محاضرات الشبكات الكهربائية

أ. هبه عبد الخالق الحسان
قسم الالكترونيك

Hiba Alsawaf

Basics of Electrical Engineering :-

Unit	Symbol	Value	Example
Tera	T	10^{12}	Tg
Giga	G	10^{9}	GHz
Mega	M	10^{6}	MW/MVA
Kilo	K	10^{3}	KΩ
centi	c	10^{-2}	cm
milli	m	10^{-3}	msec. / mA
Micro	μ	10^{-6}	μF
Nano	n	10^{-9}	nF
Pico	P	10^{-12}	PF
femto	f	10^{-15}	
* atto	a	10^{-18}	
* Zepto	z	10^{-21}	
* yocto	y	10^{-24}	

Voltage & current :-

Voltage :- (Potential difference)
is the work done to move electric charge between two ponits.

$$V = \frac{\text{work}}{\text{charge}} \quad (\underset{V}{\text{Volt}},\ \underset{J\,/\,C}{\text{Joule / coulumb}})$$

(Newton.meter / coulumb)

Current :-
المعدل الزمني لتغير الشحنات الكهربائية
is the time rate of change of electric charge

The unit of current is the ampere (A)

$i = \frac{dq}{dt}$ ← charge / ← time — in general — Hiba Alnawaf

The symbol of current is (I or i)

Power:-

We have already defined power, and we will represent it by P or p. If one joule of energy is expended in transferring one collumb of charge through the device in one second, then the rate of energy transfer is one watt. The absorbed power must be proportional both to the number of coulombs transferred per second (current) and to the energy nedded to transfer one coulomb through the element (Voltage). Thus

$$P = Vi \quad \text{(Volt. Amp.) watt}$$

$$= \frac{\text{energy}}{\text{time}}$$

$$= {}^{\text{unit}}\left(\frac{J}{s}\right) \Rightarrow \frac{Q \cdot V}{t}$$

$$= V \cdot \frac{Q}{t}$$

$$= V \cdot I$$

$$E = Q \cdot V$$

energy — charge — Potential difference

Circuit Elements :-

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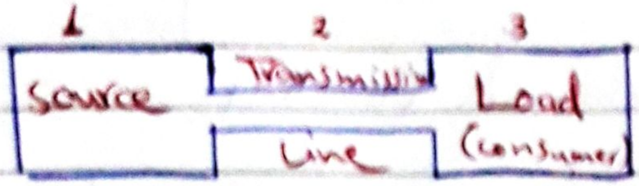
Any circuit contain the following elements :-

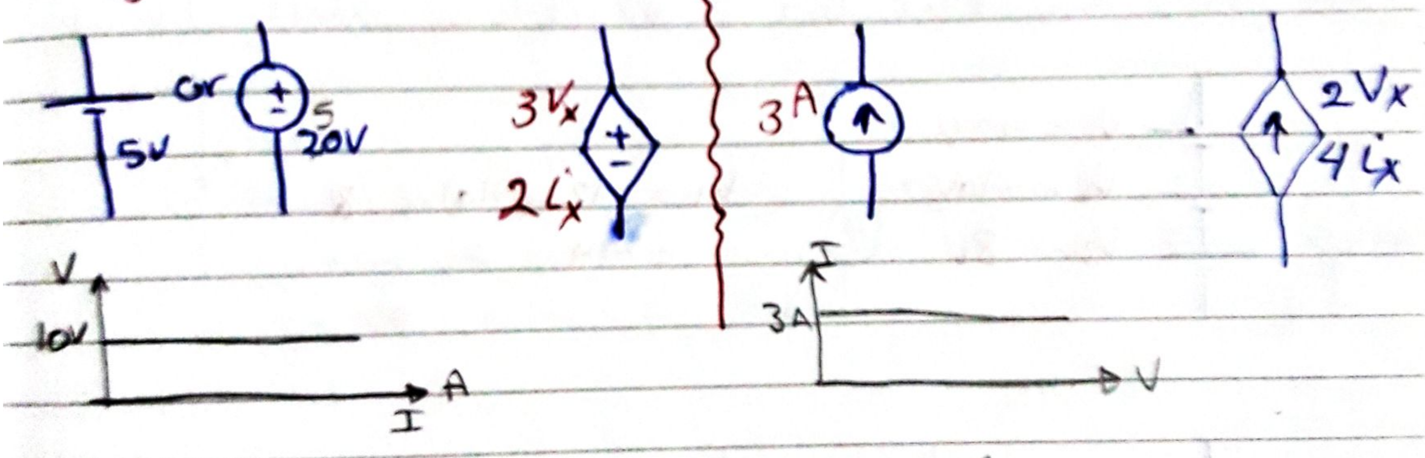
1. Source. 2. Transmission Line. 3. Load.



* Source :-

1. Voltage Source
   - Independent voltage source
   - dependent voltage source

2. Current Source
   - Independent current source
   - dependent current source



* المصدر الغير معتمد (Independent) يكتب عليه مقدار المصدر وقيمته ثابتة بغض النظر عن قيمة التيار المسحوب منه.
* تكون قيمته معلومة ثابتة.
* قيمة المصادر المعتمدة تكون معتمدة على قيمة المتغير (Vx مثلا) او الوحدة تكون Volt.

* مصدر التيار الغير معتمد يكتب عليه مقدار المصدر وقيمته ثابتة بغض النظر عن قيمة الفولتية المتكونة عنه.
* قيمة المصادر المعتمدة (التيار) تكون معتمدة على قيمة المتغير (Vx مثلا) والوحدة تكون Amper.

* المصدر المعتمد لا يستطيع ان يجهز قدرة اذا كان تيار او فولتية الا بوجود مصدر غير معتمد اي ان المصدر المعتمد مرتبط بمصدر غير معتمد.

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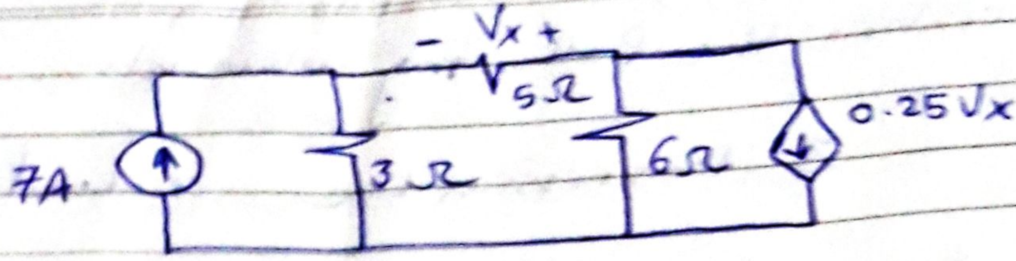
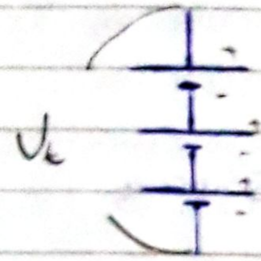


* Voltages in series & Parallel



$V_1 = 12V$
$V_2 = 10V$
$V_3 = 8V$

$V_1 = 12V$
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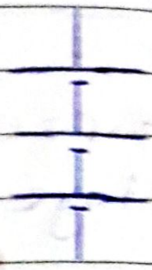
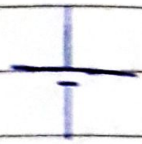
$V_3 = 8V$



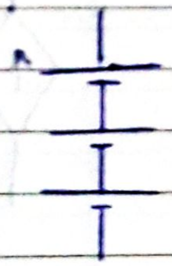
$V_1 = 12V$
$V_2 = -10V$
$V_3 = 8V$

$V_t = 12 + 10 + 8$
$= 30V$

$V_t = 12 - 10 + 8$
$= 10V$

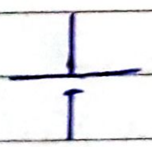


$V_t = 12 - 10 + 8$



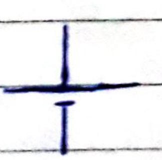
$V_1 = -12V$
$V_2 = -10V$
$V_3 = 8V$

$V_t = -12 - (-10) + 8$
$= -14$



$V_t = -14$

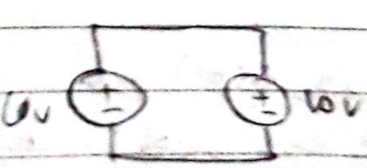
$V_t = 14$ Volt



V = zero ⇒

short

* اذا كانت مصادر الفولتية غير المعتمدة مرتبطة على التوازي لازم يجب ان ننتبه لشكل قيمتها واتجاهها.



Currents in series & parallel

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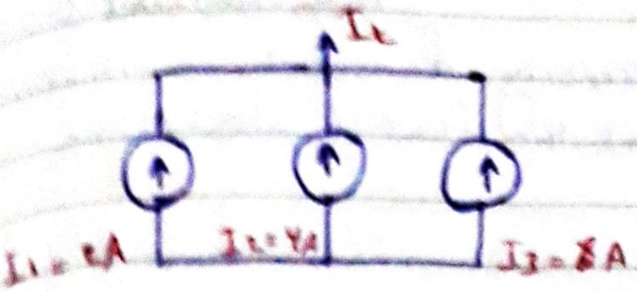
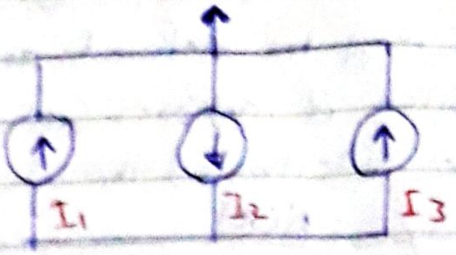
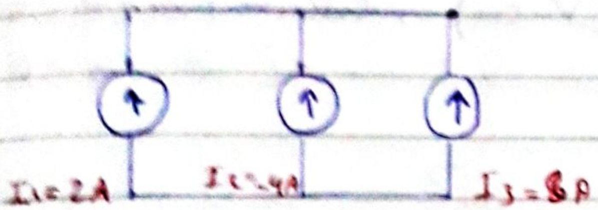
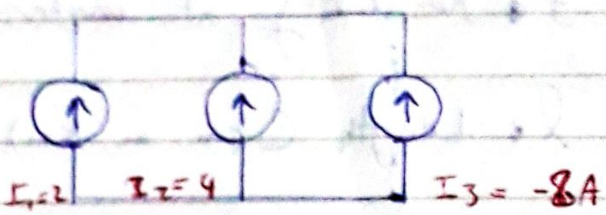




$I_t = I_1 + I_2 + I_3$
$= 2 + 4 + 8$
$= 14A$

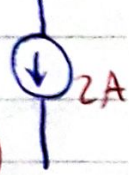
$I_t = I_1 - I_2 + I_3$
$= 2 - 4 + 8$
$= 6A$



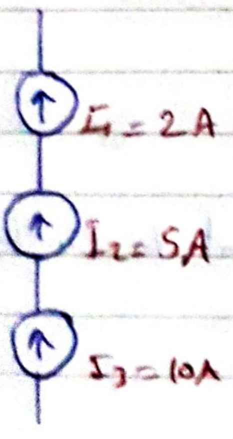


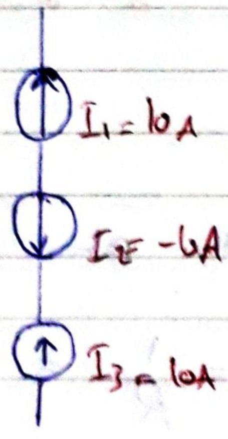
$I_t = I_1 + I_2 + I_3$
$= 2 - 4 + 8$
$= 6A$

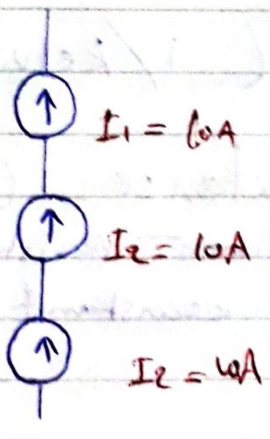
$I_t = I_1 + I_2 + I_3$
$= 2 + 4 - 8$
$= -2$ ( )



$I_t =$ Zero (open circuit)







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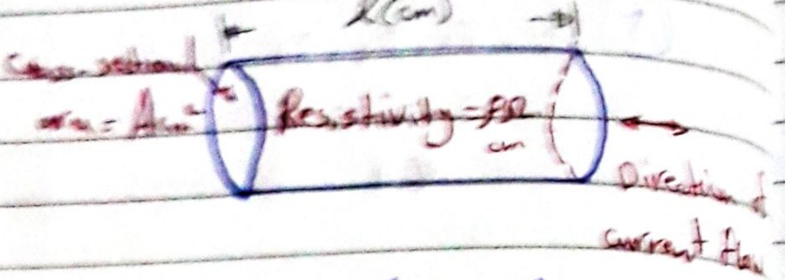
The Load:

Resistance (R):

It may be defined as the property of a substance due to which it opposes (or restricts) the flow of electricity (i.e electrons) through it.

خاصية المادة بمقاومة التيار المار
خاصية المادة في مقاومة مرور الكهرباء

$$R \propto \frac{l \rightarrow (cm, meter)}{A \rightarrow (cm^2, m^2)}$$



$$R = \rho \frac{l}{A}$$

ρ = Resistivity of material (Ω cm, Ω.m) المقاومية النوعية للمادة

l = Length (cm, m, km)

A = cross sectional Area (cm^2, m^2) مساحة المقطع العرضي

* هنالك مواد ذات معامل حراري موجب حيث تزداد مقاومتها بزيادة درجة الحرارة وهنالك مواد ذات معامل حراري سالب تقل مقاومتها بزيادة درجة الحرارة ووفقاً للقانون الآتي :-

$$R_2 = R_1 \left[1 + \alpha (T_2 - T_1) \right]$$

R_2: المقاومة بعد التغير
R_1: المقاومة قبل التغير

α : Temperature coefficient of material (constant)
المعامل الحراري للمادة

Unit (1/kelvin)

المواصلة الثانية

Conductance: (G)

For a linear resistor the ratio of current to voltage is also a constant:

$$\frac{i}{v} = \frac{1}{R} = G$$

Ohm's Law :-

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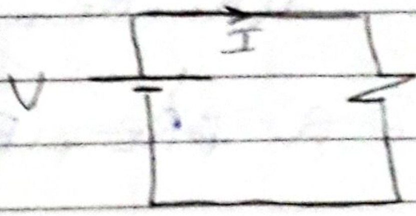
The current passing through any conductor is directly proportional to potential difference across it, on condition that the temperature is kept constant.

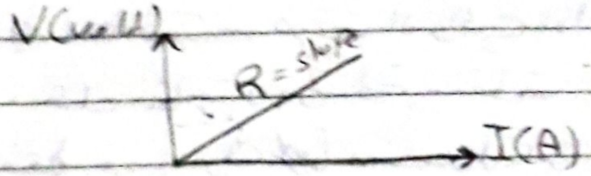
ينص قانون اوم على انه التيار المار في اي موصل يتناسب تناسبا طرديا مع فرق الجهد بشرط ان تكون درجة الحرارة ثابتة.

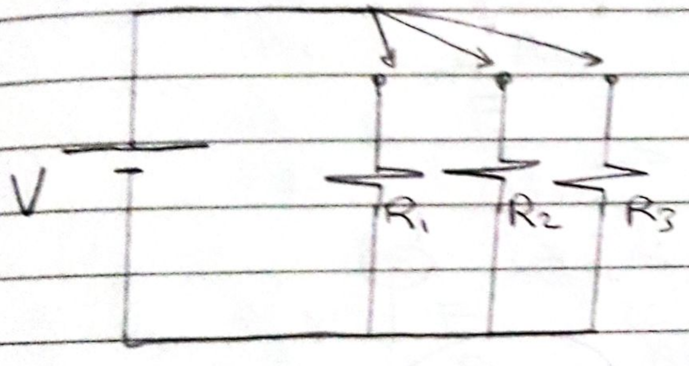
$V \propto I$

$V = RI.$

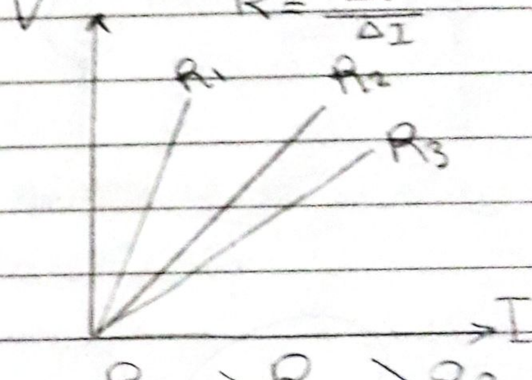
$R = \dfrac{V}{I} \quad \Omega$







$R = \dfrac{\Delta V}{\Delta I}$



$R_1 > R_2 > R_3$

Resistance in Series

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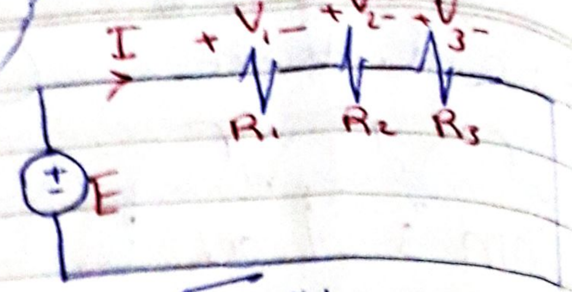


1. $I_1 = I_2 = I_3 = I$
2. $E = V_1 + V_2 + V_3$

E عبارة عن مصدر الطاقة الكهربائية وقيمتها دائماً تكون معلومة وغير مرتبطة بالتيار بينما (V) هي مولتية ناتجة عن مرور التيار في عنصر مقاومة مرتبطة مع التيار.

$V_1 = I R_1$, $V_2 = I R_2$, $V_3 = I R_3$

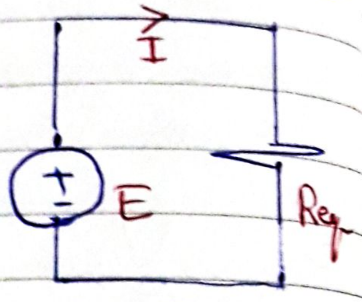
$E = V_1 + V_2 + V_3$

$I \cdot R_{eq} = I R_1 + I R_2 + I R_3$

$I R_{eq} = I(R_1 + R_2 + R_3)$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

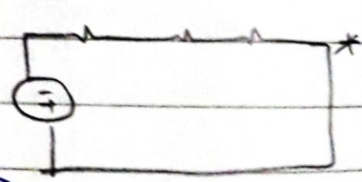
التيار يسري في الجهاز السالب الى الموجب.



$E = I \, R_{eq}$
$= I \cdot R_{eq}$

* يطلق على المقاومات صفة التوالي اذا كانت كل مقاومتين مرتبطة بنقطة واحدة فقط ويسري بهما نفس التيار.

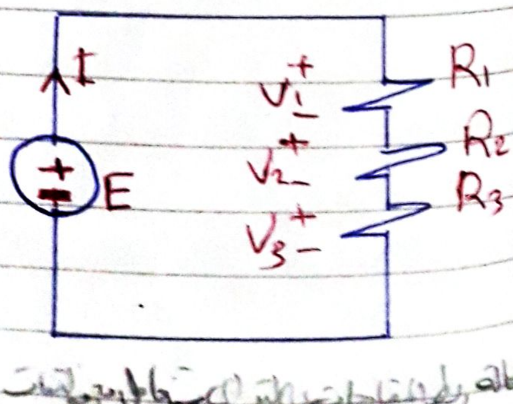
عند عكس قطبية المصدر المغذي للدارة الكهربائية فان القوانين لا تتغير بينما اتجاه التيار يتغير.



Voltage Division (Voltage Divider Rule) :-

$V_1 = I R_1$, $E = I R_{eq}$

$$I = \frac{E}{R_{eq}} = \frac{E}{R_1 + R_2 + R_3}$$



في حالة ربط المقاومات التي تشترك بمسار واحد مع بعضها

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$$\boxed{V_1 = E * \frac{R_1}{R_1 + R_2 + R_3}}$$ Volt

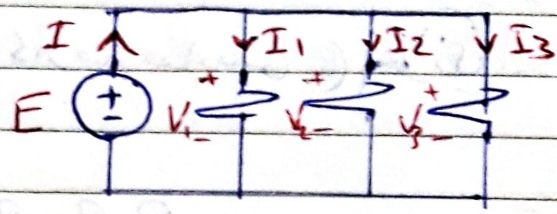
فولتية الجزء = الفولتية الكلية * (مقاومة ذلك الجزء / المقاومة المكافئة الكلية)

$$V_2 = E * \frac{R_2}{R_1 + R_2 + R_3} , \quad V_3 = E * \frac{R_3}{R_1 + R_2 + R_3}$$

Resistance in parallel :

* $V_1 = V_2 = V_3 = E$

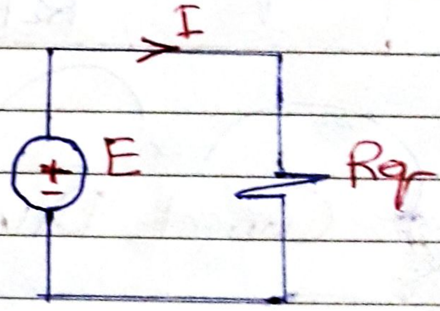
* $I = I_1 + I_2 + I_3$



$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3}$$

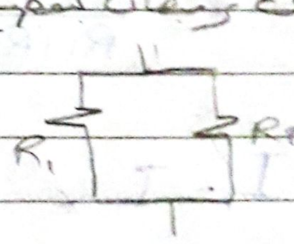


$$I = \frac{E}{R_{eq}}$$

$$I = I_1 + I_2 + I_3$$

$$\frac{E}{R_{eq}} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

المقاومتان متوازيتان يجب ان تتصلان بنقطتين



$$\frac{\cancel{E}}{R_{eq}} = \cancel{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$G_{eq} = G_1 + G_2 + G_3$$

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[illegible]

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{R_2 + R_1}{R_1 R_2}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

* المقاومة المكافئة إذا كان لدينا أكثر من مقاومتين على التوازي :

$$\boxed{R_{eq} = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2}}$$

Current Divider Rule :

$$E = V_1 = V_2$$

$$I R_{eq} = I_1 R_1$$

$$I * \frac{\cancel{R_1} R_2}{R_1 + R_2} = I_1 \cancel{R_1}$$

$$I_1 = I * \frac{R_2}{R_1 + R_2} , \quad I_2 = I * \frac{R_1}{R_1 + R_2}$$

$$\text{تيار الفرع} = \text{التيار الكلي} * \frac{\text{المقاومة المقابلة الأولى}}{\text{مجموع المقاومات المتوازية}}$$

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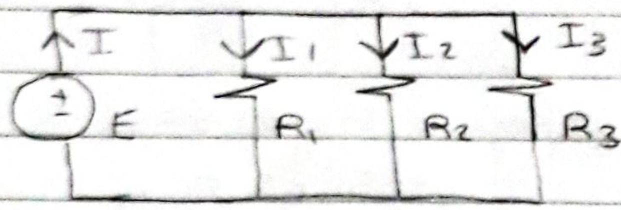
★اذا كانت لدينا اكثر من مقاومة على التوازي:

$R_{eq} = R_1 // R_2 // R_3 \text{ ---- } // R_n$

$$R_{eq} = \frac{R_1 R_2 R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$



$$I_1 = I * \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$E = V_1$

$$\frac{I}{G_{eq}} = \frac{I_1}{G_1}$$

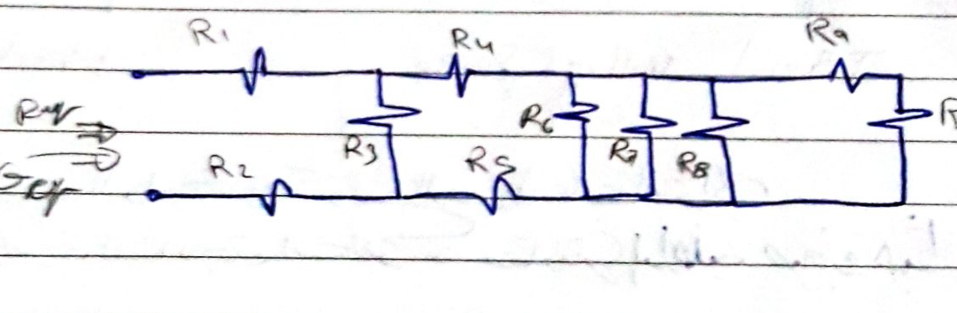
$$I_1 = I * \frac{G_1}{G_{eq}}$$

OR

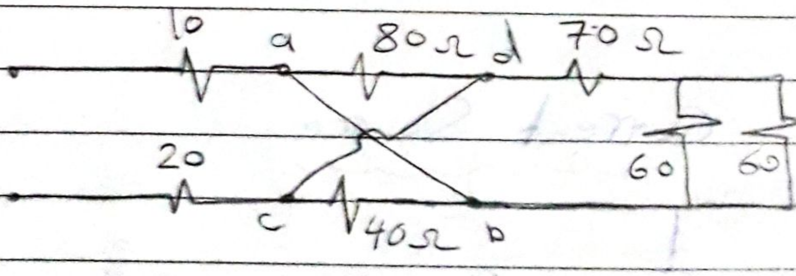
$$I_1 = I * \frac{G_1}{G_1 + G_2 + G_3}$$

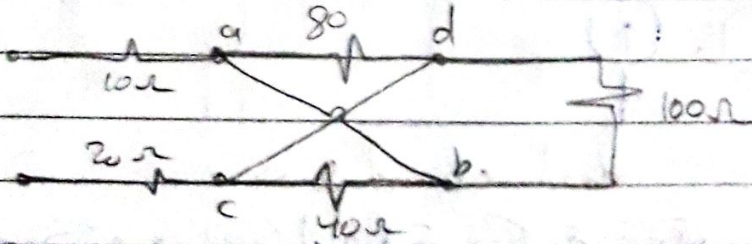
Example:-
Find R_{eq} for the circuit shown in Figure if $R = 100\ \Omega$

$R_{eq} = 269.56\ \Omega$



Example:-





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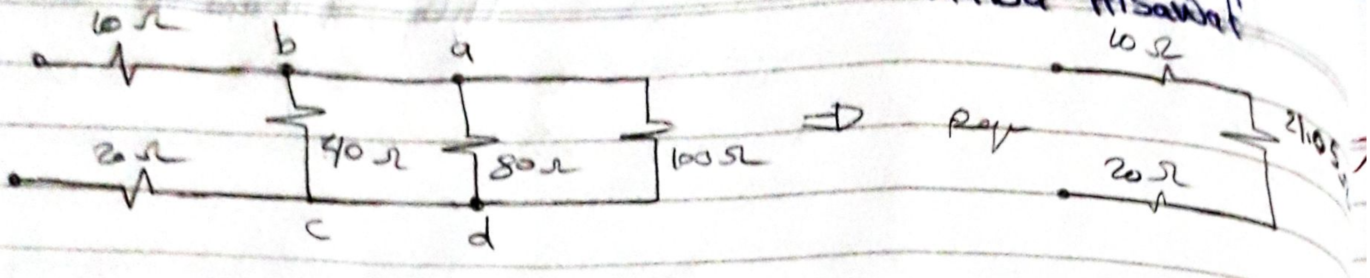
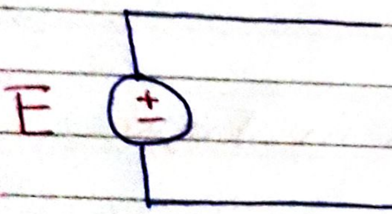


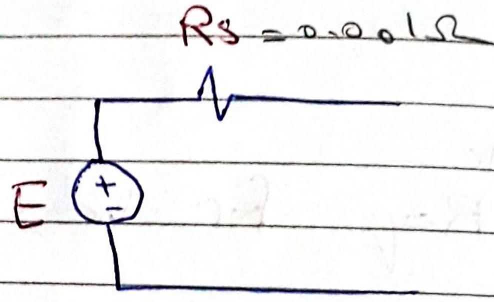
$R_{eq} = 10 + 21.05 + 20$
$= 51.05\ \Omega$

Ideal & Practical Source

* Voltage Source



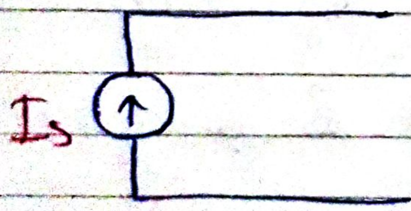
Ideal voltage source.



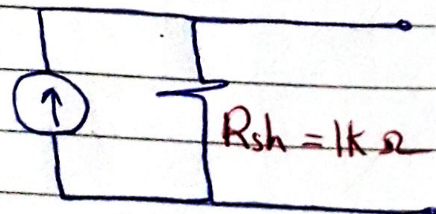
Practical voltage source

* لكي نضعه صناعي في المولدة يكون الربط على التوالي.
* يفضل في مصادر الفولتية المثالية ان تكون مقاومتها الداخلية صغيرة جداً قريبة من الصفر.

* Current Source



Ideal current source



Practical current source

* يفضل في مصادر التيار المثالية ان تكون مقاومتها الداخلية كبيرة قد تصل الى اللانهائية.
* لكي نضعها صناعي في التيار يكون الربط على التوازي.

* Voltage and Current Sources Transformation

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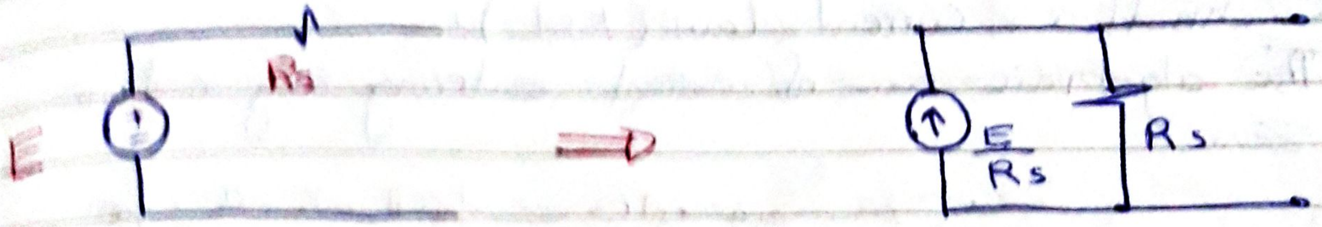
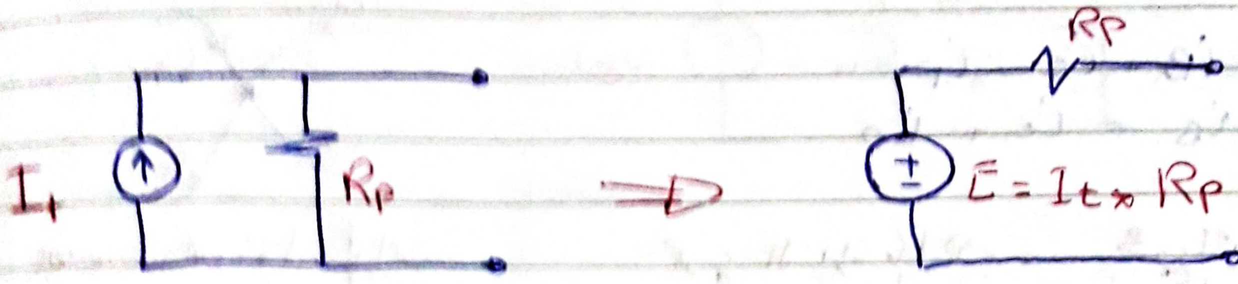
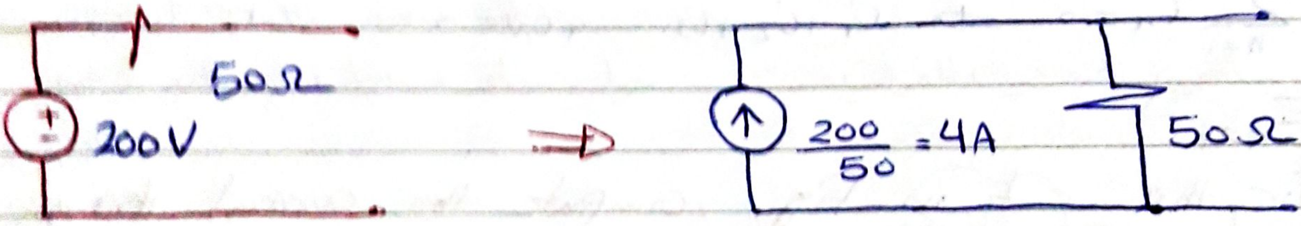
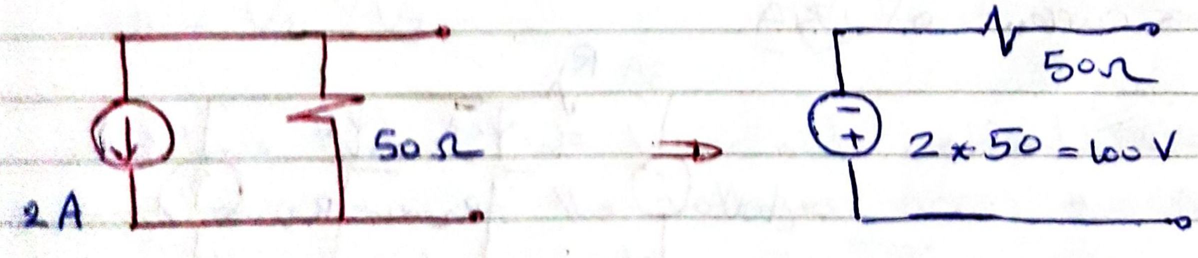




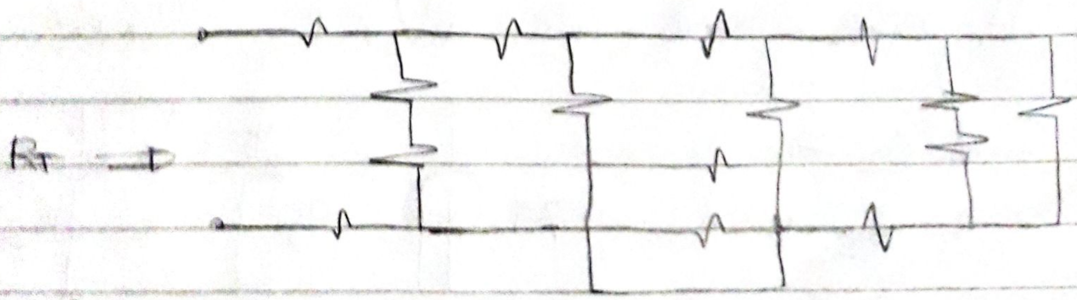
Exa:



Exa:



HW:



find RT if R= 30Ω

* Kirchhoff's Laws:-

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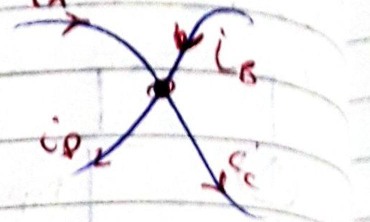
Kirchhoff's current law (KCL):-

1. Kirchhoff's current law
The algebraic sum of currents entering any node is Zero.

المجموع الجبري للتيارات في اي عقدة يساوي صفر.



$i_A + i_B - i_C - i_D = 0$

$i_A + i_B = i_C + i_D$

العقدة: هي نقطة التقاء فرعين او اكثر.

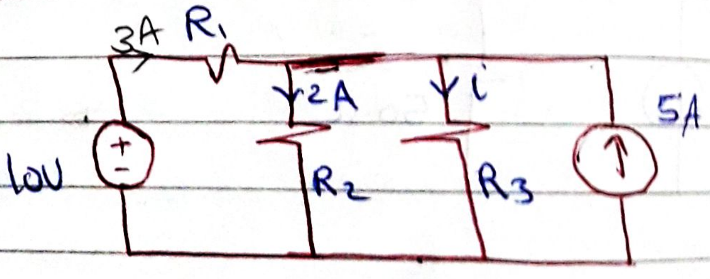
تيارات الداخلة موجبة والخارجة سالبة.

مجموع التيارات الداخلة الى العقدة = مجموع التيارات الخارجة من العقدة

$$\sum_{n=1}^{N} i_n = 0 \Rightarrow i_1 + i_2 + i_3 + \dots + i_N = 0$$

Exa:-

For the cct in Fig., compute the current through resistor R_3 if it is known that the voltage source supplies a current of 3A.



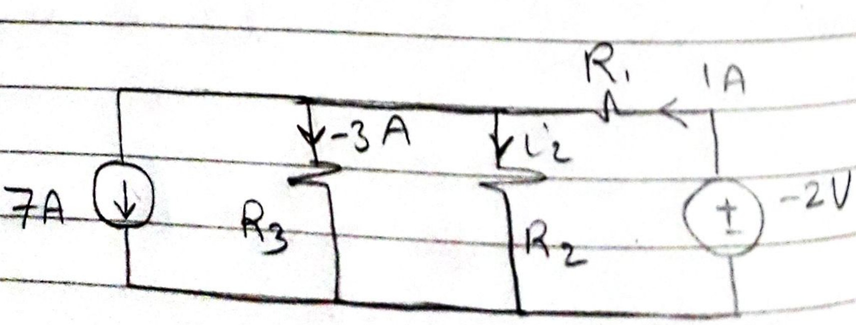
$3 + 5 = 2 + i$

$i = 8 - 2$

$i = 6A$

H.W:-

Find i_2 ?



Ex:-

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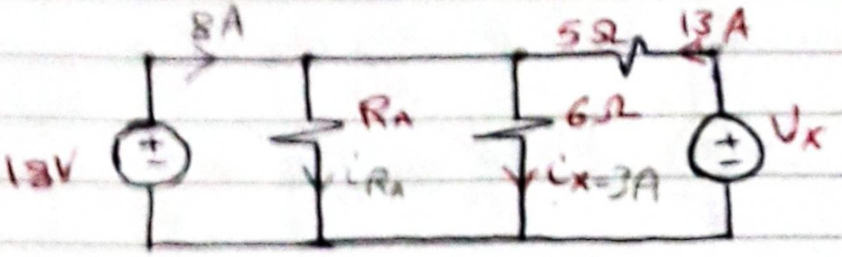
Count the number of branches and nodes in the circuit in Fig. If $i_x = 3A$ & the 18 V Source delivers 8A of current, What is the value of R_A?

$8 + 13 = 3 + i_{RA}$

$21 - 3 = i_{RA}$

$i_{RA} = 18A$



$i_{RA} = \frac{18}{18} = 1\Omega$

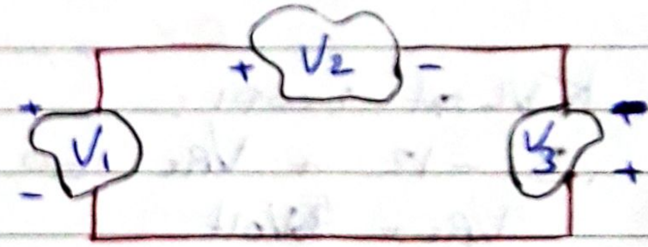
2. Kirchhoff's Voltage Law (KVL):-

- The algebraic sum of voltages around any closed path is Zero.

المجموع الجبري للفولتيات حول حلقة مغلقة تساوي صفر.

$-V_1 + V_2 - V_3 = 0$

$V_1 = V_2 - V_3$



It follows that if we trace out a closed path, the algebraic sum of the voltages across the individual elements around it must be Zero. Thus, we may write

$$V_1 + V_2 + V_3 + \text{———} + V_N = 0$$

OR

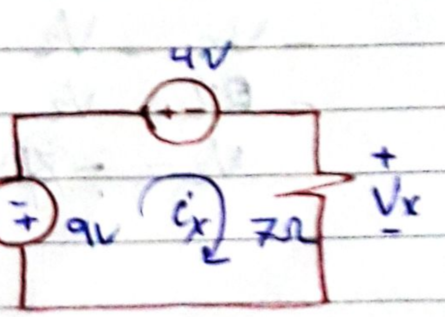
$$\sum_{n=1}^{N} V_n = 0$$

Exa:-

Determine V_x & i_x?

$+9 + 4 + V_x = 0$

$V_x = -13$ Volt



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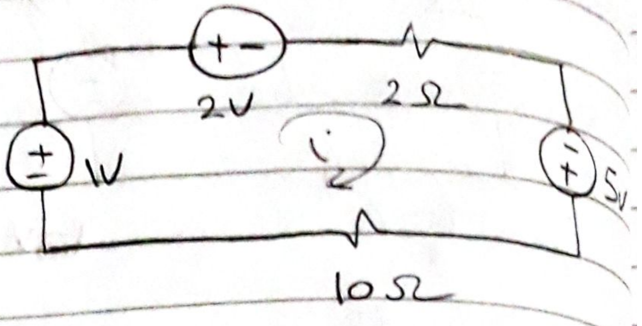
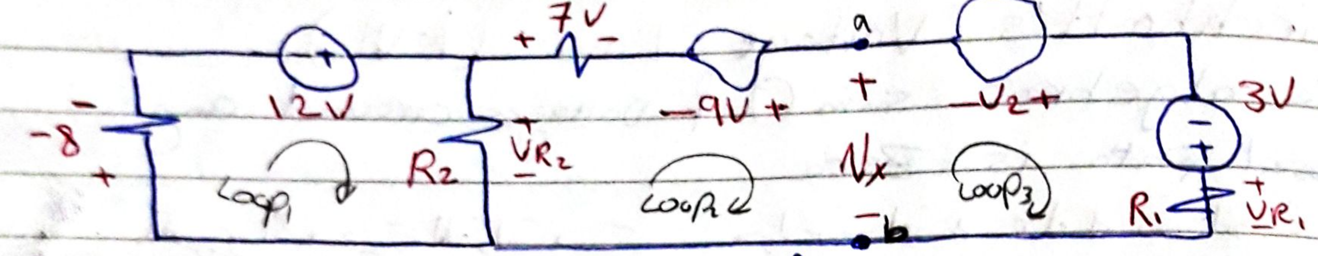
$i_x = \frac{V_x}{7} = \frac{-13}{7} \Rightarrow i_x = -1.857\ A$

H.W:-

Find i ?



Exa:-



Determine V_{R_2} & V_2, if $V_{R_1} = 1V$

- KVL at Loop1:-

$-8 - 12 + V_{R_2} = 0$

$V_{R_2} = 20\ Volt$

KVL at Loop2:-

$-V_{R_2} + 7 - 9 + V_x = 0$

$-20 + 7 - 9 + V_x = 0$

$V_x = 22\ Volt$

KVL at Loop3:-

$-V_x - V_2 - 3 + V_{R_1} = 0$

$-22 - V_2 - 3 + 1 = 0$

$V_2 = -24\ Volt$

Ex:-

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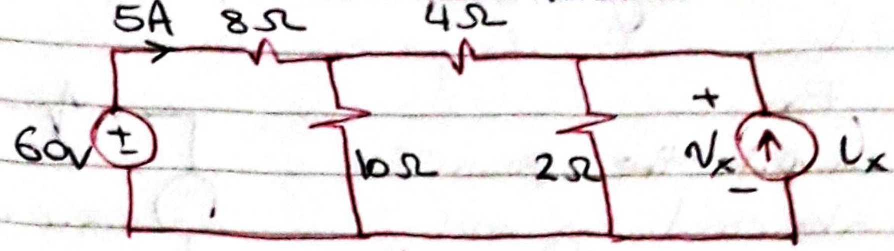
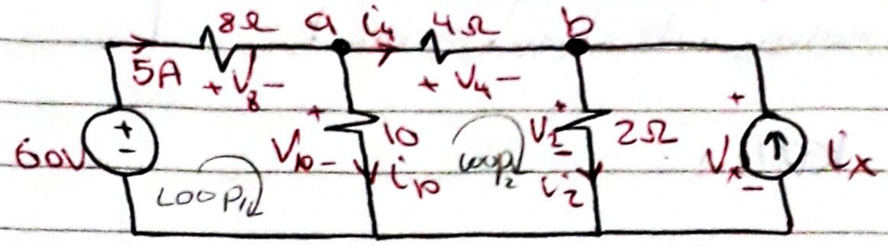
Determine v_x & i_x





KVL at Loop 1 :-

$-60 + V_8 + V_{10} = 0$

$-60 + 5 * 8 + V_{10} = 0$

$-60 + 40 + V_{10} = 0 \Rightarrow V_{10} = 20V$

$i_{10} = \frac{V_{20}}{10} = \frac{20}{10} \Rightarrow i_{10} = 2A$

KCL at node a:-

$5 = 2 + i_4 \Rightarrow i_4 = 3A$

KVL at Loop 2 :-

$-V_{10} + V_4 + V_2 = 0$

$-20 + 4 * 3 + V_2 = 0$

$-20 + 12 + V_2 = 0 \Rightarrow V_2 = 8V = V_x$

KCL at b:-

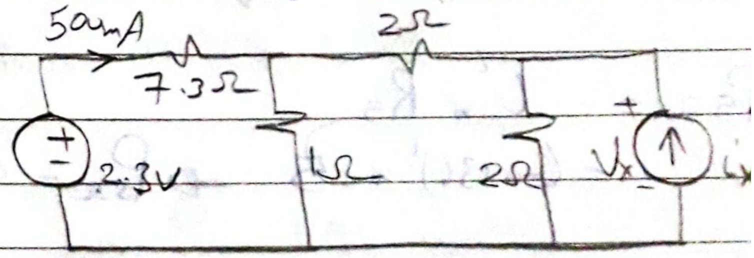
$i_x + i_4 = i_2$

$i_x + 3 = \frac{8}{2}$

$i_x = 4 - 3 \Rightarrow i_x = 1A$

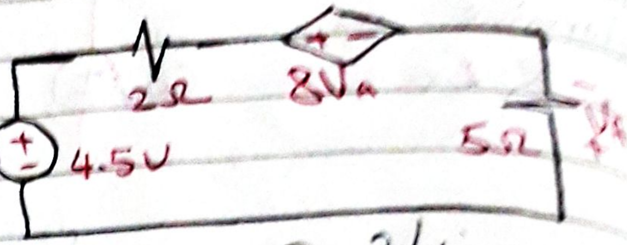
H.W:-

Find V_x ?



The single Loop circuit :-

Hiba Alsamad



Ex:-
Compute the power absorbed by each element of the circuit.

$P = V i$

$V = R i$

$P = i^2 R$

KVL :-

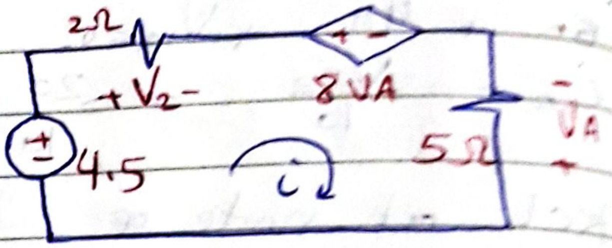
$-4.5 + V_2 + 8VA - VA = 0$

$-4.5 + V_2 + 7VA = 0$

$V_2 = 2i$

$VA = -5i$

* في حالات لدينا Loop واحدة فقط فإنه يجري التيار لعناصر الدائرة الكهربائية بحيث انه متساوي فيهن.



$-4.5 + 2i + 7(-5i) = 0$

$-4.5 + 2i - 35i = 0$

$-4.5 - 33i = 0$

$i = \frac{-4.5}{33} = -0.136\ A$

لم نقرب القيم بعد الفاصلة بدقة بهذا المثال حتى تكون النتيجة اكثر دقيقة.

$P_{4.5} = -(-0.136) * 4.5$

$= 0.612$ Watt

$P_{2R} = i^2 * R_2$

$= (-0.136)^2 * 2 \Rightarrow P_{2\Omega} = 0.0369$ Watt

$P_{8VA} = V * i$

$= 8VA * (-0.136)$

$= 8 * -5i * i$

$= -40i^2 \Rightarrow P_{8VA} = -0.739$ Watt

$P_{5\Omega} = i^2 * R_5$

$= (-0.136)^2 * 5 \Rightarrow P_{5\Omega} = 0.0924$

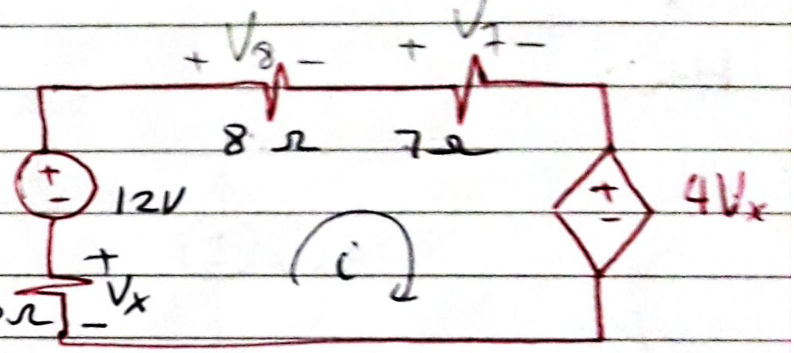
The sum of absorbed power equals the sum of the supplied power

$$\sum_{\substack{\text{all}\\\text{elements}}} P_{absorbed} = 0.612 + 0.0369 - 0.739 + 0.092$$
$$= \text{Zero}$$

$$\therefore \sum P_{absorbed} = \sum P_{supplied}$$

Ex a:-

Find the power absorbed by each of the five elements in the cct.



KVL:-

$$-V_X - 12 + V_8 + V_7 + 4V_X = 0$$

$$V_X = -30i \quad , \quad V_8 = 8i \quad , \quad V_7 = 7i$$

$$-(-30i) - 12 + 8i + 7i + 4(-30i) = 0$$

$$+30i - 12 + 15i - 120i = 0$$
$$-75i = 12$$
$$i = \frac{-12}{75} \Rightarrow i = -0.16\,A$$

$$P_{30\Omega} = i^2 * R_{30\Omega}$$
$$= (-0.16)^2 * 30 \Rightarrow P_{30\Omega} = 0.768 \text{ Watt}$$

$$P_{12V} = V * i$$
$$= -12 * (-0.16) \Rightarrow P_{12V} = 1.92 \text{ Watt}$$

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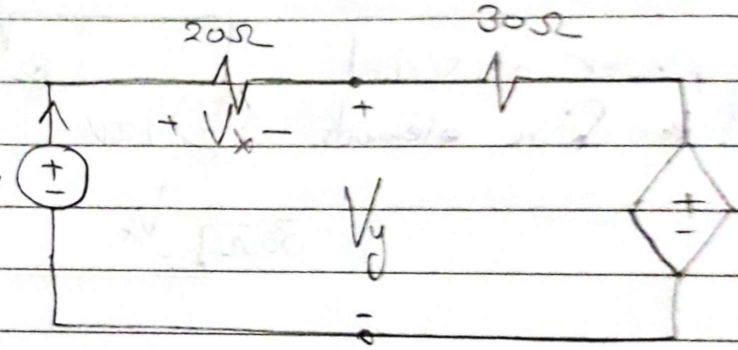
$P_{8\Omega} = i^2 * R_{8\Omega}$
$= (0.16)^2 * 8 \Rightarrow P_{8\Omega} = 0.2048$ Watt

$P_{7\Omega} = (0.16)^2 * 7 \Rightarrow P_{7\Omega} = 0.1792$ watt

$P_{dep.} = (4V_x) * (0.16)$
$= -3.072$ watt

$P_{30} + P_{12} + P_8 + P_7 + P_{dep.} = 0$
$0.768 + 1.92 + 0.2048 + 0.1792 - 3.072 = 0$ ok

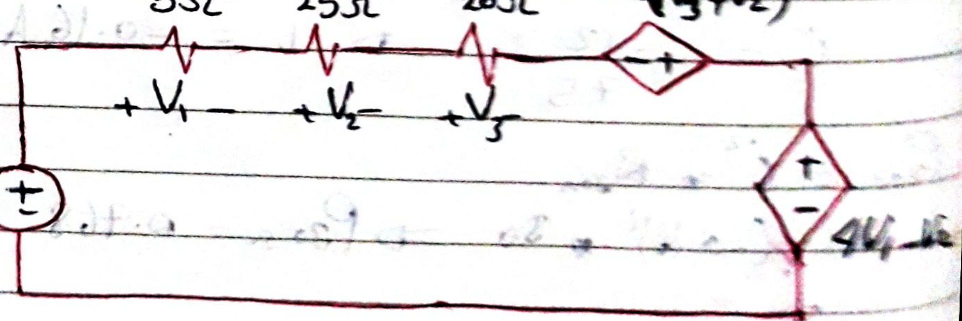
H.W:-



Find i if the dependent Voltage source is:-
a. $2V_x$
b. $1.5V_y$
c. $-25i$

Exa:-
Find the power absorbed by each of six circuit elements

$2(V_3+V_2)$
$4(V_1-V_2)$



The Single-Node-Pair Circuit

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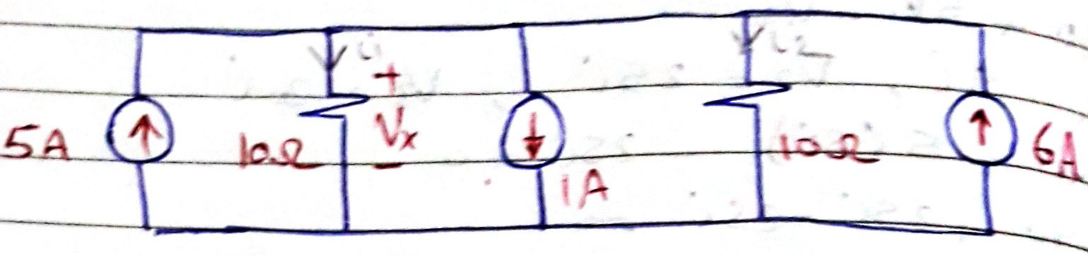
Exa:-

Find the Voltage, current and power associated with each element in the circuit in Fig.



KCL :-

$5 + 6 = i_1 + 1 + i_2$

$10 = i_1 + i_2$, $i_1 = \frac{V_x}{10}$, $i_2 = \frac{V_x}{10}$

$10 = \frac{V_x}{10} + \frac{V_x}{10}$

$10 = \frac{2V_x}{10} \Rightarrow V_x = 50V$

$i_1 = i_2 = 5A$

$P_{5A} = -5 * 50 = -250$ Watt

$P_{1A} = 1 * 50 = 50$ Watt

$P_{6A} = -6 * 50 = -300$ watt

$P_{10\Omega} = i_1^2 * R_{10\Omega}$

$= (5)^2 * 10 = 250$ watt

$P_{10\Omega} = i_2^2 * R_{10\Omega}$

$= (5)^2 * 10 = 250$ watt

$P_{5A} + P_{1A} + P_{6A} + P_{10\Omega} + P_{10\Omega} = 0$

$-\cancel{250} + 50 - 300 + 250 + \cancel{250} = 0$

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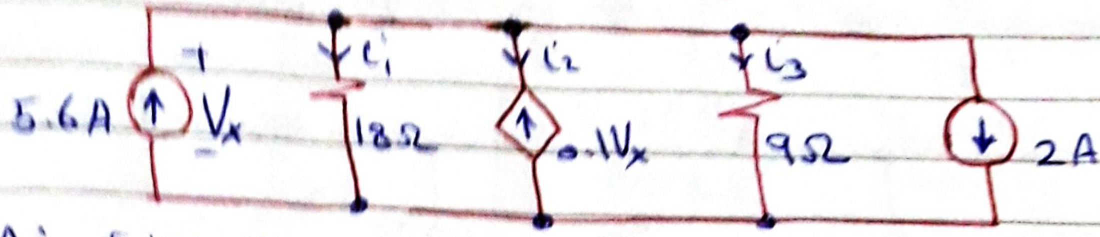
H.W:-

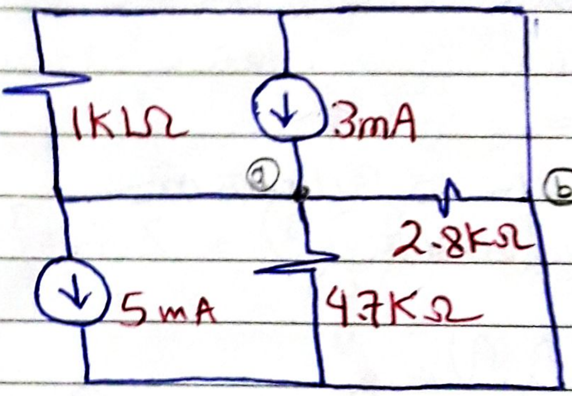
For the single node pair circuit of Fig., find i_1, i_2 & i_3.

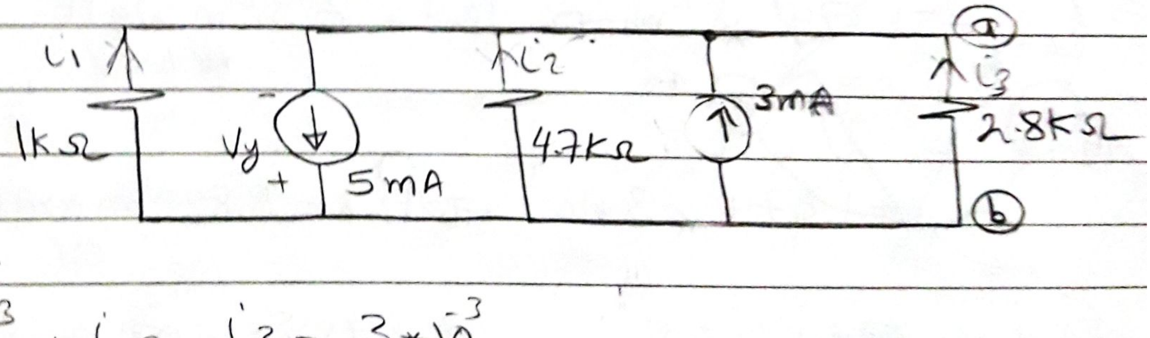


Ans: 3A, −5.4A, 6A.

Ex:-

Determine the power ~~supplied~~ absorbed by each ~~current~~ element ~~source~~ in the single-node-pair circuit & show that the sum is equal to zero





$$i_1 + 5\times10^{-3} + i_2 + i_3 = 3\times10^{-3}$$

$$\frac{V_y}{10^3} + \frac{5}{10^3} + \frac{V_y}{4.7\times10^3} + \frac{V_y}{2.8\times10^3} = \frac{3}{10^3}$$

$$V_y + 3 + 0.212V_y + 0.357V_y = 5$$

$$1.569V_y = 5 - 3$$

$$V_y = 1.274$$

Hiba Alsaeed

$$i_1 = \frac{V_y}{10^3} = \frac{1.274}{10^3} \Rightarrow i_1 = 1.274\ mA$$

$$i_2 = \frac{V_y}{4.7k} = \frac{1.274}{4.7\,10^3} \Rightarrow i_2 = 0.271\ mA$$

$$i_3 = \frac{V_y}{2.8\times10^3} = \frac{1.274}{2.8\times10^3} \Rightarrow i_3 = 0.455\ mA$$

$$P_{1k\Omega} = i_1^2 R$$

$$= (1.274\times10^{-3})^2 * 10^3 \Rightarrow P_{1k} = 1.623\ mwatt$$

$$P_{4.7k} = i_2^2 R$$

$$= (0.271\times10^{-3})^2 * 4.7\times10^3 \Rightarrow P_{4.7k} = 0.345\ m\,watt$$

$$P_{2.8k} = i_3^2 * R$$

$$= (0.455\times10^{-3})^2 * 2.8\times10^3 \Rightarrow P_{2.8k} = 0.579\ mwatt$$

$$P_{5mA} = V * i$$

$$= -1.274 * 5\times10^{-3} \Rightarrow P_{5mA} = -6.37\ m\,watt$$

$$P_{3mA} = V * i$$

$$= +1.274 * 3\times10^{-3} \Rightarrow P_{3mA} = 3.822\ m\,watt$$

$$\Sigma P = zero$$

$$P_{1k\Omega} + P_{4.7k\Omega} + P_{2.8k\Omega} + P_{5mA} + P_{3mA} = 0$$

$$1.623 + 0.345 + 0.574 - 6.37 + 3.822 = 0$$

OK

Nodal Analysis

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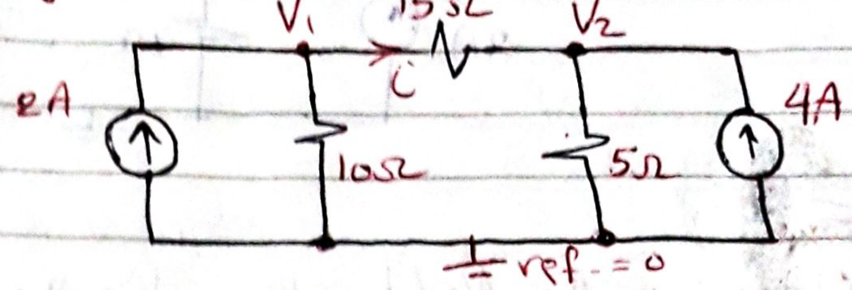
Exa:
Determine the current flowing left to right through 15Ω resistor of Fig.



KCL at node 1:

١. فرض فولتيات العقد.
٢- فرض عقدة الاساس reference node
٣. فرضنا عقدة الاساس = صفر
عدد المعادلات = عدد العقد - ١
* لكون فولتيات احد العقد تساوي صفر.

عقدة الاساس: هي العقدة التي فيها اكثر اتصال بالفروع.

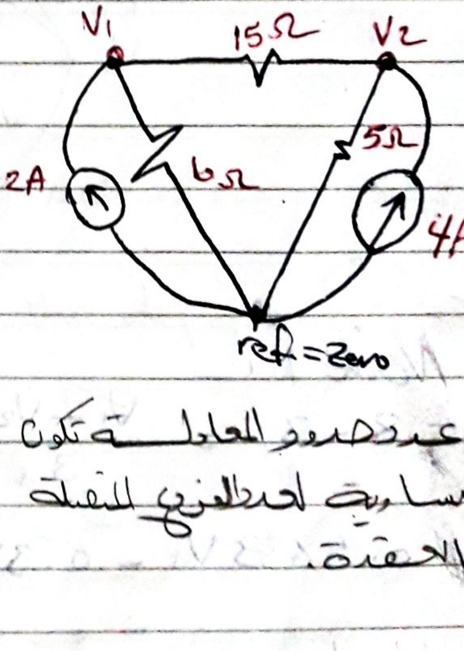
Node 1 :-

$$2 = \frac{V_1}{10} + \frac{V_1 - V_2}{15}$$

$$2 = \frac{3V_1 + 2V_1 - 2V_2}{30}$$

$$5V_1 - 2V_2 = 60 \quad \text{--(1)}$$

Node 2:



$$4 = \frac{V_2}{5} + \frac{V_2 - V_1}{15}$$

$$4 = \frac{3V_2 + V_2 - V_1}{15}$$

$$-V_1 + 4V_2 = 60 \quad \text{--(2)} \ast 5$$

* عدد حدود المعادلة تكون مساوية لعدد الفروع المتصلة بالعقدة.

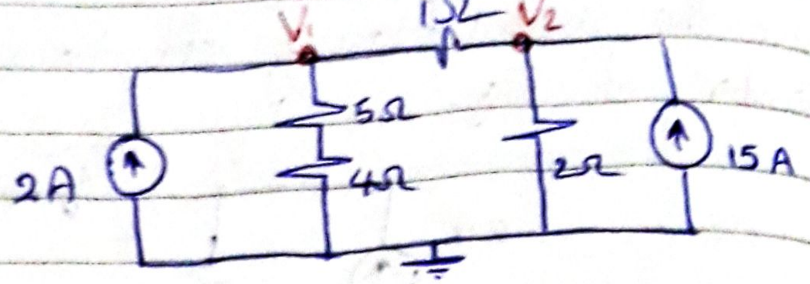
$$5V_1 - 2V_2 = 60 \quad \text{--(1)}$$
$$-5V_1 + 20V_2 = 300 \quad \text{--(2)}$$

in addition

$$18V_2 = 360$$
$$V_2 = 20 \text{ Volt} \quad \text{sub. in eq (1)}$$
$$V_1 = 20 \text{ Volt}$$

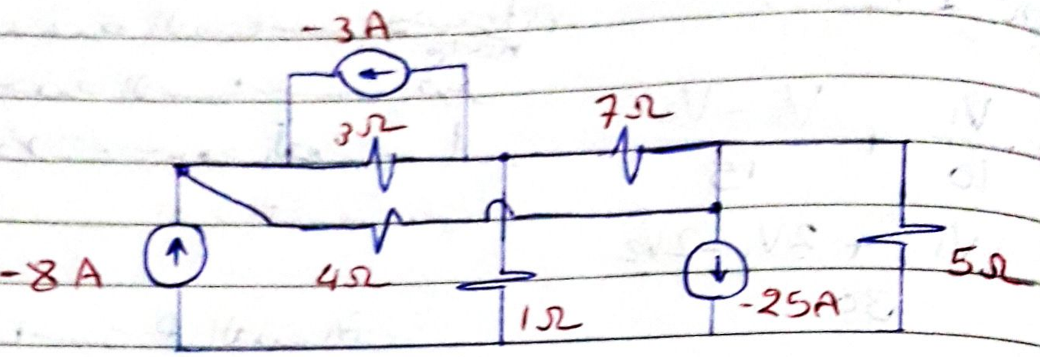
H.W:- Hiba Alsaad

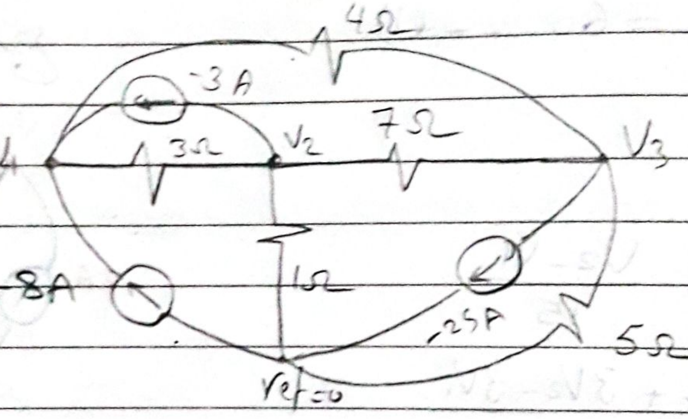
For the circuit of Fig., determine the nodal voltages V_1 & V_2.



Exa:-

Determine the Nodal Voltages for the circuit of fig., as referenced to the bottom node.





Node 1:-

$$-8-3 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4}$$

$$0.5833V_1 - 0.3333V_2 - 0.25V_3 = -11 \quad \text{---(1)}$$

Node 2:

$$0 = \frac{V_2 - V_1}{3} + \frac{V_2 - V_3}{7} + \frac{V_2}{1} + (-3)$$

$$-0.3333V_1 + 1.4762V_2 - 1.429V_3 = 3 \quad \text{---(2)}$$

Node 3:-

$$0 = \frac{V_3}{5} + \frac{V_3 - V_2}{7} + \frac{V_3 - V_1}{4} + (.25)$$

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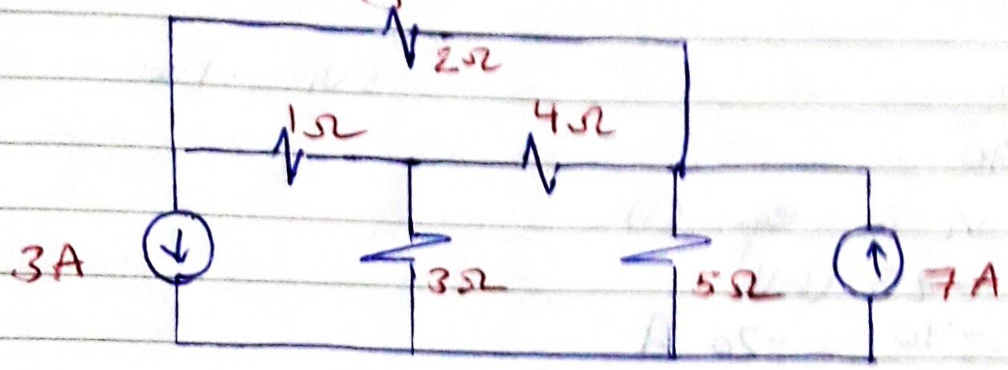
$$-0.25V_1 - 0.1429V_2 + 0.5929V_3 = 25 \quad ---(3)$$

$V_1 = 5.412$ Volt , $V_2 = 7.736$ Volt & $V_3 = 46.32$ Volt

$V_1 = \frac{1.714}{0.3167}$, $V_2 = \frac{2.45}{0.3167}$, $V_3 = \frac{14.67}{0.3167}$

H.W:-
Compute the Voltage across each current Source.



$V_{3A} = 5.235V$, $V_{7A} = 11.47V$

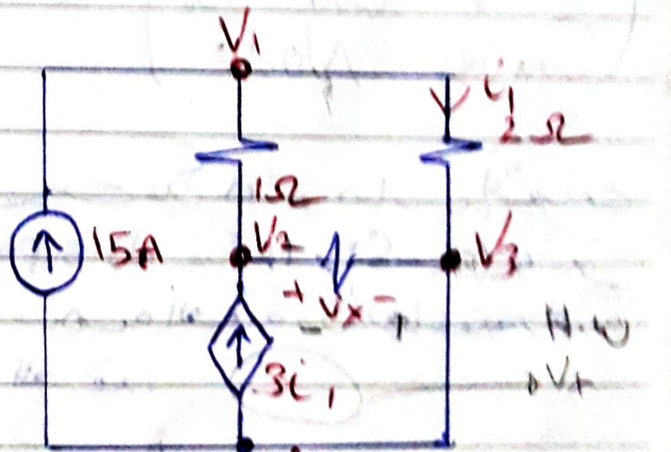
Exa:-
Determine the power supplied by the dependent Source

Node 1:-

$$15 = \frac{V_1 - V_2}{1} + \frac{V_1}{2}$$

$$15 = \frac{2V_1 - 2V_2 + V_1}{2}$$

$$3V_1 - 2V_2 = 30 \quad --- \quad (1) \ *4$$



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Node 2:

$$3i_1 = \frac{V_2 - V_1}{1} + \frac{V_2 - 0}{3} \quad , \quad i_1 = \frac{V_1}{2}$$

$$\frac{3V_1}{2} = \frac{V_2 - V_1}{1} + \frac{V_2}{3}$$

$$\frac{6V_2 - 6V_1 + 2V_2 - 9V_1}{6} = 0$$

$$-15V_1 + 8V_2 = 0 \quad --- (2)$$

$$3V_1 - 2V_2 = 30 \quad --- (1) * 4$$

$$-15V_1 + 8V_2 = 0 \quad --- (2)$$

$$12V_1 - \cancel{8V_2} = 120 \quad --- (1)$$

$$-15V_1 + \cancel{8V_2} = 0 \quad --- (2)$$

$$-3V_1 = 120$$

$V_1 = -40$

Sub. V_1 in eq. (1)

$V_2 = -75$ Volt

$$i_1 = \frac{-40}{2} = -20 \text{ A}$$

$$P_{dep.} = 3i_1 * V_2$$

$$= (3 * -20) * -75$$

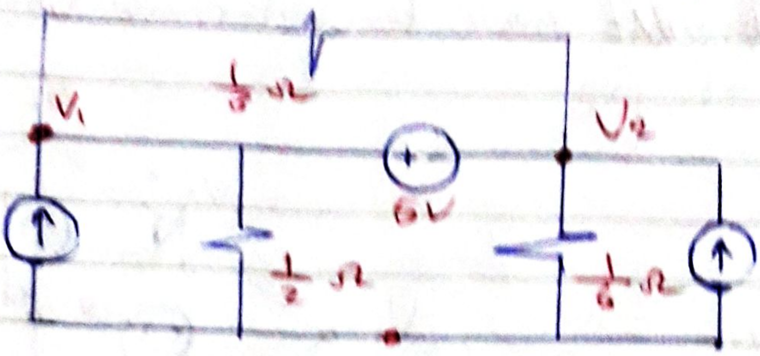
$$= 4500 \text{ watt}$$

Super Node

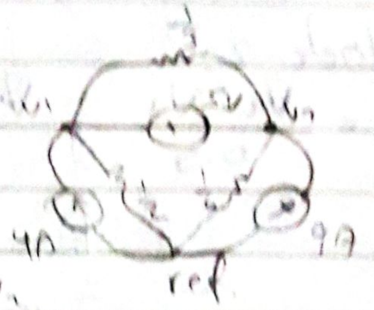
عند وجود مصدر فولتية بين عقدتين يمكن اعتبار هاتين العقدتين عقدة واحدة تسمى super Node (العقدة المتراكبة) ونبين حساب التيارات الداخلة والخارجة من هذه العقدة وكأنها عقدة واحدة دون تغيير الرموز التي اعطيت للعقد.

Hiba Alsaud

Exam
compute the voltage across each current source.



Super Node at Node 1 & 2



$V_1 - V_2 = 5$ ---(1)

$$4 + 9 = \frac{V_1 - V_2}{\frac{1}{3}} + \frac{V_1}{\frac{1}{2}} + \frac{V_2 - V_1}{\frac{1}{3}} + \frac{V_2}{\frac{1}{6}}$$

$$13 = \cancel{3V_1} - \cancel{3V_2} + 2V_1 + \cancel{3V_2} - \cancel{3V_1} + 6V_2$$

$2V_1 + 6V_2 = 13$ --- (2)

$V_1 - V_2 = 5$ -- (1) $\times 6$

$6V_1 - \cancel{6V_2} = 30$ --(1)

$2V_1 + \cancel{6V_2} = 13$ --(2)

in addition

$8V_1 = 43$

$V_1 = 5.375$ Volt, sub. V_1 in eq(1)

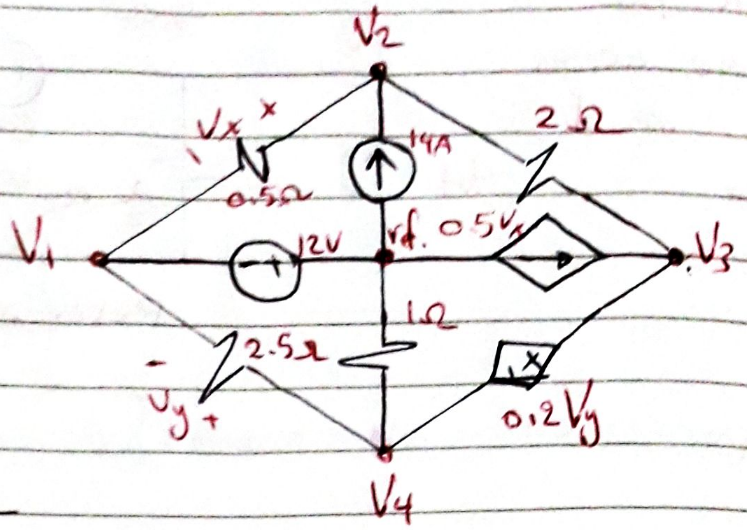
$5.375 - 5 = V_2$

$V_2 = 0.375$ Volt

Exa

Hiba Alsaud

Determine the node to reference voltages in the circuit of Fig.



Node 1:-

$V_1 = -12V$ ... (1)

Node 2:-

$$14 = \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2}$$

$$14 = \frac{4V_2 - 4V_1 + V_2 - V_3}{2}$$

$-4V_1 + 5V_2 - V_3 = 28$ ... (2)

Node 3 & Node 4 , Super node

Node (3 & 4) :-

$V_3 - V_4 = 0.2V_y$ ... (3)

$V_y = V_4 - V_1$ ... (a) , $V_3 - V_4 = 0.2(V_4 - V_1)$

$V_x = V_2 - V_1$ ... (b) $V_3 - V_4 - 0.2V_4 + 0.2V_1 = 0$ - (3) $\times 10$

Super Node at node (1) & node (2)

$$0.5V_x = \frac{V_3 - V_2}{2} + \frac{V_4 - 0}{1} + \frac{V_4 - V_1}{2.5} \Rightarrow \frac{5}{2}$$

$$\frac{1}{2}(V_2 - V_1) = \frac{5V_3 - 5V_2 + 10V_4 + 4V_4 - 4V_1}{10}$$

$5V_2 - 5V_1 = 5V_3 - 5V_2 + 10V_4 + 4V_4 - 4V_1$

$-V_1 + 10V_2 - 5V_3 - 14V_4 = 0$ ... (4)

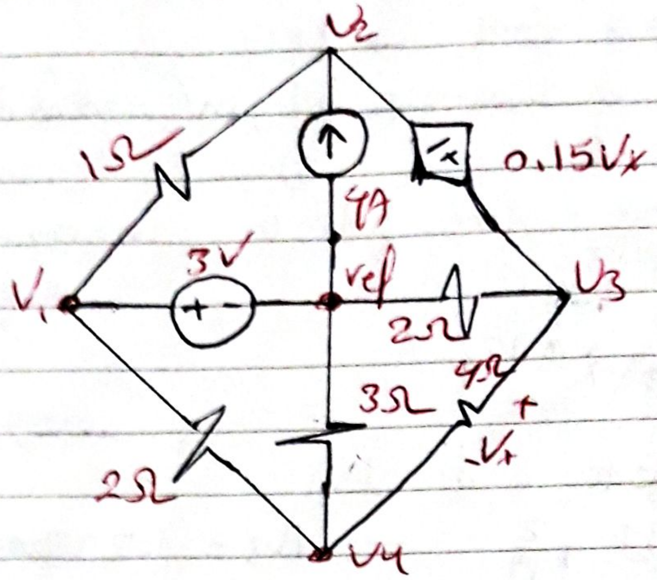
$2V_1 + V_3 - 12V_4 = 0$ ... (3)

$-4V_1 + 5V_2 - V_3 = 28$ ... (2)

Hiba Alsawaf

$V_1 = 12$ Volt , $V_2 = -4$ Volt , $V_3 = 0$, $V_4 = -2V$

H.W:—

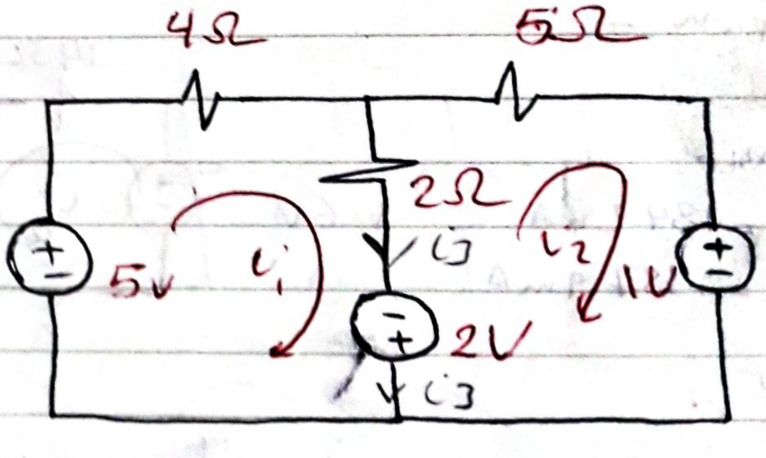


Ans:
$V_1 = 3V$
$V_2 = -2.33V$
$V_3 = -1.91V$
$V_4 = 0.945V$

Mesh Analysis:—

Exa:—

Find i_1, i_2 and then find the power supplied by 2V Source.



Mesh 1:-
$-5 + 4i_1 + 2(i_1 - i_2) - 2 = 0$
$6i_1 - 2i_2 = 7$ ——— (1)

Mesh 2:-
$2 + 2(i_2 - i_1) + 5i_2 + 1 = 0$
$-2i_1 + 7i_2 = -3$ ——— (2)

Hiba Alsaraf

$6i_1 - 2i_2 = 7$ —— (1)

$-2i_1 + 7i_2 = -3$ —— (2) $] \times 3$

$\cancel{6i_1} - 2i_2 = 7$ —— (1)

$-\cancel{6i_1} + 21i_2 = -9$ —— (2)

in addition

$19i_2 = -2$

$i_2 = \frac{-2}{19}$ A , sub. i_2 in equ. (1)

$i_1 = \frac{43}{38} = 1.132$ A

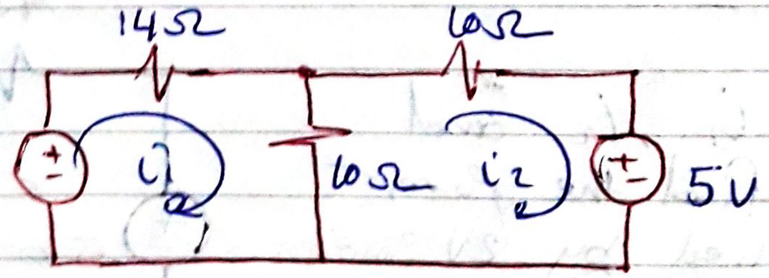
$i_3 = i_1 - i_2$

$= 1.132 + \frac{2}{19}$ $\Rightarrow$ $i_3 = 1.237$A

$P_{2V} = -(2 \times -1.237)$

$= +2.47$ watt

H.w:-



Ans:-

$i_1 = 184.2$ mA

$i_2 = -157.9$ mA

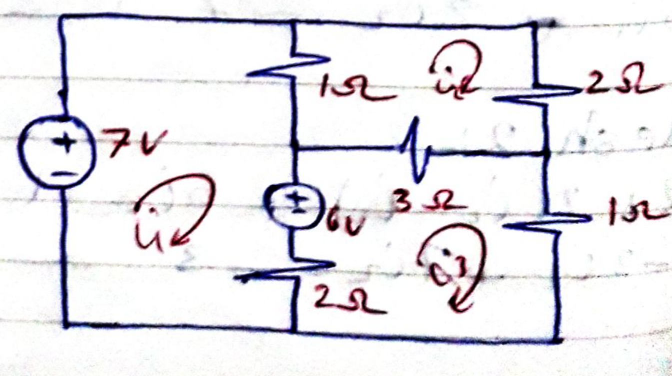
Exe:-

Use mesh analysis to determine the three mesh currents in the cct. of fig.

Mesh (1)

$-7 + 1(i_1 - i_2) + 6 + 2(i_1 - i_3) = 0$

$3i_1 - i_2 + 2i_3 = 1$ —— (1)



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Mesh (2) :-

$1(i_2-i_1)+2i_2+3(i_2-i_3)=0$

$-i_1+6i_2-3i_3=0$ ---- (2)

Mesh (3) :-

$2(i_3-i_1)-6+3(i_3-i_2)+1*i_3=0$

$-2i_1-3i_2+6i_3=6$ ---- (3)

$$i_1=\frac{\begin{vmatrix}1 & -1 & -2 & 1 & -1\\ 0 & 6 & -3 & 0 & 6\\ 6 & -3 & 6 & 6 & -3\end{vmatrix}}{\begin{vmatrix}3 & -1 & -2 & 3 & -1\\ -1 & 6 & -2 & -1 & 6\\ -2 & -3 & 6 & -2 & -3\end{vmatrix}}$$

$i_1=\frac{117}{39}$ = 3 Ampere

$i_2=\frac{78}{39}$ = 2 Ampere

$i_3=\frac{117}{39}$ = 3 Ampere

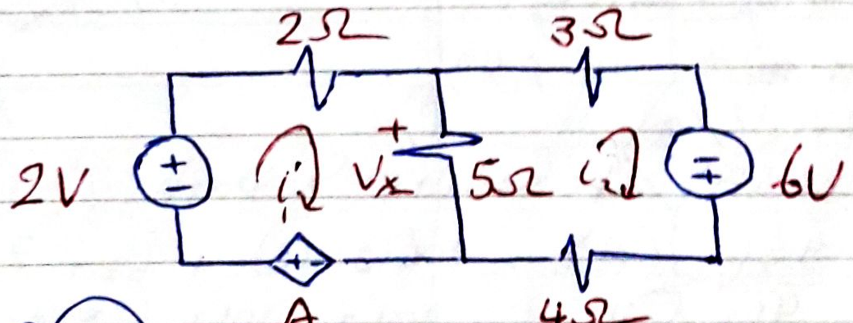
H.W:-

Determine i_1 in the cct. of fig. if the controlling quantity A is equal to (a) $2i_2$, (b) $2V_x$ -

Ans:-

(a) 1.35 A

(b) 546 mA



حالات خاصة لتطبيق التحليل الشبكي (Mesh Analysis

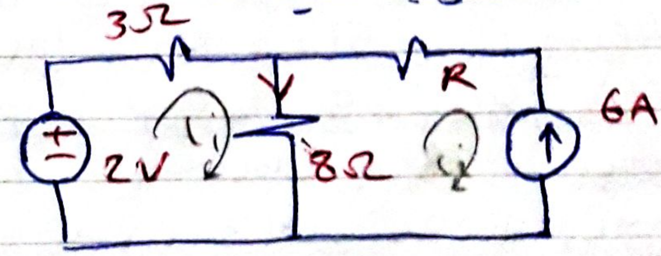
1- اذا وجد مصدر تيار وكان على فرع خارجي من فروع الدائرة الكهربائية .

Mesh (1)

$-2+3i_1+8(i_1-i_2)=0$ ---- (1)

Mesh (2)

$i_2=-6A$

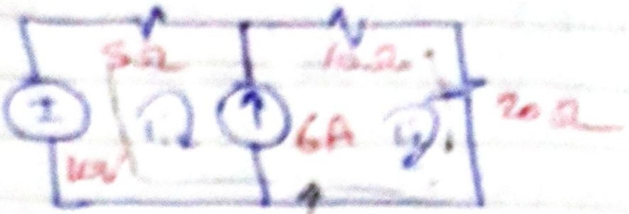


Hiba Ahmed

ع اذا وجد مصدر تيار مشترك بين حلقتين

mesh(1) & mesh(2)

$i_2 - i_1 = 6A$ —(1)



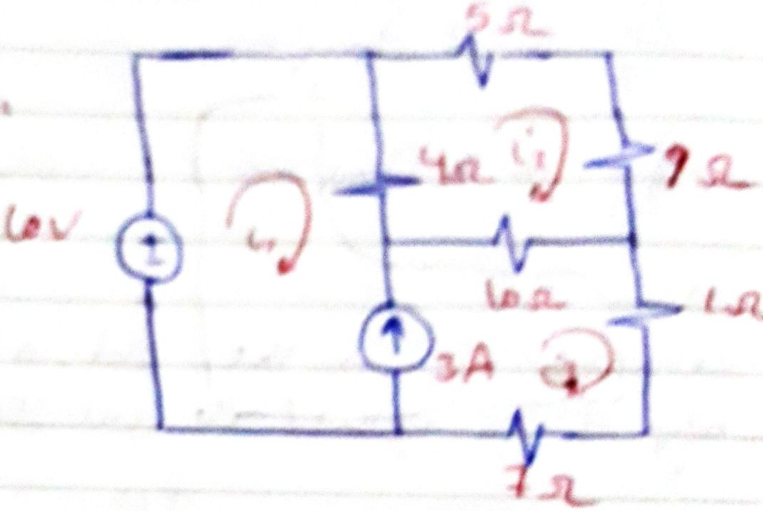
Super mesh at (1) & (2)

$-10 + 5i_1 + 10i_2 + 20i_2 = 0$

super mesh ([illegible])

The Super Mesh

Exa:-
Determine the current i_1 in Fig.



Mesh (1) & Mesh (2)

$i_3 - i_1 = 3$ —(1)

$i_3 = 3 + i_1$

Super mesh at mesh(1) & mesh(2)

$-10 + 4(i_1 - i_2) + 10(i_3 - i_2) + i_3 + 7i_3 = 0$

$-10 + 4i_1 - 4i_2 + 10i_3 - 10i_2 + 8i_3 = 0$

$4i_1 - 14i_2 + 18i_3 = 10$ —(2)

$4i_1 - 14i_2 + 18(3 + i_1) = 10$

$22i_1 - 14i_2 = -54$ —(2)

Mesh (2):-

$4(i_2 - i_1) + 5i_2 + 9i_2 + 10(i_2 - i_3) = 0$

$4i_2 - 4i_1 + 5i_2 + 9i_2 + 10i_2 - 10i_3 = 0$

$-4i_1 + 28i_2 - 10i_3 = 0$

$-4i_1 + 28i_2 - 10(3 + i_1) = 0$

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$-14 i_1 + 28 i_2 = 30$ ---- (3)

$22 i_1 - 14 i_2 = -54$ ---- (1)
$-14 i_1 + 28 i_2 = 30$ ---- (3) $] \div 2$

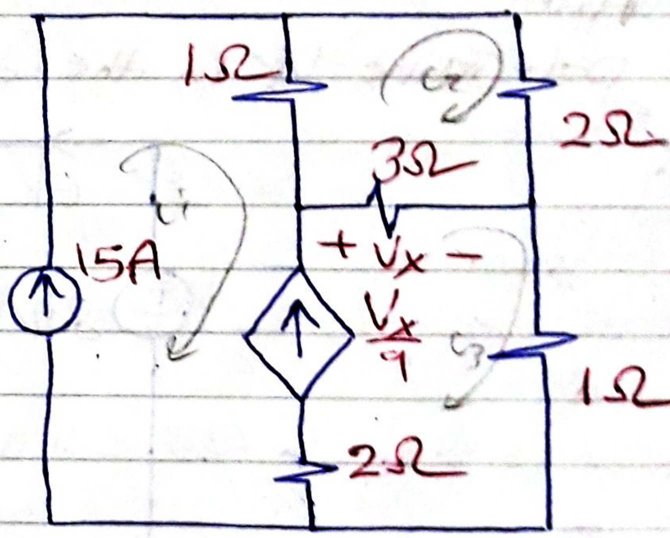
$22 i_1 - \cancel{14 i_2} = -54$
$-7 i_1 + \cancel{14 i_2} = -15$

in addition

$15 i_1 = -69$
$i_1 = -4.6$ A

Exa -
Use Mesh Analysis to find the three currents in the $\overline{cct}$.

Mesh (1):-
$i_1 = 15$ ---- (1)



Mesh (1) & Mesh (2)
$i_3 - i_1 = \frac{V_x}{9}$

$V_x = 3(i_3 - i_2)$

$i_3 - i_1 = \frac{\cancel{3}(i_3 - i_2)}{\cancel{9}_3}$

$3i_3 - 3i_1 - i_3 + i_2 = 0$
$-3 i_1 + i_2 + 2 i_3 = 0$ ---- (2)   sub. i_1 in (2)
$-3 \times 15 + i_2 + 2 i_3 = 0$ ---- (2)
$i_2 + 2 i_3 = 45$ ---- (2)

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Mesh (2):-

$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 6$

$6i_2 - 3i_3 = 15$ —— (3)

$i_2 + 2i_3 = 45$ —— (2)] * 6

$6i_2 - 3i_3 = 15$ —— (3)

$\cancel{6i_2} + 12i_3 = 270$ —— (2)

$\mp \cancel{6i_2} \pm 3i_3 = \mp 15$ —— (3)

in subtraction

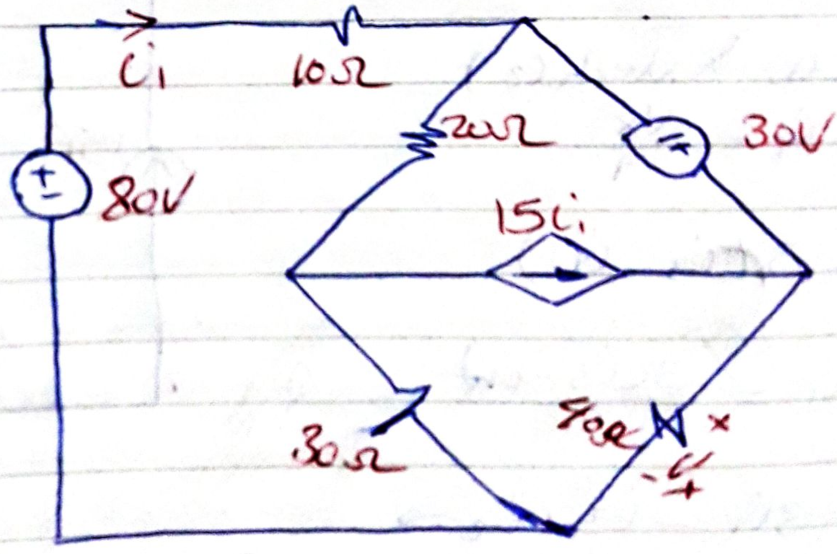
$15 i_3 = 255$

$i_3 = 17A$

$i_1 = 15A$, $i_2 = 11A$

H.W:-

Determine V_x in the cct of Fig.



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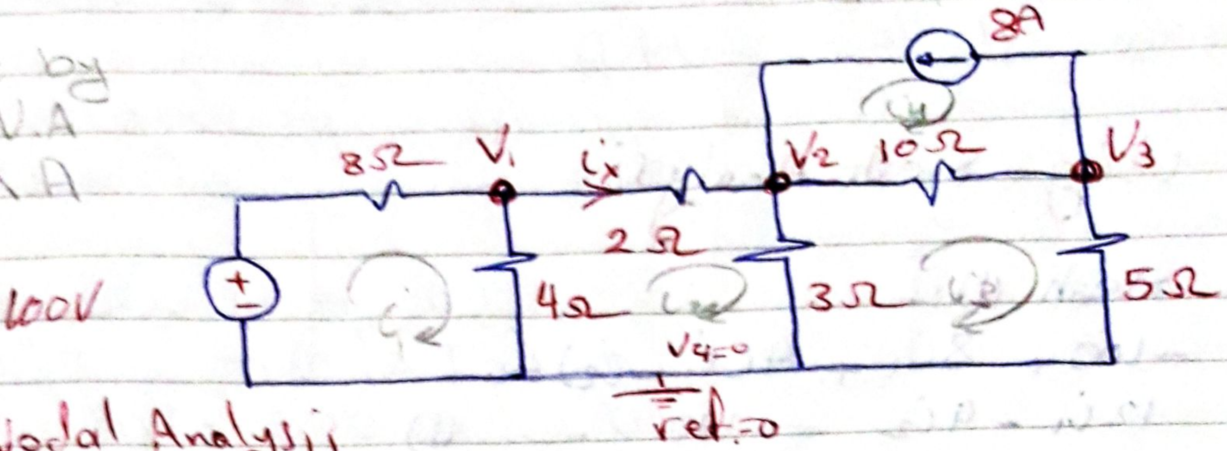
Nodal Vs. Mesh Analysis: A comparison

find i_x by using N.A and M.A



Using Nodal Analysis

Node 1:

$$\frac{V_1 - 100}{8} + \frac{V_1 - V_2}{2} + \frac{V_1}{4} = 0$$

$$\frac{V_1 - 100 + 4V_1 - 4V_2 + 2V_1}{8} = 0$$

$$7V_1 - 4V_2 = 100 \quad --- \quad (1)$$

Node 2:

$$8 = \frac{V_2 - V_1}{2} + \frac{V_2}{3} + \frac{V_2 - V_3}{10}$$

$$8 = \frac{15V_2 - 15V_1 + 10V_2 + 3V_2 - 3V_3}{30}$$

$$-15V_1 + 28V_2 - 3V_3 = 240 \quad — \quad (2)$$

Node 3:

$$\frac{V_3 - V_2}{10} + \frac{V_3}{5} + 8 = 0$$

$$\frac{V_3 - V_2 + 2V_3}{10} = -8$$

$$-V_2 - 3V_3 = -80 \quad , \quad V_2 = 80 - 3V_3 \quad — (3)$$

Sub. V_2 in eqns. (1) & (2)

$V_1 = 25.89$, $V_2 = 20.31$ Hiba Alsamad

$i_x = \dfrac{V_1 - V_2}{2} = 2.79\ A$

Using Mesh Analysis

Mesh (1):-

$-100 + 8i_1 + 4(i_1 - i_2) = 0$

$12i_1 - 4i_2 = 100 \quad \text{---} \quad (1)$

Mesh (2):-

$4(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$

$-4i_1 + 9i_2 - 3i_3 = 0 \quad \text{---} \quad (2)$

Mesh (3):-

$3(i_3 - i_2) + 10(i_3 - \overset{-8}{i_4}) + 5i_3 = 0$

$-3i_2 + 18i_3 = -80 \quad \text{---} \quad (3)$

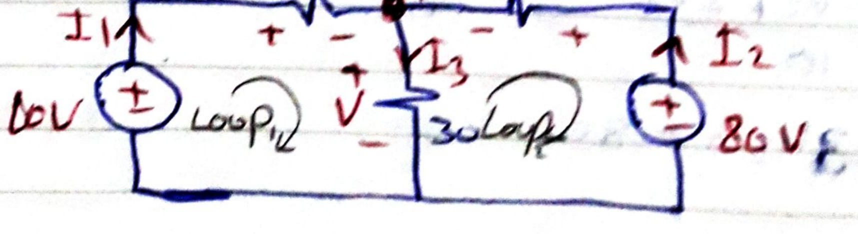
Mesh (4):-

$i_4 = -8 \quad \text{---} \quad (4)$

$\therefore i_x = i_2 = 2.79\ A$

Ex:-

Use Kirchhoff's Law to find the power by the Voltage sources.



KVL at Loop1 :-

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$-10 + 40I_1 + 30I_3 = 0$ ---- (1)

KCL at node A:-

$I_1 + I_2 = I_3$ ---- (2)

sub. eq. (2) in eq. (1)

$-10 + 40I_1 + 30(I_1 + I_2) = 0$

$70I_1 + 30I_2 = 10$ ---- (1)

KVL at Loop 2:-

$-30I_3 - 10I_2 + 80 = 0$ ---- (3)

sub. eq. (2) in eq. (3):-

$-30(I_1 + I_2) - 10I_2 + 80 = 0$

$-30I_1 - 40I_2 = -80$

$30I_1 + 40I_2 = 80$ ---- (3)

$70I_1 + 30I_2 = 10] * 40$

$30I_1 + 40I_2 = 80] * 30$

$2800I_1 + 1200I_2 = 400$ ---- (1)

$\mp 900I_1 \mp 1200I_2 = \mp 2400$ ---- (3)

بالطرح

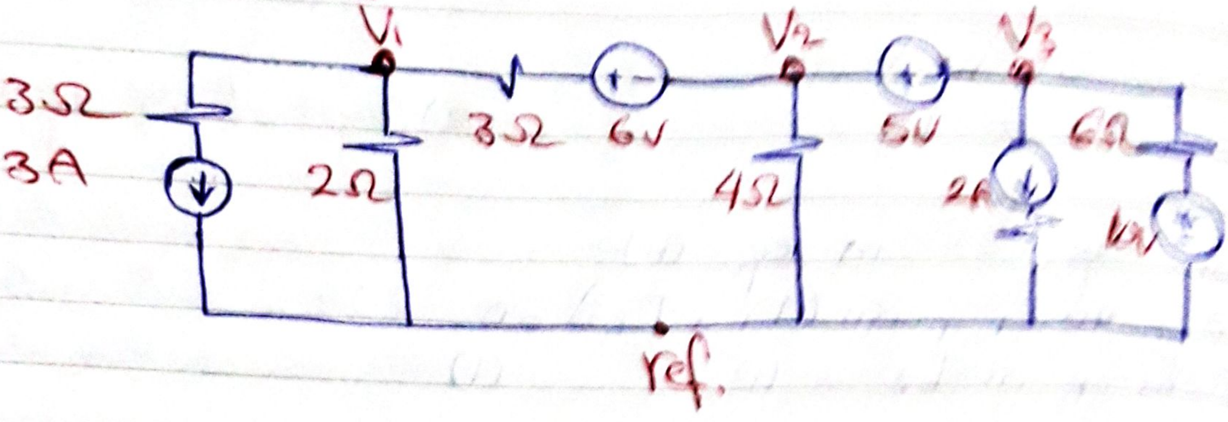
$1900I_1 = 2000 \Rightarrow I_1 = \frac{-20}{19}$ A

sub. I_1 in eq (1)

$I_2 = \frac{53}{19}$ A

Exa:-

Use Nodal Analysis to find the Power at 2Ω



Node 1:

$$0 = 3 + \frac{V_1}{2} + \frac{V_1 - V_2 - 6}{3}$$

$$0 = 3 + \frac{3V_1 + 2V_1 - 2V_2 - 12}{6}$$

$$5V_1 - 2V_2 = -18 + 12$$

$$5V_1 - 2V_2 = -6 \quad --- (1)$$

Node 2 & Node 3:-

$$V_2 - V_3 = 5 \;, V_3 = \quad --- (2)$$

SuperNode at Nodes (2 & 3):-

$$0 = \frac{V_2 - V_1 - (-6)}{3} + \frac{V_2}{4} + 2 + \frac{V_3 - 10}{6}$$

$$-2 = \frac{4V_2 - 4V_1 + 24 + 3V_2 + 2V_3 - 20}{12}$$

$$-4V_1 + 7V_2 + 2V_3 = -24 - 4$$

$$-4V_1 + 7V_2 + 2V_3 = -28 \quad --- (3)$$

$$V_2 = -3.08$$

$$V_1 = -4.7$$

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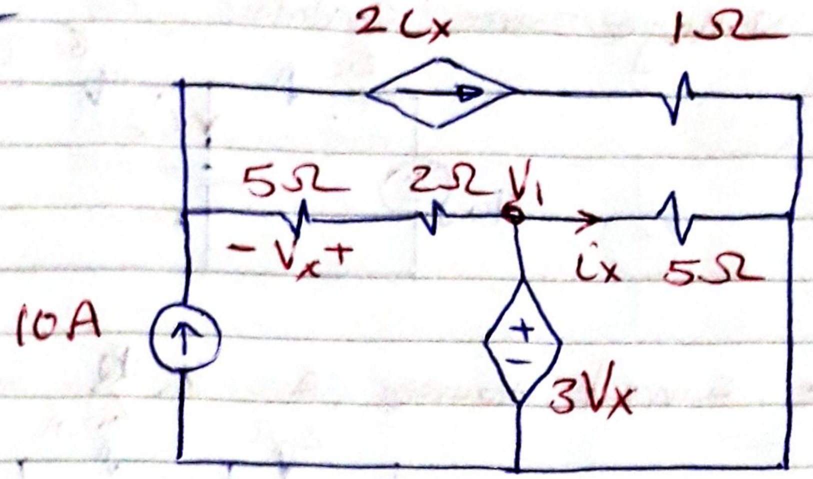
H.W:-



Determine V_1 by using Nodal Analysis.

(Linearity & Superposition)

In any linear circuit, the current passing in any branch is the algebric sum of the currents passing in the same branch from each source independently.

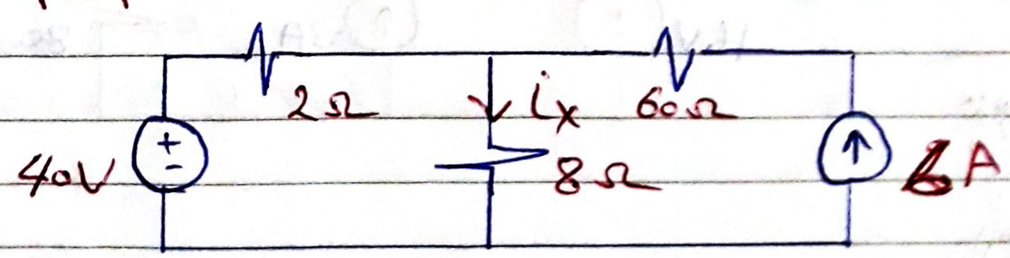
في الدارة الكهربائية التي تحتوي على اكثر من مصدر سواء كان فولتية او تيار،

تنص ان التيار المار في اي فرع من فروع الشبكة الكهربائية التي تحتوي على اكثر من مصدر غير معتمد هو عبارة عن المجموع الجبري للتيارات المارة في نفس الفرع عندما نأخذ تأثير كل مصدر على انفراد.

* عدد الدوائر = عدد المصادر غير المعتمدة عند تطبيق نظرية التراكب

Exa:-

Use superposition to determine the ix.

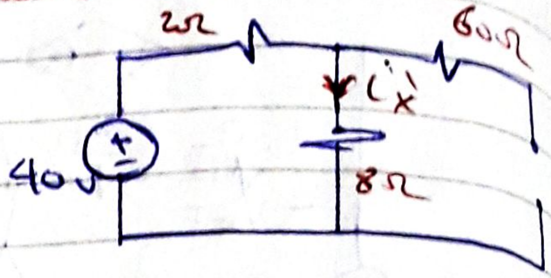


Hiba Alsamaf

* effect of the voltage source 40 Volt:

$$i_x' = \frac{40}{2+8}$$

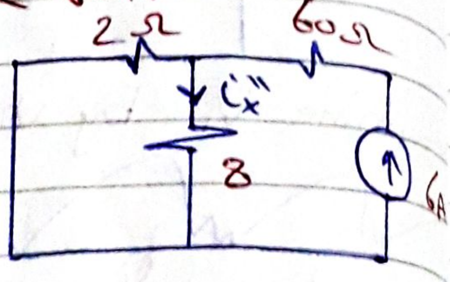
$$= 4\,A \downarrow$$



* effect of the current source 6A:-

$$i_x'' = 6 \times \frac{2}{2+8}$$

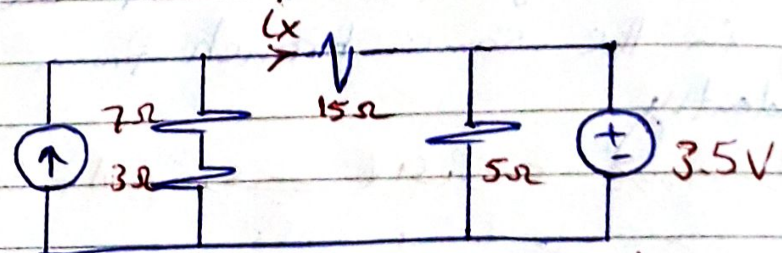
$$= 1.2\,A \downarrow$$



$i_x = 4 + 1.2 \Rightarrow 5.2\,A$

H.W:

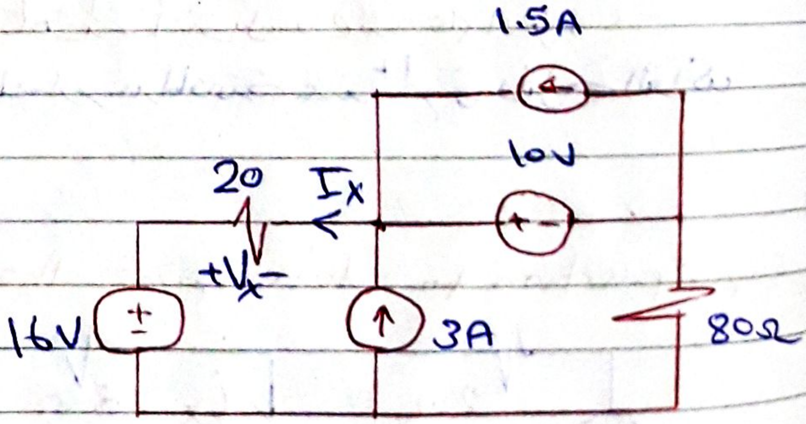
Ans:- 660mA



Use superposition to compute the current i_x.

Exa:-

Find V_x by Using superposition Theorem



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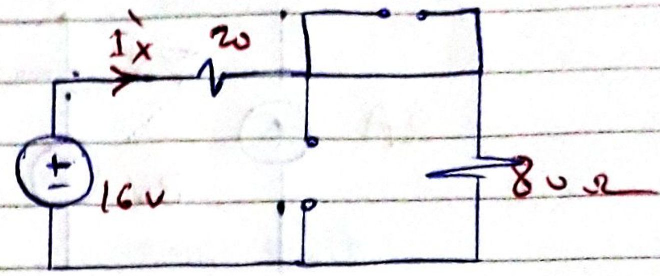
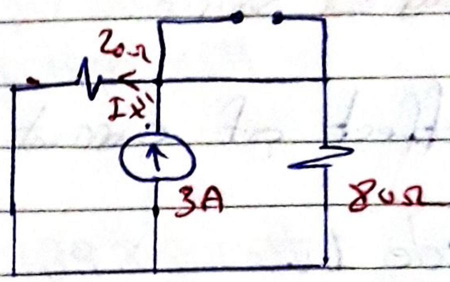
* effect of voltage source 16V:-

$$I_x' = \frac{16}{20+80} = 0.16A$$



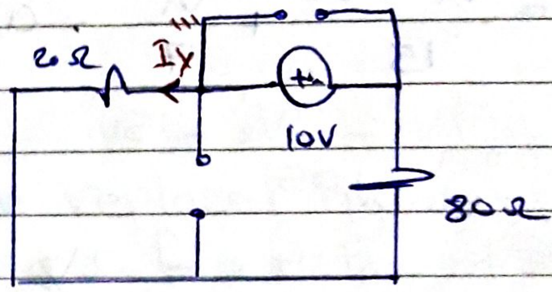
* effect of current source 3A:-

$$I_x'' = 3 * \frac{80}{20+80}$$
$$= 2.4 A$$



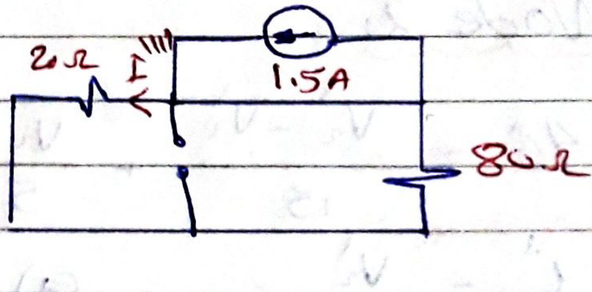
* effect of voltage source 10V:-

$$I_x''' = \frac{10}{20+80}$$
$$= 0.1 A$$



* effect of current source 1.5A:-

$$I'''' = \text{Zero}$$

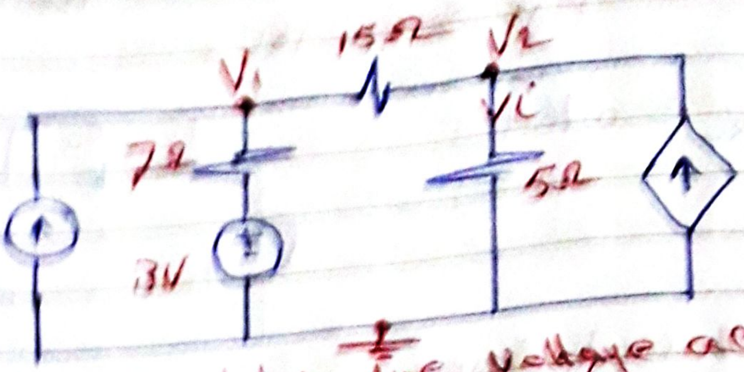


$$I_x = -0.16 + 2.4 + 0.1$$
$$= 2.34 A$$

$$V_x = -2.34 * 20$$
$$= -46.8 \text{ Volt}$$

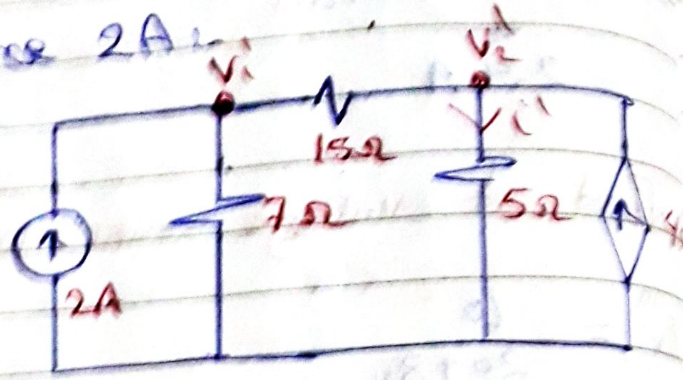
Hiba Alsaoud

Ex:



Use superposition to obtain the voltage across each current source.

* effect of current source 2A:



Node 1:

$$2 = \frac{V_1' - V_2'}{15} + \frac{V_1'}{7} \quad \text{---(1)}$$

$$2 = \frac{7V_1' - 7V_2' + 15V_1'}{105}$$

$$22V_1' - 7V_2' = 210 \quad \text{---(1)}$$

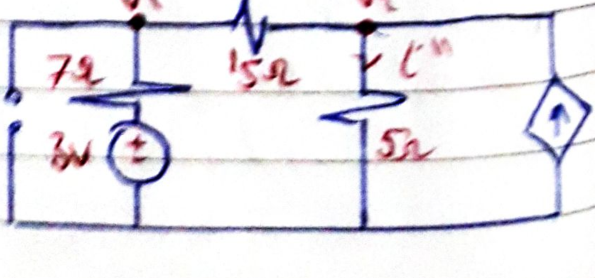
Node 2:

$$4i' = \frac{V_2' - V_1'}{15} + \frac{V_2'}{5} \quad \text{---(2)}$$

$$i' = \frac{V_2'}{5} \quad \text{---(a)}$$

$V_1' = 9.18$ Volt , $V_2' = -1.1475$ volt

* effect of the voltage source 3V:



Node 1:

$$0 = \frac{V_1'' - 3}{7} + \frac{V_1'' - V_2''}{15} \quad \text{---(1)}$$

Node 2:

$$4i'' = \frac{V_1'' - V_2''}{15} + \frac{V_2''}{5} \quad --- (a)$$

$$i'' = \frac{V_2''}{5} \quad --- (b)$$

$V_1'' = 1.968$ Volt , $V_2'' = -0.246$ Volt

$V_1 = V_1' + V_1''$

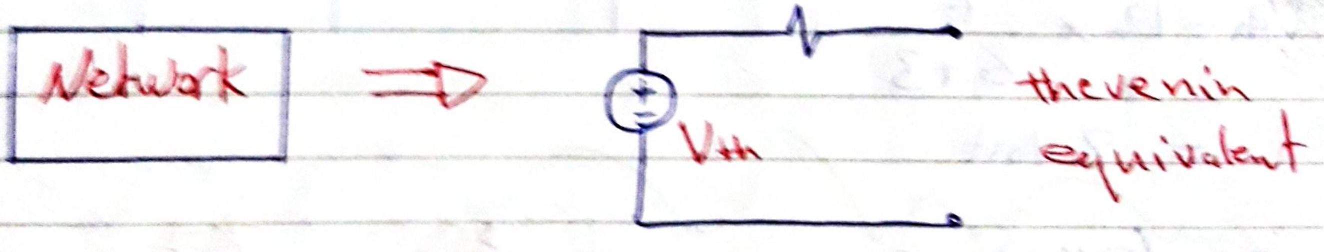
$= 9.18 + 1.968 \Rightarrow V_1 = 11.148$ volt

$V_2 = V_2' + V_2''$

$= -1.1475 - 0.246 \Rightarrow V_2 = -13935$ Volt

Thevenin's Theorem

Any electrical circuit can be replaced by an equivalent circuit, consisting of a voltage source and a series resistor.



Vth : الفولتية التي تظهر عبر الفرع المطلوب إيجاد التيار فيه بعد رفع المقاومة من الدائرة

Rth : المقاومة المكافئة التي تظهر عبر نفس الجزء المفتوح بعد التعويض عن كل مصدر فولتية بـ (short cct) والتيار بـ (open cct)

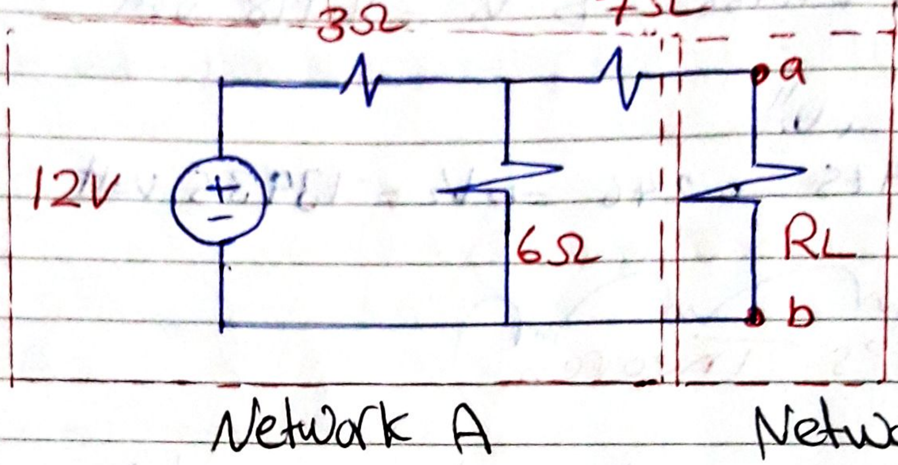
Thevenin's Theorem حالات تطبيق

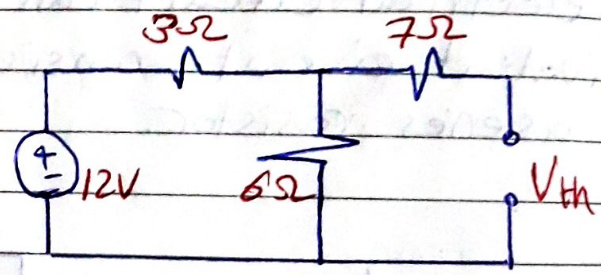
Hiba Alkawi

1. Independent Sources & Resistors

Exa:

Determine the Thevenin equivalent at (a_b), and compute the power delivered to the Load resistor R_L

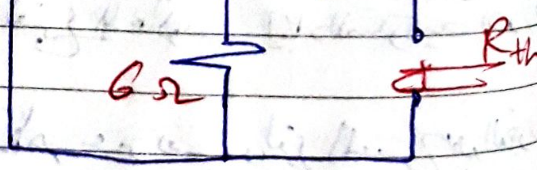




$V_{th} = V_{6\Omega} = V_{o.c}$

$V_{th} = 12 * \dfrac{6}{6+3}$

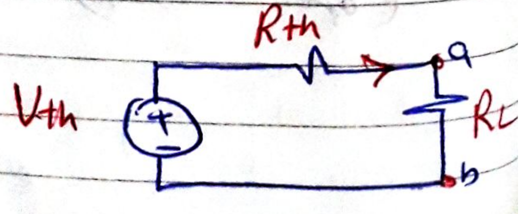
$= 8$ Volt



$R_{th} = (3 // 6) + 7$

$= \dfrac{3*6}{3+6} + 7$

$= 9\Omega$



$I = \dfrac{V_{th}}{R_{th} + R_L}$

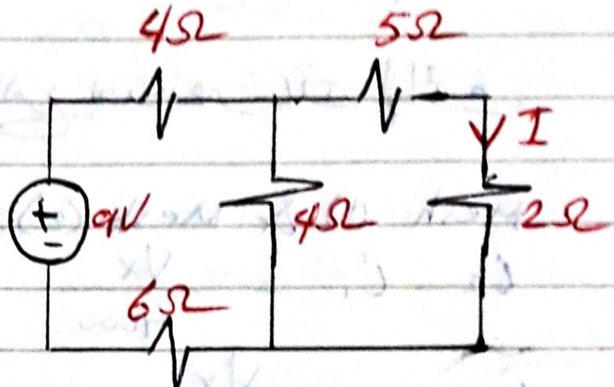
Hiba Alsawaf

If $R_L = 10$

$$I = \frac{8}{9+10} \Rightarrow I = 0.42\,A$$

$$P_L = I^2 * R_L$$
$$= (0.42)^2 * 10 \Rightarrow P_L = 1.764\ \text{Watt}$$

H.W1:-

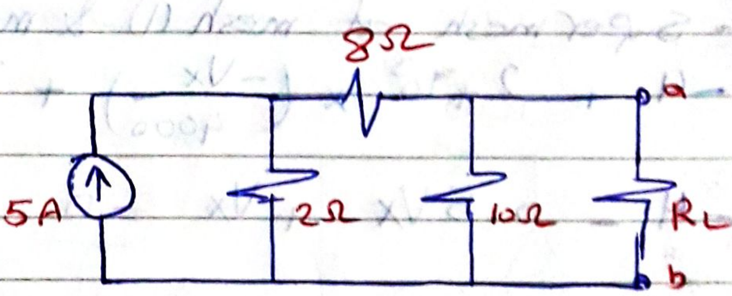


Use Thevenin's theorem to find the current through the 2Ω resistor in the cct.

Ans. $V_{th} = 2.571$, $R_{th} = 7.857$

$I = 0.2608\,A$

HoW2:-



Find the thevenin equivalent of the circuit.

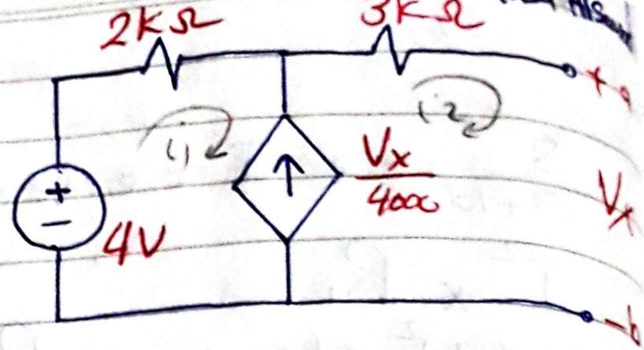
Ans. $V_{th} = 5\,\text{Volt}$, $R_{th} = 5\,\Omega$

2. Dependent Sources & Independent sources & Resistors

Exa:-

Determine the Thevenin equivalent of the circuit in Fig.

Hiba Alsa



* في حالة احتواء الدارة الكهربائية على مصدرين معتمد وغير معتمد فانه لا يمكن الغاء احد المصادر.
* اذا كانت الدارة الكهربائية فيها مصادر معتمدة وغير معتمدة فانه يتم ايجاد Rth من خلال القانون

$$V_{th} = V_{o.c}$$

$$R_{th} = \frac{V_{th}}{I_{s.c}}$$

$I_{s.c}$: التيار المار في نفس الفرع بعد عمل قصر للدارة.

Mesh (1) & Mesh (2):—

$$i_2 - i_1 = \frac{V_x}{4000} \quad , \quad i_2 = 0$$

$$i_1 = -\frac{V_x}{4000}$$

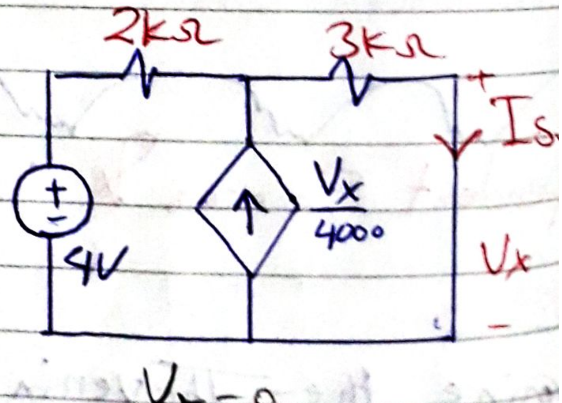
supermesh at mesh (1) & mesh (2)

$$-4 + 2*10^3 * \left(\frac{-V_x}{4000}\right) + 3*10^3 (0) + V_x = 0$$

$$-4 - 0.5 V_x + V_x = 0$$

$$V_x = 8 \text{ Volt} = V_{th}$$

$I_{s.c}$?



$$-4 + 2*10^3 I_{s.c} + 3*10^3 I_{s.c} = 0$$

$V_x = 0$

يعني التيار المعتمد اصبح opened

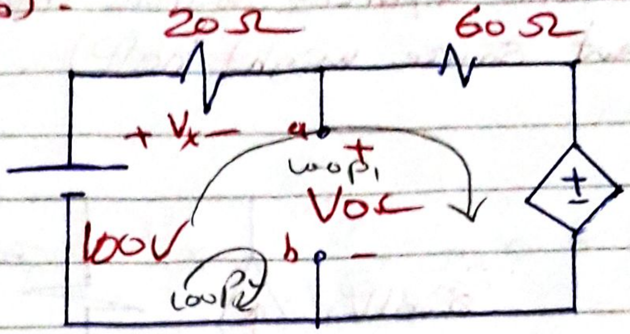
$$I_{s.c} = \frac{4}{5*10^3}$$

$$= 0.8 \text{ mA}$$

$$R_{th} = \frac{V_{o.c}}{I_{s.c}} = \frac{8}{0.8*10^{-3}} = 10k\Omega$$

Exa[illegible] Hiba Alsawaf

For the cct. shown in Fig., find the Thevenin equivalent at (a-b).



$$-100 + V_x + 60i + 6V_x = 0$$

$$-100 + 7V_x + 60i = 0 \quad , V_x = 20i$$

$$-100 + 7(20i) + 60i = 0$$

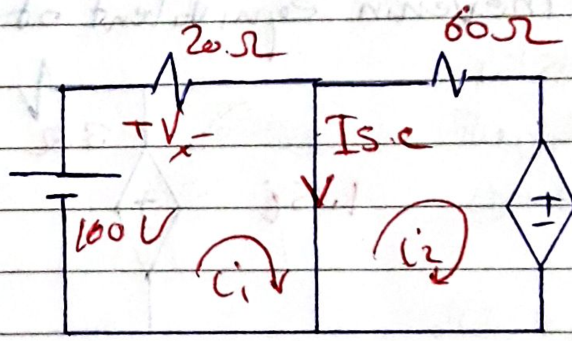
$$200i = 100$$

$$i = \frac{100}{200} \Rightarrow i = 0.5\ A$$

At Loop 2:-

$$-100 + 20 \times 0.5 + V_{o.c} = 0$$

$$V_{oc} = 90\ \text{Volt}$$



Mesh (1):-

$$-100 + 20i_1 = 0$$

$$i_1 = 5A$$

Mesh (2)

$$i_2 = 0$$

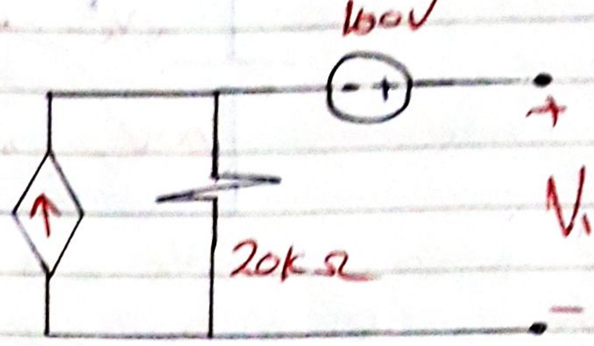
$$I_{sc} = i_1 = 5A$$

$$R_{th} = \frac{V_{o.c}}{I_{s.c}} = \frac{90}{5} = 18\ \Omega$$

Hiba Alwail

H.W:-

Find the Thevenin equivalent for the network of Fig. (Hint: a quick source transformation on the dependent source might help).



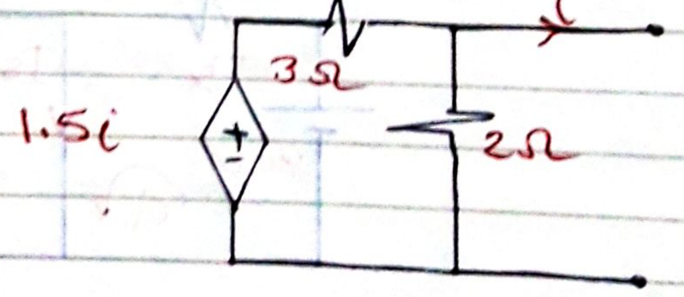
Ans. $V_{th} = -0.5025$ Volt

$R_{th} = 100.5\ \Omega$

3- Dependent Source Only

Ex.:-

Find the Thevenin equivalent of the circuit.



واذا كانت الدارة الكهربائية تحتوي فقط على مصادر معتمدة فان

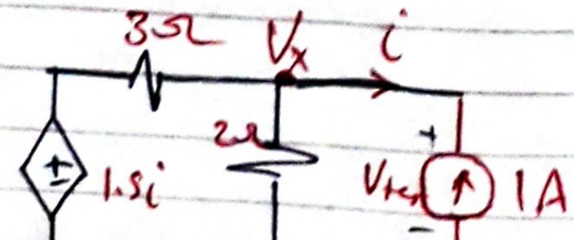
$V_{th} = 0$

والدارة المكافئة تتكون من R_{th} فقط.

في حالة هكذا دوائر اما ان نفرض مصدر فولتية بقيمة (1V) او مصدر تيار بقيمة (1A) وبالتالي فان:

$$R_{th} = \frac{1}{I_{test}} \quad \text{or} \quad R_{th} = \frac{V_{test}}{1}$$

$$R_{th} = \frac{V_{test}}{1}$$



$V_{th} = 0$

Hiba Alsawaf

$$1 = \frac{V_x - 1.5i}{3} + \frac{V_x}{2} \quad , \; i = -1A$$

$$1 = \frac{V_x + 1.5}{3} + \frac{V_x}{2}$$

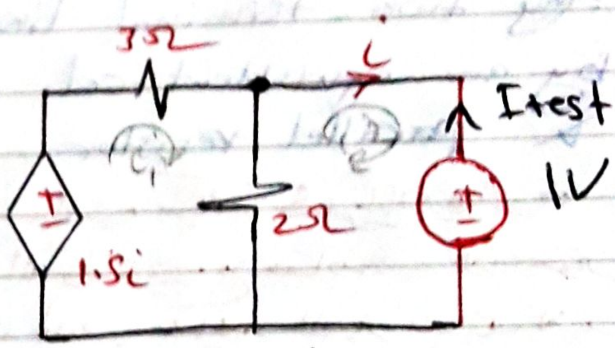
$$1 = \frac{2V_x + 3 + 3V_x}{6}$$

$$6 = 5V_x + 3 \Rightarrow V_x = \frac{3}{5} = 0.6 \text{ Volt}$$
$$= V_{test}$$

$$R_{th} = \frac{0.6}{1}$$

2// $R_{th} = \dfrac{1}{I_{test}}$

$V_{th} = 0$



Mesh (1) :-

$$-1.5i + 3i_1 + 2(i_1 - i_2) = 0$$
$$-1.5i_2 + 3i_1 + 2i_1 - 2i_2 = 0$$
$$5i_1 - 3.5i_2 = 0 \quad --- (1)$$

$i_2 = i$

Mesh (2)

$$2(i_2 - i_1) + 1 = 0$$
$$2i_2 - 2i_1 = -1 \quad \div 2$$
$$i_2 - i_1 = -0.5$$
$$i_1 = i_2 + 0.5 \quad --- (2)$$

Sub. eq. (2) in eq. (1)

$$5(i_2 + 0.5) - 3.5i_2 = 0$$
$$5i_2 + 2.5 - 3.5i_2 = 0 \Rightarrow i_2 = \frac{-2.5}{1.5} = -\frac{5}{3} A$$

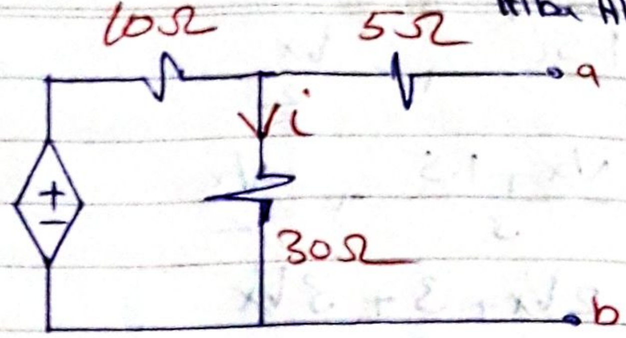
$$i_2 = -I_{test}$$
$$I_{test} = \frac{5}{3} A$$

$$R_{th} = \frac{1}{\frac{5}{3}} \Rightarrow R_{th} = \frac{3}{5} = 0.6 \, \Omega$$

H.W:

Hiba Alsawaf

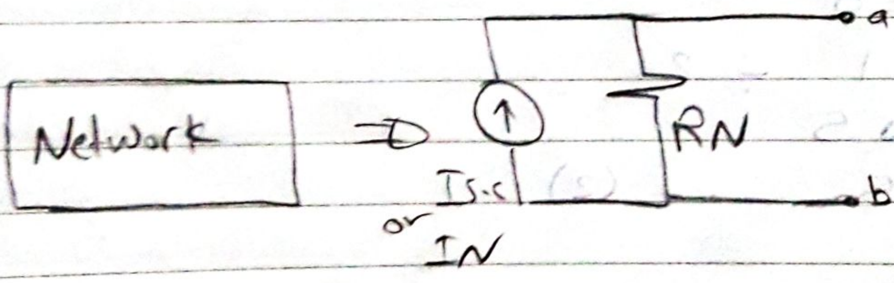
Ans. $R_{th} = 20\Omega$



Norton Theorem

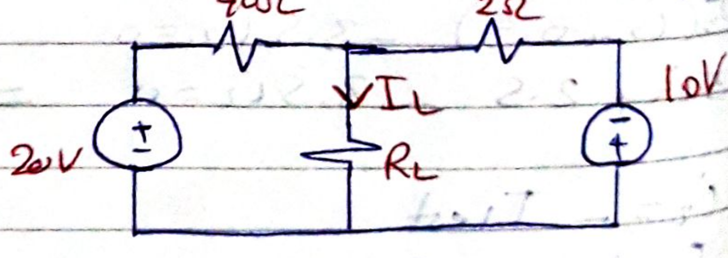
Any two-terminal linear d.c Network can be replaced by an equivdent cct. onsisting of a current Source & parallel resistor.

تنص هذه النظرية على انه يمكن التعويض عن اي شبكة كهربائية بدائرة مكافئة تتكون من مصدر تيار على التوازي مع مقاومة فقط ويكون تيار المصدر عبارة عن التيار للدائرة في الفرع المطلوب ايجاد التيار فيه بعد قصر ذلك الفرع اما المقاومة للتوازية فهي تمثل المقاومة المكافئة التي تظهر عبر نفس الفرع بعد التعويض عن مصادر الفولتية بـ short cct. ومصادر التيار بـ open cct.



Exa:

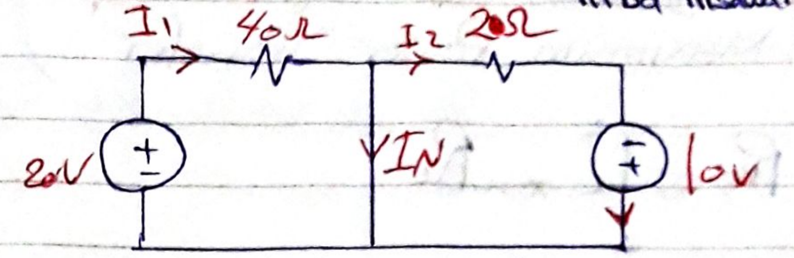
Use Norton Theorem to find I_L.



Hiba Alsaeat

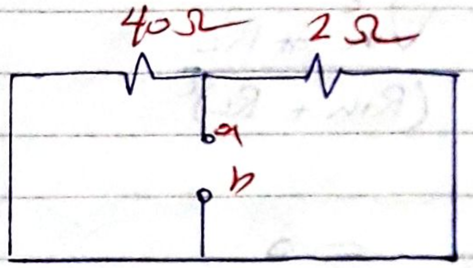
$$I_1 = \frac{20}{40} = 0.5A$$

$$I_2 = \frac{10}{2} = 5A$$

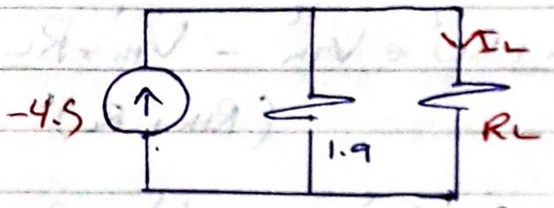


$$I_N = I_1 - I_2 \Rightarrow I_N = 0.5 - 5 = -4.5A$$

$$R_N = R_{th} = \frac{40 * 2}{40+2} = 1.9\Omega$$



$$I_L = -4.5 * \frac{1.9}{1.9+R_L}$$



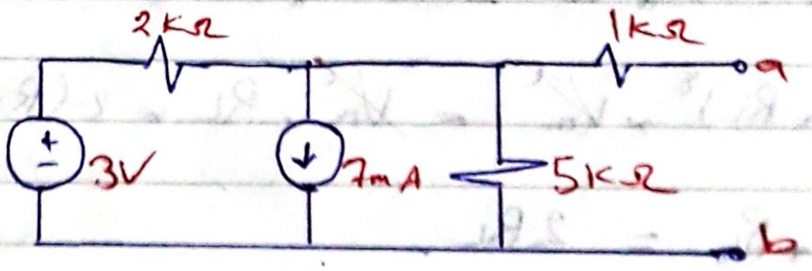
H.W:-

Ans.

$R_N = 2.428K\Omega$

$V_{th} = -7.357V$

$I_{s.c} = 3.235\ mA$

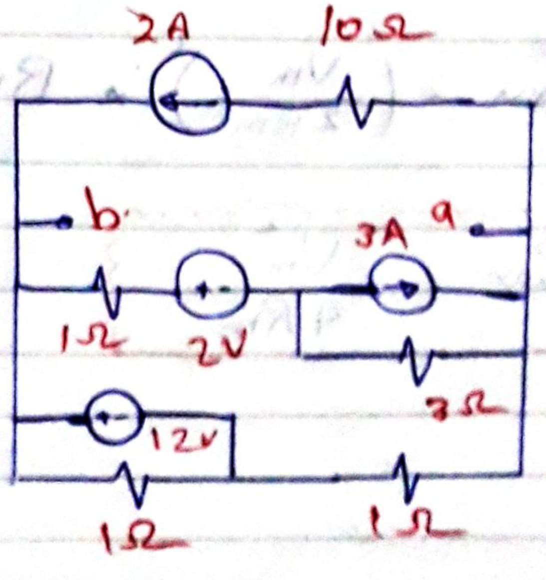


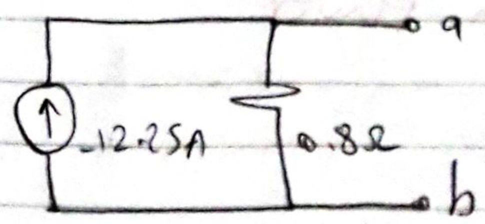
Exam

Find Norton equivalent at (a-b).

$R_N = 0.8\ \Omega$

$I_{s.c} = -12.25A$

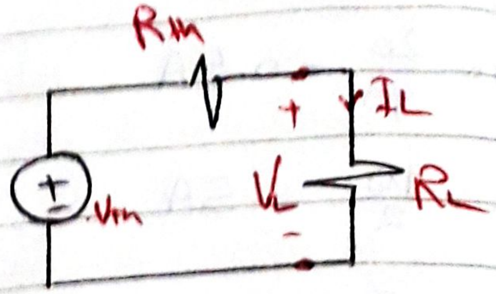




Norton equivalent

Hiba Alsawaf

Maximum Power Transfer



$$P_{RL} = I_L^2 * R_L$$

$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 * R_L$$

$$= \frac{V_{th}^2 * R_L}{(R_{th}+R_L)^2}$$

$$\frac{dP_{RL}}{dR_L} = 0$$

$$\frac{(R_{th}+R_L)^2 * V_{th}^2 - V_{th}^2 * R_L * 2(R_{th}+R_L) * 1}{(R_{th}+R_L)^4} = 0$$

$$(R_{th}+R_L)^2 * V_{th}^2 - V_{th}^2 * R_L * 2(R_{th}+R_L) = 0$$

$$(R_{th}+R_L)^{\cancel{2}} * \cancel{V_{th}^2} = \cancel{V_{th}^2} * R_L * 2\,\cancel{(R_{th}+R_L)}$$

$$R_{th} + R_L = 2R_L$$

$$\boxed{R_{th} = R_L}$$

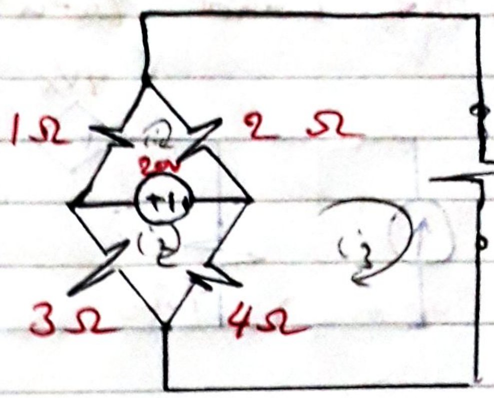
$$P_{max} = \left(\frac{V_{th}}{2R_{th}}\right)^2 * R_{th}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_{th}}}$$ maximum power

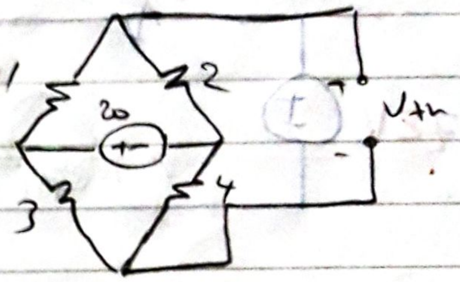
Hiba Alsawaf

Ex:-
Find maximum power transfered to R.

$V_{AB} = V_{th}$
$R_{AB} = R_{th}$



Mesh(1):-
$-20 + 1 * i_1 + 2(i_1 - \cancel{i_3}) = 0$
$i_1 = \frac{20}{3}$ A

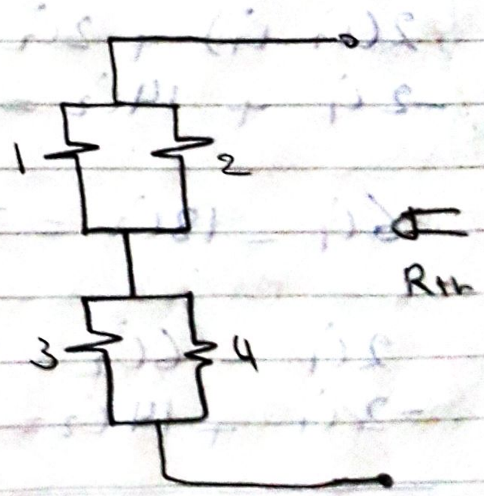


Mesh (2):-
$3i_2 + 20 + 4(i_2 - \cancel{i_3}) = 0$
$i_2 = \frac{-20}{7}$ A

Mesh (3):-
$4(\cancel{i_3} - i_2) + 2(\cancel{i_3} - i_1) + V_{th} = 0$
$-4i_2 - 2i_1 + V_{th} = 0$
$-4 * \frac{-20}{7} - 2 * \frac{20}{3} + V_{th} = 0$

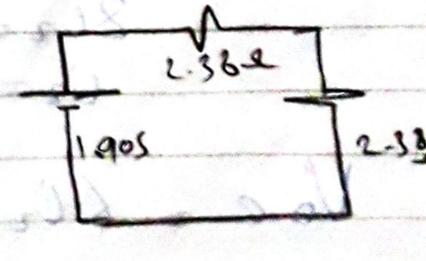
$V_{th} = 1.905$ Volt

$R_{th} = 1 \parallel 2 + 3 \parallel 4$
$= \frac{1*2}{1+2} + \frac{3*4}{3+4}$
$= \frac{2}{3} + \frac{12}{7}$



$R_{th} = 2.38\ \Omega$

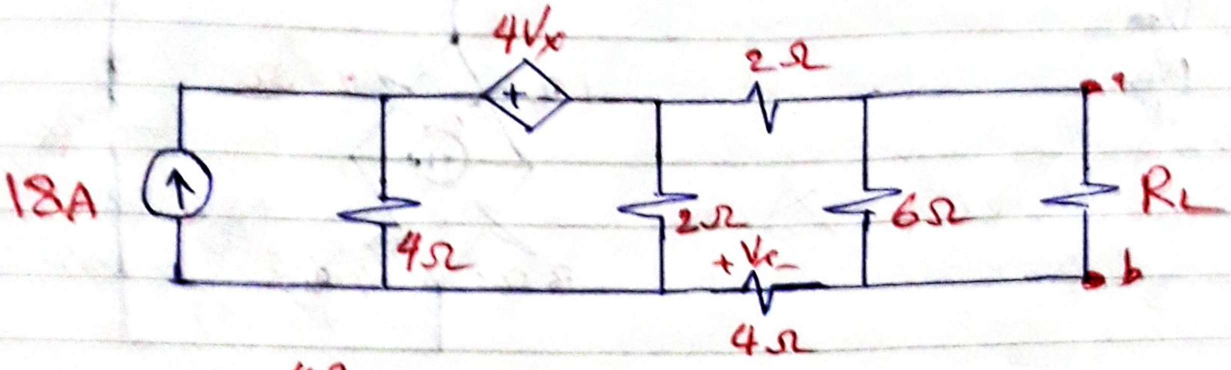
$P_{max} = \frac{V_{th}^2}{4R_{th}}$
$= \frac{(1.905)^2}{4 * 2.38} = 0.38$ Watt

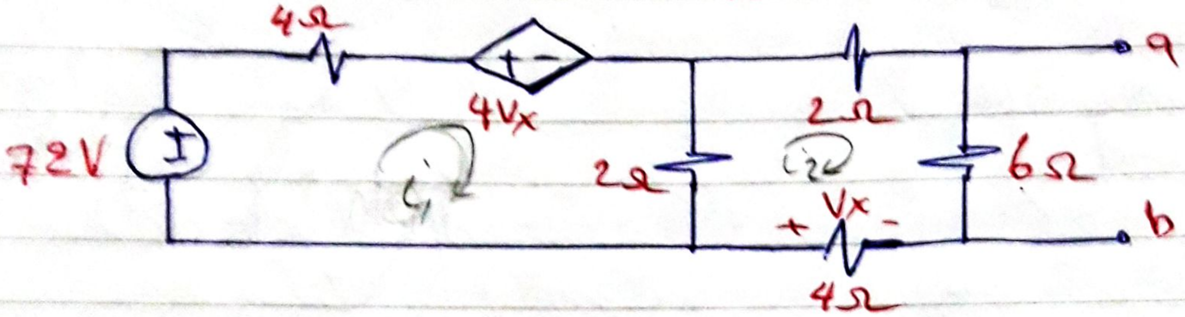


Hiba Alsawaf

Exa:_

Find the value of R_L for maximum power transfer to the Load then find P_{max}.





Mesh (1):-

$-72 + 4i_1 + 4V_x + 2(i_1 - i_2) = 0$, $V_x = -4i_2$

$-72 + 4i_1 + 4(-4i_2) + 2i_1 - 2i_2 = 0$

$6i_1 - 18i_2 = 72$ --- (1)

Mesh (2):-

$2(i_2 - i_1) + 2i_2 + 6i_2 + 4i_2 = 0$

$-2i_1 + 14i_2 = 0$ --- (2)

$6i_1 - 18i_2 = 72$ --- (1) $] \div 3$

$\cancel{2i_1} - 6i_2 = 24$ --- (1)

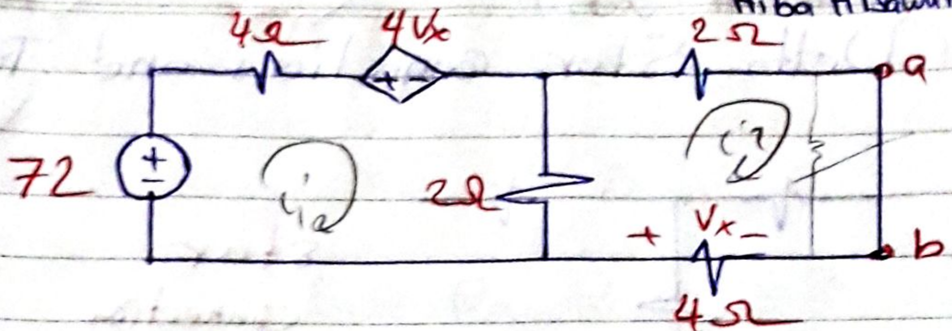
$\cancel{-2i_1} + 14i_2 = 0$ --- (2)

$8i_2 = 24 \Rightarrow i_2 = \frac{24}{8}$

$i_2 = 3\ A$

$V_{o.c} = 6i_2 = 6 * 3 = 18$ Volt

Hiba Alsawaf



Mesh (1):-

$-72 + 4i_1 + 4V_x + 2(i_1 - i_2) = 0$, $V_x = -4i_2$ ---- (a)

$-72 + 4i_1 + 4(-4i_2) + 2i_1 - 2i_2 = 0$

$6i_1 - 18i_2 = 72$] ÷ 3 ---- (1) ⇒ $2i_1 - 6i_2 = 24$ ---- (1)

Mesh (2):-

$2(i_2 - i_1) + 2i_2 + 4i_2 = 0$

$-2i_1 + 8i_2 = 0$ ---- (2)

$\cancel{2i_1} - 6i_2 = 24$ ---- (1)

$\cancel{-2i_1} + 8i_2 = 0$ ---- (2)

$2i_2 = 24 \Rightarrow i_2 = 12\ A$

$I_{s.c} = i_2 = 12A$

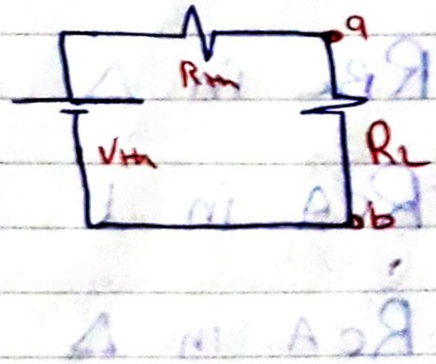
$R_{th} = \frac{V_{th}}{I_{sc}}$

$= \frac{18}{12} \Rightarrow R_{th} = 1.5\,\Omega$

$P_{max} = \frac{V_{th}^2}{4R_{th}}$

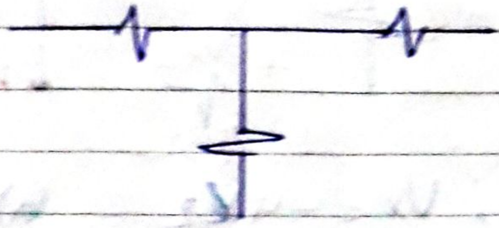
$= \frac{(18)^2}{4 \times 1.5}$

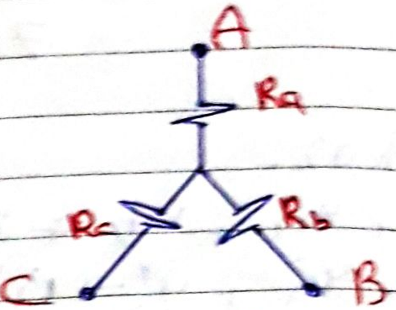
$= 54$ Watt



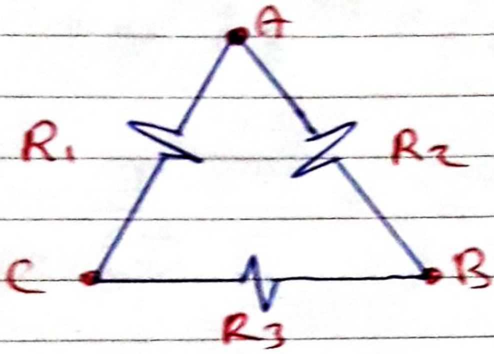
Delta_Star connections and Transformations

Hiba Alsaraf

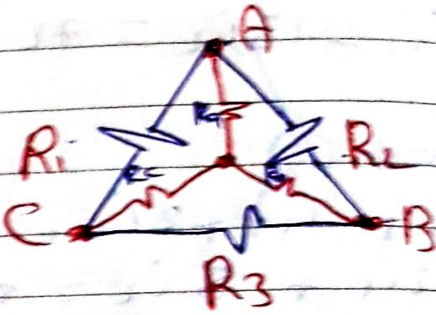




star connection



Delta connection



R_{AB} in λ = R_{AB} in Δ

R_{AB} (equivalent) in λ = $R_a + R_b$

R_{AB} (equivalent) in Δ = $\dfrac{R_2(R_1+R_3)}{R_2+R_1+R_3}$

$R_a + R_b = \dfrac{R_2(R_1+R_3)}{R_1+R_2+R_3}$ --- (1)

R_{BC} in λ = $R_b + R_c$

R_{BC} in Δ = $\dfrac{R_3(R_1+R_2)}{R_1+R_2+R_3}$ --- (2)

R_{CA} in λ = $R_c + R_a$

R_{CA} in Δ = $\dfrac{R_1(R_2+R_3)}{R_1+R_2+R_3}$ --- (3)

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Transformation $\Delta \rightarrow Y$ & $Y \rightarrow \Delta$

$\Delta \rightarrow Y$

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

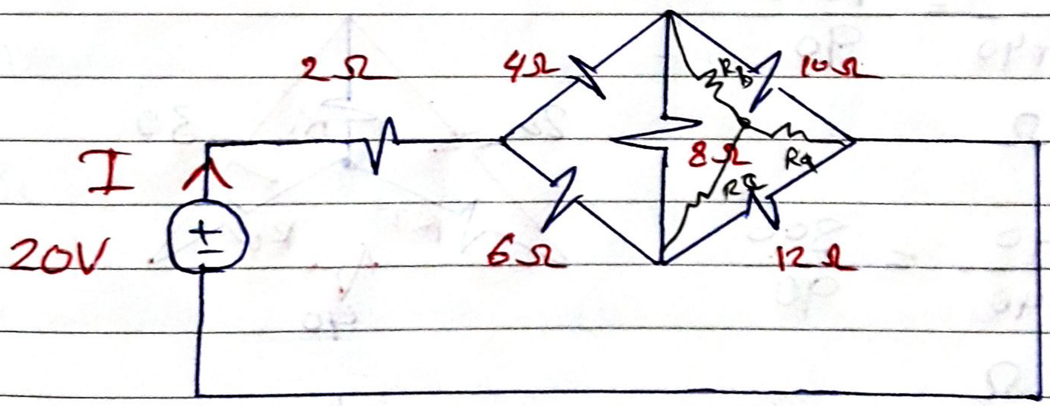
$Y \rightarrow \Delta$

$$R_1 = R_a + R_c + \frac{R_a R_c}{R_b}$$

$$R_2 = R_a + R_b + \frac{R_a R_b}{R_c}$$

$$R_3 = R_b + R_c + \frac{R_b R_c}{R_a}$$

Exa:-
Find the power supplied by 20V.



$$R_a = \frac{10 * 12}{10 + 12 + 8} = 4\,\Omega$$

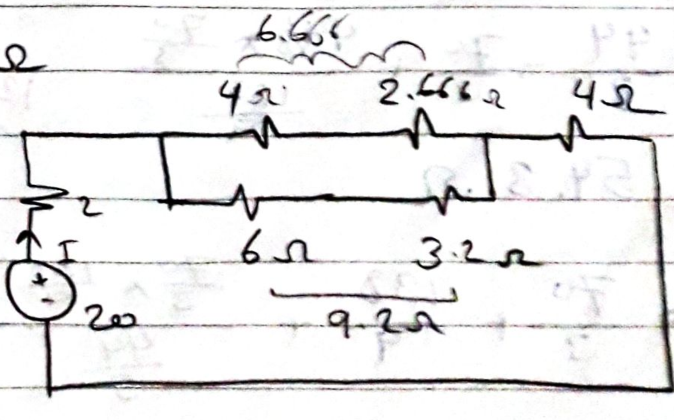
$$R_b = \frac{10 * 8}{10 + 12 + 8} = 2.666\,\Omega$$

$$R_c = \frac{8 * 12}{10 + 12 + 8} = 3.2\,\Omega$$

$$R_{eq} = 2 + 4 + \left(\frac{6.666 * 9.2}{6.666 + 9.2}\right)$$

$$= 2 + 4 + 3.86$$

$$= 9.86\,\Omega$$



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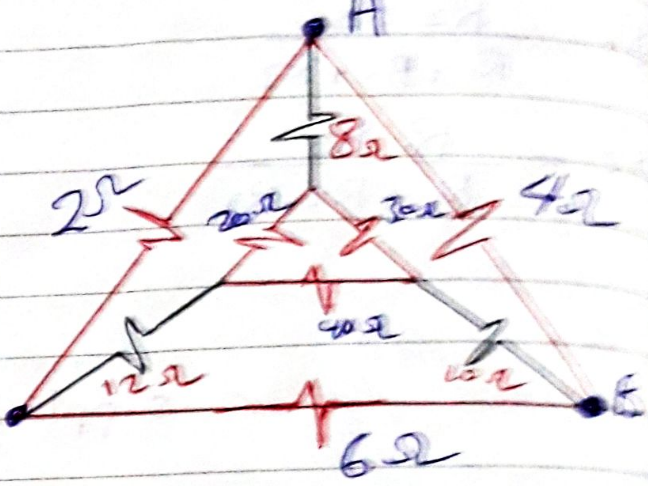
$$I = \frac{20}{9.86} = 2.028\ A$$

$$P = v * i$$

$$= 20 * 2.028$$

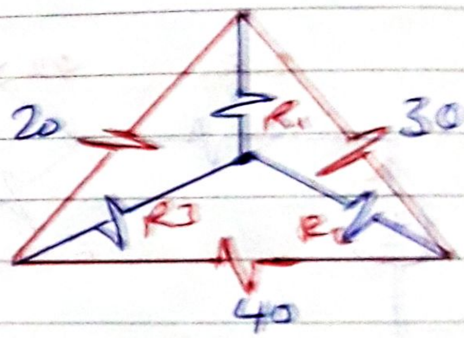
$$= 40.56\ \text{Watt}$$

Exa:
Find Req between AB & BC



$$R_1 = \frac{20 * 30}{20+30+40} = \frac{600}{9}$$

$$R_1 = \frac{20}{3}\ \Omega$$

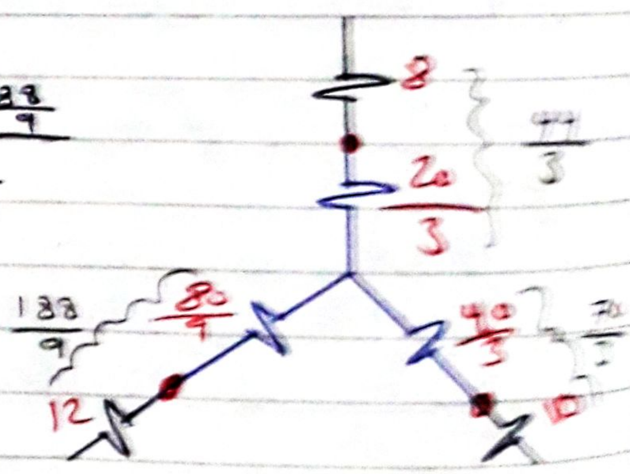


$$R_2 = \frac{30 * 40}{20+30+40} = \frac{1200}{90}$$

$$R_2 = \frac{40}{3}\ \Omega$$

$$R_3 = \frac{20 * 40}{20+30+40} = \frac{800}{90}$$

$$R_3 = \frac{80}{9}\ \Omega$$



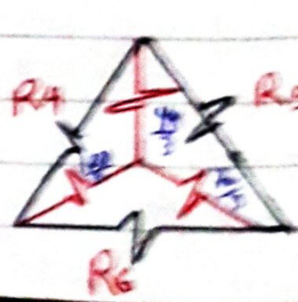
$$R_4 = \frac{44}{3} + \frac{188}{9} + \frac{\frac{44}{3} * \frac{188}{9}}{\frac{70}{3}}$$

$$= 48.6\ \Omega$$

$$R_5 = \frac{44}{3} + \frac{70}{3} + \frac{44/3 * \frac{70}{3}}{\frac{188}{9}}$$

$$= 54.3\ \Omega$$

$$R_6 = \frac{70}{3} + \frac{188}{9} + \frac{\frac{70}{3} * \frac{188}{9}}{\frac{44}{3}}$$



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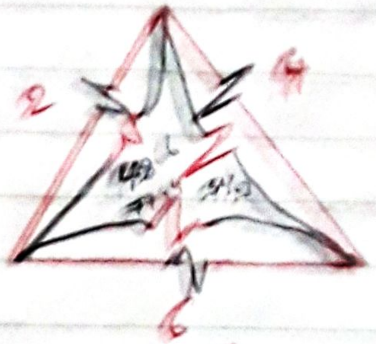
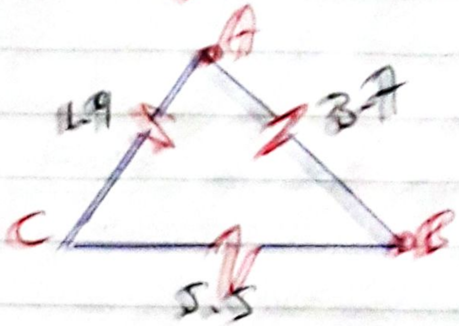
$R_6 = 77.4\ \Omega$



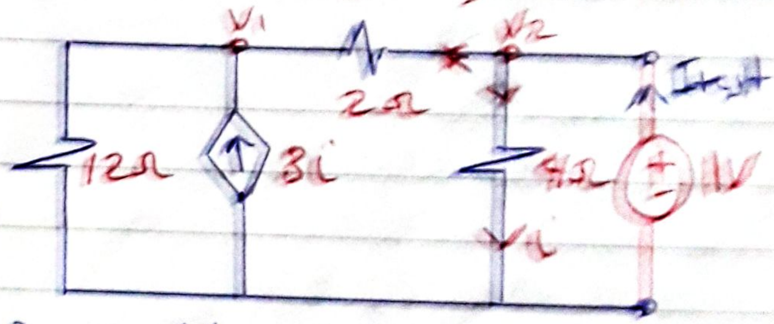
$$R_{AB} = \frac{3.7(1.9+5.5)}{3.7+1.9+5.5} = 2.46\ \Omega$$

$$R_{BC} = \frac{5.5(1.9+3.7)}{3.7+1.9+5.5} = 2.77\ \Omega$$



Exa:-
Find Thevenin equivalent at (ab)

Node 1:_



$$3i = \frac{V_1}{12} + \frac{V_1 - V_2}{2} \quad \text{---(1)}$$

$$i = \frac{V_2}{4} = \frac{-1}{4}\ A \quad \text{---(a)} \quad , V_2 = 1V$$

sub. eq. (a) in eq. (1)

$$\frac{3}{4} = \frac{V_1}{12} + \frac{V_1 - 1}{2}$$

$$-\frac{3}{4} = \frac{V_1 + 6V_1 - 6}{12} \Rightarrow 4V_1 + 24V_1 - 24 = 36$$

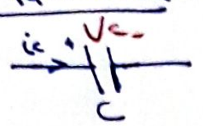
$$28V_1 = 60 \Rightarrow V_1 = 2.142\ \text{Volt}$$

شبكات (1)
المحاضرة الاولى
الفصل الثاني
Hiba Alsawaf

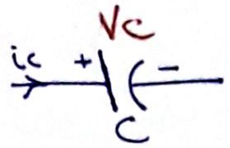
Transient in Rc circuit:-

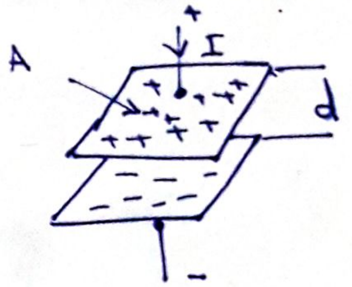
* Capacitors and Inductors

• Capacitor:-



or





$C \propto \frac{A}{d}$ measured in (F)

$C = \epsilon \frac{A}{d}$ ابسلون

المتسعة تخزن طاقة على شكل شحنات كهربائية

A: Area of two plates

d: distance between two plates

• طبعا هنالك عدة أشكال للمتسعة

ϵ: permitivity of the insulating

$\epsilon = \epsilon_o \epsilon_r$

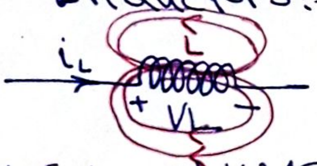
C: Capacitance in farad (coulombs/volt)

* المتسعة لاتقبل تغيير فجئي واتجاه فولتية الاطراف آنياً.

* التيار يمكن ان يتغير آنياً.

* في حالة الـ steady state، تتصرف المتسعة open cct S

* The Inductors:-



سلك او ملف
داخل الملف
هواء او حديد

الحاثة تخزن الطاقة على شكل مجال مغناطيسي

الـ core او القلب يكون اما قلب من الحديد او الهواء.

* كلما زادت مساحة المقطع وعدد اللفات ~~تزداد~~ تصبح الحاثة اكبر

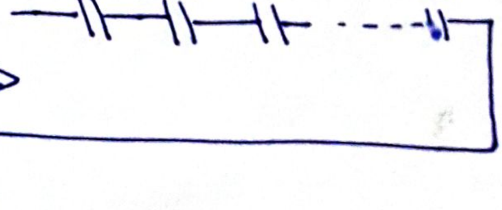
* الحاثة لاتقبل تغيير فجئي واتجاه التيار آنياً.

* في حالة الـ steady state تتصرف الحاثة S.C S

Capacitors in Series:-

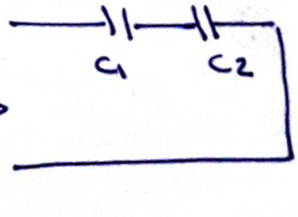
$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

$C_{eq} \Rightarrow$



$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

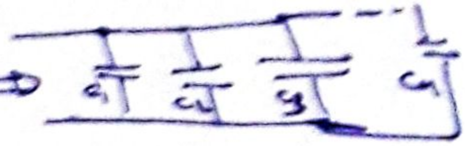
$C_{eq} \Rightarrow$



* المتسعات المربوطة على التوالي تمثل او تشبه ربط المقاومات على التوازي.

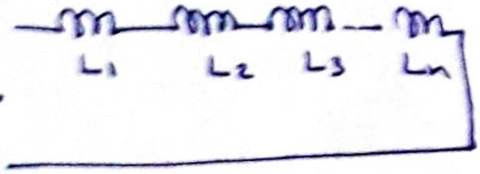
* Capacitors in parallel :- (2)   Hiba Alsaud

* $C_{eq} = C_1 + C_2 + C_3 + \dots + C_n$

$C_{eq} \Longrightarrow$ 

* في حالة الربط على التوازي تكون ا تشبه ربط المقاومات على التوالي
* نتيجة ceq تصبح اكبر في حالة الربط على التوازي

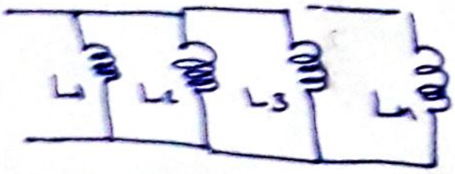
* Inductors in Series:-

$L_{eq} = L_1 + L_2 + L_3 + \dots L_n$ $L_{eq} \Longrightarrow$ 

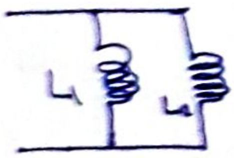
تشبه ربط المقاومات في التوالي

* Inductors in parallel:-

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}}$$

$L_{eq} \Longrightarrow$ 

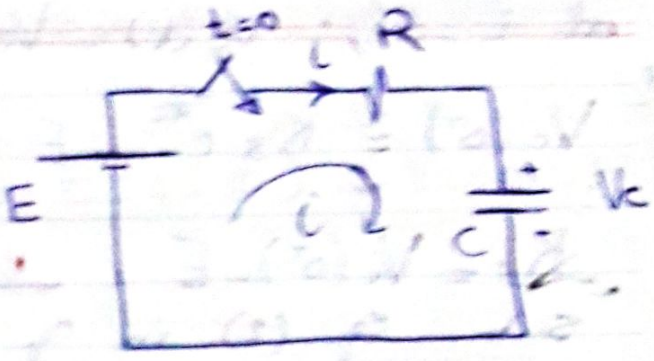
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

$L_{eq} \Longrightarrow$ 

Transient in RC circuit:-

Abo Alsaud

$q = C * V_c$



$i = \frac{dq}{dt}$

$i_c = \frac{d C V_c}{dt}$

$i_c = C \frac{d V_c}{dt}$

Apply KVL to find V_c:-

$-E + iR + V_c = 0$

$-E + RC \frac{dV_c}{dt} + V_c = 0$

$V_c - E = \frac{-RC\, dV_c}{dt} \Rightarrow \frac{dV_c}{V_c - E} = -\frac{1}{RC} dt$

Integrating both sides:-

$\int \frac{dV_c}{V_c - E} = \int -\frac{dt}{RC}$

$\ln(V_c - E) + K_1 = -\frac{1}{RC} t + K_2$

$\ln(V_c - E) = \frac{-1}{RC} t + \underbrace{(K_2 - K_1)}_{K}$

$V_c - E = e^{-\frac{t}{RC} + K}$

$V_c - E = e^{-\frac{t}{RC}} \cdot e^{K}$, Let $K_3 = e^{K}$

$V_c = K_3 e^{-\frac{t}{RC}} + E$ ——— (1)

To find K_3 from Initial Condition

at $t=0$, $V_c(t) = V_c(0^+)$ Hiba Alsaad

$$V_c(0^+) = K_3 e^{\frac{-0}{RC}} + E \Rightarrow V_c(0^+) = K_3 + E$$

$$K_3 = V_c(0^+) - E \quad \text{——} (2)$$

sub. eq. (2) in eq. (1) :-

$$V_c = [V_c(0^+) - E] e^{\frac{-t}{RC}} + E$$

$$V_c = V_c(0^+) e^{\frac{-t}{RC}} + E(1 - e^{\frac{-t}{RC}}) \quad \text{——} (3)$$

at $V_c(0^+) = 0$

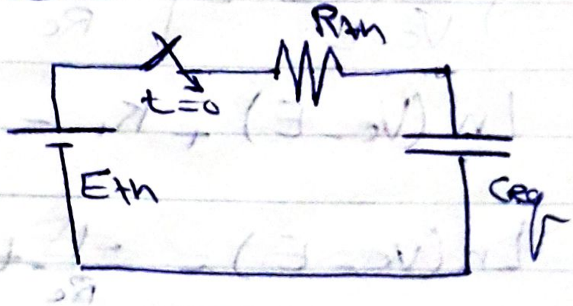
$$\boxed{V_c = E(1 - e^{\frac{-t}{RC}})} \quad \text{at } V_c(0^+) = 0$$

In general :-

$$V_c = E_{th}(1 - e^{\frac{-t}{R_{th} * C_{eq}}}) + V_c(0^+) e^{\frac{-t}{R_{th} * C_{eq}}}$$

at $t=0$, $V_c(0^+) = 0$

$$V_c = E_{th}(1 - e^{\frac{-t}{R_{th} * C_{eq}}})$$



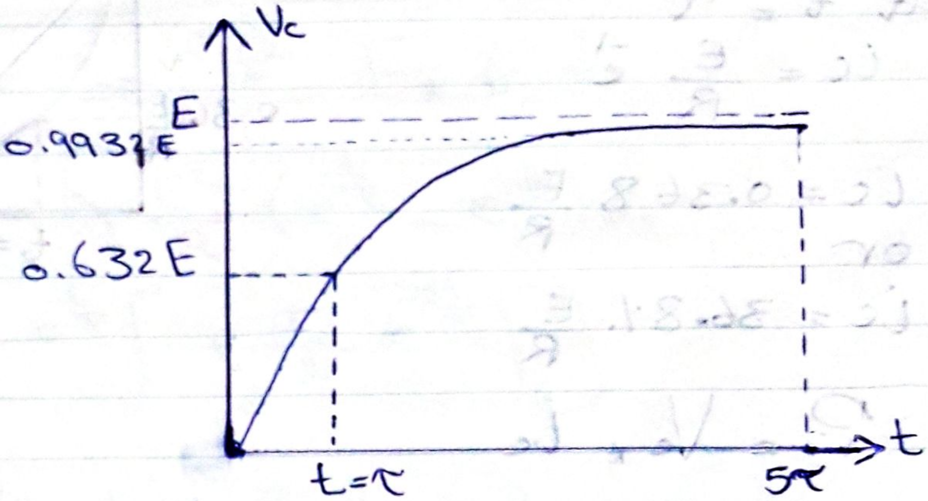
RC = ohm * Farad
= second

RC : time constant (τ) :-

$$\boxed{V_c = E(1 - e^{\frac{-t}{\tau}})}$$

at $t=0 \Rightarrow V_c=0$ Hiba Alsawaf

at $t=\tau$
$V_c = E(1 - e^{\frac{-\tau}{\tau}}) \Rightarrow V_c = E(1-e^{-1}) \Rightarrow V_c = 0.632E$



at $t = 2\tau \Rightarrow V_c = 0.86E$
at $t = 4\tau \Rightarrow V_c = 0.98E$
at $t = 5\tau \Rightarrow V_c = 0.9932E$
at $t = \infty \Rightarrow V_c = E$

* To Find i_c :-

$$i_c = C \frac{dV_c}{dt}$$

$$= C \frac{d}{dt}\left[E(1-e^{\frac{-t}{\tau}})\right]$$ for Zero inital condition $V_c(0^+)=0$

$$= CE\left(0 - e^{\frac{-t}{\tau}} * \frac{-1}{\tau}\right)$$

$$= CE * \frac{1}{\tau} * e^{\frac{-t}{\tau}} \quad , \tau = RC$$

$$i_c = \frac{E}{R} e^{\frac{-t}{\tau}}$$

Hiba Alsamaf

at $t=0 \Rightarrow i_c = \frac{E}{R}$

at $t=\infty \Rightarrow i_c = 0$

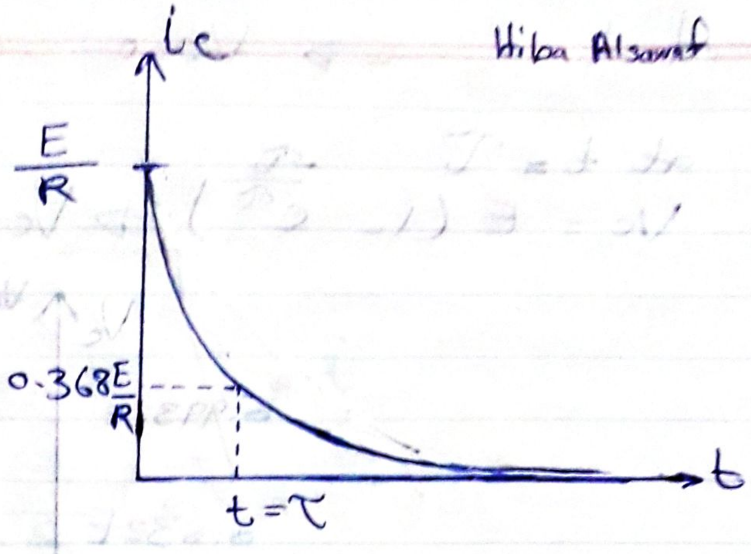
at $t = \tau$

$i_c = \frac{E}{R} e^{-1}$

$i_c = 0.368 \frac{E}{R}$

or

$i_c = 36.8\% \frac{E}{R}$



$P_c = V_c * i_c$

$= E(1-e^{\frac{-t}{\tau}}) * \frac{E}{R} e^{\frac{-t}{\tau}} \Rightarrow P_c = \frac{E^2}{R}(1-e^{\frac{-t}{\tau}}) e^{\frac{-t}{\tau}}$

$$\boxed{P_c = \frac{E^2}{R}\left(e^{-\frac{t}{\tau}} - e^{\frac{-2t}{\tau}}\right)}$$ Power of capacitance (watt)

$V_R = i * R$

$= \frac{E}{\cancel{R}} e^{\frac{-t}{\tau}} * \cancel{R} \Rightarrow \boxed{V_R = E e^{\frac{-t}{\tau}}}$

$P_R = V_R * i_R$

$= E e^{\frac{-t}{\tau}} * \frac{E}{R} e^{\frac{-t}{\tau}} \Rightarrow \boxed{P_R = \frac{E^2}{R} e^{\frac{-2t}{\tau}}}$

Power of resistance (watt)

Hiba Alsawaf

$$P_T = P_c + P_R$$

$$= \frac{E^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right) + \frac{E^2}{R} e^{-\frac{2t}{\tau}}$$

$$\boxed{P_T = \frac{E^2}{R} e^{-\frac{t}{\tau}}}$$

$$P_T = E \cdot \frac{E}{R} e^{-\frac{t}{\tau}} \Rightarrow P_T = E * i$$

$$W_c = \int_0^\infty P_c \, dt = \int_0^\infty \frac{E^2}{R}\left(e^{-\frac{t}{\tau}} - e^{-\frac{2t}{\tau}}\right) dt$$

$$W_c = \frac{E^2}{R}\left[-\tau e^{-\frac{t}{\tau}} \Big|_0^\infty + \frac{\tau}{2} e^{-\frac{2t}{\tau}} \Big|_0^\infty\right]$$

$$= \frac{E^2}{R}\left[(0 + \tau) - \frac{\tau}{2}\right]$$

$$W_c = \frac{E^2}{R} \cdot \frac{\tau}{2}$$, Where $\tau = RC$

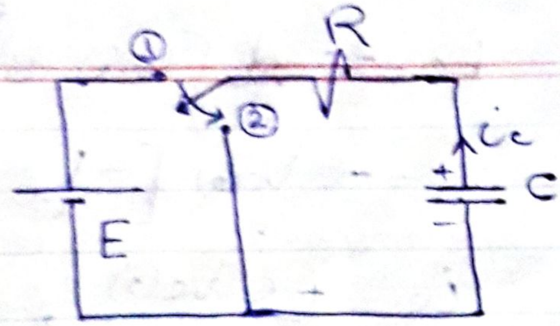
$$\boxed{W_c = \frac{1}{2} C E^2}$$ Joule at $V_c(0^+) = 0$

$$\boxed{\underset{t=0 \to \infty}{W_c(t)} = \frac{1}{2} C V_c^2(t)}$$ Joule at any time

Discharge of capacitor (source free)

Hiba Alsawaf.

~~$V_R = V_c$~~



$V_c = R * i$

$V_c = R * -C \frac{dv_c}{dt}$

$i_c = -C \frac{dv_c}{dt}$

$\frac{V_c}{dv_c} = \frac{-Rc}{dt}$

$\frac{dv_c}{V_c} = \frac{-dt}{Rc}$

Integrating both sides:-

$\int \frac{dv_c}{V_c} = \int -\frac{dt}{Rc}$

$\ln V_c + K_1 = -\frac{1}{Rc} t + K_2$

$\ln V_c = \frac{-t}{Rc} + \underbrace{(K_2 - K_1)}_{K}$

$V_c(t) = e^{\frac{-t}{Rc} + K} \Rightarrow V_c(t) = e^{\frac{-t}{RC}} \cdot e^{K}$

Let $K_3 = e^{K}$

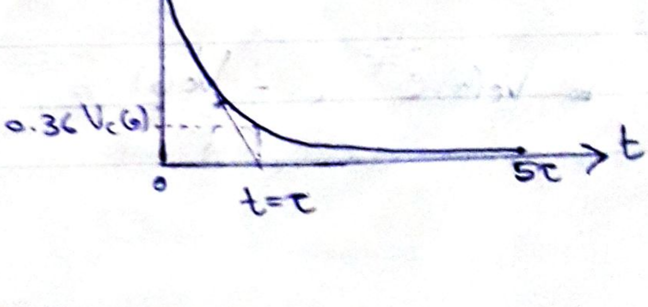
$V_c(t) = K_3 \, e^{\frac{-t}{Rc}}$, $Rc = \tau$

To find K_3 from initial condition:-

at $t = 0$, $V_c(t) = V_c(0)$

$V_c(0) = K_3$

$\boxed{\therefore V_c(t) = V_c(0) \, e^{\frac{-t}{\tau}}}$



$$i_c = -C\frac{dv_c}{dt}$$

[illegible]

$$i_c = -C\frac{d}{dt}\left[V_c(0)\, e^{-\frac{t}{\tau}}\right]$$

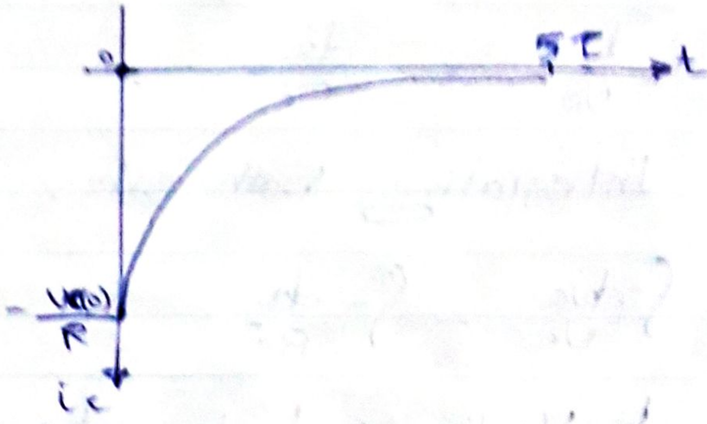
$$= -C\,V_c(0)\left[-\frac{1}{\tau}\, e^{-\frac{t}{\tau}}\right]$$

$$i_c = \frac{C\,V_c(0)}{\tau} * e^{-\frac{t}{\tau}} \quad , \tau = RC$$

$$= \frac{\cancel{C}\,V_c(0)}{R\cancel{C}} * e^{-\frac{t}{\tau}}$$

Discharge

$$\boxed{i_c = \ominus\frac{V_c(0)}{R}\, e^{-\frac{t}{\tau}}}$$



$$i_c = I_0\, e^{-\frac{t}{\tau}}$$

$$V_R = R * i$$

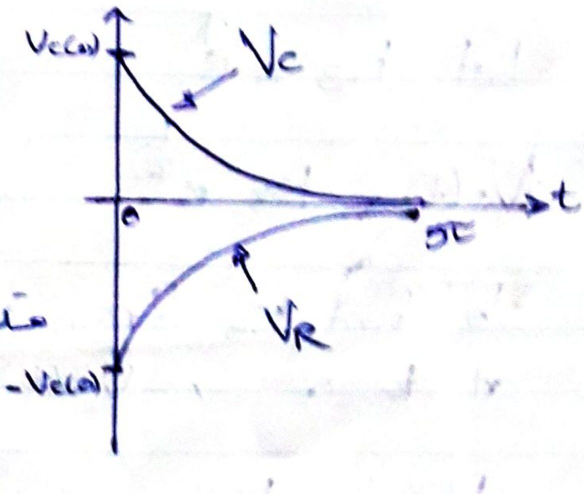
Discharge

$$= R * \frac{-V_c(0)}{R}\, e^{-\frac{t}{\tau}} \Rightarrow \boxed{V_R = \ominus V_c(0)\, e^{-\frac{t}{\tau}}}$$

$$P_R = V_R * i_R$$

$$= -V_c(0)\, e^{-\frac{t}{\tau}} * \frac{-V_c(0)}{R}\, e^{-\frac{t}{\tau}}$$

$$\boxed{P_R = \frac{V_c^2(0)}{R}\, e^{-\frac{2t}{\tau}}}$$ [illegible]



$$P_c = V_c * i_c$$

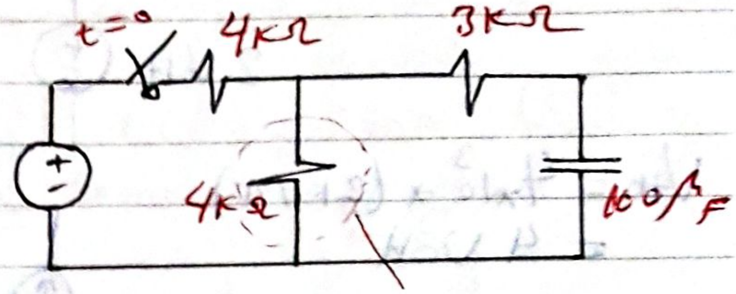
$$= V_c(0)\, e^{-\frac{t}{\tau}} * \frac{-V_c(0)}{R}\, e^{-\frac{t}{\tau}} \Rightarrow \boxed{P_c = \ominus\frac{V_c^2(0)}{R}\, e^{-\frac{2t}{\tau}}}$$ [illegible]

Transient in RC circuit

Hiba Alsawaf

Exa:-
For the circuit shown, find V_c at $t = 0.1$ sec & $t = 5$ sec. (assume zero initial condition)

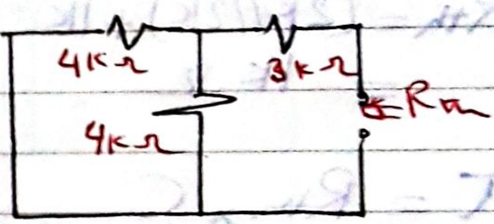
$$V_c = V_{th}\left(1 - e^{-\frac{t}{\tau}}\right)$$



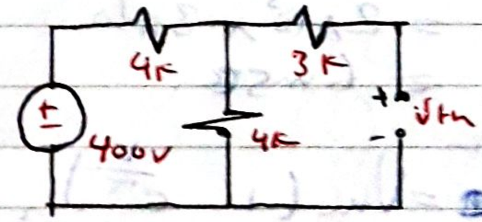
المصدر الثامن للمتحة

To find the R_{th}:-

$$R_{th} = (4K \parallel 4K) + 3K$$
$$= 5K\Omega$$



$$V_{th} = \frac{400}{8} * 4$$
$$= 200 \text{ Volt}$$



$$\tau = R_{th} * C$$
$$= 100*10^{-6} * 5*10^{3} \Rightarrow \tau = 0.5 \text{ sec.}$$

$$V_c = 200\left(1 - e^{-\frac{t}{0.5}}\right) \quad \text{for } t > 0$$

$$V_c = 200\left(1 - e^{-\frac{0.1}{0.5}}\right) \quad \text{At } t = 0.1 \text{ sec.}$$
$$= 36.25 \text{ Volt}$$

$$V_c = 200\left(1 - e^{-\frac{5}{0.5}}\right) \quad \text{At } t = 5 \text{ sec}$$
$$= 199.99 \text{ Volt}$$

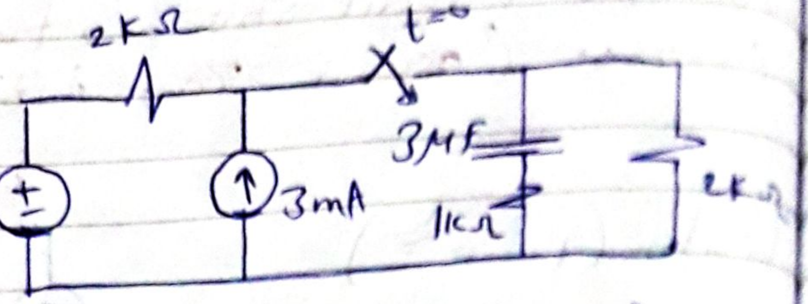
if we want to find W_c

$$W_c = \frac{1}{2} C \, \cancel{V^2(t)}$$
$$= \frac{1}{2} * 100*10^{-6} * 200 \Rightarrow W_c = 2 \text{ Joule}$$

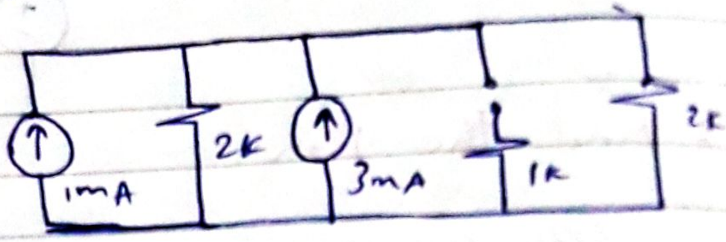
Hiba Alsarraf

Exa:-
For the cct. shown, find V_c at $t = 4$ msec

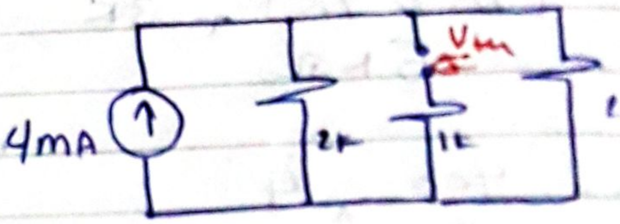
$V_c = V_{th}(1 - e^{-\frac{t}{\tau}})$



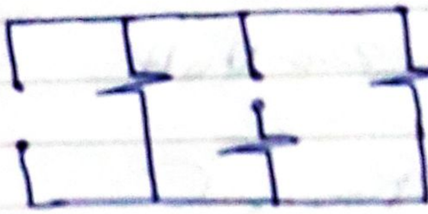
$V_{th} = 4 \times 10^{-3} \times (2K // 2K)$
$= 4$ Volt



$R_{th} = (2K // 2K) + 1K$
$= 1K\Omega$

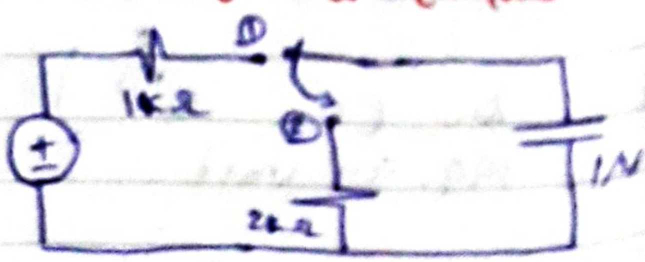


$\tau = R_{th} \times C$
$= 2 \times 10^{3} \times 3 \times 10^{-6}$
$= 6$ msec.



$V_c = V_{th}(1 - e^{-\frac{t}{\tau}})$
$= 4\left(1 - e^{-\frac{4 \times 10^{-3}}{6 \times 10^{-3}}}\right)$
$= 1.95$ Volt

Exa:-
For the cct shown of circuit if the switch was at position (1) for along time, then it is moved to position (2) at $t = 0$, Find V_c at $t = 1$ msec



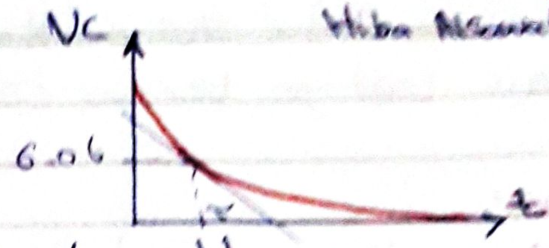
at $t < 0$
$V_{co} = 10V$
for $t > 0$
$V_c = V_{co}\, e^{-\frac{t}{\tau}}$

$\tau = RC$

معنى along time :
يعني وصلت للحالة المستقرة
(يعني ال steady state)

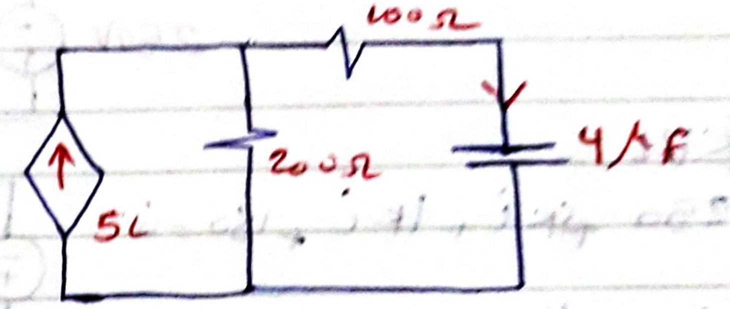
Hiba Masoud

$\tau = 2*10^{3} * 1*10^{-6}$
$= 2\,msec$



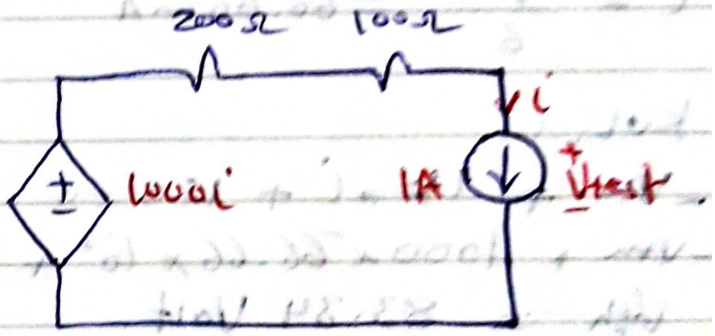
$V_c = 10\, e^{-\frac{1}{2}} \Rightarrow V_c = 6.06$ volt

Exa.-
For the cct. shown, if $i(0^+) = 2mA$, find $i(t)$, for $t>0$ & $t = 4\,msec$.



$V_{th} = 0$

$-1000i + 300*1 + V_{test} = 0$
$-1000*1 + 300 + V_{test} = 0$
$V_{test} = 700$ Volt



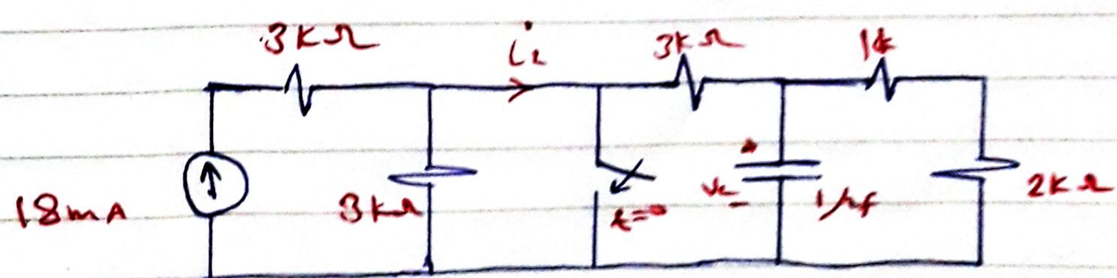
$R_{th} = \frac{V_{test}}{1}$

$= 700\,\Omega$

$\tau = R_{th}\, C$
$= 700 * 4*10^{-6} \Rightarrow \tau = 2.8\,msec$

$i_c(t) = i_o\, e^{-\frac{t}{\tau}}$
$= 2*10^{-3}\, e^{-\frac{4*10^{-3}}{2.8*10^{-3}}}$
$= 8.34\,mA$

H.W.-



if the switch is closed at t=0 after being open for along time, find the voltage of capacitor then sketch this voltage versus time

Hiba Alsawaf

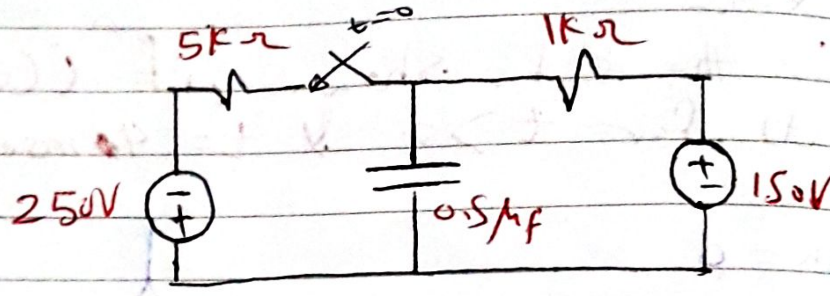
Exa:-

The switch closed at $t=0$, after being open for a several minutes. Find $V_c(t)$ for $t>0$

$V_c(0) = 150$ volt

$= 5k // 1k$

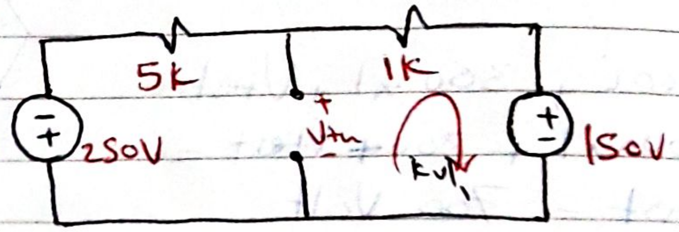
$= 0.833 k\Omega$



KVL:-

$250 + 5k\,i + 1k\,i + 150 = 0$

$i = \frac{400}{6} = 66.66 mA$



$\tau = R_{th} * C$

$= 0.833 * 10^{3} * 0.5 * 10^{-6}$

$= 0.416 msec$

KVL:-

$-V_{th} + 1k * i + 150 = 0$

$-V_{th} + 1000 * 66.66 * 10^{-3} + 150 = 0$

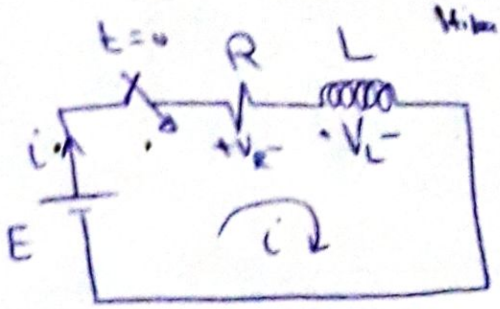
$V_{th} = 83.34$ Volt

$$V_c(t) = V_{c0}\, e^{\frac{-t}{\tau}} + V_{th}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$= 150\, e^{\frac{-t}{0.416 * 10^{-3}}} + 83.34\left(1 - e^{\frac{-t}{0.416 * 10^{-3}}}\right)$$

Transient in R-L Circuits

Mi... Alsaaf



L: Inductance (Henry)

Applying KVL:-

$$-E + V_R + V_L = 0$$

$$-E + iR + L\frac{di}{dt} = 0$$

$$iR - E = -L\frac{di}{dt}$$

$$i - \frac{E}{R} = -\frac{L}{R}\frac{di}{dt}$$

$$\int \frac{di}{i - \frac{E}{R}} = \int \frac{-R}{L}\,dt$$ Integrate both sides

$$\ln\left(i - \frac{E}{R}\right) + K_1 = \frac{-R}{L}t + K_2$$

$$\ln\left(i - \frac{E}{R}\right) = \frac{-R}{L}t + \underbrace{(K_2 - K_1)}_{K}$$

$$i - \frac{E}{R} = e^{-\frac{R}{L}t + K}$$

$$i = \frac{E}{R} + e^{-\frac{R}{L}t} \cdot e^{K}$$, Let $K_3 = e^{K}$

$$i(t) = \frac{E}{R} + K_3 e^{-\frac{R}{L}t}$$

* To Find K_3 from initial conditions

at $t=0$, $i(t) = i(0)$

$$i(0) = \frac{E}{R} + K_3 e^{0} \Rightarrow K_3 = i(0) - \frac{E}{R}$$

$$i_L(t) = \frac{E}{R} + \left(i(0) - \frac{E}{R}\right)e^{\frac{-t}{L/R}}$$

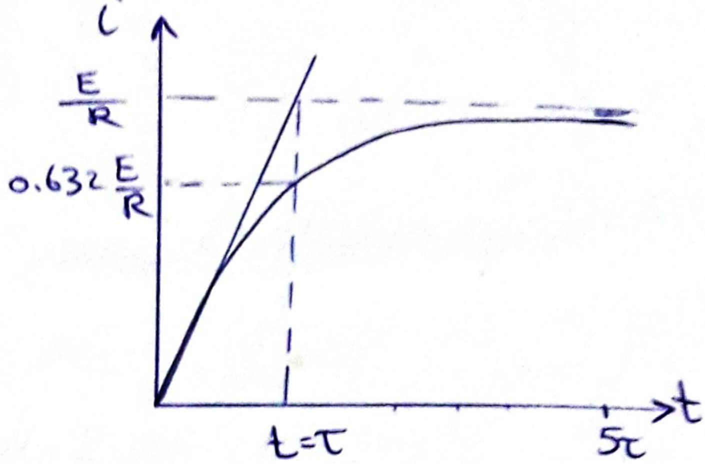
$$i_L(t) = i(0)\, e^{\frac{-t}{\tau}} + \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$$

$$\tau = \frac{L}{R} = \frac{\text{Henry}}{\text{ohm}} = \text{second}$$, τ: Time constant

at $t=0$, $i(0)=0$

Hiba Ahmad

$$\boxed{i_L(t)=\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right)} \quad \text{for} \quad 0<t<\infty$$



at $t=0$, $i_L=0$

at $t=\infty$, $i_L=\frac{E}{R}$

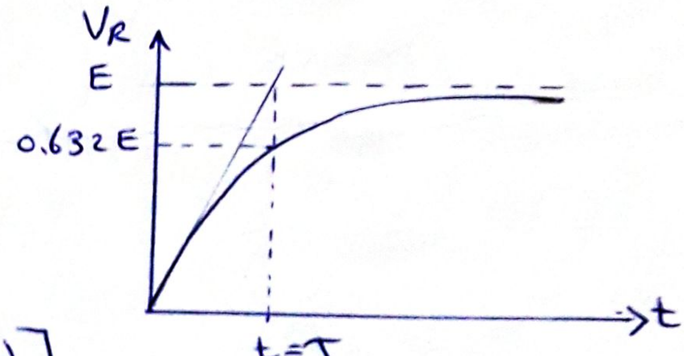
at $t=\tau$, $i_L=0.632\frac{E}{R}$

at $t=5\tau$

$i_L=0.993\frac{E}{R}$

$$V_R=i*R$$

$$=\frac{E}{\cancel{R}}\left(1-e^{\frac{-t}{\tau}}\right)*\cancel{R} \Rightarrow \boxed{V_R=E\left(1-e^{\frac{-t}{\tau}}\right)}$$



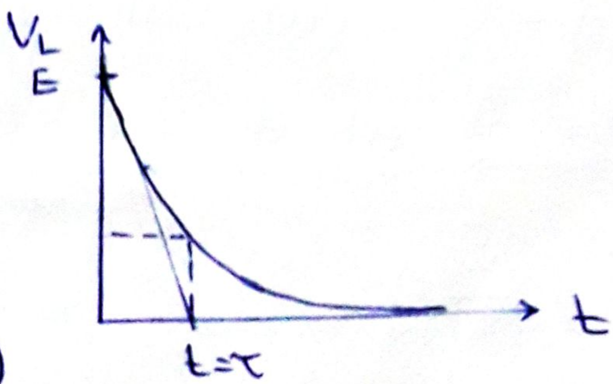
$$V_L=L\frac{di}{dt}$$

$$=L\frac{d\,\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right)}{dt}$$

$$=L*\frac{E}{R}\left[0-e^{\frac{-t}{\tau}}*\left(-\frac{1}{\tau}\right)\right]$$

$$=\frac{\cancel{L}}{\cancel{R}}E\,e^{\frac{-t}{\tau}}*\frac{1}{\cancel{\tau}} \quad , \text{ Where } \tau=\frac{L}{R}$$

$$\boxed{V_L=E\,e^{\frac{-t}{\tau}}}$$



$$P_R=V_R*i$$

$$=E\left(1-e^{\frac{-t}{\tau}}\right)*\frac{E}{R}\left(1-e^{\frac{-t}{\tau}}\right)$$

$$\boxed{P_R=\frac{E^2}{R}\left(1-e^{\frac{-t}{\tau}}\right)^2} \quad \text{Watt}$$

Hiba Alsawad

$P_L = V_L * i_L$

$= E\, e^{\frac{-t}{\tau}} * \frac{E}{R}\left(1 - e^{\frac{-t}{\tau}}\right)$

$\boxed{P_L = \frac{E^2}{R}\left(e^{\frac{-t}{\tau}} - e^{\frac{-2t}{\tau}}\right)}$ Watt

$P_T = P_{source} = P_R + P_L$

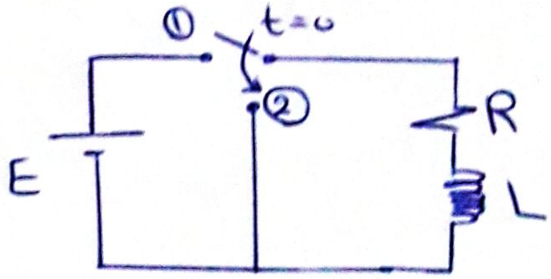
$P_T = \frac{E^2}{R}\left(1 - e^{\frac{-t}{\tau}}\right)^2 + \frac{E^2}{R}\left(1 - e^{\frac{-t}{\tau}}\right) * e^{\frac{-t}{\tau}}$

$\boxed{P_T = \frac{E^2}{R}\left(1 - e^{\frac{-t}{\tau}}\right)}$ Watt

$W_{stured} = \int_0^{\infty} P_L \cdot dt$ $t = 0 \to \infty$

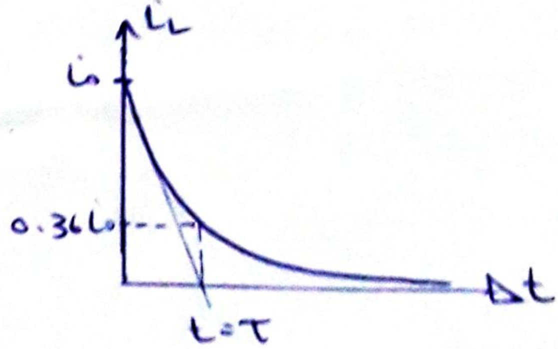
$\boxed{W_L = \frac{1}{2} L I^2}$ $t = 0 \to \infty$ Joule , where $I = \frac{E}{R}$

Current Decay:-



$\boxed{i_L = i(0)\, e^{\frac{-t}{\tau}}}$

where $i(0) = \frac{E}{R}$

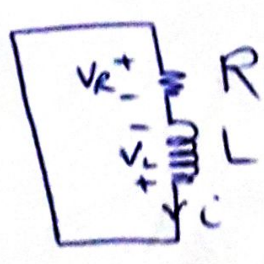


$V_R = i_L * R$

$= i_0\, e^{\frac{-t}{\tau}} * R$

$= \frac{E}{\cancel{R}}\, e^{\frac{-t}{\tau}} * \cancel{R}$

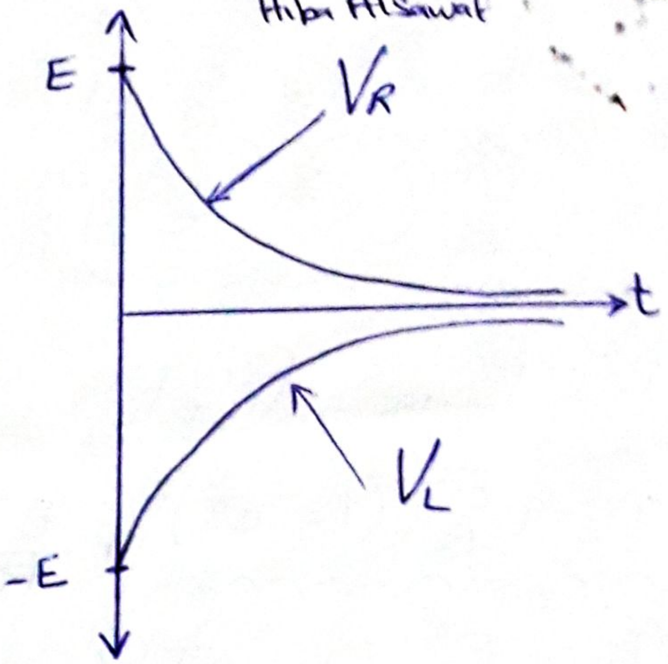
$\boxed{V_R = E\, e^{\frac{-t}{\tau}}}$



[redacted]
Hiba Alsawaf

$$V_L = -L \frac{di}{dt}$$

$$\boxed{V_L = -E\, e^{-\frac{t}{\tau}}}$$ volt



$$P_L = V_L * i$$

$$= -E\, e^{-\frac{t}{\tau}} * \frac{E}{R}\, e^{-\frac{t}{\tau}}$$

$$\boxed{P_L = -\frac{E^2}{R}\, e^{-\frac{2t}{\tau}}}$$ watt

$$P_R = V_R * i$$

$$= E\, e^{-\frac{t}{\tau}} * \frac{E}{R}\, e^{-\frac{t}{\tau}}$$

$$\boxed{P_R = \frac{E^2}{R}\, e^{-\frac{2t}{\tau}}}$$

(1)

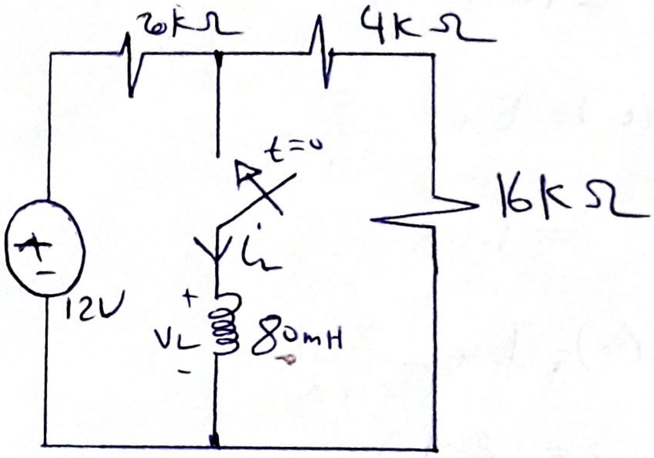
Exa:-

Hiba Alsawaf

For the cct. shown:-

a- Find the mathematical expression for the transient behavior of the current i_L & Voltage V_L after the closing of the switch.

b. Draw the resultant wave form for each. (H.W)

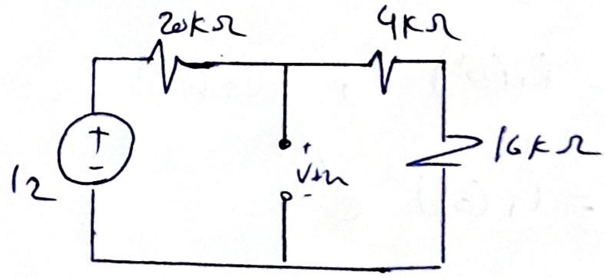


$$V_{th} = 12 * \frac{(4+16)}{4+20+16}$$

$$= 6\ V$$

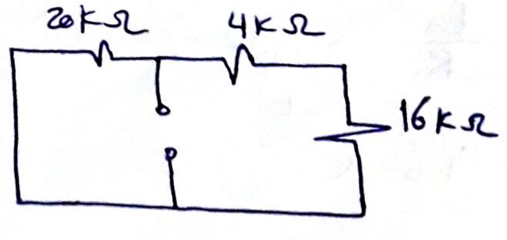
OR

$$I = \frac{12}{20+4+16} = 0.3mA$$



$$-12 + 20 * 10^3 * 0.3 * 10^{-3} + V_{th} = 0$$

$$V_{th} = 6\ V$$



$$R_{th} = 20K // (4+16)k$$

$$= 20k // 20k$$

$$= 10\ K\Omega$$

$$i_L(t) = \frac{V_{th}}{R_{th}} (1 - e^{\frac{-t}{\tau}}) \quad , \quad \tau = \frac{L}{R} = \frac{80*10^{-3}}{10*10^3} = 8\ \mu sec$$

$$i_L(t) = \frac{6}{10*10^3} (1 - e^{\frac{-t}{8*10^{-6}}}) \text{ for } t > 0$$

$$= 0.6*10^{-3} (1 - e^{\frac{-t}{8*10^{-6}}}) \text{ for } t > 0$$

$$V_L(t) = V_{th}\ e^{-\frac{t}{\tau}}$$

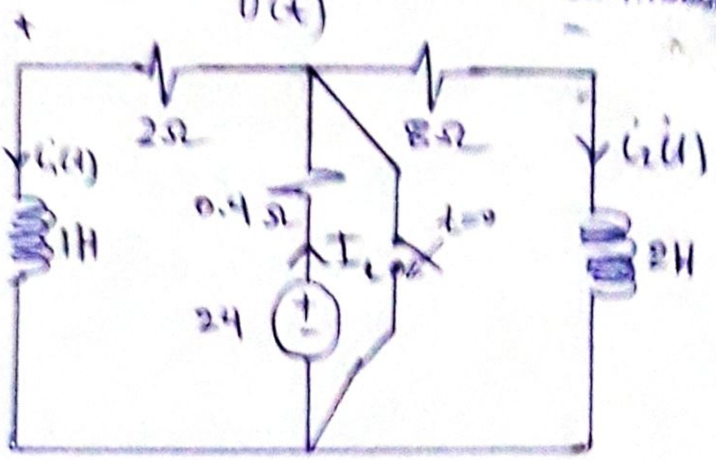
$$= 6\ e^{-\frac{t}{8*10^{-6}}} \text{ for } t > 0$$

Ex:-

Find $i_1(t)$ and $i_2(t)$
then find $v(t)$ for $t>0$.

(2)

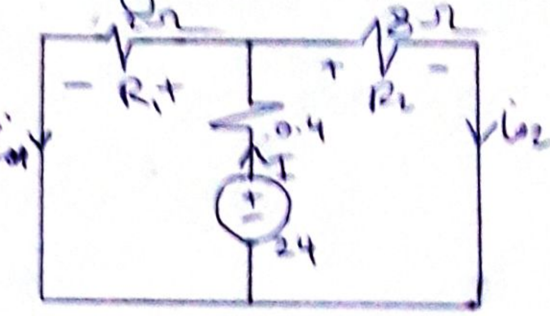
Hiba Alawad



$$I_t = \frac{24}{0.4+\frac{2*8}{2+8}}$$

$$=12A$$

At steady state before closing the switch



$$i_1(0^-) = I * \frac{8}{2+8}$$

$$=9.6A$$

$$i_2(0^-) = I * \frac{2}{2+8}$$

$$= 2.4A$$

$i_1(0^-) = i_1(0^+)$, $i_2(0^-) = i_2(0^+)$ (وتيار الملف لا يمكن ان يتغير فجأة)

$i_1(t) = i_1(0^+)\, e^{-\frac{t}{\tau_1}}$, $i_2(t) = i_2(0^+)\, e^{-\frac{t}{\tau_2}}$

$\tau_1 = \frac{L_1}{R_1} = \frac{1}{2} = 0.5\,\text{sec.}$, $\tau_2 = \frac{L_2}{R_2} = \frac{2}{8} = 0.25\,\text{sec}$

$i_1(t) = 9.6\, e^{-\frac{t}{0.5}}$ A , $i_2(t) = 2.4\, e^{-\frac{t}{0.25}}$ Amper

$$v(t) = 8\, i_2(t) - 2\, i_1(t)$$

$$= 8 * 2.4\, e^{-\frac{t}{0.25}} - 2 * 9.6\, e^{-\frac{t}{0.5}}$$

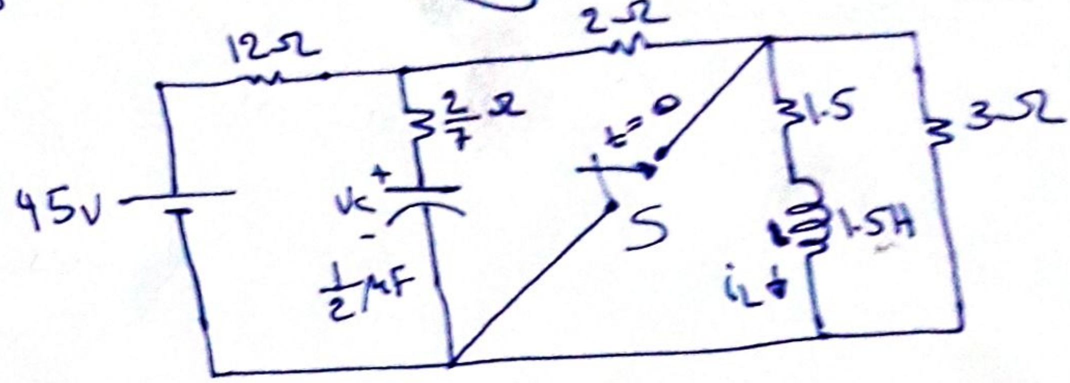
$$v(t) = 19.2\, e^{-4t} - 19.2\, e^{-2t} \text{ Volt}$$

(4)

Exa:-

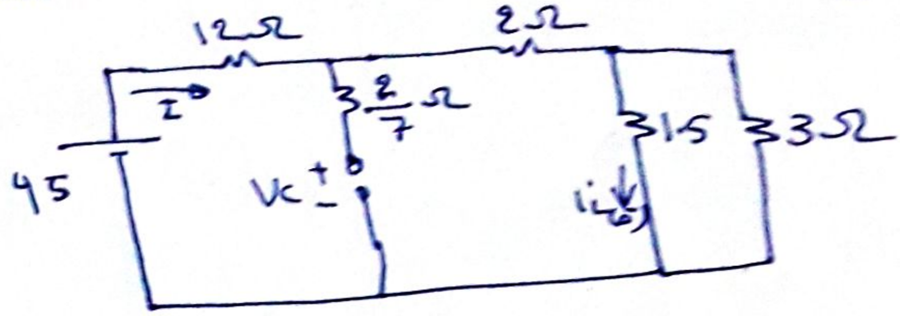
Find $V_c(t)$ & $i_L(t)$ if the switch is closed at $t=0$ after being opened for along time

Hiba Alsaraf



Solution:-

For steady state the circuit is as shown in the fig(a)



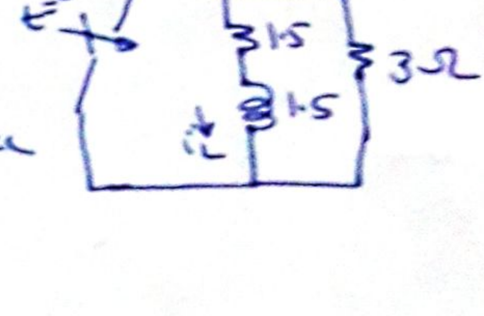
$$I = \frac{45}{12+2+(1.5//3)}$$

$$I = 3A$$

$V_c(0^-) = +45 - 12*I \Rightarrow V_c(0^-) = 45 - 12*3 = 9$ Volt

$i_L(0^-) = I * \frac{3}{1.5+3} \Rightarrow i_L(0^-) = 3* \frac{3}{4.5} = 2A = i_L(0^+)$

After the switch is closed $i_L(t)$ will decay through 1.5Ω Resistance only so—

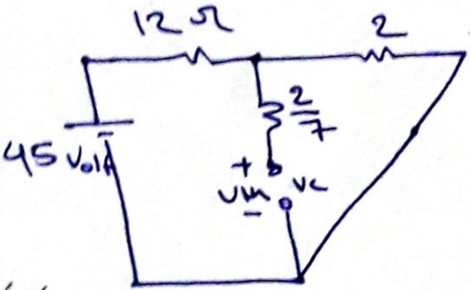


$i_L(t) = i_L(0^+)\, e^{-t/\tau_1}$, $\tau_1 = \frac{L}{R_1} = \frac{1.5}{1.5} = 1$ sec

$i_L(t) = 2\, e^{-t}$

-Hiba Alsawaf-

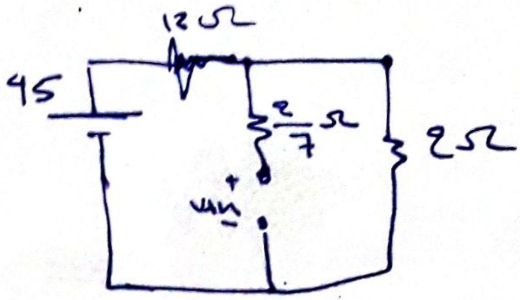
For $V_c(t)$ we find V_{th} & R_{th} at C



$$R_{th} = 12 // 2 + \frac{2}{7}$$

$$= \frac{12 * 2}{12 + 2} + \frac{2}{7} = 2\Omega$$

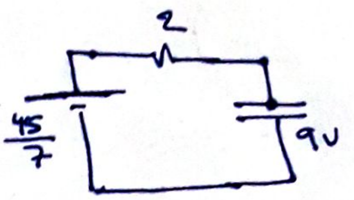
$$V_{th} = \frac{45}{12+2} * 2 = \frac{45}{7} \text{ volt}$$



$$\tau_2 = R_{th} * C = 2 * \frac{1}{2} * 10^{-6} = 1 \mu sec$$

$$V_c(t) = V_{co}\, e^{\frac{-t}{\tau_2}} + E_{th}\left(1 - e^{\frac{-t}{\tau_2}}\right)$$

$$= 9\, e^{\frac{-t}{10^{-6}}} + \frac{45}{7}\left(1 - e^{\frac{-t}{10^{-6}}}\right)$$



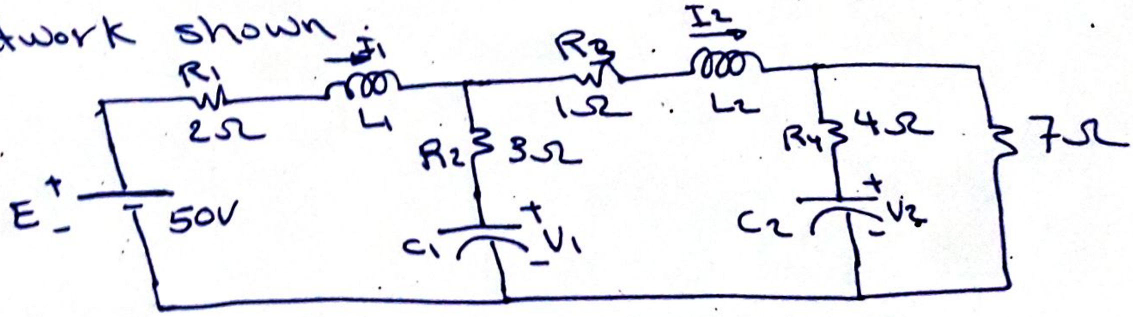
$$= 9\, e^{-10^{6}t} + 6.4\left(1 - e^{-10^{6}t}\right)$$

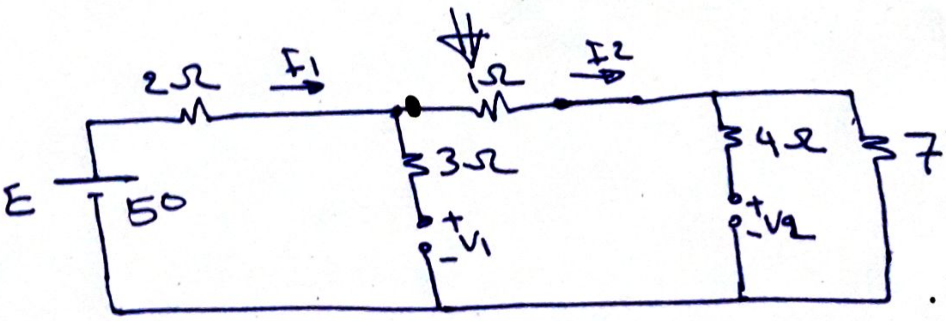
$$= 9\, e^{-10^{6}t} + 6.4 - 6.4\, e^{-10^{6}t}$$

$$V_c(t) = 6.4 + 2.6\, e^{-10^{6}t} \text{ Volt}$$

Exa:-

Find the currents I_1 and I_2 and the voltages V_1 and V_2 for the network shown.





$\cdot I_1 = I_2$

-Hiba Alsaoad-

$$I_1 = \frac{50}{2+1+7} = \frac{50}{10} = 5A$$

$$V_2 = I_2 * 7 = 5 * 7 = 35 \text{ Volt}$$

$$-50 + 5 * 2 + V_1 = 0$$

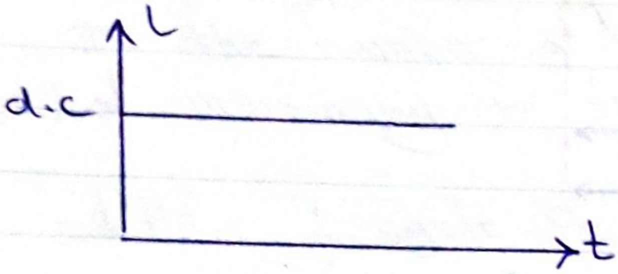
$$V_1 = 40 \text{ Volt}$$

Alternating Current (A.C)

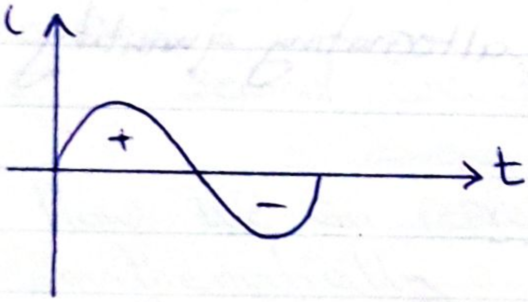
Hiba Alsawaf

Currents can be devided into two types:-

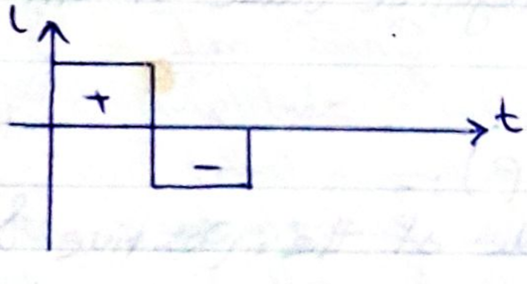
1. Direct current (D.C) التيار المستمر (ذو الاتجاه الواحد)



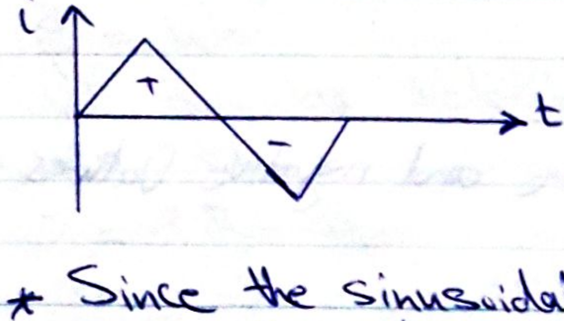
2. Alternating current (A.C) التيار المتناوب (ذو الاتجاهين)



Sinusoidal wave



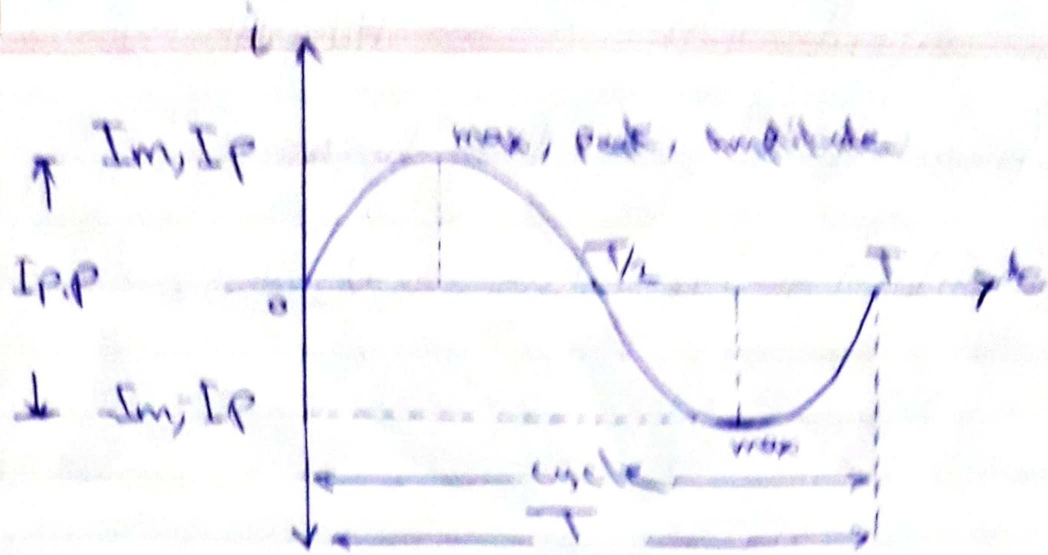
Square wave



Triangular wave

Alternating wave forms

* Since the sinusoidal A.c has wide applications in our Life, it is worth to study sinusoidal A.c in detail.



* There are some terms can be used in A.C.:-

1. Wave form :- شكل الموجة
A graph that shows how an alternating quantity varies with time.

2. Amplitude or peak value:-
قمة الموجة أو سعة الموجة
The maximum positive or negative value of an alternating quantity.

3. Peak to peak value (P.P):-
The sum of the magnitude of the positive & negative peaks.

4- Cycle :-
One complete set of positive and negative values of an alternating quantity.

5- Period (T):- فترة الموجة   Hiba AlSawaf

Is the time required by an alternating quantity to complete one cycle.

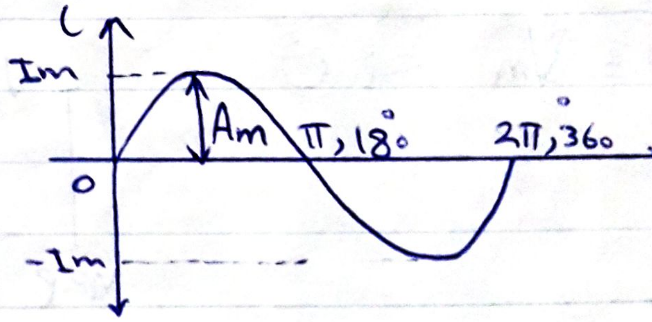
6- Frequency (f):- التردد
Is the number of cycle per second. The unit of measuring for frequency is the hertz (Hz)

1 Hz = 1 cycle per second
$= \frac{C}{S}$

$f = \frac{1}{T}$ (Hz or cycle/second
constant
T : second (sec.)

* How we can represent sinusoidal wave forms mathematically?

$i = \underbrace{Im}_{\text{Amplitude (Am)}} \underbrace{\sin\theta}_{\text{wave form}}$



i : instantenous value القيمة الآنية
θ : angular displacement الازاحة الزاوية
θ = Wt
W: angular Velocity السرعة الزاوية
W = 2πf
$= 2\pi * \frac{1}{T}$

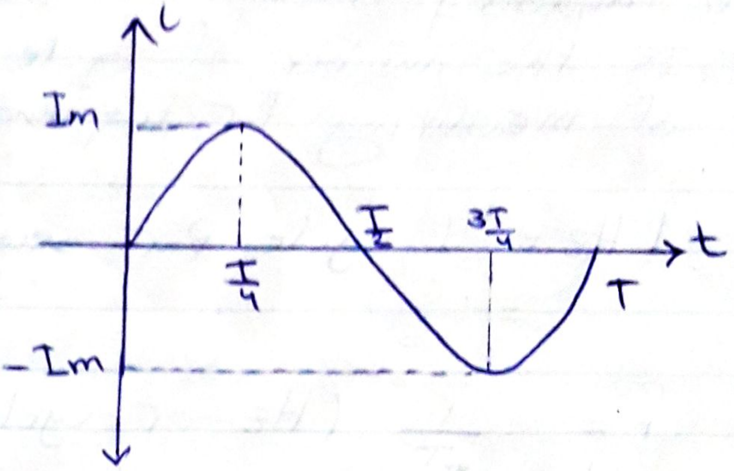
i = Im sinWt

$i = I_m \sin(2\pi f t)$ Hiba Alsawaf

$i = I_m \sin \frac{2\pi t}{T}$

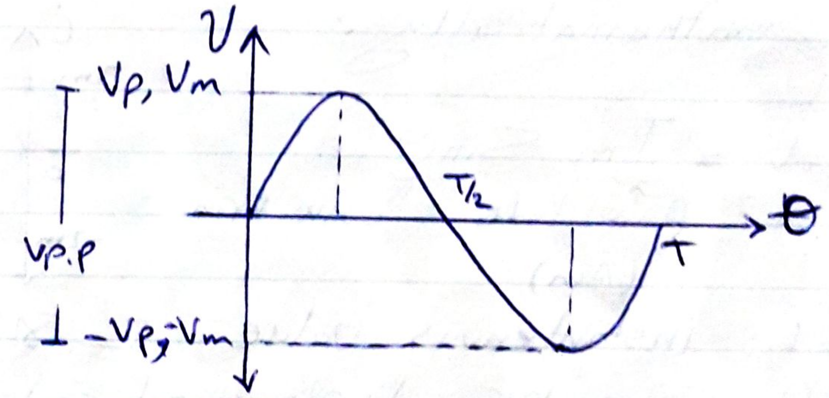
* If we want to draw i with time.

t	i
0	0
$\frac{T}{4}$	I_m
$\frac{T}{2}$	0
$\frac{3T}{4}$	$-I_m$
T	0

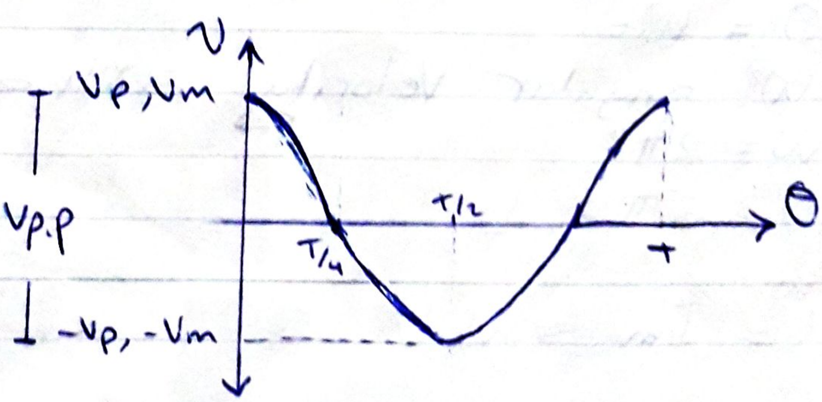


* For the Voltage :-

$v = v_m \sin\theta$



$v = v_m \cos\theta$

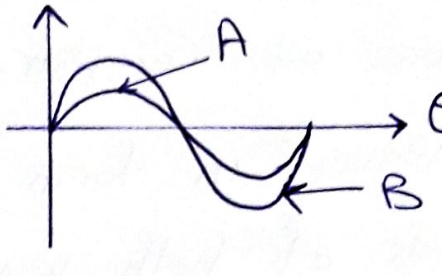


Phase of Alternating current طور التيار المتناوب

-Hiba Alsawaf-

A in phase with B

Phase angle = 0



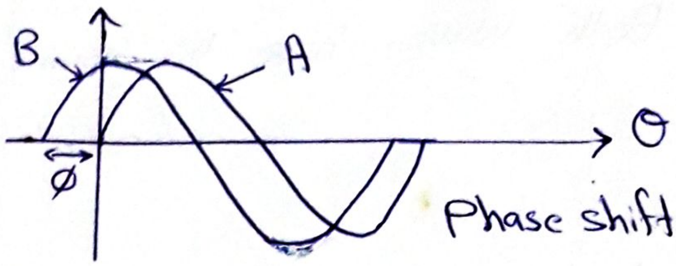
$\boxed{\theta = \omega t}$

B leads (يتقدم) A by ϕ

A lags (يتأخر) B by ϕ

$A = V_m \sin\theta$

$B = V_m \sin(\theta + \phi)$



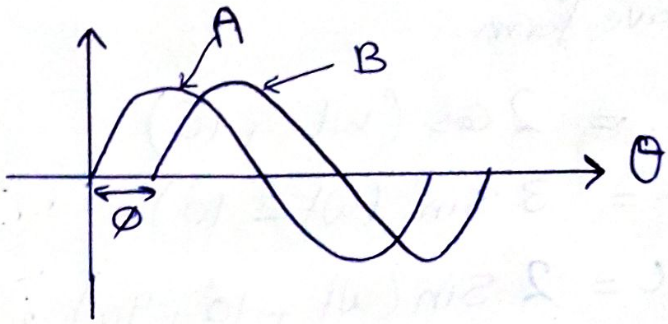
Phase shift = ϕ

$A = V_m \sin\theta$

$B = V_m \sin(\theta - \phi)$

A Leads B by ϕ

B Lags A by ϕ



* Afew additional geometric relations that may prove useful in application involving Sines or Cosines in phase relationships are the following:-

$\sin\theta = \cos(\theta - 90^\circ)$

$-\sin\theta = \sin(\theta \pm 180^\circ)$

$-\cos\theta = \cos(\theta \pm 180^\circ)$

$\cos\theta = \sin(\theta + 90^\circ)$

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

* To compare the phase between two sinusoidal waveforms mathematically.

-Hiba Alsawaf-

* Conditions of Comparison

1- Both waves in form of sine or cosine.
2- Amplitude of both wave (positive).
3- Both waves have the same frequency (f or ω).

Exa:-

What is the phase relationship between the sinusoidal wave form.

$i = 2\cos(\omega t + 10^\circ)$

$V = 3\sin(\omega t - 10^\circ)$

$i = 2\sin(\omega t + 10^\circ + 90^\circ)$

$\left.\begin{array}{l} i = 2\sin(\omega t + 100^\circ) \\ V = 3\sin(\omega t - 10^\circ) \end{array}\right\}$ i leads V by $[100-(-10)] = 110^\circ$

Exa:-

$i = -\sin(\omega t + 30^\circ)$

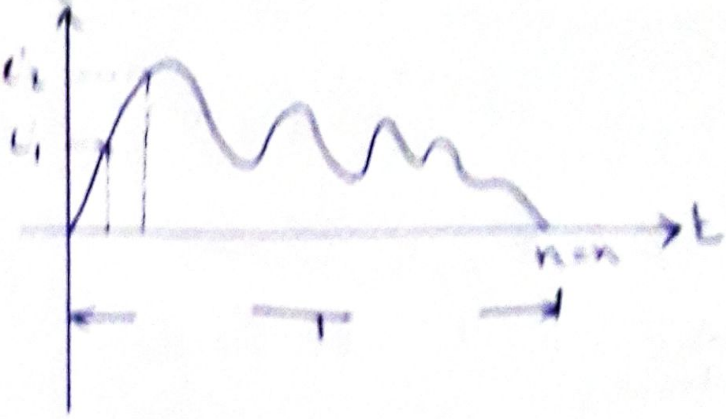
$V = 2\sin(\omega t + 10^\circ)$

$i = \sin(\omega t + 30^\circ - 180^\circ) \Rightarrow i = \sin(\omega t - 150^\circ)$

V leads i by $[10-(-150^\circ)] = 160^\circ$

• Effective Value (Root mean square value):-

القيمة الفعالة — جذر متوسط المربع — r.m.s



$$I_{average} = \frac{(i_1+i_2+ --- + i_n)}{n}$$

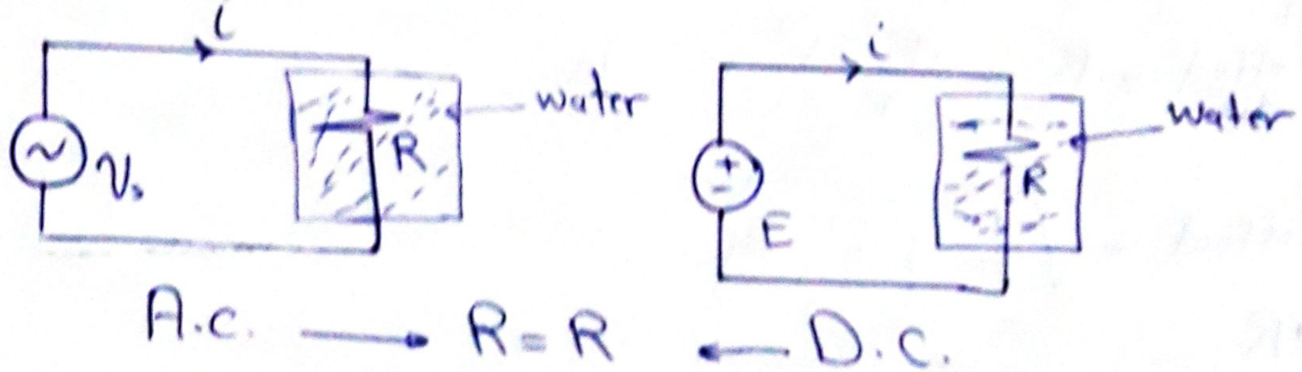
Multiply by $\frac{\Delta t}{\Delta t}$

$$I_{av} = \frac{(i_1+i_2+ --- + i_n)\Delta t}{n \Delta t}$$

$$T = n \Delta t$$

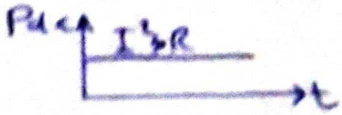
$$I_{av} = \frac{(i_1+i_2+ --- + i_n)\Delta t}{T} \quad \text{if } \Delta t \simeq 0$$

$$I_{av} = \int_0^T \frac{i}{T} dt \Rightarrow I_{av} = \frac{1}{T}\int_0^T i \, dt$$



A.C. ⟶ R = R ⟵ D.C.

* إذا كانت كمية الحرارة المتولدة من مرور تيار في المقاومة هي نفسها وتكون مساوية لكمية الحرارة المتولدة من مرور التيار المتناوب في المقاومة عندئذ يمكن القول ان التيار قيمة فعالة (القيمة الفعالة للتيار).



$$P_{av_{A.c}} = P_{av_{d.c}} \quad , \quad I_{effect_{A.c}} = I_{effect_{D.c}}$$

-Hiba Alsawaf-

* To Find $P_{av\ d.c}$:-

$$\underset{\text{instanteneous}}{P_{d.c}} = i^2_{d.c} * R$$

آني

$$P_{av} \text{ in } d.c = I^2_{effect} * R$$

* To Find P_{av} in A.c :-

$$\underset{\text{instanteneous}}{P_{a.c}} = i^2 . R$$

$$P_{av\ a.c} = \frac{1}{T} \int_0^T P_{a.c}\, dt$$

$$= \frac{1}{T} \int_0^T i^2 . R\, dt$$

$$= \frac{R}{T} \int_0^T i^2 . dt$$

Since $P_{av\ d.c} = P_{av\ a.c}$

$$I^2_{effect\ d.c} * \cancel{R} = \frac{\cancel{R}}{T} \int_0^T i^2 . dt$$

$$I_{effect} = \sqrt{\frac{1}{T} \int_0^T i^2\, dt}$$

OR

$$I_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T i^2\, dt}$$

root mean square value

* By the same way :-

$$V_{r.m.s} = \sqrt{\frac{1}{T} \int_0^T v^2 . dt}$$

Hiba Alnasrallah

[صو لنيةاوتياد]

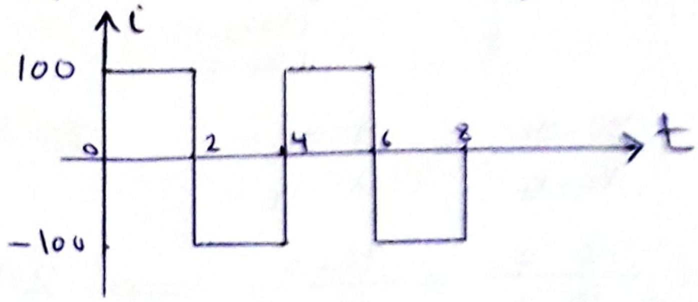
* Form Factor (kf) = $\frac{\text{r.m.s Value}}{\text{average Value}}$

عامل الشكل

* عامل الشكل لجميع الموجات يكون اكبر من الواحد ماعدا الموجة المربعة يكون مساوي للواحد.

Exa:-

For the wave form shown, Find form factor (kf)



$$kf = \frac{\text{r.m.s Value}}{\text{average Value}}$$

$$I_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T i^2 \, dt}$$

$$= \sqrt{\frac{1}{4}\int_0^4 (100)^2 \, dt} = \sqrt{\frac{(100)^2}{4}[t]_0^4} \Rightarrow I_{r.m.s} = 100$$

$$I_{av} = \frac{1}{T}\int_0^T i \, dt$$

$$= \frac{1}{4}\left[\int_0^2 100 dt + \int_2^4 100 \, dt\right]$$

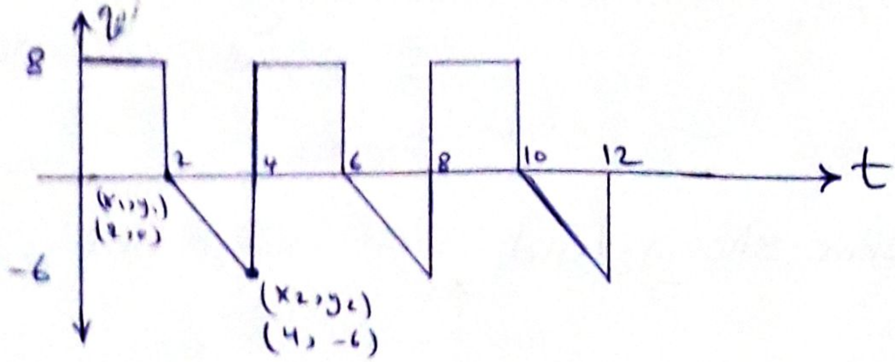
$$= \frac{100}{4}\left[[t]_0^2 + [t]_2^4\right]$$

$$= \frac{100}{4}[2-0+4-2] \Rightarrow I_{av} = 100$$

$$kf = \frac{100}{100} \Rightarrow kf = 1$$

Exa.- - Hiba Alsawaf -

For the Voltage wave form shown, find the effective valu ($V_{r.m.s}$), then find the form factor (kf).



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1}$$

$$\frac{-6-0}{4-2} = \frac{v-0}{t-2} \Rightarrow 2v = -6t + 12 \Rightarrow V = -3t + 6$$

$$v = \begin{cases} 8 & 0 < t < 2 \\ -3t+6 & 2 < t < 4 \end{cases}$$

$$V_{av} = \frac{1}{T}\int_0^T v\,dt$$

$$= \frac{1}{4}\left[\int_0^2 8\,dt + \int_2^4 (-3t+6)\,dt\right]$$

$$= \frac{1}{4}\left[[8t]_0^2 + \left[-\frac{3t^2}{2} + 6t\right]_2^4\right]$$

$$= \frac{1}{4}\left[16 + \left[\left(\cancel{-\frac{3*4^2}{2} + 6*4}\right)^{0} - \left(-\frac{3*2^2}{2} + 6*2\right)\right]\right]$$

$$= \frac{1}{4}[16 + (-6)] \Rightarrow V_{av} = 2.5$$

$$V_{r.m.s} = \sqrt{\frac{1}{4}\left[\int_0^2 (8)^2\,dt + \int_2^4 (-3t+6)^2\,dt\right.}$$

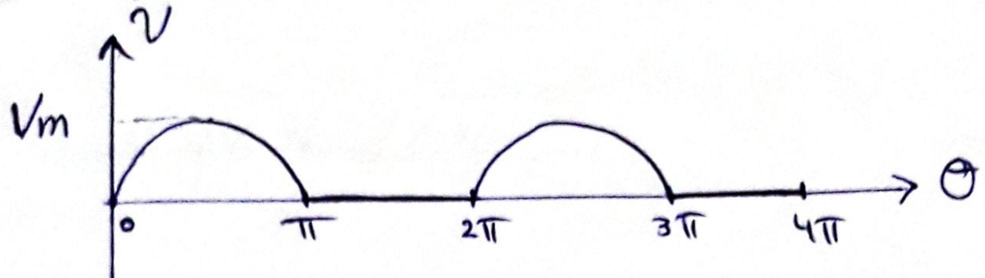
$$= \sqrt{\frac{1}{4}\left[[64t]_0^2 + \left[\frac{9t^3}{3} - \frac{36t^2}{2} + 36t\right]_2^4\right.} = 6.103$$

$$Kf = \frac{6.103}{2.5} = 2.44$$

-Hiba Alsaud-

Exa:-

For the half rectified wave form shown, find form factor.



Solution:-

$$v = \begin{cases} V_m \sin\theta & 0 < \theta < \pi \\ 0 & \pi < \theta < 2\pi \end{cases}$$

$$V_{av} = \frac{1}{T}\int_0^T v \, dt$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta \, d\theta + \cancel{\int_{\pi}^{2\pi} 0 \, d\theta}\right]$$

$$= \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-(-1-1)\right] \Rightarrow V_{av} = \frac{V_m}{\pi}$$

$$V_{r.m.s} = \sqrt{\frac{1}{T}\int_0^T v^2 \, dt}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta \, d\theta + \cancel{\int_{\pi}^{2\pi} 0 \, d\theta}\right]}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \frac{1}{2}(1-\cos 2\theta) \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{4\pi}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi} * \pi} \Rightarrow V_{r.m.s} = \frac{V_m}{2}, \quad K_f = \frac{\pi}{2} = 1.57$$

(3.14)

* $r.m.s$ = [illegible]

$$I_{r.m.s} = \frac{I_m}{\sqrt{2}} \quad \text{or} \quad V_{r.m.s} = \frac{V_m}{\sqrt{2}}$$

Phasor Diagram:- [illegible]

Exa:-

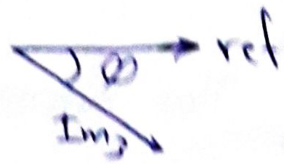
$i = I_{m_1} \sin \omega t$

Exa:-

$i = I_{m_2} \sin(\omega t + \phi)$

Exa:-

$i = I_{m_3} \sin(\omega t - \phi)$



Exa:-

For the following current, find i_{total} By:-

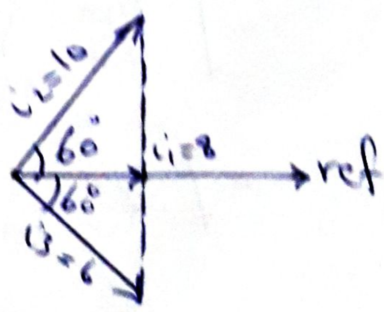
1. phasor diagram method.
2. Pure mathematical method.

$i_1 = 8 \sin 1000t$

$i_2 = 10 \sin(1000t + 60)$

$i_3 = 6 \sin(1000t - 60)$

1. By phasor diagram method



- $X_{component} = 8 + 10 \cos 60 + 6 \cos(-60)$

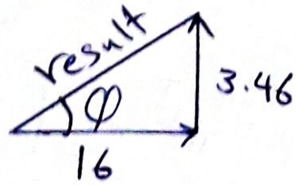
$$= 8 + 10 * \frac{1}{2} + 6 * \frac{1}{2}$$

$$= 16 \text{ Amper}$$

— Hiba Alsawaf —

$$Y_{component} = 0 + 10 \sin 60 + 6 \sin(-60)$$
$$= 10 * \frac{\sqrt{3}}{2} - 6 * \frac{\sqrt{3}}{2}$$
$$= 3.46 \text{ Amper}$$

$$I_{result} = \sqrt{(16)^2 + (3.46)^2}$$
$$= 16.36 \text{ Amper.}$$

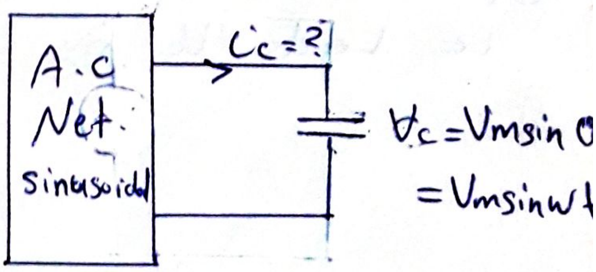


$$i_{total} = 16.36 \sin(1000t + \phi)$$

$$\phi = \tan^{-1} \frac{3.46}{16} = 12.2^\circ$$

$$i_{total} = 16.36 \sin(1000t + 12.2^\circ) \text{ Amper.}$$

1. A.c through capacitance:-



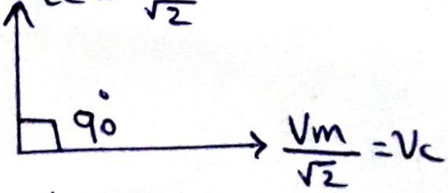
$$i_c = c \frac{dV_c}{dt}$$
$$= c \frac{d}{dt}(V_m \sin wt) \cdot \frac{w}{w}$$
$$= c\,(V_m \cos wt) * w$$
$$= \frac{V_m}{\frac{1}{wc}} * \cos wt \Rightarrow i_c = \frac{V_m}{\frac{1}{wc}} \sin(wt + 90^\circ)$$

$\frac{1}{wc}$: Capacitive reactance (Ω) الرادة السعوية

$$X_c = \frac{1}{wc} = \frac{1}{2\pi f c}$$

$$\therefore i_c = \frac{V_m}{X_c} \sin(wt + 90^\circ)$$

$$i_c = I_m \sin(wt + 90^\circ) \quad , \quad I_m = \frac{V_m}{X_c}$$



- Nima Mosaad -

$P_c = V_c * i_c$

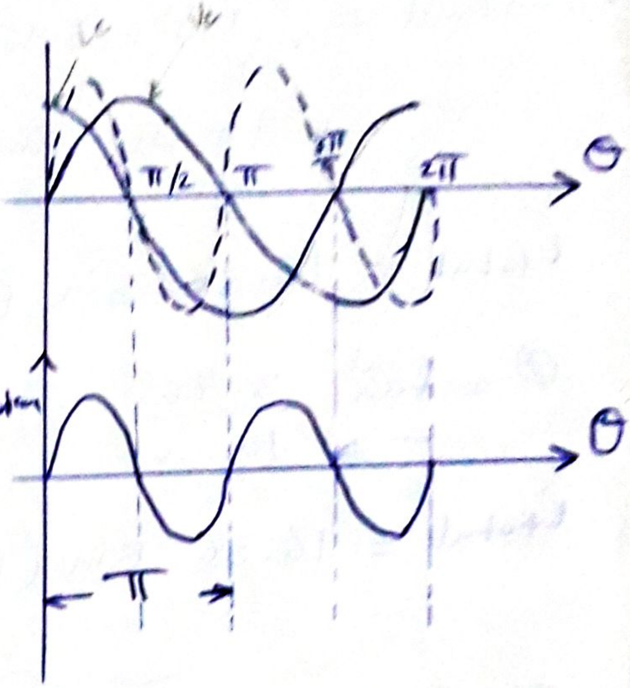
instantenous $= V_m \sin\theta * I_m \sin(\theta + 90^\circ)$

$= V_m I_m \sin\theta \cdot \sin(\theta + 90^\circ)$

$P_{c\,av} = V_m I_m * \frac{1}{\pi} \int_0^{\pi} \sin\theta \cos\theta \, d\theta$

$= \frac{V_m I_m}{\pi} \left[\frac{1}{2} \sin^2\theta \right]_0^{\pi}$

$P_{c\,av} =$ Zero



The average Power in ideal capacitor is Zero. But there is an instantenous Power.

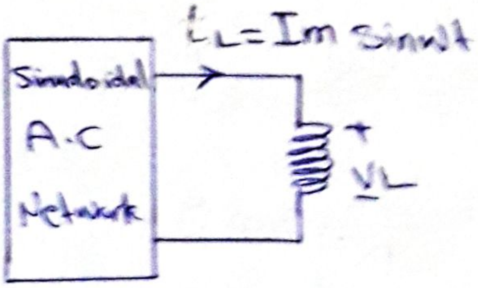
* مقادير الشحنة في I_c & V_c حالة sinusoidal waveform.

* V_c Lags i_c by 90°
or
i_c Leads V_c by 90°, Phase shift $= 90^\circ$.

(1) ②

2: A.C. through inductance :-

-Hiba Alsarraf-



$V_L = L \dfrac{di}{dt}$

$= L \dfrac{d}{dt}(I_m \sin\omega t) * \dfrac{\omega}{\omega}$

$= L\, I_m \cos\omega t * \omega$

$= \omega L\, I_m \sin(\omega t + 90^\circ)$

$V_L = V_m \sin(\omega t + 90^\circ)$, $V_m = \omega * L\, I_m$

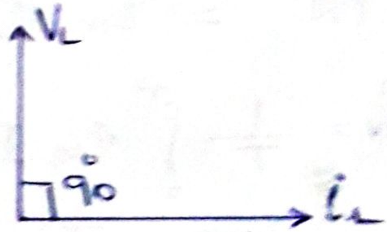
$\underset{\Omega}{X_L} = \omega * L = 2\pi f \underset{\text{Henry}}{L}$, X_L : inductance reactance

الـرادة الحثيـة

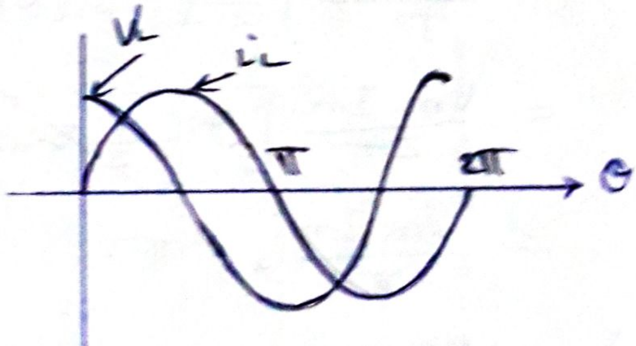
V_L Leads i_L by 90°

OR

i_L Lags V_L by 90°



Phase shift $(\phi) = 90^\circ$

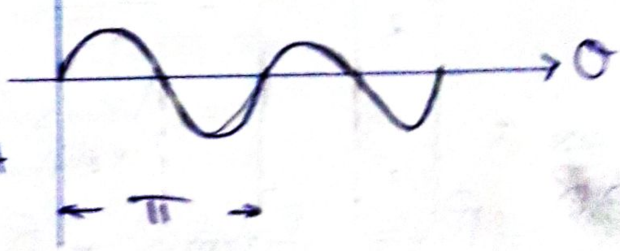


$\underset{\text{instantaneous}}{P_L} = V_L * i_L$

$= V_m I_m \sin\omega t \sin(\omega t + 90)$

$P_{av} = \dfrac{1}{T}\displaystyle\int_0^T P_L \cdot dt$

$= \dfrac{1}{T}\displaystyle\int_0^T V_m I_m \sin\omega t \cdot \sin(\omega t + 90)\, dt$

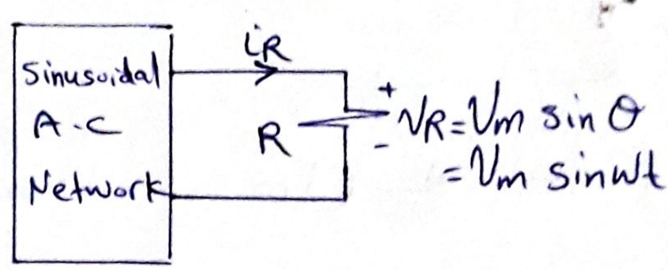


$P_{av} =$ Zero. (الملف لا يستهلك قدرة عند تغذيته بفولتية متناوبة)

* The average power in inductance is zero But there is an instantaneous Power.

3. A.c through Resistance :-

Hiba Alsawaf

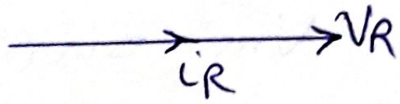


$$i_R = \frac{V_R}{R} \quad \text{(ohms Law)}$$

$$= \frac{V_m}{R} \sin\theta$$

$$i_R = I_m \sin\theta \quad , \quad I_m = \frac{V_m}{R}$$

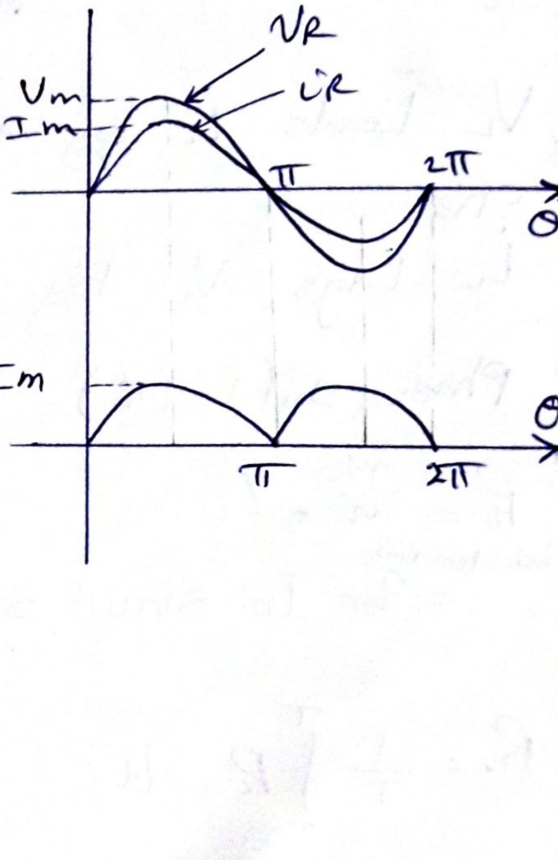
$\therefore$ i_R is inphase with V_R



$$P_{R\,\text{instantaneous}} = V_R * i_R$$

$$= V_m \sin\theta * I_m \sin\theta$$

$$P_R = V_m I_m \sin^2\theta$$



$$P_{av} = \frac{1}{T} \int_0^T V_m I_m \sin^2\theta \, d\theta$$

$$= \frac{V_m I_m}{T} \int_0^{\pi} \frac{1}{2}(1-\cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{2\pi} \left[\theta - \cancel{\frac{1}{2}\sin 2\theta}^{\,0} \right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\cancel{\pi}} * \cancel{\pi}$$

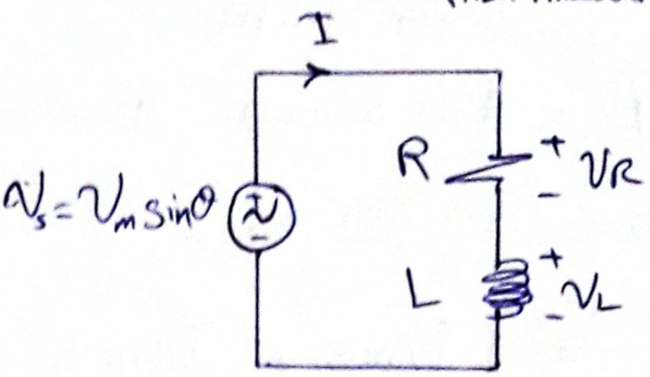
$$P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$= \underset{\text{for Resistance}}{V_{r.m.s}} * \underset{\text{for Resistance}}{I_{r.m.s}}$$

OR $P_{av} = I_{r.m.s}^2 * R$ (watt)

Hiba Alsawaf

AC in RL in series :-



Applying KVL :-

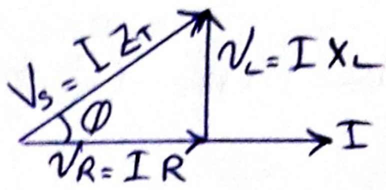
$$-V_s + V_R + V_L = 0$$

$$\vec{V_s} = \vec{V_R} + \vec{V_L}$$

$$V_R = I * R \quad , \quad V_L = I * X_L \quad , \quad X_L = 2\pi f L$$

$$V_s = I * Z_T$$

Z_T : Total impedance



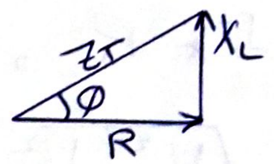
Voltage triangle

$$V_s = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{I^2 R^2 + I^2 X_L^2}$$

$$V_s = I\sqrt{R^2 + X_L^2} \Rightarrow \frac{V_s}{I} = \sqrt{R^2 + X_L^2}$$

$$Z_T = \sqrt{R^2 + X_L^2}$$



Impedance triangle

$$\phi = \tan^{-1}\frac{X_L}{R} \quad \text{OR}$$

$$\phi = \tan^{-1}\frac{V_L}{V_R}$$

In instantaneous forms:-

$$V_s = V_m \sin\theta$$

$$i = I_m \sin(\theta - \phi)$$

* Average Power:-

1) $P_{av} = P_{av_R} + \cancel{P_{av_L}}^{0}$

$$P_{av_R} = I^2 * R$$

$$= I * I * R \Rightarrow P_{av} = \frac{V}{Z} * I * R$$

$$P_{av} = V I * \frac{R}{Z} \Rightarrow P_{av} = V I \cos\phi$$

Hiba Masoud

$\cos\phi$: Power factor

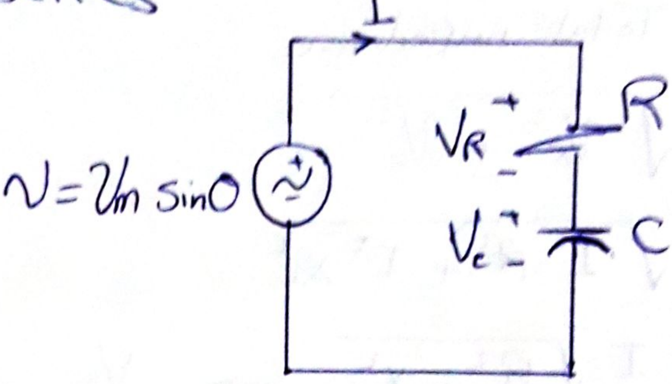
2) $P_i = V_m \sin\omega t \cdot I_m \sin(\omega t - \phi)$

$$P_{av} = \frac{V_m I_m}{2} \cos\phi$$

$$= V_{r.m.s} * I_{r.m.s} \cos\phi$$

$$P_{av} = V I \cos\phi$$

AC in RC in series



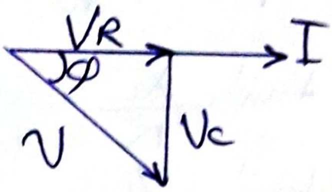
Applying KVL:-

$$-\vec{V} + \vec{V_R} + \vec{V_c} = 0$$

$$\vec{V} = \vec{V_R} + \vec{V_c}$$

$$V_R = I * R \quad , \quad V_c = I * X_c$$

$$V = \sqrt{V_R^2 + V_c^2}$$

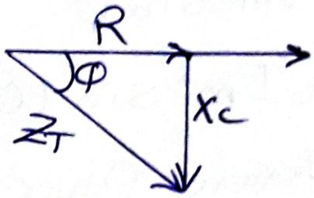


Voltage triangle

$$V = \sqrt{I^2 R^2 + I^2 X_c^2}$$

$$V = I\sqrt{R^2 + X_c^2}$$

$$\frac{V}{I} = \sqrt{R^2 + X_c^2}$$



impedance triangle

$$Z_T = \sqrt{R^2 + X_c^2}$$

$$\phi = \tan^{-1} \frac{X_c}{R}$$

Hiba Alsawaf

The instantaneous forms:-

$v = V_m \sin\theta$

$i = I_m \sin(\theta + \phi)$

* Average Power:-

$P_{av_T} = P_{av_R} + \cancel{P_{av_C}}^{0}$

$P_{av_T} = P_{av_R} \Rightarrow P_{av} = I^2 * R$ oR

$P_{av} = V_R * I_R \Rightarrow P_{av} = \frac{V_R^2}{R}$

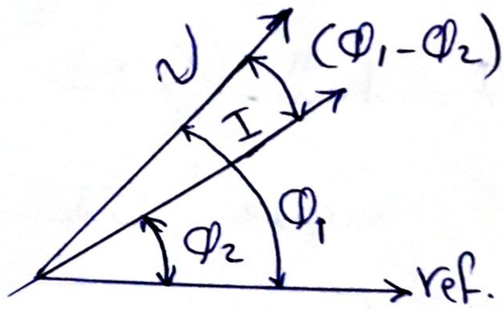
$P_{av} = I \cdot I \cdot R$

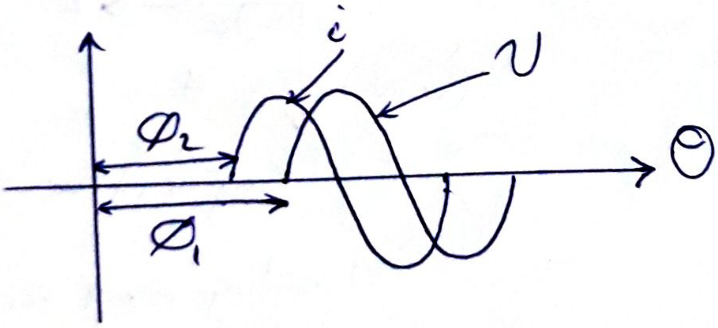
$= \frac{V}{Z} \cdot I R \Rightarrow P_{av} = V \cdot I \cdot \frac{R}{Z}$

$P_{av} = V I \cos\phi$

* In general:-

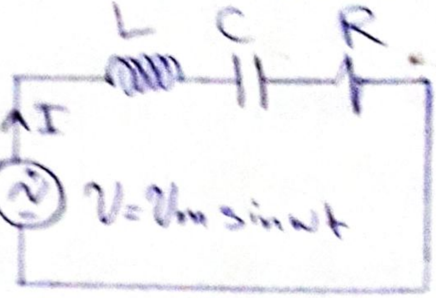
$P_{av} = V I \cos(\phi_1 - \phi_2)$



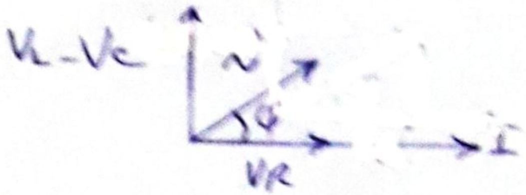


—Hiba Ahmad—

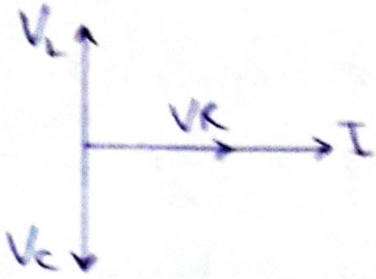
A.C in RLC circuit:-



* اذا كانت $V_c < V_L$ فسوف تكون ممانعة الدائرة حثية



(Inductive)



Lagging power factor.

$$V = \sqrt{V_R^2 + (V_L - V_c)^2}$$

$$V = I \sqrt{R^2 + (X_L - X_c)^2}$$ divided by I.

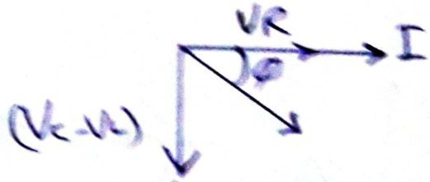
$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_c)^2} \Rightarrow Z_T = \sqrt{R^2 + (X_L - X_c)^2}$$

$$\cos\phi = \frac{V_R}{V} = \frac{R}{Z}$$

$$\phi = \tan^{-1} \frac{(X_L - X_c)}{R}$$

* اذا كانت $V_L < V_c$ فان ممانعة الدائرة ستكون سعوية:-

$$Z = \sqrt{R^2 + (X_c - X_L)^2}$$



(Capacitive)

Leading power factor

$$P_{av} = \frac{V_m I_m}{2} \cos\phi$$
$$= V_{r.m.s} \cdot I_{r.m.s} \cos\phi$$

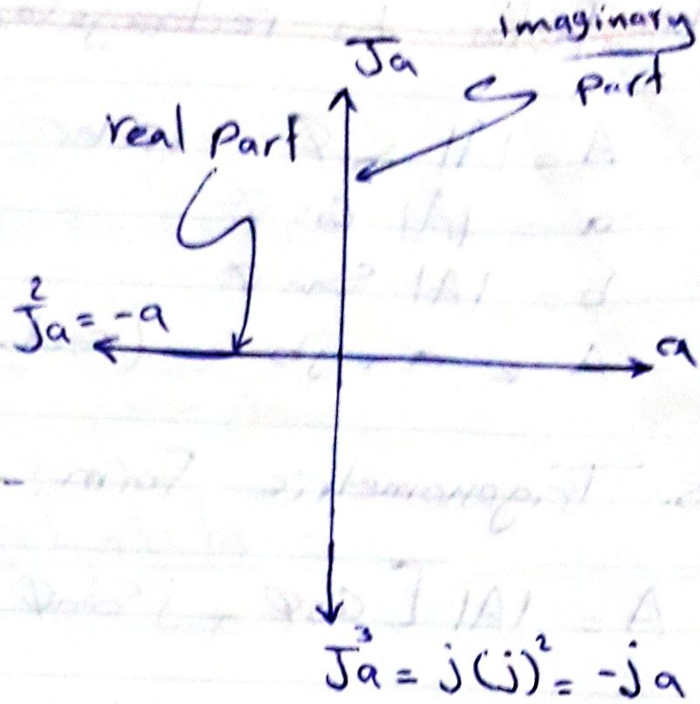
$$\phi = \tan^{-1} \frac{(X_c - X_L)}{R}$$

Hiba Alsawal

J- operator

$j = \sqrt{-1}$

$j^2 = -1$

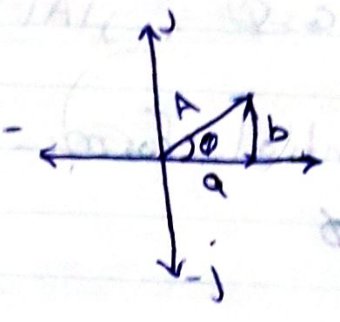


* Vector representation:-

1. Rectangular Form:-

فيه دوارية

$A = a + jb$



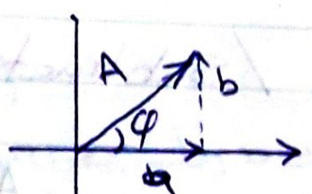
2. Polar Form:-

$A = |A| \angle \phi$

Conversion between Forms:-

a. Rectangular to polar

$|A| = \sqrt{a^2 + b^2}$



A = a+jb (rectangular)

$\phi = \tan^{-1} \frac{b}{a}$

$A = |A| \angle \phi$ polar form

– Hiba Alsawaf –

b. Polar to rectangular:-

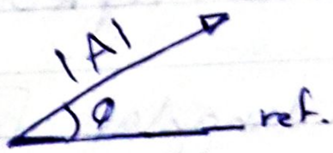
$A = |A| \angle \phi$ (Polar form)

$a = |A| \cos\phi$

$b = |A| \sin\phi$

$A = a + jb$ (rect. form)

3. Trigonometric Form:-



$A = |A| [\cos\phi + j\sin\phi]$

$= |A|\cos\phi + j|A|\sin\phi$

4. Exponential Form:-

$A = |A| e^{j\phi}$

* $e^{j\phi} = \cos\phi + j\sin\phi$
* $e^{-j\phi} = \cos\phi - j\sin\phi$
* $j = 1\angle 90^\circ$
* $-j = 1\angle -90$
* $A = a + jb$, $\underbrace{A^* = a - jb}$ conjugate

Mathematial operations

1. Addition / Subtraction

$A_1 = a_1 + jb_1$, $A_2 = a_2 + jb_2$

$B_1 = A_1 + A_2 \Rightarrow B_1 = a_1 + a_2 + j(b_1 + b_2)$

$B_2 = A_1 - A_2 \Rightarrow B_2 = a_1 - a_2 + j(b_1 - b_2)$

2. Division :-

—Hiba Alnawaf—

* In rectangular form :-

$A_1 = a_1 + jb_1 \quad , \quad A_2 = a_2 + jb_2$

$$B = \frac{A_1}{A_2} = \frac{a_1 + jb_1}{a_2 + jb_2} * \frac{a_2 - jb_2}{a_2 - jb_2}$$

$$B = \frac{a_1a_2 - ja_1b_2 + ja_2b_1 + b_1b_2}{a_2^2 + b_2^2}$$

$$= \frac{(a_1a_2 + b_1b_2) - j(a_1b_2 - a_2b_1)}{a_2^2 + b_2^2}$$

* In polar form :-

$A_1 = |A_1| \angle \phi_1 \quad , \quad A_2 = |A_2| \angle \phi_2$

$$B = \frac{|A_1| \angle \phi_1}{|A_2| \angle \phi_2} \Rightarrow B = \frac{|A_1|}{|A_2|} \angle \phi_1 - \phi_2$$

3. Multiplication :-

* In rect. form

if $A_1 = a_1 + jb_1 \quad , \quad A_2 = a_2 + jb_2$

find $B = A_1 A_2$

$$B = (a_1 + jb_1)(a_2 + jb_2)$$
$$= a_1a_2 + ja_1b_2 + ja_2b_1 - b_1b_2$$
$$= (a_1a_2 - b_1b_2) + j(a_1b_2 + a_2b_1)$$

$|B| = \sqrt{(real)^2 + (imag.)^2}$

Hiba Alsawaf

$\varphi = \tan^{-1} \dfrac{imag.}{real}$

* In Polar form :-

The magnitudes are multiplied and the angles are added.

if $A_1 = |A_1| \angle \varphi_1$, $A_2 = |A_2| \angle \varphi_2$

$B = A_1 \cdot A_2 \Rightarrow B = |A_1| \cdot |A_2| \angle \varphi_1 + \varphi_2$

4- Roots and Powers :-

if $A = |A| \angle \varphi$

find

$B = \sqrt{A} \Rightarrow B = \sqrt{|A| \angle \varphi}$

$B = \sqrt{|A|} \angle \frac{\varphi}{2}$

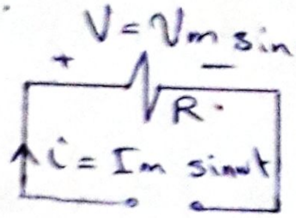
if $A = |A| \angle \varphi$, find $B = A^3$

$B = (|A| \angle \varphi)^3 \Rightarrow B = |A|^3 \angle 3\varphi$

Circuit Elements as complex numbers:-

-Hiba Alsaud-

1. Resistance:-



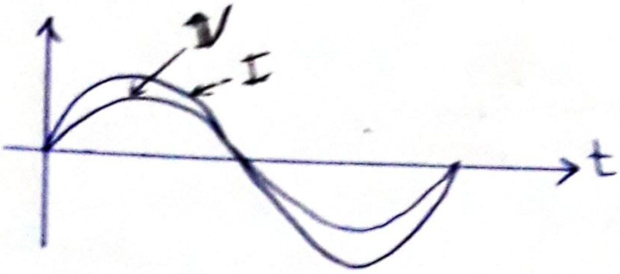
$$R = \frac{V}{I}$$

$$= \frac{|V| \angle 0}{|i| \angle 0} \Rightarrow R = R \angle 0$$

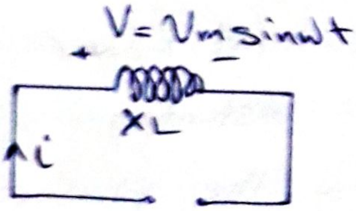
$\vec{V} = |V| \angle 0$

$\vec{I} = |i| \angle 0$

$$R = R + j0$$



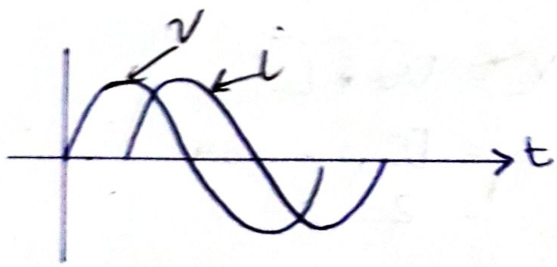
2. Inductor:-



$V = V_m \sin wt$

$i = I_m \sin(wt - 90)$

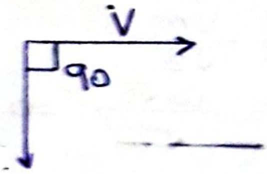
$V = |V| \angle 0 \quad , \quad I = |i| \angle -90°$



$$X_L = \frac{V}{I}$$

$$= \frac{|V| \angle 0}{|i| \angle -90}$$

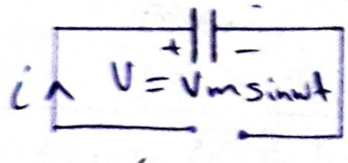
$$= X_L \angle 90 \Rightarrow X_L = j X_L$$



Lagging power factor because i Lags V.

3. Capacitor:-

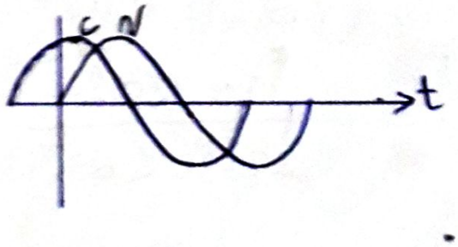
$X_C = \frac{1}{wc}$



$V = V_m \sin wt$

$I = I_m \sin(wt + 90°)$

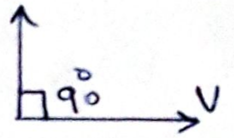
$V = |V| \angle 0 \quad , \quad I = |i| \angle 90°$



$$X_C = \frac{V}{I} \Rightarrow X_C = \frac{|V| \angle 0}{|i| \angle 90}$$

$$X_C = X_C \angle -90$$

$X_c = -j\,X_c$



Leading power factor because i Leads V by 90°.

RLC in Series:-

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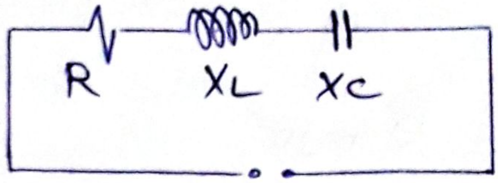


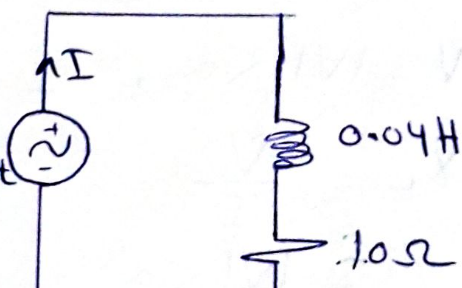
$Z_T = R + j0 + jX_L - jX_C$

$= R + j(X_L - X_C)$

Exa:-

For the Circuit shown, Find:-

(a) The circuit current , (b) V_R & V_L.
(c) The total Power consumed.
(d) The instantaneous forms for i, V_R & V_L.
(e) Draw the phasor digram.



instantaneous form

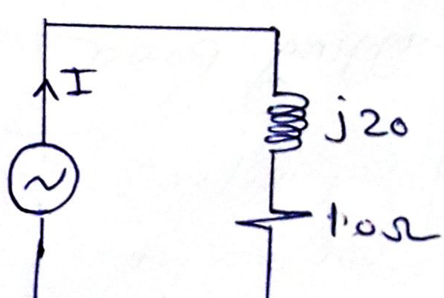
$X_L = W * L$

$= 500 * 0.04$

$= 20\ \Omega$

$V_{r.m.s} = \dfrac{V_m}{\sqrt{2}}$

$= \dfrac{10\sqrt{2}}{\sqrt{2}} \Rightarrow V_{r.m.s} = 10$ Volt



Vector form

(a) $I = \dfrac{V_s}{Z}$

$= \dfrac{10\angle 0}{10 + j20}$

$= \dfrac{10\angle 0}{\sqrt{10^2 + 20^2}\ \angle \tan^{-1}\frac{20}{10}}$

$= \dfrac{10\angle 0}{22.36\angle 63.43^\circ} \Rightarrow I = 0.44\angle -63.43^\circ$ Amper

(b) $\therefore V_R = I * R$

$= 0.44\angle{-63.43^\circ} * 10\angle 0 \Rightarrow V_R = 4.4\angle{-63.43^\circ}$ Volt

$V_L = I * jX_L$

$= 0.44\angle{-63.43^\circ} * 20\angle 90^\circ \Rightarrow V_L = 8.8\angle 26.57^\circ$ Volt

(c) $P_{av} = I^2 * R$

$= (0.44)^2 * 10 \Rightarrow P_{av} = 1.936$ Watt

* يتم أخذ التيار في حالة حساب القدرة فقط دون اتجاه.

OR $P_{av} = V I \cos\phi$

$= 10 * 0.44 \cos(0-(-63.43^\circ))$

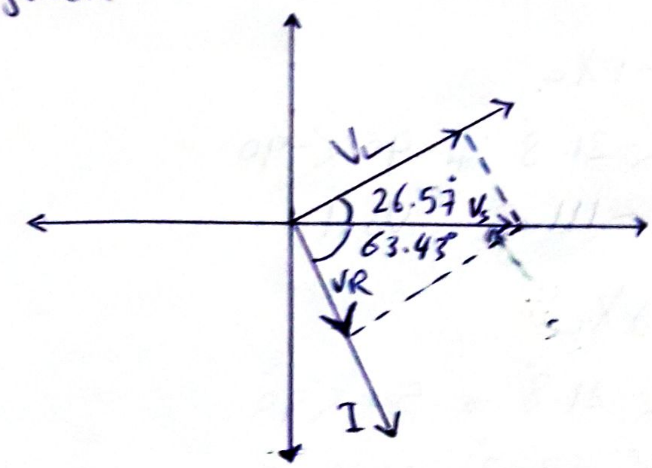
$= 1.936$ Watt

(d) $i = 0.44\sqrt{2}\sin(500t - 63.43^\circ)$ Amper

$v_R = 4.4\sqrt{2}\sin(500t - 63.43^\circ)$ Volt

$v_L = 8.8\sqrt{2}\sin(500t + 26.57^\circ)$ Volt

(e) phasor digram



H.W:-

For the cct. shown, Find:-

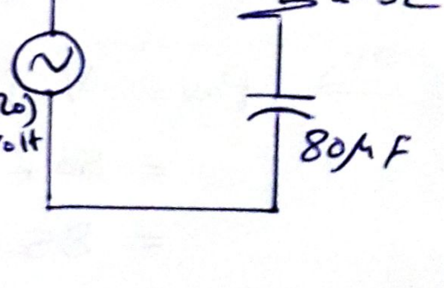
(a) The circuit current.

(b) V_R & V_C.

(c) The total power consumed.

(d) The instantaneous form for i, v_R & v_C.

(e) Sketch the phasor diagram.



Hiba Alsawaf

Exa:-

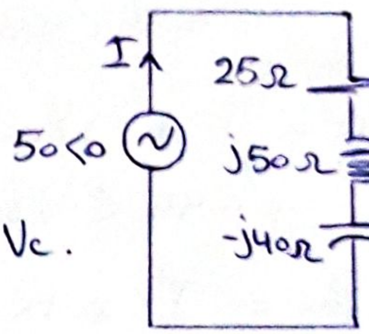
For the cct. shown, find:-

(a) The cct. current. (b) V_R, V_C & V_L.

(c) The Power consumed.

(d) The instantaneous form for i, V_R, V_L & V_C.

(e) Draw the phasor diagram.



Vector form

$Z_T = 25 + j50 - j40$

$= 25 + j10 \Rightarrow Z_T = \sqrt{25^2 + 10^2} \angle \tan^{-1} \frac{10}{25}$

$Z_T = 26.92 \angle 21.8^\circ \ \Omega$

(a) $I = \dfrac{50 \angle 0}{26.92 \angle 21.8^\circ} \Rightarrow I = 1.85 \angle -21.8^\circ \ A$

(b) $V_R = I * R$

$= 1.85 \angle -21.8^\circ * 25 \angle 0$

$= 46.25 \angle -21.8^\circ$ Volt

$V_C = I * -jX_C$

$= 1.85 \angle -21.8^\circ * 40 \angle -90$

$= 74 \angle -111.8^\circ$ Volt

$V_L = I * jX_L$

$= 1.85 \angle -21.8^\circ * 50 \angle 90^\circ$

$= 92.5 \angle 68.2^\circ$ Volt

(c) $P_{av} = I^2 * R$

$= (1.85)^2 * 25 \Rightarrow P_{av} = 85.56$ watt

OR $P_{av} = V I \cos\phi$

$= 50 * 1.85 \cos(0 - (-21.8))$

$\cong 85.56$ watt

Hiba Alsamaf

(d) The instantaneous forms:-

$i = 1.85\sqrt{2}\,\sin(\omega t - 21.8^\circ)$ Amper

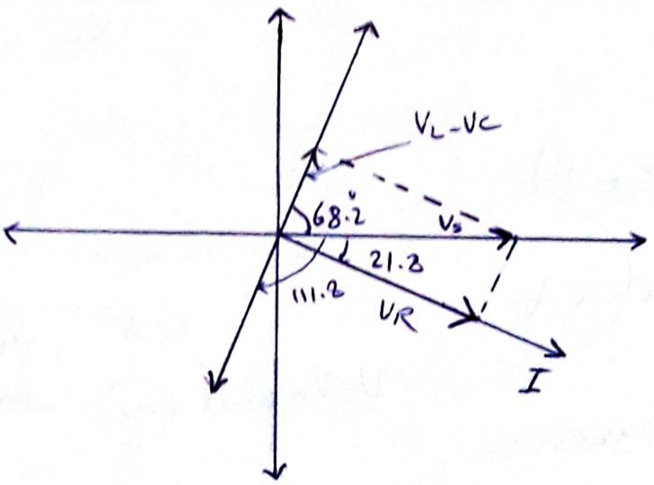
$V_s = 50\sqrt{2}\,\sin \omega t$ Volt

$V_R = 46.25\sqrt{2}\,\sin(\omega t - 21.8^\circ)$ Volt

$V_L = 92.5\sqrt{2}\,\sin(\omega t + 68.2^\circ)$ Volt

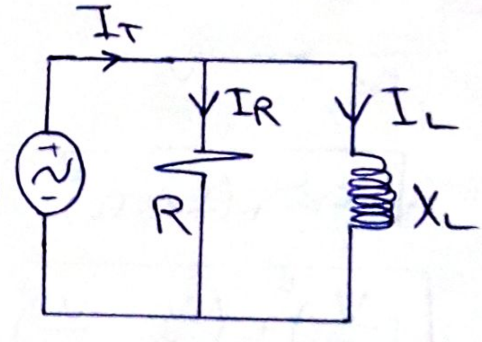
$V_C = 74\sqrt{2}\,\sin(\omega t - 111.8^\circ)$ Volt

(e)



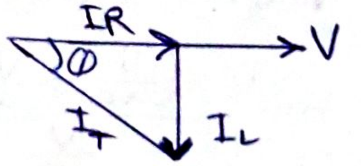
Parallel Connection:-

1) R-L circuit



$$I_T = \sqrt{I_R^2 + I_L^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2}$$



$$\phi = \tan^{-1} \frac{I_L}{I_R}$$

$$= \tan^{-1} \frac{\frac{V}{X_L}}{\frac{V}{R}} \Rightarrow \phi = \tan^{-1} \frac{R}{X_L}$$

$v = V_m \sin\theta$

$i = I_m \sin(\theta - \phi)$

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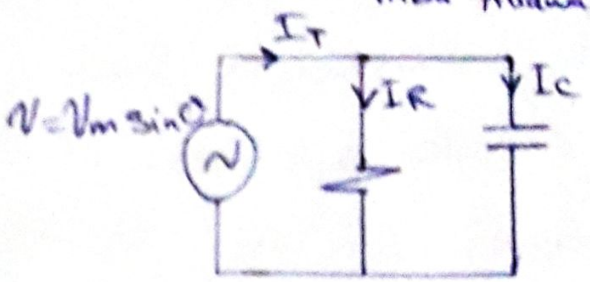
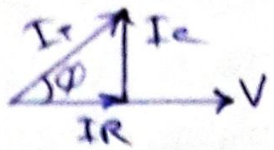
2) R-C circuit



$$I_T = \sqrt{I_R^2 + I_c^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_c}\right)^2}$$



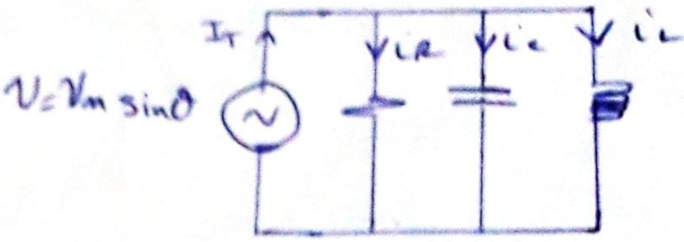
$$\phi = \tan^{-1} \frac{I_c}{I_R}$$

$$= \tan^{-1} \frac{\frac{V}{X_c}}{\frac{V}{R}} \Rightarrow \phi = \tan^{-1} \frac{R}{X_c}$$

$V = V_m \sin\theta$

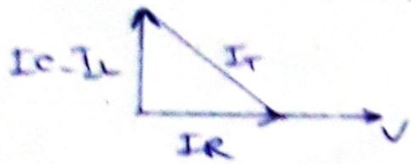
$i = I_m \sin(\theta + \phi)$

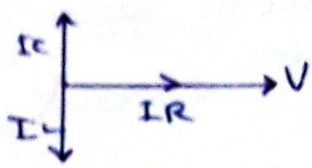
3) R-L-C circuit



* if $I_c > I_L$ then:-

capacitive cara.



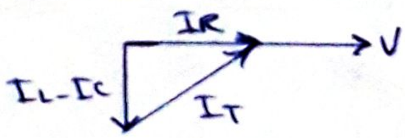


$$I_T = \sqrt{I_R^2 + (I_c - I_L)^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_c} - \frac{V}{X_L}\right)^2}$$

$$\phi = \tan^{-1} \frac{\left(\frac{V}{X_c} - \frac{V}{X_L}\right)}{\frac{V}{R}} \Rightarrow \phi = \tan^{-1} \frac{R}{X_c - X_L}$$

* if $I_L > I_c$ then Inductive cara.



$$I_T = \sqrt{I_R^2 + (I_L - I_c)^2}$$

$$= \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_c}\right)^2}$$

$$\phi = \tan^{-1} \frac{\frac{V}{X_L} - \frac{V}{X_c}}{\frac{V}{R}} \Rightarrow \phi = \tan^{-1} \frac{R}{X_L - X_c}$$

Admittance (Y) :- المسامحة

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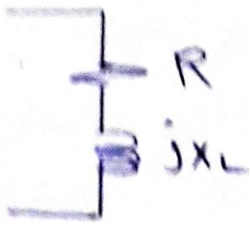
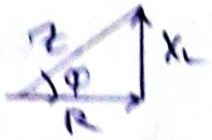
$Z = R + jX_L$

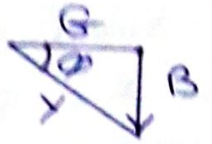


$Y = \frac{1}{Z}$

$= \frac{1}{R + jX_L} * \frac{R - jX_L}{R - jX_L}$

$= \frac{R - jX_L}{R^2 + X_L^2}$





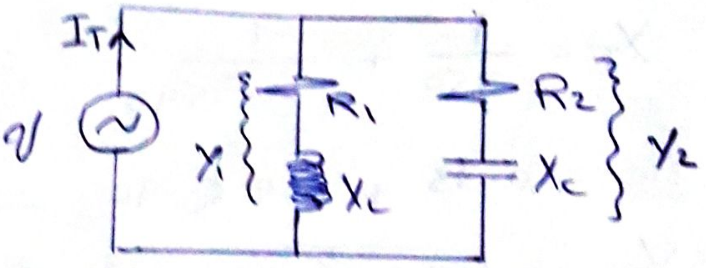
$Y = \underbrace{\frac{R}{R^2 + X_L^2}} - j \underbrace{\frac{X_L}{R^2 + X_L^2}}$

Conductance (G)   Susceptance (B)

التوصيلية

$\therefore Y = G - jB$

$Y_1 = G_1 - jB_1$



$Y_2 = \frac{1}{R - jX_c} * \frac{R + jX_c}{R + jX_c}$

$= \frac{R + jX_c}{R^2 + X_c^2}$

$I_T = \frac{V}{Z}$

$= V * Y$

$Y_2 = \frac{R}{R^2 + X_c^2} + j \frac{X_c}{R^2 + X_c^2}$

$Y_2 = G_2 + jB_2$

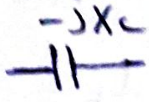
$Y = \frac{1}{R}$

$Y = \frac{1}{jX_L}$

$= \frac{-j}{X_L}$

$Y = \frac{1}{-jX_c}$

$= \frac{j}{X_c}$



$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$Y = Y_1 + Y_2$

$= (G_1 - jB_1) + (G_2 + jB_2)$

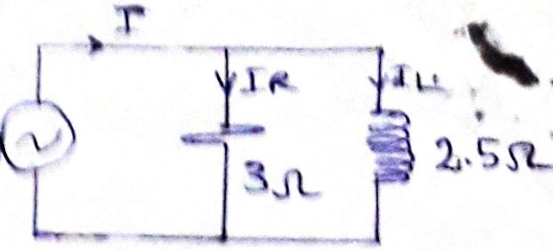
$= (G_1 + G_2) + j(B_2 - B_1)$

-Hiba Ahmed-

Ex:-

For the ckt. shown, $E = 20\sqrt{2}\sin(\omega t + 50.47°)$

find:-



instantaneous form

⇓

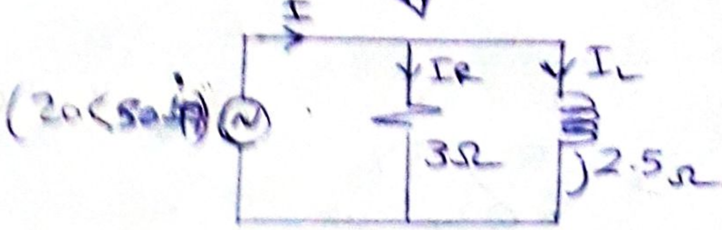
(a) The circuit current I, after calculate I_R & I_L.

(b) Verify Kirchhoff's current Law at one node.

(c) The total power.

(d) Draw the phasor diagram.

(e) The instantaneous form for i, i_R & i_L.



Vector form

$$I = \frac{E}{Z_T} = E * Y_T$$

$$Y_T = Y_1 + Y_2 \Rightarrow Y_T = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y_T = \frac{1}{3\angle 0} + \frac{1}{2.5\angle 90}$$

$$= 0.33 + 0.4\angle -90 \Rightarrow Y_T = 0.33 - j0.4$$

$$Y_T = \sqrt{(0.33)^2 + (-0.4)^2}\ \angle \tan^{-1}\frac{-0.4}{0.33}$$

$$= 0.518\angle -50.47°$$

$$\therefore I = (20\angle 50.47°) * (0.518\angle -50.47°)$$

$$= 10.36\angle 0\ A$$

$$I_R = \frac{E}{R} \Rightarrow I_R = \frac{20\angle 50.47°}{3\angle 0}$$

$$I_R = 6.667\angle 50.47°\ A$$

$$I_L = \frac{E}{jX_L} \Rightarrow I_L = \frac{20\angle 50.47°}{2.5\angle 90°} \Rightarrow I_L = 8\angle -39.53°\ A$$

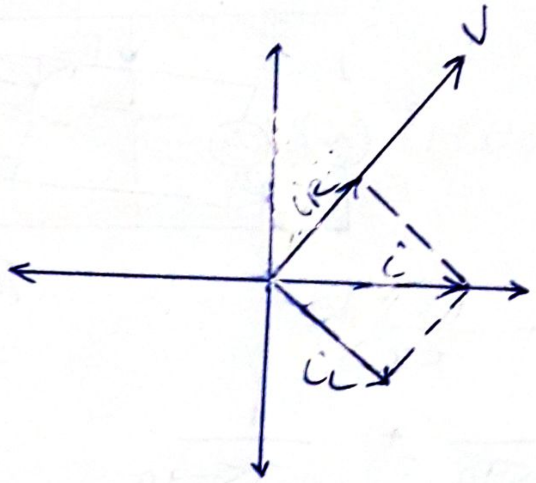
(b) H.W

(c) $P_{av_T} = P_{av_{in R}} + \cancel{P_{av_{in L}}}^{0}$

$= I_R^2 * R$

$= (6.667)^2 * 3 \Rightarrow P_{av_T} = 133.34$ watt

(d)



(e) $i = 10.36\sqrt{2}\,\sin(\omega t + 0)$ Amper

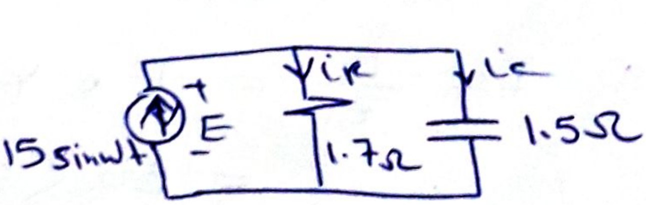
$i_R = 6.667\sqrt{2}\,\sin(\omega t + 50.47^\circ)$ Amper

$i_L = 8\sqrt{2}\,\sin(\omega t - 39.53^\circ)$ Amper

H.W:-

For the circuit shown, find:-

(a) The voltage source, after calculate i_R & i_c.



(b) Verify KCL at one node.

(c) The total Power.

(d) The instantaneous form for E, i_R & i_c.

(e) Draw the phasor diagram.

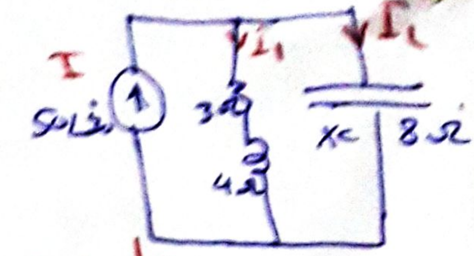
Ex:—

For the network shown, calculate:

(a) I_1 using the current divider rule.

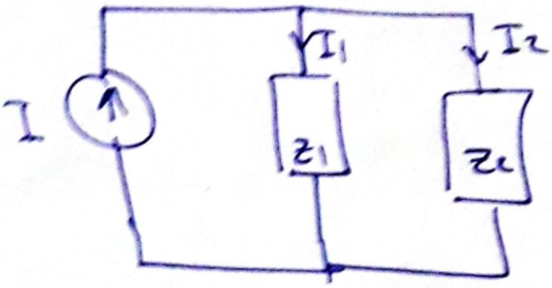
(b) Repeat part (a) for I_2

(c) verify Kirchhoff's current law at one node



(a) $Z_1 = R + jX_L$

$= 3 + j4 = 5 \angle 53.13^\circ$



$Z_2 = -jX_C = -j8 = 8 \angle -90$

using current divider rule

$$I_1 = I \frac{Z_2}{Z_1 + Z_2} = \frac{(50\angle 30^\circ) * 8\angle -90^\circ}{(-j8) + (3+j4)} = \frac{400 \angle -60}{5 \angle -53.13}$$

$I_1 = 80 \angle -6.87^\circ$

(b) $$I_2 = I \frac{Z_1}{Z_1 + Z_2} = (50\angle 30^\circ) * \frac{(5\angle 53.13)}{(5\angle -53.13)} = 50 \angle 136.26^\circ$$

(c) $I = I_1 + I_2$

$= (80 \angle -6.87) + 50 \angle 136.26$

$= (79.43 - j9.57) + (-36.12 + j34.57)$

$= 43.31 + j25$

$= 50 \angle 30^\circ$

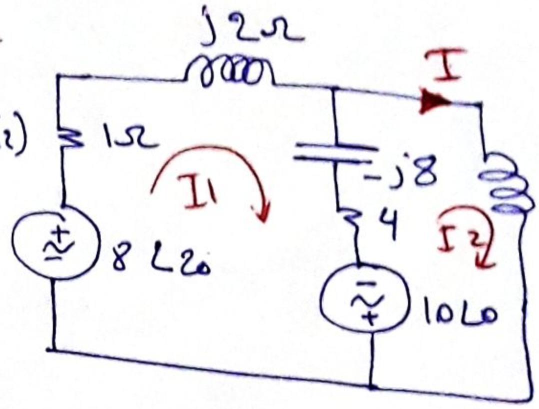
Methods of Analysis and Network theorems (ac)

1. Mesh Analysis

-Hiba Alsawaf-

Exa:-

for the cct shown find I.



Loop (1):-

$-8\angle 20^\circ + (1+j2)I_1 + (4-j8)(I_1-I_2)$

$-10\angle 0 = 0$ —— ①

Loop (2):-

$10\angle 0 + (4-j8)(I_2-I_1) + j6\,I_2 = 0$ —— ②

$I_1 + j2*I_1 + 4I_1 - j8*I_1 - 4I_2 + j8*I_2 = 17.517 + j2.736$ ①

$4I_2 - j8*I_2 - 4I_1 + j8*I_1 + j6I_2 = -10$

$(4-j2)I_2 - (4-j8)I_1 = -10$ —— ②

$(5-j6)I_1 - (4-j8)I_2 = 17.517 + j2.736$ —— ①

$$I_1 = \frac{-10-(4-j2)I_2}{-(4-j8)}$$

$$I_1 = \frac{10+(4-j2)I_2}{(4-j8)} \quad \text{——— } (3)$$

$$(5-j6)\left[\frac{10+(4-j2)I_2}{(4-j8)}\right] - (4-j8)I_2 = 17.517 + j2.736$$

$$7.8\angle{-50.19}\left[\frac{10}{8.9\angle{-63.43}} + \frac{4.47\angle{-26.56}\,I_2}{8.9\angle{-63.43}}\right] - (4-j8)I_2$$

$$= 17.517 + j2.736$$

$$8.764\angle{13.24} + (3.92\angle{-13.32})I_2 - (4-j8)I_2 =$$

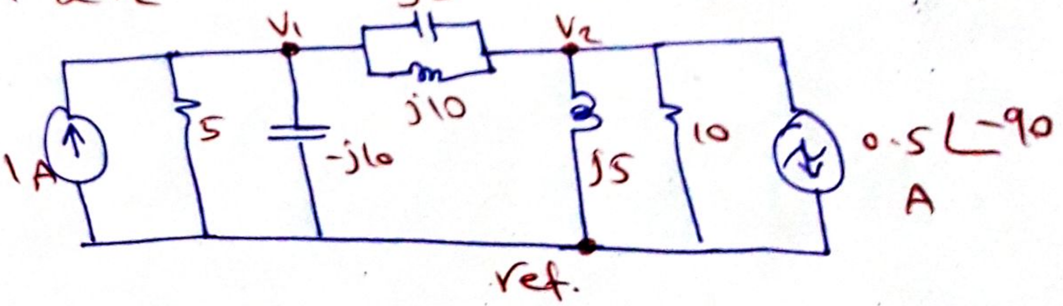
$$17.517 + j2.736$$

ans $= I_2$

$$I_2 = 1.\cancel{87}\angle{-86.}\cancel{9}$$

2. Nodal Analysis

Exa:- find V_1 & V_2



node (1):-

$$\frac{V_1}{5} + \frac{V_1}{-j10} + \frac{V_1-V_2}{-j5} + \frac{V_1 + V_2}{j10} = 1$$

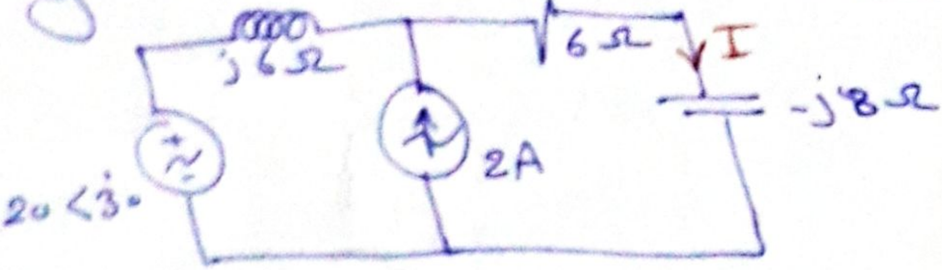
node (2):-

$$\frac{V_2-V_1}{-j5} + \frac{V_2-V_1}{j10} + \frac{V_2}{j5} + \frac{V_2}{10} + 0.5\angle{-90} = 0$$

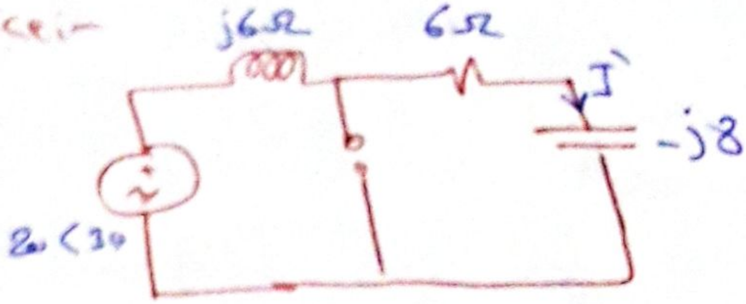
ans:- $V_1 = 2.24\angle{-63.4}\ V$ & $V_2 = 4.47\angle{116.6}\ V$

(3) Superposition theorem :—

Exa:- find I by using superposition theorem.



(1) effect of voltage source:-

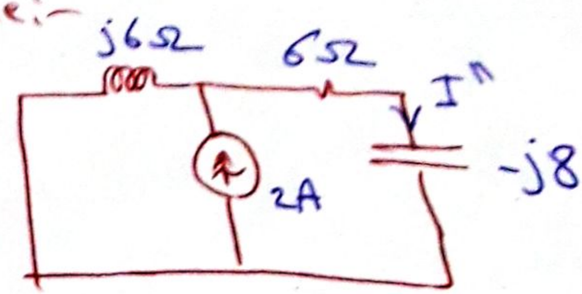


$$I' = \frac{20 \angle 30}{6 + j6 - j8}$$

$$= \frac{20 \angle 30}{6.32 \angle -18.43}$$

$$I' = 3.16 \angle 48.43 \text{ Amp.}$$

(2) effect of current source:-



$$I'' = 2 * \frac{j6}{6 - j2}$$

$$= \frac{12 \angle 90}{6.32 \angle -18.43}$$

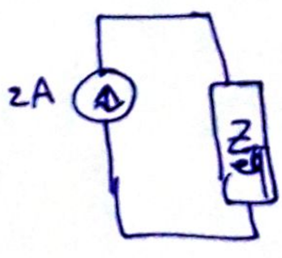
$$= 1.89 \angle 108.43$$

هنالك طريقة اخرى لايجاد I''

$$Z_{eq} = (6 - j8) // j6$$

$$= \frac{(6 - j8) * j6}{6 - j8 + j6}$$

$$= \frac{j36 + 48}{6 - j2} = \frac{60 \angle 36.86}{6.32 \angle -18.43} = 9.49 \angle 55.3$$



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$V = I * Z_{eq}$

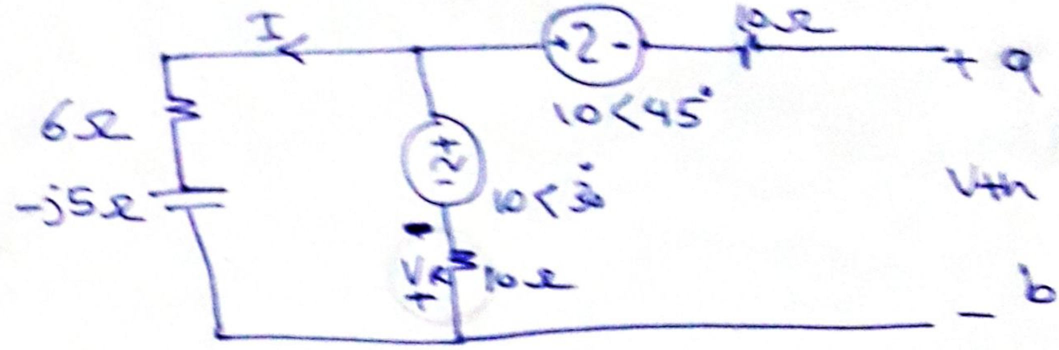
$= 2 * 9.49 \angle 55.3 = 18.98 \angle 55.3$

$$I'' = \frac{18.98 \angle 55.3}{6 - j8} = \frac{18.98 \angle 55.3}{10 \angle -53.13} = 1.89 \angle 108.43$$

④ Thevenin's Theorem:-

Exa:-

find thevenin's equivalent between a & b



to find Vth :-

Applying KVL :-

$+VR - 10 \angle 30 + 10 \angle 45^\circ + V_{th} = 0$

$V_R = I * 10$

$$I = \frac{10 \angle 30}{16 - j5} = \frac{10 \angle 30}{16.76 \angle -17.35} = 0.6 \angle 47.35 \text{ Amp.}$$

$\therefore V_R = 6 \angle 47.35$ Amp.

$V_{th} = -6 \angle 47.35 + 10 \angle 30 - 10 \angle 45^\circ$

$= -4.06 - j4.41 + 8.66 + j5 - 7.07 - j7.07 = -2.47 - j6.48$

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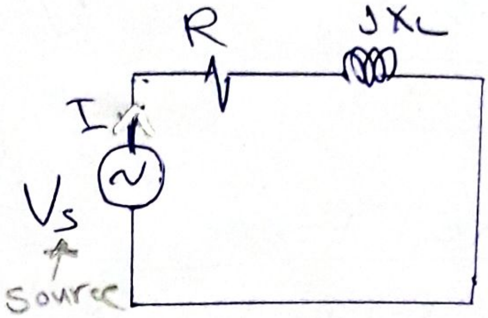
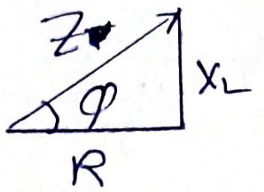
The power Triangle

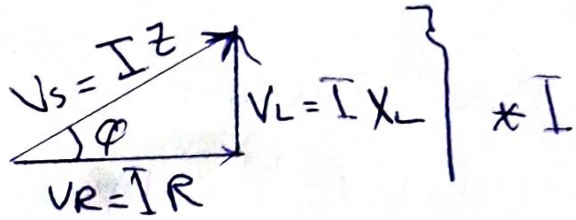
1. RL circuit

Inductive Load



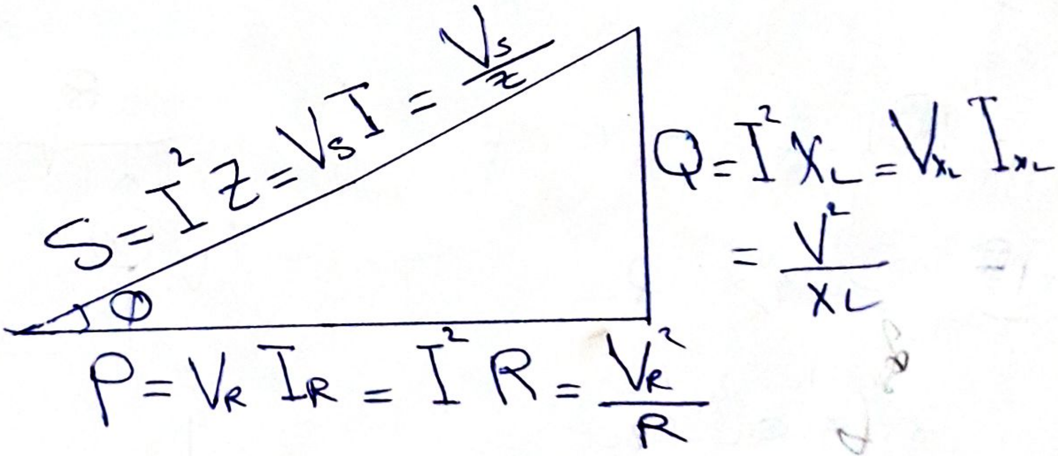


impedance triangle



voltage triangle

Power triangle



$$S = I^2 Z = V_s I = \frac{V_s^2}{z}$$

$$Q = I^2 X_L = V_{x_L} I_{x_L} = \frac{V^2}{X_L}$$

$$P = V_R I_R = I^2 R = \frac{V_R^2}{R}$$

S : Apparent Power القدرة الظاهرية

Units : (VA , KVA , MVA)

Volt Amper Kilo Volt Amper Mega Volt Amper

Q : Reactive power (القدرة الخيالية او المفاعلية)

Units : (VAR , KVAR , MVAR)

Volt Amper Reactive

P : Real Power or Active Power
القدرة الحقيقية أو القدرة الفعالة

Units : (W , KW , MW)

$S = P + jQ_L$

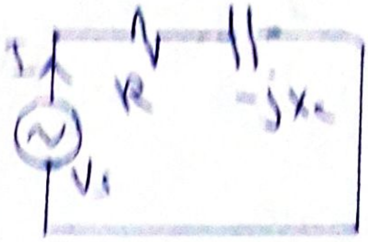
$\cos\varphi = \frac{P}{S}$ = Power factor (P.F)

معامل القدرة

$P.F = \frac{\text{Real Power}}{\text{Apparent power}}$

2. RC circuit

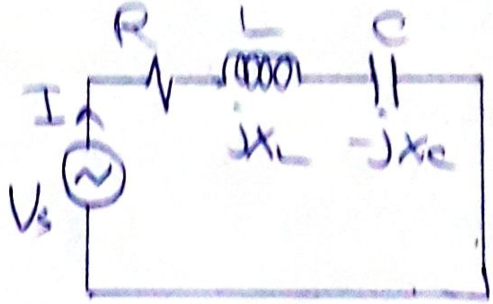
(Capacitive Load)



$S = P - jQ_c$

$P.F = \frac{P}{S}$

3. RLC circuit

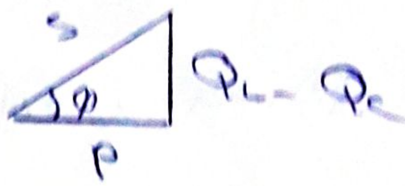


* if $Q_L > Q_c$

inductive

$Q_T = Q_L - Q_c$

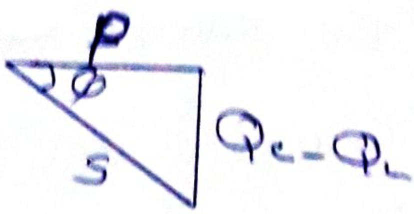
$S = P + jQ_T$



* if $Q_c > Q_L$

capacitive

$Q_T = Q_c - Q_L$



$S = P - jQ_T$

$P.F = \frac{P}{S}$

$S = \sqrt{P^2 + Q^2}$, $S = V_s I^*$

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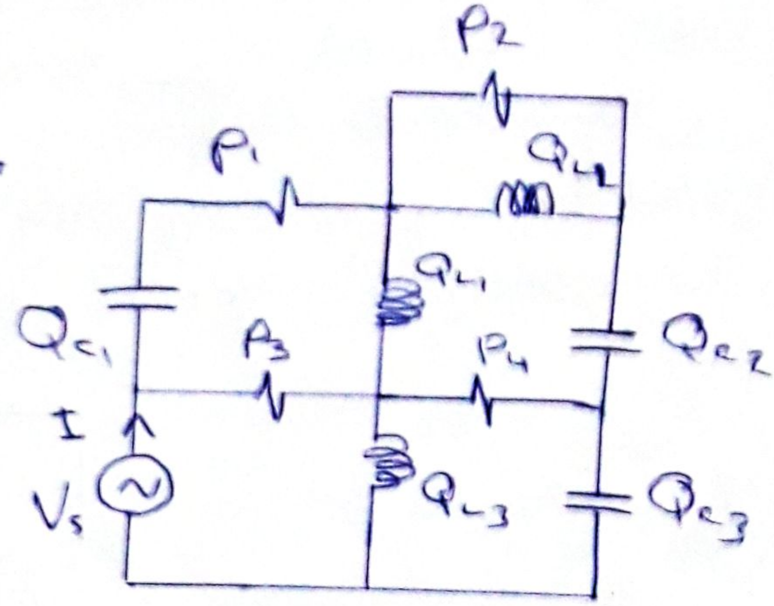
In general :-

$$P = S \cos\phi = V_s I \cos\phi = I^2 R$$

$$Q = S \sin\phi = V_s I \sin\phi = I^2 X \, (X_L \text{ or } X_C)$$

$$S = V_s I^* = I^2 Z$$

* In general :-



* عندما يطلب القدرة تجب ايجاد الـ (P) لجميع المقاومات وبعد ثم يتم جمعها جمعاً جبرياً وليس اتجاهياً.

$$P_T = \underset{\text{for resistor}_1}{P_1} + \underset{\text{for resistor}_2}{P_2} + P_3 + P_4 + \text{———}$$

* عندما يطلب القدرة الفاعلية (Q_T reactive power) يجب ايجادها لكل ملف ومتسعة على حدى.

$$Q_{LT} = Q_{L1} + Q_{L2} + Q_{L3} + \text{———}$$

$$Q_{CT} = Q_{C1} + Q_{C2} + Q_{C3} + \text{———}$$

* if $Q_{LT} > Q_{CT}$ inductive $Q_T = Q_{LT} - Q_{CT}$

$$S_T = P_T + jQ_T$$

* if $Q_{CT} > Q_{LT}$ capacitive $Q_T = Q_{CT} - Q_{LT}$

$$S_T = P_T - jQ_T$$

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Exa:-

. For the system shown:-

(a) find the total number of watts, volt-amperes ~~reactive~~ and the power factor.

(b) find I_T.

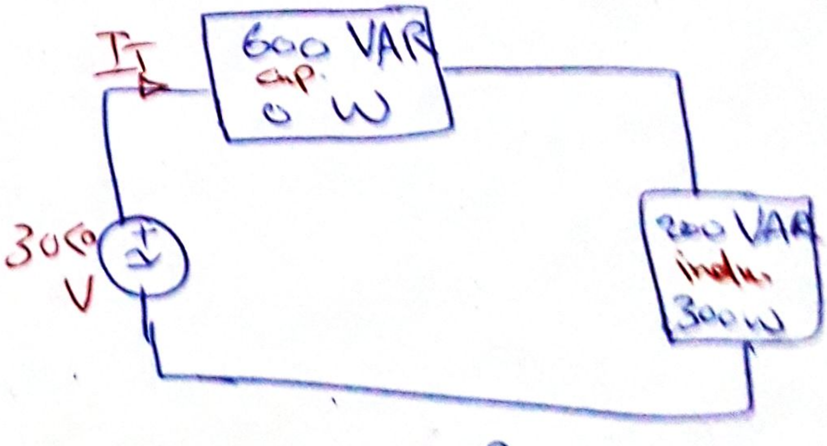
(c) Draw the power triangle.

(d) Find the type elements and their impedances with in each electircal box (assume that all elelements with in boxes in series).

(e) Find IT again using only the input voltage & the results of part (d)

(a)

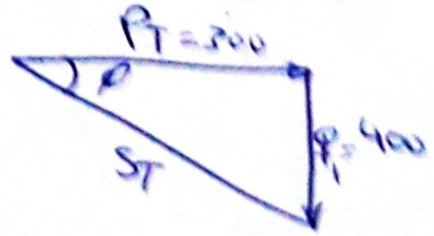
$P_T = 300 + 0 = 300W$



$Q_T = Q_C - Q_L$

$Q_T = 600 - 200$

$= 400$ VAR



$S_T = P_T + j\,Q_T$

$= 300 - j400$

$S_T = \sqrt{(300)^2 + (400)^2} = 500$ VA

$P_F = \dfrac{P}{S} = \dfrac{300}{500} = 0.6$ leading (capacitive)

التيار يسبق الفولتية

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(b) $\ddot{I}_T = ?$

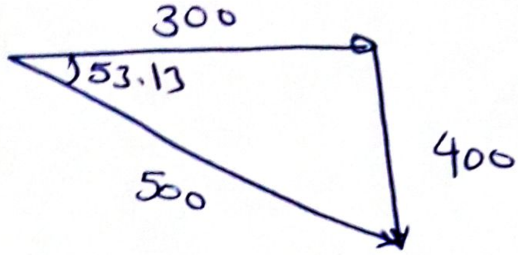
$S_T = V_s * I_s^*$

$$I_s^* = \frac{S_T}{V_s}$$

$$= \frac{500\angle -53.13}{30\angle 0} \Rightarrow I_s^* = 16.66\angle -53.13$$

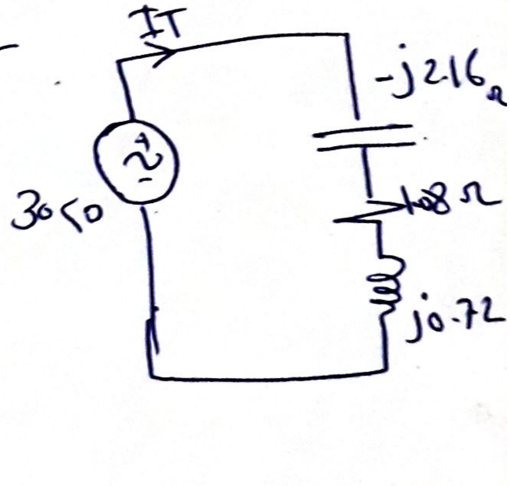
$I_s = 16.66\angle 53.13$ A

(c)



(d) $* \; P = I^2 * R$

$300 = (16.66)^2 * R \Rightarrow R = 1.08\ \Omega$



$Q_C = I^2 * X_C$

$600 = (16.66)^2 * X_C \Rightarrow X_C = 2.16\ \Omega$

$Q_L = I^2 * X_L$

$200 = (16.66)^2 * X_L \Rightarrow X_L = 0.72\ \Omega$

(e) $Z_T = 1.08 - j2.16 + j0.72$

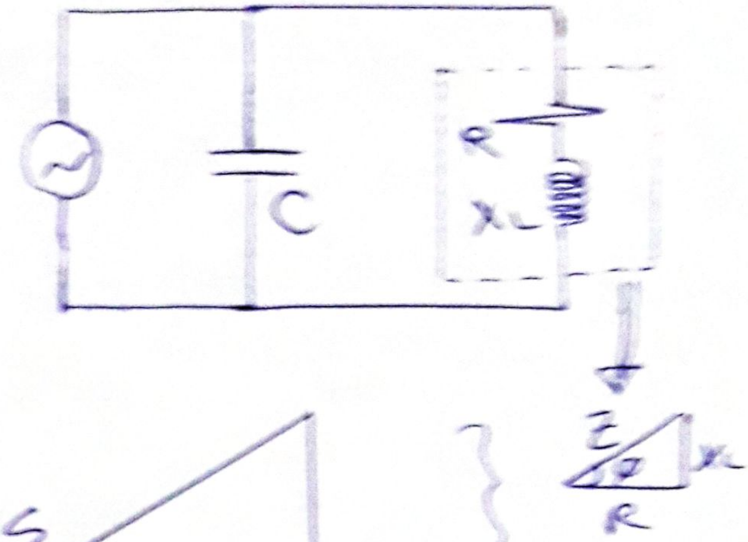
$= 1.08 - j1.44\ \Omega \Rightarrow Z_T = 1.8\angle -53.13$

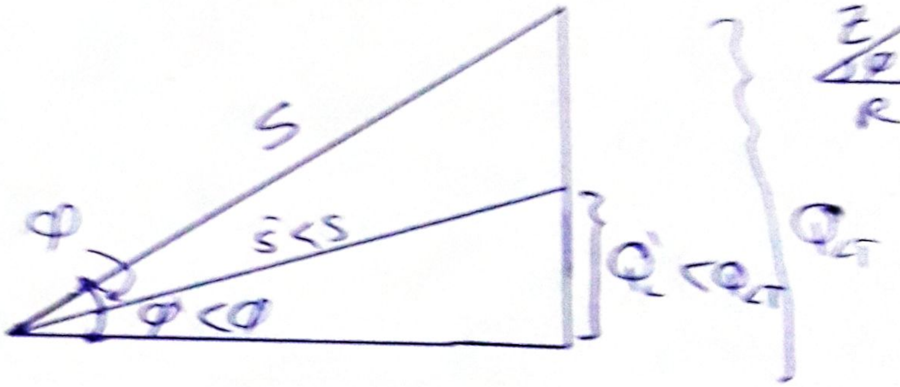
$$I_T = \frac{V_s}{Z_T} = \frac{30\angle 0}{1.8\angle -53.13} = 16.66\angle 53.13 \text{ A}$$

Power factor Correction:-

$$P.F = \cos\phi$$

$$P = V I \cos\phi$$





$$Q_C = \frac{V^2}{X_C}$$

* يكون ربط المتسعة على التوازي مع الحمل لكي تكون الفولتية ثابتة ولان الربط على التوالي يحتاج الى حمل بصيغة تربط معه.

* اذا كانت الممانعة (Z) عبارة عن مقاومة فقط فان $Q=0$ وبالتالي فان $P=S$ وبذلك فان عامل القدرة P.F يقترب من الواحد.

* اغلب الاحمال تكون ذات خصائص حثية لذلك فاننا نحتاج الى عناصر ذات خصائص سعوية لتحسين عامل القدرة.

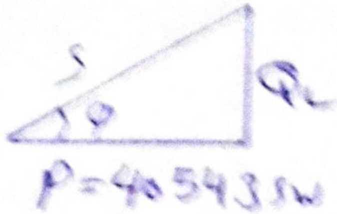
Exa:-

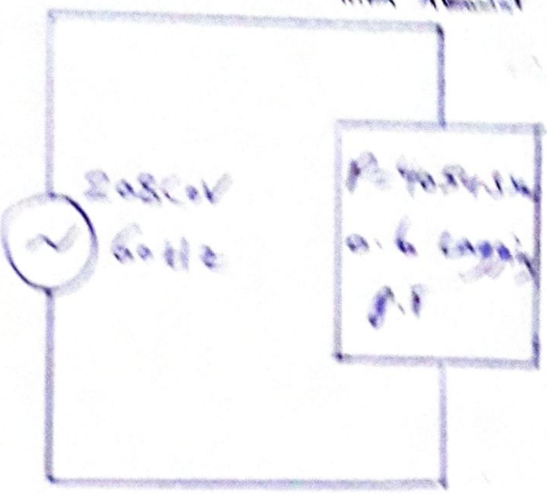
An inductive Load consumed (4054.35W) with (0.6 Lagging P.F), when it is connected to (208 V, 60 Hz) a.c voltage supply. What level of capacitance in parallel with the load you can add to raise the P.F to:-

(a) Unity P.F . (b) 0.9 Lagging P.F

(2)

(a)





$\cos\varphi = 0.6$

$\varphi = \cos^{-1} 0.6$

$\varphi = 53.13^\circ$

$Q_L = P \tan\varphi$

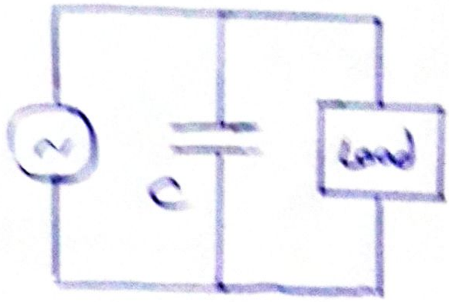
$= 4054.35 \tan(53.13^\circ)$

$= 5405.7$ VAR

$$S = \frac{P}{\cos\varphi} = \frac{4054.35}{0.6} = 6757.25 \text{ VA}$$

To get P.F = 1 ($\varphi = 0$), we must add $Q_C \geq Q_L$

$Q_C = 5405.7$ VAR $= Q_L$



$$Q_C = \frac{V_s^2}{X_C}$$

$$X_C = \frac{(208)^2}{5405.7} = 8\,\Omega$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 60 \times 8} = 331.57\,\mu F$$

Is (the total current) before adding C :-

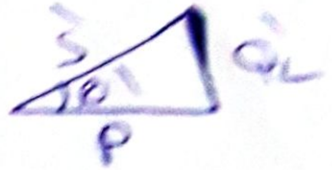
$$I_s = \frac{S}{V_s} = \frac{6757.25}{208} = 32.48 \text{ A.}$$

after adding C at P.F = 1, $S = P = 4054.35$W

$$I_s = \frac{S}{V_s} = \frac{4054.35}{208} = 19.49 \text{ A}$$

(3)

(b) 0.9 Lagging P.F



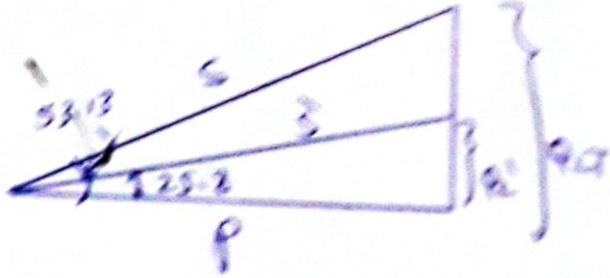
$\cos\phi' = 0.9$

$\phi' = \cos^{-1} 0.9 = 25.8^\circ$

$Q' = P \tan\phi'$

$= 4054.35 \tan(25.8)$

$= 1959.94$ VAR



$Q'_{LT} = Q_{LT} - Q'_L$

$= 5405.7 - 1959.94$

$= 3445.76$ VAR

$$Q_C = \frac{V_s^2}{X_C}$$

$$X_C = \frac{(208)^2}{(3445.76)} = 12.55\ \Omega$$

$$X_C = \frac{1}{2\pi f C} \Rightarrow C = \frac{1}{2\pi \times 60 \times 12.55} = 211.36\ \mu F$$

$$S' = \frac{P}{\cos\phi}$$

$$= \frac{4054.35}{0.9} \Rightarrow S' = 4504.8\ VA$$

$$I_s = \frac{S'}{V_s}$$

$$= \frac{4504.8}{208}$$

$= 21.65A$ (at 0.9 Lagging P.F)

(1)

Resonance in RLC circuit

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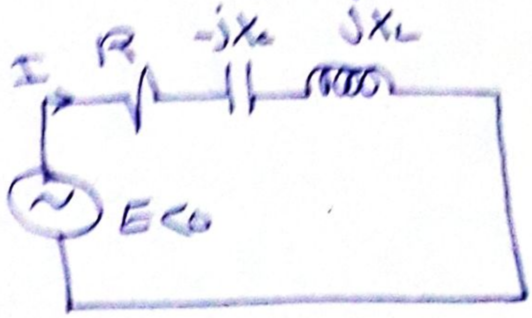
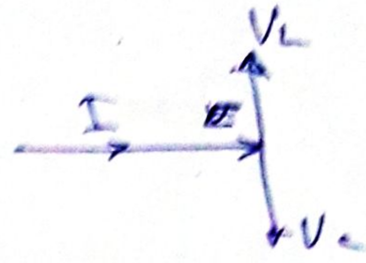
* Series Resonance:-



* In general Resonance occures when the imaginary part of Z_T is equal to zero (with the existance of C & L).



$Z_T = R + jX_L - jX_c$

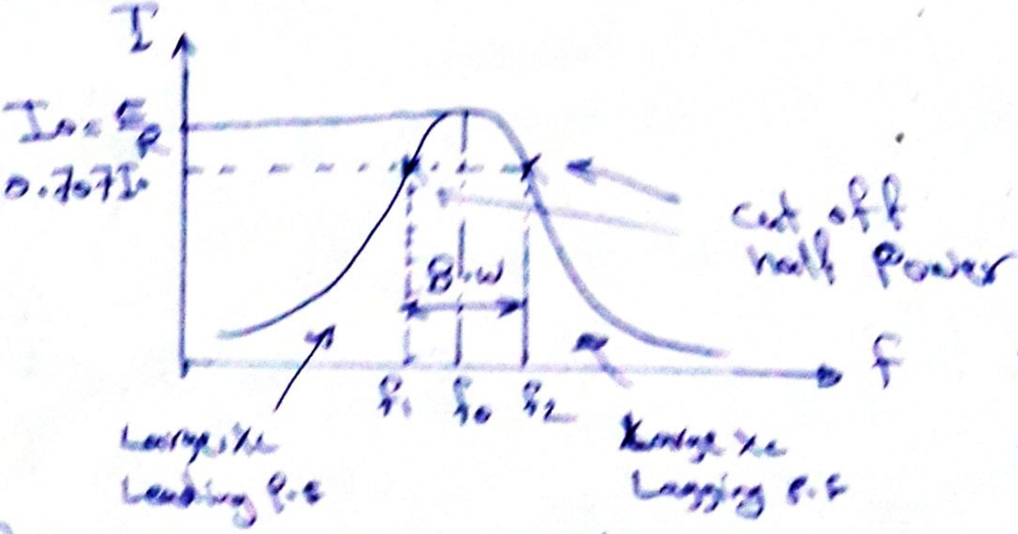
$Z_T = R + j\underbrace{(X_L - X_C)}_{zero} \Rightarrow Z_T = R$

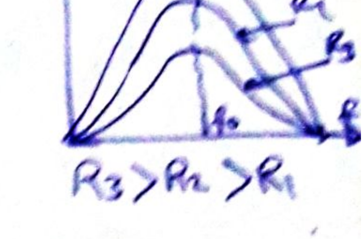
* The frequency that makes the imaginary part of Z_T equal to zero is called resonance frequency (f_o).

$X_L - X_c = 0 \Rightarrow X_L = X_c \Rightarrow$

$2\pi f_o L = \frac{1}{2\pi f_o C} \Rightarrow \left[f_o = \frac{1}{2\pi\sqrt{LC}} \ Hz \right]$

for series resonance only





At Resonance:- (Series)

1. The Imaginary part of the total impedance is zero.
2. $X_L = X_c$
3. The current is in phase with total voltage ($\phi = 0$, $\cos\phi = 1$)

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4- The total impedance is minimum, $Z_T = R$
5- The total current is maximum, $I_o = \frac{E}{R}$
6- The Power is maximum $= I^2 R$

* Quality Factor (Q_f):-

The Quality factor (Q_f) = $\frac{\text{reactive Power}}{\text{averag power (active power)}}$

$0 < Q_f < \infty$

$$\boxed{Q_f = \frac{\cancel{I^2} X_L}{\cancel{I^2} R} = \frac{WL}{R}}$$ for inductive reactance

$$\boxed{Q_f = \frac{I^2 X_C}{I^2 R} = \frac{1}{WCR}}$$ for Capacitive reactance

$Q_f = \frac{X_L}{R} = \frac{X_C}{R}$ (At resonance)

$W = 2\pi f_o \quad , \quad f_o = \frac{1}{2\pi\sqrt{LC}}$

$\therefore Q_f = \frac{WL}{R} \Rightarrow Q_f = \frac{1}{R}\sqrt{\frac{L}{C}}$

$|V_L| = |V_C|$ at resonance .....

$|V_L| = \frac{X_L * E}{Z_T} = \frac{X_L}{R} E \Rightarrow$

$$\boxed{V_L = Q_f E}$$

$V_C = \frac{X_C E}{Z_T} = \frac{X_L * E}{R} \Rightarrow \boxed{V_C = Q_f E}$

$Q_f = \frac{f_o}{f_2 - f_1} \Rightarrow Q_f = \frac{f_o}{B.W}$

B.W : Band with of the band

f_1, f_2 : Cut off frequencies = half power frequencies

$$f_{2,1} = f_o \pm \frac{B.W}{2}$$

$$B.W = \frac{R}{2\pi L} = \frac{f_o}{Q_f} = f_2 - f_1$$

* Resonance in parallel circuits

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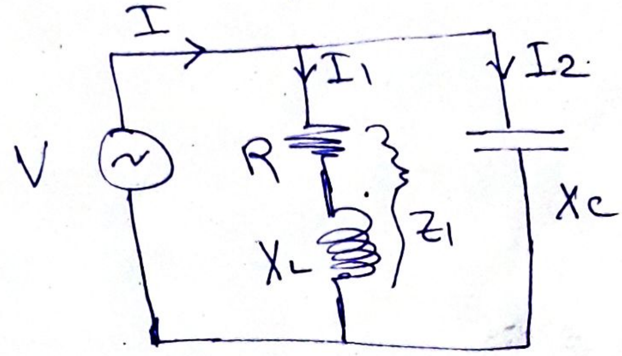


At resonance

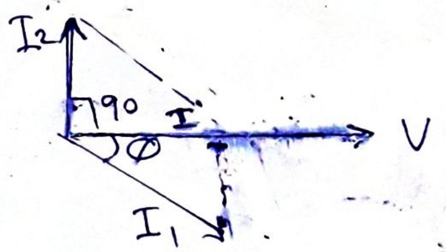
$$I_2 = I_1 \sin\varphi_1$$

$$\frac{\cancel{V}}{X_C} = \frac{\cancel{V}}{Z_1} * \frac{X_L}{Z_1}$$

$$Z_1^2 = X_L * X_C$$

$$R^2 + X_L^2 = W_o L * \frac{1}{\cancel{W_o} C}$$

$$R^2 + X_L^2 = \frac{L}{C}$$



$$X_L^2 = \frac{L}{C} - R^2$$

$$W_o^2 L^2 = \frac{L}{C} - R^2$$

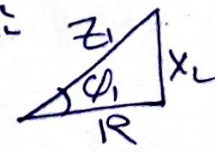
$$W_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

if we take the circut of Z_1 only.

$$|I_1| = \frac{V}{\sqrt{R^2 + X_L^2}}$$

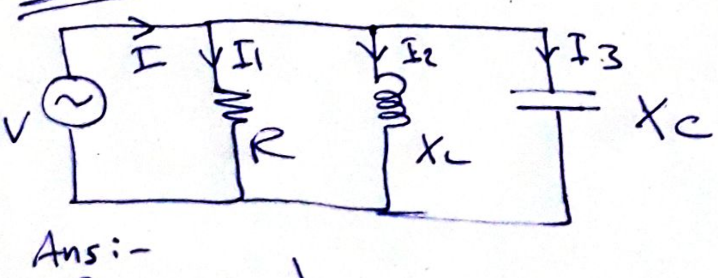
$$\varphi_1 = \tan^{-1} \frac{X_L}{R}$$

$$|Z_1| = \sqrt{R^2 + X_L^2}$$



$$\varphi_2 = 90$$

H.W.

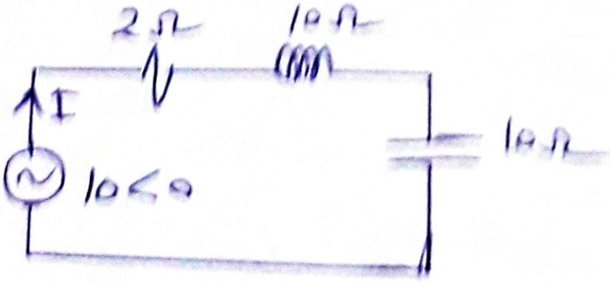


Ans:-

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

Ex:-

For the circuit shown, Find I, V_R, V_L & V_C in Phasor form. What is the quality factor of the cct. If you know that the resonance frequency is (500Hz) find the Bandwidth of the resonant cct. What is the power dissipated at half power freq.



$Z = 2 + j10 - j10$

$Z = 2$

$I = \frac{V}{Z} = \frac{10<0}{2} = 5A$

$V_R = I R$

$= 5<0 * 2<0$

$= 10<0$ Volt

$V_L = I * jX_L$

$= 5<0 * 10<90$

$= 50<90$ Volt

$V_C = I * -jX_C$

$= 5<0 * 10<-90$

$= 50<-90$ Volt

$Q_f = \frac{X_L}{R} = \frac{10}{2} = 5$

$Q_f = \frac{f_o}{B.W}$

$5 = \frac{500}{B.W} \Rightarrow B.W = 100$ Hz

$P_{max} = I^2 * R$

$= (5)^2 * 2 = 50$ watt

$P_{at\, f_1 or f_2} = \frac{1}{2} P_{max} = \frac{1}{2} * 50 = 25$ watt

Resonance in parallel ccts. :-

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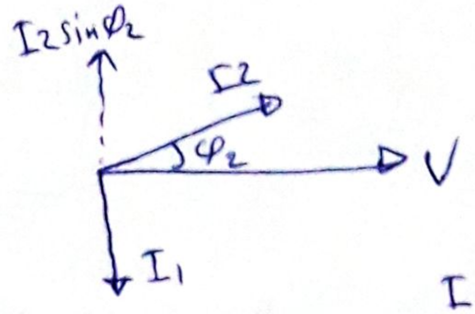
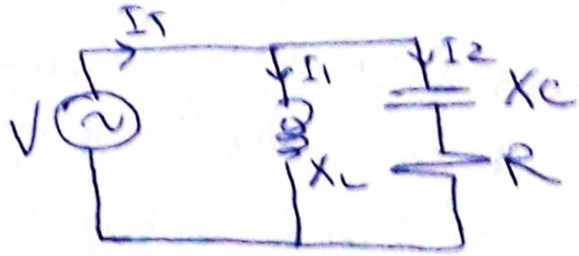
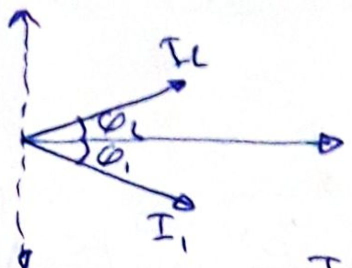
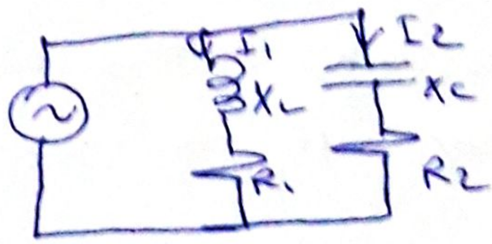




$I_1 = I_2 \sin\phi_2$





$I_1 \sin\phi_1 = I_2 \sin\phi_2$