_2 = \tan^{-1}\frac{15}{20} = 36.87°$$

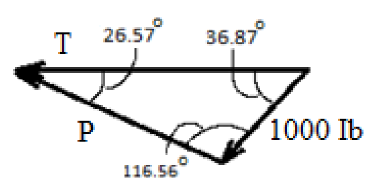
$$\frac{P}{\sin 36.87} = \frac{1000}{\sin 26.57}$$

$$P = \frac{1000 \sin 36.87°}{\sin 26.57°} = 1341.4 \text{ N}$$

$$\frac{T}{\sin 116.56} = \frac{1000}{\sin 26.57}$$

$$T = \frac{1000 \sin 116.56}{\sin 26.57} = 2000 \text{ N}$$





Example (2-17):

Three forces are applied to the bracket shown Fig. (Ex. 2-17). Find the value and direction of the force (F_3) that makes the resultant of the three forces (100 lb) directed along the positive (u-axis).

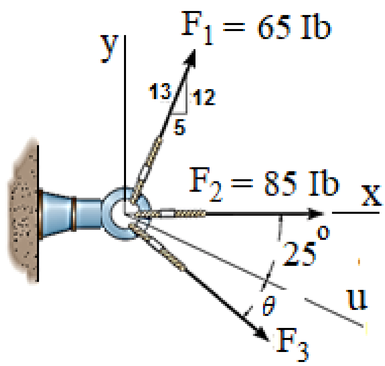


Fig. (Ex. 2-17)

Solution:

$+\uparrow \sum F_{Rx} = \sum F_x \qquad 100\cos 25° = 85 + 65\left(\frac{5}{13}\right) + F_3 \cos(25° + \theta)$

$F_3 \cos(25° + \theta) = -19.37 \quad \ldots\ldots\ldots\ldots (1)$

$+\rightarrow \sum F_{Ry} = \sum F_y \qquad -100\sin 25° = 65\left(\frac{12}{13}\right) - F_3 \sin(25° + \theta)$

$F_3 \sin(25° + \theta) = 102.26 \quad \ldots\ldots\ldots\ldots (2)$

Solving Eqs. (1) and (2):

Method (1):

From Eq. (1) $\qquad F_3 = \dfrac{-19.37}{\cos(25° + \theta)}$

Sub. in Eq. (2) $\qquad \dfrac{-19.37}{\cos(25° + \theta)} \sin(25° + \theta) = 102.26$

$-19.37 \tan(25° + \theta) = 102.26$

$(25° + \theta) = \tan^{-1}\dfrac{102.26}{-19.37} = -79.27° = 79.27°$

$\theta = 79.27 - 25 = 54.27°$

$F_3 = \dfrac{-19.37}{\cos(25° + 54.27°)} = 104 \text{ Ib}$

Method (2):

$F_3 \sin(25° + \theta) = 102.26 \quad \ldots\ldots\ldots\ldots (2)$

$F_3 \cos(25° + \theta) = -19.37 \quad \ldots\ldots\ldots\ldots (1)$

-- Division

$\tan(25° + \theta) = -5.28$

$25° + \theta = \tan^{-1} -5.28 = -79.27° = 79.27°$

$\theta = 79.27 - 25 = 54.27°$

Sub. in Eq. (1) $\qquad F_3 \cos(-79.27°) = -19.37$

$F_3 = \dfrac{-19.37}{\cos(-79.27°)} = 104 \text{ Ib}$

Example (2-18):

The forces (F_1), (F_2), and (F_3) are acted on the bracket shown in Fig. (Ex. 2-18) so that their resultant is (120 Ib) in the positive direction of the (u) axis. Find the value of the unknown force (F_1) and its direction (ϕ).

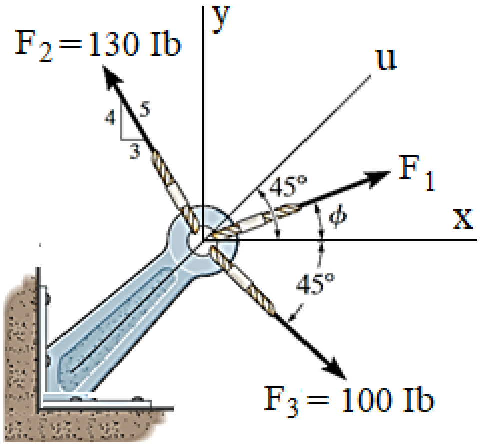


Fig. (Ex. 2-18)

Solution:

$(F_1)_x = F_1 \cos\phi$ $\qquad$ $(F_1)_y = F_1 \sin\phi$

$(F_2)_x = -130\left(\frac{3}{5}\right) = -78$ Ib $\qquad$ $(F_2)_y = 130\left(\frac{4}{5}\right) = 104$ Ib

$(F_3)_x = 100 \cos 45° = 70.7$ Ib $\qquad$ $(F_3)_y = -100 \sin 45° = -70.7$ Ib

$(F_R)_x = 120 \cos 45° = 84.84$ Ib $\qquad$ $(F_R)_y = 120 \sin 45° = 84.84$ Ib

$\overset{+}{\longrightarrow} F_{Rx} = \sum F_x$ $\qquad$ $84.84 = F_1 \cos\phi - 78 + 70.7$

$F_1 \cos\phi = 92.14$ ……………..……. (1)

$+\uparrow F_{Ry} = \sum F_y$ $\qquad$ $84.84 = F_1 \sin\phi + 104 - 70.7$

$F_1 \sin\phi = 51.54$ ……………..……. (2)

From Eq. (1): $\qquad F_1 = \dfrac{92.14}{\cos\phi}$

Sub. in Eq. (2): $\qquad \dfrac{92.14}{\cos\phi}\sin\phi = 51.54$

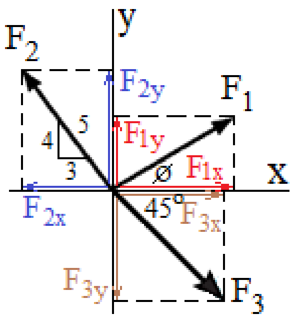
$92.14 \tan\phi = 51.54$

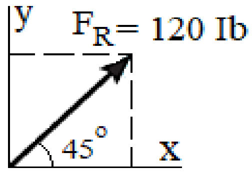
$\tan\phi = \dfrac{51.54}{92.14} = 0.56$

$\phi = \tan^{-1} 0.56 = 29.22°$

$F_1 = \dfrac{92.14}{\cos\phi}$

$= \dfrac{92.14}{\cos 29.23°} = 105.57$ Ib





Example (2-19):

If the resultant of the two forces shown in Fig. (Ex. 2-19) is directed along the positive (y-axis) and has a magnitude of (300 Ib), determine the magnitude of (F_B) and its direction (θ).

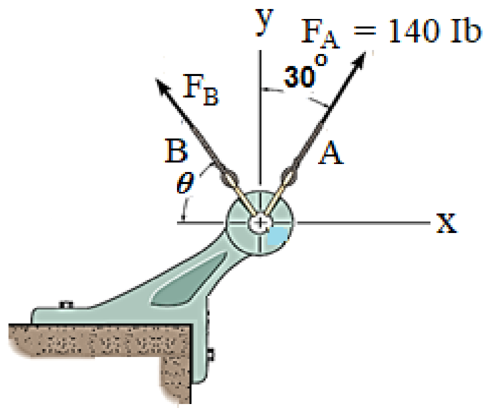


Fig. (Ex. 2-19)

Solution:

Metod (1):

$+\rightarrow F_{Rx} = \sum F_x$ $\quad 0 = 140 \sin 30° - F_B \cos\theta$

$F_B \cos\theta = 70$ ……………..……. (1)

$+\uparrow F_{Ry} = \sum F_y$ $\quad 300 = 140 \cos 30° + F_B \sin\theta$

$F_B \sin\theta = 178.76$ ..……………... (2)

$F_B \sin\theta = 178.76$ ..……………... (2)

$F_B \cos\theta = 70$ ..………………..…….. (1)

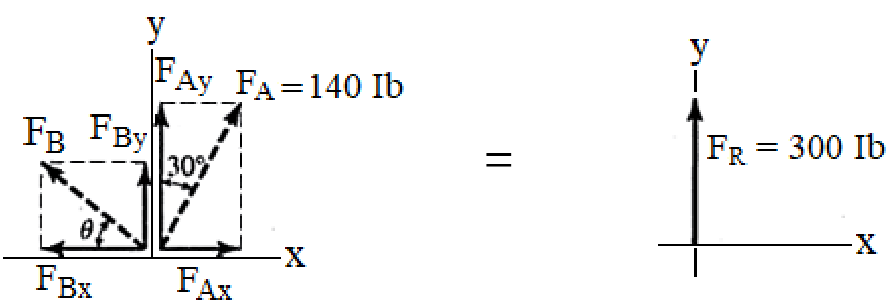
---------------------------------- Division

$\tan\theta = 2.55$

$\theta = 68.6°$

Sub. in Eq. (2): $\quad F_B \sin(68.6°) = 178.76$

$F_B = 192$ Ib



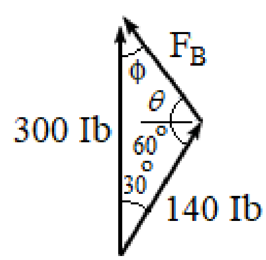
Metod (2):

$F_B = \sqrt{(300)^2 + (140)^2 - 2(300)(140)\cos 30} = 192$ Ib

$\frac{140}{\sin\phi} = \frac{192}{\sin 30°} \quad \Rightarrow \quad \phi = 21.4°$

$\theta + 60 = 180 - 30 - 21.4 = 128.6°$

$\theta = 128.6 - 60 = 68.6°$



Example (2-20):

If the forces ($F_1 = 7$ kN), (F_2) and ($F_3 = 4$ kN) acting on the bracket as shown in Fig. (Ex. 2-20). Determine the magnitude of force (F_2) so that the resultant force of the three forces is as small as possible. Then find the magnitude of the resultant force.

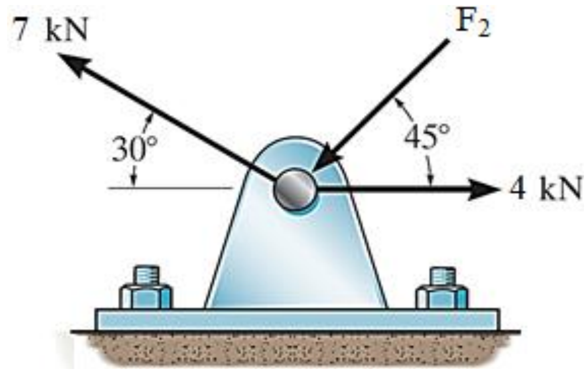


Fig. (Ex. 2-20)

Solution:

$+\rightarrow F_{Rx} = \sum F_x$

$F_{Rx} = 4 - F_2 \cos 45° - 7 \cos 30°$

$F_{Rx} = -2.06 - 0.707\, F_2$

$+\uparrow \ F_{Ry} = \sum F_y$

$F_{Ry} = -F_2 \sin 45° + 7 \sin 30°$

$F_{Ry} = 3.5 - 0.707\, F_2$

$F_R^2 = (-2.06 - 0.707\, F_2)^2 + (3.5 - 0.707\, F_2)^2$ ……………. (1)

$$2F_R \frac{dF_R}{dF} = 2(-2.06 - 0.707\, F_2)(-0.707) + 2(3.5 - 0.707\, F_2)(-0.707) = 0$$

$2(1.456 + 0.5\, F_2 - 2.475 + 0.5\, F_2) = 0$

$1.456 + 0.5\, F_2 - 2.475 + 0.5\, F_2 = 0$

$F_2 = 1.02$ kN

Sub. in Eq. (1):

$F_R^2 = [-2.06 - 0.707(1.02)]^2 + [3.5 - 0.707(1.02)]^2$

$F_R^2 = (-2.78)^2 + (2.78)^2$

$F_R = 3.93$ kN

Problems:

2-1) Find the magnitude of the resultant force acting on the plate shown in the Fig. (Pr. 2-1) and its direction, measured clockwise from the positive horizontal axis (x).

Ans.: R = 1353 Ib , ϕ = 132.8°

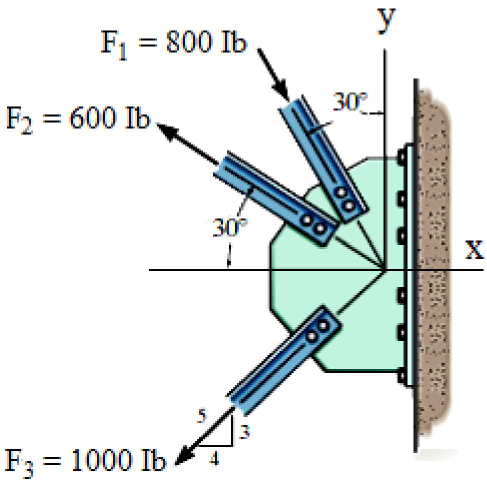


Fig. (Pr. 2-1)

2-2) Determine the magnitude of the resultant force acting on the bracket and its direction measured counterclockwise from the positive (x – axis).

Ans.: R = 2313.6 N , ϕ = 140.4°

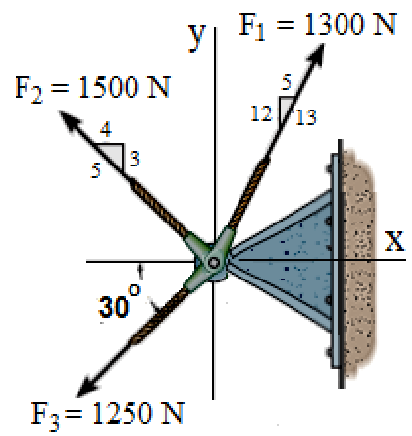
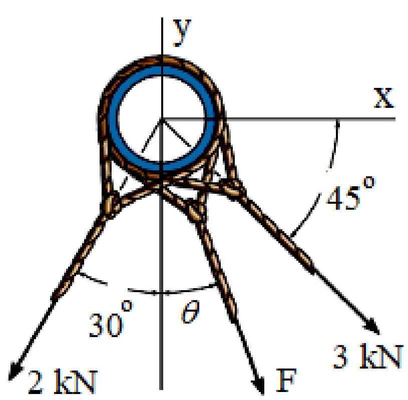


Fig. (Pr. 2-2)

2-3) A pipe pulled by three ropes with tensile forces shown in Fig. (Pr. 2-3) so that a resultant force of (5 kN) is generated. If the tensile forces in two of them are known, find the angle of the third rope (θ) so that the magnitude of the tensile force (F) in it is at its minimum. What is the magnitude of the force?

Ans.: F = 1 kN , θ = 16.12°



2-4) Determine the magnitude of the resultant force acting on the eyebolt shown in Fig. (Pr. 2-4) and its direction measured counter-clockwise from the positive (x – axis).

Ans.: F_R = 1733.82 Ib , ϕ = 93°

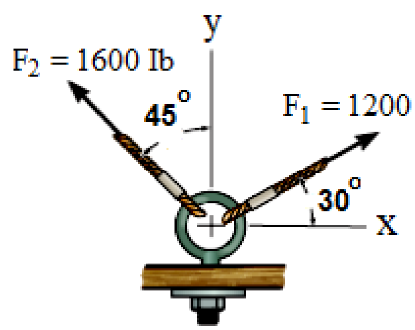


Fig. (Pr. 2-4)

Fig. (Pr. 2-3)

2-5) Determine the magnitude of the resultant force acting on the bracket shown in Fig. (Pr. 2-5) and its direction measured counter-clockwise from the negative (x – axis).

Ans.: F_R = 1083.5 N , θ = 3°

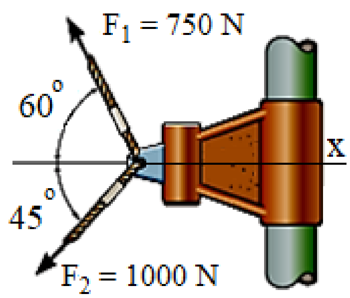


Fig. (Pr. 2-5)

2-6) Find the magnitude of the force (F) and its direction (θ) to achieve a resultant force acting the bracket shown in Fig. (Pr. 2-6), with a value of (100 N) directed along the positive (y - axis).

Ans.: F = 192 Ib , θ = 45.2°

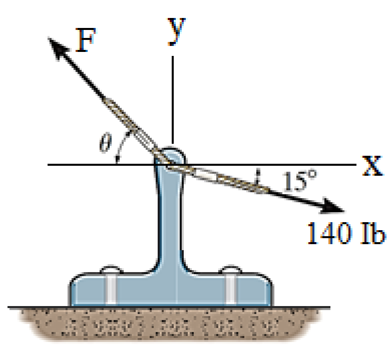


Fig. (Pr. 2-6)

2-7) A force of (4 kN) acts on the frame shown in Fig. (Pr. 2-7), so if its component acting along the member (BC) has a value of (3 kN), directed from (B) towards (C), what is the magnitude of the required angle (θ) and its component that affected along the member (AB).

Ans.: F_{AB} = 2.05 kN , θ = 32°

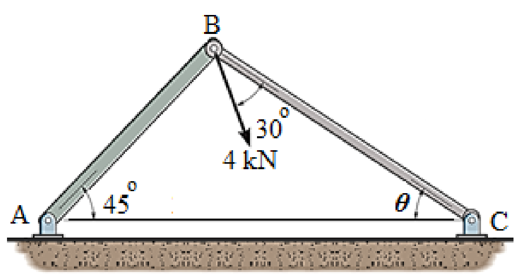


Fig. (Pr. 2-7)

2-8) Determine the angle (θ) between the two forces that act on the screw eye, so that the resultant force has a magnitude of (160 Ib).

Ans.: θ = 75.5°

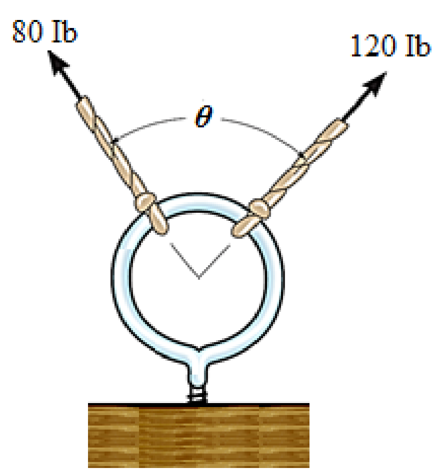


Fig. (Pr. 2-8)

2-9) Determine the angle (θ) required in the design of the struts shown in Figure (Pr. 2-9) so that the (2 kN) horizontal force has a component of (2.5 kN) directed from (A) towards (C). What is the component of the force acting along the member (AB)?

Ans.: $\theta = 62.1°$, $F_{AB} = 2.7$ kN

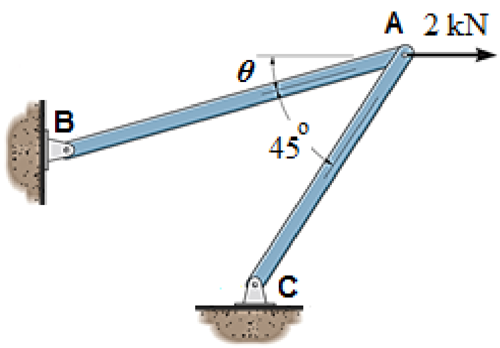


Fig. (Pr. 2-9)

2-10) Determine the angle (θ) required in the design of the struts shown in Figure (Pr. 2-10) so that the horizontal force (5 kN) has a component of (3 kN) directed from (B) towards (A).

Ans.: $\theta = 53.1°$

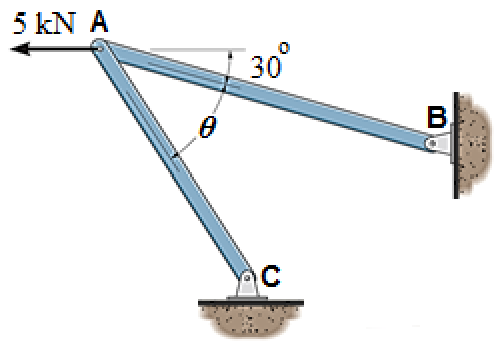


Fig. (Pr. 2-10)

2-11) Determine the magnitude of the force (F_1) and its direction (θ), if the resultant force of the three forces is (1000 N) directed (45°) counterclockwise from the positive (x – axis).

Ans.: $F_1 = 753.66$ N, $\theta = 45°$

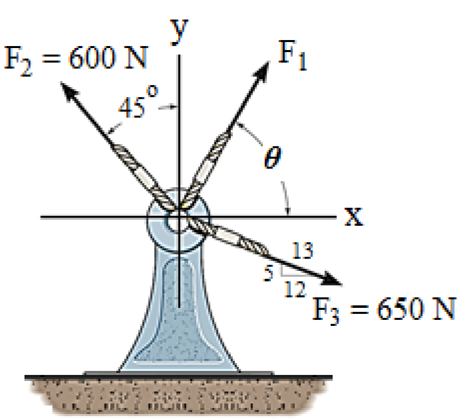


Fig. (Pr. 2-11)

2-12) Find the magnitude of the force (F_3) and its direction (θ) to achieve a resultant force affecting the bracket shown in Fig. (Pr. 2-12) with value of (200 Ib) directed along the positive (x' - axis).

Ans.: $F_3 = 178$ Ib, $\theta = 37°$

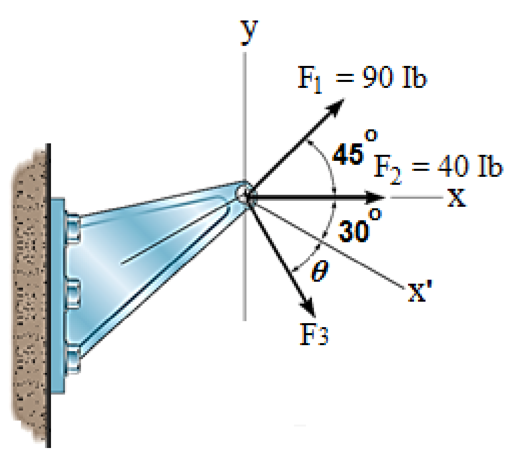


Fig. (Pr. 2-12)

2-13) Determine the magnitude of the resultant force acting on the plate shown in Fig. (Pr. 2-13) and its direction measured counter-clockwise from the posative (x- axis).

Ans.: $F_R = 433\ N$, $\theta = 183.68°$

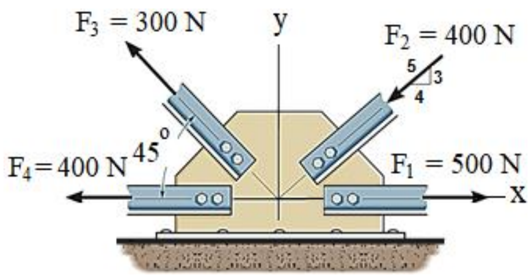


Fig. (Pr. 2-13)

2-14) The forces (F_1), (F_2), and (F_3) are applied to the brakes shown in Fig. (Pr. 2-14). What is the magnitude and direction of the resultant force, measured counter-clockwise from the positive (x- axis) of the bracket?

Ans.: $F_R = 557.48\ N$, $\theta = 67.55°$

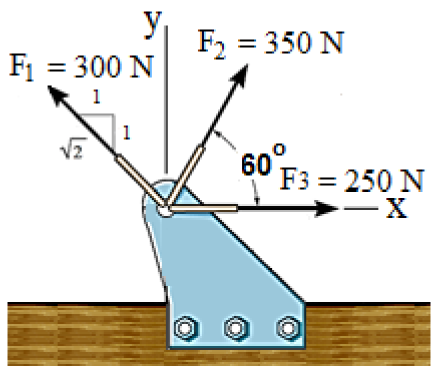


Fig. (Pr. 2-14)

2-15) Determine the magnitude of the resultant force acting on the bracket shown in Fig. (Pr. 2-15) and its direction measured clockwise from the positive (y – axis).

Ans.: $F_R = 235.79\ Ib$, $\phi = 7.06°$

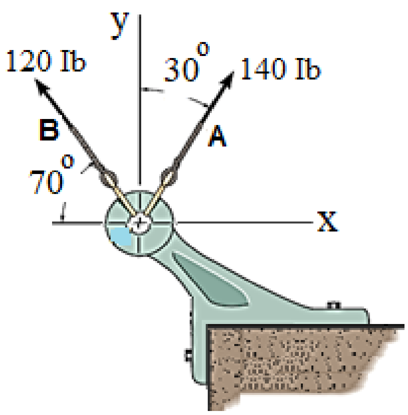


Fig. (Pr. 2-15)

2-16) If the lift force on wing cross-section (airfoil) is (600 N) and the ratio of the lift force (L) to the drag force (D) for the airfoil is (L/D = 12), compute the magnitude of the resultant force (R) and the angle (θ) which it makes with the horizontal.

Ans.: $R = 602\ N$, $\theta = 85.24°$

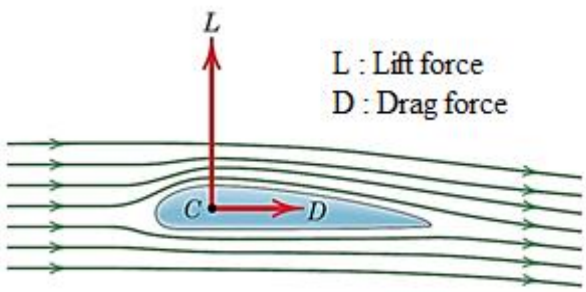


Fig. (Pr. 2-16)

2-17) Two forces applied to a bracket as shown in Fig (Pr. 2-17). Determine the resultant (F_R) of the two forces, and then find (F_R) in terms of unit vectors along the (x' – axis) and (y' – axis).

Ans.: $F_R = 520.5\ N$
$F_R = -334.57\ N\ i + 398.73\ N\ j$

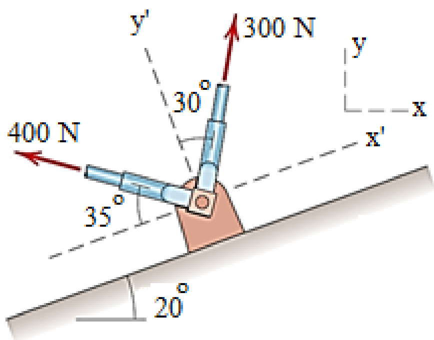


Fig. (Pr. 2-17)

2-18) In the structure shown in Fig. (Pr. 2-18), find the value of the unknown force (F) that makes the resultant (F_R) of the three forces as small as possible. Then find the value of the resultant (F_R).

Ans.: $F = 2.825\ kN$, $F_R = 2.18\ kN$

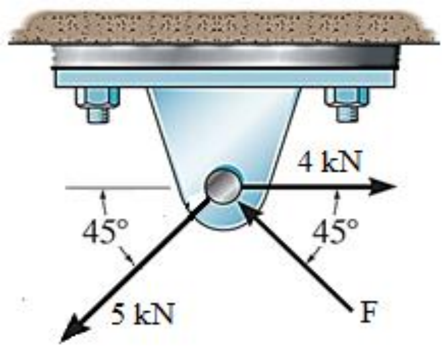


Fig. (Pr. 2-18)

2-19) The tension in the cable (AB) that prevents the bar (OA) from rotating counterclockwise about the pivot (*O*) is (150 Ib). Determine the (n) and (t) components of this force acting on point (A) of the bar.

Ans.: $F_n = 66.46\ Ib$, $F_t = 134.47\ Ib$

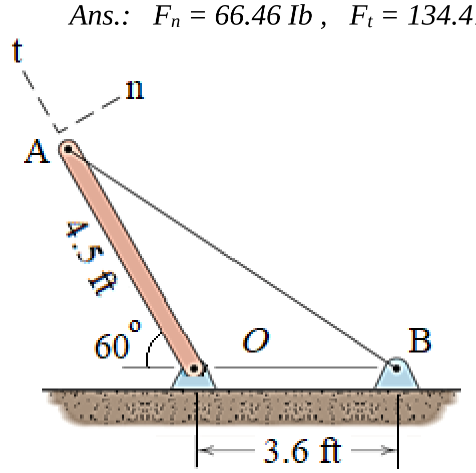


Fig. (Pr. 2-19)

2-20) Find the magnitude of (T) and its direction (θ) for which the eye bolt under a resultant of (7.5 kN) horizontally to the left.

Ans.: $T = 6.4\ k\ N$, $\theta = 38.66°$

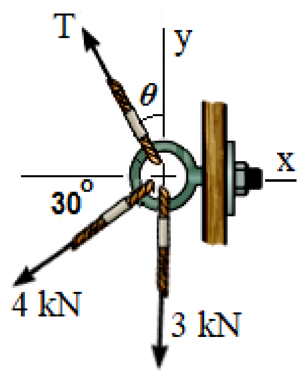


Fig. (Pr. 2-20)

CHAPTER THREE
THE MOMENTS

Definition of the moment:

When a force is applied to a specific body, the body moves in the direction of the line of action of that force. If the line of action of the force passes through the center of gravity of the body, it will move in a linear motion only by the effect of the force, but if the line of action of the force does not pass through its center of gravity, then it will move in angular motion around its center of gravity under the effect of the force in addition to the linear motion. This effect that generates angular motion is called the *moment* **(M)** of the force. Moment is also referred to as *torque*.

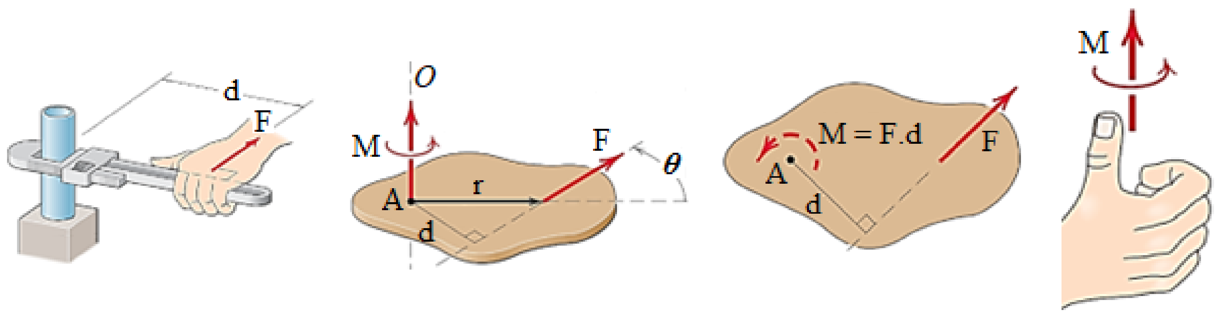


Fig. (3-1) Illustrative diagrams of moments

The diagrams in Fig. (3-1) show the effect of force (F) on a two-dimensional body in its dimensional plane. This force generates a moment that the body rotates around the axis (*O* – *O*) perpendicular to the plane of its dimensions, the magnitude of the moment is equal to the product of the magnitude of the force (F) with the perpendicular distance between this axis and the line of action of the force called the force arm (d). Therefore, the magnitude of the moment is:

$$M = F.d \quad \ldots\ldots\ldots\ldots\ldots.. \quad (3\text{-}1)$$

Methods of solution:

- **Varignon's theorem:**

  This theory states that (*the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point*).

- **Normal distance:**

  In this principle, (*the moment of any force around a given axis can be found by multiplication of the force by the normal distance between the line of action of the force and the given axis*).

- **Principle of transmissibility:**

  In this principle, (*the moment can be found by transferring the force to the horizontal axis or the vertical axis of the point where the moment required about it*).

Example (3-1):

Calculate the magnitude of the moment about the base point (*O*) of the (80 Ib) force in four different methods.

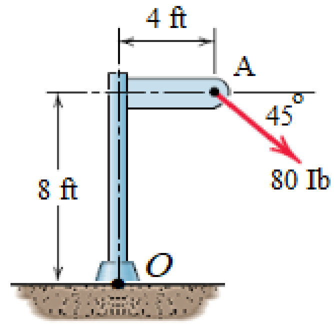


Fig. (Ex. 3-1)

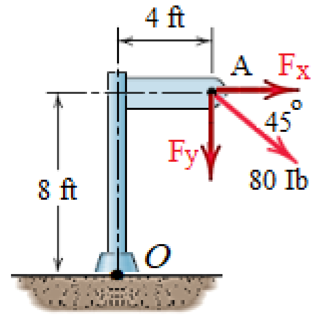
Solution:

(I) Replace the force by its rectangular (cartesian) components:

$$F_x = 80 \cos 45^o = 56.57 \text{ Ib}$$
$$F_y = 80 \sin 45^o = 56.57 \text{ Ib}$$

By using Varignon's theorem:

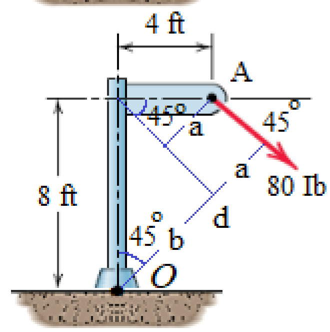
$$M_o = (-56.57 \times 8) - (56.57 \times 4) = -678.8 \text{ Ib.ft}$$
$$= 678.84 \text{ Ib.ft} \qquad \text{(C.W)}$$



(II) By using the normal distance method, where the moment arm of the (80 Ib) force is:

$$d = a + b = 4 \sin 45^o + 8 \cos 45^o = 8.485 \text{ ft}$$

$$M = Fd$$
$$M_o = -80 \times 8.484 = -678.8 \text{ Ib.ft} = 678.8 \text{ Ib.ft}$$
(C.W)

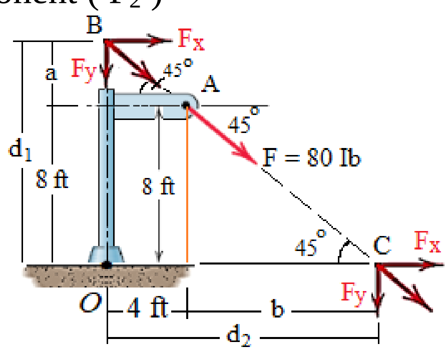


(III) By using the principle of transmissibility, The force (80 Ib) can be transferred along its line of action to point (B), the vertical component (F_2) is neglected because its line of action passes through point (*O*). The moment arm of (F_1) will be:

$$d_1 = 8 + 4 \tan 45^o = 12 \text{ ft}$$
$$F_1 = 80 \cos 45^o = 56.57 \text{ Ib}$$

The moment is:

$$M_o = -56.57 \times 12 = -678.8 \text{ Ib.ft}$$
$$= 678.8 \text{ Ib.ft} \qquad \text{(C.W)}$$



(IV) The force (80 Ib) can be transferred along its line of action to point (C), neglecting the horizontal component (F_1) because its line of action passes through point (*O*). The moment arm of (F_2) will be:

$$d_2 = 4 + 8 \cot 45^o = 12 \text{ ft} \quad , \quad F_2 = 80 \sin 45^o = 56.57 \text{ Ib}$$

The moment is:

$$M_o = -56.57 \times 12 = -678.8 \text{ Ib.ft} = 678.8 \text{ Ib.ft} \qquad \text{(C.W)}$$

Resultant moment:

In two-dimensional problems, all forces lie in one plane, let it be the (x – y) plane, as shown in Fig. (3-2), the resultant moments (M_R)$_o$ about the point (*O*) (z - axis) can be determined by finding the algebraic summation of the moments produced by each force in the system. The magnitude of the moment is positive if the direction of the moment is counterclockwise as it is directed about the positive (z) axis (outside the page) and vice versa, the magnitude of the moment is negative if the direction of the moment is clockwise.

$$\circlearrowleft + (M_R)_o = \sum F\,d\,; \qquad (M_R)_O = F_1\,d_1 - F_2\,d_2 + F_3\,d_3 \;\ldots\ldots \quad (3\text{-}2)$$

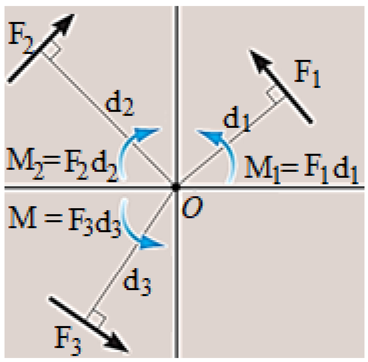


Fig. (3-2) Resultant moment

Example (3-2):

Determine the resultant moment of the four forces acting on the rod shown in Fig. (Ex. 3-2) about point (*O*).

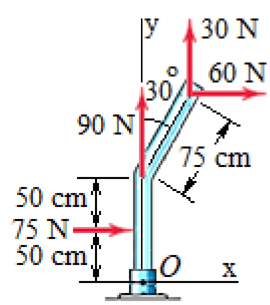


Fig. (Ex .3-2)

Solution:

$$(M_R)_o = -75 \times 0.5 + 90 \times 0$$
$$+ 30 \times 0.75 \sin 30^{\circ}$$
$$- 60 \times (1 + 0.75 \cos 30^{\circ})$$
$$= -125.22 \text{ N.m} = 125.22 \text{ N.m} \circlearrowright$$

Example (3-3):

Three forces acting on a cantilever beam as shown in Fig. (Ex. 3-3). Determine the moment about point (A) of each of the three forces and the resultant moment of these forces about the same point.

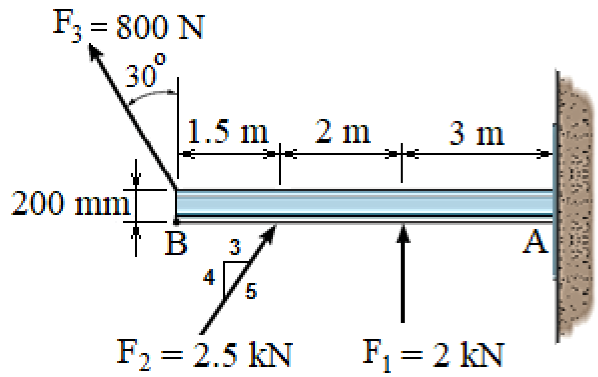


Fig. (Ex. 3-3)

Solution:

↻+ $(M_{F1})_A = -(2000)(3)$
$= -6000$ N.m $= 6$ kN.m (Clockwise)

↻+ $(M_{F2})_A = -(2500)(4/5)(5)$
$= -10000$ N.m $= 10$ kN.m (Clockwise)

↻+ $(M_{F3})_A = -(800\cos 30^\circ)(6.5) + (800\sin 30^\circ)(0.2)$
$= -4423$ N.m $= 4.4$ kN.m (Clockwise)

↻+ $(M_R)_A = 6 + 10 + 4.4 = 20.4$ kN.m (Clockwise)

Example (3-4):

Determine the moment of the forces (F_A) and (F_B) about the bolt located at point (C).

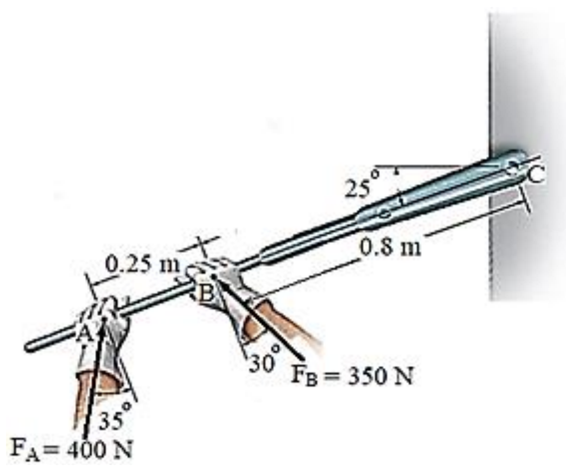


Fig. (Ex. 3-4)

Solution:

$+ M_C = -(400\cos 35°)(1.05) - (350\cos 30°)(0.8) = -586.53$ N.m
$= 586.53$ N.m ↻

Example (3-5):

The boom of the crane shown in Fig. (Ex. 3-5) can be specified at an angle (θ) confined between (0°) and (90°) within a specified elongation (x) between (0 ft) and (12 ft). If the suspended mass is (120 kg), find the moment resulting from this mass at point (B) in terms of (x) and (θ), then find the values of (x) and (θ) to achieve the maximum possible moment at point (B)? What is the value of this moment? Neglect the pulley size at point (A).

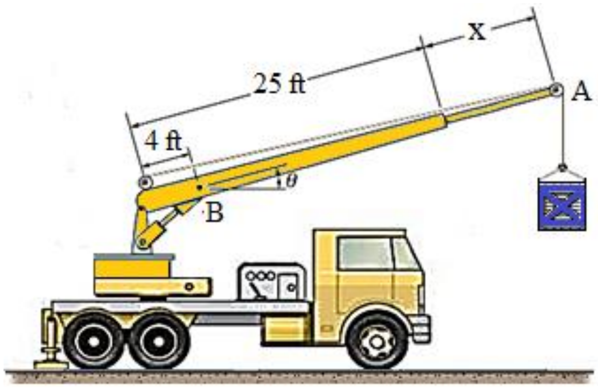


Fig. (Ex. 3-5)

Solution:

$$W = m\,g = 120 \times 9.81 = 1177.2 \text{ N} = \frac{1177.2}{4.448} = 264.66 \text{ Ib}$$

$$\circlearrowleft + M_A = -(264.66)(21 + x)\cos\theta$$
$$= (264.66\cos\theta)(21 + x) \text{ Ib.ft} \qquad (\text{Clockwise})$$

The maximum moment at (A) occurs when (θ = 0°) and (x = 12 ft).

$$\circlearrowleft + (M_A)_{max} = \{(264.66\cos 0^\circ)(21 + 12)\} \text{ Ib.ft} = 8733.78 \text{ Ib.ft} \qquad (\text{Clockwise})$$

Example (3-6):

The gate shown in Fig. (Ex. 3-6) consists of an arm with a mass of (75 kg) and a center of mass at (G_a) and a balance weight of (200 kg) with a center of mass at (G_W). Determine the magnitude and direction of the moment produced by the weights of the gate parts about point (A).

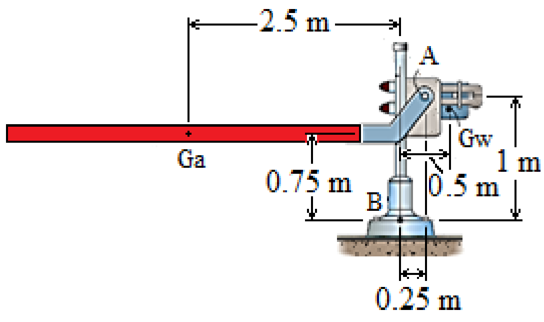


Fig. (Ex. 3-6)

Solution:

$$\circlearrowleft +(M_R)_A = \sum F\,d$$
$$(M_R)_A = (75)(9.81)(2.5 + 0.25) - (200)(9.81)(0.5 - 0.25)$$
$$= 1532.8 \text{ N.m} = 1.53 \text{ kN.m} \qquad (\text{Counterclockwise})$$

Example (3-7):

A hinged gate at point (C). pushed on both sides by two boys with two forces of different value and direction (F_A = 150 N), and (F_B = 250 N), as shown in Fig. (Ex. 3-7). Determine the moment of each force about point (C). Then indicate in which way will the gate rotate? Neglect the thickness of the gate.

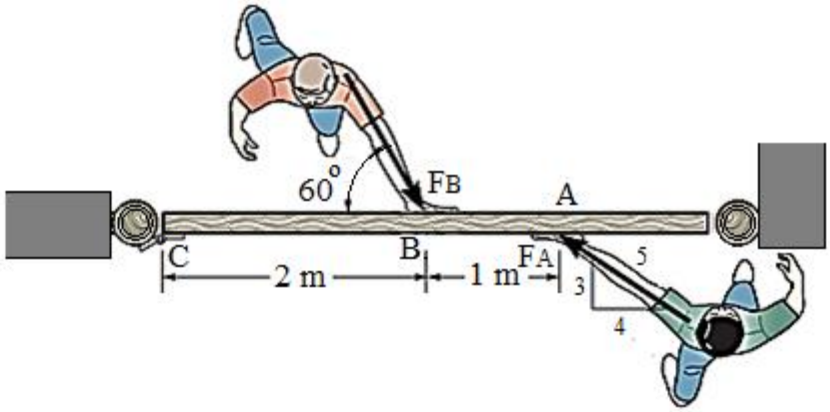


Fig. (Ex. 3-7)

Solution:

$$\circlearrowleft + (M_{FA})_C = 150 \times \frac{3}{5} \times 3 = 270 \text{ N.m} \qquad \text{(Counterclockwise)}$$

$$\circlearrowleft + (M_{FB})_C = -\,250 \sin 60^o \times 2 = -\,433 = 433 \text{ N.m} \qquad \text{(Clockwise)}$$

Since { $(M_{FB})_C > (M_{FA})_C$ }, the gate will rotate clockwise.

Example (3-8) :

A force of (100 N) is subjected to a handle of the hammer. Determine the moment of this force about the point (A).

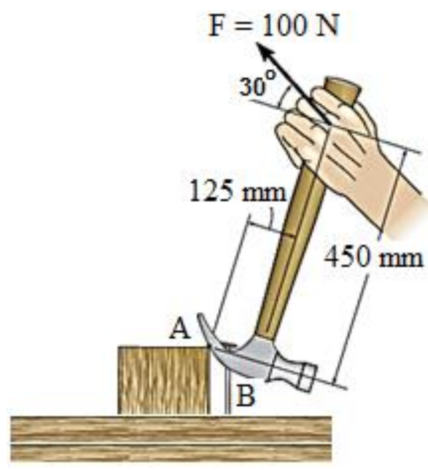
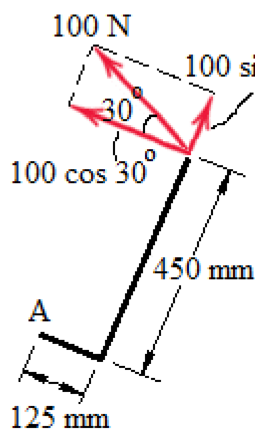


Fig. (Ex. 3-8)

Solution:



$$\circlearrowleft + \sum M_A = (100 \cos 30^o)(0.45) + (100 \sin 30^o)(0.125)$$
$$= 45.22 \text{ N.m} \qquad \text{(Counterclockwise)}$$

Example (3-9):

The figure (Ex. 3-9) shows the members of the lower arm. The weight of the forearm is (25 N) and its center of gravity is at the point (G). When the palm is carrying a mass of (4 kg), calculate the biceps tension (T) so that the resultant moments about the point (*O*) equals to zero, (equilibrium state).

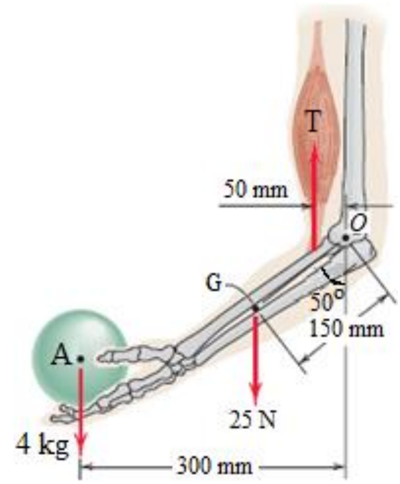


Fig. (Ex. 3-9)

Solution:

$\circlearrowleft + (\sum M_R)_o = \sum F d$

$0 = (4) (9.81) (300)$

$+ (25) (150 \sin 50)$

$- (T) (50)$

$50 T = 14644.67$

$T = 292.9 \text{ N}$

Example (3-10):

When a man tried to stand on his toes, the Achilles tendon moved with a force of (F_t = 145 Ib), and the reaction force of the ground on each of his feet was (N_f = 90 Ib). Find the resultant moments of the two forces (F_t) and (N_f) about the ankle joint (A).

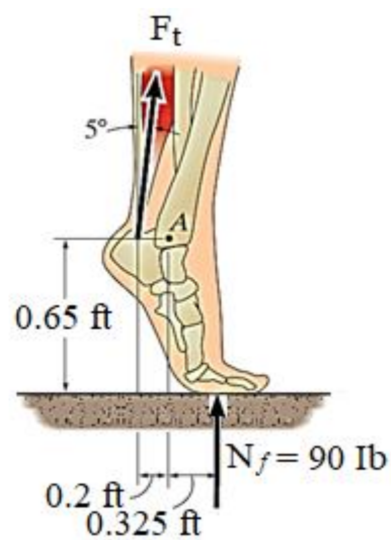


Fig. (Ex. 3-10)

Solution:

$\circlearrowleft + (\sum M_R)_A = \sum F d$

$(M_R)_A = (90) (0.325)$

$- (145 \cos 5^{\circ}) (0.2)$

$= 0.36 \text{ Ib.ft}$ (Counterclockwise)

Example (3-11):

A specialized crane in the field of electrical maintenance, the mass of its arm (AB) is (750 kg), the mass of the cage (BCD) is (100 kg), the mass of the electrician is (80 kg), and the centers of gravity are located at points (G_1), (G_2) and (G_3) respectively. Determine the moment produced by each part about the point (A), then find the resultant moments about the same point.

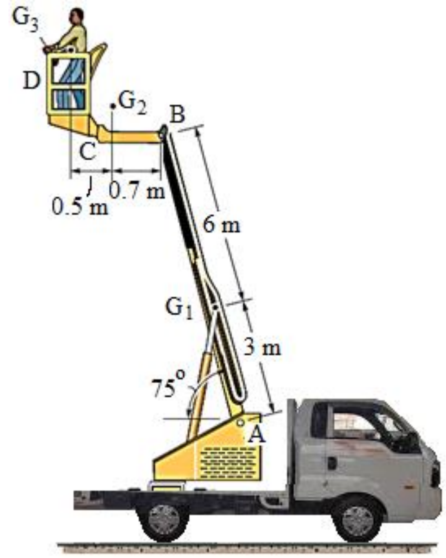


Fig. (Ex. 3-11)

Solution:

$\curvearrowleft + (M_{ar})_A = (750 \times 9.81)(3 \cos 75°) = 5712.8$ N.m

$= 5.7$ kN.m (Counterclockwise)

$\curvearrowleft + (M_c)_A = (100 \times 9.81)(9 \cos 75° + 0.7) = 2971.8$ N.m

$= 3$ kN.m (Counterclockwise)

$\curvearrowleft + (M_m)_A = (80 \times 9.81)(9 \cos 75° + 1.2) = 2769.8$ N.m

$= 2.8$ kN.m (Counterclockwise)

$\curvearrowleft + (M_R)_A = 5.7 + 3 + 2.8 = 11.5$ kN.m (Counterclockwise)

Example (3-12):

The tower crane shown in Fig. (Ex. 3-12) is used to hoist the (2000 kg) load upward at constant velocity. Its main arm (BD) has mass (1500 kg), and center of gravity at point (G_1), and the counterweight arm (BC) of mass (500 kg) and center of gravity at point (G_2), The (7000 kg) counterweight at point (C) have centers of mass at (G_3). Determine the resultant moment produced by the load and the weights of the tower crane arms and the counterweight about point (A) and about point (B).

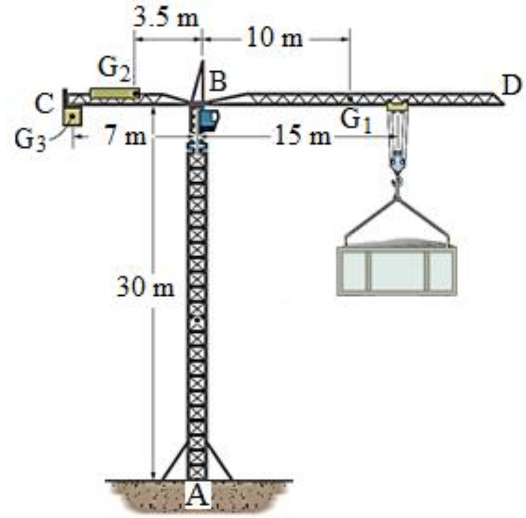


Fig. (Ex. 3-12)

Solution:

Since the moment arms of the weights and the load measured to points (A) and (B) are the same, the resultant moments produced by the load and the weights about points (A) and (B) are the same.

$$\circlearrowleft + (M_R)_A = (M_R)_B = \sum Fd$$

$$(M_R)_A = (M_R)_B = (7000)(9.81)(7) + (500)(9.81)(3.5) - (1500)(9.81)(10) - (2000)(9.81)(15)$$

$$= 480690 + 17167.5 - 147150 - 294300$$

$$= 56407.5 \text{ N.m} = 56.4 \text{ kN.m} \qquad (\text{Counterclockwise})$$

Example (3-13):

A crane with an arm length of (15 m), The towline exerts a force of (P = 3 kN) at the end of the arm. Determine the arm angle (*θ*) of the arm so that this force creates a maximum moment about point (*O*), then find the magnitude of this moment.

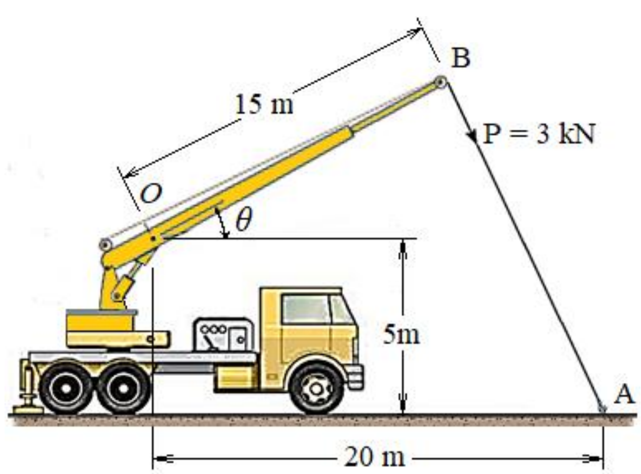
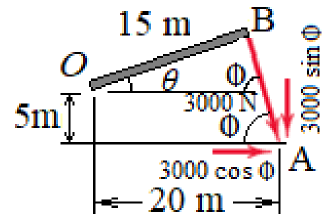


Fig. (Ex. 3-13)

Solution:



At maximum moment, OB ⊥ BA

↻ + $(M_o)_{max}$ = – (3000)(15) = – 45000 N.m

– (3000 sin ϕ)(20) + (3000 cos ϕ)(5) = – 45000

– 60000 sin ϕ + 15000 cos ϕ = – 45000 ÷ – 15000

4 sin ϕ – cos ϕ = 3

$\sin^2\phi + \cos^2\phi = 1 \Rightarrow \sin\phi = \sqrt{1-\cos^2\phi}$

$4\sqrt{1-\cos^2\phi} - \cos\phi = 3$ Let x = cos ϕ

$4\sqrt{1-x^2} - x = 3$

$4\sqrt{1-x^2} = x + 3$

$16(1-x^2) = x^2 + 6x + 9$

$16 - 16x^2 - x^2 - 6x - 9 = 0$

$-17x^2 - 6x + 7 = 0$ ÷ – 17

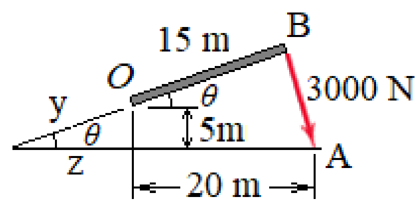
$x^2 + 0.353x - 0.412 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(0.353) \pm \sqrt{(0.353)^2 - 4(1)(-0.412)}}{2(1)} = 0.489 \quad \text{or} \quad -0.842$$

$\phi = \cos^{-1} 0.489 = 60.7^o$

$\theta = 90^o - 60.7^o = 29.3^o$

Method (2):



$(5)^2 + z^2 = y^2$

$25 + z^2 = y^2$

Similar triangles:

$$\frac{15+y}{z} = \frac{20+z}{y} \Rightarrow 15y + y^2 = 20z + z^2$$

$15(\sqrt{25+z^2}) + 25 + z^2 = 20z + z^2$

$15(\sqrt{25+z^2}) = 20z + z^2 - 25 - z^2$

$15(\sqrt{25+z^2}) = 20z - 25 \Rightarrow 225(25 + z^2) = 400z^2 - 1000z + 625$

$5625 + 225z^2 - 400z^2 + 1000z - 625 = 0$

$-175z^2 + 1000z + 5000 = 0$

$z^2 - 5.7z - 28.6 = 0 \Rightarrow (z - 8.91)(z + 3.21) = 0$

z = 8.91 m

$y = \sqrt{25+z^2} = \sqrt{25+(8.91)^2} = 10.22$ m

$\theta = \cos^{-1}\left(\frac{8.91}{10.22}\right) = 29.3^o$

The moment of couple:

A moment of couple is formed by two parallel forces that have the same magnitude, and opposite direction, and not on the same line of action, i.e. separated by a distance (d) perpendicular to their lines of action, as shown in the figures (3-3) and (3-4). Since the two forces in the couple are equal and in opposite directions, their resultant will be equal to zero, so the only effect of it is to produce a rotation or tendency of rotation in a specified direction. For example, imagine that you are driving a car with both hands on the steering wheel and you are making a turn, one hand will push the steering wheel up while the other hand pulls it down, and that produce a couple moment on the center of the steering wheel that turns it in the direction of the desired rotation of the car.

$$M = \left(\frac{d}{2} \times F \right) + \left(\frac{d}{2} \times F \right)$$

$$M = d \times F \quad \ldots\ldots\ldots\ldots \quad (3\text{-}3)$$

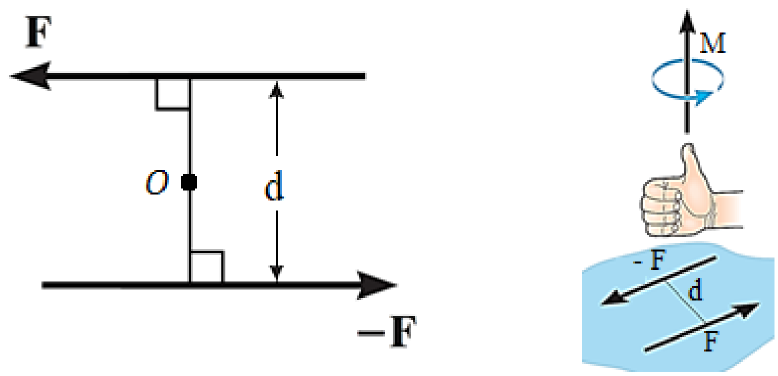


Fig. (3-3) The moment of couple

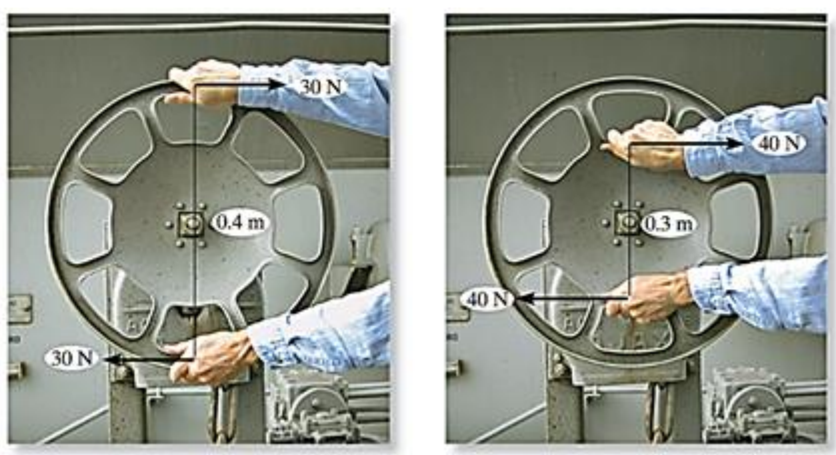


Fig. (3-4) The moment of couple

Example (3-14):

Determine the resultant couple moment of the three couples acting on the plate shown in Fig. (Ex. 3-14).

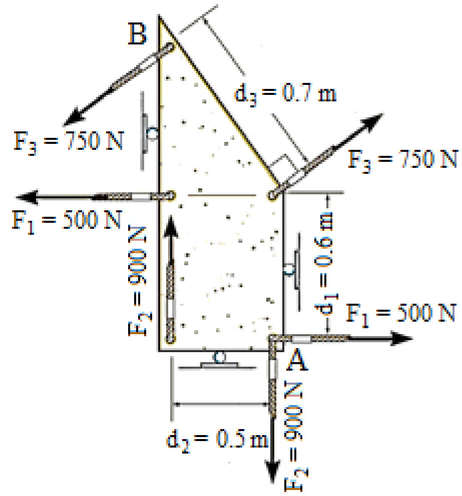


Fig. (Ex. 3-14)

Solution:

$d_1 = 0.6$ m , $d_2 = 0.5$ m , and $d_3 = 0.7$ m.

↶ $+ M_R = \sum M$

$$M_R = F_1 d_1 - F_2 d_2 + F_3 d_3$$
$$= (500)(0.6) - (900)(0.5) + (750)(0.7)$$
$$= 375 \text{ N.m} \qquad \text{(Counterclockwise)}$$

Example (3-15):

A couple moment of (7 N.m) is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces (F) exerted on the handle end and (P) exerted on the blade end.

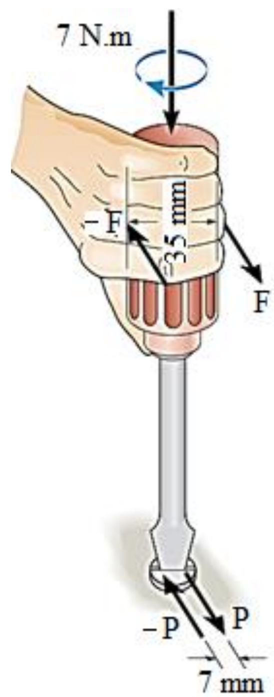


Fig. (Ex. 3-15)

Solution:

For the handle:

$M_C = F.d$ $\qquad$ $(F)(0.035) = 7$

$F = 200$ N

For the blade:

$M_C = P.d$ $\qquad$ $(P)(0.007) = 7$

$P = 1000$ N

Example (3-16):

A devices carrier wheel is subjected to the two couples. Determine the forces (F) that the bearings exert on the shaft so that the resultant couple moment on the wheel is zero.

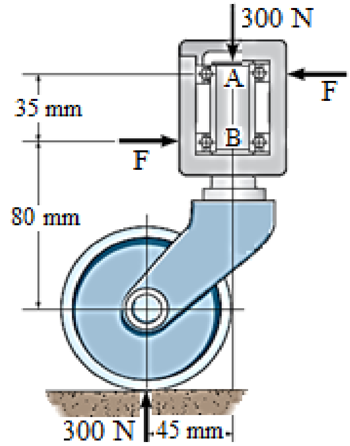


Fig. (Ex. 3-16)

Solution:

$\circlearrowleft + \sum M_C = 0 \qquad (F)(35) - (300)(45) = 0$

$35\,F = 13500$

$F = 385.7\text{ N}$

Example (3-17):

If the resultant couple moments on the frame shown in Fig. (Ex. 3-17) is (500 N.m) clockwise, determine the magnitude of the force (F).

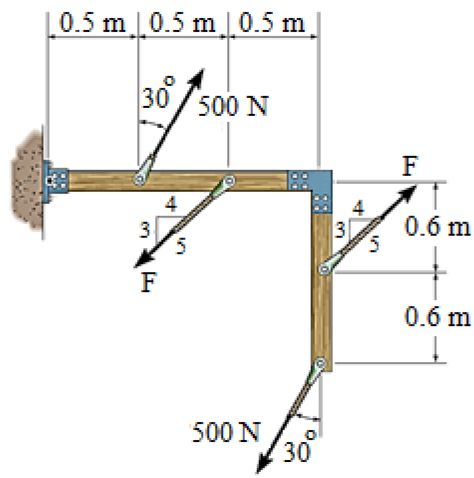
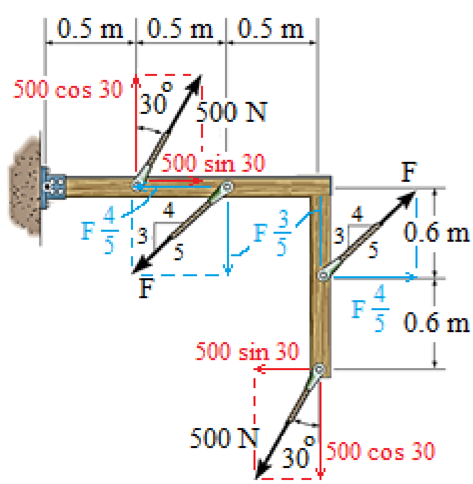


Fig. (Ex. 3-17)



Solution:

$\circlearrowleft + (M_C)_1 = (F)(4/5)(0.6) + (F)(3/5)(0.5)$

$= 0.78\,F$

$\circlearrowleft + (M_C)_2 = -(500\cos 30^o)(1) - (500\sin 30^o)(1.2)$

$= -433 - 300$

$= -733\text{ N.m} = 733\text{ N.m} \circlearrowright$

$\circlearrowleft + (M_C)_R = (M_C)_1 + (M_C)_2$

$-500 = 0.78\,F - 733$

$733 - 500 = 0.78\,F$

$0.78\,F = 233$

$F = 298.7\text{ N}$

Example (3-18):

The vertical reaction of the ground on the two main wheels of an aircraft at points (A) and (B) before the aircraft engine is running is (275 N) for each of the two wheels, and when the engine is running, the reaction is (350 N) at point (A).

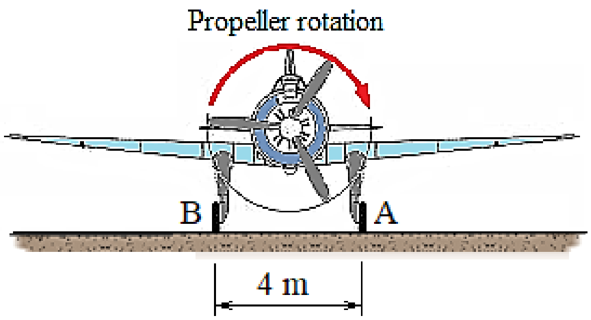


Fig. (Ex. 3-18)

The difference in reaction at point (A) is due to the torque of the dual propeller when the engine is running and its direction is clockwise, as shown in Fig. (Ex. 3-18). Find the magnitude of this torque and the magnitude of the reaction force of the ground acting at point (B) while the engine is running.

Solution:

Due to weight:

$(R_A)_W = 275$ N
$(R_B)_W = 275$ N

Due to weight and propeller couple:

$(R_A)_R = 350$ N
$(R_B)_R = ?$

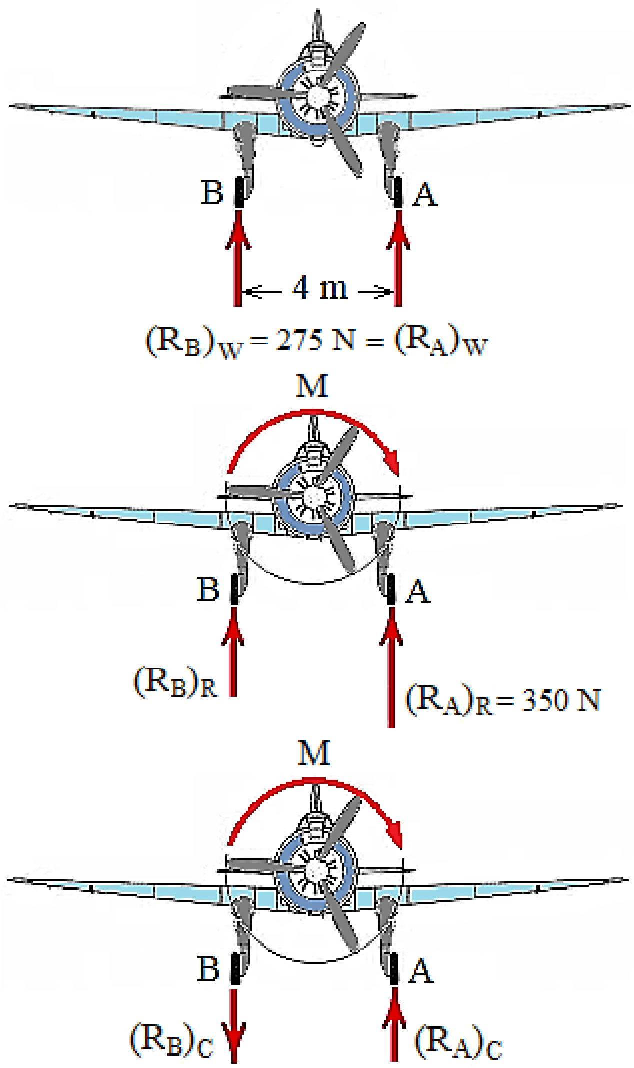
Due to propeller couple:

$(R_A)_C = 350 - 275 = 75$ N
$(R_B)_C = 75$ N

$(R_B)_R = 275 - 75 = 200$ N

$\curvearrowright + \sum M_C = 0$

$75 \times 4 = 300$ N.m



Force transformation to a line of action parallel to its line of action:

If a force moves on a certain body from one point to another point that is not on the same line of action, it is transferred in the form of a force with the same value and direction and a moment equal to the force multiplied by the distance perpendicular to the line of action of the force between the two points.

Example (3-19):

Replace the horizontal (500 N) force acting at point (A) on the lever by equivalent system consisting of a force and a couple at (*O*).

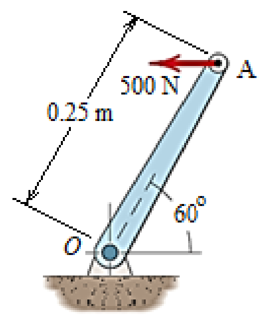


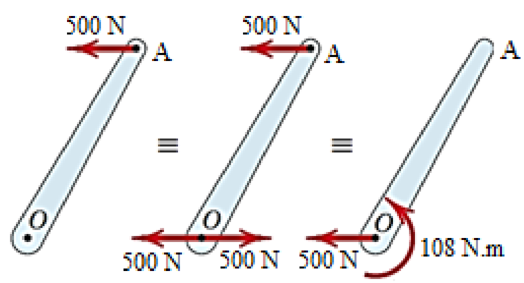
Fig. (Ex. 3-19)

Solution:

When a two forces with a value of (500 N) are applied to point (*O*) in opposite directions, so their resultant is equal to zero, so the two forces (500 N) on point (A) and opposite to it in the direction at point (*O*) generate a couple counterclockwise.

M = F.d

M = 500 × 0.25 sin 60° = 108 N.m



Thus, the force (500 N) on point (A) will be equivalent to a force of (500 N) in the same direction and a torque of (108 N.m) counterclockwise at point (*O*).

The resultant of a system of coplanar non-concurrent forces (forces and moments):

When several non-concurrent forces located in the same plane act on a body and are at specific distances from a specific point, let it be a point (*O*), then it is possible to calculate the resultant of those forces on the known point and the resultant of the moments around that point, and then these forces can be converted by a single force that is a distance away calculated from the known point, as shown in Fig. (3-5).

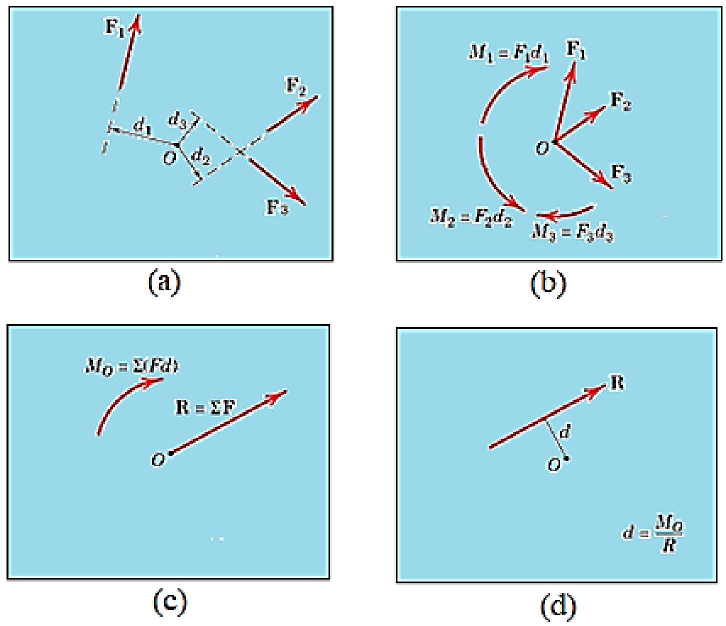


Fig. (3-5) The resultant of a system of coplanar non-concurrent forces

The resultant (value and direction) can be calculated according to the following equations:

$$R = F_1 + F_2 + F_3 + \ ..... \ = \sum F \quad .... \ (3\text{-}4)$$

$$R_x = \sum F_x \qquad R_y = \sum F_y \qquad R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} \quad ...... \ (3\text{-}5)$$

$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\sum F_y}{\sum F_x} \quad .... \ (3\text{-}6)$$

The value of the moment and the perpendicular distance (its location) can be calculated according to the following equations:

$$R = \sum F \quad .... \ (3\text{-}7)$$

$$M_o = \sum M = \sum (Fd) \quad .... \ (3\text{-}8)$$

$$Rd = M_O \quad .... \ (3\text{-}9)$$

Example (3-20):

Four forces and one couple act on the bracketed shown in Fig. (Ex. 3-20). Determine the resultant of the forces and the couple, and then indicate the point of action of the resultant on the horizontal axis with respect to the origin (*O*).

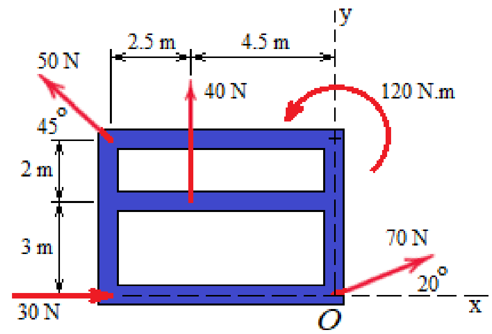


Fig. (Ex. 3-20)

Solution:

$R_x = \sum F_x$

$R_x = 30 + 70 \cos 20^o - 50 \cos 45^o = 60.4$ N

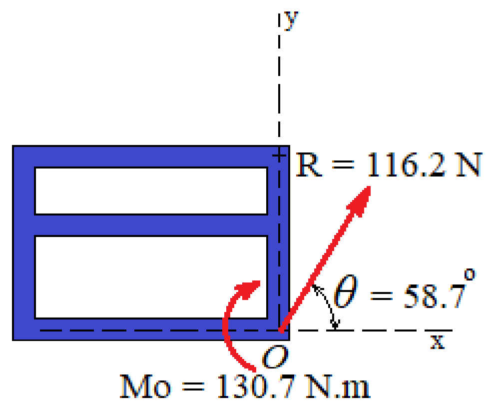
$R_y = \sum F_y$

$R_y = 40 + 70 \sin 20^o + 50 \sin 45^o = 99.3$ N

$R = \sqrt{Rx^2 + Ry^2}$

$R = \sqrt{(60.4)^2 + (99.3)^2} = 116.2$ N

$\theta = \tan^{-1} \dfrac{R_y}{R_x} \qquad \theta = \tan^{-1} \dfrac{99.3}{60.4} = 58.7^o$

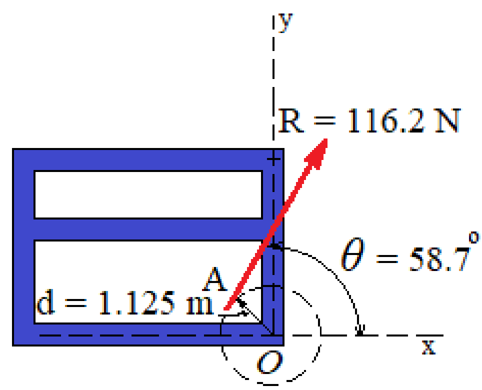


$M_o = \sum (Fd)$

$M_o = 120 - (40 \times 4.5) + (50 \cos 45^o \times 5) - (50 \sin 45^o \times 7)$

$= - 130.7 = 130.7$ N.m (C.W.)

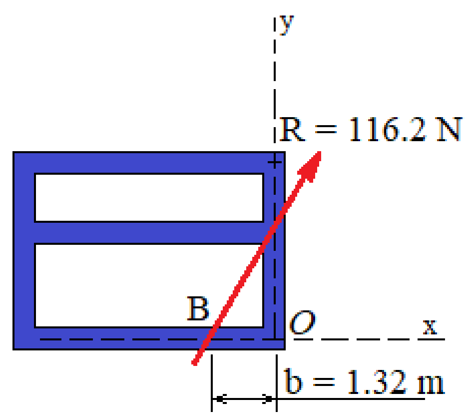
$R\,d = M_o, \quad 116.2\,d = 130.7, \quad d = 1.125$ m



$R_y\,b = M_o$

$R_y = 99.3$ N

$b = \dfrac{130.7}{99.3} = 1.32$ m



Example (3-21):

Replace the two forces acting on the bracket shown in Fig. (Ex. 3-21) by an equivalent resultant force and couple moment at point (*O*).

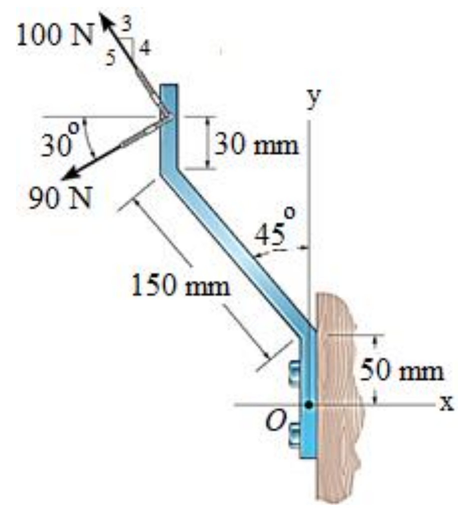
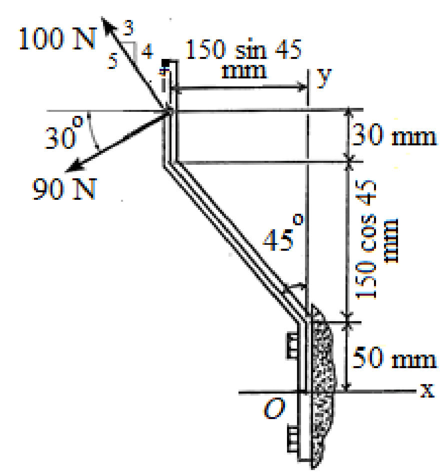


Fig. (Ex. 3-21)

Solution:

$+ \rightarrow F_{Rx} = \sum F_x \qquad F_{Rx} = -(100)(3/5) - 90 \cos 30^o = -138 = 138\ N \leftarrow$

$+ \uparrow F_{Ry} = \sum F_y \qquad F_{Ry} = (100)(4/5) - 90 \sin 30^o = 35\ N$



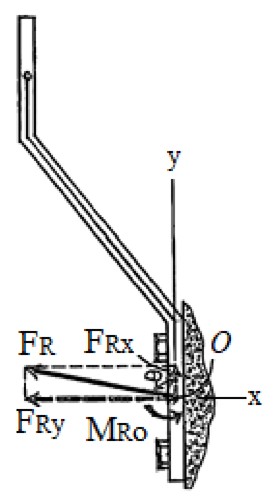
$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(138)^2 + (35)^2} = 142.4\ N$$

$$\theta = \tan^{-1} \frac{F_{Ry}}{F_{Rx}} = \tan^{-1} \frac{35}{138} = 14^o$$

||

$\circlearrowleft + M_{Ro} = \sum M_o$

$$\begin{aligned} M_{Ro} &= (90 \sin 30^o)(0.15 \sin 45^o) \\ &\quad + (90 \cos 30^o)(0.08 + 0.15 \cos 45^o) \\ &\quad - (100)(4/5)(0.15 \sin 45^o) \\ &\quad + (100)(3/5)(0.08 + 0.15 \cos 45^o) \\ &= 4.773 + 14.502 - 8.485 + 11.164 \\ &= 22\ N.m \circlearrowleft \end{aligned}$$



Example (3-22):

The bulldozer shown in Fig. (Ex. 3-22) consists of four main parts, the engine part has a mass of (2 tons) and its center of gravity (G_1), the cabin part has a mass of (0.8 ton) and its center of gravity (G_2), the drivetrain part has mass of (1.2 ton) and its center of gravity (G_3), the kit part has a mass of (1 ton) and its center of gravity (G_4). Replace the forces produced by these masses with an equivalent resultant and indicate the position of this resultant measured from point (A).

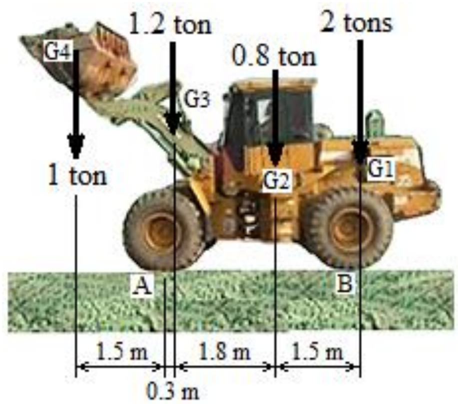


Fig. (Ex. 3-22)

Solution:

$+\uparrow F_R = \sum F_y$

$$F_R = -(1000 \times 9.81) - (1200 \times 9.81) - (800 \times 9.81) - (2000 \times 9.81)$$
$$= -49050 = 49050 \text{ N} \downarrow$$

$\curvearrowright + \; M_{RA} = \sum M_A$

$$-(49050 \times d) = (1000 \times 9.81 \times 1.5) - (1200 \times 9.81 \times 0.3) - (800 \times 9.81 \times 2.1) - (2000 \times 9.81 \times 3.6)$$

$$d = 1.548 \text{ m}$$

Problems:

3-1) Calculate the moment of the (50 Ib) force on the handle of the monkey wrench about the center of the bolt.

Ans.: M = 370.8 Ib.in (CW)

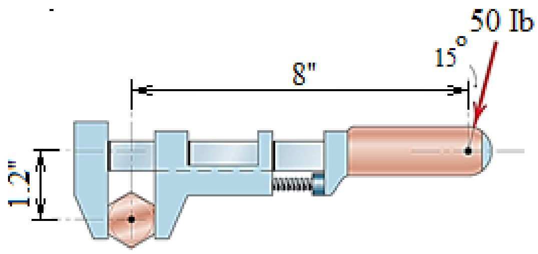


Fig. (Pr. 3-1)

3-2) A rod bent at an obtuse angle as shown in Fig. (Pr. 3-2), and a force of (25 N) is applied perpendicular to the axis of the (BC) part of it. Find the moment of this force about point (B) and about point (A).

Ans.: $M_B = 25$ *N.m (CW)*
$M_A = 46.2$ *N.m (CW)*

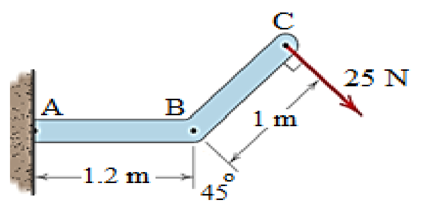


Fig. (Pr. 3-2)

3-3) The (25 Ib) force is applied to one end of the curved wrench, as shown in Fig. (Pr. 3-3). If ($\alpha = 30^o$), calculate the moment of (F) about the center (*O*) of the bolt. Determine the value of (α) which would maximize the moment about (*O*), and determine the value of this maximum moment.

Ans.: $M_o = 359.8$ *Ib.in. (CW)*
$\alpha = 33.7^o$
$(M_o)_{max} = 360$ *Ib.in. (CW)*

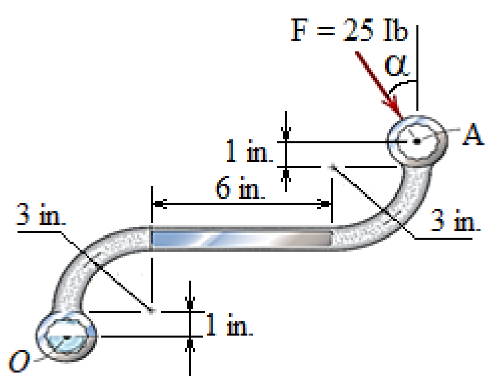


Fig. (Pr. 3-3)

3-4) For the frame shown in Fig. (Pr. 3-4). Determine the resultant moment of the three forces about point (A). Neglect the thickness of the frame.

Ans.: $M_A = 600$ *N.m (CW)*

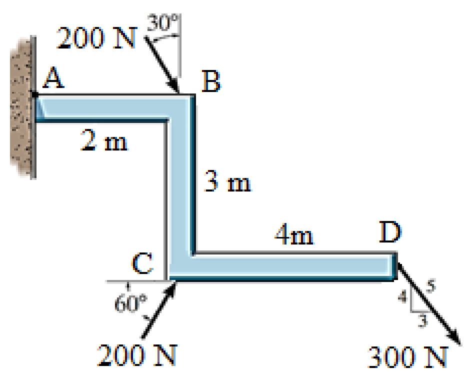


Fig. (Pr. 3-4)

3-5) The center of gravity of the gate arm, shown in Fig. (Pr. 3-5), is located at point (G_a) and the center of gravity of the counterweightv (200 kg) is located at point (G_W). If the resultant moment about point (A) is (4.6 kN.m) counter-clockwise, determine the magnitude of the gate arm mass.

Ans.: m_G = 188.5 kg

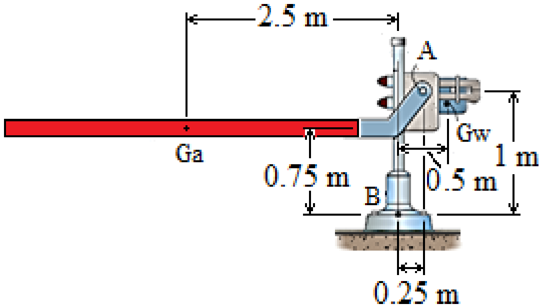


Fig. (Pr. 3-5)

3-6) The tool shown in Figure (Pr. 3-6) is used to hold the lawn mower blade while loosening the nut with a wrench. If a force of (60 N) is applied to the wrench at (B) in the direction shown in the figure, determine the moment it creates about the nut at (C). What is the magnitude of force (F) at (A) so that it creates the opposite moment about (C) ?

Ans.: M_C = 13 N.m, F = 40.2 N

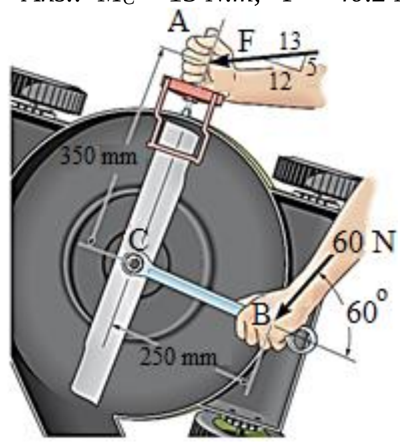


Fig. (Pr. 3-6)

3-7) Two persons push a gate from both sides of it, as shown in Fig. (Pr. 3-7). If the force applied by the person at point (B) is (F_B = 150 N), determine the magnitude of the force (F_A) required by the person at point (A) to prevent the gate from turning. Neglect the thickness of the gate.

Ans.: F_A = 144.3 N

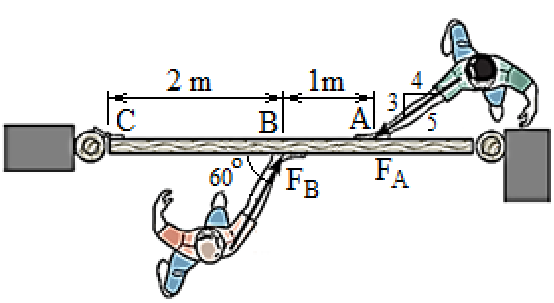


Fig. (Pr. 3-7)

3-8) To increase the torque required to unscrew the screw at point (A), the screwdriver arm is lengthened using the rod (BC) as shown in Fig. (Pr. 3-8). Determine the moment produced by the force (250 N) about the axis of the bolt at point (A).

Ans.: M_A = 102 N.m (CW)

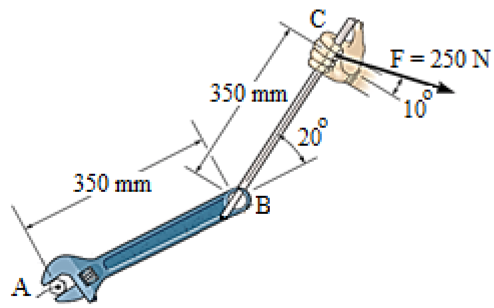


Fig. (Pr. 3-8)

3-9) In a load car (trailer), when the trailer is towed in the forward direction, a force (600 N) is applied to the trailer hitch ball, as shown in Fig. (Pr. 3-9). Determine the moment of this force force about point (*O*).

Ans.: $M_O = 167.88$ *N.m (CCW)*

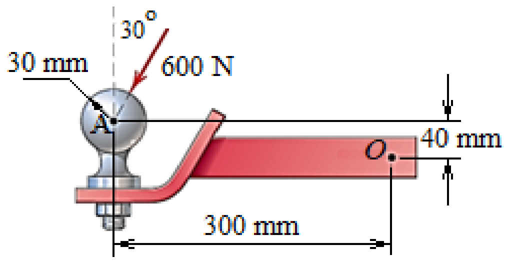


Fig. (Pr. 3-9)

3-10) In Fig. (Pr. 3-10), a force of magnitude (100 N) is exerted on an automobile parking-brake lever. Replace the force by an equivalent force – couple system at the pivot point (*O*).

Ans.: $M_O = 34.58$ *N.m (CCW)*

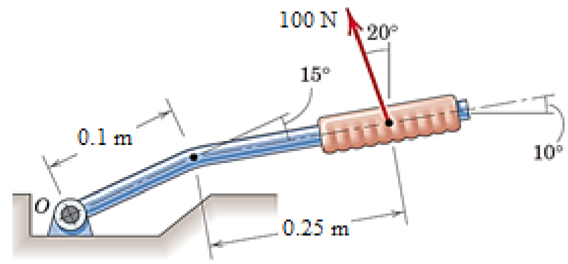


Fig. (Pr. 3-10)

3-11) During the test of the work of the aircraft, its two engines are accelerated and the direction of the two propellers is adjusted so that it results in forward thrust and rear thrust as shown in Fig. (Pr. 3-11). Calculate the friction force (F) that the ground exerts on each of the main wheels at points (A) and (B) to resist the effect of the two thrust forces. Ignore the effect of the nose wheel (C) which turns at an angle of (90º).

Ans.: $F = 5$ *kN*

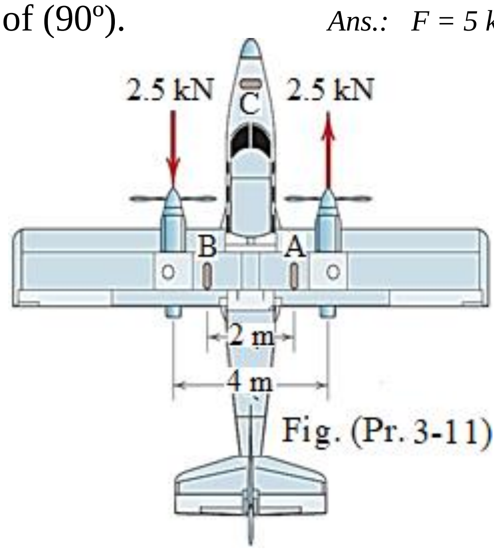


Fig. (Pr. 3-11)

3-12) The figure (Pr. 3-12) shows the top view of the entrance revolving door. Two persons approach the door at the same time and exert two forces of the same magnitudes as shown in the figure. If the resultant moment about the door pivot axis at (*O*) is (30 N.m), determine the magnitude of the force (F).

Ans.: $F = 15.5$ *N*

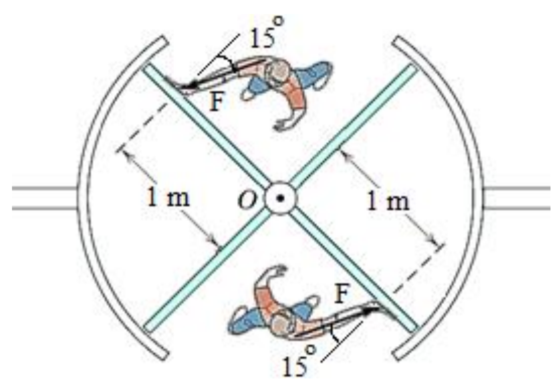


Fig. (Pr. 3-12)

3-13) Each propeller of the twin-engine ship rotates in a speed that generate thrust (300 kN). During the ship's maneuvering motion, one propeller rotates at full forward speed and the other at full rearward speed, as shown in Fig. (Pr. 3-13). Each boat applies a force of (50 kN) to the ship to counteract the action of the ship's propellers. Find the distance (x).

Ans.: X = 35 m

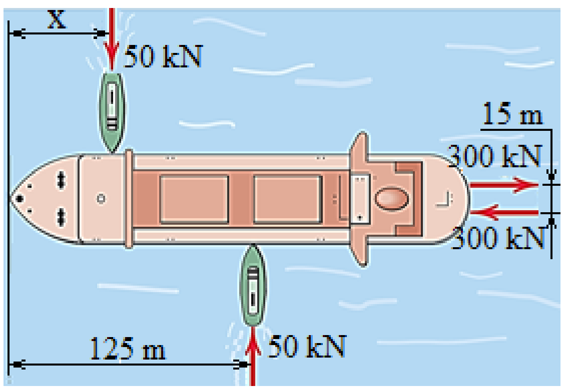


Fig. (Pr. 3-13)

3-14) The airplane shown in Fig. (Pr. 3-14) with four jet engines, each producing (100 kN) of forward thrust. At cruise flight, the engine number (3) suddenly fails. Determine and locate the resultant of the three remaining engines thrust forces.

Ans.: R = 300 kN, x = 4 m

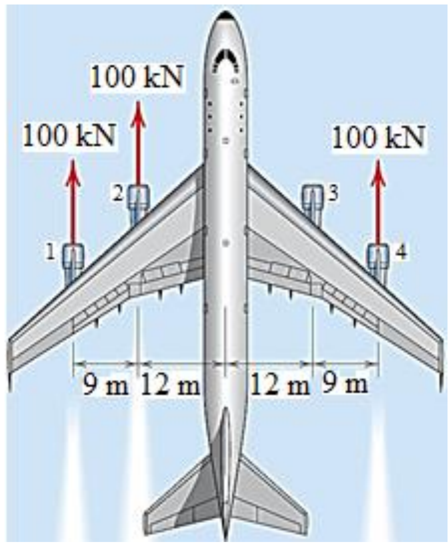


Fig. (Pr. 3-14)

3-15) In Fig. (Pr. 3-15) Two couples act on the cantilever beam. Find the resultant couple moment.

Ans.: $(M_c)_R$ = 1.8 kN. m

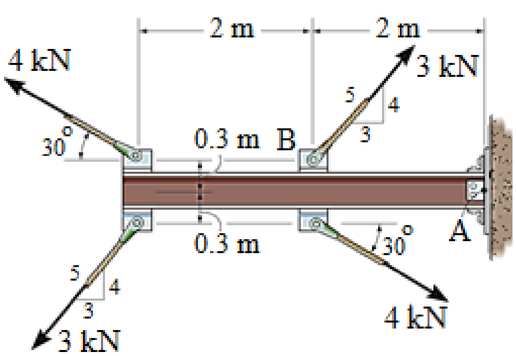


Fig. (Pr. 3-15)

3-16) Fig. (Pr. 3-16) shows a valve for opening and closing the water pipe. A man tried to open the valve by applying couple forces of (F = 100 N) on the lever of the valve. Find the couple moment of the two forces.

Ans.: Mc = 30 N. m (CW)

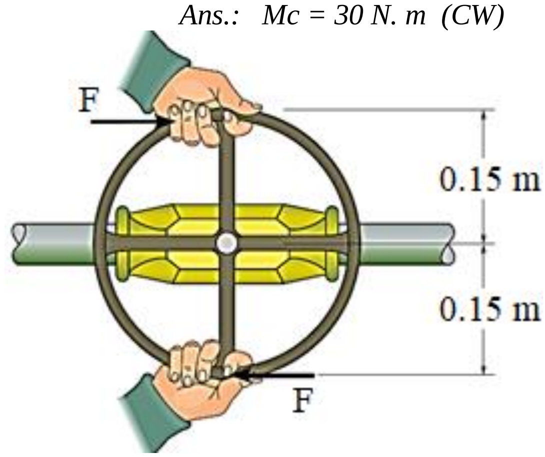


Fig. (Pr. 3-16)

3-17) If the resultant of the two forces and the couple (M) passes through point (O), determine the magnitude of the couple (M).

Ans. $M = 160.6$ N.m

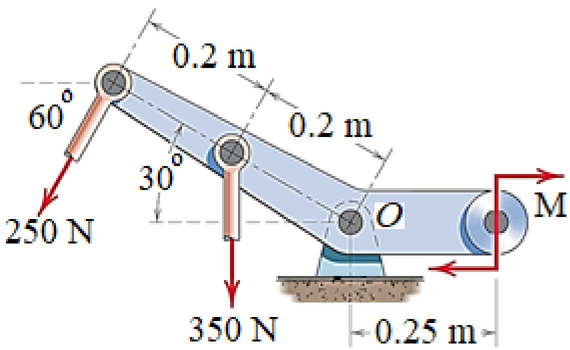


Fig. (Pr. 3-17)

3-18) The specialized truck shown in Fig. (Pr. 3-18) consists of three main parts, and its wights and center of gravity is indicated on each part. Replace the system of forces resulting from the weights of these parts with an equivalent resultant force and locate it relative to point (A).

Ans.: $F_R = 49$ kN ↓ , $d = 3$ m

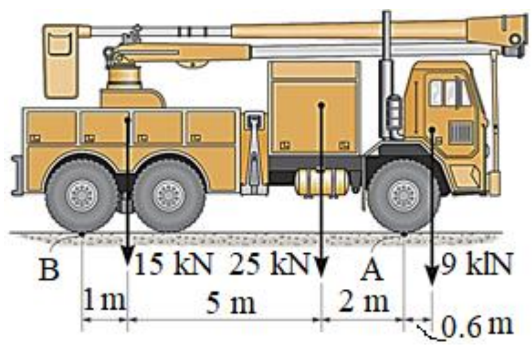


Fig. (Pr. 3-18)

3-19) Replace the forces and couple system shown in Fig. (Pr. 3-19) by an equivalent force and couple moment at point (O).

Ans.
$F_R = 2.07$ kN, $\theta = 8.5°$, $M = 10.62$ k N.m CW

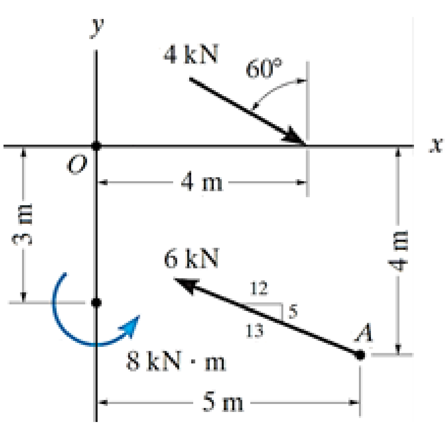


Fig. (Pr. 3-19)

3-20) Replace the force system acting on the frame shown in Fig. (Pr. 3-20) by a resultant force, and specify where its line of action intersects member (AB), measured from point (A).

Ans.: $F_R = 462$ Ib, $\theta = 50.1°$, $d = 3.07$ ft

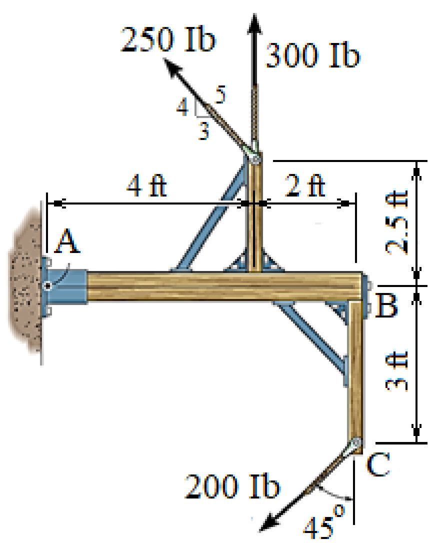


Fig. (Pr. 3-20)

CHAPTER FOUR
EQUILIBRIUM

In the previous two chapters, the study of the analysis of forces and moments on particles and rigid bodies was discussed, and how to conclude the resultant of forces in different values and directions on these rigid bodies and particles, and how to conclude the resultant of moments, and the resultant of forces and moments together.

In this chapter, the equilibrium states between these forces and moments on rigid bodies and particles will be discussed, and this topic will be divided into two parts, the first part shows the state of equilibrium in forces on particles, and the second part shows the state of equilibrium in forces and moments on rigid bodies.

A body is in equilibrium if the resultant of the forces acting on it is zero. This case is in stationary objects and objects moving in uniform motion (motion at constant velocity).

PART (1) : EQUILIBRIUM OF THE PARTICLES:

In this part of the equilibrium, the dimensions of the body are not taken into account, and the body is assumed as a point, so the body is in a state of equilibrium when the resultant of the forces acting on it is equal to zero, and the moments are not taken into consideration due to the neglect of the effect of the dimensions and the assumption of the body as a point, so the force acting on it is hypothetically concurrent. So the equilibrium equation is:

$$R = \sum F_x = \sum F_y = \sum F_z = 0 \quad \ldots\ldots\ldots\ldots \quad (4\text{-}1)$$

<u>Conditions for the equilibrium of a particle:</u>

If the particle is originally in a state of rest (without motion), it is said to be in equilibrium if it continues in its state of rest, and if it is originally in a state of uniform motion with a constant velocity and zero acceleration, it is said to be in equilibrium if it continues in its state of uniform motion without change. Most often, the term "equilibrium" or, more specifically, "static equilibrium" is used to describe a particular body at rest. To maintain equilibrium, it is necessary to apply Newton's first law of motion which is the basic law of equilibrium equations in the field of statics, and Newton's first law of motion requires that the force or the resultant of forces applied to a particle be equal to zero. This can be expressed mathematically as:

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

The free-body diagram:

The best way to explain the known and unknown forces acting on the particle, and apply the equilibrium equations to account for the unknown forces ($\sum Fx = 0$), ($\sum F_y = 0$) is to think of the particle as isolated and "free" from its surroundings. A drawing that shows a body with all the forces acting on it is called a *free body diagram* (FBD).

Procedure for drawing a free-body diagram:

In order to calculate all the forces acting on the particle when applying the equilibrium equations, the free-body diagram must be drawn first.

The following steps are necessary to create a free-body diagram.

1- We assume that the particle is isolated from its surroundings and then draw its specific shape (free body diagram).
2- Placing known and unknown forces on the particle diagram.
3- Draw the required dimensions and angles.
4- Apply the equations of equilibrium to find the unknown forces.

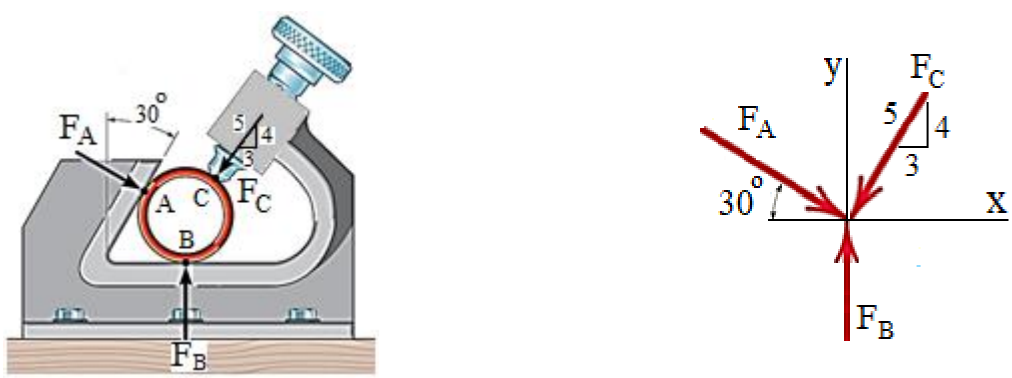


Fig. (4-1) Procedure for drawing a free-body diagram

Example (4-1):

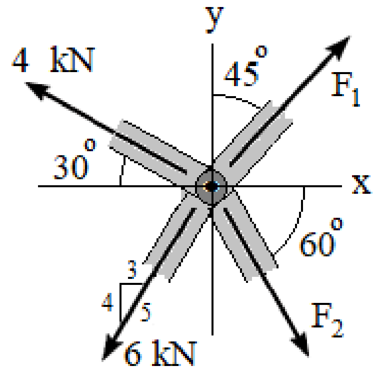


Fig. (Ex. 4-1)

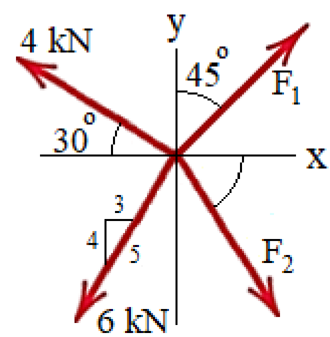
Find the magnitude of each of the two unknown forces (F_1) and (F_2) required to achieve equilibrium in the truss members shown in Fig. (Ex. 4-1), that are hinged at the (*O*) joint.

Solution:

$+\rightarrow \sum F_x = 0$

$F_1 \sin 45° + F_2 \cos 60° - 4 \cos 30° - 6 \left(\frac{3}{5}\right) = 0$

$0.707\ F_1 + 0.5\ F_2 = 7.064$ ……….…. (1)



$+\uparrow \sum F_y = 0$

$F_1 \cos 45° + 4 \sin 30° - F_2 \sin 60° - 6 \left(\frac{4}{5}\right) = 0$

$0.707\ F_1 - 0.866\ F_2 = 2.8$ ……….….. (2)

$0.707\ F_1 + 0.5\ F_2 = 7.064$
$0.707\ F_1 - 0.866\ F_2 = 2.8$
-------------------------------- Subtraction
$1.366\ F_2 = 4.264$
$F_2 = 3.12$ kN

Sub. in Eq. (1): $0.707\ F_1 + 0.5\ (3.12) = 7.064$
$0.707\ F_1 = 5.504 \quad \Rightarrow \quad F_1 = 7.78$ kN

Example (4-2):

The pipe is held in place by the vise. If the fixing bolt exerts a force of (250 N) on the pipe in the direction shown in Fig. (Ex. 4-2), determine the forces (F_A) and (F_B) that the smooth surfaces at (A) and (B) exert on the pipe.

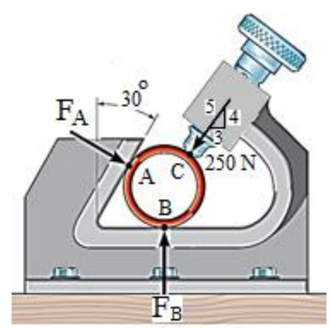
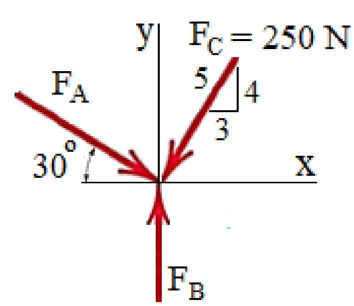


Fig. (Ex. 4-2)

Solution:

$+\rightarrow \sum F_x = 0, \qquad F_A \cos 30° - 250 \left(\frac{3}{5}\right) = 0$

$F_A = 173.2$ N



$+\uparrow \sum F_y = 0, \qquad F_B - 173.2 \sin 30° - 250 \left(\frac{4}{5}\right) = 0$

$F_B = 286.6$ N

Example (4-3):

Determine the tension in each of the ropes (AB) and (AC) used to lift a container of mass (650 kg) as a function of angle (θ). If the maximum allowable tension in each rope is (6.5 kN), find the shortest length of ropes (AB) and (AC) that can be used for lifting. Since the container's center of gravity is at point (G).

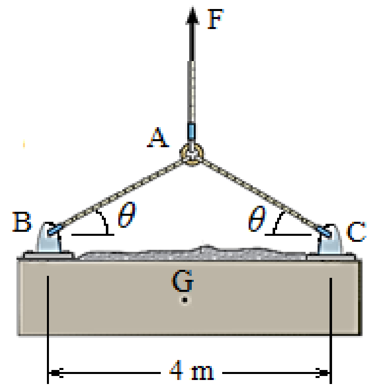
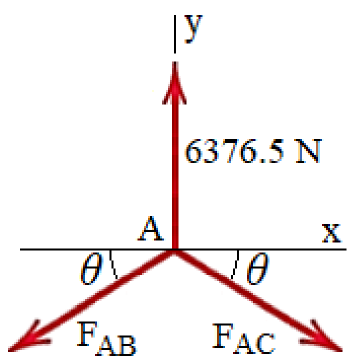


Fig. (Ex. 4-3)

Solution:

$$W = m\,g = 650 \times 9.81 = 6376.5 \text{ N}$$



$+ \rightarrow \sum F_x = 0$ $\qquad F_{AC} \cos\theta - F_{AB} \cos\theta = 0$

$$F_{AC} = F_{AB} = F$$

$+ \uparrow \sum F_y = 0$ $\qquad 6376.5 - 2\,F \sin\theta = 0$

$$2\,F \sin\theta = 6376.5$$

$$F = \frac{6376.5}{2\ \sin\theta} = \frac{3188.25}{\sin\theta}$$

Thus:

$$F_{AC} = F_{AB} = F = \frac{3188.25}{\sin\theta}\ \text{N}$$

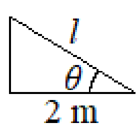
If the maximum allowable tension in the rope is (6.5 kN), then:

$$\frac{3188.25}{\sin\theta} = 6500$$

$$3188.25 = 6500 \sin\theta$$

$$\theta = \sin^{-1}\frac{3188.25}{6500} = 29.37°$$

From the geometry, ($l = \dfrac{2}{\cos\theta}$) and ($\theta = 29.37°$):



$$l = \frac{2}{\cos 29.37°} = 2.3 \text{ m}$$

Example (4-4):

The device shown in Fig. (Ex. 4-4) is used to straighten the bodies of wrecked cars. It consists of a chain that bears great forces, attached on one tip to a fixed point and connected on the other tip to the part of the car to be straightened.

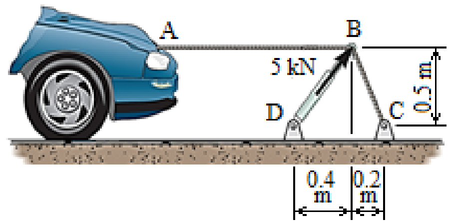
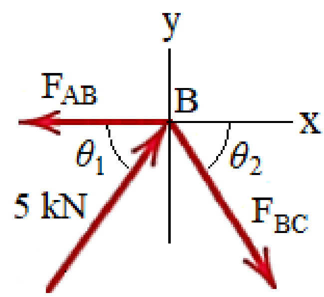


Fig. (Ex. 4-4)

A force applied by a hydraulic cylinder is applied to a point in the central part. Find the tension force of each part of the chain (AB) and (BC), if the force exerted by the hydraulic cylinder (DB) on point (B) is (5 kN).

Solution:



$$\theta_1 = \tan^{-1}\frac{0.5}{0.4} = 51.3° \quad , \quad \theta_2 = \tan^{-1}\frac{0.5}{0.2} = 68°$$

$+\uparrow \sum F_y = 0$

$$5 \sin 51.3° - F_{BC} \sin 68° = 0$$

$$F_{BC} = 4.21 \text{ kN}$$

$+\rightarrow \sum F_x = 0 \qquad 5 \cos 51.3° + 4.21 \cos 59° - F_{AB} = 0 \quad \Rightarrow \quad F_{AB} = 5.29 \text{ kN}$

Example (4-5):

In the system of wires shown in Fig. (Ex. 4-5), if the mass of the cylinder is (15 kg), Determine the required tensile force in the wires (CA) and (CB) to achieve equilibrium.

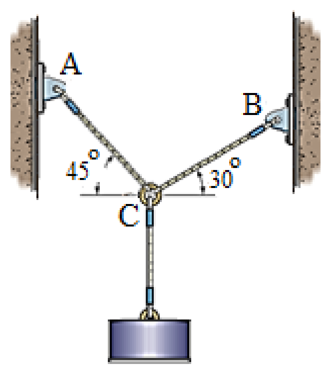


Fig. (Ex. 4-5)

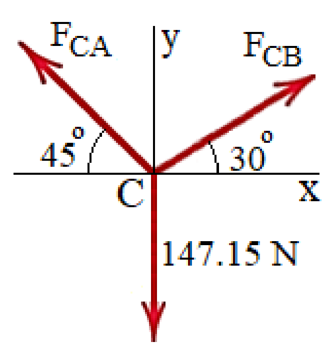
Solution:

$$W = m\,g = 15 \times 9.81 = 147.15 \text{ N}$$

$+\rightarrow \sum F_x = 0$

$$F_{CB} \cos 30° - F_{CA} \cos 45° = 0$$

$$0.866\, F_{CB} - 0.707\, F_{CA} = 0 \quad \ldots\ldots\ldots \ (1)$$



$+\uparrow \sum F_y = 0$

$$F_{CB} \sin 30° + F_{CA} \sin 45° - 147.15 = 0$$

$$0.5\, F_{CB} + 0.707\, F_{CA} - 147.15 = 0 \ \ldots\ldots \ (2)$$

From Eq. (1): $\qquad F_{CB} = 0.816\, F_{CA}$

Sub. in Eq. (2):

$$0.5\,(0.816\, F_{CA}) + 0.707\, F_{CA} - 147.15 = 0$$

$1.115\, F_{CA} = 147.15 \quad \Rightarrow \quad F_{CA} = 132 \text{ N} \qquad F_{CB} = 0.816\,(132) = 107.7 \text{ N}$

Example (4-6):

Determine the maximum weight of the flowerpot that can be supported without exceeding a cable tension of (250 N) in either cable (AB) or (AC).

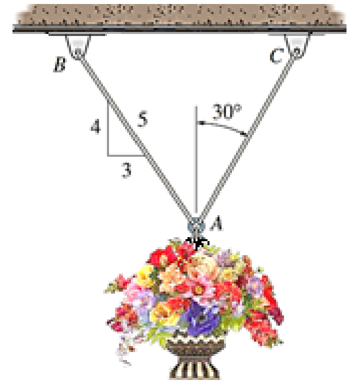


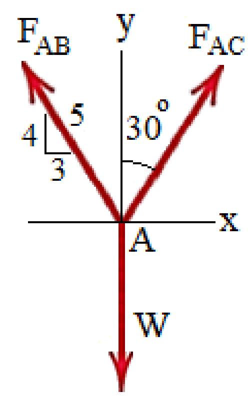
Fig. (Ex. 4-6)

Solution:

$+ \rightarrow \sum F_x = 0$

$$F_{AC} \sin 30° - F_{AB} \left(\frac{3}{5}\right) = 0$$

$$F_{AC} = 1.2\ F_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$



$+ \uparrow \sum F_y = 0$

$$F_{AC} \cos 30° + F_{AB} \left(\frac{4}{5}\right) - W = 0$$

$$0.866\ F_{AC} + 0.8\ F_{AB} = W \quad \ldots\ldots\ldots\ldots \quad (2)$$

Since ($F_{AC} > F_{AB}$) failure will occur first at cable (AC) with (F_{AC} = 250 N). Then solving Eqs. (1) and (2) yields:

$$250 = 1.2\ F_{AB} \quad \Rightarrow \quad F_{AB} = 208.33$$

$$(0.866)(250) + (0.8)(208.33) = W$$

$$W = 383.17\ \text{N}$$

Example (4-7):

Find the main tensile force (F) and the tensile force in each of the two cables (AB) and (AC) necessary to support the container which has a mass of (200 kg) and it's center of gravity located at point (G).

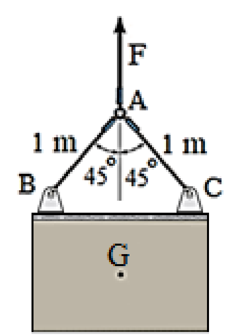


Fig. (Ex. 4-7)

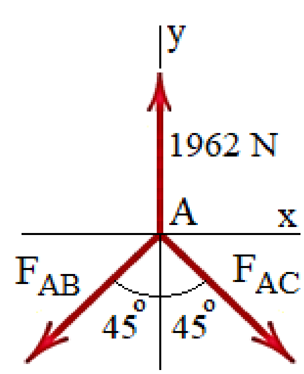
Solution:

$$F = W = m\ g = 200 \times 9.81 = 1962\ \text{N}$$

$+ \rightarrow \sum F_x = 0$

$$F_{AC} \sin 45° - F_{AB} \sin 45° = 0$$

$$F_{AC} = F_{AB}$$



$+ \uparrow \sum F_y = 0$

$$1962 - 2\ F_{AB} \cos 45° = 0$$

$$F_{AB} = F_{AC} = 1387.34\ \text{N}$$

Example (4-8):

If the mass of each of cylinders (D) and (F) is (2 kg) and the mass of cylinder (E) is (3 kg). Determine the distance (d) for equilibrium. Neglect the size of the pulleys.

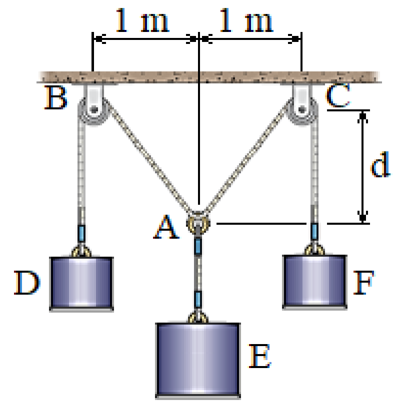


Fig. (Ex. 4-8)

Solution:

$W_D = 2 \times 9.81 = 19.62$ N
$W_E = 3 \times 9.81 = 29.43$ N
$W_F = 2 \times 9.81 = 19.62$ N

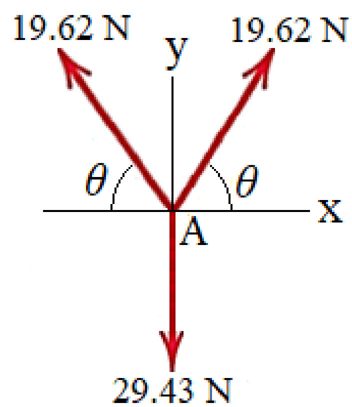
$+\uparrow \sum F_y = 0$

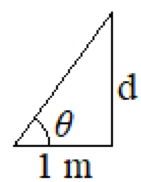
$2\,(19.62) \sin\theta - 29.43 = 0$

$\theta = \sin^{-1}(0.75) = 48.6°$

$\tan\theta = \dfrac{d}{1}$

$d = \tan 48.6°$ ٩





Example (4-9):

If you know that the maximum tension that both ropes (AB) and (AC) can withstand is (750 Ib), and that the mass of the drum is (25 slugs), find the smallest angle (*θ*) at which the drum can be lifted within the limits that the ropes withstand.

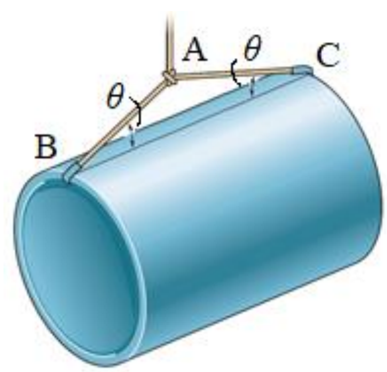


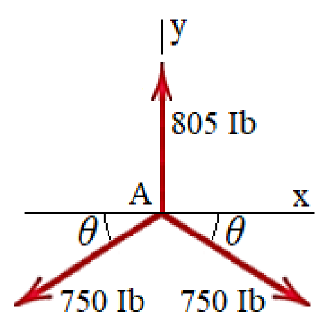
Fig. (Ex. 4-9)

Solution:

$W = m\,g = 25 \times 32.2 = 805$ Ib

$+\uparrow \sum F_y = 0$ $\quad 805 - 2(750)\sin\theta = 0$

$\theta = 32.5°$



Example (4-10):

Find the tension force in each cord in the system of cords shown in Fig. (Ex. 4-10) so that equilibrium is achieved with the load (25 kg).

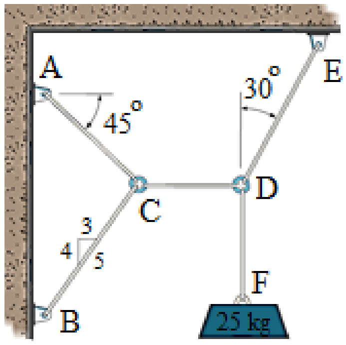
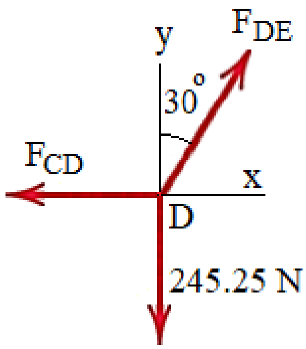


Fig. (Ex. 4-10)

Solution:

$$W = m\,g = 25 \times 9.81 = 245.25 \text{ N}$$

Equilibrium at point (D):



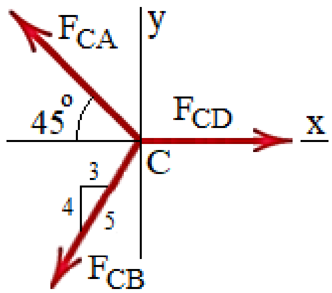
$+\uparrow \sum F_y = 0 \qquad F_{DE} \cos 30° - 245.25 = 0$

$F_{DE} = 283.2 \text{ N}$

$+\rightarrow \sum F_x = 0 \qquad 283.2 \sin 30° - F_{CD} = 0$

$F_{CD} = 141.6 \text{ N}$

Equilibrium at point (C):



$+\rightarrow \sum F_x = 0$

$141.6 - F_{CA} \cos 45° - F_{CB} \left(\frac{3}{5}\right) = 0$

$141.6 - 0.707\, F_{CA} - 0.6\, F_{CB} = 0$

$0.707\, F_{CA} + 0.6\, F_{CB} = 141.6 \quad \ldots\ldots\ldots\ldots \quad (1)$

$+\uparrow \sum F_y = 0 \qquad F_{CA} \sin 45° - F_{CB} \left(\frac{4}{5}\right) = 0$

$0.707\, F_{CA} - 0.8\, F_{CB} = 0 \quad \ldots\ldots \quad (2)$

From Eq. (2): $F_{CA} = 1.13\, F_{CB}$

Sub. in Eq. (1): $0.707\,(1.13\, F_{CB}) + 0.6\, F_{CB} = 141.6$

$1.4\, F_{CB} = 141.6$

$F_{CB} = 101.15 \text{ N}$

$F_{CA} = 1.13\,(101.15) = 114.3 \text{ N}$

Or:

$0.707\, F_{CA} + 0.6\, F_{CB} = 141.6 \quad \ldots\ldots\ldots\ldots \quad (1)$

$0.707\, F_{CA} - 0.8\, F_{CB} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$

---------------------------------- Subtraction

$1.4\, F_{CB} = 141.6 \quad \Rightarrow \quad F_{CB} = 101.15 \text{ N}$

Sub. in Eq. (1): $0.707\, F_{CA} + 0.6\,(101.15) = 141.6$

$0.707\, F_{CA} + 60.69 = 141.6$

$0.707\, F_{CA} = 141.6 - 60.69 = 80.91$

$F_{CA} = 114.3 \text{ N}$

Problems:

4-1) Find the tensile force in each wire of the wire system shown in Fig. (Pr. 4-1).

Ans.: $F_{ED} = 60.6$ *Ib* , $F_{EB} = 87.27$ *Ib*
$F_{BC} = 120.9$ *Ib*, $F_{BA} = 139.6$ *Ib*

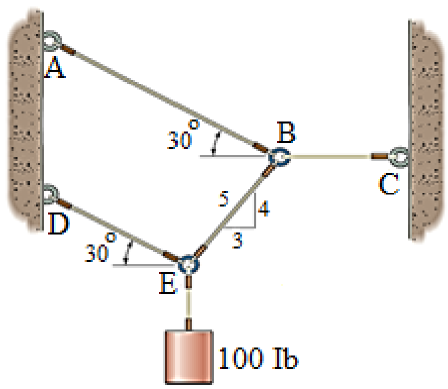


Fig. (Pr. 4-1)

4-2) Find the tensile force in each wire of the wire system shown in Fig. (Pr. 4-2).

Ans.: $F_{ED} = F_{EC} = 57.74$ *Ib*
$F_{DB} = F_{CA} = 70.7$ *Ib*, $F_{CD} = 21.13$ *Ib*

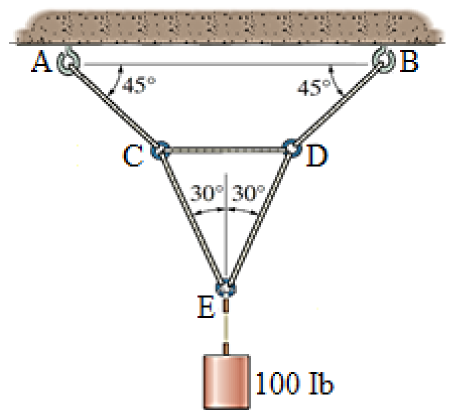


Fig. (Pr. 4-2)

4-3) The length of the wire (ABC) is (5 m). Find the distance (x) and the tension force applied in the wire (ABC) required for equilibrium with the mass of the cylinder of (100 kg). Neglect the size of the pully at (B).

Ans.: $x = 1.38$ *m*, $T = 686.87$ *N*

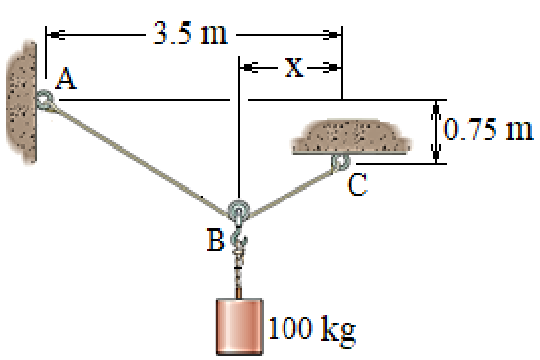


Fig. (Pr. 4-3)

4-4) The total length of the rope (ABCD) shown in Fig. (Pr. 4-4) is (6 m), determine the magnitude of the angle (θ) and the force (F) for equilibrium with the (10 kg) mass.

Ans.: $\theta = 36.87°$, $F = 61.3$ *N*

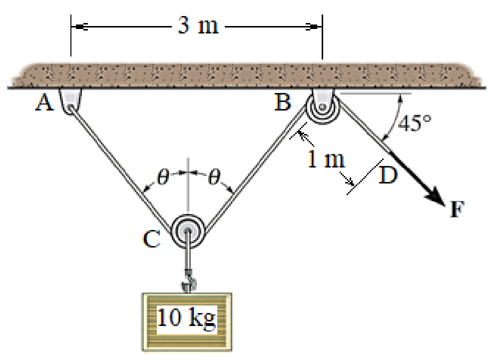


Fig. (Pr. 4-4)

4-5) A container of (100 kg) is lifted by the sling (BAC) with constant velocity. Determine the force in the sling (F), and find the value of the tension (T) of each of the robs (AB) and (AC) as a function of its orientation (θ),where ($0° \le \theta \le 90°$).

Ans.: $F = 981\ N$, $T = \dfrac{981}{2 \sin\theta}$

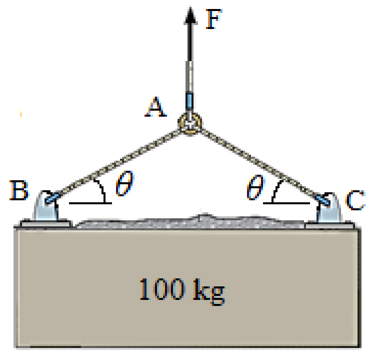


Fig. (Pr. 4-5)

4-6) The box (D) has a mass of (15 kg). If a force of (F = 80 N) is applied horizontally to the ring at (C), determine the largest dimension (d) so that the force in cable (CB) is zero.

Ans.: $d = 2.17\ m$

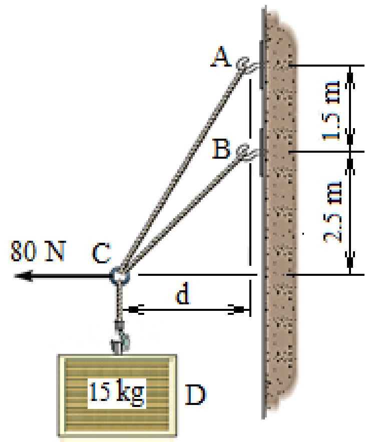


Fig. (Pr. 4-6)

4-7) Calculate the values of the tensile force (T) and the compressive force (P) in the bracket shown in Fig. (Pr. 4-7) to achieve equilibrium.

Ans.: $P = 714.3\ N$, $T = 606.2$ N

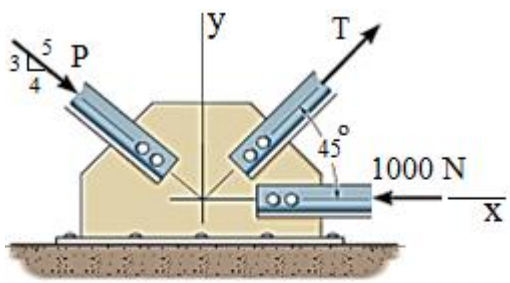


Fig. (Pr. 4-7)

4-8) An engine suspended by the system of chains shown in Fig. (Pr. 4-8). Determine the maximum weight of the engine that can be suspended without exceeding the tensile force of (500 Ib) in both chains (AB) and (AC).

Ans.: $W = 500$ *Ib*

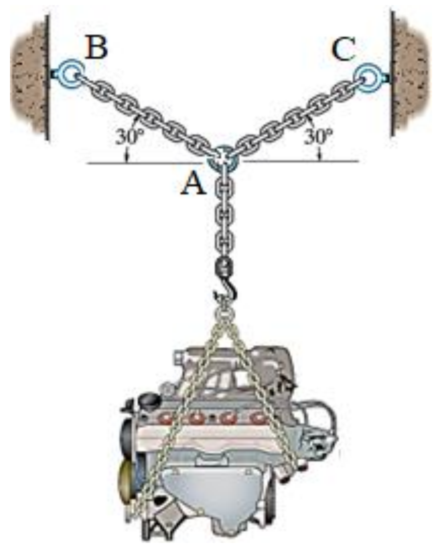


Fig. (Pr. 4-8)

4-9) In the cables arrangement shown in Fig. (Pr. 4-9), determine the tension forces in cables (AC) and (BC) caused by the weight the (25 kg) box.

Ans.: $T_{AC} = 179.6\ N$, $T_{BC} = 219.9\ N$

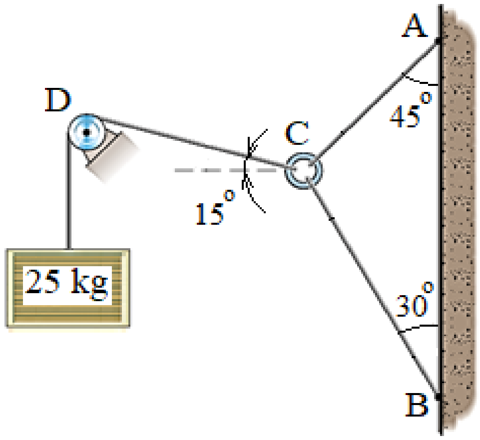


Fig. (Pr. 4-9)

PART (2): EQUILIBRIUM OF THE RIGID BODIES:

In this part of the equilibrium, the dimensions of the body are taken into account and have an effect in calculating the forces and moments applied to it, and the body is in a state of equilibrium when the sum of all forces and moments or couples applied to it is equal to zero. So the equilibrium equations will be:

$$F_R = \sum F = 0 \quad \text{……………..} \quad (4\text{-}2)$$
$$M_R = \sum M = 0 \quad \text{…………..} \quad (4\text{-}3)$$

System isolation and the free-body diagram (FBD):

A mechanical system can be virtually isolated from its surroundings by a so-called free-body diagram (FBD). A mechanical system may be a single body or a combination of bodies connected in a manner appropriate to the desired purpose of the system. The bodies may be solid or non-solid. The system may also be an identifiable fluid mass, either a liquid or a gas, or a mixture of liquids and solids.

In the statics branch of engineering mechanics, we mainly study the forces acting on solid bodies at rest.

To draw a free body diagram, you must follow these steps:

1- We assume that the rigid body is isolated from its surroundings and then draw its isolated shape (free body diagram).
2- Placing known and unknown forces on the free body diagram.
3- Replace the supports with reaction forces.
4- Draw the required dimensions and angles.
5- Applying the equations of equilibrium to find the unknown forces.

Modeling the action of forces:

The following figures shows the common types of supports used in mechanical systems and the corresponding forces of reactions on the free-body diagram (FBD) for analysis in a two-dimensional plane. Where each of the following examples shows the forces acting on the body as an isolated body from its surroundings. Newton's third law must be taken into account when replacing the supports on which the bodies rest with reaction forces, which states that for every action force there is a reaction force equal in magnitude and opposite in direction. The reaction forces that is applied to a specific body as a result of its contact with a support or with another body is always opposite to the direction of motion of the isolated body that would occur if the support or the contacting body were removed.

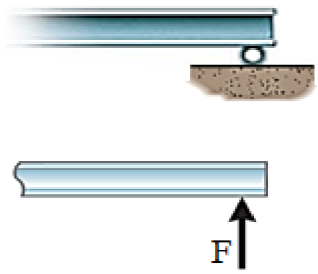


Fig. (4-2) Roller support

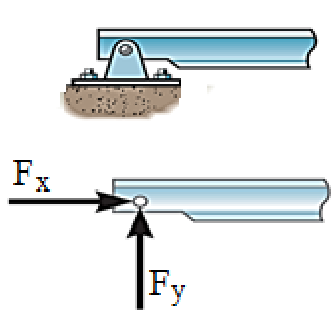


Fig. (4-3) Pin support

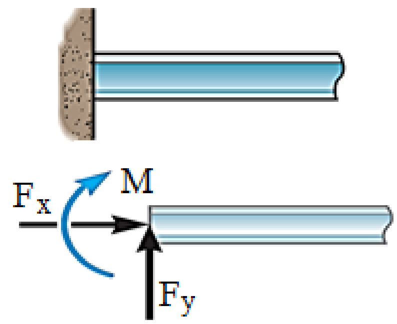


Fig. (4-4) Fixed support

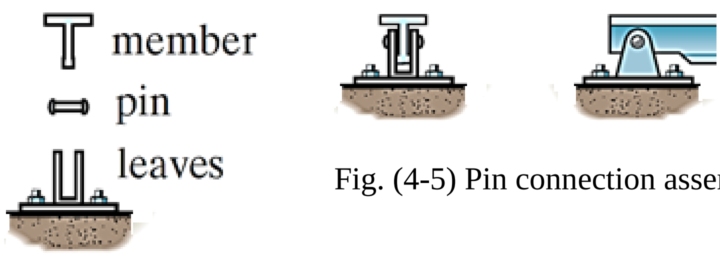


Fig. (4-5) Pin connection assembly

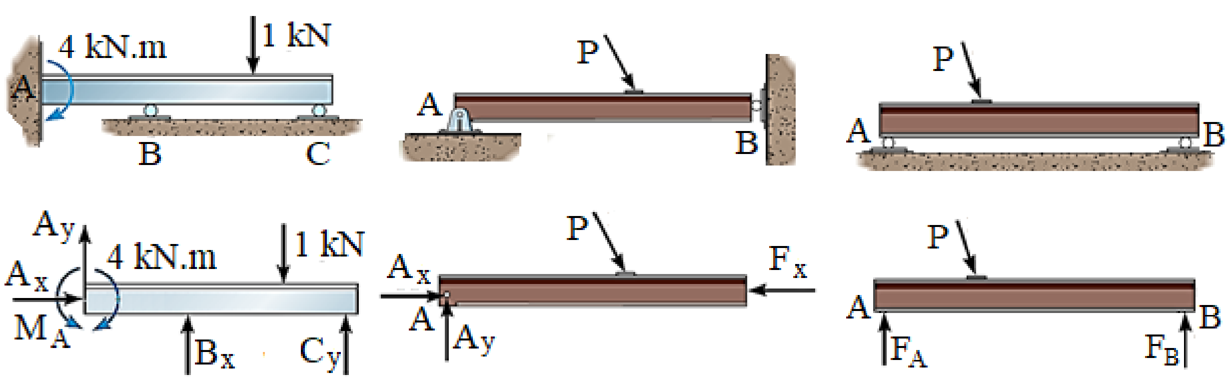


Fig. (4-6) Types of supports

Modeling the action of forces in two-dimensional analysis:

Table (4-1) Modeling the action of forces in two-dimensional analysis:

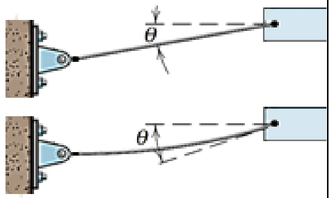
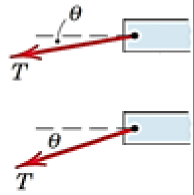
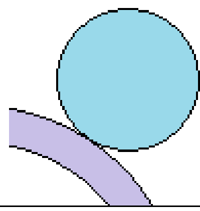
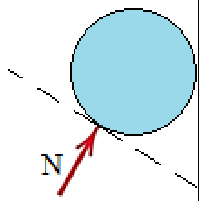
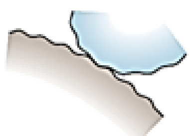
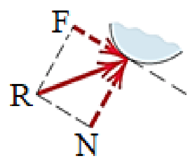
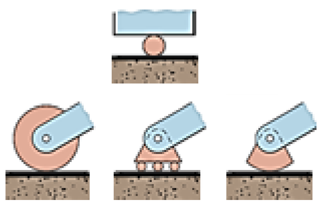
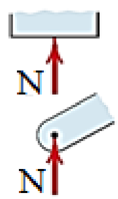
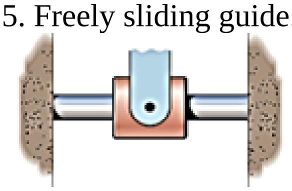
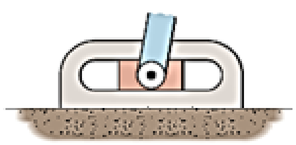
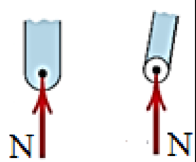
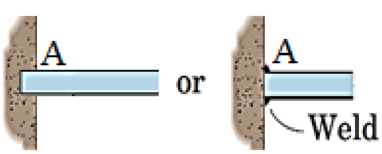
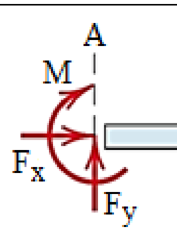
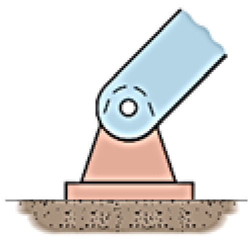
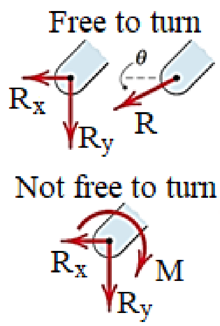
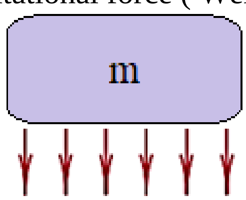
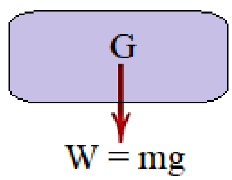
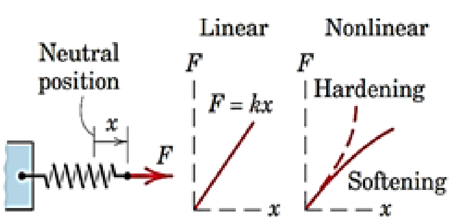
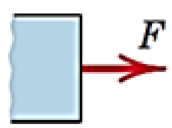
Type of contact and force origin		**Representation of forces on a body as an isolated body**	
1. Flexible cable, belt, chain, or rope. Weight of cable negligible. Weight of cable considerable.			The force that a flexible cable, belt, chain, or rope exerts on a body is represented by a tensile force starting from the body in the direction of the cable or the direction of the tangent to the cable.
2. Smooth surfaces.			Reaction force as a result of contact represented by a compressive force and is normal to the surface.
3. Rough surfaces.			Reaction force as a result of contact is inclined force (R), and it is resolved into two components, the tangential component (F) (friction force) and the normal component (N).
4. Roller supports.			Roller, rocker, or ball support represented by a compressive force normal to the supporting surface.
5. Freely sliding guide. 			Collar or slider free to move along smooth guides, can be represented by a support force normal to the guide.
6. Fixed support. 			The fixed support represented by an axial force (F_x), and a lateral force (F_y) (shear force), and a moment (M) about the fixation point to prevent the rotation.

Table (4-1) Modeling the action of forces in two-dimensional analysis:

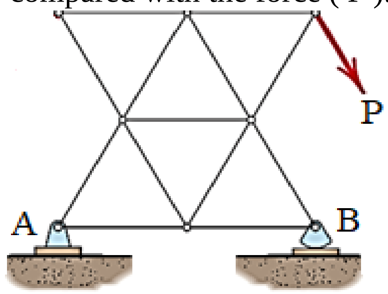
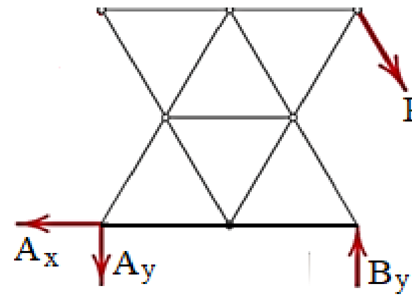
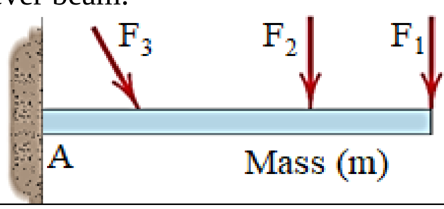
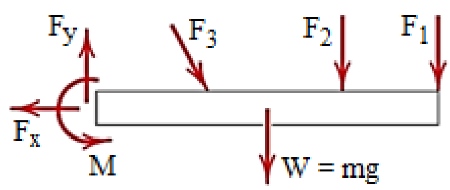
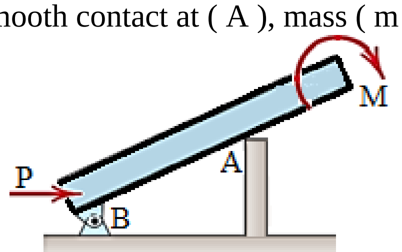
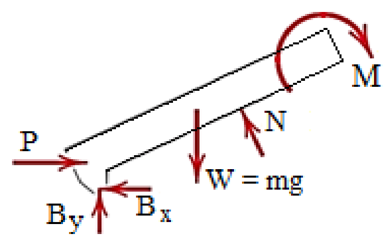
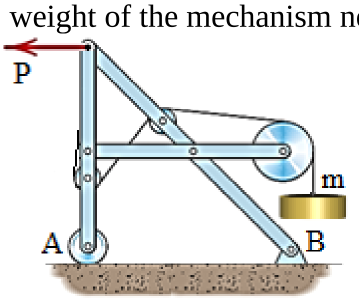
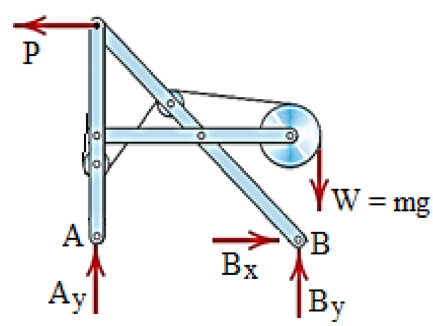
<table>
<tr><th>Type of contact and force origin</th><th colspan="2">Representation of forces on a body as an isolated body</th></tr>
<tr><td>7. Pin connection.
</td><td>
</td><td>In a free-rotational joint, the reaction force can be represented as either a horizontal (R_x) and a vertical (R_y) component, or a force (R) with its direction (θ). As for the hinge with restricted rotational motion, to the above reactions, a torque (M) about the fixation point is added.</td></tr>
<tr><td>8. Gravitational force (Weight).

</td><td>
</td><td>The resultant gravitational forces resulting from the elements of a body of mass (m) is the weight (W = mg). The weight is represented by a force directed towards the center of the earth, starting from the center of gravity of the body (G).</td></tr>
<tr><td>9. Spring force.

</td><td>
</td><td>The force of the spring results from multiplying the elongation by the stiffness of the spring, and it is a tensile force if the spring is elongated and a compressive force if it contracts.
The spring's stiffness (k) is the force required to deform the spring by an elongation per unit distance.</td></tr>
</table>

Construction of free – body diagrams:

Examples of free - body diagrams:

Table (4-2) shows four examples of mechanisms and structures with their free-body diagrams. Dimensions and magnitudes omitted as general examples. In each case the entire mechanism or structure is treated as a single body, so that internal forces do not appear. The four examples shown in the table show the known and unknown forces and the reactions of the various types of supports.

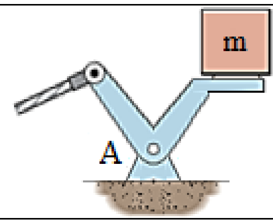
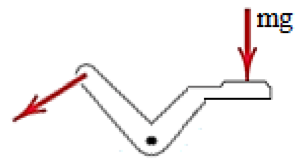
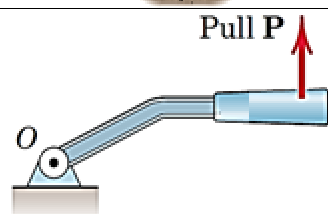
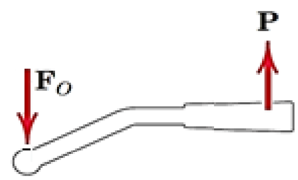
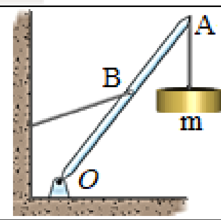
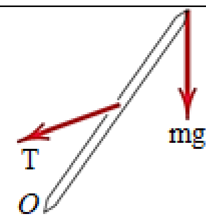
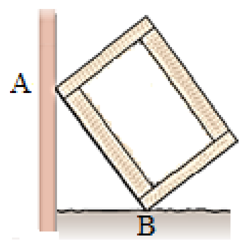
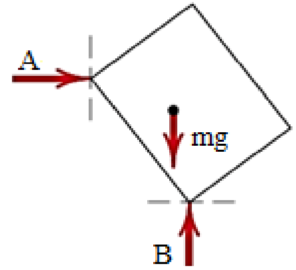
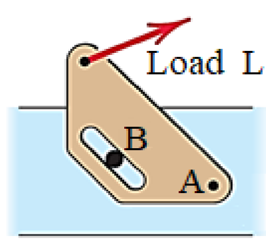
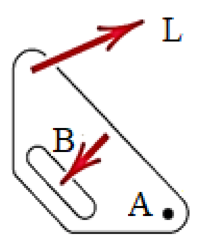
Table (4-2) Sample of free-body diagrams

<table>
<tr><th>Mechanical System</th><th>Free-Body Diagram of Isolated structure</th></tr>
<tr><td>1- Plane truss: Weight of truss assumed to be negligible compared with the force (P).

</td><td>
</td></tr>
<tr><td>2- Cantilever beam.

</td><td>
</td></tr>
<tr><td>3- Beam: smooth contact at (A), mass (m).

</td><td>
</td></tr>
<tr><td>4- System of rigid interconnected bodies analyzed as a single unit, weight of the mechanism negligible

</td><td>
</td></tr>
</table>

Exercises of free - body diagrams:

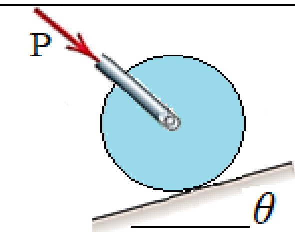
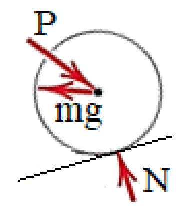
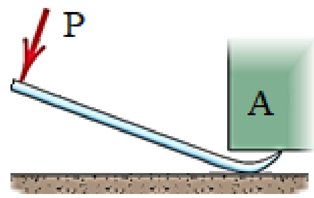
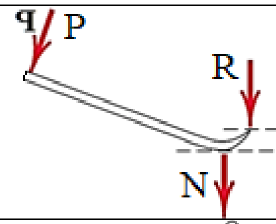
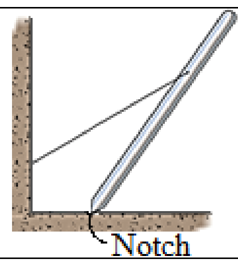
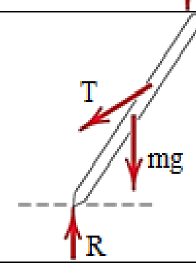
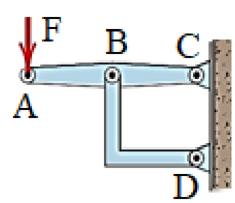
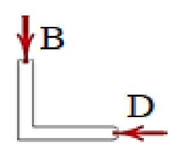
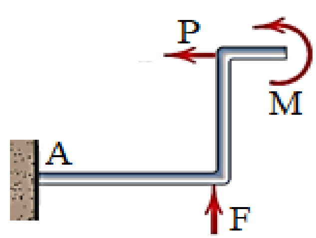
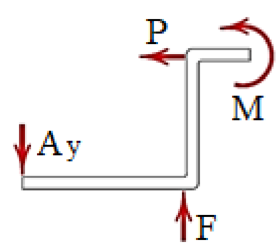
In Table (4-3), the middle column represents specific structures, the left column represents brief details of these structures, and the right column represents an incomplete free body diagrams (FBD) of the isolated body. It is required to complete the free body diagrams of the structures in the right column.
Body weights are negligible unless otherwise noted.
Dimensions and numerical values have been omitted for simplicity.

Table (4-3) Free-body diagram exercises

	Body	**Incomplete (FBD)**
a- Bell crank supporting mass (m) with pin support at (A).		
b- Control lever applying torque to the shaft at (*O*).		
c- The boom (OA), of negligible mass compared with the mass (m). The boom hinged at (*O*) and taut by cable at (B).		
d- Uniform crate of mass (m) leaning against smooth vertical wall and supported on a rough horizontal surface.		
e- Loaded bracket supported by pin connection at (A) and fixed pin in smooth slot at (B).		

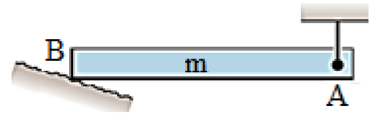
In Table (4-4), the middle column represents specific structures, the left column represents brief details of these structures, and the right column represents the free body diagrams (FBD) of the isolated body, which is incorrect or incomplete. It is required to correct or complete the free body diagrams of the structures in the right column.
Body weights are negligible unless otherwise noted.
Dimensions and numerical values have been omitted for simplicity.

Table (4-4) Free-body diagram exercises

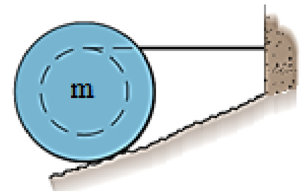
	Body	**Wrong or Incomplete (FBD)**
a- A cylinder of mass (m) is pushed up a slope inclined at an angle (θ).		
b-Pry-bar lifting body (A) which have a smooth horizontal surface. The bar rests on horizontal rough surface.		
c- Uniform pole of mass (m) being hoisted into position by cable. Horizontal supporting surface notched to prevent slipping of pole.		
d- The member (BD) in the form of a right angle is hinged with the horizontal member at the point (B).		
e- Bent rod welded to a wall at (A) and subjected to two forces and couple.		

Draw a free-body diagram for each of the structures shown below showing all the known and unknown forces, knowing that the weights of the bodies are not required unless the mass is specified.

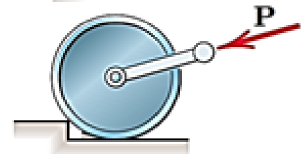
a- Uniform horizontal bar of mass (m) suspended by vertical cable at (A) and supported on a rough inclined surface at (B).



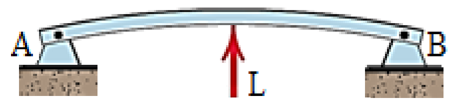
b- A notched disk of uniform mass, of mass (m), is drawn with a horizontal wire and rests on a rough surface.



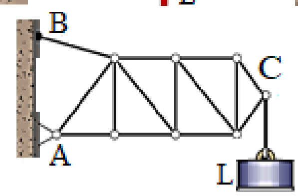
c- A disk of mass (m) is about to tip over on to the pavement due to an inclined pushing force (P).



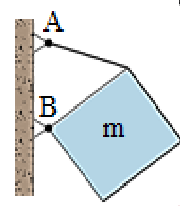
d- A horizontal bar bent due to load (L). Fixed at both ends with articulated anchors.



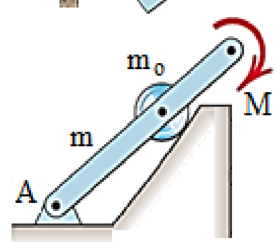
e- A truss hinged at point (A) and taut with a cable at point (B) and curing a weight (L) at point (C).



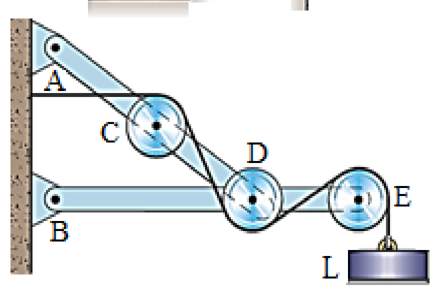
f- A plate of uniform mass, of mass (m), hinged at point (B) and taut by a wire at point (A).



g- A structure consisting of a uniform rod of mass (m) and a pulley of mass (m_o). A torque (M) is applied to it and hinged at point (A).



h- A structure consisting of rods and pulleys jointly linked with each other, and a connecting wire that carries the mass (L).



<u>Example (4-11):</u>

The smooth disk (A) with a mass of (50 kg) and the smooth disk (B) with a mass of (100 kg), a horizontal force of (F = 1000 N) was applied to the center of the disk (A). Find the normal reactions to the support surfaces at points (C), (D), and (E).

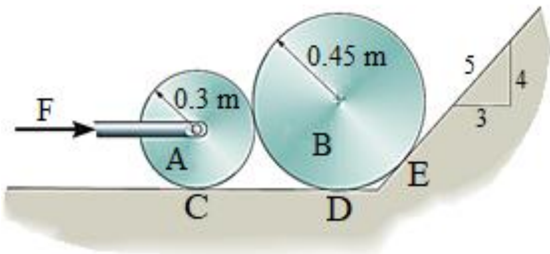
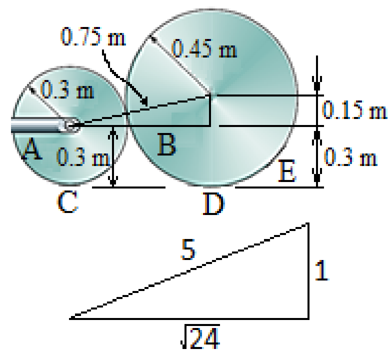


Fig. (Ex. 4-11)

Solution:



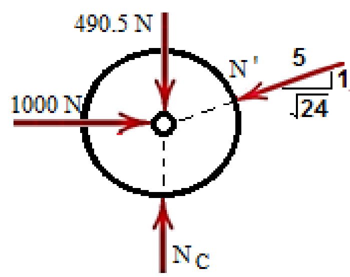
Disk (A):

$W = 50 \times 9.81 = 490.5$ N

$+ \rightarrow \sum F_x = 0$

$$1000 - N' \left(\frac{\sqrt{24}}{5}\right) = 0$$

$N' = 1020.6$ N



$+ \uparrow \sum F_y = 0$

$$N_C - 490.5 - 1020.6 \left(\frac{1}{5}\right) = 0$$

$N_C = 694.6$ N

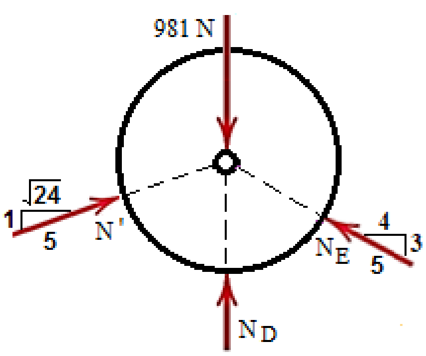
Disk (B):

$W = 100 \times 9.81 = 981$ N

$+ \rightarrow \sum F_x = 0$

$$N_E \left(\frac{4}{5}\right) - 1020.6 \left(\frac{\sqrt{24}}{5}\right) = 0$$

$N_E = 1250$ N



$+ \uparrow \sum F_y = 0$

$$1250 \left(\frac{3}{5}\right) + N_D - 981 + 1020.6 \left(\frac{1}{5}\right) = 0$$

$N_D = 26.88$ N

Example (4-12):

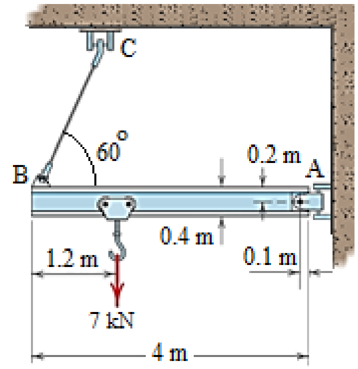


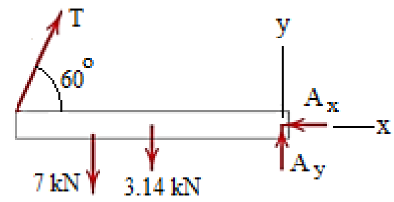
Fig. (Ex. 4-12)

For the jib crane shown in Fig. (Ex. 4-12), determine the magnitude of the tension (T) in the supporting cable and the magnitude of the reaction force on the pin at (A). The beam (AB) is a standard (0.4-m / I-beam) with a mass of (80 kg) per meter of length.

Solution:

Algebraic solution:

$W = m\,g = (80 \times 4)(9.81) = 3139.2\ N = 3.14\ kN$



$\sum M_A = 0,$

$- T \cos 60^o \times 0.2 - T \sin 60^o \times (4 - 0.1)$
$+ 7\,(4 - 1.2 - 0.1)$
$+ 3.14\,(2 - 0.1) = 0$

$- 0.1\,T - 3.38\,T + 18.9 + 5.97 = 0$

$3.48\,T = 24.87$

$T = 7.15\ kN$

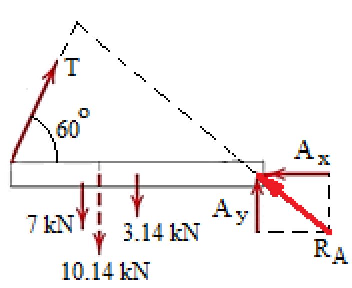
$\sum F_x = 0$ $\quad 7.15 \cos 60^o - A_x = 0 \quad A_x = 3.6\ kN$

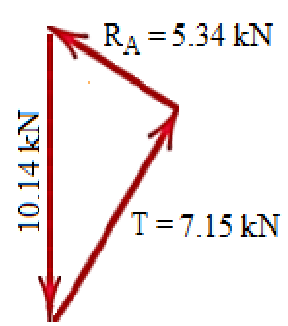
$\sum F_y = 0$ $\quad A_y + 7.15 \sin 60^o - 3.14 - 7 = 0$

$A_y = 3.95\ kN$

$R_A = \sqrt{Ax^2 + Ay^2} = \sqrt{(3.6)^2 + (3.95)^2} = 5.34\ kN$

Graphical solution:





Example (4-13):

The system shown in Fig. (Ex. 4-13) carry a cylinder of mass (10 kg). If the mass of the uniform shaft is (6 kg), and the pulley at point (D) is frictionless, determine the tension in the cable and the components of reactions at the fixed point (A).

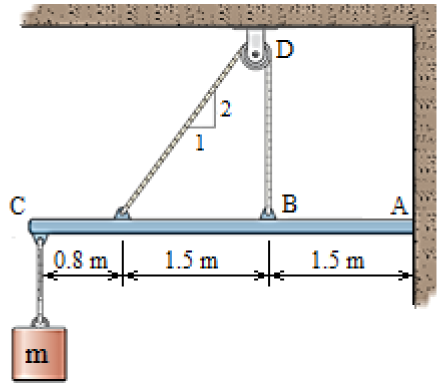
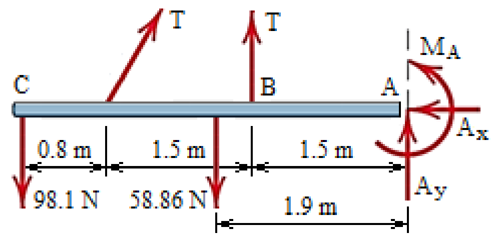


Fig. (Ex. 4-13)

Solution:

$W_{cy} = 10 \times 9.81 = 98.1$ N

$W_{sh} = 6 \times 9.81 = 58.86$ N



$\circlearrowright + \sum M_A = 0$

$$-(T)(1.5) - (T)\left(\frac{2}{\sqrt{5}}\right)(3) + (98.1)(3.8) + (58.86)(1.9) = 0$$

$4.18\ T = 484.61$

$T = 115.94$ N

$+ \rightarrow \sum F_x = 0 \qquad (115.94)\left(\frac{1}{\sqrt{5}}\right) - A_x = 0$

$A_x = 51.85$ N

$+ \uparrow \sum F_y = 0 \qquad A_y + 115.94 + (115.94)\left(\frac{2}{\sqrt{5}}\right) - 98.1 - 58.86 = 0$

$A_y = -62.68 = 62.68\ \text{N} \downarrow$

$\circlearrowright + \sum M_A = 0$

$$M_A - (115.94)(1.5) - (115.94)\left(\frac{2}{\sqrt{5}}\right)(3) + (98.1)(3.8) + (58.86)(1.9) = 0$$

$M_A - 173.91 - 311.1 + 372.78 + 111.83 = 0$

$M_A = 0$

Example (4-14):

On the lever shown in Fig. (Ex. 4-14), determine the horizontal and vertical components of reaction force at the pin at point (A) and the reaction force of the roller at point (B).

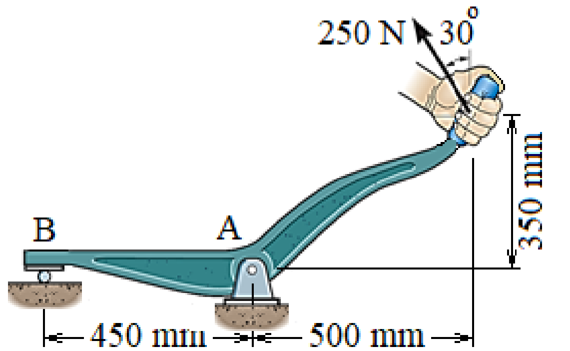
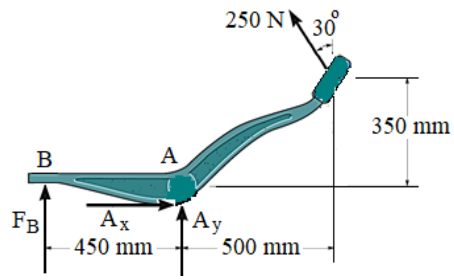


Fig. (Ex. 4-14)

Solution:



$\circlearrowleft + \sum M_A = 0$ $\quad$ $(250 \cos 30°) (0.5)$
$+ (250 \sin 30°) (0.35)$
$- (F_B) (0.45) = 0$
$152 = 0.45 F_B$
$F_B = 337.78$ N

$+ \rightarrow \sum F_x = 0$ $\quad$ $A_x - 250 \sin 30° = 0$
$A_x = 125$ N

$+ \uparrow \sum F_y = 0$ $\quad$ $A_y + 250 \cos 30° + 337.78 = 0$
$A_y = - 554.29 = 554.29$ N $\downarrow$

Example (4-15):

On the rod shown in Fig. (Ex. 4-15), determine the reactions at the supports (A) and (B).

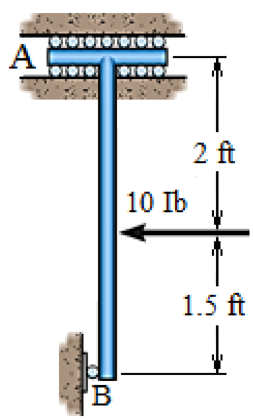
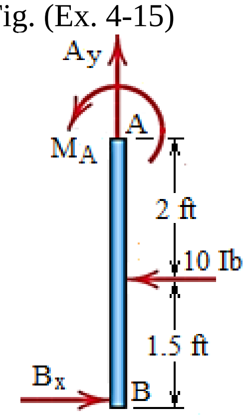


Fig. (Ex. 4-15)

Solution:



$+ \rightarrow \sum F_x = 0$ $\quad$ $B_x - 10 = 0$
$B_x = 10$ Ib

$+ \rightarrow \sum F_y = 0$ $\quad$ $A_y = 0$

$\circlearrowleft + \sum M_A = 0$ $\quad$ $M_A + 10 (3.5) - 10 (2) = 0$
$M_A = - 15 = 15$ Ib.ft $\quad$ (C.W)

Example (4-16):

Calculate the reactions of the Earth to the wheels of the bulldozer shown in Fig. (Ex. 4-16) at points (A) and (B). Consider the problem as a two-dimensional problem.

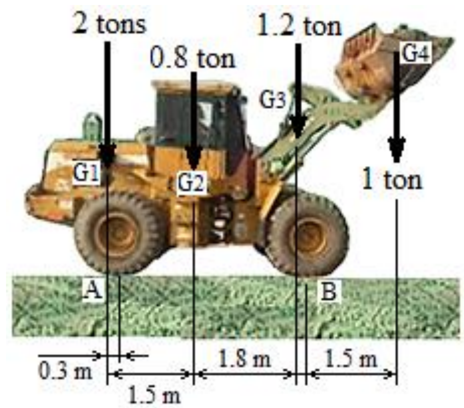


Fig. (Ex. 4-16)

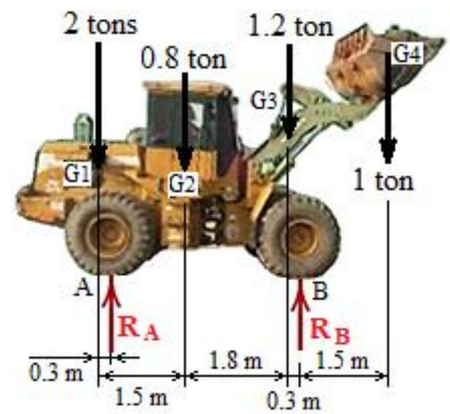
Solution:

$\circlearrowright + \sum M_B = 0$

$(2000 \times 9.81 \times 3.6) + (800 \times 9.81 \times 2.1) + (1200 \times 9.81 \times 0.3) - (1000 \times 9.81 \times 1.5) - (R_A \times 3.3) = 0$

$70632 + 16480.8 + 3531.6 - 14715 = 3.3\, R_A$

$R_A = 23009 \text{ N} = 23 \text{ kN}$



$+\uparrow \sum F_y = 0$

$R_B + 23009 - (2000 \times 9.81) - (800 \times 9.81) - (1200 \times 9.81) - (1000 \times 9.81) = 0$

$R_B = 26041 \text{ N} = 26 \text{ kN}$

Example (4-17):

On the cantilevered structure shown in Fig. (Ex. 4-17), determine the components of the support reactions at the fixed support (A).

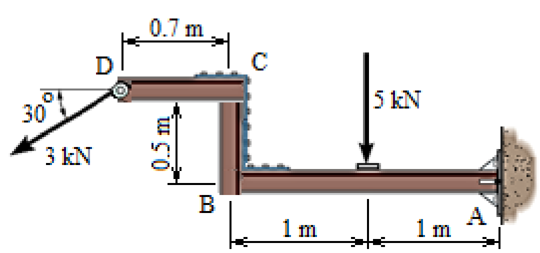
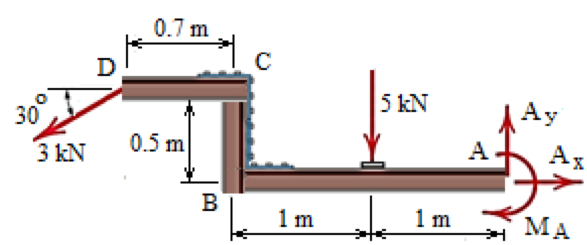


Fig. (Ex. 4-17)

Solution:



$+\rightarrow \sum F_x = 0$

$A_x - 3 \cos 30^o = 0$

$A_x = 2.6 \text{ kN}$

$+\uparrow \sum F_y = 0$

$A_y - 5 - 3 \sin 30^o = 0$

$A_y = 6.5 \text{ kN}$

$\circlearrowright + \sum M_A = 0$

$(5)(1) + (3 \cos 30^o)(0.5) + (3 \sin 30^o)(2.7) - M_A = 0 \quad \Rightarrow \quad M_A = 10.35 \text{ kN.m}$

<u>Example (4-18):</u>

Calculate the tensile force (T) in the cable that carries a mass of (300 kg) with the pulley system shown in Figure (Ex. 4-18), and find the magnitude of the total force acting on the support of the pulley (C).

All part weights are negligible compared to the load, and all pulleys rotate freely on their axis.

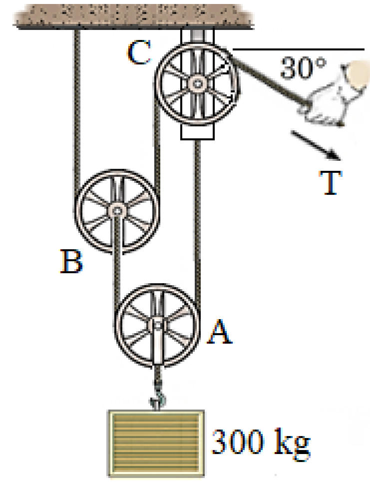
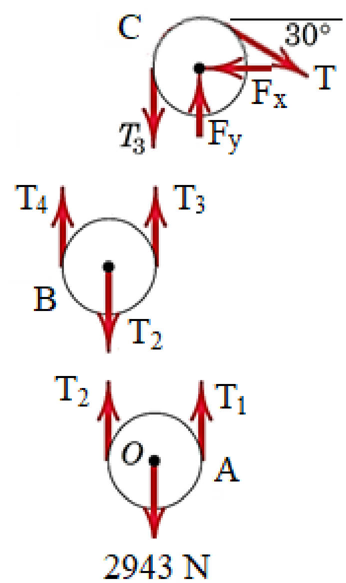


Fig. (Ex. 4-18)

Solution:

$$W = m\,g = 300 \times 9.81 = 2943 \text{ N}$$



pulley A:

$\sum M_o = 0$ $\qquad T_2\, r - T_1\, r = 0 \qquad T_1 = T_2$

$\sum F_y = 0$ $\qquad T_1 + T_2 - 2943 = 0$

$T_1 = T_2 = 1471.5$ N

pulley B:

$$T_3 = T_4 = T_2 / 2 = 735.75 \text{ N}$$

pulley C:

$T = T_3 \qquad$ or $\qquad T = 735.75$ N

Equilibrium of the pulley in the x-and y-directions requires:

$\sum F_x = 0$ $\qquad 735.75 \cos 30^\circ - F_x = 0$

$F_x = 637.2$ N

$\sum F_y = 0$ $\qquad F_y - 735.75 \sin 30^\circ - 735.75 = 0$

$F_y = 1103.6$ N

$$F = \sqrt{Fx^2 + Fy^2} = \sqrt{(637.2)^2 + (1103.6)^2} = 1274.4 \text{ N}$$

Example (4-19):

Calculate the magnitude of the force supported by the pin at point (A) under the action of the (1.5 kN) load applied to the bracket. Neglect friction in the slot.

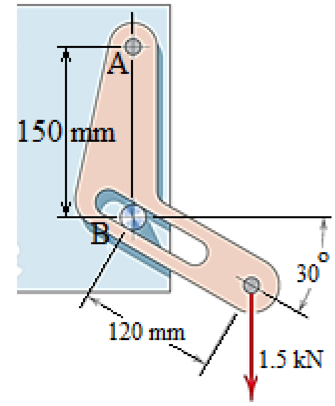


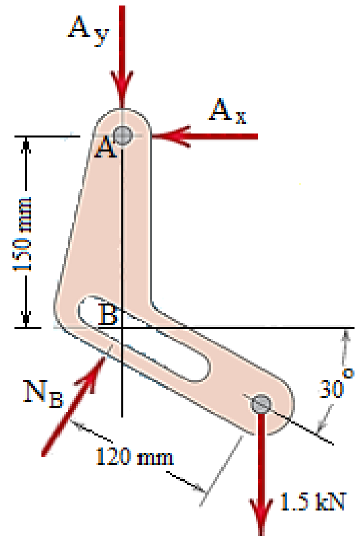
Fig. (Ex. 4-19)

Solution:

$\circlearrowright + \sum M_A = 0$

$(N_B \sin 30°) (0.15) - (1.5) (0.12 \cos 30°) = 0$

$N_B = 2.1$ kN



$+ \rightarrow \sum F_x = 0$ $\qquad 2.1 \sin 30° - A_x = 0$

$A_x = 1.05$ kN

$+ \uparrow \sum F_y = 0$

$2.1 \cos 30° - A_y - 1.5 = 0$

$A_y = 0.3$ kN

$$R_A = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{(1.05)^2 + (0.3)^2} = 1.09 \text{ kN}$$

Example (4-20):

In Fig. (Ex. 4-20), a gerl is training on the rowing machine. If she exerts a pulling force of (F = 300 N) on the handle of the machine (ABC), determine the force exerted by the hydraulic cylinder (BD) on the handle, and the horizontal and vertical components of the reaction force at the joint (C).

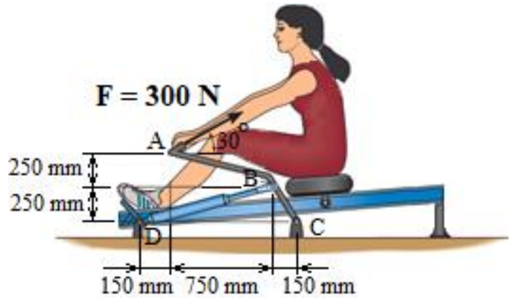
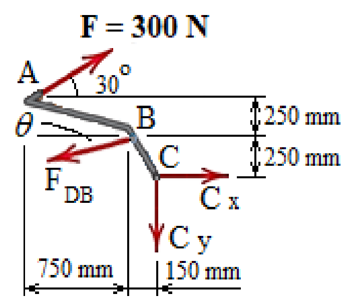


Fig. (Ex. 4-20)

Solution:

$$\theta = \tan^{-1}\frac{0.25}{0.75} = 18.4$$



$\circlearrowright + \sum M_C = 0$

$-(300\cos 30)(0.5) - (300\sin 30)(0.9)$
$+ (F_{DB}\cos 18.4)(0.25)$
$+ (F_{DB}\sin 18.4)(0.15) = 0$

$-129.9 - 135 + 0.237\,F_{DB} + 0.047\,F_{DB} = 0$

$0.284\,F_{DB} = 232.7$

$F_{DB} = 819.4\ \text{N}$

$+\rightarrow \sum F_x = 0$

$C_x + 300\cos 30 - 819.4\cos 18.4 = 0$

$C_x = 777.51 - 259.81 = 517.7\ \text{N} \rightarrow$

$+\uparrow \sum F_y = 0$

$300\sin 30 - 819.4\sin 18.4 - C_y = 0$

$C_y = 150 - 258.64 = -108.64 = 108.64\ \text{N} \uparrow$

Example (4-21):

The total mass of the floor crane and its driver is (5 tons) with the center of gravity at point (G). If the crane is to lift a box of (250 kg), determine the reaction forces of the ground to both wheels at (A) and both wheels at (B) when the boom is in the position shown in Fig. (Ex. 4-21).

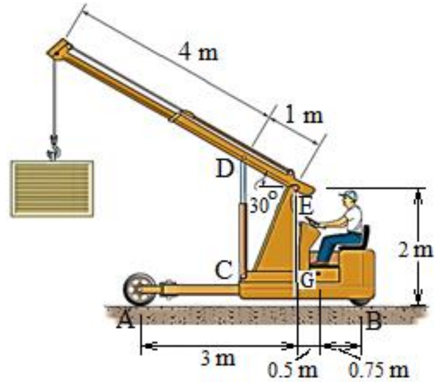
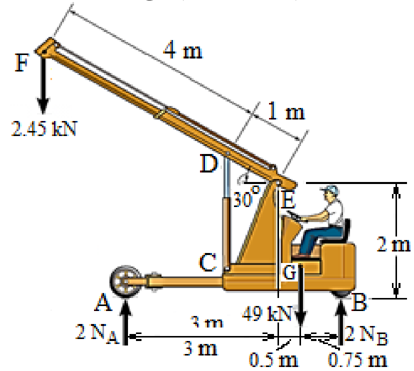


Fig. (Ex. 4-21)

Solution:

$W_{cr} = 5000 \times 9.81 = 49050 \text{ N} = 49 \text{ kN}$

$W_b = 250 \times 9.81 = 2452.5 \text{ N} = 2.45 \text{ kN}$



$\circlearrowleft + \sum M_B = 0 \qquad (2.45)(5 \cos 30 + 1.25) + (49)(0.75) - (2 N_A)(4.25) = 0$

$2 N_A = 11.86 \text{ kN} \quad \Rightarrow \quad N_A = 5.93 \text{ kN}$

$+\uparrow \sum F_y = 0 \qquad 11.86 - 49 - 2.45 + 2 N_B = 0$

$2 N_B = 39.59 \text{ kN} \ \Rightarrow \ N_A = 19.79 \text{ kN}$

Example (4-22):

The mass of the mobile crane is (60 tons) with a center of gravity at (G_1), and the mass of the boom is (15 tons) with a center of gravity at (G_2). Determine the smallest angle of tilt (θ) of the boom, without causing the crane to overturn if the suspended load is (W = 200 kN). Neglect the thickness of the tracks at (A) and (B).

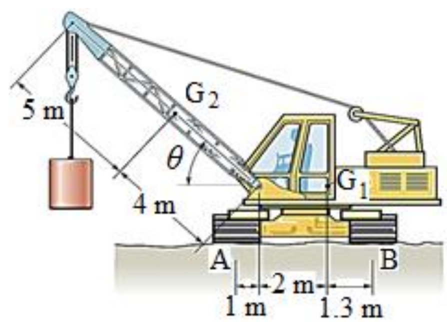
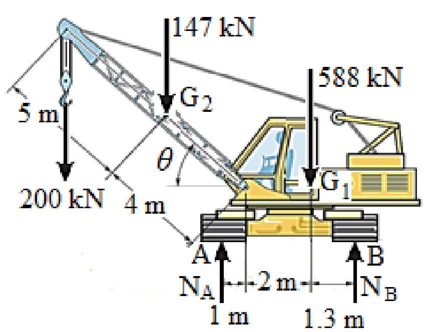


Fig. (Ex. 4-22)

Solution:

$W_c = 60000 \times 9.81 = 588600 \text{ N} = 588 \text{ kN}$

$W_b = 15000 \times 9.81 = 147150 \text{ N} = 147 \text{ kN}$



$\circlearrowleft + \sum M_A = 0$

$(200)(9 \cos\theta - 1) + (147)(4 \cos\theta - 1) + (0)(4.3) - (588)(3) = 0$

$1800 \cos\theta - 200 + 588 \cos\theta - 147 - 1764 = 0$

$2388 \cos\theta - 2111 = 0 \quad \Rightarrow \quad 2388 \cos\theta = 211$

$\cos\theta = 0.884 \quad \Rightarrow \quad \theta = \cos^{-1}(0.884) = 27.87°$

Example (4-23):

The crane shown in Fig. (Ex. 4-23) consists of three parts, which have masses of (m_1 = 1750 kg), (m_2 = 450 kg), and (m_3 = 750 kg) and centers of gravity at (G_1), (G_2) and (G_3) respectively. Determine:

(a) The reaction of the earth on each of the four wheels if the weight of the suspended load is (4 kN) and it is pulled at a constant speed.

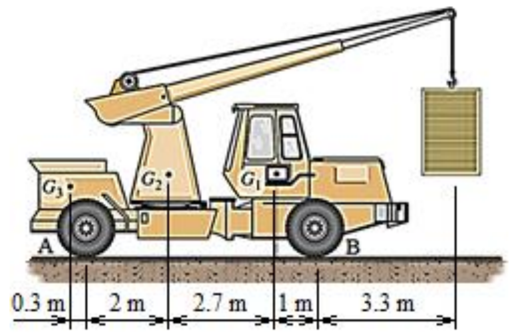


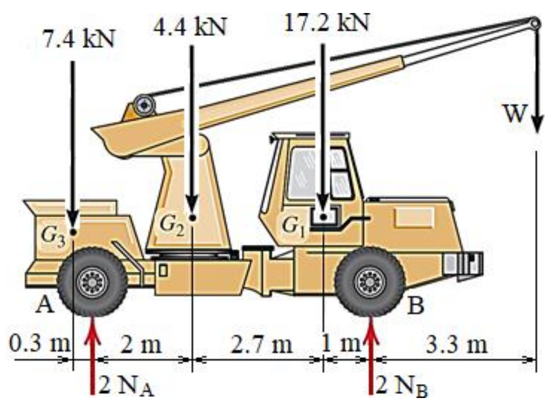
Fig. (Ex. 4-23)

(b) The maximum load the crane can lift without tipping over, When the boom held in the position shown.

Neglect the weight of the boom.

Solution:

$W_1 = 1750 \times 9.81 = 17167.5$ N = 17.2 kN
$W_2 = 450 \times 9.81 = 4414.5$ N = 4.4 kN
$W_3 = 750 \times 9.81 = 7357.5$ N = 7.4 kN



$\curvearrowright + \sum M_A = 0$

$(2N_B)(5.7) - (W)(9) - (17.2)(4.7) - (4.4)(2) + (7.4)(0.3) = 0$

$11.4\, N_B - 9W - 80.84 - 8.8 + 2.22 = 0$

$11.4\, N_B = 9W + 87.42$

$N_B = 0.79W + 7.67 \quad \ldots\ldots\ldots\ldots \quad (1)$

Using the result ($N_B = 0.79W + 7.67$):

$+\uparrow \sum F_y = 0$

$2N_A + 2N_B - W - 17.2 - 4.4 - 7.4 = 0$

$2N_A + 2\,(0.79W + 7.67) - W - 17.2 - 4.4 - 7.4 = 0$

$2N_A + 1.58W + 15.34 - W - 29 = 0$

$2N_A = -0.58W + 13.66$

$N_A = -0.29W + 6.83 \quad \ldots\ldots\ldots\ldots \quad (2)$

a) By substituting the load (W = 4 kN) in equations (1) and (2):

$N_A = -0.29(4) + 6.83 = 5.67$ kN
$N_B = 0.79(4) + 7.67 = 10.83$ kN

b) At the overturning moment of the lever, ($N_A = 0$). And from equation (2):

$0 = -0.29W + 6.83 \quad \Rightarrow \quad W = 23.55$ kN

Problems:

4-10) If the mass of the bicycle is (15 kg) with center of gravity at (G). Determine the normal reactions at (A) and (B) when the bicycle is in equilibrium.

Ans.: $R_A = 81.75\ N$, $R_B = 65.4\ N$

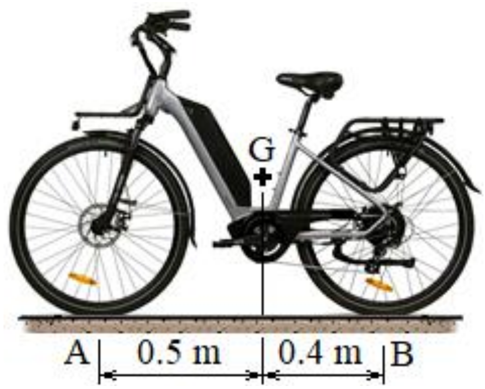


Fig. (Pr. 4-10)

4-11) The cantilever beam shown in Fig. (Pr. 4-11) is subjected to a two external forces and one couple. Compute the reactions at the support point (*O*).

Ans.:
$O_x = 1.73\ kN$, $O_y = 2.5\ kN$, $M_o = 0.5\ k\ N.m\ (CW)$

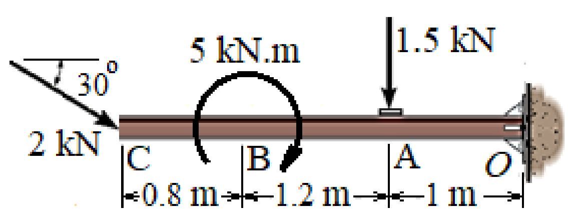


Fig. (Pr. 4-11)

4-12) The total mass of the wheel-barrow and its load is (120 kg) with center of gravity at point (G). Determine the magnitude of the minimum vertical force (F) required to lift the wheelbarrow free of the ground at point (B).

Ans.: $F = 327\ N$

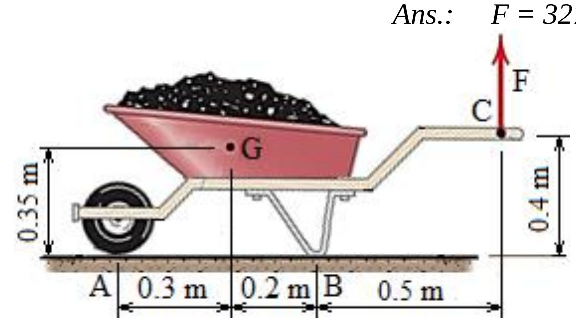


Fig. (Pr. 4-12)

4-13) The center of gravity of the (1.6 tons) pickup truck located at point (G) for the unloaded condition. If a load whose center of gravity is (0.4 m) in front of the rear wheels axle is added to the truck, determine the load weight (W_L) for which the normal reactions under the front and rear wheels are equal.

Ans.: $W_L = 4.13\ kN$

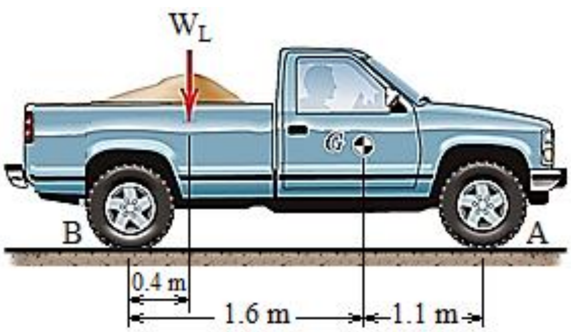


Fig. (Pr. 4-13)

4-14) The total mass of the floor crane and its driver is (5 tons) with the center of gravity at point (G). Determine the largest weight of the box that can be lifted without causing the crane to overturn when its boom is in the position shown in Fig. (Pr. 4-14).

Ans.: $W_b = 64\ kN$

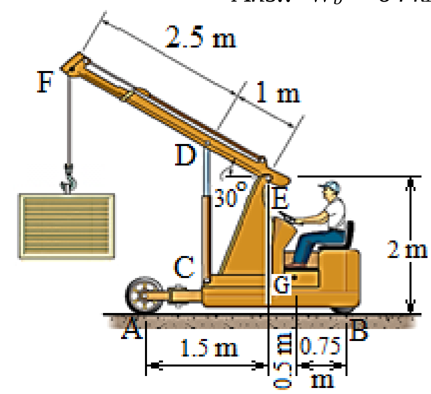


Fig. (Pr. 4-14)

4-15) Excavator weighing (400 kN) at center of gravity (G_1) and its boom weight (50 kN) at center of gravity (G_2) and its arm weight (30 kN) at center of gravity (G_3) and bucket weight (10 kN) at center of gravity (G_4). Determine the reactions at (A) and (B).

Ans.: $N_A = 238.3\ kN$, $N_B = 251.7\ kN$

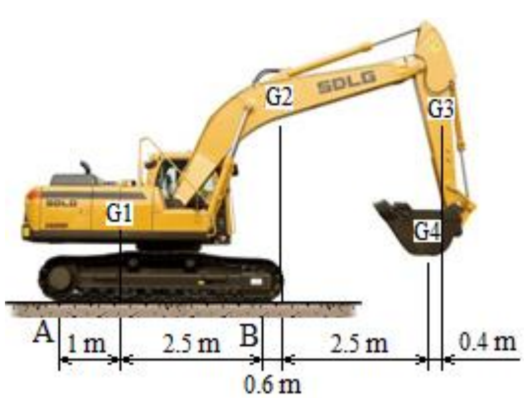


Fig. (Pr. 4-15)

4-16) The ramp shown in Fig. (Pr. 4-16) has a mass of (100 kg) and a center of gravity at (G). Determine the tension in cable (CD) needed to just start lifting the ramp, and the horizontal and vertical components of reaction force at the hinge (pin) at (A).

Ans.:
$T = 955.3\ N$, $A_x = 477.65\ N$, $A_y = 153.7\ N$

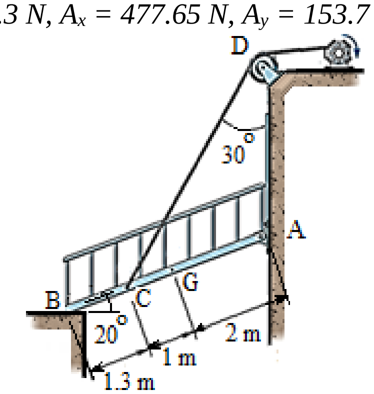


Fig. (Pr. 4-16)

4-17) The dump truck's payload container has a weight of (25 kN) and its center of gravity at point (G), it is hinged with the truck body at point (A), and the hydraulic cylinder is hinged with the truck body at point (C) and with the container hold at point (B). Find the hydraulic cylinder force (F_{CB}) required for equilibrium and the horizontal and vertical components of the reaction force at the join (A).

Ans.:
$F_{CB} = 23.03\ kN$, $A_x = 19.95\ kN$, $A_y = 13.49\ kN$

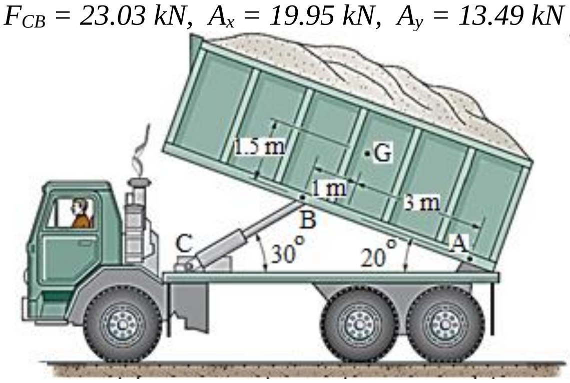


Fig. (Pr. 4-17)

4-18) A worker holds the handles of a building materials wheelbarrow upward, and pushes it forward with a force of (250 N). If the wheelbarrow and its contents have a mass of (50 kg) and center of mass at (G), determine the reactions on the tire.

Ans.: $A_y = 85.9\ N$, $B_x = 250\ N$, $B_y = 404.6\ N$

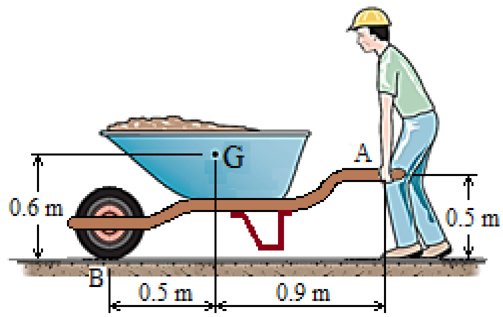


Fig. (Pr. 4-18)

4-19) The beam shown in Fig. (Pr. 4-19) carry a box of (400 kg) mass. Determine the horizontal and vertical components of reaction force at the hinge (pin) at (A) and the reaction force of the rocker (B) on the beam.

Ans.:
$N_B = 3.4\ k\ N$, $A_x = 1.7\ k\ N$, $A_y = 0.98\ k\ N$

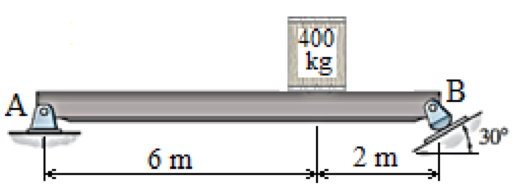


Fig. (Pr. 4-19)

4-20) The jib crane is fixed at (A) and carry a box of (250 kg) mass, as shown in Fig. (Pr. 4-20). Determine the reactions on the jib crane at point (A).

Ans.: $A_x = 1.225\ kN$, $A_y = 2.122\ kN$
$M_A = 8.73\ kN.m$ (C.W)

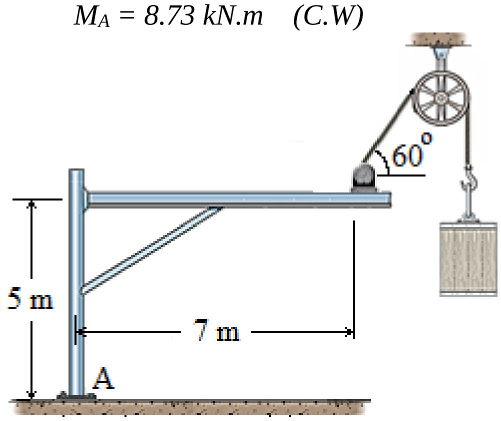


Fig. (Pr. 4-20)

CHAPTER SIX
FRICTION

In the previous chapters, it was assumed that the contact surfaces between the contacting bodies are smooth, that mean, the reaction of one body to the other is perpendicular to the contact surface. But in practical life, the contact surface is not smooth and this leads to the reaction not being perpendicular to the contact surface. And when analyzing the reaction force into two components, one of which is perpendicular to the contact surface and the second tangential to the contact surface, the tangential component is called the frictional force, and the effect of this force is usually opposite to the direction of motion.

Definition of friction:

Friction is the force that resists the motion of two connected surfaces that slide over each other. Usually the effect of this force is opposite to the direction of motion. Often there are friction forces between two rough surfaces in contact with a relative motion between them and these forces act on the body to resist its motion or tendency to move.

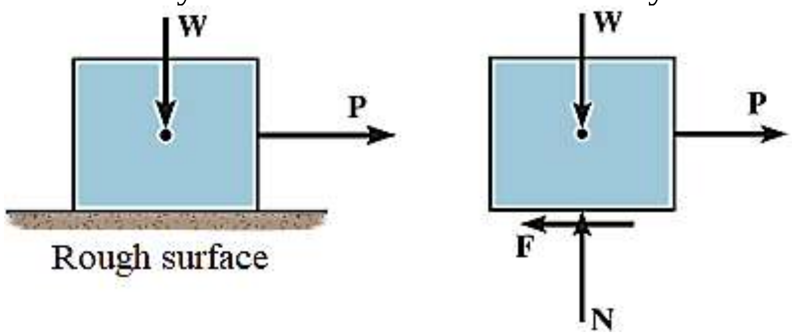


Fig. (6-1) Friction

The importance of friction and its uses:

Friction is a normal and common phenomenon that occurs in everyday life and in many industries. There are many different uses for controlling and benefiting from friction, including:

1- Heat generation: Friction can be used to generate heat, and this is what happens when you rub your hands together quickly or when an object moves quickly on another surface. This heat can be used in processes such as heating foods or operating thermal machines.
2- Movement control: Friction is used in brake systems and clutches for vehicles, trains, and many equipments to control movement and stop the body safely.
3- Taking advantage of static electricity: sliding and friction between two surfaces can generate static electric charges, and this phenomenon is used in static electricity meters and electrostatic printers.

4- Dynamic movement and surface movement: Friction can be used to transform movement between two surfaces, such as using shoes on the ground for walking and using car tires for driving, friction is also used in belt drives, wedges, and other practical life applications, as it without friction, these devices cannot performs its functions.
5- Reducing slip: Friction can be used to increase stability and reduce slip, and this is done by sports shoes or tires specially designed for off-road.
1- Manufacturing and production: Friction is used in manufacturing processes such as cutting, grinding, welding and forming to achieve the required changes in shape and size.

Disadvantages of friction:

Friction can cause wear of surfaces, and this phenomenon may damage materials and machinery. However, it can be controlled and reduced by using lubrication or using less corrosive materials.

Friction coefficient:

The region up to the starting point of the slip or the start of the slip is called the region of friction, and the value of the friction force can be calculated by the equations of equilibrium, where these force are confined between zero and its maximum value. We note that the maximum static friction value of two contact surfaces is directly proportional to the normal force (N).

$$F_s \propto N$$

$$F_s = \mu_s N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (6\text{-}1)$$

Where (μ_s) is the constant of proportionality and is known as the *coefficient of static friction*, and this equation describes the maximum value of static friction, and therefore the equation applies at the moment of starting the movement and not before or after it.

After slipping, the state of motion is accompanied by what is called kinetic friction, and the force resulting from this friction is called the kinetic friction force and is usually less than the static friction force, considering that the static friction force represents the great force of friction. We note that the value of kinetic friction of two contacting surfaces is also directly proportional to the vertical force (N).

$$F_k \propto N$$

$$F_k = \mu_k N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \quad (6\text{-}2)$$

Where (μ_k) is the constant of proportionality and is known as the *coefficient of kinetic friction*, and this equation describes the value of friction during the motion, that is, after the moment of the start of the motion.

Types of friction:

Static friction:

Static friction is the friction produced at the beginning of the motion as it tries to prevent the body from moving, and the force of the static friction approaches the maximum value of ($F_s = \mu_s N$), where (μ_s) is the *coefficient of static friction*.

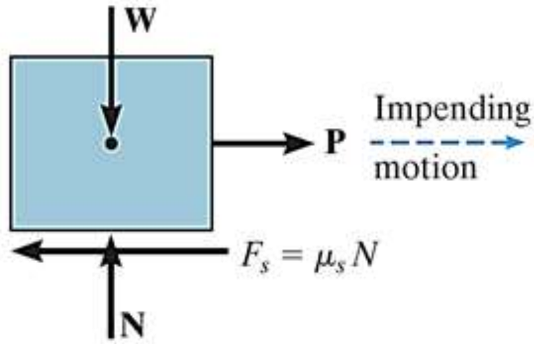


Fig. (6-2) Static friction

Kinetic friction:

Kinetic friction is the friction produced during motion and if slipping occurs, the frictional force remains constant during the period of motion and is equal to ($F_k = \mu_k N$).

Here (μ_k) is the *coefficient of kinetic friction*, It is always less than the coefficient of static friction..

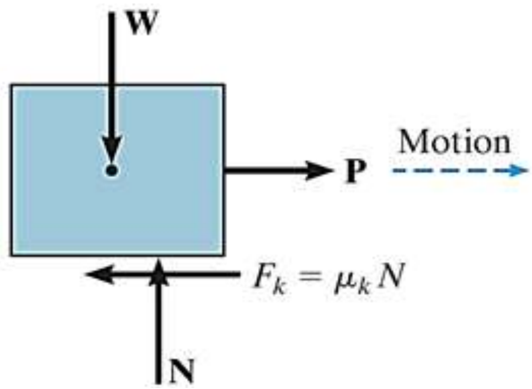


Fig. (6-3) Kinetic friction

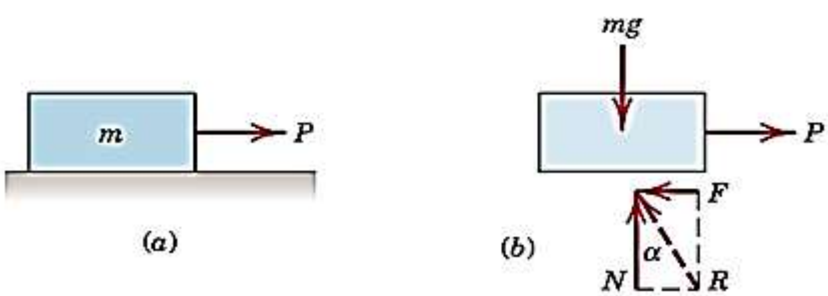


Fig. (6-4) Forces acting on a body while it is moving on a surface it is in contact with

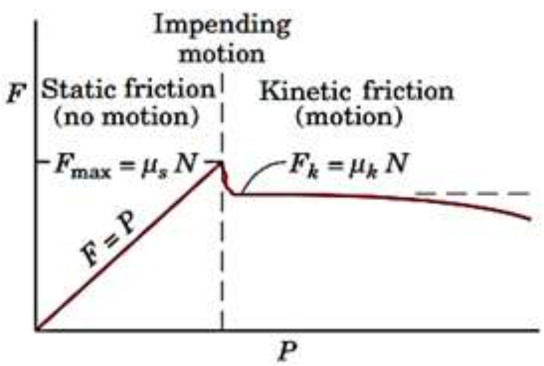


Fig. (6-5) The relationship between the force causing the motion of a body and the frictional force resisting its motion

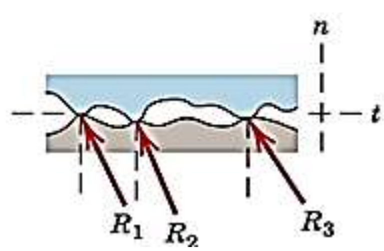


Fig. (6-6) Frictional forces resisting motion between two rough surfaces

It is necessary to draw a free-body diagram to solve the friction problem.

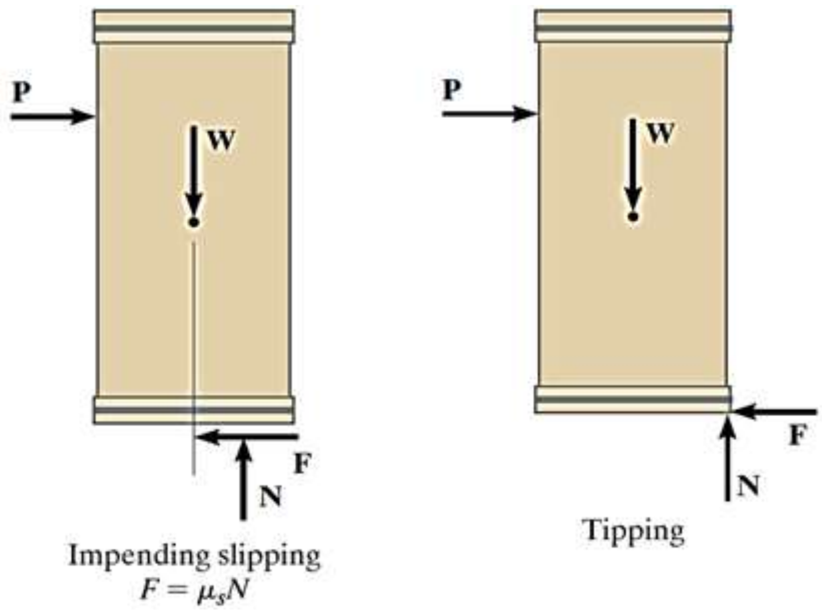


Fig. (6-7) Free body diagram

Characteristics of friction:

1- During motion, the resulting friction force between the contacting surfaces is in a direction opposite to the direction of motion or the tendency of motion for one surface with respect to the other surface (i.e. it is a force resisting the motion) and is tangent to the two surfaces in contact.
2- The normal force to obtain the maximum static frictional force (F_s) of the contacting surfaces must not be so low nor so great that it severely deforms or crushes the contacting surfaces of the bodies.
3- In general, the maximum force of static friction is greater than the force of kinetic friction for any two contacting surfaces. However, if the speed of motion between the two bodies in contact is very low, the force of kinetic friction (F_k) becomes approximately equal to the force of static friction (F_s), i.e. ($\mu_s \approx \mu_k$).
4- The force of static friction between the two contacting surfaces is proportional to the normal force when slipping on the contacting surfaces is about to occur ($F_s = \mu_s N$).
5- The force of kinetic friction between the two contacting surfaces is proportional to the normal force when sliding occurs on the contacting surface ($F_k = \mu_k N$).

Friction angle:

Fig. (6-8) shows that the direction (α) of the resultant (R) measured with a respect to the normal force (N), which is the force of the ground reaction to the body.

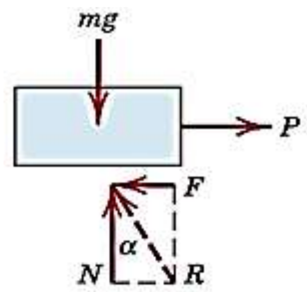


Fig. (6-8) Friction angle

$$\tan \alpha = \frac{F}{N} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6\text{-}3)$$

From the mathematical law of friction:

$$(F_s = \mu_s N) \text{ or } (F_k = \mu_k N) \text{ or } (F = \mu N) \Rightarrow \mu = \frac{F}{N} \quad \ldots\ldots\ldots\ldots \quad (6\text{-}4)$$

$$\therefore \ \mu = \tan \alpha \qquad \mu_s = \tan \alpha_s \qquad \mu_k = \tan \alpha_k \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (6\text{-}5)$$

Where:
(α): Friction angle.
(α_s): Static friction angle.
(α_k): Kinetic friction angle.

Friction laws:

1- The force of friction is directly proportional to the normal force.
2- The force of friction does not depend on the area of contacting surfaces.
3- The maximum force of static friction exceeds the force of kinetic friction.
4- The kinetic friction force does not depend on the relative motion between the contacting surfaces.
5- The coefficient of static friction increases somewhat in the case of very low pressures, and in the case of very high pressures, it increases to the extent that it causes distortions in the contacting bodies.
6- At the low relative velocities between the contacting surfaces, we notice that the coefficient of kinetic friction increases and is apparently equal to the coefficient of static friction.
7- At very high speeds, the coefficient of kinetic friction decreases significantly.
8- Normal temperature changes do not affect the coefficient of friction.

Table (6-1) Typical values of coefficient of friction for some contact surfaces

Contact surfaces	**Static coefficient of friction (μ_s)**	**Kinetic coefficient of friction (μ_k)**
Steel on Steel (dry)	0.6	0.4
Steel on Steel (greasy)	0.1	0.05
Teflon on Steel	0.04	0.04
Wood on Wood	0.5	0.2
Wood on Metal	0.6	0.2
Cast iron on Cast iron	0.4	0.3
Brass on Steel (dry)	0.5	0.4
Rubber on Concrete	0.8	0.6
Wire rope on Iron pulley (dry)	0.2	0.15
Metal on Stone	0.5	0.2
Metal on Ice		0.02
Rubber on Ice	0.2	0.05

Example (6-1):

Determine the highest value of the angle of inclination of the inclined surface (θ) with the horizontal plane before the block of mass (m) begins to slide, given that the coefficient of friction between the mass and the inclined surface is (μ).

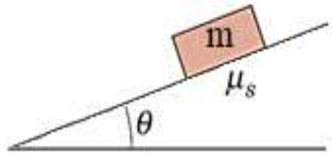
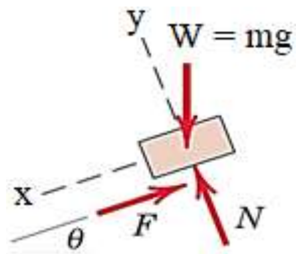


Fig. (Ex. 6-1)

Solution:

To achieve equilibrium in the directions x- and y- axes:



$\sum F_x = 0$ $\quad$ $mg \sin\theta - F = 0$ $\quad$ $F = mg \sin\theta$ ….. (1)

$\sum F_y = 0$ $\quad$ $-mg \cos\theta + N = 0$ $\quad$ $N = mg \cos\theta$ ….. (2)

Dividing Eq. (1) by Eq. (2):

$$\frac{F}{N} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta$$

Since the maximum angle occurs when ($F = F_{max} = \mu_s N$), for impending motion we have:

$\mu_s = \tan\theta_{max}$ $\quad$ or $\quad$ $\theta_{max} = \tan^{-1}\mu_s$

Example (6-2):

The uniform crate shown in Fig. (Ex. 6-2) has a mass of (30 kg). If a force (P = 100 N) is applied to the crate, show the movement status of the crate. The coefficient of static friction is ($\mu_s = 0.3$).

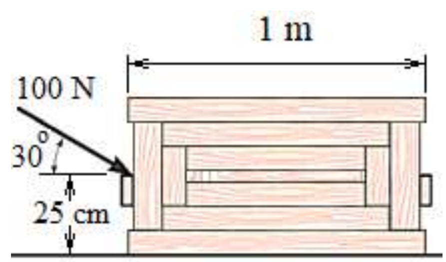
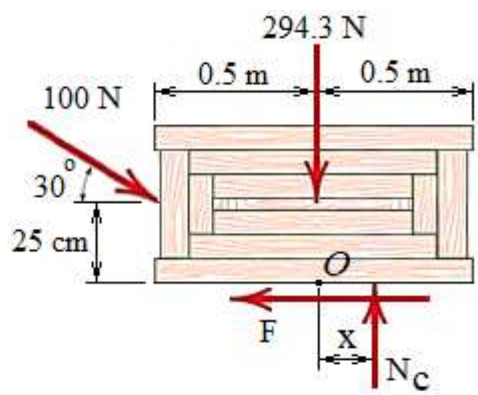


Fig. (Ex. 6-2)

Solution:



$$W = m\,g = 30 \times 9.81 = 294.3 \text{ N}$$

$+\rightarrow \sum F_x = 0$ $\quad$ $100 \cos 30° - F = 0$

$F = 86.6$ N

$+\uparrow \sum F_y = 0$ $\quad$ $-100 \sin 30° + N_C - 294.3 = 0$

$N_C = 344.3$ N

$$F_{max} = \mu_s N_C = 0.3\,(344.3) = 103.29 \text{ N}$$

Since (F = 86.6 N < 103.29 N), the crate will not slip.

Example (6-3):

If the mass of the block shown in the figure (Ex. 6-3) is (100 kg), find the magnitude and direction of the frictional force acting on the block in the case of:

a- (P = 500 N).

b- (P = 100 N).

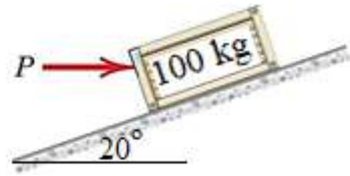
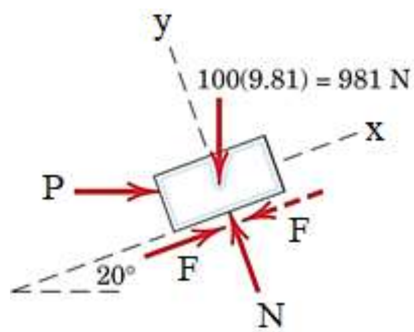


Fig. (Ex. 6-3)

The coefficient of static friction is (0.2), and the coefficient of kinetic friction is (0.17). The forces are applied when the block initially at rest.

Solution:



$+\rightarrow \sum F_x = 0 \qquad P\cos 20° + F - 981\sin 20° = 0$

$+\uparrow \sum F_y = 0 \qquad N - P\sin 20° - 981\cos 20° = 0$

Case I: $P = 500$ N

$F = -134.3 \text{ N} = 134.3 \text{ N} \swarrow$

$N = 1093$ N

$F_{max} = \mu_s N \qquad F_{max} = 0.2 \times 1093 = 219$ N

We note that the force (P) is greater than the force required for equilibrium (F), which means that the equilibrium assumption is correct. The answer will be:

$F = 134.3$ N down the plane

Case II. $P = 100$ N

Substitution into the two equilibrium equations gives**:**

$F = 242$ N

$N = 956$ N

But the maximum possible static friction force is:

$F_{max} = \mu_s N \qquad F_{max} = 0.2(956) = 191.2$ N

The force (P) is smaller than the force required for equilibrium (F). Therefore, equilibrium can be achieved, and we get the correct value of the friction force using the coefficient of kinetic friction associated with the movement down the plane. So the answer is:

$F_k = \mu_k N \qquad F = 0.17(956) = 162.5$ N up the plane

Example (6-4):

A ladder with uniform mass of (12 kg) rests against the smooth wall at (B), and the end (A) rests on the rough horizontal plane for which the coefficient of static friction is ($\mu_s = 0.3$).

Determine the angle of inclination (θ) of the ladder and the normal reaction at (B) if the ladder is about to slip.

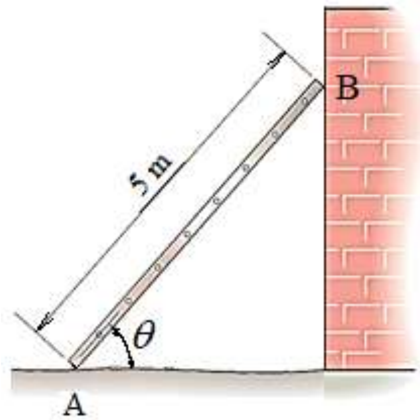
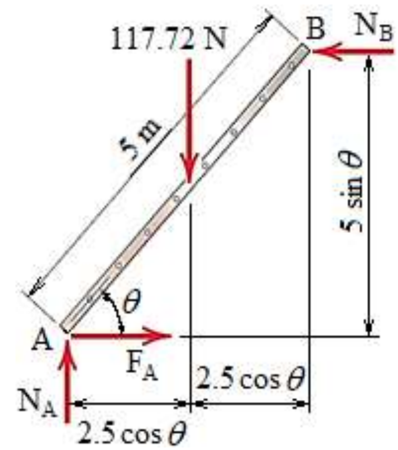


Fig. (Ex. 6-4)

Solution:



$W = m\,g = 12 \times 9.81 = 117.72$ N

$F_A = \mu_s N_A = 0.3 N_A$

$+\uparrow \sum F_y = 0 \qquad N_A - 117.72 = 0$

$N_A = 117.72$ N

$F_A = 0.3 \times 117.72 = 35.316$ N

$+\rightarrow \sum F_x = 0 \qquad 35.316 - N_B = 0$

$N_B = 35.316$ N

$\circlearrowleft + \sum M_B = 0$

$(35.316)(5 \sin\theta) - (117.72)(5 \cos\theta) + (117.72)(2.5 \cos\theta) = 0$

$176.58 \sin\theta - 588.6 \cos\theta + 294.3 \cos\theta = 0$

$176.58 \sin\theta - 294.3 \cos\theta = 0 \qquad \div \cos\theta$

$176.58 \tan\theta - 294.3 = 0$

$176.58 \tan\theta = 294.3$

$\tan\theta = 1.667$

$\theta = 59°$

Example (6-5):

Find the minimum coefficient of static friction between the shoe of the man and the surface of the ground so that the man can move the box, if you know that the coefficient of static friction between the box and the surface of the ground is ($\mu_s = 0.3$), the mass of the box is (120 kg), and the mass of the man is (75 kg).

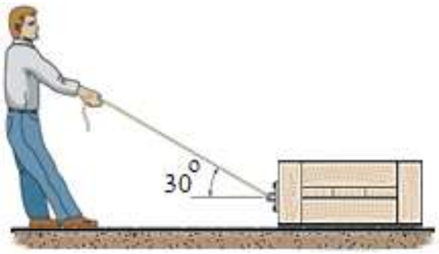


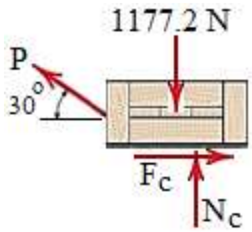
Fig. (Ex. 6-5)

Solution:

$$W_c = 120 \times 9.81 = 1177.2 \text{ N}$$
$$W_m = 75 \times 9.81 = 735.75 \text{ N}$$

For the crate:

$$F_C = \mu_s N_C = 0.3 N_C$$



$+\uparrow \sum F_y = 0$ $\quad N_C + P \sin 30° - 1177.2 = 0$

$N_C + 0.5 P - 1177.2 = 0 \quad \ldots\ldots\ldots \quad (1)$

$+\rightarrow \sum F_x = 0$ $\quad F_C - P \cos 30° = 0$

$0.3 N_C - 0.866 P = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$

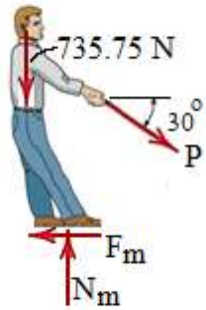
From (2) : $\quad N_C = 2.887 P$

Sub. In (1) : $\quad 2.887 P + 0.5 P - 1177.2 = 0$

$3.387 P = 1177.2 \quad \Rightarrow \quad P = 347.56 \text{ N}$

$N_C = 2.887 \times 347.56 = 1003.4 \text{ N}$

For the man:



$+\uparrow \sum F_y = 0$ $\quad N_m - 347.56 \sin 30° - 735.75 = 0$

$N_m = 909.53 \text{ N}$

$+\rightarrow \sum F_x = 0$ $\quad 347.56 \cos 30° - F_m = 0$

$F_m = 301 \text{ N}$

Thus, the required minimum coefficient of static friction between the man's shoes and the ground is given by:

$$\mu'_s = \frac{F_m}{N_m} = \frac{301}{909.53} = 0.33$$

Example (6-6):

The center of gravity of the car shown in Fig. (Ex. 6-6) is located at point (G). If the car is moving on a sideways road at a constant speed, and the coefficient of static friction between the shoulder of the road and the tires is ($\mu_s = 0.4$), determine the greatest slope the road shoulder can have without causing the car to slip or tip over if the car travels along the road shoulder at constant velocity.

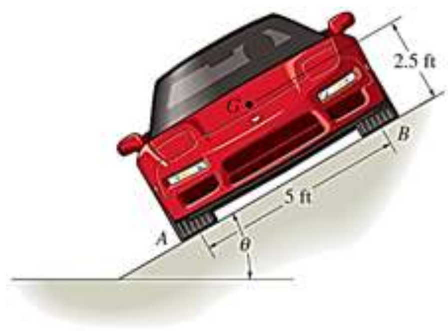
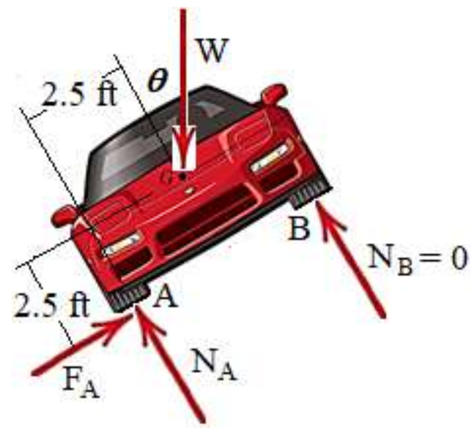


Fig. (Ex. 6-6)

Solution:

Tipping:



$\circlearrowright + \sum M_A = 0$

$$-(W\cos\theta)(2.5) + (W\sin\theta)(2.5) = 0$$

$$(W\cos\theta)(2.5) = (W\sin\theta)(2.5)$$

$$\tan\theta = 1 \qquad \Rightarrow \qquad \theta = 45°$$

Slipping:

$\nearrow + \sum F_x = 0$

$$0.4\,N - W\sin\theta = 0 \qquad \ldots\ldots\ (1)$$

$\nwarrow + \sum F_y = 0$

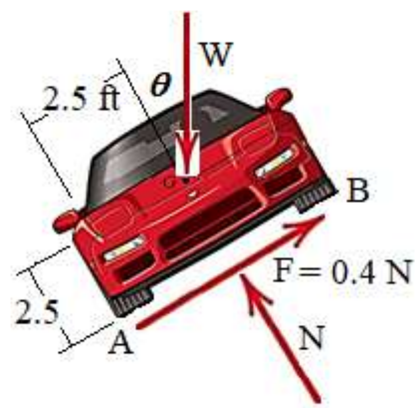
$$N - W\cos\theta = 0 \qquad \ldots\ldots\ldots\ (2)$$

------------------------- (Division)

$$0.4 - \tan\theta = 0$$

$$\tan\theta = 0.4 \qquad \Rightarrow \qquad \theta = 21.8°$$

car slips before it tips



Example (6-7):

Determine the range of mass values (m_o) so that the (100 kg) block shown in Fig. (Ex. 6-7) will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is (0.3).

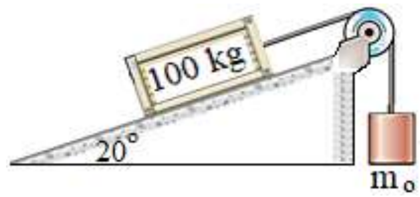
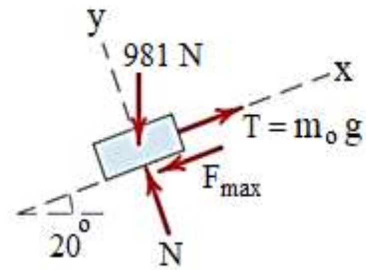


Fig. (Ex. 6-7)

Solution:

The maximum value of (m_o) will be given by the requirement for motion impending up the plane.



$$W = m\,g = 100 \times 9.81 = 981 \text{ N}$$

$+\uparrow \sum F_y = 0 \qquad N - 981 \cos 20° = 0$

$N = 922 \text{ N}$

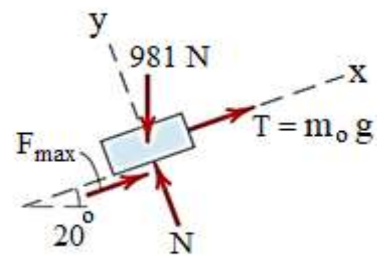
$F_{max} = \mu_s N \qquad F_{max} = 0.3 \times 922 = 277 \text{ N}$

$+\rightarrow \sum F_x = 0$

$m_o \times 9.81 - 227 - 981 \sin 20° = 0$

$m_o = 57.3 \text{ kg}$

The minimum value of (m_o) is determined when motion is impending down the plane.



Case II

$+\rightarrow \sum F_x = 0$

$m_o \times 9.81 + 227 - 981 \sin 20° = 0$

$m_o = 11.06 \text{ kg}$

Thus, (m_o) may have any value from (11.06 kg - 57.3 kg).

Example (6-8):

The parking brake system shown in Fig. (Ex. 6-8) consists of an arm hinged with a friction block at (B).

If you know that the coefficient of static friction between the wheel and the arm is (μ_s = 0.3), and a torque of (3 Ib.ft) is applied to the disc, determine whether the brake is able to hold the wheel when the force that applied to the arm:

(a) F = 6 Ib

(b) F = 14 Ib.

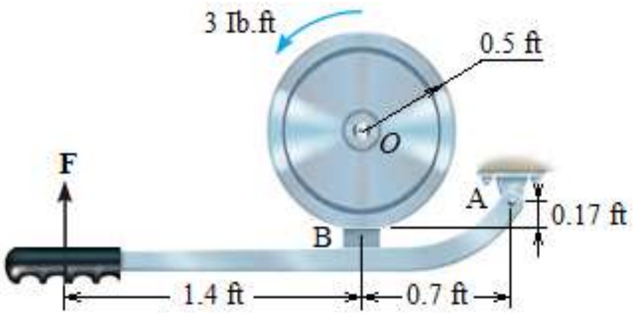


Fig. (Ex. 6-8)

Solution:

Disc:

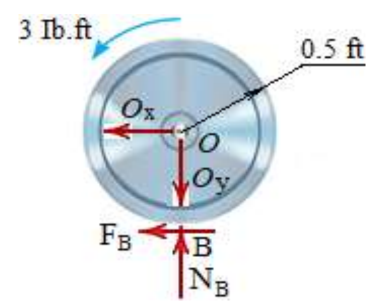
$\circlearrowleft + \sum M_o = 0$

$3 - F_B(0.5) = 0$

$F_B = 6$ Ib

$F_B = \mu_s N_B$

$N_B = \dfrac{F_B}{\mu_s} = \dfrac{6}{0.3} = 20$ Ib



Lever:

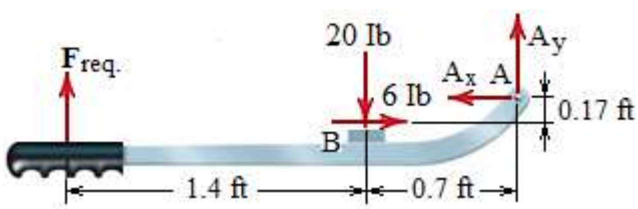
$\circlearrowleft + \sum M_A = 0$

$(20)(0.7) + (6)(0.17) - (F_{Req})(2.1) = 0$

$F_{Req.} = 7.15$ Ib

(a) F = 6 Ib < 7.15 Ib No

(b) F = 14 Ib > 7.15 Ib Yes



Example (6-9):

The mass of the tractor shown in Fig. (Ex. 6-9) is (2.5 tons) and its center of gravity is at (G). The rear wheels exert traction force at point (B), while the front wheels at point (A) are free to roll.

If the coefficient of static friction between the wheels at (B) and the ground is (μ_s = 0.45), and between the contaner of (1 ton) and the ground is (μ_s = 0.5), determine whether the contaner can be pulled without causing the rear wheels at (B) to slip or lifting the front wheels off the ground at (A).

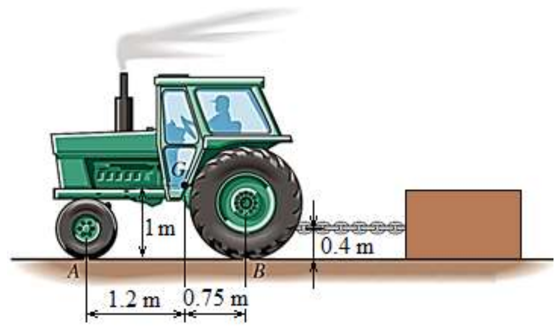
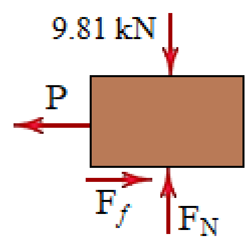


Fig. (Ex. 6-9)

Solution:

$$W_t = 2500 \times 9.81 = 24525 \text{ N} = 24.525 \text{ kN}$$
$$W_c = 1000 \times 9.81 = 9810 \text{ N} = 9.81 \text{ kN}$$

For Container:



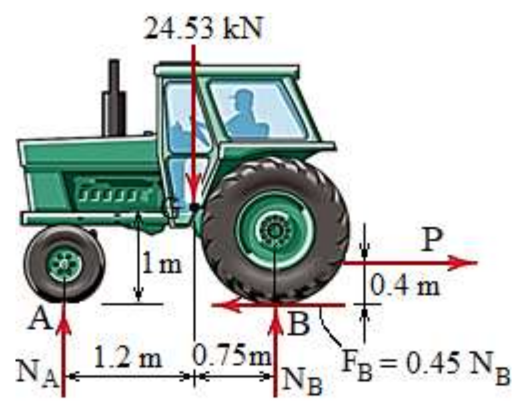
$+\uparrow \sum F_x = 0 \qquad F_N - 9.81 = 0 \qquad F_N = 9.81 \text{ kN}$

$F_f = \mu_s F_N = 0.5 \times 9.81 = 4.905 \text{ kN}$

$+\rightarrow \sum F_x = 0 \qquad 4.905 - P = 0 \qquad P = 4.905 \text{ kN}$

For Tractor:

Slipping:



$\circlearrowleft + \sum M_A = 0$

$-(24.525)(1.2) - (4.905)(0.4) + N_B\,(1.95) = 0$

$N_B = 16.1 \text{ kN}$

$+\rightarrow \sum F_x = 0$

$P = F_B = 0.45\,N_B = 0.45 \times 16.1$

$= 7.25 \text{ kN}$

Tipping ($N_A = 0$):

$\circlearrowleft + \sum M_B = 0 \qquad -(P)(0.4) + (24.525)(0.75) = 0 \qquad P = 46 \text{ kN}$

Since $P_{Required} = 4.905 \text{ kN} < 7.25 \text{ kN} < 46 \text{ kN}$

It is possible to pull the load without slipping or tipping.

Example (6-10):

In Fig. (Ex. 6-10), determine the range of mass (m) for which the (50 kg) block is in equilibrium. All wheels and pulleys have negligible friction.

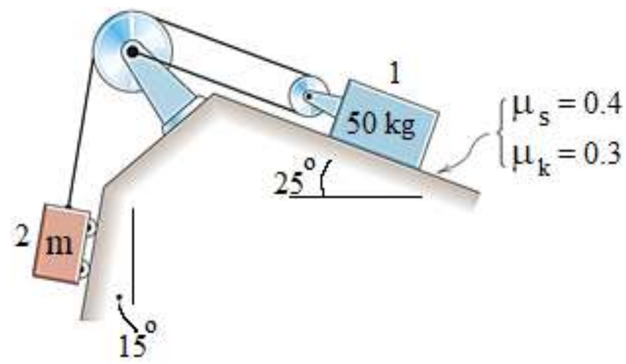
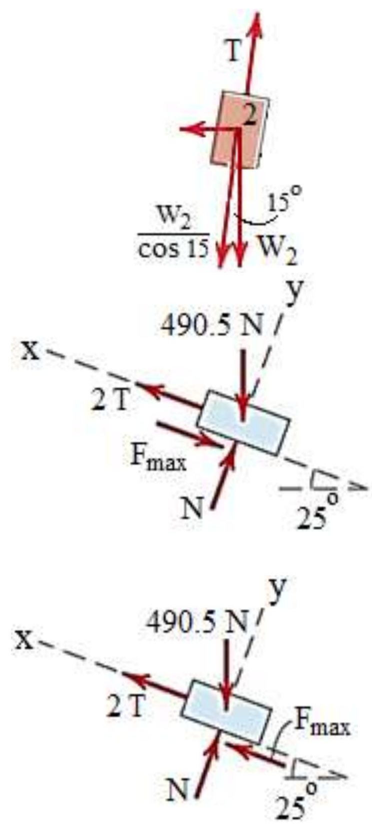


Fig. (Ex. 6-10)

Solution:



$W_1 = 50 \times 9.81 = 490.5$ N

$$T = \frac{W_2}{\cos 15}$$

$\sum F_y = 0$

$N - 490.5 \cos 25 = 0$

$N = 444.54$ N

$F_{max} = \mu_s N = 0.4 \times 444.54 = 177.82$ N

$\sum F_x = 0$

$2\,T - F_{max} - W_1 \sin 25 = 0$

$$2\left(\frac{W_2}{\cos 15}\right) - 177.82 - 490.5 \sin 25 = 0$$

$2.07\, W_2 - 177.82 - 207.29 = 0$

$2.07\, W_2 = 385.11$ N

$W_2 = 186$ N

$$m_2 = \frac{W_2}{g} = \frac{186}{9.81} = 18.96 \text{ kg}$$

$$2\left(\frac{W_2}{\cos 15}\right) + 177.82 - 490.5 \sin 25 = 0$$

$2.07\, W_2 + 177.82 - 207.29 = 0$

$2.07\, W_2 = 29.47$ N

$W = 14.24$ N

$$m_2 = \frac{W_2}{g} = \frac{14.24}{9.81} = 1.45 \text{ kg}$$

the range of mass is: (1.45 kg - 18.96 kg)

Example (6-11):

The car shown in Fig. (Ex. 6-11) is just beginning to negotiate the (15°) ramp. If the mass of the car is (1.2 ton) and it has rear-wheel drive, determine the minimum coefficient of static friction required at (B).

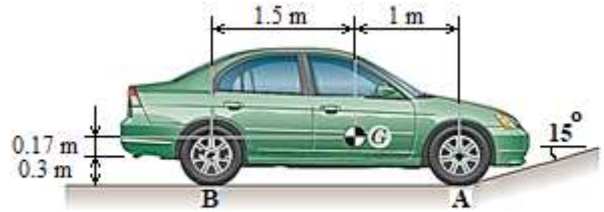
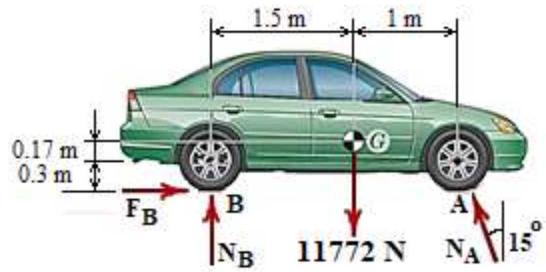


Fig. (Ex. 6-11)

Solution:

$W = m\,g = 1200 \times 9.81 = 11772$ N



$\circlearrowleft + \sum M_B = 0$

$(N_A \cos 15)(2.5) + (N_A \sin 15)(0.3) - (11772)(1.5) = 0$

$2.415\ N_A + 0.078\ N_A = 17658$

$N_A = 7083$ N

$\sum F_y = 0 \qquad 7083 \cos 15 + N_B - 11772 = 0$

$6841.65 + N_B - 11772 = 0 \qquad N_B = 4930.35$ N

$\sum F_x = 0 \qquad F_B - 7083 \sin 15 = 0 \qquad F_B = 1833.22$ N

$F_B = \mu_s N_B \quad \Rightarrow \quad \mu_s = F_B / N_B = 1833.22 / 4930.35 = 0.37$

Example (6-12):

Four bolts are used to connect the plates shown in Fig. (Ex. 6-12), each of which is tightened so that it is subjected to a tensile force of (5 kN). Find the maximum value of the force (F) that can be supported by this connection so that no slippage occurs between the plates. Note that the coefficient of static friction between each two plates is (μ_s = 0.45).

Solution:

The tension subjected to the four bolts:

$T = N = 5 \times 4 = 20$ kN

$f = \mu_s N = 0.45 \times 20 = 9$ kN

$\sum F_y = 0 \qquad f - \frac{F}{2} = 0 \qquad 9 - \frac{F}{2} = 0$

$F = 18$ kN

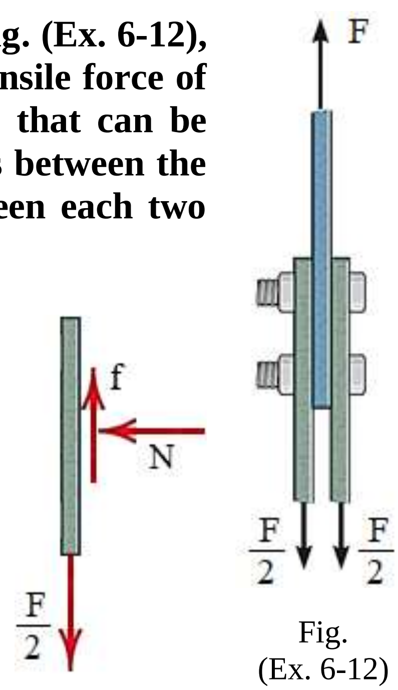


Fig. (Ex. 6-12)

Example (6-13):

A man of mass (90 kg) and the coefficient of static friction between his feet and the ground (μ_s = 0.45), uses a system of pulleys to lift a box as shown in Fig. (Ex. 6-13). Find the maximum weight (W) that the man can lift at a constant speed using the guide pulley at (A).

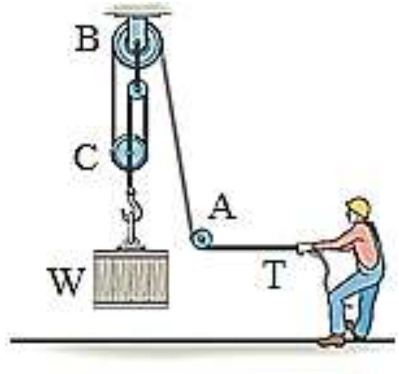


Fig. (Ex. 6-13)

Solution:

$$W_m = m_m\, g = 90 \times 9.81 = 882.9 \text{ N}$$

Bolly C:

$$3T - W = 0$$

$$T = \frac{W}{3}$$

The man:

$\sum F_y = 0$ $\quad N - 882.9 = 0$

$$N = 882.9 \text{ N}$$

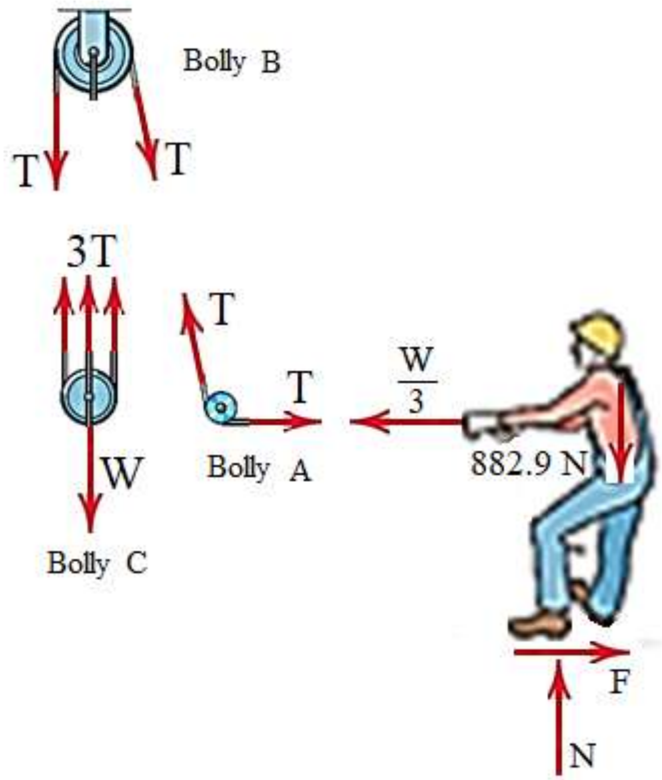
$\sum F_x = 0$ $\quad F - \dfrac{W}{3} = 0$

$$0.45\, N - \frac{W}{3} = 0$$

$$0.45\,(882.9) - \frac{W}{3} = 0$$

$$1324.35 - W = 0$$

$$W = 1192 \text{ N}$$



Example (6-14):

For equilibrium, determine the range of masses (m) of the uniform bar. Neglect friction at all bearings.

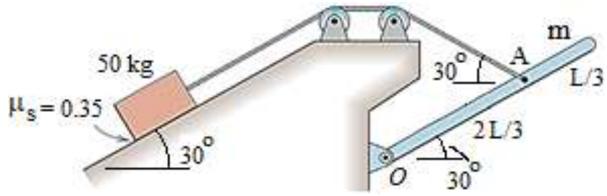
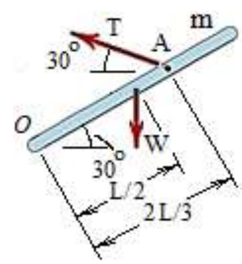


Fig. (Ex. 6-14)

Solution:

$$W_{block} = 50 \times 9.81 = 490.5 \text{ N}$$

Bar:



$$\circlearrowright + \sum M_o = 0$$

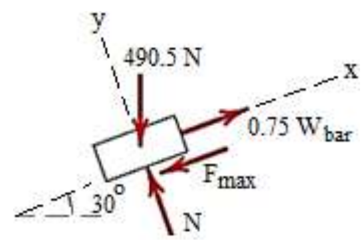
$$(W_{bar}) \left(\frac{L}{2} \cos 30 \right) - (T \sin 60) \left(\frac{2L}{3} \right) = 0$$

$$0.433 \text{ L } W_{bar} - 0.577 \text{ L T} = 0$$

$$0.433 \, W_{bar} - 0.577 \, T = 0$$

$$T = 0.75 \, W_{bar}$$

Block:



$\sum F_y = 0$ $\quad$ $N - 490.5 \cos 30 = 0$

$$N = 424.79 \text{ N}$$

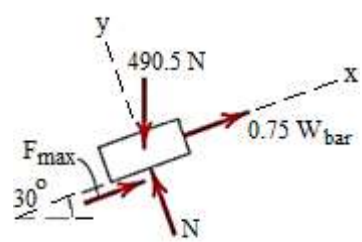
$$F_{max} = \mu N = 424.79 \times 0.35 = 148.67 \text{ N}$$

$\sum F_x = 0$

$$0.75 \, W_{bar} - 148.67 - 490.5 \sin 30 = 0$$

$$W_{bar} = 525.23 \text{ N}$$

$$m_{bar} = \frac{W}{g} = \frac{525.23}{9.81} = 53.54 \text{ kg}$$



$$0.75 \, W_{bar} + 148.67 - 490.5 \sin 30 = 0$$

$$W_{bar} = 128.77 \text{ N}$$

$$m_{bar} = \frac{W}{g} = \frac{128.77}{9.81} = 13.13 \text{ kg}$$

the range of masses is: (13.13 kg - 53.54 kg)

<u>Example (6-15):</u>

Determine the maximum angle (θ) which the adjustable arm can be tilted with the horizontal before the block of mass (m) begins to slip. The coefficient of static friction between the block and the inclined surface is (μ_s).

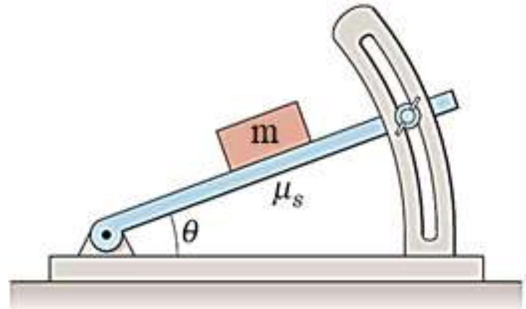
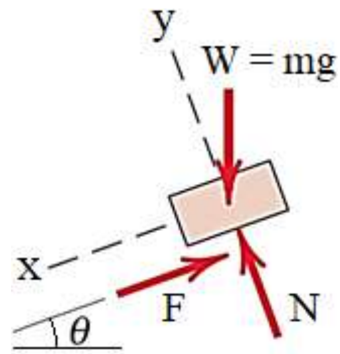


Fig. (Ex. 6-15)

Solution:



$\sum F_x = 0$

$mg\sin\theta - F = 0 \qquad F = mg\sin\theta \quad \ldots\ldots (1)$

$\sum F_y = 0$

$-mg\cos\theta + N = 0 \qquad N = mg\cos\theta \quad \ldots\ldots (2)$

Dividing Eq. (1) by Eq. (2):

$$\frac{F}{N} = \tan\theta$$

Since the maximum angle occurs when ($F = F_{max} = \mu_s N$), for impending motion we have :

$$\mu_s = \tan\theta_{max} \qquad \text{or} \qquad \theta_{max} = \tan^{-1}\mu_s$$

Problems:

6-1) The body with weight (200 N) shown in Fig. (Pr. 6-1) started to move under the action of a horizontal force (400 N), what is the coefficient of friction between the contact surfaces.

Ans.: $\mu = 0.66$

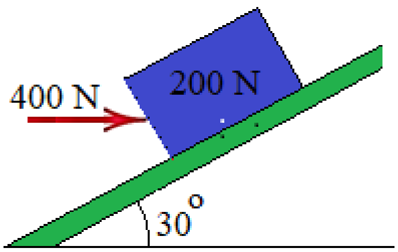


Fig. (Pr. 6-1)

6-2) The force (P = 375 N) was applied to the (100 kg) crate which was stationary before applying the force. Determine the magnitude and direction of the friction force (F) exerted by the horizontal surface on the crate.

Ans.: $F = 490.5\ N \leftarrow$
$F > P$ *No motion.*

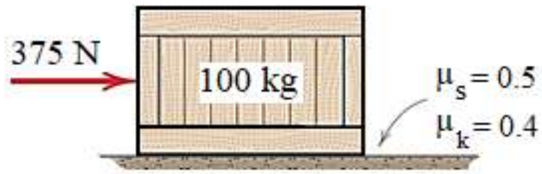


Fig. (Pr. 6-2)

6-3) The mass of the crate shown in Fig. (Pr. 6-3) is (75 kg), and the coefficients of static and kinetic friction are ($\mu_s = 0.3$) and ($\mu_k = 0.2$), respectively. Determine the friction force between the crate and the floor.

Ans.: $(F_f)\ max = 70.725\ N$
$F_f = 47.15\ N.$

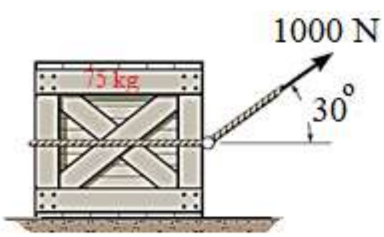


Fig. (Pr. 6-3)

6-4) Determine the limits for the weight values of the body (W) for which the (500 N) block is in equilibrium. All pulleys in the system have negligible friction.

Ans.: (15 N - 156 N)

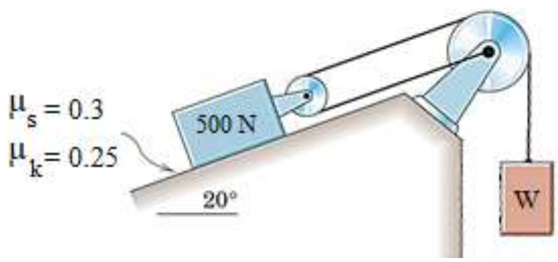


Fig. (Pr. 6-4)

6-5) Find the limits for the mass values of the body (B) that affect the body (A) which has a mass of (100 kg) so that it prevents its movement up and down the inclined surface shown in Fig. (Pr. 6-5), knowing that the value of the friction coefficient between the contact surfaces is ($\mu_s = 0.3$).

Ans.: (24 kg - 76 kg)

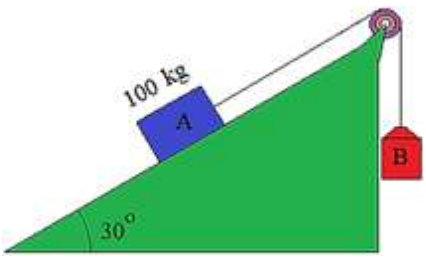


Fig. (Pr. 6-5)

6-6) The two boxes (A) and (B) shown in Fig. (Pr. 6-6) are tied with a flexible non-elongating rope that passes over a smooth (frictionless) pulley. If the masses of the boxes (A) and (B) are (200 N) and (150 N) respectively, show in which direction the motion is.

Ans.: Motion towards the box (B).

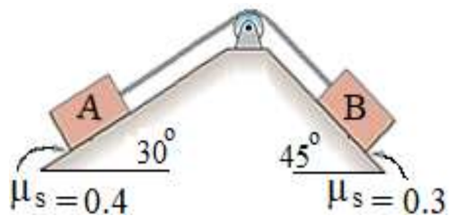


Fig. (Pr. 6-6)

6-7) The (100 lb) uniform box rests on a tile floor with coefficient of static friction of ($\mu_s = 0.25$). Determine the smallest magnitude of force (P) needed to move the box. Also, If the weight of the man pushing the box is (140 Ib), determine the smallest coefficient of static friction between his shoes and the floor so that he does not slip.

Ans.: P = 33.7 Ib, $\mu_s = 0.24$

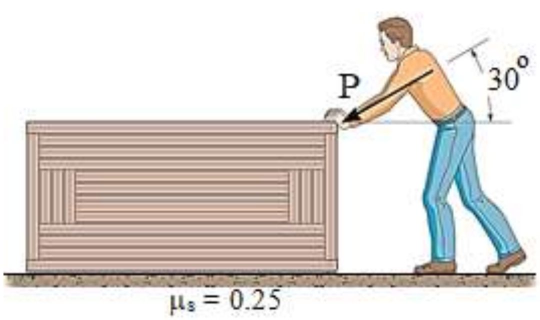


Fig. (Pr. 6-7)

6-8) The ladder of (20 ft) length has a uniform weight of (100 lb) and rests on a smooth wall at point (B). If the coefficient of static friction at point (A) is ($\mu_s = 0.4$), determine if the ladder will slip.

Ans.: The ladder will not slip.

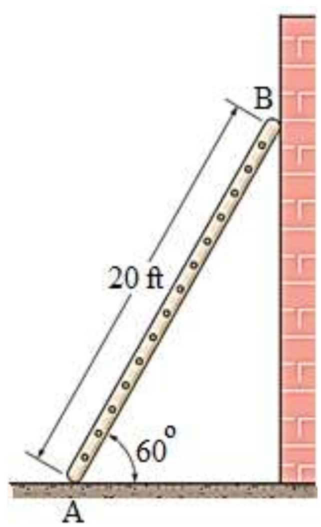


Fig. (Pr. 6-8)

6-9) To achieve equilibrium in the system shown in Figure (Pr. 6-9), determine the limits of the values of the weight (W) for the regular bar. Neglect the friction in all bearings.

Ans.: (31.5 Ib - 128.5 Ib)

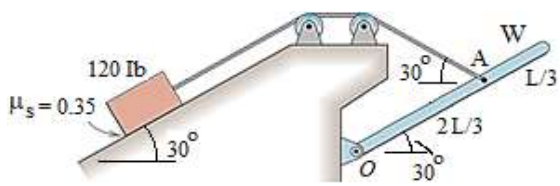


Fig. (Pr. 6-9)

6-10) The (3000 Ib) car is just beginning to negotiate the (15°) ramp. If the car has rear-wheel drive, determine the minimum coefficient of static friction required at (B).

Ans.: $\mu_s = 0.356$

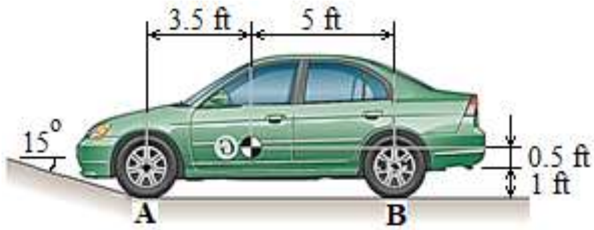


Fig. (Pr. 6-10)

6-11) The mass of the tractor shown in Fig. (Pr. 6-11) is (2.5 tons) and its center of gravity is at (G). The rear wheels exert traction force at point (B), while the front wheels at point (A) are free to roll. If the coefficient of static friction between the wheels and the ground at (B) is (μ_s = 0.45), determine whether a load (P = 6 kN) can be pulled without causing the rear wheels at (B) to slip or lifting the front wheels off the ground at (A).

Ans.: It is possible to pull the load without slipping or tipping.

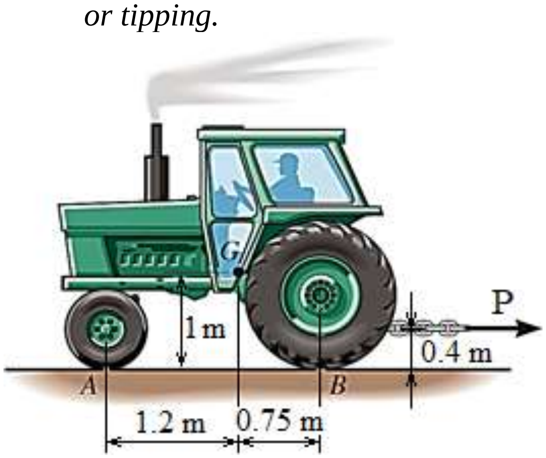


Fig. (Pr. 6-11)

6-12) A man of mass (90 kg) and the coefficient of static friction between his feet and the ground (μ_s = 0.5), uses a system of pulleys to lift a box as shown in Fig. (Pr. 6-12). Find the maximum weight (W) that the man can lift at a constant speed.

Ans.: W = 1250 N

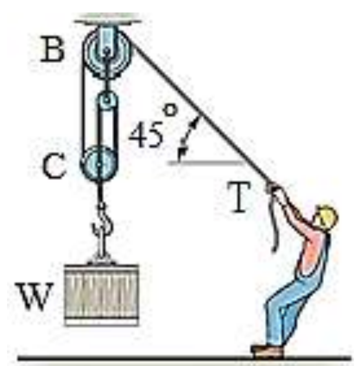


Fig. (Pr. 6-12)

CHAPTER SEVEN
CENTERS OF MASS AND CENTROIDS

Centers of mass and centroids are concepts commonly used in physics, engineering, and mathematics to describe the average position or weighted center of a distribution of mass or points in space.

Center of Mass:

The center of mass of an object is the point where the mass of the entire body is concentrated. For a system of particles, the center of mass can also be defined as the average position of all the particles or elements that make up the body.

Centroid:

The centroid is a concept specifically used in geometry to describe the center of a two-dimensional shape or a three-dimensional object with uniform density. In other words, the centroid is the average position of all the points that make up the shape (center of the length or the area or the volume of the body).

In a practical experiment, a three-dimensional body of a certain size and shape was taken and has a mass of value (m). The body is suspended by a wire in the manner shown in Fig. (7-1) from any point, let it be (A), the body will be in equilibrium under the action of the tension force of the wire and the resultant of gravitational forces of the particles of the body (W). We note that the wire tension and the resultant of gravitational forces of the body particles are on one line of action and in opposite directions. If we assume that the line of action of the forces is represented by an imaginary line. Then the body is suspended from other points such as (B) and (C) and the process is repeated, we notice the lines of action of the forces intersect at one point symbolized as (G), this point is called the center of gravity of the body.

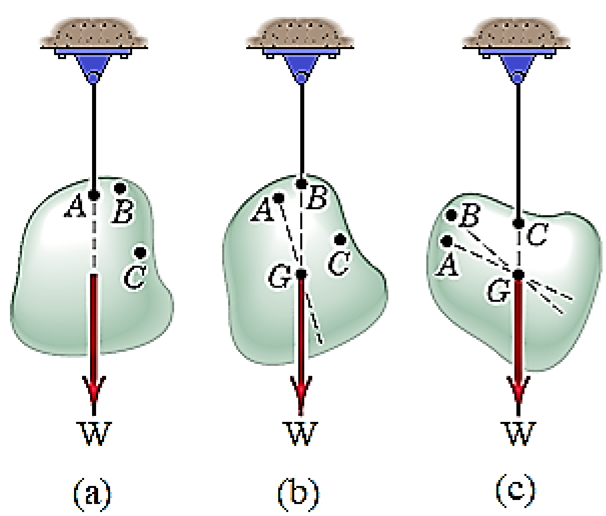


Fig. (7-1) Finding the center of gravity of the body experimentally

Mathematically, the position of the center of gravity for any body is determined by applying the *principle of moments* about an axis parallel to the gravitational forces. The moment of the gravitational forces (W) of the body about any axis is equal to the sum of the moments of the gravitational forces of the particles of this body (dW) about the same axis. The product of the gravitational forces acting on all particles is the weight of the body and is given by the integral ($W = \int dW$). If we apply the principle of moments about the axis (y) for example, then the moment about this axis of the weight of the particle is (x dW), and the sum of these moments for all particles of the body is ($\int x\,dW$). The moment resulting from the sum of the gravitational forces of the body in general must be equal to the sum of the moments resulting from the gravitational forces of its particles. As follows:

$$\bar{x}\,W = \int x\,dW \qquad \bar{y}\,W = \int y\,dW \qquad \bar{z}\,W = \int z\,dW \quad \text{.........} \quad (7\text{-}1)$$

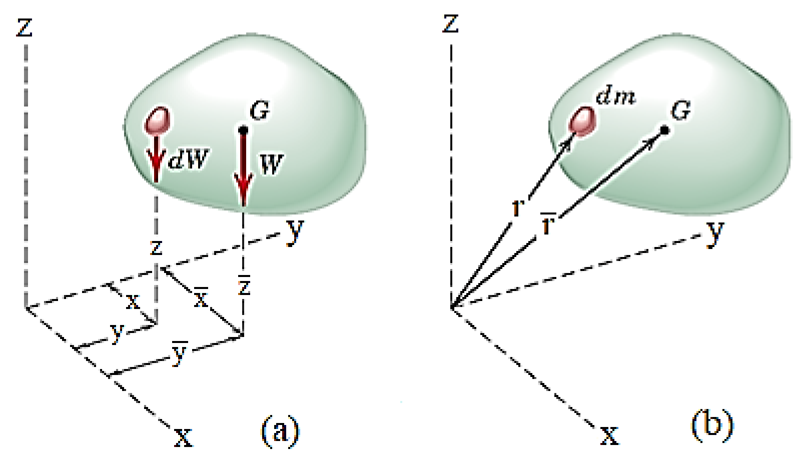


Fig. (7-2) Finding the center of gravity of the body mathematically

From the previous equations, the coordinates of the center of gravity (G) can be expressed as:

$$\bar{x} = \frac{\int x\,dW}{W} \qquad \bar{y} = \frac{\int y\,dW}{W} \qquad \bar{z} = \frac{\int z\,dW}{W} \quad \text{...........} \quad (7\text{-}2)$$

With the substitution of (W = mg) and (dW = g dm), the equations for the coordinates of the center of gravity become as follow:

$$\bar{x} = \frac{\int x\,dm}{m} \qquad \bar{y} = \frac{\int y\,dm}{m} \qquad \bar{z} = \frac{\int z\,dm}{m} \quad \text{...........} \quad (7\text{-}3)$$

Importance of centers:

The importance of defining the centers of objects in terms of (mass, weight, volume, area, and length) becomes clear during the study of the subject of strength of materials (stress analysis):

- To obtain uniform stresses on the surface of any structural or industrial section, the line of action of the resultant of the loads placed on it must pass in its center.
- Knowing the center of the area is important in determining the neutral axis, which is the imaginary line along which the stress is equal to zero.
- Knowing the centers of areas is very important in the subject of the moment of inertia, as the axis that passes through the center of the area is called a central axis, and this is very important in calculating the moment of inertia.
- Knowing the center of the area is important when it is required to use the moment of area, since the moment of area with respect to any axis is equal to the product of the area multiplied by the perpendicular distance from its center to the moment axis.

Coordinates of centers of (lines, areas, and volumes):

1- Lines:

If the body is in the form of a thin rod or wire of length (L), its cross-sectional area is (A), and its density is (ρ), then when taking an element of length (dL), the body approaches a straight segment, and ($dm = \rho \, A \, dL$). If the value of density (ρ) and area (A) are constant along the length of the rod, then the coordinates of the center of mass become the coordinates of the centroid (C) of the line segment.

$$\left.\begin{aligned} \bar{x} &= \frac{\int x \, dL}{L} \\ \bar{y} &= \frac{\int y \, dL}{L} \\ \bar{z} &= \frac{\int z \, dL}{L} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7\text{-}4)$$

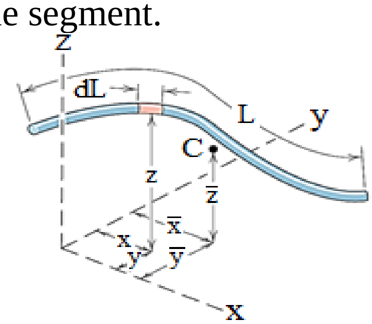


Fig. (7-3) Finding the center of gravity of the lines

2- *Area:*

When the body has a small but constant thickness (t) and a density of (**ρ**), we can model it as a surface area (A). The mass of the element becomes (dm = **ρ** t dA). If the density (**ρ**) and thickness (t) are constant over the entire area, then the coordinates of the center of mass of the body also become the coordinates of the centroid (C) of the surface area.

$$\left.\begin{aligned}\bar{x} &= \frac{\int x\, dA}{A}\\ \bar{y} &= \frac{\int y\, dA}{A}\\ \bar{z} &= \frac{\int z\, dA}{A}\end{aligned}\right\} \quad \ldots\ldots\ldots \; (7\text{-}5)$$

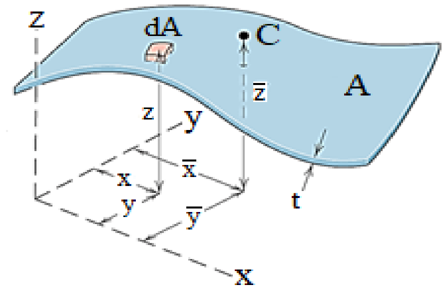


Fig. (7-4) Finding the center of gravity of the areas

3- *Volumes:*

In the case of volume (V) and density (**ρ**), the amount of mass is (dm = **ρ** dV). Density (**ρ**) can be canceled if it is constant over the entire volume, and the coordinates of the center of mass also become the coordinates of the centroid (C) of the body.

$$\bar{x} = \frac{\int x\, dV}{V} \qquad \bar{y} = \frac{\int y\, dV}{V} \qquad \bar{z} = \frac{\int z\, dV}{V} \quad \ldots\ldots\ldots \; (7\text{-}6)$$

<u>Composite bodies and figures:</u>

In the event that a body or figure can be easily divided into several parts, their centers of mass can be easily determined, as the principle of moments can be used and each part is treated as a finite element of the whole. Such a body is schematically illustrated in Fig. (7-5).

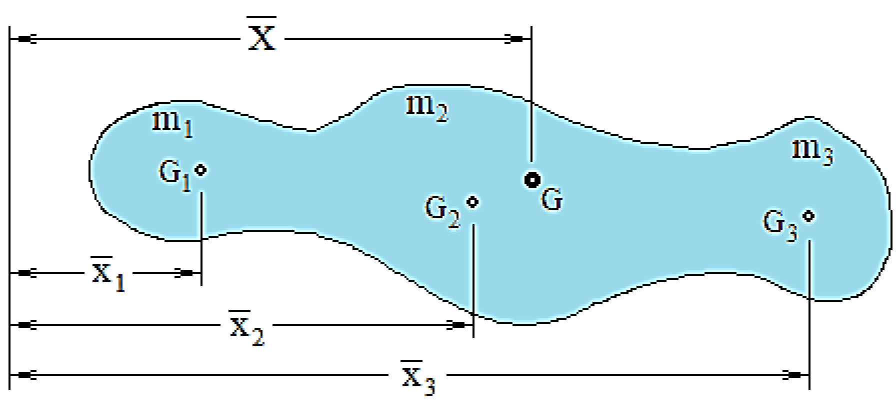


Fig. (7-5) Finding the center of gravity of the Composite bodies

Its parts have masses (m_1), (m_2) and (m_3) with the respective mass-center coordinates in the x-direction.
The moment principle gives:

$$(m_1 + m_2 + m_3) \bar{X} = m_1 \bar{x}_1 + m_2 \bar{x}_2 + m_3 \bar{x}_3 \quad \ldots\ldots\ldots \quad (7\text{-}7)$$

Where ($\bar{X}$) is the (x-coordinate) of the center of mass with respect to the total body. The same relationships apply to the other two directions.

We generalize, then, for a body of any number of parts and express the sums in condensed form to obtain the mass-center coordinates:

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} \qquad \bar{Y} = \frac{\sum m\bar{y}}{\sum m} \qquad \bar{Z} = \frac{\sum m\bar{z}}{\sum m} \quad \ldots\ldots\ldots \quad (7\text{-}8)$$

Approximation method:

The boundaries of an area or volume may not be practically expressed in terms of simple geometric shapes or as shapes that can be represented mathematically. In such cases, we must resort to the approximation method. As an example, the location of the centroid (G) of the irregular region shown in Fig. (7-5) can be determined by the approximation method.

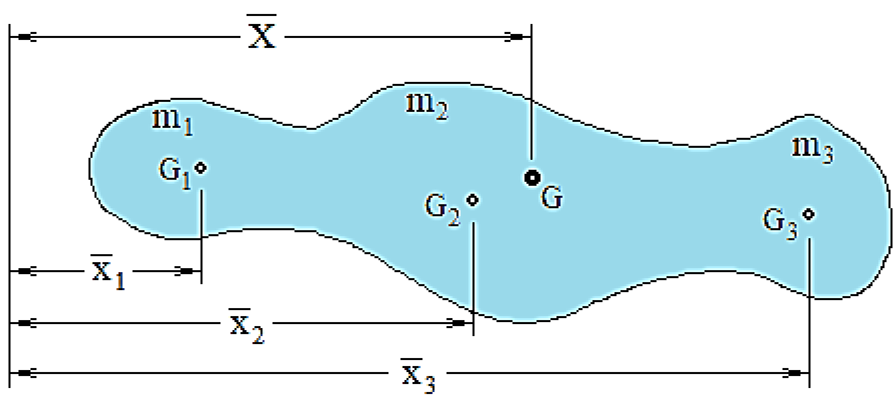


Fig. (7-5)

Procedures of this case can be applied for (masses, areas, lines, and volumes).

Common Geometric Centers:

Table (7-1) Common Geometric Centers

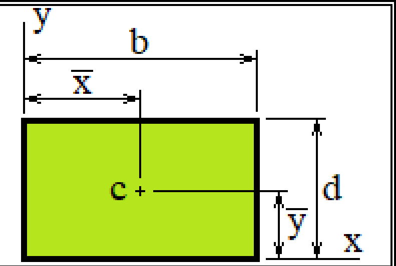
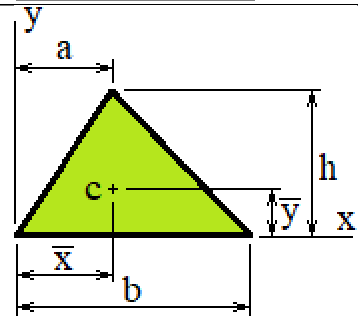
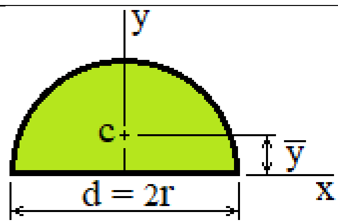
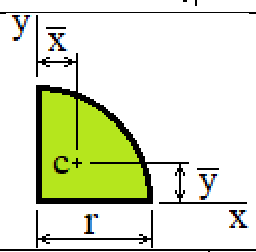
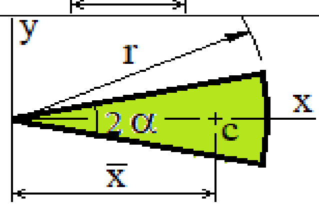
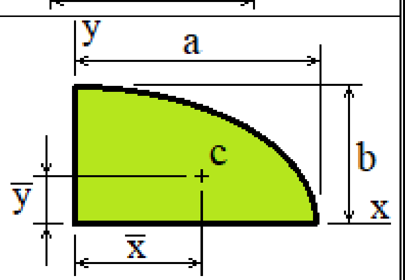
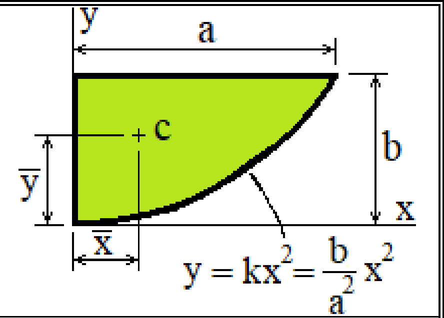
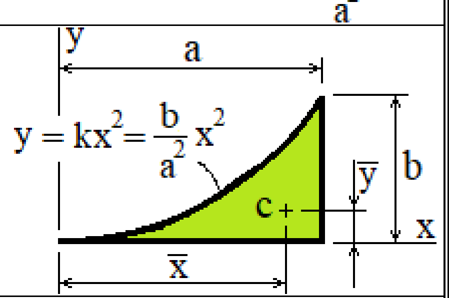
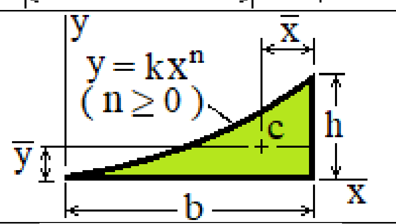
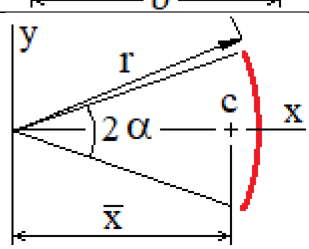
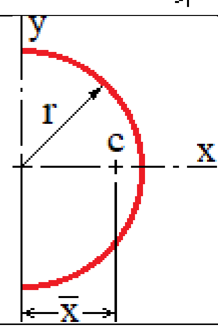
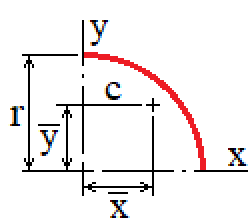
Figures		Area or Length	$\bar{x}$	$\bar{y}$
Rectangle		$b\,d$	$\frac{1}{2}\,b$	$\frac{1}{2}\,d$
Triangle		$\frac{1}{2}\,b\,h$	$\frac{a+b}{3}$	$\frac{1}{3}\,h$
Semicircle		$\frac{\pi r^2}{2}$	0	$\frac{4\,r}{3\,\pi}$ or 0.424 r
Quarter circle		$\frac{\pi r^2}{4}$	$\frac{4\,r}{3\,\pi}$ or 0.424 r	$\frac{4\,r}{3\,\pi}$ or 0.424 r
Circle sector		$r^2\,\alpha$	$\frac{2}{3}\,\frac{r\sin\alpha}{\alpha}$	0
Elliptical quadrant		$\frac{\pi\,a\,b}{4}$	$\frac{4\,a}{3\,\pi}$	$\frac{4\,b}{3\,\pi}$

Table (7-1) Common Geometric Centers

Figures		**Area or Length**	$\bar{x}$	$\bar{y}$
Parabolic area		$\frac{2\,a\,b}{3}$	$\frac{3\,a}{8}$	$\frac{3\,b}{5}$
Sub parabolic area		$\frac{a\,b}{3}$	$\frac{3\,a}{4}$	$\frac{3\,b}{10}$
Trigonometric area under the curve		$\frac{b\,h}{n+1}$	$\frac{b}{n+2}$	$\frac{n+1}{4n+2}\,h$
Circle arc		$2\,r\,\alpha$	$\frac{r\sin\alpha}{\alpha}$	0
Half circumference of a circle		$\pi\,r$	$\frac{2\,r}{\pi}$	0
A quarter of a circumference of a circle		$\frac{\pi\,r}{2}$	$\frac{2\,r}{\pi}$	$\frac{2\,r}{\pi}$

Example (7-1):

Locate the centroid of the shaded area shown in Fig. (Ex. 7-1).

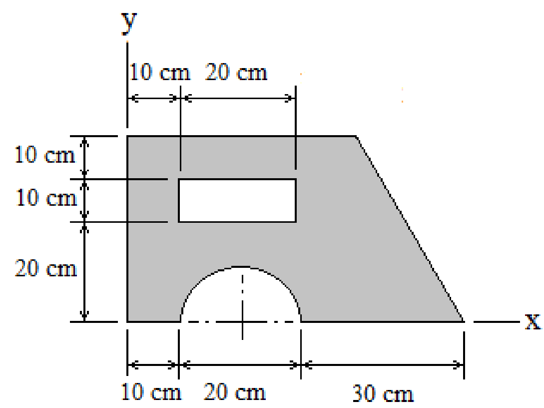
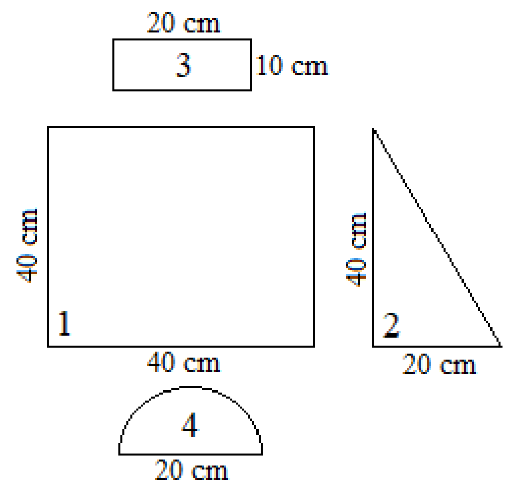


Fig. (Ex. 7-1)

Solution:

The composite area is divided into a four elementary regular shapes shown in the figure (Ex.7-1). The centroid locations of all these shapes may be obtained from the attached figure below. Note that the areas of the "holes" (parts 3 and 4) are taken as negative in the following table:



$$A_1 = 40 \times 40 = 1600 \text{ cm}^2$$
$$A_2 = \frac{1}{2} \times 20 \times 40 = 400 \text{ cm}^2$$
$$A_3 = 20 \times 10 = 200 \text{ cm}^2$$
$$A_4 = \frac{1}{2} \times 10^2 \times \pi = 157 \text{ cm}^2$$

Part	A (cm^2)	$\bar{x}$ (cm)	$\bar{y}$ (cm)	$A\bar{x}$ (cm^3)	$A\bar{y}$ (cm^3)
1	1600	20	20	32000	32000
2	400	46.67	13.33	18666	5333
3	– 200	20	25	– 4000	– 5000
4	– 157	20	4.24	– 3140	– 666
Totals	1643			43526	31667

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{43526}{1643} = 26.49 \text{ cm}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{31667}{1643} = 19.27 \text{ cm}$$

<u>Example (7-2):</u>

Locate the centroid of the plate area shown in Fig. (Ex. 7-2).

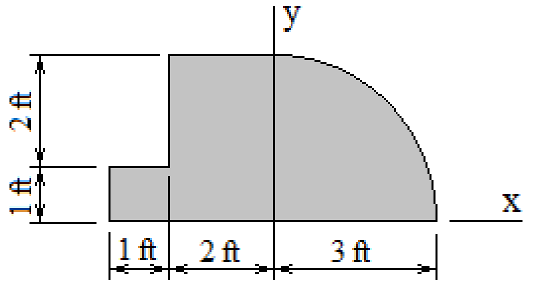


Fig. (Ex. 7-2)

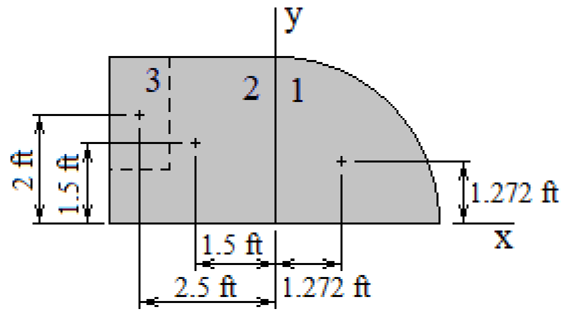
Solution:

The plate is divided into three segments as shown. The area of the small rectangle (3) is considered "negative" and it must be subtracted from the larger rectangle (2).

$$A_1 = \frac{1}{4} \times 3^2 \times \pi = 7 \text{ ft}^2$$

$$A_2 = 3 \times 3 = 9 \text{ ft}^2$$

$$A_3 = 1 \times 2 = 2 \text{ ft}^2$$



Part	**A (ft^2)**	**$\bar{x}$ (ft)**	**$\bar{y}$ (ft)**	**$A\bar{x}$ (ft^3)**	**$A\bar{y}$ (ft^3)**
1	7	1.272	1.272	8.9	8.9
2	9	– 1.5	1.5	– 13.5	13.5
3	– 2	– 2.5	2	5	– 4
$\sum$	$\sum A = 14$			0.4	18.4

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{0.4}{14} = 0.029 \text{ ft}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{18.4}{14} = 1.314 \text{ ft}$$

Example (7-3):

Locate the centroid of the wire shown in Fig. (Ex. 7-3).

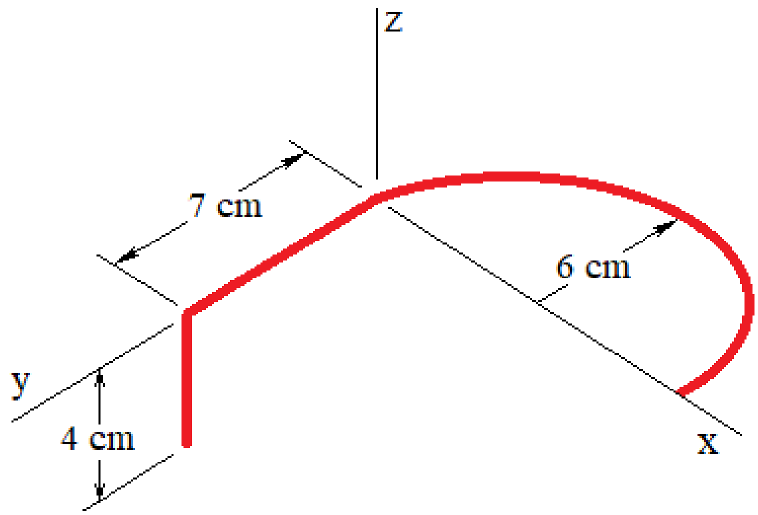
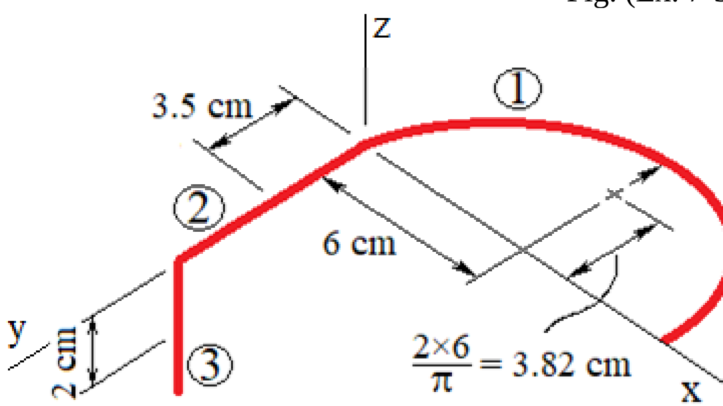


Fig. (Ex. 7-3)

Solution:



Segment	L (cm)	$\bar{x}$ (cm)	$\bar{y}$ (cm)	$\bar{z}$ (cm)	$\bar{x}$ L (cm^2)	$\bar{y}$ L (cm^2)	$\bar{z}$ L (cm^2)
1	π × 6 =18.85	6	– 3.82	0	113.1	– 72	0
2	7	0	3.5	0	0	24.5	0
3	4	0	7	– 2	0	28	– 8
Total	29.85				113.1	– 19.5	– 8

$$\bar{X} = \frac{\sum \bar{x}\, L}{\sum L} = \frac{113.1}{29.85} = 3.79 \text{ cm} \qquad \bar{Y} = \frac{\sum \bar{y}\, L}{\sum L} = \frac{-19.5}{29.85} = -0.65 \text{ cm}$$

$$\bar{Z} = \frac{\sum \bar{z}\, L}{\sum L} = \frac{-8}{29.85} = -0.27 \text{ cm}$$

Example (7-4):

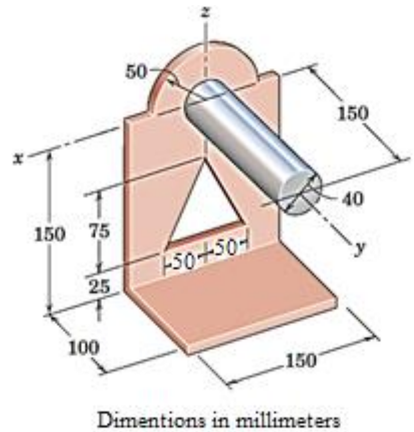


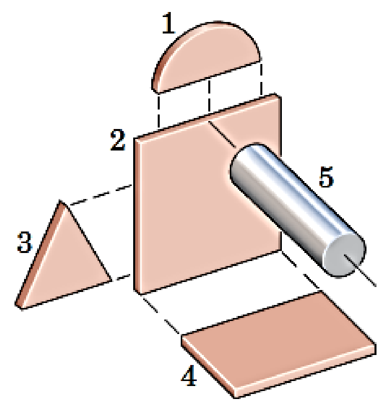
Fig. (Ex. 7-4)

In the bracket-and-shaft combination shown in Fig. (Ex. 7-4), the vertical face is made from metal plate which has a mass of (25 kg/m^2), and the material of the horizontal base has a mass of (40 kg/m^2), and the steel shaft has a density of (7.83 Mg/m^3). Locate the center of mass of the bracket-and-shaft combination.

Solution:

The composite body is composed of five elements shown in the illustration. The trigonometric part will be considered as a negative mass.

For the reference axes indicated it is clear by symmetry that the x-coordinate of the center of mass is zero.



$m_1 = 25 \times (1/2 \times \pi \times 0.05^2) = 0.098$ kg

$m_2 = 25 \times (0.15 \times 0.15) = 0.562$ kg

$m_3 = 25 \times (1/2 \times 0.1 \times 0.075) = 0.0938$ kg

$m_4 = 40 \times (0.15 \times 0.1) = 0.6$ kg

$m_5 = 7830 \times (0.15 \times \pi \times 0.02^2) = 1.476$ kg

$$\bar{z}_1 = \frac{4r}{3\pi} = \frac{4\times 50}{3\pi} = 21.2 \text{ mm}$$

$$\bar{z}_3 = -[\,150 - 25 - 1/3\,(75)\,] = -100 \text{ mm}$$

Part	m (kg)	$\bar{y}$ (mm)	$\bar{z}$ (mm)	m $\bar{y}$ (kg.mm)	m $\bar{z}$ (kg.mm)
1	0.098	0	21.2	0	2.08
2	0.562	0	– 75	0	– 42.19
3	– 0.0938	0	– 100	0	9.38
4	0.6	50	– 150	30	– 90
5	1.476	75	0	110.7	0
Total	2.642			140.7	– 120.73

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{140.7}{2.642} = 53.3 \text{ mm}$$

$$\bar{Z} = \frac{\sum m\bar{z}}{\sum m} = \frac{-120.73}{2.642} = -45.7 \text{ mm}$$

Example (7-5):

Determine the height of the centroid of the cross-sectional area of the beam shown in Fig. (Ex. 7-5) above the base. Neglect the fillets.

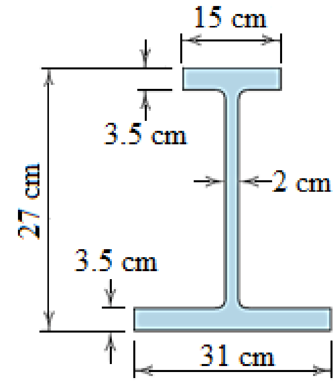
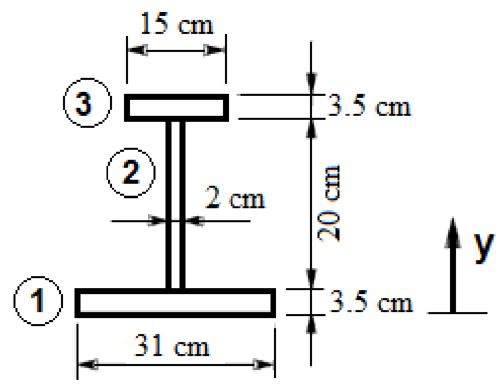


Fig. (Ex. 7-5)

Solution:

Part	A (cm²)	$\bar{y}$ (cm)	$A\bar{y}$ (cm³)
1	108.5	1.75	189.88
2	40	13.5	540
3	52.5	25.25	1325.63
Totals	201		2055.51



$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{2055.51}{201} = 10.23 \text{ cm}$$

Example (7-6):

Locate the centroid of the rectangular plate area which hollowed by a circular Sector as shown in Fig. (Ex. 7-6).

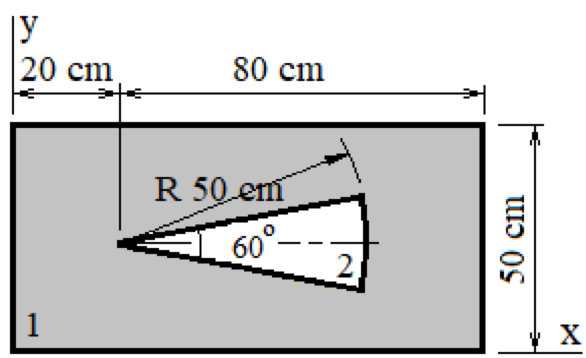


Fig. (Ex. 7-6)

Solution:

$$\alpha = 30 \times \frac{\pi}{180} = 0.5236 \text{ rad.}$$

$$A_2 = R^2\,\alpha = (50)^2 \times 0.5236$$
$$= 1309 \text{ cm}^2$$

$$\bar{x}_1 = 20 + \frac{2}{3}\frac{R\sin\alpha}{\alpha} = 20 + \frac{2}{3}\frac{50\sin 30}{0.5236} = 20 + 31.83 \text{ cm} = 51.83 \text{ cm}$$

Part	A (cm²)	$\bar{x}$ (cm)	A $\bar{x}$ (cm³)
1	5000	50	250000
2	– 1309	51.83	– 67847
Totals	3691		182153

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{182153}{3691} = 49.35 \text{ cm}$$

Example (7-7):

Locate the centroid of the plate area shown in Fig. (Ex. 7-7), which consist of three parts, Quarter Circle, Elliptical Quadrant, and rectangle.

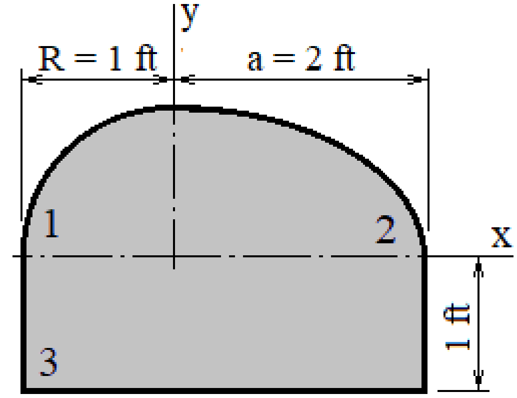


Fig. (Ex. 7-7)

Solution:

$A_1 = \dfrac{\pi\, r^2}{4} = 0.785\ ft^2$

$A_2 = \dfrac{\pi\, a\, b}{4} = 1.57\ ft^2$

$A_3 = 1 \times 3 = 3\ ft^2$

$\bar{x}_1 = -0.424\ r = -0.424\ \ ft$ $\bar{y}_1 = 0.424\ r = 0.424\ ft$

$\bar{x}_2 = \dfrac{4\, a}{3\, \pi} = 0.849\ ft$ $\bar{y}_2 = \dfrac{4\, b}{3\, \pi} = 0.4244\ ft$

Part	**A (ft^2)**	**$\bar{x}$ (ft)**	**$\bar{y}$ (ft)**	**$A\bar{x}$ (ft^3)**	**$A\bar{y}$ (ft^3)**
1	0.785	– 0.424	0.424	– 0.333	0.333
2	1.57	0.849	0.4244	1.333	0.666
3	3	0.5	– 0.5	1.5	– 1.5
Total	∑A = 5.355			2.5	– 0.501

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{2.5}{5.355} = 0.467\ ft$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{-0.501}{5.355} = -0.094\ ft$$

Example (7-8):

Locate the centroid of the wire shown in Fig. (Ex. 7-8).

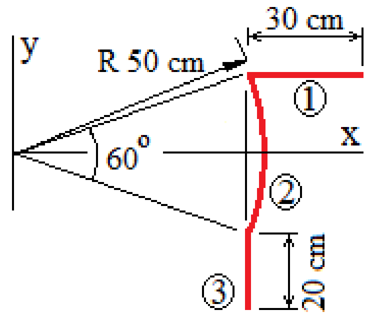


Fig. (Ex. 7-8)

Solution:

$$\alpha = 30 \times \frac{\pi}{180} = 0.52 \text{ rad}$$

$$L_2 = 2\,r\,\alpha = 2 \times 50 \times 0.52 = 52 \text{ cm}$$

$\bar{x}_1 = 50 \cos 30 + 15 = 58.3$ cm $\qquad \bar{y}_1 = 50 \sin 30 = 25$ cm

$\bar{x}_2 = \dfrac{R \sin \alpha}{\alpha} = \dfrac{50 \sin 30}{0.52} = 48.08$ cm $\qquad \bar{y}_2 = 0$

$\bar{x}_3 = 50 \cos 30 = 43.3$ cm $\qquad \bar{y}_3 = -(50 \sin 30 + 10) = -35$ cm

Segment	**L (ft)**	**$\bar{x}$ (ft)**	**$\bar{y}$ (ft)**	**L $\bar{x}$ (ft^2)**	**L $\bar{y}$ (ft^2)**
1	30	58.3	25	1749	750
2	52	48.08	0	2500	0
3	20	43.3	– 35	866	– 700
Total	102			5115	50

$\bar{X} = \dfrac{\sum L\,\bar{x}}{\sum L} = \dfrac{5115}{102} = 50.15$ cm $\qquad \bar{Y} = \dfrac{\sum L\,\bar{y}}{\sum L} = \dfrac{50}{102} = 0.49$ cm

Example (7-9):

Locate the centroid of the plate area shown in Fig. (Ex. 7-9), which consist of two parts, Parabolic part and rectangle part.

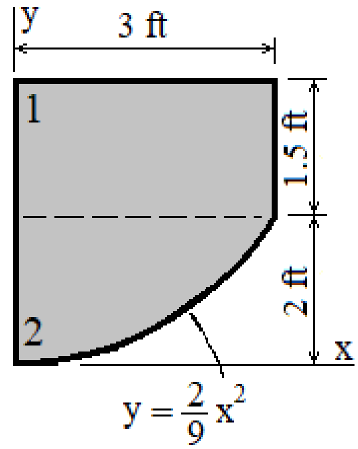


Fig. (Ex. 7-9)

Solution:

$$A_2 = \frac{2\,a\,b}{3} = \frac{2\times3\times2}{3} = 4 \text{ ft}$$

$$\bar{x}_2 = \frac{3\,a}{8} = \frac{3\times3}{8} = 1.125 \text{ ft}$$

$$\bar{y}_2 = \frac{3\,b}{5} = \frac{3\times2}{5} = 1.2 \text{ ft}$$

Part	A (ft^2)	$\bar{x}$ (ft)	$\bar{y}$ (ft)	$A\bar{x}$ (ft^3)	$A\bar{y}$ (ft^3)
1	4.5	1.5	2.75	6.75	12.375
2	4	1.125	1.2	4.5	4.8
Total	$\sum A = 8.5$			11.25	17.175

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{11.25}{8.5} = 1.32 \text{ ft}$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{17.175}{8.5} = 2.02 \text{ ft}$$

Example (7-10):

Locate the centroid of the rectangular plate area which hollowed by a trigonometric area under a curve as shown in Fig. (Ex. 7-10).

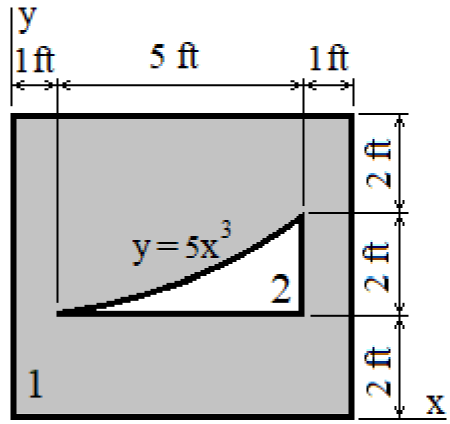


Fig. (Ex. 7-10)

Solution:

$$A_2 = \frac{1}{n+1}\ b\ h = \frac{1}{3+1} \times 5 \times 2 = 2.5\ ft^2$$

$$\bar{x}_2 = 1 + \frac{1}{n+2}\ b = 1 + \frac{1}{3+2} \times 5 = 2\ ft$$

$$\bar{y}_2 = 2 + \frac{n+1}{4n+2}\ h = 2 + \frac{3+1}{12+2} \times 2 = 2.57\ ft$$

Part	A (ft^2)	$\bar{x}$ (ft)	$\bar{y}$ (ft)	$A\bar{x}$ (ft^3)	$A\bar{y}$ (ft^3)
1	42	3.5	3	147	126
2	– 2.5	2	2.57	– 5	– 6.425
Total	$\sum A = 39.5$			142	119.575

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{142}{39.5} = 3.6\ ft$$

$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{119.575}{39.5} = 3.03\ ft$$

Example (7-11):

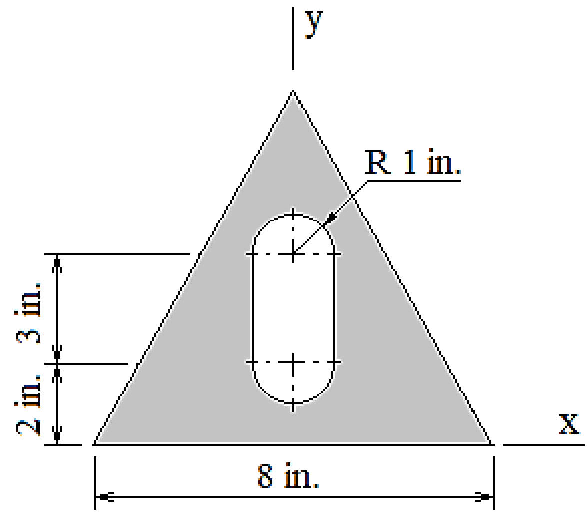


Fig. (Ex. 7-11)

For the equilateral triangle shown in Fig. (Ex. 7-11), determine the (y-coordinate) of the centroid of the shaded area.

Solution:

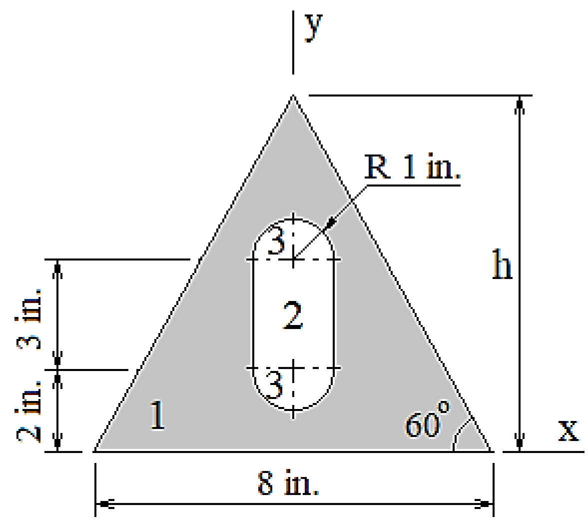
h = 8 sin 60 = 6.93 in.

$A_1 = ½ \times 8 \times 6.93 = 27.7$ in.

$A_2 = 2 \times 3 = 6$ in.

$A_3 = ½ \times 1^2 \times \pi \times 2 = 3.14$ in.

Part	**A (in.^{2})**	**$\bar{y}$ (in.)**	**$\bar{y}$A (in.^{3})**
1	27.7	2.31	64
2	– 6	3.5	– 21
3	– 3.14	3.5	– 11
Totals	18.56		32



$$\bar{Y} = \frac{\sum A\bar{y}}{\sum A} = \frac{32}{18.56} = 1.72 \text{ in.}$$

Example (7-12):

Determine the coordinates of the mass center of the welded assembly of uniform slender rods made from the same metal, as shown in Fig. (Ex. 7-12).

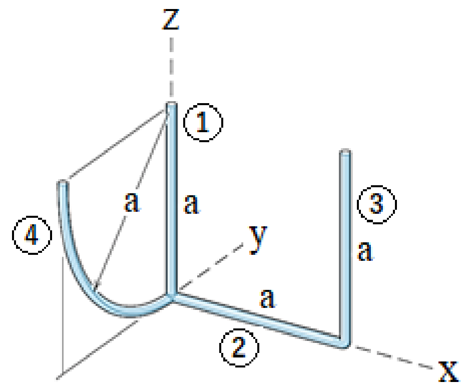


Fig. (Ex. 7-12)

Solution:

Segment	L	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x}$ L	$\bar{y}$ L	$\bar{z}$ L
1	a	0	0	$\frac{a}{2}$	0	0	$\frac{a^2}{2}$
2	a	$\frac{a}{2}$	0	0	$\frac{a^2}{2}$	0	0
3	a	a	0	$\frac{a}{2}$	a^2	0	$\frac{a^2}{2}$
4	$\frac{\pi a}{2}$	0	$-\frac{2a}{\pi}$	$a\,(1-\frac{2}{\pi})$	0	$-a^2$	$a^2(\frac{\pi}{2}-1)$
Total	$a\,(3+\frac{\pi}{2})$				$\frac{3}{2}a^2$	$-a^2$	$\frac{\pi}{2}a^2$

$$\bar{X} = \frac{\sum \bar{x}\,L}{\sum A} = \{\ \frac{3}{2}a^2\ \} / \{\ a(3+\frac{\pi}{2})\ \} = \frac{3a}{6+\pi}$$

$$\bar{Y} = \frac{\sum \bar{y}\,L}{\sum A} = \{\ -a^2\ \} / \{\ a(3+\frac{\pi}{2})\ \} = -\frac{2a}{6+\pi}$$

$$\bar{Z} = \frac{\sum \bar{z}\,L}{\sum A} = \{\ \frac{\pi}{2}a^2\ \} / \{\ a(3+\frac{\pi}{2})\ \} = \frac{\pi a}{6+\pi}$$

Example (7-13):

Find the distance (D) from the center of the Earth to the center of mass common of the Earth and the Moon.

1- **Earth radius (R) = 6371 km.**
2- **The mass of the moon earth (m_E) = 5.97×10^{24} kg.**
3- **The mass of the moon (m_M) = 7.35×10^{22} kg.**
4- **The distance between the center of the Earth and the center of the moon (d) = 384400 km**

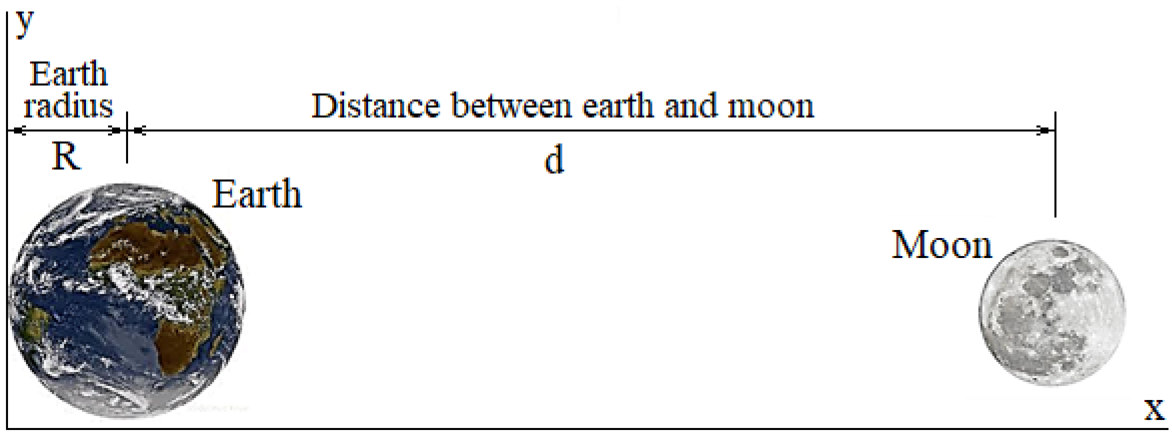


Fig. (Ex. 7-13)

Solution:

Blanet	**m (kg)**	**$\bar{x}$ (km)**	**m $\bar{x}$ (kg.km)**
Earth	5.97×10^{24}	6371	3803.5×10^{25}
Moon	7.35×10^{22}	390771	2872.2×10^{25}
Totals	0.60435×10^{25}		6675.7×10^{25}

$$\bar{x} = \frac{\sum m\bar{x}}{\sum m} = \frac{5937.7 \times 10^{25}}{0.60435 \times 10^{25}} = 11046 \text{ km}$$

$$D = \bar{x} - R = 11046 - 6371 = 4675 \text{ km}$$

<u>Example (7-14):</u>

For the motorcycle shown in Fig. (Ex. 7-14), the location of the center of gravity of each component and its mass are tabulated. If the three-wheeler motorcycle is symmetrical with respect to the (x–y plane), locate the center of mass of the motorcycle.

1. **Rear wheels mass is (9 kg).**
2. **Mechanical components mass is (40 kg).**
3. **Structuer mass is (60 kg).**
4. **Front wheel mass is (4 kg).**

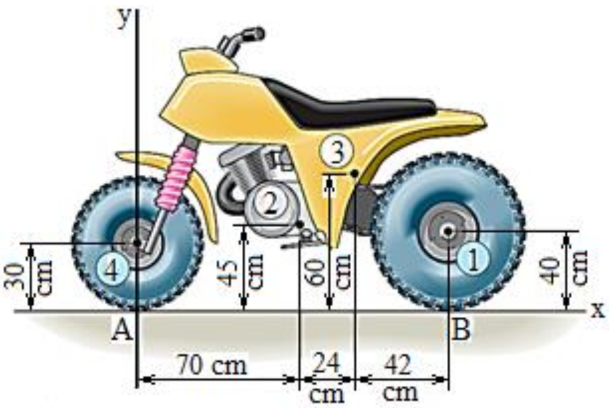


Fig. (Ex. 7-14)

Solution:

Part	**m** (kg)	$\bar{x}$ (cm)	$\bar{y}$ (cm)	**m** $\bar{x}$ (kg.cm)	**m** $\bar{y}$ (kg.cm)
1	9	136	40	1224	360
2	40	70	45	2800	1800
3	60	94	60	5640	3600
4	4	0	30	0	120
Total	113			9664	5880

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m} = \frac{9664}{113} = 85.5 \text{ cm}$$

$$\bar{Y} = \frac{\sum m\bar{y}}{\sum m} = \frac{5880}{113} = 52 \text{ cm}$$

Example (7-15):

A crane specialized in the field of electrical maintenance, the weight of the base is (2000 Ib), the weight of the wheels assembly is (1000 Ib), the weight of the arm (AB) is (1500 Ib), the weight of the cage (BCD) is (200 Ib), and the weight of the electrician is (160 Ib), and the centers of gravity are located at points (G_1), (G_2), (G_3), (G_4) and (G_5) respectively. Locate the center of weight of the crane in the (x–y plane).

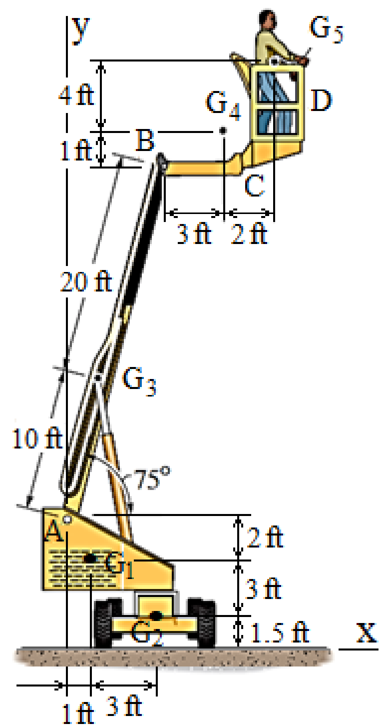


Fig. (Ex. 7-15)

Solution:

$\bar{x}_3 = 10 \cos 75 = 2.6$ ft

$\bar{y}_3 = 6.5 + 10 \sin 75 = 16.16$ ft

$\bar{x}_4 = 30 \cos 75 + 3 = 10.76$ ft

$\bar{y}_4 = 6.5 + 30 \sin 75 + 1 = 36.48$ ft

$\bar{x}_4 = 30 \cos 75 + 5 = 12.76$ ft

$\bar{y}_4 = 6.5 + 30 \sin 75 + 5 = 40.48$ ft

Part	**W (Ib)**	**$\bar{x}$ (Ib)**	**$\bar{y}$ (Ib)**	**W $\bar{x}$ (Ib.ft)**	**W $\bar{y}$ (Ib.ft)**
1	2000	1	4.5	2000	9000
2	1000	4	1.5	4000	1500
3	1500	2.6	16.16	3900	24240
4	500	10.76	36.48	5380	18240
5	160	12.76	40.48	2042	6477
Total	5160			17322	59457

$$\bar{X} = \frac{\sum W \bar{x}}{\sum W} = \frac{17322}{5160} = 3.36 \text{ ft}$$

$$\bar{Y} = \frac{\sum W \bar{y}}{\sum W} = \frac{48513}{5160} = 11.52 \text{ ft}$$

Example (7-16):

The location of the center of gravity and mass of each of the bulldozer components shown in Fig. (Ex. 7-16), are tabulated below. Locate the center of mass of the bulldozer in the (x–y plane).

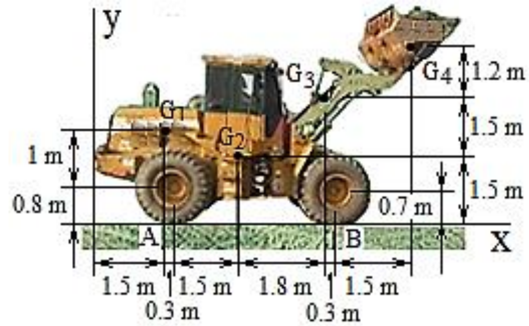


Fig. (Ex. 7-16)

1- **The bulldozer engine mass is (2 tons) at center of gravity (G_1).**
2- **The bulldozer cabin mass is (0.8 ton) at center of gravity (G_2).**
3- **The bulldozer arm mass is (1.2 ton) at center of gravity (G_3).**
4- **The bulldozer bucket mass is (1 ton) at center of gravity (G_4).**
5- **The rear wheels mass is (0.2 ton).**
6- **The front wheels mass is (0.18 ton).**

Solution:

Part	**m (ton)**	**$\bar{x}$ (m)**	**$\bar{y}$ (m)**	**m $\bar{x}$ (ton.m)**	**m $\bar{y}$ (ton.m)**
1	2	1.5	1.8	3	3.6
2	0.8	3.3	1.5	2.64	1.2
3	1.2	5.1	3	6.12	3.6
4	1	6.9	4.2	6.9	4.2
5	0.2	1.8	0.8	0.36	0.16
6	0.18	5.4	0.7	0.972	0.126
Total	5.38			19.992	12.886

$$\overline{X} = \frac{\sum m\,\bar{x}}{\sum W} = \frac{19.992}{5.38} = 3.7 \text{ m}$$

$$\overline{Y} = \frac{\sum m\,\bar{y}}{\sum W} = \frac{12.886}{5.38} = 2.4 \text{ m}$$

Example (7-17):

A regular rod in the form of a semicircle of weight (1 kN) and radius (1 m), hinged at point (A) and resting on a smooth surface at point (B). Determine the reactions at points (A) and (B).

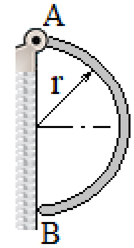
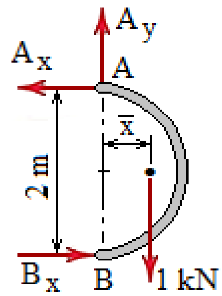


Fig. (Ex. 7-17)

Solution:



$$\bar{x} = \frac{2\,r}{\pi} = \frac{2\times 1}{\pi} = 0.64 \text{ m}$$

$\sum M_A = 0$

$(B_x \times 2) - (1 \times 0.64) = 0$

$B_x = 0.32$ kN

$\sum F_x = 0$

$0.32 - A_x = 0$

$A_x = 0.32$ kN

$\sum F_y = 0$

$A_y - 1 = 0$

$A_y = 1$ kN

$$F_A = \sqrt{{A_x}^2 + {A_y}^2} = \sqrt{0.32^2 + 1^2} = 1.05 \text{ kN}$$

$$\theta = \tan^{-1}\frac{1}{0.32} = 72.3°$$

Problems:

7-1) Determine the (x) and (y) coordinates of the centroid of the shaded area shown in Fig. (Pr. 7-1).

Ans.: $\bar{X} = 98.95\ mm$, $\bar{Y} = 68.4\ mm$

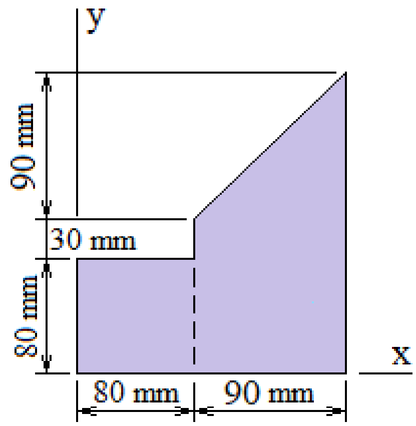


Fig. (Pr. 7-1)

7-2) Determine the (x) and (y) coordinates of the centroid of the shaded area shown area in Fig. (Pr. 7-2).

Ans.: $\bar{X} = 3\ in.$, $\bar{Y} = 2\ in.$

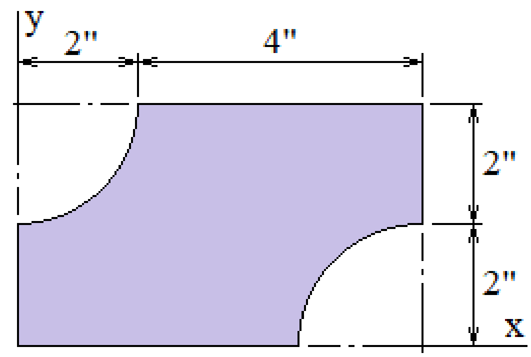


Fig. (Pr. 7-2)

7-3) Locate the centroid of the wire shown in Fig. (Pr. 7-3).

Ans.: $\bar{X} = 0.7\ ft$, $\bar{Y} = 0.7\ ft$

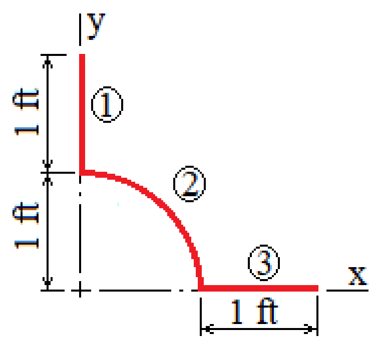


Fig. (Pr. 7-3)

7-4) Determine the coordinates of the mass center of the body which is constructed of three pieces of uniform thin plate welded together.

Ans.: $\bar{X} = 23.33\ cm$, $\bar{Y} = 8.33\ cm$
$\bar{Z} = 13.33\ cm$

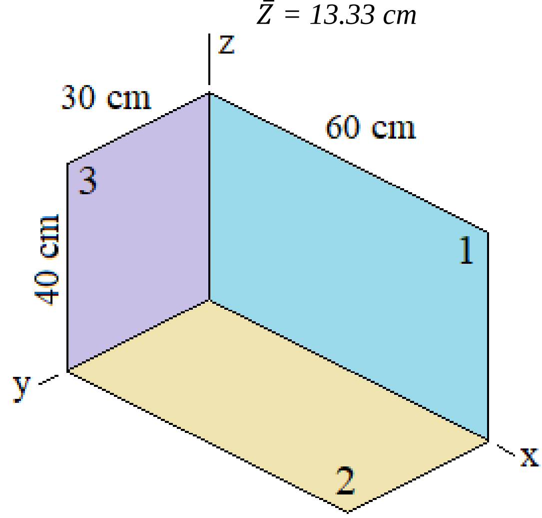


Fig. (Pr. 7-4)

7-5) Determine the (x) and (y) coordinates of the centroid of the shaded area shown in Fig. (Pr. 7-5).

Ans.: $\bar{X} = 5.4$ *in.*, $\bar{Y} = 3.5$ *in.*

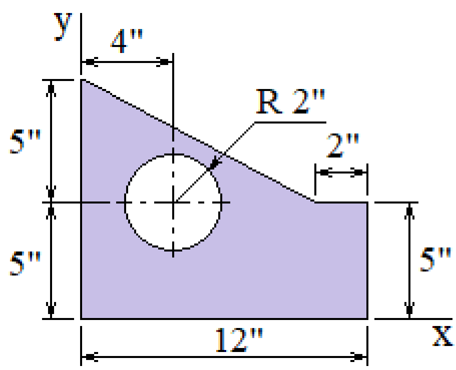


Fig. (Pr. 7-5)

7-6) Determine the y-coordinate of the centroid of the shaded area shown in Fig. (Pr. 7-6).

Ans.: $\bar{Y} = 42.64$ *mm*

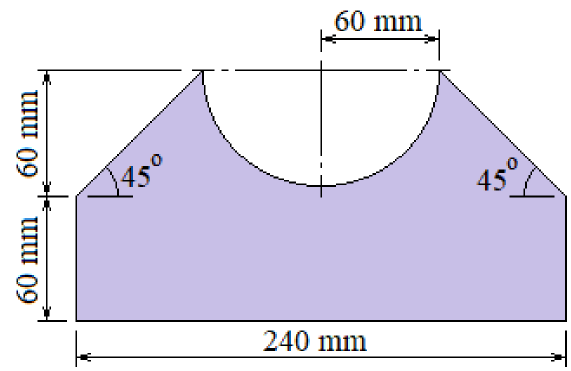


Fig. (Pr. 7-6)

7-7) Locate the centroid of the rectangular plate area which hollowed by an area under a sub parabolic area as shown in Fig. (Pr. 7-7).

Ans.: $\bar{X} = 2.43$ *ft*, $\bar{Y} = 2.53$ *ft*

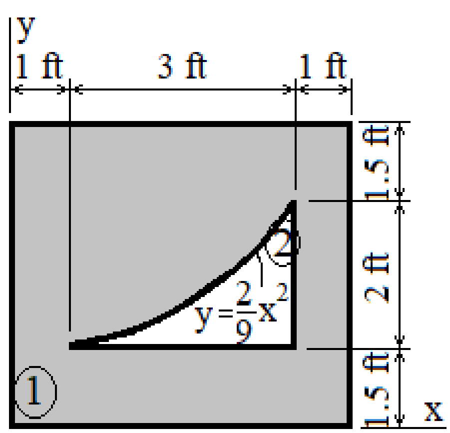


Fig. (Pr. 7-7)

7-8) Determine the distance (Y) from the bottom of the base plate to the centroid of the built-up structural section shown in Fig. (Pr. 7-8).

Ans.: $\bar{Y} = 36.11$ *mm*

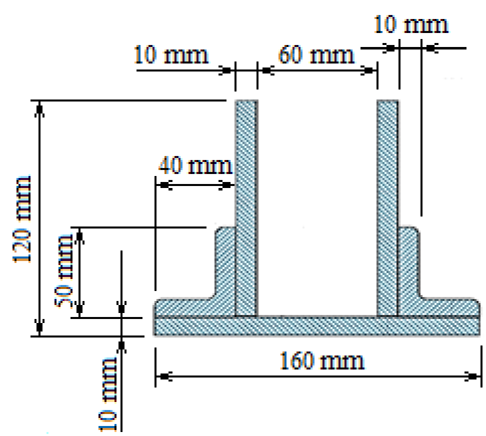


Fig. (Pr. 7-8)

7-9) Determine the coordinates of the mass center of the bracket shown in Fig. (Pr. 7-9), which is constructed from sheet metal of uniform thickness.

Ans.: $\bar{X} = 4.29$ cm, $\bar{Y} = 4.85$ cm, $\bar{Z} = -2.07$ cm.

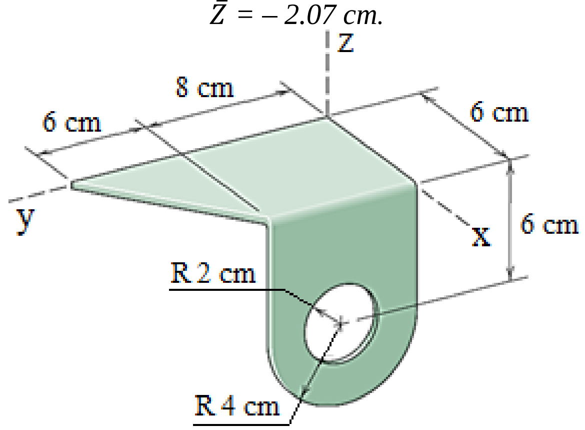


Fig. (Pr. 7-9)

7-10) Determine the (x) and (y) coordinates of the centroid of the shaded area shown in Fig. (Pr. 7-10).

Ans.: $\bar{X} = 0.85$ ft, $\bar{Y} = 0.85$ ft

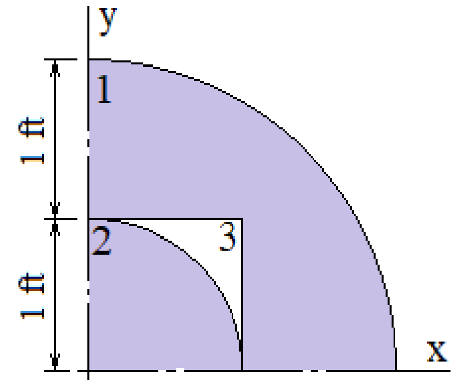


Fig. (Pr. 7-10)

7-11) For the bicycle shown in Fig. (Pr. 7-11), the location of the center of gravity of each component and its mass are tabulated in the figure. Locate the center of mass of the bicycle in the (x – y plane).

1- Front wheel mass is (2 kg).
2- Rear wheel mass is (2 kg).
3- The mass of steering assembly is (5 kg).
4- The mass of seat and driving gear assembly is (6 kg).

Ans.: $\bar{X} = 63$ cm, $\bar{Y} = 58$ cm

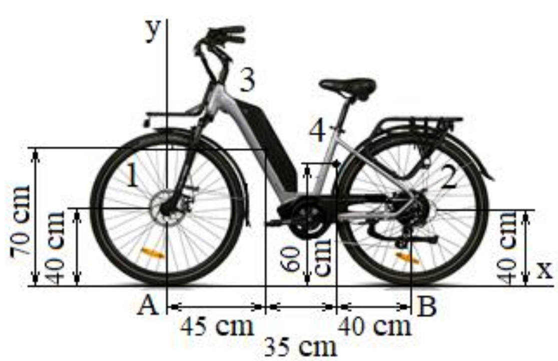


Fig. (Pr. 7-11)

7-12) For the Excavator shown in Fig. (Pr. 7-12), the location of the center of gravity of each component and its weight are tabulated in the figure. Locate the center of weight of the Excavator in the (x–y plane).

1- The Excavator weight is (400 kN) at center of gravity (G_1).
2- The Excavator boom weight is (50 kN) at center of gravity (G_2).
3- The Excavator arm weight is (30 kN) at center of gravity (G_3).
4- The bucket weight is (10 kN) at center of gravity (G_4).
5- The wheel assembly weight is (40 kN) at center of gravity (G_5).

Ans.: $\bar{X} = 2.47$ m, $\bar{Y} = 2$ m

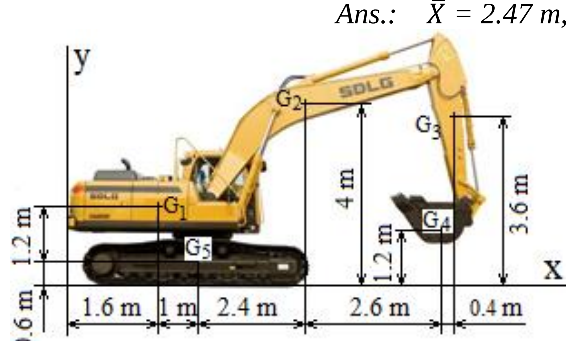


Fig. (Pr. 7-12)