<div align="center">

Nivevah University
College of Electronics Engineering
Communication Principles
Dr. Mohamad Abdulrahaman Alhabbar
2018-2019

</div>

References:

- Arabic book "Communication Principles" by Dr. Sami AbdulMawjood, Dr. Khalil Hasan Said Mariyee, and Dr. Bayez Alslevani)

- Introduction to Communication System By Stremler

- Introduction to Analog and Digital Communication System By Haykin

- Communication Systems Engineering 2nd-5th Editions by Praokis

- MATLAB software.

1   ELECTRICAL COMMUNICATION SYSTEM

Electrical communication systems are designed to send messages or information from a source that generates the messages to one or more destinations. In general, a communication system can be represented by the functional block diagram shown in Fig. 1. The information generated by the source may be of the form of voice (speech source), a picture (image source), or plain text in some particular language
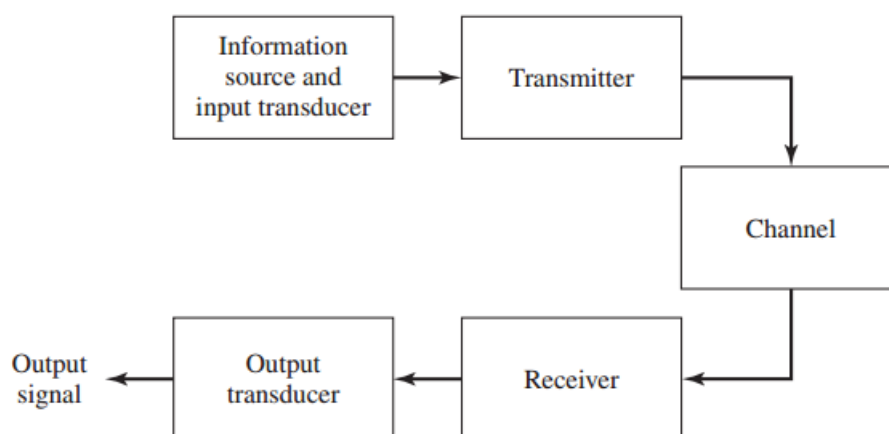

Figure 1: Functional block diagram of a communication system.

A transducer is usually required to convert the output of a source into an electrical signal that is suitable for transmission. For example, a microphone serves as the transducer that converts an acoustic speech signal into an electrical signal, and a video camera converts an image into an electrical signal. At the

destination, a similar transducer is required to convert the electrical signals that are received into a form that is suitable for the user; e.g., acoustic signals, images, etc. The heart of the communication system consists of three basic parts, namely, the transmitter, the channel, and the receiver. The functions performed by these three elements are described next.

1.1   The Transmitter:

The transmitter converts the electrical signal into a form that is suitable for transmission through the physical channel or transmission medium. For example, in radio and TV broadcast, the Federal Communications Commission (FCC) specifies the frequency range for each transmitting station. Hence, the transmitter must translate the information signal to be transmitted into the appropriate frequency range that matches the frequency allocation assigned to the transmitter. Thus, signals transmitted by multiple radio stations do not interfere with one another. Similar functions are performed in telephone communication systems where the electrical speech signals from many users are transmitted over the same wire. In general, the transmitter performs the matching of the message signal to the channel by a process called **modulation**. Usually, modulation involves the use of the information signal to systematically vary either the amplitude (AM), frequency (FM), or phase (PM) of a sinusoidal carrier. For example, in AM radio broadcast, the information signal that is transmitted is contained in the amplitude variations of the sinusoidal carrier, which is the center frequency in the frequency band allocated to the radio transmitting station. This is an example of amplitude modulation. In FM radio broadcast, the information signal that is transmitted is contained in the frequency variations of the sinusoidal carrier. This is an example of frequency modulation. Phase modulation (PM) is yet a third method for impressing the information signal on a sinusoidal carrier.

In general, carrier modulation such as AM, FM, and PM is performed at the transmitter, as indicated above, to convert the information signal to a form that matches the characteristics of the channel. Thus, through the process of modulation, the information signal is translated in frequency to match the allocation of the channel. The choice of the type of modulation is based on several factors, such as the amount of bandwidth allocated, the types of noise and interference that the signal encounters in transmission over the channel, and the electronic devices that are available for signal amplification prior to transmission. In any case, the modulation process makes it possible to accommodate the transmission of multiple messages from many users over the same physical channel. In addition to modulation, other functions that are usually performed at the transmitter are filtering of the information-bearing signal, amplification of the modulated signal, and in the case of wireless transmission, radiation of the signal by means of a transmitting antenna.

1.2   The Channel:

The communications channel is the physical medium that is used to send the signal from the transmitter to the receiver. In wireless transmission, the channel is usually the atmosphere (free space). On the other hand, telephone channels usually employ a variety of physical media, including wirelines, optical fiber cables, and wireless (microwave radio). Whatever the physical medium for signal transmission, the essential feature is that the transmitted signal is corrupted in a random manner by a variety of possible mechanisms. The most common form of signal degradation comes in the form of additive noise, which is generated at the front end of the receiver, where signal amplification is performed. This noise is often called thermal noise.

In some radio communication channels, such as the ionospheric channel that is used for long range, short-wave radio transmission, another form of signal degradation is multi-path propagation. Such signal distortion is characterized as a non-additive signal disturbance which manifests itself as time variations in the signal amplitude, usually called fading.

1.3   The Receiver:

The function of the receiver is to recover the message signal contained in the received signal. If the message signal is transmitted by carrier modulation, the receiver performs carrier demodulation in order to extract the message from the sinusoidal carrier. Since the signal demodulation is performed in the presence of additive noise and possibly other signal distortion, the demodulated message signal is generally degraded to some extent by the presence of these distortions in the received signal. As we shall see, the fidelity of the received message signal is a function of the type of modulation, the strength of the additive noise, the type and strength of any other additive interference, and the type of any non-additive interference. Besides performing the primary function of signal demodulation, the receiver also performs a number of peripheral functions, including **signal filtering and noise suppression**.

2   Transmission Lines:

Transmission line can be represented by a successive symmetrical sections (Δ_x). A uniform transmission line is a "distributed circuit" that we can describe as a cascade of identical cells with infinitesimal length. The conductors used to realize the line possess a certain series inductance and resistance. In addition, there is a shunt capacitance between the conductors, and even a shunt conductance if the medium insulating the wires is not perfect. We use the concept of shunt conductance, rather than resistance, because it is more convenient for adding the parallel elements of the shunt. We can represent the uniform transmission line with the distributed circuit below (general lossy line)
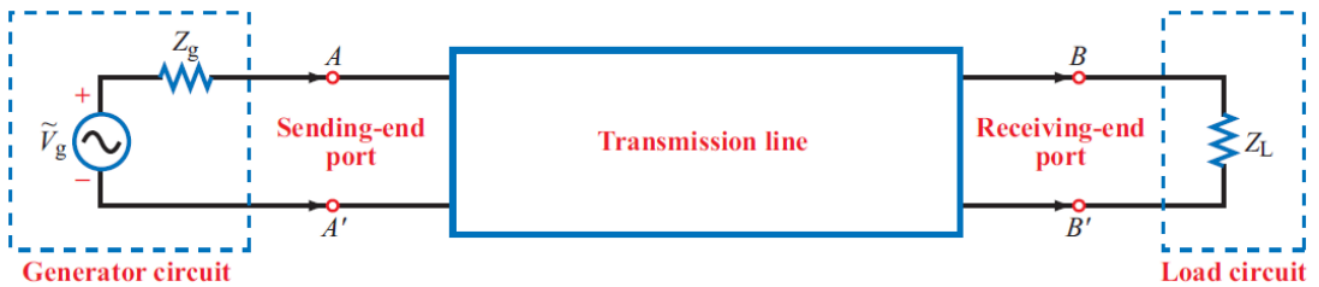
Figure 2: A transmission Line between two points A and B

Figure 3: transmission Line Types

TEM (Transverse Electromagnetic): The field distributions for the uniform transmission line are both known as transverse electromagnetic (TEM) waves.

In TEM, electric and magnetic fields are orthogonal to one another, and both are orthogonal to direction of propagation as shown.
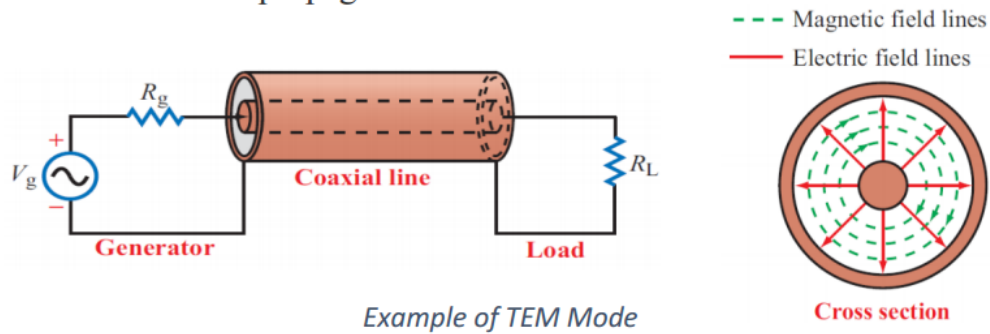

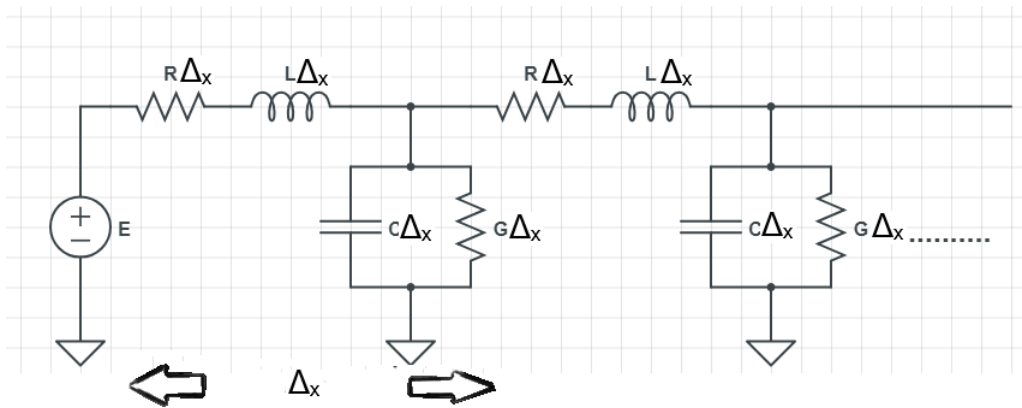Figure 4: Example of TEM mode for a coaxial TL

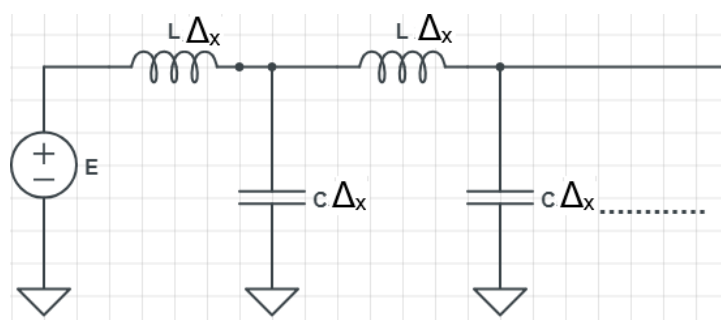Figure 5: Schematic representation of the elementary component of a lossy transmission line

Figure 6: Schematic representation of the elementary component of a lossless transmission line

The impedance parameters L, R, C, and G represent:

L = series inductance per unit length

R = series resistance per unit length

C = shunt capacitance per unit length
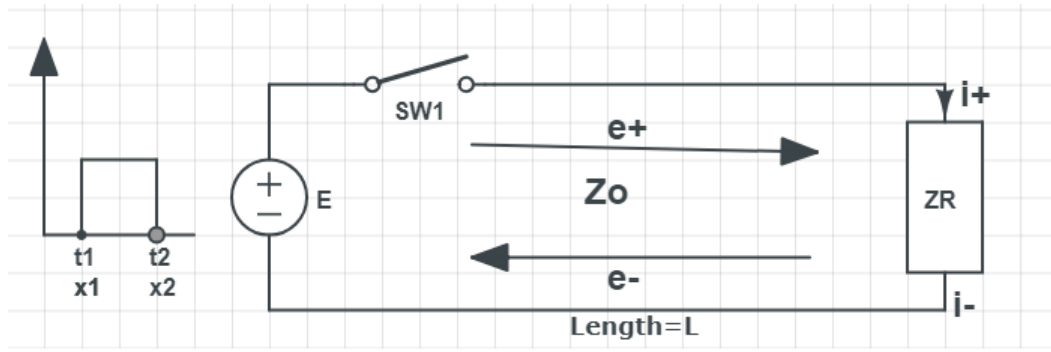
College of Electronics Engineering

Figure 7:

G = shunt conductance per unit length.

Each cell of the distributed circuit will have impedance elements with values: $L\Delta_x$, $R\Delta_x$, $C\Delta_x$, and $G\Delta_x$, where Δ_x is the infinitesimal length of the cells. If we can determine the differential behavior of an elementary cell of the distributed circuit, in terms of voltage and current, we can find a global differential equation that describes the entire transmission line. We can do so, because we assume the line to be uniform along its length. So, all we need to do is to study how voltage and current vary in a single elementary cell of the distributed circuit.

Assumption:

- TL is uniform.

- the change of voltage and current of the source does not appear instantaneously at the load.

- SW1 is closed from $t1$ to $t2$, where $\Delta t = t2 - t1$ and $\Delta x = x2 - x1$.

- e^+ represents the incident wave, while e^- represents the reflected wave from the the load to the source due to mismatch between the Z_R and Z_o.

- if $Z_R = Z_o$ (Matching state), $e^- = 0$

- The energy supplied by the source $= EI\Delta t$ measured in Jule.

- The Energy stored in the electrical field of the capacity (C) and in the magnetic field of the inductance (L).

$$EI\Delta t = \frac{1}{2}(L\Delta x)I^2 + \frac{1}{2}(C\Delta x)E^2 \tag{1}$$

and

$$\Delta x = v\Delta t \tag{2}$$

College of Electronics Engineering

where v represents the velocity of wave through a TL. Now by substituting Eq. 2 in Eq. 1, we can obtain

$$EI\Delta t = \frac{1}{2}(Lv\Delta t)I^2 + \frac{1}{2}(Cv\Delta t)E^2 \tag{3}$$

which can rewritten as

$$EI = \frac{1}{2}vLI^2 + \frac{1}{2}vCE^2 \tag{4}$$

The current from the source in the time Δt has charged the section of TL $\Delta x = v\Delta t$ to a a potential E and the charge Q can be expressed as

$$Q = I\Delta t = C\Delta x E \tag{5}$$

By substituting Eq. 5, which is $\boxed{\Delta x = v\Delta t}$, in Eq. 5

$$I\Delta t = Cv\Delta t E$$
$$I = CvE \tag{6}$$

Now, by substituting Eq. 6 in Eq. 4

$$E^2 Cv = \frac{1}{2}vL(CvE)^2 + \frac{1}{2}vCE^2$$
$$1 = \frac{1}{2}LCv^2 + \frac{1}{2} \tag{7}$$

Finally, we can write an expression for v as

$$\boxed{v = \frac{1}{\sqrt{LC}}} \quad m/s \tag{8}$$

This means that for a TL with length l the signal needs a time of $\boxed{T = \sqrt{LC} \times l}$ to reach the load.

For air dielectric $\boxed{v = c = \dfrac{1}{\sqrt{\mu_o \epsilon_o}} = 3 \times 10^8}$ m/s, where $\boxed{\mu_o = 4\pi \times 10^{-7}}$ H/m and $\boxed{\epsilon_o = 8.85 \times 10^{-12}}$ F/m. While for another dielectric with ϵ_r and μ_r the velocity would be expressed as $\boxed{v = \dfrac{c}{\sqrt{\mu_r \epsilon_r}}}$.

The general formula for the velocity can be written as:

$$\boxed{v = \frac{1}{\sqrt{\mu_s \epsilon_s}}} \tag{9}$$

College of Electronics Engineering ⁷

where $\mu_s = \mu_o \mu_r$ and $\epsilon_s = \epsilon_o \epsilon_r$ and for air $\mu_r = \epsilon_r = 1$. Moreover, ϵ_r is the relative permittivity, while μ_r is the relative permeability of the dielectric.

Now by substituting Eq. 8 in Eq. 6, this will lead to $I = C\frac{1}{\sqrt{LC}}E$, which can be rewritten as

$$\boxed{\frac{E}{I} = \sqrt{\frac{L}{C}} = Z_o}\quad \Omega \tag{10}$$

The last equation represents the characteristic impedance for lossless TL in which $R = G = 0$ as defined already. In practice, Z_o depends on R, L, C, G and frequency. Moreover, it does not depend on the length of the line.

$$Z_o = \frac{e^+}{i^+} = -\frac{e^-}{i^-} \tag{11}$$

3 Reflection in Transmission Lines

The reflection occurs in TL if the impedance of the load is not matched to the characteristic impedance of the TL Z_o, i.e. $Z_L \neq Z_o$. In this case the load will absorb part of the incident wave e^+, and another part of the signal will reflect back to the source as e^-.

As defined already, we have $Z_o = \frac{e^+}{i^+}$, and because of the reflection, the voltage and current at the load will be created and defined as

$$V_r = e_r^+ + e_r^-$$
$$I_r = i_r^+ + i_r^- \tag{12}$$

where $i_r^+ = \frac{e_r^+}{Z_o}$ and $i_r^- = -\frac{e_r^-}{Z_o}$, in which the negative sign of the current is due to the opposite direction of current for the reflected wave.

But it is known that the value of the load is defined as

$$Z_r = \frac{V_r}{I_r} = \frac{e_r^+ + e_r^-}{i_r^+ + i_r^-} = \frac{e_r^+ + e_r^-}{\frac{e_r^+}{Z_o} - \frac{e_r^-}{Z_o}} \tag{13}$$

This lead to

$$\boxed{\frac{Z_r}{Z_o} = \frac{e_r^+ + e_r^-}{e_r^+ - e_r^-}} \tag{14}$$

The voltage reflection coefficient $\boxed{K_V}$ [1] can be expressed as $\boxed{K_V = \dfrac{e_r^-}{e_r^+}}$, which

---

[1] In some books the symbol Γ is used to express the voltage reflection coefficient.

makes Eq. 14 to be

$$\boxed{\frac{Z_r}{Z_o} = \frac{1 + K_V}{1 - K_V}} \tag{15}$$

Which can be rewritten as

$$\boxed{K_V = \frac{Z_r - Z_o}{Z_r + Z_o}} \tag{16}$$

Using the same procedure, we can derive the reflection coefficient of the current which should have the same magnitude but with opposite sign, i.e.$K_I = -K_V$.

What discussed already for the load can be applied exactly to the source as well, in which the reflection coefficient from source K_S can be occurred if $Z_g \neq Z_o$

$$\boxed{K_S = \frac{Z_g - Z_o}{Z_g + Z_o}} \tag{17}$$

Example (HW/CW): A lossless transmission line with length l has a capacity C and inductance L. Its characteristic impedance is Z_o and the impedance at the load $Z_R \neq Z_o$ and the source impedance $Z_g \neq Z_o$. Write the expressions for K_V, K_i, K_g, Z_o, T in addition to the incident and reflected signal e^+ and e^- for $6T$.

Answers:

- $K_r = (Z_r - Z_0)/(Z_r + Z_o)$.

- $K_i = -K_r = -(Z_r - Z_0)/(Z_r + Z_o)$.

- $K_g = (Z_g - Z_0)/(Z_g + Z_o)$.

- $Z_0 = \sqrt{\frac{L}{C}}$.

- $T = l \times 1/v = l \times \sqrt{LC}$

- $e^+ = E \times \frac{Z_o}{Z_0 + Z_g}$ $(t = 0 \text{ to } t = \text{T})$

- $e^- = K_r e^+$. $(t = T \text{ to } t = 2\text{T})$

- $e^{++} = K_g e^-$. $(t = 2T \text{ to } t = 3\text{T})$

- $e^{--} = K_r e^{++}$. $(t = 3T \text{ to } t = 4\text{T})$

- $e^{+++} = K_g e^{--}$. $(t = 4T \text{ to } t = 5\text{T})$

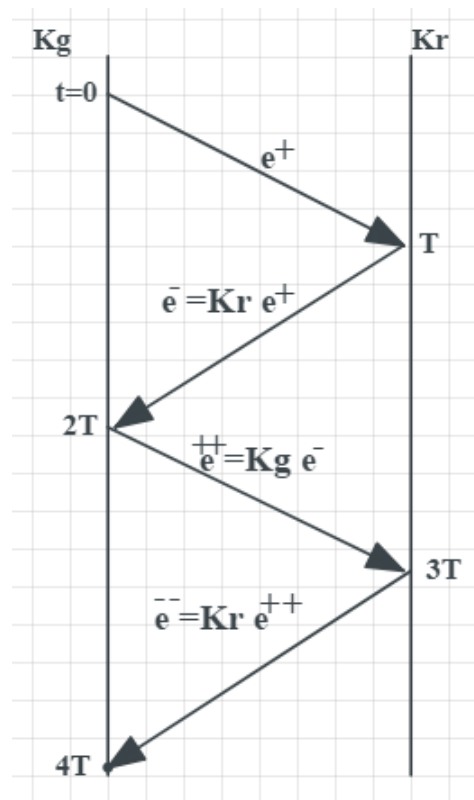- $e^{---} = K_r e^{+++}$. $(t = 5T \text{ to } t = 6\text{T})$

Figure 8:

- We will denote K_R for voltage reflection coefficient at the load, K_i for current reflection coefficient at the load, and K_g for voltage reflection coefficient at the generator.

- If $Z_R = Z_o$, this will lead to $K_R = 0$, (Matching case).

- For short circuit (SC) load, i.e. $Z_R = 0$, this will lead to $K_R = -1$.

- For open circuit (OC) load, i.e. $Z_R = \infty$, this will lead to $K_R = 1$.

Example 1: For the lossless TL shown in below with length l, let $Zg = 0$ and $Z_R = 3Z_0$. Use **Zig-zag** diagram to show the incident and reflected signals for $8T$, then draw the time-spacing diagram for the sending end E_S and I_S and receiving end E_R and I_R.
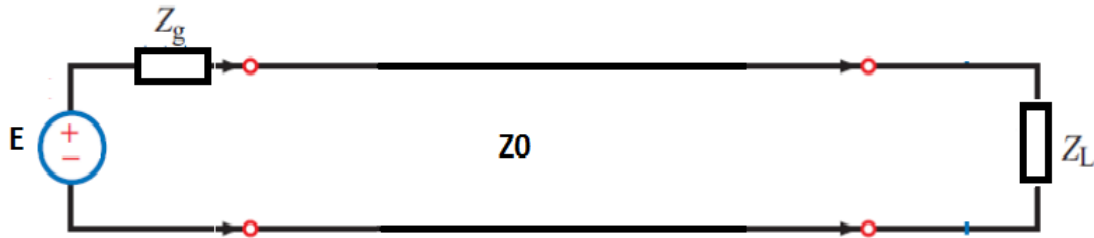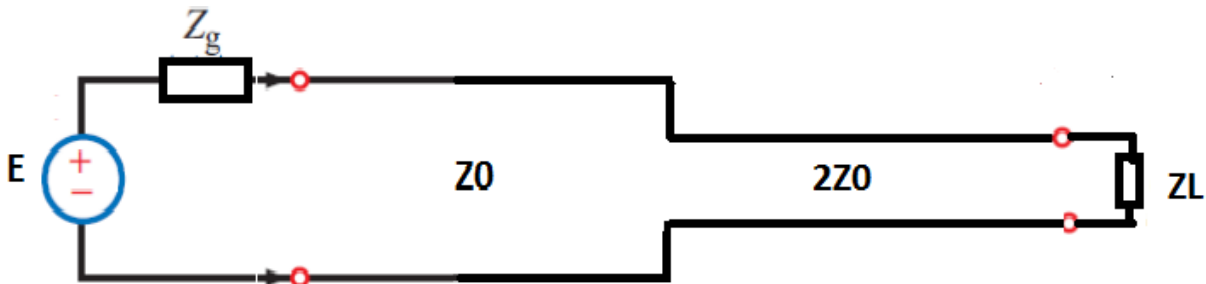

Example 2: For the lossless TL shown in below with length l, let $Zg = Z_0/3$ and $Z_R = 0$ (short circuit load). Use **Zig-zag** diagram to show the incident and reflected signals for $8T$, then draw the time-spacing diagram for the sending end E_S and I_S and receiving end E_R and I_R.
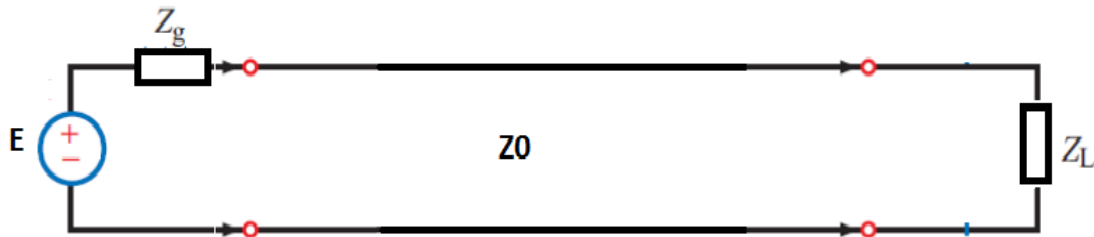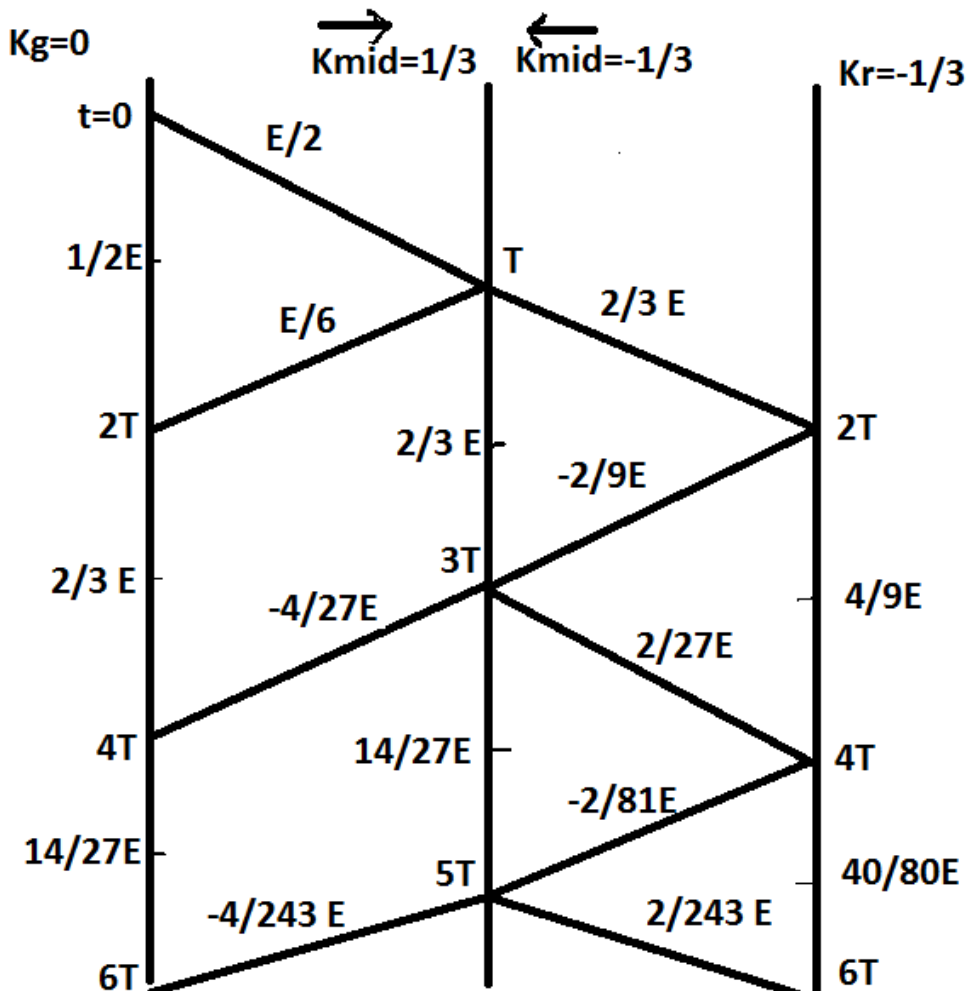

Example 3: For the lossless TL shown in below with length l, let $Zg = 6Z_0$ and $Z_R = \infty$ (open circuit load). Use **Zig-zag** diagram to show the incident and reflected signals for $8T$, then draw the time-spacing diagram for the sending end E_S and I_S and receiving end E_R and I_R.
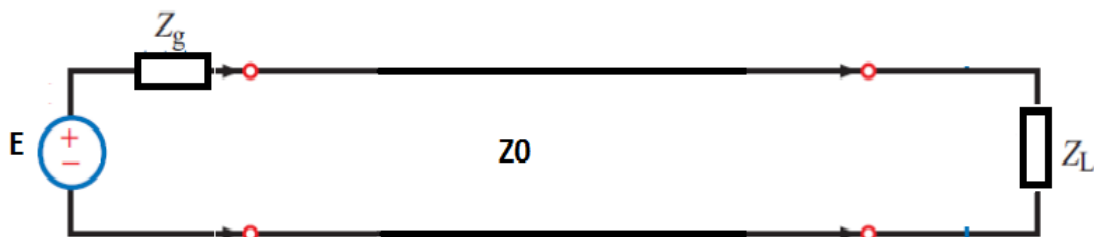
Example 4: For the lossless TL shown in below with length l and characteristic impedance Z_0, let $Zg = Z_0$ and $Z_R = Z_0$. The line in connected another lossless TL with with length l and characteristic

impedance $2Z_0$. Use Zig-zag diagram to show the incident and reflected signals for $8T$, then draw the time-spacing diagram for the sending end E_S and receiving end E_R and at the point the two lines are connected E_{mid} .


Solution:

Example 5 (HW): For the lossless TL shown in below with length l and characteristic impedance Z_0, let $Zg = Z_0$ and $Z_R = Z_0$. The line in connected another lossless TL with with length l and characteristic impedance $2Z_0$ and an impedance of Z_0 is connected between the two line as well. Use Zig-zag diagram to show the incident and reflected

signals for $8T$, then draw the time-spacing diagram for the sending end E_S and receiving end E_R and at the point the two lines are connected E_{mid} .
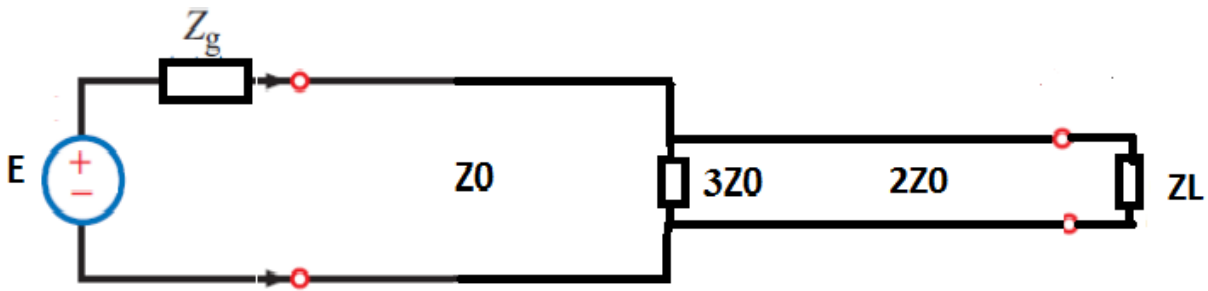
Hint:

$$K_{mid}^{\rightarrow} = \frac{(3Z_0||2Z_0)-Z_0}{(3Z_0||2Z_0)+Z_0}$$

$$K_{mid}^{\leftarrow} = \frac{(3Z_0||Z_0)-2Z_0}{(3Z_0||Z_0)+2Z_0}$$

4   Discharge of TL

An open circuit TL from both ends, charged to a voltage E through its length. At $t = 0$ the switch closed and a wave of $-E$ starts from right to left. Find E_S variation with time.
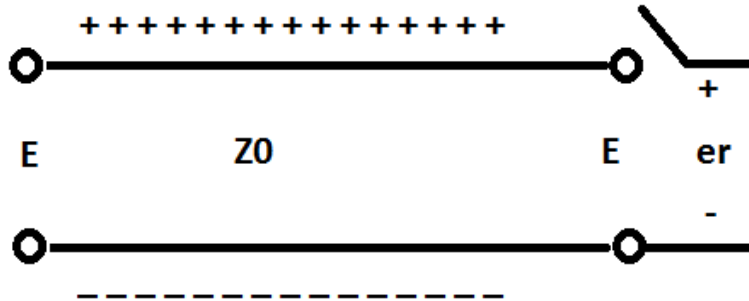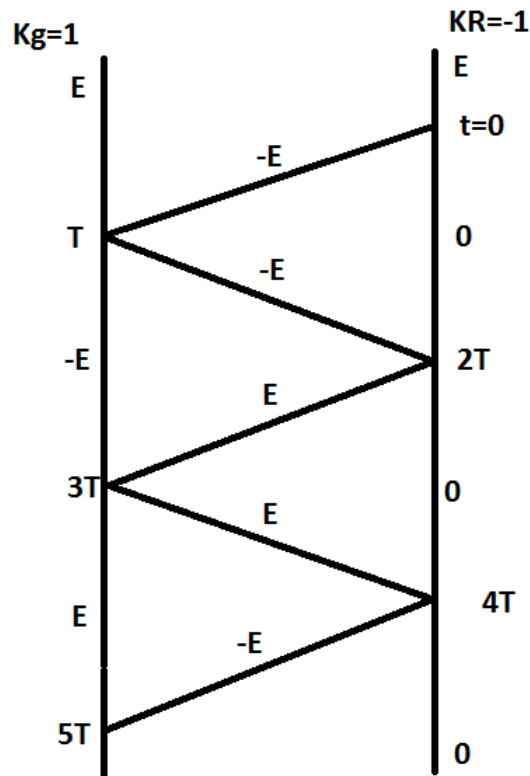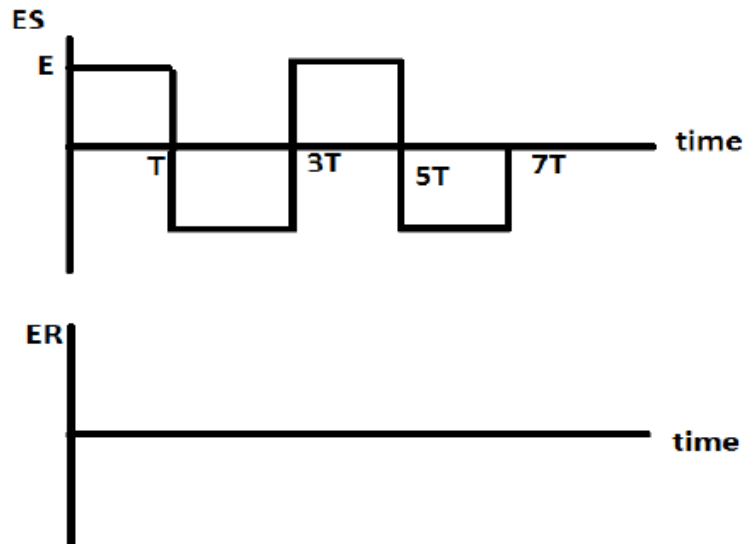

Figure 9: Discharge of TL

$$e_r = e^+ + e^-$$
$$0 = e^+ + e^-$$
$$e^- = -e^+ = -E \tag{18}$$

For the circuit shown in Fig. 9, we can see that $Kg = 1$ for open circuit source and $K_R = -1$ for short circuit load.

Example: An open circuit TL from both ends, charged to a voltage E through its length. At $t = 0$ the switch closed to connect a resistance R at the right end.

This causes a voltage e^- which originates at the switch and moves to the left along the line until the arrival of reflection. This wave produces a current through the resistance R equal to $-e^-/Z_o$ and a voltage across R is equal to $e^+ + e^- = E + e^-$. Show that the wave of voltage that is started by closing the switch is given by $\boxed{e^- = E \dfrac{-Z_o}{R + Z_o}}$
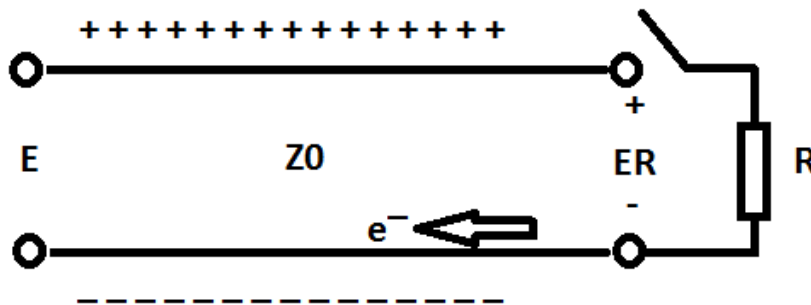
$$R = \frac{E_R}{i_R} = \frac{e^+ + e^-}{i^+ + i^-}$$

$$(19)$$

where $i^- = \frac{-e^-}{Z_o}$ and $i^+ = 0$

$$R = \frac{E_R}{i_R} = \frac{e^+ + e^-}{0 + \left(\frac{-e^-}{Z_o}\right)} \tag{20}$$
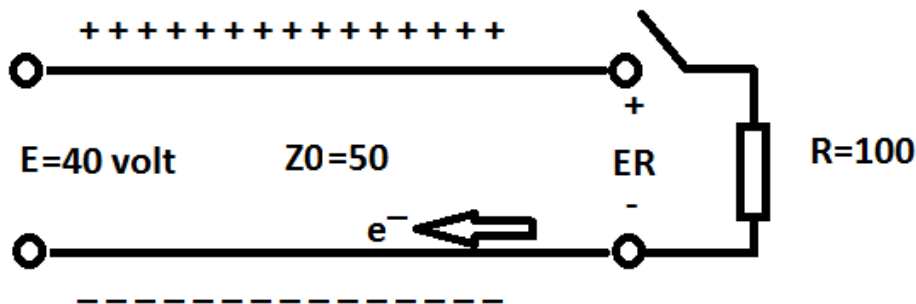
$$e^- = \frac{-Z_0}{R + Z_0} e^+ \tag{21}$$

and because $e^+ = E$

$$\boxed{e^- = \frac{-Z_0}{R + Z_0} E} \tag{22}$$

Also, we have $E_R = e^+ + e^- = E - \frac{-Z_0}{R+Z_0} E$, which leads to

$$E_R = E \frac{R}{R + Z_0} \tag{23}$$

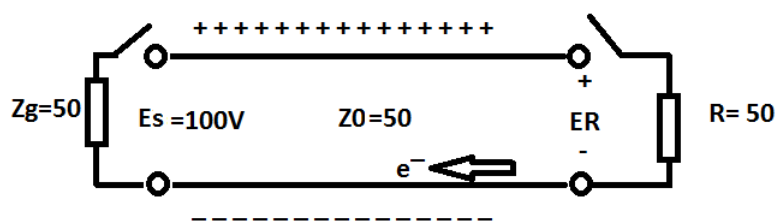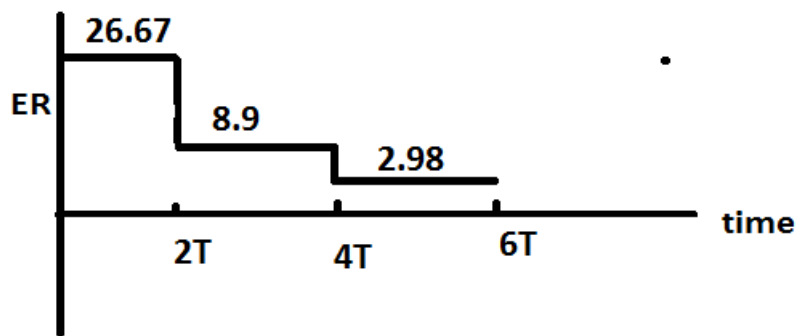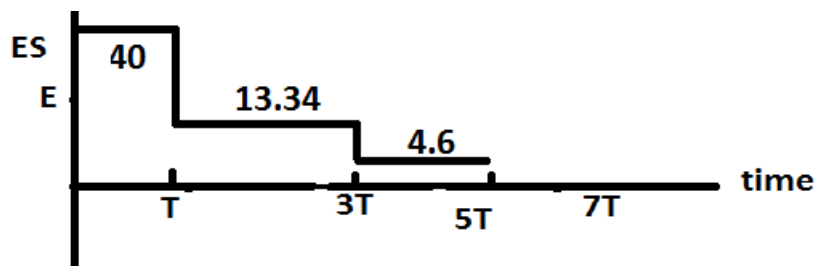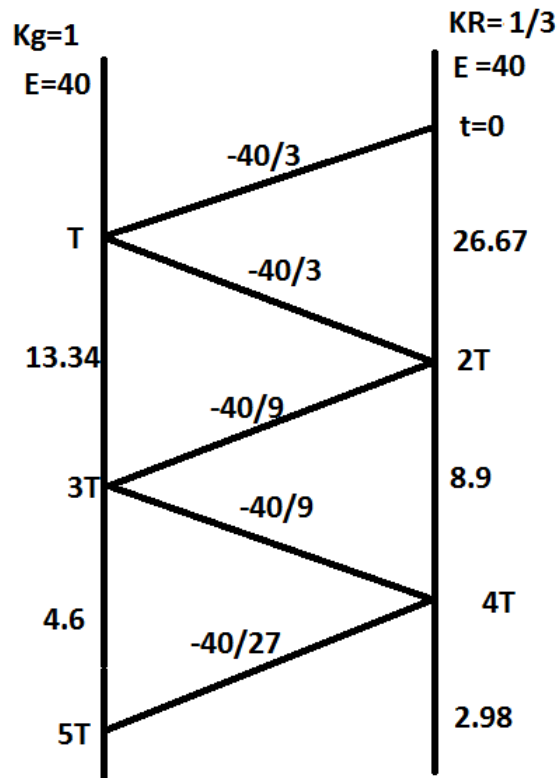**Example: Find the space-time diagram for the sending and receiving signals E_S and E_R with the time.**

$$e^- = E \frac{-Z_0}{R + Z_0}$$
$$e^- = 40 \frac{-50}{100 + 50} = \frac{-40}{3} \text{volt}$$

Also we have $Kg = 1$ and $K_R = \frac{1}{3}$ (Why?)

Example: The two switches are closed at $t = 0$, Find the space-time diagram for the sending and receiving signals E_S and E_R with the time.

Kg=1
E=40

KR= 1/3
E =40
t=0

-40/3

T

26.67

-40/3

13.34

2T

-40/9

3T

8.9

-40/9

4T

4.6

-40/27

2.98

5T

ES
E

40

13.34

4.6

time

T   3T   5T   7T

ER

26.67

8.9

2.98

time

2T   4T   6T

+ + + + + + + + + + + + + +

Zg=50   Es =100V   Z0=50   ER   R= 50

e⁻

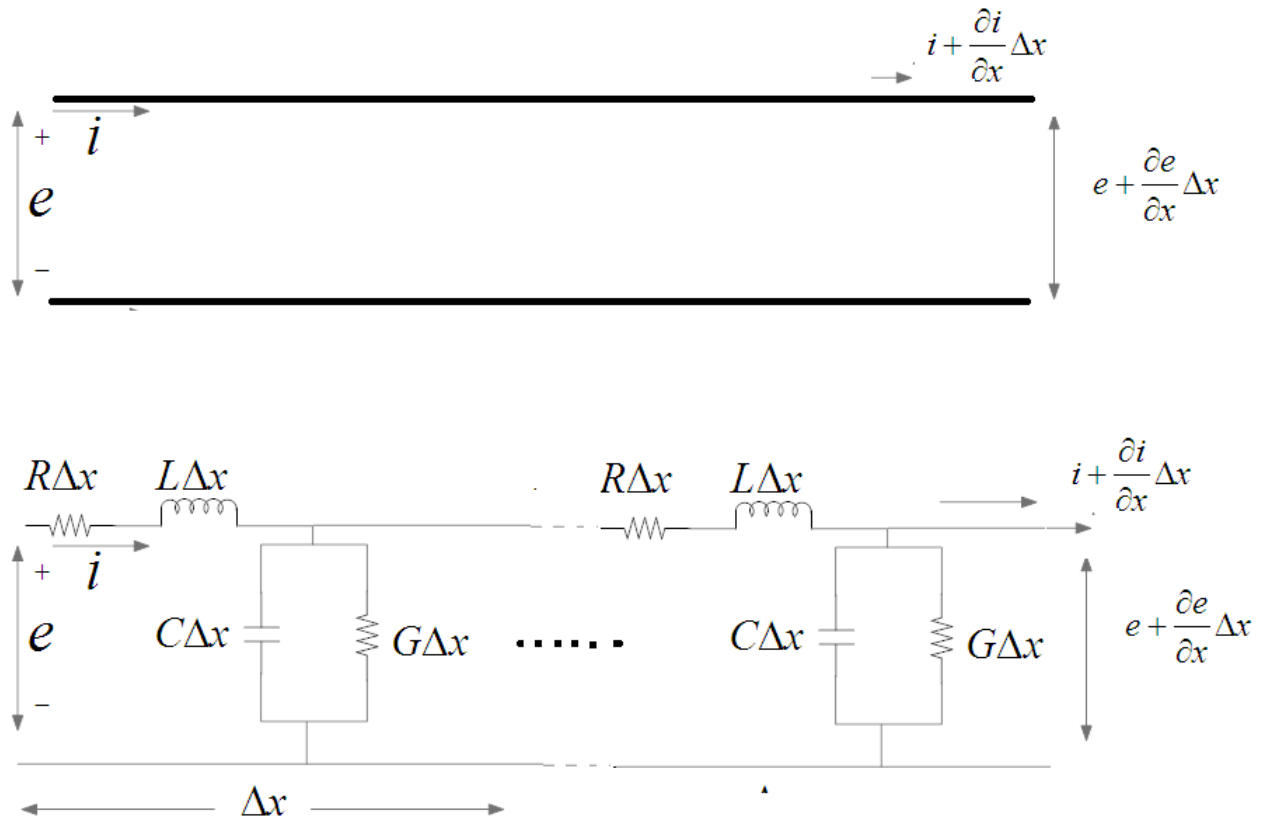## 5   AC Steady State Solution for a Uniform TL

Figure 10: Schematic representation of the elementary component of a transmission line with Variation of voltage and current along the line

$$e - (e + \frac{de}{dx}\Delta_x) = (R\Delta_x)i + (L\Delta_x)\frac{di}{dt} \tag{24}$$

Eq. 24 represents the voltage drop on the resistance and variation of voltage on inductance, which can be simplified to be as

$$-\frac{de}{dx} = Ri + L\frac{di}{dt} \tag{25}$$

Similarly for the current

$$i - (i + \frac{di}{dx}\Delta_x) = (G\Delta_x)e + (C\Delta_x)\frac{de}{dt} \tag{26}$$
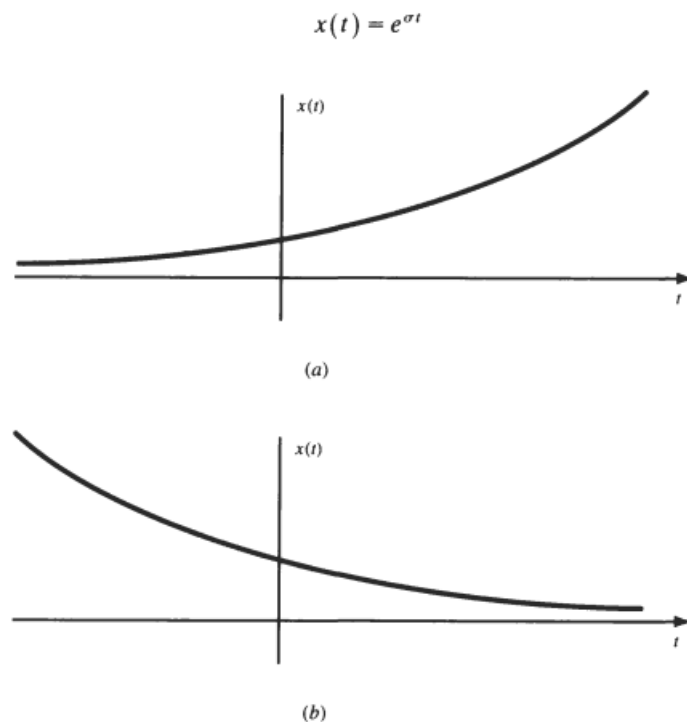
which can be simplified as

$$-\frac{di}{dx} = Ge + C\frac{de}{dt} \tag{27}$$

College of Electronics Engineering [18]

$$\frac{de}{dx} = -Ri - L\frac{di}{dt}$$
$$\frac{di}{dx} = -Ge - C\frac{de}{dt} \tag{28}$$

Assume e and i to be sinusoidal signals

$$e = \text{Real}\{(E\exp(\text{jwt})\}$$
$$i = \text{Real}\{(I\exp(\text{jwt})\} \tag{29}$$

This is because $\boxed{\exp(jwt) = \cos(wt) + j\sin(wt)}$, and $\exp(\sigma t)$ is the exponential function which increased for positive σ for and decreased when σ is negative as shown in Fig.11.

$$x(t) = e^{\sigma t}$$

(a)

(b)

Continuous-time real exponential signals. (a) $\sigma > 0$; (b) $\sigma < 0$.

Figure 11:

Now, by substituting Eq. 29 in Eq. 28, the letter can be rewritten as

$$\frac{d}{dx}(E\exp(jwt)) = -RI\exp(jwt) - L\frac{d}{dt}(I\exp(jwt))$$
$$\frac{d}{dx}(I\exp(jwt)) = -GE\exp(jwt) - C\frac{d}{dt}(E\exp(jwt)) \tag{30}$$

The last equation can be simplified more by completing the derivation of $\boxed{\frac{d}{dt}(\exp(jwt)) = jw\exp(jwt)}$, then by removing $\exp(jwt)$ from both side. These

steps lead to

$$\frac{dE}{dx} = -I(R + jwL)$$

$$\frac{dI}{dx} = -E(G + jwC) \tag{31}$$

We can now define the series impedance $Z = R + jwL$ and the shunt admittance $Y = G + jwC$ in Eq.31

$$\frac{dE}{dx} = -IZ \tag{32}$$

$$\frac{dI}{dx} = -EY \tag{33}$$

To eliminate the current I in Eq.32, we can find $\frac{d^2E}{dx^2}$ as

$$\frac{d^2E}{dx^2} = -Z\frac{dI}{dx} \tag{34}$$

and we have $\frac{dI}{dx} = -EY$, which leads to

$$\frac{d^2E}{dx^2} = ZYE \tag{35}$$

The last equation represents standard differentiation equation which can be solved as

$$E = \underbrace{V_1 \exp(-\sqrt{ZY}\,x)}_{e^+} + \underbrace{V_2 \exp(\sqrt{ZY}\,x)}_{e^-} \tag{36}$$

where V_1 and V_2 are constant and they have dimensions of voltage.

Now in order to find the current, we can substitute Eq. 36 in Eq. 32, which leads to

$$\frac{d}{dx}\left(V_1 \exp(-\sqrt{ZY}\,x) + V_2 \exp(\sqrt{ZY}\,x)\right) = -IZ \tag{37}$$

$$I = \frac{1}{\sqrt{Z/Y}}\left(V_1 e^{-\sqrt{ZY}\,x} - V_2 e^{\sqrt{ZY}\,x}\right) \tag{38}$$

which can be rewritten as

$$I = \underbrace{\frac{V_1}{\sqrt{Z_0}}\ e^{-\sqrt{ZY}\,x}}_{i^+} - \underbrace{\frac{V_2}{\sqrt{Z_0}}\ e^{\sqrt{ZY}\,x}}_{i^-} \tag{39}$$

where

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + jwL}{G + jwC}} \qquad (40)$$

which represents the characteristic impedance of the lossy TL. It is quit obvious that for lossless TL we can use Eq. 40 after substituting for $R = G = 0$ to obtain $Z_0 = \sqrt{\frac{L}{C}}$ as given already.

Also, it is important to define the propagation constant as

$$\gamma = \sqrt{ZY} = \sqrt{(R + jwL)(G + jwC)} = \alpha + j\beta \qquad (41)$$

where α is the attenuation constant in (Nep/Km), while. β represents the phase constant in (Radian/Km).

Example: TL has the following parameters
$R = 10\Omega/Km,$
$C = 0.0088\mu F/Km,$
$L = 0.0037H/Km,$ and
$G = 0.4\mu S/Km$
Find Z_0, γ, α and β
$f = 1$ **KHz.**
Answers:
$Z_0 = 663 - j134 = 696.8\angle - 11.42$
$\gamma = 0.0363\angle 78.2$
$\alpha = 0.0077$ **Nep/Km**
$\beta = 0.0363$ **rad/Km**
The students can use the MATLAB code in below to check their results:

```
R=10; L=0.0037; C=0.0088e-6; G=0.4e-6;
f=1e3
w=2*pi*f;

Z=R+j*w*L;
Y=G+j*w*C
gamma=sqrt(Z*Y);
Alpha=real(gamma)
Beta=imag(gamma)
Z0=sqrt(Z/Y)
```

College of Electronics Engineering

6   Transmission Lines with NO Reflection (Matching Scenario)

At any point at a transmission the voltage and current are expressed as given already as:

$$E(x) = \underbrace{V_1\,e^{-\gamma x}}_{e^+} + \underbrace{V_2\,e^{\gamma x}}_{e^-}$$

$$I(x) = \underbrace{\frac{1}{Z_0}V_1 e^{-\gamma x}}_{i^+} - \underbrace{\frac{1}{Z_0}V_2 e^{\gamma x}}_{i^-}$$

For the case of matching TL with load, in which the transmission line is smooth, infinite and $Z_R = Z_0$, It is known now that the incident signal e^+ will be absorbed totally by the load which leads to make the reflected signal $e^- = 0$. This lead to
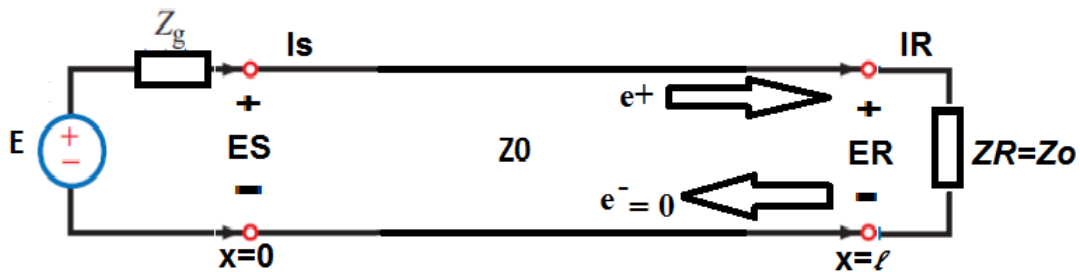

Figure 12: Matched TL

$$E(x) = V_1 e^{-\gamma x} = e^+$$

$$I(x) = \frac{1}{Z_0}V_1 e^{-\gamma x} = i^+$$

At $x = 0$

$$E_S = V_1 = e^+ = \frac{E Z_0}{Z_0 Z_g} \tag{42}$$

and the voltage at any point at the TL from $x = 0$ to $x = l$

$$E(x) = E_S\,e^{-\gamma x} = E_S\,e^{-(\alpha + j\beta)x} = E_S\,e^{-\alpha x}\,e^{-j\beta x} \tag{43}$$

$$E(x) = (E_S\,e^{-\alpha x})\,\angle{-\beta x} \tag{44}$$

Similarly for the current

$$I(x) = (I_S\,e^{-\alpha x})\,\angle{-\beta x} = (\frac{E_S}{Z_0}e^{-\alpha x})\,\angle{-\beta x} \tag{45}$$

We can now write the absolute value for the voltage and current, i.e. without taking into account the phase.

$$|E(x)| = |E_S| \ e^{-\alpha x}$$

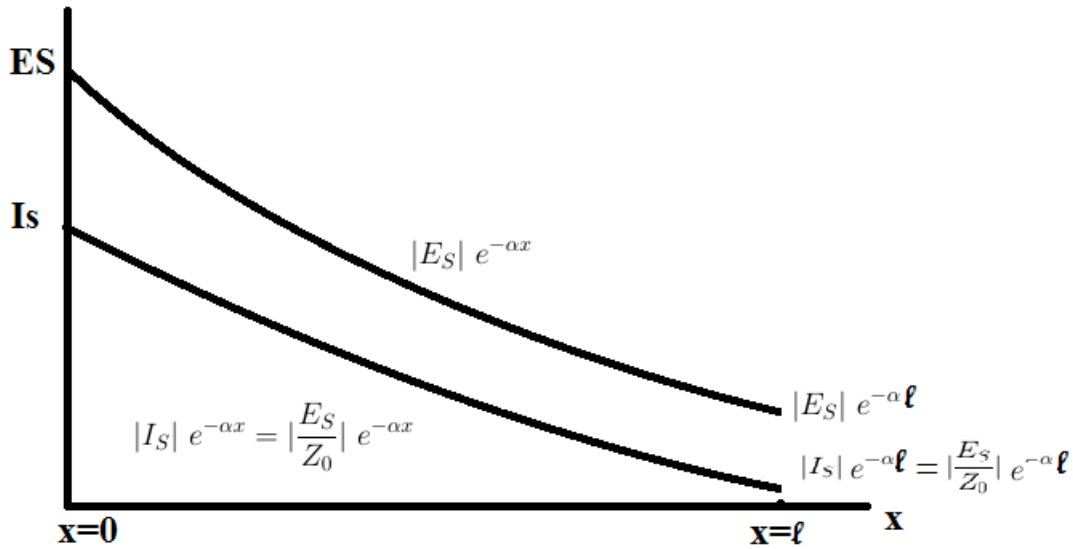$$|I(x)| = |I_S| \ e^{-\alpha x} = |\frac{E_S}{Z_0}| \ e^{-\alpha x}$$

Figure 13: The effect of the attenuation constant α on the voltage and current along a TL

7   Traveling Waves and Its Characteristics:

The prorogation constant γ affects on the amplitude by the attenuation factor α, while the effect of β is on the phase of the signal.

$$E(x) = E_S \ e^{-\gamma x} = E_S \ e^{-(\alpha + j\beta)x} = E_S \ e^{-\alpha x} \ e^{-j\beta x}$$

We assume now that we have a sinusoidal applied at the source as:

$$E_S = A\cos(wt) = AReal(e^{jwt})$$

This applied signal will be affected by propagation constant $\gamma = \alpha + j\beta$ as explained already. Therefore, the applied signal, for matched load, at any point over the TL will be expressed as:

$$
\begin{aligned}
E(x) &= E_S \ e^{-\gamma x} = E_S \ e^{-\alpha x} e^{-j\beta x} \\
&= AReal(e^{jwt})e^{-\alpha x}e^{-j\beta x} \\
&= Ae^{-\alpha x} Real(e^{j(wt - \beta x)}) \\
&= Ae^{-\alpha x} \cos(wt - \beta x) \\
&= Ae^{-\alpha x} \cos(\theta)
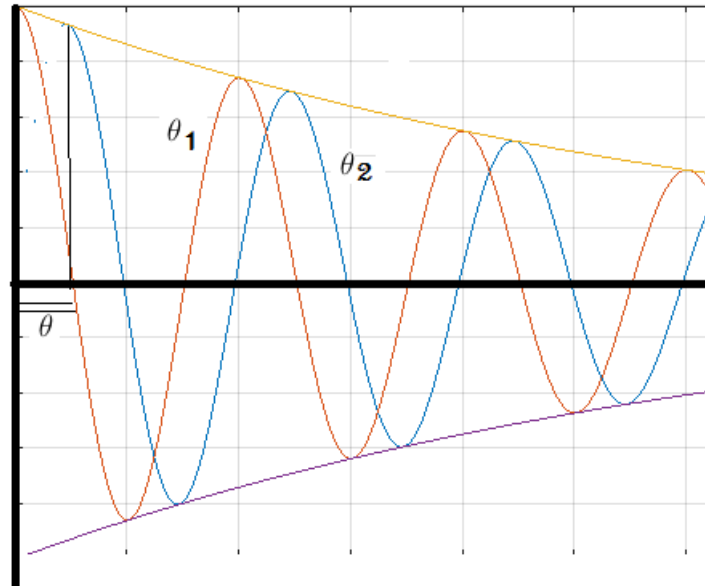\end{aligned}
$$

College of Electronics Engineering [23]

Figure 14: The phase velocity

where $\theta = wt - \beta x$.

$$\text{Phase difference} = \theta_2 - \theta_1 = \text{Constant}$$
$$(wt_2 - \beta x_2) - (wt_1 - \beta x_1) = \text{Constant}$$
$$(wt_2 - wt_1) - (\beta x_2 - \beta x_1) = \text{Constant}$$
$$wdt - \beta dx = \text{Constant}$$
$$\frac{1}{dt}(wdt - \beta dx = \text{Constant})$$
$$w - \beta \frac{dx}{dt} = 0$$
$$\frac{dx}{dt} = \frac{w}{\beta}$$

where $\frac{dx}{dt}$ is defined as phase velocity (v_p), which represents the velocity of moving the power over TL, and can be defined as

$$\boxed{v_p = \frac{w}{\beta} = \frac{2\pi f}{\beta} \quad m/sec} \tag{46}$$

The phase velocity (v_p) is also represents the product of the operating frequency (f) and the signal wave length (λ), i.e. $\boxed{v_p = f \times \lambda}$, then $v_p = f \times \lambda = \frac{w}{\beta} = \frac{2\pi f}{\beta}$, this leads to

$$\boxed{\beta = \frac{2\pi}{\lambda}} \tag{47}$$

Furthermore, the time delay or (the phase delay) which represents the time required to the wave in order to be moved a unit of length, this term can be defined

as

$$\boxed{\text{phase delay} = \frac{1}{v_{\text{p}}} = \frac{\beta}{w} \quad \text{sec}} \tag{48}$$

The delay must be constant and independent of frequency.

Example: A lossy TL has a length of 200 **Km and its characteristic impedance is** $Z_0 = 683 - j138\Omega$. **The load** $Z_R = Z_0$ **is connected to this TL and it source impedance is equal to** 500Ω **with a voltage source of** 10 **V.r.m.s, the propagation constant for this TL is measured as** $\gamma = 0.0074 + j0.0356$ **per km. Calculate** E_s, I_S, P_s **and** E_R, I_R, P_R.

The students can use the MATLAB code in below to check their results:

```
clear all; clc, close all;
gamma= 0.0074+j*0.0356; %per km
L=200; %km
Z0= 683-j*138;
ZR=Z0;
Zg=500;
Eg=10;
%%%%%%%%%%%%%%%%% Source Calculations %%%%%%%%%%%%%%%%%%%%%%%%%
Es=Eg*Z0/(Z0+Zg);
Is=Eg/(Z0+Zg);
Ps=abs(Is)^2*real(Z0)
%%%%%%%%%%%%%%%%%%% Load Calculations%%%%%%%%%%%%%%%%%%%%%%%%%%%
alpah=real(gamma);
beta=imag(gamma);
ER= Es*exp(-alpah*L)*exp(-j*beta*L);
IR=ER/ZR;
Ps=abs(IR)^2*real(ZR)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Answers:
% Ps = 48.2 mW
% PR =2.49 mW
```

8 Variation of Z_0, α, β with Frequency

As it becomes well-known now that:

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + jwL}{G + jwC}}$$

$$\gamma = \sqrt{ZY} = \sqrt{(R + jwL)(G + jwC)} = \alpha + j\beta$$

To discuss the variation of Z_0, α, β with frequency, we can take different scenarios:

- $f = 0$ **the case of D.C:**

$$Z_0 = \sqrt{\frac{R}{G}}$$

$$\gamma = \sqrt{RG} \rightarrow \alpha = \sqrt{RG} \ \text{ and } \ \beta = 0$$

- **For the case of High frequency:**
  $(wL >> R)$ and $(wC >> G)$

$$Z_0 = \sqrt{\frac{R + jwL}{G + jwC}} = \sqrt{\frac{jwL}{jwC}} = \sqrt{\frac{L}{C}}$$

$$\gamma = \sqrt{(R + jwL)(G + jwC)} = (jwL + R)^{\frac{1}{2}}(jwC + G)^{\frac{1}{2}}$$

Now , by using binomial expression to solve the last equation as

$$(a + b)^{\frac{1}{2}} = a^{\frac{1}{2}}(1 + \frac{b}{a})^{\frac{1}{2}}$$

For $a >> b \rightarrow a^{\frac{1}{2}}(1 + \frac{b}{a})^{\frac{1}{2}} \approx a^{\frac{1}{2}} + \frac{1}{2}a^{-\frac{1}{2}} \, b$

$\gamma = \left[(jwL)^{\frac{1}{2}} + \frac{1}{2}(jwL)^{-\frac{1}{2}} \, R \right] \left[(jwC)^{\frac{1}{2}} + \frac{1}{2}(jwC)^{-\frac{1}{2}} \, G \right]$

$\gamma = jw\sqrt{LC} + \frac{1}{2}\sqrt{\frac{L}{C}}G + \frac{1}{2}\sqrt{\frac{C}{L}}R + \underbrace{\frac{1}{4}jw^{-1}\frac{1}{\sqrt{LC}}RG}_{\text{neglected at High frequency}}$

$\gamma = \underbrace{\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}}_{\alpha} + \underbrace{jw\sqrt{LC}}_{\beta}$

But we have $Z_0 = \sqrt{\frac{L}{C}}$ as been found above which leads to

$\gamma = \frac{R}{2Z_0} + \frac{GZ_0}{2} + jw\sqrt{LC}$

$\alpha = \underbrace{\frac{R}{2Z_0}}_{\text{Losses due to Conductor}} + \underbrace{\frac{GZ_0}{2}}_{\text{Losses due to Isolator}} \text{ and } \beta = jw\sqrt{LC}$

We conclude from the last equation:

- α depends on R and G and it does not depend on frequency because Z_0 is constant.

- β has linear variation with frequency.

- Phase velocity v_p must be equal at all frequencies

- For distortion-less TL the equation in below should be applied.

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

But in the real world $\frac{R}{L} > \frac{G}{C}$, therefore, it required to increase L as a practical solution in order to avoid distortion in TL.
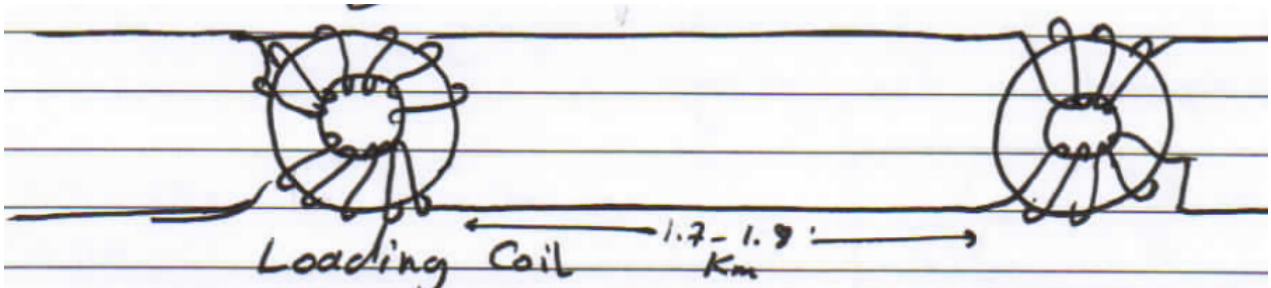
Example: For a lossy TL with $R = 10\Omega$; $L = 0.0037$H; $C = 0.0088\mu F$; $G = 0.4\mu S$.
1. Examine the TL to check if it causes distortion or it is distortion-less.
2. if it causes distortion Find the value of L_{Coil} that should be added to the value of L to make this TL dimensionless.
Answer:
$R/L = 2.7027e + 03$
$G/C = 45.4545$
The two terms are not equal, so this TL causes distortion.
The total inductance required to make this TL distortion-less can be obtained as:
$L_{Total} = R * C/G = 0.2200$ **H**
So, inductance of the loading coil can be calculated as:
$L_{Coil} = L_{Total} - L = 0.2163$ **H**.

College of Electronics Engineering [27]

Example: A telephone cable operating at frequency of 30 **KHz, and it has the following parameters** $R = 52.5\Omega/Km, G = 0, C = 0.062\mu F/Km$ **and** $L = 1.1mH/Km$**. Calculate:**

1) $\alpha, \beta, \lambda, v_p$

2) The length of the line which gives attenuation of 25 **dB.**

The Matlab code in below is only to check the results accurately by the students.

```
R=52.5; L=1.1e-3; C=0.062e-6; G=0; f=30e3; ATT=25;%dB
w=2*pi*f;
Z=R+j*w*L
Y=G+j*w*C
gamma=sqrt(Z*Y)
Alpha=real(gamma)
beta=imag(gamma)
lambda=2*pi/beta
vp=w/beta
X=log(10^(-ATT/20))/-Alpha
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Alpha =0.1955 Nep/km
beta =1.5689 rad/km
lambda =4.0049 km
vp =120.15e+03 km/sec
X =14.7196 km
```

Hint:
The relationship between linear ratios for voltage and power and these ratios in dB is calculated as

$$V_{dB} = 20 \log(V_{Linear})$$
$$P_{dB} = 10 \log(P_{Linear})$$

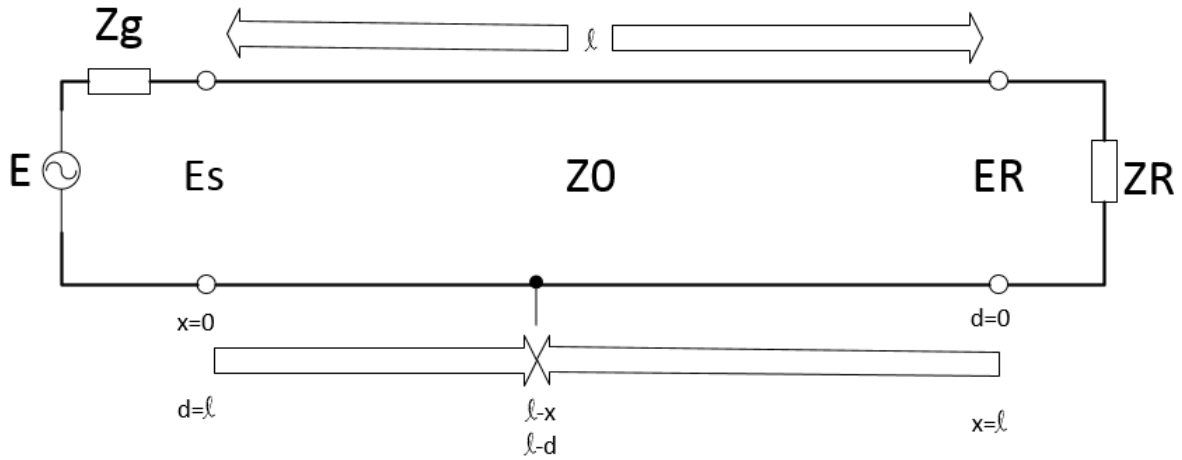while to convert these ratios back from (dB) to linear

$$V_{Linear} = 10^{\frac{V_{dB}}{20}}$$
$$P_{Linear} = 10^{\frac{P_{dB}}{10}}$$

where V represents a ratio between two terms (voltages) in volt, while, P is a ratio between two terms (power) in watt.

9   Transmission Lines with Reflection

9.1   For lossy Line:

- Let $Z_R \neq Z_0$ and $Z_g \neq Z_0$

- This means that there are several reflections form both terminals (load and source).

- At any points at this TL, we can find the voltage and current as a function of E, Z_R, Z_g and Z_0, i.e. as a function of K_R and K_g.

- The propagation constant γ will affect the voltage, power and current depending on the distance from source to load, i.e. $x = 0 \rightarrow x = \ell$.

- Let us measure the distance to the load by using the term x, while to measure the distance from load to the source we can use the term d, where $d = \ell - x$ and $x = \ell - d$.

- As we know that $K_R = \frac{Z_R - Z_0}{Z_R + Z_0}$, $K_g = \frac{Z_g - Z_0}{Z_g + Z_0}$ and $E_S = \frac{EZ_0}{Z_0 + Z_g} = e^+$.

- At point x on this TL, we can calculate E_x as:

$$E_x = \frac{EZ_0}{Z_0 + Z_g}\Big(e^{-\gamma x} + K_R\ e^{-2\gamma\ell}e^{\gamma x} + K_R K_g\ e^{-2\gamma\ell}e^{-\gamma x}$$
$$+ K_R^2 K_g\ e^{-4\gamma\ell}e^{\gamma x} + K_R^2 K_g^2\ e^{-4\gamma\ell}e^{-\gamma x} + \dots .\Big) \qquad (49)$$

- By continuing in this manner, it will be an infinite reflections at distance x which represents the A.C steady state.

- By using geometrical series Eq. 49 can be solved as

$$\boxed{E_x = \frac{EZ_0}{Z_0 + Z_g}\Big(\frac{e^{-\gamma x} + K_R\ e^{-2\gamma\ell}e^{\gamma x}}{1 - K_R K_g\ e^{-2\gamma\ell}}\Big)} \qquad (50)$$
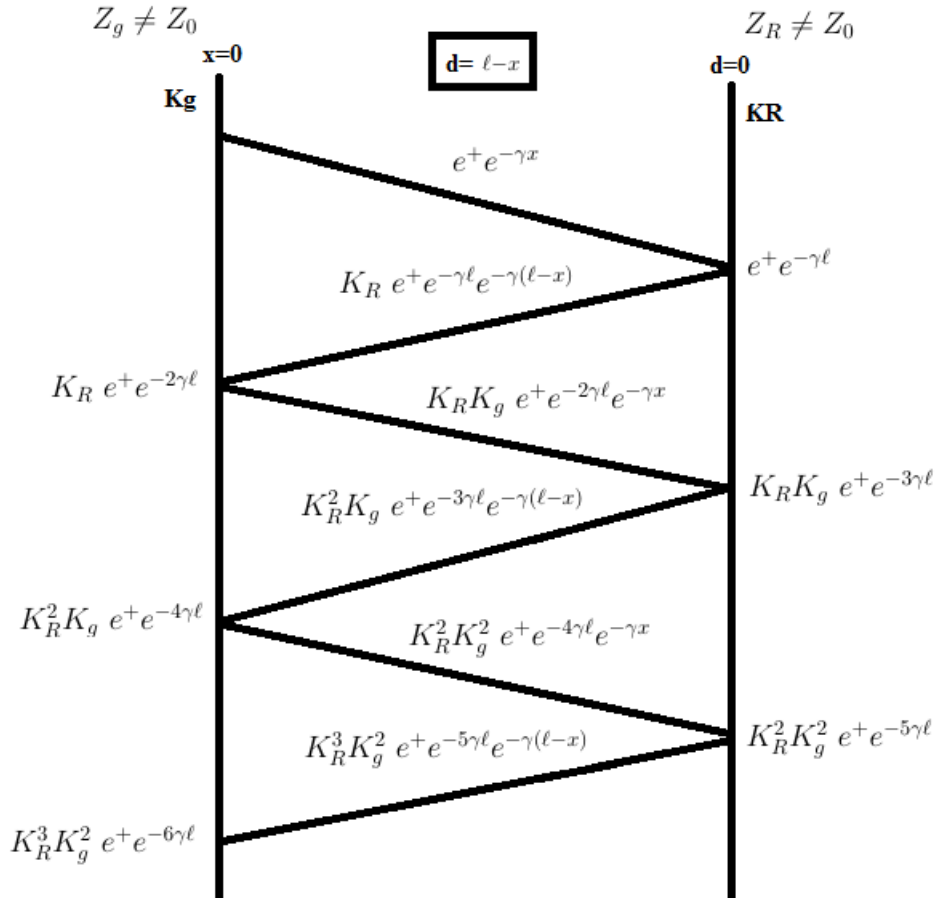
College of Electronics Engineering [29]

Figure 15: Zig-Zag Diagtram for Lossy TL with multiple reflections form both source and load.

and the current

$$\boxed{I_x = \frac{E_x}{Z_0} = \frac{E}{Z_0 + Z_g}\left(\frac{e^{-\gamma x} + K_R\, e^{-2\gamma\ell}e^{\gamma x}}{1 - K_R K_g\, e^{-2\gamma\ell}}\right)} \tag{51}$$

- Comparing Eq. 50 with $E_x = \underbrace{V_1\, e^{-\gamma x}}_{e^+} + \underbrace{V_2\, e^{\gamma x}}_{e^-}$, we can rewrite e^+ and e^- as

$$e^+ = \frac{EZ_0}{Z_0 + Z_g}\left(\frac{e^{-\gamma x}}{1 - K_R K_g\, e^{-2\gamma\ell}}\right)$$

$$e^- = \frac{EZ_0}{Z_0 + Z_g}\left(\frac{K_R\, e^{-2\gamma\ell}e^{\gamma x}}{1 - K_R K_g\, e^{-2\gamma\ell}}\right) \tag{52}$$

Also, we can write expression for V_1 and V_2 as:

$$V_1 = \frac{EZ_0}{Z_0 + Z_g}\left(\frac{1}{1 - K_R K_g\, e^{-2\gamma\ell}}\right)$$

$$V_2 = \frac{EZ_0}{Z_0 + Z_g}\left(\frac{K_R\, e^{-2\gamma\ell}}{1 - K_R K_g\, e^{-2\gamma\ell}}\right) \tag{53}$$

College of Electronics Engineering [30]

Moreover, sometimes it is required to express E and I with respect to a distance measured from load using the term d

$$E_d = \frac{EZ_0}{Z_0 + Z_g} \times \frac{\left(e^{-\gamma(\ell-d)} + K_R\ e^{-2\gamma\ell}e^{\gamma(\ell-d)}\right)}{(1 - K_R K_g\ e^{-2\gamma\ell})} \tag{54}$$

which can be rewritten as:

$$\boxed{E_d = \frac{EZ_0}{Z_0 + Z_g} \times \frac{e^{-\gamma\ell}\left(e^{\gamma d} + K_R\ e^{-\gamma d}\right)}{(1 - K_R K_g\ e^{-2\gamma\ell})}} \tag{55}$$

Similarly

$$\boxed{I_d = \frac{E}{Z_0 + Z_g} \times \frac{e^{-\gamma\ell}\left(e^{\gamma d} - K_R\ e^{-\gamma d}\right)}{(1 - K_R K_g\ e^{-2\gamma\ell})}} \tag{56}$$

9.2   For Lossless TL:

As we know that the propagation constant is defined as $\gamma = \alpha + j\beta$, and for lossless TL $\alpha = 0$, then $\gamma = j\beta$, which can be substituted in Eq. 50

$$E_x = \frac{EZ_0}{Z_0 + Z_g} \times \frac{(e^{-j\beta x} + K_R\ e^{-j2\beta\ell}e^{j\beta x})}{(1 - K_R K_g\ e^{-j2\beta\ell})}$$

$$E_x = \frac{EZ_0}{Z_0 + Z_g} \times \frac{e^{-j\beta x}(1 + K_R\ e^{-j2\beta\ell}e^{j2\beta x})}{(1 - K_R K_g\ e^{-j2\beta\ell})}$$

$$E_x = \frac{EZ_0}{Z_0 + Z_g} \times \frac{e^{-j\beta x}(1 + K_R\ e^{-j2\beta(\ell-x)})}{(1 - K_R K_g\ e^{-j2\beta\ell})}$$

$$E_x = \frac{EZ_0}{Z_0 + Z_g} \times \frac{e^{-j\beta x}(1 + K_R\ e^{-j2\beta d})}{(1 - K_R K_g\ e^{-j2\beta\ell})}$$

10   Interference and Standing Waves Patterns:

- For $Z_R \neq Z_0$, reflection occurs from load, which means that the load absorbs part of the incident signal, while another part is reflected back to the source.

- Total reflection is occurred when the load is open circuit (O.C) or short circuit (S.C).

1. Open circuit (O.C) Load:
   $Z_R = \infty \rightarrow K_R = +1$
   $E_x = e^+ + e^-$ and $I_x = i^+ + i^- \rightarrow I_x = \frac{e^+}{Z_0} - \frac{e^-}{Z_0}$
   where E_x and I_x represent the voltage and current at a distance of x away from the source, respectively. Furthermore, e^+ is in-phase with i^+, while, e^- is out of phase with i^-.

   At the load, which is open circuit here, it is known that voltage would be $E|_{x=\ell} = E_{max}$ while, the current $I|_{x=\ell} = 0$, as shown in Fig. 16.

   In Fig. 16, by moving from the load towards the source, the voltage would be decreased to minimum at $d = \frac{\lambda}{4}$ from the load, and it would be increased again to the maximum when $d = \frac{\lambda}{2}$ from the load. In contrast, the current would be increased from zero to the maximum at $d = \frac{\lambda}{4}$ from the load, and it would be decreased again to the minimum (zero) when $d = \frac{\lambda}{2}$ from the load. It is obvious that the maximum and minimum for both the voltage and current are repeated every $d = \frac{\lambda}{4}$ from the previous case.
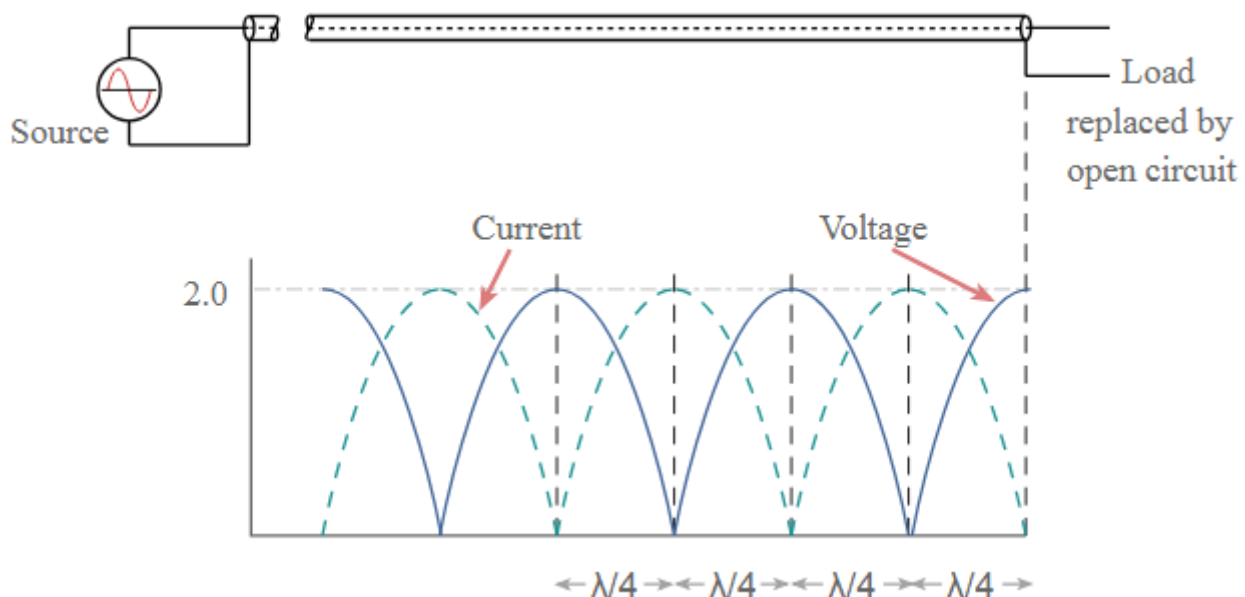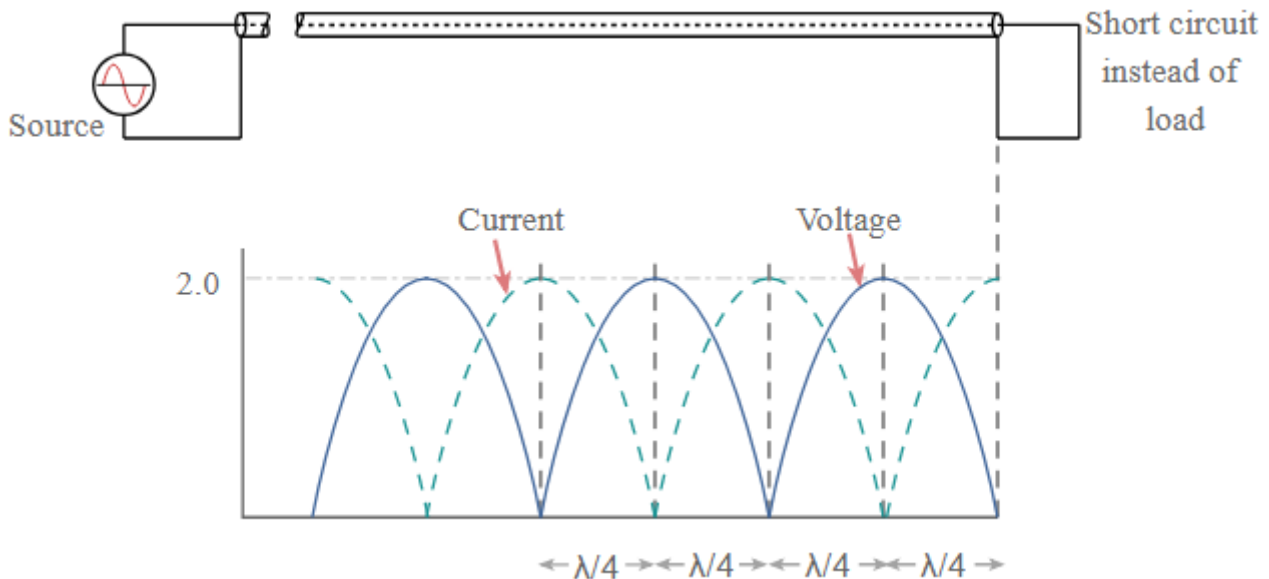

Figure 16: Voltage and current standing wave patterns for open circuit

College of Electronics Engineering [32]

2. Short circuit (S.C) Load:
   $Z_R = 0 \rightarrow K_R = -1$
   $E_x = e^+ + e^-$ and $I_x = i^+ + i^- \rightarrow I_x = \frac{e^+}{Z_0} - \frac{e^-}{Z_0}$

At the load, which is short circuit here, it is known that voltage would be $E|_{x=\ell} = 0$ while, the current $I|_{x=\ell} = I_{max}$, as shown in Fig. 17.

In Fig. 17, by moving from the load towards the source, the voltage would be increased to maximum at $d = \frac{\lambda}{4}$ from the load, and it would be decreased again to the minimum when $d = \frac{\lambda}{2}$ from the load. In contrast, the current would be decreased from maximum to the minimum (zero) at $d = \frac{\lambda}{4}$ from the load, and it would be increased again to the maximum when $d = \frac{\lambda}{2}$ from the load. It is obvious that the maximum and minimum for both the voltage and current are repeated every $d = \frac{\lambda}{4}$ from the previous case.

Figure 17: Voltage and current standing wave patterns for short circuit

11   Input Impedance of lossy Transmission Lines:

By returning to Eq. 50 and Eq. 51,

$$E_x = \frac{E_g Z_0}{Z_g + Z_0} \times \frac{(e^{-\gamma x} + k_R e^{-2\gamma l} e^{\gamma x})}{(1 - k_R k_g e^{-2\gamma l})}$$

$$I_x = \frac{E_g}{Z_g + Z_0} \times \frac{(e^{-\gamma x} - k_R e^{-2\gamma l} e^{\gamma x})}{(1 - k_R k_g e^{-2\gamma l})}$$

To simplify E_x, we can substitute $V_1 = \frac{E_g Z_o}{Z_g + Z_o} \times \frac{1}{(1 - k_R k_g e^{-2\gamma l})}$, then we can rewrite E_x as

$$E_x = V_1 (e^{-\gamma x} + k_R e^{-2\gamma l} e^{\gamma x}) \qquad (57)$$

Similarly

$$I_x = \frac{V_1}{Z_0}(e^{-\gamma x} - k_R e^{-2\gamma l} e^{\gamma x}) \qquad (58)$$

From the above equations of voltages and current at a particular point (x) in the TL, we can evaluate the impedance at that point as:

$$Z_x = \frac{E_x}{I_x} = Z_0 \frac{(e^{-\gamma x} + k_R e^{-2\gamma l} e^{\gamma x})}{(e^{-\gamma x} - k_R e^{-2\gamma l} e^{\gamma x})}$$

The input impedance can be found by substituting for $x = 0$ and $l = d + x = d$, because $(x = 0)$. In another words, if we have Z_R, the input impedance can be calculated easily by applying the last equation, this leads to

$$Z_{in} = Z_0 \frac{(1 + k_R e^{-2\gamma d})}{(1 - k_R e^{-2\gamma d})}$$

while to find the impedance at any point in the TL can be obtained by multiplying the last equation by $\left(\frac{e^{\gamma d}}{e^{\gamma d}}\right)$, or, we can return to the equations of E_d in Eq. 55 and I_d in Eq. 56, which are given previously as

$$E_d = \frac{E_g Z_0}{Z_g + Z_0} \times \frac{e^{-\gamma l}(e^{\gamma d} + k_R e^{-\gamma d})}{(1 - k_R k_g e^{-2\gamma l})}$$

$$I_d = \frac{E_g}{Z_g + Z_0} \times \frac{e^{-\gamma l}(e^{\gamma d} - k_R e^{-\gamma d})}{(1 - k_R k_g e^{-2\gamma l})}$$

From the last two equation, we can evaluate Z_d as $Z_d = \frac{E_d}{I_d}$, which can be expressed as

$$Z_d = Z_0 \frac{(e^{\gamma d} + k_R e^{-\gamma d})}{(e^{\gamma d} - k_R e^{-\gamma d})}$$

Now by substituting for K_R with its well-known derived formula as $K_R = \frac{Z_R - Z_0}{Z_R + Z_0}$, and after some straight forward simplifications we can obtain

$$Z_d = Z_0 \frac{(e^{\gamma d} + (\frac{Z_R - Z_0}{Z_R + Z_0})e^{-\gamma d})}{(e^{\gamma d} - (\frac{Z_R - Z_0}{Z_R + Z_0})e^{-\gamma d})}$$

$$Z_d = Z_0 \left[\frac{e^{\gamma d}(Z_R + Z_0) + e^{-\gamma d}(Z_R - Z_0)}{e^{\gamma d}(Z_R + Z_0) - e^{-\gamma d}(Z_R - Z_0)} \right]$$

$$Z_d = Z_0 \left[\frac{Z_R(e^{\gamma d} + e^{-\gamma d}) + Z_0(e^{\gamma d} - e^{-\gamma d})}{Z_0(e^{\gamma d} + e^{-\gamma d}) + Z_R(e^{\gamma d} - e^{-\gamma d})} \right]$$

Now by substituting $\cosh(\gamma d) = \frac{e^{\gamma d} + e^{-\gamma d}}{2}$ and $\sinh(\gamma d) = \frac{e^{\gamma d} - e^{-\gamma d}}{2}$ in the above equation, Z_d can be expressed as

$$Z_d = Z_0 \left[\frac{Z_R \cosh(\gamma d) + Z_0 \sinh(\gamma d)}{Z_0 \cosh(\gamma d) + Z_R \sinh(\gamma d)} \right]$$

Finally, as we know that $\tanh(\gamma d) = \frac{\sinh(\gamma d)}{\cosh(\gamma d)}$, Z_d can be expressed as

$$\boxed{Z_d = Z_0 \left[\frac{Z_R + Z_0 \tanh(\gamma d)}{Z_0 + Z_R \tanh(\gamma d)} \right]} \tag{59}$$

The above equation is very important as it represents the impedance at any point on the TL, for example if $d = 0$, which means that we are going to evaluate the impedance at the load, i.e. $Z_d = Z_R$. While, at the source when $d = l$,

$$Z_s = Z_0 \left[\frac{Z_R + Z_0 \tanh(\gamma l)}{Z_0 + Z_R \tanh(\gamma l)} \right].$$

Furthermore, for the lossless transmission lines $\gamma = j\beta$, and we can substitute $l = d + x$ in Eq. 57, as

$$E_x = V_1(e^{-j\beta x} + k_R e^{-j2\beta(x+d)}e^{j\beta x})$$

which leads to

$$E_x = V_1 e^{-j\beta x}(1 + k_R e^{-j2\beta d})$$

College of Electronics Engineering ³⁵
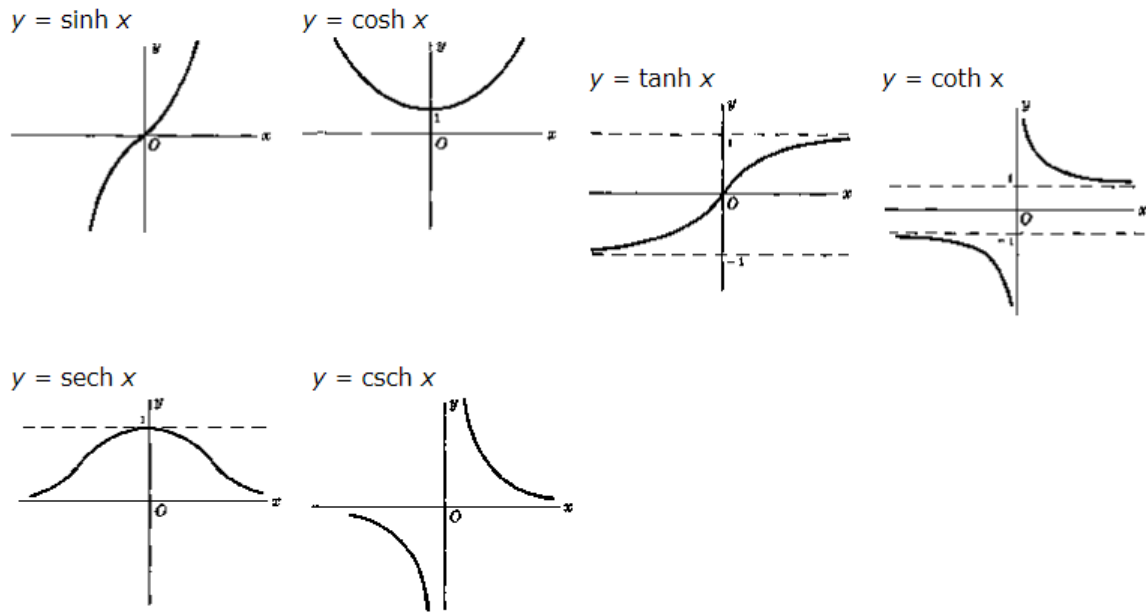
GRAPHS OF HYPERBOLIC FUNCTIONS

Figure 18:

and the current in Eq. 58 can be expressed similarly as

$$I_x = \frac{V_1}{Z_0} e^{-j\beta x}(1 - k_R e^{-j2\beta d})$$

By using the same procedure used for the lossy TL, we can find a general expression for Z_d for the lossless TL by substituting $\cosh(j\beta d) = \frac{e^{j\beta d} + e^{-j\beta d}}{2} = \cos(\beta d)$ and $\sinh(j\beta d) = \frac{e^{j\beta d} - e^{-j\beta d}}{2} = j\sin(\beta d)$, and $\tan(\beta d) = \frac{\sin(\beta d)}{\cos(\beta d)}$ in the above equation, Z_d can be expressed as

$$Z_d = Z_0 \left[\frac{Z_R + jZ_0 \tan(\beta d)}{Z_0 + jZ_R \tan(\beta d)} \right]$$

and the impedance at the source terminal can be expressed as

$$Z_s = Z_0 \left[\frac{Z_R + jZ_0 \tan(\beta l)}{Z_0 + jZ_R \tan(\beta l)} \right]$$

Note that if $Z_R = Z_0$, this will lead to $Z_s = Z_0$ as well.

College of Electronics Engineering ³⁶

Example:

A lossy TL with $Z_g = 700\Omega$, $E_{rms} = 10$volt $Z_0 = 685 - j92\Omega$, $f = 1000$Hz, $l = 100$Km, $Z_R = 2000\Omega$ and $\gamma = 0.00497 + j0.0352$ per km, Find Z_s?

Solution:

$$Z_s = Z_0 \left[\frac{Z_R + Z_0 \tanh(\gamma l)}{Z_0 + Z_R \tanh(\gamma l)} \right]$$

where

$$\tanh(\gamma l) = \frac{\sinh(\gamma l)}{\cosh(\gamma)l} = \frac{\sinh((\alpha + j\beta)l)}{\cosh((\alpha + j\beta)l)} = \frac{\sinh(0.497 + j3.52)}{\cosh(0.497 + j3.52)}$$

which can be solved separately using the formulas in below (given in the exam no need to remember them)

$$\sinh(a + jb) = \sinh(a)\cos(b) + j\cosh(a)\sin(b)$$
$$\cosh(a + jb) = \cosh(a)\cos(b) + j\sinh(a)\sin(b)$$

where $a = \alpha\ell = 0.00497 * 100 = 0.497$ and $b = \beta\ell = 0.0352 * 100 = 3.52$

$$\sinh(0.497 + j3.52) = \sinh(\underbrace{0.497}_{rad})\cos(\underbrace{3.52}_{rad}) + j\cosh(\underbrace{0.497}_{rad})\sin(\underbrace{3.52}_{rad})$$
$$= 0.518 * (-0.9293) + j1.1261 * (-0.3694)$$
$$= -0.4811 - j0.416$$
$$\cosh(0.497 + j3.52) = \cosh(\underbrace{0.497}_{rad})\cos(\underbrace{3.52}_{rad}) + j\sinh(\underbrace{0.497}_{rad})\sin(\underbrace{3.52}_{rad})$$
$$= 1.1261 * (-0.9293) + j0.5177 * (-0.3694)$$
$$= -1.0464 - j0.1913 \tag{60}$$

$$\tanh(\gamma l) = \frac{\sinh(0.497 + j3.52)}{\cosh(0.497 + j3.52)} = \frac{-0.4811 - j0.416}{-1.0464 - j0.1913} = \frac{0.636\angle -139.15}{1.063\angle -169.639}$$
$$= 0.598\angle 30.48 = 0.515 + j0.303$$

$$Z_s = (685 - j92) \left[\frac{2000 + (685 - j92)(0.515 + j0.303)}{(685 - j92) + 2000(0.515 + j0.303)} \right]$$

$$Z_s = 862.39 + j322\Omega$$

Example:

A loss-less TL with $Z_0 = 50\Omega$, $f = 10\text{MHz}$, $l = 30\text{m}$, $Z_R = 50 - j10\Omega$, Find Z_d at $d = 10m, 15m$, and $30m \rightarrow Z_s$?

Solution:

$\lambda = \frac{c}{f} = \frac{3*10^8}{10*10^6} = 30\text{m}$

$\beta = \frac{2\pi}{\lambda} = 0.2\,\text{rad} \times 180/\pi = 12^o$

1) For $d = 10m$

$\beta d = 12 \times 10 = 120^o$

$$Z_d = Z_0 \left[\frac{Z_R + jZ_0 \tan(\beta d)}{Z_0 + jZ_R \tan(\beta d)} \right]$$

$$Z_d = 50 \left[\frac{(50 - j10) + j50\tan(120^o)}{50 + j(50 - j10)\tan(120^o)} \right] = 58.75\underline{/6.69°}$$

2) For $d = 15\text{m}$

$\beta d = 12 \times 15 = 180^o$, which leads to $\tan(\beta d) = 0$

$$Z_d = 50 \left[\frac{(50 - j10) + j50 \times 0}{50 + j(50 - j10) \times 0} \right] = Z_R = 50 - j10\Omega$$

3) For $d = 30\text{m}$

$Z_s = Z_R = 50 - j10\Omega$ (Home Work)

Comment of these results?

The impedance, voltage and current have the same values every $\boxed{d = \dfrac{n\lambda}{2}}$.

College of Electronics Engineering ³⁸

12   The Complex Reflection coefficient (K_R) for Lossless TL:

For a lossless TL Z_0 is real (why?), but because in general $K_R = \frac{Z_R - Z_0}{Z_R + Z_0}$ and Z_R might be complex, this leads to make K_R complex at the load. Therefore, we can express K_R as

$$K_R = |K_R|\, e^{j\theta_R}$$

At any point on the TL the reflection coefficient can be written as

$$K = K_R\, e^{-j2\beta d}$$

which leads to

$$K = |K_R|\, e^{j\theta_R}\, e^{-j2\beta d} = |K_R|\, e^{j(\theta_R - 2\beta d)}$$
$$= K_R\, \underline{/\theta_R - 2\beta d}$$

where $|K| = |K_R|$, which means that K at any point on the TL has the same magnitude of K_R but with different phase. We can return to Eq. 57 and Eq. 58 to find E_d and I_d by applying substitution of $x = d - \ell$ in both equations to be as:

$$|E_d| = |e^+||1 + \underbrace{K_R\, e^{-j2\beta d}}_{\text{K}}| \tag{61}$$

$$|I_d| = |\frac{e^+}{Z_0}||1 - \underbrace{K_R\, e^{-j2\beta d}}_{\text{K}}| \tag{62}$$

To draw $(1 - K_R\, e^{-j2\beta d})$ for different distances away from the load, i.e. different d from $d = 0$ to $d = \ell$.

Note: It is obvious that by changing d along the TL, the phase would be changed as well, while the absolute value of K_R, i.e. $|K_R|$, stays constant. On the other hand, the term $|1 + K_R \angle(\theta_R - 2\beta d)|$ is affected and changed by any change in d as shown in Fig. 20.

College of Electronics Engineering [39]
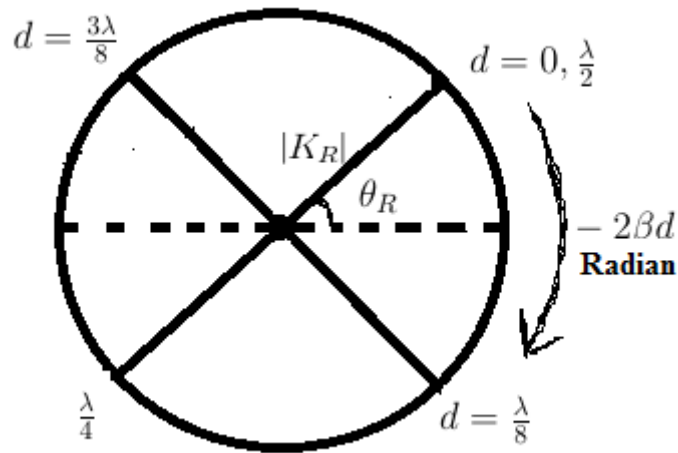
Figure 19: Drawing $K_R\ e^{-j2\beta d} = |k_R|\angle(\theta_R - 2\beta d)$ only without 1

d	$-2\beta d$	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R - 2\beta d)$
0	0	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R)$
$\frac{\lambda}{8}$	$-2\frac{2\pi}{\lambda}\frac{\lambda}{8} = -\frac{\pi}{2}$	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R - \frac{\pi}{2})$
$\frac{\lambda}{4}$	$-2\frac{2\pi}{\lambda}\frac{\lambda}{4} = -\pi$	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R - \pi)$
$\frac{3\lambda}{8}$	$-2\frac{2\pi}{\lambda}\frac{3\lambda}{8} = -\frac{3\pi}{2}$	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R - \frac{3\pi}{2})$
$\frac{\lambda}{2}$	$-2\frac{2\pi}{\lambda}\frac{\lambda}{2} = -2\pi$	$K_R\angle\theta_R$	$1 + K_R\angle(\theta_R - 2\pi)$
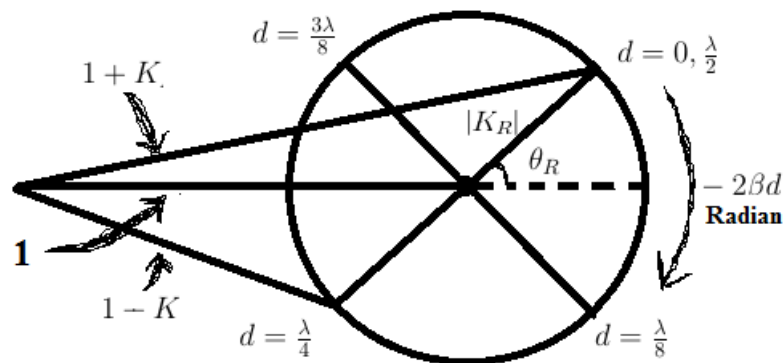
Figure 20: Phase diagram of voltage. Drawing $1 + K_R\ e^{-j2\beta d} = 1 + |k_R|\angle(\theta_R - 2\beta d)$

Example: For the circuit shown in below find $|E_d|$ at
$d = 0, \frac{\lambda}{8}, \frac{\lambda}{4}, \frac{3\lambda}{8}, \frac{\lambda}{2}, \frac{5\lambda}{8}, \frac{3\lambda}{4}, \frac{7\lambda}{8}, \lambda$?
Solution:
For OC load, it is known that $K_R = 1\angle 0$ $E_d = |e^+||1 + K_R e^{-j2\beta d}|$,
where $e^+ = E\frac{Z_0}{Z_0 + Z_g} = E$

College of Electronics Engineering
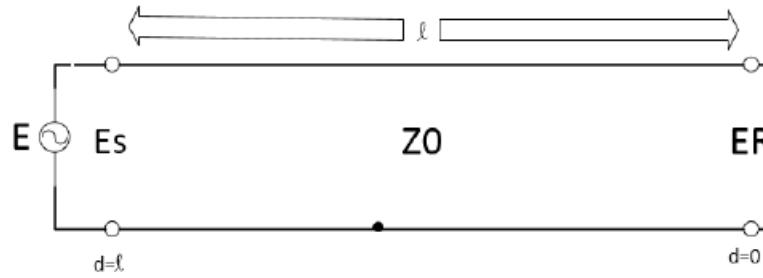
Figure 21: Phase diagram of voltage
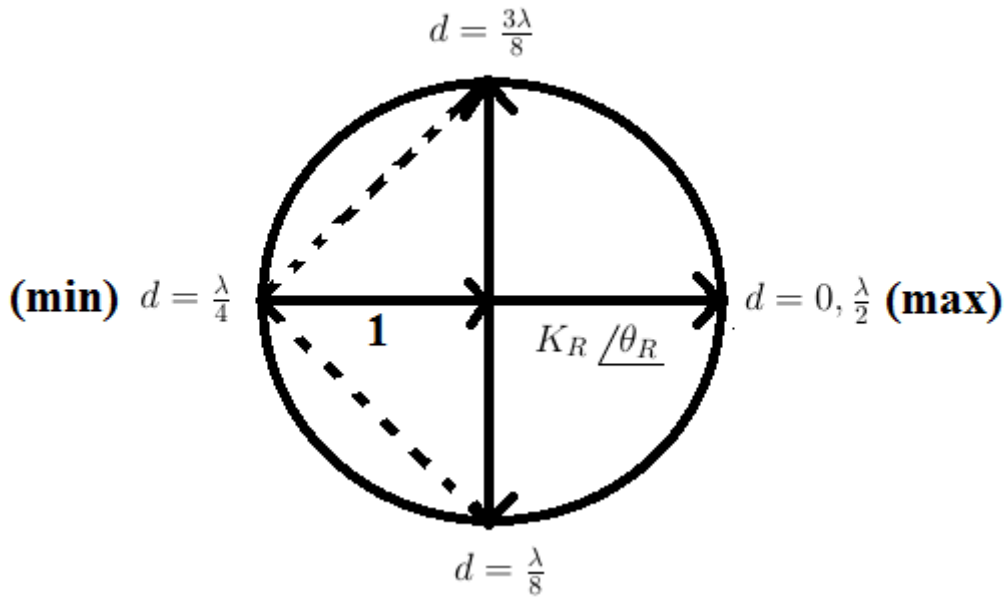
Figure 22: Phase diagram of voltage for OC load

d	$1 + k_R e^{-j2\beta d} = 1 + |k_R|\angle(\theta_R - 2\beta d)$	$|1 + k_R e^{-j2\beta d}|$
0	$1 + 1\angle 0 = 1 + 1$	2
$\frac{\lambda}{8}$	$1 + 1\angle -90^o = 1 - j$	1.414
$\frac{\lambda}{4}$	$1 + 1\angle -180^o = 1 - 1$	0
$\frac{3\lambda}{8}$	$1 + 1\angle -270^o = 1 + j$	1.414
$\frac{\lambda}{2}$	$1 + 1\angle -360^o = 1 + 1$	2

Example (H.W): For the circuit shown in below find $|E_d|$ at $d = 0, \frac{\lambda}{8}, \frac{\lambda}{4}, \frac{3\lambda}{8}, \frac{\lambda}{2}, \frac{5\lambda}{8}, \frac{3\lambda}{4}, \frac{7\lambda}{8}, \lambda$**?**

Solution:

For SC load, it is known that $K_R = -1 = 1\angle 180^o$ $E_d = |e^+||1 + K_R e^{-j2\beta d}|$, where $e^+ = E\frac{Z_0}{Z_0 + Z_g} = E$

College of Electronics Engineering ⁴¹

d	$1 + k_R e^{-j2\beta d} = 1 + \|k_R\| \angle (\theta_R - 2\beta d)$	$\|1 + k_R e^{-j2\beta d}\|$
0	$1 + 1\angle 180^o = 1 - 1$	0
$\frac{\lambda}{8}$	$1 + 1\angle(180^o - 90^o) = 1 + j$	1.414
$\frac{\lambda}{4}$	$1 + 1\angle(180^o - 180^o) = 1 + 1$	2
$\frac{3\lambda}{8}$	$1 + 1\angle(180^o - 270^o) = 1 - j$	1.414
$\frac{\lambda}{2}$	$1 + 1\angle(180^o - 360^o) = 1 - 1$	0

Figure 23: Phase diagram of voltage for SC load

13   Standing Wave Ratio (S)

$$E_{max} = |e^+| + |e^-| = |e^+|(1 + |\frac{e^-}{e^+}|) = |e^+|(1 + |K|) \tag{63}$$

$$E_{min} = |e^+| - |e^-| = |e^+|(1 - |\frac{e^-}{e^+}|) = |e^+|(1 - |K|) \tag{64}$$

E and I are always root mean squared (r.m.s) values. We can define a new parameter know as Standing Wave Ratio (SWR) which can be denoted by (S),
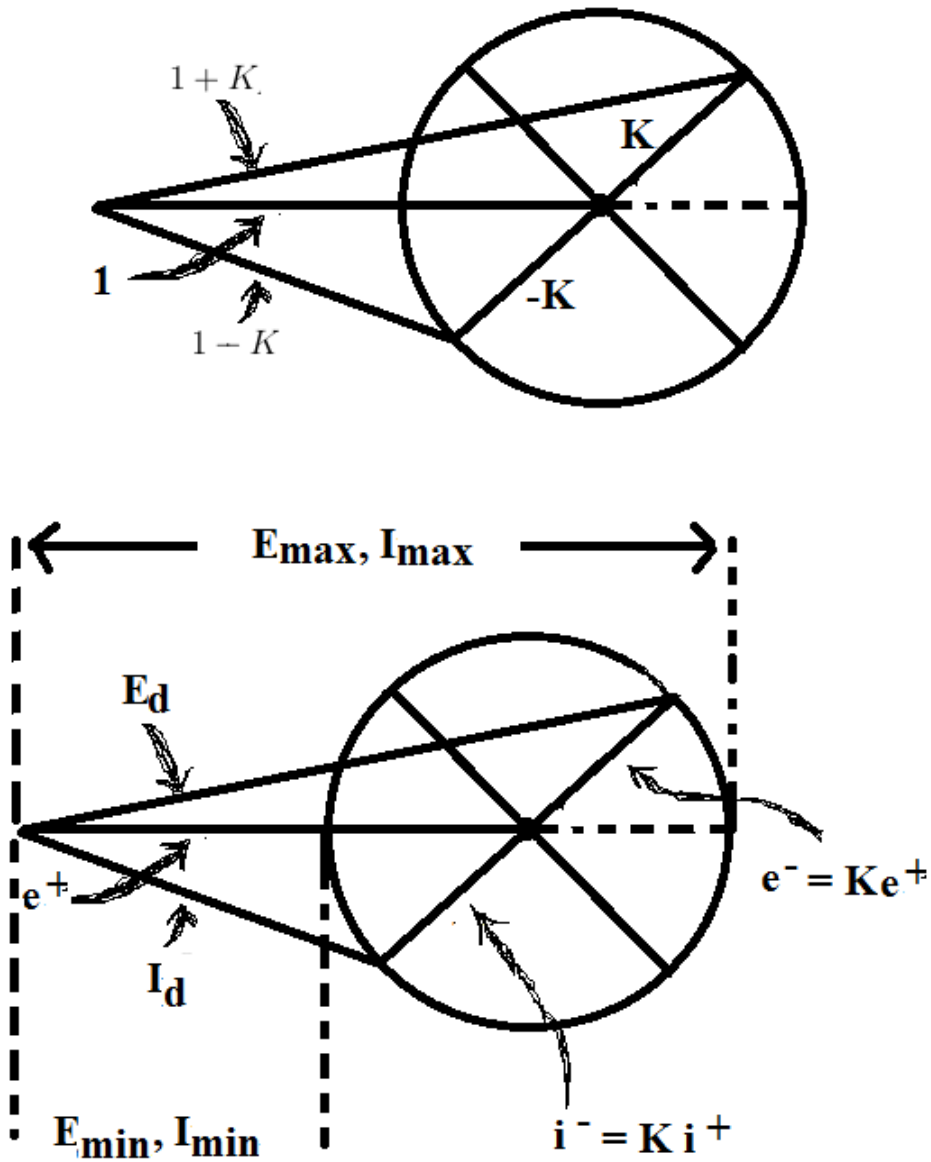
which can be expressed as:

$$S = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}} \tag{65}$$

$$S = \frac{1 + |K|}{1 - |K|} \tag{66}$$

Also if we have S we can find $|K|$ as:

$$|K| = \frac{S - 1}{S + 1} \tag{67}$$

Moreover, in order to evaluate the maximum impedance across a transmission

line we can use the expression

$$Z_{max} = \frac{|E_{max}|}{|I_{min}|} \tag{68}$$

From Eq. 65, it is obvious that $|I_{min}| = \frac{|I_{max}|}{S}$, and we know that $|I_{max}| = \frac{|E_{max}|}{Z_0}$, where the letter is defined already in Eq. 63, all these steps lead to express Z_{max} as

$$Z_{max} = \frac{|E_{max}|}{\frac{|I_{max}|}{S}} = \frac{|e^+|(1+|K|)}{\frac{|e^+|}{SZ_0}(1+|K|)} \tag{69}$$

$$\boxed{Z_{max} = SZ_0} \tag{70}$$

Similarly, to find the minimum impedance along a TL

$$Z_{min} = \frac{|E_{min}|}{|I_{max}|} \tag{71}$$

$$\boxed{Z_{min} = \frac{Z_0}{S}} \tag{72}$$

$$\boxed{\underbrace{1}_{Z_R=Z_0 \quad \text{and} \quad Z_{max}=Z_{min}} \leq \ S \ \leq \underbrace{\infty}_{OC \quad Z_{max}=SZ_0 \quad \text{and} \quad SC \quad Z_{min}=\frac{Z_0}{S}}} \tag{73}$$

14   Power Transmitted along TL

$$P = |E| \times |I| \cos\theta$$

$$P = |E_{max}| \times |I_{min}| = |E_{min}| \times |I_{max}| \tag{74}$$

in which the phase between E_{max} and I_{min} is zero, and the phase between E_{min} and I_{max} is zero too.

$$P = |e^+|(1+|K|) \times \frac{|e^+|}{Z_0}(1-|K|) = \frac{|e^+|^2}{Z_0}(1-|K|^2)$$

$$P = \underbrace{\frac{|e^+|^2}{Z_0}}_{\text{Incident Power}} - \underbrace{\frac{|e^-|^2}{Z_0}}_{\text{Reflected Power}} \tag{75}$$

15   Crank Diagram

It is one of the simplest graphical solution for lossless TL, which can be used to find the current, voltage, impedance and power at any point on a TL using graphical methods.

Example (1): if the voltage a cross a load connected to lossless TL is 10 V_{rms} and the $K_R = 0.5\angle0^o$.

1. Find the voltage at $d = \lambda/8, \lambda/4, \lambda/2$

2. Find Z_R which is at $d = (\quad)$?

Solution:

$$|E_d| = |e^+||1 + \underbrace{K_R\,e^{-j2\beta d}}_{K}|$$

$$|E_d| = |e^+||1 + K| \rightarrow |E_d| \propto |1 + K|$$
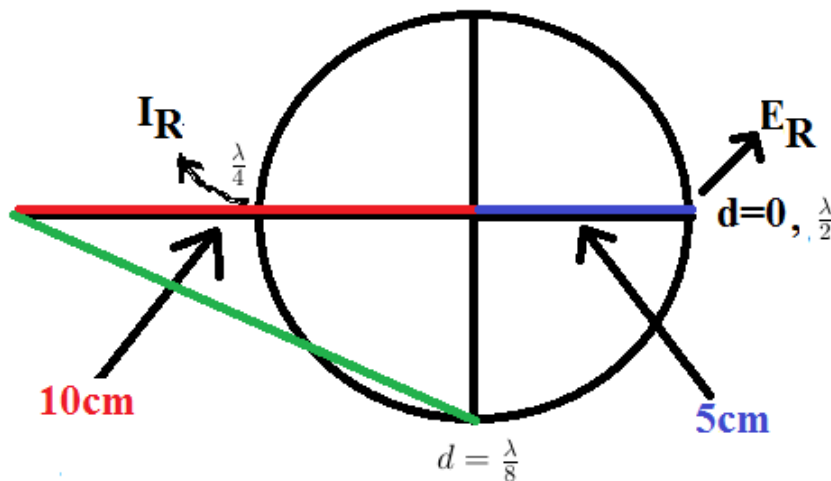
Figure 24: Blank to solved by Student

- Let $1 = 10cm \rightarrow K_R = 0.5 = 5cm$.

- Draw the line of $10cm$ to represent the 1 in the above equation.

- At $d = 0 \rightarrow E_d = E_R = 10 = 15cm \rightarrow voltage\ Scale\ (V.S) = \frac{10V}{15cm}$, while the current scale $C.S = V.S/Z_0$.

- Put the right angle to represent the angle θ_R, the angle associated with reflection coefficient, k_R, which is at $d = 0$.

- In this example, $\theta_R = 0$ as has been given.

- At $d = \frac{\lambda}{8}$, which is in the position of -90°

- $E|_{d=\frac{\lambda}{8}} = \sqrt{10^2 + 5^2} = 11.2cm \rightarrow E|_{d=\frac{\lambda}{8}} = 11.2 \times V.S = 11.2 \times \frac{10}{15} = 7.46 \; volt$

- $E|_{d=\frac{\lambda}{4}} = 5cm \times V.S = 5cm \times \frac{10}{15} = 3.33 \; volt$, which represents V_{min} .

- $E|_{d=\frac{\lambda}{2}} = 15cm \times V.S = 15cm \times \frac{10}{15} = 10 \; volt$, which represents V_{max} .

- $Z|_{d=0} = Z_R = \frac{|e^+|(1+|K|)}{\frac{|e^+|}{Z_0}(1-|K|)} = \frac{Z_0(1+|K|)}{(1-|K|)} = Z_0 \frac{15cm}{5cm} = 3Z_0$

- we can find $Z|_{d=0}$ which represents Z_R without using the graph as $\frac{Z_0(1+|K|)}{(1-|K|)} = \frac{Z_0(1+0.5)}{(1-0.5)} = 3Z_0$

- I_R can be found at the apposite direction of E_R, i.e. at $d = \frac{\lambda}{4}$. $I_R = 5cm \times C.S = 5cm \times \frac{V.S}{Z_0} = 5cm \times \frac{15}{10Z_0} = \frac{7.5}{Z_0}$.

Example (2): HW if the voltage a cross a load connected to lossless TL is 10 V_{rms} and the $K_R = 0.5\angle 45^o$.

Example (3): HW if the voltage a cross a load connected to lossless TL is 12 V_{rms} and the $K_R = 0.4\angle -45^o$.

1. Find the voltage at $d = \lambda/8, \lambda/4, \lambda/2$

2. Find Z_R.

Solution:

Example (4): CW
A lossless TL with $L = 5\mu H$ and $C = 2nF$, if the voltage a cross a load connected to lossless TL is 15 V_{rms}, and the load impedance is $Z_R = 49.8 - j58$.

1. Find K_R and the VSWR (S) for this TL.

2. Find the voltage and current scales.

3. Find the voltage and current at $d = 0, \lambda/8, \lambda/4, \lambda/2$ using Crank diagram and tabulate the results.

4. From Crank diagram confirm that S has the same value found theoretically already in (1).

5. Using the drawn Crank diagram evaluate the transmitted power.

6. Draw clearly the SWR pattern from $d = 0$ to $d = \lambda/2$ showing the maximum and minimum positions in your plot. $d_{min} = ($ $)$ and $d_{max} = ($ $)$.

7. If the load is changed to $Z_R = 50$ how would the SWR pattern look like? why? draw it clearly.

College of Electronics Engineering ⁴⁶

Solution:

1)

$Z_0 = \sqrt{\frac{L}{C}} = 50\Omega$
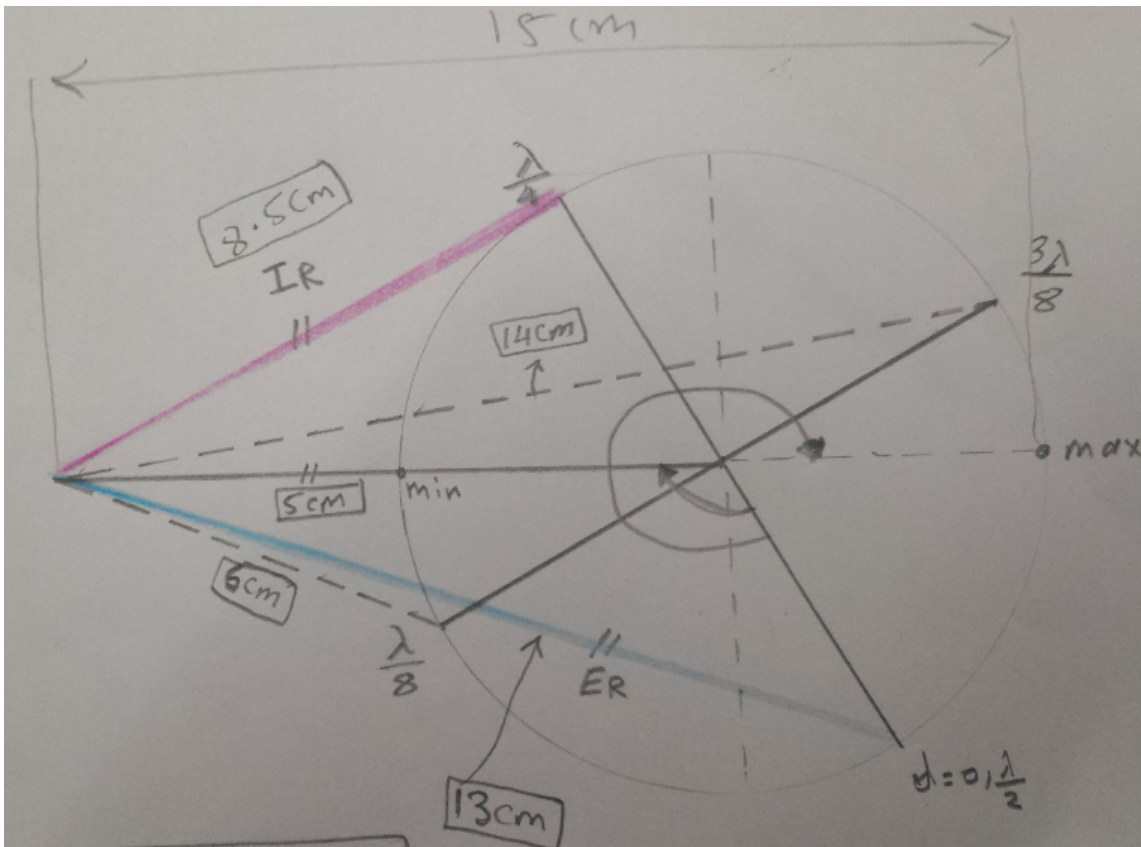
$K_R = \frac{Z_R - Z_0}{Z_R + Z_0} = 0.5\angle-60$

$S = \frac{1+|K|}{1-|K|} = \frac{1-0.5}{1+0.5} = 3$

2)

Voltage Scale (V.S)$= \frac{15volt}{13cm}$

Current Scale (C.S)$= \frac{15volt/13cm}{50\Omega}$

3)

d	Voltage (volt)	Current (Amp.)
0	13cm* V.S=	8.5cm *C.S=
$\frac{\lambda}{8}$	6cm*V.S=	14*C.S=
Min voltage position (?)	Min. Voltage= 5cm*V.S	Max. Current= 15cm*C.S
$\frac{\lambda}{4}$	8.5cm*V.S=	13cm *C.S=
$\frac{3\lambda}{8}$	14cm*V.S	6cm*C.S
Max voltage position (?)	Max. Voltage= 15cm*V.S	Min. Current= 5cm*C.S
$\frac{\lambda}{2}$	13cm* V.S=	8.5cm *C.S=

4)

$S = \frac{15cm}{5cm} = 3$

5)
$$P = E_{max} * I_{min} = 15cm * V.S * 5cm * C.S = (\qquad)Watt$$
6)
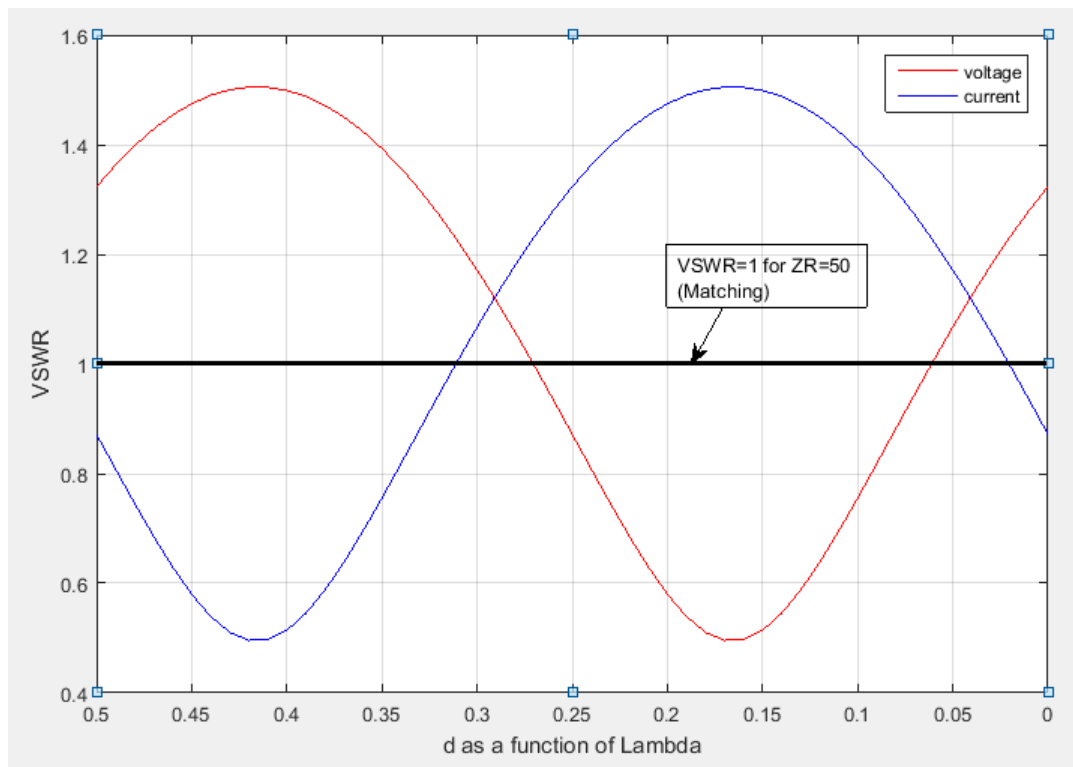
$$-2\beta d_{min} = -2\frac{2\pi}{\lambda}d_{min} = -120^o * \pi/180 \rightarrow d_{min} = \frac{\lambda}{6}$$
$$-2\beta d_{max} = -2\frac{2\pi}{\lambda}d_{max} = -300^o * \pi/180 \rightarrow d_{max} = \frac{15\lambda}{36} = 0.4167\lambda$$

7)
When $Z_R = 50$ Matching case leads to S=1 see VSWR figure above.

16   Smith Chart

Example1: CW to be solved by students
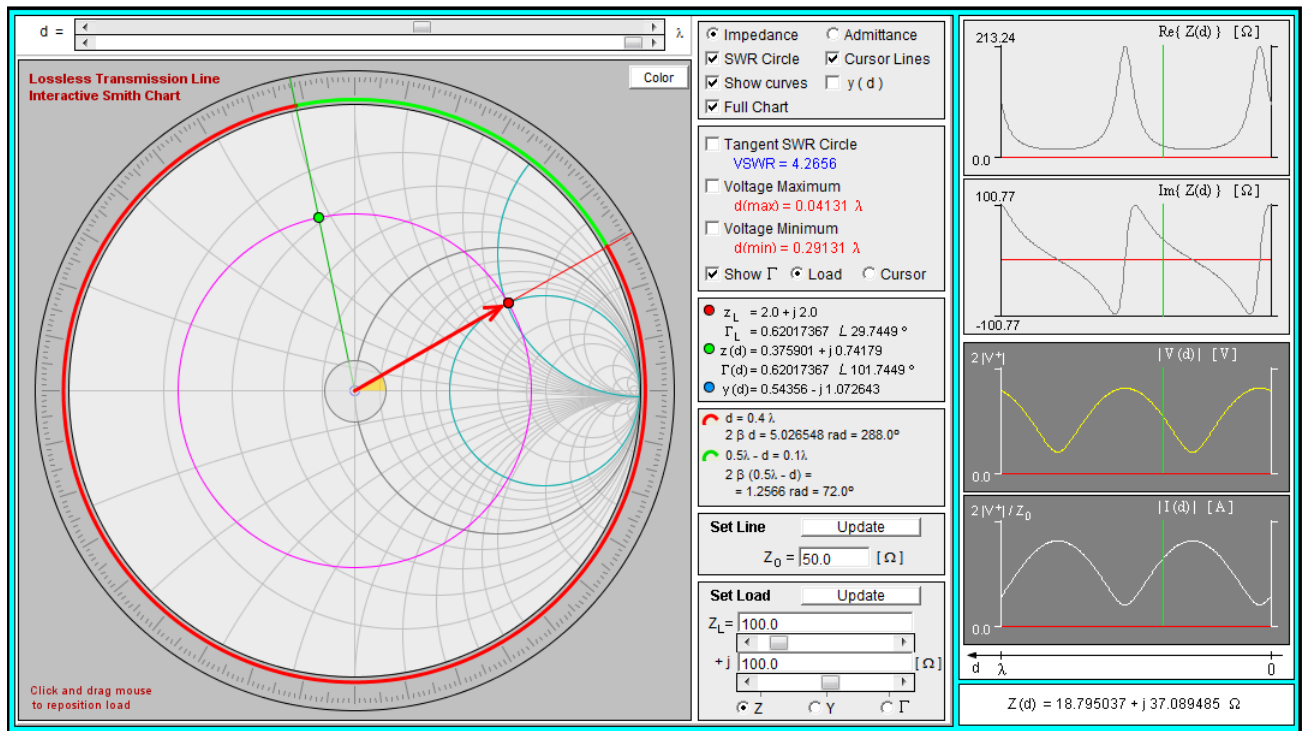
Example2:CW to be solved by students
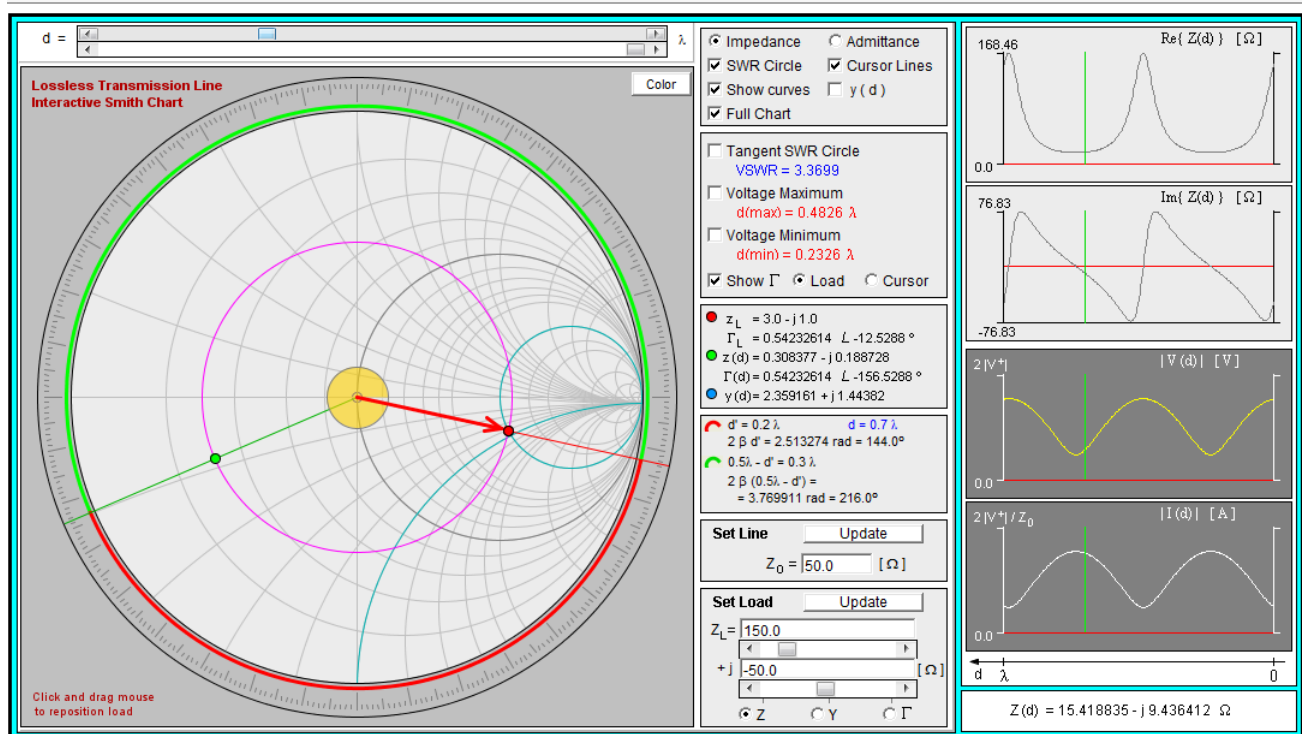
Impedance Matching:

Series Matching:
Quarter Wave Transformer for Pure Resistive Load:
For the TL shown in below, assume that $Z_R = R + jX$ and $X = 0$, which means that there is no imaginary term and the load is pure resistive. For the case of $Z_0 \neq Z_R$, the reflection from load is existed.
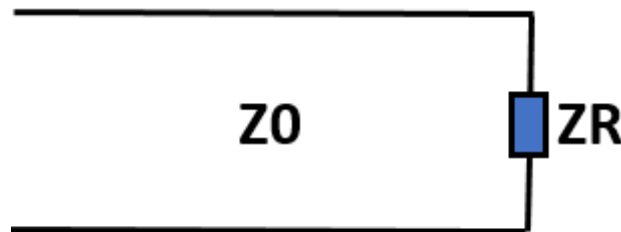
In order to remove the reflection and make the TL matched with the load without any changes in the load, we can use a Quarter wave transformer, i.e. a TL of length $(\lambda/4)$ and with a characteristic impedance of Z_{01}, in which $Z_{01} \neq Z_0$ as shown in the figure in below:
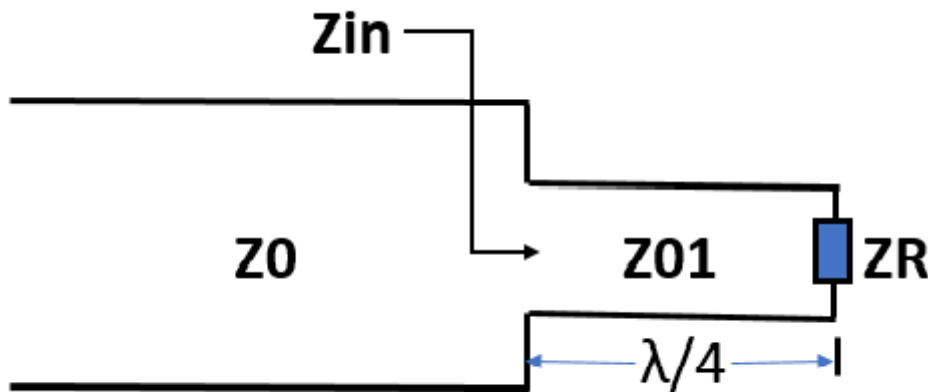
Now by applying the TL equation which is given previously as

$$Z_d = Z_{01} \left[\frac{Z_R + jZ_{01}\tan(\beta d)}{Z_{01} + jZ_R\tan(\beta d)} \right] \tag{76}$$

we can obtain

$$Z|_{d=\lambda/4} = Z_{01} \left[\frac{Z_R + jZ_{01}\tan(\pi/2)}{Z_{01} + jZ_R\tan(\pi/2)} \right] \tag{77}$$

College of Electronics Engineering [50]

Now by applying the limits theorem we can

$$Z|_{d=\lambda/4} = \frac{Z_{01}^2}{Z_R} = Z_{in}$$

In order to make the main TL of characteristic impedance (Z_0) matched to the load after adding this additional $(\lambda/4$) transformer of characteristic impedance (Z_{01}), we should satisfy $Z_{in} = Z_0$, which leads to

$$Z_{in} = \frac{Z_{01}^2}{Z_R} = Z_0$$

but we have Z_R and Z_0 already, therefore it is required to evaluate Z_{01}, which can be found as

$$\boxed{Z_{01} = \sqrt{Z_0 Z_R}}$$

Quarter Wave Transformer for Complex Load:
For the case of load that is not pure resistive, i.e. $Z_R = R + jX$ and $X \neq 0$, we could not use the same procedure explained previously. This means that the quarter wave transformer does not connected directly to the load. To solve this problem we can use smith chart to find the next position of TL that the impedance is pure resistive. This can be obtained in two positions on the TL, the maximum and the minimum voltage positions.
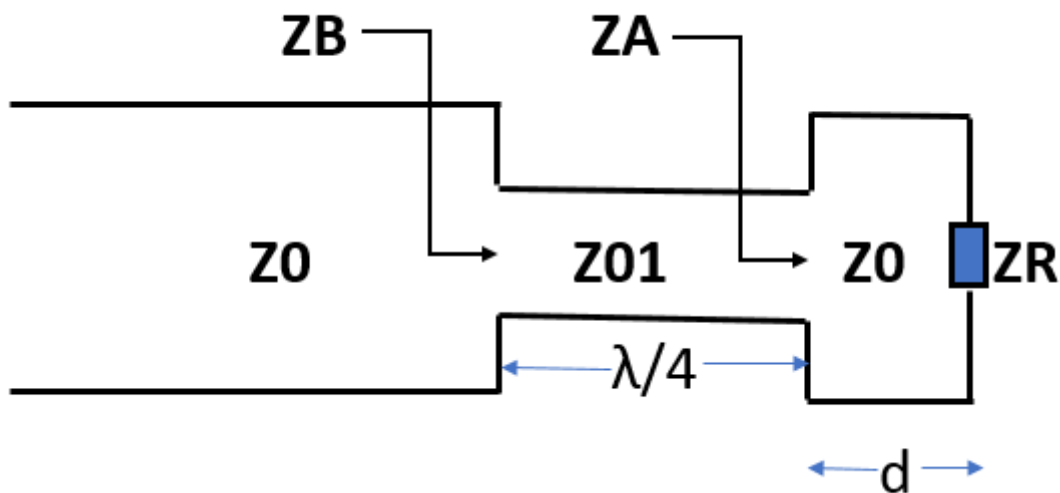
1. At Maximum Voltage:
It is well-known that at the maximum voltage the impedance would be $\boxed{Z_A = S Z_0}$. Now, it is required to find the position of d that makes the voltage at the maximum by using smith chart. This $\lambda/4$ transformer is positioned now d away from

College of Electronics Engineering ⁵¹

the load. The second important parameter is to evaluate the value of Z_{01} which satisfy matching of TL with load, i.e. $Z_B = Z_0$. This can be implemented by applying equations (76) and (77) above, which leads to

$$Z_B = \frac{Z_{01}^2}{Z_A} = Z_0$$

$$\boxed{Z_{01} = \sqrt{Z_0 Z_A} = \sqrt{Z_0 S Z_0} = \sqrt{S} Z_0}$$

It is important to note that S can be found from smith chart, or it can be calculated from the reflection coefficient.

2. At Minimum Voltage:

Using the same procedure of the maximum voltage, Z_A can be expressed as

$$\boxed{Z_A = \frac{Z_0}{S}}$$

$$Z_B = \frac{Z_{01}^2}{Z_A} = Z_0$$

$$\boxed{Z_{01} = \sqrt{Z_0 Z_A} = \sqrt{Z_0 \frac{Z_0}{S}} = \frac{Z_0}{\sqrt{S}}}$$

Example:
A TL has the following parameters: $Z_0 = 340, Z_L = 210 + j280, \lambda = 5$ meter
1) Examin this TL to find whether it is matched or not to this load. 2) if this TL is not matched, design a $\lambda/4$ transformer that satisfying matching with this load with an impedance of greater and smaller than the characteristic impedance of the original TL.
Solution:
Step 1 $\bar{Z}_R = \frac{210 + j280}{340} = 0.618 + j0.824$ This normalized load impedance is located at 0.128λ (TWG) outer on Smith chart.

To find d that makes the impedance is only real value, there are two positions, the maximum and minimum voltage positions.
At Max voltage:
$Z_{01} = \sqrt{S} * Z_0 = \sqrt{3} * 340 = 588.89 \ \Omega$ which is greater than 340 Ω
This new $\lambda/4$ transformer should be located at $d_{max} = 0.25\lambda - 0.128\lambda = 0.122\lambda$, this means that the actual position of this transformer is $d_{max} = 0.122 * 5 = 0.61$ meter.

At Min voltage: $Z_{01} = \frac{Z_0}{\sqrt{S}} = \frac{340}{\sqrt{3}} = 196.29\,\Omega$ which is smaller than $340\,\Omega$
This new $\lambda/4$ transformer should be located at $d_{min} = 0.5 - 0.128\lambda = 0.372\lambda$, this means that the actual position of this transformer is $d_{min} = 0.372 * 5 = 1.86$ meter.

Parallel Matching Using Single Stub:

Parallel Matching can be used via applying single or double stub on the TL to satisfy matching with the load. As the subs are always connected in parallel with the load on a particular TL, it is easier to deal with admittance (Y) rather than the impedance (Z). In the figure in below, it is required to satisfy

$$Y_{in} = Y_1 + Y_{ls} = 1$$

where

Y_{in} = the admittance to the left of stub.

Y_1 = the admittance to the right of stub.

Y_{ls} = input admittance of the stub.
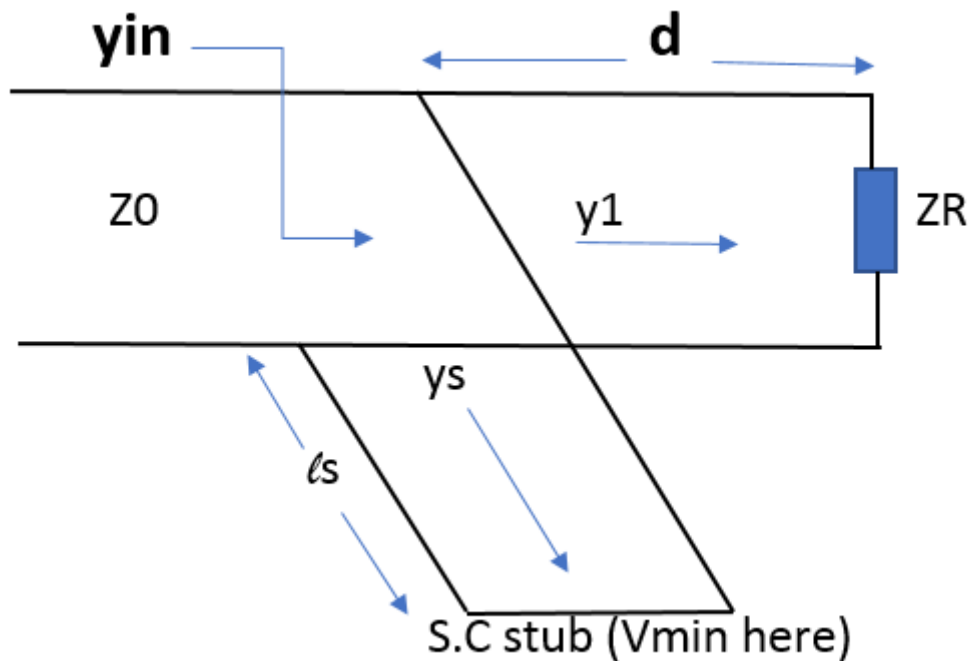
This can be satisfied by assuming :

$$Y_1 = 1 \mp b$$
$$Y_{ls} = \pm b$$

NOTE: see the website in below, it is very helpful for using smith chart and matching techniques:

http://amanogawa.com/archive/transmissionA.html

Example 1:

A TL of characteristic impedance 600Ω and a load antenna of 100Ω, and the operating frequency is 1 GHz. Design a single parallel short circuit stub that make this TL matched to this load. (Find d and l_s?) $\bar{Z}_R = \frac{100}{600} = 0.167$ $\lambda = \frac{c}{f} = \frac{3*10^8}{1*10^9} = 0.3$ meter
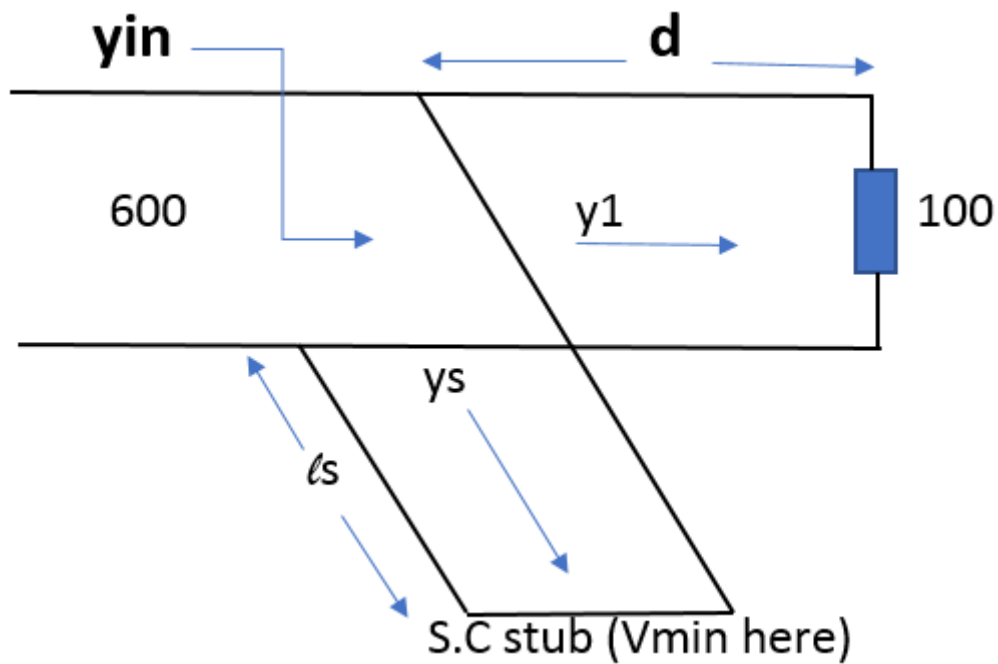
- Find $\bar{Z}_R$ on the smith chart.

- Draw a circle that its center is at 1.0 of the real part of smith chart.

- Find $\bar{Y}_R$ on the circle and find its position on the outer of smith chart (TWG).
  $\bar{Y}_R = 6$ at 0.25λ TWG

Impedance Matching
Single Stub Tuning

SHORTED STUB

$Z_0 = 600.0\ \Omega$
$\varepsilon_r = 1.0$
$Z_L = 100.0 + j\,0.0\ \Omega$
$f_o = 1.0$ GHz
$\lambda = 0.2998$ m

Go Back   Next Step   Back to Start

Step 3 - First Solution

Move along the transmission line from load to generator until a position with normalized conductance $g = 1.0$ is reached.

The first solution is at location

$d_{stub1} = 0.0617\ \lambda$ from the load

You can insert a stub at this location.

Enlarge   Color

●	z_L	$= 0.16667 + j\,0.0$
	Γ_L	$= 0.71429\ L\ 180.0°$
○	y_L	$= 6.0 + j\,0.0$
●	$z(d)$	$= 0.19355 + j\,0.39508$
	$\Gamma(d)$	$= 0.71429\ L\ 135.58469°$
●	$y(d)$	$= 1.0 - j\,2.04124$

↻ $d = 0.0617\lambda$ ¦ $2\beta\,d = 0.7752$ rad $= 44.4153°$
↻ $0.5\lambda\text{-}d = 0.4383\lambda$ ¦ $2\beta\,(0.5\lambda\text{-}d) = 5.508$ rad $= 315.5847°$

⊙ Impedance Chart   ○ Admittance Chart

Impedance Matching
Single Stub Tuning

SHORTED STUB

$Z_0 = 600.0\ \Omega$
$\varepsilon_r = 1.0$
$Z_L = 100.0 + j\,0.0\ \Omega$
$f_o = 1.0$ GHz
$\lambda = 0.2998$ m

Go Back   Next Step   Back to Start

Step 4 - First Solution

To cancel the imaginary part of the line admittance we add a stub with:

Length: $L_{stub1} = 0.4275\ \lambda$
Admittance:

$Y_{stub1} = j\,0.0034$ S   (actual)

$y_{stub1} = j\,2.04124$ (normalized)

Enlarge   Color

●	z_L	$= 0.16667 + j\,0.0$
	Γ_L	$= 0.71429\ L\ 180.0°$
○	y_L	$= 6.0 + j\,0.0$
●	$z(d)$	$= 1.0 + j\,0.0$
	$\Gamma(d)$	$= 0.0\ L\ 0.0°$
●	$y(d)$	$= 1.0 + j\,0.0$

↻ $d = 0.0\lambda$ ¦ $2\beta\,d = 0.0$ rad $= 0.0°$
↻ $0.5\lambda\text{-}d = 0.5\lambda$ ¦ $2\beta\,(0.5\lambda\text{-}d) = 6.2832$ rad $= 360.0°$

⊙ Impedance Chart   ○ Admittance Chart

College of Electronics Engineering ⁵⁶

Impedance Matching
Single Stub Tuning

SHORTED STUB

$Z_0 = 600.0 \; \Omega$

$\varepsilon_r = 1.0$

$Z_L = 100.0 + j\,0.0 \; \Omega$
$f_o = 1.0 \; GHz$
$\lambda = 0.2998 \; m$

Go Back | Next Step | Back to Start

Step 3 - Second Solution

A second stub can be inserted at the new location:

$d_{stub2} = 0.4383 \; \lambda$ from the load

●	z_L =	$0.16667 + j\,0.0$
	Γ_L =	$0.71429 \; \angle \; 180.0°$
●	y_L =	$6.0 + j\,0.0$
●	$z(d)$ =	$0.19355 - j\,0.39508$
	$\Gamma(d)$ =	$0.71429 \; \angle \; -135.58469°$
●	$y(d)$ =	$1.0 + j\,2.04124$

$d = 0.4383\lambda \; \vdots \; 2\beta\,d = 5.508 \; rad = 315.5847°$
$0.5\lambda - d = 0.0617\lambda \; \vdots \; 2\beta\,(0.5\lambda - d) = 0.7752 \; rad = 44.4153°$

○ Impedance Chart   ○ Admittance Chart

Impedance Matching
Single Stub Tuning

SHORTED STUB

$Z_0 = 600.0 \; \Omega$

$\varepsilon_r = 1.0$

$Z_L = 100.0 + j\,0.0 \; \Omega$
$f_o = 1.0 \; GHz$
$\lambda = 0.2998 \; m$

Go Back | Summary | Back to Start

Step 4 - Second Solution

To cancel the imaginary part of the line admittance we add a stub with:
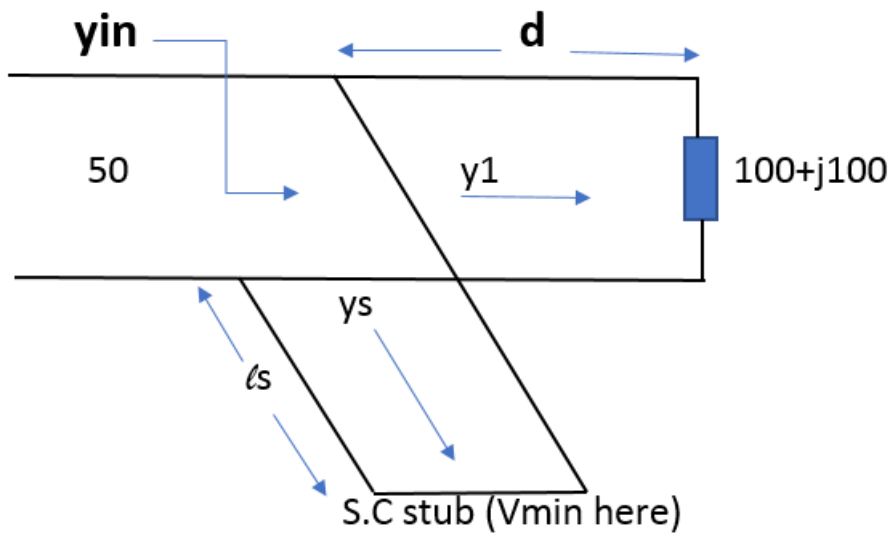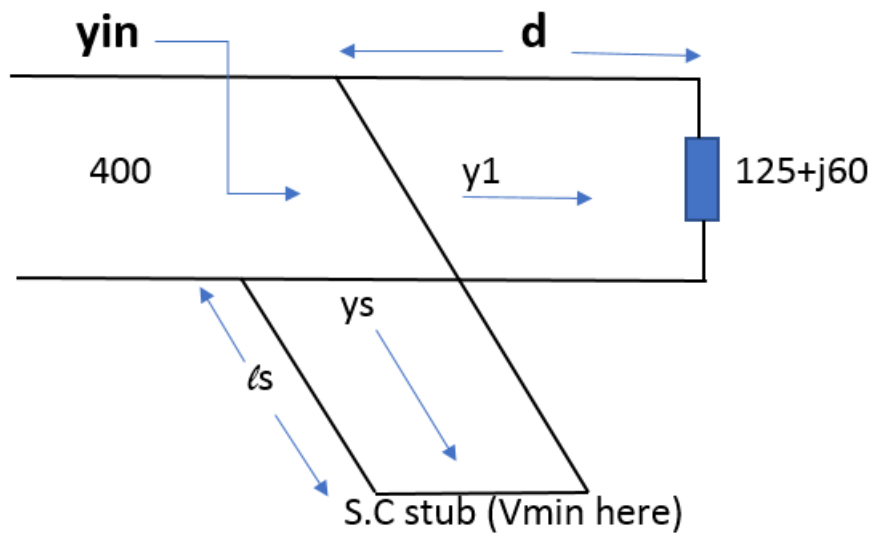
Length: $L_{stub2} = 0.0725 \; \lambda$

Admittance:

$Y_{stub2} = -j\,0.0034 \; S$ (actual)

$y_{stub2} = -j\,2.04124$ (normalized)

●	z_L =	$0.16667 + j\,0.0$
	Γ_L =	$0.71429 \; \angle \; 180.0°$
●	y_L =	$6.0 + j\,0.0$
●	$z(d)$ =	$1.0 + j\,0.0$
	$\Gamma(d)$ =	$0.0 \; \angle \; 0.0°$
●	$y(d)$ =	$1.0 + j\,0.0$

$d = 0.0\lambda \; \vdots \; 2\beta\,d = 0.0 \; rad = 0.0°$
$0.5\lambda - d = 0.5\lambda \; \vdots \; 2\beta\,(0.5\lambda - d) = 6.2832 \; rad = 360.0°$

○ Impedance Chart   ○ Admittance Chart

College of Electronics Engineering ⁵⁷

yin ⟶   ⟵ **d** ⟶

400   y1 ⟶   125+j60

ys

ℓs

S.C stub (Vmin here)

yin ⟶   ⟵ **d** ⟶

50   y1 ⟶   100+j100

ys

ℓs

S.C stub (Vmin here)

Communication Principles

Ampltiude Modulation (AM):

Need of Modulation

Practical Antenna Length

In theory the length of antenna approximately equal to wavelength of the wave.

$$\lambda = \frac{3\times10^8}{frequecny\ (Hz)}\ \text{meters}$$

For example, if the frequency of carrier wave is 20 kHz, then we need an antenna length of 15,000 meters. This is too long antenna to be constructed practically.
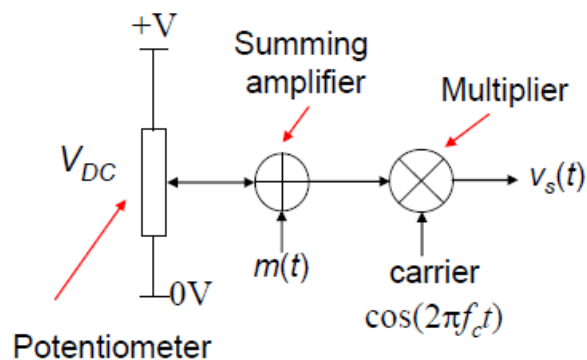
Operating Range

Greater the frequency, greater the energy possessed by it which basically helps in transmitting over large distances.

Wireless Communication

The performance of wired communication over a large distances is poor compare to modulated signal with high frequency. It is comparatively cheap.

Amplitude Modulation [1/9]

$$v_s(t) = \left(V_{DC} + m(t)\right)\cos\left(2\pi f_c t\right)$$

V_{DC} is a variable voltage, which can be set between 0 Volts and $+V$ Volts. This schematic diagram is very useful; from this all the important properties of AM and various forms of AM may be derived.

Amplitude Modulation [2/9]

Equations for DSB-AM

• Assume that our message $m(t)$ is a single tone at f_m with amplitude V_m, i.e. $m(t) = V_m \cos(2\pi f_m t)$

• Using the trigonometric identity $\cos A \cos B = \dfrac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$v_s(t) = \underbrace{V_{DC} \cos(2\pi f_c t)}_{\text{Carrier}} + \underbrace{\frac{V_m}{2}\cos(2\pi(f_c + f_m)t)}_{\text{Upper sideband}} + \underbrace{\frac{V_m}{2}\cos(2\pi(f_c - f_m)t)}_{\text{Lower sideband}}$$

Components	Carrier	Upper Sideband (USB)	Lower Sideband (LSB)
Amplitude	V_{DC}	$\dfrac{V_m}{2}$	$\dfrac{V_m}{2}$
Frequency	f_c	$f_c + f_m$	$f_c - f_m$

• This equation represents **Double Sideband Amplitude Modulation – DSBAM**

Amplitude Modulation [3/9]

Input: $V_{DC} + V_m \cos(2\pi f_m t)$ **Carrier:** $\cos(2\pi f_c t)$
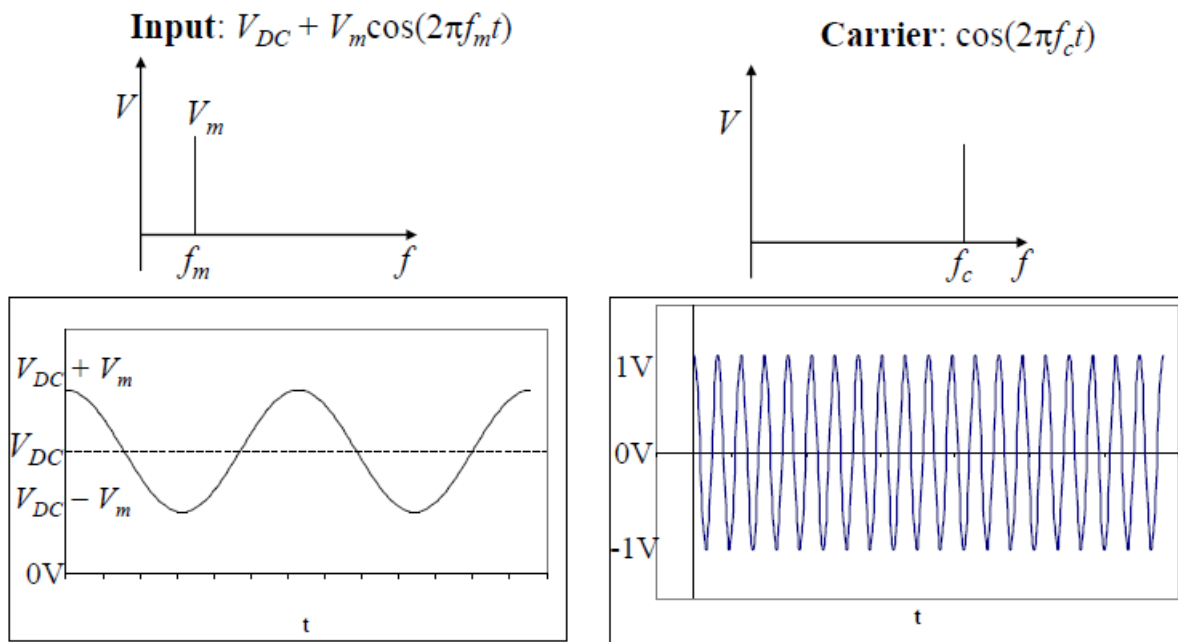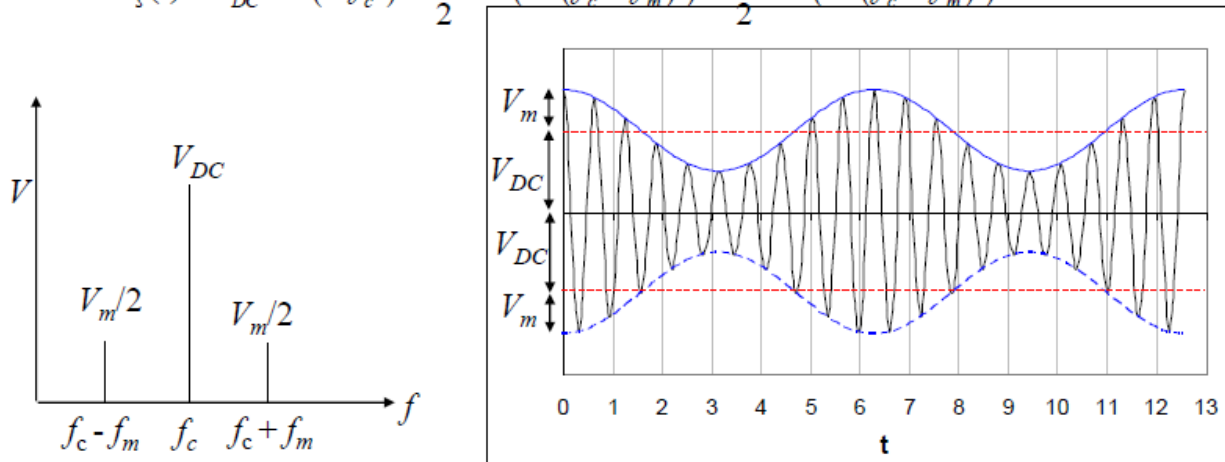
Amplitude Modulation [4/9]

Spectrum and Waveforms

The above are input signals. The diagram below shows the spectrum and corresponding waveform of the <u>output</u> signal, given by

$$v_s(t) = V_{DC}\cos(2\pi f_c t) + \frac{V_m}{2}\cos(2\pi(f_c + f_m)t) + \frac{V_m}{2}\cos(2\pi(f_c - f_m)t)$$

Amplitude Modulation [5/9]

Modulation Depth

Consider again the equation $v_s(t) = (V_{DC} + V_m\cos(2\pi f_m t))\cos(2\pi f_c t)$
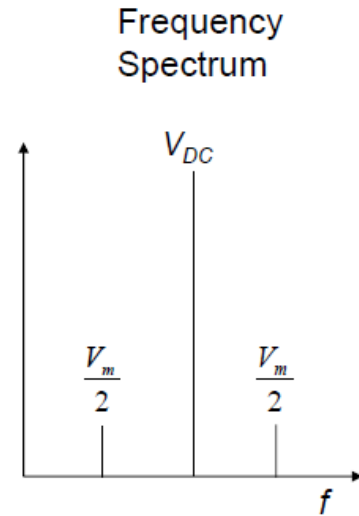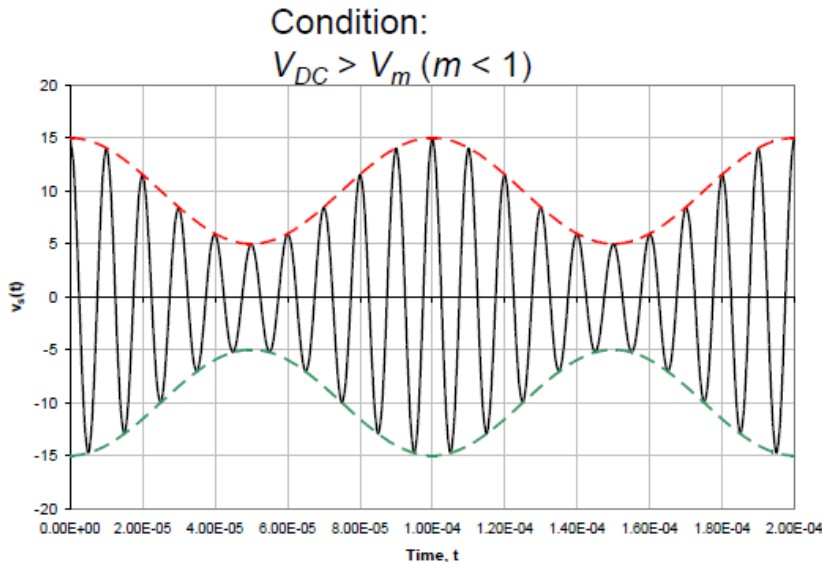
which may be written as

$$v_s(t) = V_{DC}\left(1 + \frac{V_m}{V_{DC}}\cos(2\pi f_m t)\right)\cos(2\pi f_c t)$$

The ratio $\dfrac{V_m}{V_{DC}}$ is defined as the **modulation depth**, *m*, *i.e.*

$$\text{Modulation Depth} \quad m = \frac{\text{Message amplitude}}{\text{Carrier amplitude}}$$

Amplitude Modulation [6/9]

Graphical Representation of Modulation Depth and Modulation Types

Condition:
$V_{DC} > V_m \ (m < 1)$

Frequency Spectrum

DSB-AM

Amplitude Modulation [7/9]

Graphical Representation of Modulation Depth and Modulation Types

Condition:
$V_{DC} = V_m \ (m = 1)$

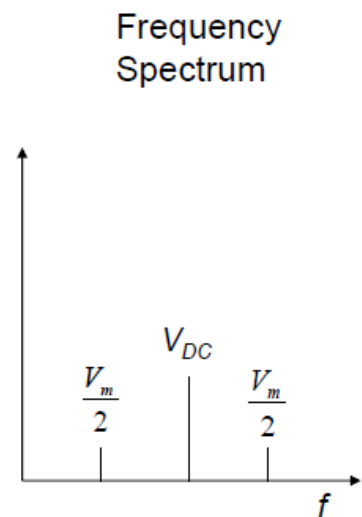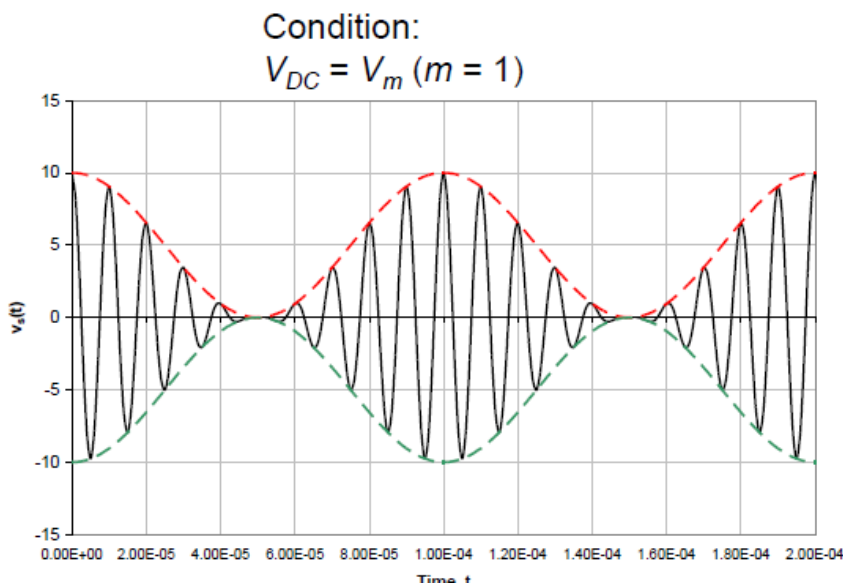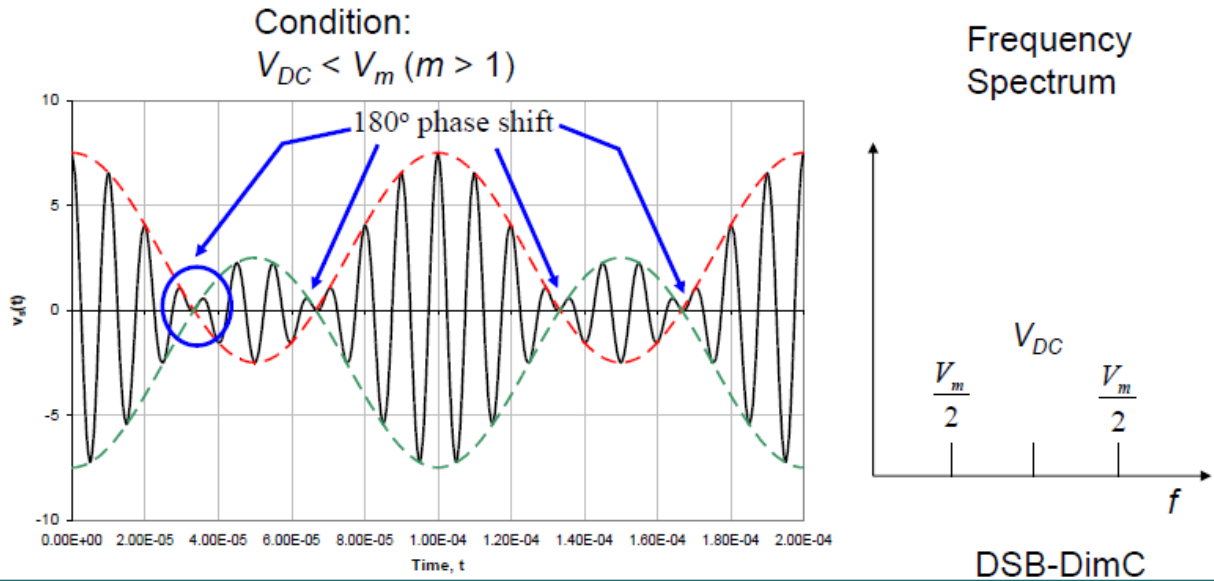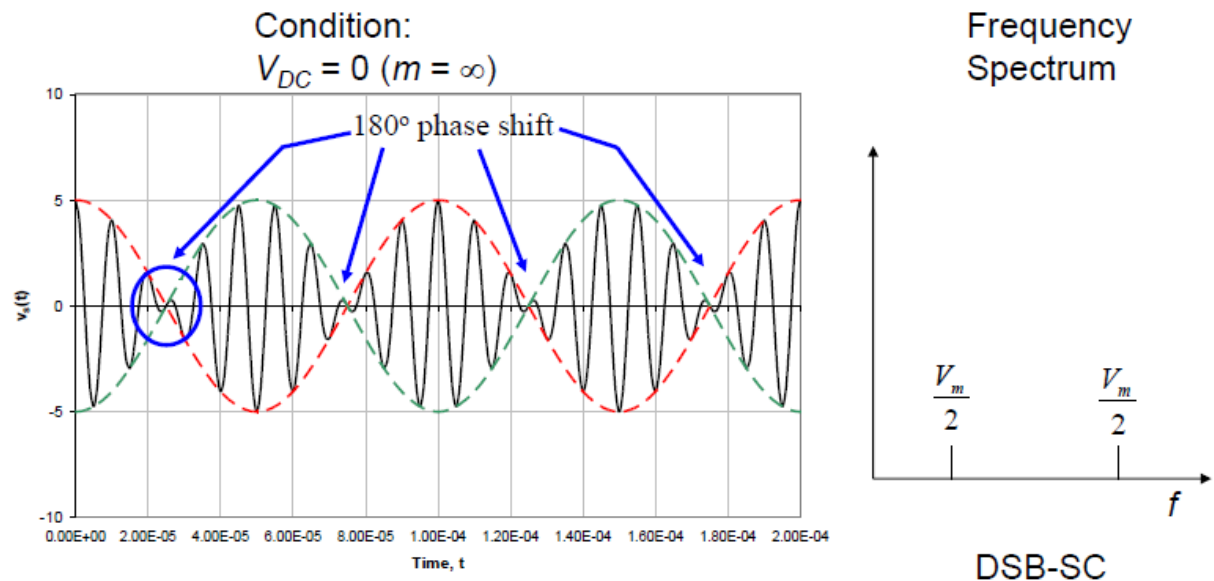Frequency Spectrum

DSB-AM

Amplitude Modulation [8/9]

Graphical Representation of Modulation Depth and Modulation Types

Amplitude Modulation [9/9]

Graphical Representation of Modulation Depth and Modulation Types

Amplitude Demodulation [1/8]
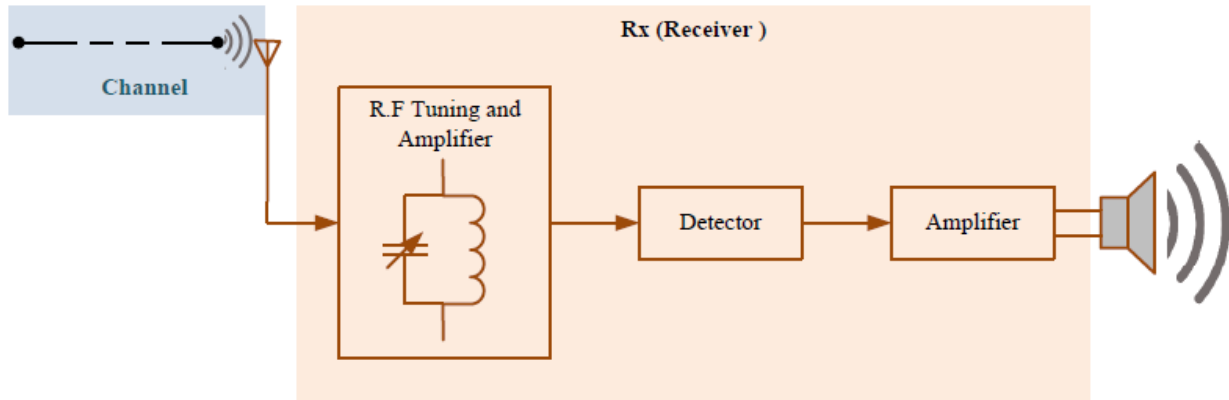
The above block diagram shows a generic radio receiver block diagram. The antenna receives radio wave from broadcasting stations, further amplified. The desired frequency is selected based on the tuned circuit. The detector, basically extract the message which can be amplified to feed into output appliances, for example speaker if it is audio message.

Amplitude Demodulation [2/8]

Envelope or Non-Coherent Detection

An envelope detector for AM is shown below:
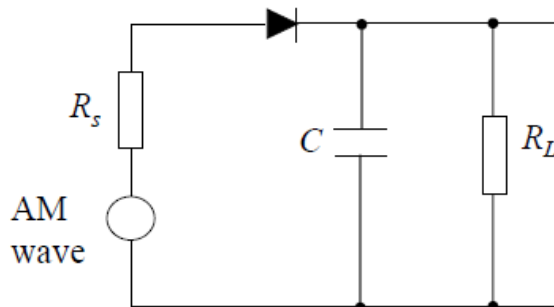
The charging time constant must be much smaller than the period of the carrier wave

$$R_s C << 1/f_c$$

The discharging time constant must be larger than the carrier wave period but smaller than the message period

$$1/f_c << R_L C << 1/f_m$$

This is obviously simple and low cost. But the AM input have $m < 1$,

Amplitude Demodulation [3/8]

Envelope or Non-Coherent Detection

For large signal inputs, ($\approx$ Volts) the diode is switched *i.e.* forward biased $\equiv$ ON, reverse biased $\equiv$ OFF, and acts as a half wave rectifier. The 'RC' combination acts as a 'smoothing circuit' and the output is $m(t)$ plus 'distortion'.
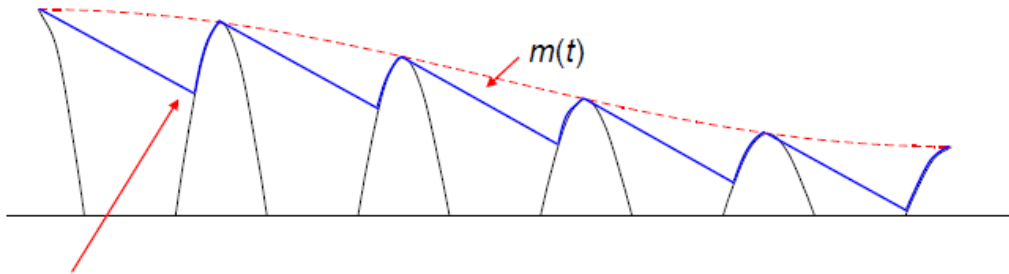
Output of the Envelope Detector, $m(t)$ + distortion

The magnitude of the slope of the RC discharge must be greater that the magnitude of the slope of the envelope

Amplitude Demodulation [4/8]

Synchronous or Coherent Demodulation

A synchronous demodulator is shown below
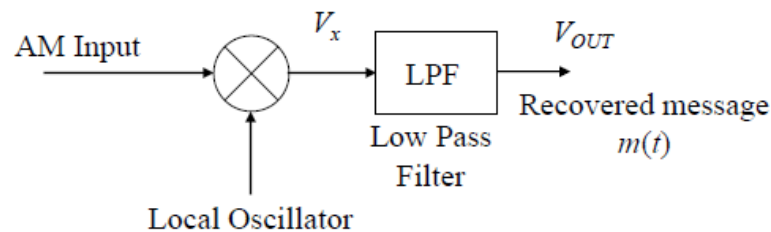
This is relatively more complex and more expensive. The Local Oscillator (LO) must be synchronised or coherent, *i.e.* at the same frequency and in phase with the carrier in the AM input signal. This additional requirement adds to the complexity and the cost.

Amplitude Demodulation [5/8]

Synchronous (Coherent) Local Oscillator

If we assume zero path delay between the modulator and demodulator, then the ideal LO signal is $\cos(2\pi f_c t)$. Note – in general there will be a path delay, say τ, and the LO would then be $\cos(2\pi f_c(t - \tau))$, *i.e.* the LO is synchronous with the carrier implicit in the received signal. Hence for an ideal system with zero path delay

Modulator Output → ⊗ → LPF → V_{OUT}

Local Oscillator
$\cos(2\pi f_c t)$
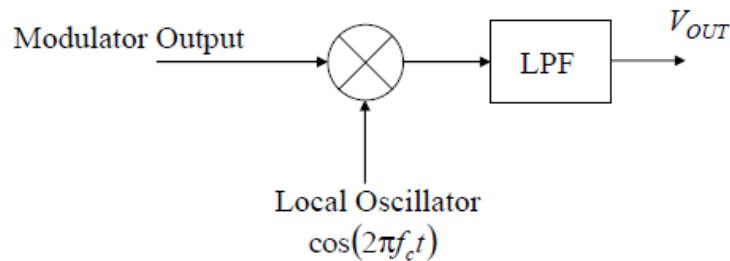
Analysing this for a DSBAM input = $(V_{DC} + m(t))\cos(2\pi f_c t)$

Amplitude Demodulation [6/8]

Synchronous (Coherent) Local Oscillator

V_X = AM input x LO

$$= (V_{DC} + m(t))\cos(2\pi f_c t)\cos(2\pi f_c t)$$

$$= (V_{DC} + m(t))\left(\frac{1}{2} + \frac{1}{2}\cos(4\pi f_c t)\right)$$

$$V_x = \frac{V_{DC}}{2} + \frac{V_{DC}}{2}\cos(4\pi f_c t) + \frac{m(t)}{2} + \frac{m(t)}{2}\cos(4\pi f_c t)$$

We will now examine the signal spectra from 'modulator to V_x'

Amplitude Demodulation [7/8]

Synchronous (Coherent) Local Oscillator

$m(t)$

V_{DC}

MOD

$\cos(2\pi f_c t)$

$V_{DC} + m(t)$

Carrier

f_c f

DSBAM

f_c f
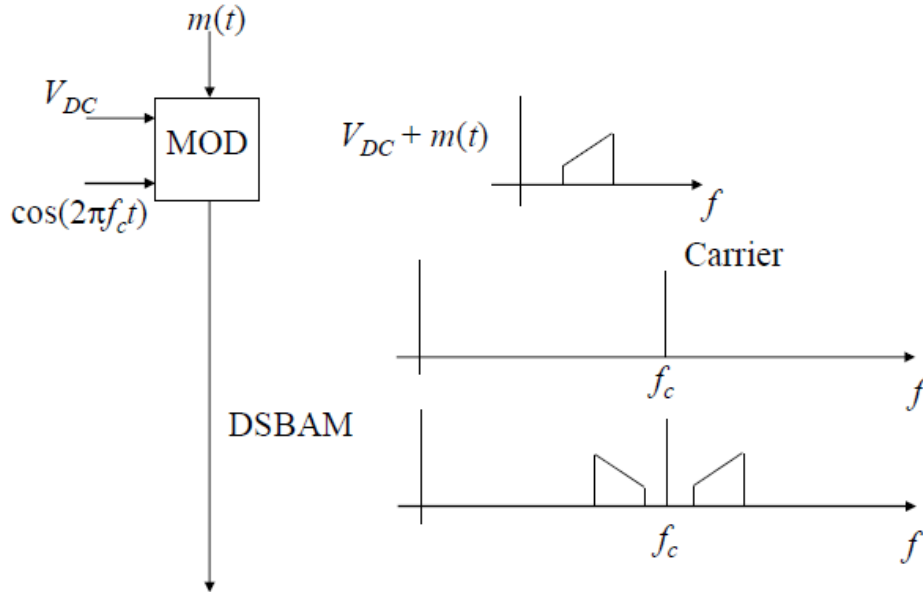
Amplitude Demodulation [8/8]

Synchronous (Coherent) Local Oscillator

LO

$\cos(2\pi f_c t)$

V_x

LO

f_c f

$m(t)/2$

V_x LPF response

$V_{DC}/2$

f_c $2f_c$ f
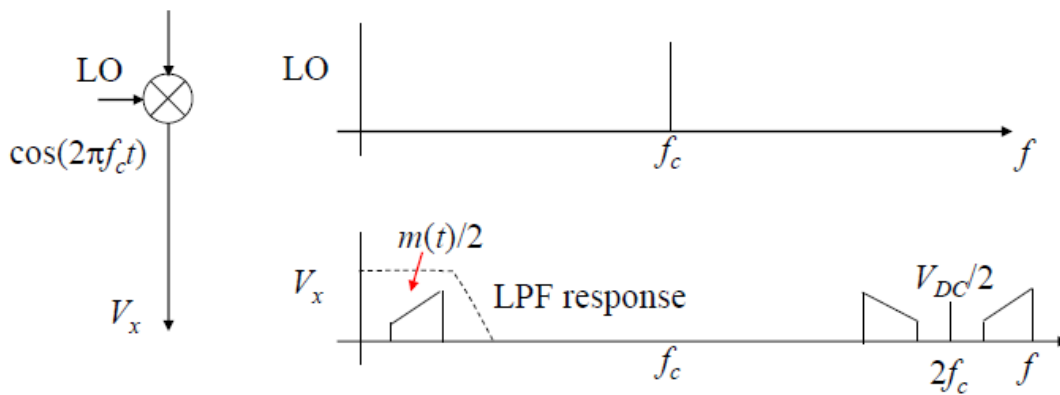
Note – the AM input has been 'split into two' – 'half' has moved or shifted up to

$$\left(\frac{m(t)}{2} + \frac{V_{DC}}{2}\right)\cos(4\pi f_c t) \quad \text{and half shifted down to baseband,} \quad \frac{V_{DC} + m(t)}{2}$$

Example:

a) An amplitude modulator is shown in **fig. 4**.

V_{DC} $V_{OUT}(t)$

$m(t)$ $\cos(4\times10^5 \pi t)$
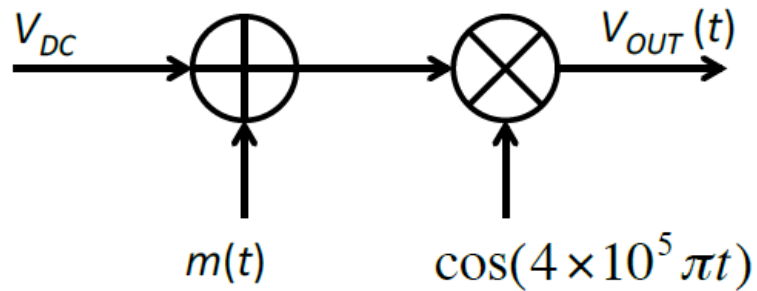
Figure 4

i)   If the message $m(t)$ is defined as $m(t) = 3\cos(2000\pi t)$ and the DC offset voltage V_{DC} is set to 6V, show that the output signal $V_{OUT}(t)$ can be expressed as

$$V_{OUT}(t) = 6\cos(4\times10^5\pi t) + 1.5\cos(4.02\times10^5\pi t)$$
$$+ 1.5\cos(3.98\times10^5\pi t)$$

ii)   Determine the modulation depth of the amplitude modulated signal $V_{OUT}(t)$.

iii)   Sketch the amplitude modulated signal $V_{OUT}(t)$ from part ii) and draw its frequency spectrum.

Angle Modulation (Phase and Frequency Modulation)

Angle modulation: Carrier angle is varied according to the slowly-varying message signal

An important feature of angle modulation:

- It can provide a better discrimination (robustness) against noise and interference than AM
- This improvement is achieved at the expense of increased transmission bandwidth
- In case of angle modulation, channel bandwidth may be exchanged for improved noise performance
- Such trade-off is *not* possible with AM

BASIC DEFINITIONS

Relationship between the angle and frequency of a sinusoidal signal

Sinusoidal carrier

$$c(t) = A_c \cos[\theta_i(t)]$$

Angle of carrier

$$\theta_i(t) \ \ [\text{rad}]$$

Instantaneous frequency of carrier

$$f_i(t) = \frac{1}{2\pi} \omega_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = \frac{1}{2\pi} \dot{\theta}_i(t) \ \ [\text{Hz}]$$

In the case of an unmodulated carrier, the angle becomes

$$\theta_i(t) = 2\pi f_c t + \phi_c$$

Phase modulation (PM)

Sinusoidal carrier: $c(t) = a(t)\cos[\theta_i(t)]$

PM: The angle $\theta_i(t)$ of carrier is varied *linearly* with the message signal $m(t)$

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

Amplitude of carrier is constant: $a(t) = A_c$

Phase-modulated waveform

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

where
- f_c denotes the carrier frequency (i.e., frequency of unmodulated signal)
- k_p is the *phase sensitivity* of the PM modulator expressed in radians per volt
- It is assumed that the angle of unmodulated carrier is zero at $t = 0$

Frequency modulation (FM)

Sinusoidal carrier: $c(t) = a(t)\cos[\theta_i(t)]$

The instantaneous frequency $f_i(t)$ of carrier is varied *linearly* with the message signal $m(t)$

$$\frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\dot{\theta}_i(t) = f_c + k_f m(t)$$

Angle of carrier

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau$$

Amplitude of carrier is constant:

$$a(t) = A_c$$

Frequency-modulated waveform

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

where
- f_c denotes the carrier frequency (i.e., frequency of unmodulated signal)
- k_f is the *frequency sensitivity* of the FM modulator expressed in Hertz per volt
- It is assumed that the angle of unmodulated carrier is zero at $t = 0$

PM and FM signal in the time domain produced by a single tone message signal
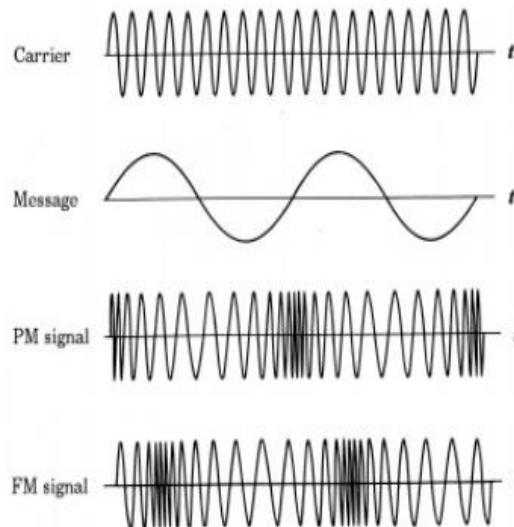
Note:
- The similarity between the angle modulated signals
- Amplitude of angle modulated signals is constant

MOST IMPORTANT PROPERTIES OF PM and FM SIGNALS – Part I

PM signal in the time domain:

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

FM signal in the time domain:

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

1. Amplitude of PM and FM signals is constant

2. Because the information is carried by the angle of carrier, a nonlinear operation (also nonlinear distortion) that preserves the angle has no influence on the angle modulation systems (i.e., it does not cause distortion). Consequently, even a **hard limiter** may be used to fix the amplitude of a PM or FM signal

MOST IMPORTANT PROPERTIES OF PM & FM SIGNALS – Part II

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)] \qquad s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$$

3. A close relationship exists between the PM and FM signals:

$$k_p = 2\pi k_f \qquad\qquad k_f = \frac{k_p}{2\pi}$$
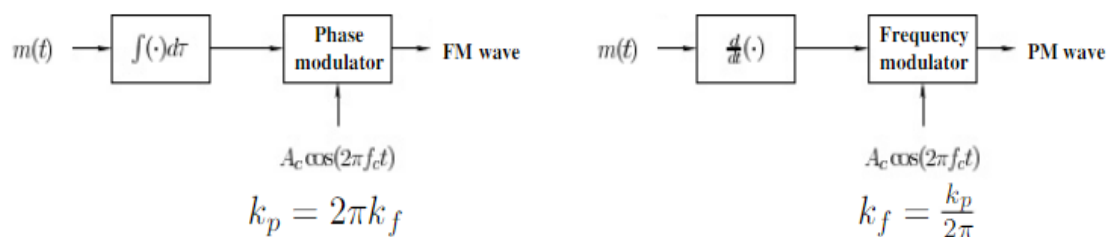
Conclusions:
- A PM/FM modulator may be used to generate an FM/PM waveform
- FM is much more frequently used than PM
- All the properties of a PM signal may be deduced from that of an FM signal
- Henceforth, in the remaining part of our studies **we deal only with FM signals**

FREQUENCY MODULATION

Goal: Determine the *spectrum* and *transmission bandwidth* of an FM signal

FM signal $s_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\right]$ is a nonlinear function of message signal $m(t)$

Angle modulations (including FM and PM) are *nonlinear modulation processes*

Consequently, spectrum of FM signal may not be determined in the *frequency domain* using Fourier transform

Empirical approach is required to determine the spectrum and transmission bandwidth of FM signal where the following single-tone sinusoidal message signal is considered

$$m(t) = A_m \cos(2\pi f_m t)$$

Definition of frequency deviation and modulation index

Let $m(t) = A_m \cos(2\pi f_m t)$ denote the single-tone message (modulating) signal

Then the instantaneous frequency of FM signal becomes

$$f_i(t) = f_c + k_f A_m \cos(2\pi f_m t) = f_c + \Delta f \cos(2\pi f_m t) \tag{1}$$

In (1), $\Delta f = k_f A_m$ is the **frequency deviation**, representing the maximum departure of instantaneous frequency of FM signal from the carrier frequency f_c

Angle of FM signal is

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau = 2\pi f_c t + \frac{\Delta f}{f_m}\sin(2\pi f_m t) = 2\pi f_c t + \beta \sin(2\pi f_m t) \tag{2}$$

In (2), $\beta = \Delta f/f_m$ is the **modulation index**, representing the maximum departure of angle of FM signal from angle $2\pi f_c t$ of unmodulated carrier

Units of frequency deviation and modulation index

$$\Delta f = k_f A_m \quad [\text{Hz}] \qquad\qquad \beta = \frac{\Delta f}{f_m} \quad [\text{rad}]$$

Recall the relationship between FM and PM signals

FM: $s = (t) = s_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$

PM: $s_{PM}(t) = A_c \cos[2\pi f_c t + k_p A_m \cos(2\pi f_m t)] \implies$!! $\beta \Leftrightarrow k_p A_m$

Two cases are distinguished:

- *Narrow-band* FM, for which $\beta = \frac{k_f A_m}{f_m} << 1$ rad
- *Wide-band* FM, for which $\beta = \frac{k_f A_m}{f_m} >> 1$ rad

The spectrum of narrow- and wide-band FM signals are completely different

Spectrum of narrow-band FM signal

Narrow-band FM signal in the time domain is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \text{where} \quad \beta << 1 \text{ rad}$$

Using a trigonometric identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ we get

$$s(t) = A_c \cos(2\pi f_c t) \cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t) \sin[\beta \sin(2\pi f_m t)] \qquad (3)$$

If the modulation index is much smaller than 1 radian then

Since $\lim\limits_{\alpha \to 0}\{\cos\alpha\} = 1,$ $\implies$ $\cos[\beta \sin(2\pi f_m t)] \approx 1$

Since $\lim\limits_{\alpha \to 0}\{\sin\alpha\} = \alpha,$ $\implies$ $\sin[\beta \sin(2\pi f_m t)] \approx \beta \sin(2\pi f_m t)$

Hence, (3) simplifies to

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

From the equation developed on the previous transparency

$$s(t) \approx A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

we get using the trigonometric identity $\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$$s(t) = s_{FM}(t) \approx A_c \cos(2\pi f_c t) + \frac{1}{2}\beta A_c \left\{ \cos[2\pi(f_c + f_m)t] - \cos[2\pi(f_c - f_m)t] \right\}$$

Recall, the single-tone modulated AM signal in the *time domain* is

$$s_{AM}(t) \approx A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \left\{ \cos[2\pi(f_c + f_m)t] + \cos[2\pi(f_c - f_m)t] \right\}$$

Comparing $s_{FM}(t)$ and $s_{AM}(t)$ we conclude:

- If $\beta = \mu$ then the only difference between AM and narrow-band FM signals is that in the latter the sign of the lower side frequency component is reversed

Spectrum and transmission bandwidth of narrow-band FM signal
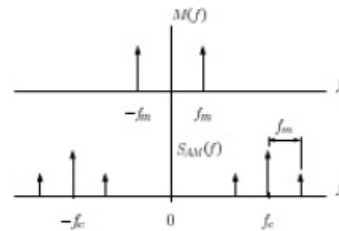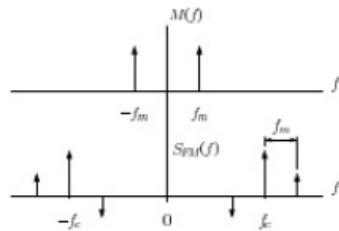
Spectrum of narrow-band FM and AM signals

Narrow-band FM: AM:
$$S_{FM}(f) = \mathcal{F}\{s_{FM}(t)\} \qquad\qquad S_{AM}(f) = \mathcal{F}\{s_{AM}(t)\}$$

Transmission bandwidth of narrow-band FM is equal to that of AM:

$$B_T = 2W$$

where W is the bandwidth of the low-pass message signal

Spectrum of wide-band FM signal

Equation giving the FM signal in the time domain was developed on page 13, see (3) on page 13

For convenience, that equation is repeated here:

$$s(t) = A_c \cos(2\pi f_c t)\cos[\beta \sin(2\pi f_m t)] - A_c \sin(2\pi f_c t)\sin[\beta \sin(2\pi f_m t)] \quad (4)$$

Recall, only periodic signals may be expanded as a trigonometric Fourier series. Fourier series and, consequently, periodic signals have a discrete spectrum

Even though $s(t)$ is not necessarily periodic, the terms $\cos[\beta \sin(2\pi f_m t)]$ and $\sin[\beta \sin(2\pi f_m t)]$ are periodic and each can be expanded as a Fourier series

It has been shown in applied mathematics that

$$\cos[\beta \sin(2\pi f_m t)] \;=\; J_0(\beta) + \sum_{n \text{ even}}^{\infty} 2J_n(\beta)\cos(2\pi n f_m t) \qquad (5)$$

$$\sin[\beta \sin(2\pi f_m t)] \;=\; \sum_{n \text{ odd}}^{\infty} 2J_n(\beta)\sin(2\pi n f_m t) \qquad (6)$$

where n is positive, and the **Bessel functions** of the first kind, of order n and argument β are defined as

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp[j(\beta \sin \lambda - n\lambda)]d\lambda$$

Substituting (5) and (6) (see page 17) into (4) (given on page 16) and expanding products of sine and cosine functions using trigonometric identities finally yields the FM signal for arbitrary modulation index β in the *time domain*

$$s(t) = A_c J_0(\beta)\cos(2\pi f_c t)$$

$$+ \sum_{n \text{ odd}}^{\infty} A_c J_n(\beta)[\cos(2\pi f_c + n2\pi f_m)t - \cos(2\pi f_c - n2\pi f_m)t]$$

$$+ \sum_{n \text{ even}}^{\infty} A_c J_n(\beta)[\cos(2\pi f_c + n2\pi f_m)t + \cos(2\pi f_c - n2\pi f_m)t]$$

This equation is valid for both the narrow-band and wide-band FM signals

Recall the Fourier transform of a cosine signal with infinite duration is

$$\mathcal{F}\{\cos(2\pi f_c t)\} = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

To get the spectrum of an FM signal, the Fourier transform of

$$s(t) = A_c J_0(\beta)\cos(2\pi f_c t)$$

$$+ \sum_{n \text{ odd}}^{\infty} A_c J_n(\beta)[\cos(2\pi f_c + n2\pi f_m)t - \cos(2\pi f_c - n2\pi f_m)t]$$

$$+ \sum_{n \text{ even}}^{\infty} A_c J_n(\beta)[\cos(2\pi f_c + n2\pi f_m)t + \cos(2\pi f_c - n2\pi f_m)t]$$

has to be calculated

Observe

- Single-tone modulated FM signal has a discrete spectrum
- Spectrum contains a carrier-frequency line plus an infinite number of sideband lines $\Longrightarrow$ Distortion-less transmission requires infinite bandwidth!
- All lines are equally spaced by the modulating frequency
- Odd-order lower sideband lines are reversed in phase

Properties of Bessel functions of the first kind

1. For small values of modulation index β the following approximations may be used (Recall, this is the case of narrow-band FM)

$$J_0(\beta) = 1 \quad \text{and} \quad J_1(\beta) = \frac{\beta}{2} \quad \text{and} \quad J_n(\beta) = 0, \ n > 2$$

2. Bessel functions $J_n(\beta)$ as a function of modulation index β
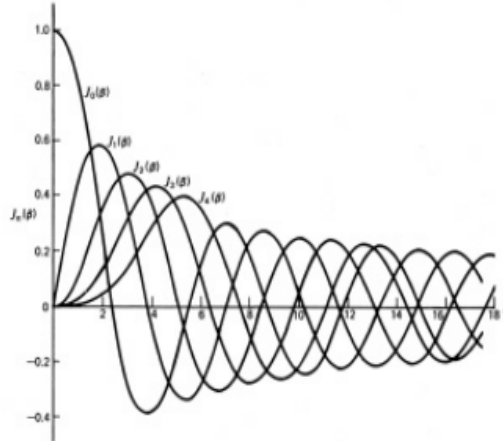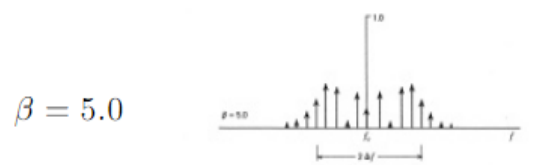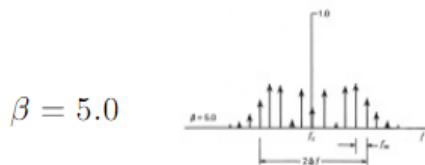

Note: At certain values of $J_n(\beta)$, the carrier disappears

Spectrum of a single-tone FM signal

Conditions:
- Single-tone sinusoidal modulation $m(t)$
- Spectra are normalized with respect to the carrier amplitude
- Magnitude of spectra is shown only for positive frequencies

$m(t)$: Frequency fixed, amplitude increased $m(t)$: Amplitude fixed, frequency decreased

$\beta = 1.0$ $\beta = 1.0$

$\beta = 2.0$ $\beta = 2.0$

$\beta = 5.0$ $\beta = 5.0$
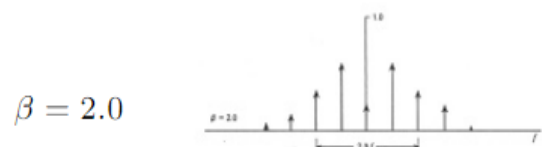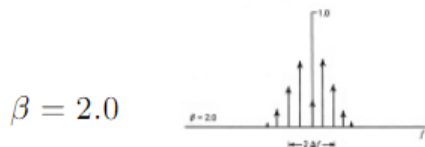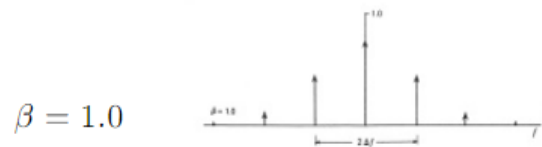
TRANSMISSION BANDWIDTH OF FM SIGNAL

In theory, an FM signal contains an infinite number of side frequencies $\implies$ Bandwidth required for distortion-free transmission is infinite in extent whether or not the message is band-limited

But implemented FM systems using finite bandwidth do exist and perform well

Explanation: Amplitude of side frequencies decays if we move away from the carrier frequency and sufficiently far away from the carrier the spectral components becomes negligible

Experiments showed that if the amplitude of side frequency components is 1 % then a distortion may not be noticed, if the amplitude of side frequency components is 10 % then a small but noticeable distortion exists

By definition: The **transmission bandwidth** of an FM signal is the separation between the two frequencies beyond which none of the side frequencies is greater than 1 % of carrier amplitude obtained when the modulation is removed

Determination of transmission bandwidth of a single-tone FM signal

In practice, the frequency deviation Δf is fixed

Carson's rule
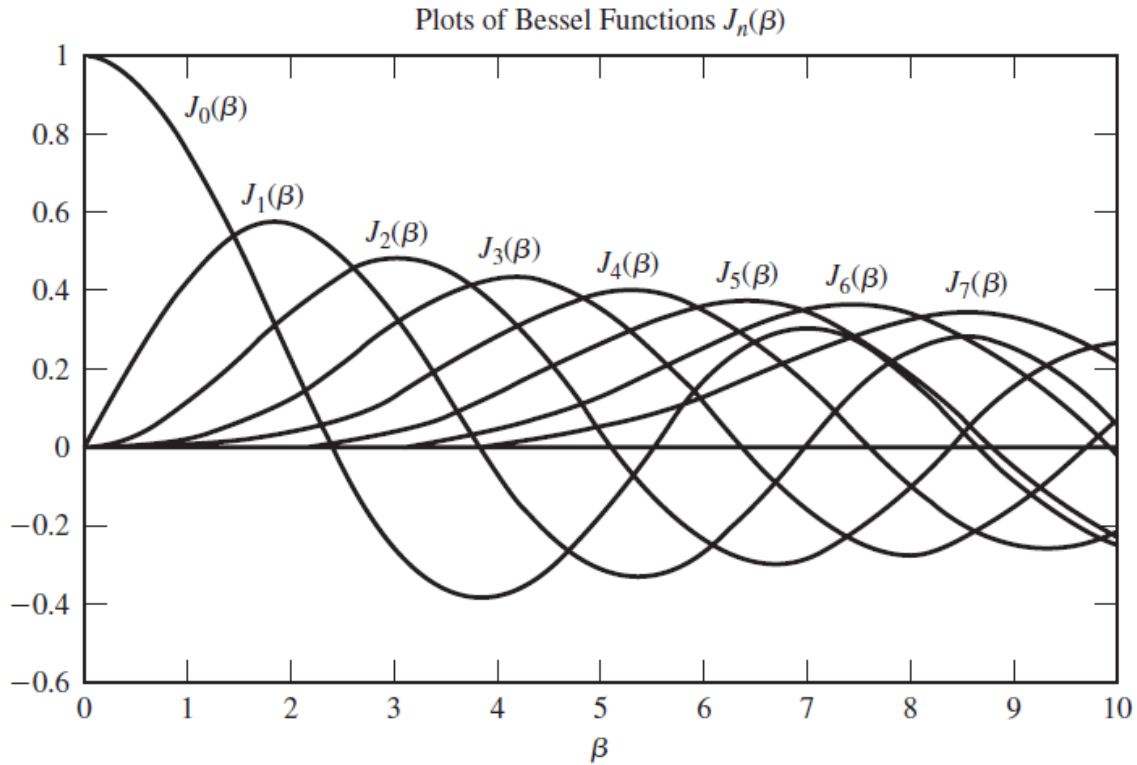
$$B_T \approx 2(\Delta f + f_m)$$

$$B_T \cong 2f_m\,(1+\beta)$$

Plots of Bessel Functions $J_n(\beta)$

TABLE 3.1 TABLE OF BESSEL FUNCTION VALUES

n	$\beta = 0.1$	$\beta = 0.2$	$\beta = 0.5$	$\beta = 1$	$\beta = 2$	$\beta = 5$	$\beta = 8$	$\beta = 10$
0	0.997	0.990	0.938	0.765	0.224	−0.178	0.172	−0.246
1	0.050	0.100	0.242	0.440	0.577	−0.328	0.235	0.043
2	0.001	0.005	0.031	0.115	0.353	0.047	−0.113	0.255
3				0.020	0.129	0.365	−0.291	0.058
4				0.002	0.034	0.391	−0.105	−0.220
5					0.007	0.261	0.186	−0.234
6					0.001	0.131	0.338	−0.014
7						0.053	0.321	0.217
8						0.018	0.223	0.318
9						0.006	0.126	0.292
10						0.001	0.061	0.207
11							0.026	0.123
12							0.010	0.063
13							0.003	0.029
14							0.001	0.012
15								0.004
16								0.001

(From Ziemer and Tranter; © 1990 Houghton Mifflin, reprinted by permission.)

Reversion:

FM -- With a sinusoidal message signal $s(t) = A_m \cos(2\pi f_m t)$:

$$s_{FM}(t) = A\cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \qquad \beta = \frac{kA_m}{f_m}$$

With a general message signal $s(t)$: $\qquad \beta = \dfrac{k\max|s(t)|}{B_s}$

$s_{FM}(t)$ can be expanded as an infinite Fourier series:

$$s_{FM}(t) = A\cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$
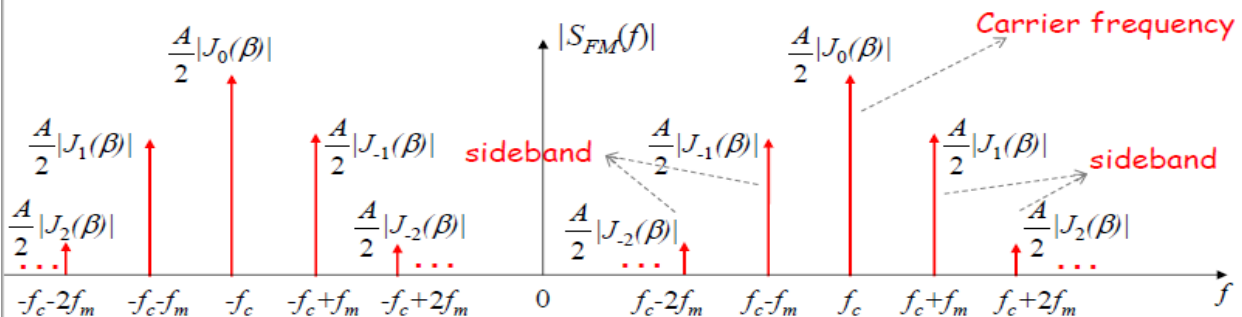$$= A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t]$$

$J_n(\beta)$ is called the Bessel Function of the first kind and of order n, which is defined by

$$J_n(\beta) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin x - nx)}dx$$

Magnitude Spectrum of FM Sinusoidal Signals

$$s_{FM}(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t] = \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)e^{j2\pi(f_c+nf_m)t} + \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)e^{-j2\pi(f_c+nf_m)t}$$

$$S_{FM}(f) = \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f - f_c - nf_m) + \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f + f_c + nf_m)$$

The bandwidth of an FM sinusoidal signal is ∞.

Power Spectrum of FM Sinusoidal Signals

$$S_{FM}(f) = \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f - f_c - nf_m) + \frac{A}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\delta(f + f_c + nf_m)$$
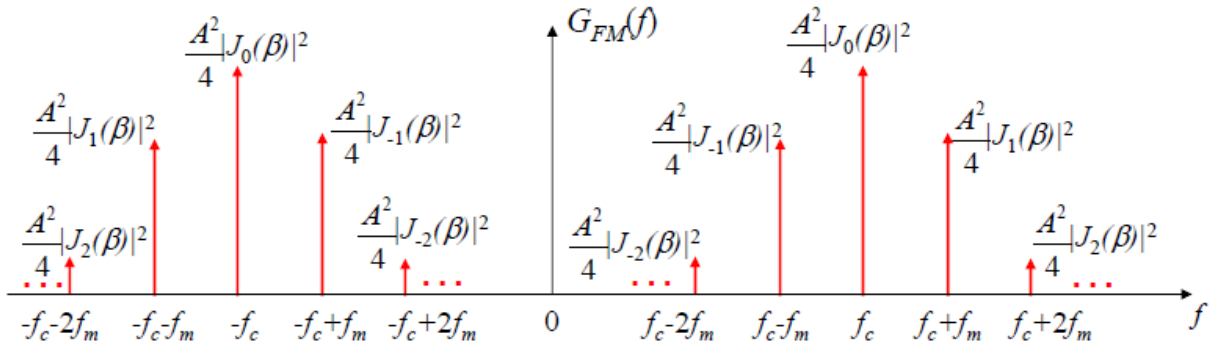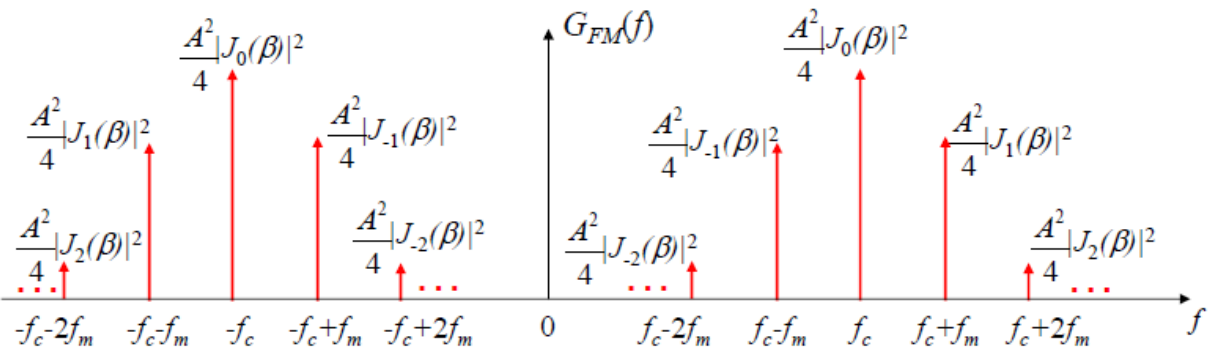
$$G_{FM}(f) = \frac{A^2}{4}\sum_{n=-\infty}^{\infty} |J_n(\beta)|^2 \,\delta(f - f_c - nf_m) + \frac{A^2}{4}\sum_{n=-\infty}^{\infty} |J_n(\beta)|^2 \,\delta(f + f_c + nf_m)$$

Power Spectrum of FM Sinusoidal Signals

$$P_t = \underbrace{\frac{A^2}{2}\cdot J_0^2(\beta)}_{\text{Power at carrier}} + \underbrace{\frac{A^2}{2}\cdot\left(\sum_{n=1}^{\infty} J_n^2(\beta) + \sum_{n=-\infty}^{-1} J_n^2(\beta)\right)}_{\text{Power at sidebands}} = \frac{A^2}{2} \quad\Rightarrow\quad \boxed{J_0^2(\beta) + 2\sum_{n=1}^{\infty} J_n^2(\beta) = 1}$$

$$s_{FM}(t) = A\cos[2\pi f_c t + \beta\sin(2\pi f_m t)] \;\Rightarrow\; P_t = \frac{A^2}{2}$$

Example:

Let the carrier be given by $c(t) = 10\cos(2\pi f_c t)$ and let the message signal be $\cos(20\pi t)$. Further assume that the message is used to frequency modulate the carrier with $k_f = 50$. Find the expression for the modulated signal and determine how many harmonics should be selected to contain 99% of the modulated signal power.

Solution   The power content of the carrier signal is given by

$$P_c = \frac{A_c^2}{2} = \frac{100}{2} = 50 \tag{3.3.29}$$

The modulated signal is represented by

$$u(t) = 10\cos\left(2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} \cos(20\pi\tau)\,d\tau\right)$$

$$= 10\cos\left(2\pi f_c t + \frac{50}{10}\sin(20\pi t)\right)$$

$$= 10\cos(2\pi f_c t + 5\sin(20\pi t)) \tag{3.3.30}$$

The modulation index is given by

$$\beta = k_f \frac{\max[|m(t)|]}{f_m} = 5 \tag{3.3.31}$$

and, therefore, the FM-modulated signal is

$$u(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\cos(2\pi(f_c + nf_m)t)$$

$$= \sum_{n=-\infty}^{\infty} 10 J_n(5)\cos(2\pi(f_c + 10n)t) \tag{3.3.32}$$

It is seen that the frequency content of the modulated signal is concentrated at frequencies of the form $f_c + 10n$ for various n. To make sure that at least 99% of the total power is within the effective bandwidth, we have to choose k large enough such that

$$\sum_{n=-k}^{n=k} \frac{100 J_n^2(5)}{2} \geq 0.99 \times 50 \tag{3.3.33}$$

This is a nonlinear equation and its solution (for k) can be found by trial and error and by using tables of the Bessel functions. Of course, in finding the solution to this equation we have to employ the symmetry properties of the Bessel function given in Equation (3.3.28). Using these properties we have

$$50\left[J_0^2(5) + 2\sum_{n=1}^{k} J_n^2(5)\right] \geq 49.5 \tag{3.3.34}$$

Starting with small values of k and increasing it, we see that the smallest value of k for which the left-hand side exceeds the right-hand side is $k = 6$. Therefore, taking frequencies

$f_c \pm 10k$ for $0 \le k \le 6$ guarantees that 99% of the power of the modulated signal has been included and only one per cent has been left out. This means that, if the modulated signal is passed through an ideal bandpass filter centered at f_c with a bandwidth of at least 120 Hz, only 1% of the signal power will be eliminated. This gives us a practical way to define the ==*effective bandwidth*== of the angle-modulated signal as being 120 Hz. Figure 3.29 shows the frequencies present in the effective bandwidth of the modulated signal.

==In general the effective bandwidth of an angle-modulated signal, which contains at least 98% of the signal power, is given by the relation==

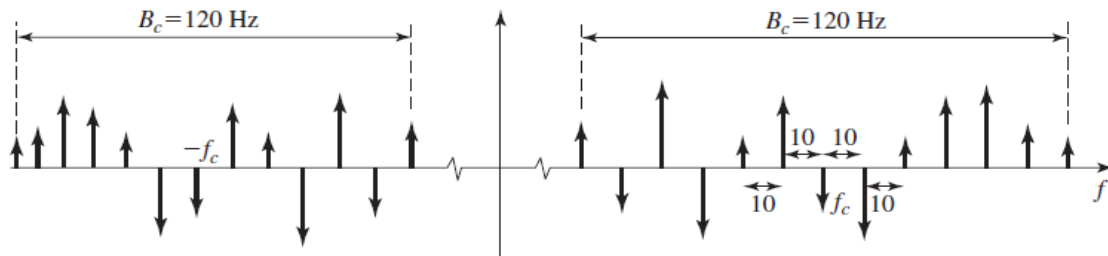$$B_c = 2(\beta + 1)f_m \tag{3.3.35}$$

Figure 3.29   The harmonics present inside the effective bandwidth of Example 3.3.2

Outline

□ Two methods of generating FM waves:

■ Direct method

■ Indirect Method: Armstrong's wideband frequency modulator

Review of frequency deviation:

Angle modulation:
$$s(t) = A_c \cos(2\pi f_c t + \phi(t))$$

Instantaneous frequency
$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt} = f_c + \frac{1}{2\pi} \frac{d\phi(t)}{dt}$$

Frequency deviation
$$\Delta f = \max \left| \frac{1}{2\pi} \frac{d\phi(t)}{dt} \right|.$$

Frequency deviation for FM signals:
$$\Delta f = k_f \max |m(t)|.$$

For example, FM radio allows 75kHz deviation to each side of the carrier.

3

Direct FM Generation

□ The carrier freq is directly varied by the input signal

□ Can be accomplished by Voltage-Controlled Oscillator (VCO), whose output frequency is proportional to the voltage of the input signal.

□ A VCO example: implemented by variable capacitor

A **varactor diode** is a capacitor whose capacitance changes with the applied voltage. Therefore, if this capacitor is used in the tuned circuit of the oscillator and the message signal is applied to it, the frequency of the tuned circuit, and the oscillator will change in accordance with the message signal.
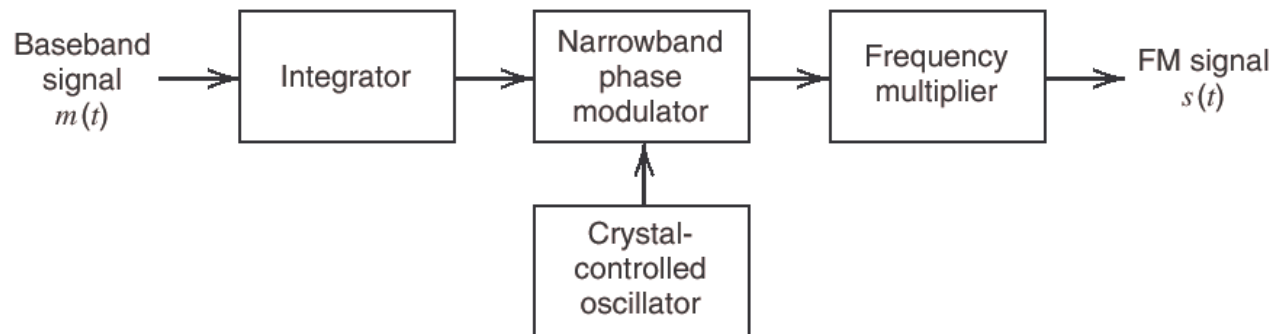
The **reactance-tube** is an inductor whose inductance varies with the applied voltage is employed and the analysis is very similar to the analysis presented for the varactor diode.

Problems of direct FM generator

- The carrier freq of VCO tends to drift away.
  - (Crystal oscillator cannot be used in direct FM: its freq is too stable, and is difficult to change.)
  - Feedback freq stabilization circuit is required:
    - The complexity is increased.
- The frequency deviation with direct FM is only about 5 KHz, too small for wideband FM:
  - Recall: the max frequency deviation in commercial FM radio is 75kHz.

Indirect Method: Armstrong Modulator

- First obtain NBFM via a NBPM circuit with crystal oscillator
- Then apply frequency multiplier
  - Increase both the carrier frequency and the freq deviation
- If necessary, use mixer to concatenate multiple multipliers
  - Mixer only changes the carrier frequency, but not the frequency deviation.
- Indirect FM is preferred when the stability of carrier frequency is of major concern (e.g., in commercial FM broadcasting)

Baseband signal $m(t)$ → Integrator → Narrowband phase modulator → Frequency multiplier → FM signal $s(t)$

Crystal-controlled oscillator

6

Recall: Narrow-band FM

□ if Δf is small:

$$u(t) = A_c \cos 2\pi f_c t \cos \phi(t) - A_c \sin 2\pi f_c t \sin \phi(t)$$

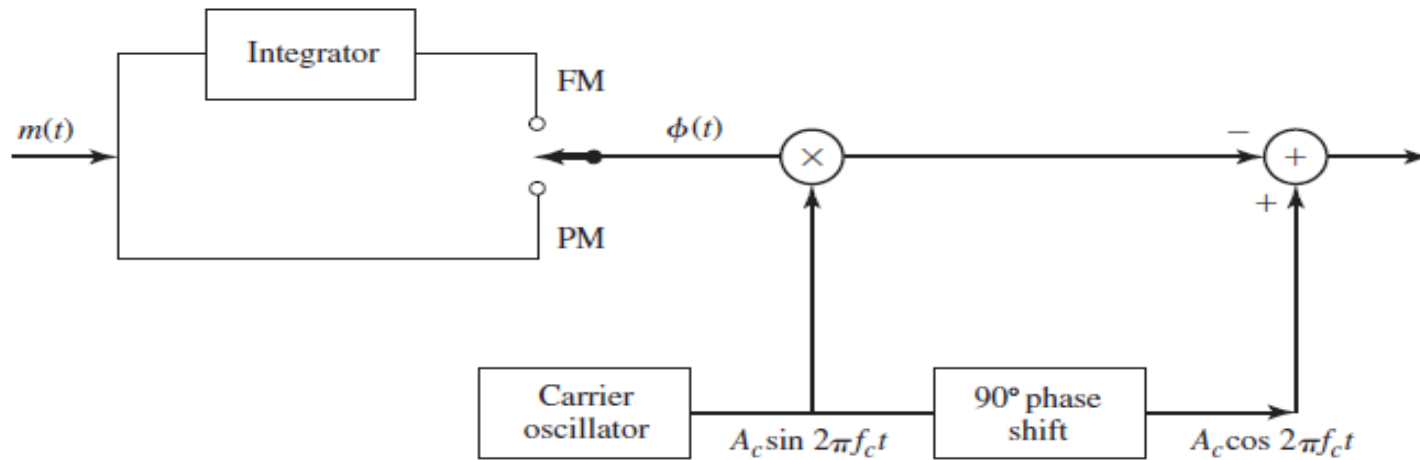$$\approx A_c \cos 2\pi f_c t - A_c \phi(t) \sin 2\pi f_c t$$

Figure 3.32   Generation of narrowband angle-modulated signal.

Crystal oscillator can be used to get stable frequency (prevent drifting)

But frequency deviation of NBFM is small.

To get larger one, use freq multiplier…

Frequency Multipliers

■ How to increase the frequency deviation ?

■ Answer: trigonometric identity!

From $s(t) = A_c \cos(2\pi f_c t + 2\pi k_f \int_0^t m(\tau) d\tau)$

If we can get the squared signal:

Power-Law Modulation. Let us consider the use of a nonlinear device such as a P-N diode which has a voltage-current characteristic as shown in Figure 3.15. Suppose that the voltage input to such a device is the sum of the message signal $m(t)$ and the carrier $A_c \cos 2\pi f_c t$, as illustrated in Figure 3.16. The nonlinearity will generate a product of the message $m(t)$ with the carrier, plus additional terms. The desired modulated signal can be filtered out by passing the output of the nonlinear device through a bandpass filter.

To elaborate on this method, suppose that the nonlinear device has an input–output (square-law) characteristic of the form

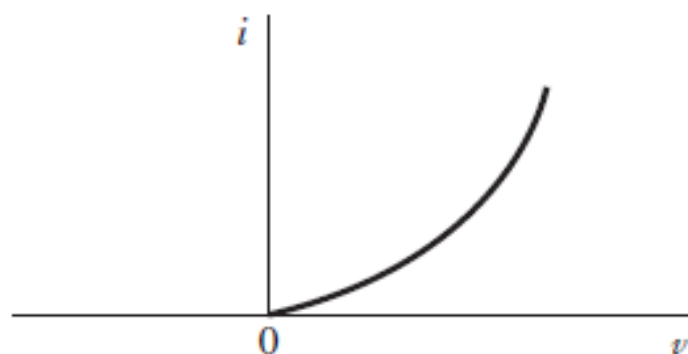$$v_0(t) = a_1 v_i(t) + a_2 v_i^2(t) \tag{3.2.29}$$

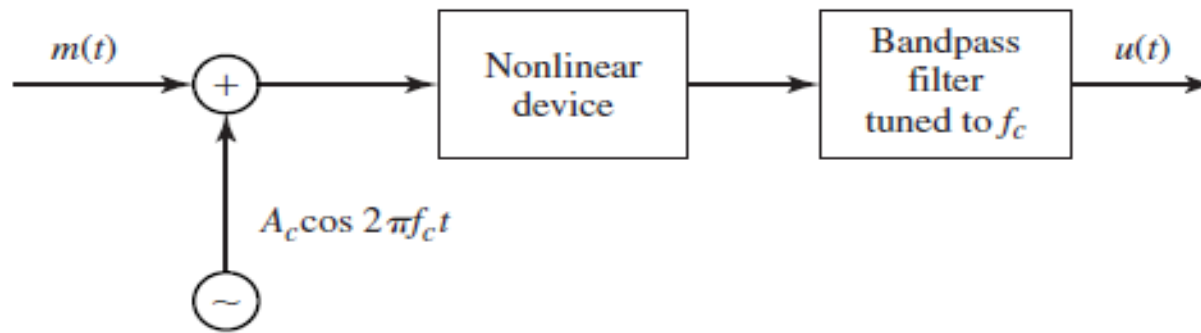Figure 3.15   Voltage-current characteristic of P-N diode.

Figure 3.16 Block diagram of power-law AM modulator.

where $v_i(t)$ is the input signal, $v_0(t)$ is the output signal, and the parameters (a_1, a_2) are constants. Then, if the input to the nonlinear device is

$$v_i(t) = m(t) + A_c \cos 2\pi f_c t, \tag{3.2.30}$$

its output is

$$v_o(t) = a_1[m(t) + A_c \cos 2\pi f_c t] + a_2[m(t) + A_c \cos \cos 2\pi f_c t]^2$$

$$= a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2 2\pi f_c t + A_c a_1 \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t$$

$$\tag{3.2.31}$$

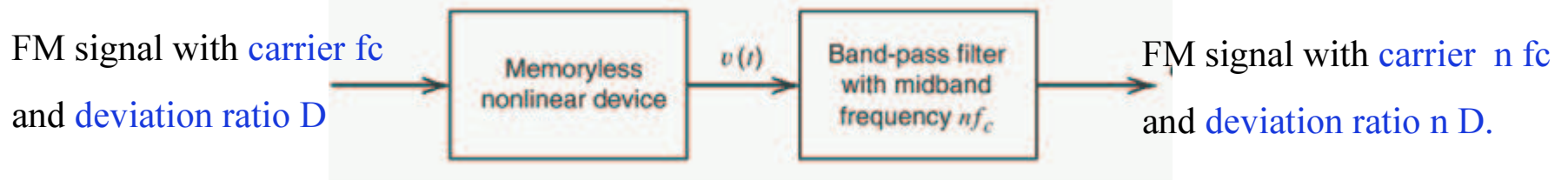The output of the bandpass filter with bandwidth $2W$ centered at $f = f_c$ yields

$$u(t) = A_c a_1 \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t \tag{3.2.32}$$

where $2a_2|m(t)|/a_1 < 1$ by design. Thus, ==the signal generated by this method is a conventional DSB AM signal.==

Frequency Multipliers

If we can get $s^3(t)$:

Freq Multipliers via Nonlinear Circuit

FM signal with carrier fc

and deviation ratio D

Memoryless nonlinear device

$v(t)$

Band-pass filter with midband frequency nf_c

FM signal with carrier n fc

and deviation ratio n D.

- A general nonlinear circuit produces

$$v(t) = a_1 s(t) + a_2 s^2(t) + ... + a_n s^n(t)$$

- The highest carrier frequency:
- The highest freq sensitivity factor:
- The bandpass filter:
  - Center:
  - Passband width:
  - In practice: n = 2, or 3.   Larger n is not efficient.
  - But can concatenate multiple stages to obtain higher orders. 10

Armstrong's Indirect FM
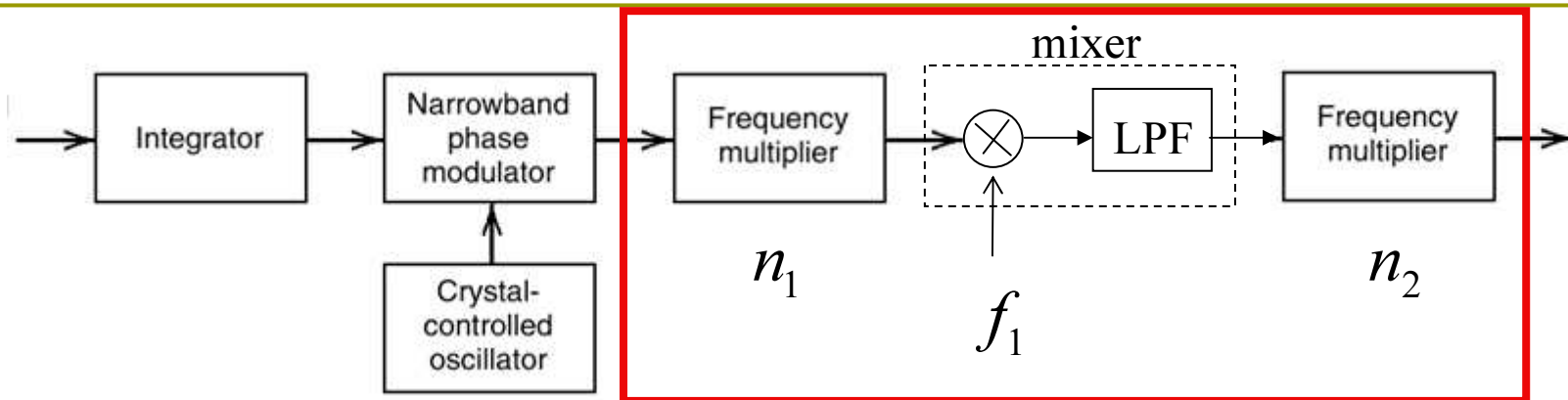
- **Two stages of multiplier and one mixer are used.**
  - Allow flexible choices of carrier freq and freq deviation.
  - The first stage multiplier amplifies both fc and Δf.
  - The mixer brings down the central freq.
  - The second stage amplifies fc and Δf again.

Example

$$n_1 = 162$$

$$n_2 = 30$$

$$f_1 = 77.97 MHz$$
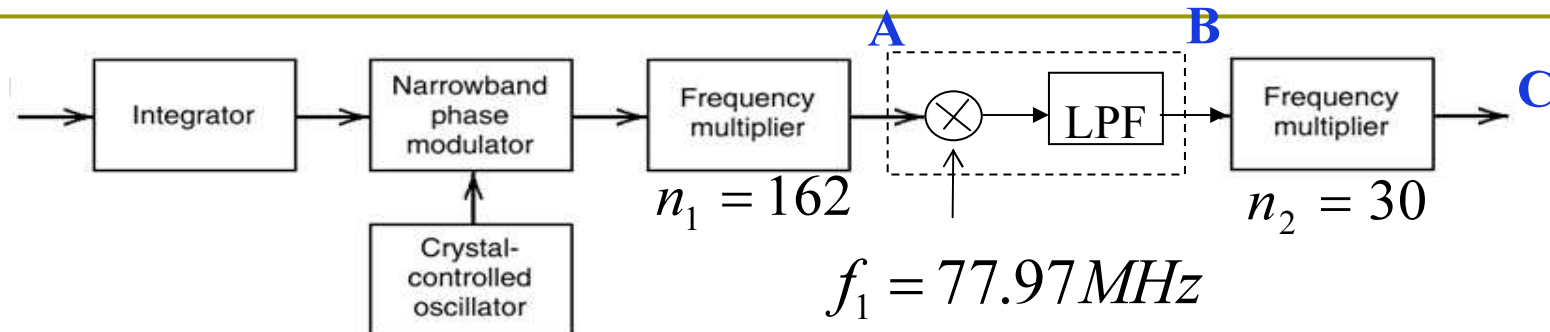
NBPM output : $f = 500 kHz, \; \Delta f = 15.432 Hz$

Find f and Δf at A, B, C.

At point A: fA=162*500KHz=81MHz

Freq. Divation=162*15.432=2500Hz

At Point B: fB=81-77.97=3.03 MHz

Freq. Divation= 2500Hz

At Point C: fC=3.03*30=90.9MHz

Freq. Divation= 2500Hz*30=75KHz

13

Example

$$n_1 = ?$$

f=8.5 MHz

$$n_2 = ?$$
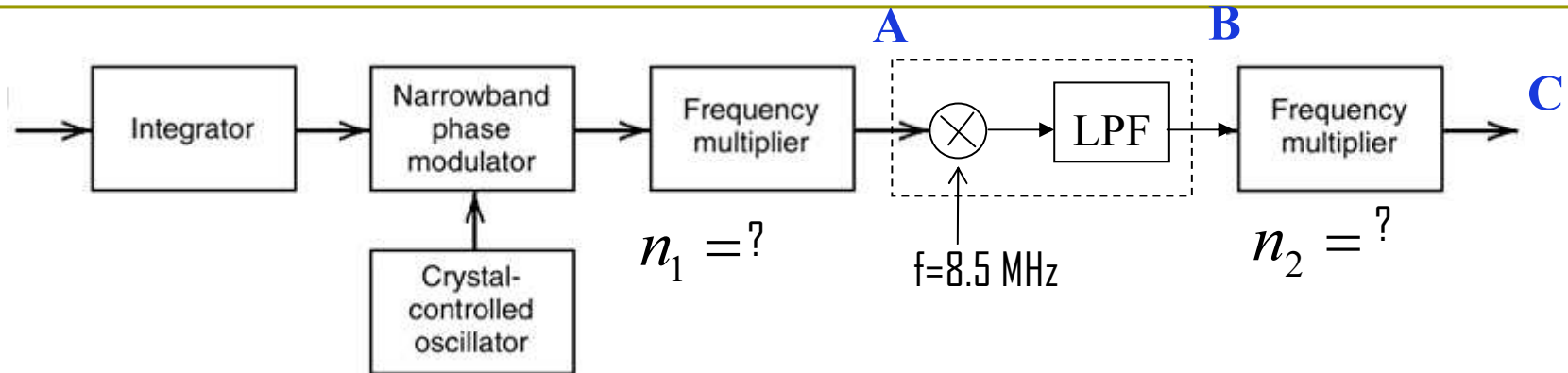
Δf at point C is 75KHz and the frequency is 100MHz while Δf at the o/p of NBFM modulator is 10HZ and the frequency is 100KHz. Find n1 and n2.

Armstrong FM transmitter

$f_{c1} = 0.1 MHz$
$\Delta f_1 = 10 Hz$

$f_{c2} = 7.5 MHz$
$\Delta f_2 = 750 Hz$

$f_{c3} = 1 MHz$
$\Delta f_3 = 750 Hz$

$f_{c4} = 100 MHz$
$\Delta f_4 = 75 KHz$

NBFM → frequency Multiplier Xn1 75 → Frequency converter → Frequency Multiplier 100 Xn2 → Power Amplifier

$$\frac{75000}{10} = 7500$$

$$= 100 \times 75$$

Local Oscillator
$f_0 Hz = 8.5 MHz$
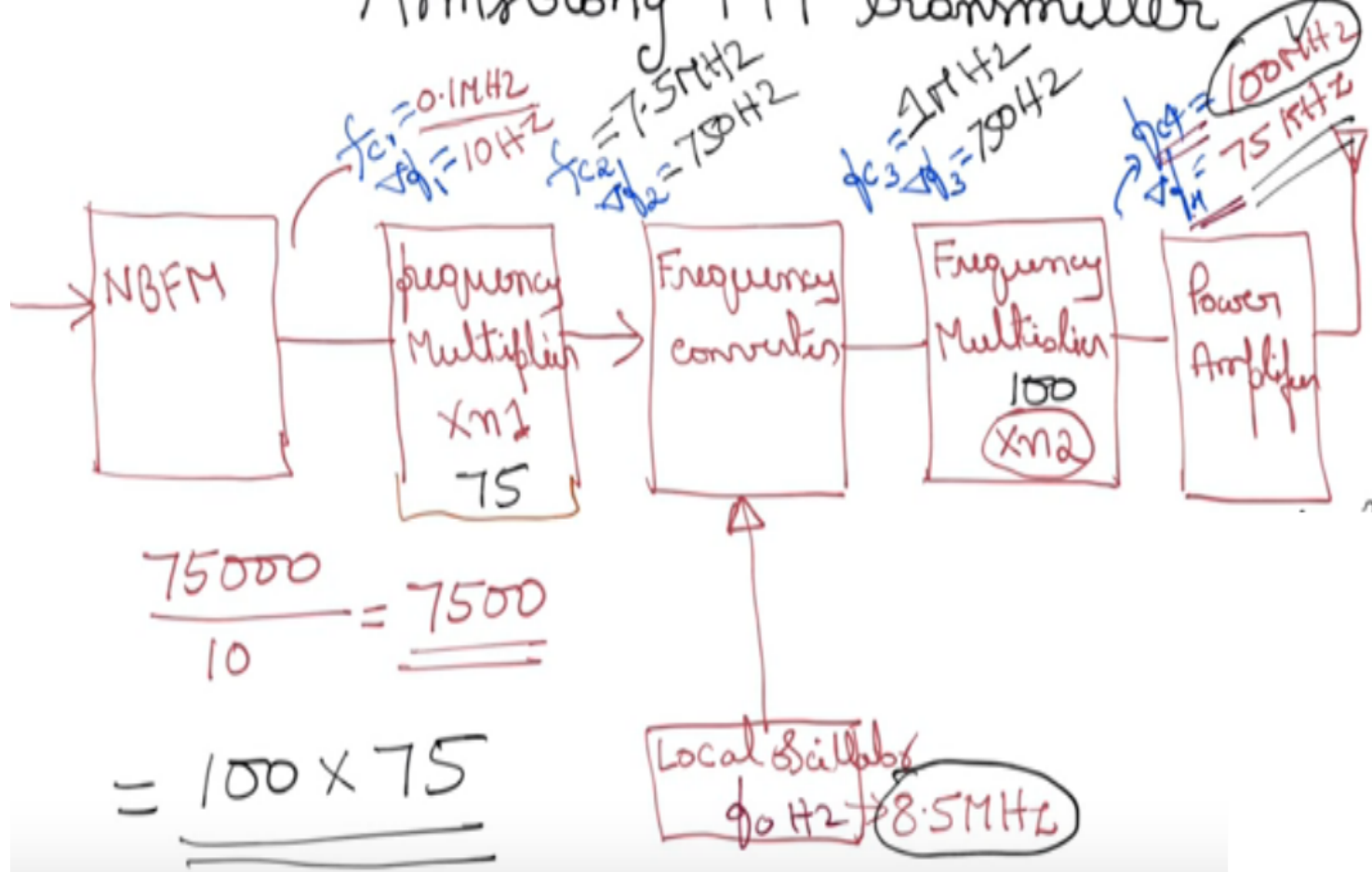
FM Demodulatation

1st Method:

FM discriminator:

- FM demodulators are implemented by generating an AM signal whose amplitude is proportional to the instantaneous frequency of the FM signal.
- Then using an AM demodulator to recover the message signal.
- To implement the first step; i.e., transforming the FM signal into an AM signal, it is enough to pass the FM signal through an LTI system whose frequency response is approximately a straight line in the frequency band of the FM signal.
- The next step is to demodulate this signal from which the message m(t) can be recovered.
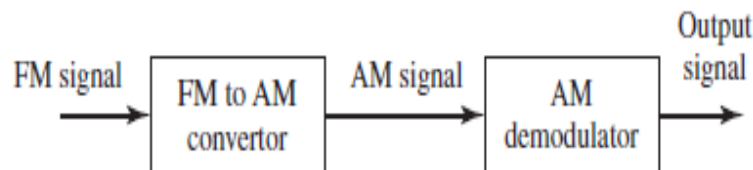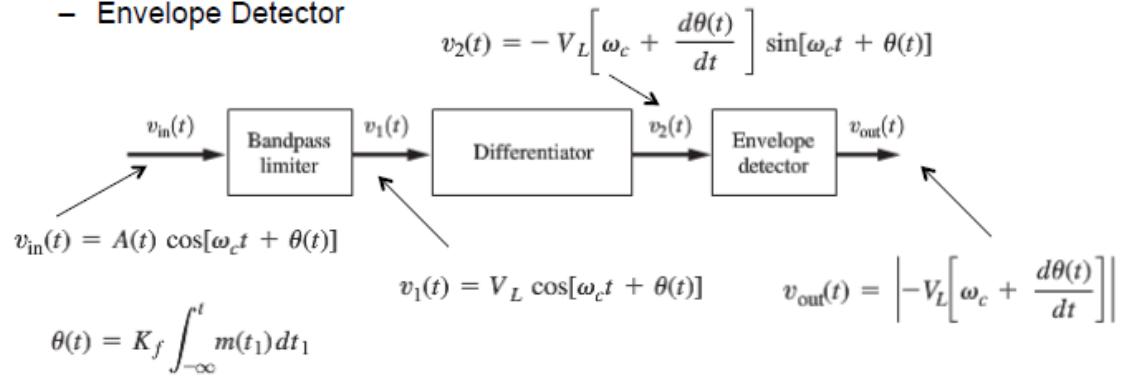

- There exist many circuits that can be used to implement the first stage (FM to AM conversion).
- One such candidate is a simple differentiator with:

$$|H(f)| = 2\pi f$$
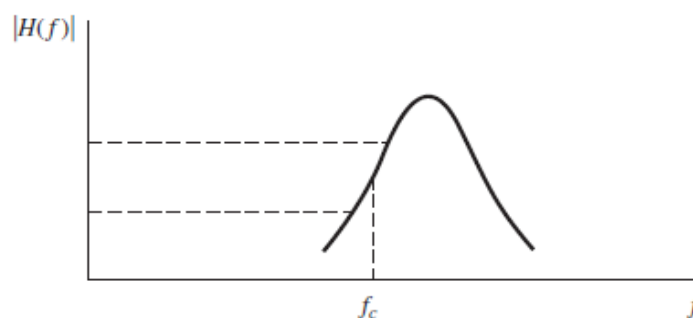
Frequency Discrimination

- Components
  - Bandpass Limiter: Consists of **Hard Limiter** & **BP Filter**
  - Discriminator (frequency discriminator gain: K_{FD} V/rad - assume unity)
  - Envelope Detector

$$v_2(t) = -V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]\sin[\omega_c t + \theta(t)]$$

$$v_{in}(t) \rightarrow \boxed{\begin{array}{c}\text{Bandpass}\\\text{limiter}\end{array}} \xrightarrow{v_1(t)} \boxed{\text{Differentiator}} \xrightarrow{v_2(t)} \boxed{\begin{array}{c}\text{Envelope}\\\text{detector}\end{array}} \xrightarrow{v_{out}(t)}$$

$$v_{in}(t) = A(t)\cos[\omega_c t + \theta(t)]$$

$$v_1(t) = V_L\cos[\omega_c t + \theta(t)] \qquad v_{out}(t) = \left|-V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]\right|$$

$$\theta(t) = K_f\int_{-\infty}^{t} m(t_1)\,dt_1$$

Note: Df=Kf
Freq. deviation sensitivity

THE OUTPUT WILL BE:

$$v_{out}(t) = V_L\left[\omega_c + \frac{d\theta(t)}{dt}\right]$$

$$v_{out}(t) = V_L\omega_c + V_L K_f m(t)$$

DC Component can be blocked
by an AC coupled circuit

Frequency Discrimination – Slope Detector

- In practice the differentiator can be approximated by a **slope detector** that has a linear frequency-to-amplitude transfer characteristic over the bandwidth BW –One drawback is that it is **narrow band**

Frequency-to-amplitude converter · Envelope detector · Tuned Circuit
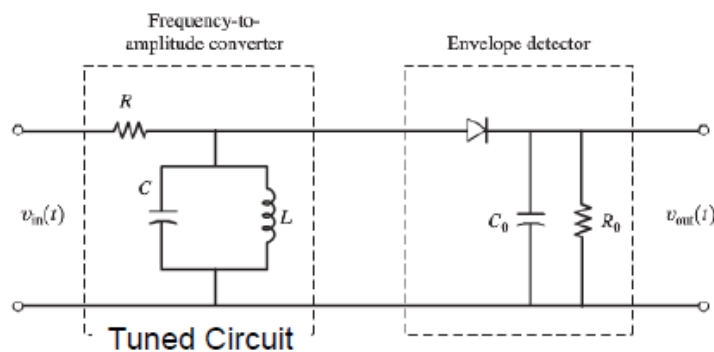
- Such a circuit can be easily implemented, but usually the linear region of the frequency characteristic may not be wide enough.
- To obtain a linear characteristic over a wider range of frequencies, usually two circuits tuned at two frequencies, f_1 and f_2, are connected in a configuration which is known as a ***balanced discriminator***.
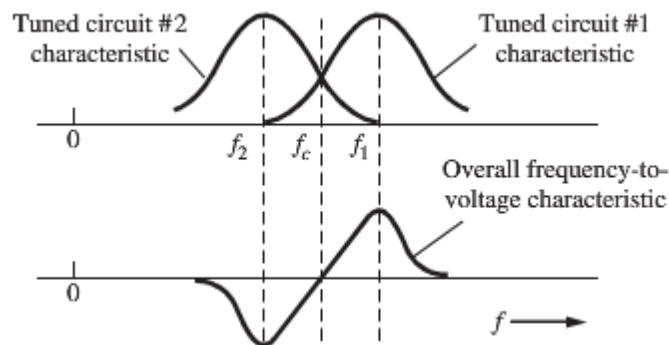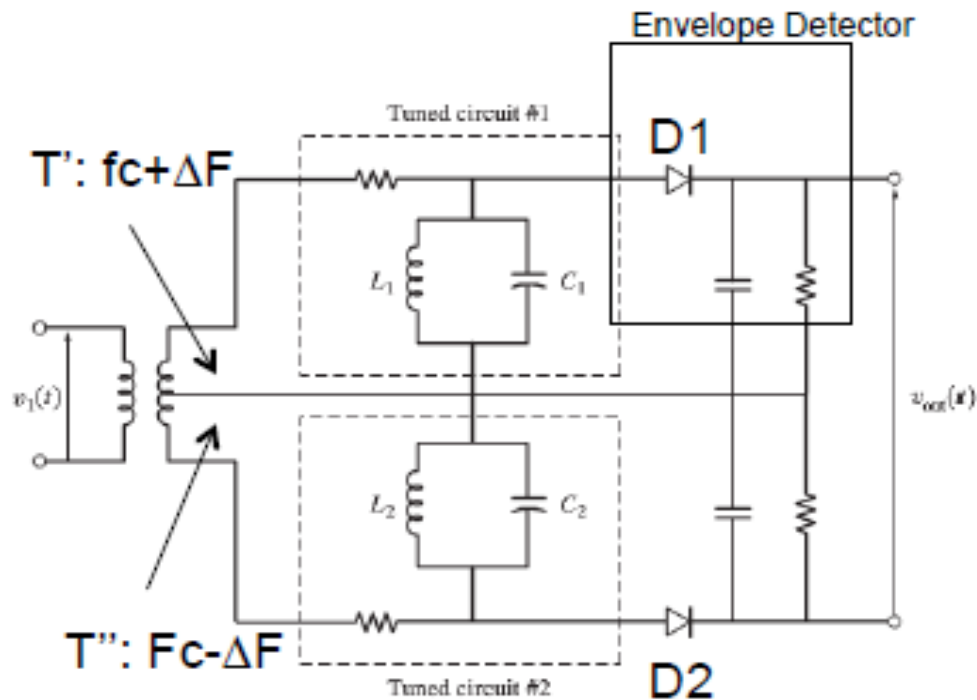

- Use two tuned circuits each set to a fixed frequency at least $f_c \pm \Delta f$, i.e. $f_1 = f_c + \Delta f$ and $f_2 = f_c - \Delta f$.
- The center-tapped transformer feeds the tuned circuits (Tuned circuits are 180 degrees out of phase).

- When fi>fc, the o/p of the upper tuned circuit has positive voltage (+ve), and it is greater than the o/p of the lower circuit which has negative voltage (-ve). Maximum (positive) voltage can be obtained across D1 in this situation.

- When fi<fc, the o/p of the upper tuned circuit has positive voltage (+ve), and it is smaller than the o/p of the lower circuit which has negative voltage (-ve). Maximum (negative) voltage can be obtained across D2 in this situation.

- When fi=fc, the o/p of the upper tuned circuit has positive voltage (+ve), and it is equal to the o/p of the lower circuit which has negative voltage (-ve). The voltages across the two diodes D1and D2 are the same, in this situation the overall circuit o/p is zero.

2nd Method:
FM demodulator with feedback (FMFB) using PLL:

- FMFB demodulator is implemented via using a phase-locked loop (PLL), as shown in below.
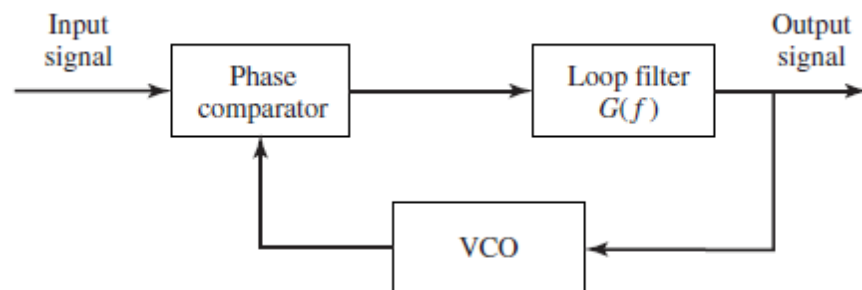
- The input to the PLL is the angle-modulated signal (we neglect the presence of noise in this discussion, i.e. noise=0).

$$u(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

where, for FM,

$$\phi(t) = 2\pi k_f \int_{-\infty}^{t} m(\tau)\, d\tau$$

- The VCO generates a sinusoid of a fixed frequency, in this case the carrier frequency fc, in the absence of an input control voltage.

- Now, suppose that the control voltage to the VCO is the output of the loop filter, denoted as v(t). Then, the instantaneous frequency of the VCO is

$$f_v(t) = f_c + k_v v(t)$$

where kv is a deviation constant with units of Hz/volt. Consequently, the VCO output may be expressed as

$$y_v(t) = A_v \sin[2\pi f_c t + \phi_v(t)]$$

where

$$\phi_v(t) = 2\pi k_v \int_0^t v(\tau)\,d\tau$$

- The phase comparator is basically a multiplier and filter that rejects the signal component centered at 2fc. Hence, its output may be expressed as

$$e(t) = \tfrac{1}{2} A_v A_c \sin[\phi(t) - \phi_v(t)]$$

where the difference, $\phi(t) - \phi_v(t) \equiv \phi_e(t)$, constitutes the phase error. The signal $e(t)$ is the input to the loop filter.

- Since the control voltage of the VCO is proportional to the message signal, v(t) is the demodulated signal.

- We observe that the output of the loop filter with frequency response G(f) is the desired message signal. Hence, the bandwidth of G(f) should be the same as the bandwidth W of the message signal. Consequently, the noise at the output of the loop filter is also limited to the bandwidth W.

- On the other hand, the output from the VCO is a wideband FM signal with an instantaneous frequency that follows the instantaneous frequency of the received FM signal.