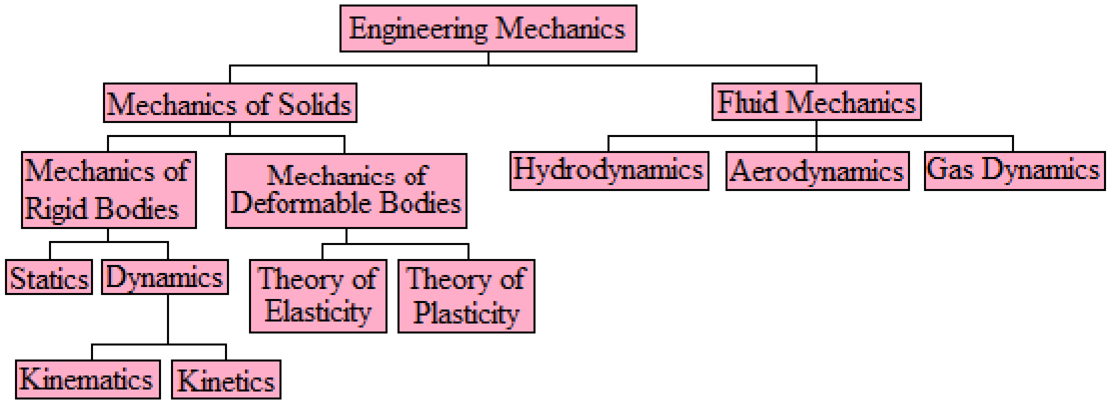
Engineering mechanics definition:

Engineering mechanics: It is the physical science that describes the state of motion of bodies (rest or motion) under the action of the forces exerted on them. It is divided into two branches, as shown in the figure below.



Fluid mechanics:

Is concerned with the behaviour of fluid at rest (fluid static) and in motion (fluid dynamic) and the interaction of fluid with other fluids or solids at the boundaries.

Fluid:

Is a substance, might be liquid or gas, that deforms continuously (flow) when subjected to a shear stress.

Solids & Liquids & Gases:

From Microscopic point of view

Solids: They are substances in which the forces of attraction between molecules are large and the separation between the molecules is small.

Liquids: These are substances in which the forces of attraction between molecules are smaller than the forces of attraction between molecules in solids, with greater spacing between molecules.

Gases: They are substances in which the forces of attraction between molecules are smaller than the forces of attraction between molecules in solids and liquids, with greater spacing between the molecules in the solid and liquid states.

From Fluid Mechanics Perspective

When shear stress is subjected to a substance:

- If the substance deforms continuously (flow), it is a fluid.
- If the substance experiences a small deformation (δl) (strain), it is a solid.

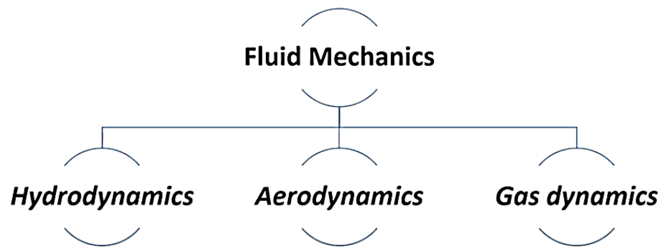
<u>Fluid mechanics departments</u>:

Fluid mechanics is divided into several categories:

- ***Hydrodynamics:***
  Deals with the flow of incompressible fluids (especially water) at low speed such as hydro power plants (Dams).

- ***Aerodynamics:***
  Deals with the flow of compressible fluids (especially air) over bodies such as aircrafts, rockets and cars.

- ***Gas dynamics:***
  Deals with the flow of fluids (especially gases) that undergoes significant density change such as the flow of natural gas through nozzles at high speed, for example the flow of natural gases in to the combustion chamber of gas turbine.



Property of Fluids:

Density (ρ):

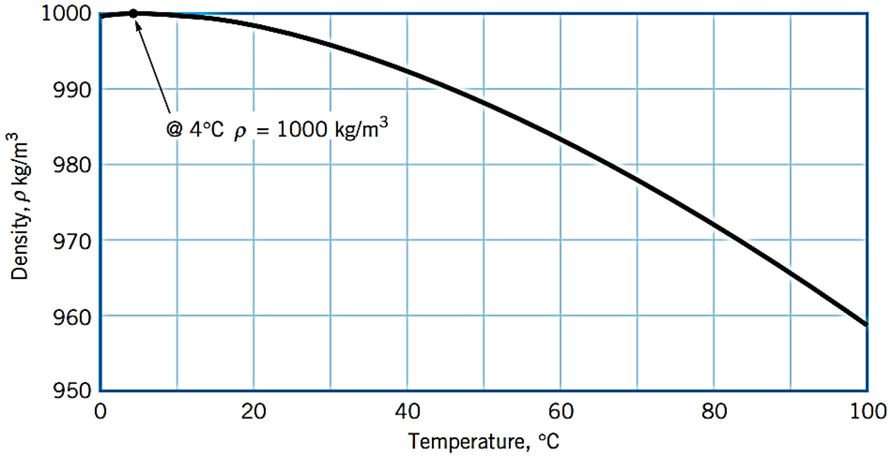
The ratio of the mass of a fluid to its volume (kg/m^3). It characterizes a mass of the fluid system.

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

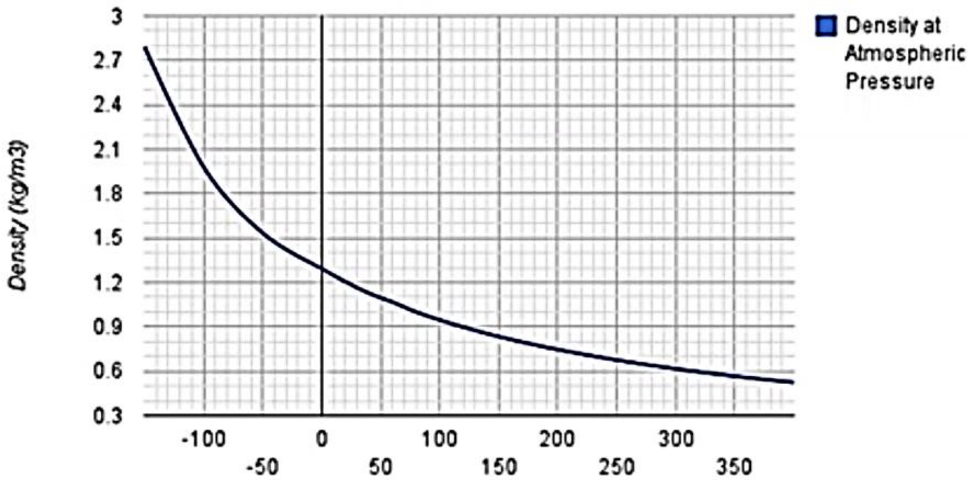
The density of the water is 1000 kg/m^3.

The variations in pressure and temperature generally have only a small effect on the value of (ρ) for liquid. In contrast to liquid, the density of a gas is strongly influenced by both pressure and temperature.

Density of water as a function of temperature



Density of gases as a function of temperature



Specific weight (γ):

The ratio of the weight of a fluid to its volume (N/m^3). It characterizes the weight of a fluid system.

$$\gamma = \frac{\text{weight of fluid}}{\text{Volume of fluid}}$$

$$\gamma = \frac{m \times g}{v} = \frac{\rho \times v \times g}{v} = \rho \times g$$

The specific weight of the water is $\boldsymbol{\gamma_{water} = \rho_{water} \times g = 1000 \times 9.81 = 9810}$ ***N/m³***.

Specific Gravity (SG):

The specific gravity of a fluid is the ratio of fluid density to the density of standard fluid (Water for liquids & air for gases).

$$SG = \frac{\text{density of fluid}}{\text{density of water}} = \frac{\rho_{liquid}}{\rho_{water}} = \frac{\gamma_{liquid}}{\gamma_{water}}$$

This give us an indication about the heaviness of the Liquids

- If SG > 1, The liquid is heavier than the water.
- If SG < 1, The liquid is lighter than the water.

Specific volume (υ):

Is defined as the volume of a fluid occupied by a unit mass. Thus, it is the reciprocal of the density.

$$\upsilon = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\text{density}} = \frac{1}{\rho}$$

Example (1) :

You have two liquids, Liquid (A) with a density of (0.9 g/ml) and Liquid (B) with a density of (1.2 g/ml). Which liquid is denser, and by how much?

Solution:

Liquid (B) is denser. The density difference is:

Density of Liquid (B) - Density of Liquid (A)

= 1.2 – 0.9 = 0.3 g/ml

Example (2):

Calculate the specific weight of water at standard conditions ($\rho = 1000\ kg/m^3$, $g = 9.81\ m/s^2$).

Solution:

Use the formula for specific weight:

$\gamma = \rho \times g = (1000\ kg/m^3) \times (9.81\ m/s^2) = 9810\ N/m^3$

So, the specific weight of water at standard conditions is ($9810\ N/m^3$).

Example (3):

Calculate the specific gravity of a liquid with a density of ($0.85\ g/cm^3$).

Solution:

Using the formula for specific gravity:

SG = (Density of the Substance) / (Density of Water)

$$SG = \frac{0.85}{1} = 0.85$$

So, the specific gravity of the liquid is (0.85).

Example (4):

If the specific gravity of a liquid is (1.2), what is its density in (kg/m^3) if the density of water at (4°C) is ($1000\ kg/m^3$)?

Solution:

Using the formula for specific gravity:

SG = (Density of the Substance) / (Density of Water)

1.2 = (Density of the Substance) / $1000\ kg/m^3$

Density of the Substance = $1.2 \times 1000\ kg/m^3 = 1200\ kg/m^3$

Example (5):

(200) litres of a certain oil weights (180 kg). Calculate the specific weight, specific gravity and specific volume of it. Is the liquid lighter or heavier than the water? Why?

Solution:

v = 200 litres m = 180 kg

$$\rho = \frac{m}{v} = \frac{180}{0.2} = 900 \text{ kg/m}^3$$

$$\gamma = \rho\, g = 900 \times 9.81 = 8829 \text{ N/m}^3$$

$$SG = \frac{\rho_{liquid}}{\rho_{water}} = \frac{900}{1000} = 0.9$$

$$\upsilon = \frac{v}{m} = \frac{200 \times 10^{-3}}{180} = 1.11 \times 10^{-3} \text{ m}^3\text{/ kg}$$

The liquid is lighter than the water because SG < 1

Example (6):

Calculate the density, specific weight and weight of one liter of petrol of specific gravity equals to (0.7).

Solution:

$$SG = \frac{\rho_{liquid}}{\rho_{water}}$$

$$0.7 = \frac{\rho_{liquid}}{1000}$$

$$\rho_{liquid} = 0.7 \times 1000 = 700 \text{ kg/m}^3$$

$$\gamma = \rho \times g = 700 \times 9.81 = 6867 \text{ N/m}^3$$

$$\gamma = \frac{w}{v}$$

$$w = \gamma\, v = 6867 \times 1 \times 10^{-3} = 6.867 \text{ N}$$

Compressibility and bulk modulus

Compressibility (C) shows how easily can the volume of a fluid change when the pressure changes.

Bulk Modulus K:

A property that is commonly used to characterize compressibility. It is defined as the ratio of pressure change to volumetric strain.

$$k = \frac{\text{Pressure change}}{\text{Volumetric strain}} = -\frac{\Delta P}{\Delta V/V}$$

The negative sign means that the increase in pressure will cause a decrease in volume.

Compressibility is reciprocal of bulk modulus.

$$C = \frac{1}{K}$$

For most purposes, liquids can be considered as incompressible fluids. For example**,** it would require a pressure of (220) bar to compress a unit volume of water by (1%).

Example (7):

Determine the bulk modulus of elasticity of a fluid which is compressed in a cylinder from a volume of (0.009 m^3) at (70 N/cm^2) pressure to a volume of (0.0085 m^3) at (270 N/cm^2) pressure. [Ans. 3.6×10^3 N/cm^2]

Solution:

$$k = -\frac{\Delta P}{\Delta V/V}$$

$$\Delta P = P - P_o = 270 - 70 = 200\ N/cm^2$$

$$\Delta V = V - V_o = 8500 - 9000 = -500\ cm^3$$

$$\frac{\Delta V}{V} = \frac{-500}{9000} = -0.055$$

$$k = -\frac{\Delta P}{\Delta V/V} = -\frac{200}{-0.055} = 3600\ N/cm^2$$

Ideal Gas Law

Gases are highly compressible in comparison to liquids, changes in gas volume (and hence density) can be directly related to changes in pressure and temperature through the ideal gas equation:

$$PV = mRT \qquad \text{or} \qquad P = \rho RT \qquad \ldots \text{(Ideal Gas Law)}$$

P: Absolute pressure ($P_{gauge} + P_{atm}$)
T: Absolute temperature (T (K) = T (ºC) + 273)
m: Mass
ρ: Density
R: Gas constant, (Air: 287 $\frac{J}{Kg.K}$)

Example (8):

A compressed air tank has a volume of (0.3 m^3). When the tank is filled with air at a gage pressure of (0.2 MPa), determine the density of the air and the weight of air in the tank. Assume the temperature is (30 °C) and the atmospheric pressure is (101.3 kPa).

Solution:

$P = \rho RT$
$(0.2 \times 10^6) + (101.3 \times 10^3) = \rho \times 287 \times (30 + 273)$
$301300 = 86961\ \rho$
$\rho = 3.46\ kg/m^3$
$\rho = \frac{m}{v}$
$m = \rho\ v = 3.46 \times 0.3 = 1.04\ kg$
$w = m\ g = 1.04 \times 9.81 = 10.2\ N$

Example (9):

A tire having a volume of (0.08 m^3) contains air at a gage pressure of (2) bar and a temperature of (30^oC). Determine the density, specific weight, specific volume and the weight of the air contained in the tire.

Solution:

$$P = \rho RT$$

$$\rho = \frac{P}{RT} = \frac{(200+101.3)\times 10^3}{287 \times (30+273)} = 3.46 \text{ kg/m}^3$$

$$\gamma = \frac{w}{V} = \rho \times g = 3.46 \times 9.81 = 33.94 \text{ N/m}^3$$

$$\gamma = \frac{W}{V} \rightarrow W = \gamma \times V = 33.94 \times 0.08 = 2.7 \text{ N}$$

$$\upsilon = \frac{1}{\rho} = \frac{1}{3.46} = 0.289 \text{ m}^3\text{/kg}$$

Problems:

1- If a liquid has a specific gravity of (0.75), what is its density in g/cm^3?

2- The density of a certain liquid is measured to be (0.8 g/cm^3). Calculate its specific gravity with respect to water (density of water = 1 g/cm^3).

3- The density of a certain liquid is measured to be (850 kg/m^3). Calculate its specific gravity with respect to water.

4- The density of water at standard conditions { usually (4°C or 39.2°F) } is approximately (1000 kg/m^3 or 1 g/cm^3).

5- The specific weight of a certain liquid is (13×10^3 N/m^3). Determine its density and specific gravity.

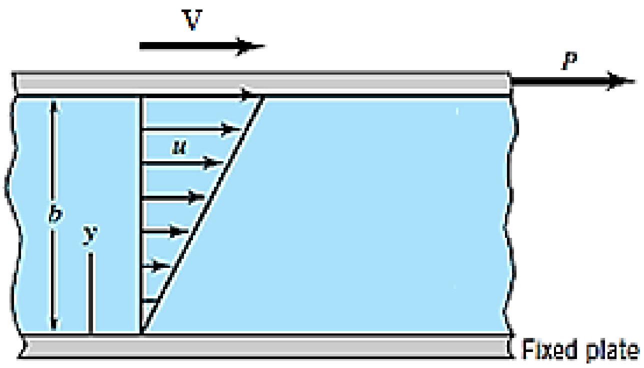
6- A tire having a volume of (0.1 m^3) contains air at a gage pressure of (4 bar) and a temperature of (30 ^oC) Determine the density of the air and the weight of the air contained in the tire. Assume the atmospheric pressure is (101.3 kPa).

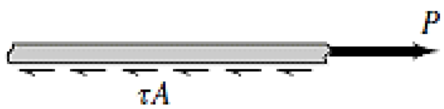
Viscosity:

The properties of density and specific weight are measures of the "heaviness" of a fluid. However, these properties are not enough to describe the behavior of the fluid. For example, two fluids such as water and oil can have approximately the same value of density but they behave differently when they flow. This is because they have different viscosity values.

Viscosity: is the property of a fluid which offers resistance to shear of one layer of fluid over another adjacent layer. Simply, it is fluid's resistance to flow.

To determine the viscosity, consider a hypothetical experiment in which a fluid is placed between two very wide parallel plates as show below:





Assumptions:

1. The bottom plate is rigidly fixed while the upper plate is free to move.
2. no-slip condition, *i.e.* fluid "sticks" to the solid boundaries.

A fluid at rest has no shearing forces. When the force (P) is applied to the upper plate. Shear stresses are developed and the fluid is now in motion. Thus, the particles of the fluid move relative to each other at different velocities and as follow:

- The fluid in contact with the bottom fixed plate has a zero velocity.
- The fluid in contact with the upper plate moves with the plate velocity (v).

This means that the fluid between the two plates moves with velocity that vary linearly as follow:

$$\frac{u}{v} = \frac{y}{b} \qquad\qquad u = v\,\frac{y}{b}$$

v: plate velocity.
u: Fluid velocity.
b: Film Thickness.
y: distance between fluid layers.

Thus, the velocity gradient (Rate of shearing stress)

$$\frac{du}{dy} = \frac{v}{b}$$

From the figure, we can conclude that the higher shear stress is on the point of the higher fluid velocity. Thus the shear stress (τ) and the velocity gradient ($\frac{du}{dy}$) can be related with a relationship:

$$\tau \propto \frac{du}{dy}$$

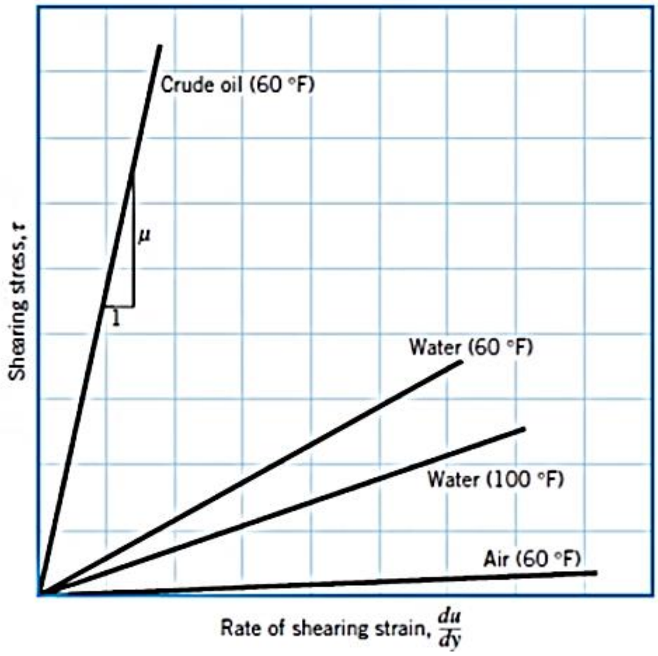
$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{b}$$

Newton Law of Viscosity

μ : The viscosity of the liquid and it is highly affected by the temperature.

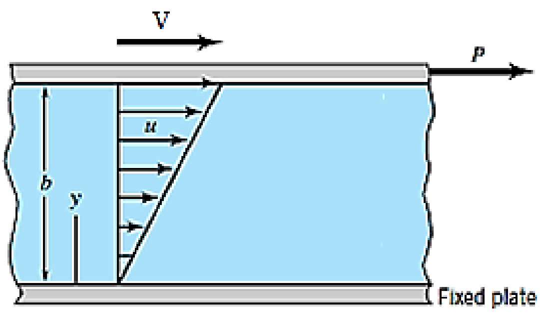
$\mu_{Water} = 0.001 \frac{N.s}{m^2} = 0.001$ Pa.s = 1 mPa.s

Viscosity varies from fluid to fluid and for a given fluid it varies with temperature.



Example – 1 :

As shown in the figure, certain oil (μ = 1.5 N.s/m^2) is used for lubrication purposes. The distance (b) between the plates is (6 mm). what is the minimum force (p) required to move the plate at (5.5 m/s). knowing that the area of the upper plate is (0.01 m^2).



Solution:

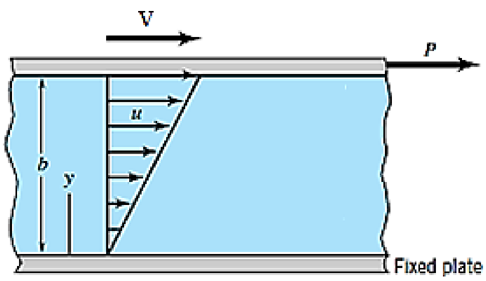
$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{v-0}{b-0} = 1.5 \times \frac{5.5}{6\times10^{-3}} = 1375 \text{ N/m}^2$$

$$\tau = \frac{P}{A} \qquad \Rightarrow \qquad P = \tau\, A = 1375 \times 0.01 = 13.75 \text{ N}$$

Example – 2 :

As shown in the figure, certain oil (μ = 0.8 N. S/m^2) is used for lubrication purposes. The distance (b) between the plates is (3 mm). An external force (p) of (18 N) is applied to the plate. Calculate the plate velocity knowing that the area of the upper plate is (0.02 m^2).



Solution:

$$\tau = \frac{P}{A} = \frac{18}{0.02} = 900 \text{ N/m}^2$$

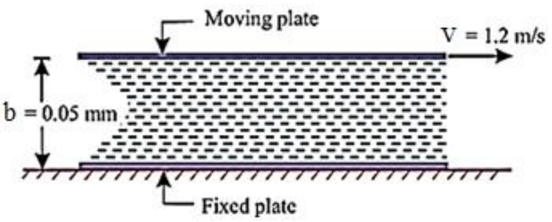
$$\tau = \mu \frac{du}{dy}$$

$$\tau = \mu \frac{u-0}{b-0} \rightarrow 900 = 0.8 \text{ x } \frac{u}{3\text{x}10^{-3}}$$

$$u = 3.375 \text{ m/s}$$

Example – 3 :

A moving plate of (0.05 mm) distant from a fixed plate moves at (1.2 m/s) and requires a shear stress of (2.2 N/m^2) to maintain this speed. Find the viscosity of the fluid between the plates.

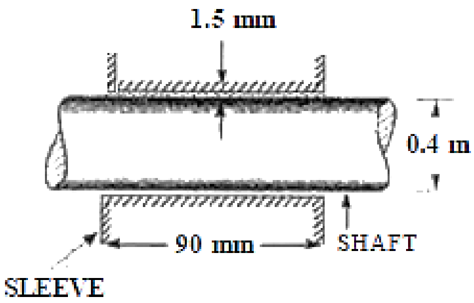


Solution:

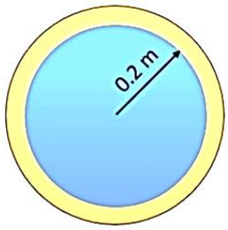
$$\tau = \mu \frac{v}{b} \quad \rightarrow \quad \mu = \frac{\tau \times y}{v} = \frac{2.2 \times 0.05 \times 10^{-3}}{1.2} = 9.16 \times 10^{-5} \text{ N.s/m}^2$$

Example – 4 :

The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is (0.6 Pa.s). The shaft is of diameter (0.4 m) and rotates at (190 r.p.m). Calculate the power lost due to shear stress knowing that the sleeve length is (90 mm) and thickness of the oil film is (1.5 mm).



Solution:



$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{b}$$

$$v = \omega \times r = N \left(\frac{rev}{min} \right) \times \frac{2\pi}{60} \times r$$

$$v = 190 \times \frac{2\pi}{60} \times 0.2 = 3.97 \text{ m/s}$$

$$\tau = \mu \frac{v}{b} = 0.6 \times \frac{3.97}{1.5 \times 10^{-3}} = 1588 \text{ N/m}^2$$

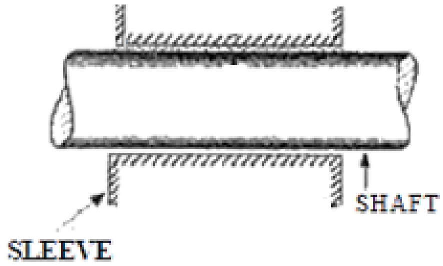
$$F_{lost} = \tau A = \tau \pi DL$$

$$F_{lost} = 1588 \times (\pi \times 0.4 \times 0.09) = 179.5 \text{ N}$$

$$P_{lost} = F_{lost} \times v = 179.5 \times 3.97 = 712 \text{ W}$$

Example – 5 :

The dynamic viscosity of an oil, used for lubrication between a shaft and sleeve is (0.7 Pa.s). The sleeve inner diameter is (0.55 m) and the shaft is of diameter (0.5 m) and rotates at (200 r.p.m). Calculate the power lost due to shear stress knowing that the sleeve length is (100 mm).



Solution:

$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{b}$$

$$v = \omega \times r = N \left(\frac{rev}{min} \right) \times \frac{2\pi}{60} \times r$$

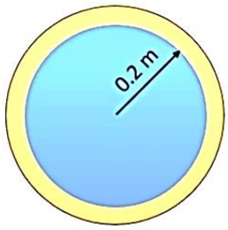
$$v = 200 \times \frac{2\pi}{60} \times 0.25 = 5.24 \text{ m/s}$$

$$\tau = \mu \frac{v}{b} = 0.7 \times \frac{5.25}{0.025} = 146.6 \text{ N/m}^2$$

$$F_{lost} = \tau A = \tau \pi DL$$

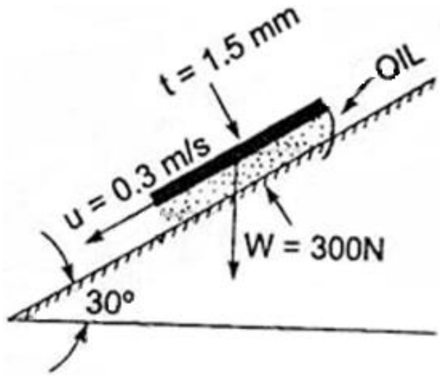
$$F_{lost} = 146.6 \times (\pi \times 0.5 \times 0.1) = 23.03 \text{ N}$$

$$P_{lost} = F_{lost} \times v = 23.03 \times 5.24 = 120.67 \text{ W}$$



Example – 6 :

Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size (0.8 m × 0.8 m) and an inclined plane with angle of inclination (30°) as shown in the figure. The wight of the square plate is (300 N) and it slides down the inclined plane with a uniform velocity of (0.3 m/s). The thickness of the oil film is (1.5 mm).



Solution:

$$\tau = \frac{F}{A} = \frac{300 \cos 30}{0.8 \times 0.8} = 234.375 \text{ N/m}^2$$

$$\tau = \mu \frac{v}{b}$$

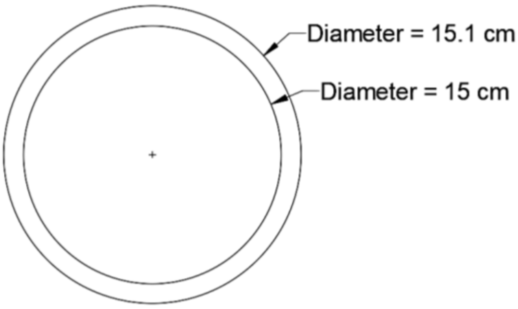
$$234.375 = \mu \frac{0.3}{1.5 \times 10^{-3}} = 200 \mu$$

$$\mu = \frac{234.375}{200} = 1.18 \text{ Pa.s}$$

Example – 7 :

A (15 cm) diameter vertical cylinder rotates concentrically inside another cylinder of diameter (15.1 cm). Both cylinders are (25 cm) high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of (12 N.m) is required to rotate the inner cylinder at (100 r.p.m.), determine the viscosity of the fluid.

Solution:



$$\tau = \mu \frac{du}{dy} = \mu \frac{v}{b}$$

$$v = \omega \times r = N\left(\frac{rev}{min}\right) \times \frac{2\pi}{60} \times r$$

$$v = 100 \times \frac{2\pi}{60} \times 0.075 = 0.785 \text{ m/s}$$

$$T = F \times r$$

$$F = \frac{T}{r} = \frac{12}{0.075} = 160 \text{ N}$$

$$\tau = \frac{F}{A} = \frac{F}{\pi DL} = \frac{160}{\pi \times 0.15 \times 0.25} = 1359 \text{ N/m}^2$$

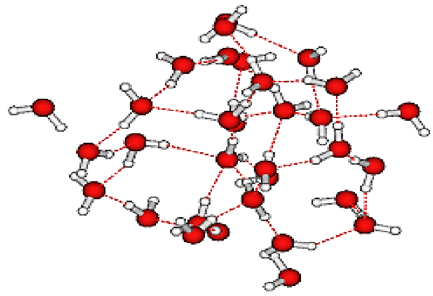
$$\tau = \mu \frac{v}{b} \quad \rightarrow \quad \mu = \frac{\tau \times b}{v} = \frac{1359 \times 0.0005}{0.875} = 0.77 \text{ N.s/m}^2$$

Viscosity & Temperature:

The viscosity of liquids **decreases** with the **increase** of temperature while the viscosity of gases **increases** with the temperature **increase**.

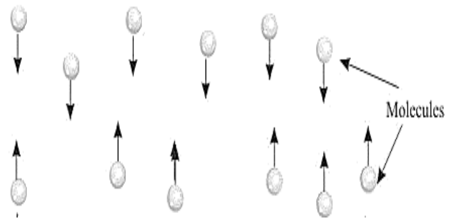
This difference in the effect of temperature on the viscosity of liquids and gases can again be traced back to the **difference in molecular Structure**.

The liquid molecules are **closely spaced**, with strong cohesive forces between molecules. The resistance to relative motion between adjacent layers of fluid (viscosity) is related to these **intermolecular forces**. As the temperature increases, these cohesive forces are reduced and hence result in reduction in viscosity.



In gases, the molecules are widely spaced and intermolecular forces negligible. In this case resistance to relative motion arises due to the exchange of momentum of gas molecules between adjacent layers. With the increase in temperature, molecular momentum transfer increases and hence viscosity increases.

Molecular momentum transfer: is the drag forces between the molecules.



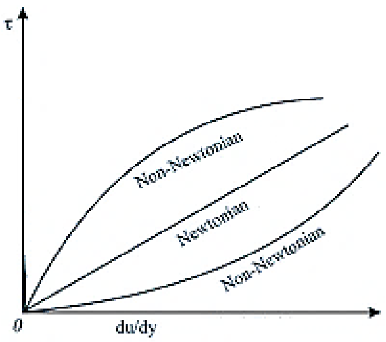
Newtonian and non-Newtonian fluids:

- ***Newtonian Fluids***:

  The shearing stress is linearly related to the velocity gradient. Fortunately, most common fluids, both liquids and gases are Newtonian.

- ***Non-Newtonian Fluids:***

  The shearing stress is not linearly related to the velocity gradient.



<u>Viscosity Grade (Multi – Grade Oil):</u>

In most vehicles, the temperature range the oil is exposed to can be wide, ranging from cold temperatures in the winter before the vehicle is started up to hot operating temperatures when the vehicle is fully warmed up. Thus, the oil will have high viscosity when cold and a lower viscosity at the engine's operating temperature.

The difference in viscosities is too large between the extremes of temperature. To bring the difference in viscosities closer together, special polymer additives called viscosity index improvers (VIIs) are added to the oil. These additives are used to make the oil a multi-grade oil.

The Society of Automotive Engineering (SAE) designation for multi-grade oils includes two viscosity grades. For example, (10W - 40) designates a common multi-grade oil. The first number (10 W) represents oil's viscosity at cold temperature and the second number (40) describes its viscosity at (100 °C). Note that both numbers are grades and not viscosity values. The viscosity value equivalent to each grade is as follow:

SAE 10 = 65 m Pa. sec

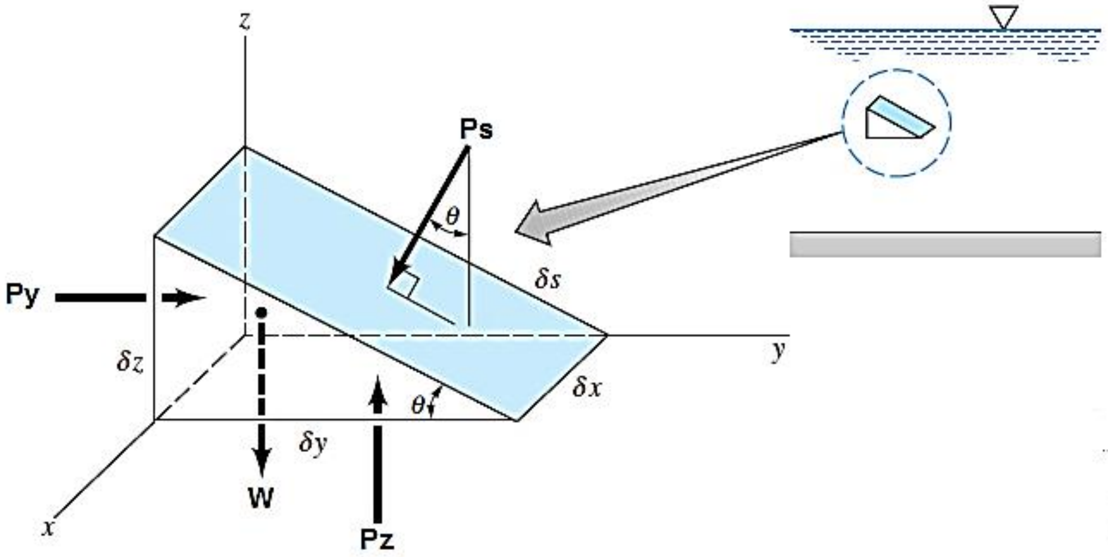
SAE 40 = 320 m Pa. sec

Fluid Static:

The fluid is either at rest or moving in such a manner so that there is no relative motion between adjacent particles. In both cases, there will be no shearing stress in the fluid and the only forces that develop on the surfaces of the particles will be due to the pressure.

Pascal's Law (Pressure at a Point):

'The pressure acting on a point in a fluid at rest is the same in all directions'.
To prove that, consider a **small triangular slice** of fluid within a fluid mass.



Since the fluid at rest (there are no shearing stresses), the only external forces acting on the slice are due to the pressure (P_y , P_z and P_s) and the weight (W).

Applying Newton's 2nd law of motion (F = m.a) in the (y) and (z) directions respectively:

$$\sum F_y = m.a_y \quad \Rightarrow \quad P_y\, \delta_x\, \delta_z - P_s\, \delta_x\, \delta_s \sin\theta = \rho\, v\, a_y$$

$$P_y\, \delta_x\, \delta_z - P_s\, \delta_x\, \delta_s \sin\theta = \rho \frac{\delta_x\, \delta_y\, \delta_z}{2} a_y$$

$$\delta_z = \delta_s \sin\theta$$

$$P_y\, \delta_x\, \delta_s \sin\theta - P_s\, \delta_x\, \delta_s \sin\theta = \rho \frac{\delta_y\, \delta_x\, \delta_s \sin\theta}{2} a_y$$

$$P_y - P_s = \rho \frac{\delta_y}{2} a_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\sum F_z = m.a_z \quad \Rightarrow \quad P_z\, \delta_x\, \delta_y - P_s\, \delta_x\, \delta_s \cos\theta - \gamma v = \rho\, v\, a_z$$

$$P_z\, \delta_x\, \delta_y - P_s\, \delta_x\, \delta_s \cos\theta = (\rho\, a_z + \gamma)\, \frac{\delta_x\, \delta_y\, \delta_z}{2}$$

$$\delta_y = \delta_s \cos\theta$$

$$P_z\, \delta_x\, \delta_s \cos\theta - P_s\, \delta_x\, \delta_s \cos\theta = (\rho\, a_z + \gamma)\, \frac{\delta_z\, \delta_x\, \delta_s \cos\theta}{2}$$

$$P_z - P_s = (\rho\, a_z + \gamma)\, \frac{\delta_z}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Since we are really interested in what is happening at a point, therefore ($\delta_x\ \delta_y\ \delta_z$) are **really small** and can be considered **zero**.

Eq. (1) becomes: $P_y - P_s = 0 \quad \Rightarrow \quad P_y = P_s$

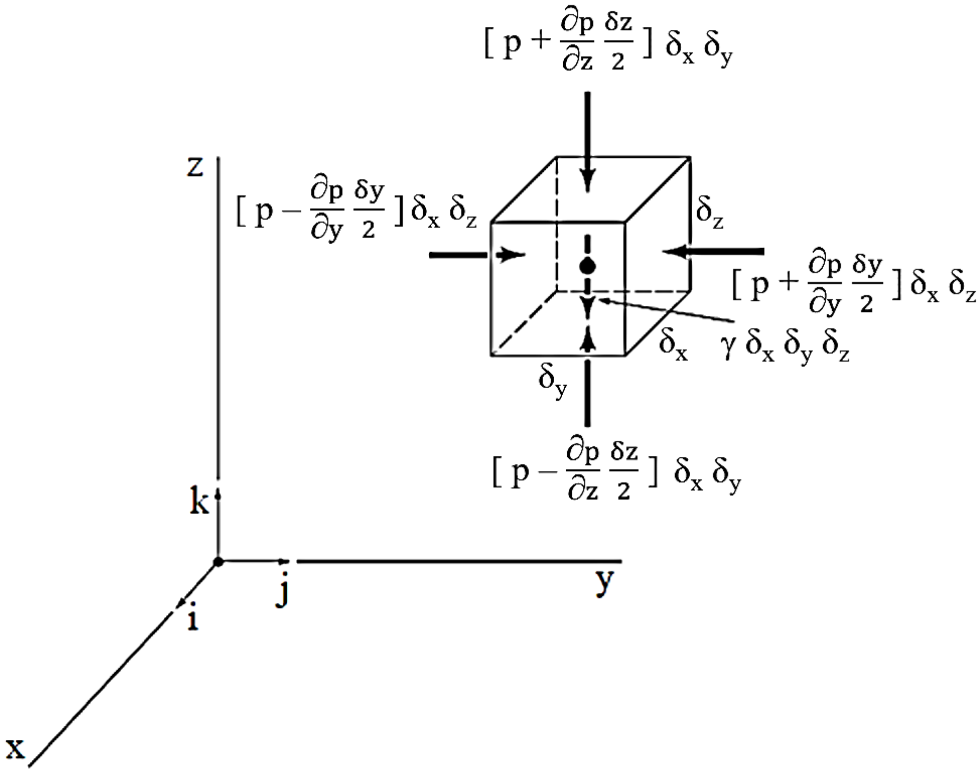
Eq. (2) becomes: $P_z - P_s = 0 \quad \Rightarrow \quad P_z = P_s$

Then $P_y = P_z = P_s$

The angle was arbitrarily chosen so we can conclude that the pressure at a point in a fluid at rest is **independent of direction** as long as there are no shearing stresses present.

Hydrostatic Pressure Law:

To answer the question of how the pressure changes from point to point, consider a rectangular element of fluid as follow:



The pressure at the center of the element is (P), then the average pressure on the various faces can be expressed in Taylor series expansion as shown above.

The resultant surface force in the (y) direction is:

$$F_y = [\, p - \frac{\partial p}{\partial y}\frac{\delta y}{2} \,]\, \delta_x\, \delta_z - [\, p + \frac{\partial p}{\partial y}\frac{\delta y}{2} \,]\, \delta_x\, \delta_z$$

$$F_y = p\, \delta_x\, \delta_z - \frac{\partial p}{\partial y}\frac{\delta y}{2}\, \delta_x\, \delta_z - p\, \delta_x\, \delta_z - \frac{\partial p}{\partial y}\frac{\delta y}{2}\, \delta_x\, \delta_z$$

$$F_y = - \frac{\partial p}{\partial y}\, \delta_x\, \delta_y\, \delta_z$$

Similarly, for the (x) and (z) directions the resultant surface forces are:

$$F_x = - \frac{\partial p}{\partial x}\, \delta_x\, \delta_y\, \delta_z \qquad , \qquad F_z = - \frac{\partial p}{\partial z}\, \delta_x\, \delta_y\, \delta_z$$

The resultant surface force acting on the element can be expressed in vector form as

$$F_R = F_X i + F_Y j + F_Z k$$

$$F_R = -[\ \frac{\partial p}{\partial x} \delta_x\ \delta_y\ \delta_z\ ]\ i - [\ \frac{\partial p}{\partial y} \delta_x\ \delta_y\ \delta_z\ ]\ j - [\ \frac{\partial p}{\partial z} \delta_x\ \delta_y\ \delta_z\ ]\ k$$

$$F_R = -[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ]\ \delta_x\ \delta_y\ \delta_z$$

Also, the weight of the element can be expressed in vector form as:

$$W = -\gamma\ \delta_x\ \delta_y\ \delta_z$$

Newton's second law, applied to the fluid element, can be expressed as:

$$\sum F = m.a$$

$$-[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ]\ \delta_x\ \delta_y\ \delta_z - \gamma\ \delta_x\ \delta_y\ \delta_z\ k = \rho\ \delta_x\ \delta_y\ \delta_z\ a$$

$$-[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ] - \gamma k = \rho\ a$$

For a fluid at rest, a = 0

$$-[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ] = k\ \gamma$$

$$[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ] = -k\ \gamma$$

$$[\ \frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k\ ] = 0\ i + 0\ j - k\ \gamma$$

$$\frac{\partial p}{\partial x} = 0 \quad , \quad \frac{\partial p}{\partial y} = 0 \quad , \quad \frac{\partial p}{\partial z} = -\gamma$$

These equations show that the pressure **does not** depend on **(x) or (y) directions**. Thus, when we move from point to point in a horizontal plane, the pressure does not change and it **depends** only on **(z) direction.**

Since (p) depends only on **(z) direction**, the equation can be written as an ordinary differential equation.

$$\frac{dp}{dz} = -\gamma$$ This is the fundamental equation for fluids at rest that can be used to determine how pressure changes with elevation:

$$dp = -\gamma\, dz$$

$$\int_{P0}^{P1} dp = -\gamma \int_{Z0}^{Z1} dz$$

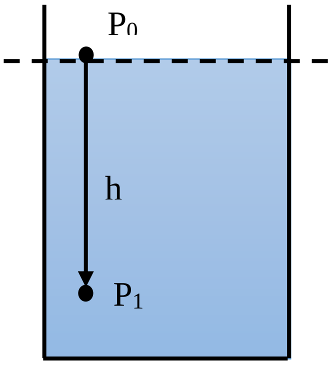
$$P_1 - P_0 = -\gamma\,(Z_1 - Z_0)$$

$$P_1 = P_0 - \gamma\,(-h - 0)$$

$$P_1 = P_0 + \gamma\, h$$

P_0 can be taken as zero (open to atmosphere)

$$P_1 = \gamma\, h = \rho g h$$



Example:

Calculate the pressure due to a column of (0.3) of :

a) A water.
b) An oil of specific gravity (0.8).
c) A mercury of specific gravity (13.6).

Take density of water (1000 kg/m^3).

Solution:

a) For water : ρ = 1000 kg/m^3

$$P = \rho g h = 1000 \times 9.81 \times 0.3 = 2943\ N/m^2$$

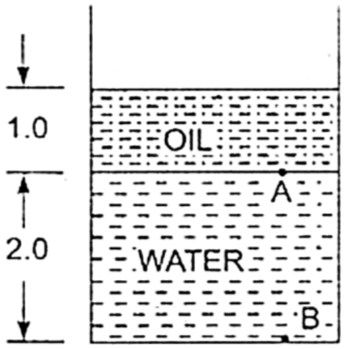
b) For oil of specific gravity (0.8) :

Example:

Example:

An open tank contains water upto a depth of (2 m) and above it an oil of specific gravity of (0.9) for a depth of (1 m). Find the pressure intensity:

a) At the interface of the two liquids.
b) The bottom of the tank.



Solution:

$$SG = \frac{\rho_{oil}}{\rho_{water}}$$

$$\rho_{oil} = SG \times \rho_{water} = 0.9 \times 1000 = 900 \ kg/m^3$$

a) At the interface of the two liquids.

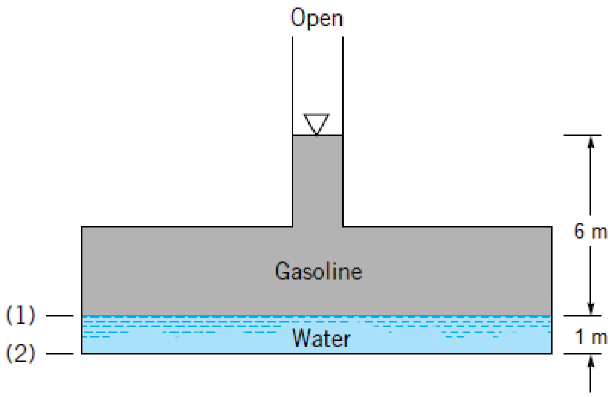
$$P_1 = \rho_{oil} \, gh_1 = 900 \times 9.81 \times 1 = 8892 \ N/m^2$$

b) The bottom of the tank.

$$P_2 = P_1 + \rho_{water} \, gh_2 = 8892 + (1000 \times 9.81 \times 2) = 28500 \ Pa$$

Example:

Because of a leak in a buried gasoline storage tank, water has seeped in to the depth. If the specific gravity of the gasoline is (0.68). Determine the gauge pressure at the gasoline-water interface (1) and at the bottom of the tank (2).



Solution:

$$SG = \frac{\gamma_{gasoline}}{\gamma_{water}}$$

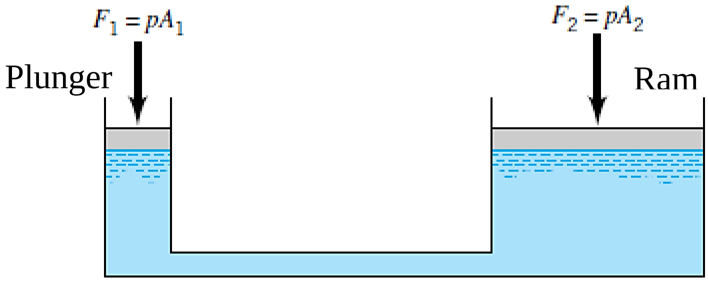
$$\gamma_{gasoline} = 0.68 \times 9810 = 6670.8 \ N/m^3$$

$$P_1 = \gamma_{gasoline} \, h_1 = 6670.8 \times 6 = 40024.8 \ Pa$$

$$P_2 = P_1 + \gamma_{water} \, h_2 = 40024.8 + (9810 \times 1) = 49834.8 \ Pa$$

Application on Hydrostatic Pressure Law:

The equality of pressures at same elevations throughout a system is important for the operation of hydraulic jacks, lifts and presses.



The pressure (p) acting on the faces of both pistons is the same. Therefore, the Ram area (A_2) can be made much larger than Plunger area (A_1) and hence a large mechanical advantage can be developed.

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \frac{A_2}{A_1} F_1$$

That is, a small force applied at the smaller piston can be used to develop a large force at the larger piston.

Example:

A hydraulic press has a ram of (20 cm) diameter and a plunger of (3 cm) diameter. It is used for lifting a weight of (10 kN). Find the force required at the plunger.

Solution:

$$A_r = \pi \frac{{D_r}^2}{4} = \pi \frac{(0.2)^2}{4} = 0.0314 \text{ m}^2$$

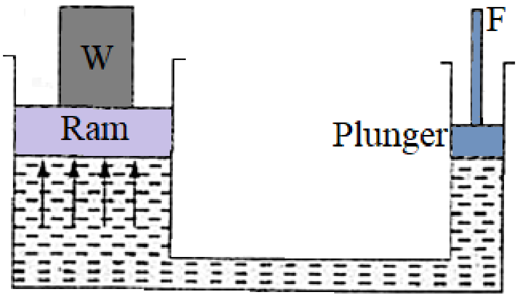
$$A_p = \pi \frac{{D_p}^2}{4} = \pi \frac{(0.03)^2}{4} = 0.0007 \text{ m}^2$$

$$F_2 = \frac{A_2}{A_1} F_1 \quad , \quad F_p = \frac{A_p}{A_r} F_r$$

$$F_p = \frac{0.0007}{0.0314} \times 10000 = 225 \text{ N}$$

Example:

A hydraulic press has a ram of (30 cm) diameter and a plunger of (4.5 cm) diameter. Find the wight lifted by the hydraulic press when the force applied at the plunger is (500 N).



Solution:

Diameter of ram, $D = 30$ cm $= 0.3$ m

Diameter of plunger, $d = 4.5$ cm $= 0.045$ m

Force on plunger, $F = 500$ N

$$A_r = \frac{\pi\, D_r{}^2}{4} = \frac{\pi\,(0.3)^2}{4} = 0.07068\ m^2$$

$$A_p = \frac{\pi\, D_p{}^2}{4} = \frac{\pi\,(0.045)^2}{4} = 0.00159\ m^2$$

$$F_p = \frac{A_p}{A_r}\, F_r \quad , \quad F = \frac{A_p}{A_r}\, W \quad \Rightarrow \quad W = F\,\frac{A_r}{A_p}$$

$$W = 500 \times \frac{0.07068}{0.00159} = 22226\ N = 22.2\ kN$$

Pressure (Gauge, Absolute and Atmospheric):

Pressure:

Is defined as the normal force per unit area. The (SI) unit is (N/m^2) or Pascal (Pa).

$$P = \frac{\text{Force}}{\text{Area over which the force is applied}}$$

Gauge pressure:

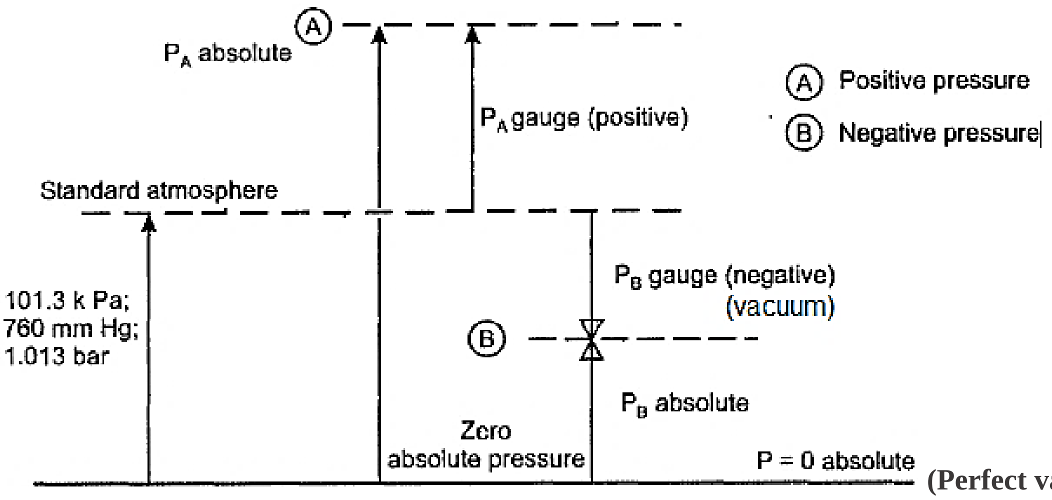
Is the pressure measured by a manometer relative to the atmospheric pressure.

Atmospheric pressure:

Is the pressure caused by the weight of the atmosphere (air).

Absolute pressure:

Is the pressure obtained relative to a perfect vacuum condition.

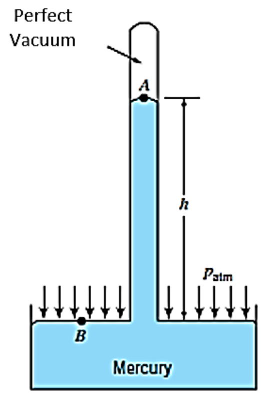


(Perfect vacuum)

$$(\mathbf{P_A})_{\text{absolute}} = \mathbf{P_{atm}} + (\mathbf{P_A})_{\text{gauge}}$$

$$(\mathbf{P_B})_{\text{absolute}} = \mathbf{P_{atm}} - (\mathbf{P_B})_{\text{Vacuum}}$$

The **measurement of the Atmospheric Pressure** is usually accomplished with a ***mercury barometer***. A tube is initially filled with mercury and then turned upside down in a container of mercury. The column of mercury will come to an **equilibrium position** (@ 760 mm) where the weight of the fluid **balances** the force due to the atmospheric pressure.



Measurement of Pressure:

Manometers:

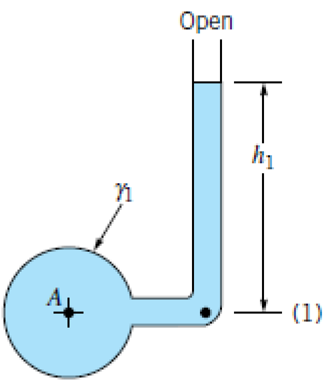
Is a standard technique for measuring pressure which involves the use of liquid columns in vertical or inclined tubes. The common types of manometers are:

- Piezometer tube.
- U-tube manometer.
- Differential manometer.
- Inclined-tube manometer.

Piezometer Tube Manometer:

The simplest type of manometer. It consists of a vertical tube opened at the top and attached to the container in which the pressure is desired to be measured.

$$P_A = P_1 = \gamma_1 h_1$$

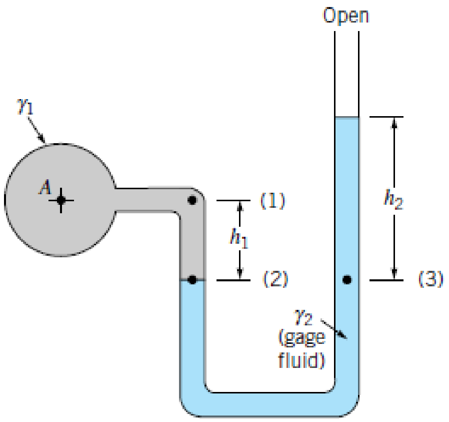


Advantages and Disadvantages:

1. Very simple and accurate.
2. Measure **liquid pressure** only.
3. Only suitable if the pressure in the container is **greater** than atmospheric pressure.
4. Pressure must be relatively small so that the **height** of the column is reasonable.

U-Tube Manometer:

To overcome the difficulties in piezometer tube, U-tube manometer is used which includes an additional liquid (gauge fluid).



$$P_A + \gamma_1 h_1 - \gamma_2 h_2 = 0 \quad \text{(See the note below)}$$

$$P_A = \gamma_2 h_2 - \gamma_1 h_1$$

Note: Pressure ***increases*** **(+)** as we move ***downward*** and ***decreases*** **(-)** as we move ***upward***.

Advantages:

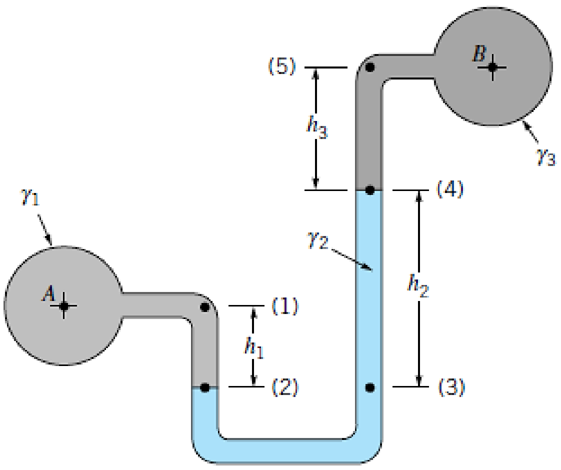
- Measure **gas** and **liquid** pressures.
- Adjustable column height for different pressure ranges.

How do we adjust the measurement range?

If the pressure (P_A) is **large**, then a **heavy gauge fluid** (such as mercury) can be used and a reasonable column height (not too long) can still be maintained. Alternatively, if the pressure (P_A) is **small**, a **lighter gage fluid** (such as water) can be used so that a relatively large column height (readable) can be achieved.

Differential manometer:

It is basically a U-type manometer connected from both sides with a gauge liquid in between.

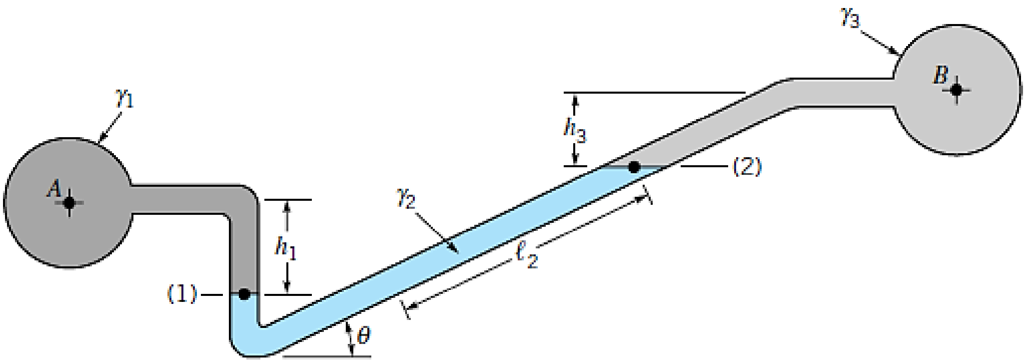


$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

$$P_A - P_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$

Inclined-tube manometer

The inclined-tube manometer is often used to measure **small differences in gas pressures.**



$$P_A + \gamma_1 h_1 - \gamma_2 L_2 \sin\theta - \gamma_3 h_3 = P_B$$

$$P_A - P_B = \gamma_2 L_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

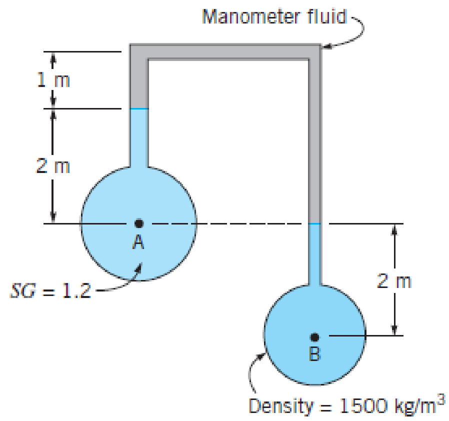
The contributions of the gas columns ($\gamma_1 h_1$ & $\gamma_3 h_3$) can be neglected which gives:

$$L_2 = \frac{P_A - P_B}{\gamma_2 sin\theta}$$

For a given pressure difference, the **sensitivity** can be changed by **adjusting** the manometer's angle. Thus, for relatively small angles the differential reading along the inclined tube (L_2) can be large even for small pressure differences.

Example:

The pressure at point (B) is (20 kPa) greater than at point (A). Determine the specific weight of the manometer fluid.



Solution:

$$P_A - (\gamma_A \times 2) + (\gamma_m \times 2) + (\gamma_B \times 2) = P_B$$

$$P_B - P_A = -(\gamma_A \times 2) + (\gamma_m \times 2) + (\gamma_B \times 2)$$

$$P_B - P_A = 2\,\gamma_m + 2(\gamma_B - \gamma_A)$$

$$SG_A = \frac{\gamma_A}{\gamma_{water}}$$

$$\gamma_A = SG_A \,.\, \gamma_{Water} = 1.2 \times 9810 = 11772 \text{ N/m}^3$$

$$\gamma_B = \rho_B \times g = 1500 \times 9.81 = 14715 \text{ N/m}^3$$

$$P_B - P_A = 2\,\gamma_m + 2(\gamma_B - \gamma_A)$$

$$20000 = 2\,\gamma_m + 2(14715 - 11772)$$

$$20000 = 2\,\gamma_m + 5886$$

$$2\,\gamma_m = 20000 - 5886 = 14114$$

$$\gamma_m = 7057 \text{ N/m}^3$$

Example:

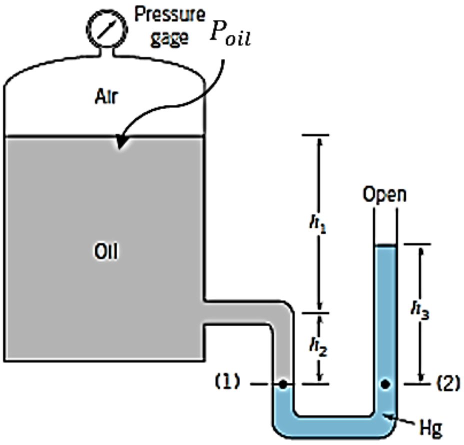
A closed tank contains compressed air and oil (SG = 0.9). A U-tube manometer using mercury (SG = 13.6) is connected to the tank. For column heights:

(h_1 = 140 cm),

(h_2 = 24 cm),

(h_3 = 36 cm),

determine the pressure reading of the gage.



Solution:

$$P_{Air} + \gamma_{oil}\, h_1 + \gamma_{oil}\, h_2 - \gamma_{me}\, h_3 = 0$$

$$P_{Air} = \gamma_{me}\, h_3 - \gamma_{0il}\, h_1 - \gamma_{0il}\, h_2$$

$$\gamma_{oil} = SG_{oil} \,.\, \gamma_{Water} = 0.9 \times 9810 = 8829 \text{ N/m}^3$$

$$\gamma_{me} = SG_{me} \,.\, \gamma_{Water} = 13.6 \times 9810 = 133416 \text{ N/m}^3$$

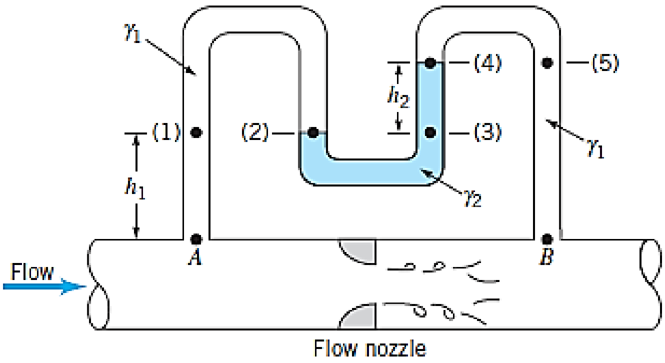
$$P_{Air} = (\ 133416 \times 0.36\ ) - (\ 8829 \times 1.4\ ) - (\ 8829 \times 0.24\ )$$

$$= 48029.76 - 12360.6 - 2118.96 = 33550 \text{ Pa} = 33.55 \text{ kPa}$$

Example:

Determine the value of the pressure drop (ΔP) created by the nozzle shown. For:

($\gamma_1 = 9.8\ KN/m^3$),
($\gamma_2 = 15.6\ KN/m^3$),
($h_1 = 1\ m$)
($h_2 = 0.5\ m$).



Solution:

$$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1 (h_1 + h_2) = P_B$$

$$P_A - \gamma_1 h_1 - \gamma_2 h_2 + \gamma_1 h_1 + \gamma_1 h_2 = P_B$$

$$P_A - \gamma_2 h_2 + \gamma_1 h_2 = P_B$$

$$P_A + h_2 (\gamma_1 - \gamma_2) = P_B$$

$$P_A - P_B = h_2 (\gamma_1 - \gamma_2)$$

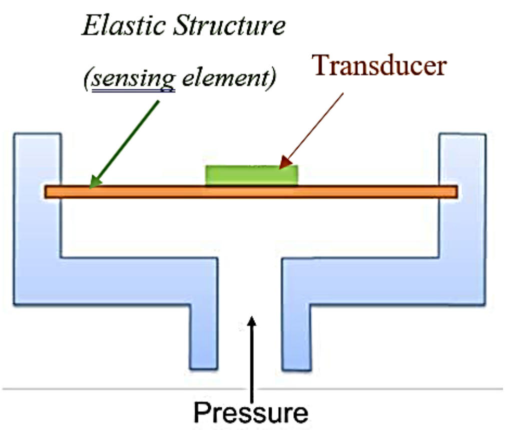
$$\Delta P = 0.5 (15.6 - 9.8) = 2.9\ kPa$$

Pressure Sensor:

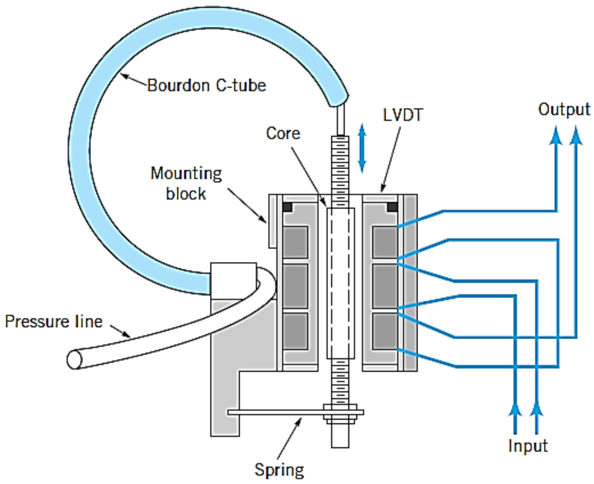
Although manometers are widely used, they have some disadvantages:

- They are not well suited for measuring very high pressures.
- Not suitable for pressures that are changing rapidly with time.
- They require the measurement of one or more column heights which can be time consuming.

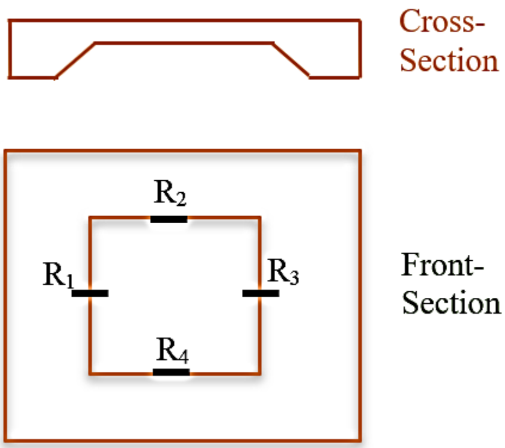
To overcome these problems, other types of pressure measuring instruments have been developed. Most of these instruments make use of the idea that when a pressure acts on an elastic structure, the structure will deform and this deformation can be related to the magnitude of the pressure.



The most familiar device is the Bourdon Tube Pressure Sensor. The essential mechanical element in this devise is the hollow, elastic curved tube Bourdon tube which is connected to the pressure source. As the pressure within the tube increases the tube tends to straighten. This deformation can be translated into an electrical output using linear variable differential transformer (LVDT).



Because of the relatively high stiffness of the Bourdon tube**, it cannot respond to rapid changes in pressure**. To overcome this difficulty, the sensing element is replaced by a thin, elastic diaphragm. Also, a strain gauge is attached on the surface of the diaphragm. As the pressure changes, the diaphragm deflects and this deflection can be sensed by strain gage and converted into electrical signal.



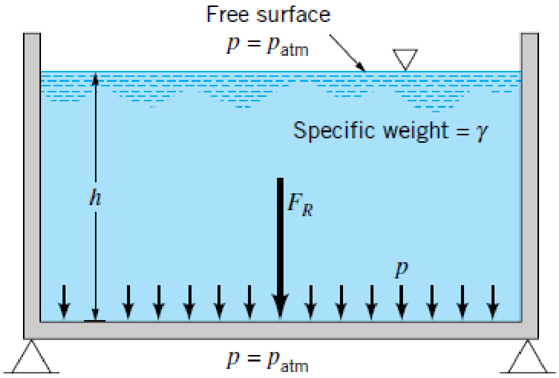
Hydrostatic Force on Submerged Surfaces:

When a surface is submerged (مغمور) in a fluid, forces develop perpendicularly to the surface of contact. The calculations of these forces are important in the design of **storage tanks**, **dams, gates, etc.**

Force on Horizontal Surfaces:

In horizontal surfaces, the pressure is constant and uniformly distributed. Therefore, the magnitude of the resultant force is simply equal to:

$$F_R = P.A = \gamma h.A$$



Location:

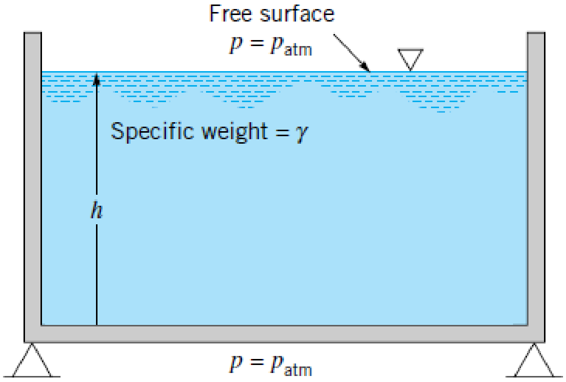
It acts through the centroid of the surface area.

Force on Vertical Surfaces:

In horizontal surfaces, the pressure will vary **linearly** with depth Therefore, the magnitude of the resultant force can be calculated from the average pressure:

$$F_R = P_{av}.A = \frac{P_{Max.} - P_{Min.}}{2} A$$

$$F_R = \frac{P_{Max.}}{2} A = \frac{\gamma h}{2} A$$

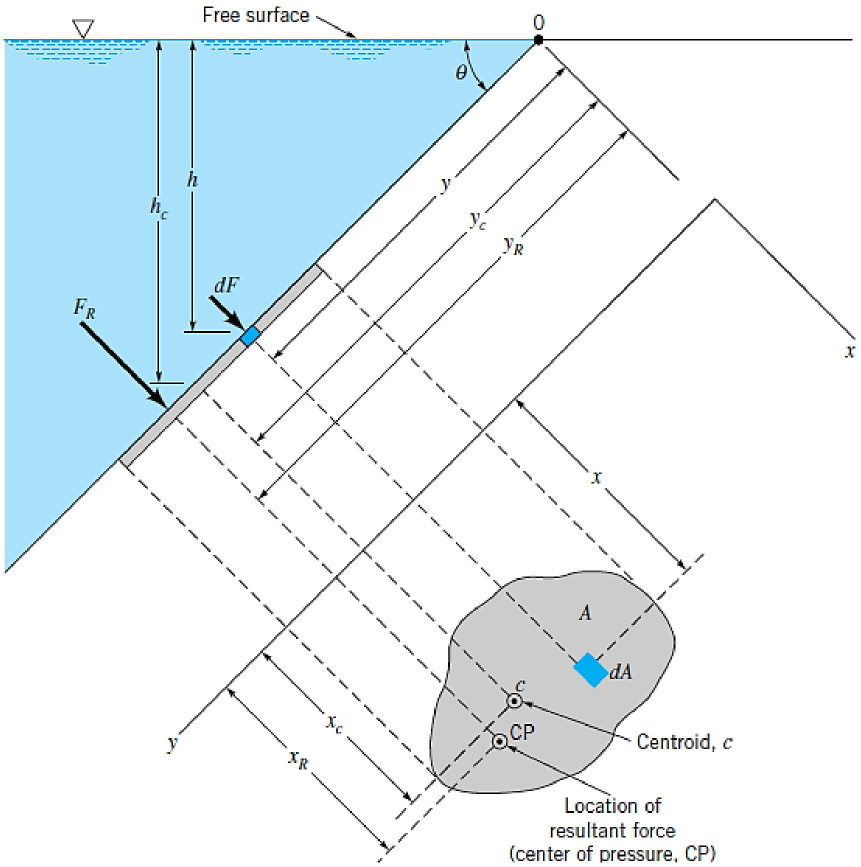


Location:

It acts through the centroid of the area formed by the pressure on the surface (Center of Pressure)

Force on Inclined Surfaces:

In more general case where the surface is inclined, the pressure **will also vary linearly** with depth. To calculate the **magnitude** and **location** of the resultant force, we follow the following procedure:



- The *x–y* coordinate system is defined in the origin (*O*) and *y* is directed along the surface passing through the centroid.
- The entire surface is divided into a number of small parallel strips (dA). The force on small strip (dF) is calculated as follow:

$$dF = \gamma h \, dA$$

- The magnitude of the resultant force can be found by summing these small forces over the entire surface:

$$F_R = \int_A \gamma h \, dA = \int_A \gamma \, y \sin\theta \, dA$$

Where: ($h = y \sin\theta$) For constant (γ) and (θ).

$$F_R = \gamma \sin\theta \int_A y \, dA$$

- The integral Part is the ***first moment of the area*** *and it is equal to* ($A.y_C$). Thus, the resultant force can be written as:

$$F_R = \gamma A y_c \sin\theta$$
$$h_c = y_c \sin\theta$$
$$F_R = \gamma h_c A$$

h_c : the vertical distance from the **fluid surface** to the **centroid of the area**.

The ***location (Cp)*** of the resultant force can be determined by taking moments around the origin (*O*) in the X and Y directions:

$$F_R y_R = \int_A y \, dF$$

$$dF = \gamma h \, dA$$
$$h = y \sin\theta$$
$$dF = \gamma y \sin\theta \, dA$$

$$\gamma A y_c \sin\theta \; y_R = \int_A \gamma \sin\theta \; y^2 \, dA$$

$$y_R = \frac{\int_A \gamma^2 \, dA}{y_c A}$$

The integral part is the second moment of area (I_x) with respect to the free surface (x – axis).

$$y_R = \frac{I_x}{y_c A}$$

The parallel axis theorem can be used to determine (I_x).

$$I_x = I_{xc} + A y_c^2$$

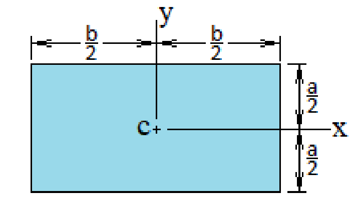
Where (I_{xc}) is the second moment of area with respect to the *centroid axis*.

$$y_R = \frac{I_x}{y_c A} + y_c$$

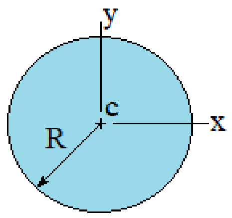
The (x) coordinate (x_R) can be determined in a similar manner by summing moments about the (y) axis.

$$x_R = \frac{I_{xyc}}{y_c A} + x_c$$

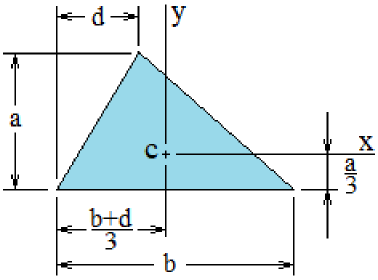
The geometric properties (area, Centroid & moment of area) of some common shapes are given below:



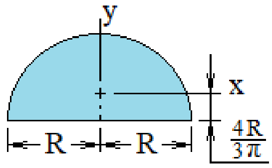
$$A = ba$$
$$I_{xc} = \frac{1}{12} ba^3$$
$$I_{yc} = \frac{1}{12} ab^3$$
$$I_{xyc} = 0$$



$$A = \pi R^2$$
$$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$$
$$I_{xyc} = 0$$



$$A = \frac{ab}{2}$$
$$I_{xc} = \frac{ba^3}{36}$$
$$I_{xyc} = \frac{ba^2}{72}(b-2d)$$

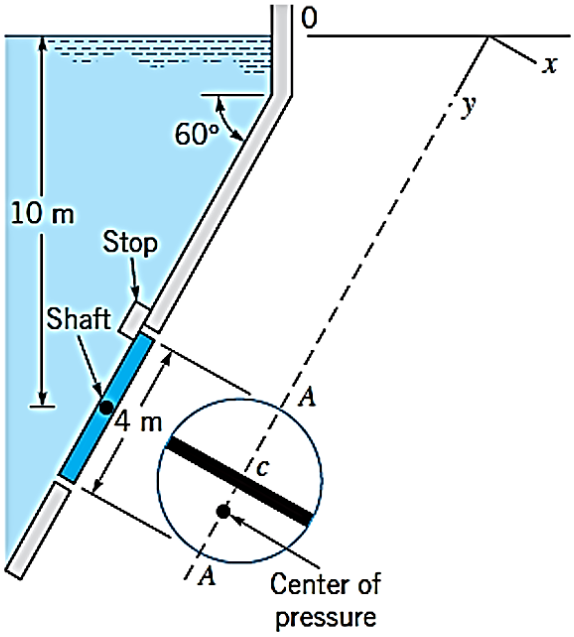


$$A = \frac{\pi R^2}{2}$$
$$I_{xc} = 0.1098R^4$$
$$I_{yc} = 0.3927R^4$$
$$I_{xyc} = 0$$

Example 1:

A (4 m) diameter circular gate is located in the inclined (60º) wall of a large reservoir containing water. The gate is mounted on a shaft along its horizontal diameter. For a water depth of (10 m) above the shaft determine:

1. **The magnitude and location of the resultant force exerted on the gate by the water.**
2. **The moment that would have to be applied to the shaft to open the gate.**



Solution:

$F_R = \gamma \, h_c \, A$

$F_R = 9810 \times 10 \times (\frac{\pi}{4} \times 4^2)$

$= 1.23$ MN

To locate the point (Cp) through which (F_R) acts

$y_R = \frac{I_{xc}}{y_c A} + y_c$

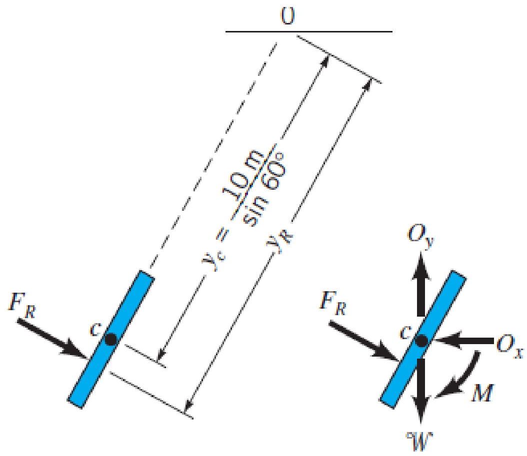
$I_{xc} = \frac{\pi R^4}{4} = \frac{\pi (2)^4}{4} = 12.566 \text{ m}^4$

$y_c = \frac{10}{\sin 60} = 11.55$ m

$A = \pi r^2 = \pi (2)^2 = 12.566 \text{ m}^2$

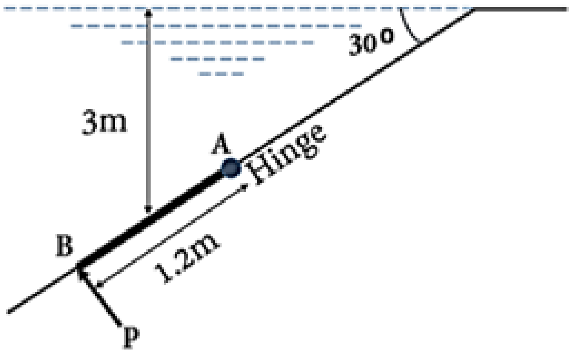
$y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{12.566}{(11.55)(12.566)} + 11.55 = 11.64$ m

$M_c = F_R \times (y_R - y_c) = 1.23 \times (11.64 - 11.55) = 0.11$ MN.m



Example 2:

An inclined rectangular gate (AB) (1.2 m) height and (3.5 m) width is installed to control the discharge of water. The end (A) is hinged. Determine the force (P) normal to the gate applied at (B) to open it.



Solution:

$F_R = \gamma\, h_c\, A$

$A = 1.2 \times 3.5 = 4.2\ m^2$

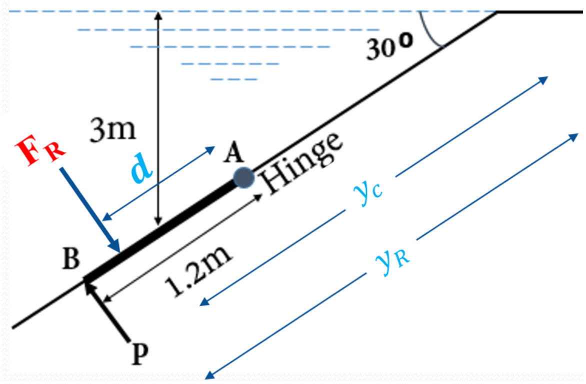
$F_R = 9810 \times 3 \times 4.2 = 123606\ N$

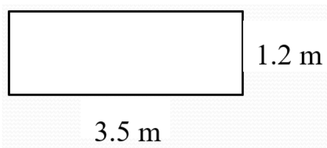
$$y_R = \frac{I_{xc}}{y_c\, A} + y_c$$

$$I_{xc} = \frac{1}{12} b a^3 = \frac{1}{12} (3.5)(1.2)^3 = 0.504\ m^4$$

$$y_c = \frac{3}{\sin 30} = 6\ m$$

$$y_R = \frac{I_{xc}}{y_c\, A} + y_c = \frac{0.504}{(6)(4.2)} + 6 = 6.02\ m$$





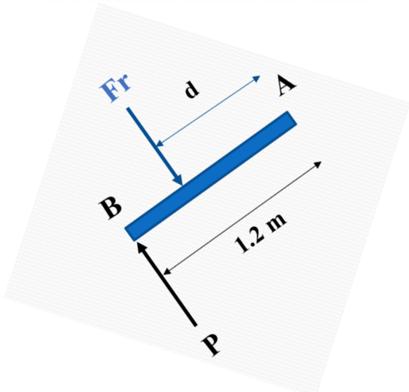
$\sum M_A = 0$

$P \times 1.2 = F_R \times d$

$P \times 1.2 = 123606 \times [\ (y_R - y_c) + 0.6\ ]$

$P \times 1.2 = 123606 \times [\ (6.02 - 6) + 0.6\ ]$

$P = 63863\ N$



Elementary Fluid Dynamics

As was discussed earlier, Fluid Static is the situations where the fluid is considered to be ***stationary (Rest)***. In contrast, Fluid Dynamics studies and analyses fluid ***in motion (Flow)*** along a streamline.

Flow Classification

The flow of the fluid particles can be classified as follow:

Flow
- ***Viscous***: is a flow with a considerable amount of viscosity (Real Flow $\mu \neq 0$).
- ***Non-viscous***: is a flow of inviscid fluid (Ideal Flow $\mu = 0$).

Flow
- ***Laminar Flow***: each particle of the fluid follows a smooth path, usually at low velocities, which never interfere with one another.
- ***Turbulent Flow***: Turbulent .

Flow
- ***Uniform***: at a given instant, the fluid particles have the same velocity at **every point** along the streamline.
- ***Non-uniform***: at a given instant, velocity **is not** the same at every point along the streamline.

Flow
- ***Steady***: at a given point, the velocity may differ from other points along the streamline but ***DO NOT*** change with time.
- ***Unsteady***: at a given point along the streamline, the conditions change with time.

Combining the last two types above, we can classify any flow as:

- *Steady uniform flow.*
- *Steady non-uniform flow.*
- *Unsteady uniform flow.*
- *Unsteady non-uniform flow.*

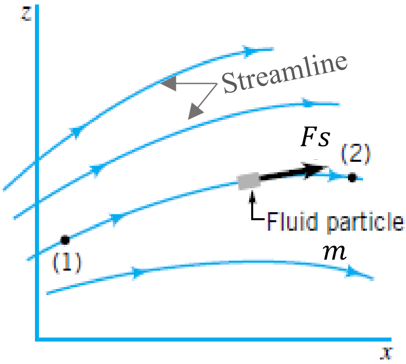
Equations of motion

As a fluid particle moves from one location to another along the streamline, it usually experiences acceleration and deceleration. This **dynamic behaviour** of the fluid flow is analysed using several equations of motion such as:

1. *Newton 2nd law of motion*

According to Newton's second law of motion, the net force $\boldsymbol{F_S}$ acting on a fluid element in the streamline direction (***S***) is equal to mass ***m*** of the fluid element multiplied by the acceleration in the S-direction. Mathematically:

$$\mathbf{F_S} = \mathbf{m} \cdot \mathbf{a_S}$$



In fluid flow, the following ***forces*** are present:

- F_G, gravity force.
- F_P, pressure force.
- F_V, force due to viscosity.
- F_C, force due to compressibility.
- F_R, force due to turbulence.

Thus, $F_S = F_G + F_P + F_V + F_C + F_R$

2. *Euler equation of motion*

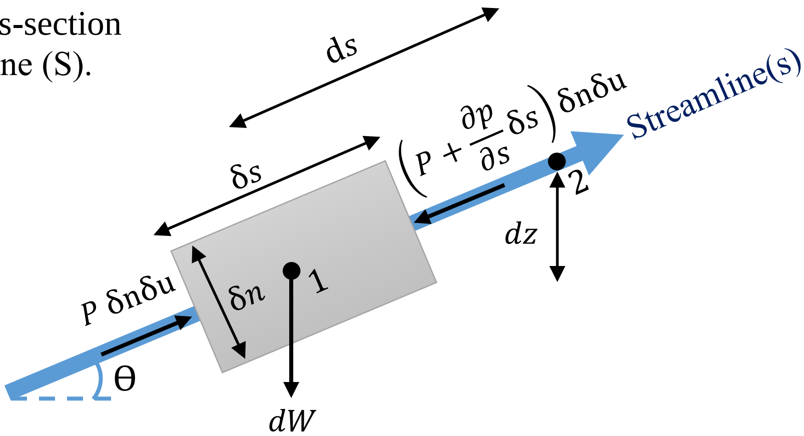
The following assumptions are made in the derivation of Euler's equation:

- The flow is ideal, *i.e.*, viscosity is zero ($F_V = 0$).
- The flow is steady, *i.e.*, velocity change is zero.
- The flow is incompressible ($F_C = 0$).
- The flow is non-rotational ($F_R = 0$).

Therefore, only forces due to **gravity** and **pressure** are taken into consideration.

$$\mathbf{F_S = F_G + F_P}$$

Consider a fluid element of cross-section δnδu and length δS along the streamline (S).



The forces acting on the element are:

1. Pressure force (p δnδu) in the direction of flow.
2. Pressure force $\left(P + \frac{\partial p}{\partial s}\delta s\right)$ δnδu opposite to the direction of flow.
3. Weight of element dW = *specific weight x volume* = $\rho g\ \delta n \delta u \delta s$.

$\sum$ **F$_s$ = m a$_s$**

$$P\delta n\delta u - \left(P + \frac{\partial p}{\partial s}\delta s\right)\delta n\delta u - \rho g\ \delta n\delta u\delta s\ \mathrm{Sin}\theta = \rho\ \delta n\delta u\delta s\, .\, a_s$$

The acceleration (a_s) is the rate change of velocity V(s) in the (S) direction, *Therefore*

$$a_s = \frac{dv(s)}{dt}$$

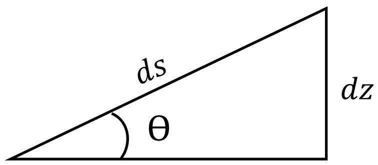
The velocity may change from point to point, *Thus*

$$a_s = \frac{\partial v}{\partial s}\frac{ds}{dt}, \quad \text{where } \frac{ds}{dt} = v$$

$$a_s = v\frac{\partial v}{\partial s}$$

$$-\frac{\partial P}{\partial s}\delta n\delta u\delta s - \rho g\ \delta n\delta u\delta s\ \mathrm{Sin}\theta = \rho\ \delta n\delta u\delta s\, .\, v\frac{\partial v}{\partial s}$$ dividing by (ρ δnδuδs)

$$-\frac{\partial P}{\rho\ \partial s} - g\ \mathrm{Sin}\theta = v\frac{\partial v}{\partial s}$$ $$\sin\theta = \frac{dz}{ds}$$



$$\frac{\partial P}{\rho\ \partial s} + g\frac{dz}{ds} + v\frac{\partial v}{\partial s} = 0$$ dividing by (g)

$$\frac{1}{\rho\, g}\frac{\partial P}{\partial s} + \frac{V}{g}\frac{\partial v}{\partial s} + \frac{dz}{ds} = 0$$ Euler equation of motion.

3. Bernoulli equation

Bernoulli equations can be obtained by rearranging and integrating Euler equation derived above and as follow:

$$\frac{1}{\rho\, g}\frac{dP}{ds} + \frac{V}{g}\frac{dv}{ds} + \frac{dz}{ds} = 0$$ Multiplying by (ds)

$$\frac{dP}{\rho\, g} + \frac{V\ dv}{g} + dz = 0$$ By integration

$$\frac{P}{\rho\, g} + \frac{V^2}{2g} + Z = \text{constant along the streamline}$$

Therefore, for **any two points (1&2**) on a streamline in steady, inviscid and incompressible flow the Bernoulli equation can be applied in the form:

$$\boxed{\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + Z_2}$$ *Bernoulli equation of motion where:*

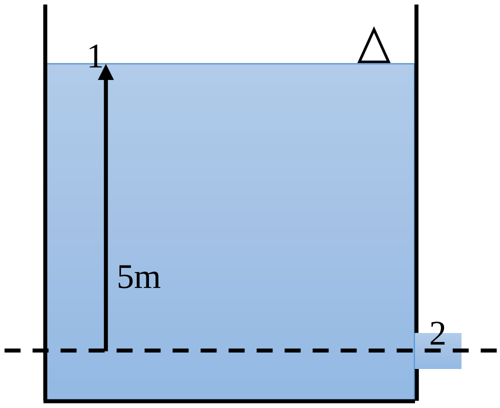
$\frac{P}{\rho\, g}$ Is the pressure energy per unit weight of the fluid or pressure head

$\frac{V^2}{2g}$ Is the kinetic energy per unit weight of the fluid or kinetic head

Z Is the potential energy of the fluid or potential head

Example:

A large tank, opened to the atmosphere, is filled with water to a height of (5m). A tab near to the bottom is opened and water flows from the smooth and rounded outlet. Determine the water velocity at the outlet.



Solution:

$$\frac{P_1}{\rho\ g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho\ g} + \frac{V_2^2}{2g} + Z_2$$

Both (1&2) are in contact with the atmosphere,

$P_1 = P_2 = 0$ Also, $z_1 = 5m$ and $z_2 = 0.$

The tank is large, (v1) is approximately zero.

$$z_1 = \frac{V_2^2}{2g}$$

$$5 = \frac{V_2^2}{2g} \quad \rightarrow \quad V_2 = \sqrt{10\ g} = 9.9\ m/s$$

Flow Rate:

It is a measure at which fluid is flowing per unit time.

Flow Rate

Mass flow rate ($\dot{m}$) is the mass of fluid flowing per unit time

$$\dot{m} = \frac{\text{mass}}{\text{time}} \quad (\text{ kg/s })$$

Volume flow rate (Q) is the volume of fluid flowing per unit time

$$Q = \frac{\text{volume}}{\text{time}} \quad (\text{ m}^3\text{/s })$$

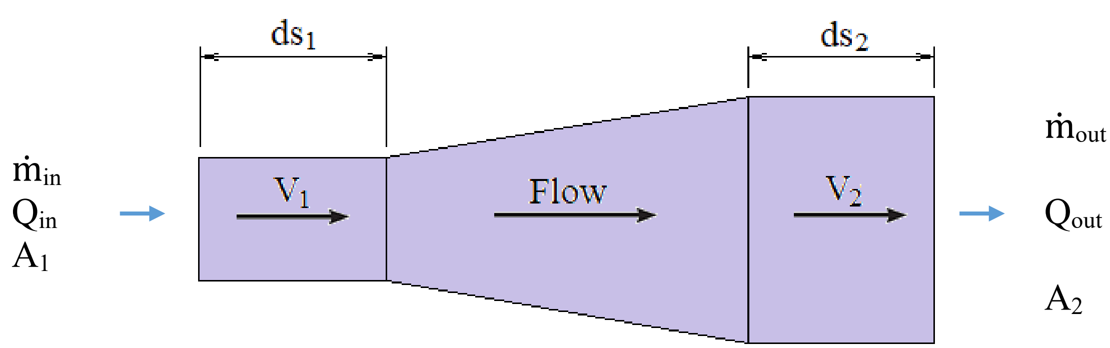
For example, an empty bucket weighs (2 kg). After (7) seconds of collecting water the bucket weighs (8 kg), then:

$$\text{Mass flow rate} = \dot{m} = \frac{\text{mass of fluid in bucket}}{\text{time taken to collect the fluid}} = \frac{8-2}{7} = 0.857 \text{ kg/s}$$

Continuity Equation:

According to the **conservation of mass**, matter cannot be created or destroyed but it can change to different forms. Thus:

Mass entering per unit time = Mass leaving per unit time



$\dot{m}_{in} = \dot{m}_{out}$

$\rho_1 A_1 \frac{ds_1}{dt} = \rho_2 A_2 \frac{ds_2}{dt}$

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

For incompressible fluid, $\rho_1 = \rho_2$

$A_1 V_1 = A_2 V_2 = Q$ volume flow rate

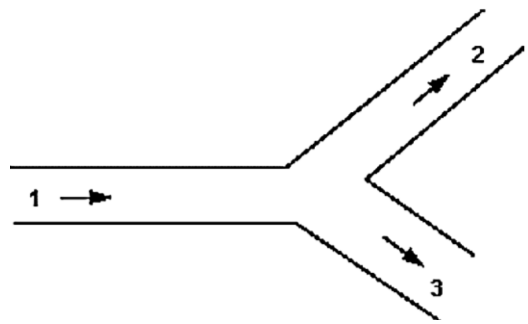
$Q_{in} = Q_{out}$

$\dot{m} = \rho Q$

Similarly, in the case of junctions or flow dividers, as shown below:

$Q_1 = Q_2 + Q_3$

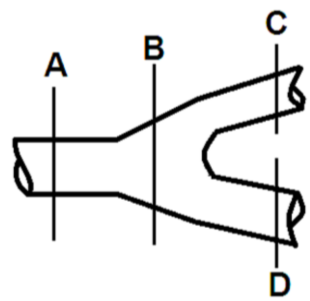
$A_1 V_1 = A_2 V_2 + A_3 V_3$



Example (1):

The pipe has a diameter of (1.2 m) at (A), (1.5 m) at (B) and (0.8 m) at (C). The discharge at (C) is ($Q_A / 3$) and the velocities at (A) & (D) are (3.5 m / s) and (2.5 m / s) respectively. If the flow is steady incompressible, Determine the:

1. Discharge at (A).
2. The velocity at (B) & (C).
3. The diameter at (D).



Solution:

$$Q_A = A_A\, V_A = \left(\frac{\pi}{4} \times 1.2^2 \right) \times 3.5$$

$$Q_A = 3.95 \text{ m}^3/\text{s}$$

$$Q_A = Q_B = A_B\, V_B$$

$$V_B = \frac{Q_B}{A_B} = \frac{3.95}{\frac{\pi}{4} \times 1.5^2} = 2.23 \text{ m/s}$$

$$Q_C = \frac{Q_A}{3} = \frac{3.95}{3} = 1.32 \text{ m}^3/\text{s}$$

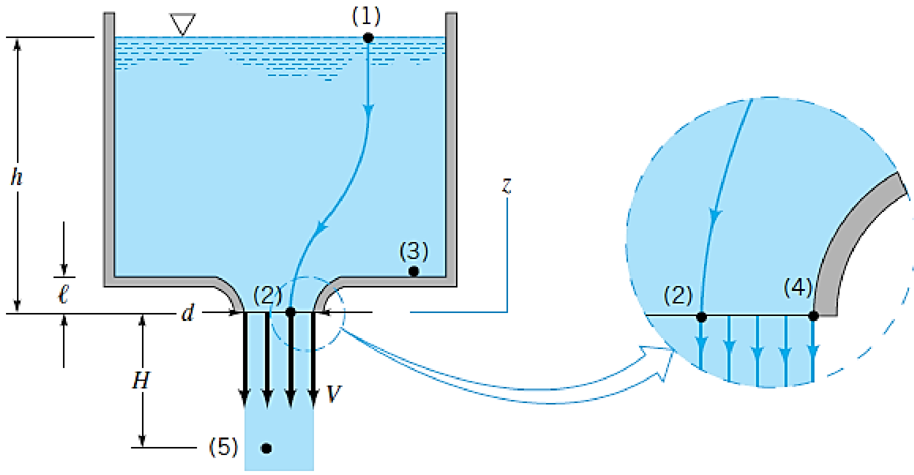
$$V_C = \frac{Q_C}{A_C} = \frac{1.32}{\frac{\pi}{4} \times 0.8^2} = 2.62 \text{ m/s}$$

$$Q_D = Q_B - Q_C = 3.95 - 1.32 = 2.63 \text{ m}^3/\text{s}$$

Application on Bernoulli Equation:

a- Free jets:

Bernoulli's equation can be applied to the flow of a liquid from large tanks, as is shown:



Applying Bernoulli's equation between point (1 & 2):

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + z_2$$

Both streamlines (1 & 2) in contact with the atmosphere, ($P_1 = P_2 = 0$). Also ($Z_1 = h$) and ($Z_2 = 0$).

The reservoir is large, (V_1) is approximately equal to zero. Thus,

$$h = \frac{V_2^2}{2g} \qquad \rightarrow \qquad V_2 = \sqrt{2gh}$$

The stream continues to fall as a free jet with zero pressure,
Applying Bernoulli's equation between point (1 & 5):

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + z_1 = \frac{P_5}{\rho\, g} + \frac{V_5^2}{2g} + z_5$$

$$h + H = \frac{V_5^2}{2g}$$

The Speed increases according to $V_5 = \sqrt{2g(h+H)}$

The pressure at point (3) can be obtained by:
Applying Bernoulli's equation between point (1 & 3):

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + z_1 = \frac{P_3}{\rho\, g} + \frac{V_3^2}{2g} + z_3$$

$P_1 = 0$. Also, $Z_1 = h - \ell$, and $Z_3 = 0$.

The reservoir is large, (V_1) and (V_3) is approximately equal to zero. Thus,

$$\frac{P_3}{\rho\, g} = h - \ell$$

$$P_3 = \rho g\ (h - \ell\ )$$

$$P_3 = \gamma\ (h - \ell\ )$$

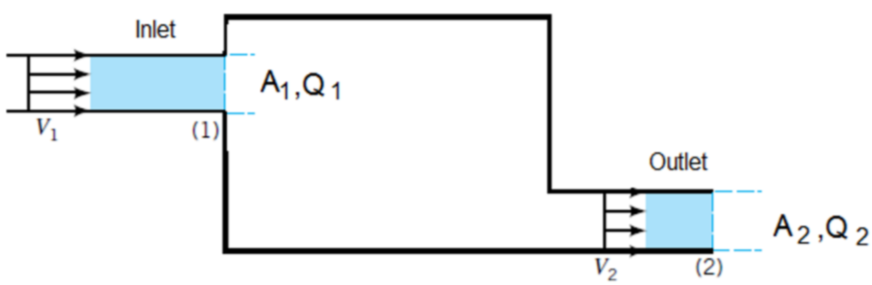
Or by simply applying the hydrostatic field pressure law,

$$P_3 - \gamma\ (h - \ell\ ) = 0$$

$$P_3 = \gamma\ (h - \ell\ )$$

b- Flow in pipes:

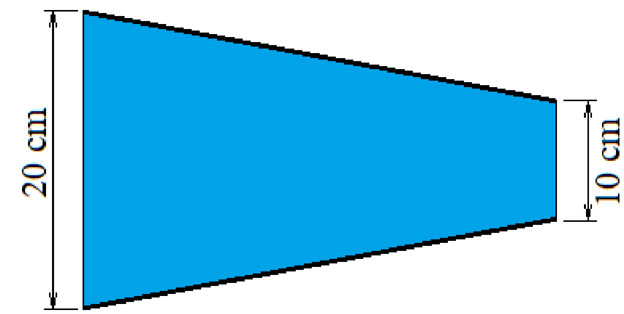
As the fluid flows within a pipe of variable dimeter, the velocity changes because the flow area is different from one section to another. In these situations, we use the continuity equation along with the Bernoulli equation.



$Q_1 = Q_2$
$A_1\, V_1 = A_2\, V_2$

Example (2):

The water is flowing through a pipe having diameters (20 cm) and (10 cm) at sections (1) and (2) respectively. The rate of flow through pipe is (35 liters/s). If the pressure at section (1) is (39.24 N/cm^2), find the intensity of the pressure at section (2).



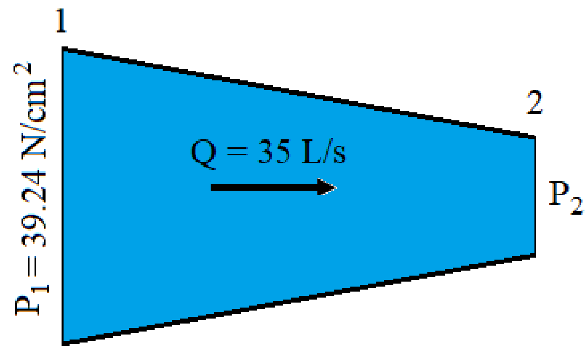
Solution:

Applying the continuity equation:

$$Q = A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{Q}{A_1} = \frac{0.035}{\frac{\pi}{4} \times 0.2^2} = 1.11 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.035}{\frac{\pi}{4} \times 0.1^2} = 4.456 \text{ m/s}$$



Applying Bernoulli's equation:

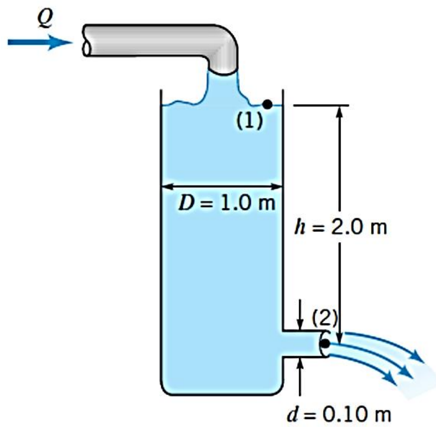
$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{39.24 \times 10^4}{9810} + \frac{1.11^2}{2\times 9.81} + 0 = \frac{P_2}{9810} + \frac{4.456^2}{2\times 9.81} + 0$$

$$P_2 = 38.3 \times 10^4 \text{ N/m}^2 = 38.3 \text{ N/cm}^2$$

Example (3):

A stream of water of diameter (0.1 m) flows steadily from a tank of diameter (D = 1 m) as shown. Determine the flowrate (Q) needed from the inflow pipe so that the depth remains constant, (h = 2 m).



Solution:

Applying Bernoulli's equation between point (1 & 2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Both streamlines (1 & 2) in contact with the atmosphere,

($P_1 = P_2 = 0$). Also, ($Z_1 = h$) and ($Z_2 = 0$)

$$\frac{V_1^2}{2g} + h = \frac{V_2^2}{2g} \qquad \times 2g$$

$$V_1^2 + 2gh = V_2^2$$

In order to keep the depth constant, ($Q_{in} = Q_{out}$)

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \left(\frac{A_2}{A_1}\right) V_2$$

$$V_1 = \left(\frac{d}{D}\right)^2 V_2$$

$$\left(\frac{d}{D}\right)^4 V_2^2 + 2gh = V_2^2$$

$$2gh = V_2^2 - \left(\frac{d}{D}\right)^4 V_2^2$$

$$2gh = \left[1 - \left(\frac{d}{D}\right)^4\right] V_2^2$$

$$V_2 = \sqrt{\frac{2gh}{1-\left(\frac{d}{D}\right)^4}} = \sqrt{\frac{2\times 9.81\times 2}{1-\left(\frac{0.1}{1}\right)^4}} = 6.26 \text{ m/s}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$Q = \left(\frac{\pi}{4} \times o.1^2\right) \times 6.26 = 0.0492 \text{ m}^3/\text{s}$$

Practical Applications of Bernoulli's Equation

Flow Rate Measurement:

An effective way to measure the **flowrate** through a pipe is to place some type of restriction within the pipe and to measure the pressure difference between the high-pressure and low-pressure sections. Several measuring devices use Bernoulli principle to measure fluid **velocities and flowrates**.

Continuity Equation:

$$A_1 V_1 = A_2 V_2$$

$$V_1 = \frac{A_2}{A_1} V_2$$

Bernoulli's equation:

$$\frac{P_1}{\rho g} + \frac{{V_1}^2}{2g} = \frac{P_2}{\rho g} + \frac{{V_2}^2}{2g} \qquad \times g$$

$$\frac{P_1}{\rho} + \frac{{V_1}^2}{2} = \frac{P_2}{\rho} + \frac{{V_2}^2}{2}$$

$$\frac{{V_2}^2}{2} = \frac{P_1}{\rho} + \frac{{V_1}^2}{2} - \frac{P_2}{\rho}$$

$$V_2^2 = 2 \left[\frac{P_1}{\rho} + \frac{{V_1}^2}{2} - \frac{P_2}{\rho} \right]$$

$$V_2^2 = \frac{2P_1}{\rho} + V_1^2 - \frac{2P_2}{\rho}$$

$$V_2^2 = \frac{2P_1}{\rho} + \left(\frac{A_2}{A_1} \right)^2 V_2^2 - \frac{2P_2}{\rho}$$

$$V_2^2 - \left(\frac{A_2}{A_1} \right)^2 V_2^2 = \frac{2P_1}{\rho} - \frac{2P_2}{\rho}$$

$$\left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] V_2^2 = \frac{2}{\rho} (P_1 - P_2)$$

$$V_2^2 = \frac{2(P_1 - P_2)}{\rho[1 - (\frac{A_2}{A_1})^2]} \qquad \rightarrow \qquad V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (\frac{A_2}{A_1})^2]}}$$

$$Q = A_2 V_2 \qquad \rightarrow \qquad Q = A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho[1 - (\frac{A_2}{A_1})^2]}}$$

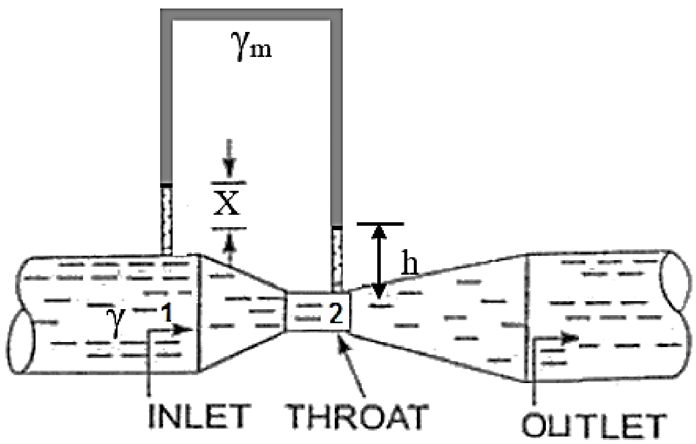
1. ***Venturimeter:***

is a device used for measuring **the rate of a flow** of a fluid flowing through a pipe. It consists of three parts:

a- Short converging part

b- Throat

c- Diverging part



γ_m : Specific weight of the Manometer's liquid.
γ : Specific weight of the liquid flowing in the pipe.

Applying Bernoulli's equation at sections (1) and (2), we get:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

The pipe is horizontal, thus $Z_1 = Z_2$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{P_1}{\gamma} - \frac{P_2}{\gamma} + \frac{V_1^2}{2g}$$

$$\frac{V_2^2}{2g} = \frac{P_1 - P_2}{\gamma} + \frac{V_1^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

To find ($P_1 - P_2$), we apply the hydrostatic pressure law:

$$P_1 - \gamma (h + x) + \gamma_m x + \gamma h = P_2$$
$$P_1 - \gamma h - \gamma x + \gamma_m x + \gamma h = P_2$$
$$P_1 - \gamma x + \gamma_m x = P_2$$
$$P_1 - P_2 = \gamma x - \gamma_m x$$
$$P_1 - P_2 = x (\gamma - \gamma_m)$$

Apply in (1)

$$\frac{{V_2}^2}{2g} = \frac{x(\gamma-\gamma_m)}{\gamma} + \frac{{V_1}^2}{2g} \qquad \times 2g$$

$$V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right) + V_1^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

To find (V_1), we apply the continuity equation:

$$A_1 V_1 = A_2 V_2$$
$$V_1 = \frac{A_2}{A_1} V_2$$

Apply in (2):

$$V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right) + \left(\frac{A_2}{A_1} \right)^2 V_2^2$$
$$V_2^2 - \left(\frac{A_2}{A_1} \right)^2 V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right)$$
$$\left[1 - \left(\frac{A_2}{A_1} \right)^2 \right] V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right)$$
$$\left[1 - \frac{{A_2}^2}{{A_1}^2} \right] V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right)$$
$$\left[\frac{{A_1}^2 - {A_2}^2}{{A_1}^2} \right] V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right)$$
$$V_2^2 = 2gx \left(1 - \frac{\gamma_m}{\gamma} \right) \left[\frac{{A_1}^2}{{A_1}^2 - {A_2}^2} \right]$$
$$V_2 = \sqrt{2gx \left(1 - \frac{\gamma_m}{\gamma} \right) \left[\frac{{A_1}^2}{{A_1}^2 - {A_2}^2} \right]}$$
$$V_2 = \frac{A_1}{\sqrt{{A_1}^2 - {A_2}^2}} \sqrt{2gx \left(1 - \frac{\gamma_m}{\gamma} \right)}$$

$$Q = A_2\,V_2 = \frac{A_1\,A_2}{\sqrt{{A_1}^2 - {A_2}^2}}\,\sqrt{2gx\,(1 - \frac{\gamma_m}{\gamma})}$$

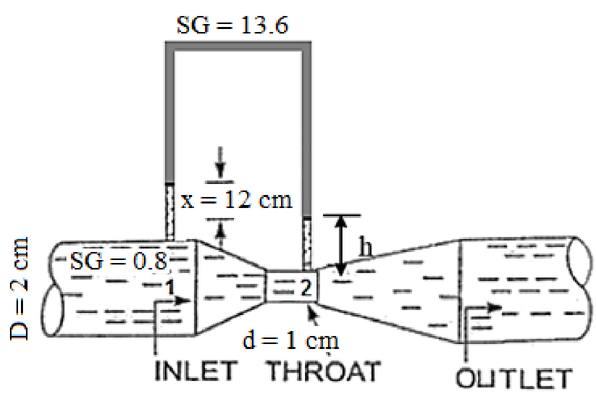
The above formula is correct when the Manometer liquid is lighter than the liquid flowing in the pipe ($\gamma_m < \gamma$). Therefore, the term $\left(1 - \frac{\gamma_m}{\gamma}\right)$ is positive. If the Manometer liquid is heavier than the liquid flowing through the pipe, the formula can be written as:

$$Q = \frac{A_1\,A_2}{\sqrt{{A_1}^2 - {A_2}^2}}\,\sqrt{2gx\,(\frac{\gamma_m}{\gamma} - 1)}$$

Example (1):

An oil of (SG = 0.8) is flowing through a Venturimeter having inlet diameter (2 cm) and throat diameter (1 cm). The oil-mercury differential manometer shows a reading of (x = 12 cm). Calculate the theoretical discharge of oil through the horizontal Venturimeter given that (SG) of mercury is (13.6).

Solution:



$$A_1 = \frac{\pi}{4} \times 2^2 = 3.14\ cm^2$$

$$A_2 = \frac{\pi}{4} \times 1^2 = 0.785\ cm^2$$

$$Q = \frac{A_1\,A_2}{\sqrt{{A_1}^2 - {A_2}^2}}\,\sqrt{2gx\,(\frac{\gamma_m}{\gamma} - 1)}$$

$$Q = \frac{3.14 \times 0.785}{\sqrt{(3.14)^2 - (0.785)^2}}\,\sqrt{2 \times 981 \times 12\,(\frac{13.6}{0.8} - 1)}$$

$Q = 497\ cm^3/s = 0.497$ L/s

Example (2):

An oil of (SG = 0.8) is flowing through a Venturimeter having inlet diameter (20 cm) and throat diameter (10 cm). The oil-mercury differential manometer shows a reading of (x = 15 cm). Calculate the theoretical discharge of oil through the horizontal Venturimeter given that (SG) of mercury is (13.6).

Solution:

$$A_1 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$$A_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

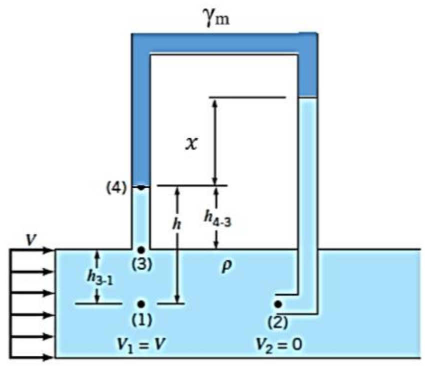
$$Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gx\left(\frac{\gamma_m}{\gamma} - 1\right)}$$

$$Q = \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \sqrt{2 \times 981 \times 15\left(\frac{13.6}{0.8} - 1\right)}$$

$Q = 55634 \text{ cm}^3/\text{s} = 55.634 \text{ L/s}$

2. *Static Pitot tube:*

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that the velocity of flow at the stagnation point becomes zero.



Applying Bernoulli's equation at sections (1) and (2), we get:

$$\frac{P_1}{\rho g} + \frac{V_1^{\,2}}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^{\,2}}{2g} + Z_2$$

The pipe is horizontal, hence $Z_1 = Z_2$ $\qquad V_2 = 0$

$$\frac{P_1}{\rho g} + \frac{V_1^{\,2}}{2g} = \frac{P_2}{\rho g}$$

$$\frac{V_1^{\,2}}{2g} = \frac{P_2}{\rho g} - \frac{P_2}{\rho g}$$

$$\frac{V_1^{\,2}}{2g} = \frac{P_1 - P_2}{\rho g} \quad \rightarrow \quad V_1 = \sqrt{2g\left(\frac{P_2 - P_1}{\gamma}\right)}$$

To find ($P_2 - P_1$), we apply the hydrostatic pressure law:

$$P_2 - \gamma\,(h + x) + \gamma_m\,x + \gamma\,h = P_1$$

$$P_2 - \gamma\,h - \gamma\,x + \gamma_m\,x + \gamma\,h = P_1$$

$$P_2 - \gamma\,x + \gamma_m\,x = P_1$$

$$P_2 - P_1 = \gamma\,x - \gamma_m\,x \quad \rightarrow \quad P_2 - P_1 = x\,(\gamma - \gamma_m)$$

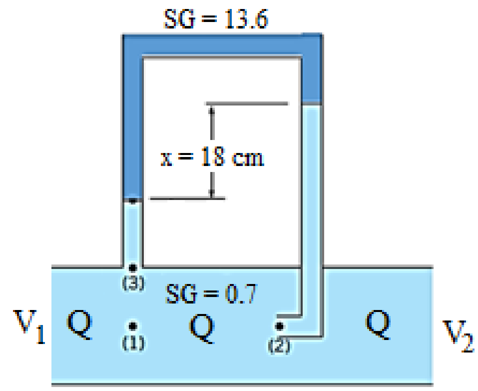
$$V_1 = \sqrt{2gx\left(\frac{\gamma - \gamma_m}{\gamma}\right)} = \sqrt{2gx\left(1 - \frac{\gamma_m}{\gamma}\right)}$$

$$Q = A_1\,V_1 = A_1\sqrt{2gx\left(1 - \frac{\gamma_m}{\gamma}\right)}$$

Example (3):

A pitot tube is inserted in a pipe of (10 cm) diameter. The pressure difference measured by a mercury (SG = 13.6) differential manometer gives a reading of (18 cm) of mercury. Find the velocity and the flowrate of the flow of an oil of (SG = 0.7).

Solution:



$$V_1 = \sqrt{2gx\,(\frac{\gamma_m}{\gamma} - 1)}$$

$$= \sqrt{2 \times 981 \times 18\,(\frac{13.6}{0.7} - 1)}$$

$$= 806 \text{ cm/s}$$

$$Q = A_1 V_1 = A_1 \sqrt{2gx\,(\frac{\gamma_m}{\gamma} - 1)}$$

$$= \frac{\pi}{4} \times 10^2 \times 806 = 63330 \text{ cm}^3/\text{s}$$

$$= 63.33 \text{ L}^3/\text{s}$$

Pneumatic & Hydraulic Systems:

Fluid Power: Power generated by an effective pumped or compressed fluids to provide force and motion to mechanisms. This force may be in the form of pushing, pulling, lifting or cutting. Fluid power includes hydraulics, which involves liquids, and pneumatics which uses air.

Advantages of Fluid Power:

Hydraulics and pneumatics systems have a number of favorable characteristics:

- Very high power to weight ratio.
- Self-lubricating and cooling.
- Low Initial cost, especially Pneumatic System.
- Motion can be transmitted via fluid without the need for complicated systems of gears, cams, and levers.
- The forces generated are transmitted over large distances with small loss.
- It can provide smooth, flexible and uniform action without vibration.
- It provides variable motions in both rotary and straight-line.
- Fluid power systems are economical to operate.

Disadvantages of Fluid Power:

- Leaks must be prevented. This is a serious problem with the high pressure obtained in many fluid power installations.
- Movement of the fluid within the lines and components can cause friction against the containing surfaces which can lead to serious losses in efficiency.
- Fluid must be kept clean, clogging can cause series damages.

Pneumatic or Hydraulic?

- In general, when the application requires a low amount of power and only fairly accurate control, a pneumatic system may be used.
- If the application requires a great amount of pressure and/or extremely accurate control, a hydraulic system should be used.

Pneumatic	Hydraulic
Power to weight ratio is lower than the Hydraulic System	Very High Power to weight ratio
Relatively cheap	More expensive than pneumatic
Can exhaust to atmosphere	Mess from oil leaks
Temperature has less effect	Oil property changes with temperatures
Safe in potentially explosive environment	Danger from oil leaks

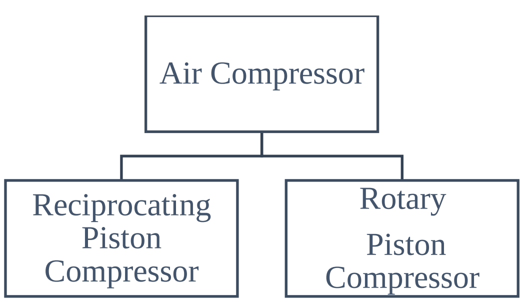
Pneumatic systems components:

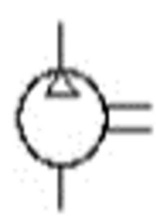
The Pneumatic system consists of:

- Air compressors.
- Air filter, dryer and separator.
- Air reservoir (tanks).
- Regulator, relief, check and control valves.
- Actuation cylinders.

Air compressors:

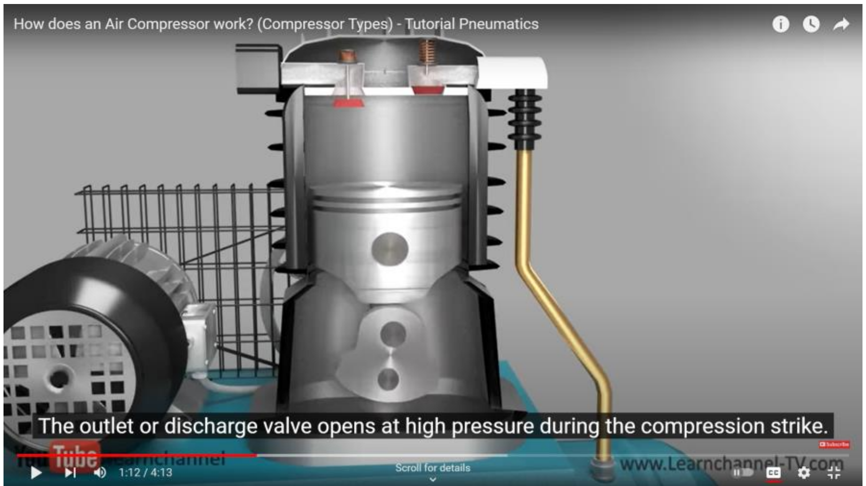
Rises the air pressure from atmospheric value to the desired level. Pneumatic components are designed for a maximum operating pressure of (800 to 1000 Kpa) (8-10 bar).

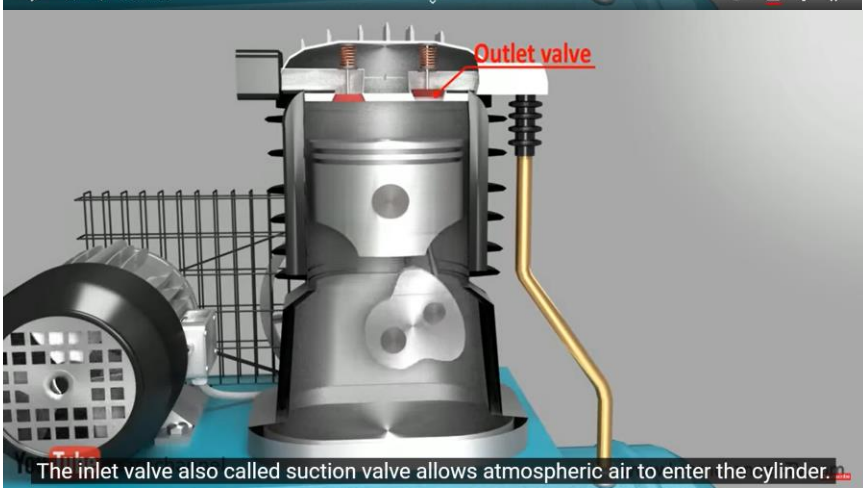


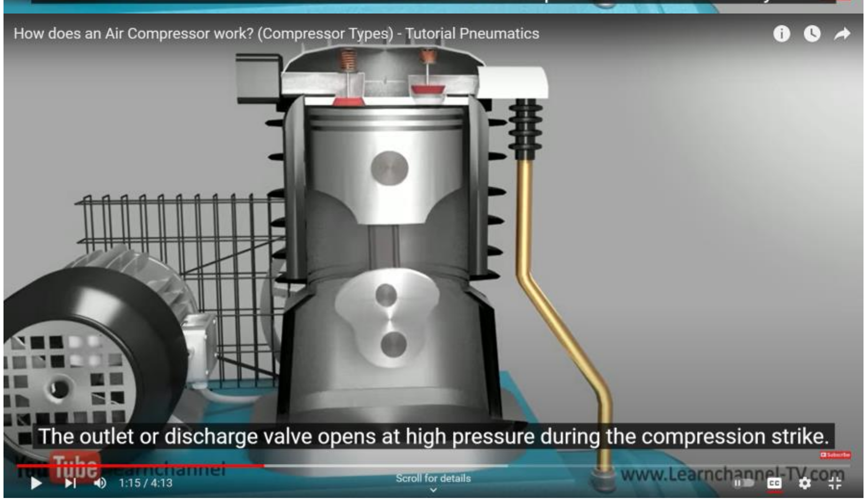


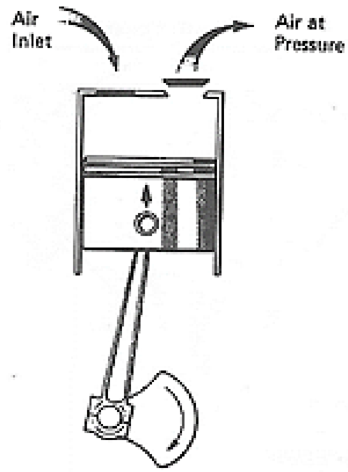
Reciprocating piston compressors:

Single acting piston Produces up to (400 Kpa) (4 bar).

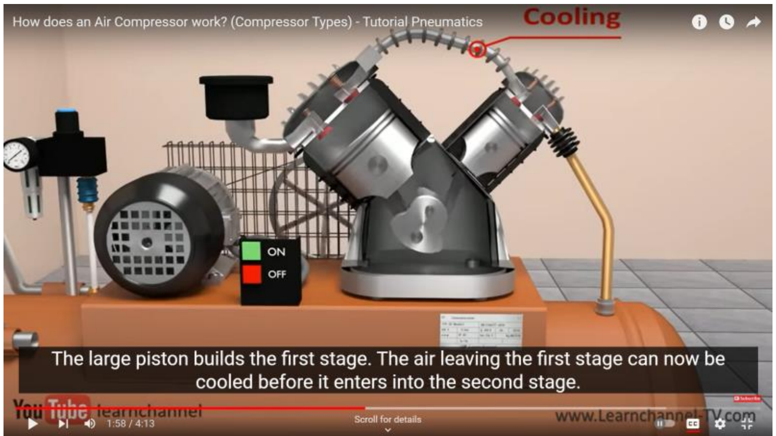


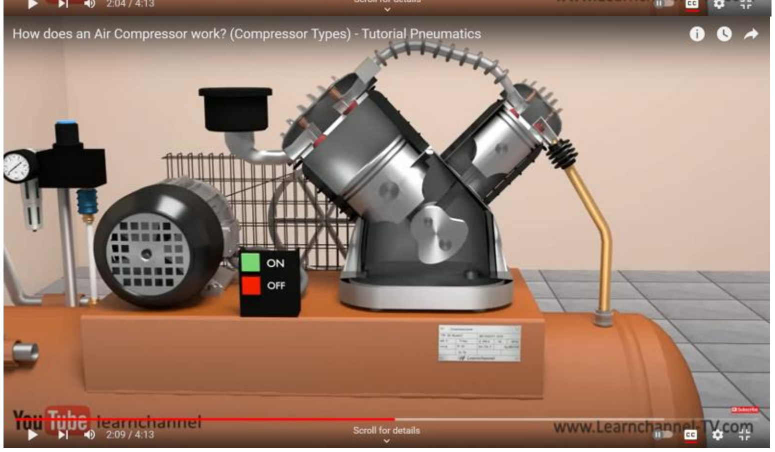


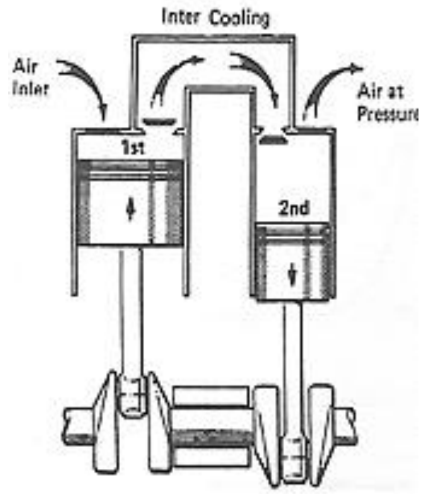




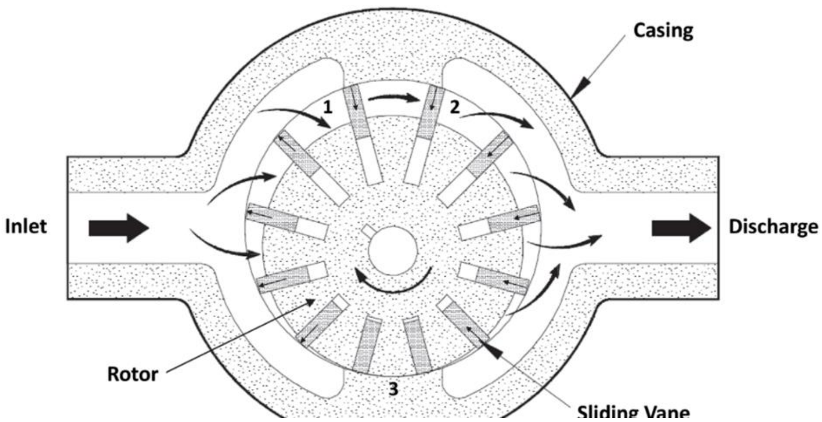
Double acting piston produces up to (800 Kpa) (8 bar).

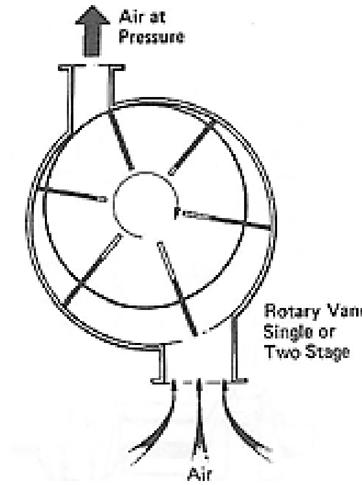


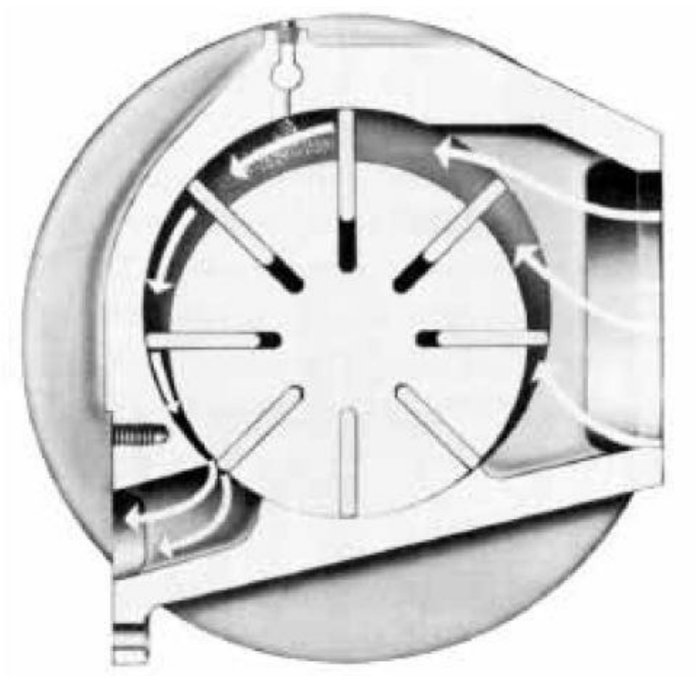




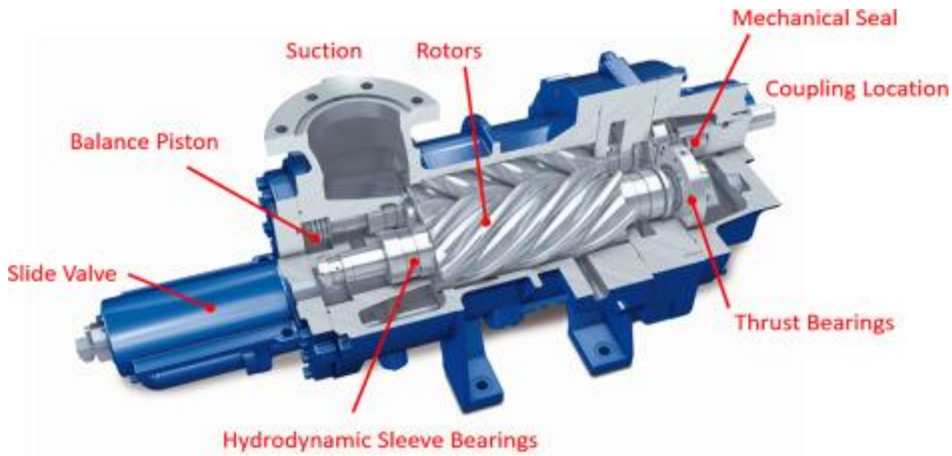
Rotary Vane Compressor:

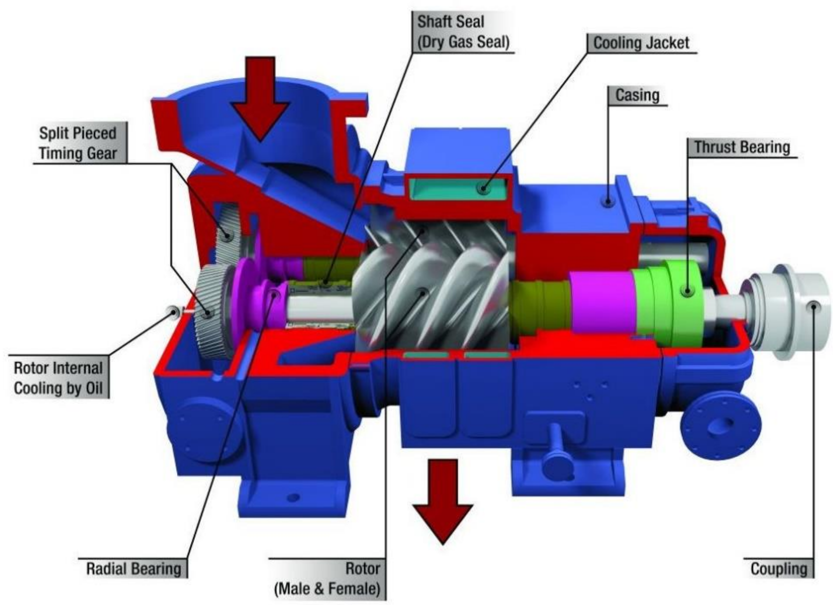


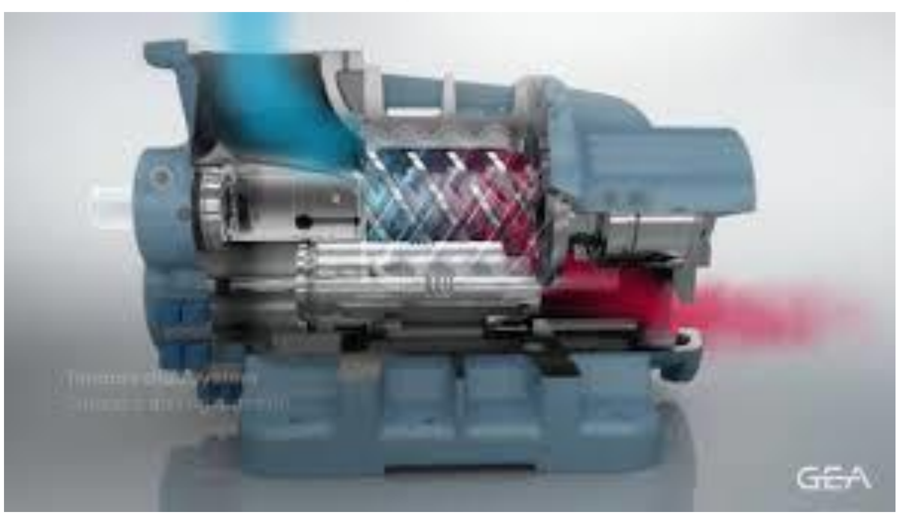


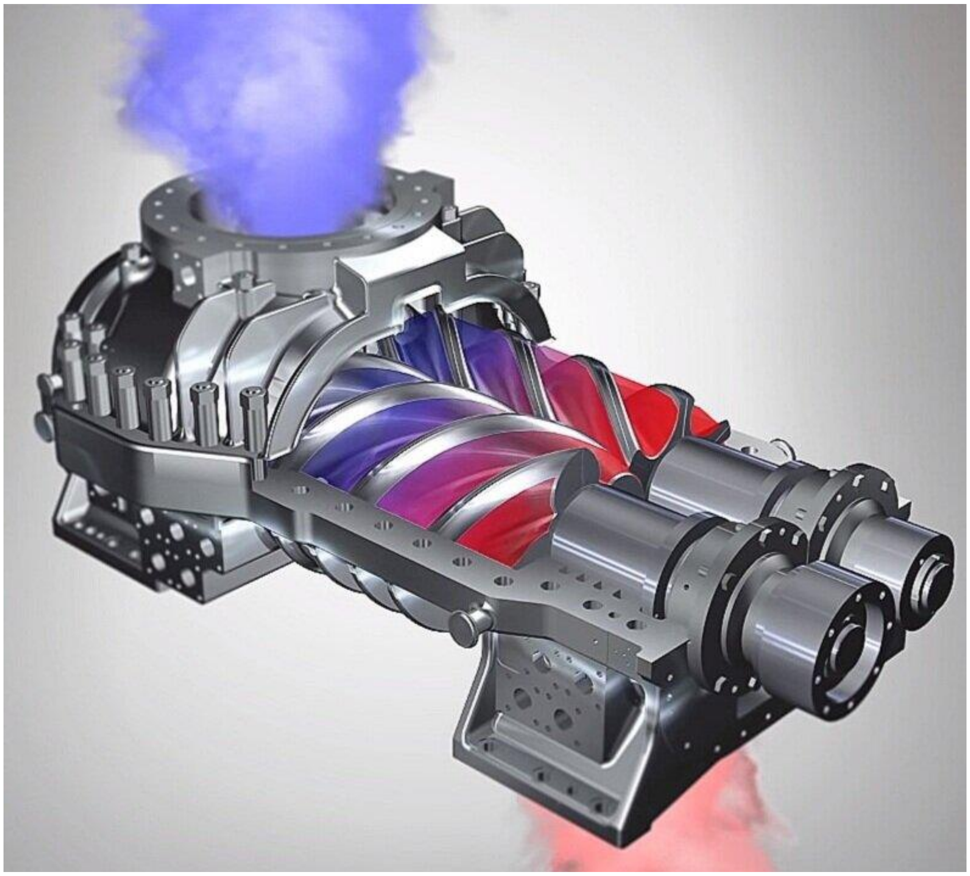


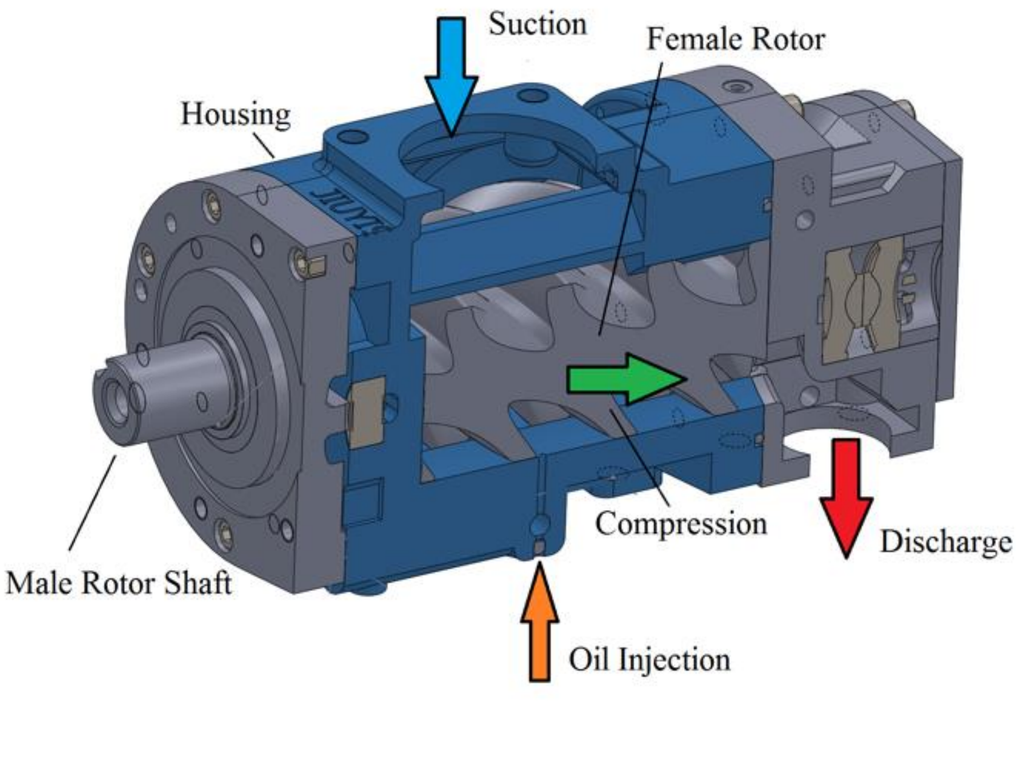
Screw Compressor:





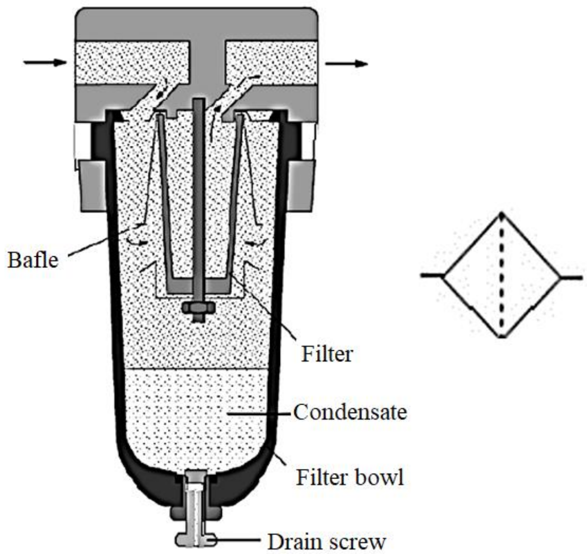


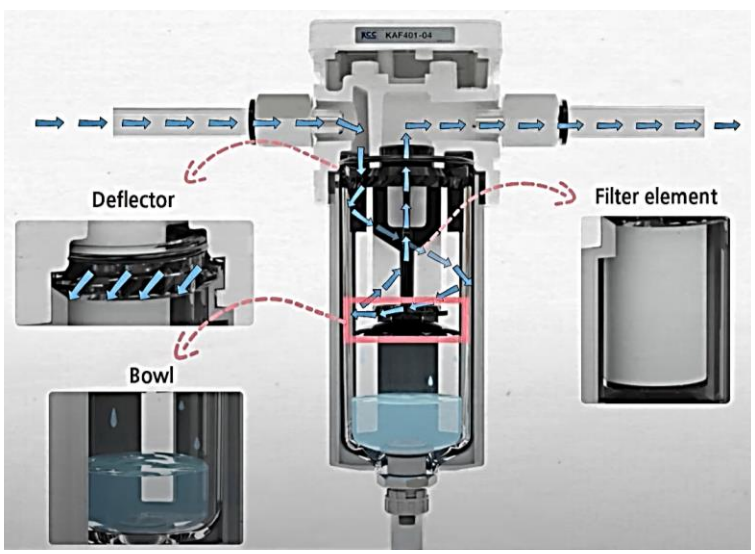



Suction
Female Rotor
Housing
Male Rotor Shaft
Compression
Discharge
Oil Injection

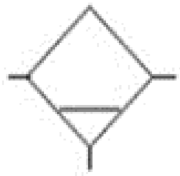
Air preparation: involves the following:

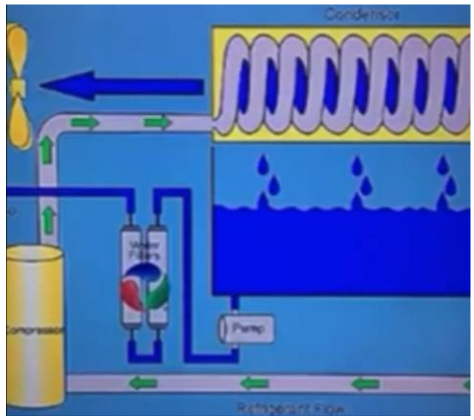
1- **A pneumatic filter:** is a device which removes contaminants from a compressed airstream. One characteristic of compressed-air filters is the size. The size of the filter element indicates the minimum particle size which can be filtered out of the compressed air. Typical (5) microns.



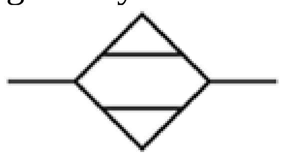


2- **Water separators:** separate out the mist of water droplets which are sometimes suspended in air by centrifugal means.

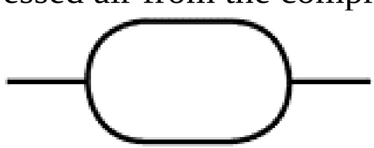




3- **Pneumatic Dryer:** An extra stage to dry out the compressed water using silica gel.

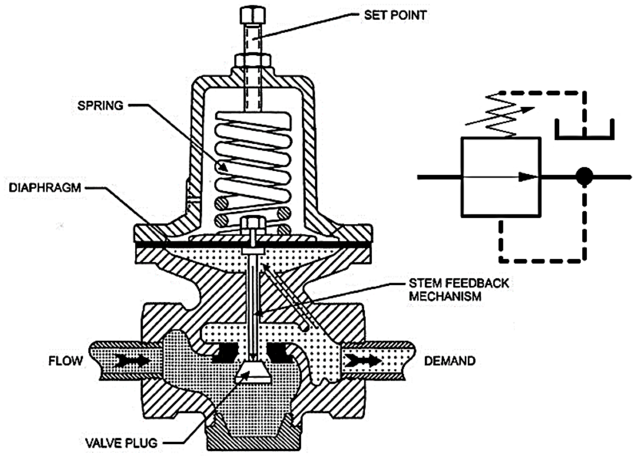


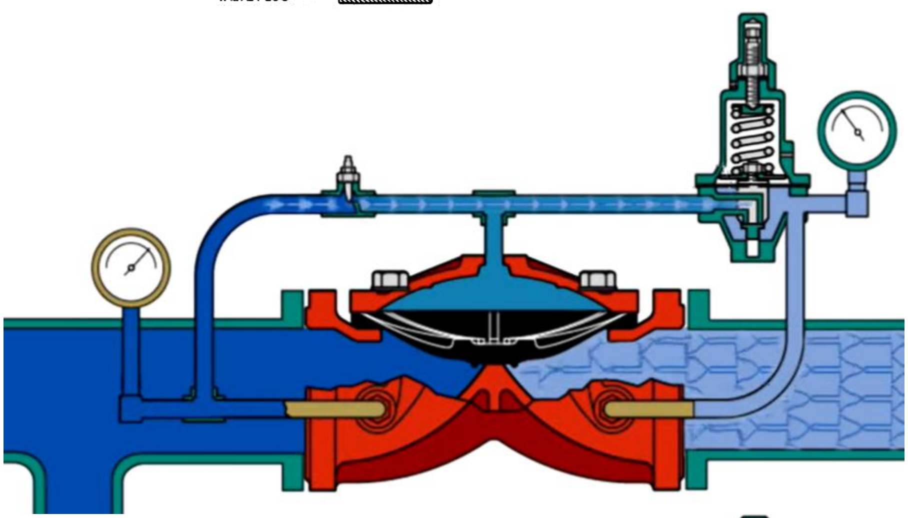
4- **Reservoirs:** store the compressed air from the compressor.

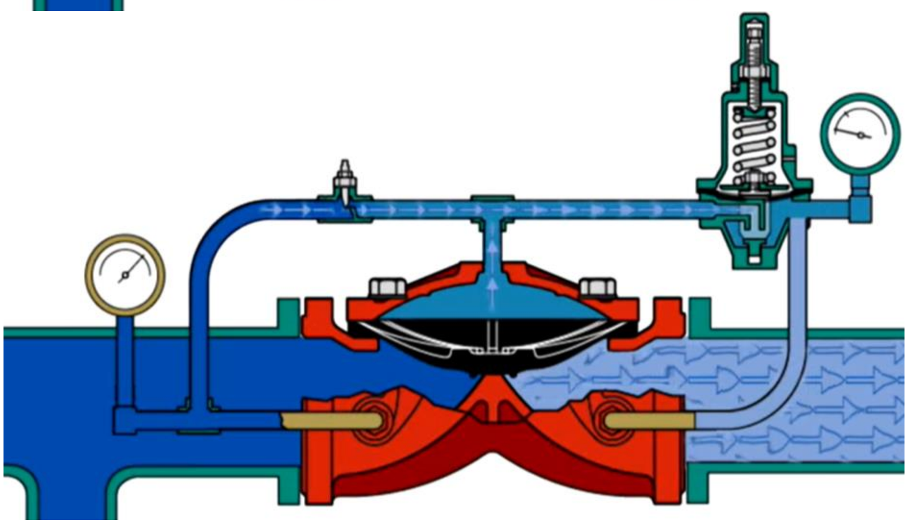


Pneumatic Valves:

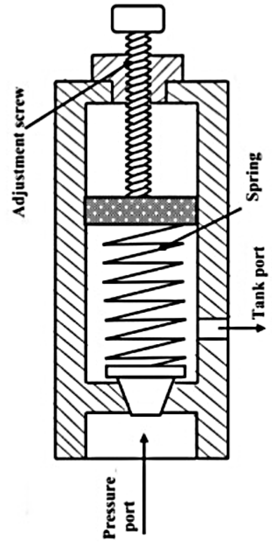
Pressure regulator valve: is a control valve that reduces the input pressure of a fluid to a desired value at its output keeping the operating pressure constant, regardless of pressure fluctuations or air consumption in the system as long as the input pressure at the pressure regulator is higher than the output pressure.

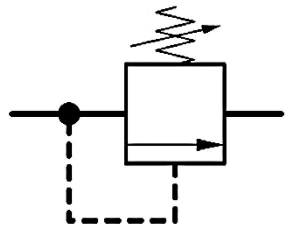


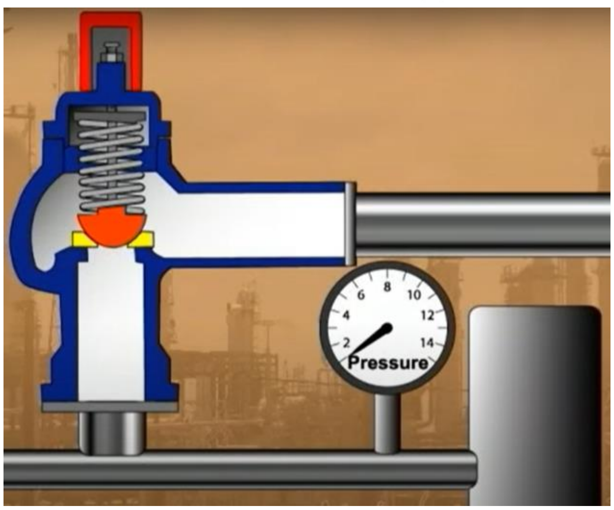




Pressure relief valve: When the set maximum pressure is reached, the pressure-relief valve opens and the air is exhausted to atmosphere.







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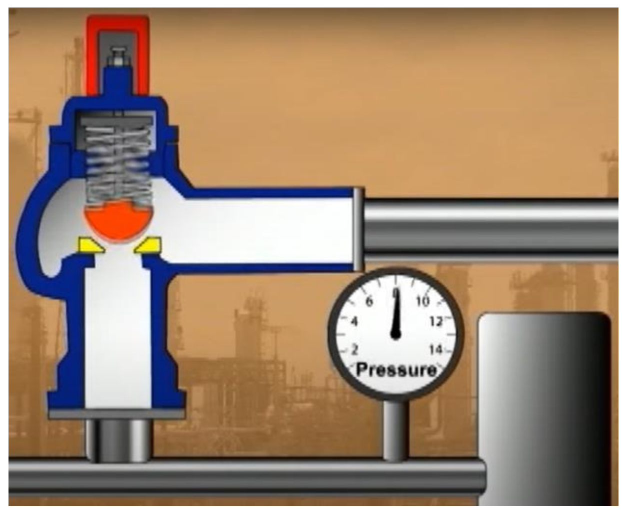

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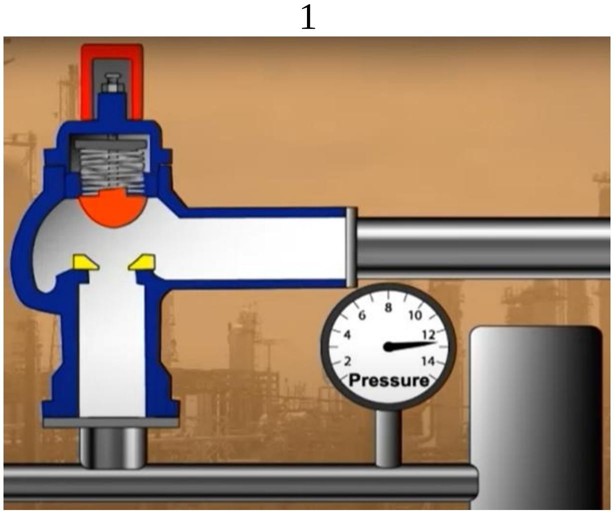

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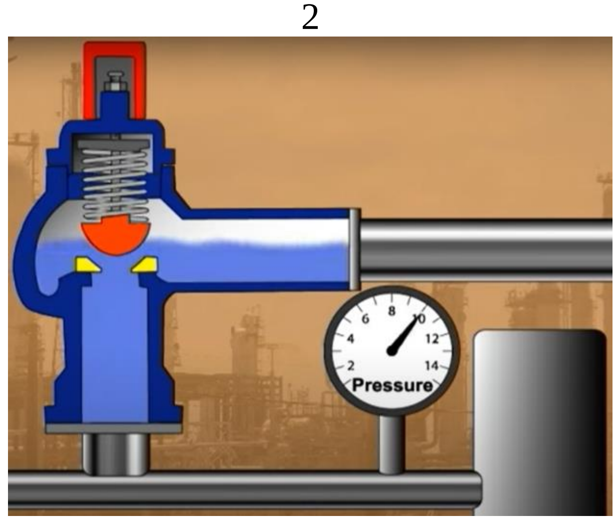
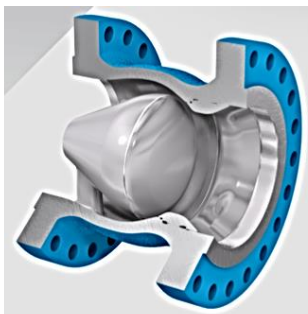
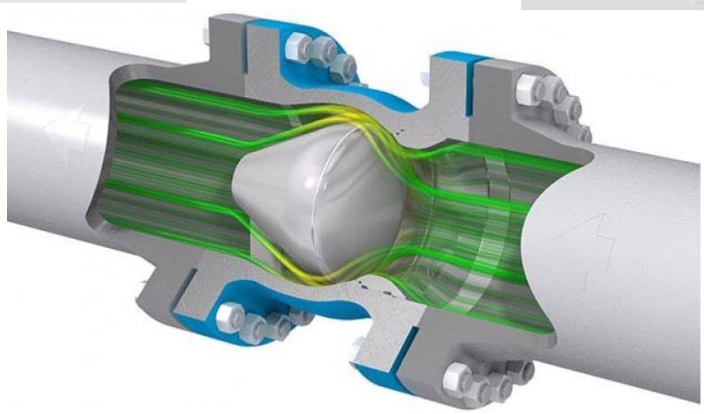


3

Pressure check valve: on-return valve or one-way valve is a valve that normally allows fluid (liquid or gas) to flow through it in only one direction.





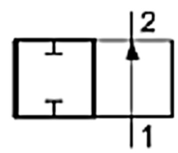
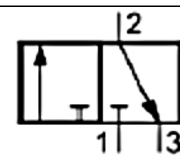
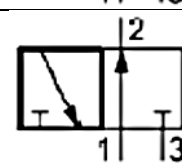
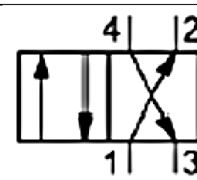
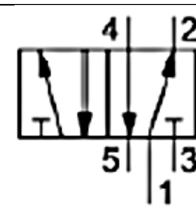
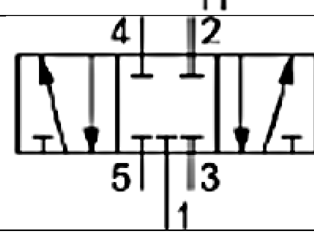
Directional control valves:

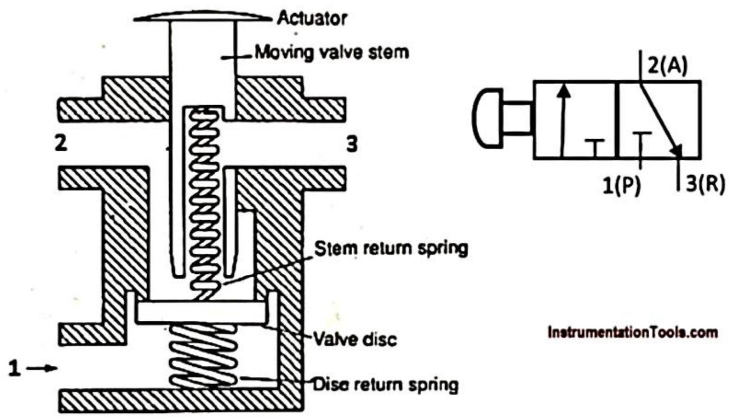
Directional control valves are devices that control the direction of fluid flow in the system as well as stopping it, thus determining the direction of movement of actuators, whether cylinders or hydraulic motors.

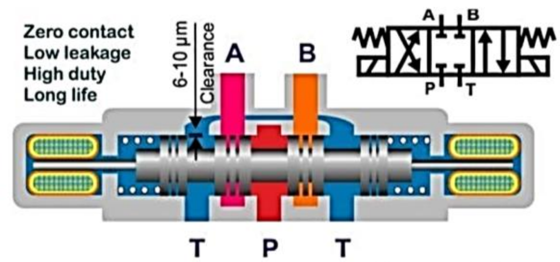
Directional control valves are named according to the number of ports in the valve and the number of switching positions. They are named by two numbers separated by a sign (/). Example: Directional control valve (3/4), is a directional control valve with four ports and three switching positions.

The directional control valve is characterized according to:

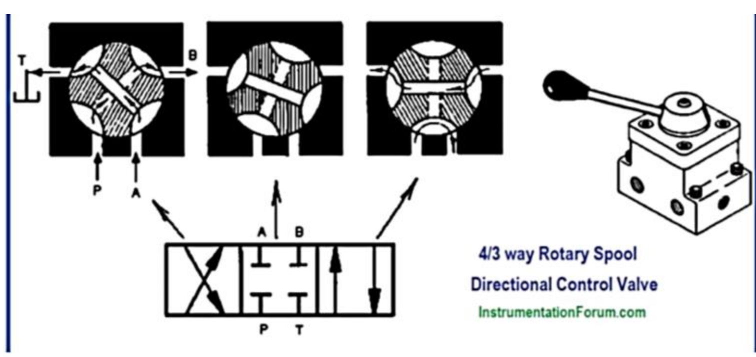
- Number of switching positions & Ports (2-way, 3-way, etc.).
- Method of actuation (Manual, mechanical, electrical).

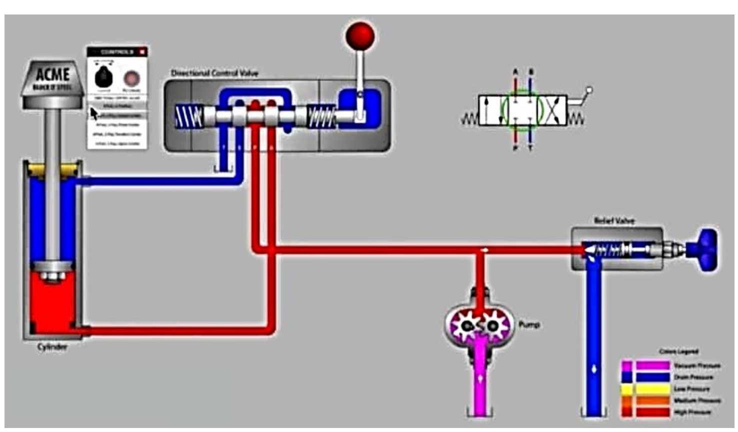
Number of ports عدد المنافذ ↑ 2/2 – Way directional control valve, normally open. ↓ Number of positions عدد المواقع	
3/2 – Way directional control valve, normally closed.	
3/2 – Way directional control valve, normally open.	
3/2 – Way directional control valve Flow from 1 → 2 and from 4 → 3	
5/2 – Way directional control valve Flow from 1 → 2 and von 4 → 5	
5/3 – Way directional control valve Mid position clpsed	



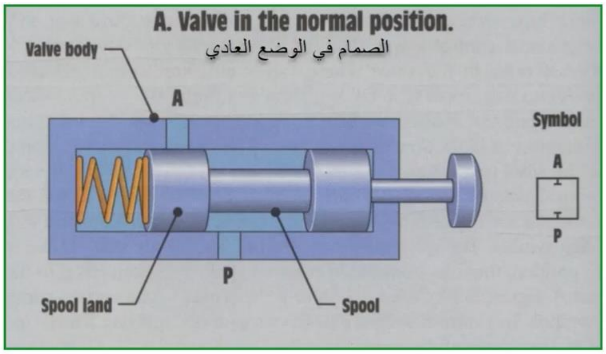


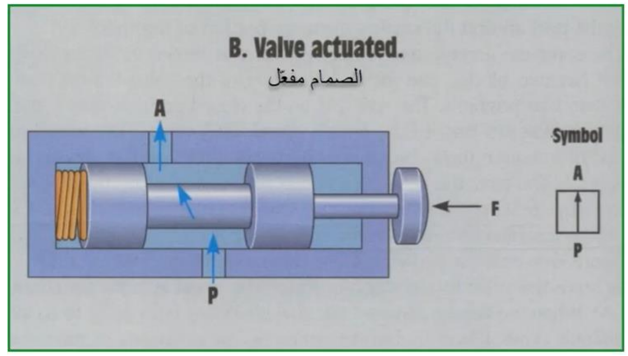
ARABIC 2. Sliding pool *\ شكل SEQ شكل
directional control valve (4/3)

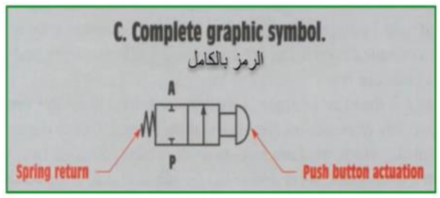


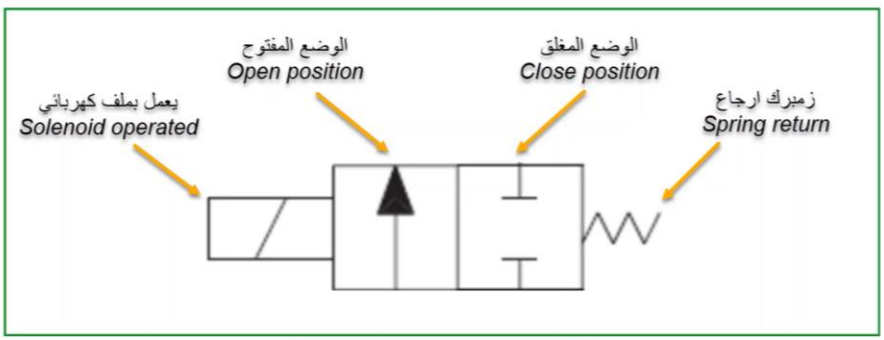


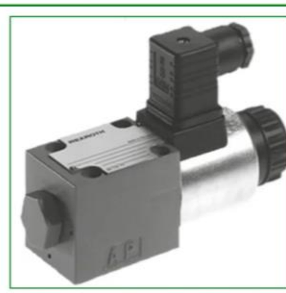
2/2 – Way directional control valve:



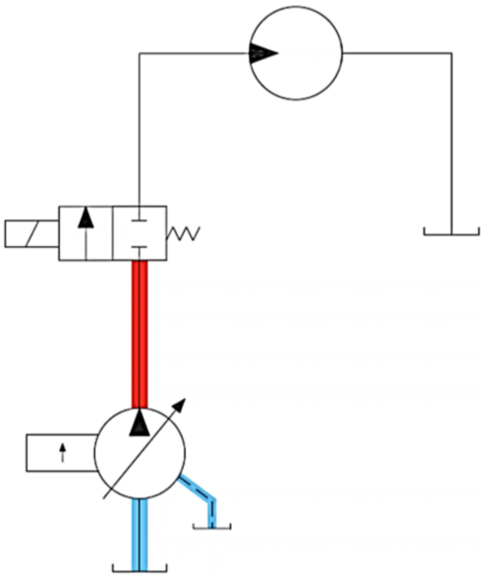


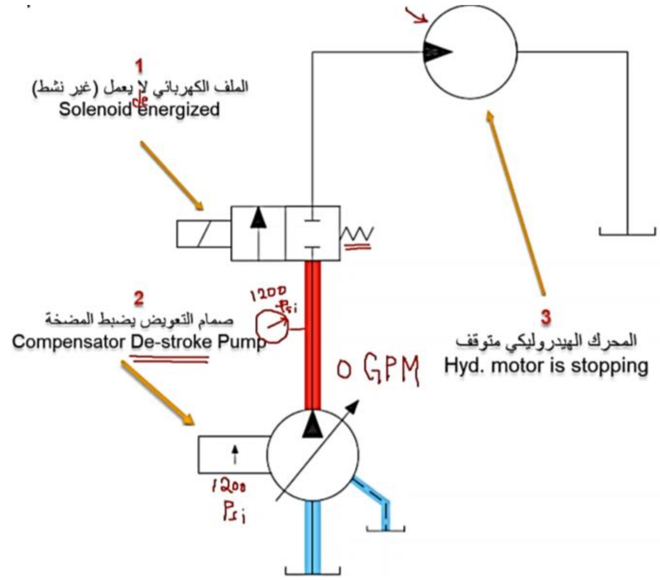




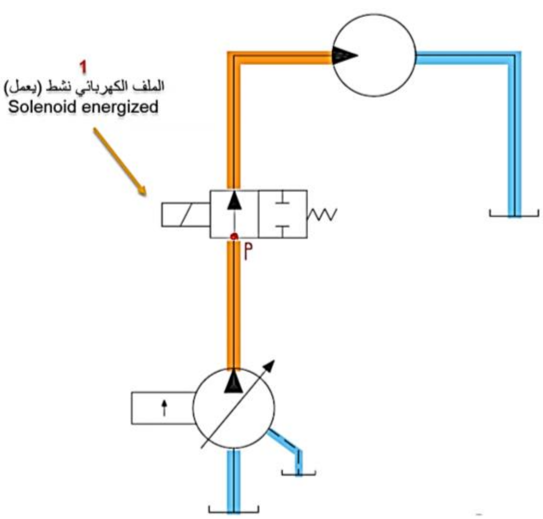


صمام تحكم توجيهي مغلق في الوضع العادي تشغيل زر انضغاطي
DCV 2/2, Normally closed, Push button actuation

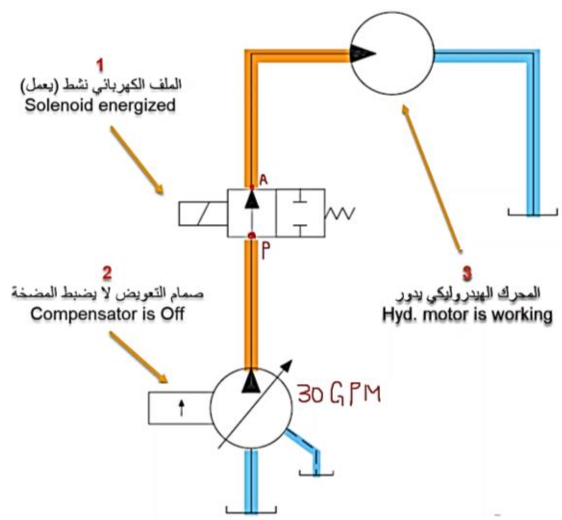




1
الملف الكهربائي لا يعمل (غير نشط)
de
Solenoid energized
1200 Psi
2
صمام التعويض يضبط المضخة
Compensator De-stroke Pump
0 GPM
1200 Psi
3
المحرك الهيدروليكي متوقف
Hyd. motor is stopping

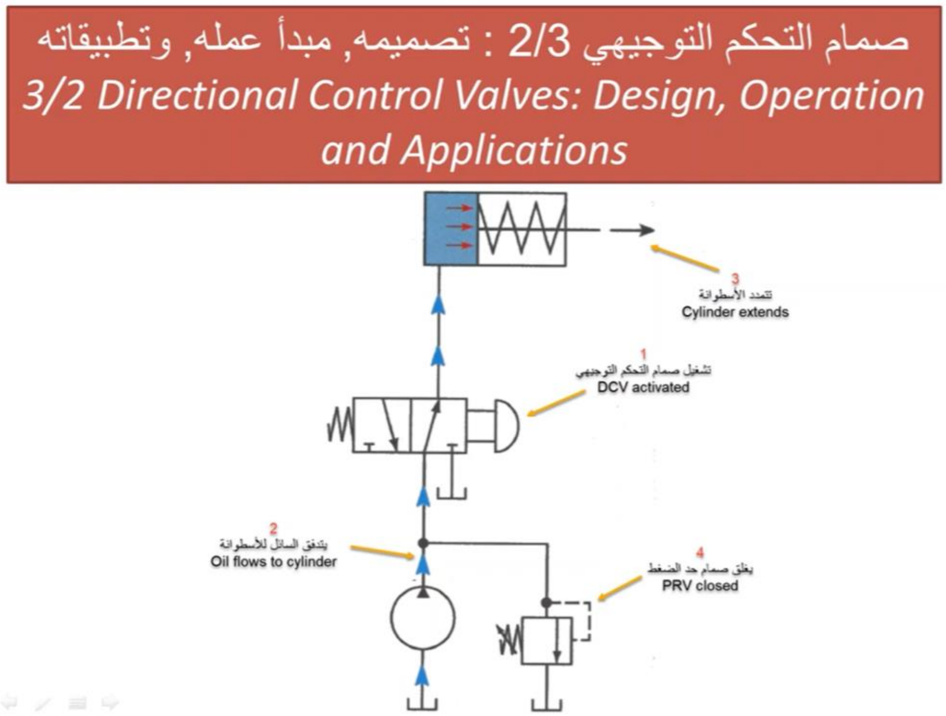


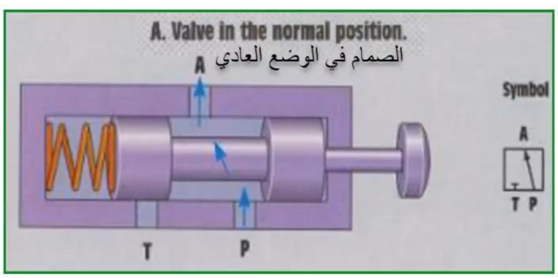
1
الملف الكهربائي نشط (يعمل)
Solenoid energized
P

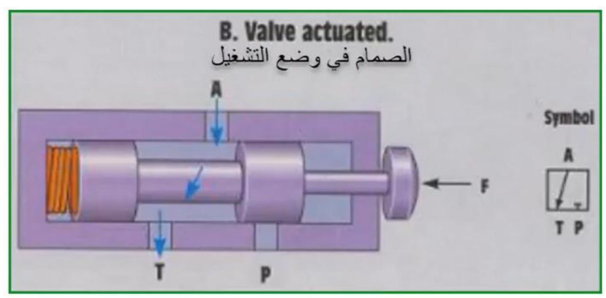


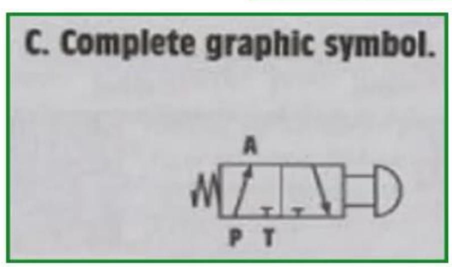
1
الملف الكهربائي نشط (يعمل)
Solenoid energized
A
P
2
صمام التعويض لا يضبط المضخة
Compensator is Off
30 GPM
3
المحرك الهيدروليكي يدور
Hyd. motor is working

3/2 – Way directional control valve:

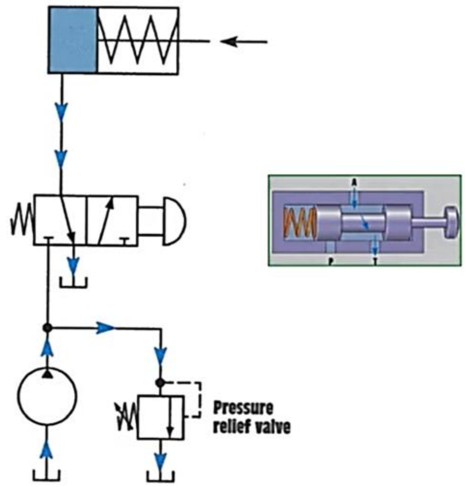


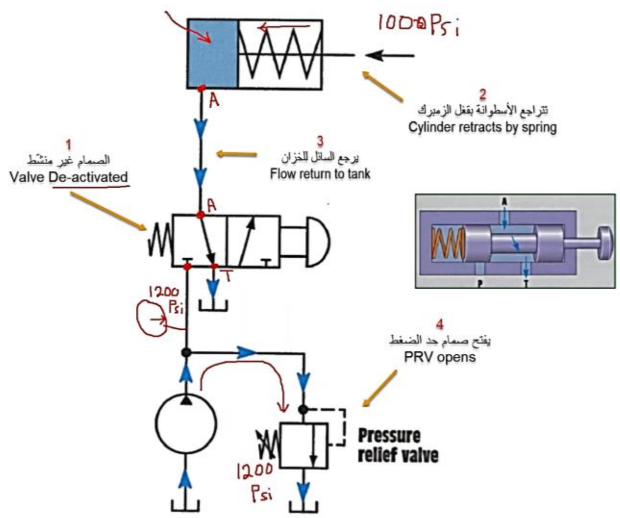


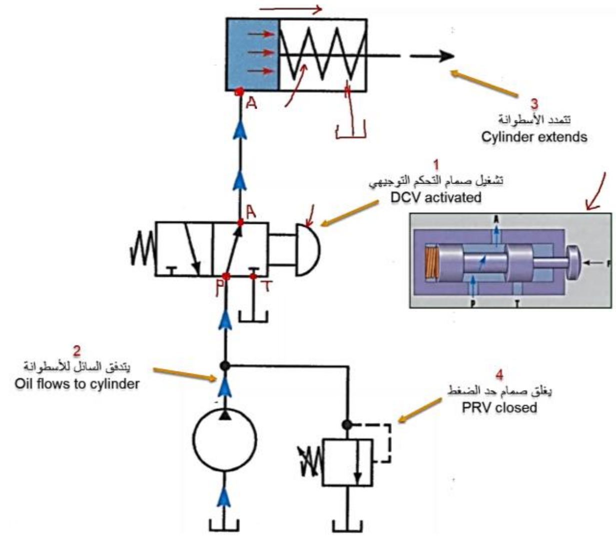




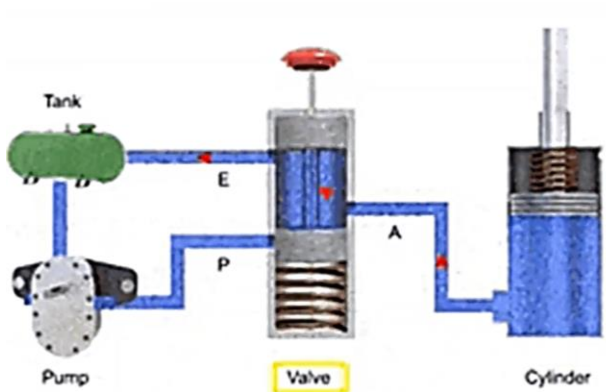
P: Pump
T: Tank
A: Actuator



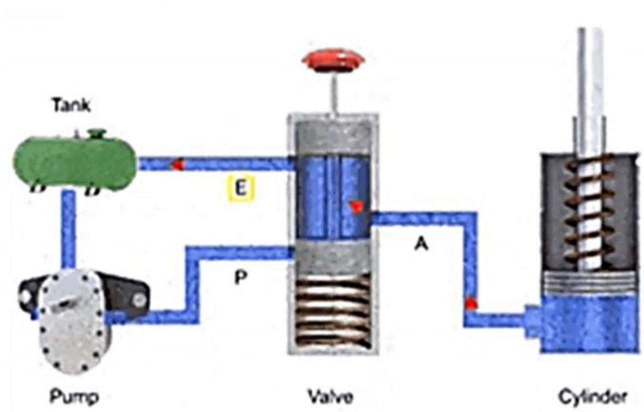




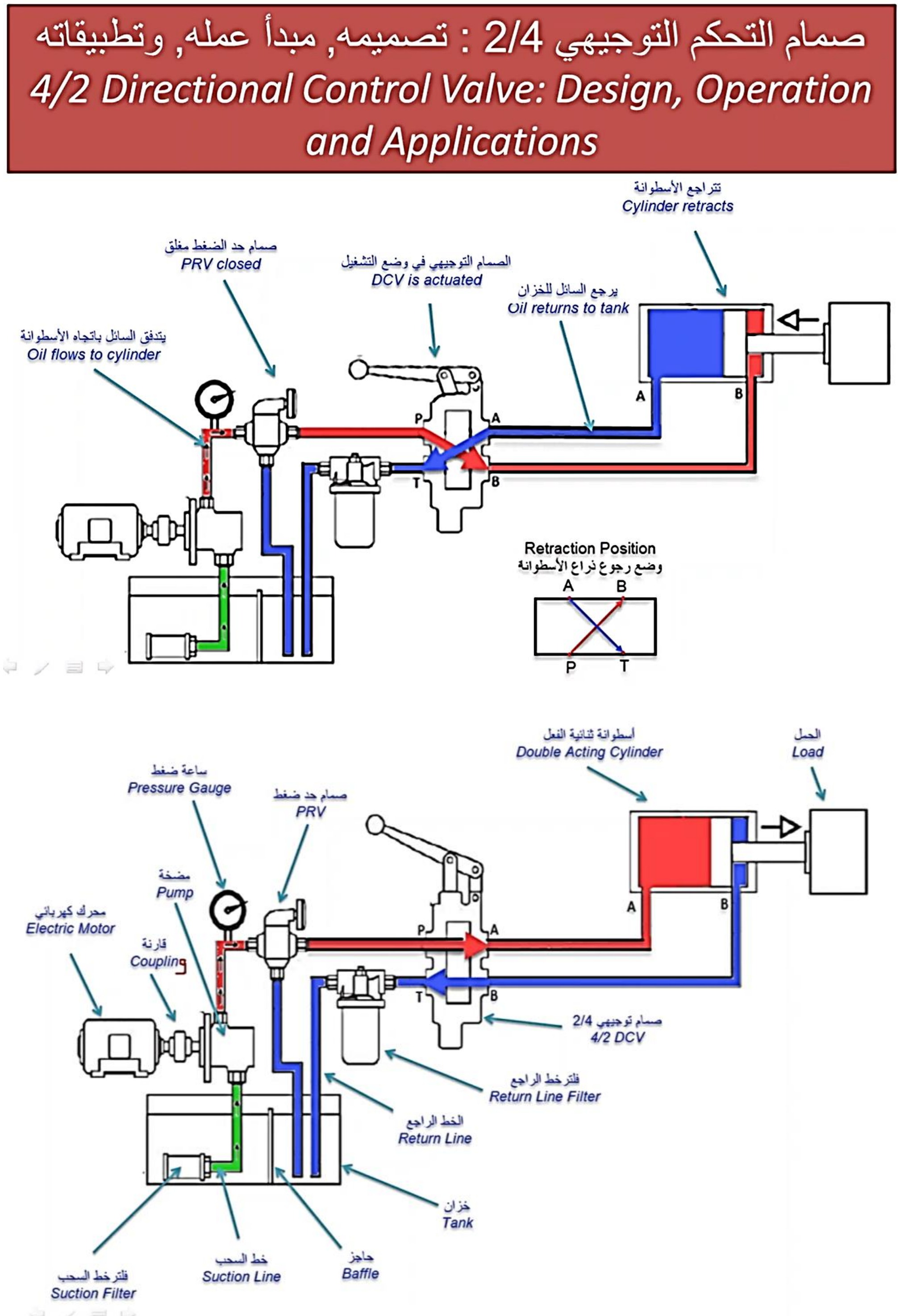
3 - Way, 2 Position Valve



3 - Way, 2 Position Valve



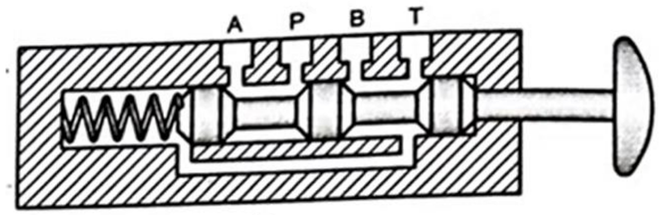
4/2 – Way directional control valve:



4/2 – Sliding Spool valve:

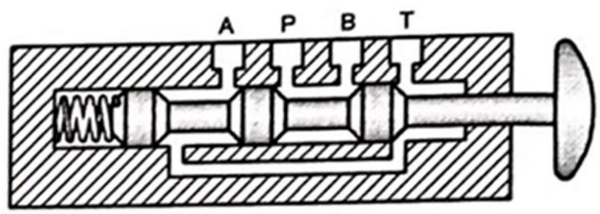
- It has a spring – loaded spool inside the valve body.

- In spool position as shown in Fig. (a), there is connection from (P) to (A) and from (B) to (T). working fluid flows to cap end port of cylinder, and comes out from rod end port. Hence the double acting cylinder extends.



(a)

- When the palm button is pressed, the spool position is as shown in Fig. (b), there is connection from (P) to (B) and from (A) to (T). working fluid flows to rod end port of cylinder, and comes out from cap end port. Hence the double acting cylinder retracts.

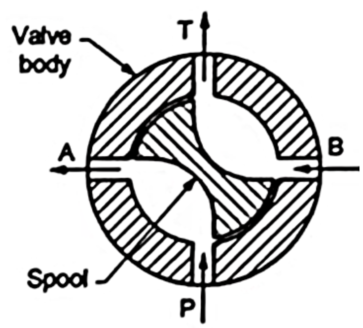


(b)

4/2 – Rotary Spool valve:

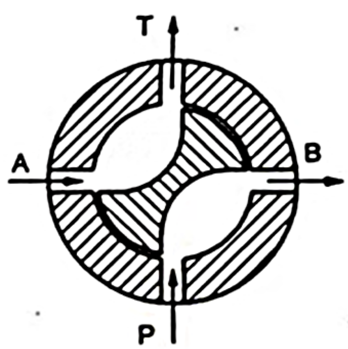
- It has a rotary spool inside the valve body. The spool is rotated through (90^o) to operate the valve.

- In spool position as shown in Fig. (a), there is connection from (P) to (A) and from (B) to (T). working fluid flows to cap end port of cylinder, and comes out from rod end port. Hence the double acting cylinder extends.



(a)

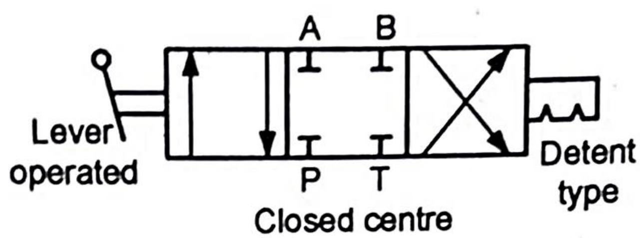
- When the spool is rotated through (90^o) as shown in Fig. (b), there is connection from (P) to (B) and from (A) to (T). working fluid flows to rod end port of cylinder, and comes out from cap end port. Hence the double acting cylinder retracts.



(b)

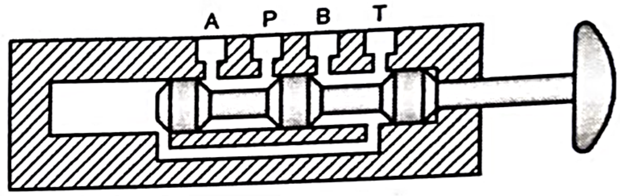
4/3 – Way directional control valve:

- This valve is used to operate double acting cylinder and bidirectional motor.
- It has four ports namely.
- Pump port or inlet port (P).
- Cylinder port (A).
- Cylinder port (B).
- Tank port (T).
- It has three positions of its spool.
- In first position, there is connection from (P) to (A) and from (B) to (T), hence the cylinder / motor moves in one direction.
- In the other position of spool, there is connection from (P) to (B) and from (A) to (T), hence the cylinder / motor runs in opposite direction.
- In middle position of spool, the cylinder or motor stops, it will not move in any direction.



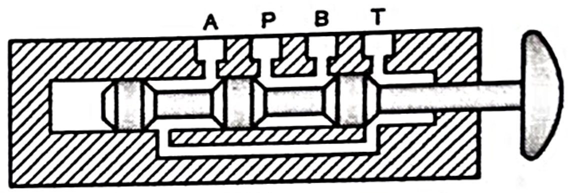
4/3 – Sliding Spool valve (Closed Center Mid-Position):

- In first position of spool position as shown in Fig. (a), there is connection from (P) to (B) and from (A) to (T). working fluid flows to cap end port of cylinder, and comes out from rod end port. Hence the double acting cylinder extends.



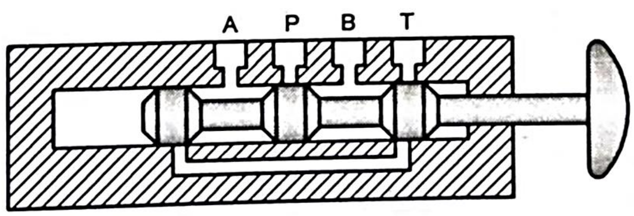
(a)

- In second position of spool position as shown in Fig. (b), there is connection from (P) to (A) and from (B) to (T). working fluid flows to rod end port of cylinder, and comes out from cap end port. Hence the double acting cylinder retracts.



(b)

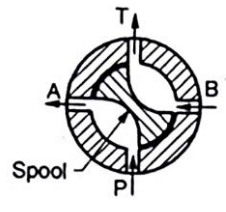
- When the spool is kept in the middle position, as shown in Fig. (c), all ports are closed, and hence the actuator stops. This is closed center middle position.



(c)

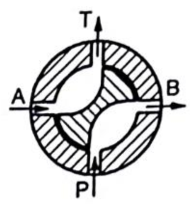
4/3 – Rotary Spool valve (Closed Center Mid-Position):

- In spool position as shown in Fig. (a), there is connection from (P) to (A) and from (B) to (T). working fluid flows to cap end port of cylinder, and comes out from rod end port. Hence the double acting cylinder extends.



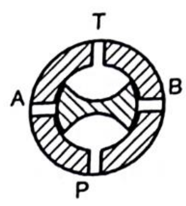
(a)

- When the spool is rotated through (90^o) as shown in Fig. (b), there is connection from (P) to (B) and from (A) to (T). working fluid flows to rod end port of cylinder, and comes out from cap end port. Hence the double acting cylinder retracts.



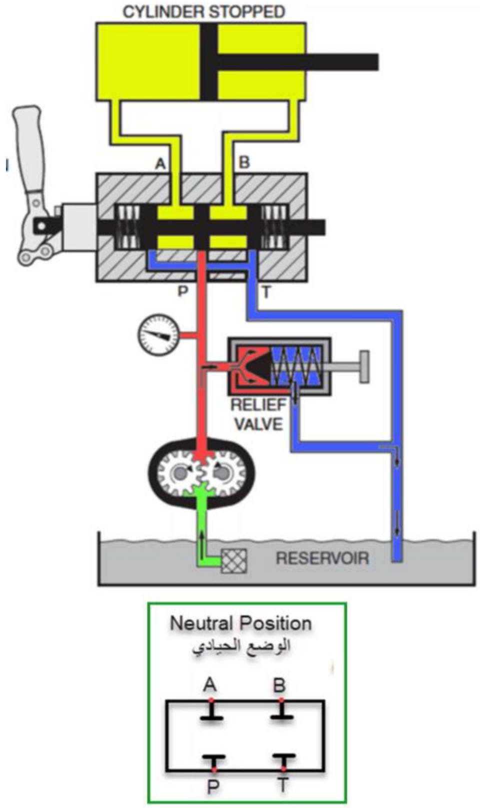
(b)

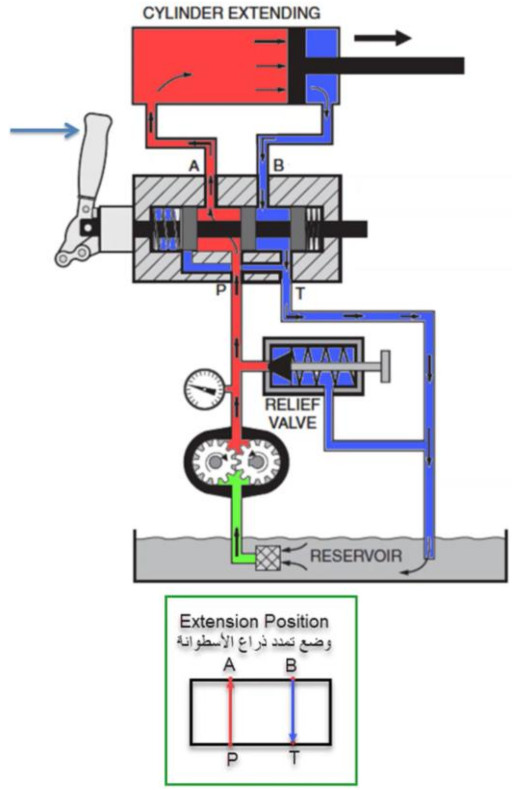
- Middle position is shown in Fig. (c), all ports are closed, and hence the actuator stops. This is closed center middle position.

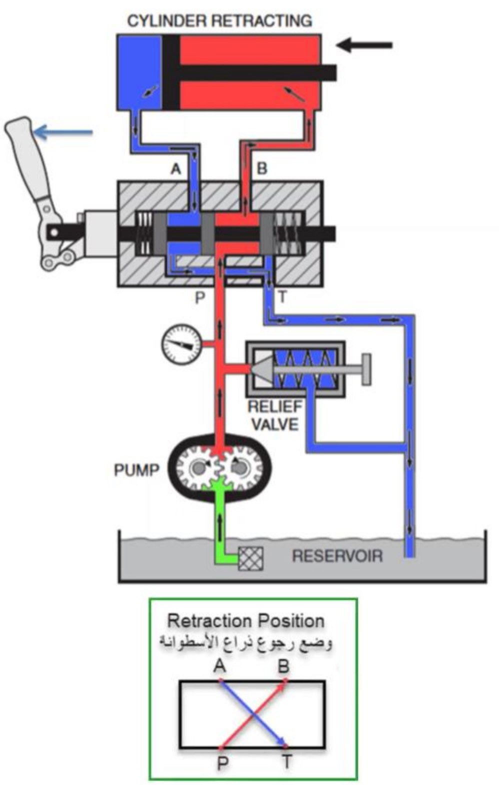


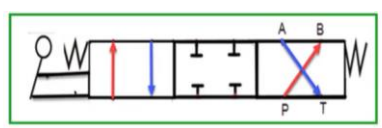
(c)

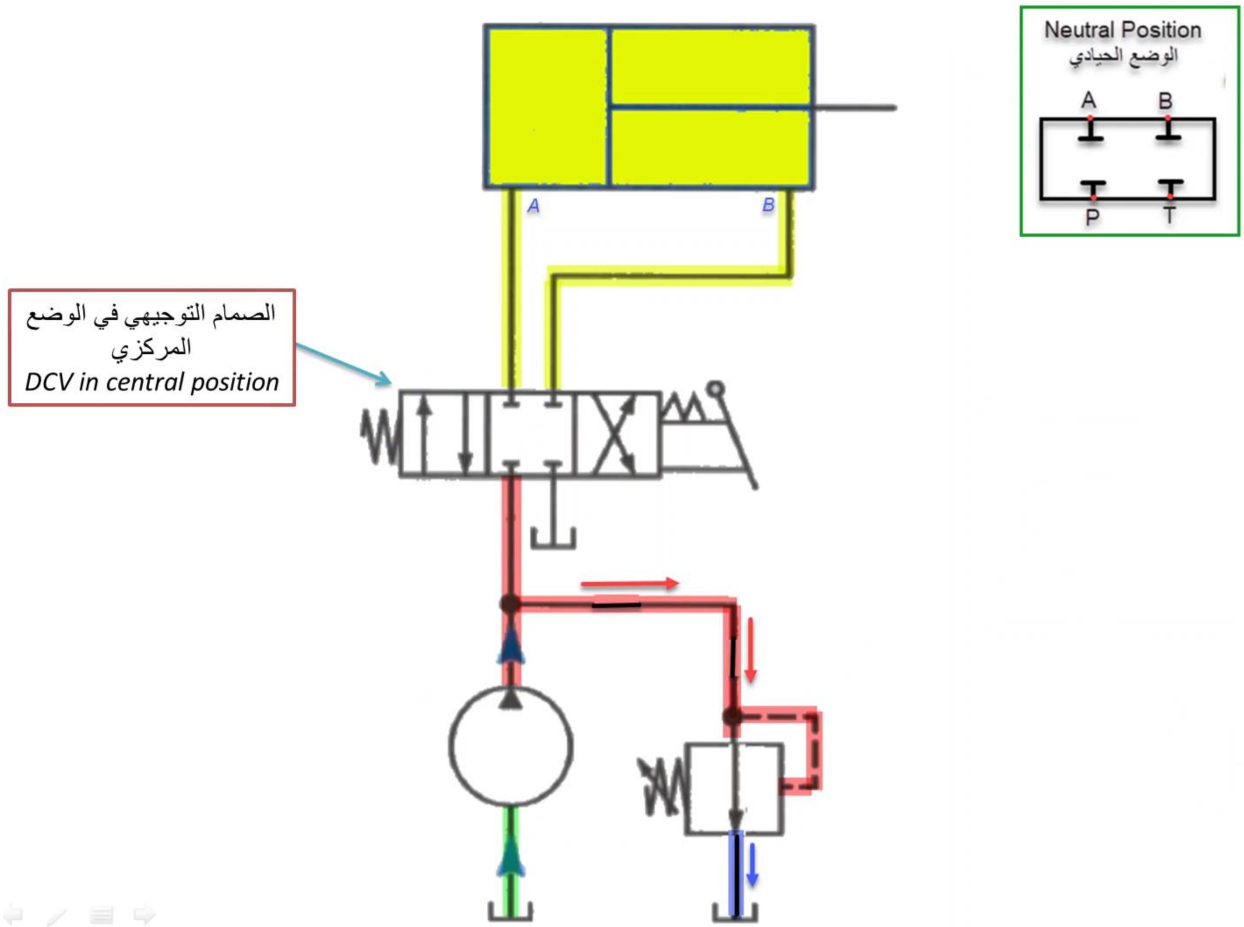
5/3 – Way directional control valve:

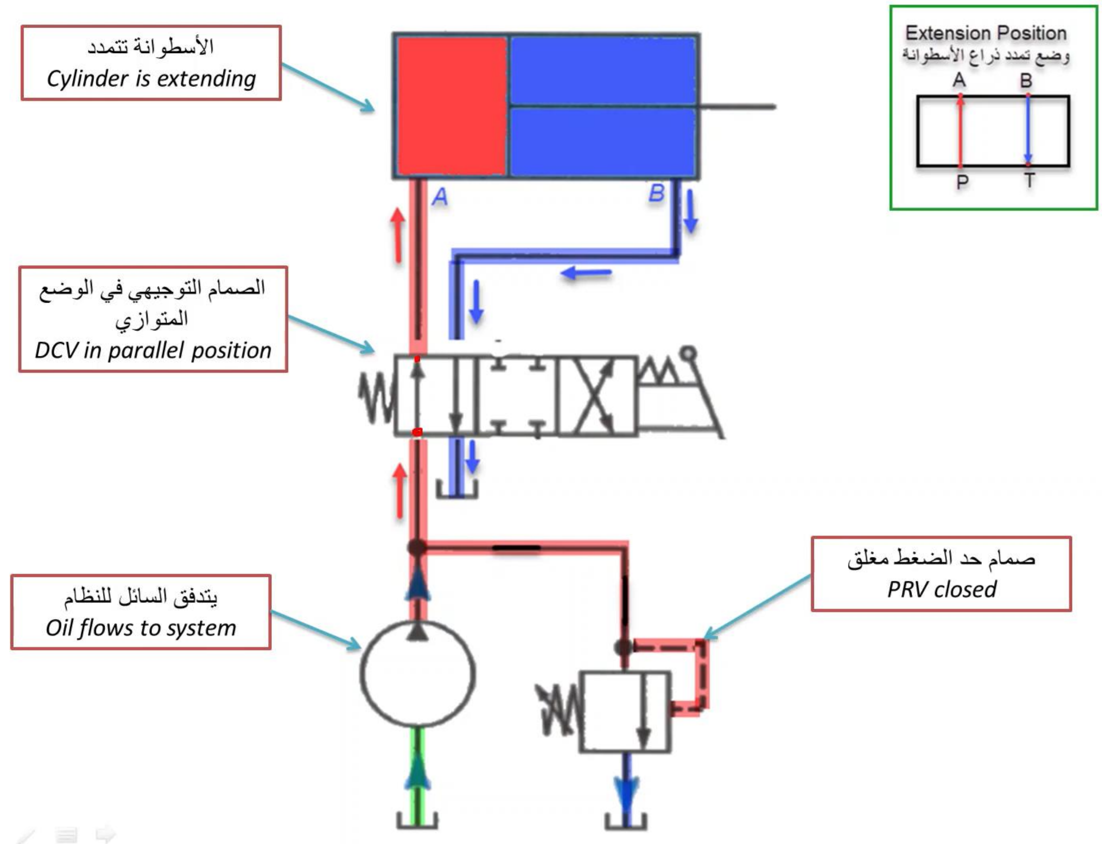




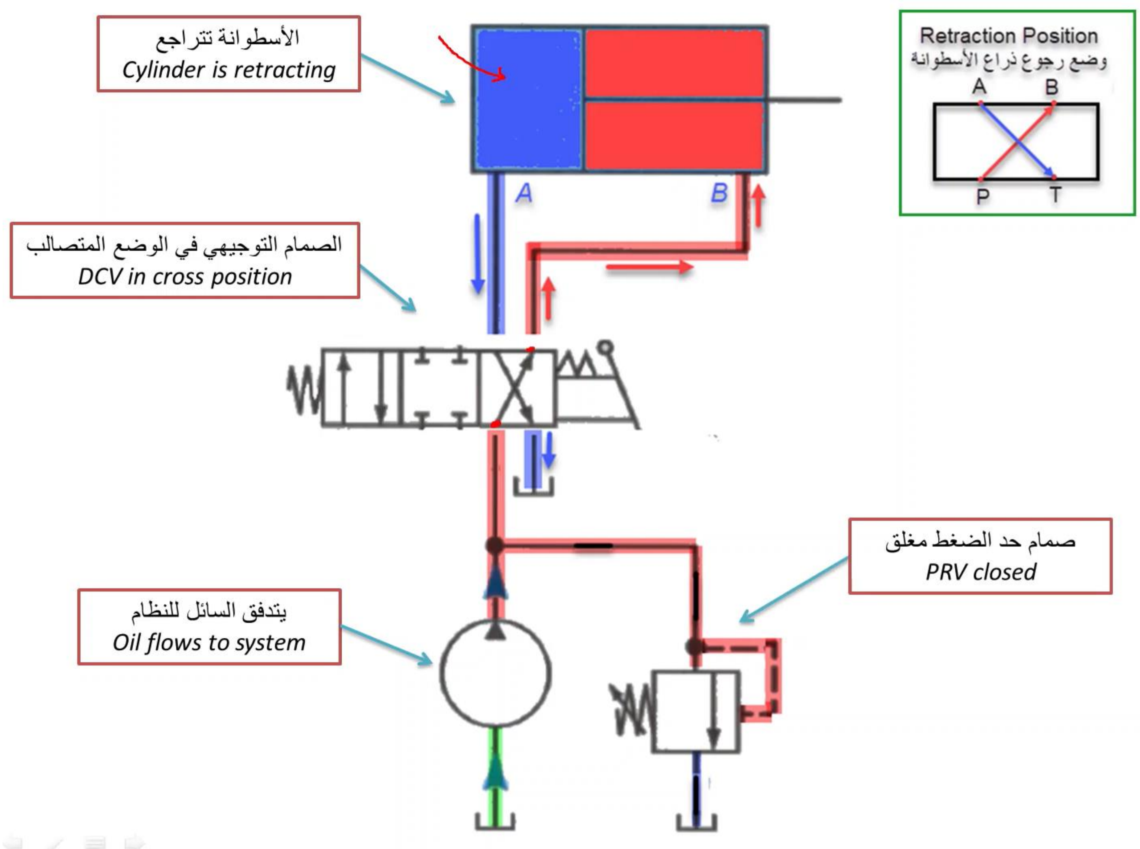





Neutral Position
الوضع الحيادي
A
B
P
T
A
B
الصمام التوجيهي في الوضع المركزي
DCV in central position



الأسطوانة تتمدد
Cylinder is extending
الصمام التوجيهي في الوضع المتوازي
DCV in parallel position
يتدفق السائل للنظام
Oil flows to system
صمام حد الضغط مغلق
PRV closed
Extension Position
وضع تمدد ذراع الأسطوانة
A
B
P
T



الأسطوانة تتراجع
Cylinder is retracting
الصمام التوجيهي في الوضع المتصالب
DCV in cross position
يتدفق السائل للنظام
Oil flows to system
صمام حد الضغط مغلق
PRV closed
Retraction Position
وضع رجوع ذراع الأسطوانة
A
B
P
T